Influence of Supply Chain Network Topology on the Evolution of Firm Strategies

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ABSTRACT: This study investigates the influence of the topological structure of a supply chain network (SCN) on the evolution of cooperative and defective strategies adopted by the individual firms. First, a range of topologies representative of SCNs was generated using a fitness-based network growth model, which enabled cross comparisons by parameterising the network topologies with the power law exponent of their respective degree distributions. Then, the inter-firm links in each SCN were considered as repeated strategic interactions and were modelled by the Prisoner’s Dilemma game to represent the self-interested nature of the individual firms. This model is considered an agent-based model, where the agents are bound to their local neighbourhood by the network topology. A novel strategy update rule was then introduced to mimic the behaviour of firms. In particular, the heterogeneously distributed nature of the firm rationality was considered when they update their strategies at the end of each game round. Additionally, the payoff comparison against the neighbours was modelled to be strategy specific as opposed to accumulated payoff comparison analysis adopted in past work. It was found that the SCN topology, the level of rationality of firms and the relative strategy payoff differences are all essential elements in the evolution of cooperation. In summary, a tipping point was found in terms of the power law exponent of the SCN degree distribution, for achieving the highest number of co-operators. When the connection distribution of an SCN is highly unbalanced (such as in hub and spoke topologies) or well balanced (such as in random topologies), more difficult it is to achieve higher levels of co-operation among the firms. It was concluded that the scale-free topologies provide the best balance of hubs firms and lesser connected firms. Therefore, scale-free topologies are capable of achieving the highest proportion of co-operators in the firm population compared to other network topologies.

KEY WORDS: supply chain network; game theory; repeated games; Prisoner’s Dilemma; network science

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1.0 Introduction

Modern SCNs are highly complex systems comprising of multiple entangled supply chains. Therefore, an SCN can be viewed as a structured population of supply chain firms, characterised by a specific topology based on the way they are interconnected with each other. Recent advances in network science has paved the way towards topological modelling of SCNs (Thadakamalla et al., 2004; Zhao et al., 2011a; Kim et al., 2015; Orenstein, 2016). In particular, there is growing research interest in the influence of the topological structure of a system on the evolution of strategies adopted by the entities, under repeated interactions. In early literature, authors have argued that the density of a network (Coleman, 1992) along with the way individual entities engage in repeated relational exchanges (Krackhardt and Hanso, 1993) determine the emergence of trust and social norms over time. Therefore, in contrast to the traditional buyer-supplier dyad view, the network-based topological view of SCNs offers a more realistic framework for analysing the evolutionary nature of supply chain relationships (Nair et al., 2009).

In an SCN, when firms adopt cooperative strategies, it indicates commitment to pursuing long-term relations with their partners (Li et al., 2013) and as a result can foster behaviours such as timely sharing of knowledge and information on inventory levels for collaborative planning, forecasting, and replenishment (Fliedner, 2003); cooperative advertising (SeyedEsfahani et al., 2011); and joint research and development on initiatives such as cost cutting and component redesign (Huang et al., 2002). Such cooperative behaviours increase the total benefits, such as the profits (in both short and long run) and shareholder value, for all firms in the SCN (Hendricks and Singhal, 2003; Chan and Chan, 2010).

Similar cooperative behaviours underpin the success of the traditional Japanese supply chain transaction practice known as ‘Keiretsu’ which seeks to nurture long term relationships between firms, based on trust and goodwill (Matous and Todo, 2015). Keiretsu relationships typically occur in asymmetrical relationships, where one organisation uses its dominant position to govern and maintain the relationships through close and stable business collaborations between its partners. Although Keiretsu represents a sharp contrast to the Western style arm’s length and contractual relationships between firms, it has been successfully implemented by many Japanese businesses. For example, Toyota has adopted Keiretsu practices where instead of abandoning suppliers when others offer lower prices, it
provides support by organising ‘study groups’ and dispatching engineers to train the suppliers to improve process efficiencies and reduce prices (Aoki and Lennerfors, 2013).

Although there are clear benefits to cooperative behaviour, firms do not always act cooperatively with their trading partners, mainly due to differing self-interests between the trading partners. Such ‘inter-firm rivalry’ (Park and Ungson, 2001) can lead firms to adopt defective strategies and behave opportunistically by not readily sharing information, skills and processes (Dyer and Nobeoka, 2000; Fawcett and Magnan, 2001). Under more extreme situations, firms could increase the prices at the times of product shortages (Nair et al., 2009). Therefore, failures in spreading cooperation in an SCN due to inter-firm rivalry, has significant monetary implications to all firms, as it leads to performance ‘glitches’ which could cripple the capability of the system to timely and cost-effectively supply the customer demands (Hendricks and Singhal, 2003, 2005).

Since the local decisions of individual entities in an SCN has global consequences, it is of practical importance to understand how different forms of interdependence structures (i.e. topologies) could influence the evolution of various strategies adopted by firms. Over the past two decades, networked game theory has been increasingly used to understand the constraints placed by the topological structure of a community on the cooperative and non-cooperative decision making processes of individuals (Santos et al., 2006; Kasthurirathna et al., 2015, Roman & Brede, 2017). Evolutionary game theory (Smith, 1973) provides a theoretical framework capable of addressing the issue of cooperation among self-interested and unrelated individuals. Under this framework, social dilemmas are formalised at the most basic level as two-person games, where each player can either choose to cooperate or to defect (Cimini and Sánchez, 2014). The Prisoner’s Dilemma (PD) game formalised by Axelrod (1984) symbolises a typical situation in which two parties may not cooperate even if it is in both their best interests to do so. In such situations, although mutual cooperation gives the best outcome for both players, the highest individual benefit is derived by defecting. Each player therefore has to assess the probability of the other defecting (i.e. the level of trust in the other player).

Previous work in the area of networked games have investigated how various strategies spread across a given network topology when players repeatedly interact with their local neighbourhood and update their strategies based on various ‘update rules’. In particular, the
evolution of strategies over multiple iterations has been studied to identify the evolutionarily stable strategy (ESS).

The relationship dynamics between individual firms which comprise an SCN are subject to a large number of variables and therefore such systems are difficult to model for the long term. However, past studies involving networked games have adopted simple yet intuitive strategy update rules for each player reflecting the temporal adjustments of behaviour of firms during their interactions with other firms in the SCN. These update rules conceptually capture how firms may change their purchasing/service level agreements or contract clauses, after each interaction, when dealing with other firms in the long run. Past studies have confirmed that evolution of strategies strongly depends on the update rule used. Therefore, it is important to adopt an update rule that sufficiently captures the expected behaviour of the agents. Broadly speaking, update rules specify how fast trust is gained or lost in response to the decisions made by the other player in the past.

In this work, we argue that the topology of a SCN influences the level of information available to individual firms and this in turn impacts the ability of the firms to maximise the level of utility they gain by adopting best strategy when interacting with others. A strategy can be considered as a discrete choice that an autonomous agent makes to maximize its payoff. The central hypothesis in discrete choice theory is that agents (be they humans or firms) maximise their utilities between a finite number of disjoint alternatives (Hensher et al., 2005). However, this is based on the assumption that they have sufficient abilities to carry out perfect optimisation in their choices (Huang et al., 2013). For example, the Nash Equilibrium (NE) concept in non-cooperative game theory assumes that all players in a system are fully rational and as such they adopt strategies to maximise their own utility in the absence of knowledge about the strategies of the other players. In a mixed strategy context, players would update their strategy choice probabilities in the light of the observed strategy choices of the other player(s).

In real-world settings, the behaviour of agents has been observed to deviate from those predicted by the NE (Haile et al., 2008). Indeed, the rationality of agents in the real world, are bounded by the level of information at hand, cognitive capacity and computational time available (Kasthurirathna and Piraveenan, 2015). Due to limitations in their ability to behave rationally, non-optimal decision making by agents has been ascribed to ‘bounded rationality’ (Simon, 1956).
In this study, by using each firm’s topological degree within the SCN as an indicator for its ability to behave rationally, a topologically distributed rationality model is formulated to represent the bounded rationality of firms in SCNs. This topologically distributed rationality consideration is then carried over to devise a novel strategy update rule which sufficiently mimics the behaviour of self-interested firms in an SCN. The Log Normal Fitness Attachment (LNFA) growth model (discussed in section 2.1.1) has enabled comparison of ESSs across network topologies in a more continuous way by parameterising network topologies based on the power law exponent of their respective degree distributions.

The remainder of this manuscript is organized as follows: Section 2 provides a background (including relevant literature) of the key concepts used in this study, Section 3 presents the details of the methodology in terms of simulation design, Section 4 presents the simulation results, Section 5 includes a discussion of the results and Section 6 concludes the manuscript.

2.0 Background

This section provides a brief overview of the key concepts used in this study.

2.1 Network modelling of supply chains

Due to the increasingly complex and interconnected nature of the global businesses, recent research has focussed on modelling SCNs as complex systems using network science concepts. Following on from the influential work published by Thadakamalla and colleagues in 2004 (Thadakamalla et al., 2004), which utilised a network science lens to investigate the robustness of various SCN topologies, a large number of theoretical research papers have appeared in this area (Surana et al., 2005; Borgatti and Li, 2009; Xuan et al., 2011; Zhao et al., 2011; Wen and Guo, 2012; Yi et al., 2013; Li, 2014; Mari et al., 2015; Kim et al., 2015). Most of these studies have theoretically formulated plausible and generalizable growth mechanisms underlying the firm partnering process in SCN formation. Subsequently, the network topologies generated based on various growth models have been studied in depth for their topological characteristics, such as robustness and efficiency.

Complementing the early theoretical work, recent data driven studies have revealed a number of interesting general topological features of SCNs in various industries. It is evident from the data driven studies in literature that SCNs tend to have node degree distributions which follow power-law (Brintrup et al., 2015; Perera et al., 2017). The degree distribution, $P_k$ of such networks is approximated with power-law as follows;
\[ P_k \sim k^{-\gamma} \]  

(1)

where \( k \) is the degree of the node and \( \gamma \) is the power-law exponent.

Although in literature, the networks whose degree distributions follow power-law is generally referred to as scale-free networks, in this study we reserve the term ‘scale-free’ to identify networks within a specific range of power law exponents. In particular, we refer to networks whose power-law exponent lies between 2 and 3 as scale-free networks. When the power law exponent is below 2 or above 3, such networks are referred to as hub-and spoke and random networks, respectively, as outlined in Figure 1.

In many real world networks, it has been found that the power-law exponent of the degree distribution lies between 2 and 3 (Barabasi, 2014). The growth mechanisms underlying such scale-free networks have been related to some form of preferential attachment, most notably the Barabasi-Albert (BA) model, which is known to generate scale-free networks with \( \gamma = 3 \).

### 2.1.1 Fitness based network growth models

Several growth models have been proposed in the literature for generating scale-free networks by fitness-based attachment (Caldarelli et al., 2002; Smolyarenko, 2014; Bell et al., 2017). For instance, Ghadge et al (2010) used log-normally distributed fitness distributions to simulate a range of power-law networks by varying the shape parameter \( \sigma \) of the log-normal fitness distribution. When \( \sigma \) is zero, all nodes have the same fitness and therefore at the time a new node joins the network, it chooses an existing node as a neighbour with equal probability, replicating the random graph model with an exponential degree distribution. When \( \sigma \) is increased beyond a certain threshold, a very few nodes will contain very high fitness while the overwhelming majority of nodes have very low levels of fitness. As a result, the majority of new connections are made to a single or very few nodes with high fitness. The resulting network therefore resembles a monopolistic or “winner-takes-all” scenario. Between the above two extremes lies a spectrum of power-law networks (Nguyen and Tran, 2012).
2.2 **Evolutionary networked game theory**

Game theory has historically been used as an effective tool to model complex systems that consist of multiple self-interested entities under various decision making scenarios (Simon, 1959). Evolutionary game theory, in particular, is concerned with how contending strategies evolve over time in a population of players. The concept of evolutionary stability in evolutionary game theory is equivalent to the concept of NE in static game theory. Since NE is considered to be a static equilibrium, evolutionary stability represents a dynamic equilibrium of a strategy over time. A strategy is considered to be evolutionarily stable if it is capable of dominating over any other strategy.

Over the past two decades, networked evolutionary game theory has been increasingly used to understand the constraints placed by the topological structure of a community on the decision making process of individuals who repeatedly interact with each other. In particular, iterated PD game (which is the static PD game iterated over multiple time steps over a population of players) has received much attention. In such iterated game scenarios, each player would play the PD game with its neighbours in each time step and update their strategies by inspecting the past performance of their neighbours.

### 2.2.1 Prisoners dilemma

The PD is a fundamental paradigm in game theory that demonstrates why two parties may not cooperate even if it is in both their best interests to do so. In this particular game, which was popularised by Albert W. Tucker (Poundstone, 1992), the only objective for each party is maximising their own payoff, without any concern for the other parties’ payoff. In this situation, each party is tempted to unilaterally defect when they believe that the other

<table>
<thead>
<tr>
<th>Scale Parameter of the Lognormal Distribution</th>
<th>$\sigma = 5$</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Law Exponent</td>
<td>$\gamma &lt; 2$</td>
<td>$2 \leq \gamma &lt; 3$</td>
<td>$\gamma &gt; 3$</td>
</tr>
<tr>
<td>Description of the Topology</td>
<td>Hub and spoke topology</td>
<td>Scale-free topology</td>
<td>Random topology</td>
</tr>
</tbody>
</table>

Figure 1. Network topology under various regimes of power-law exponent

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6
party will collaborate. Unilateral defection (assuming the other party cooperates), always pays more than mutual collaboration or mutual defection (Dawes, 1980). Therefore, the unique NE for this game occurs when both parties defect, even though the benefit derived by each party would be greater if they both cooperate.

The PD is a two-player, non-zero sum game that can be applied to model the conflict between individual and collective interests in a myriad of domains, in particular in economical situations such as inter-firm relationships (Zagare, 1984; Li et al., 2013). PD scenarios do indeed occur often in buyer-seller relationships within SCNs, since firms prefer maximizing their own profit over the profit of the SCN (Viswanathan and Piplani, 2001). The NE arises because neither player trusts the other enough to cooperate. To try to avoid this, measures are required that build trust by sufficiently incentivising cooperation (or penalising defection).

The following section provides an overview of various supply chain scenarios that are analogous to PD.

- Between suppliers and manufacturers: Terwiesch et al. (2005) present the difficulties in contract establishment, between a supplier and a buyer, based on shared forecasts. This scenario, which resembles a PD, is presented in Figure 2 under the payoff matrix (a).
- Between manufacturers and retailers: Allen (2017) presents a PD type scenario between manufacturers and retailers in the context of sharing consumer data. The payoff matrix for this scenario is presented in Figure 2 (b).
- Between manufacturers and distributors: Günther et al. (2008) discuss the advantages of adopting RFID (Radio Frequency Identification) tags to improve material tracking in warehouses. In light of the positive outlook of this technology, the authors present a PD type scenario as the key reason why RFID is not widespread in manufacturing sector than it is today. This scenario is presented in Figure 2 under the payoff matrix (c).
- Between distributors and retailers: Gao et al. (2006) discuss vertical cooperation (cooperation between upstream and downstream members) in SCNs. A PD type scenario is presented where a manufacturer and a retailer have to negotiate the optimal order quantity and delivery frequency. The payoff matrix for this scenario is presented in Figure 2 (d).
### Figure 2: Scenarios analogous to Prisoner’s Dilemma game, at various stages of a supply chain

#### (a) SUPPLIER

<table>
<thead>
<tr>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MANUFACTURER</strong> Cooperate</td>
<td>Manufacturer provides a genuine forecast and the supplier trusts and meets the forecast, thus improving the reliability of the supply chain as a whole.</td>
</tr>
<tr>
<td><strong>MANUFACTURER</strong> Defect</td>
<td>Manufacturer provides an intentionally inflated forecast, however the supplier waits until a firm purchase order is submitted (supplier incurs cost of delay).</td>
</tr>
<tr>
<td><strong>Retailer</strong> Cooperate</td>
<td>Manufacturer provides an intentionally inflated forecast, the supplier trusts the inflated forecast (supplier faces the cost of inventory and cancellation).</td>
</tr>
<tr>
<td><strong>Retailer</strong> Defect</td>
<td>Manufacturer provides an intentionally inflated forecast, supplier discounts the forecasts and waits until firm purchase order is submitted. The supply chain faces inefficiencies.</td>
</tr>
</tbody>
</table>

#### (b) MANUFACTURER

<table>
<thead>
<tr>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Retailer</strong> Cooperate</td>
<td>Both parties collaborate by sharing the consumer data which provides them an end-to-end view of the value chain. This reduces the overall supply chain system cost by rooting out operational inefficiencies.</td>
</tr>
<tr>
<td><strong>Retailer</strong> Defect</td>
<td>Manufacturer establishes a direct relationship with consumers by forming consortia. As a result, retailers lose sales.</td>
</tr>
<tr>
<td><strong>Retailer</strong> Cooperate</td>
<td>Retailer does not share consumer data and continues develops valuable consumer insights. They market in store more effectively than the manufacturer and offer consumers integrated solution bundles. Manufacturer loses market power.</td>
</tr>
<tr>
<td><strong>Retailer</strong> Defect</td>
<td>Both retailers and manufacturers vertically integrate and continue to compete against each other. In order to gather more capabilities, both parties spend significant amounts. The market now becomes twice as competitive.</td>
</tr>
</tbody>
</table>
Figure 2 (continued): Scenarios analogous to Prisoner’s Dilemma game, at various stages of a supply chain

(c) **MANUFACTURER**

<table>
<thead>
<tr>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturer does not allocate any staff time for the development of RFID and expects the distributor to commit and develop the RFID.</td>
</tr>
<tr>
<td></td>
<td>Distributor does not allocate any staff time for the development of RFID and expects the manufacturer to commit and develop the RFID.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Distributor</th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both parties commit to the development and implementation of RFID (Radio-frequency identification) to improve the productivity of the supply chain as a whole.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neither party is willing to commit staff time for the development and implementation of RFID. The overall supply chain continues to face inefficiencies.</td>
<td></td>
</tr>
</tbody>
</table>

(d) **DISTRIBUTOR**

<table>
<thead>
<tr>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Retailer demands high frequency of smaller orders (retailer avoids inventory costs but the distributor cannot achieve economies of scale through batch delivery).</td>
</tr>
<tr>
<td></td>
<td>Neither party is willing to compromise. As a result, the supply chain may not be able to meet the customer demand.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both parties negotiate and establish a medium order frequency, so it benefits the entire supply chain as a whole.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Distributor prefers low frequency of larger orders (to achieve economies of scale through batch delivery but the retailer incurs inventory costs).</td>
<td></td>
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</table>
Figure 3 shows the typical four parameter payoff matrix representation of the PD game where C and D denote corporate and detect strategies and the payoffs satisfy the inequality $T > R > P > S$.

![Figure 3: Prisoner’s Dilemma game payoff matrix](image)

However, following standard practice, in this study, the single parameter representation of the PD game has been used (Nowak and May, 1992; Szabo and Toke, 1998), as illustrated in Figure 4. Here the $\beta$ parameter represents the temptation to defection and is usually constrained between $1 < \beta \leq 2$. In particular, the larger the value of $\beta$, the more incentive there is from defection.

![Figure 4: The single parameter representation of the Prisoner’s Dilemma game payoff matrix](image)
In Figure 4, C and D denote cooperate and defect strategy adopted by each player. $\pi_{nm}^i$ represents the payoff to player i, when the first player adopts strategy n and the second player adopts strategy m.

### 2.2.2 Strategy update rules

In order to represent the imitative nature of agents constrained within a network topology, various strategy update rules have been adopted in the literature, where at the end of each iteration of a game (i.e. an interaction), a given player $i$ adopts a new strategy by copying the strategy of another player $j$ from its local neighbourhood $K_i$, with a probability $p(s_{i}^{t+1} \leftarrow s_{j}^{t})$. Table 1 summarises some key strategy update rules used in literature (Cimini and Sánchez, 2014).

**Table 1: Summary of strategy update rules used in literature**

<table>
<thead>
<tr>
<th>Update Rule</th>
<th>Descriptions</th>
</tr>
</thead>
</table>
| **Proportional Imitation (Helbing, 1992)** | $p(s_{i}^{t+1} \leftarrow s_{j}^{t}) = \begin{cases} \frac{\pi_{j}^{t} - \pi_{i}^{t}}{\Phi_{q}} & \text{if } \pi_{j}^{t} > \pi_{i}^{t} \\ 0 & \text{otherwise} \end{cases}$
| where $\Phi_{q} = \max(K_{i}, K_{j})(\max(R,T) - \min(P,S))$ so that $p(.) \in [0,1]$ |
| **Fermi rule (Szabó and Toke, 1998)** | $p(s_{i}^{t+1} \leftarrow s_{j}^{t}) = \frac{1}{1 + e^{-\mu(\pi_{j}^{t} - \pi_{i}^{t})}}$ |
| A neighbour $j$ is randomly chosen from the local neighbourhood $K_i$ and $i$ imitates $j$ based on a probability proportional to their normalised accumulated payoff differences in the previous round of the game. Note that $R, T, P, S$ represent the payoff values in the typical PD game (as per Figure 3). |
| **Death-Birth rule, inspired by Moran dynamics (Moran, 1962)** | $p(s_{i}^{t+1} \leftarrow s_{j}^{t}) = \frac{\pi_{j}^{t} - \varphi}{\sum_{j=0}^{N} (\pi_{j}^{t} - \varphi)}$ |
| where $N_{i}$ is the set of players including $i$ and its neighbours and $\varphi = \max_{j \in N} K_{j} \min(0,S)$ so that $p(.) \in [0,1]$ |
Player $i$ imitates the strategy of one of its neighbours or itself with a probability proportional to the accumulated payoffs in the previous round of the game. Note that $S$ represents a payoff value in the PD game (as per Figure 3).

<table>
<thead>
<tr>
<th>Unconditional Imitation or “Imitate the Best” (Nowak and May, 1992)</th>
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<tbody>
<tr>
<td>$p(s_i^{t+1} \leftarrow s_j^t) = 1$ if $\pi_j^t = \max_{K \in N_i} \pi_{K}^t$</td>
</tr>
<tr>
<td>where $N_i$ is the set of players including $i$ and its neighbours</td>
</tr>
<tr>
<td>Each player $i$ deterministically imitates the best off neighbour $j$ with the largest accumulated payoffs from the previous rounds of the game.</td>
</tr>
</tbody>
</table>

In this study, particular attention is given to the Fermi Rule (see Table 1) since in the case of two behavioural strategies only (such as in the PD game considered here), it has been used to model the evolution of corporation in complex networks (Santos et al., 2006).

### 2.2.3 Rationality of firms

NE in a non-cooperative decision making environment is a state (set of decisions) where no player can improve his or her payoff by a unilateral change of strategy. However, it has been observed that in many real-world situations, the equilibrium states of the players deviate substantially from those predicted by the NE (Haile et al., 2008). A key reason for a deviation is non-perfect or bounded rationality of the players involved. Indeed in the real world, the players involved will not be perfectly rational due to the limitations in information availability, computational time and cognitive capacity (Gigerenzer and Selten, 2002). Since these limitations vary from player to player, one could expect players in a system to respond heterogeneously, where at one extreme a player would make user optimal decisions while at the other extreme a player would make seemingly arbitrary decisions.

Inspired by the work of Kasthurirathna and Piraveenan (2015), it is argued here that there could be a correlation between the quantity of business interactions of a particular firm (indicated by the topological node degree in the SCN) and the level of its rationality. Indeed this argument has also been made and validated in social science fields such as social learning theory (Bandura, 2001) and social cognitive theory (Dunbar, 1998), where a player with a relatively high amount of social interactions is deemed to have access to more up-to-date information, compared to a player with a lower amount of social interactions. Similarly, it can be argued that in SCNs, firms with more interactions with other firms tend to behave more rationally, due to their superior market access and intelligence.
Early studies such as Burkhardt and Brass (1990) and Ibarra (1993) note that a firm’s power and influence is derived from its structural position in its surrounding network. Some researchers have also associated the firm’s network position with issues such as innovation adoption (Burt, 1980; Ibarra, 1993), brokering (Pollock et al., 2004) and creation of alliances (Gulati, 1999). Therefore, it can be reasonably argued that from an SCN perspective, the relative position of individual firms with respect to one another influences both strategy and behaviour (Borgatti and Li, 2009). Kim et al. (2011) notes that firms with higher node degrees, in terms of contractual relationships in SCNs, also influence the operational decisions or strategic behaviour of other firms in the SCN more. Furthermore, Uzzi (1996) stipulates that how well a firm is linked to its business network determines its level of access to the benefits (such as information) circulating in the network which correlates with competitive advantage. Accordingly, it can be reasonably assumed that the rationality of a firm is positively correlated with the amount of inter-firm interactions they are involved in.

Furthermore, the level of interactions would also depend on its ‘weight’ for various attributes, such as time spent on the relationship, amount of materials/information exchanged and so on, between each pair of firms. Indeed the correlation between the rationality of a firm and its amount of interactions with other firms could be linear or non-linear. Therefore, this relationship has been modelled through a generic function $f$, which uses the weighted degree of a node (or simply the degree, if all link weights are considered to be equal) as an input, as follows;

$$\lambda_i = r \cdot f \left( \sum_{j=1}^{n} w_{ij} \right)$$

(1)

In the above formulation, $\lambda_i$ represents the rationality of firm $i$ and $r$ can be considered the ‘SCN rationality parameter’ which is a common system wide constant which controls the responsiveness of rationality to degree. $w_{ij}$ is the weight of the link connecting firm $i$ with each neighbour $j$, and $n$ is the number of neighbours that firm $i$ has.

In this study, the function $f$ has been modelled as a simple linear function for simplicity and computational efficiency. Also, this function facilitates the $[0: \infty)$ range of
possible rationality values of \( \lambda_i \). Empirical data from experiments would be required to specify a more accurate functional form for \( f \).

In the above model, a firm will behave completely randomly if the SCN rationality parameter, \( r \) is set to zero or when the firm is fully disconnected from the SCN (i.e. the degree is zero). At the other extreme, as \( r \) approaches infinity or when the degree of the firm is extremely large, the firm will display rational behaviour as predicted by the NE.

### 3.0 Methodology

In the methodology adopted here, several revisions have been made to the ‘general Fermi rule’. In particular, this ‘revised Fermi rule’ acknowledges that picking a neighbour at the end of each game round (for copying a strategy) and subsequently imitating the neighbour’s strategy, are two distinct operations, each involving the rationality of the central player.

- In the general Fermi rule, the players calculate their accumulated payoffs at the end of each game round. In the revised Fermi rule adopted in this study, the accumulated payoff calculation has been made strategy specific (since when deciding to adopt a strategy, a player would be interested in how well that particular strategy has paid off in the past and this information is not captured when comparing accumulated payoffs). In particular, in the revised Fermi rule, the players calculate the average payoff they have received so far, by using the current strategy (i.e. the strategy adopted at time \( t \)).
- In the general Fermi rule, a neighbour for copying the strategy is selected by players in a random fashion. In the revised Fermi rule, each player selects a neighbour based on a probability proportional to the product of that player’s rationality and the neighbour’s average payoff based on the current strategy. This represents the fact that highly rational (more connected) firms are more likely to select a better off neighbour.
- In the general Fermi rule, the parameter \( \mu \) is set as the selection intensity and is constant for all players. As \( \mu \to \infty \), player \( i \) adopts the strategy of its better off neighbour \( j \) deterministically. However, for any finite value of \( \mu \), there exists a probability that \( i \) copies the strategy of its neighbour \( j \) who gained less. In contrast, in
the revised Fermi rule, the strategy update probability is calculated as the product of each player’s rationality and the difference between the average payoff (with the current strategy) of itself and the average payoff (with the current strategy) of the selected neighbour. This method enables capturing the heterogeneity across players in relation to their rationality levels (i.e. more rational players will adopt the strategy of their better off neighbour with a higher probability).

The following steps were followed in the simulation experiments carried out.

**Step 1: Generating an ensemble of network topologies with varying power law exponents**

The spectrum of network topologies studied here (from hub and spoke to random to scale free regimes) have been parameterised by the power law exponents ($\gamma$) of the degree distributions for comparison purposes. Therefore, an ensemble of network topologies representative of SCNs were generated with varying power law exponents, using the LNFA method described in Section 2.1.1. In particular, by varying the shape parameter ($\sigma$) of the lognormal distribution, 100 networks (each with 1,000 nodes) were generated at each of the following power law exponents: $\gamma = 1.5$, $\gamma = 2$, $\gamma = 2.5$, $\gamma = 3$, $\gamma = 4$ and $\gamma = 5$.

**Step 2: Modelling rationality of firms**

Using the generic function presented in Eq. 1, the rationality of each firm was modelled as a monotonically increasing linear function of its degree (with link weights set to unity), with the SCN rationality parameter $r$ (which controls the responsiveness of rationality to degree) set to 0.1, 1.0 and 10.0 in separate experiments.

**Step 3: Allocate Co-operators and Defectors**

At the beginning of each experiment, each strategy $C$ (cooperate) or $D$ (defect) was uniformly distributed in the network, i.e., the probability of each strategy is 50%. Note that previous studies have confirmed that this arrangement has negligible impacts on the evolutionary outcome, suggesting the uniqueness of the outcome.

**Step 4: Play the PD game, then calculate and assign the payoffs**

The PD game is played once by each player with the neighbours (the game parameter $\beta$ of the PD game was tested at three values in separate experiments: $\beta = 1.2$, $\beta = 1.5$ and $\beta =$
Then, calculate and assign the average payoffs for each agent based on their current strategy as follows;

\[
\langle \pi_i \rangle = \sum_{j=K_i} \frac{\pi_i(i,j)}{n_i}
\]  

(2)

Here, \(\langle \pi_i \rangle\) is the average payoff player \(i\) received with their current strategy (i.e. strategy at time \(t\)), \(\pi_i(i,j)\) is the payoff of player \(i\) received against player \(j\) where \(K_i\) is the set of \(i\)'s neighbours and \(n_i\) is the number of iterations (i.e. game rounds) played by player \(i\) with the current strategy at time \(t\).

**Step 5: Select a neighbour (from whom the strategy will be updated)**

For each node, select a neighbour based on a probability proportional to the product of the node’s rationality and that neighbour’s average payoff based on the current strategy.

\[
\tau_{ij} \propto \lambda_i \cdot \langle \pi_j \rangle, \text{where } j \in k_i
\]

Where \(\tau_{ij}\) is the probability of selection for a neighbour of node \(i\), \(\lambda_i\) is the rationality of node \(i\) (as calculated based on its degree using Eq. 1), \(\langle \pi_j \rangle\) is the average payoff neighbour \(j\) received with their current strategy (i.e. strategy at time \(t\)).

**Step 6: Update the strategy**

Update the strategy of player \(i\), using a probability calculated based on the difference between the average payoff of itself and the average payoff of the neighbour selected (from the above step) based on the following Fermi rule – note that the rationality of player \(i\) is also considered here (in particular, the system wide parameter \(\mu\) in the Fermi rule as shown in Table 5.1, is replaced with the player specific rationality parameter \(\lambda_i\)).

\[
p(s_i^{t+1} \leftarrow s_j^t) = \frac{1}{1 + \exp(-\lambda_i(\langle \pi_j \rangle - \langle \pi_i \rangle))}
\]  

(3)

**Step 7: Iteratively play the game**

Repeat steps 4 to 6 above for \(N\) iterations. In networks, the level of cooperation is generally measured by cooperation frequency, \(P_c\) (i.e. proportion of co-operators in the
Therefore, in this study, the fluctuations in cooperation frequency has been recorded in each experiment, at each time step (i.e. end of each game round). In the simulation experiments carried out in this study, \( N \) was set at 1,000.

4.0 Results

Figures 5, 6 and 7 illustrate the proportion of co-operators recorded at each time step at three different values of PD game parameter \((\beta)\), for a range of network topologies with varying power law exponents \((\gamma)\), when the network rationality parameter \((\gamma)\) is set to 0.1. Each value presented in these plots has been obtained by 100 averages.

Figure 8 illustrates the proportion of co-operators \((P_c)\) recorded at convergence (for the cases where convergence was not achieved, the average of co-operators between iterations 500 and 1,000 has been considered) at three different values of SCN rationality parameter \((r)\), for a range of network topologies with varying power law exponents \((\gamma)\) at three different values of PD game parameter \((\beta)\).
Figure 5: Proportion of co-operators recorded across iterations for network topologies comprising degree distributions with varying power law exponents ($\gamma$), for PD game parameter, $\beta$ at 1.2, 1.5 and 1.8 with SCN rationality parameter, $r = 0.1$
Figure 6: Proportion of co-operators recorded across iterations for network topologies comprising degree distributions with varying power law exponents ($\gamma$), for PD game parameter, $\beta$ at 1.2, 1.5 and 1.8 with SCN rationality parameter, $r = 1.0$
Figure 7: Proportion of co-operators recorded across iterations for network topologies comprising degree distributions with varying power law exponents ($\gamma$), for PD game parameter, $\beta$ at 1.2, 1.5 and 1.8 with SCN rationality parameter, $r = 10$
Figure 8: Proportion of co-operators recorded at convergence at various SCN rationality parameter ($r$) values for networks with various power law exponents ($\gamma$), for PD game parameter, $\beta$ at 1.2, 1.5 and 1.8
5.0 Discussion

From our results, it is evident that there exists a strong connection between the topology of the SCN (i.e. the manner in which firms are interconnected with each other) and evolution of cooperation across the firm population comprising the SCN. Note that all experiments were carried out with an initial random distribution of 50% co-operators and 50% defectors. Past studies have confirmed that the initial distribution of strategies have negligible impact on the long run evolutionary outcome (Santos et al., 2006).

The proposed revised Fermi strategy update rule works as follows; (1) the players calculate the average payoff they have received so far, by using the current strategy, (2) each player selects a neighbour based on a probability proportional to the product of that player’s rationality and the neighbour’s average payoff based on the current strategy, and (3) the strategy update probability is calculated as the product of each player’s rationality and the difference between the average payoff (with the current strategy) of itself and the average payoff (with the current strategy) of the selected neighbour.

First, it is worth noting that the power law exponent ($\gamma$) of each network provides information about the heterogeneity of the degree distribution – in particular, $\gamma<2$ represents hub and spoke topologies characterised by significantly large hubs, while $2<\gamma<3$ indicates scale free topologies, again characterised by numerous moderate sized hubs. As $\gamma$ is increased beyond 3, the topology of the network becomes indistinguishable from a random network with no distinct hubs (i.e. more homogenous degree distribution). Refer to Figure 1 for a graphical illustration of the above identified regimes of network topologies.

When the SCN topologies are characterised predominantly by the presence of hubs, it implies the presence of firms with a disproportionately high number of connections with other firms. These hub firms generally receive higher payoffs than average firms, since they play more instances of the game and also they are capable of being more rational (i.e. better informed from their experiences with multiple interactions) so that they are capable of adopting the strategy, which gives higher payoffs, in each game.

From a strategy dynamics perspective, the strategy of the hubs will generally remain stable (i.e. they will rarely copy the strategy of their less-earning neighbours). This phenomenon is particularly evident in hub and spoke systems, which include significantly large hubs compared to other topologies. Consequently, the neighbours will ultimately copy
the strategy of the hubs (who have a clear systematic payoff advantage), resulting in pockets of defectors spread around hubs (Santos et al., 2006).

The above phenomenon cannot be expected from networks lacking hubs (i.e. those topologies with $\gamma>3$). However, the results indicate that in some instances, the network topologies with $\gamma>3$ are also pulled towards the defective state (i.e. $P_c<0.5$) across time steps. The reason for this is two-fold; (1) the increase in PD game parameter, $\beta$ will encourage more firms to defect as it represents the temptation to defect (i.e. how much more a firm will receive against the other firm, by defecting) and, (2) the increase in SCN rationality parameter, $r$ which controls the responsiveness of rationality to degree – with a sufficiently large $r$, even the lesser connected firms are likely to choose the rational strategy (i.e. defection) in the PD game.

From the results, it can be seen that as $\beta$ is increased (regardless of the SCN rationality parameter, $r$), the hub and spoke topologies (i.e. topologies with lower $\gamma$) do not achieve full convergence and are consistently pulled (one by one) towards fully defective state (with $\gamma=1.5$ topology being closest to majority defective state across all scenarios). The lack of evolutionary stability of these topologies is due to the mistakes made by non-hubs, since for any finite value of $\lambda$ in the proposed revised Fermi rule, there exists a probability that the player copies the strategy of one of its neighbours who gained less.

Similarly, investigating the plots across Figure 5, 6 and 7, it is evident that as the PD game parameter, $\beta$ is increased, all network topologies (regardless of $\gamma$) are pulled closer towards full defection state (since defection is the rational strategy in the PD game).

To complement the above discussion, let’s focus on the results presented in Figure 8. It can be clearly seen that the proportion of co-operators at convergence reduces drastically as the SCN rationality parameter, $r$ is increased. Also, it is clear that when the SCN rationality ($r$) and/or temptation to defect ($\beta$) is low, hub and spoke topologies can also achieve majority cooperation. As the SCN rationality ($r$) and/or temptation to defect ($\beta$) is increased, the topologies which lack hubs are favourable in terms of achieving majority co-operators in the system. The general trend across these plots (in Figure 8) indicates a tipping point for achieving high proportion of co-operators in the firm population under various topologies. In general, the scale free regime ($2<\gamma<3$) achieves the highest level of cooperation followed by the random ($\gamma>3$) and the hub and spoke regimes ($\gamma<2$). This result can be explained by the
hub structure present in each of the above topologies. The hub and spoke topologies achieve the lowest cooperators compared to other topologies, since they include significantly large hub firms. From a firm level perspective, a hub is likely to defect their partners in PD interactions (since defection is the rational strategy and the hubs are capable of being more rational). A defector hub is best surrounded by cooperators since this type of interactions will maximize the defector hub’s payoffs. Therefore, a defector hub can exploit their neighbors. In doing so, the number of cooperators in the neighborhood of the defector hub will reduce in subsequent interactions (i.e. iterations of the PD game). This results in higher levels of defectors at convergence for hub and spoke topologies compared to other topologies. This result explains why Keiretsu (the traditional Japanese supply chain transaction practice) has been proven successful in many practical situations in Japan. Keiretsu relationships typically occur in asymmetrical relationships, where one organisation uses its significantly more powerful position to govern and maintain the relationships through close and stable business collaborations between its partners. By nurturing long term relationships between firms, based on trust and goodwill, cooperation easily spreads across all firms in the SCN.

Compared to hub and spoke topologies, the scale free topologies include moderate sized hubs with a much less distinct payoff advantage against their neighbors. When the payoff of these moderate sized hubs become comparable to that of their cooperating neighbors, strategy invasion can occur. Once a hub adopts a cooperative strategy, it will maximize the number of cooperating neighbors, since when both players cooperating leads to the highest payoff (for both players) under the PD game. Therefore, scale-free topologies are capable of achieving the highest proportion of co-operators in the firm population compared to other topologies. At the opposite end, the random topologies do not include any hubs. Therefore, there would be no firms in the system with clear payoff advantages. This leads to coexistence of firms with defective and co-operative strategies. However, since co-operation by both players achieves the highest payoff for both players under the PD game, more firms adopt co-operation than defection, leading to the presence of a higher proportion of co-operators in the population compared to the hub and spoke topology.
6.0 Conclusions

This paper presented a simulation study which evaluates the influence of the SCN topology in the evolution of cooperation across firms. By considering the inter-firm links as strategic interactions modelled as PD games, a range of topologies representative of SCNs were tested and compared by parameterising the topologies to the power law exponent of their respective degree distributions. Additionally, the general Fermi rule used in literature was revised in the simulations to realistically represent the firm level behaviours in an SCN. In particular, the proposed revised Fermi rule takes into account the heterogeneity in the rationality levels across the firms in the SCN and uses strategy specific payoff comparisons when selecting neighbours and updating the strategy at the end of each game round.

Based on the results, it is evident that the firm population structure within an SCN (i.e. the SCN topology), the level of rationality of firms and the payoff differences (indicated by the game parameter of the PD game) are all essential elements in evolution of cooperation.

In summary, a tipping point was found in terms of the power law exponent of the SCN degree distribution, for achieving the highest number of co-operators. The hub and spoke topologies (represented by lower power law exponents) are not able to achieve majority co-operators due to hub firms implementing the more rational strategy (in the PD game, this is defection) and maintaining this strategy across time periods and spreading it locally within their neighbourhood. At the opposite end, when the hub are lacking in the SCN (random topologies characterised by higher power law exponents), firms with clear payoff advantages cannot be expected. This leads to coexistence of firms with defective and co-operative strategies. However, since co-operation by both players achieves the highest payoff for both players under the PD game, more firms adopt co-operation than defection, leading to the presence of a higher proportion of co-operators in the population compared to the hub and spoke topology. The highest proportion of co-operators is achieved by the SCNs characterised by scale free topologies (with power law exponent in the range of 2-3). These systems include moderate sized hubs with a much less distinct payoff advantage against their neighbors (compared to hub and spoke topologies). When the payoff of these moderate sized hubs become comparable to that of their cooperating neighbors, strategy invasion can occur. Once a moderate sized hub adopts a cooperative strategy, it will maximize the number of cooperating neighbors, since when both players cooperate, it leads to the highest payoff (for
both players) under the PD game. Therefore, scale-free topologies are capable of achieving the highest proportion of co-operators in the firm population compared to other topologies.

In all cases, it was found that the number of co-operator firms are reduced when; (1) the PD game parameter ($\beta$) is increased (which tempts the firms to defect since it enables achieving higher payoffs than their partners); and (2) the SCN rationality parameter, $r$ is increased (this controls the responsiveness of rationality to degree and with a sufficient large $r$, even the lesser connected firms are likely to choose the rational strategy of defection in the PD game).

In this study, rationality of each firm, within each SCN, has been modelled as a monotonically increasing linear function of its degree. Future studies could estimate a more accurate fit for this generic function. Additionally, more qualitative research is required to understand the behavioural basis underlying the strategy adoption by firms, so that accurate update rules can be developed for various contexts.
7.0 References