STRENGTH DESIGN OF CRUCIFORM STEEL COLUMNS

NICHOLAS S TRAHAIR

RESEARCH REPORT R921
OCTOBER 2011

ISSN 1833-2781

SCHOOL OF CIVIL ENGINEERING

THE UNIVERSITY OF SYDNEY

THE UNIVERSITY OF SYDNEY

SCHOOL OF CIVIL ENGINEERING

STRENGTH DESIGN OF CRUCIFORM STEEL COLUMNS

RESEARCH REPORT R921

N S TRAHAIR

October 2011

ISSN 1833-2781

Copyright Notice

School of Civil Engineering, Research Report R921
Strength Design of Cruciform Steel Columns
N S Trahair BSc BE MEngSc PhD DEng
October 2011

ISSN 1833-2781

This publication may be redistributed freely in its entirety and in its original form without the consent of the copyright owner.

Use of material contained in this publication in any other published works must be appropriately referenced, and, if necessary, permission sought from the author.

Published by:
School of Civil Engineering
The University of Sydney
Sydney NSW 2006
Australia

This report and other Research Reports published by the School of Civil Engineering are available at http://sydney.edu.au/civil

ABSTRACT

Very different strengths are predicted by two different methods of designing steel cruciform columns. Both methods require design against local and flexural buckling, and while one method also requires design against torsional buckling, the other does not.

Investigations of the elastic local and torsional buckling and post-buckling of cruciforms columns show that these two modes are virtually identical.

The first yield and inelastic buckling approaches often used to formulate methods of designing columns against flexural buckling are extended to the torsional buckling design of cruciforms. These extensions show that it is sufficient to use local buckling design to guard against torsional buckling.

It is found that design methods which make separate checks against local and torsional buckling are unnecessarily severe, and are equivalent to making the same strength reduction twice. Instead, it is sufficient to ignore the torsional buckling of cruciforms provided design checks are made against local buckling as well as flexural buckling.

KEYWORDS

Buckling, Columns, Cruciforms, Design, Flexure, Post-buckling, Steel, Torsion, Yield

TABLE OF CONTENTS

1 INTRODUCTION

Very different strengths are predicted by two different methods of designing steel cruciform columns (Fig. 1). Both methods require design against local and flexural buckling, and while one method [1, 2] also requires design against torsional buckling, the other [3] does not. This second method might seem optimistic, because cruciform columns have very low torsional stiffness and are susceptible to torsional buckling. Instead, it relies on the local buckling design check to guard against torsional failure.

The methods of [1, 2] use a unified approach to column buckling to allow for torsional buckling. In this unified approach, the common method of designing against flexural buckling is extended to torsional (and flexural-torsional) buckling by replacing the elastic flexural buckling load in the design formulations by the elastic torsional (and flexural-torsional) buckling load. When this method is applied to low stiffness cruciforms, it produces significant reductions below the section capacity (as governed by yielding and local buckling effects). These reductions do not occur with the second method [3].

The purposes of this paper are to compare these two different methods and to find reasons for preferring one method over the other.

Firstly, the torsional and local buckling and post-buckling behaviour of cruciform columns are reviewed and investigated. Secondly, the bases for the design of columns against local and flexural buckling are reviewed. Thirdly, the two torsional design methods are compared, and the justifications that are needed for these are discussed. These are investigated by extending the first yield and inelastic buckling design bases for flexural buckling to torsional buckling.

2 ELASTIC TORSIONAL BUCKLING

The elastic torsional buckling resistance N_{oz} of a simply supported doubly symmetric column of length L is given by [4-6]

$$N_{oz} = (GJ + \pi^2 EI_w / L^2) / r_0^2 \tag{1}$$

in which GJ is the uniform torsional rigidity, EI_w is the warping rigidity and

$$r_0^2 = (I_x + I_y) / A \tag{2}$$

in which I_x, I_y are the principal axis second moments of area and A is the area of the cross-section. For thin-walled open sections the torsion section constant

$$J = \sum bt^3 / 3 \tag{3}$$

is small, while for concurrent sections (such as angles, tees, and cruciforms) the warping section [6]

$$I_w = b^3 t^3 / 9 \tag{4}$$

is very small and often neglected.

The variations of the dimensionless torsional buckling loads N_{oz}/N_y of cruciforms with b/t = 10, 20, 30 and f_y = 235 N/mm^2 with the modified minor axis flexural slenderness

$$\lambda_{oy} = \sqrt{(N_y / N_{oy})} \tag{5}$$

are shown in Fig. 2, in which the squash load is

$$N_y = Af_y \tag{6}$$

and the minor axis elastic flexural buckling load is

$$N_{oy} = \pi^2 EI_y / L^2 \tag{7}$$

Also shown in Fig. 2 is the variation of the dimensionless minor axis flexural buckling load N_{oy}/N_y.

3 ELASTIC LOCAL BUCKLING

3.1 LOCAL BUCKLING

The elastic local buckling load of a cruciform column may be expressed as

$$N_{ol} = Af_{ol} \tag{8}$$

in which the local buckling stress [5] is given by

$$f_{ol} = \frac{\pi^2 E}{12(1-\nu^2)} \frac{k}{(b/t)^2} \tag{9}$$

in which ν is Poissons'ratio (commonly taken as 0.3 for metals) and the buckling coefficient is

$$k = \frac{6(1-\nu)}{\pi^2} + \left(\frac{b}{L}\right)^2 = 0.4255 + \left(\frac{b}{L}\right)^2 \tag{10}$$

When k is approximated by 0.4255, then the dimensionless local buckling load may be expressed as

$$N_{ol} / N_y = 1/\lambda_{ol}^2 \tag{11}$$

in which the local buckling modified slenderness is

$$\lambda_{ol} = \sqrt{\frac{N_y}{N_{ol}}} = \frac{b/t}{18.09}\sqrt{\frac{f_y}{235}} \tag{12}$$

The variation of the dimensionless local buckling load N_{ol}/N_y with the local buckling slenderness λ_{ol} is shown in Fig. 3.

3.2 COMPARISON WITH TORSIONAL BUCKLING

The local buckling load given by Equations 8-10 may be transformed to

$$N_{ol} = \left(GJ + \frac{\pi^2 EI_w / L^2}{(1-\nu^2)} \right) / r_0^2 \tag{13}$$

by using

$$G = \frac{E}{2(1+\nu)} \tag{14}$$

and $\nu = 0.3$. This is almost identical to Equation 1 for the torsional buckling load N_{oz}, the difference being the (1- ν^2) term in (13). This term is a Poisson's ratio effect which is important in two dimensional plates but negligible in one dimensional beams. The variations of the dimensionless local buckling loads N_{ol}/N_y of cruciforms with b/t = 10, 20, 30 and f_y = 235 N/mm^2 with the modified minor axis slenderness λ_{oy} are shown in Fig. 2.

4 POST-BUCKLING BEHAVIOUR

4.1 TORSIONAL POST-BUCKLING

There are torsional post-buckling reserves of strength which result from redistributions of axial stress or other secondary effects, but these are commonly ignored. An analysis of the post-buckling strengths N_{pz} of cruciforms (with negligible I_w) is made in Appendix A where it is shown that the dimensionless post-buckling strength is given by

$$\frac{N_{pz}}{N_y} = \frac{5}{9}\frac{N_{oz}}{N_y} + \frac{4}{9} \tag{15}$$

The variation of the dimensionless torsional post-buckling strength N_{pz}/N_y with the modified width-thickness ratio

$$\frac{b/t}{18.09}\sqrt{\frac{f_y}{235}} = \sqrt{(N_y / N_{ol})} = \lambda_{ol} \tag{16}$$

is shown in Fig. 3.

4.2 LOCAL POST-BUCKLING

There are local post-buckling reserves of strength which also result from redistributions of axial stress. An analysis similar to that in Appendix 3 for torsional post-buckling may be made for local post-buckling. A simple approximation [7] for this is given by

$$\frac{N_{pl}}{N_y} = \frac{18.09}{b/t}\sqrt{\frac{235}{f_y}} = \frac{1}{\lambda_{ol}} \tag{17}$$

The variation of this dimensionless local post-buckling strength N_{pl}/N_y with the modified local buckling slenderness λ_{ol} is shown in Fig. 3. It can be seen that this approximate local post-buckling strength is close to the torsional post-buckling strength of Equation 15.

5 DESIGN AGAINST LOCAL AND FLEXURAL BUCKLING

5.1 DESIGN AGAINST LOCAL BUCKLING

A column is first designed against local buckling [7] by comparing the width-thickness ratio (b/t) of its plate elements against design code yield slenderness limits λ_{ey}. These limits are based on elastic local buckling stresses f_{ol} which have been modified to account for the effects of material and geometric imperfections. For a lightly welded cruciform with a yield stress of f_y = 235 N/mm^2, the limiting width-thickness ratio of [1] corresponds approximately to b/t = 15, compared with the value of 18.09 at which the elastic buckling stress is equal to the yield stress. If the width-thickness ratio is less than 15, then the cross-section is fully effective and the section capacity is

$$N_s = N_y \tag{18}$$

If not, then the section capacity is reduced to

$$N_s = N_y\left(\frac{15}{b/t}\right)\sqrt{\frac{235}{f_y}} \tag{19}$$

which is based on the local post-buckling strength. The variation of N_s/N_y with the modified local buckling slenderness λ_{ol} for a cruciform is shown in Fig. 3.

5.2 DESIGN AGAINST FLEXURAL BUCKLING

A column is designed against flexural buckling [7] by reducing the section capacity N_s by a reduction factor χ which depends principally on the relative magnitude of the section capacity N_s and the elastic minor axis flexural buckling load N_{oy}, as expressed by the modified slenderness

$$\lambda_{ey} = \sqrt{N_s / N_{oy}} \tag{20}$$

Thus

$$N_d / N_s = \chi = fn(\lambda_{ey}) \tag{21}$$

in which $fn(\lambda_{ey})$ allows for the effects of geometrical imperfections and residual stresses.

The effects of geometrical imperfections are usually allowed for by using the load which causes first yield in a column with initial crookedness as the nominal strength. Thus, for example [7],

$$\frac{N_{fy}}{N_s} = \left[\frac{1+(1+\lambda_{ey}^2/4)/\lambda_{ey}^2}{2}\right] - \sqrt{\left\{\left[\frac{1+(1+\lambda_{ey}^2/4)/\lambda_{ey}^2}{2}\right]^2 - \frac{1}{\lambda_{ey}^2}\right\}} \tag{22}$$

as shown in Fig. 4. This method ignores the negative effects of residual stresses, and the positive effects of post-yielding and strain-hardening.

The effects of residual stresses are usually allowed for by using the inelastic tangent modulus buckling load as the nominal strength. Thus, for example [7],

$$\frac{N_i}{N_s} = 1 - \lambda_{ey}^2/4 \quad \text{while} \quad \lambda_{ey} < \sqrt{2} \tag{23}$$

as shown in Fig. 4. This method ignores the negative effects of geometrical imperfections, and the positive effects of the tangent modulus theory and strain-hardening.

Design codes usually modify one or other of these methods in the light of experimental evidence. The variation of N_d/N_s with λ_{ey} according to [1] for lightly welded cruciforms is also shown in Fig. 4.

6 DESIGN AGAINST TORSIONAL BUCKLING

6.1 METHODS OF DESIGN

The method of [1, 2] for designing against torsional (and flexural-torsional) buckling is to use the same form for the slenderness reduction factor χ as for flexural buckling, but with the modified flexural slenderness λ_{ey} replaced by the modified flexural-torsional slenderness

$$\lambda_{eft} = \sqrt{(N_s / N_{oft})} \tag{24}$$

in which N_{oft} is the lowest elastic flexural-torsional buckling load. The effect of this on the variations of the dimensionless cruciform design strengths N_d/N_s according to [1] with the modified flexural slenderness λ_{ey} is shown in Fig. 5 by the solid lines.

The method of [3] is to not to make any specific reductions in strength to allow for torsional buckling. The effect of doing this on the design of cruciforms according to [1] is shown by the dashed line in Fig. 5. It can be seen that using the method of [1] to allow for torsional buckling leads to significant reductions in the design strength, even for cruciforms with low b/t ratios and correspondingly high torsional resistances.

6.2 DISCUSSION

A rational explanation needs to be found for or against the significant reductions shown in Fig. 5. Possible explanations might be derived from the rationales used for the methods of designing against flexural buckling. These include the effects of first yield on columns with initial crookedness, and the effects of residual stresses on inelastic buckling.

6.3 FIRST YIELD STRENGTHS

The use of first yield for design against flexural buckling [7] is based on the assumption of an initial crookedness of the same form as the buckling mode. This crookedness is magnified by incipient flexural buckling effects, and additional normal stresses are generated which add to the normal stresses due to the axial load, leading to early first yield. The corresponding geometrical imperfection for cruciforms that fail by torsional buckling is initial twist, which will be increased by incipient torsional buckling effects, so that torsional shear stresses will be developed. The combination of these with the normal axial stresses will lead to early yield.

An analysis of the effects of initial twist on the first yield of cruciform columns is given in Appendix B. The dimensionless first yield loads N_{fy}/N_s are shown in Fig. 5. For the cruciform with b/t = 10 (which according to [1] is fully effective against local buckling), the torsional first yield load is virtually equal to the section capacity N_s, and first yield is governed by initial crookedness and flexural buckling effects.

For the cruciform with b/t = 20, the torsional first yield load is noticeably lower than the torsional buckling load, but substantially greater than the design strength of [1] predicted by using $N_o = N_{oz}$. For the cruciform with b/t = 30, the first yield load is slightly lower than the torsional buckling load, but substantially greater than the design strength of [1] predicted by using $N_o = N_{oz}$.

It can be seen that there is no first yield justification for the substantial reductions of the method of [1] shown in Fig. 5.

It may be noted that first yield loads do not allow for the significant post-buckling reserves of strength that occur at low slendernesses, as shown in Fig. 3, and so it can be expected that the first yield loads will provide conservative estimates of the torsional buckling strength.

6.4 INELASTIC BUCKLING

The use of inelastic buckling for design against flexural buckling [7] is based on the assumption of residual normal stresses, which when combined with normal stresses due to the axial load lead to early yield and reductions in the effective modulus below the Young's modulus E, and corresponding reductions in the buckling resistance. This early yield will also cause reductions in the shear modulus below the elastic value G, and corresponding reductions in the torsional buckling resistance.

An analysis of the inelastic torsional buckling of cruciform columns is given in Appendix C. The reduced inelastic buckling loads are compared in Fig. 3 with the first yield loads. It can be seen that they are generally a little lower for low slendernesses, but not markedly lower than the local buckling strengths.

These inelastic buckling loads do not allow for the significant post-buckling reserves of strength that occur at low slendernesses, as shown in Fig. 3, and so it can be expected that the inelastic buckling loads will provide conservative estimates of the torsional buckling strength. It can be concluded that there is no inelastic buckling justification for the substantial reductions of the method of [1] shown in Fig. 5.

7 CONCLUSIONS

The principal justification for the method of designing columns against torsional, flexural, or flexural-torsional buckling by adapting the methods for flexural buckling is that this corrects for the inability of the flexural buckling method to allow for the low flexural-torsional buckling resistances of some types of section, such as lipped channels and lipped angles. It provides a seemingly unified common method of design for a complete range of cross section types.

However, the application of this proposal to cruciform columns leads to significant reductions in their low slenderness design strengths, which cannot be justified by modifying for torsional buckling either the first yield or the inelastic buckling approach often used for the design of columns against flexural buckling. This paper has shown that torsional and local buckling and post-buckling analyses of cruciform columns lead to virtually identical results, so that design against local buckling can be regarded as simultaneously designing against torsional buckling.

This virtual identity between torsional and local buckling in cruciform columns leads to the conclusion that the method of [1] for designing against torsional buckling allows for this twice, once in designing against torsional buckling, and a second time in designing against local buckling. This leads to the significant strength reductions predicted by the method.

On the other hand, the investigations in this paper of the effects of post-buckling, first yield and inelastic buckling show that it is appropriate to ignore the effects of torsional buckling, since these are accounted for by the allowances made for local buckling, as in the design method of [3].

8 REFERENCES

[1] BSI. Eurocode 3: Design of Steel Structures: Part 1-1: General Rules and Rules for Buildings, BS EN 1993-1-1. British Standards Institution, London, 2005.

[2] AISC. Specification for Structural Steel Buildings. American Institute of Steel Construction, Chicago, 2010.

[3] SA. AS 4100-1998 Steel Structures. Standards Australia, Sydney, 1998.

[4] Wagner, H. Verdrehung und knickung von offenen profilen (Torsion and buckling of open sections). 25th Anniversary Publication, Technische Hochschule, Danzig, 1936; Translated as Technical Memorandum No. 87, National Committee for Aeronautics.

[5] Timoshenko, SP, and Gere, JM. Theory of Elastic Stability. 2nd ed., McGraw-Hill, New York, 1961.

[6] Trahair, NS. Flexural-Torsional Buckling of Structures. E & FN Spon, London, 1993.

[7] Trahair, NS, Bradford, MA, Nethercot, DA, and Gardner, L. The Behaviour and Design of Steel Structures to EC3. Taylor and Francis, London, 2008.

[8] Trahair, NS. Inelastic lateral buckling of beams. Beams and Beam-Columns: Stability and Strength, Applied Science Publishers, 1983; 35-69.

[9] Trahair, NS. Inelastic buckling of monosymmetric I-beams. Research Report R921, School of Civil Engineering, University of Sydney, September 2011.

9 NOTATION

9.1 SUBSCRIPTS

o, i	Elastic or inelastic buckling
l, ft, x, y	Buckling mode
m	Maximum value

9.2 PRINCIPAL NOTATION

A	Area of cross section
b	Leg width
E	Young's modulus of elasticity
f	Stress
f_b	Notional stress at the end of a leg
f_e	Equivalent von Mises stress
f_r	Residual stress
f_y	Yield stress
G	Shear modulus of elasticity
I_x, I_y	Second moments of area about x, y axes
I_w	Warping section constant
J	Uniform torsion constant
k	Local buckling coefficient
L	Column length
N	Axial compression
N_d	Design strength
N_{fy}	First yield load
N_{pl}	Local post- buckling load
N_{pz}	Torsional post- buckling load
N_s	Design section capacity
N_{sz}	Torsional post- buckling strength
N_y	Squash load
r_0	Polar radius of gyration
t	Leg thickness
v, w	Displacements in y, z directions
W	A stress resultant of axial stresses
w_f	Displacement due to axial straining
w_s	Axial shortening
x, y	Principal axis coordinates
x_y	Value of x for which $f = f_y$
z	Distance along column
χ	Design slenderness reduction factor
δ	Initial crookedness
ϕ	Twist rotation
ϕ_0	Initial twist rotation
λ_o	Modified slenderness
λ_{eft}	Design modified slenderness for flexural-torsional buckling
λ_{ey}	Design modified slenderness for flexural buckling
ν	Poisson's ratio
θ	Maximum twist rotation
θ_0	Maximum initial twist rotation
τ	Shear stress

APPENDIX A - TORSIONAL POST-BUCKLING

After torsional buckling, a simply supported cruciform undergoes twist rotations

$$\phi = \phi_m \sin \pi z / L \tag{A1}$$

as shown in Fig. A1. It is assumed that the axial end load N acts through rigid end platens so that the end displacements w are constant. These displacements are combinations of those due to elastic axial straining and to axial shortening caused by the twist rotations.

The shortening displacements are

$$w_s = \frac{1}{2}\int_0^L \left(\frac{dv}{dz}\right)^2 dz \tag{A2}$$

in which

$$v = x\phi \tag{A3}$$

whence

$$w_s = \frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{x^2}{2} \tag{A4}$$

The displacements due to axial straining are

$$w_f = w - w_s \tag{A5}$$

so that the elastic compression stresses are

$$f = Ew_f / L = Ew/L - \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{x^2}{2} \tag{A6}$$

The axial compression force is

$$N = \int_A f dA = EAw/L - \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{4b^3 t}{6} \tag{A7}$$

so that

$$f = \frac{N}{A} + \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\left(\frac{b^2}{6} - \frac{x^2}{2}\right) \tag{A8}$$

and the maximum compression stress is

$$f_m = \frac{N}{A} + \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{b^2}{6} \tag{A9}$$

First yield at $N = N_{fy}$ occurs when $f_m = f_y$ so that

$$N_{fy} = N_y - \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{Ab^2}{6} \tag{A10}$$

The stresses f cause torsional post-buckling at $N=N_{pz}$ when

$$GJ = \int_A f(x^2 + y^2) dA \tag{A11}$$

whence

$$GJ = N_{pz} r_0^2 + \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2} 4\left(\frac{b^5}{18} - \frac{b^5}{10}\right) t \tag{A12}$$

so that

$$N_{pz} = N_{oz} + \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{2}{45}\frac{4b^5 t}{r_0^2} \tag{A13}$$

If the post-buckling strength is taken as the value of N_{pz} which causes first yield so that

$$N_{sz} = N_{pz} = N_{fy} \tag{A14}$$

then

$$N_{sz} = N_{oz} + (4/5)(N_y - N_{sz}) \tag{A15}$$

after using

$$r_0^2 = b^2/3 \tag{A16}$$

for a cruciform section. Thus

$$\frac{N_{sz}}{N_y} = \frac{5}{9}\frac{N_{oz}}{N_y} + \frac{4}{9} \tag{A17}$$

APPENDIX B – FIRST YIELD OF TWISTED CRUCIFORMS

The first yield load N_{fy} of a simply supported cruciform column with initial twists

$$\phi_0 = \phi_{0m} \sin(\pi z / L) \qquad \text{(B1)}$$

may be determined by solving the torsional equilibrium equation

$$GJ\phi' - EI_w \phi''' = Nr_0^2 (\phi' + \phi_0') \qquad \text{(B2)}$$

in which ' indicates differentiation with respect to the distance z along the column. If Equation B1 and the solution

$$\phi = \phi_m \sin(\pi z / L) \qquad \text{(B3)}$$

are substituted into Equation B2, then ϕ_m can be obtained from

$$\frac{\phi_m}{\phi_{0m}} = \frac{N / N_{oz}}{1 - N / N_{oz}} \qquad \text{(B4)}$$

in which

$$N_{oz} = \frac{GJ + \pi^2 EI_w / L^2}{r_0^2} \qquad \text{(B5)}$$

is the torsional buckling load. The maximum shear stress is given by

$$\tau = G\phi_m t_m = G\frac{\pi\phi_m}{L} t \qquad \text{(B6)}$$

The normal stress due to the axial load

$$f = N / A \qquad \text{(B7)}$$

and the torsional shear stress τ may be combined as an equivalent von Mises stress

$$f_e = \sqrt{f + 3\tau^2} \qquad \text{(B8)}$$

First yield occurs when

$$f_e = f_y \qquad \text{(B9)}$$

so that

$$N_{fy} = A\sqrt{(f_y^2 - 3\tau^2)} \qquad \text{(B10)}$$

The maximum initial twist may be taken as

$$\phi_{0m} = \frac{L/1000}{2b} \qquad \text{(B11)}$$

which is consistent with the maximum initial crookedness of L/1000 often assumed for first yield in columns that fail by flexural buckling.

APPENDIX C – INELASTIC TORSIONAL BUCKLING

The inelastic torsional buckling of a cruciform column with the normal residual stresses

$$f_r = -0.3 f_y (1 - 2x/b) \tag{C1}$$

shown in Fig. 1c for one leg may be analysed by using a reduced shear modulus in the yielded regions shown.

If the applied compressive load is defined by a notional stress f_b at the end of the leg, then the stress distribution is given by

$$\begin{aligned} f &= f_b - 0.3 f_y (1 - 2x/b) \quad \text{while} \quad 0 \le x \le x_y \\ f &= f_y \quad \text{while} \quad x_y \le x \le b \end{aligned} \tag{C2}$$

in which x_y is given by

$$\frac{x_y}{b} = \frac{(1.3 - f_b / f_y)}{0.6} \tag{C3}$$

and the axial compression by

$$N_i = f_y bt \left(\frac{f_b}{f_y} - 0.3 \frac{(b - x_y)^2}{b^2} \right) \tag{C4}$$

The stresses f and f_r have a stress resultant W [4] which is given by

$$W = \int_A (f - f_r)(x^2 + y^2) dA \tag{C5}$$

whence

$$W = f_y b^3 t \left\{ \frac{1}{3} \left(\frac{f_b}{f_y} - 1.3 \right) \left(\frac{x_y}{b} \right)^3 + \frac{3}{20} \left(\frac{x_y}{b} \right)^4 + \frac{1}{3} \right\} \tag{C6}$$

When the column twists, this stress resultant exerts a disturbing torque [4-6] which is resisted by the inelastic torsional stiffness

$$(GJ)_i = \frac{(Gx_y + G_i (b - x_y))t^3}{3} \tag{C7}$$

in which the elastic and inelastic [6, 8, 9] shear moduli for steel may be taken as

$$\begin{aligned} G &= 76923 \text{ MPa} \\ G_i &= 20761 \text{ MPa} \end{aligned} \tag{C8}$$

For inelastic torsional buckling

$$W = (GJ)_i \tag{C9}$$

This equation can be solved iteratively for the inelastic buckling load N_i which corresponds to a given set of values of b, t, and f_y.

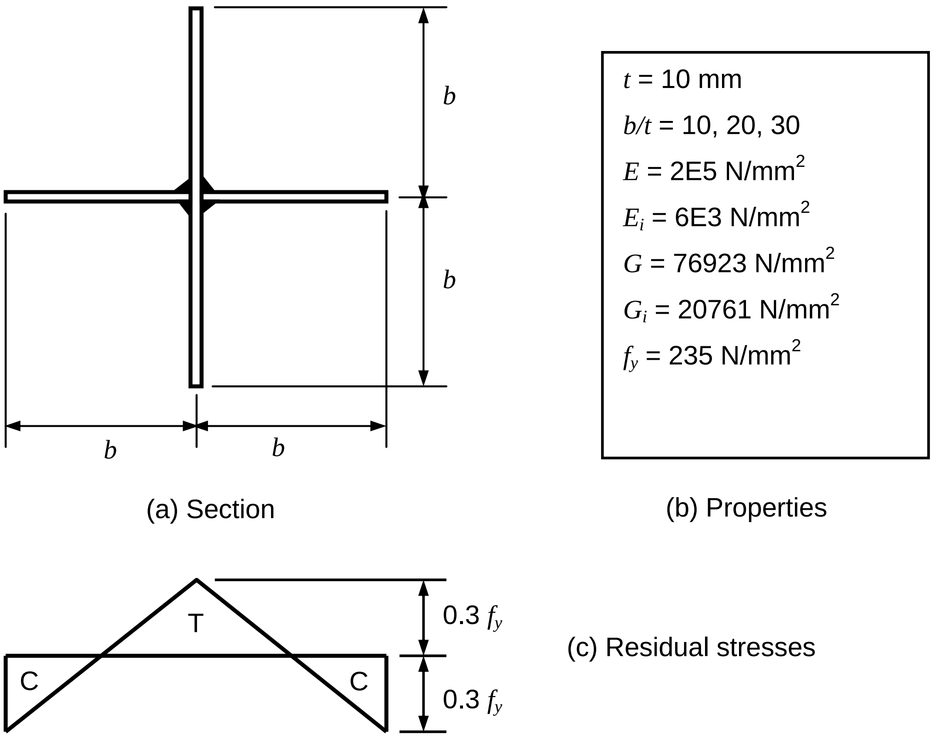

Fig. 1 Cruciform Section and Properties

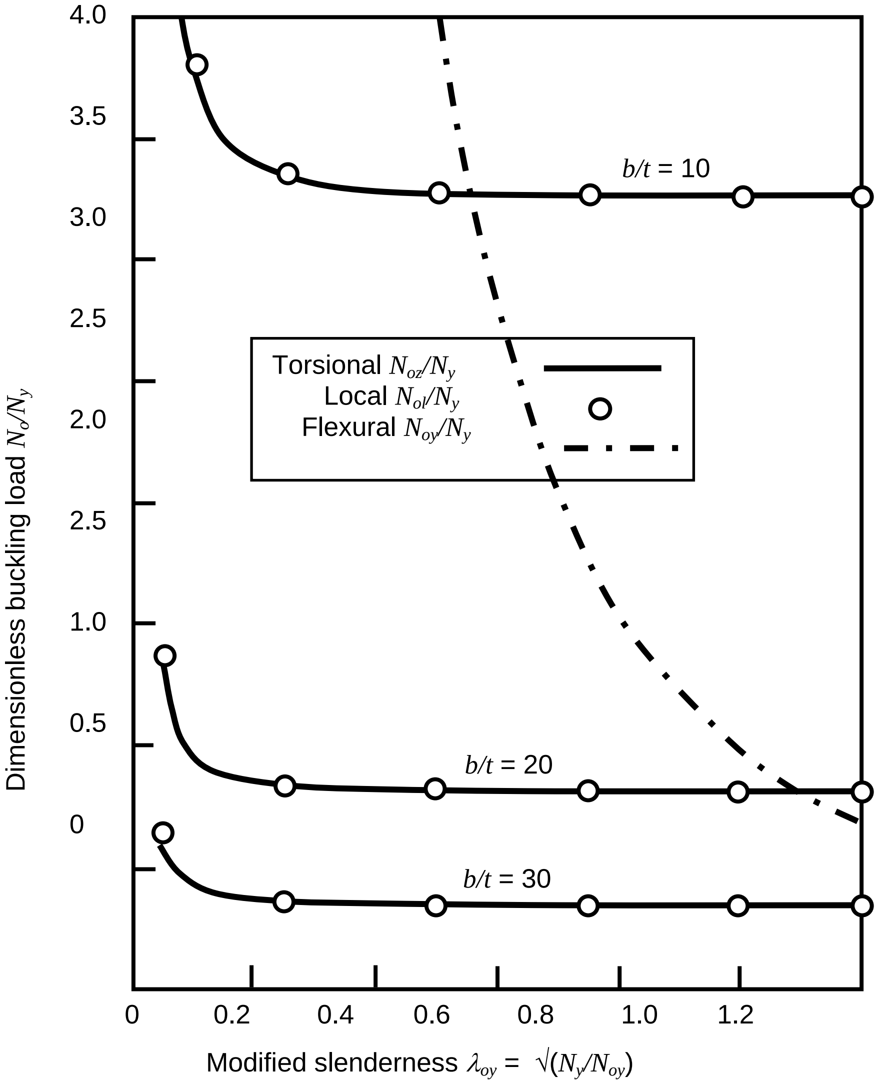

Fig. 2 Torsional and Local Buckling Loads

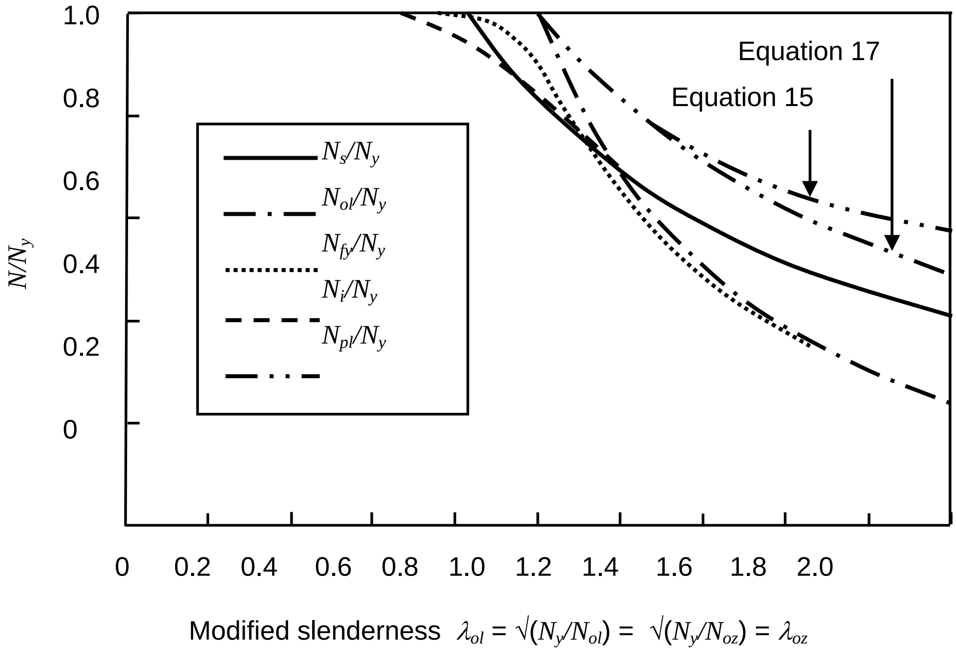

Fig. 3 Local Buckling and Torsional Strengths

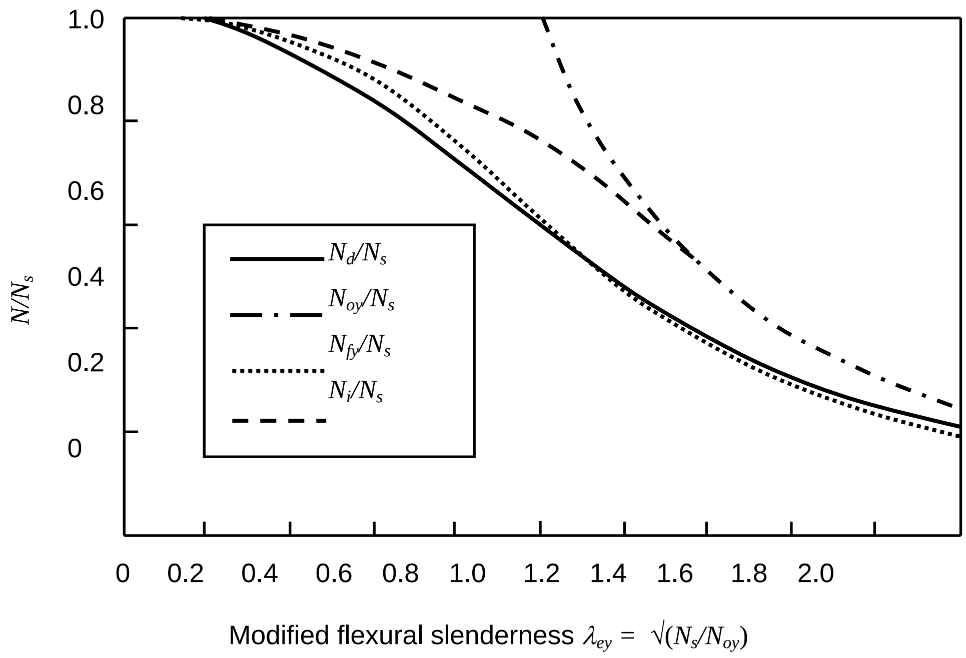

Fig. 4 Design Against Flexural Buckling

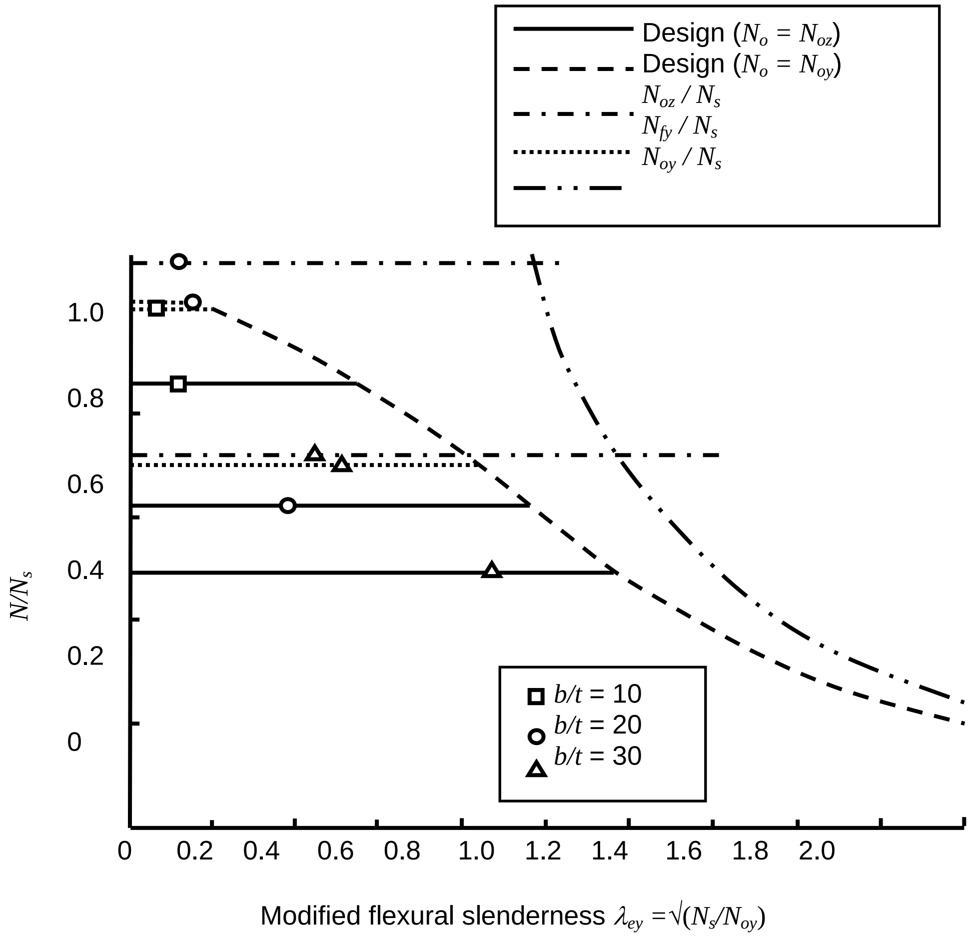

Fig. 5 Torsional Design [1] and First Yield

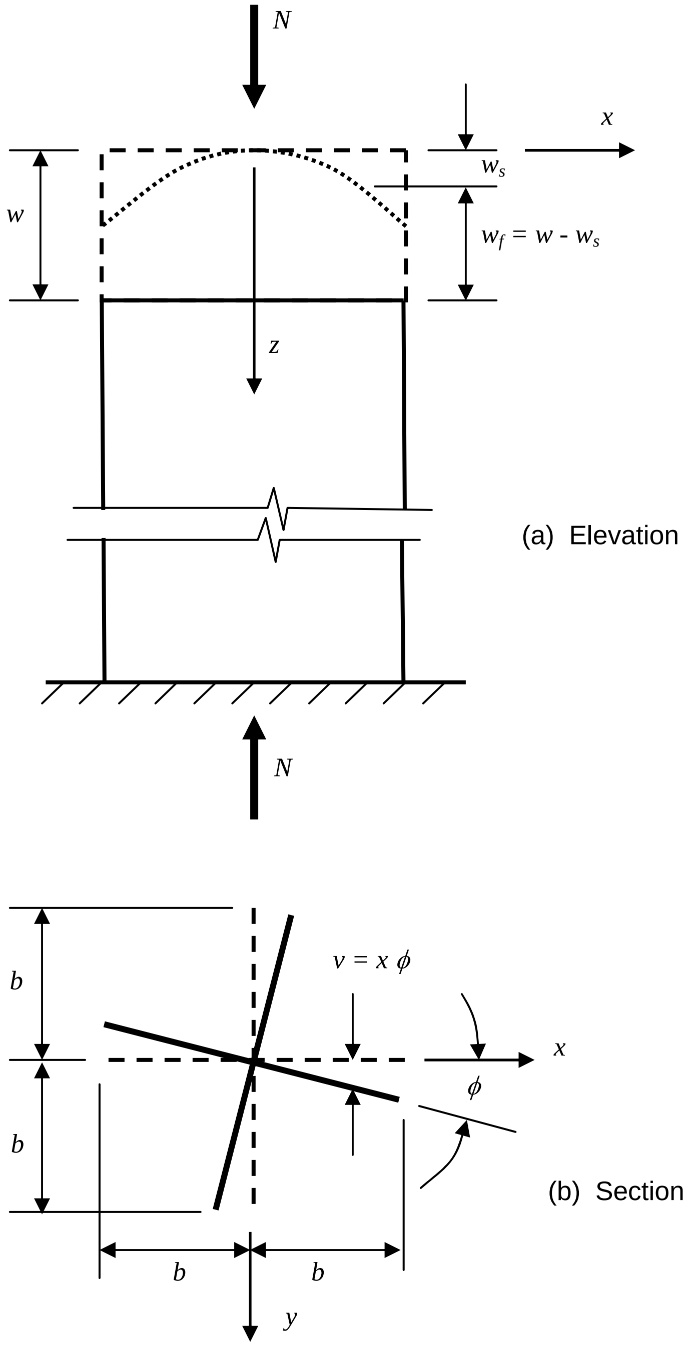

Fig. A1 Torsional Post-Buckling