STEEL CANTILEVER STRENGTH BY INELASTIC LATERAL BUCKLING

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ABSTRACT

Methods used for the design of steel beams supported at both ends are not well suited for the design of cantilevers against lateral buckling. The end restraints are very different for cantilevers, and the maximum displacements and twist rotations take place at the free ends, instead of near mid-span. Consequently, their buckling modes are very different to those of supported beams. The methods of allowing for the effects of the moment distribution on the elastic and inelastic buckling of supported beams use a mean of the moment distribution which is weighted to allow for the maximum deformations being near mid-span. These methods are clearly inappropriate for cantilevers whose deformations are greatest at the free ends.

Lateral buckling design methods for cantilevers are modifications of the methods for supported beams, but are of doubtful accuracy, and may be over conservative. In some cases there is little or no design guidance.

This paper summarizes information on the effects of the moment distribution and load height on the elastic buckling of cantilevers which can be used in the method of design by buckling analysis. It then extends a method of designing supported beams by inelastic buckling analysis to allow for the effects of the moment distribution on the inelastic buckling of cantilevers. This extended method is then used to provide improved design methods for cantilevers which are consistent with those for simply supported beams. A worked example is summarized.

KEYWORDS

Analysis, cantilevers, design, inelasticity, lateral buckling, steel.

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1 INTRODUCTION

Methods used for the code design [1 - 4] of steel beams supported at both ends are not well suited to the design of cantilevers and overhanging beams against flexural-torsional (lateral) buckling. The end restraints are very different for cantilevers and overhanging beams, and the maximum displacements and twist rotations take place at the free ends, instead of near mid-span. Consequently, their buckling modes are very different to those of supported beams. The code methods of allowing for the effects of the moment distribution on the elastic and inelastic buckling of supported beams use a mean of the moment distribution which is weighted to allow for the maximum deformations being near mid-span. These methods are clearly inappropriate for cantilevers whose deformations are greatest at the free ends.

Some design codes [1, 2] give methods for the design of cantilevers which are modifications of their methods for supported beams. These modifications are of doubtful accuracy, especially those for the effect of load height. Further, there are no treatments of the beneficial effect of moment distribution on the inelastic buckling of cantilevers. Other codes provide little [3] or no [4] guidance on the design of cantilevers against lateral buckling.

The purpose of this paper is to provide sufficient information to allow the rational design of steel cantilevers against lateral buckling. This requires the provision of information on the effects of moment distribution and load height on elastic lateral buckling, and on the effect of moment distribution on inelastic buckling and cantilever strength, and the development of a method of using this information in the design process.

Because cantilever design follows logically from the design methods for supported beams, the elastic and inelastic buckling and design strengths of supported beams are first reviewed, followed by a review of the deficiencies of current methods of cantilever design. Following this, information on the effects of moment distribution and load height on the elastic lateral buckling of cantilevers and overhanging beams is collected together. The effects of the moment distribution on the inelastic lateral buckling and strength of cantilevers are then investigated by adapting a method used for supported beams. The findings of this investigation are then used to develop a rational method for design, which is illustrated by a worked example.

2 LATERAL BUCKLING DESIGN OF SUPPORTED BEAMS

2.1 ELASTIC LATERAL BUCKLING

Most design codes [1 - 4] base their design procedures on the elastic lateral buckling moment of a simply supported beam in uniform bending M_{yz} given by [5, 6]

$$M_{yz} = \sqrt{\frac{\pi^2 EI_y}{L^2}\left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \tag{1}$$

in which E and G are the Young's and shear moduli of elasticity, I_y, J, and I_w are the minor axis second moment of area, the uniform torsion section constant, and the warping section constant, and L is the length of the beam.

The effects of the bending moment distribution on the maximum moment M_c at elastic lateral buckling are allowed for by using

$$M_c = \alpha_m M_{yz} \tag{2}$$

in which α_m is a moment modification factor which is often approximated [1,2, 4] using a weighted mean of the actual moment distribution. Values of α_m obtained by using the computer program PRFELB [6 - 8] are shown in Fig. 1 for beams with either central concentrated load or a single end moment. It can be seen that these are practically independent of the torsion parameter

$$K = \sqrt{\frac{\pi^2 EI_w}{GJL^2}} \tag{3}$$

The strengthening effects of minor axis end restraints are often approximated [1, 2] by replacing the span length L by an effective length

$$l = k_r L \tag{4}$$

in which (often) conservative values are used for the restraint factor k_r.

The effects of load height which reduce the beam buckling resistance when they act above the shear centre are sometimes approximated [1,2] by using

$$l = k_l k_r L \tag{5}$$

in which (hopefully) conservative values are used for the load height factor k_l.

2.2 INELASTIC LATERAL BUCKLING

Residual stresses induced in beams during manufacture often cause yielding to occur before the elastic lateral buckling moments M_c are reached. The reduced inelastic buckling moments M_I have been studied by a number of researchers including [9 - 11]. Conservative approximations [12] given by

$$\frac{M_I}{M_p} = 0.7 + \frac{0.3(1 - 0.7M_p / M_c)}{(0.61 - 0.3\beta_m + 0.07\beta_m^2)} \le 1.0 \tag{6}$$

(in which M_p is the full plastic moment) of the predictions [11] for beams with unequal end moments M and $\beta_m M$ are shown in Fig. 2. It can be seen that the greatest reductions below the elastic buckling moments occur for uniform bending (β_m = –1), for which yielding is constant along the beam. On the other hand, the reductions are smallest for beams in double curvature bending (β_m = 1), for which yielding is confined to the end regions of the beam, where it is comparatively unimportant, because the buckling deformations here are small.

2.3 STRENGTH

The strengths of beams in uniform bending are reduced below their elastic lateral buckling resistances not only by yielding and residual stresses, but also by the effects of geometrical imperfections such as initial crookedness and twist, and by local buckling effects. Local buckling effects are allowed for by reducing the full plastic moment M_p to the section moment capacity M_{sx}.

Geometrical imperfections initiate early yielding, as indicated by the first yield uniform bending moments M_y shown non-dimensionally in Fig. 3 by the variations of M_y / M_p with the modified slenderness $\lambda = \sqrt{(M_p / M_{yz})}$ given by [12]

$$\frac{1.121M_y}{M_p} = \frac{1.25 + 1.121M_{yz} / M_p}{2} - \sqrt{\left(\frac{1.25 + 1.121M_{yz} / M_p}{2}\right)^2 - \frac{1.121M_{yz}}{M_p}} \tag{7}$$

in which 1.121 is the assumed value of the section shape factor

$$S = Z_p / Z_e \tag{8}$$

in which Z_p and Z_e are the plastic and elastic section moduli. These first yield moments omit the effects of residual stresses and inelastic behaviour, and require modification before they can be used to determine the nominal design moment capacities M_b.

2.4 CODE DESIGN

2.4.1 EC3[3]

The modification of the European code EC3 [3] is given by

$$\frac{M_b}{M_{sx}} = \frac{1/f}{\Phi + \sqrt{\Phi^2 - \beta\lambda^2}} \le 1,\ 1/\lambda^2 \tag{9}$$

in which

$$\Phi = \{1 + \alpha(\lambda - \lambda_0) + \beta\lambda^2\} / 2 \tag{10}$$

in which λ is the modified slenderness given by

$$\lambda = \sqrt{M_{sx} / M_c} \tag{11}$$

and

$$f = 1 - (1 - \sqrt{1/\alpha_m})\{1 - 2(\lambda - 0.8)^2\} / 2 \le 1 \tag{12}$$

is a factor which allows for the effect of non-uniform bending on inelastic buckling (its effect on elastic buckling is allowed for by the use of the elastic buckling moment M_c in λ). For compact ($M_{sx} = M_p$) rolled I-beams with $2 \le h/b \le 3.1$ (in which h is the overall depth and b is the flange width), $\beta = 0.75$, $\alpha = 0.49$, and $\lambda_0 = 0.4$.

The values of M_b / M_p for beams in uniform bending ($\alpha_m = 1$) are compared in Fig. 3 with the first yield values M_y / M_p and the inelastic buckling values M_I / M_p. It can be seen that while the first yield moment M_y does not reach the plastic moment M_p at low slendernesses, the nominal moment capacity M_b does. At high slendernesses, the nominal moment capacity is reduced below the inelastic and elastic buckling moments to account for geometrical imperfections, as is the first yield moment M_y. Values of M_b / M_p for beams in non-uniform bending are shown in Fig. 4.

2.4.2 AS4100 [1]

The Australian code AS4100 [1] uses a lower bound fit to test results for beams in near uniform bending reviewed in [13]. The AS4100 formulation is

$$\frac{M_b}{M_{sx}} = \alpha_m \alpha_s \le 1 \tag{13}$$

in which

$$\alpha_s = 0.6 \left\{ \sqrt{\left[\left(\frac{M_{sx}}{M_{yz}} \right)^2 + 3 \right]} - \frac{M_{sx}}{M_{yz}} \right\} \le 1 \tag{14}$$

For uniform bending ($\alpha_m = 1$), these equations produce predictions which are a little lower than those of the EC3, as shown in Fig. 3. Their predictions for non-uniform bending are shown in Fig. 5.

2.4.3 AISC [4]

The AISC specification [4] ignores geometrical imperfections, and bases its nominal moment capacities on inelastic and elastic buckling predictions. The AISC formulation for uniform bending leads to values of M_b / M_p which are not uniquely determined by the value of λ, as are those of the EC3 and the AS4100, but vary with the beam section. For the beam section shown in Fig. 6, the AISC nominal design capacities may be approximated by

$$\frac{M_b}{M_{sx}} = \alpha_m (1.19 - 0.35\lambda\sqrt{\alpha_m} - 0.08\lambda^2 \alpha_m) \le 1, \frac{M_c}{M_{sx}} \tag{15}$$

For uniform bending ($\alpha_m = 1$), this equation produces predictions which are much closer to the inelastic buckling predictions than those of the EC3 and the AS4100, as shown in Fig. 3. Its predictions for non-uniform bending are shown in Fig. 7.

2.5 DESIGN BY BUCKLING ANALYSIS

It was noted in Section 2.1 above that some design codes [1, 2] give advice on the elastic lateral buckling of beams which is of somewhat doubtful accuracy, especially with respect to the effects of load height, while all codes are limited in the amount of advice on elastic buckling that they can give. This difficulty is avoided in the Australian code AS4100 [1] which explicitly allows the alternative method of design by buckling analysis [14], in which accurate values of the elastic buckling moment M_c are used directly in place of $\alpha_m M_{yz}$ in the design process to determine the nominal design capacity M_b. Thus computer programs such as PRFELB [6–8] can be used to obtain accurate values of M_c which account properly for the effects of load height and end restraints.

The European code EC3 [3] gives no advice on elastic lateral buckling, but requires the direct use of M_c in the design process. Thus EC3 implicitly requires the use of the method of design by buckling analysis. The AISC specification [4] gives no advice on the effects of load height.

3 CODE DESIGN OF CANTILEVERS

3.1 DESIGN

Code methods of designing cantilevers against lateral buckling are generally inadequate. The AISC specification [4] does not deal with cantilevers, while the British standard BS5950 [2] provides no allowance for different moment distributions. The European code EC3 [3] requires the use of the elastic buckling moment M_c but gives no advice on how this may be determined. Further, the factor f of Equation 12 used to allow for the effect of non-uniform bending on inelastic buckling can only be used for supported beams. While the Australian code AS4100 [1] provides approximations for the effects of moment distribution and load height on the elastic buckling of cantilevers, there is no allowance for the effect of non-uniform bending on inelastic buckling.

Information on elastic lateral buckling which can be used in the design of cantilevers is summarized in Section 3.2 below. A method of allowing for the effect of non-uniform bending on the inelastic buckling of cantilevers is developed in Section 4 following.

3.2 ELASTIC LATERAL BUCKLING

3.2.1 Cantilevers with end moments

The elastic buckling moment of a cantilever with an end moment that rotates about an axis parallel to the original cantilever axis [6] may be determined from

$$\frac{M_c L}{\sqrt{EI_y GJ}} = \frac{\pi}{2}\sqrt{1+\frac{K^2}{4}} \tag{16}$$

The approximate elastic buckling moment of a cantilever with an end moment that does not rotate [6] may be determined using the conservative equation

$$\frac{M_c L}{\sqrt{EI_y GJ}} = 1.6 + 0.9K \tag{17}$$

The variations of $\alpha_m = M_c / M_{yz}$ determined using these equations with the torsion parameter K are shown in Fig. 1. It can be seen that these are not as constant as those for supported beams.

3.2.2 Cantilevers with end loads

The approximate elastic buckling moment QL of a cantilever with an end load Q [6, 15] may be determined using the conservative equation

$$\frac{QL^2}{\sqrt{EI_y GJ}} = 11\left\{1+\frac{1.2\varepsilon}{\sqrt{1+(1.2\varepsilon)^2}}\right\} + 4(K-2)\left\{1+\frac{1.2(\varepsilon-0.1}{\sqrt{1+[1.2(\varepsilon-0.1)^2}}\right\} \tag{18}$$

in which

$$\varepsilon = \frac{y_Q}{L}\sqrt{\frac{EI_y}{GJ}} = \frac{2y_Q}{d}\frac{K}{\pi} \tag{19}$$

is a dimensionless load height parameter in which y_Q is the load height below the shear centre and d is the distance between flange centroids. Less accurate approximations are given in [1,2].

The variations of $\alpha_m = M_c / M_{yz}$ determined using these equations for centroidal loading (ε = 0) with the torsion parameter K are shown in Fig. 1. It can be seen that these are less constant than those for supported beams.

3.2.3 Cantilevers with distributed loads

The approximate elastic buckling moment $qL^2/2$ of a cantilever with a uniformly distributed load q [6, 15] may be determined using the conservative equation

$$\frac{qL^3}{2\sqrt{EI_yGJ}} = 27\left\{1+\frac{1.4(\varepsilon-0.1)}{\sqrt{1+[1.4(\varepsilon-0.1)]^2}}\right\}+10(K-2)\left\{1+\frac{1.3(\varepsilon-0.1}{\sqrt{1+[1.3(\varepsilon-0.1)^2]}}\right\} \quad (20)$$

The variations of $\alpha_m = M_c / M_{yz}$ determined using this equation for centroidal loading (ε = 0) with the torsion parameter K are shown in Fig. 1. It can be seen that these are even less constant than those for end loads.

It can be concluded that using values of α_m in Equation 2 will produce inaccurate values of the elastic buckling moment M_c. Despite this, the Australian code AS4100 [1] uses conservative values of α_m = 0.25, 1.25, and 2.25 for cantilevers with end moments, end loads and distributed loads, respectively. An approximate method of determining α_m for other moment distributions is given in [16].

3.2.4 Overhanging beams

An overhanging beam consists of a cantilever which is continuous with a supported span. Lateral buckling may occur either in the cantilever or in the supported span, or simultaneously in both. The first case has been studied [6, 15], and approximations developed for the maximum elastic buckling moments under either end loads or uniformly distributed loads.

The elastic lateral buckling of overhanging monorails under bottom flange end loads whose bottom flanges are free to deflect laterally at the exterior supports has also been studied [17, 18].

4 DESIGN BY INELASTIC BUCKLING ANALYSIS.

4.1 DESIGN OF BEAMS

A method of designing supported beams by inelastic buckling analysis was developed in [19] as a first attempt to produce an advanced method of designing frame structures against lateral buckling [20]. This method has been used [19] to study the effects of moment distribution, load height, and end restraints on the design strengths of simply supported beams.

In this method, reduced elastic moduli γE, γG are used in an elastic lateral buckling analysis to determine a reduced buckling moment M_{IB}. The reduced moduli are derived from the nominal lateral buckling design strengths M_b for simply supported beams in uniform bending, and so include allowances for the effects of yielding, residual stresses and geometrical imperfections. For beams in uniform bending, the reduced moments M_{IB} are equal to the design moments M_b.

The reduced moduli decrease as the bending moment increases, and so when they are applied to beams with non-uniform moment distributions, there are greater reductions in the high moment regions and smaller or no reductions in the low moment regions. The method thus takes account of the effect of the moment distribution on inelastic lateral buckling. A similar approach was used in [10] for the effects of residual stresses only, and in [21].

The reduction factors γ for the AS4100 [1] were derived [19] by setting the reduced buckling moments γM_{yz} for simply supported beams in uniform bending equal to the nominal design moments M_b determined from Equations 13 and 14 with α_m = 1, which led to

$$\gamma_{AS} = \frac{M_b}{M_{yz}} = 0.9 - \frac{1}{1.2}\left(\frac{M_b}{M_{sx}}\right)^2 \quad (21)$$

The variation of γ_{AS} with M_b / M_{sx} is shown in Fig. 8.

Also shown in Fig. 8 is the variation of γ_{EC} with M_b / M_{sx} determined using the EC3 [3] Equations 9-12 with α_m = 1, β = 0.75, α = 0.49, and λ_0 = 0.4. This variation can be closely approximated by

$$\gamma_{EC} = 1.12 - 0.4(M_b / M_p) - 0.56(M_b / M_p)^2 \leq 1.0 \tag{22}$$

It can be seen that this variation is higher than that for the AS4100, indicating that the EC3 is more optimistic for simply supported beams in uniform bending.

Values of γ_{AISC} determined using the AISC [4] Equation 15 with α_m = 1 are also shown in Fig. 8. These can be closely approximated by

$$\gamma_{AISC} = 1.32 + 0.4(M_b / M_p) - 1.5(M_b / M_p)^2 \leq 1.0 \tag{23}$$

It can be seen that this variation is higher than that for the EC3, indicating that the AISC is even more optimistic for simply supported beams in uniform bending.

The method of design by inelastic buckling analysis has been used to determine the nominal EC3, AS4100, and AISC design strengths M_{IB} of simply supported beams with either a central concentrated load at the shear centre or a single end moment. For this, the computer program PRFELB [7,8] was used, the section properties shown in Fig. 6 were assumed, and the reduced elastic moduli γE, γG were averaged over the length of each of the 10 or more elements into which each beam length was divided. The results are shown in Fig. 9. It can be seen that the values of M_{IB} / M_b are a little less than 1.0 for the AS4100 and the AISC, and a little greater for the EC3.

4.2 DESIGN OF CANTILEVERS

It is proposed here that the method of designing supported beams by inelastic buckling analysis [19] should be extended to cantilevers by using the same variations of the reduction factors γ as those shown in Fig. 8. While there is no experimental justification for this, neither is there for the application to cantilevers by the EC3 [3] and the AS4100 [1] of the formulations of Equations 9-11 or 13 and 14 for the design strengths of supported beams. For cantilevers in uniform bending, the use of this extension to determine reduced strengths M_{IB} by inelastic buckling analysis leads to values equal to the cantilever design strengths of [1,3], and so include the same allowances for the effects of residual stresses and geometrical imperfections.

When this method to cantilevers with non-uniform moment distributions, the reduced moduli decrease as the bending moment increases, and there are greater reductions in the high moment regions and smaller or no reductions in the low moment regions. The method thus takes account of the effect of the moment distribution on the inelastic lateral buckling of cantilevers.

This method of design by inelastic buckling analysis has been used to determine the nominal EC3, AS4100, and AISC [4] design strengths M_{IB} of cantilevers with either an end load or a uniformly distributed load. Again, the section properties shown in Fig. 6 were assumed, and the reduced elastic moduli γE, γG were averaged over the length of each of the 10 or more elements into which each cantilever length was divided. The results are shown in Figs 4, 5, and 7. In general, these results may be approximated by using the formulations for supported beams, provided that the elastic buckling moments M_c are calculated separately and approximate values of α_m are used only to allow approximately for the effect of moment distribution on inelastic buckling.

For the EC3, the results shown in Fig. 4 for both top and bottom flange loading suggest that safe approximations for the design strength can be obtained by using α_m = 2.3 for end loads or 3.6 for uniformly distributed loads in Equations 9 – 12. These values of α_m are quite high, as a result of the relative conservatism of the allowances that (the EC3's) Equation 12 makes for the effects of non-uniform bending. In view of the corresponding 10% overestimates shown in Fig. 8 for simply supported beams, EC3 designers may want to reduce these values of α_m in order to achieve consistency. Reducing α_m = 2.3 to 1.4 and 3.6 to 2.0 will reduce the values of M_{IBEC} / M_p shown in Fig. 4 by 10% approximately.

For the AS4100, the results shown in Fig. 5 for both top and bottom flange loadings suggest that safe approximations for the design strength M_{IBAS} can be obtained by using α_m = 1.23 for end loads or 1.42 for uniformly distributed loads in

$$\frac{M_{IBAS}}{M_{sx}} = 0.6\alpha_m \left\{ \sqrt{\lambda^4 \alpha_m^2 + 3} - \lambda^2 \alpha_m \right\} \leq 1 \tag{24}$$

This equation is a modification of the AS4100 Equations 13 and 14 for which M_{yz} (for simply supported beams in uniform bending) is replaced by M_c / α_m in which M_c is the cantilever elastic buckling moment, which may be obtained by using Section 3.2 above.

For the AISC, the results shown in Fig. 7 for both top and bottom flange loadings suggest that safe approximations for the design strength M_{IBAISC} can be obtained by using α_m = 1.24 for end loads or 1.3 for uniformly distributed loads.

5 WORKED EXAMPLE

5.1 EXAMPLE

A cantilever with the section and material properties shown in Fig. 7 is 2.0 m long and has concentrated load applied at the free end at the top flange. Determine the nominal design strengths using the design by inelastic buckling results for the EC3 [3], the AS4100 [1] and AISC [4].

5.2 ELASTIC BUCKLING

Using (3), K = 3.14
Using (18), ε = –1.00
Using (17), $QL^2/\sqrt{(EI_yGJ)}$ = 3.48, so that $M_c = QL$ = 640 kNm.
$M_p = f_y Z_p$ = 498 kNm, so that λ = 0.882 using (11).

5.3 EC3 DESIGN

Using α_m = 1.4 and (12), f = 0.924
Using (10), Φ = 0.910
Using (9), M_{IBEC}/M_p = 0.711, so that M_{IBEC} = 384 kNm.

5.4 AS4100 DESIGN

Using α_m = 1.23 and (24), M_{IBAS}/M_p = 0.754 so that M_{IBAS} = 375 kNm.

5.5 AISC DESIGN

Using α_m = 1.24 and (15), M_{IBAISC}/M_p = 0.954 so that M_{IBAISC} = 475 kNm.
This is significantly higher than the values of 384 kNm and 375 kNm for the EC3 and the AS4100.

5.6 COMPARISON

The variations according to the EC3, AS4100, and AISC of M_{IB}/M_p with λ are compared in Fig. 10. The values shown by the heavier lines are similar for the EC3 and the AS4100, but much lower than those of the AISC. Also shown in Fig. 10 (by the lighter lines) are the corresponding values obtained by ignoring the increases caused by the effect of the moment distribution on inelastic buckling (by using the elastic buckling values of M_c and α_m = 1.0). It can be seen that there are significant advantages to be gained by using the values of α_m predicted by inelastic buckling analyses which take into account the effect of the moment distribution.

6 CONCLUSIONS

The lateral buckling strengths of cantilevers are very different from those of supported beams because of the very different restraint conditions and buckling modes. Because of this, design methods for supported beams require significant modification before they can be efficient for cantilevers. Design codes provide little or no design guidance for cantilevers, especially on the effects of load height and moment distribution.

This paper develops a method for the efficient design of cantilevers which is consistent with the methods used in the design of supported beams. Available information on the effects of moment distribution and load height on the elastic buckling of cantilevers is summarized, and a method of allowing for the effect of moment distribution on the inelastic buckling of cantilevers is developed by adapting the methods used for supported beams.

The EC3 [3], AS4100 [1], and AISC [4] design rules for the lateral buckling of supported beams are reviewed to show the ways in which these codes allow for elastic buckling, geometrical imperfections, residual stresses, moment distribution and load height. For these codes, the same formulation is used to allow for the effects of moment distribution on both elastic and inelastic buckling. However, it is found that this method is unsuitable for the elastic buckling of cantilevers, and that these effects need to be considered separately, by using the elastic buckling moment M_c, and a separate factor α_m for inelastic buckling.

A method of design by inelastic buckling analysis developed for supported beams is assessed in relation to the EC3, AS4100, and AISC codes to show how this accounts for the effects of moment distribution, residual stresses and geometrical imperfections. This method is then extended to cantilevers to allow for the effect of moment distribution on inelastic buckling, and values of the necessary factors α_m for use in design code formulations are determined.

A worked example of the EC3, AS4100, and AISC design of a cantilever with a top flange end load is summarized, and the results compared. In all cases, significant benefits are found. The AISC nominal design strengths are significantly higher than those of the EC3 and the AS4100, primarily because the AISC strength formulations ignore geometrical imperfections and are more optimistic with respect to residual stresses. The EC3 formulation is very conservative for non-uniform bending, while the AS4100 predictions for uniform bending are a little lower than those of the EC3.

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APPENDIX II NOTATION

b	flange width
d	distance between flange centroids
E	Young's modulus of elasticity
f	EC3 factor for non-uniform bending
f_y	yield stress
G	shear modulus of elasticity
h	overall depth of beam
I_w	warping section constant
I_y	second moment of area about the y axis
J	torsion section constant
k_l, k_r	effective length factors
l	effective length
L	member length
M	applied moment
M_b	nominal member moment capacity
M_c	elastic buckling moment
M_I	inelastic buckling moment
M_{IB}	moment capacity determined by inelastic buckling analysis
M_p	full plastic moment
M_{sx}	major axis section moment capacity
M_y	actual first yield moment
M_{yz}	elastic buckling moment of a beam in uniform bending
q	intensity of uniformly distributed load
Q	concentrated load
S	shape factor
t_f, t_w	flange and web thicknesses
y_Q	distance of load below shear centre
Z_e, Z_p	elastic and plastic major axis section moduli
α	EC3 imperfection factor
α_m	moment modification factor
α_s	AS4100 beam slenderness reduction factor
β	EC3 correction factor
β_m	end moment ratio
γ_{AS}, γ_{EC}	inelastic modulus reduction factors
ε	dimensionless load height
λ_c	modified slenderness
λ_0	EC3 factor for plateau length
Φ	EC3 factor (Equation 10)

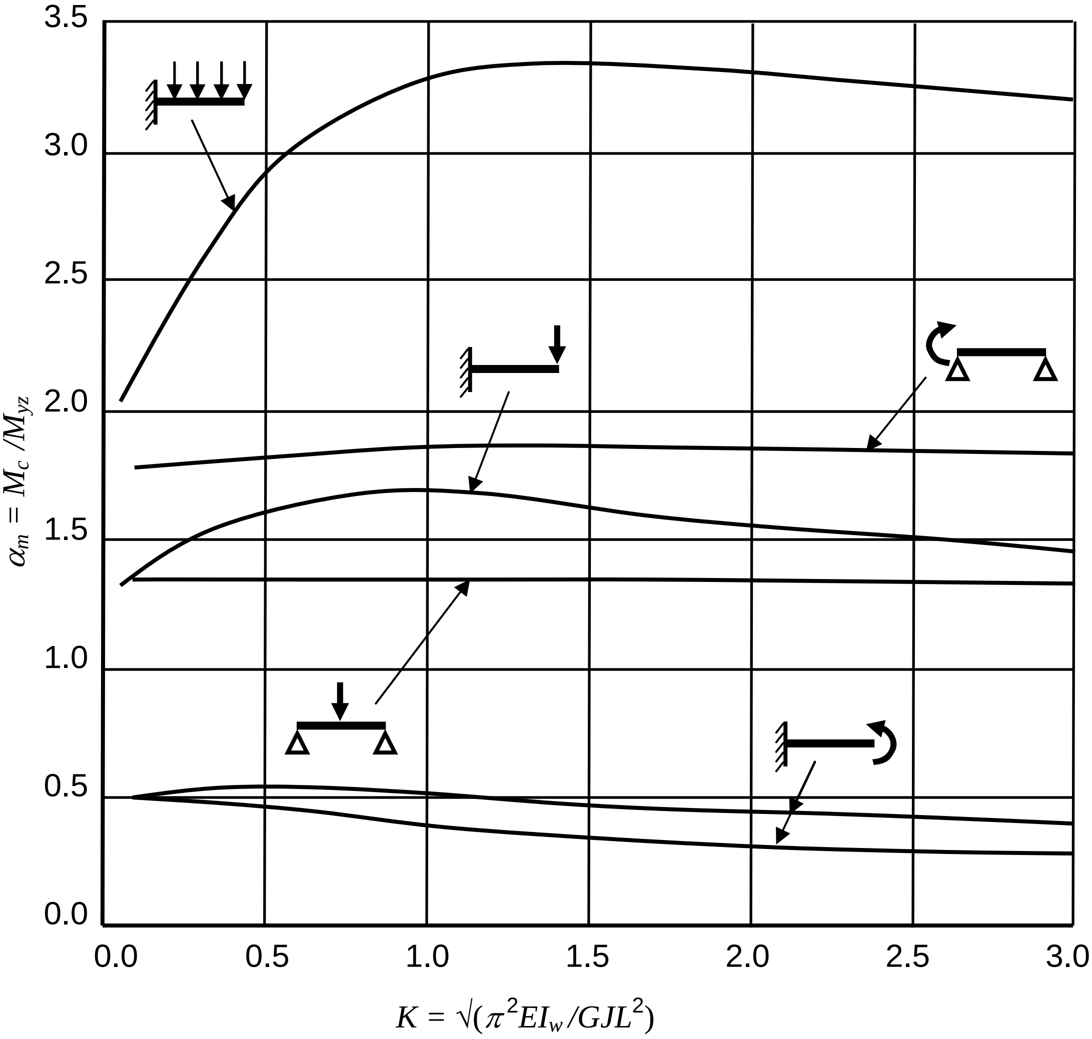


Fig. 1 Elastic Buckling Values of α_m

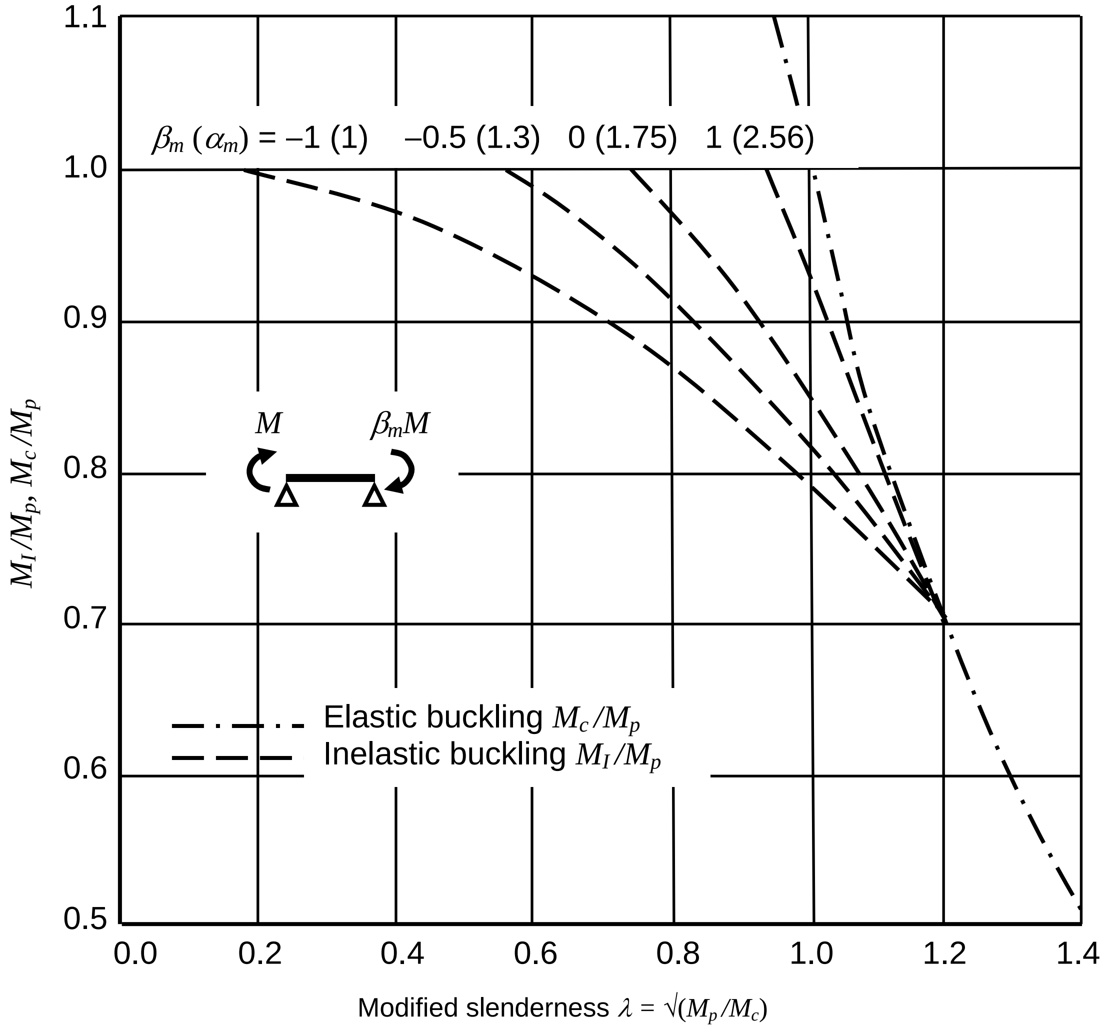


Fig. 2. Inelastic Buckling

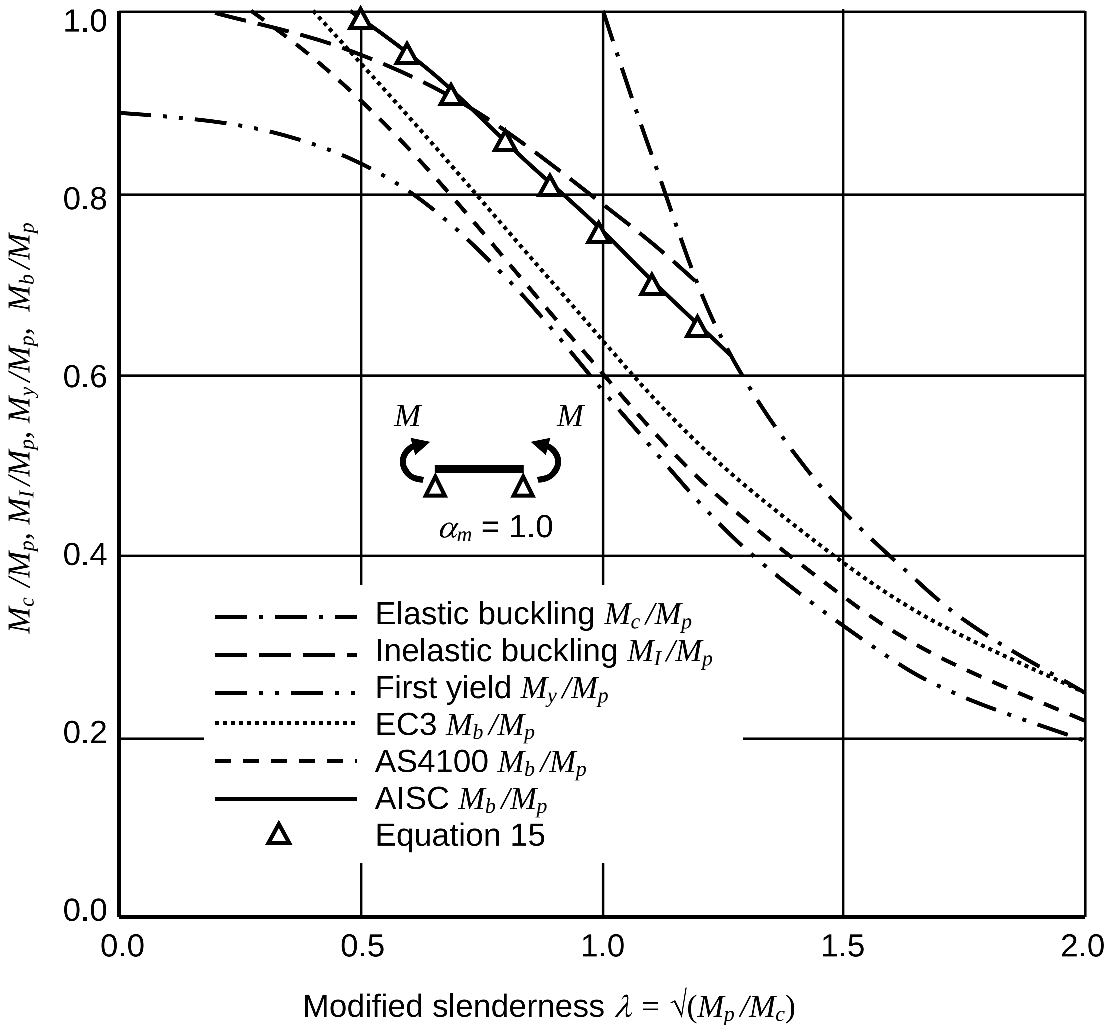


Fig. 3. Buckling, Yielding, and Nominal Design Capacities

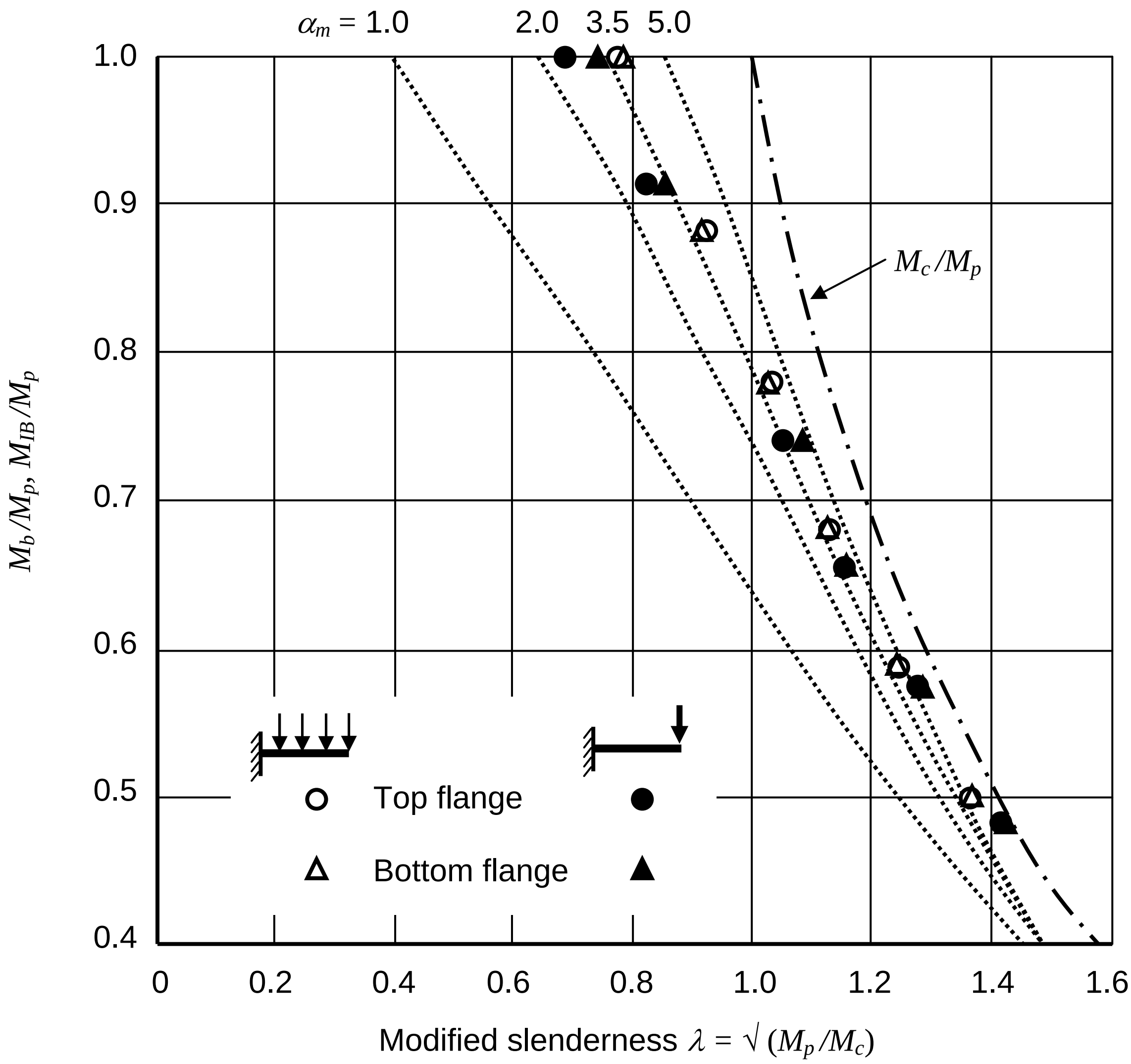


Fig. 4. EC3 and Cantilever Strengths by Inelastic Buckling

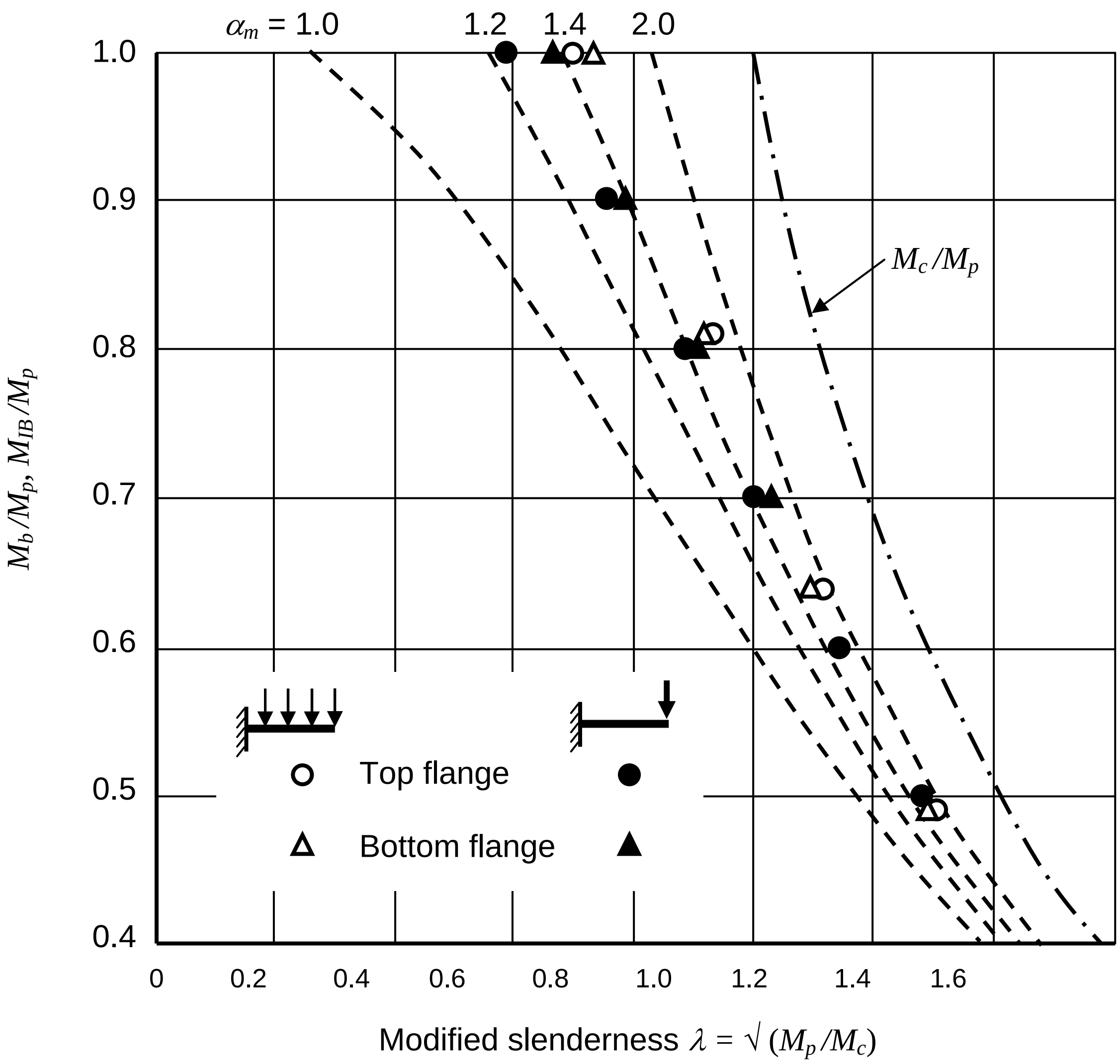
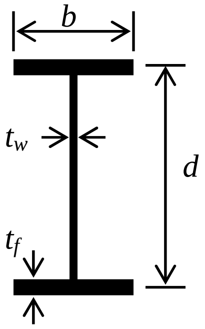


Fig. 5. AS4100 and Cantilever Strengths by Inelastic Buckling



b = 190 mm	Z_e = 1460 E3 mm^3
t_f = 14.5 mm	Z_p = 1660 E3 mm^3
d = 442.5 mm	I_y = 16.6 E6 mm^4
t_w = 9.1 m	J = 530 E3 mm^4
	I_w = 815 E9 mm^6

E = 2E5 MPa, G = 76923 MPa, f_y = 300 MPa

Fig. 6. Section Properties

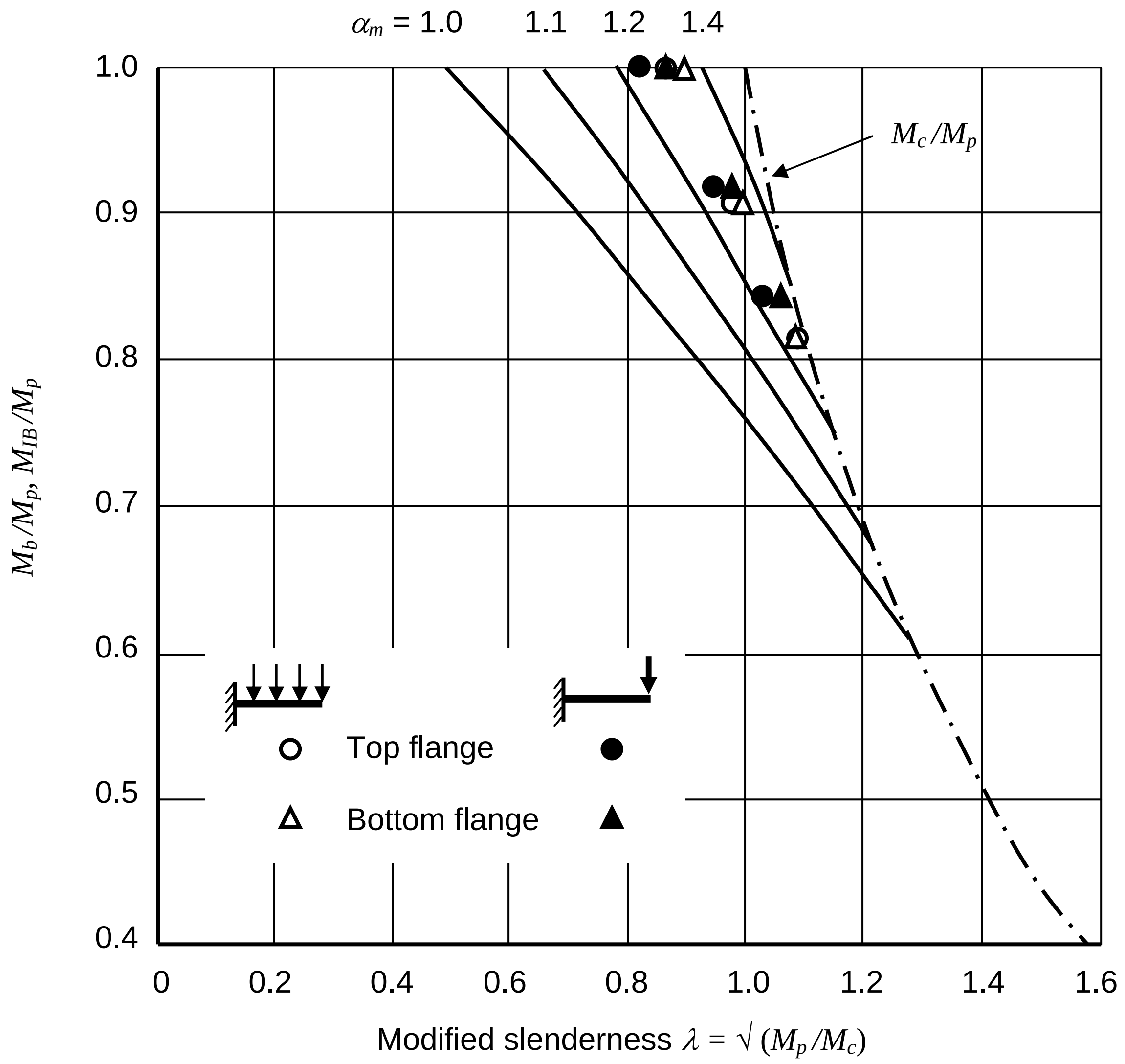


Fig. 7. AISC and Cantilever Strengths by Inelastic Buckling

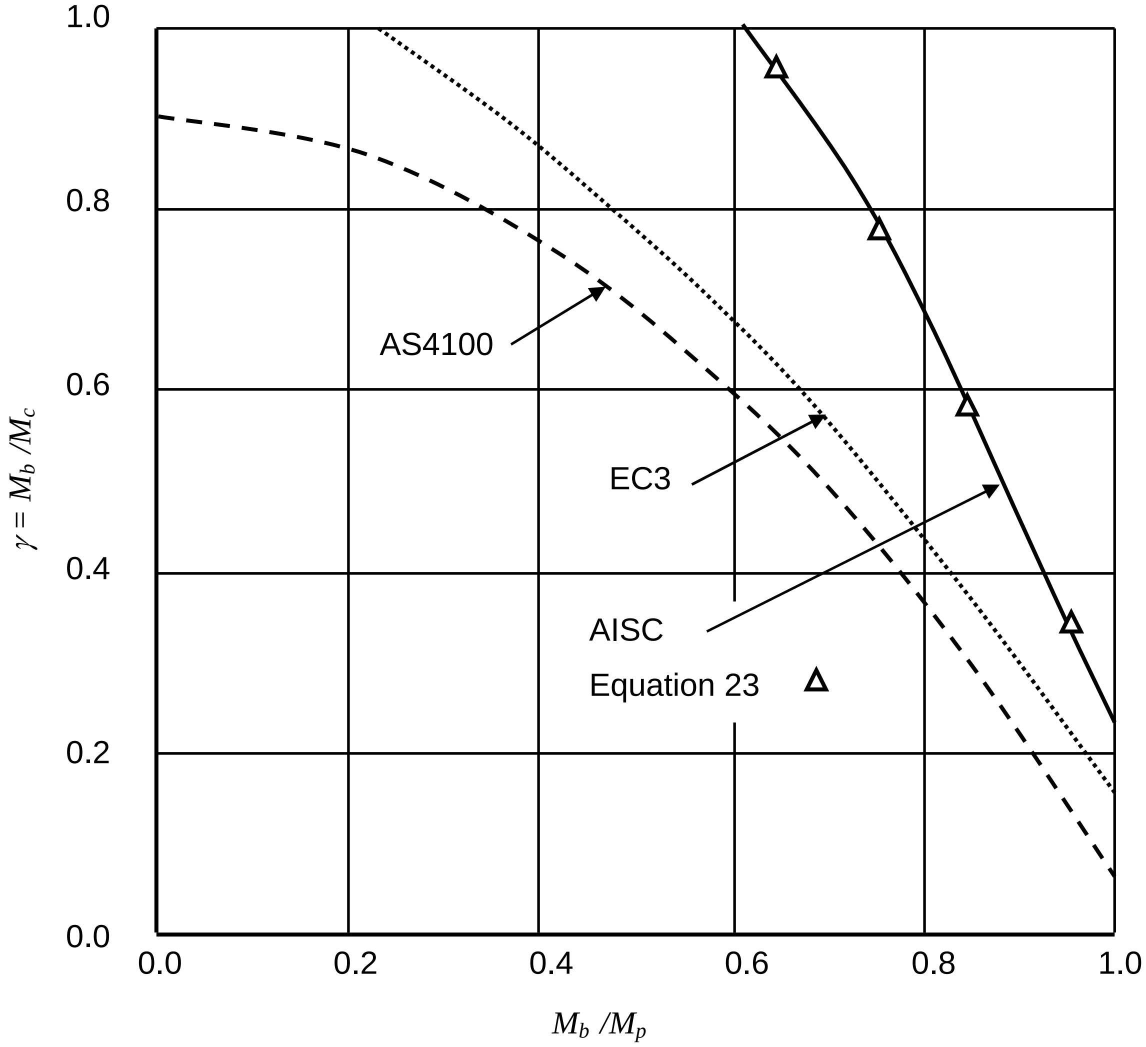


Fig. 8. Reduced Elastic Moduli Factors

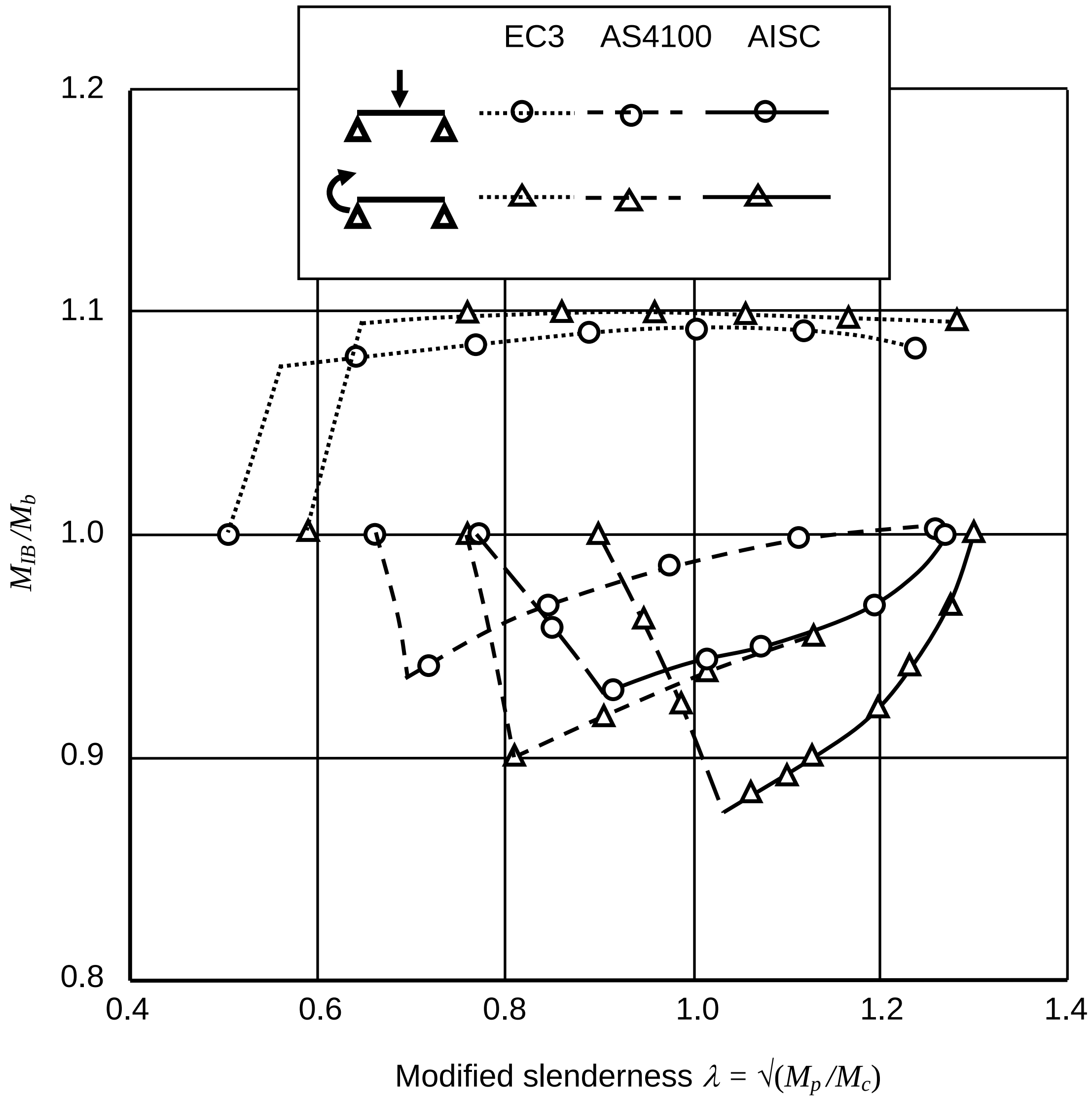


Fig. 9. Beam Design by Inelastic Buckling Analysis

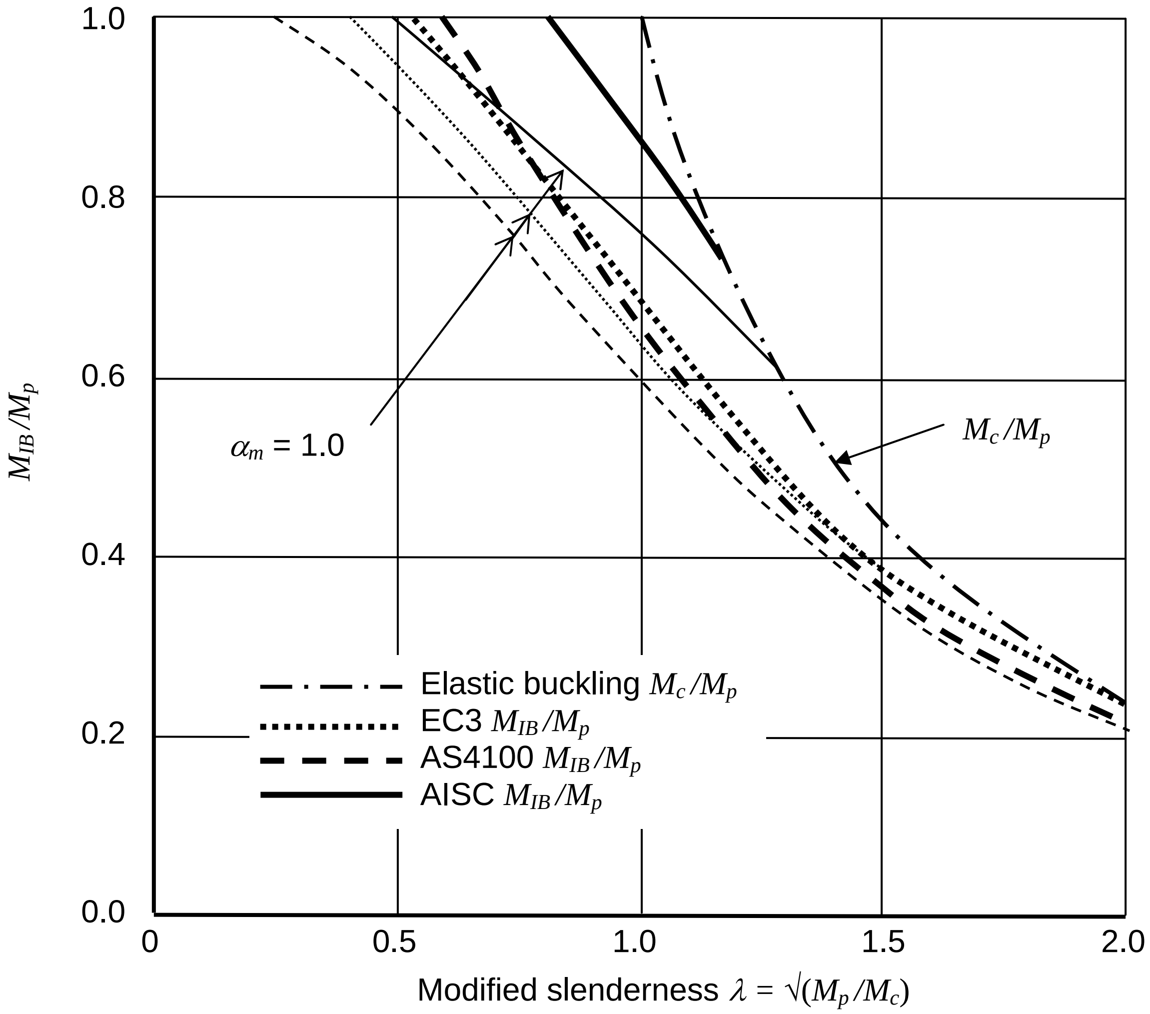


Fig. 10. Example Nominal Design Strengths

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