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**Buckling and Torsion of Steel Angle
Section Beams**

Research Report No R833

N S Trahair BSc BE MEngSc PhD DEng

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Abstract:

Although steel single angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion, their behaviour may be extremely complicated, and the accurate prediction of their strengths very difficult. Further, many design codes do not have any design rules for torsion, while some recommendations are unnecessarily conservative, or are of limited application, or fail to consider some effects which are thought to be important.

This paper is one of a series on the behaviour and design of single angle section steel beams. Two previous papers have studied the biaxial bending behaviour of restrained beams, a third has studied the lateral buckling of unrestrained beams, and a fourth the biaxial bending of unrestrained beams. In each paper, simple design methods have been developed.

In this present paper, an approximate method of predicting the second-order deflections and twist rotations of steel angle section beams under major axis bending and torsion is developed. This method is then used to determine the approximate maximum biaxial bending moments in such beams, which are then used with the section moment capacity proposals of the first paper of the series and the lateral buckling proposals of the third paper to approximate the member capacities.

Keywords:

Angles, beams, bending, buckling, design, elasticity, member capacity, moments, section capacity, steel, torsion.

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1 INTRODUCTION

Although the geometry and loading of single angle section steel beams are usually comparatively simple as shown in Fig. 1, their behaviour may be extremely complicated, and the accurate prediction of their strengths very difficult.

A single angle section beam is commonly loaded eccentrically in a plane inclined to the principal planes (Fig. 1), so that the beam undergoes primary bending and shear about both principal axes, torsion, and bearing at the supports. Even in the very unusual cases when only one of these actions occurs, the primary fully plastic or first yield capacities may be reduced by local or lateral buckling effects, or increased by stiffening resistances which become important at large rotations (Farwell and Galambos, 1969; Pi and Trahair, 1995). When all of these actions occur, as they usually do, there are first- and second-order interactions between them, some of which are very difficult to predict.

This paper is one of a series on the behaviour and design of single angle section steel beams. The first of these (Trahair, 2002a) considered the first-order (small deformation) elastic analysis of the biaxial bending (without torsion) of angle section beams including the effects of elastic restraints, and developed proposals for the section moment capacities which approximate the effects of full plasticity in compact sections, first yield in semi-compact sections, and local buckling in slender sections. A companion paper (Trahair, 2002b) developed proposals for the bearing, shear, and uniform torsion section capacities. The proposals of these two papers may be used to design steel single angle section beams which are laterally restrained as shown in Fig. 2a, so that lateral buckling or second-order effects are unimportant.

A third paper (Trahair, 2003a) considered the lateral buckling strengths of unrestrained steel single angle section beams which are loaded in the major principal plane as shown in Fig. 2b, so that there are no primary minor axis bending or torsion effects. That paper developed an approximate method of predicting the elastic second-order twist rotations of an equal angle beam in uniform bending, whose accuracy was investigated subsequently in a study (Trahair, 2003c) of the elastic non-linear large rotation behaviour of beams under non-uniform torsion. The approximate predicted rotations were used with the earlier formulations (Trahair, 2002a) of the fully plastic biaxial bending moment capacities to investigate the lateral buckling strengths of angle section beams with initial twist rotations which approximated the effects of geometrical imperfections (initial crookedness and twist) and residual stresses. The investigation included simple design proposals which approximated the predicted lateral buckling strengths.

A fourth paper (Trahair, 2003b) considered the biaxial bending of unrestrained steel angle section beams which are loaded through the shear centre as shown in Fig. 2c, so that there are no primary torsion actions. This paper also developed an approximate method of analyzing the elastic second-order twist rotations of an equal angle section beam in uniform bending, and used the predicted rotations to develop simple approximations for the biaxial bending design strengths of equal or unequal angle section beams.

The present paper is an extension of the third paper (Trahair, 2003a) on the lateral buckling strengths of single angle section beams (Fig. 2b). In this extension, the beam is considered to be loaded in a plane parallel to the major principal plane but eccentric from the shear centre as shown in Fig. 2d, so that there are primary major axis bending and torsion actions, but no primary minor axis bending actions. Again, an approximate analysis is developed for predicting the elastic second-order twist rotations of an equal angle section beam under idealized loading, and used to predict the effects of torsion on beam lateral buckling strengths and to develop simple proposals for the design of beams under major axis bending and torsion. This paper may serve as a starting point for research into the general biaxial bending and torsion of unrestrained single angle steel beams, such as that shown in Fig. 1.

2 MEMBER DESIGN METHODOLOGY

The uniform bending and torsion of unrestrained simply supported equal angle steel beams is considered in the following sections. An approximate elastic non-linear analysis of the small twist rotations of beams with initial twists is used to predict the maximum principal plane bending moments. Compact beams (Trahair, 2002a) are considered to have failed when these maximum moments reach the fully plastic moment combinations, and semi-compact beams (Trahair, 2002a) when these moments reach the first yield combinations.

This simplistic method is an extension of a first yield method of strength prediction, which takes approximate account of the additional strength beyond first yield of compact beams which can reach full plasticity. Similar methods have been used to predict the lateral buckling and biaxial bending strengths of single angle beams (Trahair, 2003a,b). The method apparently ignores the effects of residual stresses and initial crookedness which cause early yielding and reduce strength. It also makes small rotation approximations which generally overestimate the principal plane moments. These are compensated for by using initial twists which are increased sufficiently so that the small rotation analysis will predict the lateral buckling design strengths proposed in Trahair (2003a).

This method also ignores the reductions in the torque resultants of eccentrically applied loads which occur at finite rotations. For example, the load eccentricity shown in Fig. 3a decreases towards zero as the twist rotation θ of the equal angle section increases towards 45° . A different example is shown in Fig. 3b, where the point of application of the eccentric load (which does not rotate) moves suddenly towards the shear centre at the leg junction when twist rotation commences. It is conservative to ignore these reductions.

The failure moments predicted by this method are used to develop a simple method of designing unrestrained equal angle beams against major axis bending and torsion which combines the lateral buckling design strengths M_b of beams bent in the major axis principal plane with the first-order maximum twist rotations θ_1 .

The capacities of equal angle section beams to resist bearing, shear, and uniform torsion may be checked separately by comparing the appropriate design actions (which may be determined by a simple first-order analysis of the beam) with the corresponding design capacities recommended in Trahair (2002b).

3 BUCKLING AND TORSION OF EQUAL ANGLE BEAMS

An elastic simply supported equal angle section beam of length L and initial twist

$$\phi_0 = \theta_0 \sin(\pi z / L) \quad (1)$$

is shown in Fig. 4. The beam has equal and opposite major end moments M causing uniform bending $M_x = M$ in the yz principal plane and a uniformly distributed torque per unit length m . The small deformation differential equations of equilibrium for biaxial bending and torsion are

$$\begin{Bmatrix} -EI_x v'' \\ EI_y u'' \\ GJ\phi' \end{Bmatrix} = \begin{bmatrix} 1 & -u' \\ -(\phi + \phi_0) & -v' \\ u' & 1 \end{bmatrix} \begin{Bmatrix} M_x \\ M_z \end{Bmatrix} \quad (2)$$

in which

$$M_z = m L / 2 - m z \quad (3)$$

is the variation of the axial torque caused by the distributed torque m , E and G are the Young's and shear moduli of elasticity, I_x and I_y are the second moments of area about the principal x , y axes, J is the torsion section constant, u and v are the shear centre deflections in the x , y directions, ϕ is the angle of twist rotation, and ' indicates differentiation with respect to the distance z along the beam.

In Equations 2, the left hand sides represent the internal resistances to bending and torsion, while the right hand sides represents the first- and second-order actions resulting from the applied actions M_x and M_z and the small deflections u , v and twist rotations ϕ . The third of Equations 2 omits a large twist rotation resistance $EI_n(\phi')^3/2$ (Trahair, 2003c) because the non-linear "Wagner" section constant I_n is quite small for equal angle sections ($I_n = b^5 t / 90$, rather than the erroneous $8 b^5 t / 45$ used in Trahair (2003a), in which b is the leg length and t the thickness of the section).

The first-order solutions v_1 , u_1 , ϕ_1 of Equations 2 which satisfy the boundary conditions for simple supports are obtained by ignoring the terms u' , v' , and $(\phi + \phi_0)$ on the right hand sides, whence

$$v_1 = \delta_{v1}(4z/L - 4z^2/L^2) \quad (4a)$$

$$u_1 = \delta_{u1}(4z/L - 4z^2/L^2) \quad (4b)$$

$$\phi_1 = \theta_1(4z/L - 4z^2/L^2) \quad (4c)$$

in which

$$\delta_{v1} = ML^2 / 8EI_x \quad (5a)$$

$$\delta_{u1} = 0 \quad (5b)$$

$$\theta_1 = mL^2 / 8GJ \quad (5c)$$

Approximate second-order solutions may be obtained by assuming that

$$M_z u' = M_z v' = 0 \quad (6a)$$

$$v_2 / \delta_{v2} = u_2 / \delta_{u2} = \phi_2 / \theta_2 = \sin(\pi z / L) \quad (6b)$$

and by making the replacements

$$\left\{ \begin{array}{l} M \sin(\pi z / L) \\ (mL/2) \cos(\pi z / L) \\ M\theta_2 \sin(\pi z / L) \\ M(\pi / L) \delta_{u2} \cos(\pi z / L) \end{array} \right\} \quad \text{for} \quad \left\{ \begin{array}{l} M \\ M_z \\ M\phi \\ Mu' \end{array} \right\} \quad (7)$$

Equations 6a are based on the finding of Trahair and Teh (2001) that the effects of these second-order moment components of applied torques are small and can be ignored. The sine wave shape approximations of Equations 6b are close to the first order shapes of Equations 4. The replacements of Equations 7 incorporate sine or cosine shapes for terms which are either constant or else are approximately of sine or cosine shape.

Thus

$$\delta_{v2} = M / (\pi^2 EI_x / L^2) \quad (8a)$$

$$\delta_{u2} = \theta_2 M / (\pi^2 EI_y / L^2) \quad (8b)$$

$$\theta_2 = \frac{mL^2 / 2\pi GJ + (M^2 / M_{yz}^2) \theta_0}{1 - M^2 / M_{yz}^2} \quad (8c)$$

in which

$$M_{yz} = \sqrt{(\pi^2 EI_y GJ / L^2)} \quad (9)$$

The approximate Equations 8 disagree slightly with the first-order solutions of Equations 5 when the second-order quantities are ignored. Agreement can be obtained by adjusting Equations 8 to

$$\delta_{v2} = \delta_{v1} \quad (10a)$$

$$\delta_{u2} = \theta_2 M / (\pi^2 EI_y / L^2) \quad (10b)$$

$$\theta_2 = \frac{\theta_1 + (M^2 / M_{yz}^2) \theta_0}{1 - M^2 / M_{yz}^2} \quad (10c)$$

The bending moments are greatest at mid-span, and can be obtained using Equations 2 and 6a as

$$M_{x2} = M \quad (11a)$$

$$M_{y2} = -M(\theta_2 + \theta_0) \quad (11b)$$

4 FULLY PLASTIC BUCKLING AND TORSION STRENGTHS OF EQUAL ANGLE BEAMS

4.1 Fully Plastic Moment Combinations.

The combinations of principal axis moments M_{px} , M_{py} which cause full plasticity of an equal angle are given by the fully plastic interaction equation (Trahair, 2002a)

$$\pm M_{py} / M_{pym} = 1 - (M_{px} / M_{pxm})^2 \quad (12)$$

in which the principal axis full plastic moments M_{pxm} , M_{pym} are given by

$$M_{pxm} = 2 M_{pym} = f_y b^2 t / \sqrt{2} \quad (13)$$

in which f_y is the yield stress.

4.2 Elastic Lateral Buckling and Lateral Buckling Design Proposals.

4.2.1 Elastic lateral buckling

The value of the major axis uniform bending moment at elastic buckling M_{yz} is given by Equation 9 (Trahair, 1993, 2003a).

4.2.2 Lateral buckling design strength

It has been proposed (Trahair, 2003a) that the nominal design lateral buckling moment capacity M_b of an angle section beam should be obtained from

$$M_b = M_{sxm} \quad (\lambda_e \leq \lambda_{ex}) \quad (14a)$$

$$M_b = M_{sxm} - (M_{sxm} - M_{sym}) \frac{(\lambda_e - \lambda_{ex})}{(\lambda_{ey} - \lambda_{ex})} \quad (\lambda_{ex} \leq \lambda_e \leq \lambda_{ey}) \quad (14b)$$

$$M_b = M_{sym} \quad (\lambda_{ey} \leq \lambda_e) \quad (14c)$$

in which

$$\lambda_{ex} = 0.99 - \frac{0.22}{(\alpha_m - 0.7)} \quad (15)$$

$$\lambda_{ey} = \sqrt{M_{sxm} / M_{sym}} \quad (16)$$

$$\lambda_e = \sqrt{M_{sxm} / M_{quy}} \quad (17)$$

as shown in Fig. 5, in which M_{sxm} and M_{sym} are the major and minor axis maximum section moment capacities, α_m is a moment modification factor which allows for the variation of the bending moment distribution (Trahair, 1993, 2003a), and M_{quy} is the maximum moment in the beam at elastic buckling (Trahair, 2003a). For a simply supported compact equal angle in uniform bending, $M_{sxm} = M_{pxm}$, $M_{sym} = M_{pym}$, $M_{quy} = M_{yz}$ and $\alpha_m = 1$.

The modified slenderness limit λ_{ex} in Equation 15 is an approximation for the value of $\sqrt{(M_{sxm} / M_{yz})}$ at which $M_b = M_{sxm}$ according to the Australian design code AS 4100 (SA, 1998). Equation 14a uses the major axis section capacity M_{sxm} for low slenderness beams ($\lambda_e \leq \lambda_{ex}$), while Equation 14c uses the minor axis section capacity M_{sym} for high slenderness beams ($\lambda_{ey} \leq \lambda_e$), which is based on the finding of Trahair (2003a) that the moment capacity is never less than M_{sym} . Equation 14b provides a simple linear interpolation between M_{sxm} and M_{sym} for beams of intermediate slenderness ($\lambda_{ex} \leq \lambda_e \leq \lambda_{ey}$), which provides a close but conservative approximation to the predictions of Trahair (2003a).

4.3 Equivalent Initial Twists.

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 11 to determine the buckling and torsion strengths of equal angle section beams. Such initial twists will also predict the lateral buckling design strengths of unbraced beams bent in their major axis principal plane. The magnitudes θ_0 of these initial twists of compact beams have been determined in Trahair (2003b), and are closely approximated by

$$\theta_0 = -0.1116 + 0.3612 \lambda_e + 0.3551 \lambda_e^2 - 0.3935 \lambda_e^3 \geq 0 \quad (18)$$

4.4 Member Actions for Full Plasticity

The values of the dimensionless applied moment M / M_{pxm} and the maximum first-order elastic twist rotation θ_1 at which the second-order principal axis moments M_{x2} , M_{y2} cause full plasticity can be determined by combining Equations 10c, 11, 12, and 13 as

$$(\theta_1 + \theta_0) = \frac{\{1 - (M / M_{pxm})^2\} \{1 - \lambda_e^4 (M / M_{pxm})^2\}}{2M / M_{pxm}} \quad (19)$$

in which θ_0 is given by Equation 18, and by solving iteratively. The variations of M / M_{pxm} with λ_e for given values of θ_1 are shown in Fig. 5. For this figure, the values of M / M_{pxm} which are less than 0.5 have been replaced by 0.5. This is consistent with the use of dimensionless lateral buckling strengths M_b / M_{pxm} which are never less than 0.5 (Trahair, 2003a). This use is appropriate because in the worst case when the twist rotation reaches 90° , the applied moment will act about the minor principal axis, in which case the resistance will be equal to the minor axis full plastic moment $M_{pym} = 0.5 M_{pxm}$.

The values of M / M_{pxm} shown in Fig. 5 for $\theta_1 = 0$ are very close to the dimensionless lateral buckling strengths of Equations 14-17, demonstrating the ability of the equivalent initial twists of Equation 18 to represent the effects of residual stresses and initial crookednesses and twists. For the special case of $\lambda_e = 0$ and $\theta_0 = 0$, the dimensionless values of M / M_{pxm} vary with the maximum first order elastic twist rotation θ_1 according to

$$(M / M_{pxm})_0 = \sqrt{(\theta_1^2 + 1)} - \theta_1 \quad (20)$$

Simple but conservative approximations of the values of M / M_{pxm} and θ_1 at full plasticity can be obtained by using

$$\frac{M}{M_{pxm}} = \left(\frac{M}{M_{pxm}} \right)_0 \left(\frac{M_b}{M_{pxm}} \right) \geq \left(\frac{M_{pym}}{M_{pxm}} \right) \quad (21)$$

as shown in Fig. 5.

5 FIRST YIELD BUCKLING AND TORSION STRENGTHS OF EQUAL ANGLE SECTION BEAMS

5.1 First Yield Moment Combinations

The combinations of principal axis moments M_{yx} , M_{yy} which cause first yield of an equal angle are given by the interaction equations (Trahair, 2002a)

$$\pm M_{yy} / M_{yyym} = 1 - M_{yx} / M_{yxm} \quad (22)$$

in which the principal axis first yield moments M_{yxm} , M_{yyym} are given by

$$M_{yxm} = 2 M_{yyym} = f_y b^2 t (\sqrt{2} / 3) \quad (23)$$

5.2 Equivalent Initial Twists

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 11 to determine the buckling and torsion strengths of equal angle section beams. The magnitudes θ_0 of the initial twists of semi-compact unbraced beams bent in their major axis principal plane which will predict their lateral buckling design strengths have been determined in Trahair (2003b), and are closely approximated by

$$\theta_0 = -0.0358 + 0.0499 \lambda_e + 0.4262 \lambda_e^2 - 0.3142 \lambda_e^3 \geq 0 \quad (24)$$

5.3 Member Actions for First Yield

The values of the dimensionless applied moment M / M_{yxm} and the maximum first-order elastic twist rotation θ_1 at which the second-order principal axis moments M_{x2} , M_{y2} cause first yield can be determined by combining Equations 10c, 11, 22, and 23 as

$$(\theta_1 + \theta_0) = \frac{\{1 - M / M_{yxm}\} \{1 - \lambda_e^4 (M / M_{yxm})^2\}}{2M / M_{yxm}} \quad (25)$$

in which θ_0 is given by Equation 24, and by solving iteratively. The variations of M / M_{yxm} with λ_e for given values of θ_1 are shown in Fig. 6. For this figure, the values of M / M_{yxm} which are less than 0.5 have again been replaced by 0.5, as were the corresponding values for full plasticity in Fig. 5.

The values of M / M_{yxm} shown in Fig. 6 for $\theta_1 = 0$ are very close to the dimensionless lateral buckling strengths of Equations 14-17, demonstrating the ability of the equivalent initial twists of Equation 24 to represent the effects of residual stresses and initial crookednesses and twists. For the special case of $\lambda_e = 0$ and $\theta_0 = 0$, the dimensionless values of M / M_{yxm} vary with the maximum first order elastic twist rotation θ_1 according to

$$(M / M_{yxm})_0 = 1 / (1 + 2 \theta_1) \quad (26)$$

Simple but conservative approximations of the values of M / M_{yxm} and θ_1 at first yield can be obtained by using

$$\frac{M}{M_{yxm}} = \left(\frac{M}{M_{yxm}} \right)_0 \left(\frac{M_b}{M_{yxm}} \right) \geq \left(\frac{M_{yym}}{M_{yxm}} \right) \quad (27)$$

as shown in Fig. 6.

6 LOCAL BUCKLING EFFECTS

6.1 Section Classification and Moment Capacities

The effects of local buckling on the section moment capacities of angle section beams has been discussed in Trahair (2002a). In that paper, sections were classified as being plastic, compact, semi-compact or slender (BSI, 2000, Trahair et al, 2001) by comparing their long leg plate slendernesses

$$\lambda = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (28)$$

with limiting slenderness values.

A plastic section must have sufficient rotation capacity to maintain a plastic hinge until a plastic collapse mechanism develops. A plastic section satisfies (Trahair, 2002a)

$$\lambda \leq 12 \quad (29)$$

A compact section must be able to form a plastic hinge. A compact section satisfies

$$12 < \lambda \leq 16 \quad (30)$$

The nominal section moment capacity M_{sx} of a plastic or compact section is equal to its fully plastic capacity M_{pxm} , so that

$$M_{sx} = M_{pxm} \quad (31)$$

A slender section has its moment capacity reduced below the first yield moment M_{yxm} by local buckling effects. A slender section satisfies

$$26 < \lambda \quad (32)$$

The nominal section moment capacity of a slender section M_{sx} is approximated by

$$M_{sx} = M_{yxm} (\lambda_y / \lambda)^2 \quad (33)$$

A semi-compact section must be able to reach the first yield moment, but local buckling effects may prevent it from forming a plastic hinge. A semi-compact section satisfies

$$16 < \lambda \leq 26 \quad (34)$$

The nominal section moment capacity M_{sx} of a semi-compact section is approximated by the linear interpolation between the full plastic and first yield capacities given by

$$M_{sx} = M_{pxm} - (M_{pxm} - M_{yxm}) \frac{(\lambda - 16)}{10} \quad (35)$$

6.2 Buckling and Torsion Strengths

The effects of local buckling on the buckling and torsion strengths of equal angle section beams can be approximated as shown in Fig. 7, by using

$$\frac{M}{M_{sx}} = \left(\frac{M}{M_{sx}} \right)_0 \left(\frac{M_b}{M_{sx}} \right) \geq \left(\frac{M_{sy}}{M_{sx}} \right) \quad (36)$$

in which $(M/M_{sx})_0$ for plastic and compact sections is given by the value of $(M/M_{pxm})_0$ obtained from Equation 20, and for semi-compact and slender sections by the value of $(M/M_{yxm})_0$ from Equation 26. The general Equation 36 reduces to Equation 21 for plastic and compact sections ($\lambda \leq 16$), and to Equation 27 for sections with $\lambda = 26$.

7 UNEQUAL ANGLE BEAMS

7.1 Plastic and Compact Beams

It is proposed that the buckling and torsion strengths of plastic and compact unequal angle beams should be approximated by using Equation 36 developed for equal angle beams with two modifications.

The first modification is for the maximum moment M_{quy} at elastic lateral buckling which is used in Equations 14-17 for the nominal design lateral buckling capacity M_b . Approximations for the values of M_{quy} for unequal angle section beams are given in Trahair (2003a).

The second modification is associated with the need to replace Equations 12 and 13 for the combinations of the full plastic principal axis moment combinations M_{px} and M_{py} for equal angles. The combinations of $M_{px} / f_y b^2 t$ and $M_{py} / f_y b^2 t$ for unequal angles with $0.5 \leq \beta \leq 1.0$ (in which β is the leg length ratio) shown in Fig. 8 were obtained from the formulations in Trahair (2002a). In this figure, the dashed lines correspond to the approximations proposed for the cases where the angle section capacity cannot reach full plasticity. It can be seen that the fully plastic moment combinations for moments of opposite sign are less than those for the same sign. The principal axis values M_{pxm} and M_{pym} of M_{px} and M_{py} may be approximated using (Trahair, 2003a)

$$M_{pxm} / f_y b^2 t = 0.337\beta^2 - 0.001\beta + 0.371 \quad (37)$$

$$M_{pym} / f_y b^2 t = -0.075\beta^2 + 0.546\beta - 0.117 \quad (38)$$

The dimensionless applied moments $(M/M_{pxm})_0$ for which first-order rotations θ_1 cause full plasticity in zero slenderness beams ($\lambda_e = 0$) can be obtained by using Fig. 8 and the first-order relationships

$$(M_{x1})_0 = M \quad (39a)$$

$$(M_{y1})_0 = -M\theta_1 \quad (39b)$$

The variations of $(M/M_{pxm})_0$ with θ_1 for unequal angle beams are shown by the solid and dashed lines in Fig. 9. They may be approximated by using

$$(M/M_{pxm})_0 = 1 + a_1 \theta_1 + a_2 \theta_1^2 + a_3 \theta_1^3 \quad (40)$$

in which either

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} 0.5900 & 2.424 & -4.103 & 2.085 \\ -6.245 & 25.35 & -31.48 & 12.88 \\ -6.300 & 24.38 & -30.57 & 12.57 \end{bmatrix} \begin{Bmatrix} 1 \\ \beta \\ \beta^2 \\ \beta^3 \end{Bmatrix} \quad (41a)$$

when M_x and M_y are of the same sign (θ_1 negative), as shown in Fig. 10b, or

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} -8.810 & 20.35 & -19.16 & 6.615 \\ 15.91 & -42.49 & 43.04 & -15.96 \\ -7.566 & 19.86 & -19.79 & 7.428 \end{bmatrix} \begin{Bmatrix} 1 \\ \beta \\ \beta^2 \\ \beta^3 \end{Bmatrix} \quad (41b)$$

when M_x and M_y are of opposite sign (θ_1 positive), as shown in Fig. 10a.

The signs of M_x and M_y depend on the attitude of the unequal angle, the sense of the twist rotation θ_1 , and the principal axis directions x and y , as shown in Fig. 10. With the short leg down, the y axis initially vertically down, an eccentric load q through the legs causes a positive (clockwise) rotation θ_1 as shown in Fig. 10a. In this case, the applied moment M has components M_x and M_y of opposite sign, so that the smaller values of $(M/M_{pxm})_0$ obtained using Equation 41b with Equation 40 are appropriate. When the short leg is up and the y axis is initially vertically up as shown in Fig. 10b, then an eccentric load through the legs causes a negative (anticlockwise) rotation, and the applied moment M has components of the same sign, in which case the larger values of $(M/M_{pxm})_0$ obtained using Equation 41a with Equation 40 are appropriate.

The values of $(M/M_{pxm})_0$ given by Equations 40 and 41 should be used for $(M/M_{sx})_0$ in Equation 36 to approximate the buckling and torsion strengths of plastic or compact unequal angle beams.

7.2 Semi-Compact and Slender Beams

7.2.1 First Yield Capacities

Approximations for the buckling and torsion strengths of semi-compact and slender unequal angle beams may be developed in a similar manner to that used above for plastic and compact beams, provided the full plastic moment combinations are replaced by first yield combinations which are modified appropriately to account for inelastic and elastic local buckling effects.

The combinations of the first yield principal axis moment combinations $M_{yx} / f_y b^2 t$ and $M_{yy} / f_y b^2 t$ for unequal angles with $0.5 \leq \beta \leq 1.0$ shown in Fig. 11 were obtained from the formulations in Trahair (2002a). It can be seen that the first yield moment combinations for moments of opposite sign are generally less than those for the same sign. The principal axis values M_{yxm} and M_{yym} of M_{yx} and M_{yy} may be approximated using

$$M_{yxm} / f_y b^2 t = 0.577\beta^2 - 0.460\beta + 0.354 \quad (42)$$

$$M_{yym} / f_y b^2 t = 0.276\beta^2 - 0.068\beta + 0.028 \quad (43)$$

The dimensionless applied moments $(M/M_{yym})_0$ for which first-order rotations θ_1 cause first yield in zero slenderness beams ($\lambda_e = 0$) can be obtained by using Fig. 11 and the first-order relationships of Equations 39. The variations of $(M/M_{yym})_0$ with θ_1 for unequal angle beams are shown by the solid lines in Fig. 12. They may be approximated by using

$$\left(\frac{M}{M_{sx}} \right)_0 = \left| \frac{b_1}{1 - b_2\theta_1} \right| \leq \left| \frac{b_3}{1 + b_4\theta_1} \right| \quad (44)$$

in which

$$\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} = \begin{bmatrix} 1.00 & 0 & 0 & 0 \\ -2.27 & 2.39 & -0.510 & -1.62 \\ 1.88 & -0.930 & 0.880 & -0.830 \\ -27.0 & 68.0 & -65.3 & 22.3 \end{bmatrix} \begin{Bmatrix} 1 \\ \beta \\ \beta^2 \\ \beta^3 \end{Bmatrix} \quad (45)$$

and $M_{sx} = M_{yym}$.

7.2.2 Capacities of Semi-Compact and Slender Beams

The dimensionless applied moments $(M/M_{sx})_0$ for which first-order rotations θ_1 cause first yield in zero slenderness angle beams ($\lambda_e = 0$) need to be modified for semi-compact and slender angles to allow for inelastic or elastic local buckling effects. This may be done by replacing the major axis first yield capacity M_{yym} used for M_{sx} in Equation 44 by the appropriate section capacity M_{sx} determined using Equation 35 for semi-compact angles or Equation 33 for slender angles. The modified values of $(M/M_{sx})_0$ should be used in Equation 36 to approximate the buckling and torsion strengths of semi-compact or slender unequal angle beams.

8 EXAMPLE

8.1 Problem

A 150 x 100 x 12 unequal angle beam is shown in Fig. 13. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of $b = 144$ mm, $\beta b = 94$ mm, and $t = 12$ mm are shown in Fig. 13b. The unbraced beam is simply supported over a span of $L = 6$ m, and has a design uniformly distributed vertical load of $q^* = 6$ kN/m acting parallel to the long leg with an eccentricity of $e = 47$ mm from the shear centre at the leg junction, as shown in Fig. 13b.

The first-order analysis of the beam, the determination of the lateral buckling design strength, and the check of the buckling and torsion capacity are summarised below. The determination of the bearing, shear, and torsion capacities is summarised in Trahair (2002 b).

8.2 Elastic Analysis

The design major axis bending moments are

$$M_x^* = (q^* L^2 / 8) \cos \alpha = 24.7 \text{ kNm}, \text{ and } M_y^* = -(q^* L^2 / 8) \sin \alpha = -10.9 \text{ kNm}.$$

The design torque is $M_z^* = (q^* e L / 2) = 0.846 \text{ kNm}$. This is much less than the design capacity of $\phi M_u = 2.31 \text{ kNm}$ (Trahair, 2002b).

8.3 Lateral Buckling Design Strength

The angle section has been shown to be compact (Trahair, 2002a).

The elastic buckling moment and the lateral buckling strength calculated in Trahair (2003a) are $M_{quy} = 30.6 \text{ kNm}$ and $M_b = 25.0 \text{ kNm}$.

8.4 Buckling and Torsion Capacity

Using Equation 5c, the maximum first-order twist rotation is

$$\theta_1 = 0.6 \times 47 \times (6E3)^2 / (8 \times 8E4 \times 0.1371E6) = 0.016 \text{ rad.}, \text{ which is small.}$$

$$\beta = 94 / 144 = 0.653$$

Using Fig. 10, the moment components of M_x^* are of opposite sign.

Using Equations 41b, $a_1 = -1.850$, $a_2 = 2.07$, $a_3 = -0.969$.

Using Equation 40, $(M / M_{pxm})_0 = 0.972$, so that the effect of torsion is small in this case.

Using Trahair (2003a) or Equations 37 and 38, $M_{pxm} = 38.4 \text{ kNm}$, $M_{pym} = 15.5 \text{ kNm}$.

Using Equation 36 with $M_{sx} = M_{pxm}$ and $M_{sy} = M_{pym}$,

$$M = 0.972 \times 25.0 = 24.3 \text{ kNm} > 15.5 \text{ kNm} \text{ so that } \phi M = 21.9 \text{ kNm (using } \phi = 0.9).$$

This is less than the major axis design moment $M_x^* = 24.7 \text{ kNm}$ and the beam is inadequate, even if the minor axis moment $M_y^* = -10.9 \text{ kNm}$ is ignored.

9 CONCLUSIONS

This paper develops a rational, consistent, and economical design method for determining the buckling and torsion strength of an eccentrically loaded unbraced steel angle section beam, and illustrates its use in a design example.

An approximate small rotation non-linear elastic analysis is used to predict the maximum moments in equal angle beams in uniform bending and torsion. The beams have initial twists. The magnitudes of the initial twists are chosen so that the predicted strengths of beams bent in the major principal plane are equal to recent recommendations for the lateral buckling strengths (Trahair, 2002c).

The buckling and torsion strengths of equal angle beams are predicted by assuming either full plasticity or first yield at the maximum moment section, and simple design approximations are developed. The effects of local buckling are considered using proposed definitions (Trahair, 2002a) of the section capacities of plastic, compact, semi-compact, and slender sections. These definitions are then used with the simple fully plastic and first yield design approximations to develop a simple general design approximation for equal angle beams. This design approximation is formulated in terms of the maximum first-order moment M and angle of twist rotation θ_1 , and can be used for equal angle beams under general loading which acts in a plane parallel to the major axis principal plane.

The fully plastic and first yield behaviours of unequal angle section beams are then considered, and design approximations are developed from those for equal angle beams.

Proposals have been made elsewhere (Trahair, 2002b) for checking the bearing, shear, and torsion capacities of angle section beams.

APPENDIX 1 REFERENCES

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APPENDIX 2 NOTATION

a_{1-3}	constants
b	long leg length
b_{1-4}	constants
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_y	yield stress
G	shear modulus of elasticity
I_x, I_y	second moments of area about the x, y principal axes
I_n	non-linear Wagner section constant
J	torsion section constant
L	span length
M	applied end moment
M_b	lateral buckling moment strength
M_{px}, M_{py}	values of M_x, M_y at full plasticity
M_{pxm}, M_{pym}	principal axis values of M_{px}, M_{py}
M_{quy}	maximum moment at elastic lateral buckling
M_{sx}, M_{sy}	principal axis section moment capacities
M_x, M_y	moments about the x, y principal axes
M_x^*, M_y^*	design moments about the x, y principal axes
M_{yz}	uniform bending elastic buckling moment
M_{x2}, M_{y2}	second-order moments about the x, y principal axes
M_{yx}, M_{yy}	values of M_x, M_y at first yield
M_{yxm}, M_{yym}	principal axis values of M_{yx}, M_{yy}
m	intensity of uniformly distributed torque
q	intensity of uniformly distributed load
q^*	design intensity of uniformly distributed load
t	leg thickness
u, v	shear centre deflections parallel to the x, y principal axes
u_1, v_1	first-order deflections
u_2, v_2	second-order deflections
x, y	principal axes
X, Y	rectangular (geometric) axes
X_c, Y_c	X, Y distances from centroid to shear centre
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
α_m	moment modification factor
β	leg length ratio
β_x	monosymmetry section constant
δ_{u1}, δ_{v1}	maximum first-order deflections
δ_{u2}, δ_{v2}	maximum second-order deflections
λ	long leg local buckling slenderness
λ_e	modified slenderness for beam lateral buckling
$\lambda_{ex}, \lambda_{ey}$	beam lateral buckling slenderness limits
ϕ	angle of twist rotation, or capacity factor
ϕ_0	initial angle of twist rotation
ϕ_1	first-order angle of twist rotation
ϕ_2	second-order angle of twist rotation
θ_0	maximum value of ϕ_0
θ_1	maximum value of ϕ_1
θ_2	maximum value of ϕ_2

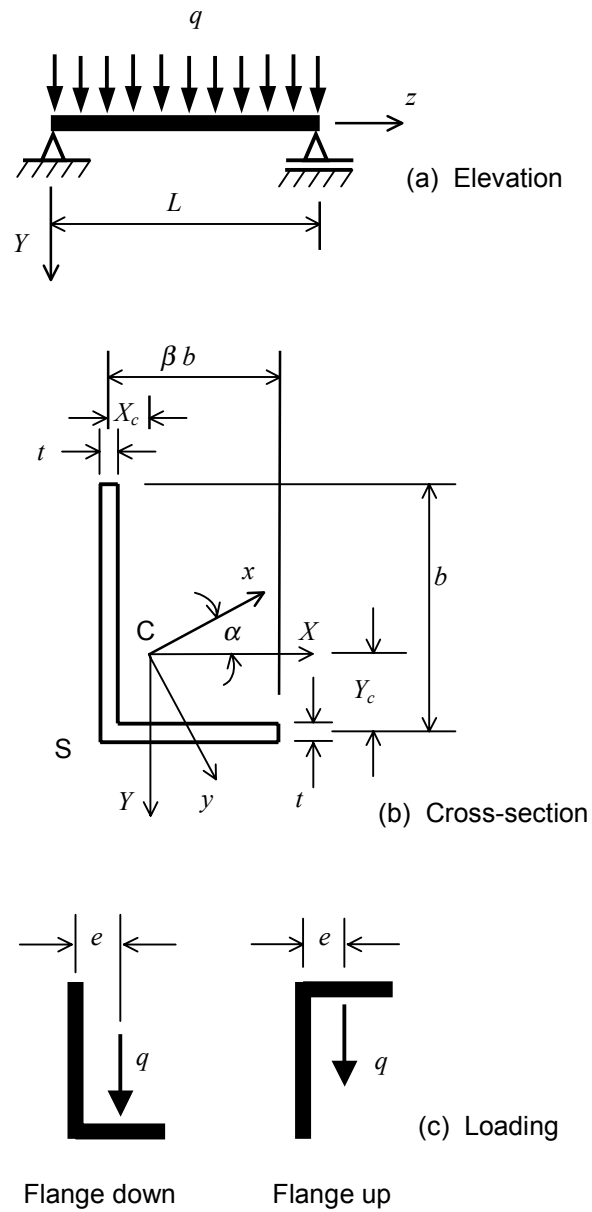


Fig. 1. Eccentrically Loaded Angle Section Beam

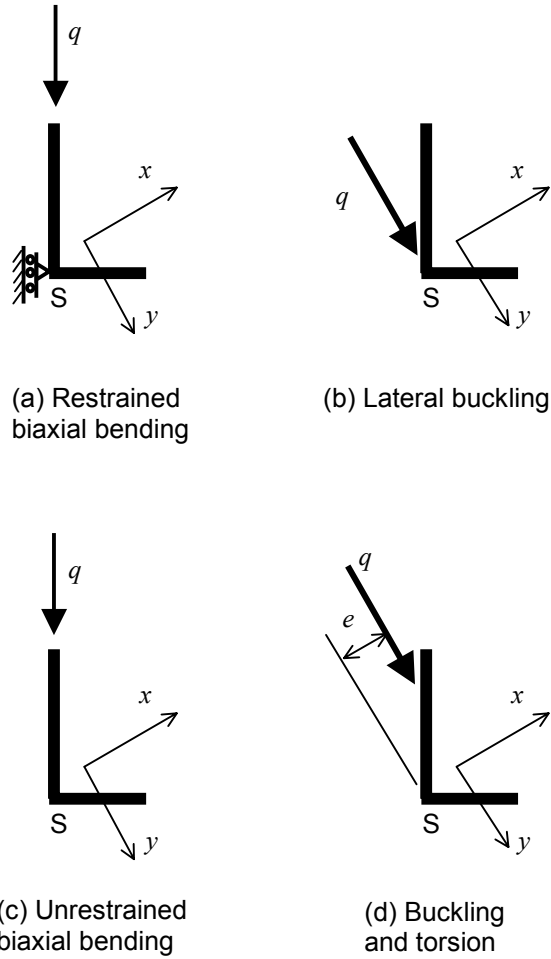
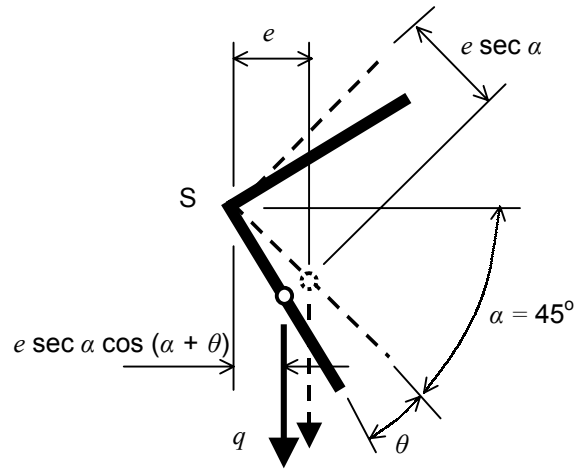
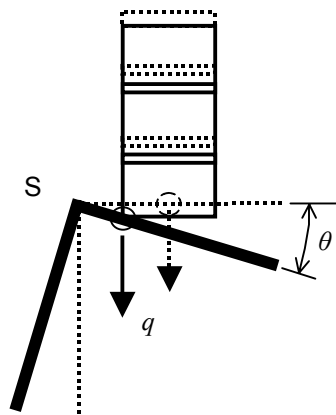


Fig. 2. Single Angle Beam Behaviour



(a) Movement of point of application



(b) Change of point of application

Fig. 3. Reduction of Load Eccentricity

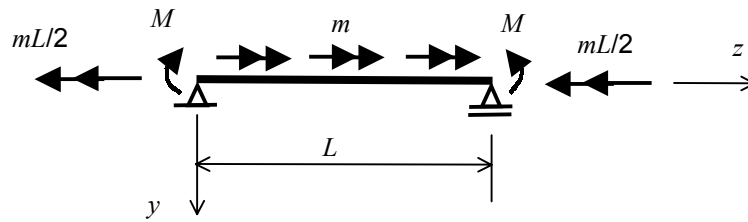


Fig. 4. Simply Supported Equal Angle in Uniform Bending and Torsion

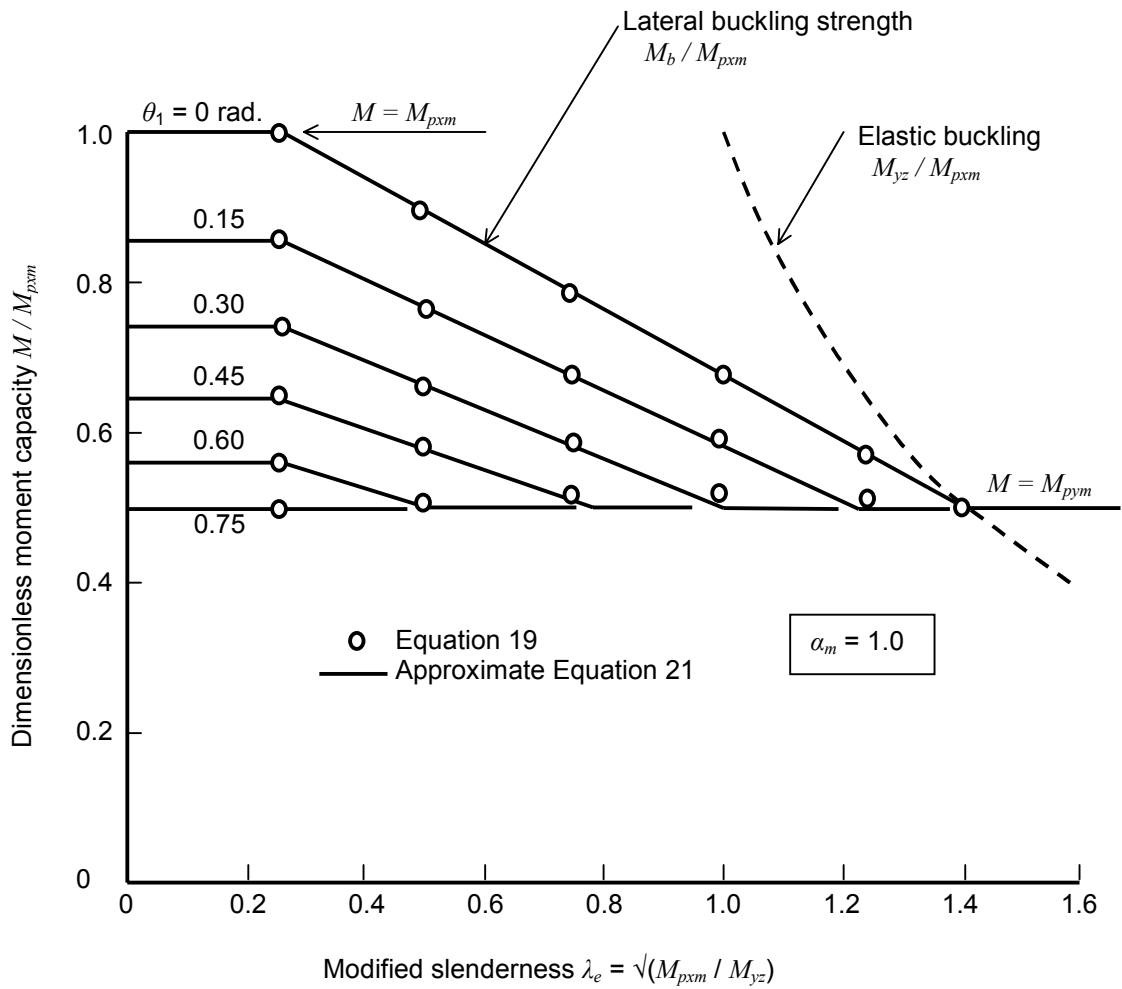


Fig. 5. Fully Plastic Buckling and Torsion Strengths of Equal Angle Beams

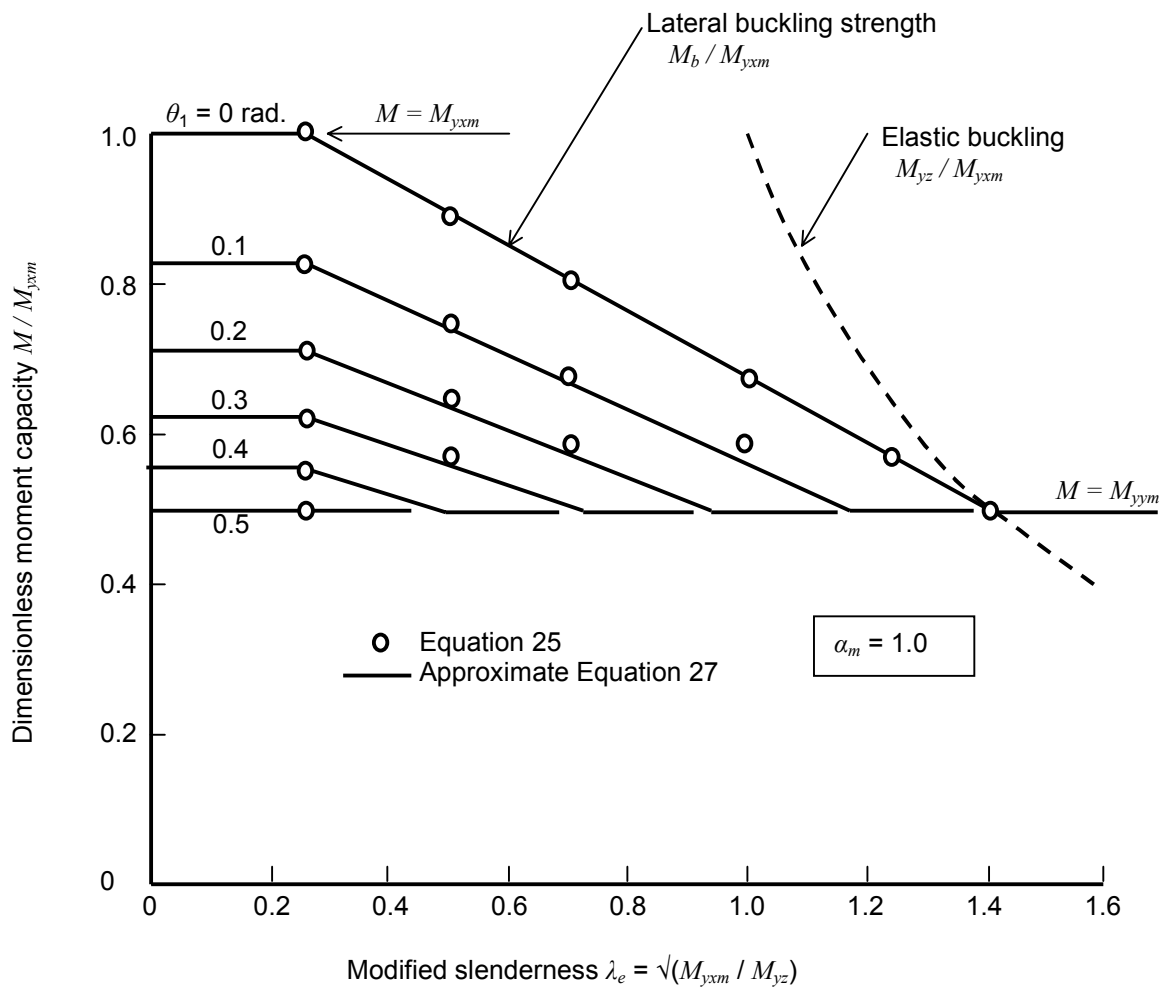


Fig. 6. First Yield Buckling and Torsion Strengths of Equal Angle Beams

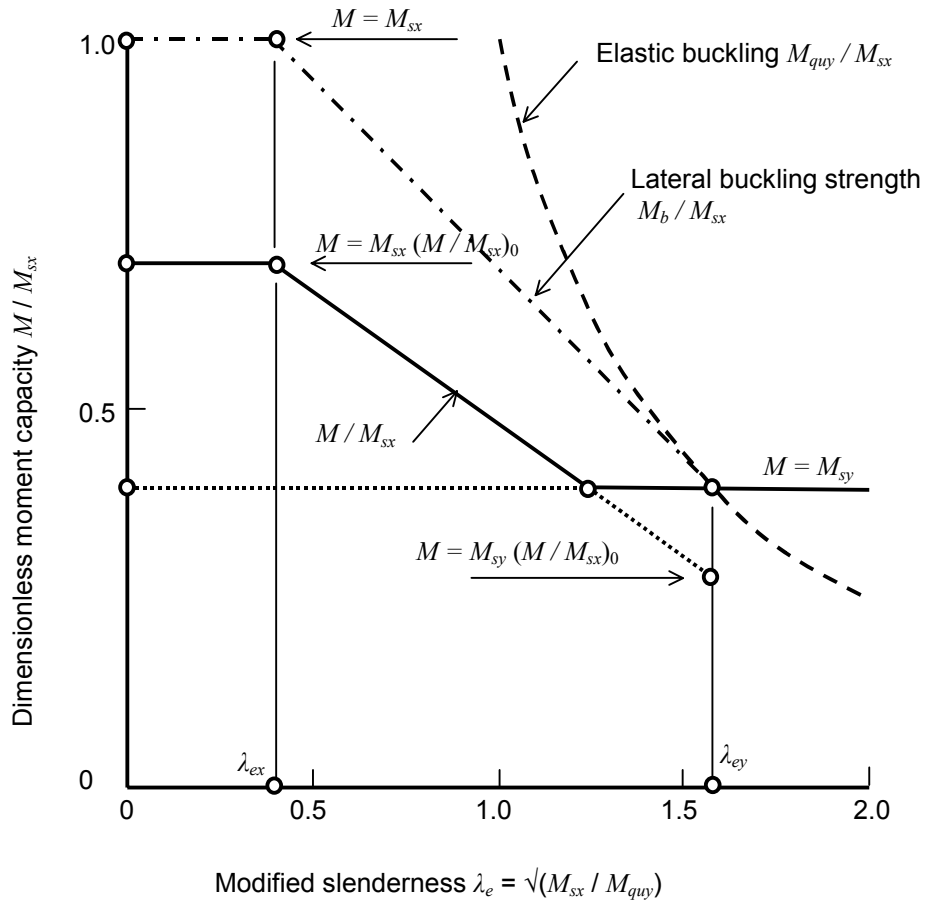


Fig. 7. Proposed Design Strengths

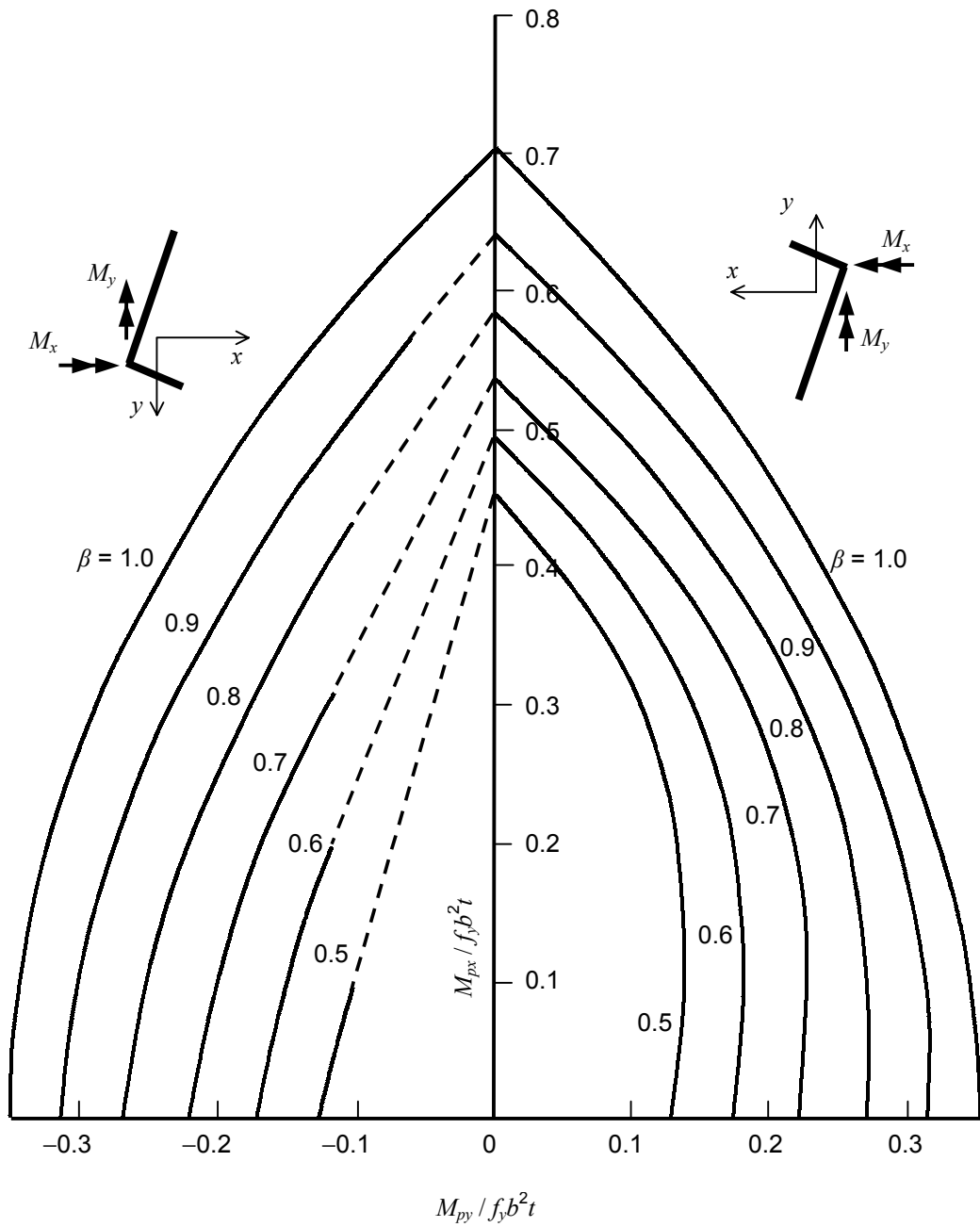


Fig. 8. Fully Plastic Moment Combinations for Unequal Angles

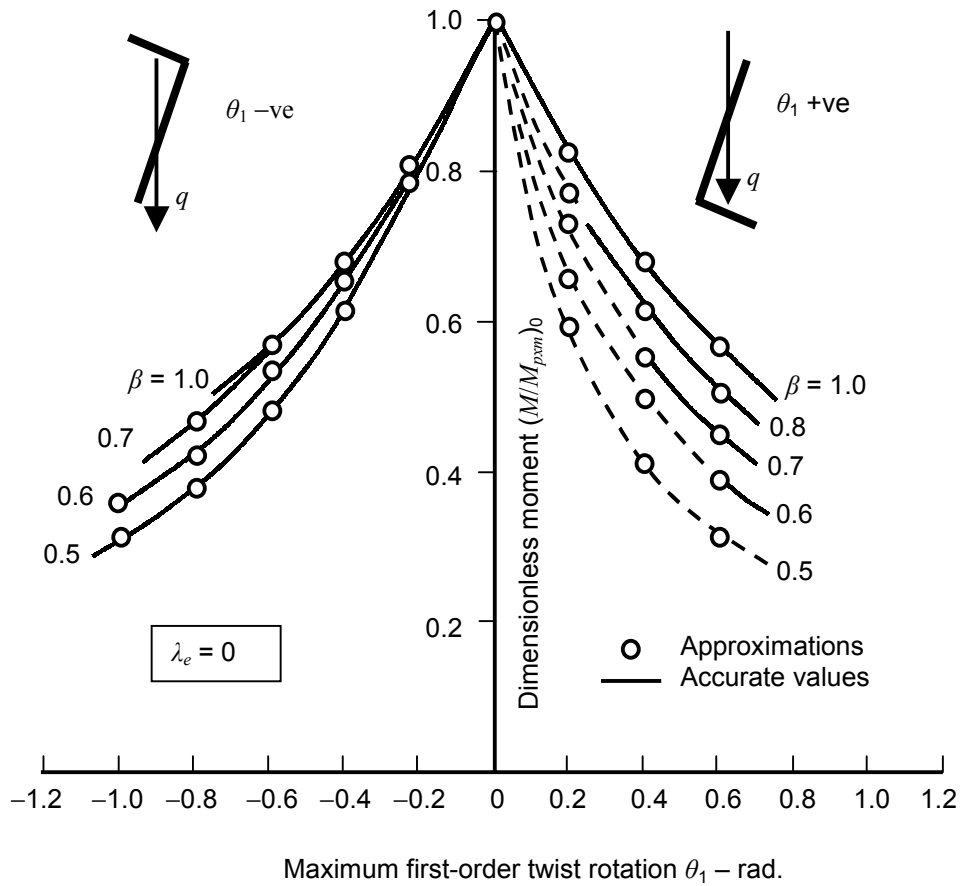
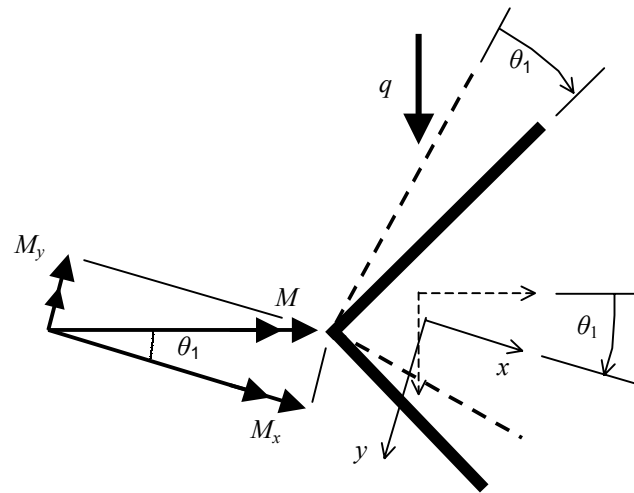
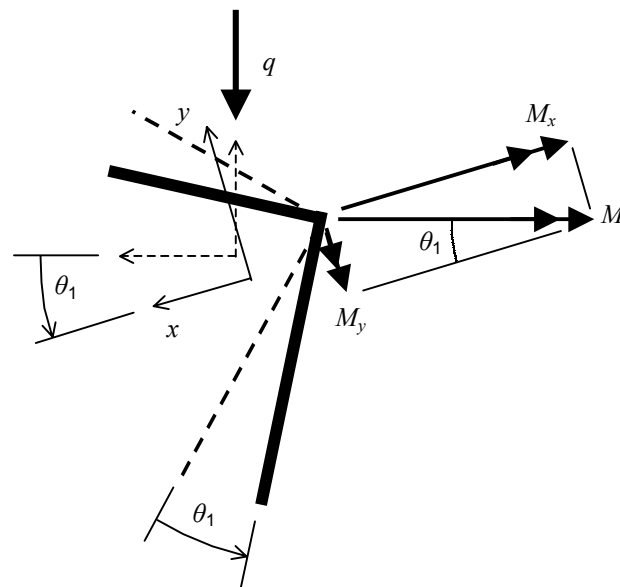


Fig. 9. Fully Plastic Strengths for Zero Slenderness



(a) Components of opposite sign
 $\theta_1 +ve, M_x +ve, M_y -ve$



(b) Components of same sign
 $\theta_1 -ve, M_x -ve, M_y -ve$

Fig. 10. Signs of Moment Components

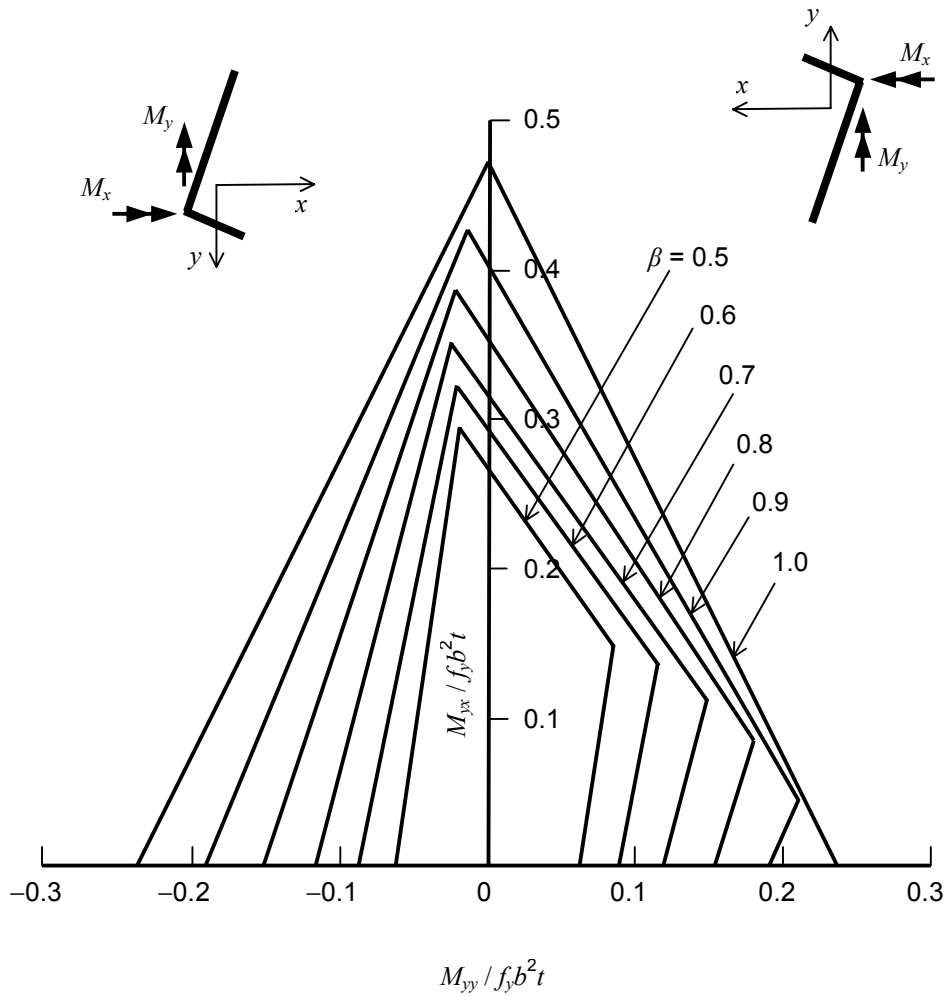


Fig. 11. First Yield Moment Combinations for Unequal Angles

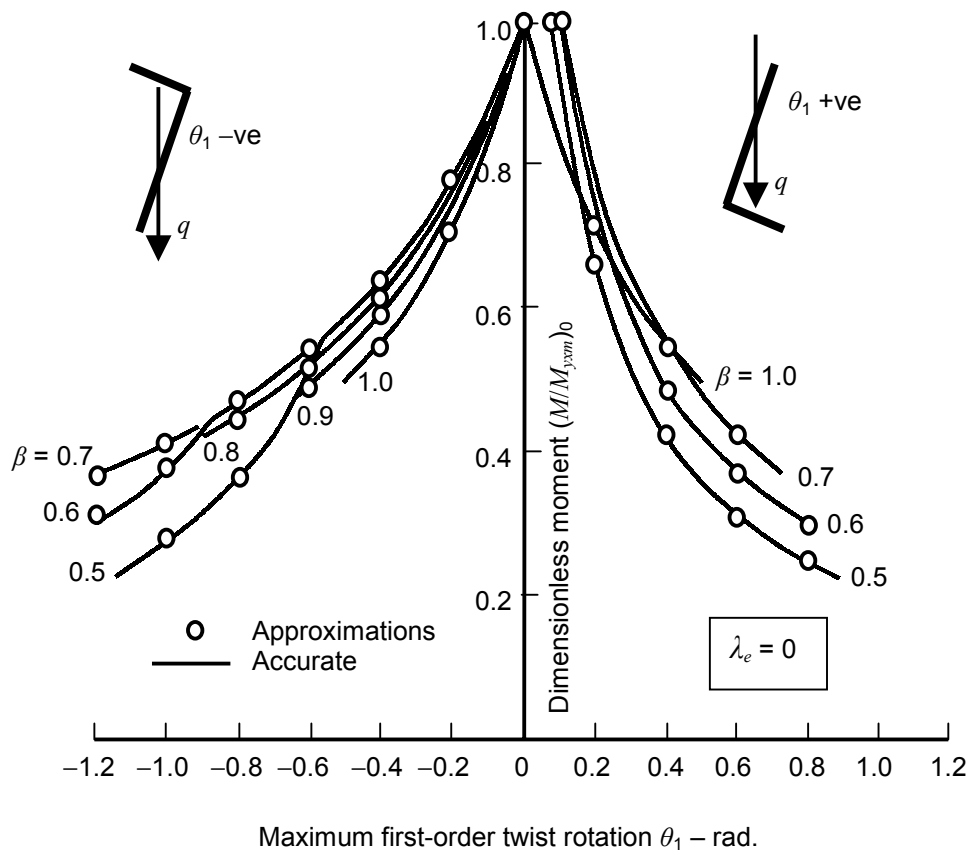
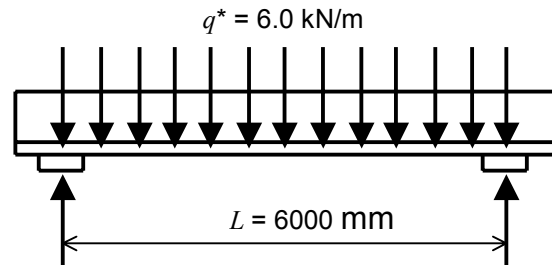


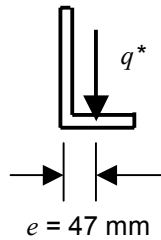
Fig. 12. First Yield Strengths for Zero Slenderness



(a) Elevation

150 x 100 x 12 UA
 ($b \times \beta \times t = 144 \times 94 \times 12$)

$E = 200,000 \text{ MPa}$
 $G = 80,000 \text{ MPa}$
 $f_y = 300 \text{ MPa}$



$\alpha = 23.91^\circ$
 $I_y = 1.314 \text{ E6 mm}^4$
 $J = 0.1371 \text{ E6 mm}^4$
 $y_o = 32.30 \text{ mm}$
 $\beta_x = -78.33 \text{ mm}$

(b) Section

Fig. 13. Example