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Lateral Buckling Strengths of Steel Angle Section Beams

Research Report No R812

By

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Abstract:

The design of unbraced steel angle section beams against biaxial bending and torsion requires the ability to predict their lateral buckling strengths when bent about the major principal axis. Present design code formulations of lateral buckling strength are based on research on doubly symmetric I-section beams, and may be inappropriate for mono-symmetric and asymmetric angle section beams.

This paper presents close approximations for the elastic lateral buckling moments of equal and unequal angle section beams loaded by uniformly distributed loads which act away from the shear centre.

Small and large twist rotation elastic analyses of angle section beams with initial twist rotations are used with interaction equations for the biaxial bending section capacities to investigate present design code formulations for lateral buckling strength. It is found that these code formulations are unnecessarily conservative, especially at high slendernesses, where the moment capacities predicted by them may be less than the minor axis section capacity instead of greater. An improved formulation is proposed which is simpler and more economic than the present formulations.

The use of the elastic lateral buckling approximations in the improved design moment capacity formulation is demonstrated in a worked example.

Keywords: angles, beams, bending, buckling, design, elasticity, member capacity, moments, section capacity, steel.

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1 INTRODUCTION

This paper is concerned with the design of unbraced steel angle section beams against lateral buckling. Previous related papers were concerned with the design of braced angle section beams against biaxial bending (Trahair, 2001a) and against bearing, shear, and torsion (Trahair, 2001b).

Although unbraced angle section beams which are bent about their major principal axis may fail by buckling laterally and twisting, this is rare in practice because angle section beams are seldom loaded in this way. Much more commonly, the applied loads act out of the principal planes and eccentrically from the shear centre, as shown in Fig. 1, so that they cause simultaneous biaxial bending about both principal axes and torsion. Despite this, the general practice for designing unbraced beams against biaxial bending is to consider the separate failure modes of either in-plane bending or lateral buckling under bending about the major principal axis alone, and in-plane bending about the minor principal axis, and then to use an interaction equation to combine the two principal axis strengths. For this reason, it is appropriate to consider the lateral buckling strengths of angle section beams, even though these are rarely sufficient to check the adequacy of beams under practical loading.

Design codes (AISC, 2000a,b; BSI, 2000; SA, 1998) provide interaction equations for designing against biaxial bending which are largely derived from research on doubly symmetric I-section beams, which may be inappropriate for angle section beams, which are either mono-symmetric or asymmetric. While the lateral buckling rules of these codes allow for mono-symmetry effects in beams under uniform bending, they appear to be in error for non-uniform bending with loads acting away from the shear centre.

Some previous research studies (Leigh and Lay, 1969, 1970a; Thomas and Leigh, 1973; and Go et al; 1991) have analysed the elastic biaxial bending and torsion of angle section beams loaded as shown in Fig. 1, and have been used to develop safe load or design capacity tables (Leigh and Lay, 1970b; AISC, 1999). However, these tables cannot be modified for different loading conditions.

Research studies by Earls and Galambos (1997, 1998) and Earls (2001a-e) have been largely analytical, and directed towards finding width-thickness and length-width limits for angle section beams which can be designed plastically, and are therefore only relevant to beams with sufficient bracing to prevent lateral buckling.

Surprisingly, there has been comparatively little specific research into the lateral buckling of angle section beams, although Earls (1999) has studied numerically the ability of unbraced equal angles bent about their major axis to reach full plasticity. This is probably because of the wealth of information available on the effects of mono-symmetry, moment distribution, and load height on elastic lateral buckling (Timoshenko and Gere, 1961; CRCJ, 1971; Trahair, 1993). However, information on unequal angles is either scattered or incomplete.

The first purpose of this paper is therefore to provide close approximations for the elastic lateral buckling moments of angle section beams with distributed loads acting away from the shear centre, and this is done in Section 2 following. The second purpose is to develop a rational and economic method for the design of unbraced angle section beams against lateral buckling which uses these elastic buckling approximations, and this is done in Section 3. Finally, a worked example illustrating this method is provided in Section 4.

2 ELASTIC LATERAL BUCKLING

The value of equal and opposite end moments M_{yz} that cause the elastic lateral buckling of a simply supported equal angle section beam of span L to buckle elastically by deflecting laterally u and twisting ϕ is given by (Timoshenko and Gere, 1961; Trahair, 1993)

$$M_{yz} = \sqrt{(\pi^2 EI_y GJ / L^2)} \quad (1)$$

in which E and G are the Young's and shear moduli of elasticity, I_y is the minor axis second moment of area, and J is the torsion section constant.

The corresponding value for an unequal angle section beam is given by (Timoshenko and Gere, 1961; Trahair, 1993)

$$M_{yzu} = M_{yz} \left\{ \sqrt{[1 + (\beta_x P_y / 2M_{yz})^2]} + \beta_x P_y / 2M_{yz} \right\} \quad (2)$$

in which

$$P_y = \pi^2 EI_y / L^2 \quad (3)$$

and β_x is a monosymmetry section constant given by

$$\beta_x = \frac{1}{I_x} \int_A y(x^2 + y^2) dA - 2y_o \quad (4)$$

in which A is the cross-sectional area, I_x is the major axis second moment of area, and y_o is the y coordinate of the shear centre at the intersection of the angle legs. The signs of y_o and β_x for two different angle section attitudes are illustrated in Fig. 2. The variation of M_{yzu} / M_{yz} with $\beta_x P_y / 2M_{yz}$ is shown in Fig. 3.

The elastic lateral buckling of simply supported mono-symmetric beams under uniformly distributed loads q has been investigated by Anderson and Trahair (1972) and Wang and Kitipornchai (1986), and approximate expressions have been developed (Trahair, 1993) for predicting the value of the maximum moment $M = qL^2/8$ at elastic buckling, which are of somewhat variable accuracy.

For this present paper, the elastic buckling of simply supported angle section beams with uniformly distributed loads and $0.5 \leq \beta \leq 1$ (in which β is the angle section leg length ratio) has been investigated using the computer program PRFELB (Papangelis et al, 1997, 1998). The maximum moment M_{qu} at the elastic lateral buckling of a simply supported unequal angle section beam under uniformly distributed loads q acting at the shear centre is closely approximated by

$$M_{qu} = 1.13M_{yz} \left(1 + 0.57 \frac{\beta_x P_y}{2M_{yz}} \right) \quad (5)$$

as shown in Fig. 3. It can be seen in Fig. 3 that the variation of $M_{qu} / 1.13M_{yz}$ with $\beta_x P_y / 2M_{yz}$ is quite different to that of M_{yzu} / M_{yz} for beams in uniform bending.

The maximum moment M_{qey} at the elastic lateral buckling of a simply supported equal angle section beam under a uniformly distributed load q acting at a distance y_q below the shear centre is closely approximated by

$$M_{qey} = 1.13M_{yz} \left\{ \sqrt{1 + \left(\frac{0.43y_q P_y}{M_{yz}} \right)^2} + \frac{0.43y_q P_y}{M_{yz}} \right\} \quad (6)$$

as shown in Fig. 4.

The effect of a distributed load q acting at a distance $(y_q - y_o)$ below the shear centre of a simply supported unequal angle section beam is not so easily approximated, but fortunately, either the load in practice acts near the shear centre, or else the effect of load height is small (as in the case of long span beams). In these cases, approximations for the maximum moments at elastic buckling can be obtained using

$$M_{quy} = M_{qu} \left\{ \sqrt{1 + \left(\frac{0.43(y_q - y_o)P_y}{M_{yz}} \right)^2} + \frac{0.43(y_q - y_o)P_y}{M_{yz}} \right\} \quad (7)$$

except for beams with negative values of β_x and positive values of $(y_q - y_o)$, as shown in Fig. 4.

The use of Equations 1-7 may be simplified by using approximations for the variations with the leg length ratio β of the section properties α (the inclination of the x principal axis to the X rectangular axis), I_y , J , y_o and β_x . These variations (for the range of $0.5 \leq \beta \leq 1$) have been determined using the program THIN-WALL (Papangelis and Hancock, 1997) (which uses the thin-walled assumption), and are shown in dimensionless form in Fig. 5, together with the close approximations

$$\alpha^\circ = -5.4 + 34.4 \beta + 16 \beta^2 \quad (8)$$

$$12 I_y / b^3 t = -0.373 + 1.053 \beta + 0.320 \beta^2 \quad (9)$$

$$3 J / 2 b t^3 = 0.5(1 + \beta) \quad (10)$$

$$y_o / b = 0.436 - 0.116 \beta - 0.320 \beta^2 \quad (11)$$

$$-\beta_x / b = 0.9 + 0.1 \beta - \beta^2 \quad (12)$$

3 METHODOLOGY FOR LATERAL BUCKLING DESIGN STRENGTH

3.1 General

The most common methodology for designing a steel beam uses a simple first-order elastic analysis of the beam to determine the design actions. The adequacy of the beam is then assessed by comparing these design actions with code rules (AISC, 1994a,b; BSI, 2000; SA, 1998) for the design section and member capacities. The design section capacities take account of yielding and local buckling of the section, while the design member capacities include an approximation for the lateral buckling strength which makes allowance for residual stresses and initial crookednesses and twists. These design capacities may be based on experimental evidence or on advanced theoretical analyses, but often these are incomplete, in which case intuitive formulations may be used.

Much less common is the method of design by advanced analysis (SA, 1998), in which the analysis includes the effects of material (yielding) and geometrical (buckling) non-linearities on the load-deformation behaviour of a structure with representative residual stresses and initial crookednesses and twists. The assessment of the adequacy of the structure then simplifies to a consideration of whether the analysis will show that the structure can reach equilibrium under the design loads. Design by advanced analysis was originally permitted only for plane frames composed of doubly symmetric compact sections (no local buckling) and with sufficient bracing to prevent lateral buckling, although there are concerted efforts underway to extend advanced analysis to structures which may fail by local or lateral buckling. The only examples of advanced analyses of angle section beams appear to be those used by Earls and Galambos (1997, 1998) and Earls (2001a-e) to develop design rules for determining whether an angle section beam has sufficient rotation capacity that it can be designed by using simple plastic analysis.

The two design methodologies outlined above represent two extremes. In the first (common code design), the analysis is as simple as possible, with all the complications of non-linear behaviour, residual stresses and initial crookednesses and twists and interactions approximated in the code rules for the design capacities. In the second methodology (design by advanced analysis), all the complications are accounted for by the advanced analysis, allowing the assessment of structural adequacy to be enormously simplified.

In this paper, a different methodology is used, in which approximate second-order elastic analyses are used which allow for geometric non-linearities (lateral buckling effects) in angle section beams with initial crookednesses and twists (which are increased to allow also for the effects of residual stresses). These analyses are used to determine the maximum moments in the beam. The adequacy of the beam is then assessed by comparing the maximum moments with design moment section capacities (Trahair, 2001a) which account for yielding and local buckling. This method is then used to develop an improved formulation for the lateral buckling design strengths of angle section beams which may be used in conjunction with a simple first order analysis for the design moments.

The capacities of the beam to resist bearing, shear, and torsion are checked separately by comparing the appropriate design actions (which may also be determined by a simple first-order analysis of the beam) with the corresponding design capacities recommended in Trahair (2001b).

3.2 Small Rotation Analysis

The small rotation differential equilibrium equations for the elastic bending and torsion of a simply supported angle section beam with an initial twist rotation ϕ_0 under equal and opposite end moments M are (Trahair, 1993; Trahair et al, 2001)

$$E I_y u'' = -M (\phi + \phi_0) \quad (13a)$$

$$G J \phi' = M u' \quad (13b)$$

in which $' \equiv d / dz$.

$$\text{When} \quad \phi_0 = \theta_0 \sin \pi z / L \quad (14)$$

$$\text{then} \quad \phi = \theta \sin \pi z / L \quad (15)$$

The maximum twist rotations θ are given by (Trahair et al, 2001)

$$\frac{\theta}{\theta_o} = \frac{(M/M_{yz})^2}{1-(M/M_{yz})^2} \quad (16)$$

and the maximum principal axis moments are

$$M_x = M \quad (17a)$$

$$M_y = \frac{-M\theta_o}{1-(M/M_{yz})^2} \quad (17b)$$

3.3 Section Moment Capacities of Angle Sections

The nominal section moment capacities M_{sx} , M_{sy} (AISC, 1994a,b; BSI, 2000; SA, 1998) vary with the classification of the section. BS 5950 (BSI, 2000) classifies sections as being plastic, compact, semi-compact, or slender. A plastic section is able to form and maintain a plastic hinge while a plastic mechanism develops, a compact section is able to form a plastic hinge, a semi-compact section has a moment capacity which is reduced below the full plastic moment by inelastic local buckling effects, and a slender section has a moment capacity which is reduced below the first yield moment by elastic local buckling effects.

A section is classified by comparing the width-thickness slenderness ratio of a critical compression element of the section of the form of

$$\lambda_t = \frac{b}{t} \sqrt{\frac{f_y}{250}} \quad (18)$$

with plastic, compact and yield limits λ_p , λ_c , and λ_y , in which b and t are the width and thickness of the element and f_y is the yield stress.

Recommendations for the classification of angle sections are made in Trahair (2001a) for which the slenderness λ_t is based on the length b of the long leg. Plastic beams satisfy

$$\lambda_t \leq \lambda_p \quad (19)$$

in which the plastic limit is $\lambda_p = 12$ for strong axis bending and $\lambda_p = 10$ for weak axis bending, while compact sections satisfy

$$\lambda_p < \lambda_t \leq \lambda_c \quad (20)$$

in which the compact limit is $\lambda_c = 16$ for strong axis bending and $\lambda_c = 14$ for weak axis bending.

The nominal section moment capacities M_{sx} , M_{sy} of both plastic and compact beams are equal to their full plastic capacities M_{pxm} , M_{pym} . The full plastic moments M_{pX} , M_{pY} about the rectangular axes of equal angle sections may be obtained from (Trahair, 2001a)

$$\frac{M_{pX}}{f_y b^2 t} = \frac{1}{2} - \{1 - \gamma_2\}^2 \quad (21a)$$

$$\frac{M_{pY}}{f_y b^2 t} = -\frac{1}{2} + \gamma_2^2 \quad (21b)$$

in which $\gamma_2 b$ is the distance from the intersection of the legs of the angle section to the point where the plastic neutral axis intersects the horizontal leg. It may be convenient to replace the rectangular (geometric) axis moments M_{pX} , M_{pY} by their equivalents M_{px} , M_{py} referred to the principal axes x , y . Thus for equal angle sections

$$M_{px} = (M_{pX} - M_{pY}) / \sqrt{2} \quad (22a)$$

$$M_{py} = (M_{pX} + M_{pY}) / \sqrt{2} \quad (22b)$$

Equation 21 and 22 can be used to derive the equal angle fully plastic interaction equations

$$\pm M_{py} / M_{pym} = 1 - (M_{px} / M_{pxm})^2 \quad (23)$$

in which

$$M_{pxm} = 2 M_{pym} = f_y b^2 t / \sqrt{2} \quad (24)$$

3.4 Design Strengths

The lateral buckling strength of an angle section beam is reduced below its elastic buckling moment by the effects of yielding, residual stresses, and initial crookednesses and twist rotations (Trahair and Bradford, 1998). In the Australian code AS 4100 (SA, 1998), the nominal lateral buckling strength M_{ba} is given by

$$M_{ba} = \alpha_m \alpha_s M_{sx} \quad (25)$$

in which α_m is a moment modification factor which allows for the bending moment distribution ($\alpha_m = 1.0$ for equal and opposite end moments, or 1.13 for uniformly distributed load), and

$$\alpha_s = 0.6 \left\{ \sqrt{\left[\left(\frac{M_{sx}}{M_{yz}} \right)^2 + 3 \right]} - \frac{M_{sx}}{M_{yz}} \right\} \leq 1.0 \quad (26)$$

is a slenderness reduction factor, as shown in Fig. 6.

3.5 Equivalent Initial Twist Rotations

The lateral buckling strength of a compact equal angle section beam with equal and opposite end moments M and initial twist rotations ϕ_0 can be investigated by first estimating the maximum moments M_x , M_y given by Equations 17 and then comparing these with the section moment capacities (Trahair, 2001a), such as those of Section 3.3.

This method requires a knowledge of the value of the central initial twist rotation θ_0 . Values of θ_0 which lead to lateral buckling strengths which are equal to those of the Australian code AS 4100 (SA, 1998) and given by Equations 25 and 26 can be obtained iteratively, by finding the values of θ_0 for which the maximum moments given by Equations 17 are equal to the nominal section capacities, such as those given in Section 3.3. The values of θ_0 determined in this way are artificial, because they allow also for the effects of initial crookedness and residual stresses.

These values of θ_0 (for compact equal angle section beams) are shown in Fig. 7, and are approximated by

$$\theta_0 = 0.4 \left(\sqrt{\frac{M_{sx}}{M_{yz}}} - 0.3 \right) \geq 0 \quad (27)$$

The lateral buckling strength predictions obtained by using this approximation are compared with those of AS 4100 in Fig. 6.

It can be seen in Fig. 7 that at high values of $\sqrt{(M_{sx} / M_{yz})}$, very large values of θ_0 are required to obtain predictions that match the AS 4100 lateral buckling strengths. In this case, the values of θ_0 (and of θ) are so high as to invalidate the assumption of small rotations made in Section 3.2. As a result, the small rotation strength predictions are conservative. There are two reasons for this conservatism. The first is the omission of a non-linear term proportional to $(\phi')^3$ from the resistance to torsion. The second is the use of the two approximations $\sin \phi \approx \phi$ and $\cos \phi \approx 1$ in both the differential equations for minor axis bending and torsion (Equations 13) and in the moment components (Equation 17).

While a large rotation analysis may be more realistic, it should be noted that this will lead to strength predictions which are always higher than the minor axis section capacity M_{sy} , indicating that the AS 4100 strength predictions may be very conservative at high values of $\sqrt{(M_{sx} / M_{yz})}$ (Trahair, 1997). An approximate large rotation analysis is given in Section 3.6 below.

3.6 Large Rotation Analysis

3.6.1 Non-linear uniform torsion

The differential equilibrium equation for elastic non-linear uniform torsion (Woolcock and Trahair, 1974, Pi and Trahair, 1995) is

$$G J \phi' + E I_{pp} (\phi')^3 / 2 = M_z \quad (28)$$

in which M_z is the torque at z , and I_{pp} is a section constant defined by

$$I_{pp} = \int r^4 dA - \{ \int r^2 dA \}^2 / A \quad (29)$$

in which r is the distance from a point (x, y) in the cross-section to the axis of twist rotation at the shear centre. For an unequal angle section $b \times \beta b \times t$,

$$I_{pp} = \frac{b^5 t (4 + 9\beta - 10\beta^3 + 9\beta^5 + 4\beta^6)}{45(1 + \beta)} \quad (30)$$

which reduces to

$$I_{pp} = 8 b^5 t / 45 \quad (31)$$

for an equal angle ($\beta = 1$).

The non-linear term $E I_{pp} (\phi')^3 / 2$ arises as a result of the torque resultant of longitudinal stresses induced to prevent helical shortening of the longitudinal fibres of the member during twisting. These stresses are similar to those which stiffen I-section beams at large twist rotations (Farwell and Galambos, 1969; Pi and Trahair, 1995).

The effect of the non-linear term $E I_{pp} (\phi')^3 / 2$ on the end twist rotations of an equal angle member of length L , with $\pi^2 E I_{pp} / G J L^2 = 5$ (which corresponds, for example, to $E / G = 2.5$, $L / b = 11.5$, and $b / t = 10$), and which is prevented from rotating at one end and loaded by a torque T at the other, is shown in Fig. 8.

3.6.2 Elastic behaviour

Approximate non-linear differential equilibrium equations for the elastic large rotation behaviour of a simply supported equal angle section beam under equal and opposite end moments M can be obtained from the small rotation Equations 13 as

$$E I_y u'' = -M \sin(\phi + \phi_0) \quad (32a)$$

$$G J \phi' + E I_{pp} (\phi')^3 / 2 = M u' \quad (32b)$$

in which the initial twist rotation ϕ_0 is given by Equation 14 and $M u'$ is an approximation for the torque component $M \sin u'$ of the applied moment M .

The deflection u can be eliminated from these equations, leading to

$$\frac{M^2 \sin(\phi + \phi_0)}{E I_y G J} + \left\{ 1 + \frac{3 E I_{pp}}{2 G J} (\phi')^2 \right\} \phi'' = 0 \quad (33)$$

Exact solutions of Equation 33 are difficult to obtain, but approximate solutions can be obtained by replacing Equation 33 by the “average” equation

$$\frac{M^2}{E I_y G J} \int_0^L \sin(\phi + \phi_0) dz + \int_0^L \phi'' dz + \frac{3 E I_{pp}}{2 G J} \int_0^L (\phi')^2 \phi'' dz = 0 \quad (34)$$

by using the approximation

$$\sin(\phi - \phi_0) \approx (\phi - \phi_0) - 0.1655 (\phi - \phi_0)^3 + 0.0074 (\phi - \phi_0)^5 \quad (35)$$

($0 \leq (\phi - \phi_0) \leq \pi / 2$), and by assuming Equation 15, whence

$$\left(\frac{M}{M_{yz}} \right)^2 = \frac{\theta + (\pi^2 E I_{pp} / 2 G J L^2) \theta^3}{(\theta + \theta_0) - 0.1103 (\theta + \theta_0)^3 + 0.0039 (\theta + \theta_0)^5} \quad (36)$$

Solutions of three versions of Equation 36 are shown in Fig. 9 for the case of $\pi^2 E I_{pp} / G J L^2 = 5$, and compared with the small rotation solutions. It can be seen that while the large rotation post-buckling twist rotations commence at $M = M_{yz}$ and increase rapidly at first, they slow down at higher values of the applied moment.

3.6.3 Design strengths

The elastic twist rotations of Equation 36 can be used to find the maximum principal axis moments

$$M_x = M \cos (\theta + \theta_0) \quad (37a)$$

$$M_y = -M \sin (\theta + \theta_0) \quad (37b)$$

If the methodology proposed in Section 3.1 is used, then at failure

$$M_x = M_{px} \quad (38a)$$

$$M_y = M_{py} \quad (38b)$$

Combining Equations 37 and 38 with the plastic interaction Equations 23 and 24 leads to

$$\frac{M}{M_{yz}} = \frac{M_{pxm}}{M_{yz}} \frac{1}{(1 - \sin(\theta + \theta_0))} \quad (39)$$

When the values of $\sqrt{(M_{pxm} / M_{yz})}$ and θ_0 are known, then Equations 36 and 39 can be solved iteratively for θ and

$$\frac{M}{M_{pxm}} = \frac{M}{M_{yz}} \frac{M_{yz}}{M_{pxm}} \quad (40)$$

The variation of M / M_{pxm} with $\sqrt{(M_{pxm} / M_{yz})}$ for values of θ_0 given by Equation 27 is shown in Fig. 6. The values of M / M_{pxm} are always greater than the values of α_s (Equation 26) used in AS 4100 (SA, 1998), and decrease from 1.0 at $\sqrt{(M_{pxm} / M_{yz})} = 0.3$ to M_{pym} / M_{pxm} ($= 0.5$) at $\sqrt{(M_{pxm} / M_{yz})} = 1.4$, and then remain constant. This behaviour occurs because as $(\theta + \theta_0)$ approaches $\pi / 2$, M approaches $-M_{py}$ (Equations 37b and 38b), and agrees with the intuitive proposition that the moment capacity will reach a minimum when the total twist rotation reaches $\pi / 2$ so that the applied moment acts about the minor principal axis. It supports the proposal made by Trahair (1977) that the nominal moment capacity at high values of $\sqrt{(M_{pxm} / M_{yz})}$ should be taken as M_{pym} .

3.6.4 Proposed design method

It is proposed that the nominal design lateral buckling moment capacity M_b of an angle section beam should be obtained from

$$M_b = M_{sx} \quad (\lambda_e \leq \lambda_x) \quad (41a)$$

$$M_b = M_{sx} - (M_{sx} - M_{sy}) \frac{(\lambda_e - \lambda_x)}{(\lambda_y - \lambda_x)} \quad (\lambda_x \leq \lambda_e \leq \lambda_y) \quad (41b)$$

$$M_b = M_{sy} \quad (\lambda_y \leq \lambda_e) \quad (41c)$$

in which

$$\lambda_x = 0.99 - \frac{0.22}{(\alpha_m - 0.7)} \quad (42)$$

$$\lambda_y = \sqrt{M_{sx} / M_{sy}} \quad (43)$$

$$\lambda_e = \sqrt{M_{sx} / M_{quy}} \quad (44)$$

as shown in Fig. 10.

The modified slenderness limit λ_x is an approximation for the value of $\sqrt{(M_{sx} / \alpha_m M_{yz})}$ ($\approx \sqrt{(M_{sx} / M_{quy})}$) at which $M_{ba} = M_{sx}$ according to AS 4100 (SA, 1998), in which M_{ba} is given by Equations 25 and 26.

Equation 41a uses the major axis section capacity M_{sx} for low slenderness beams ($\lambda_e \leq \lambda_x$), while Equation 41c uses the minor axis section capacity M_{sy} for high slenderness beams ($\lambda_y \leq \lambda_e$), which is based on the finding of Section 3.6.3 that the moment capacity is never less than M_{sy} . Equation 41b provides a simple linear interpolation between M_{sx} and M_{sy} for beams of intermediate slenderness ($\lambda_x \leq \lambda_e \leq \lambda_y$), which provides a close but conservative approximation to the large twist rotation predictions shown in Fig. 6.

For plastic and compact sections, approximations for the section capacities $M_{sx} = M_{pxm}$ and $M_{sy} = M_{pym}$ can be obtained from

$$M_{pxm} / f_y b^2 t = 0.337 \beta^2 - 0.001 \beta + 0.371 \quad (45)$$

$$M_{pym} / f_y b^2 t = -0.075 \beta^2 + 0.546 \beta - 0.117 \quad (46)$$

as shown in Fig. 11, in which M_{pxm} , M_{pym} are the full plastic moments about plastic neutral axes parallel to the elastic x , y principal axes. These approximations were derived from the accurate solutions of Trahair (2001a).

It can be seen in Fig. 10 that for very slender beams ($\lambda_y < \lambda_e$), the nominal moment capacity M_b exceeds the elastic lateral buckling moment M_{quy} . While such a beam will be adequate for strength, it may be unserviceable, in that large twist rotations may occur. The serviceability of such a beam under its serviceability design loads should be checked separately. Serviceability design loads are usually significantly less than the strength design loads.

4 EXAMPLE

4.1 Problem

A 150 x 100 x 12 unequal angle beam is shown in Fig. 12. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of $b = 144$ mm, $\beta b = 94$ mm, and $t = 12$ mm are shown in Fig. 12b. The unbraced beam is simply supported over a span of $L = 6$ m, and has a design uniformly distributed vertical load of $q^* = 6$ kN/m acting parallel to the long leg and with an eccentricity of $e = 47$ mm from the shear centre at the leg junction, as shown in Fig. 12b.

The first-order analysis of the beam is summarised in Section 4.2 below, the determination of the elastic buckling moment of the beam in Section 4.3, and of the lateral buckling design strength in Section 4.4. The checking of the moment, bearing, shear, and torsion capacities are summarised in Trahair (2001a, b).

4.2 Elastic Analysis

The design major axis bending moment is

$$M_x^* = (q^* L^2 / 8) \cos \alpha = 24.7 \text{ kNm.}$$

4.3 Moment at Elastic Buckling

$M_{yz} = 28.1$ kNm (Equation 1) and $P_y = 72.1$ kN (Equation 3).

$$\beta_x P_y / 2 M_{yz} = -0.10$$

For uniformly distributed load, $\alpha_m = 1.13$

$$M_{qu} = 29.9 \text{ kNm (Equation 5).}$$

$$(y_q - y_o) = e \sin \alpha = 19.0 \text{ mm and } (y_q - y_o) P_y / M_{yz} = 0.05$$

$$M_{quy} / M_{qu} = 1.02 \text{ (Fig. 4) and } M_{quy} = 30.6 \text{ kNm.}$$

4.4 Lateral Buckling Design Strength

The angle section has been shown to be compact (Trahair, 2001a).

Using Equations 45 & 46, $M_{sx} = M_{pxm} = 38.4$ kNm and $M_{sy} = M_{pym} = 15.5$ kNm.

$\lambda_x = 0.48$ (Equation 42), $\lambda_y = 1.57$ (Equation 43), and $\lambda_e = 1.12$ (Equation 44).

$M_b = 25.0$ kNm (Equation 41b), so that $\phi M_b = 22.5$ kNm (using $\phi = 0.9$).

This is less than the design moment $M_x^* = 24.7$ kNm, and the beam is inadequate, even if the minor axis moment $M_y^* = (q^* L^2 / 8) \sin \alpha$ and the design torque $M_z^* = (q^* L / 2) e$ are ignored.

5 CONCLUSIONS

This paper has developed a set of close approximations for the elastic lateral buckling of equal and unequal angle section beams with distributed loads which may act away from the shear centre, and has given section property approximations which help in the evaluation of the elastic buckling moments. It was found that a closed form solution for mono-symmetric beams under uniform bending can not be adapted for unequal angle section beams with uniformly distributed loads.

A small rotation elastic analysis of equal angles was used with interaction equations for the biaxial bending section moment capacities to determine the equivalent initial twist rotations implied by the Australian design code AS 4100 (SA, 1998). It was found that these become very large at high beam slendernesses. A large twist rotation elastic analysis was used with these equivalent initial twist rotations to determine approximate lateral buckling strengths which account for the post-buckling reserves which are significant for very slender beams. In particular, it was shown that the member strength reaches a minimum value equal to the minor axis section moment capacity when the beam's twist rotation reaches 90° , instead of the zero value implied in design codes. It was noted, however, that the design of very slender beams may be governed by serviceability considerations, as large twist rotations may develop under the service loads.

A simple but economic proposal for the lateral buckling design strengths of unbraced angle section beams was based on the results of the large twist rotation analysis. The use of these proposed strengths was demonstrated by a worked example.

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APPENDIX 2 NOTATION

A	area of cross-section
b	long leg length
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
G	shear modulus of elasticity
f_y	yield stress
I_{pp}	section property (Equation 29)
I_x, I_y	second moments of area about the x, y principal axes
L	span length
M	applied moment
M_b	proposed nominal lateral buckling moment capacity
M_{ba}	AS 4100 nominal lateral buckling moment capacity
M_{pX}, M_{pY}	fully plastic moment components about the X, Y axes
M_{px}, M_{py}	fully plastic moment components about the x, y axes
M_{pxm}, M_{pym}	fully plastic moments about the x, y axes
M_{qey}	equal angle elastic buckling moment for distributed load away from the shear centre
M_{qu}	unequal angle elastic buckling moment for distributed load at the shear centre
M_{quy}	unequal angle elastic buckling moment for distributed load away from the shear centre
M_{sx}, M_{sy}	nominal section moment capacities
M_x, M_y	moments about the x, y principal axes
M_{yz}	equal angle elastic buckling moment for uniform bending
M_{yzu}	unequal angle elastic buckling moment for uniform bending
M_x^*, M_y^*	design moments about the x, y principal axes
M_z	torque
M_z^*	design torque
P_y	minor axis compression buckling load
q	intensity of uniformly distributed load
q^*	design intensity of uniformly distributed load
r	distance from point (x, y) to axis of twist rotation
T	end torque
t	leg thickness
u	deflection in the x direction
X, Y	rectangular (geometric) axes
x, y	principal axes
X_c, Y_c	X, Y distances from shear centre to centroid
y_o	shear centre coordinate
y_q	distance of load below shear centre
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
α_m	moment modification factor

α_s	slenderness reduction factor
β	leg length ratio
β_x	mono-symmetry parameter
γ_2	leg length ratio defining the plastic neutral axis position
$\lambda_c, \lambda_p, \lambda_y$	compact, plastic, and yield local buckling slenderness limits
λ_e	beam lateral buckling slenderness
λ_l	long leg local buckling slenderness
λ_x, λ_y	beam lateral buckling slenderness limits
ϕ	capacity factor, or twist rotation
ϕ_0	initial twist rotation
θ	central twist rotation
θ_0	initial central twist rotation

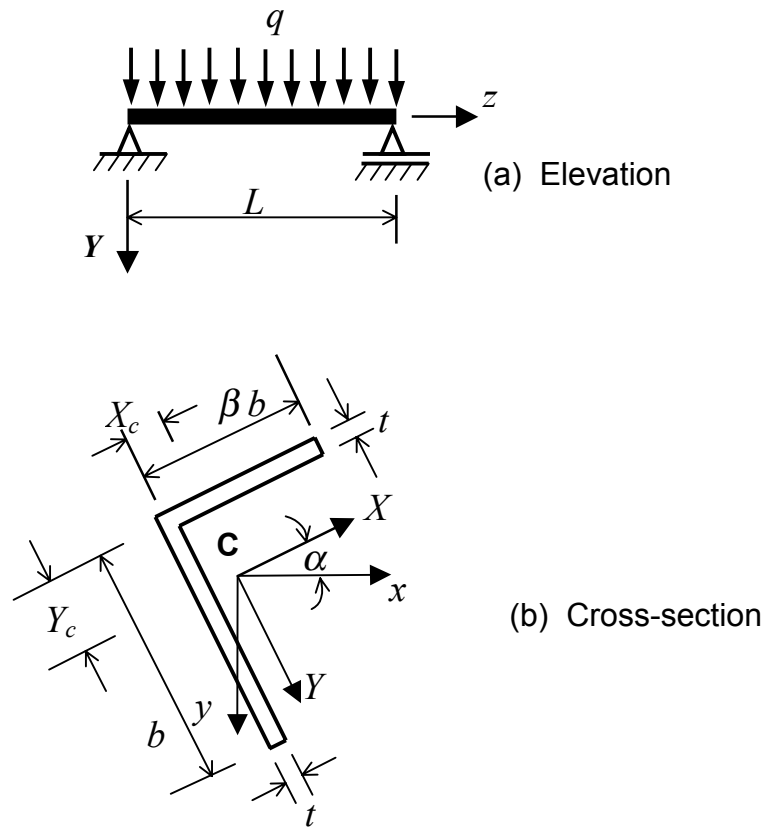


Fig. 1. Simply Supported Angle Section Beam

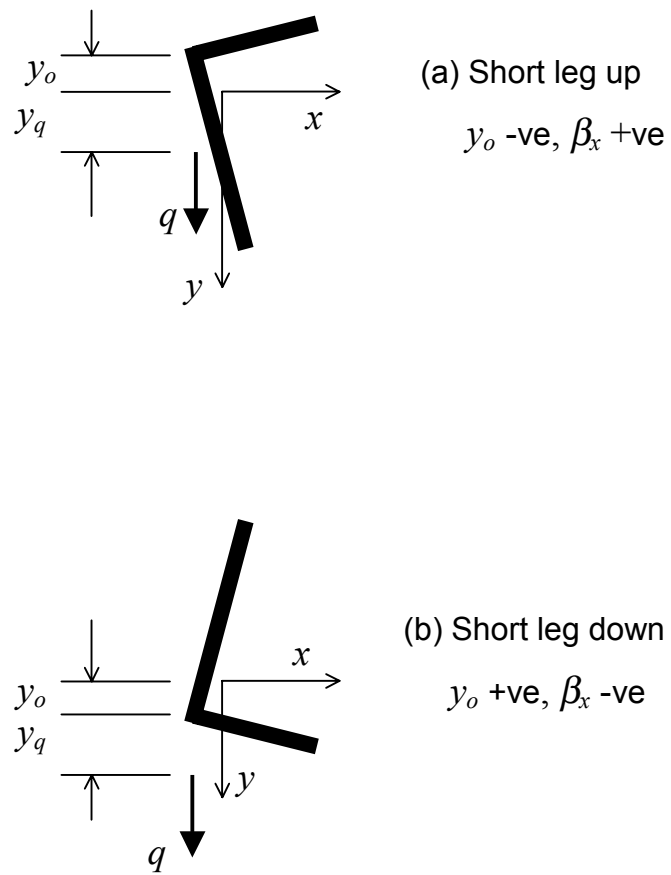


Fig. 2. Attitudes of Angle Sections and Loads

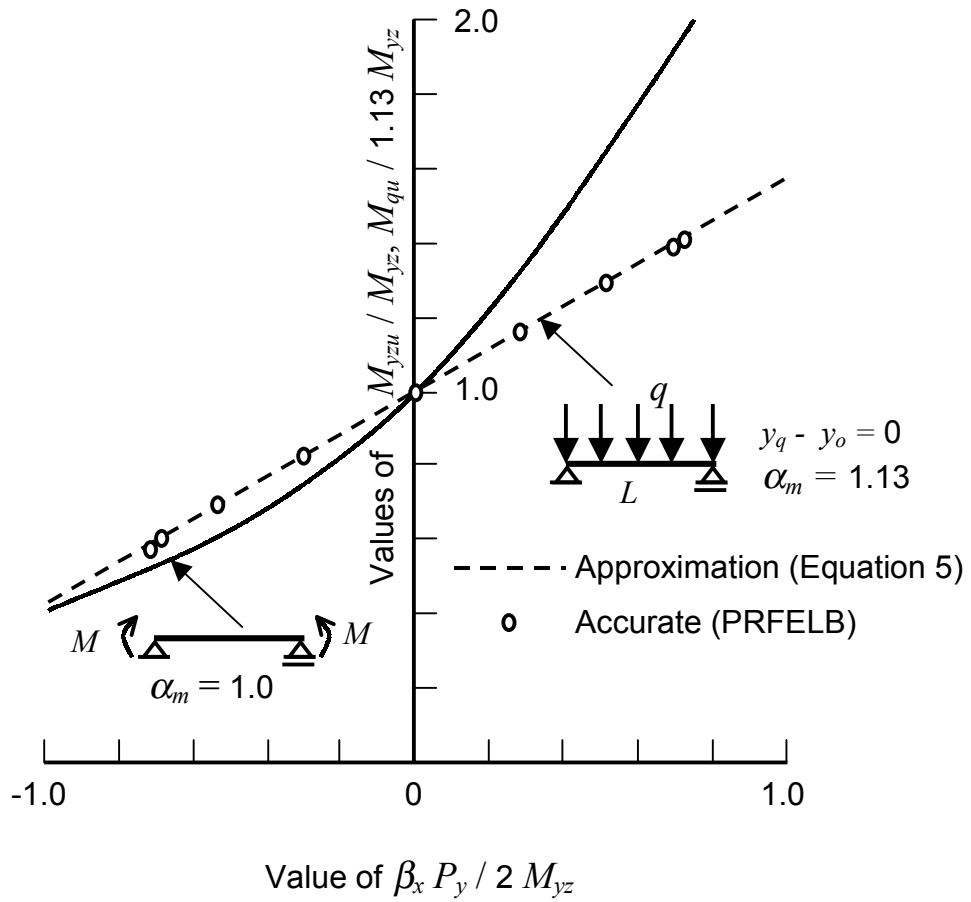


Fig. 3. Elastic Buckling of Unequal Angle Beams

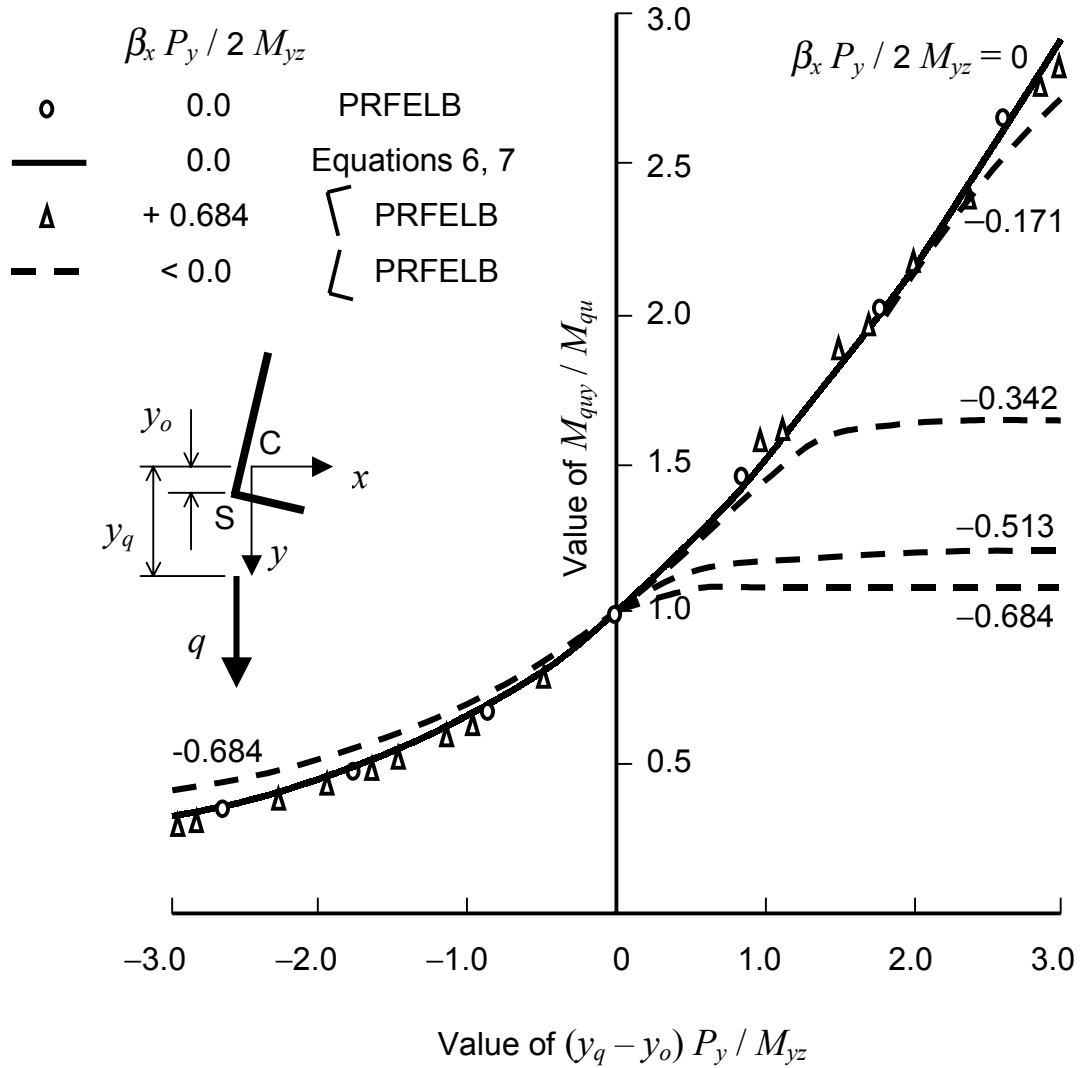


Fig. 4. Effect of Load Height on Elastic Buckling of Angle Beams

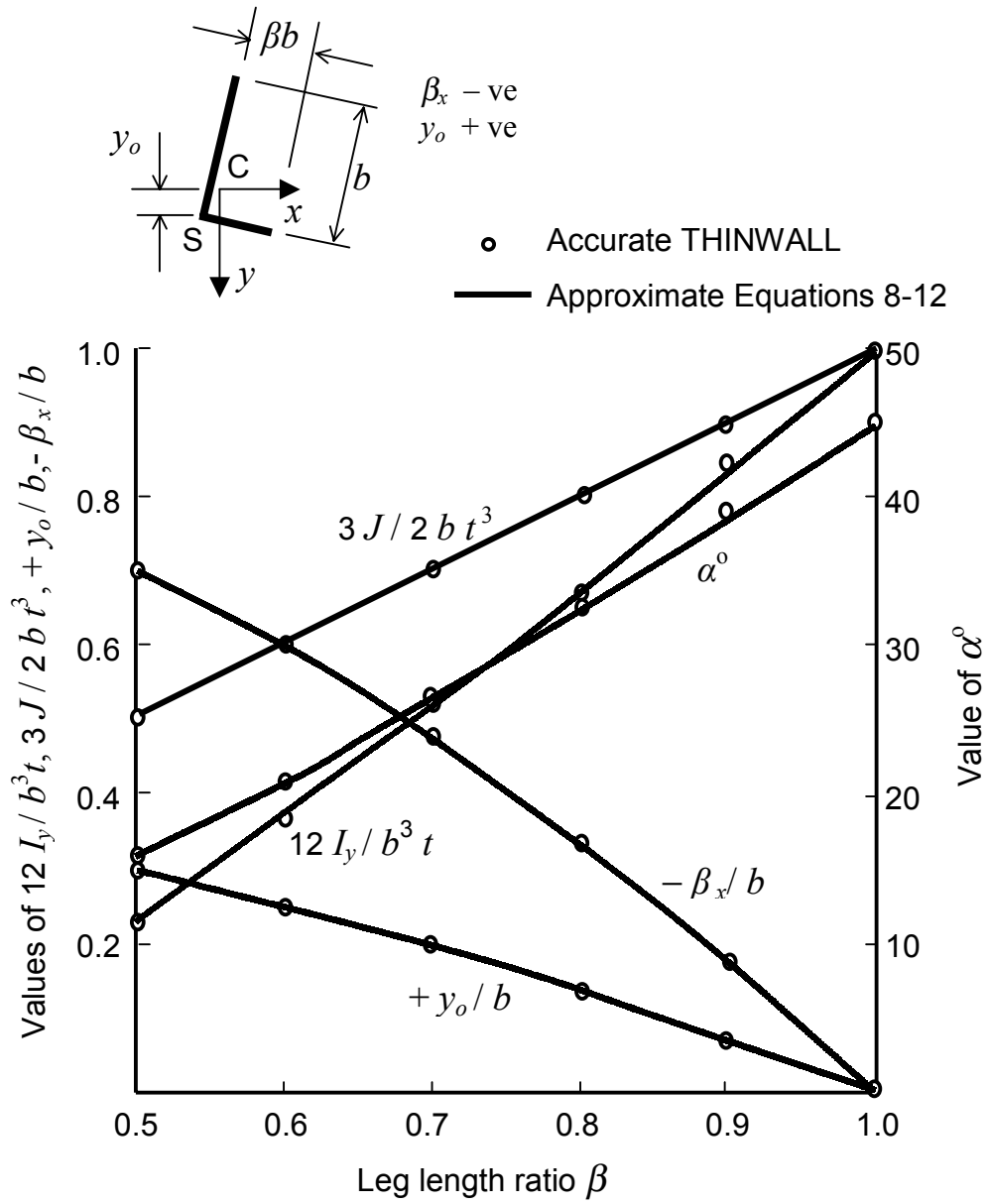


Fig. 5. Properties of Angle Sections

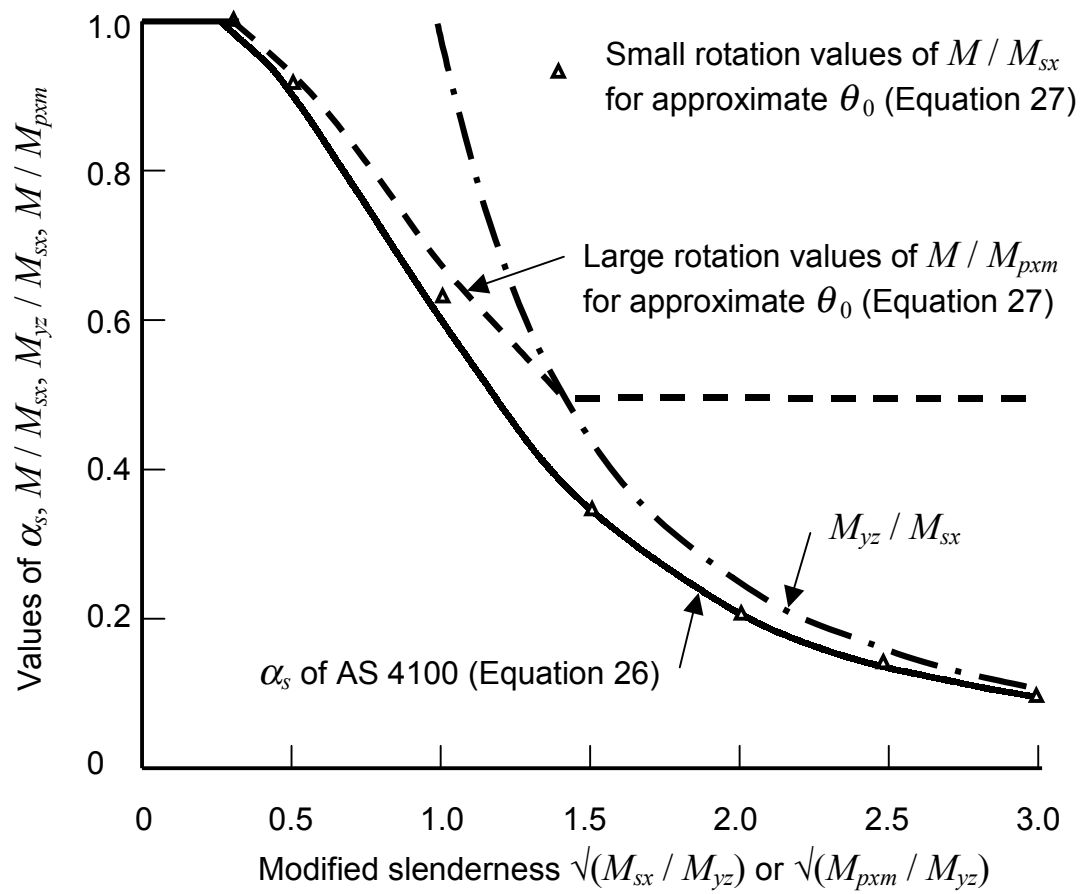


Fig. 6. Strengths of Equal Angle Beams in Uniform Bending

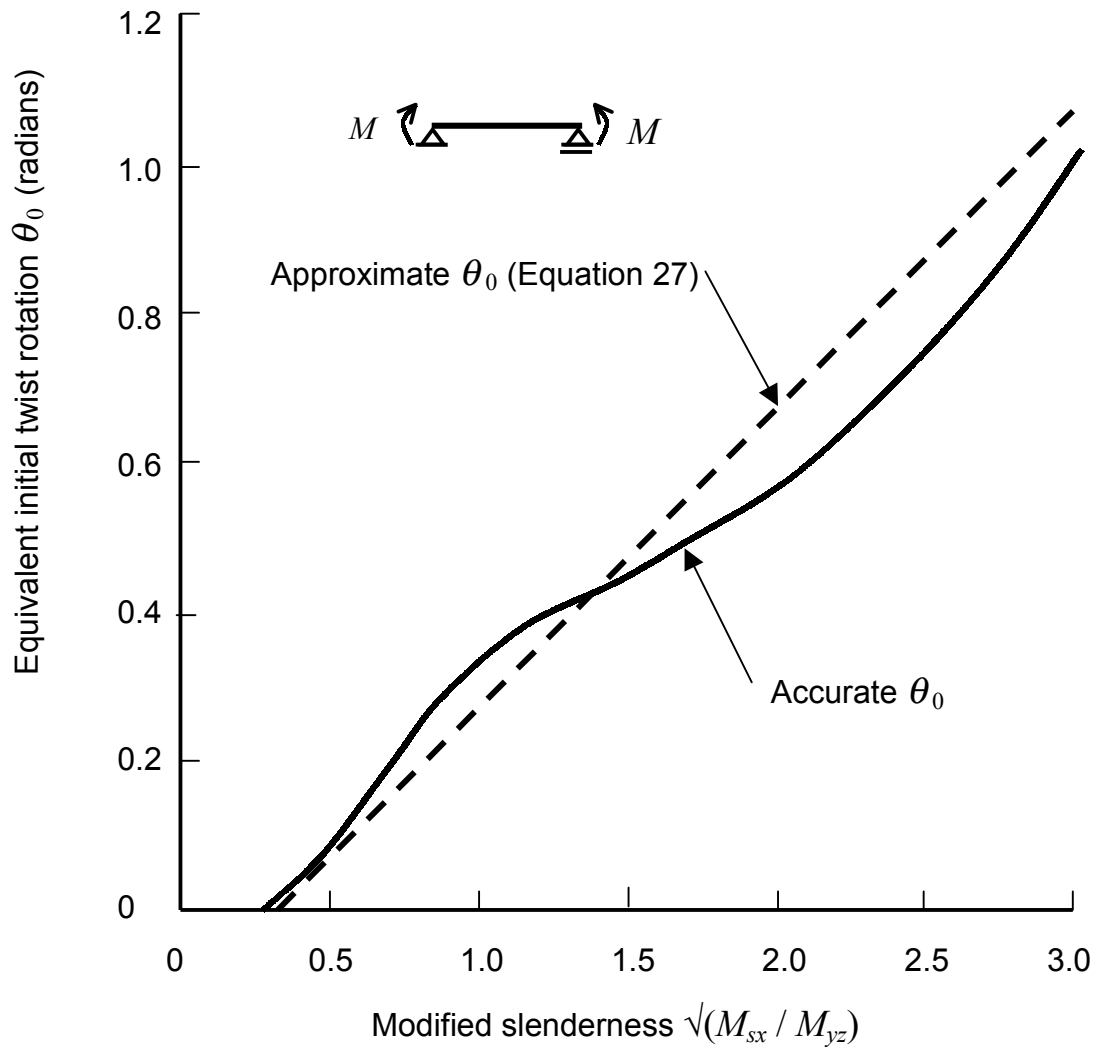


Fig. 7. Equivalent Initial Twist Rotations

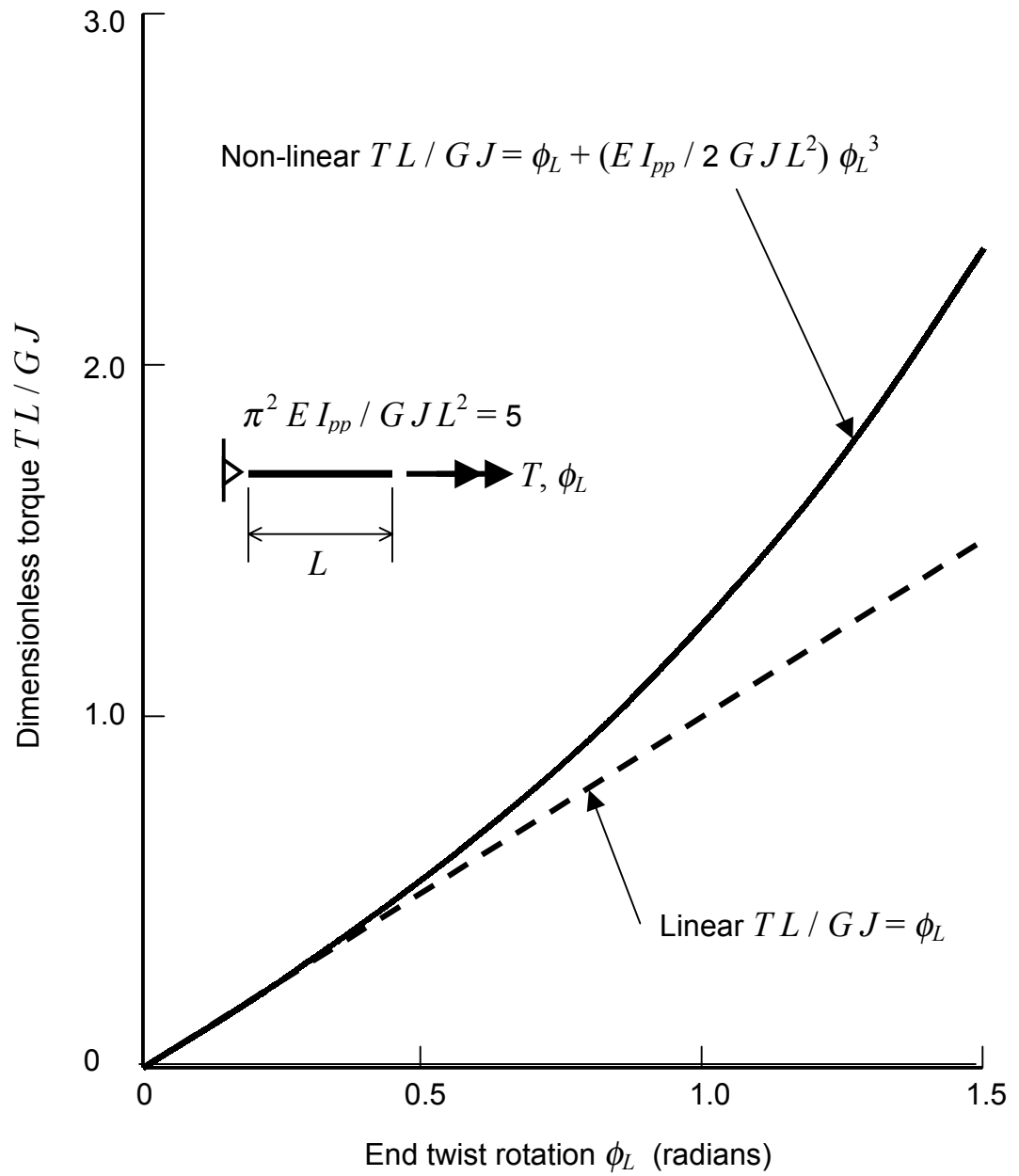


Fig. 8. Non-Linear Uniform Torsion

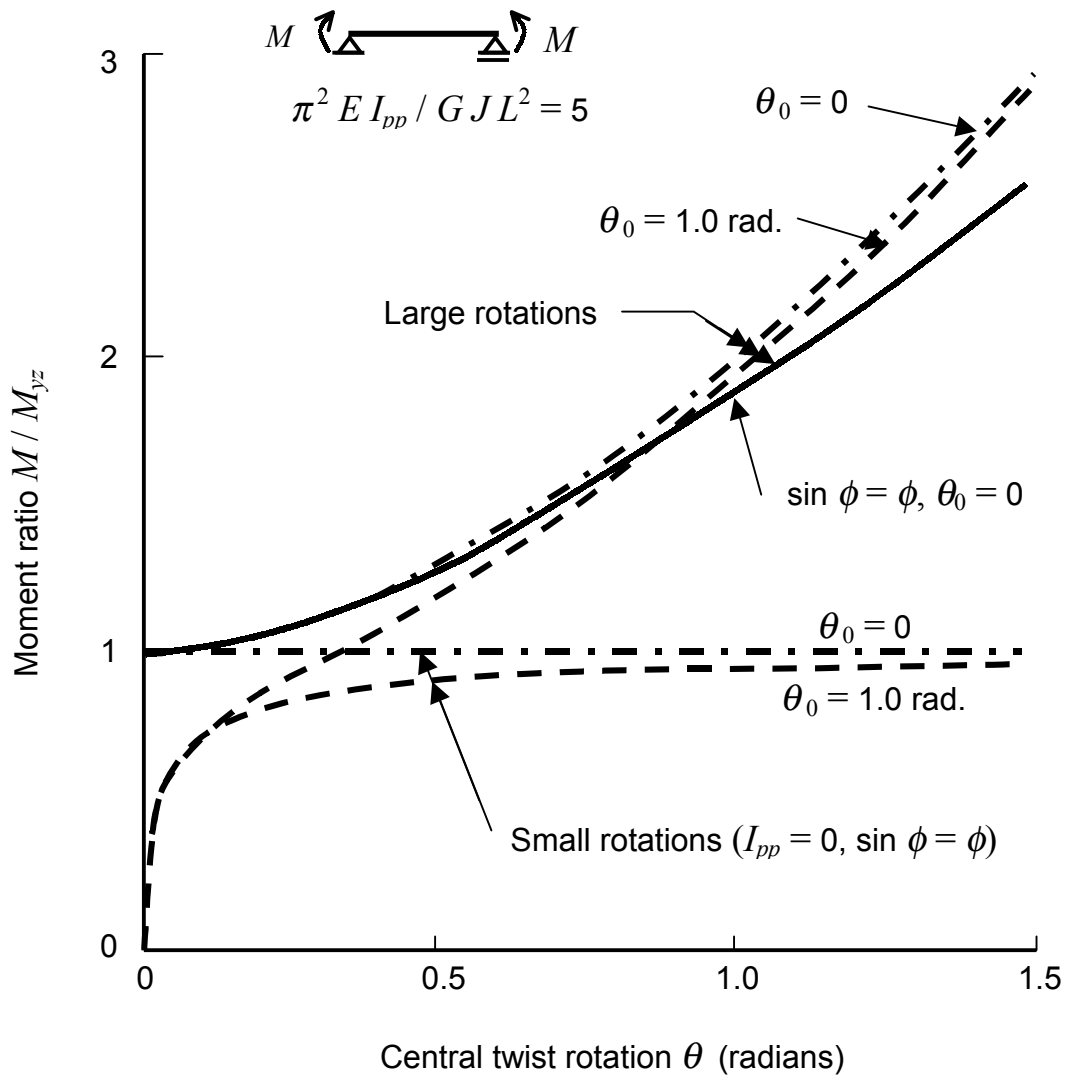


Fig. 9. Elastic Behaviour of an Equal Angle Beam in Uniform Bending

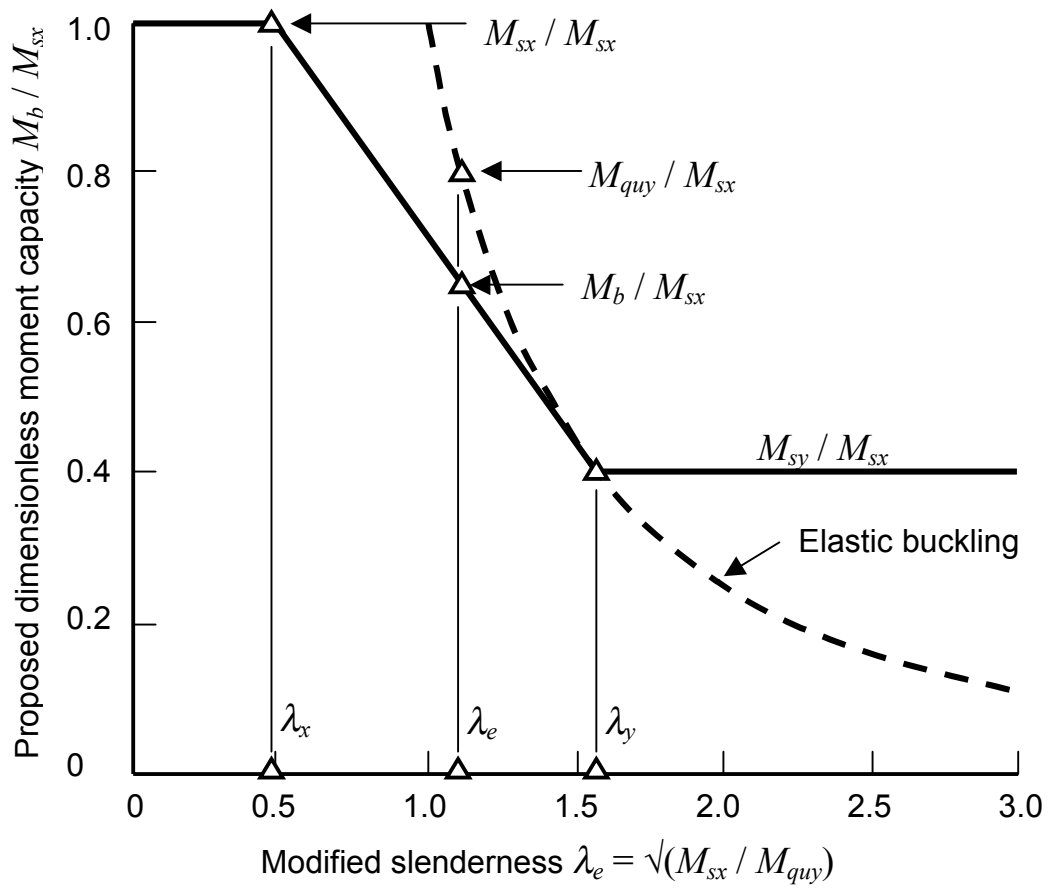


Fig. 10. Proposed Design Method

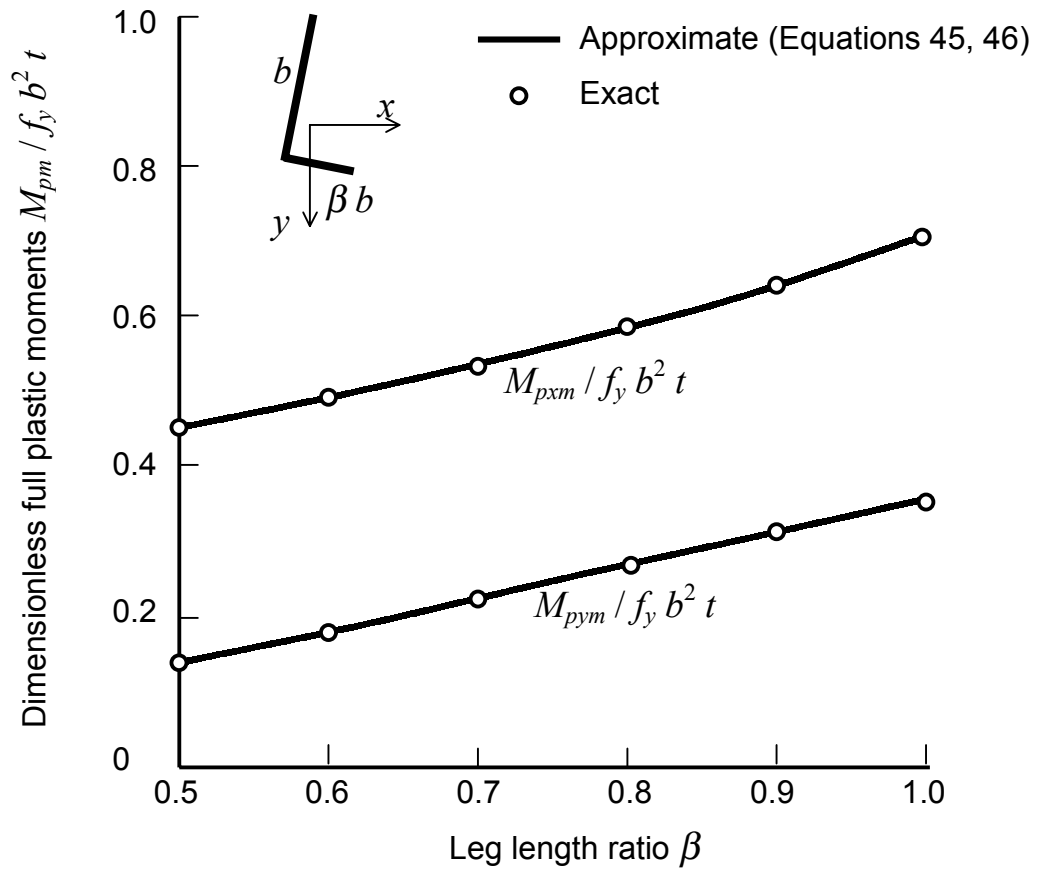
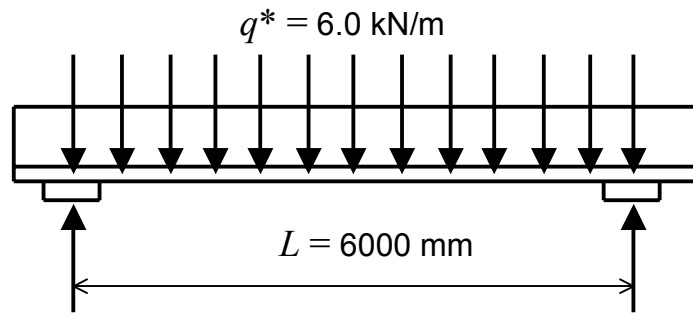
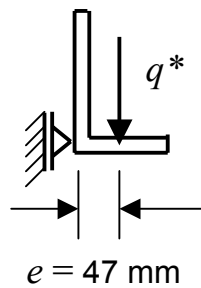


Fig. 11. Principal Axis Full Plastic Moments



(a) Elevation

150 x 100 x 12 UA
 ($b \times \beta b \times t = 144 \times 94 \times 12$)



$E = 200,000 \text{ MPa}$
 $G = 80,000 \text{ MPa}$
 $f_y = 300 \text{ MPa}$

$\alpha = 23.91^\circ$
 $I_y = 1.314 \text{ E6 mm}^4$
 $J = 0.1371 \text{ E6 mm}^4$
 $y_o = 32.30 \text{ mm}$
 $\beta_x = -78.33 \text{ mm}$

(b) Section

Fig. 12 Example