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Research Report No R824

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This paper develops a simple advanced method of designing steel members against out-of-plane failure, in which reduced elastic moduli are used in an out-of-plane buckling analysis to model the effects of high moment, residual stresses and geometrical imperfections on yielding. The reduced moduli are derived from the basic beam and column strength curves of the Australian steel code AS4100 (SA, 1998).

The strengths predicted for simply supported beams in uniform bending are exactly the same as those of AS4100, while those for simply supported columns are extremely close. The strengths predicted for simply supported beam-columns with equal and opposite end moments are a little higher than the less conservative predictions of AS4100, and are very close to the basic beam and column strengths when these are plotted against a consistent generalized slenderness.

The strengths predicted for simply supported beams under double curvature bending are somewhat less than those of the AS4100 method of design by buckling analysis, while those for beams with central concentrated loads acting at or away from the shear centre are very close, and those for end restrained beams under uniform bending and for sway columns are generally a little higher.

While the method has been developed from and compared with the Australian code AS4100, it may be modified for any other modern code for the design of steel structures. It may be more widely applied to two-dimensional frames with in-plane loading, as part of a simple method of advanced analysis in which separate assessments are made of the in-plane and out-of plane strengths.

Keywords: Advanced analysis, beams, beam-columns, columns, design, imperfections, lateral buckling, residual stresses, steel, strength, yield.

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Department of Civil Engineering, Research Report R824
Out-of-Plane Advanced Analysis of Steel Structures

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Published by:
Department of Civil Engineering
The University of Sydney
Sydney NSW 2006
AUSTRALIA

September 2002

http://www.civil.usyd.edu.au

INTRODUCTION

In current structural steel design standards such as the Australian steel structures standard AS4100 (SA, 1998), the design of members against out-of-plane (lateral) buckling is founded on basic beam and column design curves which reduce the full section strength. This empirical approach is achieved by using slenderness reduction factors which were determined using the results of tests on full-scale members with practical values of residual stress and geometric imperfections (Trahair and Bradford, 1998).

The allowances made in AS4100 for other effects such as moment gradient, transverse loads acting away from the centroid, and elastic end restraints in beams, and elastic end restraints in columns require the use of moment modification factors and effective length factors in the design equations. For beam-columns, the beam and column strengths must be combined in empirical interaction equations which are of somewhat variable accuracy, and are often unnecessarily conservative.

With the advent of computer elastic buckling analysis programs for both in–plane and out-of-plane buckling which allow for these effects, it should not be necessary to use such artificial devices in the design process. However, the problem remains as to how to incorporate the effects of realistic residual stresses and geometric imperfections so that the buckling analyses will reproduce the basic beam and column strength design curves.

One way to achieve this is to perform inelastic buckling analyses in which the elastic moduli E, G are reduced to allow for imperfections and residual stresses as well as yielding. There has been some initial work in this area (Trahair and Kitipornchai, 1972 and Trahair, 1993) for the residual stresses in perfectly straight members, which follows from the earlier work of Galambos (1968). However, these analyses do not take account of geometric imperfections. More recently, similar reductions in the Young's modulus E have been used to reduce the in-plane stiffnesses used in simple advanced analyses of the second-order in-plane bending of steel frames (Kim and Chen, 1996a,b). The suggestion to reduce the out-of-plane stiffnesses by reducing the elastic moduli was made by Trahair and Chan (2002), who proposed that in the simple advanced analysis of two-dimensional frames with in-plane loading only, the out-of-plane analysis should be separated from the in-plane analysis. Similar suggestions and proposals have been made by Wongkaew and Chen (2002).

This paper provides a simple advanced out-of-plane analysis method in which the elastic moduli E, G are reduced within a finite element elastic flexural-torsional buckling analysis as implemented in the computer program PRFELB (Papangelis et al, 1993, 1998). The method can easily account for all the effects cited above without the need for effective lengths, moment modification factors, and interaction equations. The method is general and can reproduce the basic beam and column strength design curves of existing design standards. It can also be used to determine rational out-of-plane strengths for beam-columns, as an alternative to the conventional use of empirical interaction equations. In this paper, the Australian Standard AS4100 design curves for beams and columns have been used, although the curves of any design standard could be substituted.

The advantage of the method is that design software for steel structures can use the finite element buckling analysis directly without empirical adjustments of the results. The resulting inelastic buckling strengths accurately reproduce the current code basic design strengths for beams and columns, and closely approximate code predictions for other more complex cases.

The inelastic buckling method of this paper also provides a simple method of advanced analysis of two-dimensional steel frames against out-of-plane buckling. The examples analysed in this paper demonstrate that such an advanced analysis method can be consistent with the basic member design rules of current standards, while providing a rational and consistent method of designing a wide range of structures against out-of-plane failure.

BEAM STRENGTH

Nominal Strength

The AS4100 (SA, 1998) nominal lateral buckling moment strength M_b of a compact doubly symmetric I-section beam is given by

$$M_b = \alpha_m \, \alpha_s \, M_{px} \tag{1}$$

in which $M_{px} = f_y S_x$ is the major axis full plastic moment in which f_y is the yield stress and S_x is the plastic section modulus, α_m is a moment modification factor which allows for non-uniform moment distributions (α_m = 1.0 for uniform bending), and α_s is a slenderness reduction factor which allows for the effects of elastic buckling, initial crookedness and twist, and residual stresses, and which is given by (Trahair, 1993)

$$\alpha_s = 0.6\left\{\sqrt{\left[\left(\frac{M_{px}}{M_{yz}}\right)^2 + 3\right]} - \frac{M_{px}}{M_{yz}}\right\} \le 1.0 \tag{2}$$

in which M_{yz} is the elastic buckling moment of a simply supported beam in uniform bending given by

$$M_{yz} = \sqrt{\frac{\pi^2 E I_y}{L^2}\left(GJ + \frac{\pi^2 E I_w}{L^2}\right)} \tag{3}$$

in which E and G are the Young's and shear moduli of elasticity, I_y, J, and I_w are the minor axis second moment of area, the uniform torsion section constant, and the warping section constant, and L is the length of the beam.

The variation of the dimensionless AS4100 basic beam strength M_b / M_{px} with the modified slenderness $\sqrt{(M_{px} / M_{yz})}$ is shown in Fig. 1.

Reduced Elastic Moduli

The nominal beam moment strength M_b may also be obtained by making an inelastic lateral buckling analysis using inelastic moduli $E_I = \gamma_{IM} E$ and $G_I = \gamma_{IM} G$ which are reduced below their elastic values E and G to allow for the effects of initial crookedness and twist and residual stresses in reducing the elastic lateral buckling moments M_{yz} to the nominal strengths M_b. The reduction factor γ_{IM} is then obtained by setting the uniform bending inelastic buckling moment

$$M_I = \sqrt{\frac{\pi^2 \gamma_{IM} E I_y}{L^2}\left(\gamma_{IM} GJ + \frac{\pi^2 \gamma_{IM} E I_w}{L^2}\right)} \quad (4)$$

equal to M_b so that

$$M_b = M_I = \gamma_{IM} M_{yz} \quad (5)$$

whence

$$\gamma_{IM} = 0.9 - \frac{1}{1.2}\left(\frac{M}{M_{px}}\right)^2 \quad while \quad \left(\frac{M}{M_{px}}\right) \leq 1.0 \quad (6)$$

in which M is the bending moment at the cross-section.

These values of γ_{IM} are shown in Fig. 2 and compared with simplified versions γ_{IE}, γ_{IG} (Trahair, 1993) of the approximations used by Trahair and Kitipornchai (1972) for the influence of residual stresses (i.e. without initial crookedness and twist) on the inelastic lateral buckling of I-beams in uniform bending. The differences between γ_{IM} and γ_{IE} and γ_{IG} are due to the effects of initial crookedness and twist. When the values of γ_{IM} given by Equation 6 are used in Equation 5, then the AS4100 basic strengths shown in Fig. 1 are obtained.

This suggests that nominal strengths which are consistent with the design procedures of AS4100 can be predicted for other beams with different loading and restraint conditions by using an inelastic buckling analysis with reduced moduli $\gamma_{IM} E$ and $\gamma_{IM} G$ obtained using Equation 6.

Moment Gradient

The effectiveness of the inelastic buckling method of predicting the nominal strengths of beams under moment gradient has been investigated for the extreme example of a simply supported beam with equal end moments M which cause double curvature bending. The inelastic buckling of such a beam may be analysed by using the finite element computer program PRFELB (Papangelis et al, 1993, 1998) with a sufficiently large number of uniform property elements to approximate the variations of γ_{IM} with the moment along the beam. Inelastic buckling predictions for 250UB37.3 section beams (BHP, 1998) obtained using 20 elements are shown in Fig. 3. The properties of the 250UB37.3 are A = 4750 mm^2, I_x = 55.7E6 mm^4, I_y = 5.66E6 mm^4, J = 158E3 mm^4, I_w = 85.2E9 mm^6 and S_x = 486E3 mm^3, and the material properties are E = 2E5 MPa, G = 8E4 MPa, and f_y = 250 MPa, in which A is the area of the cross-section and I_x is the major axis second moment of area.

Also shown in Fig. 3 are the nominal strengths of AS4100 obtained by using the method of design by buckling analysis specified in Clause 5.6.4, in which the nominal strength is given by Equations 1 and 2 with M_{yz} replaced by

$$M_{oa} = M_{ob} / \alpha_m \qquad (7)$$

in which M_{ob} is the maximum moment at elastic buckling and the moment modification factor α_m is obtained from

$$\alpha_m = M_{os} / M_{yz} \qquad (8)$$

in which M_{os} is the maximum moment at elastic buckling of a beam without lateral rotation end restraints and with any transverse loads applied at the centroid (the values of M_{ob} and M_{os} may be obtained by using PRFELB, for example). It can be seen in Fig. 3 that the inelastic buckling predictions of the beam strengths are a little lower than the somewhat empirical values of AS4100.

Transverse Loads Acting At or Away From the Shear Centre

The effectiveness of the inelastic buckling method of predicting the strengths of beams with loads at different distances y_Q below the shear centre has been investigated for a simply supported beam with a central concentrated load Q, for which the maximum moment is $M_m = Q\,L / 4$. The inelastic buckling of such a beam may also be analysed by using the finite element computer program PRFELB (Papangelis et al, 1993, 1998) with a sufficiently large number of elements to approximate the variations of γ_{IM} with the moment along the beam. Inelastic buckling strength predictions for 250UB37.3 section beams (BHP, 1998) of varying length with γ_{IM} = 0.6 obtained using 20 elements are plotted in Fig. 4 against the dimensionless load height $2y_Q / h$ in which $h = 2\sqrt{(I_w / I_y)}$.

Also shown in Fig. 4 are the dimensionless nominal strengths M_b / M_{px} of AS4100 obtained by using the method of design by buckling analysis. It can be seen that the inelastic predictions of the beam strengths are very close to the values of AS4100.

Elastic End Restraints

The effectiveness of the inelastic buckling method of predicting the nominal strengths of beams with elastic end restraints has been investigated for a simply supported beam in uniform bending with equal elastic restraints against minor axis end rotation acting at each end of each flange.

The elastic buckling moment of such a beam is given by (Trahair, 1993)

$$M_{yzr} = \sqrt{\frac{\pi^2 EI_y}{(kL)^2}\left(GJ + \frac{\pi^2 EI_w}{(kL)^2}\right)} \qquad (9)$$

in which the elastic effective length factor k satisfies

$$\frac{\alpha_R L}{EI_y} = -\frac{\pi}{2k}\cot\frac{\pi}{2k} \qquad (10)$$

in which α_R is the stiffness of each elastic restraint.

The inelastic buckling of such a beam may also be analysed by using the finite element computer program PRFELB (Papangelis et al, 1993, 1998). Inelastic buckling predictions for 250UB37.3 section beams (BHP, 1998) with γ_{IM} = 0.6 obtained using 20 elements are shown in Fig. 5.

Also shown in Fig. 5 are the dimensionless nominal strengths M_b / M_{px} of AS4100 obtained by using the method of design by buckling analysis. It can be seen that the inelastic predictions of the beam strengths are a little higher than the values of AS4100. This is because elastic restraints are more effective for inelastic beams with reduced values of E_I than for elastic beams. It may be concluded that the AS4100 method of design by buckling analysis is a little conservative for beams with elastic end restraints.

COLUMN STRENGTH

Nominal Strength

The AS4100 (SA, 1998) minor axis flexural buckling nominal compression strength N_{cy} of a fully effective (against local buckling) I-section column is given by

$$N_{cy} = \alpha_c N_Y \qquad (11)$$

in which $N_Y = A f_y$ is the squash load in which A is the cross-sectional area, and α_c is a slenderness reduction factor which allows for the effects of elastic buckling, initial crookedness, and residual stresses, and which depends on the type of cross-section (there are are five groups of these in AS4100) and on the modified slenderness

$$\lambda_c = \sqrt{(N_Y / N_y)} \qquad (12)$$

in which

$$N_y = \pi^2 EI_y / L^2 \quad (13)$$

is the elastic minor axis flexural buckling load of the column. The variation of the dimensionless AS4100 basic column strength N_{cy} / N_Y with the modified slenderness λ_c for the central cross-section group is shown in Fig. 1.

Reduced Elastic Moduli

The nominal column compression strength N_{cy} may also be obtained by making an inelastic lateral buckling analysis using an inelastic modulus E_{IN} = γ_{IN} E which is reduced below its elastic value E to allow for the effects of initial crookedness and residual stresses in reducing the elastic flexural buckling load N_y to the nominal strength N_{cy}. The reduction factor γ_{IN} may be obtained by setting the inelastic buckling load

$$N_I = \frac{\pi^2 \alpha_{IN} EI_y}{L^2} \quad (14)$$

equal to N_{cy} so that

$$N_{cy} = N_I = \gamma_{IN} N_y \quad (15)$$

The variation of γ_{IN} obtained by this method with the value of N_{cy} / N_Y is shown in Fig. 2 and compared with the corresponding variation of γ_{IM} with M_b / M_{px} for beams. It can be seen that these variations are quite similar. The close approximation for γ_{IN} of

$$\gamma_{IN} = 0.98 - 0.57\,(N / N_Y) + 0.59\,(N / N_Y)^2 - 0.99\,(N / N_Y)^3 \quad (16)$$

in which N is the axial compression at the section is also shown in Fig. 2.

When the approximate values of γ_{IN} given by Equation 16 are used in Equation 15, then the close approximations to the AS4100 basic strengths shown in Fig. 1 are obtained.

Elastic End Restraints

The effectiveness of the inelastic buckling method of predicting the nominal strengths of columns with elastic end restraints has been investigated for an unbraced column with an elastic restraint against minor axis end rotation acting at the free end.

The elastic buckling load of such a column is given by

$$N_{yr} = \frac{\pi^2 EI_y}{(kL)^2} \tag{17}$$

in which the elastic effective length factor k satisfies

$$\frac{\alpha_R L}{EI_y} = \frac{\pi}{k} \tan \frac{\pi}{k} \tag{18}$$

in which α_R is the stiffness of the elastic restraint.

The inelastic buckling of such a column may be analysed by using the finite element computer program PRFELB (Papangelis et al, 1993, 1998). Inelastic buckling predictions for 250UB37.3 section columns (BHP, 1998) with γ_{IN} = 0.6 obtained using 20 elements are shown in Fig. 6.

Also shown in Fig. 6 are the dimensionless nominal strengths N_{cy} / N_Y of AS4100 obtained by using a method of design by buckling analysis. It can be seen that the inelastic predictions of the beam strengths are a little higher than the values of AS4100. This is because elastic restraints are more effective for inelastic columns with reduced values of E_I than for elastic columns. It may be concluded that the method of design by buckling analysis when used with AS4100 is a little conservative for columns with elastic end restraints, just as it was for beams with elastic end restraints.

BEAM-COLUMN STRENGTH

Nominal Strength

The general design rules of AS4100 (SA, 1998) for a beam-column under axial compression and in-plane bending can be used to formulate a lesser nominal out-of-plane moment strength M_{on} of

$$M_{on} = M_b\,(1 - N / N_{cy}) \tag{19}$$

although a more accurate and economical expression can be derived from a higher tier design rule (Clause 8.4.4) for beam-columns which are without transverse loads and which are fully effective against local buckling.

Reduced Elastic Moduli

The values of γ_{IM} for bending moment are close to those of γ_{IN} for axial compression, and so a reasonably simple compromise for a moment reduction factor γ_{IMN} for beam-columns can be obtained by using

$$\gamma_{IMN} = \frac{\gamma_{IN} + \gamma_{IM}(N_Y / N)(M / M_{px})}{1 + (N_Y / N)(M / M_{px})} \tag{20}$$

for the reduced modulus $E_I = \gamma_{IMN} E$, while the value of $\gamma_{IM}\, G$ may be used for G_I. In this, γ_{IM} is the value determined from Equation 6 using M / M_{pxr} for M / M_{px} and γ_{IN} is the value determined from Equation 16 using N / N_{Yr} for N / N_Y, in which M_{pxr} and N_{Yr} are the reduced values of M and N at full plasticity. AS4100 uses the full plastic approximation

$$M_{pxr} / M_{px} = 1.18\,(1 - N_{Yr} / N_Y) \leq 1 \tag{21}$$

Uniform Bending

Elastic behaviour

For a simply supported beam-column with axial compression loads N and equal and opposite end moments M, the first-order bending moments are amplified by second-order effects to a maximum moment approximated by

$$M_m = M / (1 - N / N_x) \tag{22}$$

in which

$$N_x = \pi^2 E I_x / L^2 \tag{23}$$

is the in-plane column elastic buckling load.

Elastic lateral buckling occurs when M and N satisfy (Vacharajittiphan et al, 1974, Trahair, 1993)

$$\left(\frac{M}{M_{yz}}\right)^2 = \left(1 - \frac{N}{N_x}\right)\left(1 - \frac{N}{N_y}\right)\left(1 - \frac{N}{N_z}\right) \tag{24}$$

in which

$$N_z = \frac{(GJ + \pi^2 E I_w / L^2)}{(I_x + I_y) / A} \tag{25}$$

is the torsional buckling load of the column.

The term $(1 - N / N_x)$ in Equation 24 allows for increases caused by the pre-buckling in-plane deflections as well as the decreases caused by the amplification of the moment from M to M_m. If the effects of the pre-buckling deflections are ignored, as they usually are, then Equation 24 may be replaced by

$$\left(\frac{M_m}{M_{yz}}\right)^2 = \left(1 - \frac{N}{N_y}\right)\left(1 - \frac{N}{N_z}\right) \tag{26}$$

Inelastic buckling strength approximations

The inelastic buckling of a simply supported beam-column may be analysed by using the finite element computer program PRFELB (Papangelis et al, 1993, 1998). Approximate inelastic buckling strength predictions for 250UB37.3 beam-columns with equal and opposite end moments are shown in Fig. 7 for the case where $M_m / M_{px} = N / N_Y$ so that $M_{pxr} / M_{px} = N_{Yr} / N_Y = 0.541$ (after using Equation 21). For these approximations, it was conservatively assumed that the in-plane bending moment was constant and equal to the maximum amplified value M_m given by Equation 22.

It can be seen that the inelastic buckling strength approximations are significantly higher than the very conservative values obtained from Equation 19, but only a little higher than the more accurate higher tier approximations derived from AS4100. It is of interest that the inelastic buckling beam-column strength approximations are very close to the AS4100 basic beam lateral buckling strength curve shown in Fig. 7 and therefore also to the AS4100 basic column flexural buckling strength curve (see Fig. 1).

CONCLUSIONS

This paper develops a simple advanced method of designing steel members against out-of-plane failure, in which reduced elastic moduli are used in an out-of-plane buckling analysis to simulate the effects of high moment, residual stresses and geometrical imperfections on yielding. The reduced moduli are derived from the basic beam and column strength curves of the Australian steel code AS4100 (SA, 1998). The method is verified by comparing its predictions of member strength with those of AS4100.

The strengths predicted for simply supported beams in uniform bending are exactly the same as those of AS4100, while those for simply supported columns are extremely close.

The strengths predicted for simply supported beams under the extreme moment gradient of double curvature bending are somewhat less than those of the AS4100 method of design by buckling analysis, which themselves are somewhat higher than those of the simpler AS4100 method of allowing for non-uniform bending. The predictions for beams with central concentrated loads acting at or away from the shear centre are very close to those of the AS4100 method of design by buckling analysis, which itself is more rational and consistent than the simpler AS4100 method of allowing for load height by using empirical effective length factors.

The strengths predicted for end-restrained beams under uniform bending are generally higher than those of the AS4100 method of design by buckling analysis, because the AS4100 procedure is based on the relative stiffness of the elastic restraints compared with that of the elastic beam, and so under-estimates the relative stiffness compared with that of the inelastic beam. Similar differences were found for end-restrained unbraced columns.

The strengths predicted for simply supported beam-columns with equal and opposite end moments are significantly higher than the more conservative predictions of AS4100, and a little higher than the less conservative predictions. The strengths predicted are very close to the basic beam and column strengths when these are plotted against a consistent generalized slenderness.

While the method has been developed from and compared with the Australian code AS4100, it may be modified for any other modern code for the design of steel structures.

The method may be more widely applied to two-dimensional frames with in-plane loading, as part of a simple method of advanced analysis in which separate assessments are made of the in-plane and out-of plane strengths.

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APPENDIX II NOTATION

A	cross-sectional area
E	Young's modulus of elasticity
E_I	inelastic Young's modulus
E_{IN}	column inelastic Young's modulus
f_y	yield stress
G	shear modulus of elasticity
G_I	inelastic shear modulus
h	$= \sqrt{(I_w / I_y)}$
I_w	warping section constant
I_x, I_y	second moments of area about the x, y axes
J	torsion section constant
k	effective length factor
L	member length
M	applied moment
M_b	nominal member moment capacity
M_I	inelastic buckling moment
M_m	maximum bending moment
M_{oa}	$= M_{ob} / \alpha_m$
M_{ob}	maximum moment at elastic buckling
M_{on}	nominal out-of-plane moment capacity
M_{os}	modified value of M_{ob}
M_{px}	full plastic moment about the x axis
M_{pxr}	reduced full plastic moment about the x axis
M_{yz}	elastic buckling moment of a beam in uniform bending
M_{yzr}	value of M_{yz} for a beam with end restraints
N	axial compression
N_{cy}	column minor axis nominal strength
N_I	column inelastic buckling load
N_x, N_y	column elastic flexural buckling loads
N_{yr}	value of N_z for a column with end restraints
N_Y	column squash load
N_{Yr}	reduced column squash load
N_z	column elastic torsional buckling load
Q	beam transverse load
S_x	major axis plastic section modulus
x, y	cross-section principal axes
y_Q	distance of load below shear centre
z	distance along a member
α_c	column slenderness reduction factor
α_m	moment modification factor
α_R	restraint stiffness
α_s	beam slenderness reduction factor
γ_{IE}, γ_{IG}	inelastic modulus reduction factors
γ_{IM}, γ_{IN}	beam and column inelastic modulus reduction factors
γ_{IMN}	beam-column inelastic modulus reduction factor
λ_c	column modified slenderness

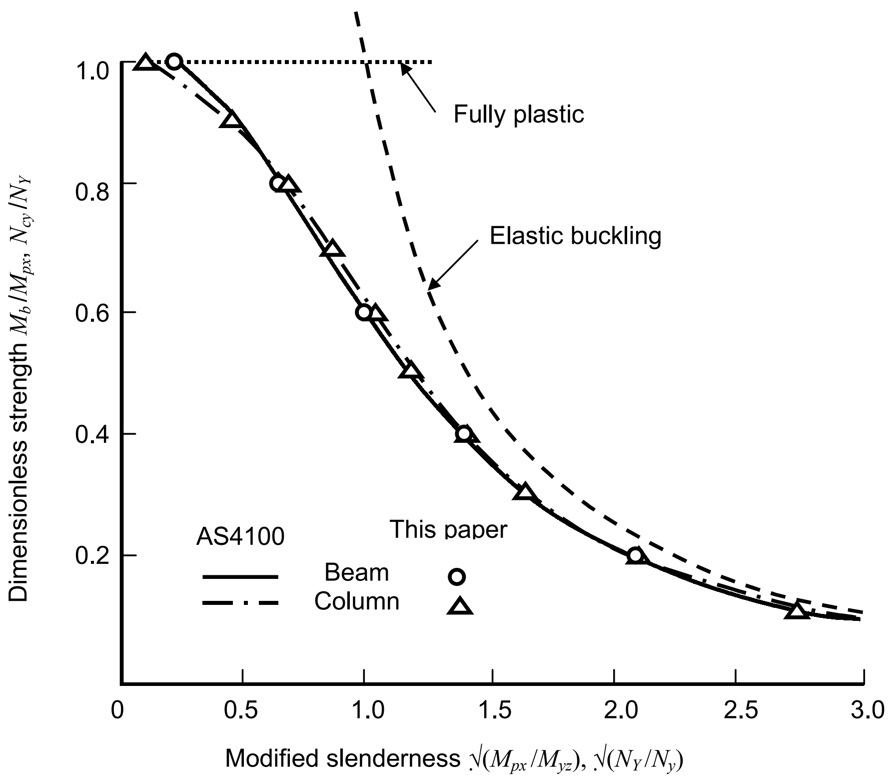

Fig. 1. Basic Beam and Column Strengths of AS4100

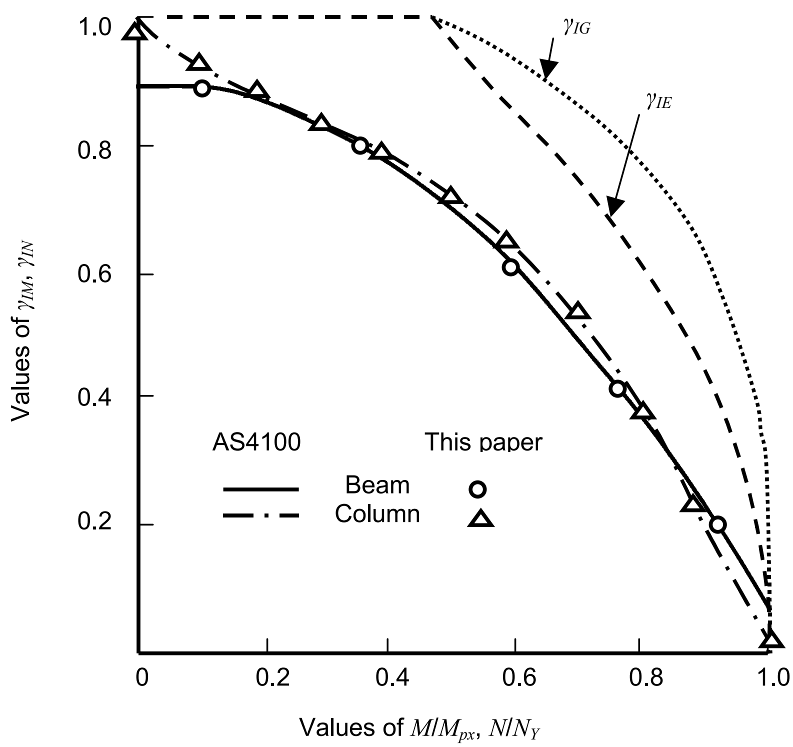

Fig. 2 Reduced Elastic Moduli Factors for Beams and Columns

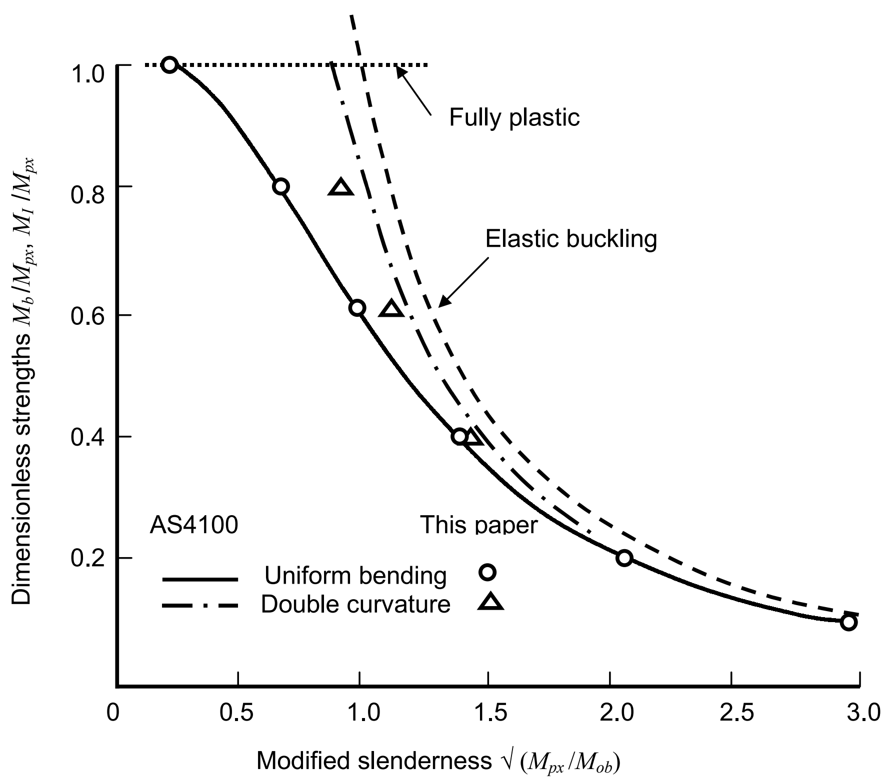

Fig. 3. Strengths of Beams in Double Curvature Bending

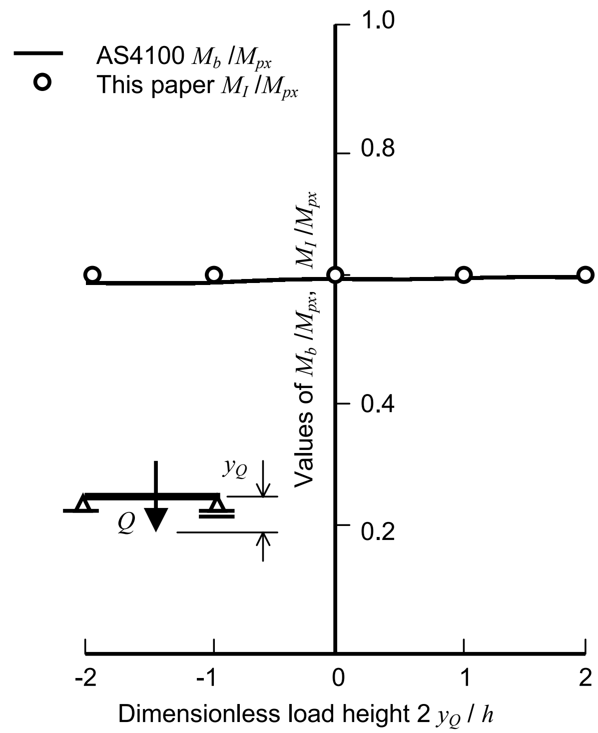

Fig. 4. Effect of Load Height on Strengths of Beams with Central Loads

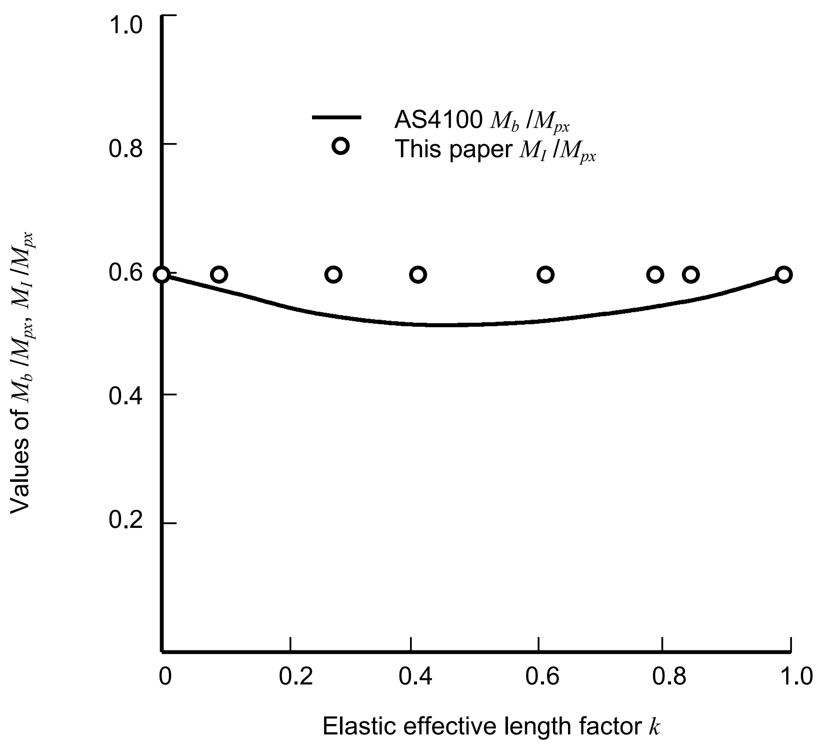

Fig. 5 Effect of End Restraints on Uniform Bending Beam Strength

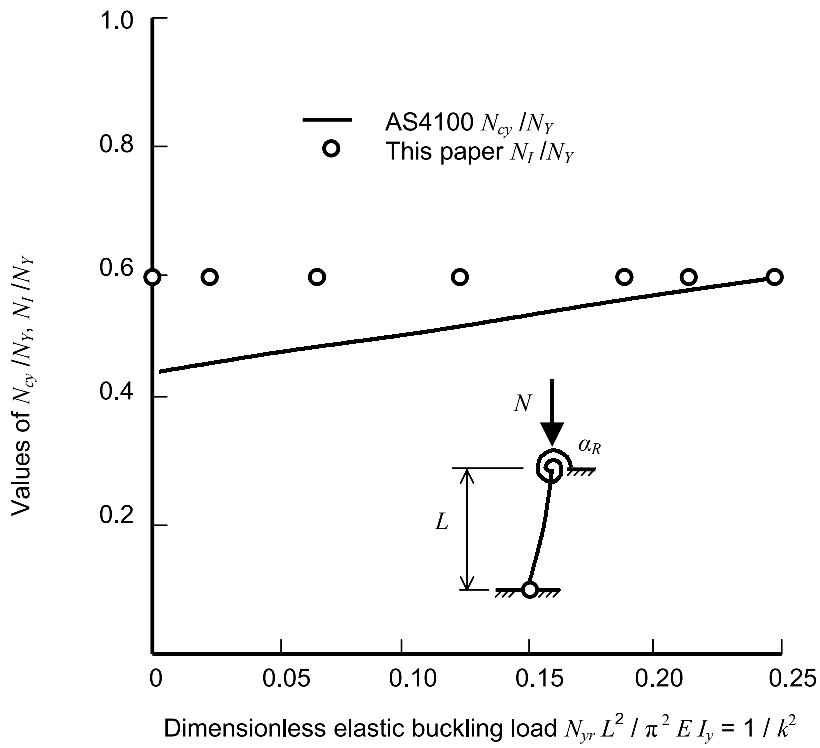

Fig. 6 Effect of End Restraint on Unbraced Column Strength

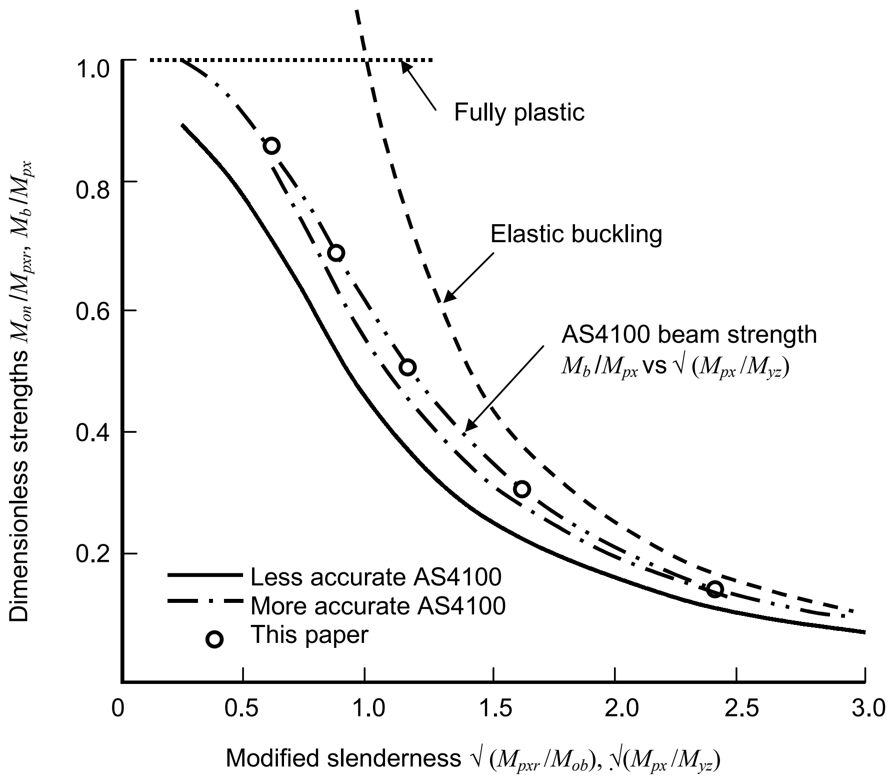

Fig. 7. Strengths of Beam-Columns in Uniform Bending