



The University of Sydney

Department of Civil Engineering
Sydney NSW 2006
AUSTRALIA

<http://www.civil.usyd.edu.au/>

Centre for Advanced Structural Engineering

**Design of Slender Angle Section Beam-
Columns by the Direct Strength Method**

Research Report No R838

**Kim JR Rasmussen MScEng PhD
M Shamim Hossain BE**

May 2004



The University of Sydney

Department of Civil Engineering
Centre for Advanced Structural Engineering
<http://www.civil.usyd.edu.au>

Design of Slender Angle Section Beam-columns by the Direct Strength Method

Research Report No. R838

Kim JR Rasmussen MScEng PhD
M Shamim Hossain BE

May 2004

Abstract:

This report is concerned with the application of the Direct Strength Method to equal angle section beam-columns with locally unstable legs. In contrast to existing design methods, which independently determine the compression and bending capacities and use an interaction equation to combine these, the Direct Strength Method determines the elastic local buckling stress for the actual stress distribution resulting from the combined action of compression and bending, and incorporates the elastic buckling stress into a direct strength equation for beam-columns.

In applying the method to equal leg angles, the torsional buckling mode is ignored when determining the overall buckling capacities, since it is accounted for through the local buckling mode, and the shift of the effective centroid is incorporated as an additional loading eccentricity. The shift in the effective centroid resulting from local buckling is determined from the actual stress distribution, as obtained using Stowell's classical solution, in place of an effective cross-section. The predicted strengths are conservative compared to tests on slender equal angle columns, and are shown to accurately predict the variation in load with applied loading eccentricity.

Keywords:

Direct Strength Method, equal angle section, beam-columns, local buckling, flexural buckling, flexural-torsional buckling, design, shift of the effective centroid.

Copyright Notice

Department of Civil Engineering, Research Report R838 Design of Slender Angle Section Beam-columns by the Direct Strength Method

© 2004 Kim JR Rasmussen and M Shamim Hossain
k.rasmussen@civil.usyd.edu.au

This publication may be redistributed freely in its entirety and in its original form without the consent of the copyright owner.

Use of material contained in this publication in any other published works must be appropriately referenced, and, if necessary, permission sought from the author.

Published by:
Department of Civil Engineering
The University of Sydney
Sydney, NSW, 2006
AUSTRALIA

May 2004

<http://www.civil.usyd.edu.au>

Table of contents

| | | |
|------------|---|----|
| 1 | Introduction..... | 4 |
| 2 | The DSM for beam-columns | 5 |
| 3 | Application to equal angle beam-columns with slender legs | 6 |
| 4 | Local buckling load for combined compression and bending | 7 |
| 5 | Summary of procedure..... | 8 |
| 6 | Tests on equal angles with slender legs | 9 |
| 7 | Comparison of the DSM with tests..... | 9 |
| 8 | Conclusions..... | 10 |
| Appendix I | References..... | 10 |

1 Introduction

The Direct Strength Method (Schafer and Pekoz 1998) is rapidly gaining acceptance in the Australian and American research communities as a reliable and efficient method for taking advantage of advanced software in the design of cold-formed steel structures. The method combines the elastic local and/or distortional buckling stresses of the entire cross-section, as determined from a rational buckling analysis, with the yield stress to define a slenderness, and uses direct strength equations to determine the ultimate limit state buckling capacity. It presents a competitive alternative to existing effective section methods as it obviates lengthy effective width calculations. The method has been formulated for the design for local and distortional buckling as well as overall flexural and flexural-torsional buckling, and a draft specification (AISI 2002) now exists for the design of compression and flexural members by the Direct Strength Method (DSM). Hancock et al. (1994) used the same approach in formulating direct strength equations for distortional buckling.

Schafer (2003) recently presented a paper outlining how the DSM may also be used for the design of beam-columns. The underlying idea being that the local and distortional modes of buckling can be accurately predicted using software for any combination of compression and bending, and hence, the method has the potential of leading to more accurate solutions for combined compression and bending than current strength predictions, which disassociate the calculations for pure compression and pure bending and combine these through an interaction equation. Schafer (2003) considered short lengths of lipped channel section beam-columns and accounted for local and distortional buckling. Duong and Hancock (2004) extended the approach to long lipped channel beam-columns, thus considering the influence of 2nd order bending (moment amplification) in determining the overall capacity.

Because the DSM dispenses with effective width calculations, it does not produce an effective cross-section representing the stiffness of the section in the presence of local buckling. In current specifications, the eccentricity of loading is measured from the centroid of the *effective* cross-sections in recognition of the fact a concentrically loaded member may undergo bending as the stiffness of the section changes with the development of local buckling. Notwithstanding the facts that a) local buckling does *not* induce overall bending of columns fixed against rotations at the ends (Rasmussen and Hancock 1993; Young and Rasmussen 1998) and b) the shift of the effective centroid may be inaccurately predicted when based on the effective cross-section (Young and Rasmussen 1999), the DSM has been criticised (Rusch and Lindner 2001) because it cannot account for the shift of the effective centroid. For the same reason, the effect of the shift of the effective centroid was not considered in Duong and Hancock's application of the DSM to long lipped channel sections.

In this report, the DSM is applied to plain equal angle section beam-columns with locally unstable legs. This type of member does not have a distortional buckling mode but the overall buckling behaviour requires particular attention because the local buckling mode is identical to the torsional mode at short column lengths, as discussed in detail in a previous report (Rasmussen 2003) on the design of slender equal angle columns. Furthermore, this type of member is particularly sensitive to the effect of the shift of the effective centroid when free to rotate flexurally at the ends. Consequently, the formulation presented herein incorporates an expression for the shift of the effective centroid. The results are compared with tests on equal angle columns with slender legs subject to varying eccentricity.

2 The DSM for beam-columns

In outlining the DSM for beam-columns, it is illustrative to first review the method for compression members. According to the draft specification (AISI 2002), the strength of a column subject to local and overall buckling shall be calculated by firstly determining the overall buckling strength as,

$$N_{ne} = \begin{cases} \left[0.658 \lambda_c^2 \right] N_Y & \text{for } \lambda_c \leq 1.5 \\ \left[\frac{0.877}{\lambda_c^2} \right] N_Y & \text{for } \lambda_c > 1.5 \end{cases} \quad (1)$$

where N_Y is the squash load and λ_c is the column slenderness,

$$N_Y = A F_Y \quad (2)$$

$$\lambda_c = \sqrt{\frac{N_Y}{N_e}}. \quad (3)$$

In equations (2-3), A is the gross area, F_Y is the yield stress, and N_e is the least of the flexural, torsional and flexural-torsional buckling loads. Denoting the local buckling load, as obtained from a rational buckling analysis (eg a Finite Strip analysis), by N_{cr} , the column strength is obtained as,

$$N_n = \begin{cases} N_{ne} & \text{for } \lambda_n \leq 0.776 \\ \left[(1 - 0.15 / \lambda_n) / \lambda_n \right] N_{ne} & \text{for } \lambda_n > 0.776 \end{cases} \quad (4)$$

where

$$\lambda_n = \sqrt{\frac{N_{ne}}{N_{cr}}}. \quad (5)$$

Rewriting the slenderness (λ_n) as,

$$\lambda_n = \sqrt{\frac{N_{ne}}{N_{cr}}} = \sqrt{\frac{\frac{N_{ne}}{N_Y}}{\frac{N_{cr}}{N_Y}}} = \sqrt{\frac{r_{ne}^N}{r_{cr}^N}} \quad (6)$$

it follows that the slenderness can be expressed in terms of r_{ne}^N and r_{cr}^N , which are vertical distances in the $(M/M_Y, N/N_Y)$ -plane, as shown in Fig. 1. Likewise, the strength equation can be expressed as,

$$N_n^N = r_n^N N_{ne} \quad (7)$$

where

$$r_n^N = \begin{cases} 1 & \text{for } \lambda_n \leq 0.776 \\ \left[(1 - 0.15 / \lambda_n) / \lambda_n \right] & \text{for } \lambda_n > 0.776 \end{cases} \quad (8)$$

This procedure can be extended to beam-columns by recognising that the distances r_{ne}^N and r_{cr}^N are to be replaced by radial distances r_{ne} and r_{cr} , as also shown in Fig. 1, where

$$r_{ne} = \left[\left(\frac{N_{one}}{N_Y} \right)^2 + \left(\frac{M_{one}}{M_Y} \right)^2 \right]^{\frac{1}{2}} \quad (9)$$

$$r_{cr} = \left[\left(\frac{N_{ocr}}{N_Y} \right)^2 + \left(\frac{M_{ocr}}{M_Y} \right)^2 \right]^{\frac{1}{2}}. \quad (10)$$

In equations (9,10), M_Y is the first yield moment, M_{one} is the overall bending capacity and M_{ocr} is the moment causing local buckling under combined compression and bending, as obtained from a rational buckling analysis. The slenderness is now defined in terms of the radial distances as,

$$\lambda_n = \sqrt{\frac{r_{ne}}{r_{cr}}}. \quad (11)$$

It will be assumed that the strength equation (8) derived for compression members is also applicable to beam-columns. Having obtained the radial distance r_n from eqn. (8), the strength is obtained from

$$r_n = \left[\left(\frac{N_{on}}{N_Y} \right)^2 + \left(\frac{M_{on}}{M_Y} \right)^2 \right]^{\frac{1}{2}} \quad (12)$$

where the moment (M_{on}) may be expressed in terms of the axial force (N_{on}) and an eccentricity (e),

$$M_{on} = e N_{on} \quad (13)$$

thus allowing (M_{on} , N_{on}) to be obtained from eqns (12, 13).

It remains to determine the overall capacities (N_{one} , M_{one}). It will be assumed that these can be obtained using the conventional linear interaction curve,

$$\frac{N_{one}}{N_{ne}} + \frac{|M_{one}|}{M_{ne}} = 1 \quad (14)$$

where N_{ne} and M_{ne} are the pure axial and flexural capacities respectively, as shown in Fig. 1, and M_{one} is the amplified 2nd order moment determined as described in the following section.

3 Application to equal angle beam-columns with slender legs

As mentioned in the Introduction, the design of equal angles requires particular attention because the local buckling mode is identical to the torsional mode. It leads to excessively conservative results if the overall buckling strengths (N_{ne} , M_{ne}) are determined considering torsional buckling while also accounting for local buckling in the direct strength equation (8). Consequently, as also proposed in Popovic et al. (1999) and adopted in Rasmussen (2003), the torsional buckling mode will be ignored in determining the overall buckling capacities (N_{ne} , M_{ne}). Specifically:

- In calculating the overall compression strength (N_{ne}) according to eqns (1-3), the elastic buckling stress (N_e) will be taken as the minor axis flexural buckling stress (N_{ey}),

$$N_{ey} = \frac{\pi^2 EI_y}{L_{ey}^2} \quad (15)$$

where E is the elastic modulus, L_{ey} is the effective length for buckling about the minor y -axis, and I_y is the second moment of area about the y -axis.

- The flexural strength (M_{ne}) will be calculated without considering flexural-torsional buckling, and hence taken as the first-yield moment,

$$M_{en} = M_{Yt} = Z_{yt} F_Y \quad \text{or} \quad M_{en} = M_{Yh} = Z_{yh} F_Y \quad (16)$$

where M_{yt} and M_{yh} are the moments for first yield at the tips and heel respectively, as appropriate, obtained using the section moduli Z_{yt} and Z_{yh} ,

$$Z_{yt} = \frac{I_y}{c_t} \quad \text{and} \quad Z_{yh} = \frac{I_y}{c_h} \quad (17)$$

In eqn. (17), c_t and c_h are the distances to the tips and heel of the sections respectively, as shown in Fig. 2.

As also mentioned in the Introduction, the shift of the effective centroid produces significant eccentricities of loading in equal angles with slender legs and cannot be ignored in design. However, for an equal angle, the eccentricity can be determined from Stowell's solution (Stowell 1949) without resorting to effective widths. As shown in Rasmussen (2003), the eccentricity produced by the shift of the effective centroid can be estimated as,

$$e_0 = \begin{cases} 0 & \text{for } \lambda_{py} \leq 1.22 \\ \frac{5}{16\sqrt{2}} \frac{\lambda_{py} - 1.22}{\lambda_{py} - 0.22} & \text{for } \lambda_{py} > 1.22 \end{cases} \quad (18)$$

where λ_{py} is the plate slenderness calculated at the yield stress,

$$\lambda_{py} = \sqrt{\frac{F_y}{F_{cr}}} \quad (19)$$

$$F_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (20)$$

In equation (20), t and b are the thickness and flat width of the legs respectively, as shown in Fig. 3, and k is the plate buckling coefficient taken as 0.425.

Denoting the eccentricity of loading by e_p , as shown in Fig. 2, the total eccentricity of the applied force relative to the effective centroid is $e_p + e_0$. Accounting for 2nd order effects, at the centre of the column, the eccentricity of the applied force relative to the effective centroid is,

$$e = \frac{e_p + e_0}{1 - N_{on} / N_{Ey}} \quad (21)$$

where $N_{Ey} = N_{ey}$ is the Euler load for buckling about the minor y -axis given by eqn. (15). Thus, the overall buckling moment (M_{one}), local buckling moment (M_{ocr}), and direct strength buckling moment (M_{on}) are,

$$M_{one} = e N_{one} \quad (22)$$

$$M_{ocr} = e N_{ocr} \quad (23)$$

$$M_{on} = e N_{on} \quad (24)$$

4 Local buckling load for combined compression and bending

The local buckling loads (M_{ocr} , N_{ocr}) depend on the ratio of axial force to moment, or the angle (θ) of the radius r_{cr} to the M -axis, as shown in Fig. 1. The angle θ is related to the eccentricity (e) through,

$$\tan \theta = \frac{N / N_Y}{M / M_Y} \quad (25)$$

where

$$M_Y = Z_y F_Y \quad (26)$$

$$Z_y = \min(Z_{yt}, Z_{yh}) \quad (27)$$

Thus,

$$\theta = \tan^{-1} \left(\frac{Z_y}{e A} \right) \quad (28)$$

In general, a rational buckling analysis will be used to determine M_{ocr} and N_{ocr} for given eccentricity (e). However, in the case of an equal angle, the local buckling stress can be determined with good accuracy using eqn. (20) in combination with the expressions for the plate buckling coefficient (k) given in Eurocode3, Part 1.3, and Appendix D of AS/NZS4600. These expressions operate on the stress ratio (ψ) and distinguish between compression and tension at the edges of the unstiffened element. In applying the expressions, the stresses at the tip (f_t) and corner (f_c) are firstly calculated for a unit force,

$$f_t^1 = \frac{1}{A} + \frac{e c_t}{I_y} \quad (29)$$

$$f_c^1 = \frac{1}{A} - \frac{e c_c}{I_y} \quad (30)$$

where c_t and c_c are the distances from the centroid to the tip and corner respectively, as shown in Fig. 2. The expressions for k and calculations of buckling stresses ($f_{t,cr}$, $f_{c,cr}$) are summarised in Table 1. Having obtained the buckling stress at the tip ($f_{t,cr}$) for given value of eccentricity (e), the buckling load (N_{ocr}) is obtained as

$$N_{ocr} = \frac{f_{t,cr}}{f_t^1} \quad (31)$$

and the buckling moment (M_{ocr}) is obtained from eqns (23, 31).

5 Summary of procedure

The direct strength calculation proceeds through the following steps.

1. Calculate the shift of the effective centroid (e_0) according to eqns (18-20) and set a value of loading eccentricity (e_p).
2. Assume a value of axial strength (N_{on}) and calculate the amplified eccentricity (e) according to eqn. (21). In a design situation, the eccentricity would be calculated for the design action rather than the strength (N_{on}).
3. Calculate the overall buckling strengths (M_{one} , N_{one}) according to eqns (1-3, 14, 16, 22).
4. Calculate the local buckling loads (M_{ocr} , N_{ocr}) according to eqns (23, 29-31) and Table 1.
5. Calculate the radial distances (r_{ne} , r_{cr}) using eqns (9, 10) and the slenderness (λ_n) using eqn. (6) with r_{ne}^N and r_{cr}^N replaced by r_{ne} and r_{cr} respectively.
6. Calculate the direct strength r_n using eqn. (8), and subsequently the axial strength from,

$$N_{on} = \frac{r_n}{\left[\left(\frac{1}{N_Y} \right)^2 + \left(\frac{e}{M_Y} \right)^2 \right]^{\frac{1}{2}}} \quad (32)$$

7. If this value does not match the assumed value, return to (2) and repeat the calculation for a new value of N_{on} until convergence is achieved. This iteration is not required in a design situation where the design action is used to determine the amplified eccentricity.
8. Having achieved convergence, calculate the flexural strength (M_{on}) using eqn. (24).

6 Tests on equal angles with slender legs

Compression tests on equal angles with slender legs have been reported by Wilhoite et al. (1984) and Popovic et al. (1999). The tests are summarised as follows:

The angles tested by Wilhoite et al. (1984) were brake-pressed from high strength steel plates. The measured dimensions and measured value of yield stress for the flats (F_Y) are shown in Table 2. Three lengths of pin-ended long columns were tested chosen so as to produce nominal L/r_y -values of 60, 90 and 120. The pin-ended columns were nominally loaded through the centroid of the gross cross-section ($e_p=0$). However, a small clearance was built into the pin-ended bearings to avoid locking, and an eccentricity of loading may have been induced as a result of this clearance. The bearings were manufactured to a tolerance that ensured the induced loading eccentricity would not exceed 1/1000 of the length of the longest columns. The test strengths and column lengths are shown in Table 3, as obtained from Fig. 18 of Wilhoite et al. (1984).

The angles tested by Popovic et al. (1999) were cold-rolled and in-line galvanised to produce a nominal yield stress value of 350 MPa. The measured dimensions and mechanical properties are shown in Table 2. Because of the cold-rolling, the yield stress varied across the cross-section with the highest values obtained at the corners. The measured yield stress value shown in Table 2 was obtained from the middle of the flat part of the legs.

Ten pin-ended columns were tested with L_e/r_y -values ranging from 46 to 130, where the effective length (L_e) is the sum of the specimen length and the total lengthwise dimension of the end bearings. The columns were loaded with a nominal eccentricity of 1/1000 of the column length relative to the gross section centroid. Two tests were performed at each length, one where the load was applied eccentrically towards the tips of the flanges, (positive eccentricity causing increased compression at the free edges), and one where the load was applied eccentrically towards the corner (negative eccentricity). Consistently, the lowest test strength was obtained when the eccentricity caused increased compression at the free edges. The geometric imperfections were measured on all specimens, as detailed in Popovic et al. (1996), indicating an average overall minor axis flexural imperfection of $L/1305$. The test strengths and column lengths are shown in Table 4.

7 Comparison of the DSM with tests

The direct strength calculations are compared with the Wilhoite et al. test strengths in Table 3 and the Popovic et al. test strengths in Table 4. Interaction diagrams and N_{on} vs e_p curves for two lengths of columns of the two test series are shown in Figs 4-11. In the interaction diagrams shown in Figs 4, 6, 8 and 10, the (a)-figures show moments (M_{one} , M_{ocr} , M_{on}) calculated according to eqns (21-23), while the (b)-figures show moments calculated as the

product of (N_{one} , N_{ocr} , N_{on}) and e_p . Test strengths are included in the N_{on} vs e_p curves shown in Figs 5, 7, 9 and 11.

It follows from Tables 3 and 4 that the direct strength calculations are conservative. The mean ratios of test strength to design strength are 1.40 and 1.14 for the Wilhoite et al. and Popovic et al. test series respectively. The corresponding coefficients of variation are 0.157 and 0.103 for the Wilhoite et al. and Popovic et al. test series respectively. Overall, the comparison is good for the specimens tested by Popovic et al., and the drop in strength with applied loading eccentricity (e_p) follows closely the experimental observations, as shown in Figs 9 and 11.

The direct strengths calculated for the longer specimens of those tested by Wilhoite et al. are very conservative, as shown in Table 3. This is partly attributed to the fact that the shift of the effective centroid is calculated at yield rather than, say, the applied stress (N_{one}/A), and partly to possible end restraints present in the bearings during testing. As shown in Fig. 7, the test strength of one of the long columns ($L_e=1636$ mm) exceeded the Euler load by 6.5%.

8 Conclusions

The report presents the application of the Direct Strength Method to equal angle section beam-columns with slender legs. The method involves determining the local buckling stress for the applied loading of combined compression and bending, which, in general, requires a rational buckling analysis but for an equal angle can be achieved from readily available analytic expressions for unstiffened elements under stress gradient. The method combines a slenderness (λ_n), determined in terms of radial distances for local and overall buckling (r_{ne} , r_{cr}), with a direct strength equation to determine the direct strength radial distance (r_n) from which the axial and flexural capacities can be obtained. The method includes 2nd order effects in determining the maximum moment and incorporates an eccentricity of loading to account for the effect of the shift of the effective centroid.

Since for slender section equal angles, the local and torsional buckling modes are identical, the overall buckling capacities (N_{one} , M_{one}) are determined by ignoring the torsional buckling mode. This approach is consistent with that presented (Rasmussen 2003) in a previous study on the design of slender section equal angle columns.

The design strengths are compared with tests of slender section equal angle columns subjected to varying loading eccentricities. The predicted strengths are conservatively predicted with a mean value of test strength to design strength of 1.14 for tests performed by Popovic et al. (1999) and 1.40 for tests performed by Wilhoite et al. (1984). The design strengths obtained by Popovic et al. accurately predict the variation in load with applied loading eccentricity.

Appendix I: References

- AISI (2002). *Design Manual for Direct Strength Method of Cold-formed Steel Design*, Draft. Washington, DC, American Iron and Steel Institute.
- Duong, H. and G. Hancock (2004). Recent Developments in the Direct Strength Design of Thin-Walled Members. *Proceedings of the International Workshop on Recent Advances and Future Trends in Thin-Walled Structures Technology*. Ed. J. Loughlan. Loughborough.

- Hancock, G., Y. Kwon, et al. (1994). "Strength Design Curves for Thin Walled Sections Undergoing Distortional Buckling." *Journal of Constructional Steel Research* **31**(2-3): 169-186.
- Popovic, D., G. Hancock, et al. (1996). Axial Compression Tests of Duragal Angles. *Research Report R730*, Department of Civil Engineering, University of Sydney.
- Popovic, D., G. Hancock, et al. (1999). "Axial Compression Tests of Cold-formed Angles." *Journal of Structural Engineering, American Society of Engineers* **125**(5): 515-523.
- Rasmussen, K. (2003). Design of Angle Columns with Locally Unstable Legs. *Research Report R830*, Department of Civil Engineering, University of Sydney. Available at <http://www.civil.usyd.edu.au/publications/>.
- Rasmussen, K. and G. Hancock (1993). "The Flexural Behaviour of Fixed-ended Channel Section Columns." *Thin-walled Structures* **17**(1): 45-63.
- Rusch, A. and J. Lindner (2001). "Remarks to the Direct Strength Method." *Thin-walled Structures* **39**: 807-820.
- Schafer, B. (2003). Advances in direct strength design of thin-walled members. *Advances in Structures*. Eds G.J. Hancock, M.A. Bradford, T.J. Wilkinson, B. Uy and K.J.R. Rasmussen. Lisse, Balkema. **1**: 333-339.
- Schafer, B. and T. Pekoz (1998). Direct strength Prediction of Cold-formed Steel Members using Numerical Elastic Buckling Solutions. *Thin-walled Structures, Research and Developments*. Eds N. Shanmugam, J. Liew and V. Thevendran. New York, Elsevier: 127-144.
- Stowell, E. (1949). Compressive Strength of Flanges. *Technical Note No. 1556*, National Advisory Committee for Aeronautics (NACA).
- Wilhoite, G., R. Zandonini, et al. (1984). "Behaviour and Strength of Angles in Compression: An Experimental Investigation". *ASCE Annual Convention and Structures Congress*. San Francisco.
- Young, B. and K. Rasmussen (1999). "Shift of Effective Centroid of Channel Columns." *Journal of Structural Engineering, American Society of Engineers* **125**(5): 524-531.
- Young, B. and K. J. R. Rasmussen (1998). "Tests of Fixed-ended Plain Channel Columns." *Journal of Structural Engineering, American Society of Engineers* **124**(2): 131-139.

Tables

Table 1: Plate buckling coefficient and buckling stress equations for unstiffened elements under stress gradient.

| | | | | |
|--|------------------------|------------------------------------|--|----------------------------|
| | $\psi = f_t^1 / f_c^1$ | $k = \frac{0.578}{\psi + 0.34}$ | $f_{c,cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$ | $f_{t,cr} = \psi f_{c,cr}$ |
| | $\psi = f_c^1 / f_t^1$ | $k = 0.57 - 0.21\psi + 0.07\psi^2$ | $f_{t,cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$ | $f_{c,cr} = \psi f_{t,cr}$ |
| | $\psi = f_t^1 / f_c^1$ | $k = 1.7 - 5\psi + 17.1\psi^2$ | $f_{c,cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$ | $f_{t,cr} = \psi f_{c,cr}$ |
| | $\psi = f_c^1 / f_t^1$ | $k = 0.57 - 0.21\psi + 0.07\psi^2$ | $f_{t,cr} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$ | $f_{c,cr} = \psi f_{t,cr}$ |

Table 2: Geometric and material properties of the Wilhoite et al. (1984) and Popovic et al. (1999) cross-sections

| Section | B | t | r_i | A | I_x | I_y | J | x_0 | F_Y | E |
|-----------------|------|------|-------|-----------------|-----------------|-----------------|-----------------|-------|-------|--------|
| | mm | mm | mm | mm ² | mm ⁴ | mm ⁴ | mm ⁴ | mm | MPa | MPa |
| Wilhoite et al. | 69.3 | 3.00 | 3.0 | 401 | 311200 | 74150 | 1203 | 24.2 | 465 | 203000 |
| Popovic et al. | 50.8 | 2.30 | 2.6 | 224 | 93830 | 22150 | 396 | 17.7 | 396 | 203000 |

Table 3: Test strengths reported by (Wilhoite et al. 1984); $N_Y=186\text{kN}$.

| Test | L_e | e_p | N_u | N_u/N_Y | N_{on} | N_u/N_{on} |
|---------|-------|-------|-------|-----------|----------|--------------|
| | mm | Mm | kN | | kN | |
| 1 | 823 | 0 | 72.5 | 0.388 | 61.5 | 1.179 |
| 2 | 823 | 0 | 72.5 | 0.388 | 61.5 | 1.179 |
| 3 | 1227 | 0 | 58.3 | 0.312 | 46.1 | 1.265 |
| 4 | 1227 | 0 | 60.1 | 0.322 | 46.1 | 1.304 |
| 5 | 1227 | 0 | 65.0 | 0.348 | 46.1 | 1.41 |
| 6 | 1636 | 0 | 48.4 | 0.259 | 32.7 | 1.48 |
| 7 | 1636 | 0 | 52.1 | 0.279 | 32.7 | 1.59 |
| 8 | 1636 | 0 | 59.2 | 0.317 | 32.7 | 1.81 |
| Average | | | | | | 1.40 |
| COV | | | | | | 0.157 |

Table 4: Test strengths reported by (Popovic et al. 1999); $N_Y=88.7\text{kN}$.

| Test | L_e | e_p | N_u | N_u/N_Y | N_{on} | N_u/N_{on} |
|---------|-------|-------|-------|-----------|----------|--------------|
| | mm | mm | kN | | kN | |
| 1 | 459 | 0.46 | 41.7 | 0.466 | 39.5 | 1.06 |
| 2 | 458 | -0.51 | 47.2 | 0.528 | 44.3 | 1.07 |
| 3 | 676 | 0.70 | 35.2 | 0.394 | 33.0 | 1.07 |
| 4 | 676 | -0.67 | 40.1 | 0.448 | 38.8 | 1.03 |
| 5 | 862 | 0.88 | 30.9 | 0.345 | 27.3 | 1.13 |
| 6 | 863 | -0.86 | 47.5 | 0.531 | 33.4 | 1.42 |
| 7 | 1088 | 1.07 | 25.1 | 0.280 | 21.1 | 1.19 |
| 8 | 1088 | -1.08 | 32.1 | 0.359 | 26.5 | 1.21 |
| 9 | 1285 | 1.29 | 17.7 | 0.198 | 16.7 | 1.06 |
| 10 | 1286 | -1.33 | 24.7 | 0.276 | 21.6 | 1.14 |
| Average | | | | | | 1.14 |
| COV | | | | | | 0.103 |

Figures

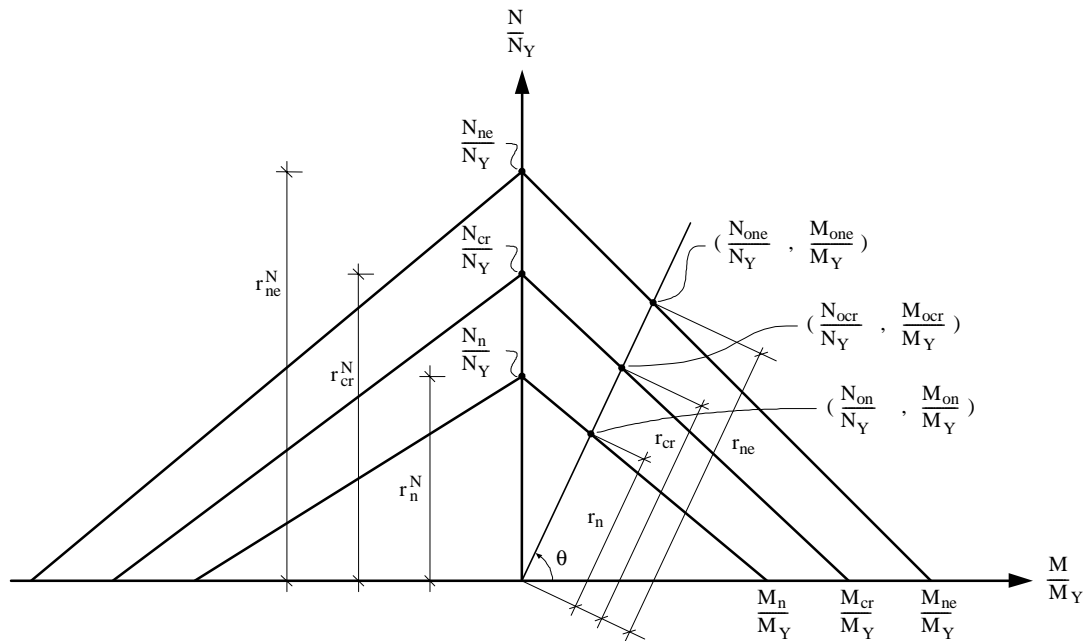


Figure 1: Simply supported angle column

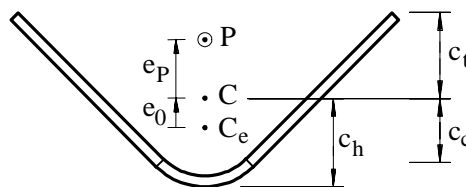


Figure 2: Loading eccentricities and distances to extreme fibres

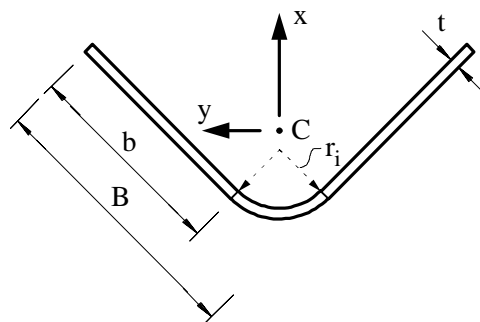


Figure 3: Nomenclature

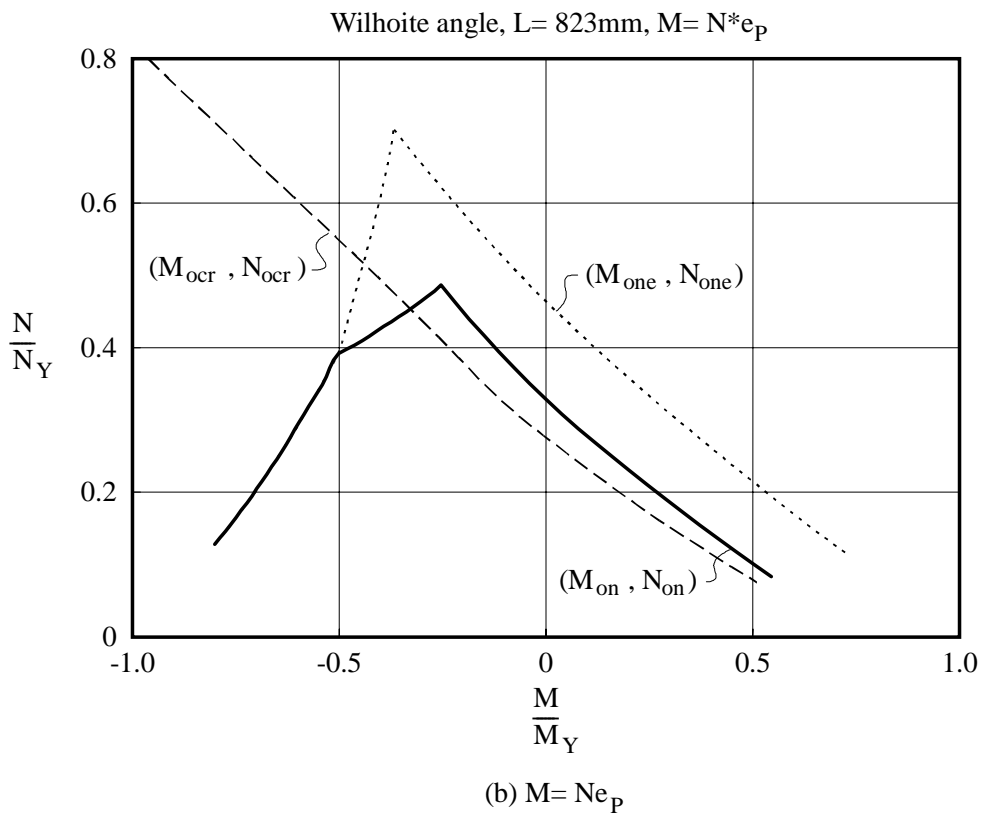
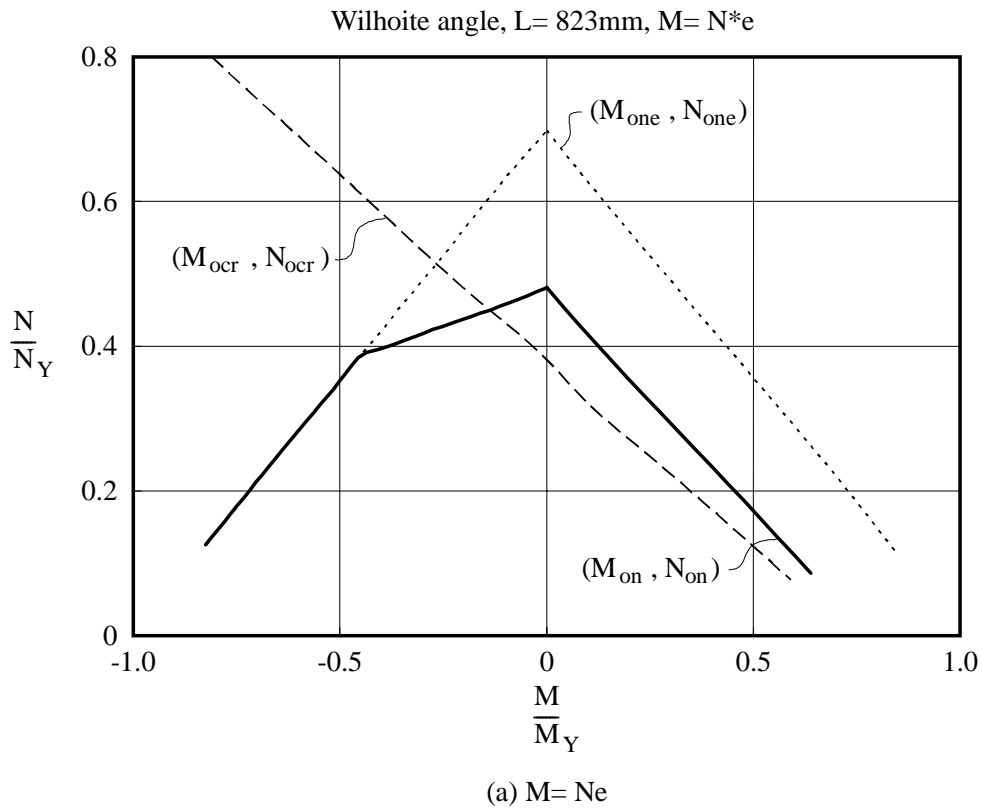


Figure 4: Interaction diagrams for equal angles tested by Wilhoite et al. (1984), $L=823$ mm.

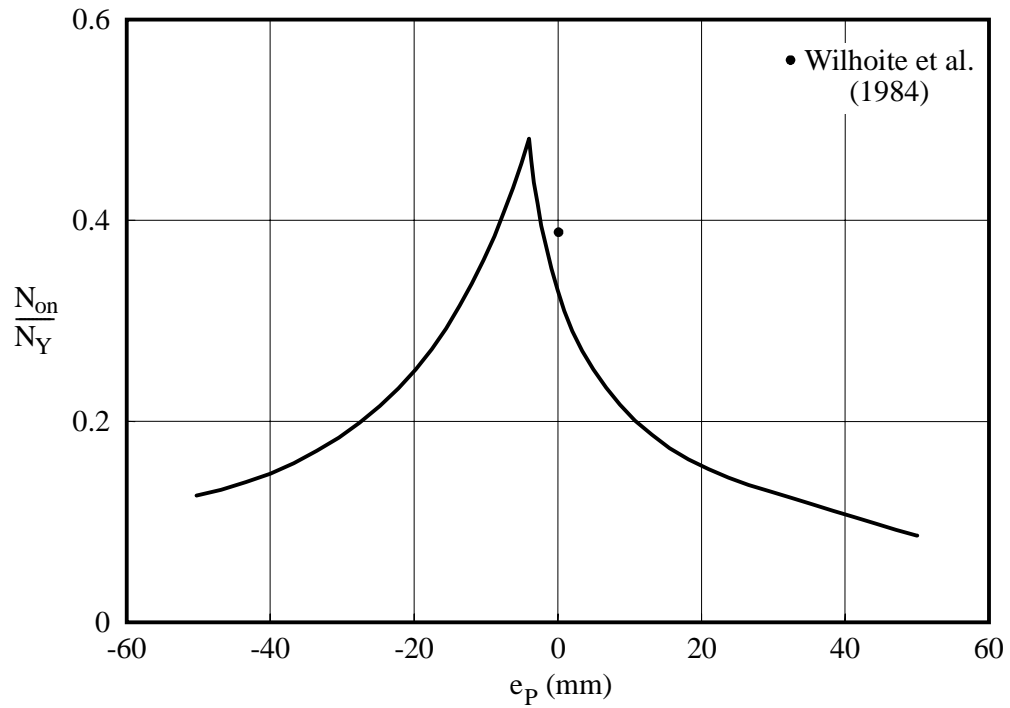


Figure 5: Strength vs eccentricity of loading for equal angles tested by Wilhoite et al. (1984), L=823 mm.

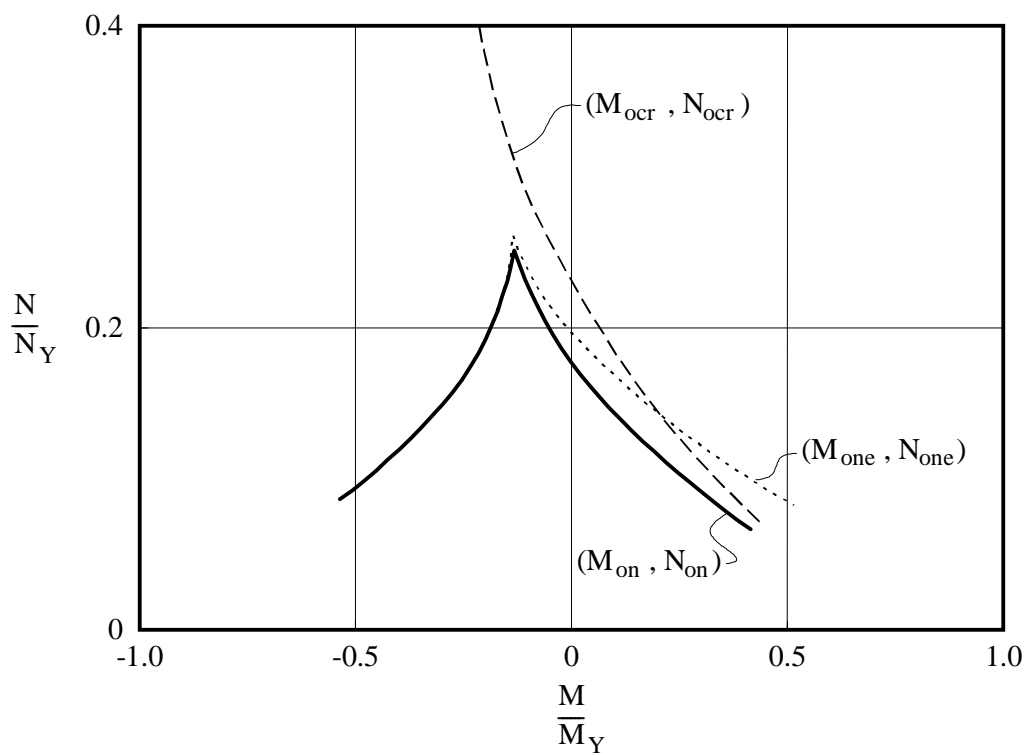
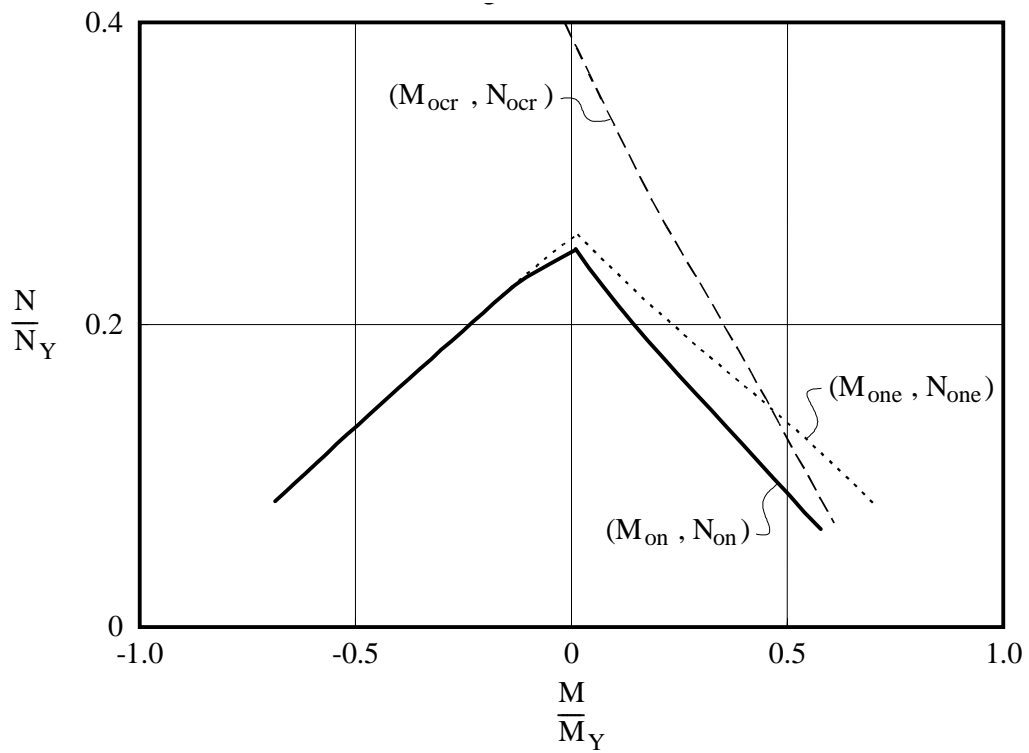


Figure 6: Interaction diagrams for equal angles tested by Wilhoite et al. (1984), $L=1636$ mm.

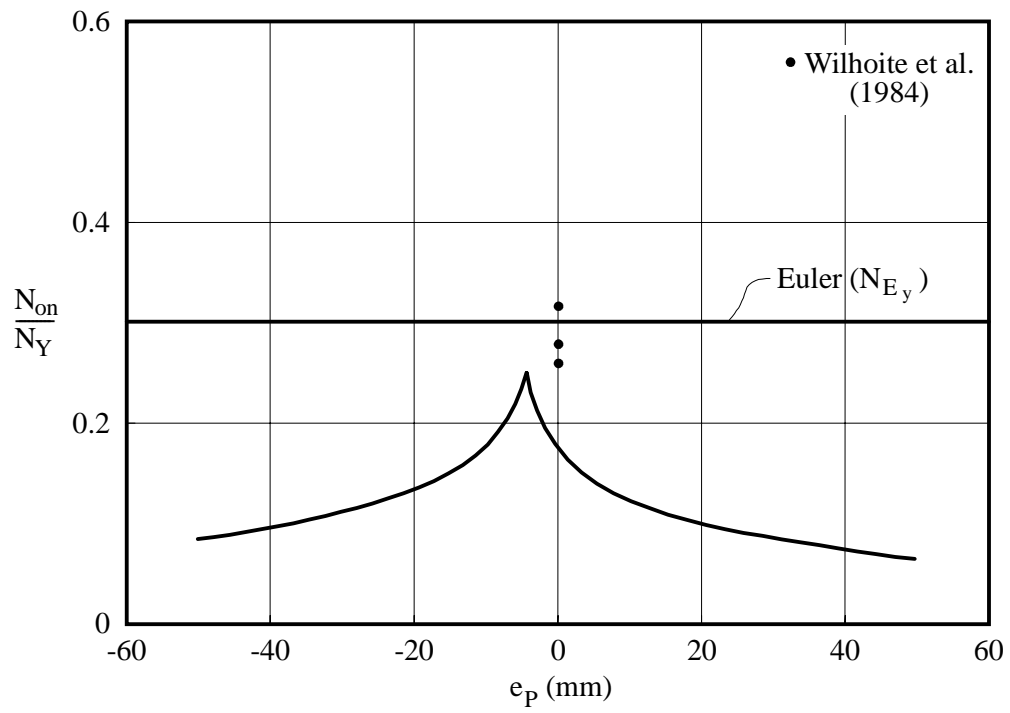
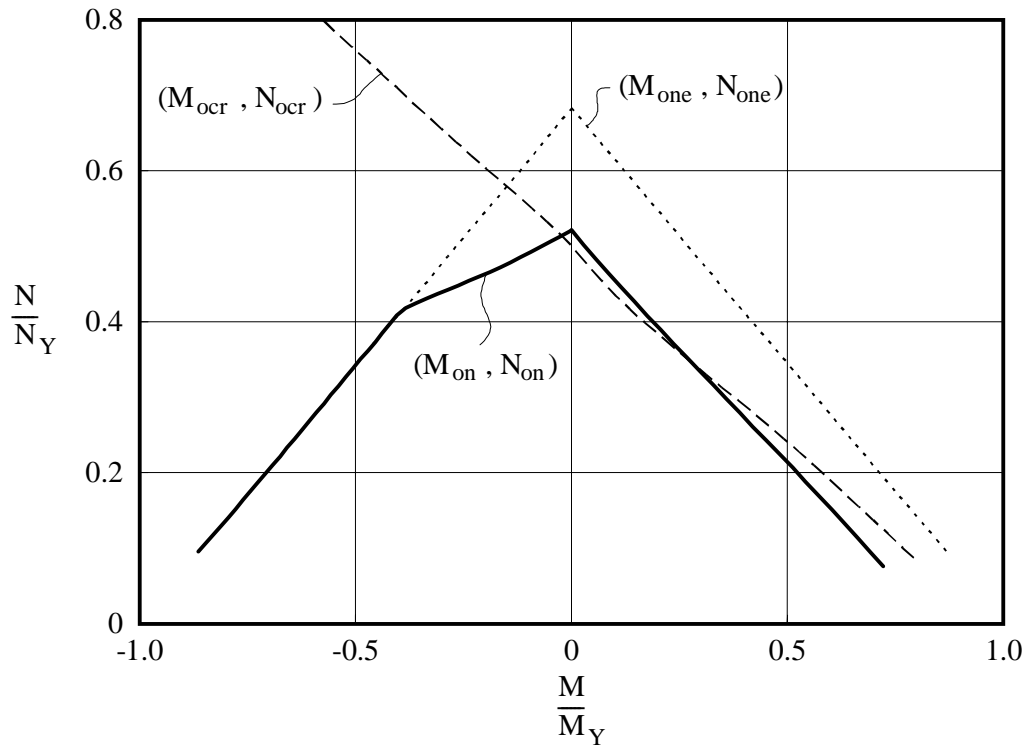
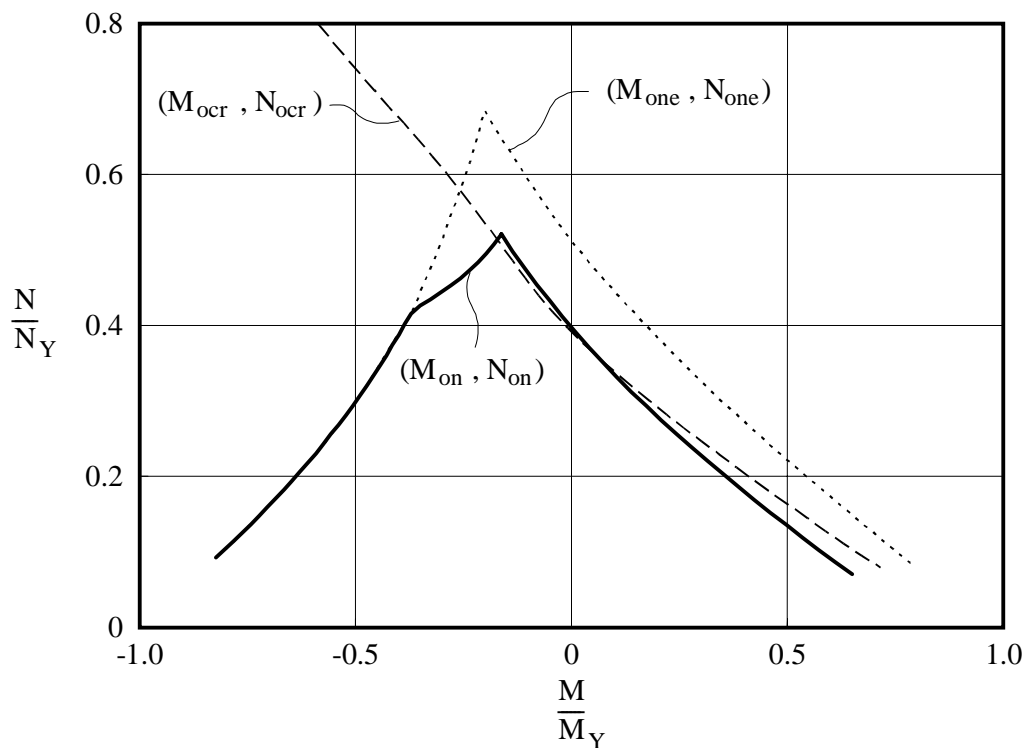


Figure 7: Strength vs eccentricity of loading for equal angles tested by Wilhoite et al. (1984), $L=1636$ mm.

(a) $M=Ne$ (b) $M=Ne_p$ **Figure 8: Interaction diagrams for equal angles tested by Popovic et al. (1999), $L=676$ mm.**

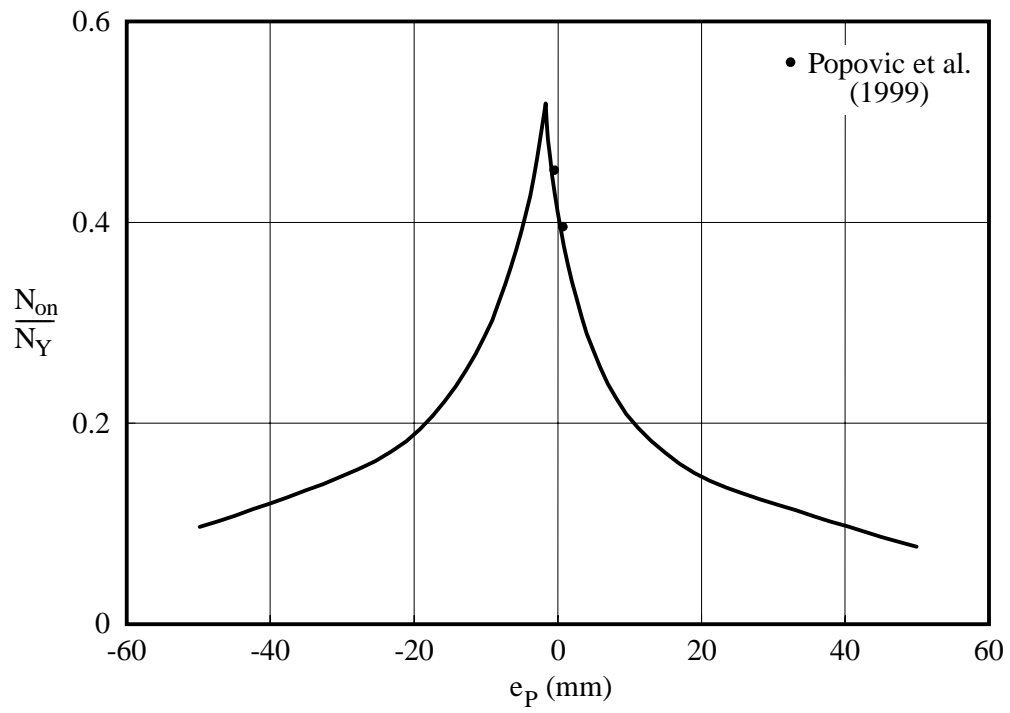


Figure 9: Strength vs eccentricity of loading for equal angles tested by Popovic et al. (1999), $L=676$ mm.

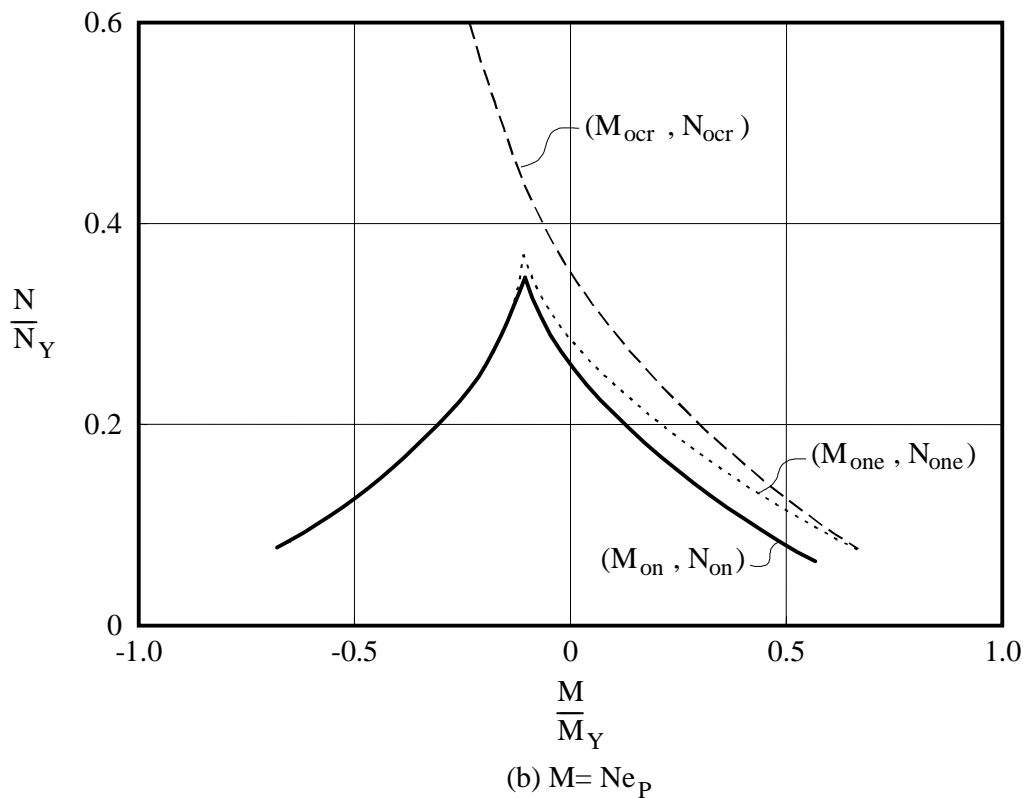
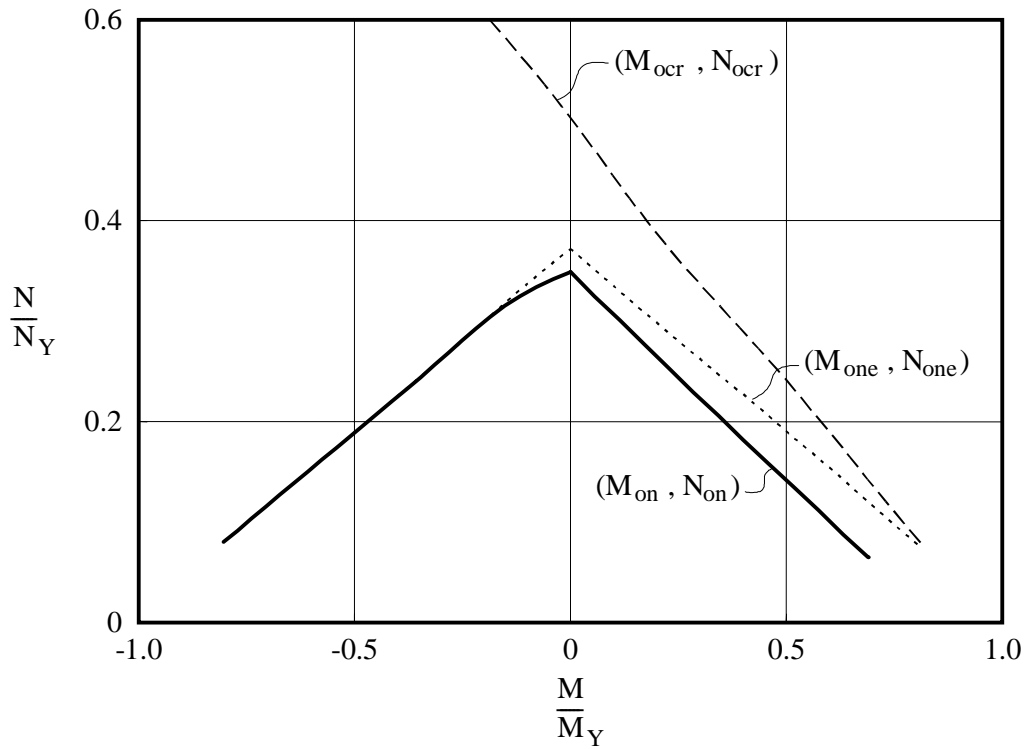


Figure 10: Interaction diagrams for equal angles tested by Popovic et al. (1999), $L=1088$ mm.

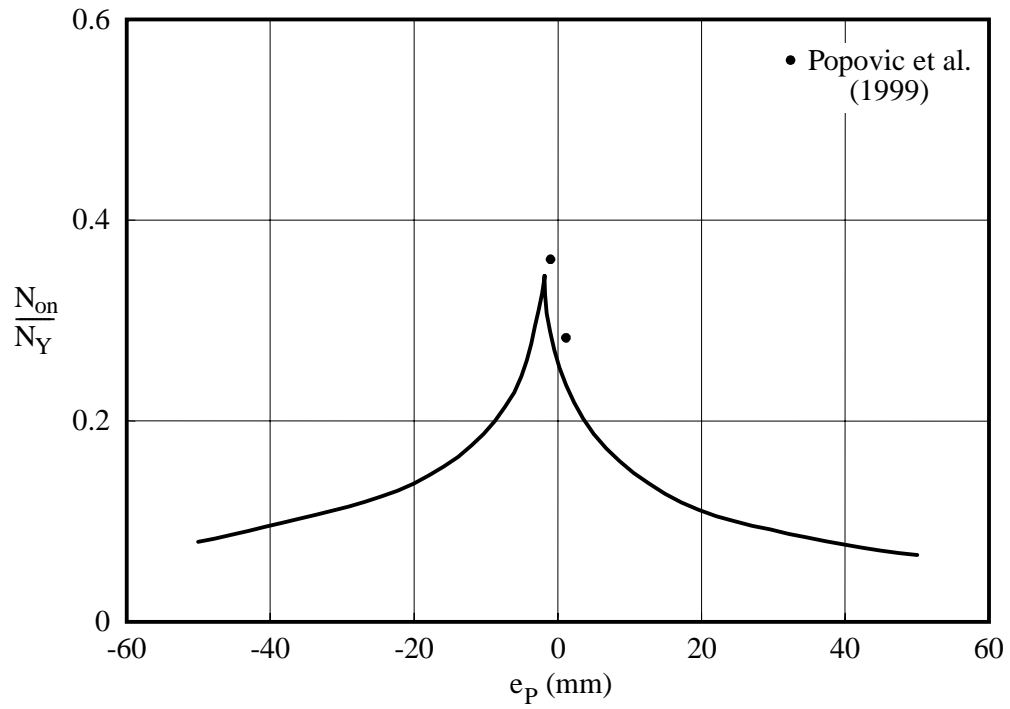


Figure 11: Strength vs eccentricity of loading for equal angles tested by Popovic et al. (1999), $L=1088$ mm.