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Research Report No R862

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Abstract:

In this study, a new analytic solution of the wave interaction with a porous structure over a sandy seabed is derived. A uniform water depth is assumed, and the layer of the rigid porous medium is placed over an infinite thickness of poro-elastic sand bottom. The potential theory with the inertial and damping effects in the porous flow is considered in the rigid porous region, while the soil consolidation theory is adopted in the sand region. A new complex dispersion relationship involving the parameters of the rigid porous and poro-elastic medium is obtained. The analytic solutions can be simplified to the special cases, such as the wave interaction with the porous structure over an impermeable bottom or only interaction with the poro-elastic medium. The results indicate that the wave decay is highly dependent upon the thickness of the porous structure, the soil stiffness and their permeability. The increase of the thickness of the porous structure will shorten the wave length of the surface waves regardless of the coarse or fine sand. The pore pressure of the fine sand is larger than that of coarse sand, both decay with the distance of the wave progressing. It is also found that the increase of the thickness of the porous structure will effectively reduce the pore pressure in the sand.

Keywords:
wave interaction; porous structure; poro-elastic medium; complex dispersion relationship; wave decay; soil stiffness; permeability; pore pressure

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1. Introduction#

Recently, a submerged porous structure has been a common coastal structure worldwide. The porous structure is able to absorb and dissipate the wave energy when the wave transmits over it. On the other hand, the submerged porous structure is served no visible in the coastal region and it has the function of ecological restoration in the coastal area. Thus, it is increasing regarded by coastal engineers for use against the beach erosion and the coastal hazard. Most previous researches have paid attention for interactions between waves and porous structure but an impermeable rigid bottom was considered. In fact, it is commonly to have sandy bed beneath a porous structure, which will have strong interaction among waves and these two porous mediums. However, to date, no research considers the poro-elastic seabed in such a problem.

Numerous numerical models for the wave interaction with the rigid porous structure have been cariied out, based on the Navier-Stokes equations (Van Gent et al., 1994; Troch and Rouck, 1998; Liu et al., 1999; Hsu et al., 2002; Hur and Mizutani 2003). Alternatively, Sollit and Cross (1972) proposed a potential theory to simplify the specific problem, in which the nonlinear drag in the porous structure was linearized by applying Lorentz's condition of equivalent work. Since then the potential theory was adopted in many investigations for solving such boundary value problems (Madsen, 1974; Lee,1987; Rojanakamthorn et al., 1989; Dalrymple et al., 1991; Chen et al., 2006). However, all these have been limited to a rigid impermeable bottom. In recent, Tsai et al. (2006) presented a newly mild slope equation to investigate the wave transformation over a submerged permeable breakwater on a rigid porous slope seabed.

Numerous investigations for wave-induced soil response in a porous seabed have been carried out since the 1970s. Yamamoto et al (1978) is one of the classic examples, which considered both pore fluid and soil to be compressible. Later, Hsu et al. (1993) further extended to the case of a three-dimensional short-crested wave system. However, these investigations considered wave pressure to penetrate directly into the soil bottom but have not responsed the interaction of waves and porous seabed. Recently, Jeng (2000) and Lee and Lan (2002) introduced mass continuity

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condition at water and soil interface to examine the effects of porous seabed on the wave characteristics, the energy decay and soil response.

In this study, an analytical solution for ocean wave propagating on a porous structure over sandy seabed will be derived. Unlike previous investigations, the seabed is considered as a poro-elastic medium. In the new solution, the interaction among waves, porous structures and sandy beds will be considered. The general solutions reduced to some special cases will also be presented for the porous structure on a rigid impermeable seabed, waves propagating over a poro-elastic seabed and waves propagating over an impermeable seabed. Based on the model, the effects of thickness of the porous structure and soil characteristics on the wave damping, variation of the wavelength and pore pressure will be examined.

2. Theoretical Formulations

2.1 Governing equations

The physical problem modelled in this study is shown in Figure 1, in which ocean waves over a uniform water depth of h_2 and a rigid porous structure of h_p on top of an infinite sandy seabed. As shown in Figure 1, three regions are formed:

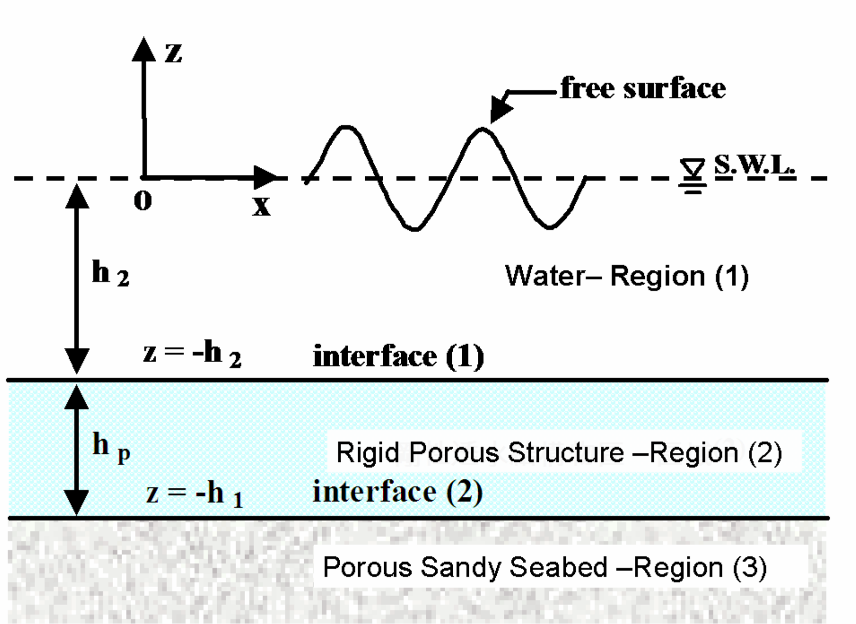

Fig. 1 Definition sketch of wave propagation on a porous structure over sand bottom.

Region (1): water field ($-h_2 \le z \le \eta$)

Assuming the fluid is inviscid and incompressible, the velocity potential (ϕ) satisfies the *Laplace*'s equation:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0, \tag{1}$$

The linear wave dynamic pressure (p) can be determined by

$$p = -\rho\frac{\partial\phi}{\partial t} - \rho g z, \tag{2}$$

where ρ is the density of water, and g is the gravitational acceleration.

Region (2): rigid porous medium field ($-h_1 \le z \le -h_2$)

Assuming the rigid porous structure is homogeneous isotropic media, and the flow obeys the linearized unsteady Bernoullis' equation (Sollitt and Cross, 1972). Thus, the velocity potential also satisfies the Laplace's equation,

$$\nabla^2\phi_p = 0. \tag{3}$$

In the rigid porous region, the relation of pressure (p_p) and velocity potential (ϕ_p) is given by

$$C_r\frac{\partial\phi_p}{\partial t} + \frac{p_p}{\rho} + gz + f_p\omega\phi_p = 0, \tag{4}$$

where C_r is the inertia coefficient in the region (2), ω is the wave frequency ($=2\pi/T$, T is the wave period). In (4), f_p is a non-dimensional friction factor, assuming the work done by friction forces in the porous fluid is a constant within one wave period, which can be expressed as

$$f_p = \frac{1}{\omega} \frac{\int\limits_V \int\limits_0^T \left[\frac{n_p^2 \nu}{k_p} \left| \vec{u}_p \right|^2 + \frac{n_p^3 C_{fp}}{\sqrt{k_p}} \left| \vec{u}_p \right|^3 \right] dt dV}{\int\limits_V \int\limits_0^T k_p \left| \vec{u}_p \right|^2 dt dV}, \tag{5}$$

where V is the volume, n_p is the porosity of porous media, ν is viscosity of fluid, $\vec{u}_p$ is the seepage velocity of pore fluid in rigid porous media, k_p is the permeability coefficient of rigid porous media, and C_{fp} is the turbulent damping coefficient.

Region (3): poro-elastic medium field ($-\infty \le z \le -h_1$)

Considering the porous seabed is homogeneous sandy seabed, in which the pore fluid and soil are compressible and linear elastic soil behaviour. Thus, the soil satisfies the Hooke's law, and porous flow is governed by Darcy's law. Thus, the governing equation is given by

$$\nabla^2 p_s - \frac{\gamma_w n_s \beta}{K_p} \frac{\partial p_s}{\partial t} = \frac{\gamma_w}{K_p} \frac{\partial \varepsilon}{\partial t}, \tag{6}$$

where p_S is the wave-induced pore water pressure, K_p is the soil permeability, γ_w is the specific weight of pore water, n_s is the porosity, ε is the volume strain of the soil skeleton and β is the compressibility of the pore water. The volume strain and the compressibility are defined by

$$\varepsilon = \frac{\partial \xi}{\partial x} + \frac{\partial \chi}{\partial z} \quad \text{and} \quad \beta = \frac{1}{K_w} + \frac{1-S}{P_{wo}}, \tag{7}$$

where ξ and χ are the soil displacements in the *x*- and *z*-directions, respectively. K_w is the bulk modulus of pore fluid, S_r is the degree of saturation, and P_{wo} is the absolute pressure. Based on Hooke's law and the effective stress concept, the equilibrium equations of a poroelastic medium in the *x* and *z* directions are given as

$$G\nabla^2\xi + \frac{G}{1-2\mu}\frac{\partial \varepsilon}{\partial x} = \frac{\partial p_s}{\partial x}, \tag{8}$$

$$G\nabla^2\chi + \frac{G}{1-2\mu}\frac{\partial \varepsilon}{\partial z} = \frac{\partial p_s}{\partial z}, \tag{9}$$

in which G and μ are the shear modulus and Poisson's ratio for the soil, respectively.

2.2 Boundary conditions

To solve the above governing equations, the following boundary conditions are required:

(a) *Free surface boundary conditions:*

Considering the small amplitude waves, the velocity potential must satisfy the dynamic and kinematic free surface boundary conditions, i.e.

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad \text{and} \quad \eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}, \qquad z = 0 \tag{10}$$

where η is the surface water elevation, which can be expressed as

$$\eta = ae^{i\Psi}, \quad \Psi = kx - \omega t, \tag{11}$$

in which a is the amplitude of incident waves, ω is wave frequency, and k is the wave number, Ψ is the wave phase function.

(b) *Interface of water and porous structure:*

At the interface of water and rigid porous structure, the pressure gradient and mass flux must be continuous, i.e.,

$$\phi = \alpha\phi_p \quad \text{and} \quad \frac{\partial \phi}{\partial z} = n_p \frac{\partial \phi_p}{\partial z}, \qquad z = -h_2, \tag{12}$$

where $\alpha = C_r + if_p$.

(c) *Interface of porous structure and porous sandy bed:*

At the interface of porous structure and sandy bed, the vertical effective normal stress (σ_z) and shear stress (τ_{xz}) are assumed to be zero due to the ignorance of viscous shear stress, and the mass and pressure must be continuous, i.e.,

$$\sigma_z = \tau_{xz} = 0, \qquad z = -h_1, \tag{13}$$

$$-\rho\alpha\frac{\partial\phi_p}{\partial t}=p_s\,,\qquad z=-h_1\,, \tag{14}$$

$$n_p\frac{\partial\phi_p}{\partial z}=-\frac{K_p}{\gamma_w}\frac{\partial p_s}{\partial z}+\frac{\partial\chi}{\partial t}\,,\qquad z=-h_1\,. \tag{15}$$

The relationships between effective stresses and soil displacements are expressed as

$$\sigma_z=2G\left[\frac{\partial\xi}{\partial z}+\frac{\mu}{1-2\mu}\varepsilon\right], \tag{16}$$

$$\tau_{xz}=G\left[\frac{\partial\chi}{\partial x}+\frac{\partial\xi}{\partial z}\right]. \tag{17}$$

(d) *Bottom boundary condition:*

At the bottom of infinite seabed, the pore pressure and displacements are vanish, i.e.,

$$\xi\,,\ \chi\ \text{and}\ p_S\rightarrow 0\ \text{ as }\ z\rightarrow-\infty\,, \tag{18}$$

3. Analytical Solutions

3.1 General solutions

Based on (1) and (10), the velocity potential in wave field can be expresses as

$$\phi=-\frac{iag}{\omega}\left\{\cosh kz+\frac{\omega^2}{gk}\sinh kz\right\}e^{i\Psi}\,. \tag{19}$$

For the velocity potential in rigid porous structure (ϕ_p), based on (3) and (12), we have

$$\phi_p=-\frac{iag}{\omega}\left\{A_p\cosh kz+B_p\sinh kz\right\}e^{i\Psi}\,, \tag{20}$$

where

$$A_p=\frac{1}{\alpha}\cosh^2 kh_2-\frac{1}{n_p}\sinh^2 kh_2+\frac{\omega^2}{2gk}\left(\frac{1}{n_p}-\frac{1}{\alpha}\right)\sinh 2kh_2\,, \tag{21a}$$

$$B_p=\frac{\omega^2}{gk}\left(\frac{1}{n_p}\cosh^2 kh_2-\frac{1}{\alpha}\sinh^2 kh_2\right)-\frac{1}{2}\left(\frac{1}{n_p}-\frac{1}{\alpha}\right)\sinh 2kh_2\,. \tag{21b}$$

Following up Hsu et al. (1993), the general solution for the pore pressure and soil displacement can be obtained from (6) and (13)-(15) as

$$p_s = P_* F_1(z) e^{i\Psi}, \tag{22a}$$

$$\xi = \frac{iP_*}{2Gk} F_2(z) e^{i\Psi}, \tag{22b}$$

$$\chi = \frac{P_*}{2Gk} F_3(z) e^{i\Psi}, \tag{22c}$$

where

$$P_* = \alpha\rho g a\left(A_p \cosh kh_1 - B_p \sinh kh_1\right), \tag{23}$$

$$F_1(z) = (1 - C_p)e^{k(h_1+z)} + C_p e^{\delta(h_1+z)}, \tag{24a}$$

$$F_2(z) = \{[\lambda + k(h_1 + z)]C_1 - (1 + 2\mu - 2\mu k/\delta)C_2\}e^{k(h_1+z)} + (k/\delta)C_2 e^{\delta(h_1+z)}, \tag{24b}$$

$$F_3(z) = [C_o + k(h_1 + z)C_1]e^{k(h_1+z)} + C_2 e^{\delta(h_1+z)}, \tag{24c}$$

$$C_p = \frac{(1-\mu)(\delta + k)\lambda}{(1-2\mu)(\delta - \delta\mu + k\mu + k\lambda)}, \tag{25a}$$

$$C_o = -(1+\lambda)C_1 - (1 + 2\mu - 2\mu k/\delta)C_2, \tag{25b}$$

$$C_1 = \frac{\delta - \delta\mu + k\mu}{\delta - \delta\mu + k\mu + k\lambda}, \tag{25c}$$

$$C_2 = \frac{k\delta\lambda}{(\delta - k)(\delta - \delta\mu + k\mu + k\lambda)}, \tag{25d}$$

$$\delta^2 = k^2 - \frac{i\omega\gamma_w}{K_p}\left[n_s\beta + \frac{1-2\mu}{2G(1-\mu)}\right], \tag{26a}$$

$$\lambda = \frac{(1-2\mu)Gn_s\beta}{Gn_s\beta + 1 - 2\mu}, \tag{26b}$$

in which the wave number (k) is an unknown parameter to be determined.

3.2 Wave dispersion relation

In this study, we consider the interaction between rigid porous structure and porous sandy bed. Thus, we substitute the velocity potential for rigid porous structures (ϕ_p), pore water pressure (p_s) and vertical displacement (χ) into the boundary condition (15), from which the new wave dispersion relation can be derived as

$$\frac{\omega^2}{gk}=\frac{n_p[\text{shp}][\text{ch2}]+\alpha[\text{chp}][\text{sh2}]+\hat{\alpha}(n_p[\text{chp}][\text{ch2}]+\alpha[\text{shp}][\text{sh2}])}{n_p[\text{shp}][\text{sh2}]+\alpha[\text{chp}][\text{ch2}]+\hat{\alpha}(n_p[\text{chp}][\text{sh2}]+\alpha[\text{shp}][\text{ch2}])} \qquad (27)$$

where $[\text{shp}]=\sinh(kh_p)$, $[\text{chp}]=\cosh(kh_p)$, $[\text{sh2}]=\sinh(kh_2)$, $[\text{ch2}]=\cosh(kh_2)$, and the non-dimensional parameter, $\hat{\alpha}$ is defined by

$$\hat{\alpha}=\frac{\alpha}{n_p}\left[\frac{-iK_p\omega}{gk}(k-C_pk+C_p\delta)+\frac{\rho\omega^2}{2Gk^2}(C_o+C_2)\right] \qquad (28)$$

It is noted that the properties of rigid porous structure (e.g., α, n_p, … etc.) and poro-elastic seabed (e.g., G, K_p and μ etc.) are included in the new wave dispersion relation (27), which is a complex form.

Equation (27) can be re-arranged as

$$\frac{\omega^2}{gk}=\frac{n_p\tanh kh_p+\alpha\tanh kh_2+\hat{\alpha}(n_p+\alpha\tanh kh_p\tanh kh_2)}{n_p\tanh kh_p\tanh kh_2+\alpha+\hat{\alpha}(n_p\tanh kh_2+\alpha\tanh kh_p)} \qquad (29)$$

Then, substituting (27) or (29) into (15), we have the velocity potential of water region, given as

$$\phi=-\frac{iag}{\omega}\frac{n_p[\text{shp}][\text{shz2}]+\alpha[\text{chp}][\text{chz2}]+\hat{\alpha}(n_p[\text{chp}][\text{shz2}]+\alpha[\text{shp}][\text{chz2}])}{n_p[\text{shp}][\text{sh2}]+\alpha[\text{chp}][\text{ch2}]+\hat{\alpha}(n_p[\text{chp}][\text{sh2}]+\alpha[\text{shp}][\text{ch2}])}e^{i\Psi} \qquad (30)$$

in which $[\text{shz2}]=\sinh k(z+h_2)$ and $[\text{chz2}]=\cosh k(z+h_2)$. Substituting (27) into (21a) and (21b), we have

$$A_p=\frac{[\text{chp}][\text{ch2}]+[\text{shp}][\text{sh2}]+\hat{\alpha}([\text{shp}][\text{ch2}]+[\text{chp}][\text{sh2}])}{n_p[\text{shp}][\text{sh2}]+\alpha[\text{chp}][\text{ch2}]+\hat{\alpha}(n_p[\text{chp}][\text{sh2}]+\alpha[\text{shp}][\text{ch2}])}, \qquad (31a)$$

$$B_p=\frac{[\text{chp}][\text{sh2}]+[\text{shp}][\text{ch2}]+\hat{\alpha}([\text{shp}][\text{sh2}]+[\text{chp}][\text{ch2}])}{n_p[\text{shp}][\text{sh2}]+\alpha[\text{chp}][\text{ch2}]+\hat{\alpha}(n_p[\text{chp}][\text{sh2}]+\alpha[\text{shp}][\text{ch2}])}. \qquad (31b)$$

Thus we have the velocity potential in the region of the rigid porous structure, given as

$$\phi_p=-\frac{iag}{\omega}\frac{[\text{chz1}]+\hat{\alpha}[\text{shz1}]}{n_p[\text{shp}][\text{sh2}]+\alpha[\text{chp}][\text{ch2}]+\hat{\alpha}(n_p[\text{chp}][\text{sh2}]+\alpha[\text{shp}][\text{ch2}])}e^{i\Psi}, \qquad (32)$$

in which $[\text{shz1}] = \sinh k(z+h_1)$, $[\text{chz1}] = \cosh k(z+h_1)$.

Finally, substituting (31a) and (31b) into (23), we have the amplitude of wave pressure at the seabed surface as

$$P_* = \frac{\alpha \rho g a_o}{n_p[\text{shp}][\text{shz2}] + \alpha[\text{chp}][\text{chz2}] + \hat{\alpha}\left(n_p[\text{chp}][\text{shz2}] + \alpha[\text{shp}][\text{chz2}]\right)}. \quad (33)$$

Introducing (33) into (22a)-(22c), the pore pressure and soil displacements in sandy bed can be further obtained.

3.3 Special cases

The above general solutions can be reduced to the following special cases.

(a) Case I: Porous structure on a rigid impermeable seabed

Let the interface of porous structure and sandy bed ($z = -h_1$) to be an impermeable bottom, that is, ignoring the porous sandy bed. This leads to $\hat{\alpha} = 0$, from which velocity potentials and wave dispersion relations become

$$\phi = -\frac{iag}{\omega} \frac{n_p[\text{shp}][\text{shz2}] + \alpha[\text{chp}][\text{chz2}]}{n_p[\text{shp}][\text{sh2}] + \alpha[\text{chp}][\text{ch2}]} e^{i\Psi}, \quad (34)$$

$$\phi_p = -\frac{iag}{\omega} \frac{[\text{chz1}]}{n_p[\text{shp}][\text{sh2}] + \alpha[\text{chp}][\text{ch2}]} e^{i\Psi}, \quad (35)$$

$$\frac{\omega^2}{gk} = \frac{n_p \tanh kh_p + \alpha \tanh kh_2}{n_p \tanh kh_p \tanh kh_2 + \alpha}. \quad (36)$$

It is noted that (34)-(36) are the solution of waves over a porous structure on an impermeable seabed, which is identical to the previous solution proposed by Rojanakamthorn et al. (1989).

(b) Case II: Waves propagating over a porous seabed without a rigid porous structure

Considering the case of wave propagating over a porous seabed without a porous structure, i.e., substituting $h_p = 0$, and $n_p = 1$, $\alpha = 1$ and $h_1 = h_2$ into (29), (30) and (33), we have

$$\phi = -\frac{iag}{\omega}\frac{[\text{chz1}] + \hat{\alpha}[\text{shz1}]}{[\text{ch1}] + \hat{\alpha}[\text{sh1}]}e^{i\Psi}, \tag{37}$$

$$P_* = \frac{\rho g a_o}{[\text{chz1}] + \hat{\alpha}[\text{shz1}]}, \tag{38}$$

$$\frac{\omega^2}{gk} = \frac{\hat{\alpha} + \tanh kh_1}{1 + \hat{\alpha}\tanh kh_1}, \tag{39}$$

which is identical to the results of Tsai and Chen (2002).

(c) Case III: Waves propagating over an impermeable seabed without porous mediums

Now, we further consider the traditional case of waves propagating over an impermeable seabed. Let non-dimensional variable $\hat{\alpha} = 0$ into (37) and (39), the velocity potential and wave dispersion relation become

$$\phi = -\frac{iag}{\omega}\frac{[\text{chz1}]}{[\text{ch1}]}e^{i\Psi} = -\frac{iag}{\omega}\frac{\cosh k(h_1 + z)}{\cosh kh_1}e^{i\Psi}, \tag{40}$$

$$\frac{\omega^2}{gk} = \tanh kh_1, \tag{41}$$

where k is now a real variable. It is noted that (40) and (41) is the conventional solution for waves over a rigid impermeable seabed bottom.

4. Theoretical Results and Discussions

The aim of this paper is to examine the effects of a porous structure and sandy seabed on the wave damping, wavelength and variation of pore water pressure. Two sandy beds are considered, and their soil characteristics are given in Table 1. The values of the inertial coefficient, the friction factor and the porosity of the rigid porous structure are set as $C_r = 1.0$, $f_p = 0.5$ and $n_p = 0.5$, respectively.

Table 1. Values of soil parameters used in the computation.

parameters	coarse sand	sine sand
permeability K_p (m/s)	1×10^{-2}	5×10^{-4}
shear modulus G (N/m^2)	1×10^{7}	5×10^{6}
porosity n_s	0.4	0.4
Poisson's ratio μ	0.333	0.333
saturation degree S_r	100%	100%

4.1 Effects of porous medium on the wave damping

In the new wave dispersion relation (29), the wave number is a complex variable, which can be expressed as $k = k_r + ik_i$. Then, the water surface elevation, η, can be further rewritten as

$$\eta = a_o e^{-k_i x} e^{i(k_r x - \omega t)} . \tag{42}$$

It is noted that wave profiles are affected by the porous medium with an amplitude of $a_o e^{-k_i x}$. That is, the wave amplitude decreases exponentially. The imagery part of wave number (k_i) represents the wave damping factor, while the real part (k_r) is the wave number.

Figure 2 illustrates the decay of the wave amplitudes in saturated sandy beds for various thickness of porous structure ($h_p / h_1 = 0$, 1/8, 1/4, 1/2). The figure clearly indicates that the wave amplitudes decrease exponentially as the number of wave cycle increases. This decaying phenomenon is more obvious in coarse sand. It is noted that wave damping is insignificant in fine sand without a porous structure (i.e., $h_p / h_1 = 0$). It is also found that the phenomenon of wave damping become more significant as the thickness of the porous structure (h_p) increases. However, as shown in Figure 2, the relative difference of wave damping effects in coarse and fine sand becomes less obvious as h_p increases. For example, the wave damping rate is almost identical in coarse and fine sand when $h_p / h_1 = 1/2$.

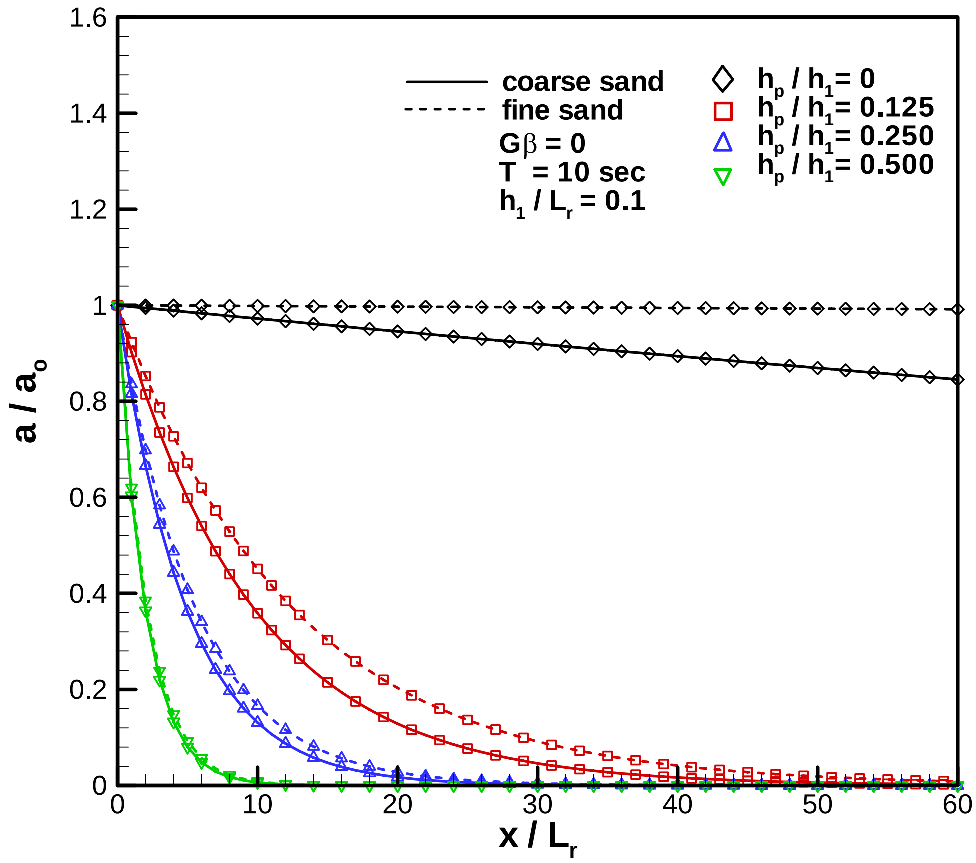

Fig. 2 Variations of wave amplitudes for various thickness of porous structure

It is well known that soil stiffness ($G\beta$) is an important factor in the evaluation of the wave-induced soil response. Figure 3 demonstrates the significant influence of soil stiffness on the decay of wave amplitudes. However, the influences of soil stiffness will becomes less, if a porous structure is placed on the sandy bed, comparing results of $h_p / h_1 = 0$ and 1/8 in Figure 3.

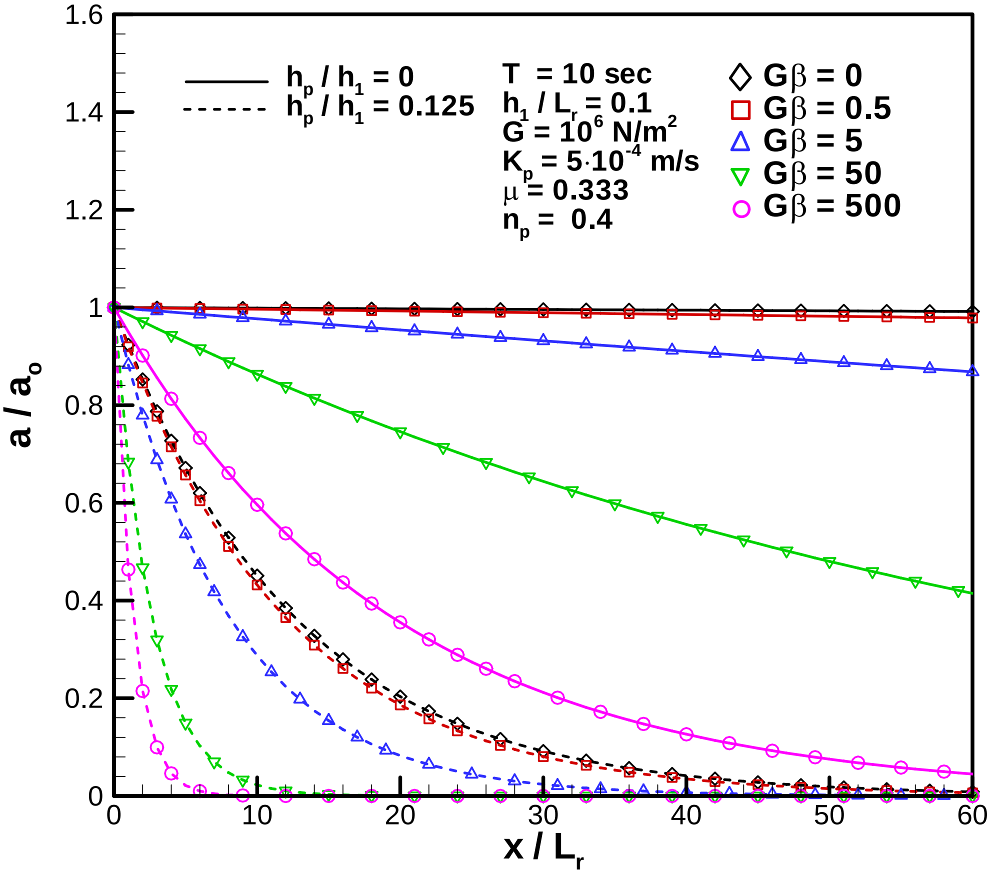

Fig. 3 Influences of soil stiffness to the decay of wave amplitude.

Soil permeability is another dominant factor which must be considered in the estimation of the wave-induced seabed response. In the example presented in Figure 4, various values of soil permeability are considered. It can be concluded from the figure that wave damping effect becomes more significant in coarse seabed (i.e., higher soil permeability).

When wave propagating over a porous structure and sandy beds, the energy damping per unit area and time can be expressed as

$$\varepsilon_D = -\frac{\partial (E c_g)}{\partial x} = 2 k_i E c_g \tag{43}$$

where c_g is the group velocity, E is the energy density of unit area, which is given by

$$E = \frac{1}{2} \rho g a_o^2 e^{-2k_i x} . \tag{44}$$

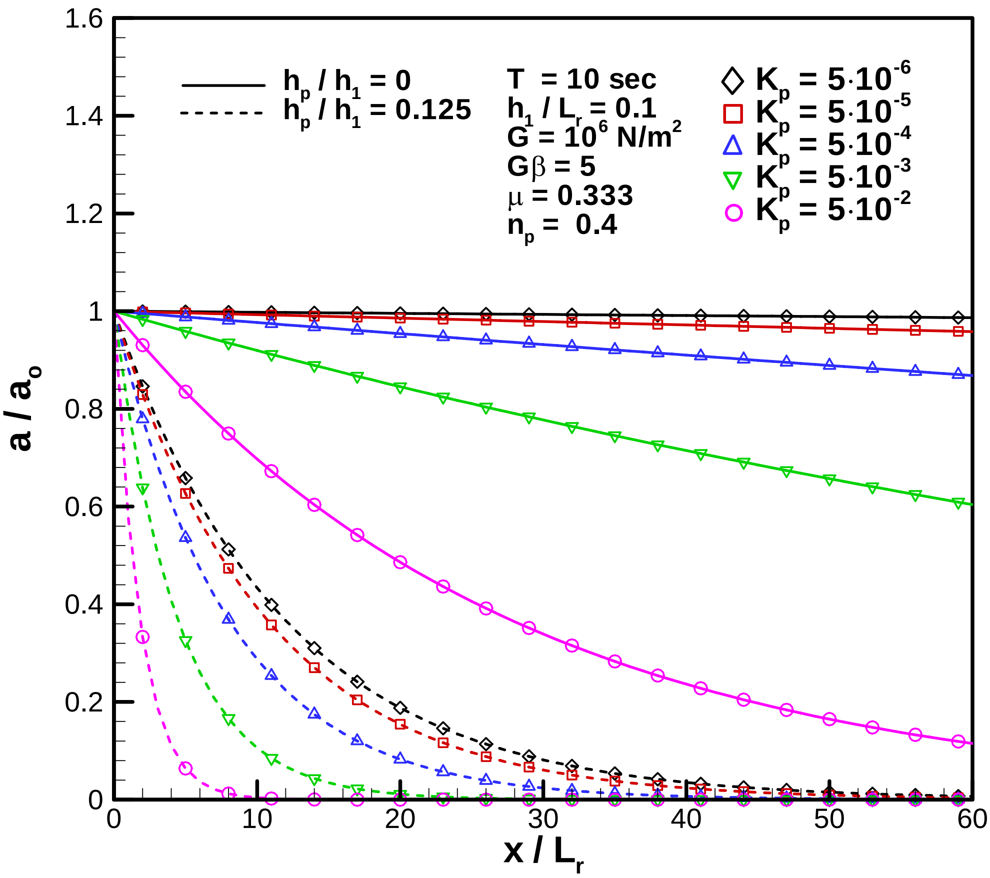

Fig. 4 Influences of soil permeability to the decay of wave amplitude.

The distribution of energy decay (ε_D / Ec_g) versus $k_o d$ (in which k_o is the wave number without damping) for various values of h_p / h_1 is presented in Figure 5. The figure indicates that ε_D decreases ad $k_o d$ increases. This implies that the wave energy is ignorable in deep water. Generally speaking, the energy decaying is significant in coarse sand for various thickness of a porous structure. The difference between coarse and find sands becomes less as h_p / h_1 increases.

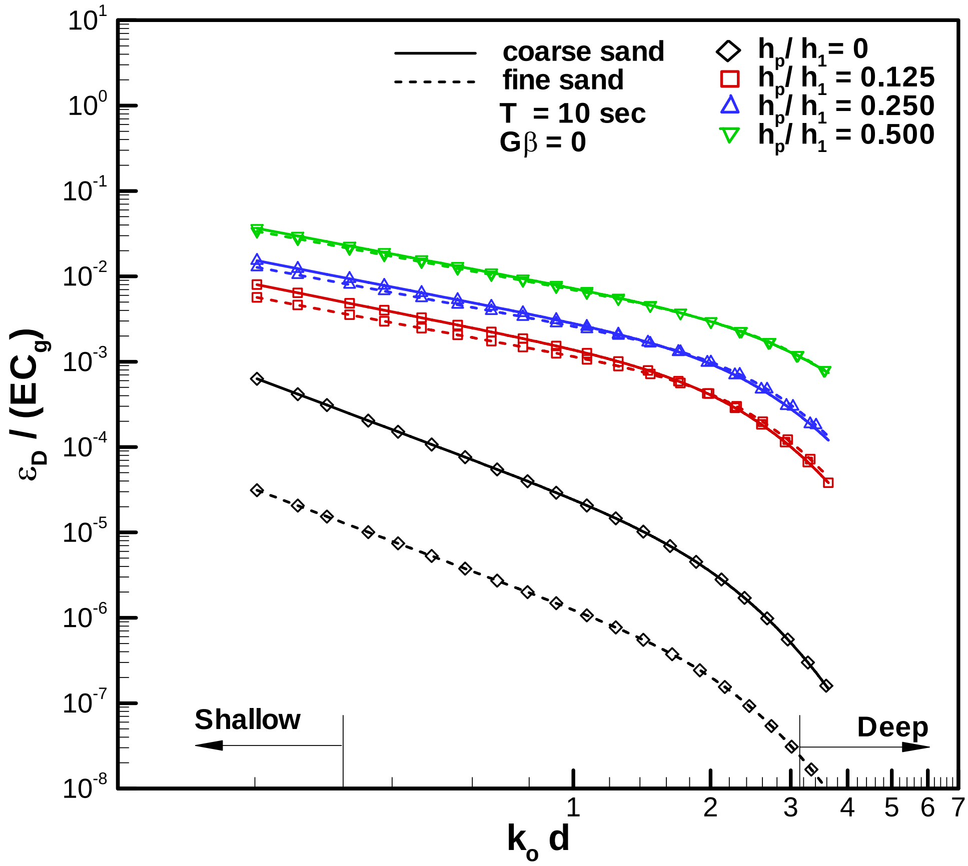

Fig. 5 Distributions of wave energy decay versus the relative water depth.

4.2 Effects of porous medium on the wavelength

Basically, the wavelength is determined by wave dispersion relation with given wave period and water depth. When ocean waves propagate over a porous medium, damping effects have been observed due to the interaction at the interface. Figure 6 illustrates the effects of porous medium on the wavelength. As shown in the figure, the wavelength decreases when wave varies from deep water condition to shallow water condition. This trend is more significant in fine sand, compare with coarse sand. The figure also indicates that the wavelength decreases as the thickness of the porous seabed increases. However, the relative difference of the variations of the wavelengths in both sandy beds does not affected by h_p, as shown in Figure 6.

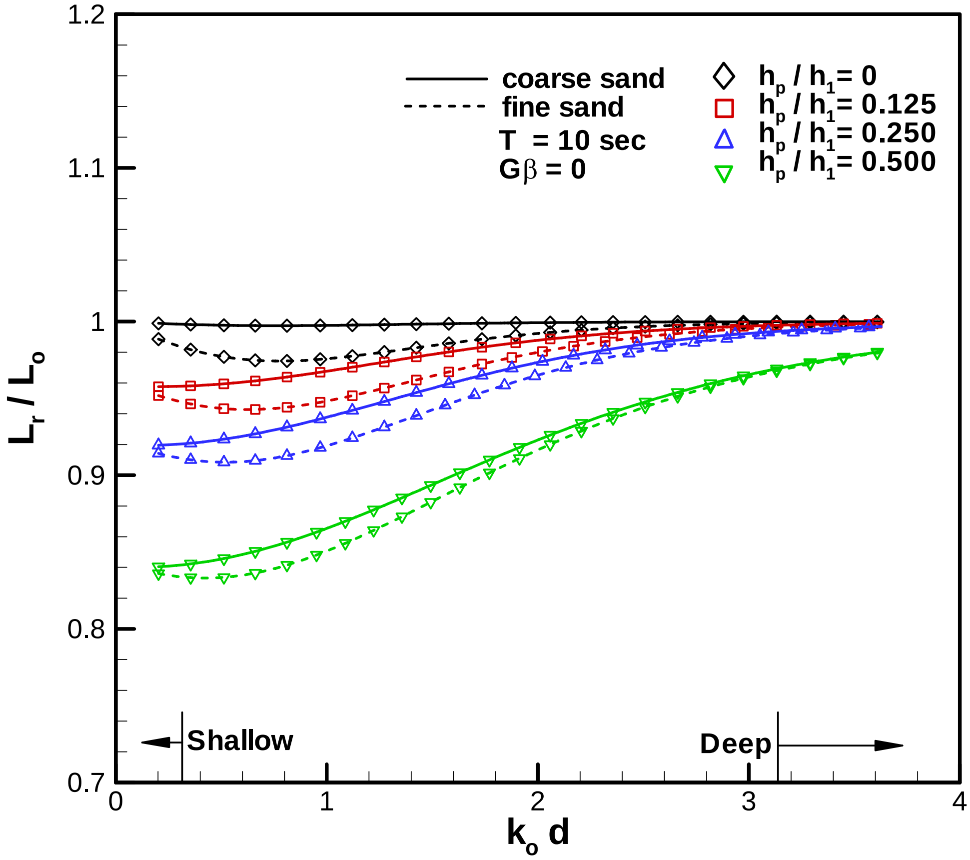

Fig. 6 Effects of porous medium on the wavelength.

4.3 Effects of porous medium on the variation of wave pressure

To further investigate the waves, a porous structure and sandy bed interactions, herein, we focus on the variation of the pore pressures at the interface of two porous medium ($z = -h_1$). To make an appropriate comparison, we use the amplitude of dynamic wave pressure at a rigid impermeable seabed (p_o) as a reference. The relative differences of pressure amplitude ($\Delta p = |p_s| - p_o$) versus $k_o d$ for various h_p / h_1 are plotted in Figure 7. As shown in the figure, the relative difference of pressure amplitudes due to wave-porous structure-seabed interaction increases from deep water to intermediate water, and then decrease to shallow water. This is because that the wave dynamic pressure at the seabed bottom is almost zero in deep water; and it then becomes significant in intermediate water region. Thus, the interaction become stronger and Δp increases. When the waves move into a shallow water region, the wave motion in the horizontal direction becomes more significant than the vertical direction at the seabed bottom. Thus, the interaction is weakened, and Δp decreases gradually. It is also observed from Figure 7 that the change of Δp in fine sand is more significant than that in coarse sand.

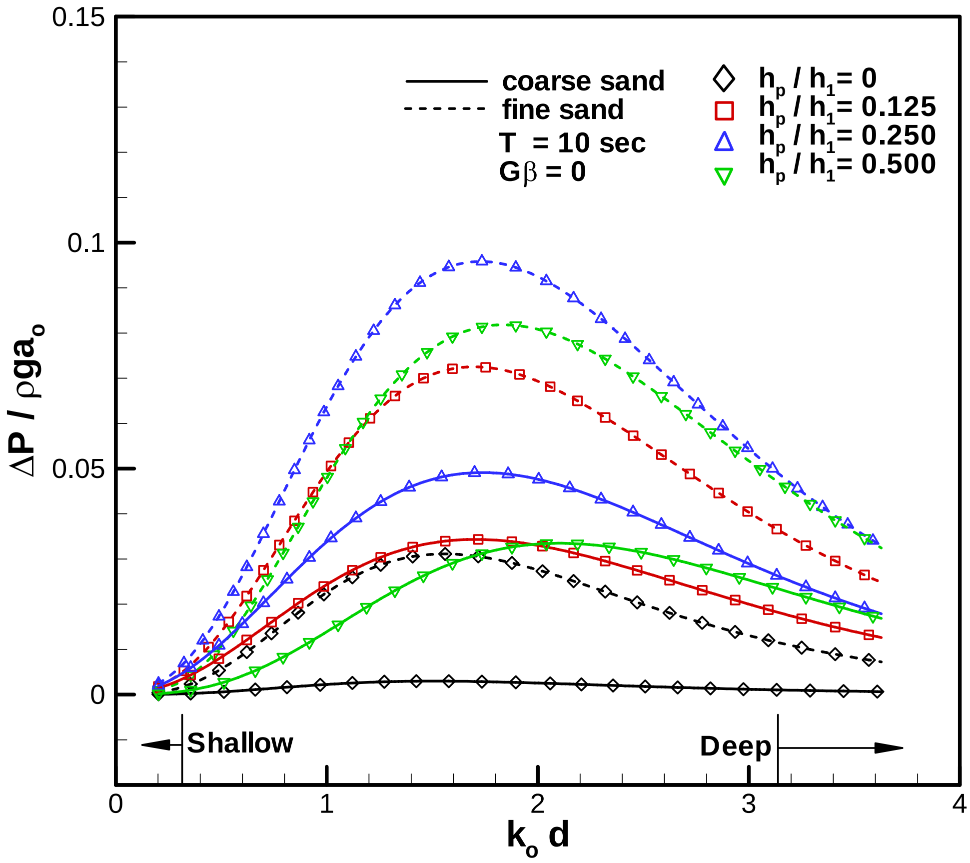

Fig. 7 Effects of porous medium on the wave pressure.

The vertical distribution of wave-induced dynamic pore pressure ($|p|/p_o$) at $x/L_r = 0$ versus soil depth (z/h_1) in saturated seabed is illustrated in Figure 8. Generally speaking, $|p|/p_o$ decreases as soil depth increases. For example, $|p|/p_o$ is less than 0.3 at $z/h_1 = -3$. It is also found that the ratio of pressure amplitude ($|p|/p_o$) decreases as h_p/h_1 increases.

Figure 9 illustrates the vertical distribution of $|p|/p_o$ versus soil depth at different x/L_r in saturated sandy beds. As shown in the figure, $|p|/p_o$ in fine sand is greater than that in coarse sand. This trend becomes more obvious as x/L_r increases. However, the pressure amplitude decreases as x/L_r increases (see Figure 9).

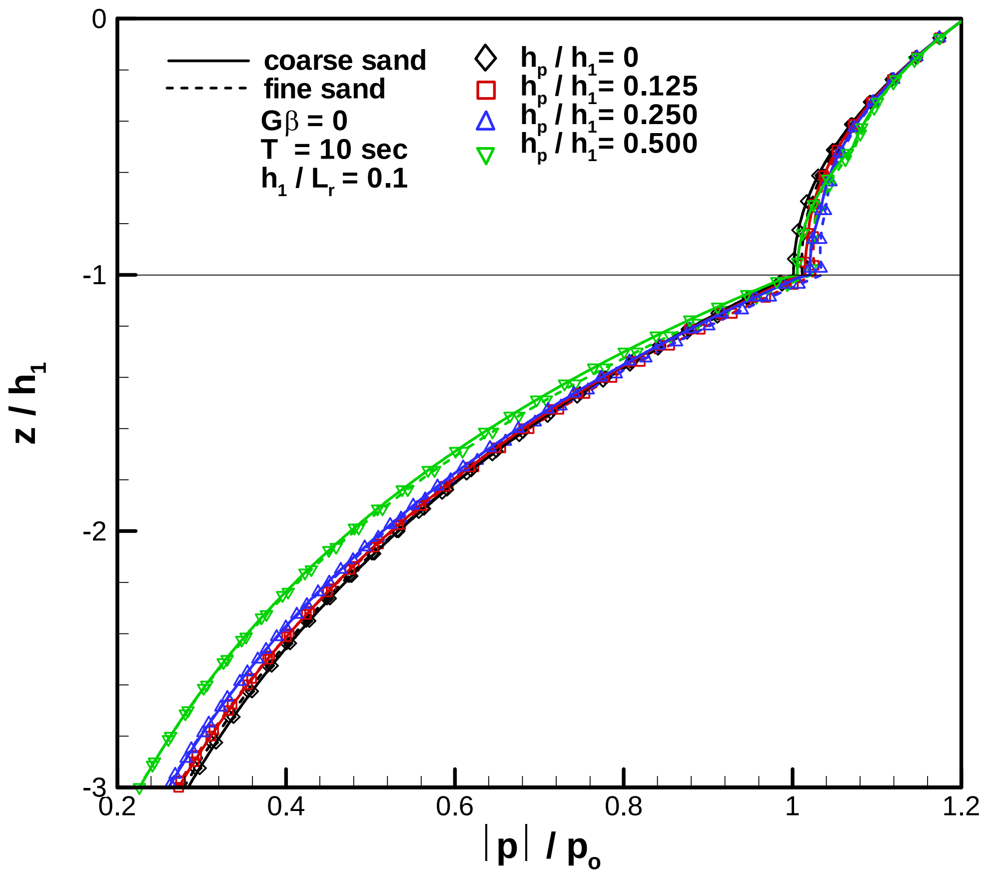

Fig. 8 Vertical distribution of wave-induced pore pressure.

5. Conclusions

For concerning the wave interaction with a submerged porous structure over a sandy seabed, a new analytic solution was presented in this paper. In the theoretical formulations, the submerged structure was considered as a homogeneous rigid porous medium, in which a simplified potential theory with inertial and damping effect was adopted to govern the porous flow in this region. While the sandy seabed was considered as a poro-elastic medium that the soil consolidation theory involving a compressible pore fluid in a compressible porous medium was applied for the general solutions in this region. By introducing the continuity conditions of mass flux and pressure at each interface, the analytic solutions were obtained for the velocity potentials in the water and submerged porous medium, as well as for the pore pressure in the soil. A complex wave dispersion relationship was newly obtained in this paper for the wave interaction with the porous structure and the sandy seabed. These analytical solutions were validated to simplify to special cases, such as the wave interaction with the porous structure over an impermeable bottom, or only interaction with the poro-elastic medium.

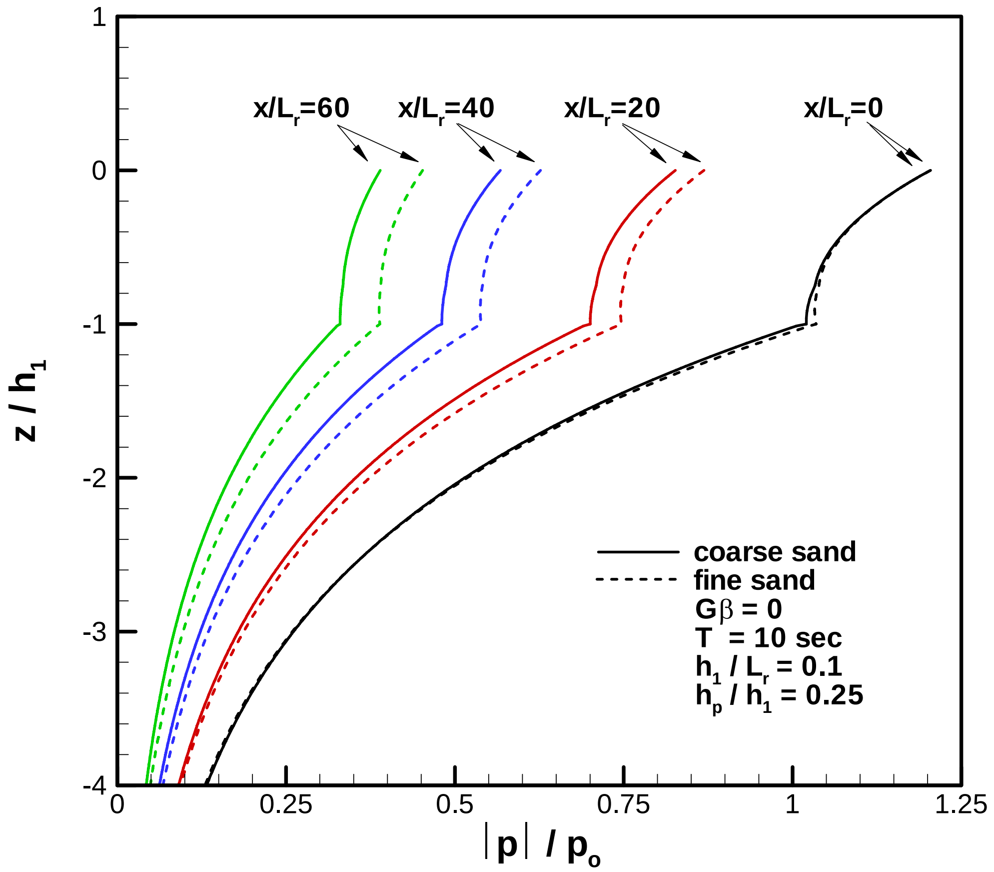

Fig. 9 Vertical distribution at different x/L_r.

The effects of the porous structure and sandy seabed on the wave damping, wavelength and variation of pore water pressure were discussed in this paper. The theoretical results showed that the wave decay is obvious in high-permeability coarse sand if the thickness of the porous structure is less than the half of the water depth. But the wave induced pore pressure in fine sand is greater than that in coarse sand. Like as the wave decay, the increase of the thickness of the porous structure will also reduce the pore pressure.

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