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Steel Equal Angle Section Beams**

Research Report No R852

N S Trahair BSc BE MEngSc PhD DEng

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Abstract:

Although steel single angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion, their behaviour may be extremely complicated, and the accurate prediction of their strengths very difficult. Further, many design codes do not have design rules for torsion, while some recommendations are unnecessarily conservative, or are of limited application, or fail to consider some effects which are thought to be important.

This paper is one of a series on the behaviour and design of single angle section steel beams. Two previous papers have studied the biaxial bending and torsion of restrained beams, a third has studied the lateral buckling of unrestrained beams, a fourth the biaxial bending of unrestrained beams, and a fifth and sixth the buckling and torsion of unrestrained beams. In each paper, simple design methods have been developed.

In this present paper, an approximate method of predicting the second-order deflections and twist rotations of steel equal angle section beams under biaxial bending and torsion is developed. This method is then used to determine the approximate maximum biaxial bending moments in such beams, which are then used with the section moment capacity proposals of the first paper of the series and the lateral buckling proposals of the third paper to approximate the member capacities.

Keywords: angles, beams, bending, buckling, design, elasticity, member capacity, moments, section capacity, steel, torsion.

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INTRODUCTION

Although the geometry and loading of single angle section steel beams are usually comparatively simple as shown in Fig. 1, their behaviour may be extremely complicated, and the accurate prediction of their strengths very difficult.

A single angle section beam is commonly loaded eccentrically in a plane inclined to the principal planes (Fig. 1), so that the beam undergoes primary bending and shear about both principal axes, torsion, and bearing at the supports. Even in the very unusual cases when only one of these actions occurs, the primary fully plastic or first yield capacities may be reduced by local or lateral buckling effects, or increased by stiffening resistances which become important at large rotations (Farwell and Galambos, 1969; Pi and Trahair, 1995). When all of these actions occur, as they usually do, there are first- and second-order interactions between them, some of which are very difficult to predict.

This paper is one of a series on the behaviour and design of single angle section steel beams. The first of these (Trahair, 2002a) considered the first-order (small deformation) elastic analysis of the biaxial bending (without torsion) of angle section beams including the effects of elastic restraints, and developed proposals for the section moment capacities which approximate the effects of full plasticity in compact sections, first yield in semi-compact sections, and local buckling in slender sections. A companion paper (Trahair, 2002b) developed proposals for the bearing, shear, and uniform torsion section capacities. The proposals of these two papers may be used to design steel single angle section beams which are laterally restrained as shown in Fig. 2a, so that lateral buckling or second-order effects are unimportant.

A third paper (Trahair, 2003) considered the lateral buckling strengths of unrestrained steel single angle section beams which are loaded in the major principal plane as shown in Fig. 2b, so that there are no primary minor axis bending or torsion effects. That paper developed an approximate method of predicting the elastic second-order twist rotations of an equal angle beam in uniform bending, whose accuracy was investigated subsequently in a study (Trahair, 2005c) of the elastic non-linear large rotation behaviour of beams under non-uniform torsion. The approximate predicted rotations were used with the earlier formulations (Trahair, 2002a) of the biaxial bending moment capacities to investigate the lateral buckling strengths of angle section beams with initial twist rotations which approximated the effects of geometrical imperfections (initial crookedness and twist) and residual stresses. The investigation included simple design proposals which approximated the predicted lateral buckling strengths.

A fourth paper (Trahair, 2004) considered the biaxial bending of unrestrained steel angle section beams which are loaded through the shear centre as shown in Fig. 2c, so that there are no primary torsion actions. This paper also developed an approximate method of analyzing the elastic second-order twist rotations of an equal angle section beam in uniform bending, and used the predicted rotations to develop simple approximations for the biaxial bending design strengths of equal or unequal angle section beams.

A fifth and a sixth paper (Trahair, 2005a, b) extended the third paper (Trahair, 2003) on the lateral buckling strengths of single angle section beams (Fig. 2b). In this extension, the beam was considered to be loaded in a plane parallel to the major principal plane but eccentric from the shear centre as shown in Fig. 2d, so that there were primary major axis bending and torsion actions, but no primary minor axis bending actions. Again, an approximate analysis was developed for predicting the elastic second-order twist rotations of an equal angle section beam under idealized loading, and used to predict the effects of torsion on beam lateral buckling strengths and to develop simple proposals for the design of beams under major axis bending and torsion.

This present paper is an extension of the fifth paper (Trahair, 2005a) on the lateral buckling and torsion of single equal angle section beams (Fig. 2d). In this extension, the beam is considered to be loaded in a plane inclined to the major principal plane and eccentric from the shear centre as shown in Fig. 1, so that there are primary biaxial bending and torsion actions. Again, an approximate analysis is developed for predicting the elastic second-order twist rotations of the equal angle beam, and used to predict the effects of biaxial bending and torsion on beam strengths and to develop simple proposals for design.

2 MEMBER DESIGN METHODOLOGY

The uniform bending and torsion of unrestrained simply supported equal angle steel beams is considered in the following sections. An approximate elastic non-linear analysis of the small twist rotations of beams with initial twists is used to predict the maximum principal plane bending moments. Compact beams (Trahair, 2002a) are considered to have failed when these maximum moments reach the fully plastic moment combinations, and beams which are just semi-compact (Trahair, 2002a) when these moments reach the first yield combinations.

This simplistic method is an extension of a first yield method of strength prediction, which takes approximate account of the additional strength beyond first yield of compact beams which can reach full plasticity. Similar methods have been used to predict the lateral buckling, biaxial bending, and buckling and torsion strengths of single angle beams (Trahair, 2003, 2004, 2005a,b). The method apparently ignores the effects of residual stresses and initial crookedness which cause early yielding and reduce strength. It also makes small rotation approximations which generally overestimate the principal plane moments. These are compensated for by using initial twists which are increased sufficiently so that the small rotation analysis will predict the lateral buckling design strengths proposed in Trahair (2003).

This method also ignores the reductions in the torque resultants of eccentrically applied loads which occur at finite rotations (Trahair 2005a). It is conservative to ignore these reductions.

The failure moments predicted by this method are used to develop a simple method of designing unrestrained equal angle beams against biaxial bending and torsion which combines the lateral buckling and biaxial bending design strengths with the first-order maximum twist rotations θ_1 .

The capacities of equal angle section beams to resist bearing, shear, and uniform torsion may be checked separately by comparing the appropriate design actions (which may be determined by a simple first-order analysis of the beam) with the corresponding design capacities recommended in Trahair (2002b).

3 APPROXIMATE ELASTIC ANALYSIS

An elastic simply supported equal angle section beam of length L and initial twist

$$\phi_0 = \theta_0 \sin (\pi z / L) \quad (1)$$

is shown in Fig. 3. The beam has equal and opposite end moments M_x , M_y causing uniform bending in the yz , xz principal planes and a uniformly distributed torque per unit length m . The small deformation differential equations of equilibrium for biaxial bending and torsion are

$$\begin{Bmatrix} -EI_x v'' \\ EI_y u'' \\ GJ\phi' \end{Bmatrix} = \begin{bmatrix} 1 & (\phi + \phi_0) & -u' \\ -(\phi + \phi_0) & 1 & -v' \\ u' & v' & 1 \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} \quad (2)$$

in which

$$M_z = m L / 2 - m z \quad (3)$$

is the variation of the axial torque caused by the distributed torque m , E and G are the Young's and shear moduli of elasticity, I_x and I_y are the second moments of area about the principal x , y axes, u and v are the shear centre deflections in the x , y directions, ϕ is the angle of twist rotation, and ' indicates differentiation with respect to the distance z along the beam.

In Equations 2, the left hand sides represent the internal resistances to bending and torsion, while the right hand sides represents the first- and second-order actions resulting from the applied actions M_x , M_y , and M_z and the small deflections u , v and twist rotations ϕ . The third of Equations 2 omits a large twist rotation resistance $EI_n(\phi')^3/2$ (Trahair, 2005c) because the non-linear "Wagner" section constant $I_n = b^5 t / 90$ is quite small for equal angle sections, in which b is the leg length and t the thickness of the section. It also omits a torque component associated with the monosymmetry of the angle about its minor principal axis.

The first-order solutions v_1 , u_1 , ϕ_1 of Equations 2 which satisfy the boundary conditions for simple supports are obtained by ignoring the terms u' , v' , and $(\phi + \phi_0)$ on the right hand sides, whence

$$v_1 = \delta_{v1}(4z/L - 4z^2/L^2) \quad (4a)$$

$$u_1 = \delta_{u1}(4z/L - 4z^2/L^2) \quad (4b)$$

$$\phi_1 = \theta_1(4z/L - 4z^2/L^2) \quad (4c)$$

in which

$$\delta_{v1} = M_x L^2 / 8EI_x \quad (5a)$$

$$\delta_{u1} = -M_y L^2 / 8EI_y \quad (5b)$$

$$\theta_1 = mL^2 / 8GJ \quad (5c)$$

Approximate second-order solutions may be obtained by assuming that

$$M_z u' = M_z v' = 0 \quad (6a)$$

$$v_2 / \delta_{v2} = u_2 / \delta_{u2} = \phi_2 / \theta_2 = \sin(\pi z / L) \quad (6b)$$

and by making the replacements

$$\begin{array}{ll} M_x \sin(\pi z / L) & M_x \\ M_y \sin(\pi z / L) & M_y \\ (mL/2) \cos(\pi z / L) & M_z \\ M_x \theta_2 \sin(\pi z / L) & \text{for } M_x \phi \\ M_y \theta_2 \sin(\pi z / L) & M_y \phi \\ M_x (\pi / L) \delta_{u2} \cos(\pi z / L) & M_x u' \\ M_y (\pi / L) \delta_{v2} \cos(\pi z / L) & M_y v' \end{array} \quad (7)$$

Equations 6a are based on the finding of Trahair and Teh (2001) that the effects of these second-order moment components of applied torques are small and can be ignored. The sine wave shape approximations of Equations 6b are close to the first order shapes of Equations 4. The replacements of Equations 7 incorporate sine or cosine shapes for terms which are either approximately of sine or cosine shape or else are constant.

Thus

$$\delta_{v2} = \{M_x + (\theta_2 + \theta_0)M_y\} / (\pi^2 EI_x / L^2) \quad (8a)$$

$$\delta_{u2} = -\{M_y - (\theta_2 + \theta_0)M_x\} / (\pi^2 EI_y / L^2) \quad (8b)$$

$$\theta_2 + \theta_0 = \frac{mL^2 / 2\pi GJ + \theta_0 - M_x M_y (1/M_{yz}^2 - 1/M_{xz}^2)}{1 - M_y^2 / M_{xz}^2 - M_x^2 / M_{yz}^2} \quad (8c)$$

in which

$$M_{yz} = \sqrt{(\pi^2 EI_y GJ / L^2)} \quad (9a)$$

$$M_{xz} = \sqrt{(\pi^2 EI_x GJ / L^2)} \quad (9b)$$

The approximate Equations 8 disagree slightly with the first-order solutions in Equations 5 when the second-order quantities (including $1 / M_{yz}$ and $1 / M_{xz}$) are ignored. Agreement can be obtained by adjusting Equations 8 to

$$\delta_{v2} = \delta_{v1} \{1 + (\theta_2 + \theta_0)M_y / M_x\} \quad (10a)$$

$$\delta_{u2} = \delta_{u1} \{1 - (\theta_2 + \theta_0)M_x / M_y\} \quad (10b)$$

$$\theta_2 + \theta_0 = \frac{\theta_1 + \theta_0 - M_x M_y (1/M_{yz}^2 - 1/M_{xz}^2)}{1 - M_y^2 / M_{xz}^2 - M_x^2 / M_{yz}^2} \quad (10c)$$

The bending moments are greatest at mid-span, and can be obtained using Equations 2 and 6b as

$$M_{x2} = M_x + M_y (\theta_2 + \theta_0) \quad (11a)$$

$$M_{y2} = M_y - M_x (\theta_2 + \theta_0) \quad (11b)$$

4 FULLY PLASTIC STRENGTHS

4.1 Fully Plastic Moment Combinations.

The combinations of principal axis moments M_{px} , M_{py} which cause full plasticity of an equal angle are given by the fully plastic interaction equation (Trahair, 2002a)

$$\pm M_{py} / M_{pym} = 1 - (M_{px} / M_{pxm})^2 \quad (12)$$

in which the principal axis full plastic moments M_{pxm} , M_{pym} are given by

$$M_{pxm} = 2 M_{pym} = f_y b^2 t / \sqrt{2} \quad (13)$$

in which f_y is the yield stress.

4.2 Elastic Lateral Buckling and Lateral Buckling Design Proposals.

4.2.1 Elastic lateral buckling

The value of the major axis uniform bending moment at elastic buckling M_{yz} is given by Equation 9a (Trahair, 1993, 2003).

4.2.2 Lateral buckling design strength

It has been proposed (Trahair, 2003) that the nominal design lateral buckling moment capacity M_b of an angle section beam should be obtained from

$$M_b = M_{sx} \quad (\lambda_e \leq \lambda_{ex}) \quad (14a)$$

$$M_b = M_{sx} - (M_{sx} - M_{sy}) \frac{(\lambda_e - \lambda_{ex})}{(\lambda_{ey} - \lambda_{ex})} \quad (\lambda_{ex} \leq \lambda_e \leq \lambda_{ey}) \quad (14b)$$

$$M_b = M_{sy} \quad (\lambda_{ey} \leq \lambda_e) \quad (14c)$$

in which

$$\lambda_{ex} = 0.99 - \frac{0.22}{(\alpha_m - 0.7)} \quad (15)$$

$$\lambda_{ey} = \sqrt{M_{sx} / M_{sy}} \quad (16)$$

$$\lambda_e = \sqrt{M_{sx} / M_{quy}} \quad (17)$$

in which M_{sx} and M_{sy} are the major and minor axis maximum section moment capacities, α_m is a moment modification factor which allows for the variation of the bending moment distribution (Trahair, 1993, 2003), and M_{quy} is the maximum moment in the beam at elastic buckling (Trahair, 2003). For a simply supported compact equal angle in uniform bending, $M_{sx} = M_{pxm}$, $M_{sy} = M_{pym}$, $M_{quy} = M_{yz}$ and $\alpha_m = 1$.

The modified slenderness limit λ_{ex} in Equation 15 is an approximation for the value of $\sqrt{M_{sx} / M_{yz}}$ at which $M_b = M_{sx}$ according to the Australian design code AS 4100 (SA, 1998). Equation 14a uses the major axis section capacity M_{sx} for low slenderness beams ($\lambda_e \leq \lambda_{ex}$), while Equation 14c uses the minor axis section capacity M_{sym} for high slenderness beams ($\lambda_{ey} \leq \lambda_e$), which is based on the finding of Trahair (2003) that the moment capacity is never less than M_{sy} . Equation 14b provides a simple linear interpolation between M_{sx} and M_{sy} for beams of intermediate slenderness ($\lambda_{ex} \leq \lambda_e \leq \lambda_{ey}$), which provides a close but conservative approximation to the predictions of Trahair (2003).

4.3 Equivalent Initial Twists.

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 11 to determine the biaxial bending and torsion strengths of equal angle section beams. Such initial twists will also predict the lateral buckling design strengths of unbraced beams bent in their major axis principal plane. The magnitudes θ_0 of these initial twists of compact beams have been determined in Trahair (2004), and are closely approximated by

$$\theta_0 = -0.1116 + 0.3612 \lambda_e + 0.3551 \lambda_e^2 - 0.3935 \lambda_e^3 \geq 0 \quad (18)$$

4.4 Member Actions for Full Plasticity

The variation of the second-order maximum moment combinations M_{x2} , M_{y2} obtained from Equations 10c and are shown in Fig. 4 for $\lambda_e = 1.0$, $M_x/M_y = 2$ and $\theta_1 = -0.3$ (for negative θ_0 , which increases the effect of negative θ_1). Fig. 5a shows a related situation where gravity loading q causes positive moments M_x and M_y , but negative rotations θ_1 . This causes the weaker xz principal plane to rotate towards the plane of loading, leading to a decrease in strength. This decrease in strength is shown by the intersection of the M_{x2} , M_{y2} curve in Fig. 4 with the positive full plasticity curve M_{px} , M_{py} obtained from Equations 12 and 13. This intersection point (circled) leads to a point 1 (squared) which defines the first-order moments M_x , M_y at plastic failure. The point 1 is also shown in Fig. 6, on a curve which shows the variation of the first-order moment combinations M_x , M_y at plastic failure with the moment ratio M_x/M_y .

Figure 4 also shows the variation of the second-order maximum moment combinations M_{x2} , M_{y2} for $\lambda_e = 1.0$, $M_x/M_y = 2$ and $\theta_1 = +0.3$ (and negative θ_0 , which decreases the effect of positive θ_1). A related situation is shown in Fig. 5b where gravity loading q causes positive moments M_x and M_y , but positive rotations θ_1 . This causes the stronger yz principal plane to rotate towards the plane of loading, leading to an increase in strength. This increase is shown by the intersection of the M_{x2} , M_{y2} curve in Fig. 4 with the positive full plasticity curve M_{px} , M_{py} . This intersection point (circled) leads to a point 2 (squared) which defines the first-order moments M_x , M_y at plastic failure. The point 2 is also shown in Fig. 6, on a second curve which shows part of the variation of the first-order moment combinations M_x , M_y at plastic failure with the moment ratio M_x/M_y for positive θ_1 .

Finally, Fig. 4 also shows another variation of the second-order maximum moment combinations M_{x2} , M_{y2} for $\lambda_e = 1.0$, $M_x/M_y = 2$ and $\theta_1 = +0.3$, this time for positive θ_0 , which increases the effect of positive θ_1 . This time the M_{x2} , M_{y2} curve intersects the negative full plasticity curve M_{px} , M_{py} . This intersection point (circled) leads to a point 3 (squared) which defines a possible combination of the first-order moments M_x , M_y at plastic failure. The point 3 is also shown in Fig. 6, on a third curve which shows part of the variation of the first-order moment combinations M_x , M_y at plastic failure with the moment ratio M_x/M_y for positive θ_1 . It can be seen that in this case the moment combination of point 3 is greater than that of point 2, and so does not define the strength. However, the strength is defined by similar points 3 at higher values of M_x/M_y .

The curves shown in Fig. 6 demonstrate that the strength of the angle section beam depends on the sense of the rotation θ_1 . When this causes the cross-section stronger principal plane yz to rotate towards the plane of bending, the strength is increased, and vice versa. A similar finding was made by Goh et al (1991).

4.5 Approximate Solutions

Approximate solutions for the first-order moment combinations $(M_x)_p / M_{pxm}$, $(M_y)_p / M_{pxm}$ at plastic failure for positive M_x / M_y and negative θ_1 can be obtained by using

$$\frac{(M_x)_p}{M_{pxm}} = \left\{ 1 - \left(1 - \frac{M_{b\theta p^-}}{M_{pxm}}\right) \frac{\psi}{\pi/2} + 0.12 \left(1 - \frac{\psi}{\pi/2}\right) \frac{\theta_1}{0.75} \right\} \frac{(1 - \cos \psi)}{\sin \psi} \quad (19)$$

in which $M_{b\theta p^-} / M_{pxm}$ is the greater of 0.5 and

$$\frac{M_{b\theta p^-}}{M_{pxm}} = \frac{M_{b0p}}{M_{pxm}} + 2 \left(1 - \frac{M_b}{M_{pxm}}\right) \left(\frac{M_{b2p}}{M_{pxm}} - \frac{M_{b0p}}{M_{pxm}}\right) \leq 1.0 \quad (20)$$

in which

$$\frac{M_{b0p}}{M_{pxm}} = 1.0 + 0.97\theta_1 + 0.37\theta_1^2 \quad (21a)$$

$$\frac{M_{b2p}}{M_{pxm}} = 0.5 + 0.35\theta_1 + 0.13\theta_1^2 \quad (21b)$$

and

$$\psi = \tan^{-1}(M_x / M_y) \quad (22)$$

Approximate solutions for positive M_x / M_y and positive θ_1 can be obtained by using the lesser of the value obtained from Equations 19 – 22 (with positive values of θ_1) and the greater of the value obtained from Equations 19 – 22 by using

$$\theta_1 = -\theta_1 \quad (23)$$

(i.e. by changing the sign of θ_1 from positive to negative) and the value obtained from

$$\frac{(M_x)_p}{M_{pxm}} = \frac{M_{b\theta p^+}}{M_{pxm}} \frac{1}{(1 - 0.8(\cot \psi) M_{b\theta p^+} / M_{pxm})} \quad (24)$$

in which $M_{b\theta p^+} / M_{pxm}$ is obtained from Equations 20, 21 using the negative value of θ_1 given by Equation 23.

These approximations are compared with the more accurate solutions in Figs 6 and 7.

5 FIRST YIELD STRENGTHS

5.1 First Yield Moment Combinations

The combinations of principal axis moments M_{yx} , M_{yy} which cause first yield of an equal angle are given by the interaction equations (Trahair, 2002a)

$$\pm M_{yy} / M_{yyM} = 1 - M_{yx} / M_{yxM} \quad (25)$$

in which the principal axis first yield moments M_{yxM} , M_{yyM} are given by

$$M_{yxM} = 2 M_{yyM} = f_y b^2 t (\sqrt{2} / 3) \quad (26)$$

5.2 Equivalent Initial Twists

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 11 to determine the buckling and torsion strengths of equal angle section beams. The magnitudes θ_0 of the initial twists of semi-compact unbraced beams bent in their major axis principal plane which will predict their lateral buckling design strengths have been determined in Trahair (2004), and are closely approximated by

$$\theta_0 = -0.0358 + 0.0499 \lambda_e + 0.4262 \lambda_e^2 - 0.3142 \lambda_e^3 \geq 0 \quad (27)$$

5.3 Member Actions for First Yield

Solutions for the first-order moment combinations M_x / M_{yxM} , M_y / M_{yxM} at first yield obtained from Equations 10c, 11, and 25 are shown in Figs 8 and 9.

Approximate solutions for the first-order moment combinations $(M_x)_y / M_{yxM}$, $(M_y)_y / M_{yxM}$ at first yield for positive M_x / M_y and negative θ_1 can be obtained by using

$$\frac{(M_x)_y}{M_{yxM}} = \frac{M_{b\theta y-}}{M_{yxM}} \left\{ \frac{1}{1 + (2.5 M_{b\theta y-} / M_{yxM} - 0.5) \cot \psi} \right\} \leq \frac{M_{yx}}{M_{yxM}} \quad (28)$$

in which $M_{b\theta y-} / M_{yxM}$ is the greater of 0.5 and

$$\frac{M_{b\theta y}}{M_{yxM}} = \frac{M_{b0y}}{M_{yxM}} + 2 \left(1 - \frac{M_b}{M_{yxM}} \right) \left(\frac{M_{b2y}}{M_{yxM}} - \frac{M_{b0y}}{M_{yxM}} \right) \leq 1.0 \quad (29)$$

in which

$$\frac{M_{b0y}}{M_{yxM}} = 1.0 + 1.73\theta_1 + 1.54\theta_1^2 \quad (30a)$$

$$\frac{M_{b2y}}{M_{yxM}} = 0.5 + 0.49\theta_1 + 0.38\theta_1^2 \quad (30b)$$

Approximate solutions for positive M_x/M_y and positive θ_1 can be obtained by using the lesser of the value obtained from Equations 28 – 30 (with positive values of θ_1) and the greater of the value obtained from Equations 28 – 30 by using

$$\theta_1 = -\theta_1 \quad (31)$$

(i.e. by changing the sign of θ_1 from positive to negative) and the value obtained from

$$\frac{(M_x)_y}{M_{yxm}} = \frac{M_{b\theta y+}}{M_{yxm}} \left\{ \frac{1}{1 - (2.5M_{b\theta y+} / M_{yxm} - 0.5) \cot \psi} \right\} \leq \frac{M_{yx}}{M_{yxm}} \quad (32)$$

in which $M_{b\theta y+}/M_{yxm}$ is obtained from Equations 29, 30 using the negative value of θ_1 given by Equation 31.

These approximations are compared with the more accurate solutions in Figs 8 and 9. The curves shown in these figures again demonstrate that the strength of the angle section beam depends on the sense of the rotation θ_1 . When this causes the cross-section stronger principal plane yz to rotate towards the plane of bending, the strength is increased, and vice versa.

6 LOCAL BUCKLING EFFECTS

6.1 Section Classification and Moment Capacities

The effects of local buckling on the section moment capacities of angle section beams have been discussed in Trahair (2002a). In that paper, sections were classified as being plastic, compact, semi-compact or slender (BSI, 2000, Trahair et al, 2001) by comparing their long leg plate slendernesses

$$\lambda = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (33)$$

with limiting slenderness values.

A plastic section must have sufficient rotation capacity to maintain a plastic hinge until a plastic collapse mechanism develops. A plastic angle section satisfies (Trahair, 2002a)

$$\lambda \leq 12 \quad (34)$$

A compact section must be able to form a plastic hinge. A compact section satisfies

$$12 < \lambda \leq 16 \quad (35)$$

The nominal section moment capacity M_{sx} of a plastic or compact section is equal to its fully plastic capacity M_{pxm} , so that

$$M_{sx} = M_{pxm} \quad (36)$$

A slender section has its moment capacity reduced below the first yield moment M_{yxm} by local buckling effects. A slender angle section satisfies

$$26 < \lambda \quad (37)$$

The nominal section moment capacity of a slender angle section M_{sx} may be approximated by

$$M_{sx} = M_{yxm} (26 / \lambda)^2 \quad (38)$$

A semi-compact section must be able to reach the first yield moment, but local buckling effects may prevent it from forming a plastic hinge. A semi-compact angle section satisfies

$$16 < \lambda \leq 26 \quad (39)$$

The nominal section moment capacity M_{sx} of a semi-compact angle section may be approximated by the linear interpolation between the full plastic and first yield capacities given by

$$M_{sx} = M_{pxm} - (M_{pxm} - M_{yxm}) \frac{(\lambda - 16)}{10} \quad (40)$$

6.2 Biaxial Bending and Torsion Strengths

The effects of local buckling on the biaxial bending and torsion strengths of equal angle section beams may be approximated by using

$$M_x = (M_x)_p \quad (41a)$$

for plastic and compact sections,

$$M_x = (M_x)_p - \{(M_x)_p - (M_x)_y\}(\lambda - 16)/10 \quad (41b)$$

for semi-compact beams, or

$$M_x = (M_x)_y (26 / \lambda)^2 \quad (41c)$$

for slender beams. These equations are adaptations of Equations 36, 40, and 38 for the section capacities.

7 EXAMPLE

7.1 Problem

A 125 x 125 x 12 equal angle beam is shown in Fig. 10. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of $b = 119$ mm and $t = 12$ mm are shown in Fig. 10b. The unbraced beam is simply supported over a span of $L = 6$ m, and has a design uniformly distributed vertical load of $q^* = 6$ kN/m acting parallel to a leg with an eccentricity of $e = 62.5$ mm from the shear centre at the leg junction, as shown in Fig. 10b.

The first-order analysis of the beam, the determination of the lateral buckling design strength, and the check of the biaxial bending and torsion capacity are summarised below.

7.2 Elastic Analysis

The design major axis bending moments are

$$M_x^* = (q^* L^2 / 8) \cos \alpha = 19.1 \text{ kNm} \text{ and } M_y^* = (q^* L^2 / 8) \sin \alpha = 19.1 \text{ kNm}.$$

The maximum design torque is $M_z^* = q^* e L / 2 = 1.13$ kNm. This is much less than the design capacity (Trahair, 2002b) of $\phi M_u = 0.9 f_y b t^2 / 2 = 2.31$ kNm.

The torque per unit length is $m^* = q^* e = 375$ Nmm/mm. The maximum first-order twist rotation (Equation 5c) is $\theta_1 = 0.146$ rad.

7.3 Lateral Buckling Design Strength

The leg plate slenderness (Equation 33) is $\lambda = 10.9 < 12$, and so the section is plastic (Equation 34).

The major axis section capacity (Equations 36 and 13) is $M_{sx} = 36.0$ kNm.

The elastic lateral buckling moment (Trahair 2003) can be calculated using Equation 9a for $M_{yz} = 35.1$ kNm, $P_y = \pi^2 E I_y / L^2 = 107.1$ kN, $\alpha_m = 1.13$, and $y_q = e \cos \alpha = 44.2$ mm, so that

$$M_{quy} = \alpha_m M_{yz} \left\{ \sqrt{1 + \left(\frac{0.43 y_q P_y}{M_{yz}} \right)^2} + \frac{0.43 y_q P_y}{M_{yz}} \right\} = 42.1 \text{ kNm}$$

The lateral buckling strength can be calculated using Equations 14-17, whence $\lambda_{ex} = 0.478$, $\lambda_{ey} = \sqrt{2}$, $\lambda_e = 0.926$, and $M_b = 27.4$ kNm.

7.4 Biaxial Bending and Torsion Capacity

M_x , M_y and θ_1 are all positive, and $M_b/M_{pxm} = 0.761$.

Using Equations 19-22 with negative θ_1 leads to $M_{b0p}/M_{pxm} = 0.866$, $M_{b2p}/M_{pxm} = 0.452$, $M_{b0p+}/M_{pxm} = 0.668 > 0.5$, whence $(M_x)_p/M_{pxm} = 0.350$.

Using Equations 19-22 with positive θ_1 leads to $M_{b0p}/M_{pxm} = 1.150$, $M_{b2p}/M_{pxm} = 0.554$, $M_{b0p+}/M_{pxm} = 0.865$, whence $(M_x)_p/M_{pxm} = 0.391 > 0.350$.

Using Equations 20, 21 with negative θ_1 leads to $M_{b0p}/M_{pxm} = 0.866$, $M_{b2p}/M_{pxm} = 0.452$, $M_{b0p+}/M_{pxm} = 0.668$ as before, and using Equations 22 and 24 leads to $(M_x)_p/M_{pxm} = 1.434 > 0.391$.

Thus $(M_x)_p/M_{pxm} = 0.391$ and $\phi(M_x)_p = 12.7$ kNm. This is less than the design moment of $M_x^* = 19.1$ kNm, and so the beam is inadequate.

7.5 Inverted Beam

If the beam in Fig. 10b is inverted so that the horizontal flange is up, then θ_1 is negative and $y_q = -44.2$ mm,

In this case, $M_{quy} = 37.5$ kNm and $M_b/M_{pxm} = 0.731$.

Using Equations 19-22, $(M_x)_p/M_{pxm} = 0.345$ and $\phi(M_x)_p = 11.2$ kNm, which is less than $\phi(M_x)_p = 12.7$ kNm for the original beam.

8. CONCLUSIONS

This paper develops a rational, consistent, and economical design method for determining the biaxial bending and torsion strength of an eccentrically loaded unbraced steel equal angle section beam.

An approximate small rotation non-linear elastic analysis is used to predict the maximum moments in equal angle beams in uniform biaxial bending and torsion. The beams have initial twists, the magnitudes of which are chosen so that the predicted strengths of beams bent in the major principal plane are equal to recent recommendations for the lateral buckling strengths (Trahair, 2003).

The biaxial bending and torsion strengths of equal angle beams are predicted by assuming either full plasticity or first yield at the maximum moment section, and simple design approximations are developed. It was found that the strength of an angle section beam depends on the sense of the twist rotation. When this causes the cross-section stronger principal plane yz to rotate towards the plane of bending, the strength is increased, and vice versa.

The effects of local buckling are considered using proposed definitions (Trahair, 2002a) of the section capacities of plastic, compact, semi-compact, and slender sections. These definitions are then used with the fully plastic and first yield design approximations to develop simple design approximations for equal angle beams. These design approximations can be used for equal angle beams under general loading.

Proposals have been made elsewhere (Trahair, 2002b) for checking the bearing, shear, and torsion capacities of angle section beams.

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APPENDIX 2 NOTATION

b	long leg length
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_y	yield stress
G	shear modulus of elasticity
I_x, I_y	second moments of area about the x, y principal axes
I_n	non-linear Wagner section constant
J	torsion section constant
L	span length
M_b	lateral buckling moment strength
$M_{b\theta p}, M_{b\theta y}$	values of M_b reduced for twist rotation θ_1
M_{px}, M_{py}	values of M_x, M_y at full plasticity
M_{pxm}, M_{pym}	principal axis values of M_{px}, M_{py}
M_{quy}	maximum moment at elastic lateral buckling
M_{sx}, M_{sy}	principal axis section moment capacities
M_x, M_y	moments about the x, y principal axes
$(M_x)_p, (M_y)_p$	moments about the x, y principal axes at plastic failure
$(M_x)_y, (M_y)_y$	moments about the x, y principal axes at first yield failure
M_x^*, M_y^*	design moments about the x, y principal axes
M_{xz}, M_{yz}	uniform bending elastic buckling moments
M_{x2}, M_{y2}	second-order moments about the x, y principal axes
M_{yx}, M_{yy}	values of M_x, M_y at first yield
M_{yxm}, M_{yym}	principal axis values of M_{yx}, M_{yy}
M_z	torque
m	intensity of uniformly distributed torque
q^*	design intensity of uniformly distributed load
t	leg thickness
u, v	shear centre deflections parallel to the x, y principal axes
u_1, v_1	first-order deflections
u_2, v_2	second-order deflections
x, y	principal axes
X, Y	rectangular (geometric) axes
X_c, Y_c	X, Y distances from centroid to shear centre
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
α_m	moment modification factor
β	leg length ratio
δ_{u1}, δ_{v1}	maximum first-order deflections
δ_{u2}, δ_{v2}	maximum second-order deflections
λ	long leg local buckling slenderness
λ_e	modified slenderness for beam lateral buckling
$\lambda_{ex}, \lambda_{ey}$	beam lateral buckling slenderness limits
ϕ	angle of twist rotation, or capacity factor
ϕ_0	initial angle of twist rotation
ϕ_1	first-order angle of twist rotation
ϕ_2	second-order angle of twist rotation
θ_0	maximum value of ϕ_0
θ_1	maximum value of ϕ_1
θ_2	maximum value of ϕ_2
ψ	$= \tan^{-1}(M_x / M_y)$

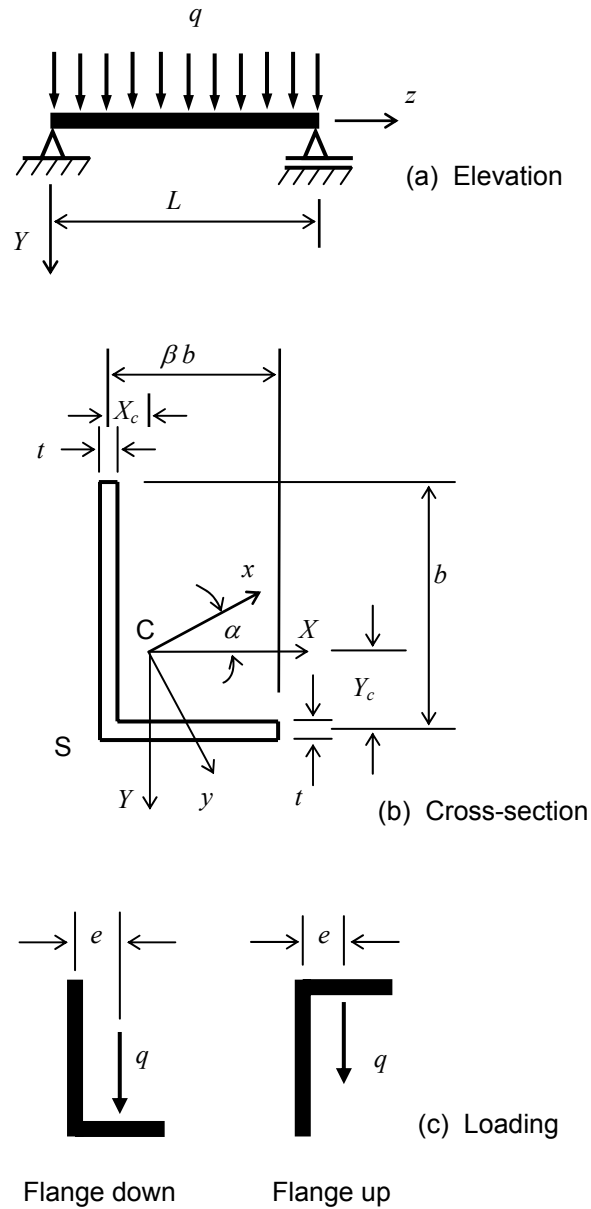
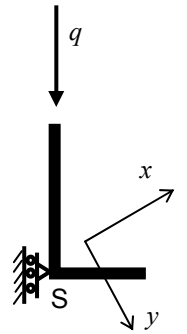
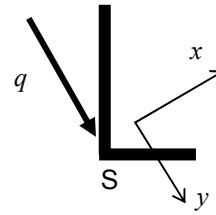


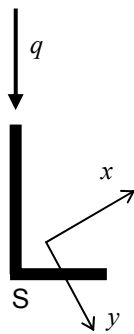
Fig. 1. Eccentrically Loaded Angle Section Beam



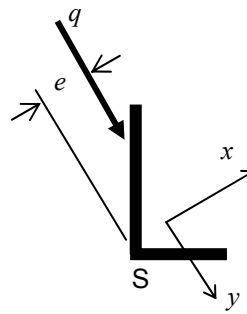
(a) Restrained biaxial bending



(b) Lateral buckling



(c) Unrestrained biaxial bending



(d) Buckling and torsion

Fig. 2. Single Angle Beam Behaviour

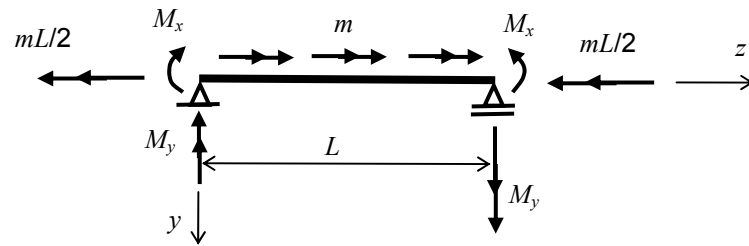


Fig. 3. Simply Supported Equal Angle in Biaxial Bending and Torsion

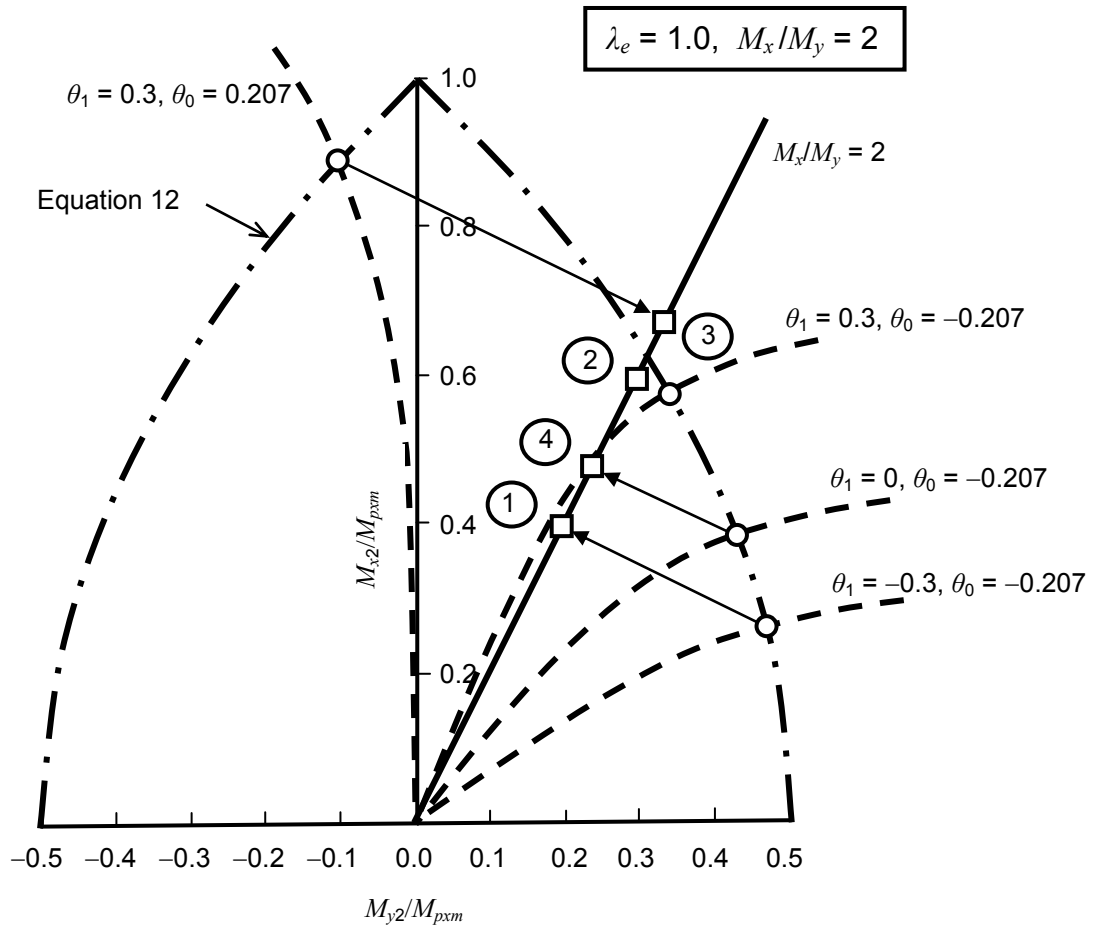
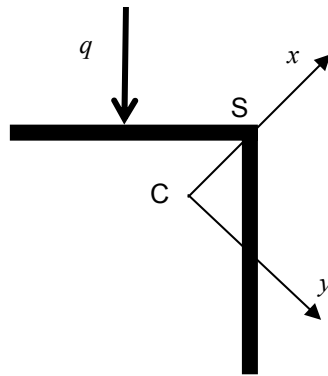
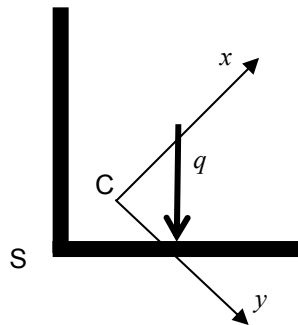


Fig. 4 Second-Order Moment Combinations



(a) M_x, M_y +ve, θ_1 -ve



(b) M_x, M_y +ve, θ_1 +ve

Fig. 5 Moment and Rotation Senses

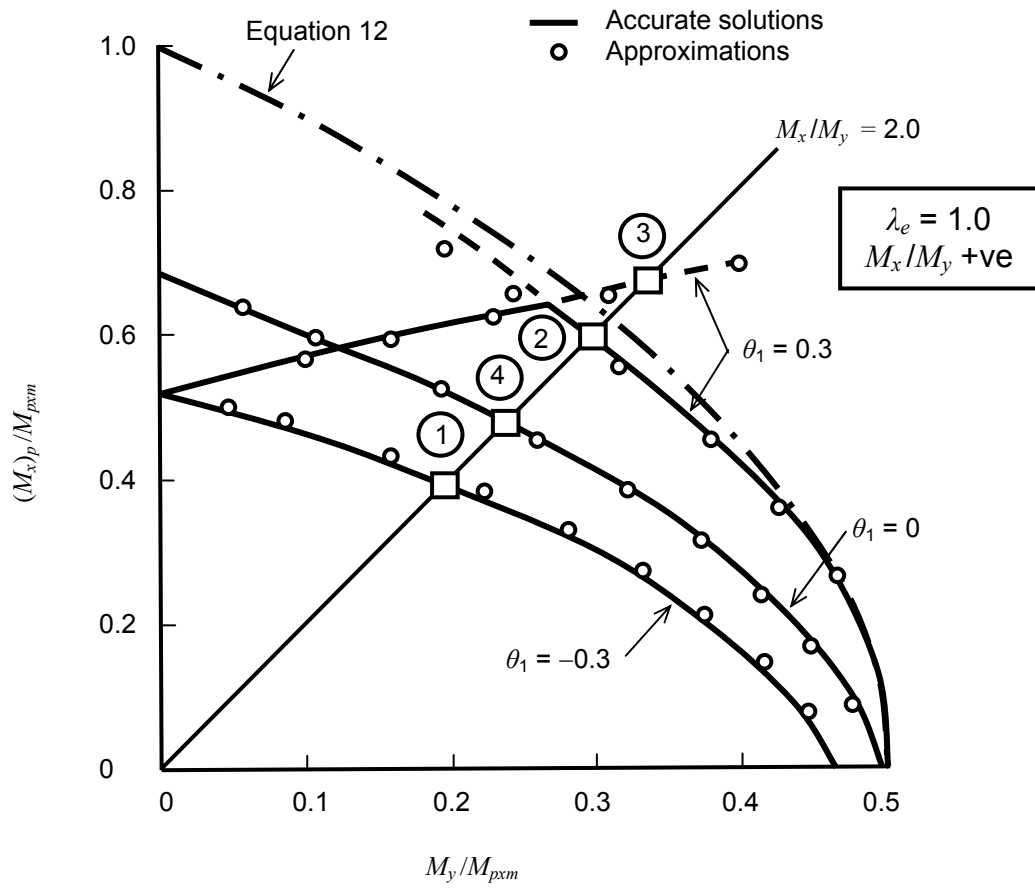


Fig. 6 First-Order Moments at Plastic Failure for $\lambda_e = 1.0$

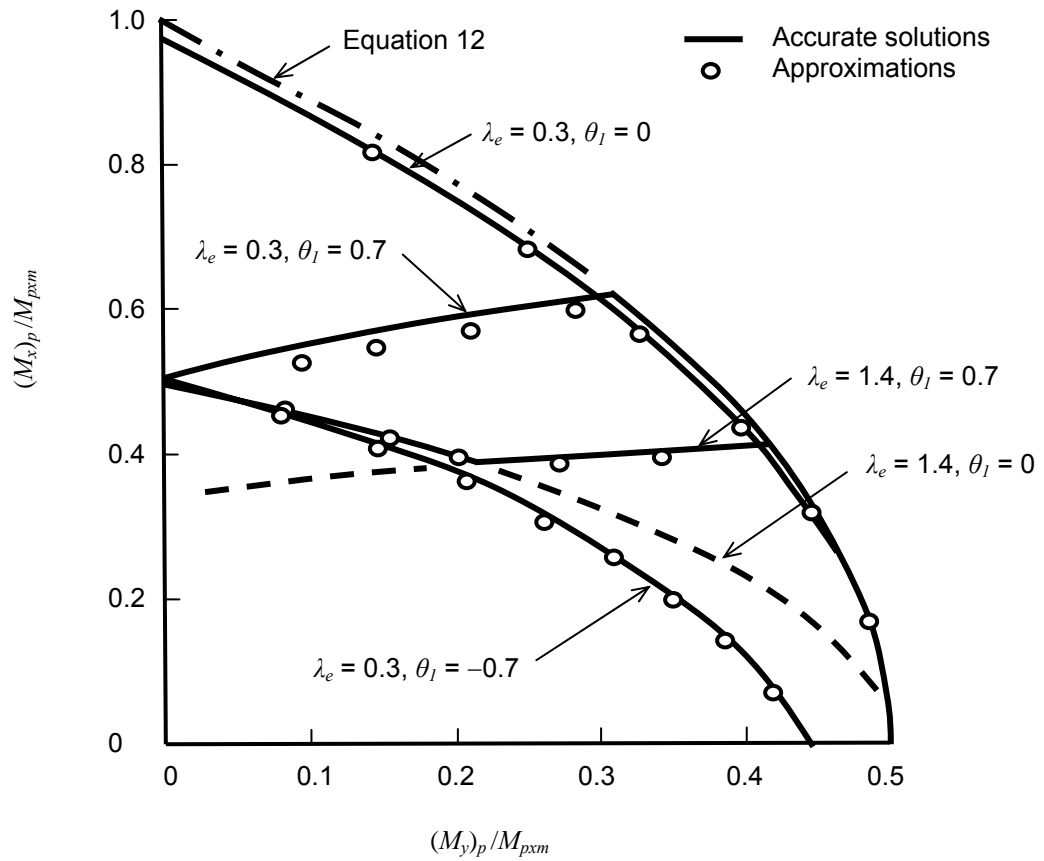


Fig. 7 First-Order Moments at Plastic Failure for $\lambda_e = 0.3, 1.4$

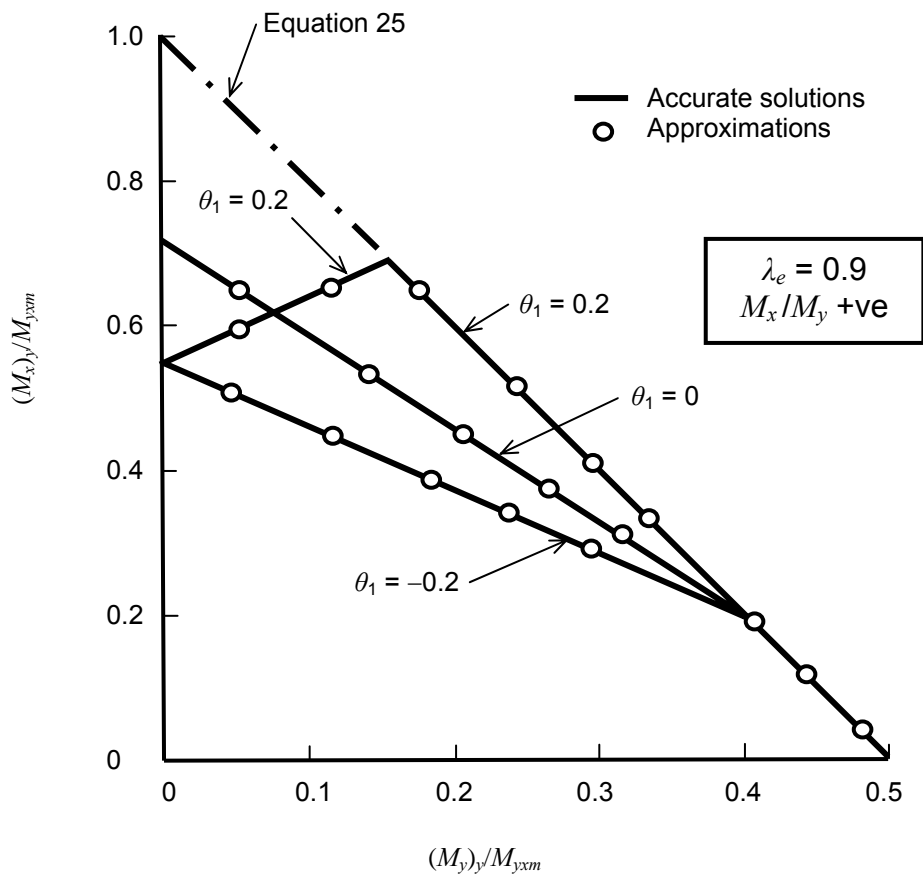


Fig. 8 First-Order Moments at First Yield Failure for $\lambda_e = 0.9$

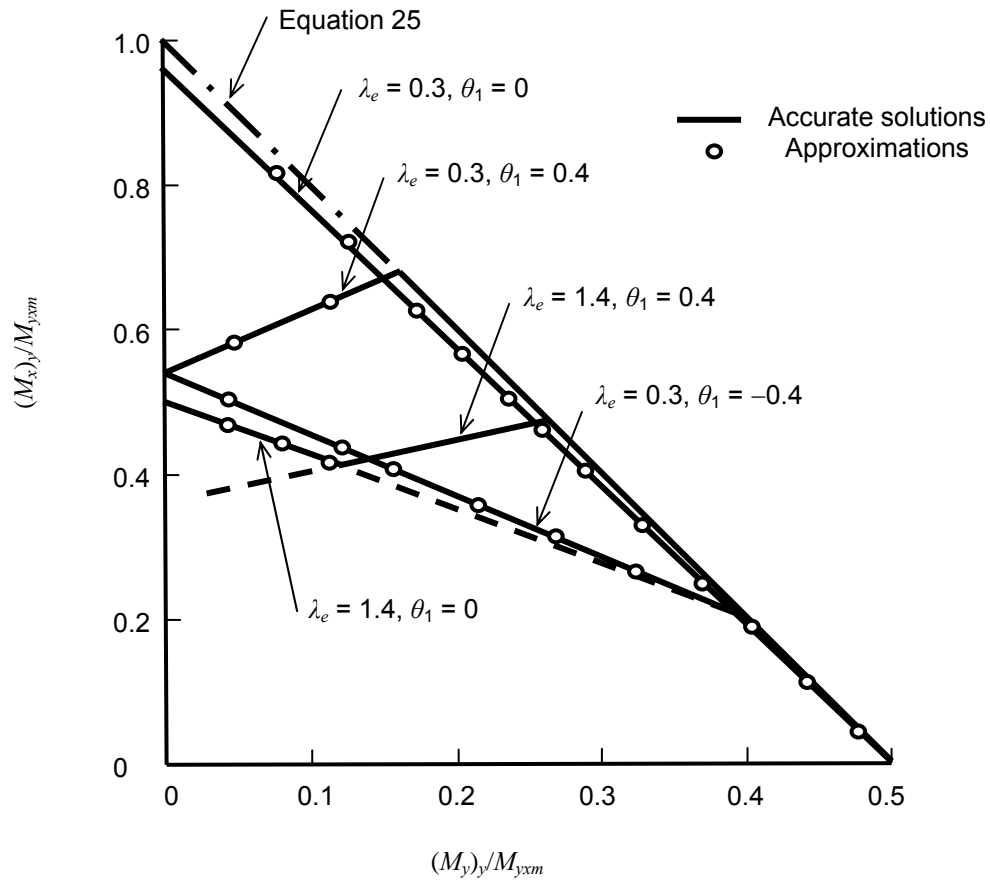
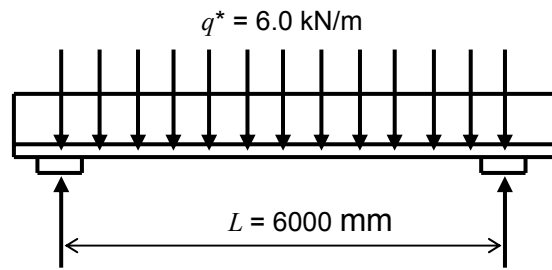


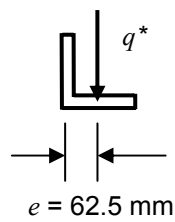
Fig. 9 First-Order Moments at First Yield Failure for $\lambda_e = 0.3, 1.4$



(a) Elevation

125 x 125 x 12 EA
 ($b \times b \times t = 119 \times 119 \times 12$)

$E = 200,000 \text{ MPa}$
 $G = 80,000 \text{ MPa}$
 $f_y = 300 \text{ MPa}$



$\alpha = 45^\circ$
 $I_y = 1.953 \text{ E6 mm}^4$
 $J = 0.1440 \text{ E6 mm}^4$
 $X_c = -44.19 \text{ mm}$

(b) Section

Fig. 10. Example