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Research Report No R821

By

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Abstract:

The report presents a plate strength formulation applicable to metal in general, notably metals with nonlinear stress strain curves, such as aluminium and stainless steel alloys. The formulation is based on a generalised Winter-curve featuring two material-dependent constants. Analytic expressions are derived for these constants so that they can be determined for given material properties. The material properties are assumed to be expressed in terms of the Ramberg-Osgood parameters.

The generalised formulation allows the plate strength equation to be determined for a given alloy, requiring only the Ramberg-Osgood parameters. The strength curves obtained using the generalised formulation are shown to compare well with finite element results.

The strength equations apply to uniformly compressed plates simply supported along all four edges. The validity ranges for the material properties are $3 \leq n \leq 100$ and $0.001 \leq e \leq 0.003$ where $e = \sigma_{0.2}/E_0$. The plates are assumed to be free of residual stress.

Keywords:

Plates, nonlinear metals, design, strength, plasticity, finite elements.

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1 Introduction

This report concerns the strength of plates in uniform compression made from metals with non-linear mechanical properties, such as stainless steel, aluminium or brass. By "non-linear" is meant a metal with low proportionality stress and extensive strain-hardening capacity. Metals of this kind do not have a yield plateau as do ordinary hot-rolled carbon structural steels, which for compression design purposes can be modeled as elastic-perfectly-plastic. However, the mechanical response of many modern cold-formed structural shapes of carbon steel, including tubulars, is becoming increasingly nonlinear because of residual stresses introduced during cold-forming. The strength of the component plates of such sections can also be determined using the strength curve formulation developed in this report.

In the absence of a yield plateau, it is common practice to define an equivalent yield stress for nonlinear metals. This is usually chosen as the 0.2 % proof stress, defined as the stress with a *plastic* strain of 0.2%, as shown in Fig. 1. It is also common practice to represent the stress-strain curve by a Ramberg-Osgood curve (Ramberg and Osgood 1943), which is defined in terms of the initial elastic modulus (E_0), a proof stress and a parameter (n) that defines the sharpness of the knee of the stress strain-curve. If the proof stress is chosen as the 0.2 % proof stress ($\sigma_{0.2}$), the Ramberg-Osgood takes the form,

$$\varepsilon = \frac{\sigma}{E_0} + 0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^n . \tag{1}$$

Often the proportionality stress (σ_p) is defined as the 0.01 % proof stress ($\sigma_{0.01}$), as shown in Fig. 1. The n-parameter can then be determined by requiring that the Ramberg-Osgood curve coincides with the measured stress-strain curve at the 0.01 % and 0.2 % proof stresses,

$$n = \frac{\ln(20)}{\ln(\sigma_{0.2} / \sigma_{0.01})} . \tag{2}$$

The Ramberg-Osgood curve is known to accurately represent actual stress-strain curves for stresses less than the 0.2 % proof stress when n is determined using eqn. (2).

The nonlinear stress-strain curve implies a loss of material stiffness at stresses beyond the proportionality stress (σ_p). The stiffness is measured as the slope of the stress-strain curve which changes from the initial modulus (E_0) below the proportionality stress to the tangent modulus (E_t) at stresses beyond the proportionality stress, as shown in Fig. 1. The loss of stiffness affects the buckling load of a section and needs to be taken into account when determining its strength.

Since the strength of a section or member, such as a plate or column, depends on the stress-strain curve, and the stress-strain curve can be defined in terms of the Ramberg-

Osgood parameters (E_0, n, $\sigma_{0.2}$), it should be possible to define the strength directly in terms of the Ramberg-Osgood parameters. This reasoning was used in Rasmussen and Rondal (1997b) to develop generic equations for the strength of metal columns failing by flexure. The generic equations were subsequently used for obtaining design curves for stainless steel (Rasmussen and Rondal 1997a) and aluminium (Rasmussen and Rondal 2000) columns. The column curve formulation developed in Rasmussen and Rondal (1997b) was implemented in the recently published Australian standard for cold-formed stainless steel structures (AS/NZS4673 2001).

The present report presents a study similar to that in Rasmussen and Rondal (1997b) for uniformly compressed metal plates simply supported along all four edges. First, finite elements analyses are performed to obtain strength curves for a large number of combinations of Ramberg-Osgood parameters. A generalised Winter curve in the form of

$$\chi = \alpha / \lambda - \beta / \lambda^2 \tag{3}$$

is then used to express the nondimensional plate strength (χ), where λ is the plate slenderness, and α and β are assumed to be functions of the Ramberg-Osgood parameters. Functional relationships are derived for α and β, allowing generic strength curves to be obtained by combining these relationships with eqn. (3).

2 Historical Background

The inelastic buckling of plates was studied extensively in the 1940s and 1950s following studies of inelastic column buckling. Stowell (1948) is to be credited with deriving the governing equation for the inelastic buckling of a rectangular plates in uniform compression,

$$D^* \left[\left(1 - \tfrac{3}{4}\kappa\right) \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right] + t\sigma_x \frac{\partial^2 w}{\partial x^2} = 0 \tag{4}$$

where w is the plate buckling deflection (see Fig. 2), t is the plate thickness and,

$$D^* = \frac{E_s t^3}{9} \tag{5}$$

$$\kappa = 1 - \frac{E_t}{E_s}. \tag{6}$$

In eqns (5,6), E_s is the secant modulus, see Fig. 1. Founded in Shanley's work, (which proved that the column load increases in the post-buckling range), Stowell based his theory on a deformation plasticity model assuming no stress reversal. When recognising that for plastic buckling, Poisson's ratio (ν) is taken as 0.5, and for elastic buckling $E_s = E_t = E_0$, eqn. (4) is seen to be the equivalent of St. Venant's equation for the elastic buckling of plates.

However, tractable solutions of equation (4) are not easily achieved and for the purpose of producing design equations, Bleich (1952) proposed that the equation could be simplified using,

$$D\left[\tau^2\frac{\partial^4 w}{\partial x^4}+2\tau\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]+t\sigma_x\frac{\partial^2 w}{\partial x^2}=0 \quad (7)$$

where D is the elastic plate flexural rigidity,

$$D=\frac{E_0 t^3}{12\,(1-\nu^2)} \quad (8)$$

and τ is a plasticity reduction factor, which Bleich suggested could be taken as

$$\tau=\sqrt{\frac{E_t}{E_0}}. \quad (9)$$

Bleich solved eqn. (7) for several support conditions and obtained the following expression for the minimum inelastic buckling stress,

$$\sigma_{cr}=\frac{k\pi^2 E_0\tau}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad (10)$$

where *k* is the plate buckling coefficient which is independent of τ and takes the same values as for elastic buckling.

Equation (10) can be compared to equivalent tangent modulus buckling stress equation for columns,

$$\sigma_{E_t}=\frac{\pi^2 E_0\tau}{(L_e/r)^2} \quad (11)$$

in which L_e is the effective length, *r* is the radius of gyration, and the plasticity reduction factor (τ) is given by,

$$\tau=\frac{E_t}{E_0}. \quad (12)$$

By comparing eqns (9,12), it can be seen that since $E_t/E_0 \leq 1$, the plasticity reduction factor for plates is greater than that for columns at a given stress. Consequently, the general conclusion can be drawn that the inelastic buckling of plates is less affected by gradual yielding than columns.

Furthermore, while the buckling of columns can be assumed to represent the ultimate load for all practical purposes, slender plates are post-buckling stable and can support loads in excess of the inelastic buckling stress. It follows from eqn. (10) that when the buckling stress reduces below the proportionality stress, the tangent modulus approaches the initial elastic modulus ($E_t \rightarrow E_0$), and so the buckling stress becomes the elastic buckling stress. Since the elastic buckling stress is a conservative estimate of plate strength, equation (10) becomes increasingly conservative for predicting plate strength as the slenderness increases. Partly for this reason, and partly because the use of plasticity reduction factors leads to an iterative calculation, eqn. (10) is not suitable for general design.

3 Material Model

As mentioned in the Introduction, the stress-strain curve of nonlinear metals can be accurately represented by the Ramberg-Osgood curve. However, the accuracy and range of applicability of the fit depend strongly on the stresses used to determine the *n*-parameter. When, as is common practice, the *n*-parameter is determined using the 0.01 % and 0.2 % proof stress values according to eqn. (2), the Ramberg-Osgood approximation can only be assumed to be accurate for stresses up to and slightly beyond the 0.2 % proof stress. While this stress range generally is adequate for numerical analyses determining the strength of columns, it may prove too limited for the analysis of plates, which are post-buckling stable and may reach significantly higher strains prior to reaching the ultimate load than columns. This observation applies particularly to low values of *n*.

To overcome this difficulty, Rasmussen (2001) suggested that the full stress-strain curve be broken into two parts, one for $\sigma \le \sigma_{0.2}$ and one for $\sigma > \sigma_{0.2}$. The first part is the standard Ramberg-Osgood curve determined on the basis of the 0.01 % and 0.2 % proof stress values, and the second is a transformed Ramberg-Osgood curve which ensures compatibility in slope at $\sigma_{0.2}$ and goes through the ultimate tensile strength of the measured stress-strain curve:

$$\varepsilon = \begin{cases} \dfrac{\sigma}{E_0} + 0.002\left(\dfrac{\sigma}{\sigma_{0.2}}\right)^n & \text{for } \sigma \le \sigma_{0.2} \\ \dfrac{\sigma - \sigma_{0.2}}{E_{0.2}} + \varepsilon_u\left(\dfrac{\sigma - \sigma_{0.2}}{\sigma_u - \sigma_{0.2}}\right)^m + \varepsilon_{0.2} & \text{for } \sigma > \sigma_{0.2} \end{cases} \tag{13}$$

In eqn. (13), σ_u and ε_u are the ultimate tensile stress and strain respectively, $\varepsilon_{0.2}$ and $E_{0.2}$ are the strain and tangent to the stress strain curve at $\sigma_{0.2}$, and *m* is a parameter equivalent to *n* which determines the shape of the stress-strain curve in the range $\sigma_{0.2} \le \sigma \le \sigma_u$. Expressions for $\varepsilon_{0.2}$ and $E_{0.2}$ can be readily obtained from the first part of the stress-strain curve,

$$\varepsilon_{0.2} = \frac{\sigma_{0.2}}{E_0} + 0.002 \tag{14}$$

$$E_{0.2} = \frac{E_0}{1 + 0.002n/e} \tag{15}$$

where *e* is the nondimensional 0.2 % proof stress,

$$e = \frac{\sigma_{0.2}}{E_0}. \tag{16}$$

Rasmussen (2001) showed that the following equations could be used with good approximation for determining *m*, σ_u and ε_u:

$$m = 1 + 3.5\frac{\sigma_{0.2}}{\sigma_u} \tag{17}$$

$$\frac{\sigma_u}{\sigma_{0.2}} = \frac{1 - 0.0375(n-5)}{0.2 + 185e} \tag{18}$$

$$\varepsilon_u = 1 - \frac{\sigma_{0.2}}{\sigma_u}. \tag{19}$$

It follows from eqns (13-19) that the complete stress-strain curve can be obtained from the Ramberg-Osgood parameters (E_0, n, $\sigma_{0.2}$).

The necessity of using a separate expression for the stress-strain curve in the range $\sigma>\sigma_{0.2}$ is demonstrated in Fig. 3. The figure compares FE plate strength curves for the two material models corresponding to:

1. Extrapolating the standard Ramberg-Osgood expression (eqn. (1)) beyond $\sigma_{0.2}$.
2. Using the full-range stress-strain curve (eqn. (13)).

The FE model is defined in the section following. The strength ($\sigma_u/\sigma_{0.2}$) on the vertical axis of Fig. 3 is the average ultimate stress ($\sigma_u=P_u/A$) nondimensionalised *wrt* the 0.2 % proof stress ($\sigma_{0.2}$). The strength curves are shown for a low value of n=3. The standard Winter equation (eqn. (24)) is included in the figure, representing the strength of carbon steels ($n\rightarrow\infty$), which do not suffer from material softening prior to reaching the yield stress. The strength curves based on an extrapolated Ramberg-Osgood curve generally lie above the Winter curve, which is physically unsound. When using the full-range stress-strain curve (eqn. (13)), the strength curves lie below the Winter curve as expected. It is apparent that the plate strength can be seriously overestimated if the standard Ramberg-Osgood expression is extrapolated beyond $\sigma_{0.2}$. The error increases as n decreases.

While eqns (17-19) were obtained on the basis of measured stress-strain curves for stainless steel alloys, they will be assumed to also apply to other types of nonlinear metals in this report. This is justified on the basis of the sensitivity study included in Rasmussen (2001) which showed that the stress-strain curve defined by eqn. (13) is insensitive to small changes in σ_u and ε_u for strains up to 2 %, which covers the practical strain range for metal plates in compression. For instance, changes in σ_u of 20% and ε_u of 40 % led to average changes in stress at 2 % strain of 6 % and 2 % respectively.

4 Finite Element Results

4.1. The FE Model

It was shown in Rasmussen et al. (2002) that accurate FE models for nonlinear materials can be achieved by using the standard multi-linear facility incorporated in most FE packages, including Abaqus. The plasticity model used in Rasmussen et al (2002) was based on flow theory with isotropic hardening. The same study showed that for moderate anisotropy, good agreement between tests and FE analysis can be achieved by assuming isotropic hardening, provided the stress-strain curve is obtained from a compression test of a coupon cut in the same direction as the direction of loading.

Based on the results of the previous study (Rasmussen et al. 2002), the present report assumes the material properties are those for compression corresponding to the direction of loading. The material properties are assumed to be defined in terms of the Ramberg-Osgood parameters (E_0, n, $\sigma_{0.2}$). The multi-linear material data is obtained by first constructing the entire stress-strain curve up to the ultimate tensile stress using eqns (13-19), then converting from engineering stress and strain to true stress and strain using the well-known expressions,

$$\sigma_t = \sigma_e(1+\varepsilon_e) \tag{20}$$

$$\varepsilon_{tp} = \ln(1+\varepsilon_e) - \frac{\sigma_t}{E_o} \tag{21}$$

where the subscripts t and e refer to "true" and "engineering" respectively, and ε_{tp} is the true plastic strain. Finally, the true stress-strain curve thereby obtained is approximated by use of a multi-linear curve with typically 20 line segments.

The general purpose FE package Abaqus (Hibbitt et al. 1997) has been used for the present study. The plates were assumed to be square with the loaded edges uniformly compressed and the longitudinal edges free to pull in, as shown in Fig. 4. Using symmetry, only half of the plate was modeled. (It would have been possible to model only a quarter of the plate but the indata files were intended to be used also for plates with one free edge which require modeling of the full width of the plate). Based on a convergence study, the mesh density was chosen as 50 elements in the longitudinal and transverse directions. The 4-node reduced integration shell element 4SR of the Abaqus element library was used for all calculations. In terms of choice of element and mesh density, the FE model is nearly identical that used in Rasmussen et al. (2002) which produced agreement with tests on stainless steel plates to within a few percent. The FE model can therefore be expected to be accurate.

The width of the plate was 100 mm in all analyses, while the thickness was varied to produce a set of pre-determined plate slenderness values,

$$\lambda = \sqrt{\frac{\sigma_{0.2}}{\sigma_{cr}}} \tag{22}$$

where σ_{cr} is the elastic critical stress,

$$\sigma_{cr} = \frac{4\pi^2 E_0}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 . \tag{23}$$

A geometric imperfection was introduced into the model, assumed to be in the shape of the elastic buckling mode, as obtained from an elastic buckling analysis. The mode resembled closely the classical double sine curve. The magnitude of the geometric imperfection was taken as a tenth of the thickness in all models. The plates were assumed to be free of residual stress.

4.2. FE Results

As discussed in Rasmussen and Rondal (1997b), when presented in non-dimensional form, the plate strength curves are functions of only two nondimensional parameters, rather than all three Ramberg-Osgood parameters. The parameters have been chosen as the *n*-parameter and the nondimensional 0.2 % proof stress (*e*) defined by eqn. (16).

Strength curves were determined for all (*n*,*e*)-permutations of the *n*- and *e*-values shown in Table 1. For each (*n*,*e*)-combination, the plate strength was determined for the slenderness values, λ=0.5, 0.75, 1, 1.25, 1.5, 2, 2.5, and 3. The strength curves thus obtained are shown in Figs 5, 6, 7, 8 and 9 for *e*=0.001, 0.0015, 0.002, 0.0025, and 0.003 respectively. Each curve shows four strength curves corresponding to *n*-values of 3, 5, 10 and 100, where *n*=100 is representative of an elastic-perfectly-plastic material, such as carbon steel. The strength (*s*) shown on the vertical axis is the average ultimate stress (σ_u) nondimensionalised *wrt* the 0.2 % proof stress ($\sigma_{0.2}$), where the ultimate stress is the maximum load obtained in the FE analysis divided by the cross-sectional area (*bt*).

Figures 5-9 also include the standard Winter curve,

$$\chi = 1/\lambda - 0.22/\lambda^2 \tag{24}$$

which is generally known to accurately represent the strength of residual stress-free carbon steel plates. This result is confirmed by the present study by the agreement between the Winter-curve and the *n*=100 curves shown in Figs 5-9. It follows from Figs 5-9 that the lower the value of *n*, the lower the nondimensional strength. It is also clear that the influence of *n* is less for high values of nondimensional yield stress (*e*) than it is for lower values.

Figures 10-13 present the same strength curves but ordered such that *n* is the same for each figure while *e* varies. It follows from Figs 10-13 that the nondimensional strength is dependent on the proof stress for low values of *n* but hardly so for large values of *n*. These results are consistent with those obtained for columns (Rasmussen and Rondal 1997b).

5 Strength Equations

Having obtained plate strength curves for a wide range of *n*- and *e*-values, analytical approximations to these curves have been derived by adopting a generalised Winter-curve in the form of eqn. (3) as a general column curve expression and then determining analytical functions for α and β in terms of *n* and *e*.

Overall good agreement with the FE plate strengths could be achieved by assuming the α and β functions to be linear in *e*. The following expressions were obtained,

$$\alpha = \begin{cases} 0.92 + 0.07\tanh\left(\dfrac{n-3}{2.1}\right) - \left(0.026\exp[-0.55(n-3)] + 0.019\right)\left(6 - 2000e\right) & 3 \le n \le 10 \\ \alpha_{10} + (1-\alpha_{10})\dfrac{n-10}{90} & 10 < n \le 100 \end{cases} \tag{25}$$

$$\beta = \begin{cases} 0.18 + 0.045\tanh\left(\dfrac{n-3}{2.5}\right) - \left(0.01\exp[-1.6(n-3)] + 0.005\right)\left(6 - 2000e\right) & 3 \le n \le 10 \\ \beta_{10} + (0.22 - \beta_{10})\dfrac{n-10}{90} & 10 < n \le 100 \end{cases} \qquad (26)$$

where α_{10} and β_{10} are the values of α and β calculated at n=10 respectively,

$$\alpha_{10} = 0.9898 - 0.01955(6 - 2000e) \qquad (27)$$

$$\beta_{10} = 0.2247 - 0.005(6 - 2000e) \qquad (28)$$

Figures 14 and 15 show the variations of α and β as functions of n for various values of e. The variation is linear in n between n=10 and n=100.

The strength curves obtained using eqns (3, 25-2) are compared with the FE strength curves in Figs 16, 17 and 18 for e=0.001, e=0.002 and e=0.003 respectively. Appendix A contains the comparison for all combination of n and e shown in Table 1. To quantify the agreement, the ratio of FE strength (s) to design strength (χ) is shown in Table 2 for all (n,e)-combinations and all analysed values of λ. Values of this ratio (s/χ) greater than unity represent conservative design strength predictions. The ratios are also shown in Fig. 19 for selected values of n and e. It appears from Table 2 and Fig. 19 that the most conservative (s/χ=1.13) and most optimistic (s/χ=0.91) predictions are found for (n,e,λ)=(3,0.0015,3) and (n,e,λ)=(5,0.001,1.25) respectively. The mean and coefficient of variation of the ratio based on all values shown in Table 2 are 1.01 and 0.039 respectively, indicating overall good agreement and a small scatter.

6 Applications

The presented strength curve formulation (eqns (3, 25-26)) can be used to obtain design curves for metal plates of any alloy provided its Ramberg-Osgood parameters are known. For instance, the American Society of Civil Engineers Standard for Cold-formed Stainless Steel Structural Members (ANSI/ASCE-8 1991) provides values of the Ramberg-Osgood parameters for a range of stainless steel alloys. For annealed AISI 304 austenitic stainless steel loaded in compression in the longitudinal direction, the Ramberg-Osgood parameters are given as (E_0, n, $\sigma_{0.2}$) = (195 GPa, 4, 195 MPa). Substituting n=4 and e=195 MPa/195 GPa=0.001 into eqns (27, 28), α and β can be obtained as 0.815 and 0.169 respectively. An accurate expression for the strength curve for annealed AISI 304 alloy can thus be written in the form,

$$\chi = 0.815/\lambda - 0.169/\lambda^2 \qquad (29)$$

Similar plate design equations can be obtained for other metals, provided their Ramberg-Osgood parameters are known.

7 Conclusions

The report presents a general formulation capable of determining strength curves for metal plates. The formulation defines the strength curve as a generalised Winter-curve (eqn. (3)) involving two material-dependent constants α and β. Expressions are given (eqns (25, 26)) for calculating α and β for given values of the Ramberg-Osgood parameters (E_0, n, $\sigma_{0.2}$). The formulation allows strength curves to be obtained for a given metal provided the Ramberg-Osgood parameters of the metal are known. It is valid for n-values greater than or equal to 3, and $e=\sigma_{0.2}/E_0$ values between 0.001 and 0.003.

The plate strength formulation is based on a fit to finite element strength curves. The finite element model has been developed in a previous report (Rasmussen et al. 2002) and shown to produce ultimate loads with an accuracy of a few percent compared to tests on stainless steel plates. The plate strength formulation can therefore be considered to be accurate.

The formulation can be used to produce design strength curves for metals with nonlinear stress-strain curves, such as stainless steel and aluminium alloy plates. The plates considered in this report are assumed to be simply supported along all four edges.

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n	3	5	10	100		
e	0.001	0.0015	0.002	0.0025	0.003	
(*n*,*e*)	(3, 0.001), (3, 0.0015),… (3, 0.003), (5, 0.001), (5,0.0015),…, (100, 0.003)					

Table 1: Values of *n* and *e*, and (*e*, *n*)-combinations

	e=0.001				*e*=0.0015				*e*=0.002				*e*=0.0025				*e*=0.003			
λ	n =3	n =5	n = 10	n = 100	n =3	n =5	n = 10	n = 100	n =3	n =5	n = 10	n = 100	n =3	n =5	n = 10	n = 100	n =3	n =5	n = 10	n = 100
0.5	0.97	1.01	0.98	0.98	1.00	1.04	1.00	0.99	0.99	1.01	1.02	0.99	0.98	0.99	1.02	1.00	0.95	0.97	1.02	1.01
0.75	0.98	0.97	0.98	1.00	1.04	1.01	1.00	1.00	1.05	1.01	1.00	1.01	1.04	1.00	1.00	1.01	1.03	1.00	1.00	1.02
1	0.94	0.93	0.99	1.04	1.00	0.95	0.99	1.04	1.00	0.96	0.99	1.03	1.00	0.96	0.99	1.04	0.98	0.96	0.98	1.04
1.25	0.96	0.909	0.98	1.00	1.00	0.95	0.97	0.99	1.01	0.95	0.96	1.00	0.98	0.95	0.96	1.01	0.98	0.95	0.95	1.01
1.5	0.98	0.913	0.97	0.99	1.02	0.95	0.97	0.99	1.02	0.96	0.97	0.99	1.01	0.97	0.97	0.99	0.98	0.95	0.96	1.01
2	1.03	0.94	1.00	1.01	1.06	0.99	1.01	1.01	1.06	1.00	1.01	1.00	1.05	0.99	1.00	1.03	1.02	0.99	1.00	1.04
2.5	1.07	0.97	1.02	1.02	1.12	1.04	1.02	1.03	1.09	1.03	1.03	1.06	1.13	1.06	1.04	1.03	1.09	1.05	1.03	1.06
3	1.08	1.01	1.06	1.06	1.13	1.04	1.06	1.06	1.07	1.03	1.05	1.00	1.09	1.04	1.03	1.08	1.06	1.02	1.04	1.06

Table 2: Values of s/χ for the complete set of FE analyses. Mean(s/χ)=1.01 and COV(s/χ)=0.039

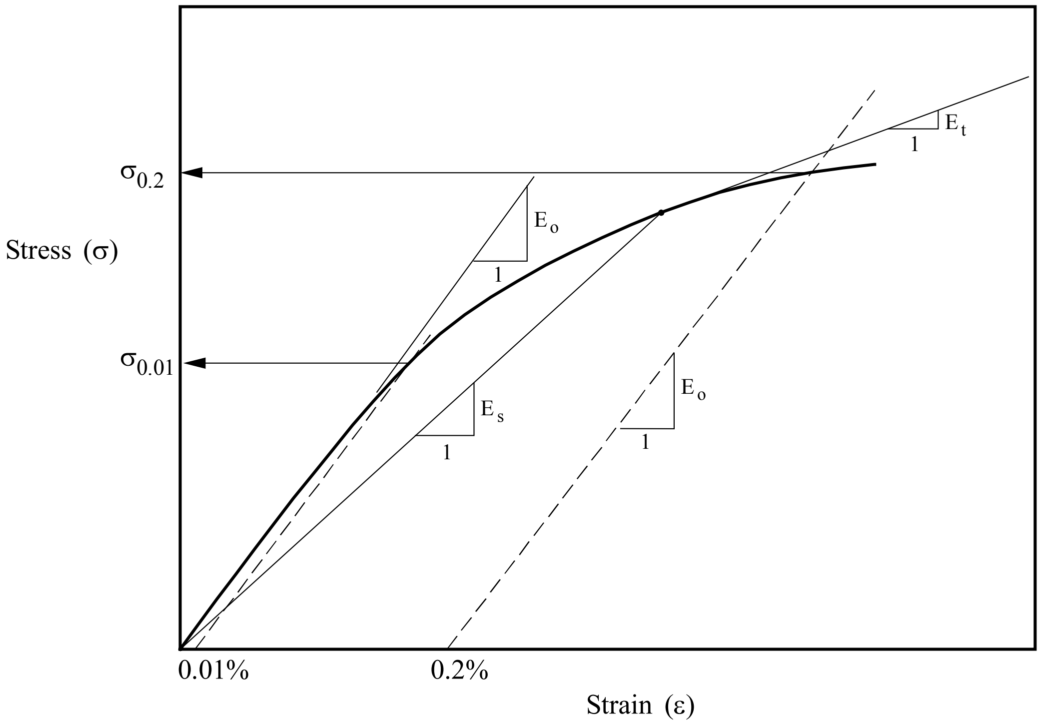

Figure 1. Initial (E_0), tangent (E_t), and secant (E_s) moduli

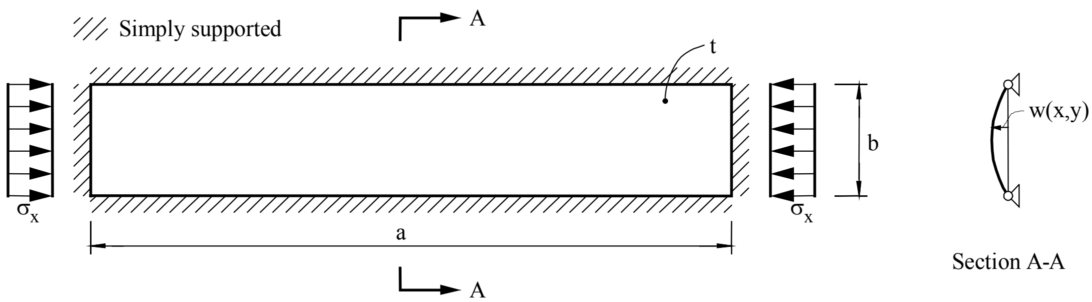

Figure 2: Buckling of rectangular plate under uniform compression

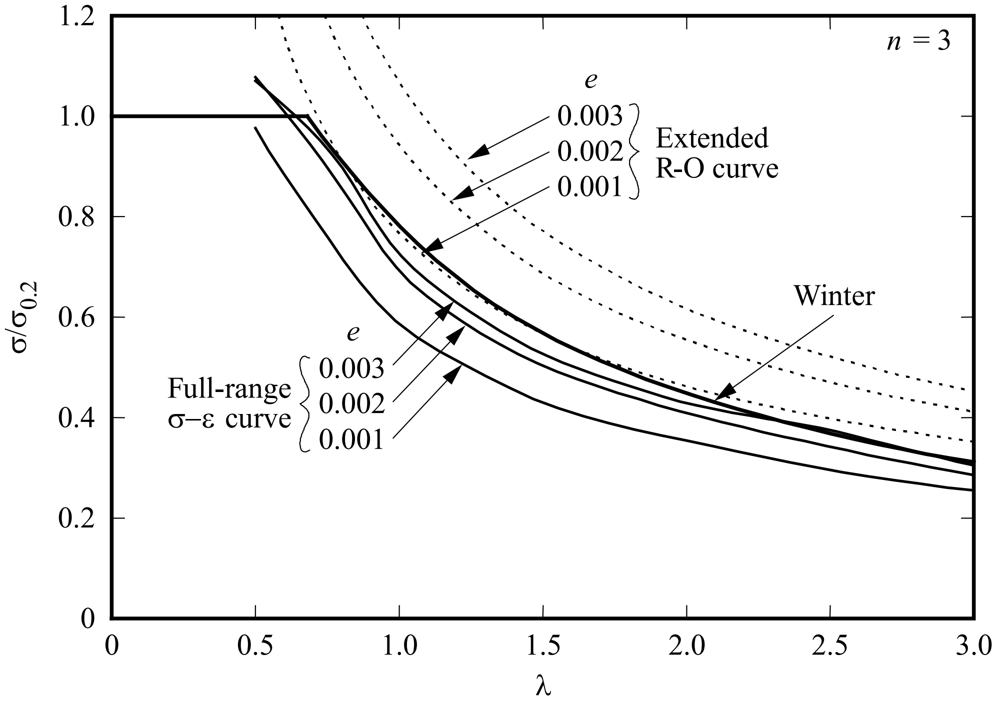

Figure 3: Plate strength curves for different material models

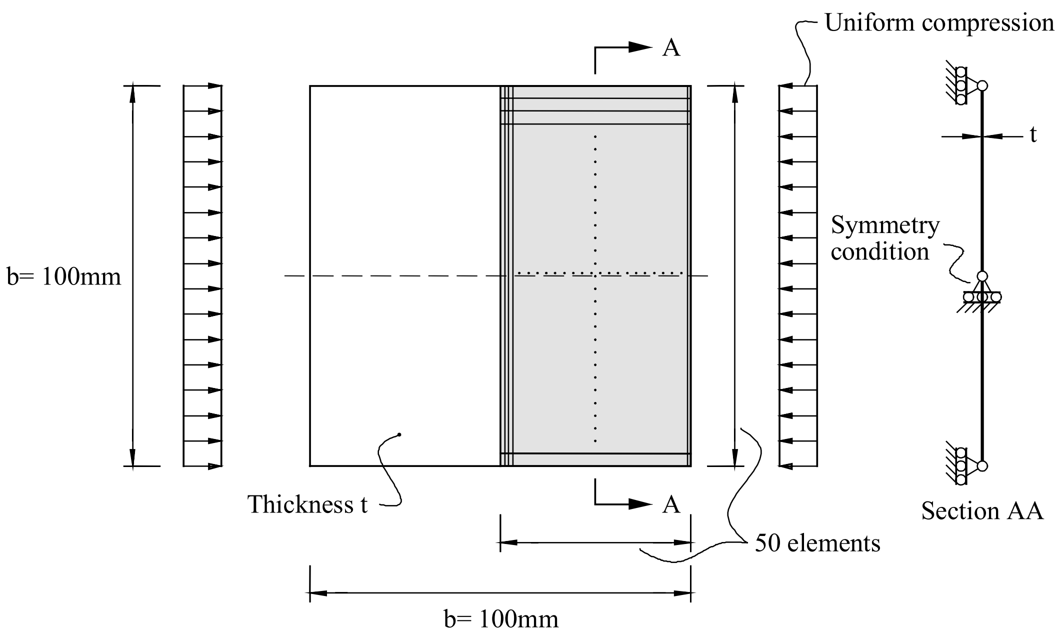

Figure 4: Finite element model (only half of the plate is modeled)

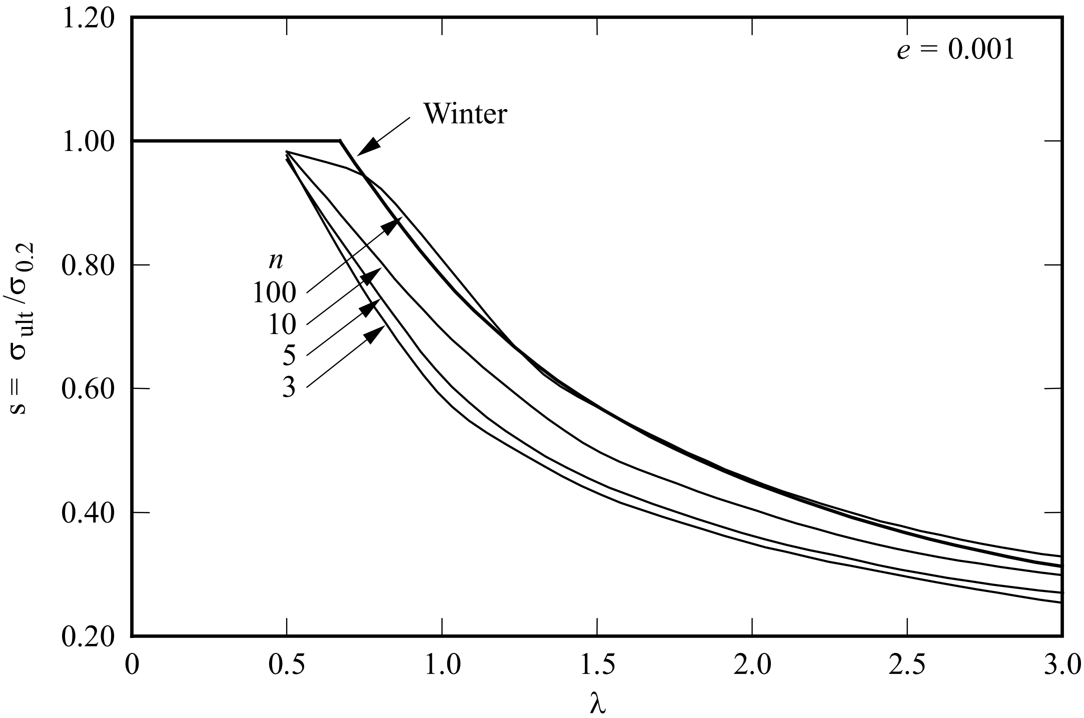

Figure 5: *e*=0.001

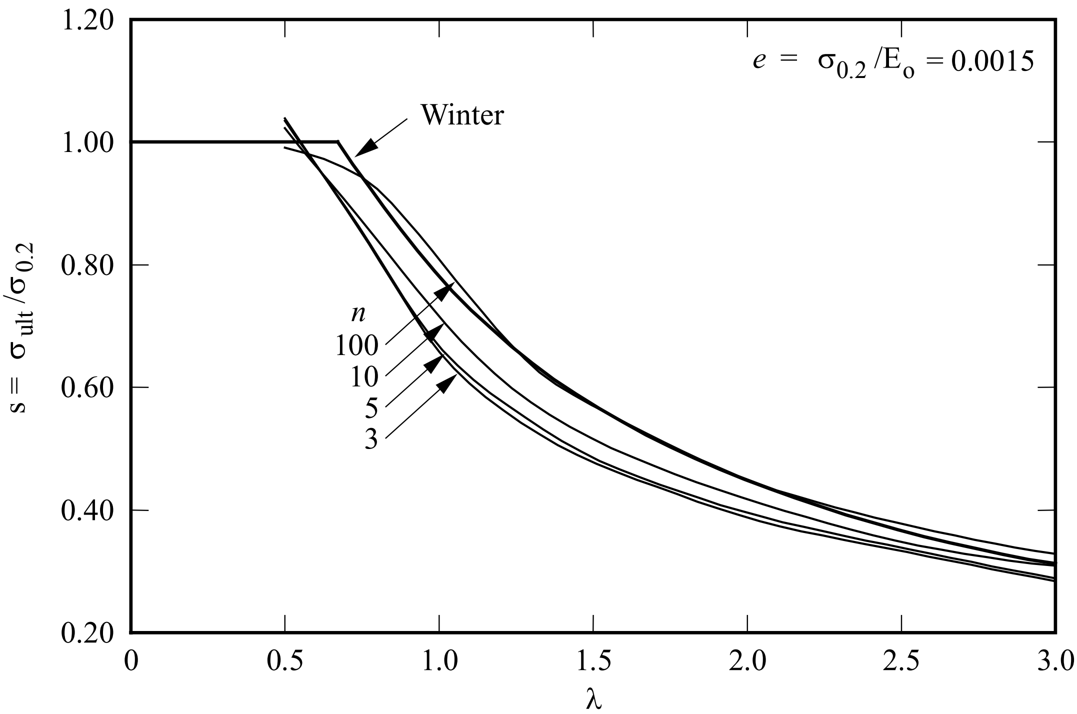

Figure 6: *e*=0.0015

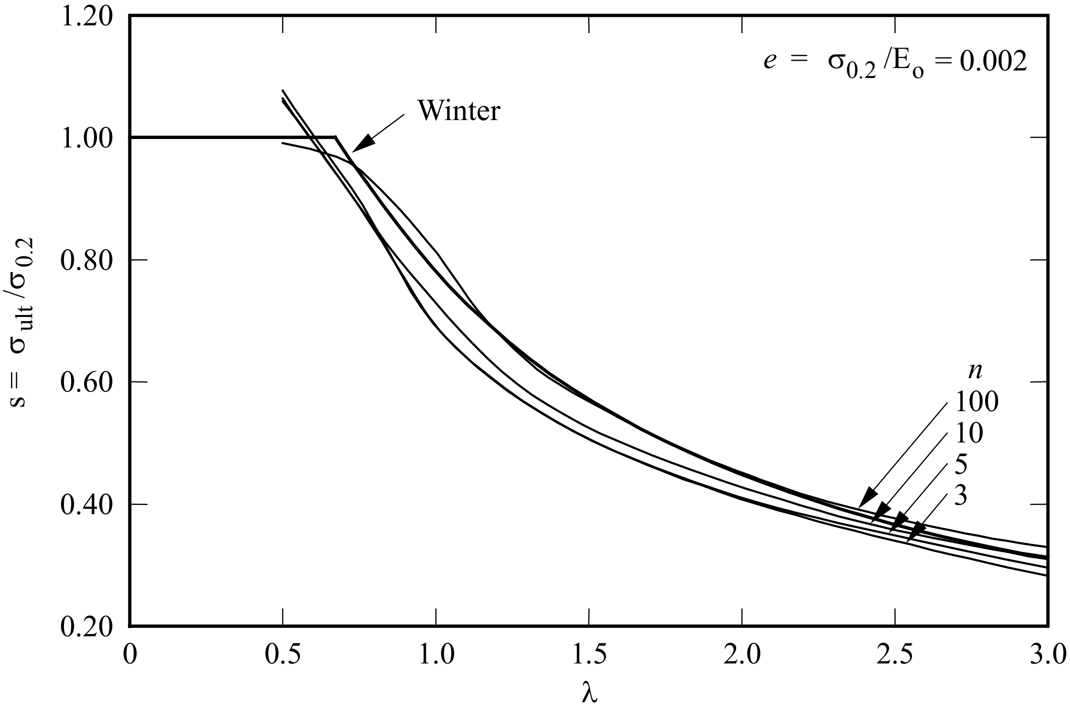

Figure 7: *e*=0.002

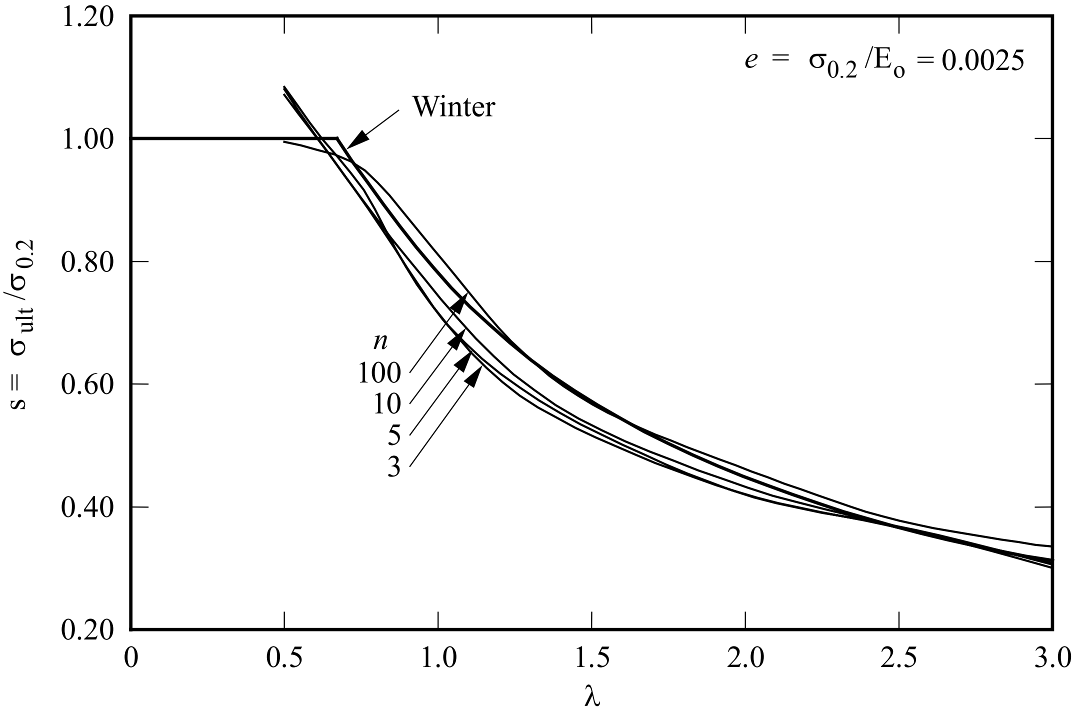

Figure 8: *e*=0.0025

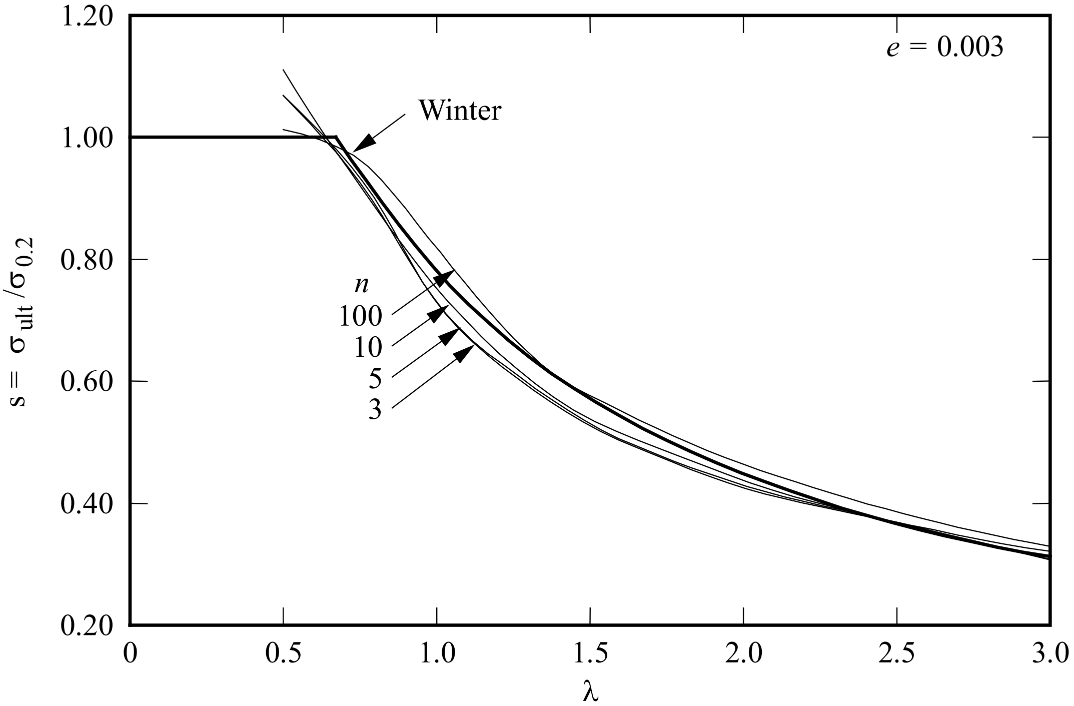

Figure 9: *e*=0.003

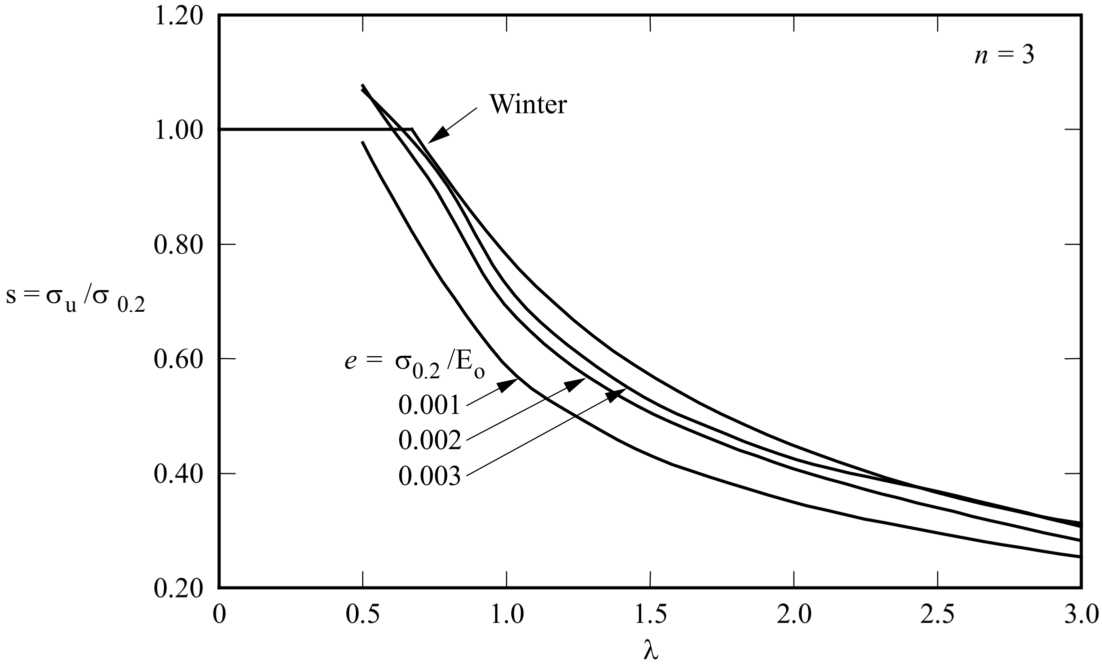

Figure 10: *n*=3

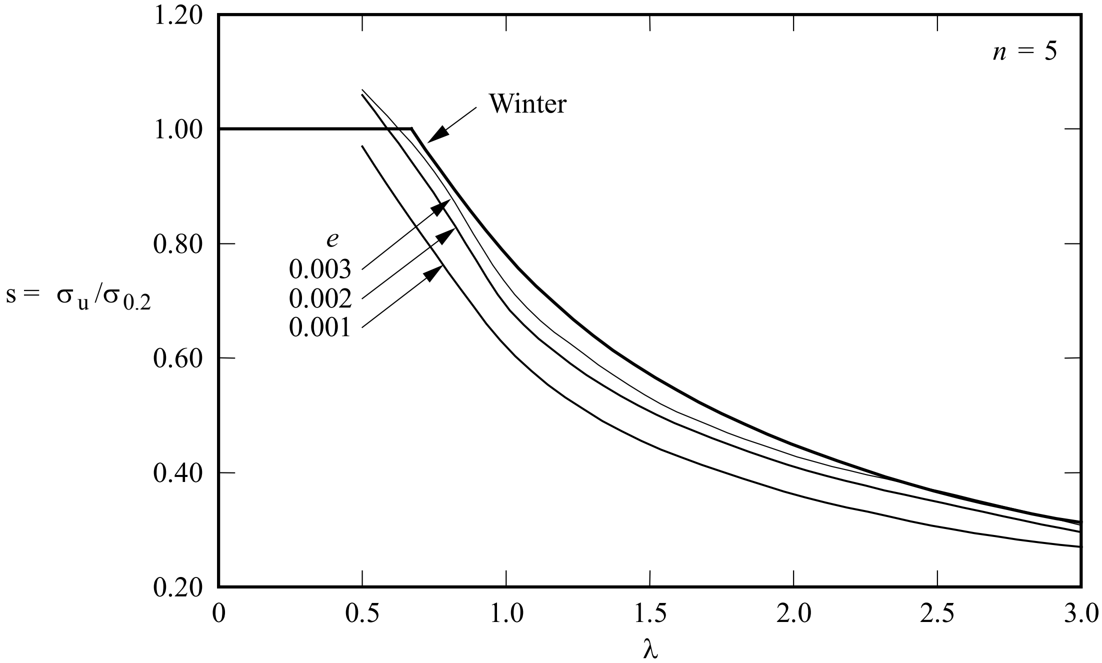

Figure 11: *n*=5

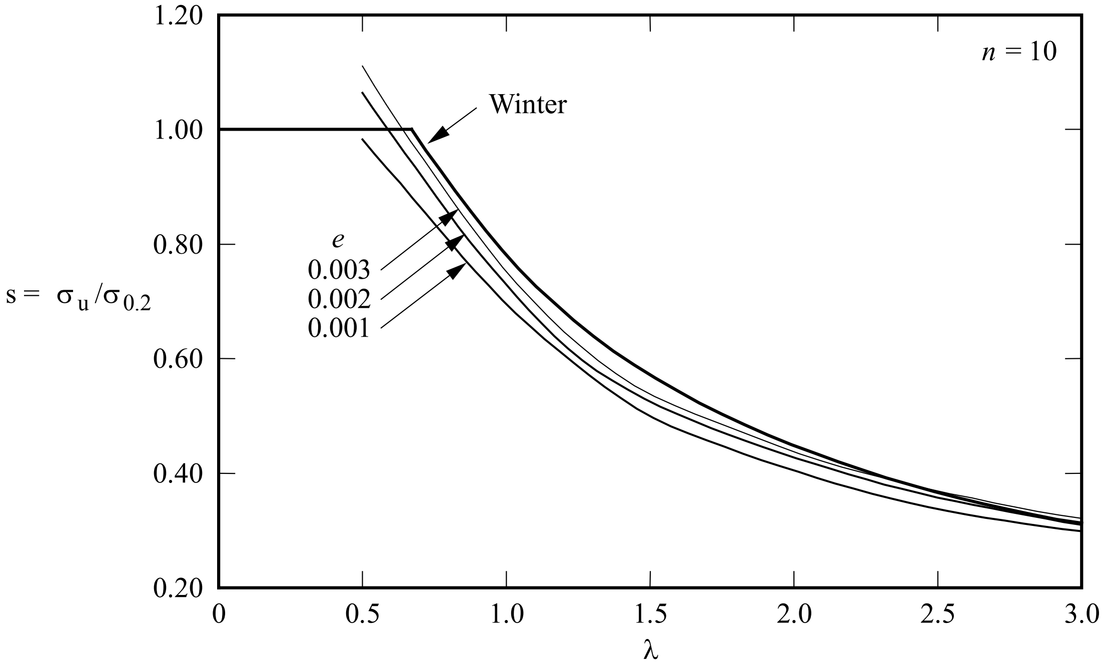

Figure 12: *n*=10

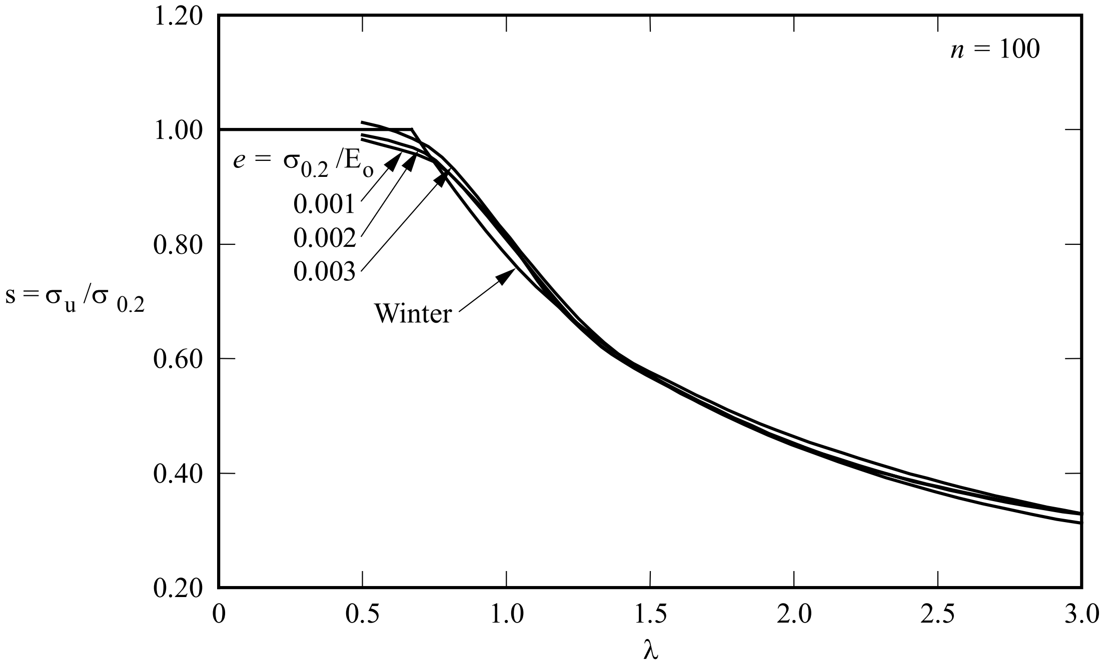

Figure 13: *n*=100

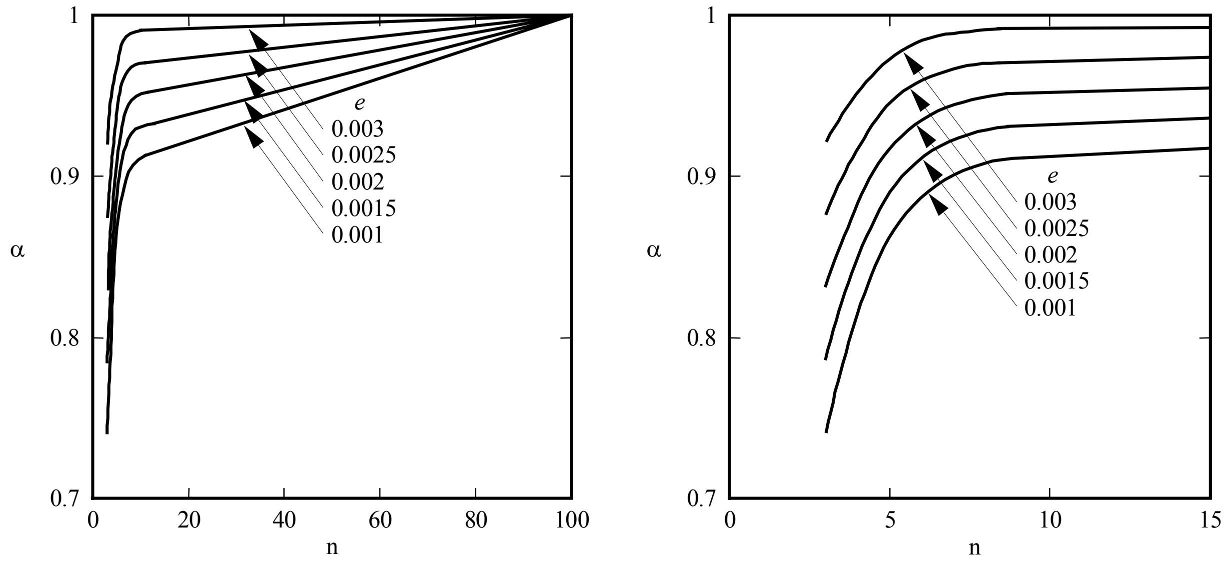

Figure 14: α vs n

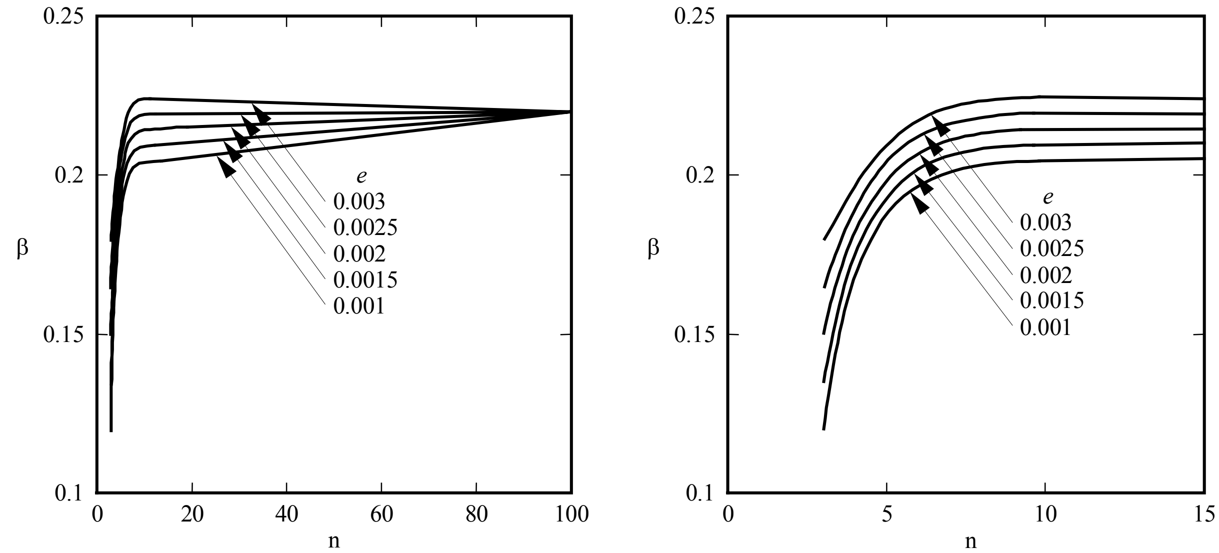

Figure 15: β vs n

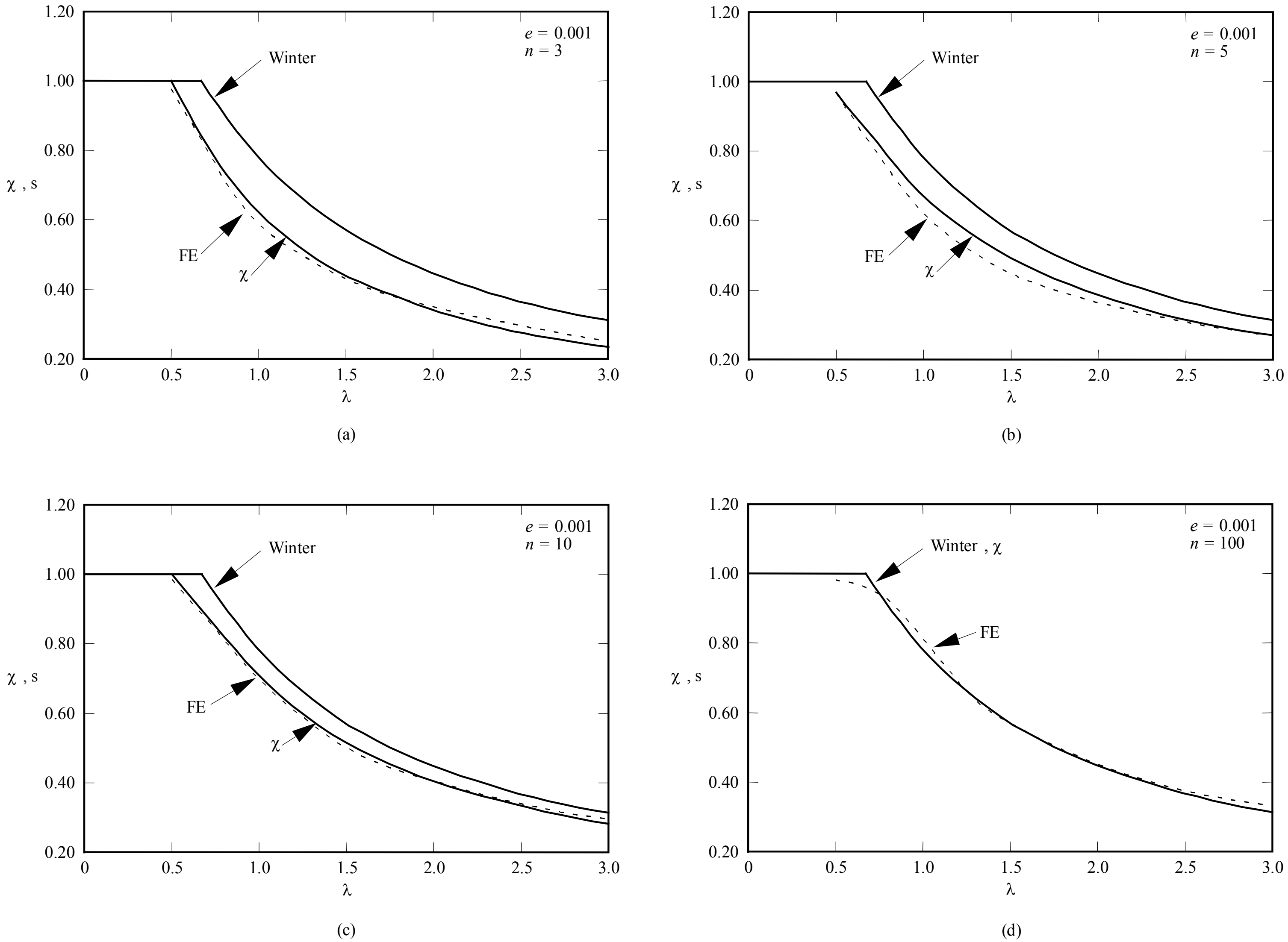

Figure 16: FE strengths and fitted plate strength curves for *e*=0.001

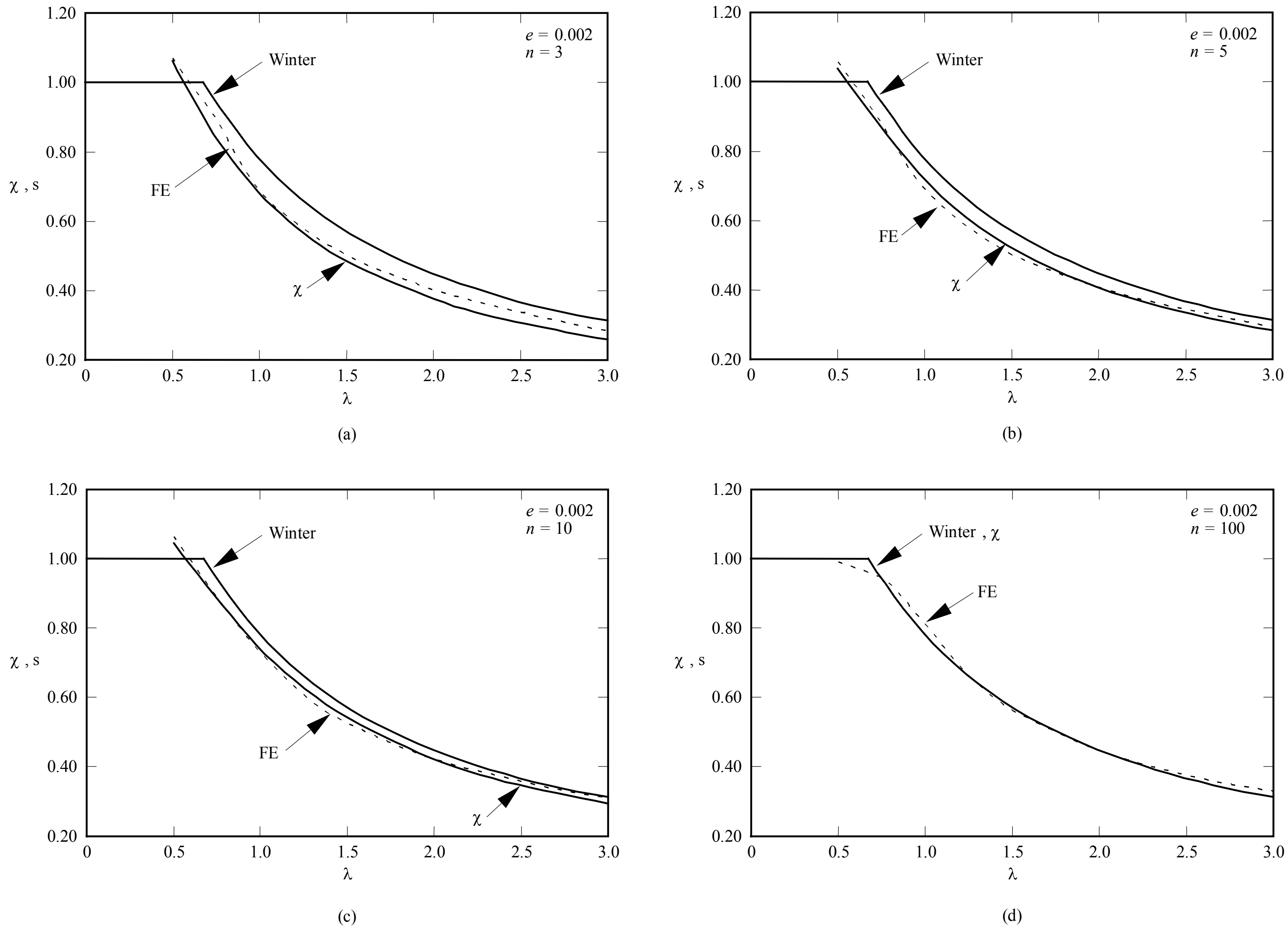

Figure 17: FE strengths and fitted plate strength curves for *e*=0.002

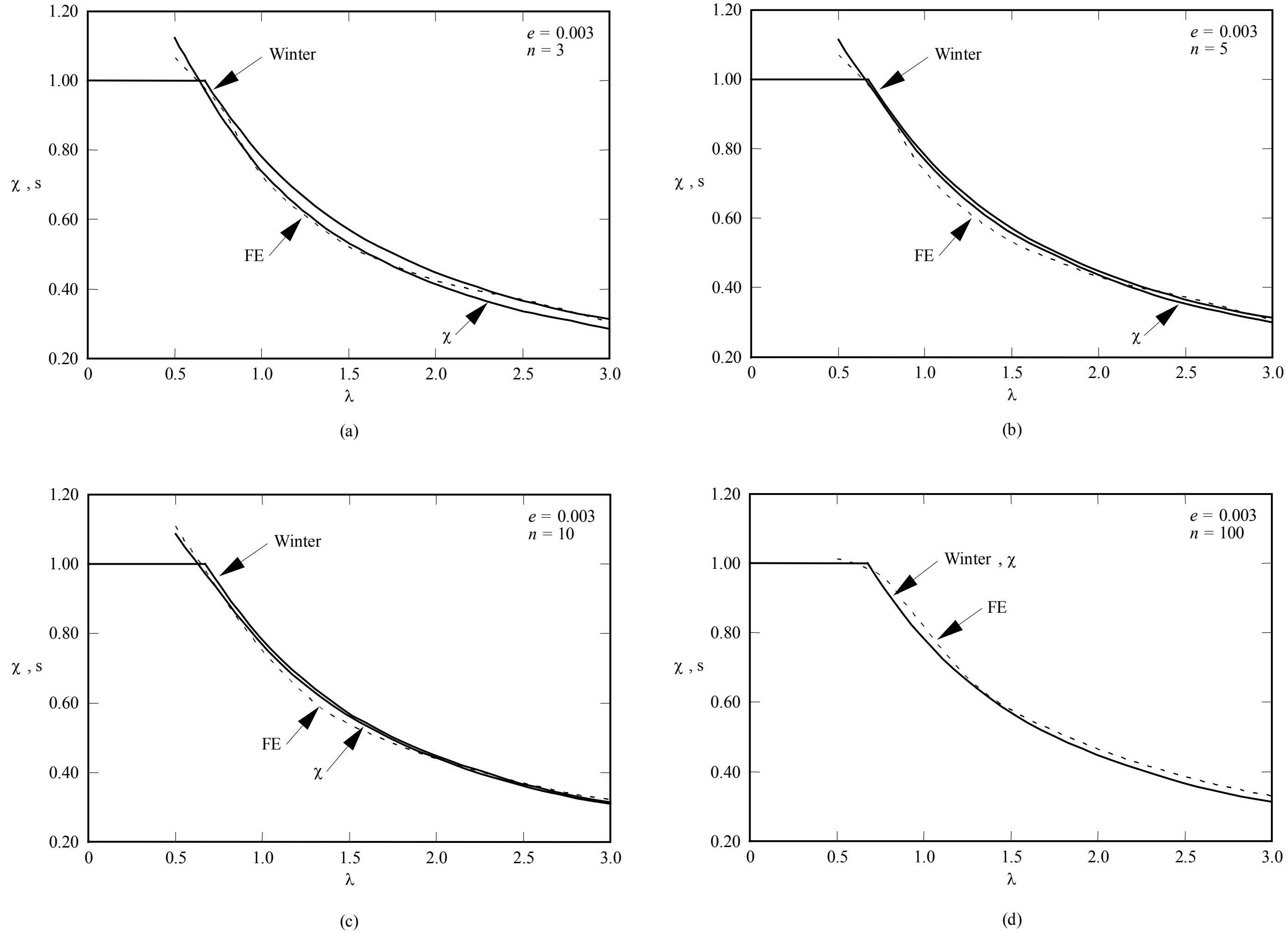

Figure 18: FE strengths and fitted plate strength curves for *e*=0.003

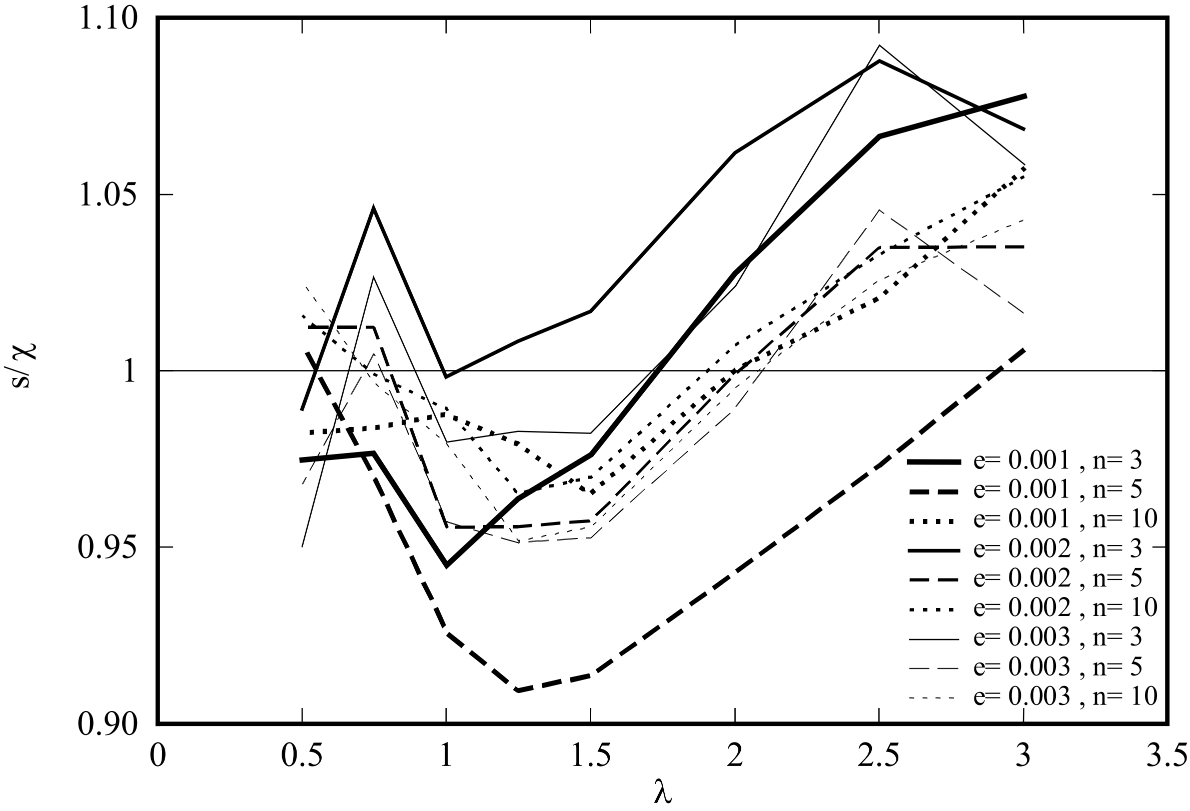

Figure 19: Variations of s/χ

Appendix A: Comparison between finite element strengths and generalised plate strength curves

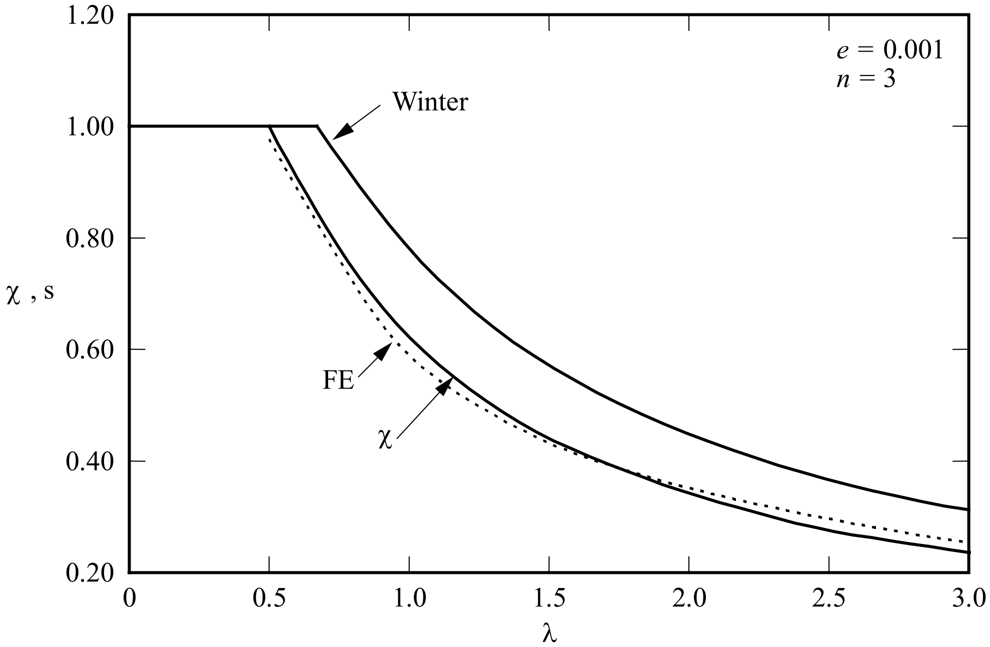

Figure A1: *e*=0.001, *n*=3

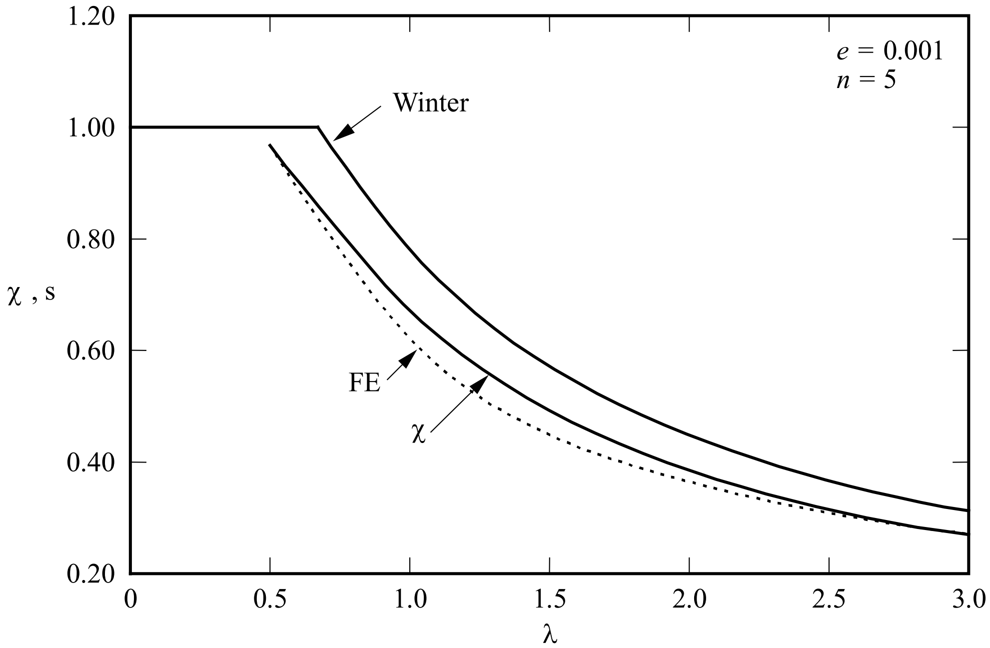

Figure A2: *e*=0.001, *n*=5

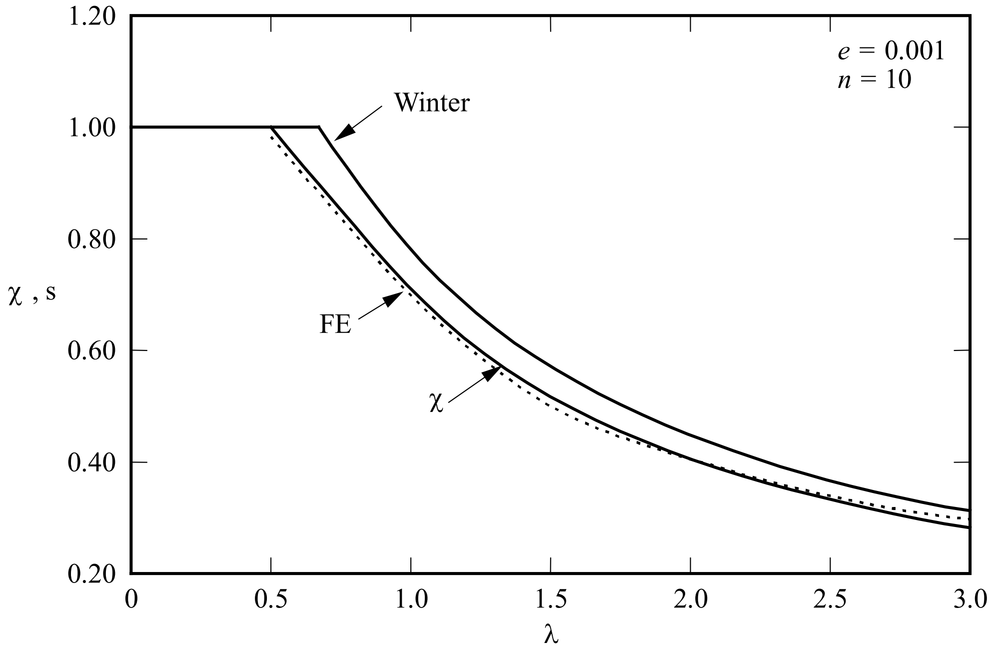

Figure A3: *e*=0.001, *n*=10

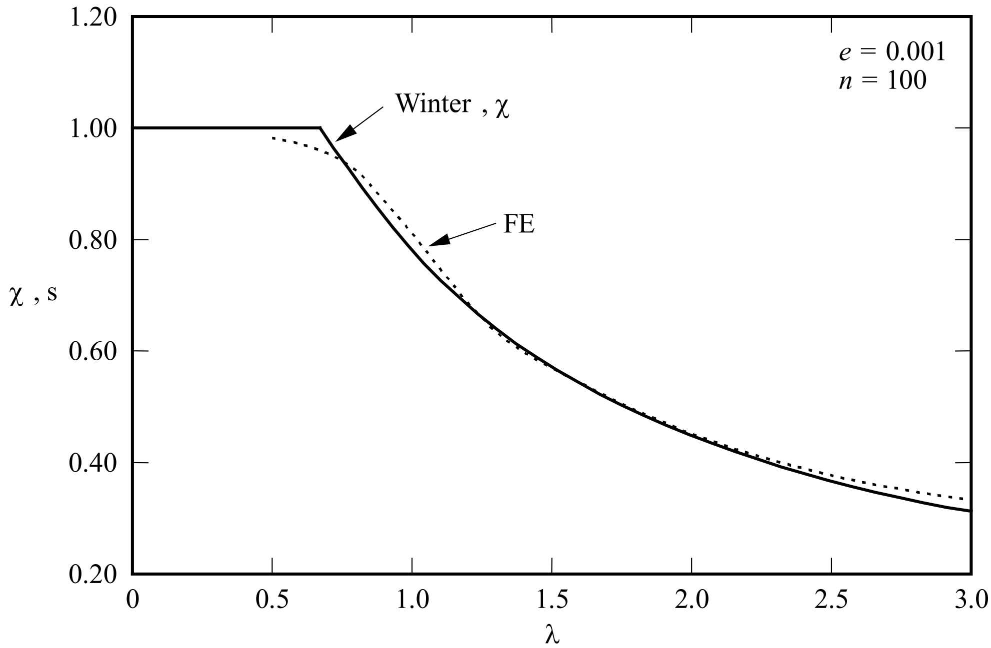

Figure A4: *e*=0.001, *n*=100

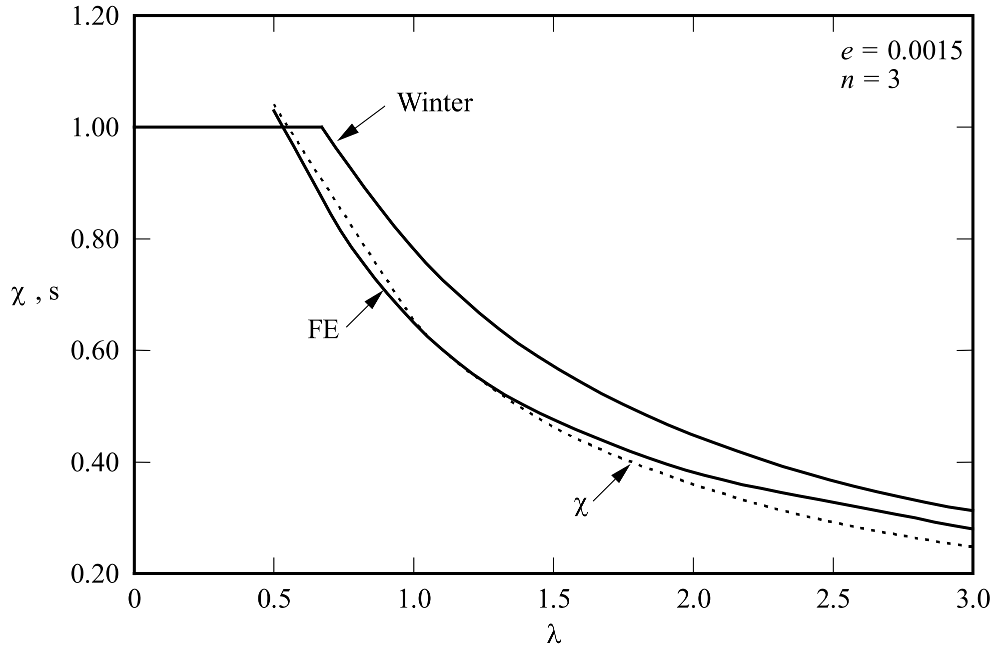

Figure A5: *e*=0.0015, *n*=3

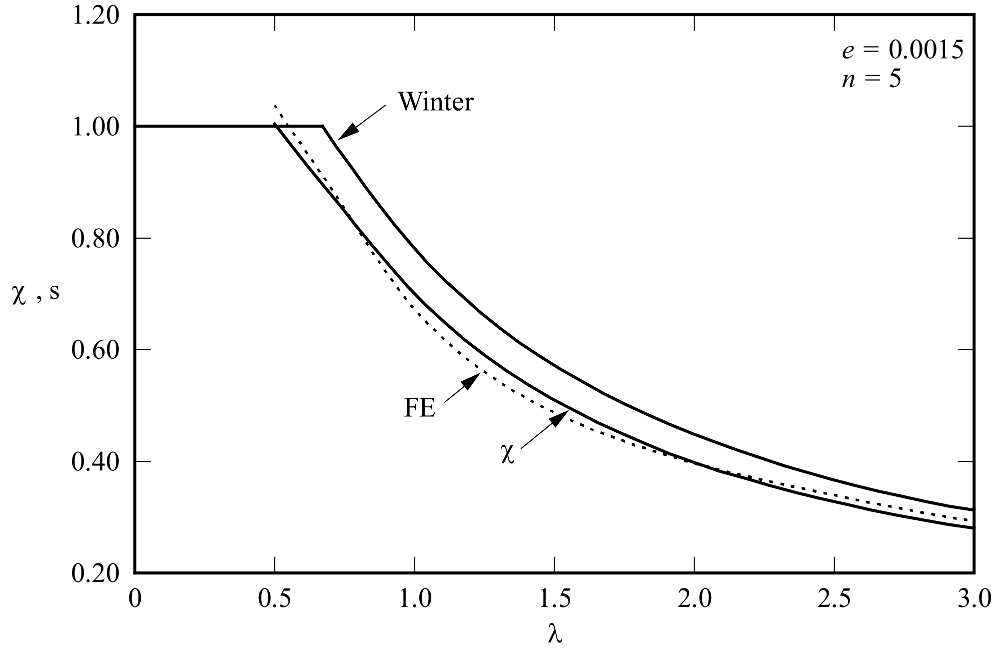

Figure A6: *e*=0.0015, *n*=5

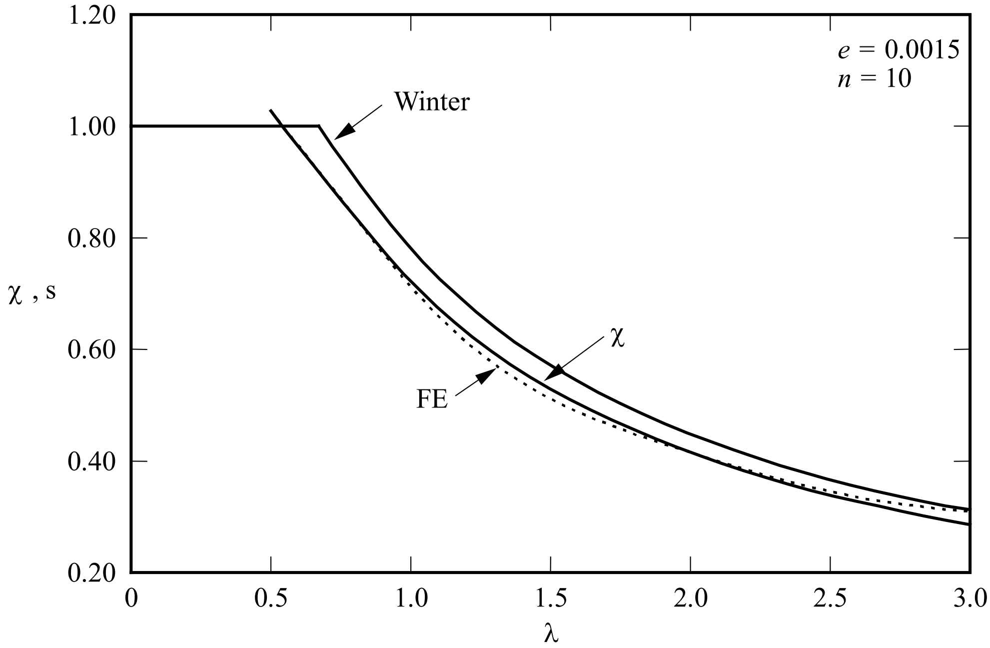

Figure A7: *e*=0.0015, *n*=10

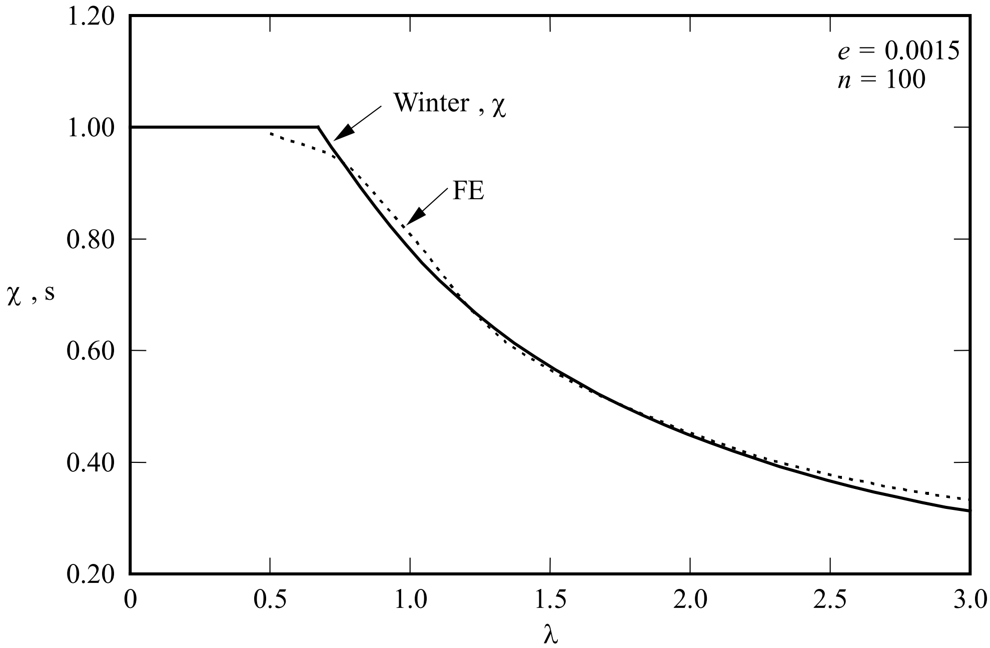

Figure A8: *e*=0.0015, *n*=100

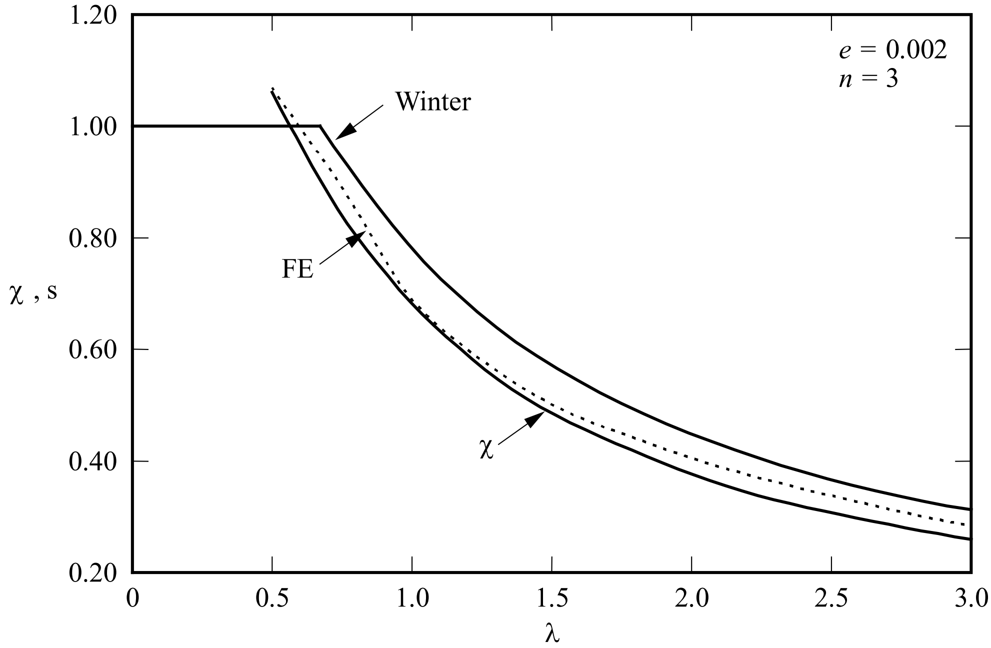

Figure A9: *e*=0.002, *n*=3

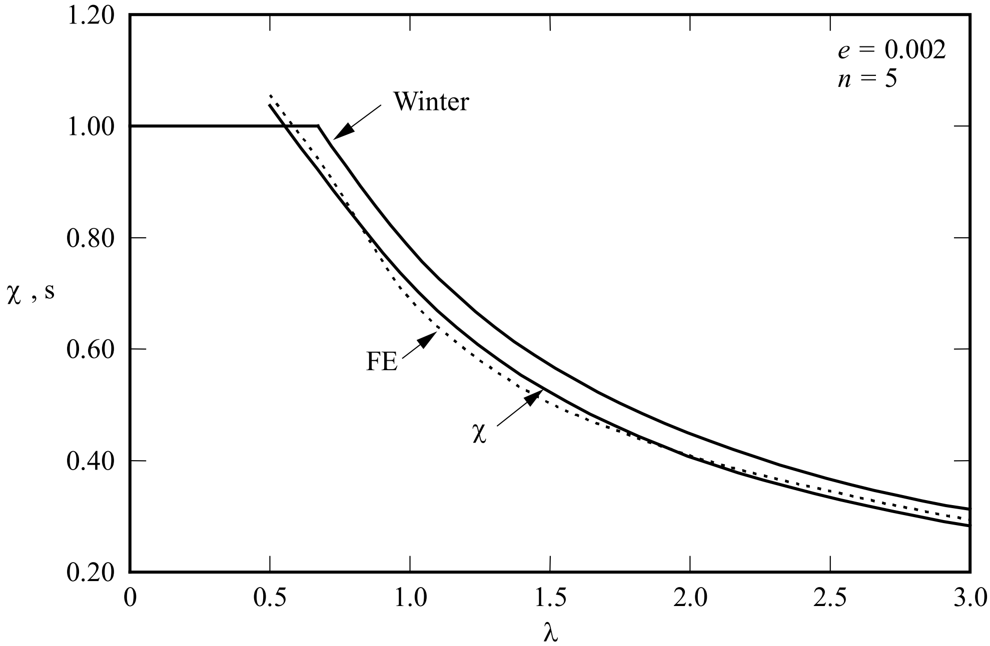

Figure A10: *e*=0.002, *n*=5

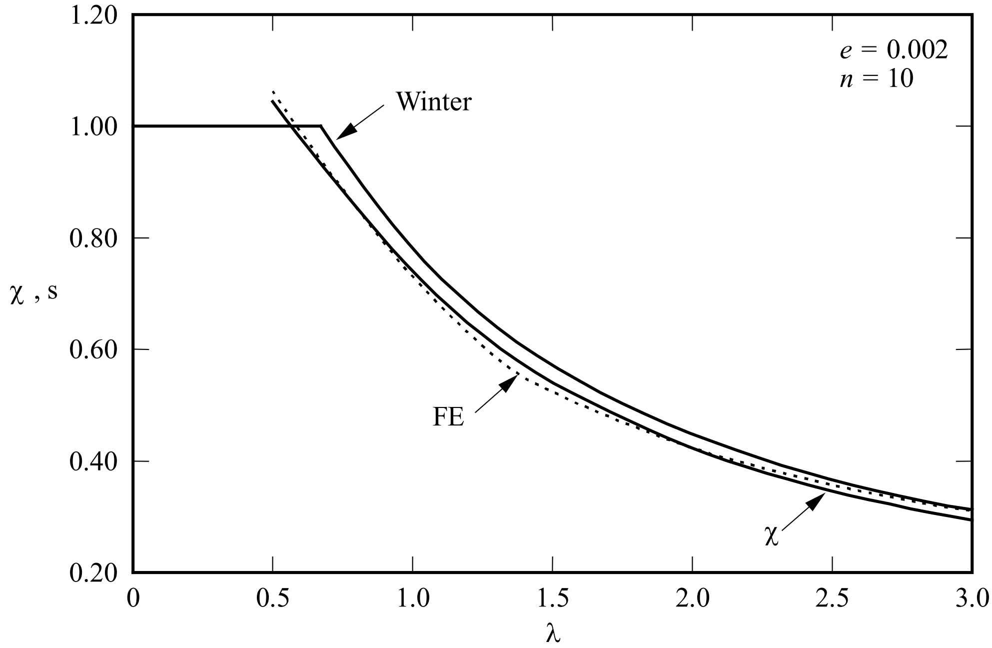

Figure A11: *e*=0.002, *n*=10

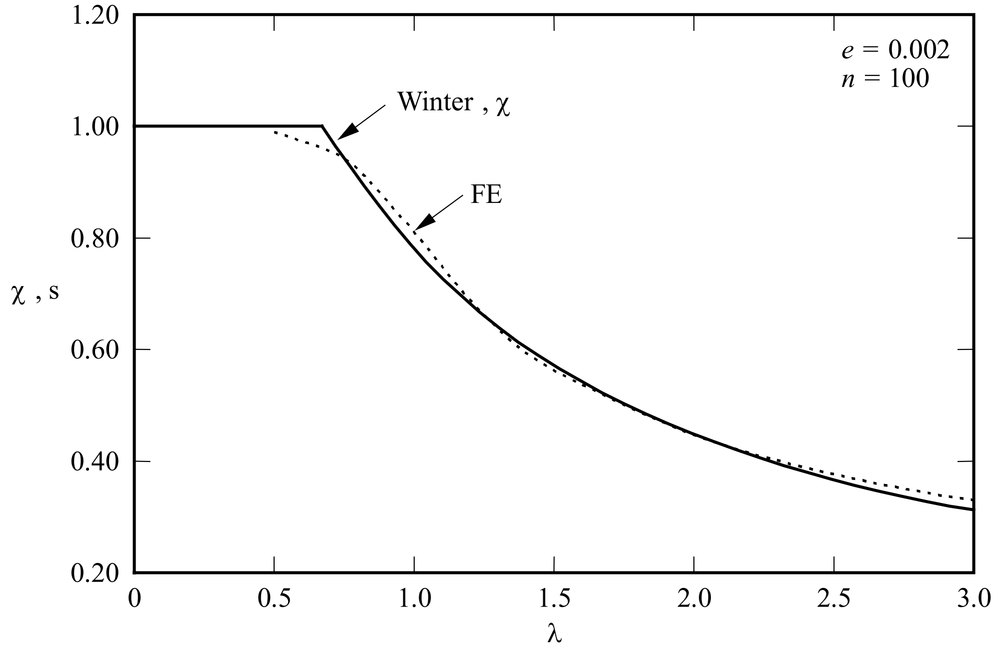

Figure A12: *e*=0.002, *n*=100

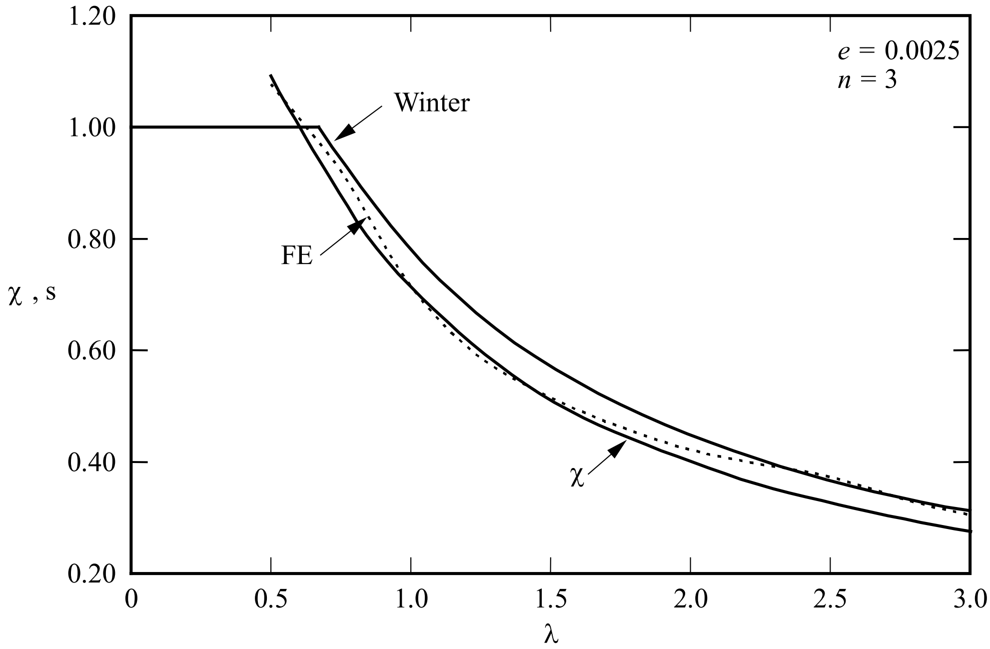

Figure A13: *e*=0.0025, *n*=3

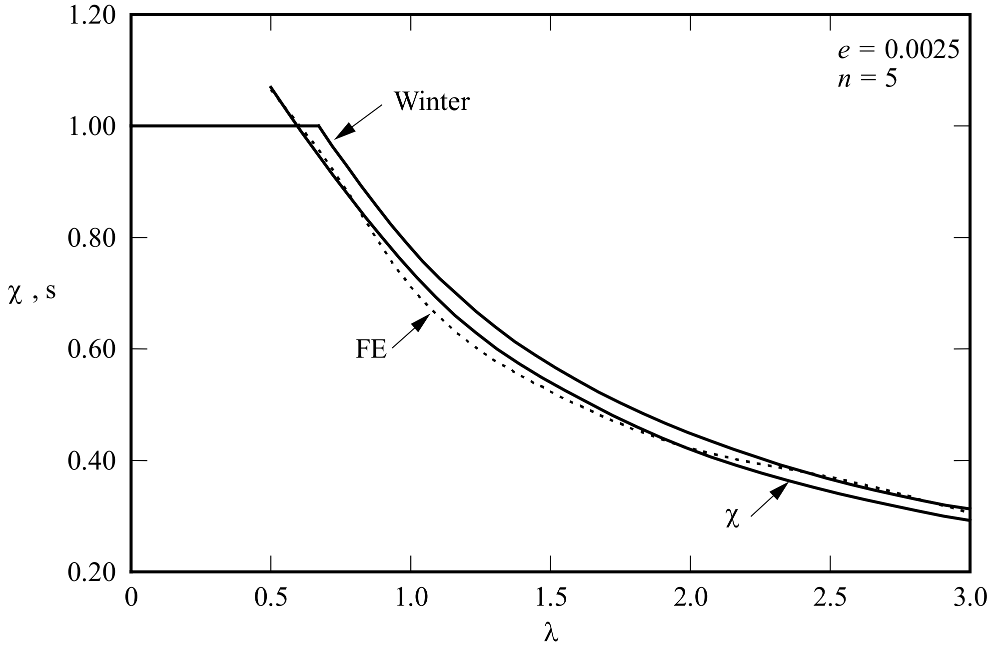

Figure A14: *e*=0.0025, *n*=5

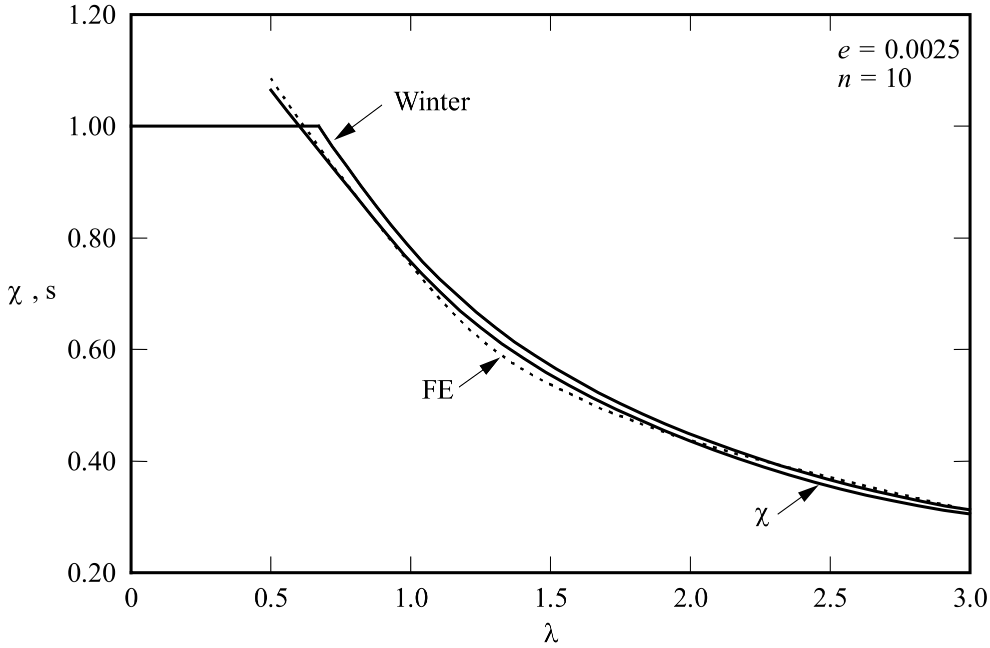

Figure A15: *e*=0.0025, *n*=10

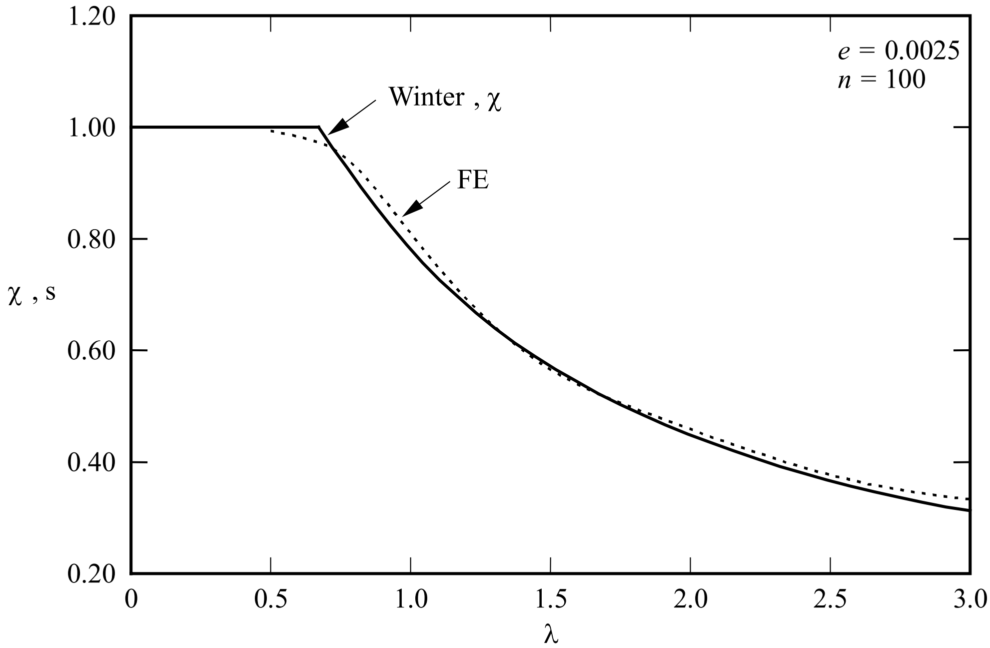

Figure A16: *e*=0.0025, *n*=100

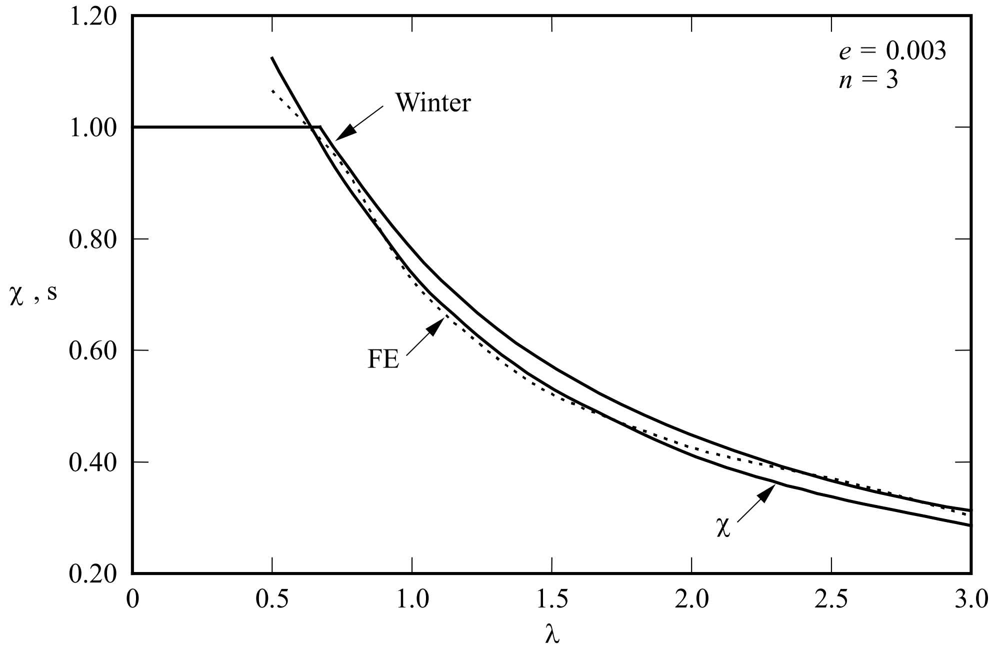

Figure A17: *e*=0.003, *n*=3

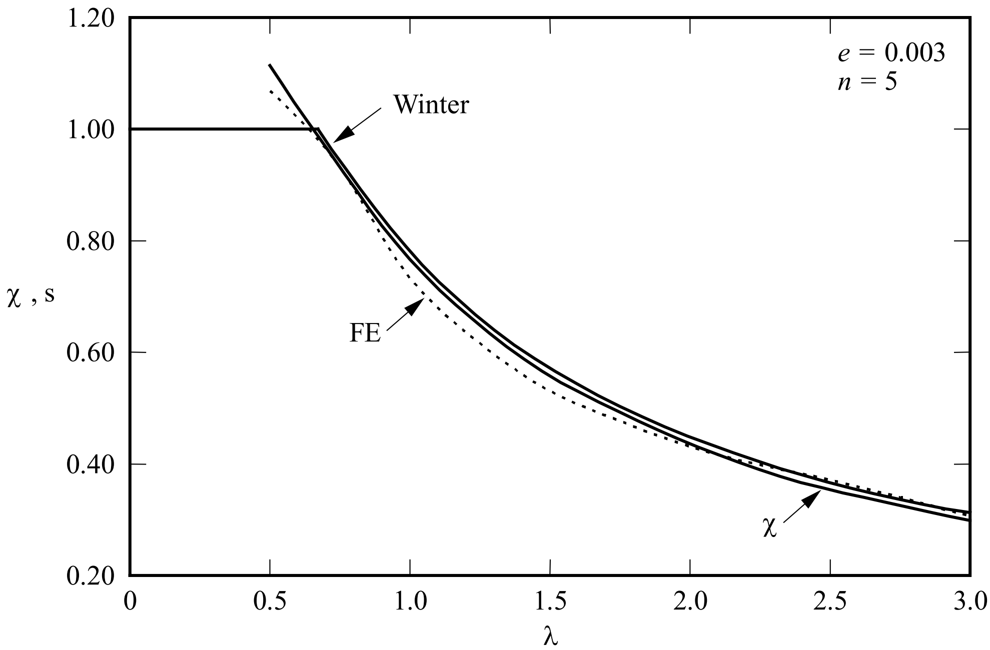

Figure A18: *e*=0.003, *n*=5

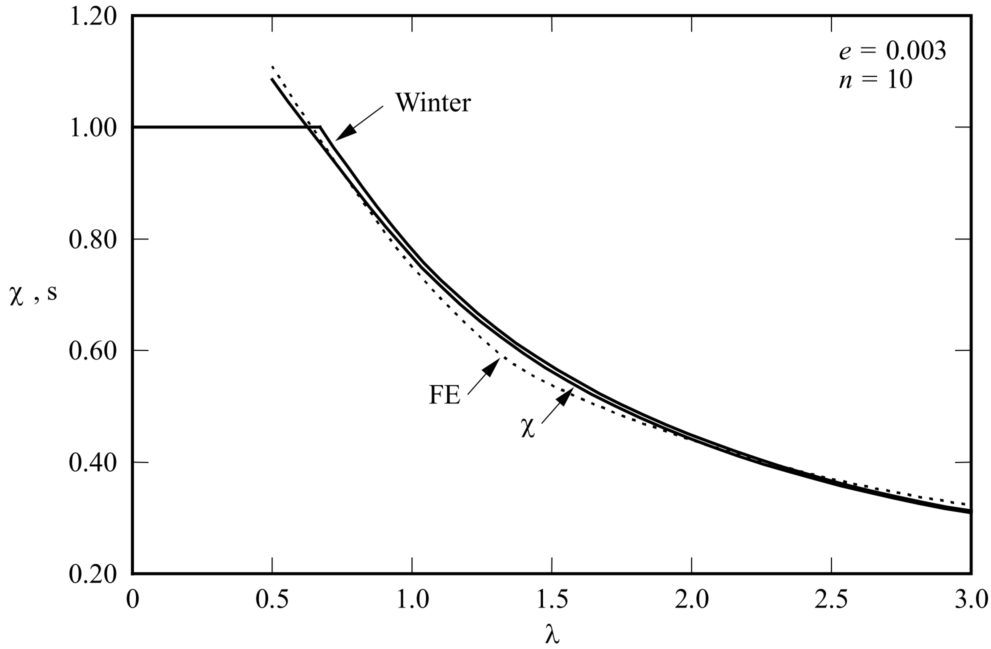

Figure A19: *e*=0.003, *n*=10

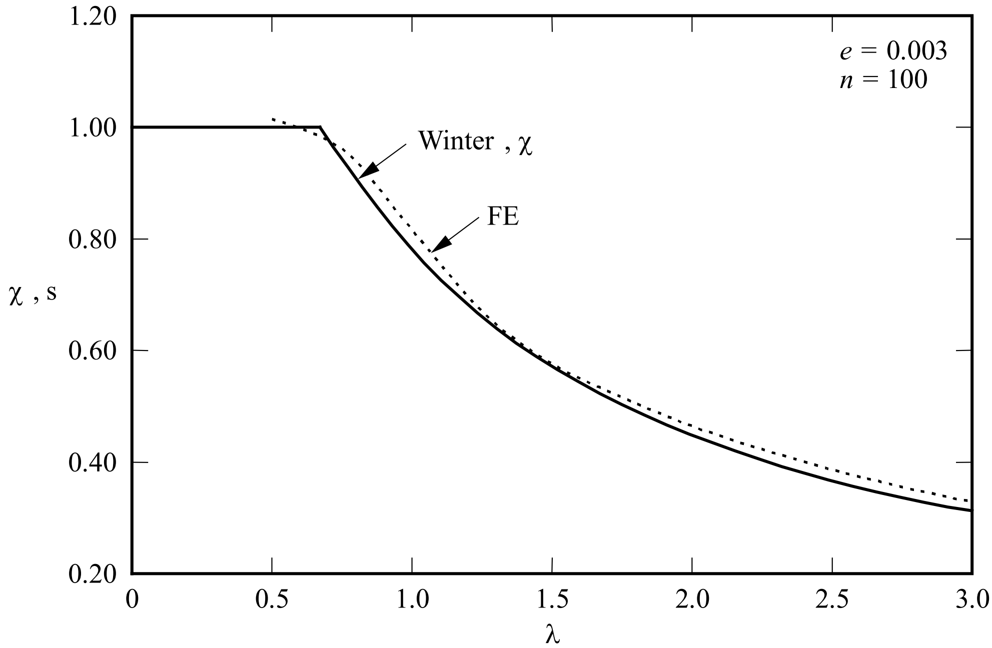

Figure A20: *e*=0.003, *n*=100