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Research Report No R823

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Abstract:

Attempts to extend practical advanced analyses of the in-plane behaviour of 2-D steel frames under in-plane loading to out-of-plane behaviour have been largely unsuccessful because of difficulties in modeling the effects of yielded zones, load heights, interactions between twist and axial force and moments, and end warping restraints.

Practical advanced analyses of out-of-plane behaviour need to be able to account for the influences of moment and axial force distributions, load heights, and end restraints on elastic and inelastic lateral buckling, and to be consistent with the code formulations of beam and column out-of-plane strengths.

It is proposed in this paper that the advanced analysis of 2-D frames for which local buckling is prevented be simplified by first carrying out an in-plane analysis using one of the presently available plastic hinge methods, and then by using a practical advanced analysis of the out-of-plane behaviour which is based on an inelastic lateral buckling analysis which includes allowances for residual stresses and initial crookednesses and twists. The paper makes a number of suggestions of how to develop such a practical method of advanced out-of-plane analysis for 2-D frames for which local buckling is prevented.

Keywords:
Advanced analysis, frames, design, elasticity, imperfections, lateral buckling, plasticity, steel, strength.

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1. INTRODUCTION

Advanced analysis methods, first permitted in the 1990 edition of the Australian steel structures standard AS4100 [1], simplify the design of two-dimensional (2-D) steel frames for which local and lateral buckling are prevented. However, attempts to extend these to 2-D frames which may fail by lateral buckling [2] have been unsuccessful for a number of reasons.

Advanced analysis methods [3, 4, 5] are intended to provide accurate predictions of frame behaviour. When these are available, then the frame can be assumed to be satisfactory if the analysis shows that it can remain in stable equilibrium under the design loading. Thus advanced analysis allows the designer to by-pass code design rules, which are generally approximate and somewhat variable in their accuracy, may not be applicable to some special structures like space frames and pre-tensioned trusses, and are often difficult to interpret appropriately.

A hierarchy of advanced methods of analysis is shown in Table 1, ranging from those for 2-D frames under 2-D loading which fail in-plane by yielding (2F-2L-IP-Y) to 3-D frames under 3-D loading which fail by the combined effects of yielding under biaxial bending and torsion with local buckling (3F-3L-BBT-LB). Almost all frames are 3-D with 3-D loading, but it is for frames that can be considered as 2-D with 2-D loading that advanced methods of analysis were first developed. Even these were limited to frames for which local (LB) and lateral (FTB) buckling are prevented, so that failure is in-plane (IP) by excessive yielding caused by the combination of primary actions (first-order moments and axial forces) and second-order moment components resulting from in-plane instability effects. The extension of these methods to 3-D frames for which local and lateral buckling are not important, such as for 3-D frames with compact tubular members, is straightforward.

Two types of advanced analysis have been developed for 2-D frames for which local and lateral buckling are prevented. Plastic zone methods allow for geometric (in-plane instability) and material (yielding) non-linearities, residual stresses, and geometrical imperfections, and account for the spread of the yielded zones through the cross-sections and along the members of the frames [6, 4]. However, while this method is extremely accurate, it is computationally very complex, wasteful, and slow. Plastic zone methods have been used to develop benchmark solutions for testing the more efficient but less accurate plastic hinge methods [3, 8, 9, 10, 11], in which yielding effects are concentrated at a few cross-sections, and residual stress and geometrical imperfection effects are approximated by using reduced stiffnesses. The computational efficiency of the plastic hinge method is enhanced when the effects of geometrical non-linearities can be accurately allowed for by using one element per member [12, 13]. Some attempts have been made to extend these plastic hinge methods to allow for semi-rigid rather than rigid joints between members [14, 15, 16].

Attempts [17] have also been made to extend the plastic hinge methods to frames whose members may undergo local buckling (2F-2L-IP-LB in Table 1). However, these cannot be applied generally unless the post-local buckling behaviour (PLB) can be predicted, and so they are usually limited to frames whose design loads do not exceed those which cause the section local buckling strength to be first reached.

Table 1. Advanced Methods of Analysis

Method	Frame	Loading	Failure	Mode	Special
2F-2L-IP-Y	2-D	2-D	IP	Y	IPB
2F-2L-IP-LB	2-D	2-D	IP	LB	PLB
2F-2L-OOP-FTB	2-D	2-D	OOP	FTB	Y, W
2F-3L-BBT-Y	2-D	3-D	BBT	Y	Torsion
2F-3L-BBT-LB	2-D	3-D	BBT	LB	PLB
3F-3L-BBT-Y	3-D	3-D	BBT	Y	Torsion
3F-3L-BBT-LB	3-D	3-D	BBT	LB	PLB

F Í Frame
L Í Loading
D Í Dimensional
IP Í In-Plane
OOP Í Out-Of-Plane
BBT Í Biaxial Bending and Torsion
Y Í Yielding
LB Í Local Buckling
FTB Í lateral Buckling
IPB Í In-Plane Buckling
PLB Í Post Local Buckling
W Í Warping Restraint

There have been recent attempts to develop advanced analyses either of the out-of-plane failure of 2-D frames with in-plane loading, or of the biaxial bending and torsion of 3-D frames under 3-D loading which might be expected to be able to analyse out-of-plane failure.

Some of the 3-D analyses are restricted, such as those of [17] and [18], which omit warping torsion and member out-of-plane buckling. The 3-D analysis in [19] appears to be the most advanced, being a plastic zone analysis which includes warping torsion, member out-of-plane buckling, and an awareness of the need to consider warping continuity at joints.

There are a number of simpler advanced analyses which use line elements instead of considering the spread of plasticity in plastic zones. The most advanced of these appears to be that in [11]. Although this appears to ignore warping torsion, it does include a rigorous approach to the second-order effects associated with axial force and bending moment. What appears to be a hybrid approach to the effects of lateral buckling in a 3-D second-order plastic hinge analysis is used in [20] and [21], in which axial force second-order effects are considered directly, but bending moment second-order effects are approximated by using reduced full plastic moments.

A more logical method is used in [22], in which the out-of-plane analysis of a 2-D frame under in-plane loading is separated from the in-plane analysis. An inelastic buckling analysis using reduced elastic moduli is then used to check the out-of-plane strength.

A common shortcoming among these methods of advanced analysis is the absence of a thorough verification of the method, in which the predictions of the method for examples dominated by out-of-plane buckling behaviour are compared either with available plastic zone advanced analyses or with accepted design code approximations. An exception is that in [23] which verified a method of predicting member out-of-plane strength by comparison with design code predictions for beams with moment gradient or off-axis loads or end restraints, simply supported braced or elastically restrained unbraced columns, and beam-columns.

It can be said that attempts to extend advanced analysis methods to 2-D frames which may fail by lateral buckling out of the plane of the frame (2F-2L-OOP-FTB in Table 1) have been somewhat less than satisfactory for two reasons. The first is associated with difficulties in modeling the effects of yielded zones on the lateral buckling behaviour, and while accurate advanced analyses have been made for individual members [24, 25], the extension of these to frames leads to prohibitively long and time consuming computations. The second reason is that it is not easy to include the effects of warping torsion and to model accurately the warping restraint conditions at joints [26, 27].

The purpose of this paper is to set out what these difficulties are, so that future attempts to develop advanced analyses for the lateral buckling of 2-D frames for which local buckling is prevented (2F-2L-OOP-FTB) can systematically assess these difficulties and develop methods of overcoming them.

2. IN-PLANE ADVANCED ANALYSIS OF 2-D FRAMES

2.1 Methods of Analysis

In the plastic zone method of advanced analysis, the cross-section of a compact member is divided into a mesh of finite sub-areas. The section tangent stiffness is formed by summing the elastic properties of these sub-areas, and the section resisting force and moment are determined by summing the effects of the resisting forces of the sub-areas. Iterations are carried out to dissipate the unbalanced forces between the applied forces and the internal resistances using the Newton-Raphson family of methods. This procedure is considered to be 'exact' by many researchers [28].

In the plastic hinge method of advanced analysis, yielding is concentrated at a small number of locations. Springs which model the section tangent stiffness are used to simulate the effect of plastic hinges. When the moment at a section is smaller than the plastic moment at the section, the spring stiffness is infinite. When the moment at the section reaches the plastic moment, the tangent spring stiffness is zero so that no further moment can be taken by the section. The moment at the section is then kept constant and equal to the section plastic moment. The plastic hinge method avoids the need to divide a cross-section into many sub-areas, and so it is simpler to use and computationally much more efficient than the plastic zone method.

The results of the two methods are similar, except when the bending moment is nearly uniform with a wide spread of plasticity. In most practical cases, the plastic hinge method is considered to be adequate.

When dealing with the design of steel structures in practice, three important considerations are acceptable accuracy, simplicity and efficiency. The advanced methods of analysis are of significantly higher accuracy than the conventional linear analysis method with design based on prescriptive design code rules.

Some advanced analyses which need to use at least two elements per member to determine elastic buckling loads are less efficient than linear analyses which use one element per member. In some cases when a higher accuracy is required for analysis of structures where the location of plastic hinge is not at the two ends, more than two elements are needed. For example, the most critical section of a propped cantilever column is not at the mid-span nor at the end, and more elements are needed to locate the point of maximum moment. Greater efficiency is achieved by using a single element per member which closely models the moment variation along the member. This greater efficiency is essential for large practical frames with many members in terms of computer time, since, unlike a linear analysis, an advanced analysis involves many iterations and factorisations of the tangent stiffness matrix. The simplicity of the linear elastic method is maintained by advanced analyses which use the same discretisation of the frame without further member division into more elements, since it is a normal practice for the designer to first carry out a linear analysis for the checking of proper modeling before undertaking the advanced analysis.

With the advance in the power of personal computers, many researchers consider the size problem posed by large structures will no longer be an issue, so that the use of more than one element per member will be satisfactory. This is partly due to their use of very simple 2-D frames of less than twenty members for their worked examples.

An added advantage of using fewer and more accurate elements is an improved numerical stability during iterations. When a greater number of elements is used, the stiffness matrix is larger, which leads to greater truncation errors and a higher possibility of divergence during iterations, especially for very large structures.

2.2 Calibration

Column buckling strength curves form the basis for code design against instability effects in 2-D steel structures, and so it is natural and desirable to calibrate an advanced analysis method against these buckling curves. For many codes, the simple Perry-Robertson formula [29] is used, with different curves used for columns with different geometrical properties, rolling processes, etc.

Possibly the severest but simplest test of a single element per member analysis is to use it to model elastic flexural buckling under various boundary conditions. Further, it should also contain an initial imperfection in its formulation so that the buckling strength can be calibrated against code curves. The element formulated in [12, 30] is curved and of sufficient accuracy to predict the buckling strength loads with an error of less than 1%. For example, an imperfection of v_{mo} equal to 1/1000 of member length should be used for hot-rolled hollow sections to order to obtain a very close lower bounds to the “a” buckling strength curve of BS5950 [31] shown in Fig.1.

3. ELASTIC LATERAL BUCKLING

3.1 Beams

The elastic lateral buckling resistance of a beam under in-plane bending depends principally on the elastic moduli, the beam's section properties and length, the bending moment distribution, the height of the loads above the shear centre, and the restraints. For a simply supported doubly symmetric I-section beam, the values of the equal and opposite end moments M which cause buckling are given by [32, 33]

$$M = M_{yz} = \sqrt{\frac{\pi^2 EI_y}{L^2}\left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \tag{1}$$

in which E and G are the Young's and shear moduli of elasticity, I_y is the minor y axis second moment of area, I_w and J are the warping and torsion section constants, and L is the length of the beam. Dimensionless values of M are shown in Fig. 2. The lateral buckling of beams of monosymmetric and asymmetric cross-section is discussed in [33].

For other bending moment distributions, the maximum moment at elastic buckling may be approximated by

$$M = \alpha_m M_{yz} \tag{2}$$

in which the moment modification factor α_m depends on the bending moment distribution caused by the beam loading. For beams with unequal end moments M and βM, α_m may be approximated by using

$$\alpha_m = 1.75 + 1.05\,\beta + 0.3\,\beta^2 \le 2.5 \tag{3}$$

Dimensionless values of M calculated using this approximation are also shown in Fig. 2. For uniformly distributed load, α_m = 1.13 and for central concentrated load α_m = 1.35. Values of α_m for some other loading arrangements are given in [33].

For beams with central concentrated loads which act at distances y_Q below the shear centre, the approximate maximum moment at elastic buckling may be obtained from

$$\frac{M}{\alpha_m M_{yz}} = \sqrt{\left[1 + \left(\frac{0.4\alpha_m y_Q}{M_{yz}/P_y}\right)^2\right]} + \frac{0.4\alpha_m y_Q}{M_{yz}/P_y} \tag{4}$$

in which $$P_y = \pi^2 E\, I_y / L^2 \tag{5}$$

as shown in Fig. 3.

For beams with elastic end restraints against minor axis rotation and warping, the equal and opposite end moments which cause elastic buckling may be approximated by

$$M = \sqrt{\frac{\pi^2 EI_y}{(k_y L)^2}\left(GJ + \frac{\pi^2 EI_w}{(k_w L)^2}\right)} \tag{6}$$

in which k_y and k_w are minor axis rotation and warping effective length factors which vary between 1.0 (zero end restraints) and 0.5 (rigid end restraints), as shown in Fig. 4. Other types of restraint are restraints against lateral deflection and against twist rotation, and may act at any point along the beam. Restraints may be continuous as well as concentrated. The analysis of and effects of restraints are discussed in [33].

The elastic lateral buckling resistance of a beam under in-plane bending depends to a lesser extent on pre-buckling and post-buckling effects and on distortion of the cross-section. Pre-buckling in-plane deflections transform the beam into a "negative" arch and increase the resistance to buckling. This effect is small except for sections with large values of I_y (comparable with the major axis second moment of area I_x), and is usually ignored. Pre-buckling effects have been studied in [34 - 36].

Post-buckling effects generally increase the strength of a beam, but the effects are very small, except for very slender indeterminate beams. These effects are also usually ignored. Post-buckling effects were studied in [37].

Distortion of the cross-section causes only a small reduction in the resistance to lateral buckling, except when the web is very flexible and the flanges have high torsional stiffness, as for a hollow flange beam. In this case, bending of the flexible web allows relative lateral displacement of the flanges without a full mobilization of their torsional stiffnesses, and the effective torsion section constant of the beam is reduced. Distortional buckling is discussed in [38, 39].

3.2 Beam-Columns

The elastic lateral buckling moment of a simply supported beam-column under axial loads P may be approximated by using [34, 33]

$$\left(\frac{M}{M_{yz}}\right)^2 = \left(1 - \frac{P}{P_x}\right)\left(1 - \frac{P}{P_y}\right)\left(1 - \frac{P}{P_z}\right) \tag{7}$$

in which

$$P_x = \pi^2 E I_x / L^2 \tag{8}$$

is the major axis flexural buckling load of the column, and

$$P_z = \frac{(GJ + \pi^2 EI_w / L^2)}{(I_x + I_y)/A} \tag{9}$$

is the torsional buckling load of the column, in which A is the cross-sectional area.

The term (1 – P / P_x) in Equation 7 allows for an increase in the maximum in-plane moment to M / (1 – P / P_x). It also allows for the effects of pre-buckling deflections, which are often ignored. This term is often small, and is commonly ignored, as it is in Fig. 5. The elastic buckling of other beam-columns is discussed in [33].

3.3 Frames

There appears to have been very few studies of the elastic lateral buckling of 2-D frames, but some of these are discussed in [33]. A finite element method of lateral buckling analysis is presented in [33], which forms the basis of the computer program PRFELB [40, 41]. This can be used to predict the elastic buckling of 2-D frames for which the warping restraint conditions at the joints can be approximated as being unrestrained, or continuous, or prevented.

More generally, the warping deformations will be elastically restrained. At the right angle joint shown in Fig. 6, warping rotations of the flanges of one member correspond to distortional rotations of the flanges of the perpendicular member, and the bimoment acting on the first member corresponds to distortional moments acting on the flanges of the other. Thus it could be expected that a rational finite element method of analysing the lateral buckling of frames with joints of this type would include distortional degrees of freedom. This would have the advantage of automatically allowing for the often neglected effects of general distortion on lateral buckling.

A less precise method of allowing for the warping conditions at joints would be to approximate the relative stiffness of the warping restraints acting at the end of each member framing into the joint. Some approximations have been developed in [26, 27]. The effects of these restraints may be allowed for in the computer program PRFELB [40, 41].

4. INELASTIC LATERAL BUCKLING

4.1 Inelastic Buckling

The inelastic lateral buckling moments of compact beams are reduced below the elastic buckling moments because yielding reduces the effective flexural, torsional, and warping rigidities $(EI_y)_e$, $(GJ)_e$, $(EI_w)_e$ [42, 33]. The buckling moment reductions depend not only on the extent of yielding of the cross-section (which is related to the magnitude of the maximum moment as it increases from the first yield moment towards the full plastic moment M_{px}), but also on the distribution of yielding along the beam length (which depends on the bending moment distribution). The greatest reduction occurs for uniform bending for which the yielded regions extend along the whole length of the beam, while yielding near mid-span (as under central concentrated load) causes greater reductions than does yielding near the supports (as in beams in double curvature bending). The inelastic buckling moments M_I in statically determinate simply supported I-beams with unequal end moments M and βM were analysed in [42] and approximated by

$$\frac{M_I}{M_{px}} = 0.7 + \frac{0.3(1 - 0.7M_{px} / \alpha_m M_{yz})}{(0.61 - 0.3\beta + 0.07\beta^2)} \leq \frac{\alpha_m M_{yz}}{M_{px}} \tag{10}$$

This approximation is shown in Fig. 7.

In indeterminate continuous beams, there is an inelastic in-plane moment redistribution which affects inelastic lateral buckling, in some cases favourably [43].

4.2 Strength

The strength of a real beam is reduced below its elastic and inelastic buckling moments by the effects of residual stresses and initial crookedness and twist. Design rules (such as those of the Australian code AS4100 [1]) for beam lateral buckling strengths are usually based on considerations of all of these effects and on test results [29]. For the AS4100, the nominal moment strength of a compact beam is given by

$$M_b = \alpha_m \, \alpha_s \, M_{px} \tag{11}$$

in which the slenderness reduction coefficient α_s is given by

$$\alpha_s = 0.6\left\{\sqrt{\left[\left(\frac{M_{px}}{M_{yz}}\right)^2 + 3\right]} - \frac{M_{px}}{M_{yz}}\right\} \leq 1 \tag{12}$$

These strengths are also shown in Fig. 7.

These strength approximations may be unnecessarily conservative for very slender beams, for which it has been suggested [44, 45] that the minimum moment strength of a compact beam is equal to its minor axis full plastic moment M_{py}, corresponding to the situation where the twist rotation of the maximum moment section of the beam approaches 90^o.

5. PLASTIC ZONE OUT-OF-PLANE ADVANCED ANALYSIS OF 2-D FRAMES

5.1 Beams

Plastic zone advanced analyses have been made of the out-of-plane behaviour of simply supported hot-rolled compact I-beams with residual stresses and initial crookedness and twist [24, 25]. Small shears due to bending and warping were ignored, but large deflections and rotations were allowed for, so that pre- and post-buckling effects could be included. Tri-linear (elastic, plastic, strain-hardening) material properties were assumed with associated flow and isotropic hardening rules and the von Mises yield criterion, and the possibility of unloading was allowed for. Similar analyses were made of simply supported cold-formed channel and zed beams [46, 47].

These analyses were made for research purposes, and are unsuitable for the practical design of steel structures by advanced analysis because of their complexity and demands on computer capacity and time. Nevertheless, the results of these analyses provide suitable benchmarks for testing more practical methods of advanced analysis.

There does not appear to have been any plastic zone advanced analyses made of indeterminate beams.

5.2 Beam-Columns and Frames

Plastic zone advanced analyses have been made of the in-plane behaviour of simply supported beam-columns [25].

However, there appears to have been few corresponding analyses made of the out-of-plane behaviour of beam-columns [48], and none of frames. Such analyses need to be developed so that they can be used to provide benchmark solutions for plastic hinge advanced analyses.

6. PRACTICAL OUT-OF-PLANE ADVANCED ANALYSIS OF 2-D FRAMES

6.1 General

The following sub-sections discuss a number of proposals that could be considered for the development of a practical out-of-plane advanced analysis of 2-D frames for which local buckling is prevented.

6.2 Separation of In-Plane and Out-of Plane Analyses

Strictly speaking, a 2-D frame in which the members have initial crookednesses in both principal planes and initial twists undergoes biaxial bending and torsion. However, it appears at present to be too difficult to develop a practical method of advanced analysis for biaxial bending and torsion.

In view of the difficulty in dealing rigorously with all modes of failure, it is therefore suggested that the analysis should be simplified by separating it into two stages, first a plastic hinge advanced in-plane analysis, and then an advanced out-of-plane analysis. The bases for this suggestion are first the similar separation that can be made when the members are perfectly straight and untwisted, and second the corresponding separation that is allowed by codes such as AS4100 [1] for routine design, in which separate checks are made of the in-plane and out-of-plane member capacities. Such a separation has been made in [22, 23].

This proposed separation has the advantage that presently available plastic hinge advanced methods of in-plane analysis of 2-D frames for which local buckling is prevented can be used for the first stage, so that only the second stage for the out-of-plane analysis needs to be developed. Further, the moment and axial force distributions found from the in-plane analysis can be used directly in the out-of-plane analysis. Similar procedures are used in the computer program PRFELB [40, 41] for the elastic lateral buckling of 2-D frames.

In the typical design procedure, a known structure is checked to determine if it can reach equilibrium under a known set of loads. The first stage would be to carry out the plastic hinge advanced in-plane analysis, to check whether the structure can resist the loads by in-plane behaviour. If it cannot, then the structure is inadequate and must be modified. If it can, then the second stage out-of-plane buckling analysis would be carried out, to check whether the structure has sufficient out-of-plane resistance. If it does not, then the structure is inadequate and must be modified. If it does, then the structure is adequate.

This proposed separation can be easily implemented by activating the out-of-plane check when the Newton Raphson incremental-iterative in-plane analysis has converged.

6.3 Advanced Out-of-Plane Analysis

The advanced out-of-plane analysis proposed is of a rather different nature to the plastic hinge in-plane analysis. The in-plane analysis is essentially an elastic bending analysis of the structure under the primary actions of the in-plane loads, with modifications that allow for the reduction of the cross-section capacity caused by axial force and yielding, and for the reductions in flexural rigidity caused by residual stresses, yielding, initial crookedness, and in-plane stability effects.

However, for the out-of-plane analysis, there are no primary out-of-plane actions, and the dominant behaviour is one of lateral buckling of the members out of the plane of the frame. Consequently, it is appropriate for the out-of-plane analysis to be a buckling analysis rather than a bending analysis. If this is to be the case, then all out-of-plane initial crookednesses and twists must be removed (so that a buckling analysis is appropriate), but their effects must be allowed for in some way. Such a buckling analysis has been used in [22, 23].

The out-of-plane buckling analysis may be formulated as a finite element eigen-value problem using

$$\{\Delta\}^{T}\,[K_L - \lambda\, K_G]\,\{\Delta\} = 0 \qquad (13)$$

in which $[K_L]$ is the out-of-plane linear stiffness matrix, $[K_G]$ is the out-of-plane stability (or the geometric stiffness) matrix for an initial set of axial forces and moments, λ is the buckling load factor, and $\{\Delta\}$ is the vector of the out-of-plane deformations. The stiffness matrix will include the effects of any reductions caused by yielding (Fig. 7), while the stability matrix will allow for the moment distribution (Fig. 2) and any load height effects (Fig. 3), and also for the inelastic redistributions of the axial forces and bending moments. Continuity between appropriate out-of-plane nodal deformations will account for the effects of end restraints (Fig. 4).

The use of an eigen-value buckling analysis of Equation 13 is simple to conduct because it avoids the complications of finite and large twist rotations which violate the vectorial addition assumption and which often lead to numerical difficulties.

The use of a suitable element in a finite element buckling analysis will automatically allow for the effects of the distribution of the axial forces and in-plane moments on both elastic (Fig. 2) and inelastic (Fig. 7) buckling, and also for the effects of end restraints.

Some suggestions for the development of a practical advanced out-of plane buckling analysis are made in the following sub-sections.

6.4 One Element per Member

It has already been noted that the significant practical advantage that the plastic hinge method of advanced in-plane analysis has over the plastic zone method is that it only requires one element per member, thereby greatly reducing computer storage and time limitations that would otherwise make advanced analyses unsuitable for medium to large structures. There would be a similar practical advantage if the out-of plane buckling analysis is able to model structural behaviour adequately using one element per member.

The development of a suitable element for a one element per member analysis of out-of-plane buckling may prove to be a significant challenge, since finite element computer programs which use cubic deformation fields for elastic lateral buckling analysis (such as PRFELB [40, 41]) typically require at least 2 and often 4 elements per member to achieve satisfactory accuracy. On the other hand, it has noted [49] that cubic elements are not as inefficient as some have thought, and that future increases in computer power may reduce the desirability of using one element per member.

The availability of a robust one element per member will make for significant convenience both in data preparation and in analysis. It seems likely that, at least in the near future, designers will still want to compare the results of the advanced analysis with those of a linear analysis in order to make sure the results are not unexpected. The use of one element per member will facilitate the comparison.

It is suggested that the element formulation should be simplified by ignoring the effects of distortion, since these will usually be very small. This will mean that the effects of end warping restraints (Fig. 6) will have to be allowed for by using approximations such as those of [26, 27]. This is because the transfer of warping bimoments from one member to another is joint detail dependent, and a significant research project will be needed to quantify more accurately the transfers for different joint types. This problem is even more complicated in three-dimensional frames with members intersecting at oblique angles.

6.5 Ignore Pre- and Post-Buckling Effects

Elastic pre- and post-buckling effects are known to increase the out-of-plane strengths of beams, and in any case, these effects are often small. It is therefore suggested that these effects should conservatively be ignored, thus simplifying the out-of-plane buckling analysis.

6.6 Allow for Imperfections

The effects of material (residual stresses) and geometrical (initial crookedness and twist) imperfections should be approximated simply in the out-of-plane analysis. One method may be to reduce the effective moduli E_e, G_e below the elastic values E and G in a similar way to that used in [50, 21] in a plastic hinge method of advanced in-plane analysis.

There may be some limitations on the application of this method to in-plane analysis, as for example for some slender members where the bowing effect is significant, in which case neglecting geometrical shortening due to bowing cannot simulate the geometrical change and the subsequent force re-distribution. For example, when a slender compression member is near buckling, its axial stiffness reduces and most of its compression force is shed to adjacent members. This force re-distribution process varies with the magnitude and sign of axial force and is difficult to model rationally by modifying the Young's modulus. A typical example of a structure encountering this difficulty is the force determination in tension and compression members in the cross bracing of a frame. However, there is no reason for such a limitation on the out-of-plane analysis, provided such redistributions are modeled accurately in the in-plane analysis.

6.7 Allow for Reductions Caused by Yielding

Yielding reduces both the capacity of the cross-section to resist axial force and moment, and the member flexural, torsional, and warping stiffnesses. The reduction in the cross-section capacity will have already been modeled in the in-plane analysis, as will the reduction in the in-plane flexural stiffness, and so it is only the elastic out-of-plane rigidities EI_y, GJ, EI_w that need to be reduced. One method may be to further reduce the effective moduli E_e, G_e in a similar way to that used for the in-plane flexural rigidity in [50, 21] in a plastic hinge method of advanced in-plane analysis.

6.8 Calibration Against Design Codes and Benchmark Solutions

Any analysis will need to be tested against benchmark solutions. It is suggested that this should be done first for a representative range of elastic lateral problems for beams and beam-columns. The computer program PRFELB [40, 41] can be used to obtain these benchmark solutions. Following this, the testing of the element for inelastic lateral buckling can be carried out through comparisons with inelastic buckling approximations such as those of Fig. 7. The results may also be tested against available advanced plastic zone solutions such as those in [25, 48]. Following this, tests may be made for any frames for which benchmark out-of-plane solutions exist.

The advanced method of out-of-plane analysis also needs to provide solutions which are consistent with appropriate code formulations for column and beam out-of-plane strengths, such as those of AS4100 [1]. Thus the allowances made for residual stresses and initial crookednesses and twists need to be adjusted so as to obtain agreement with the code formulations.

7. CONCLUSIONS

Attempts to extend practical advanced analyses of the in-plane behaviour of 2-D frames to out-of-plane behaviour have been less than satisfactory because of difficulties in modeling the effects of yielded zones, load heights, interactions between twist and axial force and moments, and end warping restraints.

While there are difficulties in developing a rigorous method of advanced analysis of the biaxial bending and torsion of steel structures, a simplified approximate method for the out-of-plane strengths of 2-D frames with in-plane loading will still have significant advantages over conventional methods based on linear in-plane analysis and semi-empirical code member strength rules which are of somewhat variable accuracy and limited application.

It is proposed in this paper that the advanced analysis of 2-D frames for which local buckling is prevented be simplified by first carrying out an in-plane analysis using one of the presently available plastic hinge methods, and then by using a practical advanced analysis of the out-of-plane behaviour which is based on an inelastic lateral buckling analysis which includes allowances for residual stresses and initial crookednesses and twists.

This paper first reviews the in-plane and out-of-plane analyses of 2-D frames. The elastic and inelastic out-of-plane buckling of beams and beam-columns is then summarized, and also the design out-of-plane strengths of beams. A number of suggestions are then made for the development of a practical method of advanced analysis of the out-of-plane behaviour of 2-D frames under 2-D loading. These include the development of an element which will be sufficiently accurate that only one element per member will be required, and allowances for residual stresses and initial crookednesses and twists and for the reductions in the out-of-plane stiffnesses caused by yielding. It is suggested that the element be simplified by ignoring distortion and pre- and post-buckling effects.

Practical advanced analyses of out-of-plane behaviour need to be able to account for the influences of moment and axial force distributions, load heights, and end restraints on elastic and inelastic lateral buckling, and to be consistent with the code formulations of beam and column out-of-plane strengths. Suggestions are also made for testing and benchmarking the proposed method of advanced analysis against available plastic zone analysis results and design code strength formulations. The advantages of the suggested design approach should be demonstrated for a number of practical problems.

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9. NOTATION

A	cross-sectional area
E	Young's modulus of elasticity
E_e	effective Young's modulus of elasticity
f_y	yield stress
G	shear modulus of elasticity
G_e	effective shear modulus of elasticity
I_w	warping section constant
I_x, I_y	second moments of area
J	torsion section constant
$[K_G]$	out-of-plane stability matrix
$[K_L]$	out-of-plane stiffness matrix
k_w, k_y	warping and minor axis restraint effective length factors
L	member length
M	bending moment
M_b	nominal member moment capacity
M_I	inelastic buckling moment
M_{px}, M_{py}	full plastic moments
M_{yz}	elastic uniform bending buckling moment
P	axial compression force
P_x, P_y, P_z	column buckling loads
v_{mo}	initial crookedness
x, y	cross-section principal axes
y_Q	distance of load below centroid
z	distance along a member
Z	elastic section modulus
a_m	moment modification factor
a_s	slenderness reduction factor
b	ratio of end moments
l	buckling load factor
$\{D\}$	vector of nodal out-of-plane deformations

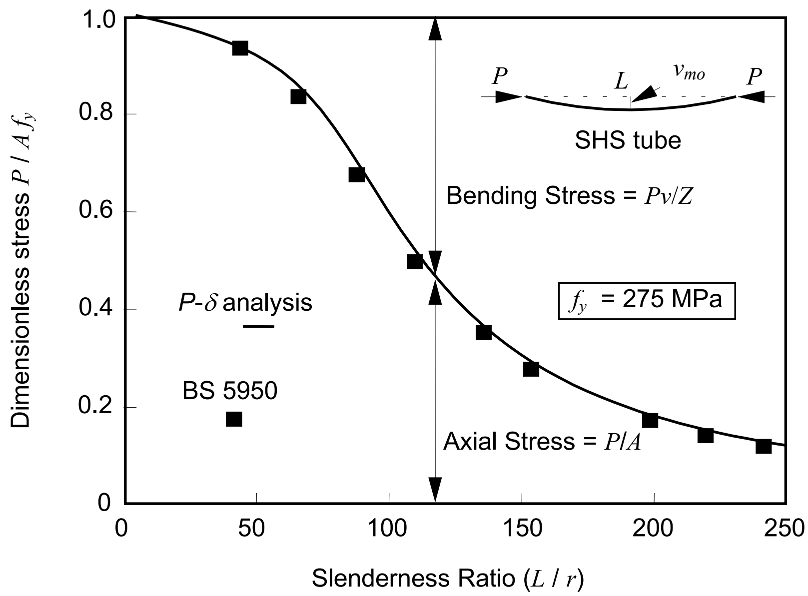


Fig. 1. Comparison Between P-δ Analysis of a Strut and BS5950

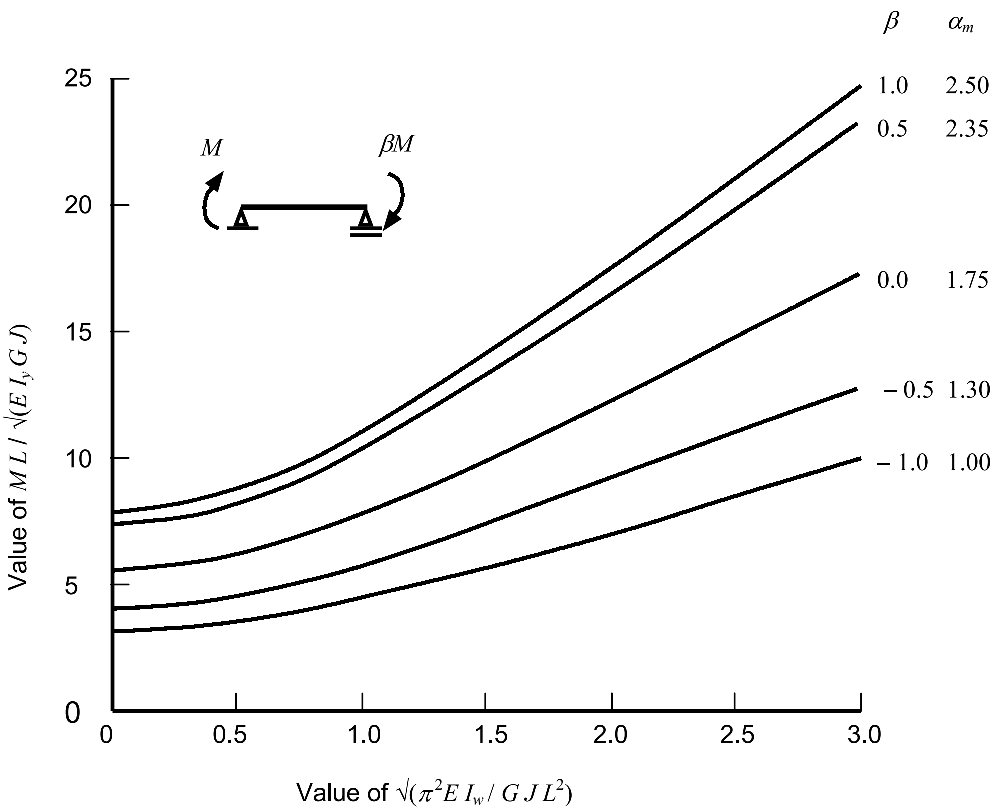


Fig. 2. Effect of Moment Distribution on Beam Elastic Buckling

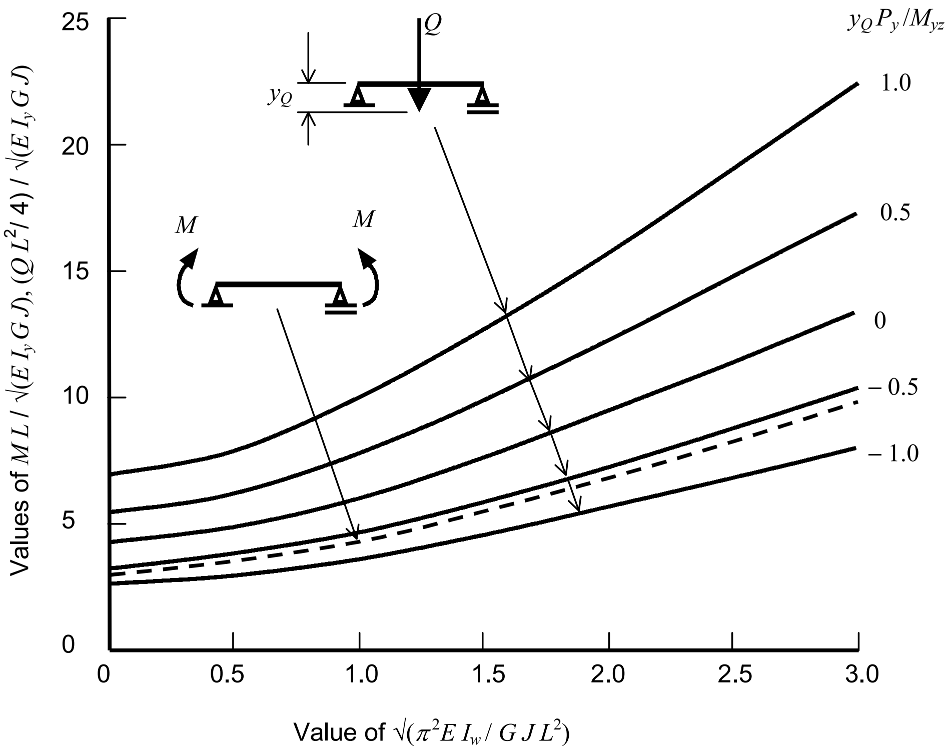


Fig. 3. Effect of Load Height on Beam Elastic Buckling

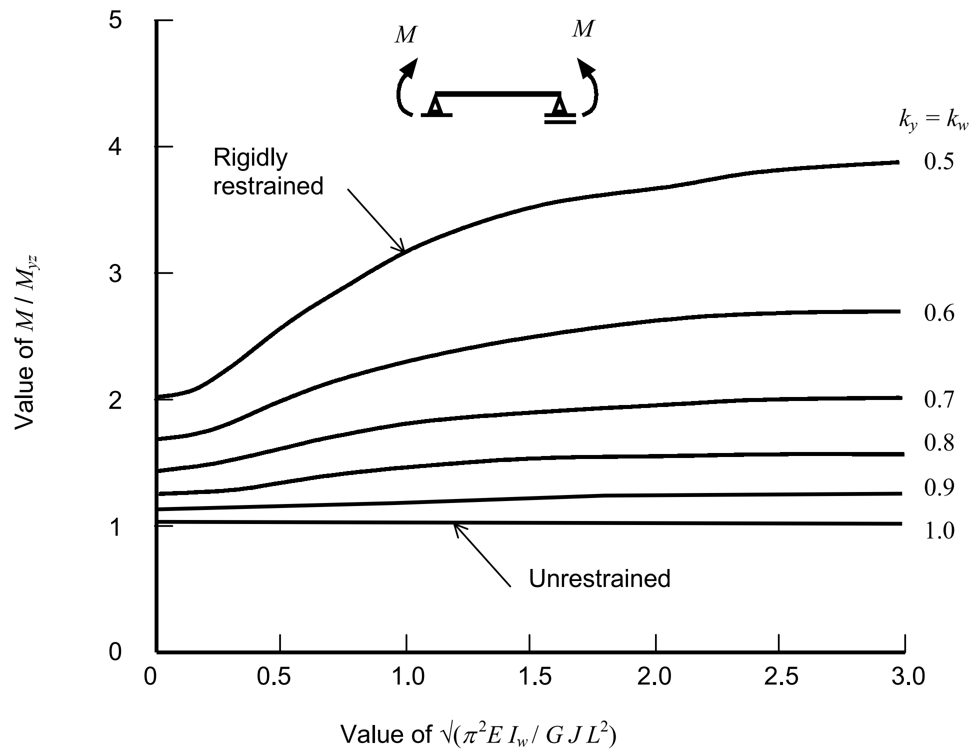


Fig. 4. Effect of End Restraints on Beam Elastic Buckling

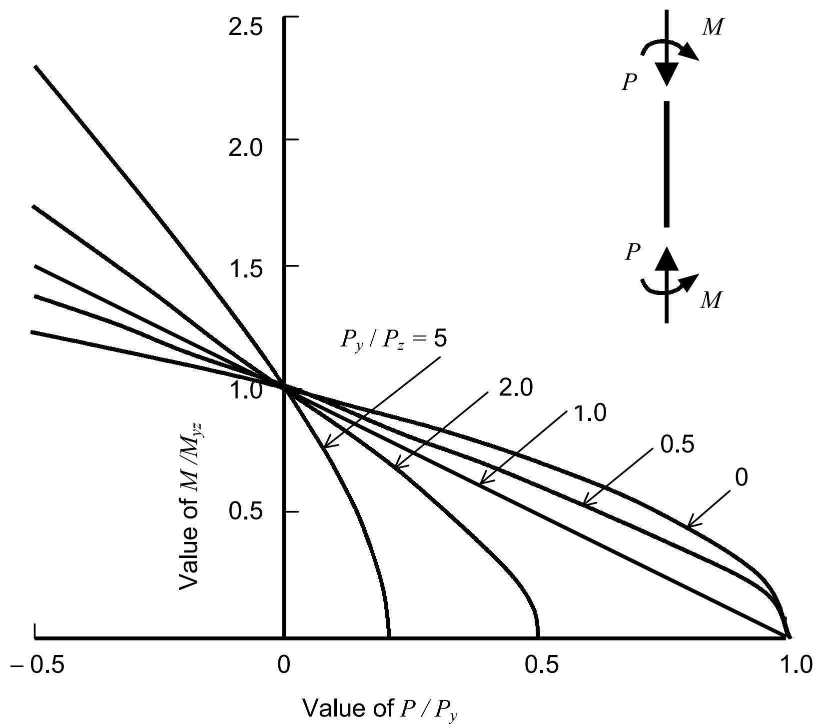
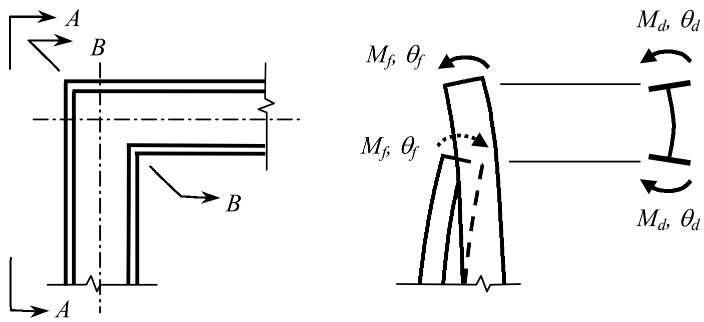


Fig. 5. Effect of Axial Load on Elastic Buckling



(a) Unstiffened joint (b) Flange warping View A-A (c) Section distortion Section B-B

Fig. 6. Warping and Distortion at an I-Section Joint

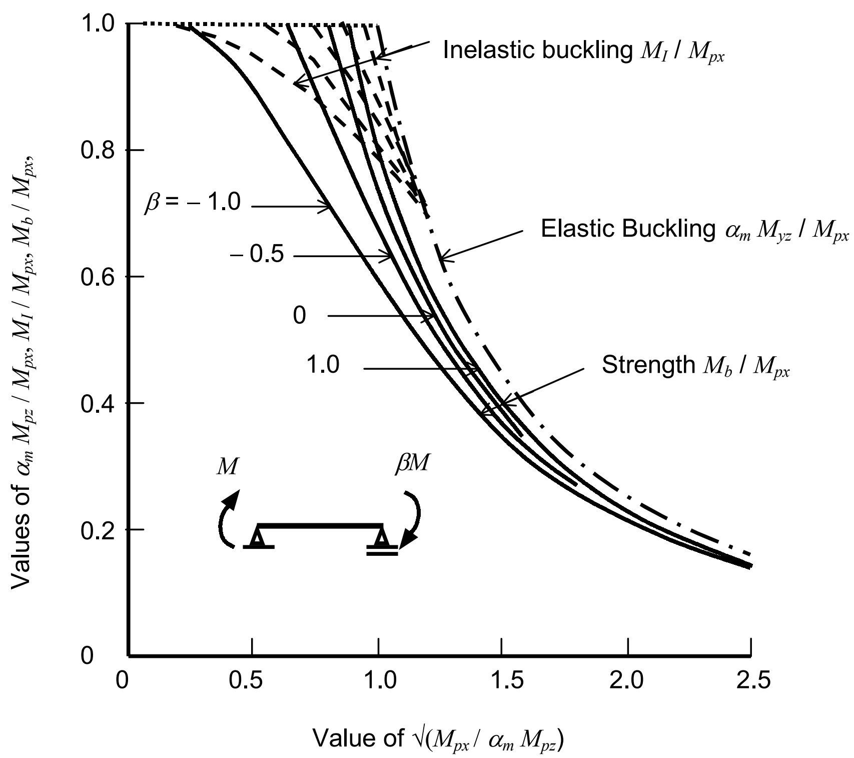


Fig. 7. Inelastic Lateral Buckling Moments and Strengths of Beams