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Steel Angle Section Beams**

Research Report No R822

N S Trahair BSc BE MEngSc PhD DEng

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Abstract:

The loads applied to angle beams usually act out of the principal planes so that they cause simultaneous biaxial bending about both principal axes. The general practice for designing unbraced beams against biaxial bending is to consider the separate failure modes of either in-plane bending or lateral buckling under bending about the major principal axis, and in-plane bending about the minor principal axis, and then to use an interaction equation to combine the two principal axis strengths. However, the interaction equations provided by many codes for designing against biaxial bending are largely derived from research on doubly symmetric I-section beams, which may be inappropriate for angle section beams, while the lateral buckling rules of these codes appear to be in error for non-uniform bending with loads acting away from the shear centre.

This paper investigates the biaxial bending of unbraced steel angle beams. The biaxial bending of compact equal angles in uniform bending is considered first, and a simple interaction equation is developed for their design which utilises recent proposals for lateral buckling design. A corresponding interaction equation is developed for the design of semicompact and slender equal angles. Suggestions are then made for extending these to the biaxial bending of unequal angles under general shear centre loading, and finally, a worked example illustrating the method is provided.

Keywords:

angles, beams, biaxial bending, buckling, design, elasticity, member capacity, moments, steel, torsion.

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INTRODUCTION

This paper is concerned with the member design of unbraced steel angle section beams against biaxial bending. Previous related papers were concerned with the section design of braced angle beams against biaxial bending (Trahair, 2002a) and against bearing, shear, and torsion (Trahair, 2002b), and the member design of unbraced beams against lateral buckling (Trahair, 2002c).

The loads applied to steel angle beams usually act out of the principal planes and eccentrically from the shear centre as shown in Fig. 1, so that they cause simultaneous biaxial bending about both principal axes and torsion. However, most design codes do not provide general guidance on design against torsion, nor on the specific design of angle beams against biaxial bending and torsion, and so there is a need to develop a rational set of rules for the general design of angle section beams. Part of such a set must be rules for the biaxial bending of angle section beams which are loaded through the shear centre so that there are no primary torsion actions. The purpose of this paper is to provide suggestions for the design of such beams.

The general practice for designing unbraced beams against biaxial bending is to consider the separate failure modes of either in-plane bending or lateral buckling under bending about the major principal axis, and in-plane bending about the minor principal axis, and then to use an interaction equation to combine the two principal axis strengths. However, the interaction equations provided by many design codes such as those of AISC (2000a,b), BSI (2000), and SA (1998) for designing against biaxial bending are largely derived from research on doubly symmetric I-section beams, which may be inappropriate for angle section beams, which are either mono-symmetric or asymmetric. In addition, while the lateral buckling rules of these codes allow for mono-symmetry effects in beams under uniform bending, they appear to be in error for non-uniform bending with loads acting away from the shear centre. A recent study (Trahair, 2002c) has developed proposals for the lateral buckling design of steel angle section beams which are loaded in the major axis principal plane, at or away from the shear centre.

This paper investigates the biaxial bending of unbraced steel angle beams. The biaxial bending of compact equal angles in uniform bending is considered first, and a simple interaction equation is developed for their design which utilises the recent lateral buckling design proposals (Trahair, 2002c). A corresponding interaction equation is developed for the design of semi-compact and slender equal angles. Suggestions are then made for extending these to the biaxial bending of unequal angles under general shear centre loading. Finally, a worked example illustrating the method is provided.

FULLY PLASTIC BIAxIAL BENDING STRENGTHS OF EQUAL ANGLE BEAMS

General

The biaxial bending of compact simply supported equal angle beams in uniform bending is considered in this section. An approximate elastic non-linear analysis of the twist rotations of beams with initial twists is used to predict the maximum principal plane bending moments. When these maximum moments reach the fully plastic moment combinations, the beams are considered to have failed.

This simplistic method is an extension of a first yield method of strength prediction, which takes approximate account of the additional strength beyond first yield of compact beams which can reach full plasticity. It apparently ignores the effects of residual stresses and initial crookedness which cause early yielding and reduce strength. This is compensated for by using initial twists which are increased sufficiently so that the analysis will predict the lateral buckling design strengths proposed in Trahair (2002c).

The failure moments predicted by this method are used to develop a simple interaction equation for the biaxial bending of compact equal angle beams which combines the lateral buckling design strengths of beams bent in the major axis principal plane with the full plastic moments of beams bent about the minor principal axis.

The capacities of an equal angle section beam to resist bearing and shear may be checked separately by comparing the appropriate design actions (which may be determined by a simple first-order analysis of the beam) with the corresponding design capacities recommended in Trahair (2001b).

Elastic Non-Linear Analysis For Small Rotations.

An elastic simply supported equal angle section beam of length L and initial twist

$$\phi_0 = \theta_0 \sin \pi z / L \quad (1)$$

in which θ_0 is the mid-span value of ϕ_0 and z is the distance along the beam, is shown in Fig. 2. The beam has equal and opposite end moments M_x, M_y which cause uniform bending in the yz, xz principal planes. The small rotation non-linear differential equations of equilibrium for biaxial bending and torsion are

$$-EI_x v'' = M_x + M_y(\phi + \phi_0) \quad (2a)$$

$$EI_y u'' = M_y - M_x(\phi + \phi_0) \quad (2b)$$

$$GJ\phi' = M_x u' + M_y v' \quad (2c)$$

in which E is the Young's modulus of elasticity, G is the shear modulus, I_x and I_y are the second moments of area about the x, y principal axes, J is the torsion section constant, u and v are the shear centre displacements parallel to the x, y principal axis directions, ϕ is the angle of twist rotation, and ' indicates differentiation with respect to the distance z . In these equations, the left hand sides represent the internal resistances to bending and torsion, while the right hand sides represent the first- and

second-order actions resulting from the applied actions M_x and M_y and the small deflections and twist rotations.

The rotations u' , v' may be eliminated from Equation 2c by using Equations 2a and 2b, whence

$$GJ\phi'' = \frac{M_x}{EI_y} \{M_y - M_x(\phi + \phi_0)\} - \frac{M_y}{EI_x} \{M_x + M_y(\phi + \phi_0)\} \quad (3)$$

Approximate solutions ϕ_2 of this equation may be obtained by assuming that

$$Z_2 / e_2 = \sin dz / L \quad (4a)$$

$$M_x M_y = M_x M_y \sin dz / L \quad (4b)$$

whence

$$\theta_2 = \frac{\theta_0 \left(\frac{M_x^2}{M_{yz}^2} + \frac{M_y^2}{M_{xz}^2} \right) - M_x M_y \left(\frac{1}{M_{yz}^2} - \frac{1}{M_{xz}^2} \right)}{\left(1 - \frac{M_x^2}{M_{yz}^2} - \frac{M_y^2}{M_{xz}^2} \right)} \quad (5)$$

in which

$$M_{yz} = \dot{A}(d^2 EI_y GJ / L^2) \quad (6a)$$

$$M_{xz} = \dot{A}(d^2 EI_x GJ / L^2) \quad (6b)$$

It should be noted that the greatest absolute values of e_2 will correspond to negative values of e_0 . The second-order bending moments are greatest at mid-span, and can be obtained using Equations 2a and 2b as

$$M_{x2} = M_x + M_y(\theta_2 + \theta_0) \quad (7a)$$

$$M_{y2} = M_y - M_x(\theta_2 + \theta_0) \quad (7b)$$

Fully Plastic Moment Combinations.

The combinations of principal axis moments M_{px} , M_{py} which cause full plasticity of a compact equal angle are given by the fully plastic interaction equations (Trahair, 2002a)

$$\pm M_{py} / M_{pym} = 1 - (M_{px} / M_{pxm})^2 \quad (8)$$

in which the principal axis full plastic moments M_{pxm} , M_{pym} are given by

$$M_{pxm} = 2 M_{pym} = f_y b^2 t / \sqrt{2} \quad (9)$$

in which f_y is the yield stress and b and t are the leg length and thickness of the equal angle section. These combinations are shown by the solid curve in Fig. 3.

Elastic Lateral Buckling and Lateral Buckling Design Proposals.

Elastic lateral buckling

The value of the major axis uniform bending moment at elastic buckling M_{yz} is given by Equation 6a (Trahair, 2002c).

Lateral buckling design strength

It has been proposed (Trahair, 2002c) that the nominal design lateral buckling moment capacity M_b of an angle section beam should be obtained from

$$M_b = M_{sxm} \quad (\lambda_e \leq \lambda_{ex}) \quad (10a)$$

$$M_b = M_{sxm} - (M_{sxm} - M_{sym}) \frac{(\lambda_e - \lambda_{ex})}{(\lambda_{ey} - \lambda_{ex})} \quad (\lambda_{ex} \leq \lambda_e \leq \lambda_{ey}) \quad (10b)$$

$$M_b = M_{sym} \quad (\lambda_{ey} \leq \lambda_e) \quad (10c)$$

in which

$$\lambda_{ex} = 0.99 - \frac{0.22}{(\alpha_m - 0.7)} \quad (11)$$

$$\lambda_{ey} = \sqrt{M_{sxm} / M_{sym}} \quad (12)$$

$$\lambda_e = \sqrt{M_{sxm} / M_{quy}} \quad (13)$$

as shown in Fig. 4, in which M_{sxm} and M_{sym} are the major and minor axis maximum section moment capacities, α_m is a moment modification factor which allows for the variation of the bending moment distribution (Trahair, 2002c), and M_{quy} is the maximum moment in the beam at elastic buckling. For a simply supported compact equal angle in uniform bending, $M_{sxm} = M_{pxm}$, $M_{sym} = M_{pym}$, $M_{quy} = M_{yz}$ and $\alpha_m = 1$.

The modified slenderness limit λ_{ex} in Equation 11 is an approximation for the value of λ_e (M_{sxx} / M_{yz}) at which $M_b = M_{sxx}$ according to the Australian design code AS 4100 (SA, 1998). Equation 10a uses the major axis section capacity M_{sxx} for low slenderness beams ($\lambda_e < \lambda_{ex}$), while Equation 10c uses the minor axis section capacity M_{syy} for high slenderness beams ($\lambda_{ey} < \lambda_e$), which is based on the finding of Trahair (2002c) that the moment capacity is never less than M_{syy} . Equation 10b provides a simple linear interpolation between M_{sxx} and M_{syy} for beams of intermediate slenderness ($\lambda_{ex} < \lambda_e < \lambda_{ey}$), which provides a close but conservative approximation to the predictions of Trahair (2002c).

Equivalent Initial Twists.

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 7 to determine the biaxial bending strengths of equal angle section beams. Such initial twists will also predict the lateral buckling design strengths of unbraced beams bent in their major axis principal plane. Thus the magnitudes θ_0 of the initial twist of a compact beam can be determined by using $M_y = 0$ in Equation 5 and substituting the second-order moments of Equations 7 into the fully plastic interaction equations of Equations 8 and 9, whence

$$\theta_2 + \theta_0 = \frac{\theta_0}{1 - M_b^2 / M_{yz}^2} \quad (14)$$

and

$$M_b (\theta_2 + \theta_0) / M_{pym} = 1 - (M_b / M_{pxm})^2 \quad (15)$$

so that

$$\theta_0 = \frac{\{1 - (M_b / M_{pxm})^2 \lambda_e^4\} \{1 - (M_b / M_{pxm})^2\}}{2M_b / M_{pxm}} \quad (16)$$

The variation of θ_0 with the modified slenderness λ_e is shown by the circled points in Fig. 5, together with the close approximation

$$\theta_0 = -0.1116 + 0.3612 \lambda_e + 0.3551 \lambda_e^2 - 0.3935 \lambda_e^3 \quad (17)$$

Maximum Moments At Full Plasticity.

An example of the variations of the maximum moments in an equal angle beam having a modified slenderness of $\lambda_e = 0.835$ so that $M_b / M_{pxm} = 0.75$ and $\theta_0 = -0.212$ rad. is shown in Fig. 6 for first-order biaxial bending moments defined by $M_x / M_y = 2.414$. The variations of the first-order moments M_x, M_y are shown by the dotted straight line, and those of the elastic second-order moments M_{x2}, M_{y2} by the curved dashed line. The curved line reaches the solid line representing the fully plastic moment combinations of Equations 8 and 9 when $M_{x2} = 0.475 M_{pxm}$, $M_{y2} = 0.388 M_{pxm}$, (the circled Point A) corresponding to first-order moments of $M_x = 0.54 M_{pxm}$, $M_y = 0.225 M_{pxm}$ (the squared Point B).

The results for similar determinations of the values of the first-order moment combinations for which the elastic second-order moment combinations cause full plasticity are shown in Figs 3 and 6 for first-order bending moments defined by $M_x / M_y = \infty, 2.414, 1.0,$ and 0 .

Proposed Design Interaction Equation.

Close approximations for the first-order moment combinations M_x, M_y shown in Fig. 6 whose second-order moment combinations M_{x2}, M_{y2} cause full plasticity can be obtained by reducing the full plastic moments M_{px}, M_{py} to M_x, M_y given by

$$\frac{M_x}{M_{px}} = \frac{M_y}{M_{py}} = 1 - \left(1 - \frac{M_b}{M_{pxm}} \right) \left(\frac{\psi}{\pi / 2} \right) \quad (18)$$

in which

$$\psi = \tan^{-1} (M_{px} / M_{py}) \quad (19)$$

and M_b is the appropriate lateral buckling moment strength obtained from Equations 10-13.

This approximation is shown by the dash-dot lines in Figs 3 and 6, and is compared with the first-order moment combinations for which the approximate second-order moment combinations cause full plasticity (the squared points). It is suggested that this approximation can also be used for compact equal angle beams with general loading through the shear centre by using the moment modification factor α_m and maximum moment M_{quy} at elastic buckling given by Trahair (2002c) in the determination of the lateral buckling strength M_b .

FIRST YIELD BIAxIAL BENDING STRENGTHS OF EQUAL ANGLE BEAMS

General

The first yield of simply supported equal angle beams in uniform biaxial bending is considered in this section. An approximate elastic non-linear analysis of the twist rotations of beams with initial twists is used to predict the maximum principal plane bending moments. The small elastic second-order rotations θ_2 are given by Equation 5, and the maximum second-order moments by Equations 7.

When these maximum moments reach the moment combinations corresponding to the first yield moment combinations, the beams are considered to have failed. The failure moments predicted by this method are used to develop a simple interaction equation for the first yield biaxial bending strengths of equal angle beams which combines the lateral buckling design strengths of beams bent in the major axis principal plane with the first yield moments of beams bent about the minor principal axis.

First Yield Moment Combinations

The combinations of principal axis moments M_{yx} , M_{yy} which cause first yield of an equal angle are given by the interaction equations (Trahair, 2002a)

$$\pm M_{yy} / M_{yyM} = 1 - M_{yx} / M_{yxM} \quad (20)$$

in which the principal axis first yield moments M_{yxM} , M_{yyM} are given by

$$M_{yxM} = 2 M_{yyM} = f_y b^2 t (\lambda^2 / 3) \quad (21)$$

These combinations are shown by the solid curve in Fig. 7.

Equivalent Initial Twists

It is desirable that the initial twist ϕ_0 of Equation 1 should be sufficiently large that it will represent the effects of residual stresses and initial crookednesses and twists on the strengths of real beams when it is used with the elastic second-order predictions of Equations 7 to determine the biaxial bending strengths of equal angle section beams. Such initial twists will also predict the lateral buckling design strengths of unbraced beams bent in their major axis principal plane. Thus the magnitude θ_0 of the initial twist of a compact beam can be determined by using $M_y = 0$ in Equation 5 and substituting the second-order moments of Equations 7 into the first yield interaction equations of Equations 20 and 21, which leads to Equation 14 and

$$M_b (\theta_2 + \theta_0) / M_{yyM} = 1 - M_b / M_{yxM} \quad (22)$$

so that

$$\theta_0 = \frac{\{1 - (M_b / M_{yxM})^2 \lambda_e^4\} \{1 - M_b / M_{yxM}\}}{2M_b / M_{yxM}} \quad (23)$$

The variation of θ_0 with the modified slenderness λ_e is shown by the triangled points in Fig. 5, together with the close approximation

$$\theta_0 = -0.0358 + 0.0499 \lambda_e + 0.4262 \lambda_e^2 - 0.3142 \lambda_e^3 \quad (24)$$

Maximum Moments At First Yield

The values of the first-order moment combinations for which the elastic second-order moment combinations cause first yield are shown by the squared points in Fig. 7 for first-order bending moments defined by $M_x / M_y = \infty, 2.414, 1.0$, and 0.

Proposed Design Interaction Equation.

Close approximations for the first-order moment combinations M_x, M_y shown in Fig. 7 whose second-order moment combinations M_{x2}, M_{y2} cause first yield can be obtained by reducing the first yield moments M_{yx}, M_{yy} to M_x, M_y given by

$$\frac{M_x}{M_{yxm}} = \frac{M_b}{M_{yxm}} - \left(2.5 \frac{M_b}{M_{yxm}} - 0.5 \right) \frac{M_y}{M_{yxm}} \leq \left(1 - \frac{M_y}{M_{yym}} \right) \quad (25)$$

This approximation is shown by the dash-dot lines in Fig. 7, and is compared with the first-order moment combinations for which the approximate second-order moment combinations cause first yield (the squared points). It is suggested that this approximation can also be used for equal angle beams with general loading through the shear centre by using the moment modification factor α_m and maximum moment M_{quy} at elastic buckling given by Trahair (2002c) in the determination of the lateral buckling strength M_b .

LOCAL BUCKLING EFFECTS

Section Classification and Capacities

The effects of local buckling on the section moment capacities of angle section beams have been discussed in Trahair (2002a). In that paper, sections are classified as being plastic, compact, semi-compact or slender by comparing their long leg plate slendernesses

$$\lambda = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (26)$$

with the limiting slenderness values given in Table 1.

Table 1 Local Buckling Slenderness Limits

Bending moment	Plastic limit λ_p	Compact limit λ_c	Yield limit λ_y
M_x	12	16	26
M_y	10	14	23

A plastic section must have sufficient rotation capacity to maintain a plastic hinge until a plastic collapse mechanism develops. A plastic section satisfies

$$\lambda < \lambda_p \quad (27)$$

in which λ_p is the plasticity limit. A compact section must be able to form a plastic hinge. A compact section satisfies

$$\lambda_p < \lambda < \lambda_c \quad (28)$$

in which λ_c is the compact limit. The nominal section moment capacity M_s of a plastic or compact section is equal to its fully plastic capacity M_p , so that

$$M_s = M_p \quad (29)$$

A slender section has its moment capacity reduced below the first yield moment by local buckling effects. A slender section satisfies

$$\lambda > \lambda_y \quad (30)$$

in which λ_y is the yield limit. The nominal section moment capacity of a slender section M_s is approximated by

$$M_s = M_y \left(\lambda_y / \lambda \right)^2 \quad (31)$$

in which M_y is the first yield capacity.

A semi-compact section must be able to reach the first yield moment, but local buckling effects may prevent it from forming a plastic hinge. A semi-compact section satisfies

$$\lambda_c < \lambda < \lambda_y \quad (32)$$

The nominal section moment capacity M_s of a semi-compact section is given by the linear interpolation between the full plastic and first yield capacities of

$$M_s = M_p - (M_p - M_y) \frac{(\lambda - \lambda_p)}{(\lambda_y - \lambda_p)} \quad (33)$$

Biaxial Bending Strengths

The approximation of the biaxial bending strengths of compact equal angle beams ($\lambda < \lambda_c$) by the modified full plasticity interaction equation of Equation 18 is demonstrated in Fig. 3.

It is suggested that the biaxial bending strengths of semi-compact and slender equal angle beams ($\lambda_y < \lambda$) may be approximated by further modifying the first yield interaction equation of Equation 25 to

$$\frac{M_x}{M_{sxm}} = \frac{M_b}{M_{sxm}} - \left(2.5 \frac{M_b}{M_{sxm}} - 0.5 \right) \frac{M_y}{M_{sxm}} \leq \left(1 - \frac{M_y}{M_{sym}} \right) \quad (34)$$

in which M_{sxm} is obtained from Equation 33 or 31 and M_b includes an allowance for the effect of local buckling through the use of reduced capacities M_{sxm} and M_{sym} in Equation 10.

BIAXIAL BENDING OF UNEQUAL ANGLE BEAMS

Some dimensionless full plastic and first yield moment combinations (Trahair, 2002a) for an extreme unequal angle section $b \times 0.5b \times t$ are shown in Fig. 8. These combinations are point symmetric about the origin rather than symmetric about the axes as a result of the asymmetry of the unequal angle section. Also shown are indications of the correspondences between the regions of the figure and the directions of the resultant loads which cause the moment combinations. The directions of the corresponding moment vectors are perpendicular to the load directions. It can be seen that the full plastic combinations are substantially larger than the first yield combinations.

It is suggested that the first-order moment combinations M_x , M_y whose second-order moment combinations M_{x2} , M_{y2} cause failure may be approximated by using the greater of the sets of values obtained from

$$\frac{M_x}{M_{sx}} = \frac{M_y}{M_{sy}} = 1 - \left(1 - \frac{M_b}{M_{sxm}}\right) \left\{ 1.4 \left(\frac{2\psi}{\pi}\right) - 0.4 \left(\frac{2\psi}{\pi}\right)^2 \right\} \quad (35)$$

and

$$\frac{M_x}{M_{sxm}} = \frac{M_b}{M_{sxm}} \left(1 - \frac{M_y}{M_{sym}}\right) \quad (36)$$

These approximations are shown in Fig. 8 for the extreme cases where $M_b = M_{sym}$ for uniform bending of unequal angle section beams with $\beta = 0.5$. They will be conservative if applied to equal angle section beams instead of Equation 18 (for compact beams) or 34 (for semi-compact or slender beams).

EXAMPLE

Problem

A 150 x 100 x 12 unequal angle beam is shown in Fig. 9. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of $b = 144$ mm, $l_b = 94$ mm, and $t = 12$ mm are shown in Fig. 9b. The unbraced beam is simply supported over a span of $L = 6$ m, and has a design uniformly distributed vertical load of $q^* = 6$ kN/m acting parallel to the long leg and through the shear centre at the leg junction, as shown in Fig. 9b.

The first-order analysis of the beam, the determination of the elastic buckling moment of the beam and the lateral buckling design strength, and the check of the biaxial bending capacity are summarised below. The checking of the bearing and shear capacities are summarised in Trahair (2002 b).

Elastic Analysis

The design major axis bending moments are

$$M_x^* = (q^* L^2 / 8) \cos U = 24.7 \text{ kNm}, \quad M_y^* = (q^* L^2 / 8) \sin U = 10.9 \text{ kNm}.$$

Moment at Elastic Buckling

The elastic buckling moment calculated in Trahair (2002c) for a load acting at 19.0 mm below the shear centre is $M_{quy} = 30.6$ kNm. However, in the present example, the load acts at the shear centre, and so the elastic buckling moment decreases to $M_{quy} = 29.9$ kNm.

Lateral Buckling Design Strength

The angle section has been shown to be compact (Trahair, 2002a).

The lateral buckling moment strength calculated in Trahair (2002c) for $M_{quy} = 30.6$ kNm is $M_b = 25.0$ kNm. However, in the present example with a reduced value of $M_{quy} = 29.9$ kNm, the lateral buckling moment strength decreases to $M_b = 24.7$ kNm.

Biaxial Bending Capacity

Using Trahair (2002c), $M_{pxm} = 38.4$ kNm, $M_{pym} = 15.5$ kNm.

Using Trahair (2002a), $M_{pX} = 27.4$ kNm and $M_{pY} = 0$ kNm.

Thus $M_{px} = 25.0$ kNm, $M_{py} = 11.1$ kNm, $\psi = 1.154$ rad., and $2\psi / \pi = 0.734$.

Using Equation 35, $M_x = 25.0 \times \{1 - (1 - 24.7 / 38.4) (1.4 \times 0.734 - 0.4 \times 0.734^2)\}$
 $= 17.8$ kNm

Using Equation 36, $M_x = 24.7 \{1 - M_x \times (11.1 / 25.0) / 15.5\} = 14.5$ kNm < 17.8 kNm.

Thus $M_x = 17.8$ kNm and $\phi M_x = 16.0$ kNm (using $\phi = 0.9$).

This is less than the design moment $M_x^* = 24.7$ kNm, and the beam is inadequate.

CONCLUSIONS

This paper develops rational, consistent, and economical design methods for determining the biaxial bending strengths of unbraced steel angle section beams loaded through the shear centre, and illustrates their use in a design example.

An approximate small rotation non-linear elastic analysis is used to predict the maximum moments in equal angle beams in uniform bending. The beams have initial twists. The maximum strengths are assumed to be reached when the maximum predicted moments cause either full plasticity or first yield of the cross-section (Trahair, 2002a). The magnitudes of the initial twists are chosen so that the predicted strengths of beams bent in the major principal plane are equal to recent recommendations for the lateral buckling strengths (Trahair, 2002c).

The biaxial bending strengths of compact section beams are predicted assuming full plasticity at the maximum moment section, and a simple design approximation is developed. The biaxial bending strengths of semi-compact and slender section beams are based on predictions assuming first yield at the maximum moment section, and a simple design approximation is developed. These design approximations are formulated so that they can be used for equal angle beams under general loading.

The first yield and fully plastic behaviour of unequal angle section beams is then considered, and conservative design approximations are developed from those for equal angle section beams.

Proposals have been made elsewhere (Trahair, 2002b) for checking the bearing and shear capacities of angle section beams.

APPENDIX 1 REFERENCES

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APPENDIX 2 NOTATION

b	long leg length
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_y	yield stress
G	shear modulus of elasticity
I_x, I_y	second moments of area about the x, y principal axes
J	torsion section constant
L	span length
M_b	lateral buckling moment strength
M_{px}, M_{py}	values of M_x, M_y at full plasticity
M_{pxm}, M_{pym}	maximum values of M_{px}, M_{py}
M_{quy}	maximum moment at elastic lateral buckling
M_s	nominal section moment capacity
M_{sx}, M_{sy}	values of M_x, M_y at section capacity
M_{sxm}, M_{sym}	maximum values of M_{sx}, M_{sy}
M_x, M_y	moments about the x, y principal axes
M_x^*, M_y^*	design moments about the x, y principal axes
M_{xz}, M_{yz}	uniform bending elastic buckling moments
M_{x2}, M_{y2}	second-order moments about the x, y principal axes
M_{yx}, M_{yy}	values of M_x, M_y at first yield
M_{yxm}, M_{yyym}	maximum values of M_{yx}, M_{yy}
q	intensity of uniformly distributed load
q^*	design intensity of uniformly distributed load
t	leg thickness
u, v	shear centre deflections parallel to the x, y principal axes
x, y	principal axes
X, Y	rectangular (geometric) axes
X_c, Y_c	X, Y distances from centroid to shear centre
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
α_m	moment modification factor
β	leg length ratio
β_x	monosymmetry section constant
λ	long leg local buckling slenderness
$\lambda_c, \lambda_p, \lambda_y$	compact, plasticity, and yield local buckling slenderness limits
λ_e	modified slenderness for beam lateral buckling
$\lambda_{ex}, \lambda_{ey}$	beam lateral buckling slenderness limits
ϕ	angle of twist rotation, or capacity factor
ϕ_0	initial angle of twist
ϕ_2	second-order angle of twist rotation
θ_0	maximum value of ϕ_0
θ_2	maximum value of ϕ_2
ψ	angle between x axis and resultant moment

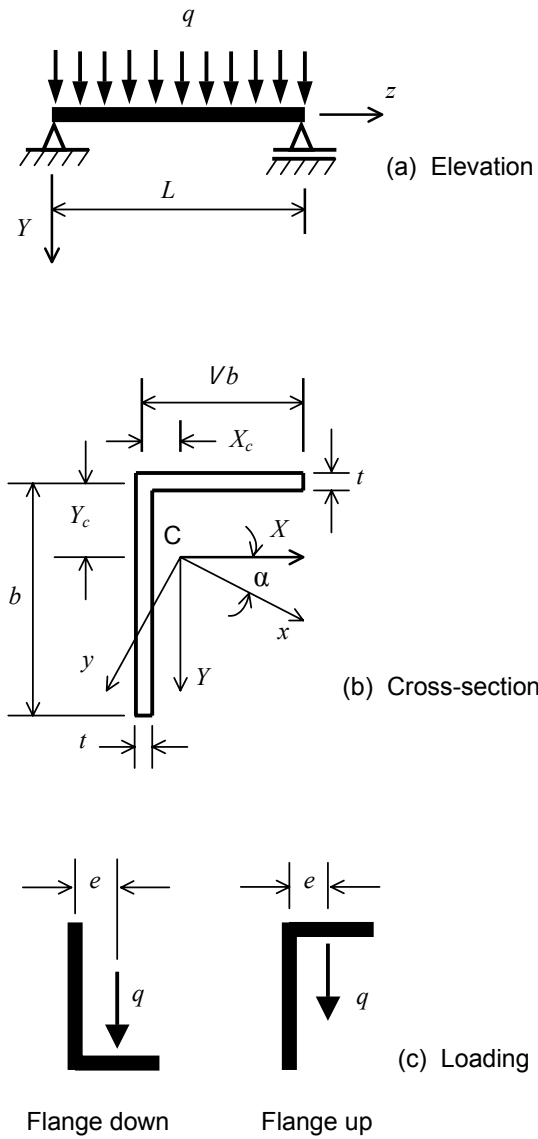


Fig. 1. Eccentrically Loaded Angle Section Beam

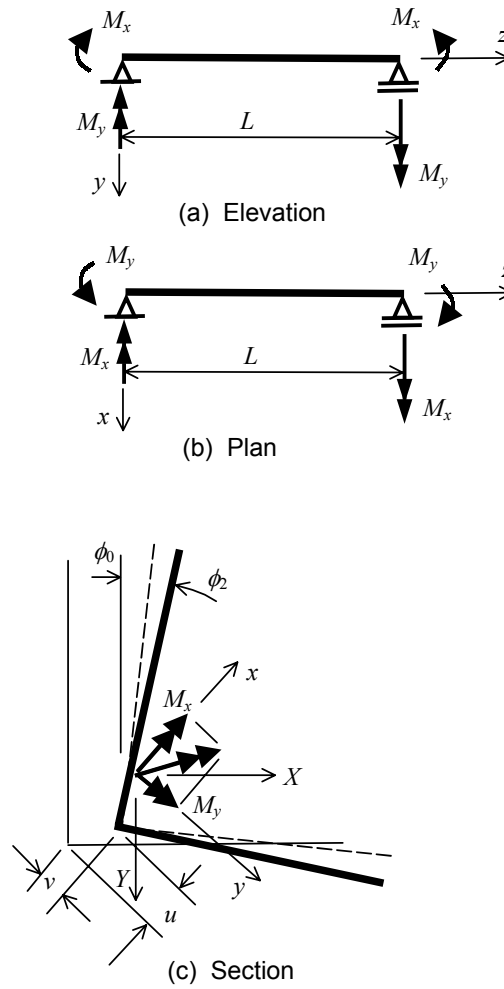


Fig. 2. Simply Supported Equal Angle in Uniform Bending

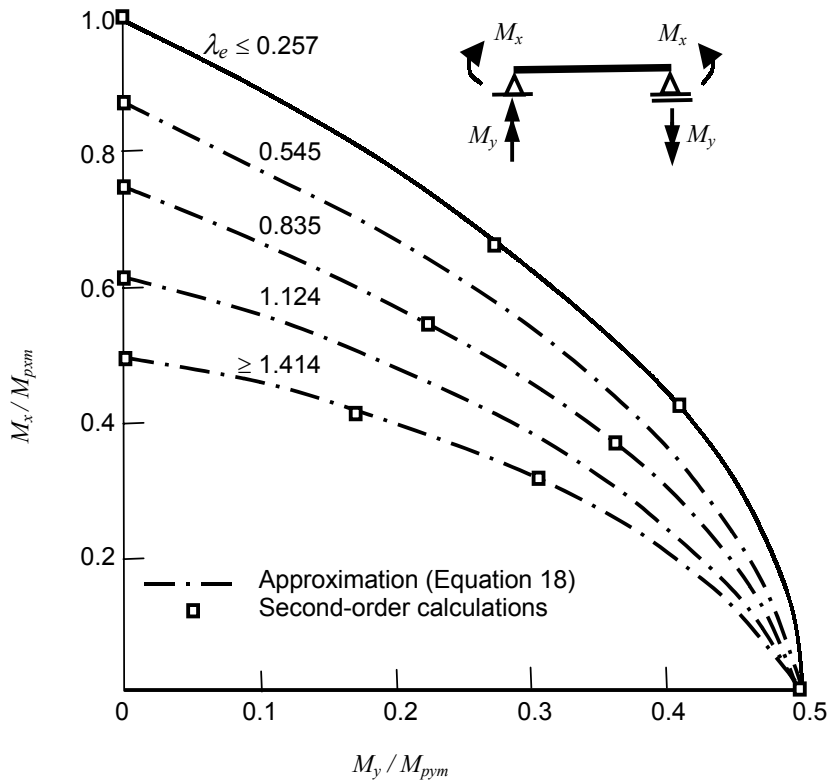


Fig. 3. Biaxial Bending Strengths of Compact Equal Angles

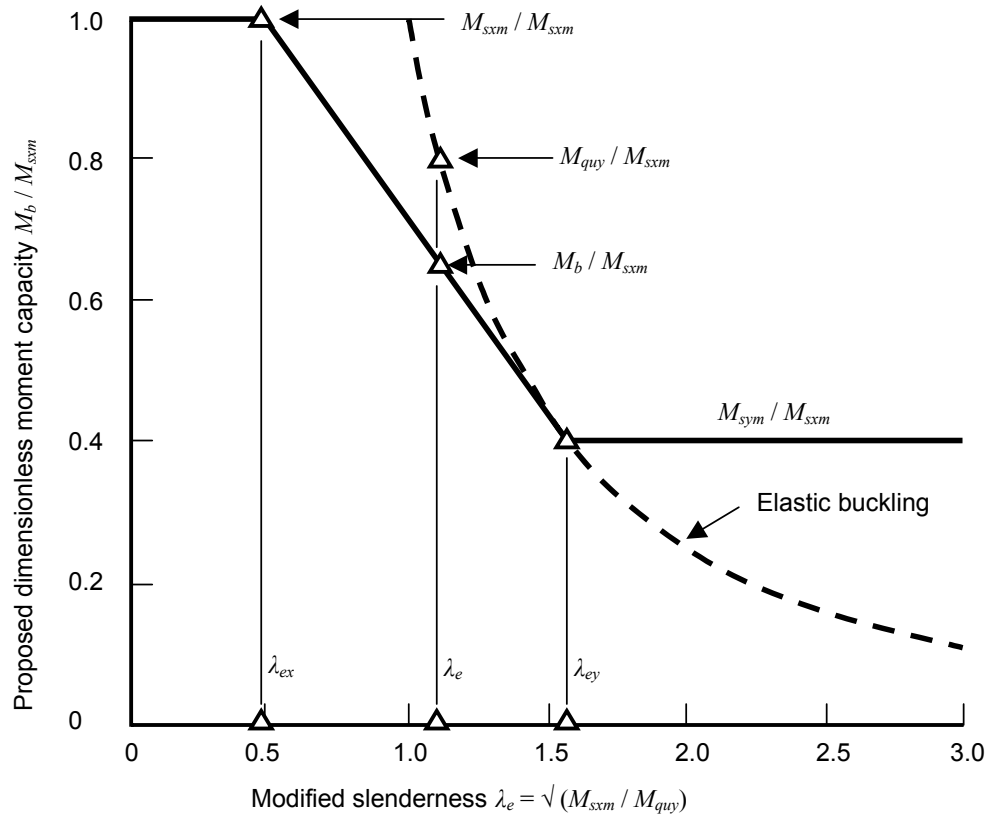


Fig. 4. Lateral Buckling Design Strengths

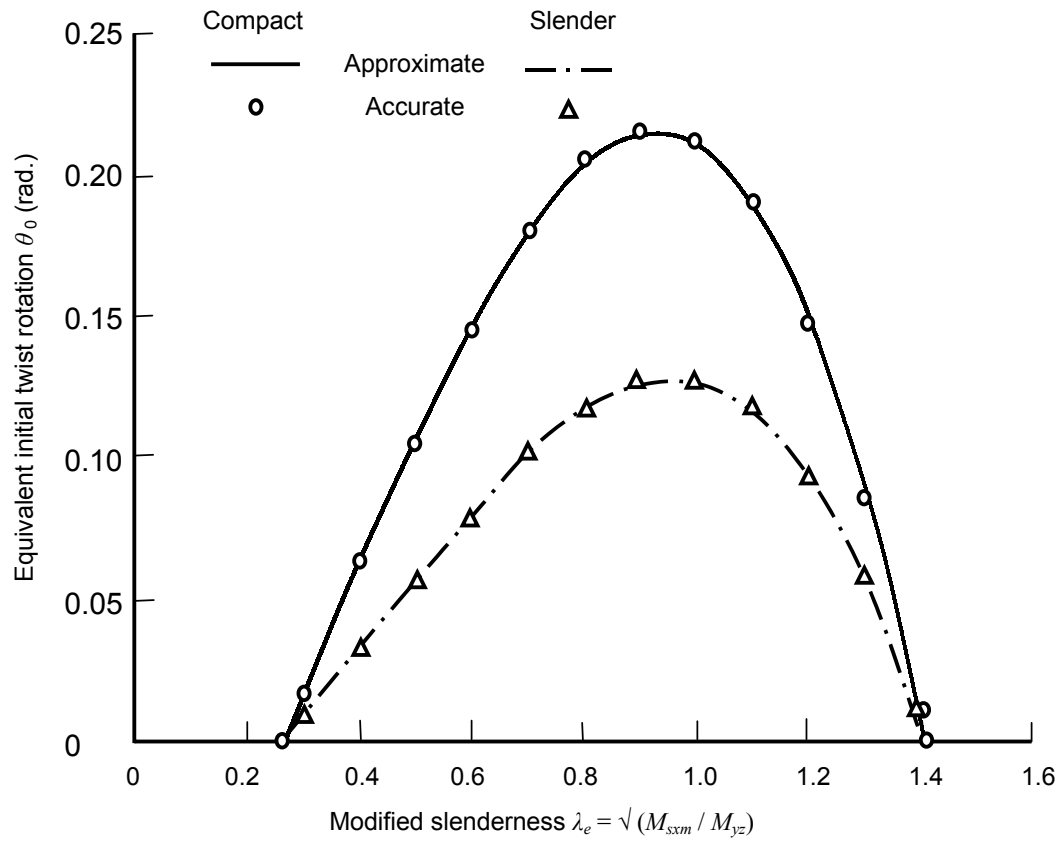


Fig. 5. Equivalent Initial Twist Rotations

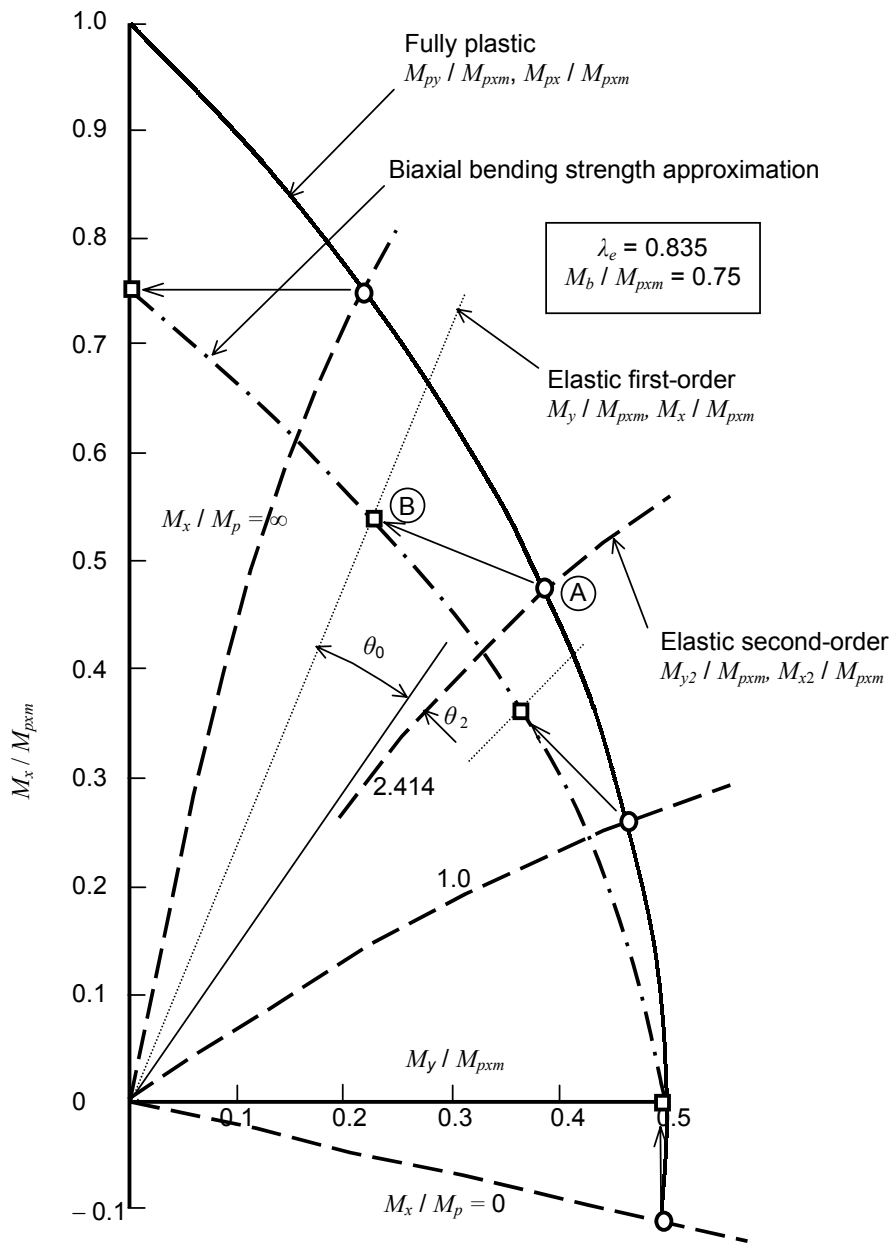


Fig. 6. Determination of Biaxial Bending Strengths of Compact Equal Angles

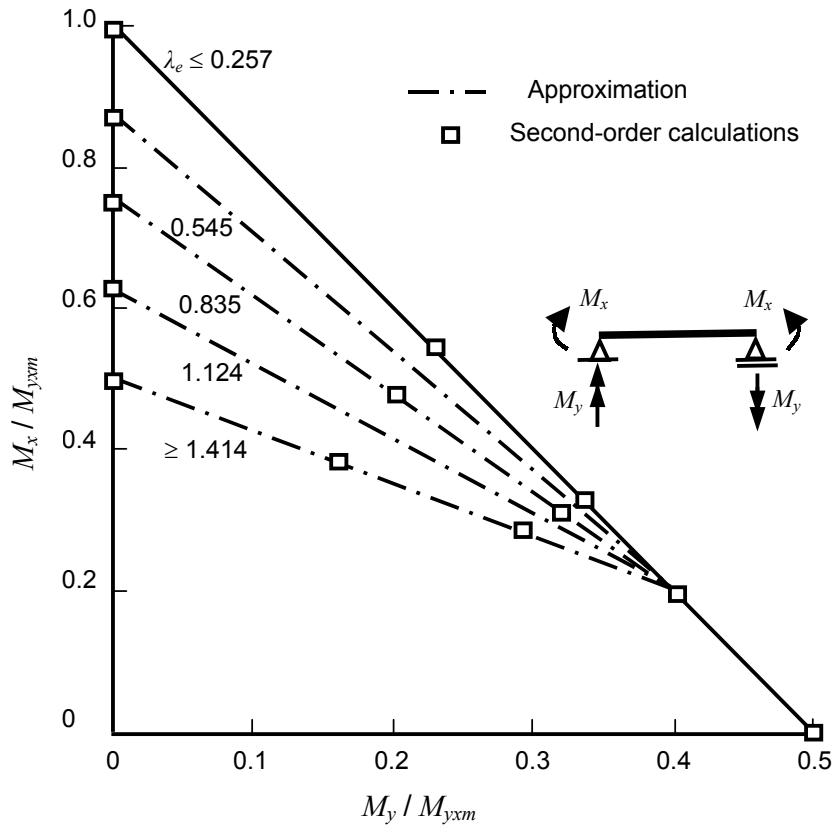


Fig. 7. Biaxial Bending First Yield Strengths of Equal Angles

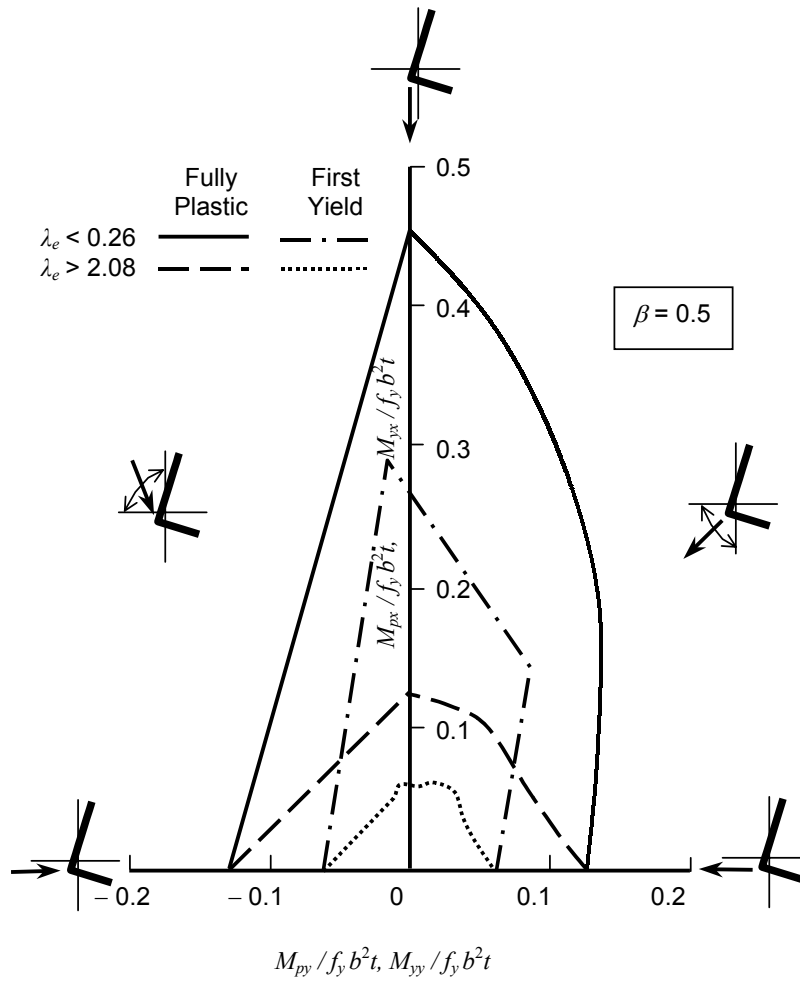
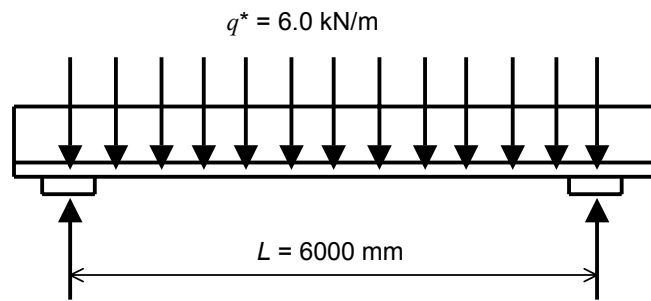


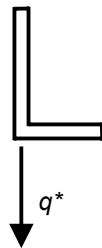
Fig. 8. Moment Combinations for an Unequal Angle



(a) Elevation

150 x 100 x 12 UA
 ($b \times \beta b \times t = 144 \times 94 \times 12$)

$E = 200,000 \text{ MPa}$
 $G = 80,000 \text{ MPa}$
 $f_y = 300 \text{ MPa}$



$\alpha = 23.91^\circ$
 $I_x = 7.548 \text{ E6 mm}^4$
 $I_y = 1.314 \text{ E6 mm}^4$
 $J = 0.1371 \text{ E6 mm}^4$
 $y_o = 32.30 \text{ mm}$
 $\beta_x = 78.33 \text{ mm}$

(b) Section

Fig. 9 Example