The University of Sydney

Department of Civil Engineering
Sydney NSW 2006
AUSTRALIA

http://www.civil.usyd.edu.au/

Centre for Advanced Structural Engineering

On the Design of Damaged Steel Columns

Research Report No R846

N S Trahair BSc BE MEngSc PhD DEng
K Kayvani BSc MEngSc PhD

June 2005

The University of Sydney

Department of Civil Engineering
Centre for Advanced Structural Engineering
http://www.civil.usyd.edu.au/

On the Design of Damaged Steel Columns

Research Report No R846

N S Trahair BSc BE MEngSc PhD DEng
K Kayvani BSc MEngSc PhD
June 2005

Abstract:

This paper explores a number of situations where columns with out-of-tolerance crookedness or which have been damaged may still be designed, despite the implications of many codes that they must be replaced, and suggests rules for their design.

The case of a column whose crookedness is out-of-tolerance is first examined, and two design methods are suggested. In the first method, the column is treated in a similar way to that used for the basis of the BS5950 column design method by allowing for the excess crookednesses. In the second method, the column is designed as a straight beam-column with design moments equal to those resulting from the first-order analysis of an imperfect structure whose geometry includes the excess crookednesses.

Following this, the case is considered of a column damaged by unexpected bending which leaves an out-of-tolerance permanent set. It is concluded that the residual stresses caused by the damaging bending moments may often be ignored, in which case the damaged column can be designed for its increased crookedness by using either of the methods proposed for columns with out-of-tolerance crookedness. The straightening of the damaged column is also considered. It is found that the residual stresses which follow relaxation after straightening may also be ignored and the column designed in the usual way.

Finally, the case is analysed of a force-fitted column which has excessive crookedness locked in during its connection to other members of a structure. It is found that the force-fitting deflection can be regarded as an initial crookedness, so that the column can be designed as an out-of-tolerance column..

Keywords:

buckling, columns, crookedness, damage, design, imperfection, steel.

Copyright Notice

Department of Civil Engineering, Research Report R846
On the Design of Damaged Steel Columns

N.Trahair@civil.usyd.edu.au

Published by:
Department of Civil Engineering
The University of Sydney
Sydney NSW 2006
AUSTRALIA

June 2005

http://www.civil.usyd.edu.au

Introduction.

While many steel design codes [1, 2, 3] provide rules for the design of columns whose crookednesses lie within given tolerances, there is no guidance for columns which have been damaged or whose crookednesses exceed the given tolerances. The implication of these codes is that such columns must be replaced by columns which are within tolerance.

However, this is wasteful, as damaged or out-of-tolerance columns will still have some strength, and may be able to resist the design loads. The purpose of this paper is to explore a number of situations where damaged or out-of-tolerance columns may still be designed, and to suggest rules for their design.

A review is first provided of the bases of rules for the design resistances of in-tolerance columns, and the influence of initial crookedness and residual stresses on these. Then the case of a column whose crookedness is out-of-tolerance is examined, and two design methods are suggested. Following this, the case is considered of a column damaged by unexpected loading which leaves an out-of-tolerance permanent set. Finally, the case is analysed of a force-fitted column which has excessive crookedness locked in during its connection to other members of a structure.

The representative calculations made for this paper have been carried out for columns with the properties shown in Fig. 1, in which E is the Young's modulus of elasticity, p_y is the yield strength, and b and t are the width and thickness of a rectangular section which models a typical I-section of a column failing about the minor axis.

Review of Column Strength Design Rules.

While perfectly straight columns without residual stresses fail either by general yielding at the squash load P_y or elastically at the buckling load P_E, the presence of initial crookedness or residual stresses causes premature yielding and consequent reductions in strength [4].

One early method allowing for these reductions was to assume a suitably large initial crookedness and use the load P_{pr} at which first yield occurs (the Perry-Robertson load) as the design resistance. Another early method was to assume a representative residual stress distribution and use the inelastic (tangent modulus) buckling load P_I as the design resistance. In both methods, the magnitudes of the initial crookedness or residual stresses need to be adjusted so that the resulting design resistances provide reasonable representations of the real column resistances determined by experiment. The variations of typical dimensionless nominal column resistances P_{cy} / P_y with modified slenderness $\sqrt{(P_y / P_E)}$ are compared with dimensionless elastic and inelastic buckling loads in Fig. 2.

The Perry-Robertson first yield load P_{pr} can be obtained from [4]

$$\frac{P_{pr}}{P_y} = \frac{P_E / P_y}{\phi^*_{pr} + \sqrt{(\phi^{*\,2}_{pr} - P_E / P_y)}} \tag{1}$$

in which

$$\phi^*_{pr} = \frac{1 + (\eta_{pr} + 1) P_E / P_y}{2} \tag{2}$$

in which

$$\eta_{pr} = \frac{\delta_0 b}{2r^2} \tag{3}$$

In these equations, δ_0 is the initial maximum crookedness, r is the section radius of gyration, P_E is the elastic buckling load

$$P_E = \pi^2 EI/L_E^2 \tag{4}$$

and P_y is the squash load

$$P_y = Ap_y \tag{5}$$

in which A is the area and I is the second moment of area of the section, and L_E is the effective length.

For this paper, it has been assumed that the design resistance P_{cy} of a steel column is given by the value of P_{pr} obtained from Equation 1 by replacing η_{pr} of Equation 3 by

$$\eta_c = \frac{3.5}{1000}\sqrt{\frac{\pi^2 E}{p_y}}\left\{\sqrt{\frac{P_y}{P_E}} - 0.2\right\} \tag{6}$$

which corresponds to Strut Curve b of BS5950 [2]. The variation of the dimensionless design resistance P_{cy} / P_y with the modified slenderness $\sqrt{(P_y / P_E)}$ is shown in Figs 2 and 3.

It should be noted that the design resistances obtained in this way incorporate allowances for the effects of residual stresses and initial crookedness. These resistances are equal to the first yield loads of columns with effective initial crookednesses δ_e which correspond to

$$\frac{1000\delta_e}{L_E}\frac{b}{4r} = 1.75\sqrt{\frac{P_E}{P_y}}\left\{\sqrt{\frac{P_y}{P_E}} - 0.2\right\} \tag{7}$$

The variation of the dimensionless effective crookedness $(1000\,\delta_e / L_E)\,(b/4r)$ with the modified slenderness $\sqrt{(P_y / P_E)}$ is shown in Fig. 4.

Some design codes [2, 3] provide tolerance limits for initial crookedness which must be satisfied if their code design resistance rules are used. For this paper, it has been assumed that the design code tolerance limit δ_t for initial crookedness is given by [2]

$$1000\ \delta_t / L_E = 1 \tag{8}$$

Residual Stresses.

Residual stresses are induced in columns by uneven cooling after hot-rolling or welding (in the case of fabricated columns). Residual stresses may be modified by partial straightening [5, 6] by gagging and rotarising, by force fitting, by heat curving [7, 8], or when a column is damaged or straightened.

In this paper, residual stresses are replaced by equivalent initial crookednesses.

Initial Crookedness.

The initial crookedness of a column is developed during its manufacture, fabrication, and erection. The manufacturing process includes hot-rolling, cooling, and any subsequent partial straightening [5, 6]. Fabrication processes may include cutting, drilling, bolting, and welding. Erection processes such as welding and bolting may change the crookedness by forcing the member to fit with other members. The initial crookedness of a column may be changed by heat curving [7, 8], or if the column is damaged, such as when unexpected transverse loads cause inelastic bending deflections which are not completely recovered after unloading. The initial curvature of a damaged column may be further changed by remedial straightening.

For this paper, it has been assumed that the effective crookedness δ_e of Equation 7 is equal to the sum of an allowance δ_r for residual stresses and the tolerance limit δ_t so that

$$\delta_e = \delta_t + \delta_r \tag{9}$$

as shown in Fig. 4. It should be noted that at low modified slendernesses $\sqrt{(P_y / P_E)} < 0.2$, the values of δ_r are negative, as a consequence of the positive values of δ_t and the zero values of δ_e (corresponding to $P_{cy} = P_y$).

Stress-Strain Relationships.

The stress-strain ($p - \varepsilon$) relationship for structural steel under increasing strain is often idealised by the tri-linear curve shown in Fig. 5, which consists of elastic (when the stress p is less that the yield strength p_y and the strain ε is less than the yield strain $\varepsilon_y = p_y / E$), plastic, and strain-hardening regions (when the strain ε is greater than the strain-hardening strain $\varepsilon_s \approx 11\ \varepsilon_y$), in which the rates of change of the stress-strain ratio are equal to the elastic Young's modulus, zero, and the strain-hardening modulus

$$E_s \approx 0.03\ E \tag{10}$$

respectively.

When unloading (reducing strain ε) takes place from the plastic or strain-hardening regions, this is idealized as taking place elastically, as shown in Fig. 5, until the stress is reduced by $2p_y$, beyond which the steel behaves plastically at first and then by strain-hardening. This behaviour is commonly referred to as kinematic hardening.

Columns With Out-of-Tolerance Crookedness.

The designer of a column with out-of-tolerance crookedness will question whether the column must be replaced by an in-tolerance column, or whether a reduced strength can be determined which may allow the column to be used. Depending on the answers to these questions, the column may be able to be used, or may have to be replaced. The following material suggests two methods of determining the strengths of an out-of-tolerance column so that it can be decided whether the column can be used or must be replaced.

Design using increased crookedness.

The simplest method of designing a column with an out-of-tolerance crookedness δ_0 is to modify the code design resistance by replacing the code effective initial crookedness δ_e by an increased effective crookedness given by

$$\delta_i = \delta_e + \delta_0 - \delta_t \qquad (11)$$

This is equivalent to replacing η_{pr} of Equation 3 by

$$\eta_c = \frac{3.5}{1000}\sqrt{\frac{\pi^2 E}{p_y}}\left\{\sqrt{\frac{P_y}{P_E}}\left[1+\frac{2}{3.5}\frac{b}{4r}\left(\frac{1000\delta_0}{L_E}-1\right)\right]-0.2\right\} \qquad (12)$$

Examples of the reduced design resistances of simply supported columns with δ_0 / δ_t = 3 and 5 determined in this way are shown in Fig. 3.

Design using imperfect structure analysis.

Another method of designing a column with an out-of-tolerance crookedness δ_0 is to use the code design resistance for an in-tolerance column, but to analyse the structure with an imperfection corresponding to the excess crookedness ($\delta_0 - \delta_t$). In this case, the column will have design moments M_y acting as well as the design axial loads F_c, and must be designed as a beam-column. For example, the design moment caused by an axial force F_c on a simply supported column is

$$M_y = F_c\,(\delta_0 - \delta_t) \qquad (13)$$

The column is adequate when F_c and M_y satisfy the BS5950 [2] requirement

$$\frac{F_c}{P_{cy}} + \frac{M_y}{p_y Z_y} \le 1 \tag{14}$$

in which Z_y is the elastic section modulus. The reduced design axial capacities for simply supported columns with δ_0 / δ_t = 3 and 5 determined in this way are shown in Fig. 3. They are a little higher than those obtained by using increased crookednesses, because BS5950 [2] allows the use of first-order moments in Equation 14, whereas higher second-order moments are incorporated in Equations 1, 2, and 12.

Damaged and Straightened Columns.

Figure 6 shows four typical deflected shapes and stress distributions associated with a simply supported rectangular section column damaged by excessive bending and then straightened. Stress distribution 1 corresponds to inelastic bending, and is defined by the depth b_1 of the remaining elastic region. Stress distribution 2 corresponds to the relaxation to zero of the bending moments M_1 which caused stress distribution 1. This distribution has maximum residual stresses p_{2b} at $y = b/2$ and p_{2b1} at $y = b_1/2$. The designer of such a column will question the residual stresses induced by the excessive bending, the effect of the resulting deflections on the strength of the column, and whether the column will be satisfactory if it is straightened. Depending on the answers to these questions, the column may be able to be used, or straightened, or may have to be replaced. The following material suggests how these answers may be obtained.

Stress distribution 3 corresponds to a reversal of the damaging bending moment which occurs during a straightening process, and is defined by the depth b_3 of the remaining elastic region. Stress distribution 4 corresponds to the case for which a relaxation of stress distribution 3 is sufficient to completely remove the crookedness caused by the original excessive bending. This distribution has maximum residual stresses p_{4b} at $y = b/2$ and p_{4b3} at $y = b_3/2$. It is shown in Appendix 1 that the value of b_3/b for the complete removal of crookedness satisfies

$$2\left(\frac{b_3}{b}\right)^3 - \left\{3 + 2\left(\frac{b}{b_1}\right) + \left(\frac{b_1}{b}\right)^2\right\}\left(\frac{b_3}{b}\right) + 4 = 0 \tag{15}$$

The variation of b_3 / b with b_1 / b is shown in Fig. 7.

It is also shown in Appendix 1 that the maximum strains corresponding to these four stress distributions can be obtained from

$$\varepsilon_1 / \varepsilon_y = b / b_1 \tag{16a}$$

$$\varepsilon_2 / \varepsilon_y = b / b_1 - \{3 - (b_1 / b)^2\} / 2 \tag{16b}$$

$$\varepsilon_3 / \varepsilon_y = b / b_1 - 2b / b_3 \tag{16c}$$

$$\varepsilon_4 / \varepsilon_y = 0 \tag{16d}$$

The corresponding maximum deflections δ can be obtained from

$$\frac{\delta_{1,2,3,4}}{L} = \frac{\varepsilon_{1,2,3,4}}{4}\frac{L}{b} \tag{17}$$

in which L is the column length. It is also shown in Appendix 1 that the maximum stresses shown in Fig. 7 can be obtained from

$$p_{2b} / p_y = 1 - \{3 - (b_1 / b)^2\} / 2 \tag{18a}$$

$$p_{2b1} / p_y = 1 - \{3 - (b_1 / b)^2\}(b_1 / b) / 2 \tag{18b}$$

$$p_{4b} / p_y = 1/2 - (b_3 / b)^2 + (b_1 / b)^2 / 2 \tag{18c}$$

$$p_{4b3} / p_y = -1 + 3(b_3 / b)/2 - (b_3 / b)^2 + (b_1 / b)^2 (b_3 / b) / 2 \tag{18d}$$

The variations of these normalized strains and non-dimensional stresses with b_1 / b are shown in Fig. 7.

It can be seen from Fig. 7 that there are substantial reductions in the stresses following the relaxation from the damaging bending moment, to residual values which are consistent with those implied by the design resistance formulation of Equations 1, 2, and 6. Further, the extreme fibre residual stress p_{2b} following the relaxation is of opposite sign to that caused by the damaging moment. This sign reversal is favourable, in that it will delay yielding when the column is loaded. While the interior maximum stress p_{2b1} is larger than p_{2b} at low values of b_1 / b, subsequent loading will have a lesser effect there than at the extreme fibre. It can be concluded that the residual stresses caused by the damaging bending moments can be ignored, and the damaged column designed by accounting for its increased crookedness δ_2 by using either of the methods described above for columns with out-of-tolerance crookedness.

It can also be seen from Fig. 7 that the residual stresses which follow relaxation after straightening are even less than those following relaxation after damage. These can be ignored and the column designed in the usual way.

Some caution should be exercised if the column is to be used in a situation which requires the material to be highly ductile, as excessive bending will reduce the ductility of the steel. Situations requiring high ductility include those where fatigue or brittle fracture must be considered, but the largely compressive stresses of columns will reduce their susceptibility to these types of failure. Another such situation is where seismic design demands large rotational ductility.

It is not easy to suggest simple limits for the inelastic bending of steel members which will ensure that their material ductility will not be reduced, although a not uncommon recommendation [6, 8, 9] is that the strain-hardening strain ε_s should not be exceeded. It is shown in Appendix 1 that for members damaged by uniform bending that this limit is equivalent to a deflection δ_2 after relaxation which satisfies

$$\frac{1000\delta_2}{L} \approx \frac{0.8}{b/4r}\frac{L}{r} \tag{19a}$$

when p_y = 275 N/mm^2, E = 205,000 N/mm^2, and ε_s = 11 ε_y.

For members damaged by central concentrated load, this limit is equivalent to

$$\frac{1000\delta_2}{L} \approx \frac{0.03}{b/4r}\frac{L}{r} \tag{19b}$$

Force-Fitted Columns.

The unloaded column shown in Fig. 8 has an initial crookedness given by

$$u_0 = 16\delta_0(z/L - z^2/L^2)^2 \tag{20}$$

whose shape is very close to the buckled shape of a fixed-ended column. The column is forced by the connection of its ends to rigid restraints into a deflected position $u_0 + u_1$ defined by

$$u_1 = 4\delta_1(z/L - z^2/L^2) \tag{21}$$

in which the central magnitude is given by

$$\delta_1 = M_1 L^2 / (8 E I) \tag{22}$$

in which $-M_1$ are the end moments exerted by the restraints. Subsequently, the axial force is increased from 0 to P, the column deflections are increased to $(u_0 + u_1 + u)$, and the end moments change from $-M_1$ to M.

The designer of such a column will question the stresses induced by the force-fitting, and the effect of the resulting deflections on the strength of the column. Depending on the answers to these questions, the column may be able to be used, or may have to be replaced. The following material suggests how these answers may be obtained.

It is shown in Appendix 2 that the total central deflection $\delta_T = \delta_0 + \delta_1 + \delta$ is given by

$$\delta_T = \delta_0\left\{\frac{24}{(\mu L/2)^3}\frac{(1-\cos\mu L/2)}{\sin\mu L/2} - \frac{12}{(\mu L/2)^2}\right\} + \delta_1\frac{2}{\mu L/2}\frac{(1-\cos\mu L/2)}{\sin\mu L/2} \tag{23}$$

in which μ can be obtained from

$$\mu L/2 = \pi\sqrt{(P/P_E)} \tag{24}$$

in which

$$P_E = 4\pi^2 EI/L^2 \tag{25}$$

is the elastic buckling load of the fixed-ended column.

The variations of the dimensionless total central deflection δ_T / (δ_0 + δ_1) with the dimensionless load P / P_E for δ_1 / δ_0 = 0 and 10 are shown in Fig. 9. The values for δ_1 / δ_0 = 0 and 10 are very close, and indicate that the force-fitting deflection δ_1 can be regarded as an initial crookedness. Also shown in Fig. 9 are the variations of the dimensionless end and central moments M / P_E (δ_0 + δ_1) and - $M_{L/2}$ / P_E (δ_0 + δ_1). Again, the values for δ_1 / δ_0 = 0 and 10 are very close, and confirm that the force-fitting deflection δ_1 can be regarded as an initial crookedness. Thus a force-fitted column can be designed as usual if the force-fitting deflection is not out-of-tolerance, or otherwise as suggested earlier for out-of-tolerance columns. Finally, it can be seen that the force-fitting moments M_1 = $(M)_{P=0}$ are generally quite small by comparison with the maximum moments at high values of P, and therefore the stresses caused by them can be ignored.

Conclusions.

This paper has explored a number of situations where columns with out-of-tolerance crookedness or which have been damaged may still be designed, despite the implications of many codes that they must be replaced, and has suggested rules for their design.

The case of a column whose crookedness is out-of-tolerance was first examined, and two design methods were suggested. In the first method, the column was treated in a similar way to that used for the basis of the BS5950 [2] column design method. In this method the code effective crookedness was increased by the excess of the actual crookedness over the code tolerance. This method included the same residual stress allowance as that which is incorporated in the code effective crookedness.

In the second method, the column was designed as a straight beam-column with design moments equal to those resulting from the first-order analysis of an imperfect structure whose geometry includes the excess crookednesses. This method predicted slightly higher strengths than the first method.

Following this, the case was considered of a column damaged by unexpected bending which leaves an out-of-tolerance permanent set. It was found that the initial residual stresses caused by excessive bending were substantially reduced when the bending was relaxed, and that the extreme fibre stresses were reversed in sign, so that they were likely to delay failure. It was concluded that the residual stresses caused by the damaging bending moments may be ignored, and that the damaged column may be designed for its increased crookedness by using either of the methods proposed for columns with out-of-tolerance crookedness.

The straightening of the damaged column was also considered. It was found that the residual stresses which follow relaxation after straightening are even less than those following the initial relaxation after damage, and it was suggested that these can be ignored and the column designed in the usual way.

Finally, the case was analysed of a force-fitted column which has excessive crookedness locked in during its connection to other members of a structure. It was found that the force-fitting deflection can be regarded as an initial crookedness, so that the column can be designed as usual if the force-fitting deflection is not out-of-tolerance, or otherwise as suggested earlier for out-of-tolerance columns

It can be concluded that many columns with out-of-tolerance crookednesses resulting from their manufacture or subsequent damage by bending or by force-fitting need not be replaced, but may be designed for reduced strengths using methods which are consistent with those of the BS5950 [2] for in-tolerance columns.

References

1. American Institute of Steel Construction.: *Load and Resistance Factor Design Specification for Structural Steel Buildings*, (1999), AISC, Chicago.

2. British Standards Institution.: *BS5950 Structural Use of Steelwork in Building. Part 1:2000. Code of Practice for Design in Simple and Continuous Construction: Hot Rolled Sections, (*2000), BSI, London.

3. Standards Australia.: *AS 4100-1998 Steel Structures,* (1998), SA, Sydney.

4. Trahair, NS, Bradford, MA, and Nethercot, DA.: *The Behaviour and Design of Steel Structures to BS5950*, (2001), 3rd British edition, Spon Press, London.

6. Gergess, A and Sen, R.: 'Fabrication Aids for Cold Straightening I-Girders', *Engineering Journal*, American Institute of Steel Construction, (2004), **41** (2), 69 - 84.

7. Lay, MG.: *Structural Steel Fundamentals*, (1982), Australian Road Research Board, Melbourne, Australia.

8. Avent, RR.: 'Heat-Straightening of Steel: Fact and Fable', *Journal of Structural Engineering*, ASCE, (1989), **115** (11), 2773 – 2793.

9. Avent, RR and Mukai, DJ.: 'What You Should Know About Heat Straightening Repair of Damaged Steel', *Engineering Journal*, American Institute of Steel Construction, (2001), **38** (1), 27 – 49.

Notation

A Area of cross-section
A, B Constants of integration
b Depth of rectangular cross-section
b_1, b_3 Section depths defining inelastic stress distributions
$c_{1,2,3}$ Components of δ_1
E, E_s Young's and strain-hardening moduli of elasticity
F_c Axial load
I Second moment of area
L Length of column
L_E Effective length
M End moment
$M_{L/2}$ Moment at mid-height
M_p Fully plastic moment
M_y Moment about y axis
$M_{1,3}$ End moments
P Axial load
P_{cy} Compression resistance about y axis
P_E Elastic buckling load
P_I Inelastic buckling load
P_{pr} Perry-Robertson first yield load
P_y Squash load
p Stress
$p_{2b,2b1,2b3}$ Stresses in inelastic stress distribution 2
p_{3b} Stress in inelastic stress distribution 3
$p_{4b,4b3}$ Stresses in inelastic stress distribution 4
p_y Yield strength
r Radius of gyration
t Thickness of rectangular section
u Additional deflection
u_0 Initial crookedness
u_1 Force-fitting deflection
x, y Principal axes
Z_y Section modulus about y axis
z Distance along column

δ Central additional deflection
δ_e Effective crookedness
δ_i Increased crookedness
δ_r Crookedness allowance for residual stresses
δ_T Central total deflection
δ_0 Central initial crookedness
δ_1 Central force-fitting deflection
$\delta_{1,2,3,4}$ Central deflections
ε Strain
ε_s Strain-hardening strain
ε_y Yield strain
$\varepsilon_{1,2,3,4}$ Extreme fibre strains
η_c Code imperfection parameter
η_{pr} Perry-Robertson imperfection parameter
μ See Equation 24
ϕ^*_{pr} See Equation 2

Appendix 1 – Damaged and Straightened Columns.

Uniform Bending

Figure 6 shows four typical deflected shapes and stress distributions associated with a rectangular section column damaged by excessive bending and then straightened. Stress distribution 1 corresponds to inelastic bending under a moment M_1, and is defined by the depth b_1 of the remaining elastic region. At $y = b_1$, the strain is equal to the yield strain ε_y, and so the extreme fibre strain ε_1 is given by

$$\varepsilon_1 / \varepsilon_y = b / b_1 \tag{26}$$

M_1 and b_1 are related by

$$M_1 / M_p = 1 - (b_1 / b)^2 / 3 \tag{27}$$

in which

$$M_p = p_y b^2 t / 4 \tag{28}$$

is the full plastic moment of the section.

Stress distribution 2 of Fig. 6 corresponds to the relaxation to zero from the bending moment M_1 which caused stress distribution 1. This relaxation takes place elastically as the strains reduce and changes the stresses by

$$p = -\frac{12 M_1 y}{b^3 t} \tag{29}$$

so that stress distribution 2 has maximum residual stresses p_{2b} at $y = b / 2$ and p_{2b1} at $y = b_1 / 2$ given by

$$p_{2b} / p_y = 1 - \{3 - (b_1 / b)^2\} / 2 \tag{30a}$$

$$p_{2b1} / p_y = 1 - \{3 - (b_1 / b)^2\} (b_1 / b) / 2 \tag{30b}$$

The extreme fibre strain ε_2 can be determined from

$$\frac{\varepsilon_2}{\varepsilon_y} = \frac{\varepsilon_1}{\varepsilon_y} - \frac{3}{2} \frac{M_1}{M_p} \tag{31}$$

whence

$$\varepsilon_2 / \varepsilon_y = b / b_1 - \{3 - (b_1 / b)^2\} / 2 \tag{32}$$

Stress distribution 3 is the result of a reversal of the damaging bending moment to $-M_3$, which corresponds to an extreme fibre strain $-\varepsilon_3$ and a remaining elastic region defined by the depth b_3. The stress p_{3b3} at $y = b_3$ can be obtained from

$$\frac{p_{3b3}}{p_y} = \frac{p_{2b3}}{p_y} - \frac{(\varepsilon_2 - \varepsilon_3)}{\varepsilon_y}\frac{b_3}{b} \tag{33}$$

in which

$$\frac{p_{2b3}}{p_y} = 1 - \frac{3-(b_1/b)^2}{2}\frac{b_3}{b} \tag{34}$$

is the dimensionless stress of stress distribution 2 at b_3. Setting the stress p_{3b3} equal to the yield strength p_y allows the extreme fibre strain $-\varepsilon_3$ to be expressed in terms of

$$-\varepsilon_3/\varepsilon_y = 2b/b_3 - b/b_1 \tag{35}$$

The moment resultant $-M_3$ of stress distribution 3 can be expressed in terms of

$$-M_3/M_p = 1 - 2(b_3/b)^2/3 + (b_1/b)^2/3 \tag{36}$$

Stress distribution 4 corresponds to the case for which the relaxation of stress distribution 3 is sufficient to completely remove the crookedness caused by the original excessive bending. This relaxation takes place elastically as the strains reduce and changes the stresses by

$$p = \frac{12M_3 y}{b^3 t} \tag{37}$$

so that stress distribution 4 has maximum residual stresses p_{4b} at $y = b/2$ and p_{4b3} at $y = b_3/2$ given by

$$\frac{p_{4b}}{p_y} = \frac{1}{2} - \left(\frac{b_3}{b}\right)^2 + \frac{1}{2}\left(\frac{b_1}{b}\right)^2 \tag{38a}$$

$$\frac{p_{4b3}}{p_y} = -1 + \frac{3}{2}\left(\frac{b_3}{b}\right) - \left(\frac{b_3}{b}\right)^2 + \frac{1}{2}\left(\frac{b_1}{b}\right)^2\left(\frac{b_3}{b}\right) \tag{38b}$$

The extreme fibre strain ε_4 is given by

$$\frac{\varepsilon_4}{\varepsilon_y} = \frac{\varepsilon_3}{\varepsilon_y} - \frac{3}{2}\frac{M_3}{M_p} \tag{39}$$

For the complete removal of the crookedness,

$$\varepsilon_4 / \varepsilon_y = 0 \tag{40}$$

which is satisfied when

$$2\left(\frac{b_3}{b}\right)^3 - \left\{3 + 2\left(\frac{b}{b_1}\right) + \left(\frac{b_1}{b}\right)^2\right\}\left(\frac{b_3}{b}\right) + 4 = 0 \tag{41}$$

The maximum deflections δ corresponding to the extreme fibre strains can be obtained from

$$\frac{\delta_{1,2,3,4}}{L} = \frac{\varepsilon_{1,2,3,4}}{4}\frac{L}{b} \tag{42}$$

In the special case for which the extreme fibre strain is equal to the strain-hardening strain ε_s = 11 ε_y, b/b_1 = 11, M_1/M_p = 0.997 and $\varepsilon_2 / \varepsilon_y$ = 9.50. This leads to

$$\frac{1000\delta_2}{L} \approx \frac{0.8}{b/4r}\frac{L}{r} \tag{43}$$

when p_y = 275 N/mm^2 and E = 205,000 N/mm^2.

Central Concentrated Load

Figure 10 shows the bending moment and extreme fibre strain distributions and the deflected shape of a simply supported rectangular section member with a central concentrated load which causes a maximum extreme fibre strain of ε_s = 11 ε_y. At mid-span, b_1/b = 1 / 11, and M/M_p = 362 / 363. The member is elastic-plastic for 0 ≤ 2z/L ≤ 60 / 181 and elastic for 60 / 181 ≤ 2z/L ≤ 1, in which z is the distance from the mid-length. The dimensionless extreme fibre strain varies according to

$$\frac{\varepsilon}{\varepsilon_y} = \frac{11}{\sqrt{(1+724z/L)}} \qquad \textit{while} \qquad 0 \le 2z/L \le 60/181 \tag{44a}$$

$$\frac{\varepsilon}{\varepsilon_y} = \frac{181}{121}(1-2z/L) \qquad \textit{while} \qquad 60/181 \le 2z/L \le 1 \tag{44b}$$

and the curvature u'' is given by

$$u'' = 2\varepsilon / b$$

(45)

in which u is the deflection and ‘ indicates differentiation with respect to the distance z along the member.

The central deflection δ_1 can be determined from

$$\delta_1 = c_1 + c_2 + c_3 \tag{46}$$

in which the components c_1, c_2, and c_3 shown in Fig. 10c can be obtained from

$$c_1 = \frac{2}{b}\int_0^{30L/181}\int_0^{z} \varepsilon \, dz \, dz \qquad = 0.12871 \frac{2}{b}\left(\frac{L}{2}\right)^2 \varepsilon_y \tag{47a}$$

$$c_2 = \frac{L}{2}\left(\frac{120}{181}\right)\frac{2}{b}\int_0^{30L/181} \varepsilon \, dz \quad = 0.60774\left(\frac{121}{181}\right)\frac{2}{b}\left(\frac{L}{2}\right)^2 \varepsilon_y \tag{47b}$$

$$c_3 = \frac{2}{b}\int_{30L/181}^{L/2}\int_{30L/181}^{z} \varepsilon \, dz \, dz = 0.14897 \frac{2}{b}\left(\frac{L}{2}\right)^2 \varepsilon_y \tag{47c}$$

so that

$$\delta_1 = 0.68395 \frac{2}{b}\left(\frac{L}{2}\right)^2 \varepsilon_y \tag{48}$$

When p_y = 275 N/mm^2, E = 205,000 N/mm^2, this deflection can be expressed non-dimensionally as

$$\frac{1000\delta_1}{L} = \frac{0.11469}{b/4r}\frac{L}{r} \tag{49}$$

If the applied load is subsequently reduced to zero, then the member relaxes elastically, so that the central deflection reduces to

$$\delta_2 = \delta_1 - \frac{(4M/L)L^3}{48EI} \tag{50}$$

Using M/M_p = 362 / 363 and Equation 49 leads to

$$\frac{1000\delta_2}{L} = \frac{0.03108}{b/4r}\frac{L}{r} \tag{51}$$

for the dimensionless deflection after relaxation.

Appendix 2 – Force-Fitted Columns.

The unloaded column shown in Fig. 8 has an initial crookedness given by

$$u_0 = 16\delta_0 (z/L - z^2/L^2)^2 \tag{52}$$

whose shape is very close to the buckled shape of a fixed-ended column. The column is forced by the connection of its ends to rigid restraints into a deflected position $u_0 + u_1$ defined by an additional deflection of

$$u_1 = 4\delta_1 (z/L - z^2/L^2) \tag{53}$$

in which the central magnitude is given by

$$\delta_1 = M_1 L^2 / (8\, E\, I) \tag{54}$$

in which $-M_1$ are the end moments exerted by the restraints. Subsequently, the axial force is increased from 0 to P, the column deflections are increased to $(u_0 + u_1 + u)$, and the end moments change from $-M_1$ to M.

The differential equation for equilibrium of the deflected position is given by

$$E\, I\, (u_1'' + u'') = M - P\,(u_0 + u_1 + u) \tag{55}$$

in which ' indicates differentiation with respect to the distance z along the column. The general solution of this equation is

$$u = A\sin\mu z + B\cos\mu z + \frac{M + M_1}{P} - u_0 - u_1 + \frac{16\delta_0}{(\mu L)^2}\left(2 - \frac{12z}{L} + \frac{12z^2}{L^2} - \frac{24}{(\mu L)^2}\right) - \frac{8\delta_1}{(\mu L)^2} \tag{56}$$

in which A and B are constants of integration.

When the boundary conditions $(u)_{z=0,L} = 0 = (u')_{z=0}$ are substituted, the constants A and B and the indeterminate end moment M can be evaluated using

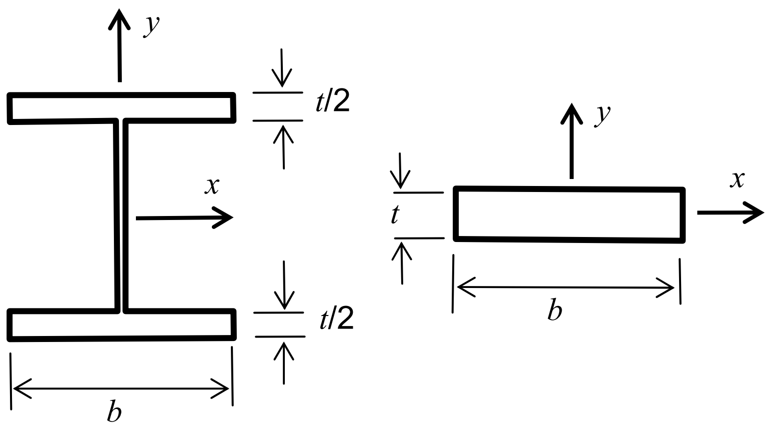
$$A = \frac{192\delta_0}{(\mu L)^3} + \frac{4\delta_1}{(\mu L)} \tag{57a}$$

$$B = -A\left\{\frac{\sin\mu L}{(\cos\mu L - 1)}\right\} \tag{57b}$$

$$\frac{M}{P} = A\left\{\frac{\sin\mu L}{(\cos\mu L - 1)}\right\} - \frac{32\delta_0}{(\mu L)^2}\left(1 - \frac{12}{(\mu L)^2}\right) + \frac{8\delta_1}{(\mu L)^2} - \frac{M_1}{P} \tag{57c}$$

The total central deflection $\delta_T = \delta_0 + \delta_1 + \delta$ can then be determined as

$$\delta_T = \delta_0\left\{\frac{24}{(\mu L/2)^3}\frac{(1-\cos\mu L/2)}{\sin\mu L/2} - \frac{12}{(\mu L/2)^2}\right\} + \delta_1 \frac{2}{\mu L/2}\frac{(1-\cos\mu L/2)}{\sin\mu L/2} \tag{58}$$



(a) I-section

(b) Equivalent rectangular section

E = 205,000 N/mm^2

p_y = 275 N/mm^2

(c) Material properties

Fig. 1 Representative Column and Properties

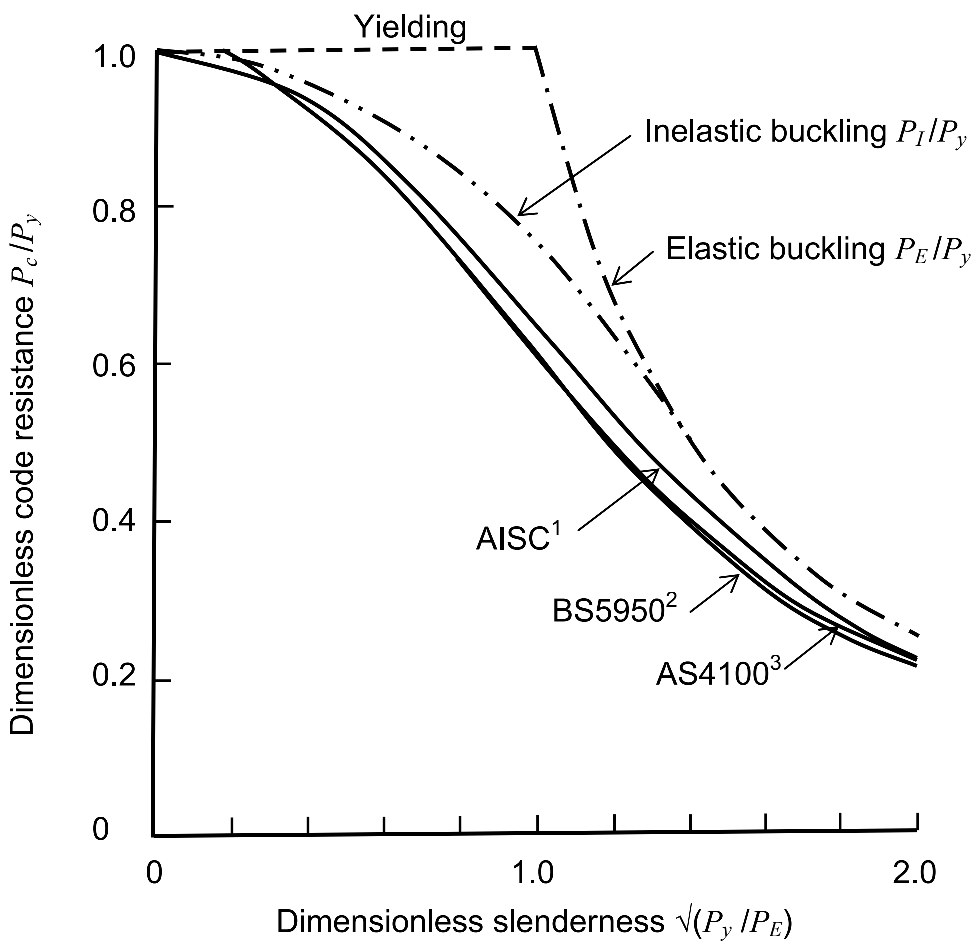


Fig. 2 Design Code Column Resistances

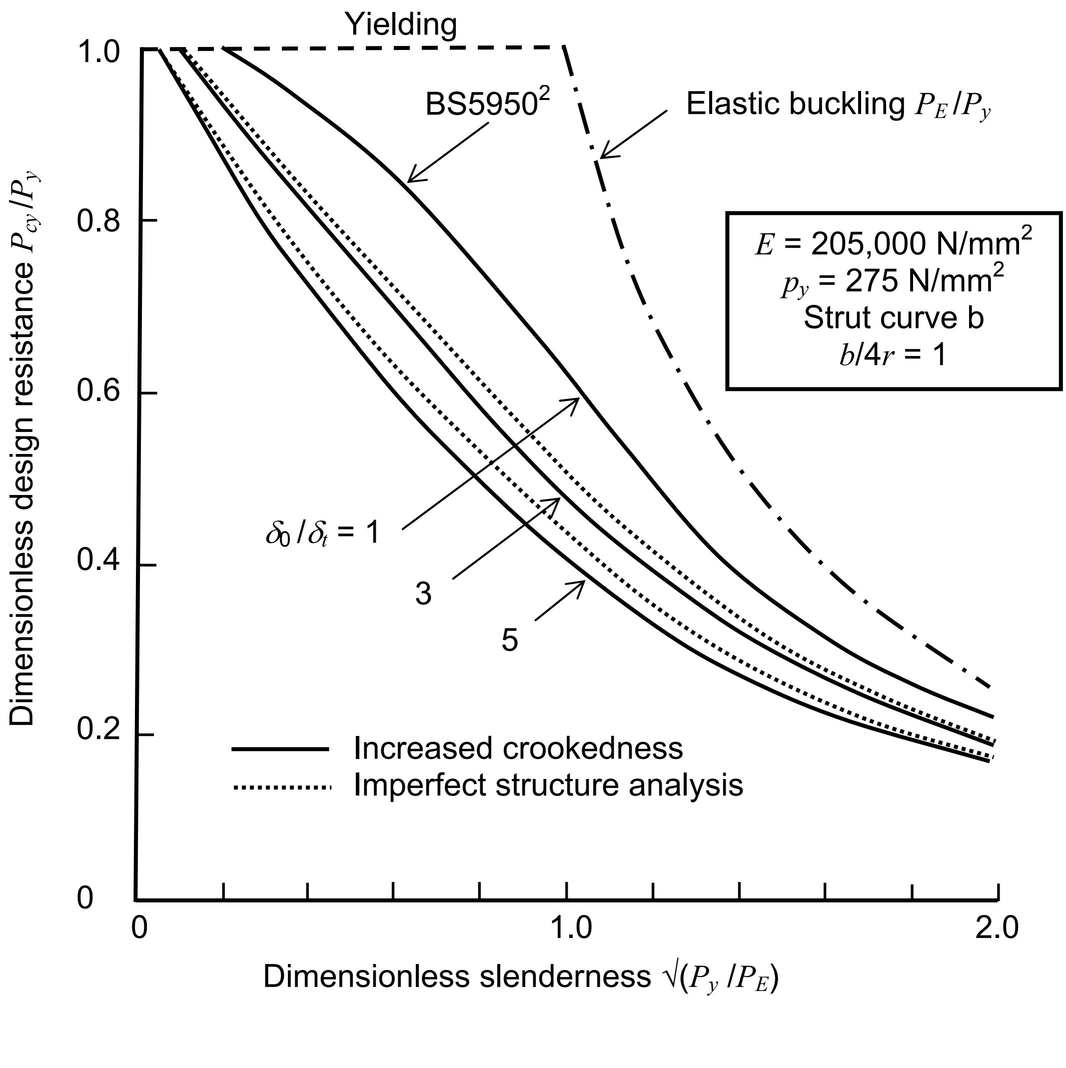
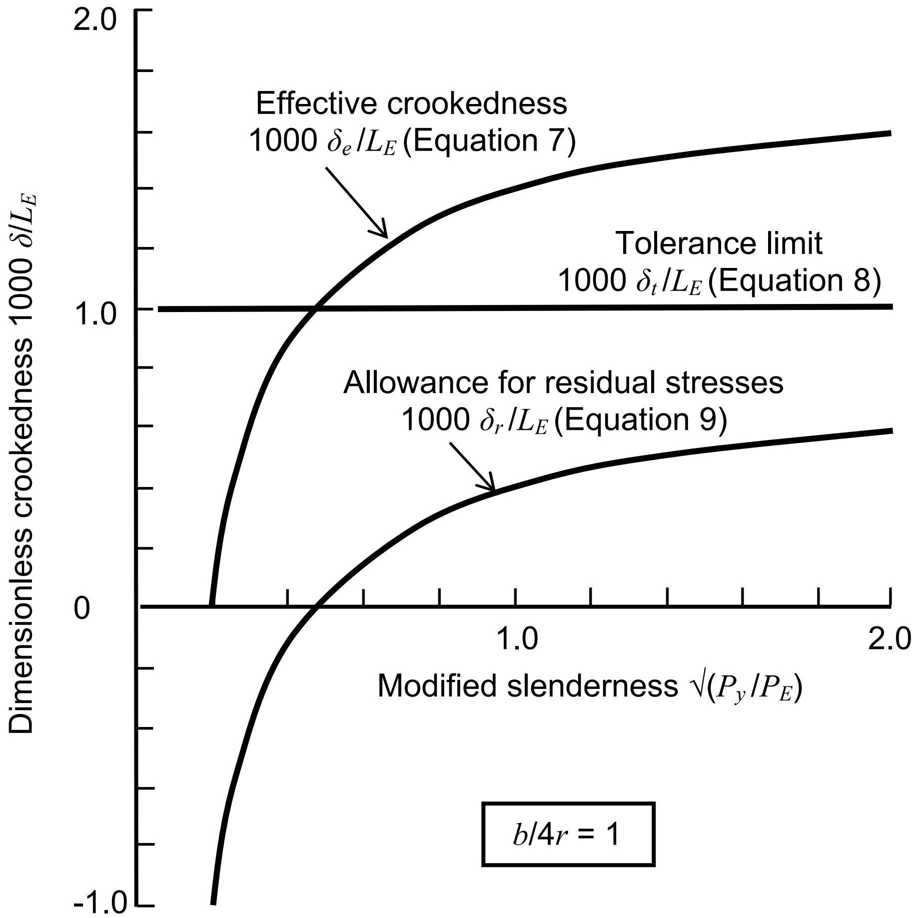


Fig. 3 Design Resistances of Out-of-Tolerance Columns



Fig, 4 Dimensionless Crookednesses of BS5950

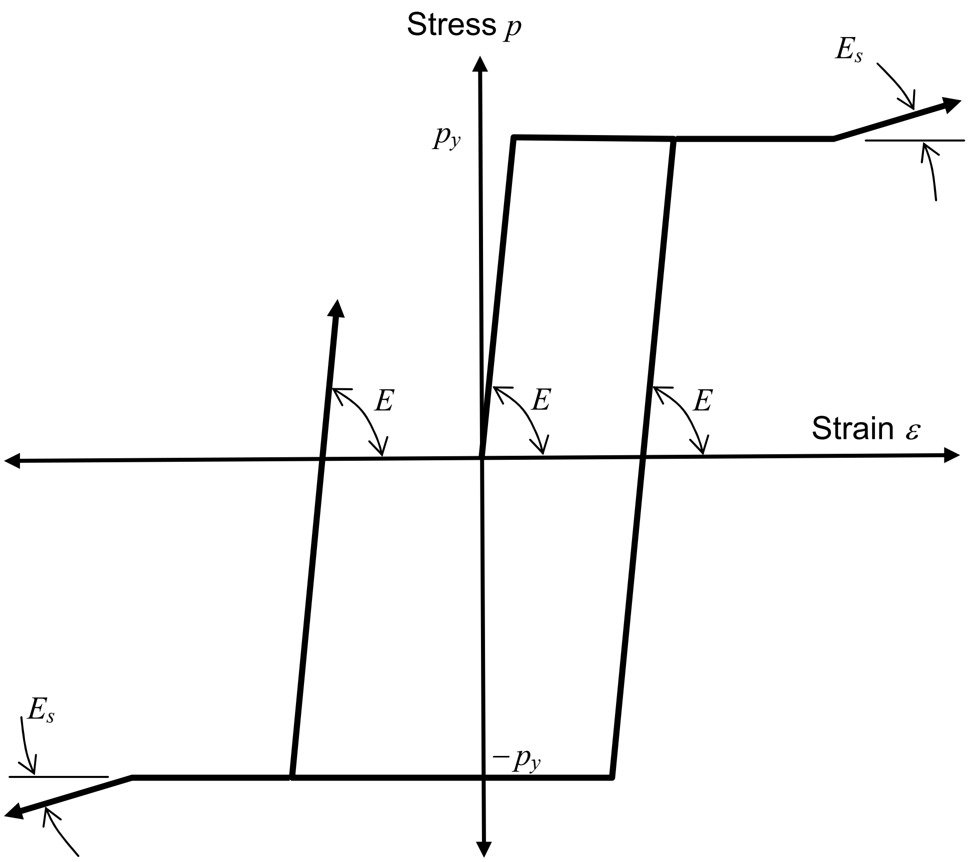
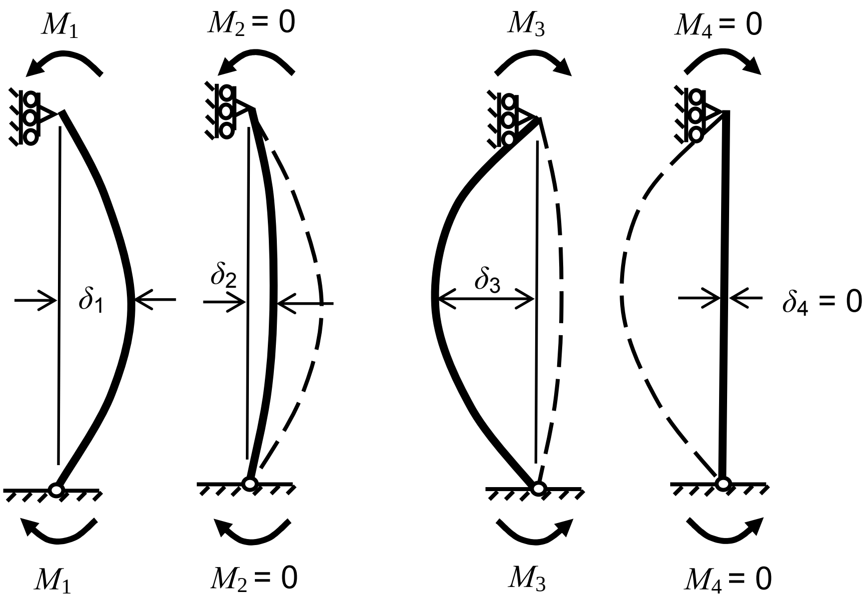
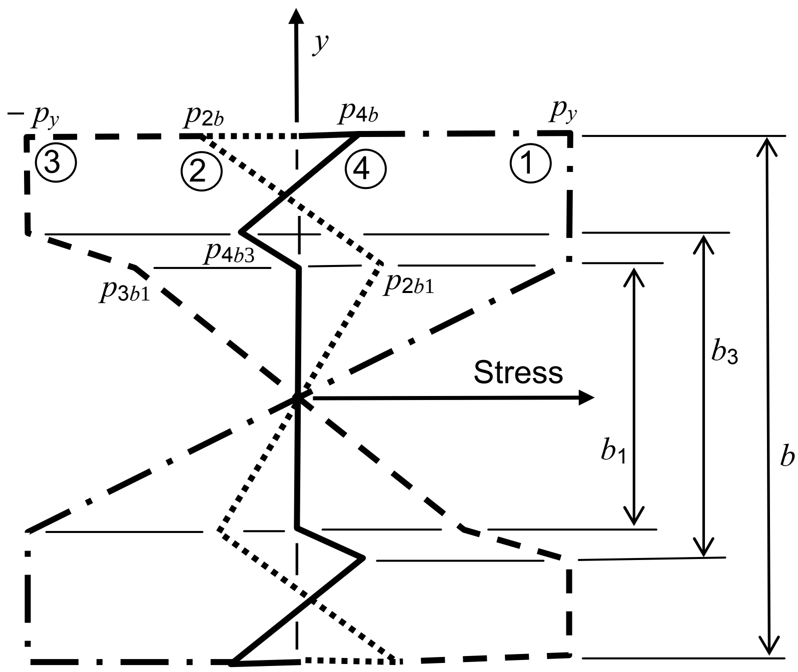


Fig. 5 Idealised Stress-Strain Relationships



(a) Deflected shapes



(b) Stress distributions

Fig. 6 Damaged and Straightened Columns

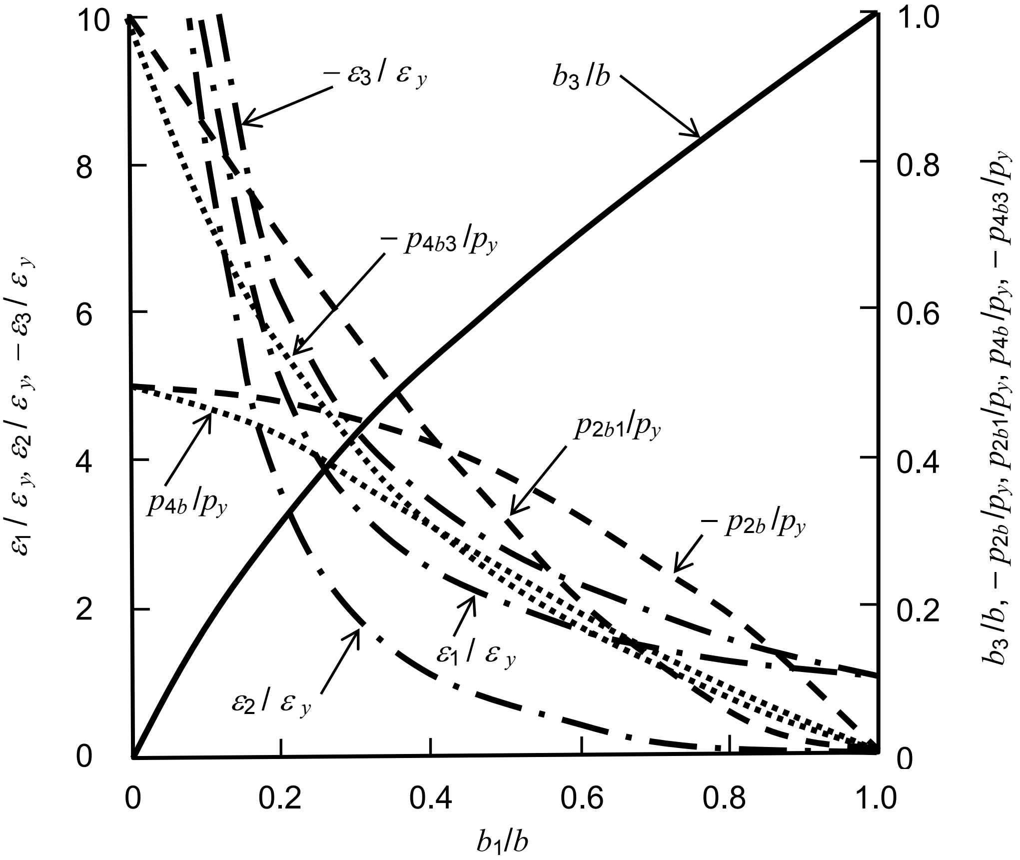


Fig. 7 Stresses and Strains in Damaged Columns

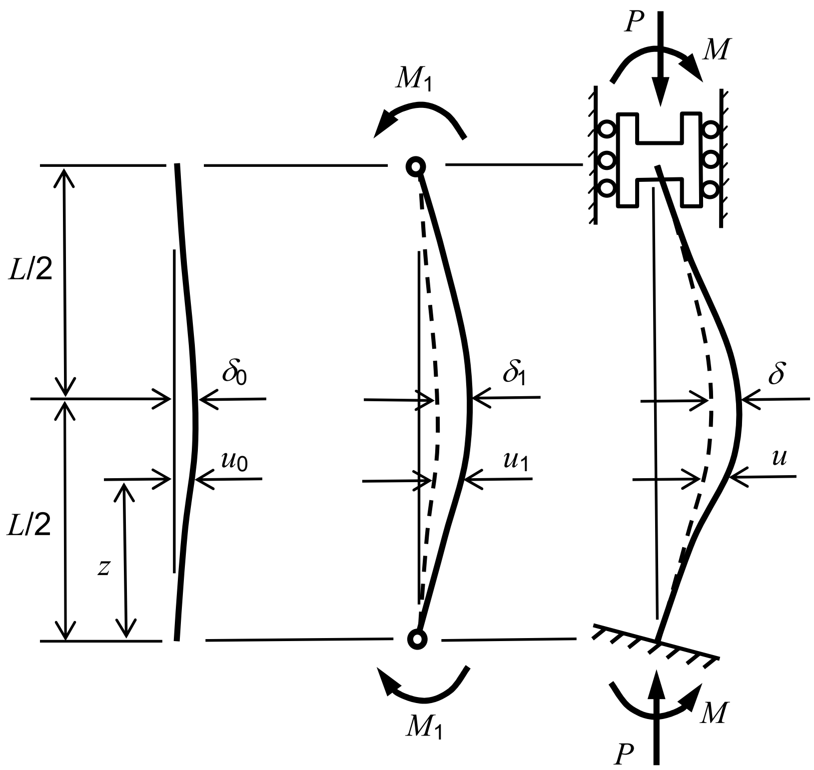


Fig. 8 Force-Fitted Column

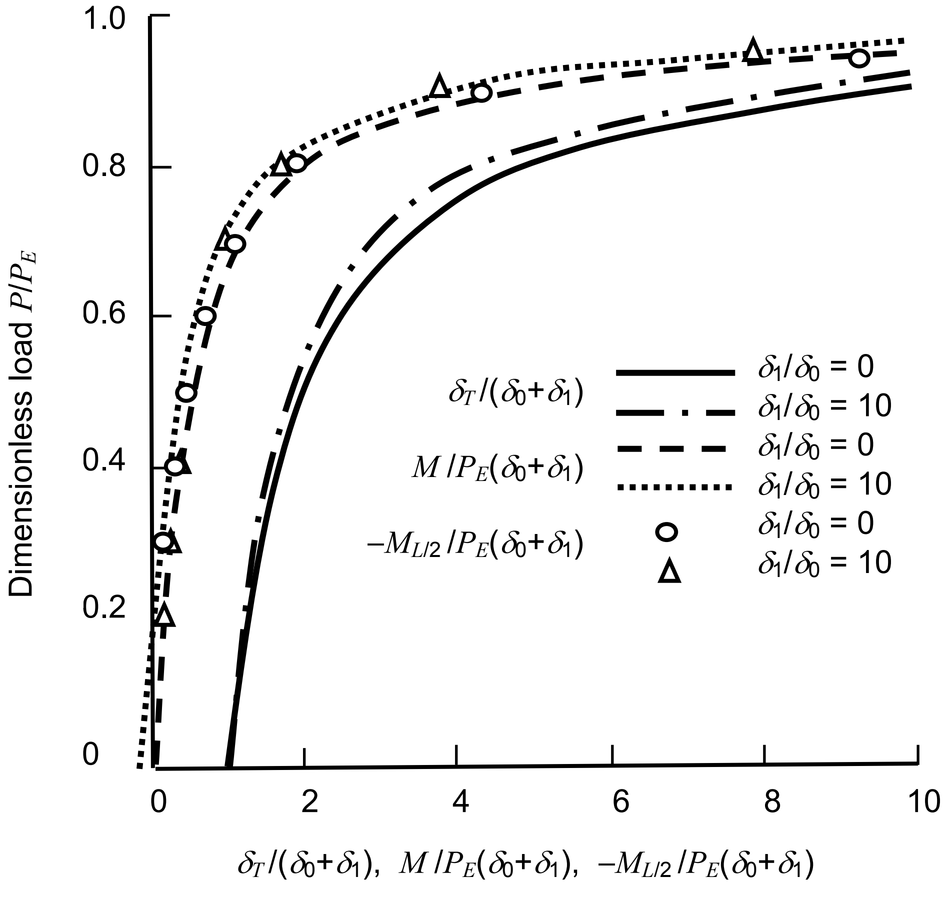
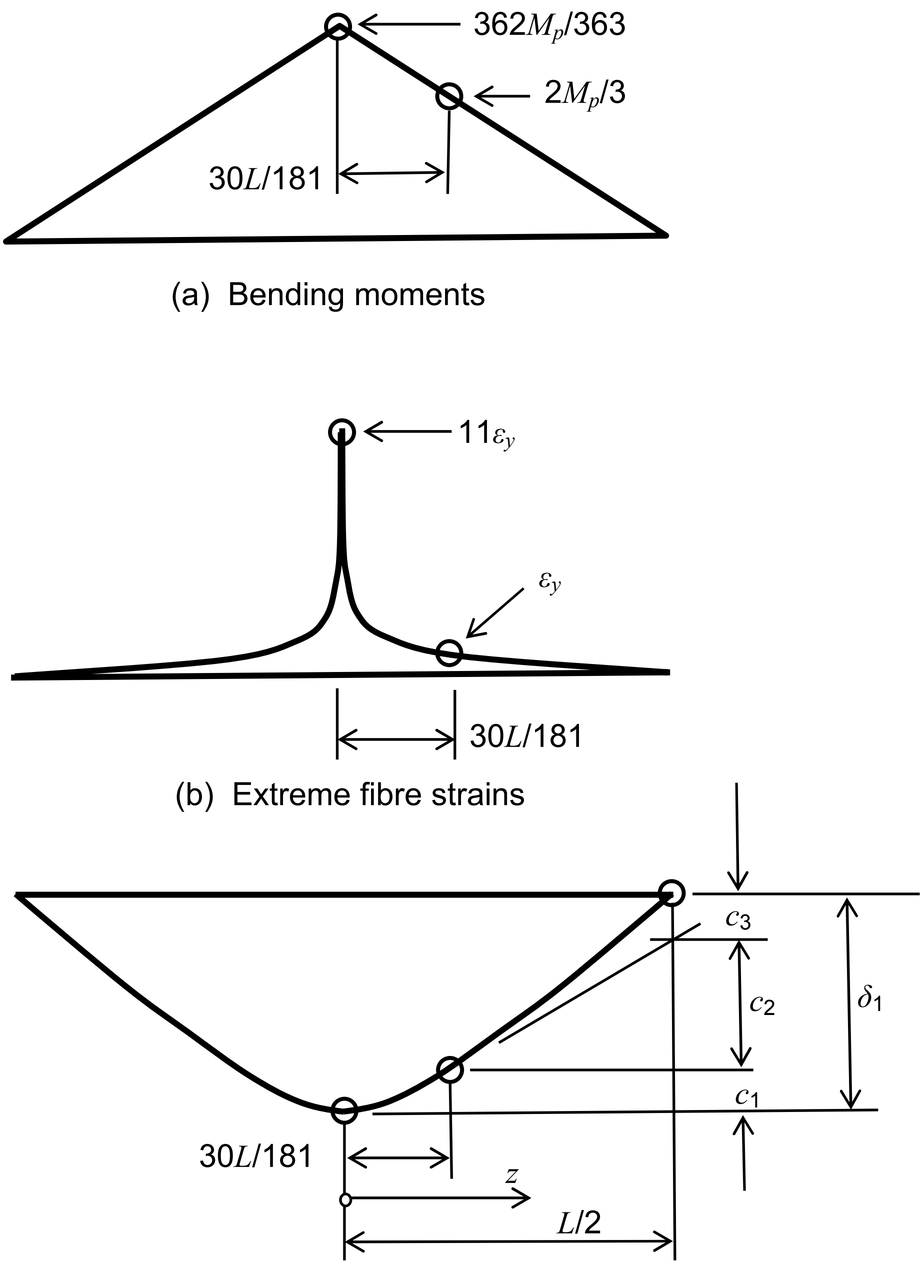


Fig. 9 Deflections and Moments in Force-Fitted Columns



(c) Deflections

Fig. 10 Inelastic Bending Under Central Concentrated Load