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Bending, Shear and Torsion Capacities of Steel Angle Sections

Research Report No R810

By

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Abstract:
Steel angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion. However, many design codes do not have any design rules for torsion, while some recommendations are unnecessarily conservative, or are of limited application, or fail to consider some effects which are thought to be important. In this paper, proposals are developed for the section capacities of angle sections under bearing, shear, and uniform torsion.

In a companion paper, consideration is given to the first-order elastic analysis of the biaxial bending of angle section beams, including the effects of restraints, and proposals are developed for the section moment capacities of angle sections under biaxial bending.

The proposals in this and the companion paper can be used to design steel angle section beams which are laterally restrained so that lateral buckling or second-order effects are unimportant.

Keywords: angles, beams, bearing, buckling, design, plasticity, section capacity, shear, steel, torsion, yielding.

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1 I INTRODUCTION

Steel angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion, as shown in Fig. 1. However, many design codes (BSI, 2000, SA, 1998) do not have any design rules for torsion, while some recommendations are unnecessarily conservative (AISC, 1993a,b), or are of limited application, or fail to consider some effects which are thought to be important. This and a companion paper (Trahair, 2001) develop an economical approximate method of designing restrained angle section beams under biaxial bending and torsion which is consistent with the philosophy of current design codes.

The behaviour of steel angle sections under biaxial bending and torsion is more complex than that of doubly symmetric sections under uniaxial bending and torsion, for which recent research (Pi and Trahair, 1994a,b) has established a better understanding of behaviour and suggested logical methods of design. The complexity arises from the monosymmetric or asymmetric nature of angle sections, as well as from the common loading condition in which loading parallel to but eccentric from one of the section legs causes biaxial bending about the principal axes and torsion.

The development of a better understanding of the behaviour of steel angle section beams requires special consideration of their loading and restraint, and of the analysis of their elastic behaviour. Firstly, horizontal restraints of beams with vertical loads acting in the plane of one leg induce significant horizontal forces which modify the elastic stress distribution, as indicated in Fig. 2b. These horizontal forces and their effects on the stress distribution need to be accounted for in the elastic analysis of the beam. Secondly, angle section beams are often loaded eccentrically from the shear centre at the intersection of the legs as shown in Fig. 1c, in which case significant torsion actions may result. These torsion actions need to be accounted for in the analysis.

The strengths of steel angle section beams are related to their section capacities to resist bending, bearing, shear, and torsion actions, and to their member capacities to resist the interactions between biaxial in-plane bending, out-of-plane buckling, and torsion. Very short span beams under distributed loading may fail at the supports, where the shear stresses induced by shear forces and uniform torques are greatest, while long span beams often fail near mid-span, where the normal stresses induced by biaxial bending moments are greatest.

Although the resistances of I-section webs to shear and bearing actions have been thoroughly investigated, the resistances of angle legs to these actions appear not to have been studied. The principal difference is that while both edges of an I-section web can be modeled as being simply supported laterally by the flanges, each leg of an angle section beam has one edge free. In addition, the elastic shear stress distribution in an I-section web is nearly uniform, but that in an angle section web is very non-uniform. Because of the lack of information on angle section legs, the following sections outline speculative proposals for designing angle section legs against shear and bearing which are adapted from design rules for I-section webs. Proposals are also made for design against uniform torsion. These proposals are generally more economic than those of the AISC Design Specification (AISC, 1993a,b), one of the few design codes with specific rules for angle section beams.

Consideration is given to the first-order elastic analysis of the biaxial bending of angle section beams, including the effects of restraints in a companion paper, and proposals are developed for the moment capacities of angle sections under biaxial bending which approximate the effects of full plasticity in compact sections, first yield in non-compact sections, and local buckling in slender sections.

2 SHEAR CAPACITY

2.1 General

In designing an angle section for shear, each leg should be checked separately for the shear acting on it. Thus the resultant design shear V^* acting at a section should be divided into its components V_X^*, V_Y^* acting on the individual legs, as shown in Fig. 3. Each leg can then be checked for yielding (if it is stocky), or for buckling (if it is slender).

2.2 Yield Capacity

The plastic shear capacity V_p of a stocky leg of an angle section is equal to the area of the leg times the shear yield stress, while the other leg makes no contribution. Thus for a leg b x t

$$V_p = \tau_y\, b\, t \tag{1}$$

in which the shear yield stress is given by

$$\tau_y = f_y / \sqrt{3} \tag{2}$$

in which f_y is the normal yield stress.

The first yield shear capacity V_y of a stocky leg is reduced below its plastic capacity in proportion to the ratio of the resultant of the first yield stress distribution in the leg to that of the full plastic distribution. The first yield stress distribution in a leg is parabolic, and when it varies from zero at one edge either to a maximum or to zero at the other, then this ratio is equal to 1 / 1.5, so that for a leg b x t

$$V_y = \tau_y\, (b\, t / 1.5) \tag{3}$$

A shear design method for a stocky angle section leg may be obtained by adapting a method used for unstiffened webs, such as that of the Australian code AS 4100 (SA, 1998). For this code, the design of a stocky web d x t with a nearly uniform shear stress distribution is governed by

$$V^* \leq \phi V \tag{4}$$

in which ϕ is the capacity factor (= 0.9) and V is the plastic capacity equal to

$$V_p = 0.6 f_y d t \tag{5}$$

in which 0.6 f_y is an approximation for the shear yield stress $\tau_y = f_y / \sqrt{3}$ ($\approx 0.577 f_y$) and dt is the web area. The design of a stocky web with a non-uniform shear stress distribution is again governed by Equation 4, but with the plastic capacity reduced to

$$V_y = 0.6 f_y d t \{2 / (0.9 + f_{vm} / f_{va})\} \leq 0.6 f_y dt \tag{6}$$

in which f_{vm} and f_{va} are the maximum and average elastic shear stresses in the web. For a web with f_{vm} / f_{va} = 1.5, this becomes

$$V_y = 0.5 f_y d t \tag{7}$$

In this, the term 0.5 f_y may be thought of as a reduced shear yield stress which allows for a less than complete stress redistribution from first yield to full plasticity.

Adapting Equation 7 for a stocky angle section leg b x t leads to a leg shear yield capacity of

$$V = 0.5 f_y b t \tag{8}$$

for use in the design inequality of Equation 4.

The AISC Specification (AISC, 1993a) limits the elastic shear stress in an angle to ϕ x 0.6 f_y , which corresponds to a reduction of the leg yield capacity to $V = 0.4 f_y b t$ when the ratio of the resultants of the fully plastic to the first yield stress distributions is equal to 1.5. The AISC Commentary (AISC, 1993b) advises that $V = 0.6 f_y b t$ may be used when there is no torsion action present.

2.3 Buckling Capacity

There appears to be no knowledge of the elastic buckling of a slender unstiffened angle section leg under either a uniform or a non-uniform shear stress distribution. However, the elastic buckling stress of a slender unstiffened I-section web d x t under a nearly uniform shear stress distribution is usually taken as the value

$$\tau_e = \frac{\pi^2 E}{12(1-\nu^2)} \frac{k_s}{(d/t)^2} \tag{9}$$

for a web simply supported on all four edges, in which k_s = 5.35 is the elastic buckling coefficient. The value of k_s needs to be adjusted to allow for the angle leg being free along one longitudinal edge, instead of simply supported as is assumed for the I-section web. For this purpose, the ratio 0.425 / 4.0 of the elastic buckling coefficients of plates under uniform compression may be used (Timoshenko and Gere 1961, Bulson 1970, Trahair et al 2001), so that

$$k_s = 5.35 \text{ x } 0.425 / 4.0 = 0.568 \qquad (10)$$

In this case, the elastic shear buckling stress τ_e for a steel with E = 200,000 MPa and v = 0.3 is equal to the shear yield stress τ_y when

$$\frac{d}{t}\sqrt{\frac{f_y}{250}} = \sqrt{\left\{\frac{\pi^2 \times 200000 \times (f_y/250) \times 0.568}{12 \times (1-0.3^2) \times f_y/\sqrt{3}}\right\}} \approx 27 \qquad (11)$$

This approximate treatment is derived for a uniform shear stress distribution, and is likely to be conservative for the non-uniform elastic shear stress distribution that occurs in angle section legs.

A shear design method for a slender angle section leg b x t may be obtained by combining the elastic shear buckling stress of Equation 8 with the design capacity of a stocky angle section leg given by Equation 7, so that

$$V_b = 0.5 f_y bt \left\{\frac{27}{(b/t)\sqrt{(f_y/250)}}\right\}^2 \leq 0.5 f_y bt \qquad (12)$$

The division between stocky and slender angle section legs is then defined by Equation 11, with d / t replaced by b / t.

The AISC Specification (AISC, 1993a) gives no guidance for the design of slender angle legs against shear buckling, probably because most practical angle legs have $(b / t)\sqrt{(f_y / 250)} < 27$, and so are not slender.

3 BEARING CAPACITY

Bearing at a support is traditionally considered separately from other actions such as shear and bending. Bearing of a short span angle section beam is only likely to require special design consideration when the bearing reaction induces compression stresses in an unstiffened leg, as shown in Fig. 4, in which case the possibilities of bearing yielding or buckling of the leg need to be considered.

There appears to be no knowledge of the buckling resistance of such an unstiffened leg, but an approximate design method may be developed from one of the code methods used for an unstiffened web of an I-section beam. For example, the Australian code AS 4100 (SA, 1998) assumes that the design bearing reaction R_b^* is dispersed at 1:2.5 through the flange and at 1:1 to the web centreline. The web width b_{bb} so determined defines a compression member of area $b_{bb}\, t$ and effective length d (the web depth), whose design buckling capacity is taken as the design bearing buckling capacity ϕR_{bb} of the web.

This method may be adapted for the leg b x t of an angle section beam by defining the corresponding column width b_{bb} as shown in Fig. 8 and using a column effective length of L_e = 2.2 b to allow for the free top edge of the leg. The design buckling capacity of this column may then be taken as the bearing buckling capacity of the leg.

The nominal bearing yield capacity R_{by} of AS 4100 is given by

$$R_{by} = 1.25\, b_{by}\, t\, f_y \tag{13}$$

in which b_{by} is the yield bearing width defined by a dispersion of the bearing reaction R_b^* at 1:2.5 through the leg, as shown in Fig. 4.

The AISC Specification (AISC, 1993a) gives no guidance for the design of angle legs against bearing yielding or buckling, probably because bearing in angles is rarely important.

4 UNIFORM TORSION CAPACITY

The plastic uniform torque M_{up} of an angle section beam b x βb x t is given by (Trahair et al, 2001)

$$M_{up} = \tau_y\, b(1 + \beta)\, t^2 / 2 \tag{14}$$

while the first yield torque is given by

$$M_{uy} = \tau_y\, b\, (1 + \beta)\, t^2 / 3 \tag{15}$$

If the shear yield stress $\tau_y \approx 0.577\, f_y$ is reduced to $0.5\, f_y$ as it was for shear in Section 2.2, then the design uniform torque M_u^* should satisfy

$$M_u^* \le \phi\, M_u \tag{16}$$

in which the nominal uniform torsion capacity M_u is given by

$$M_u = 0.5\, f_y\, b(1+\beta)\, t^2 / 2 \tag{17}$$

5 COMBINED SHEAR AND TORSION

When shear and uniform torsion act at the same section, then the design actions should satisfy

$$\frac{V_X^*}{\phi V_X} + \frac{V_Y^*}{\phi V_Y} + \frac{M_u^*}{\phi M_u} \le 1 \tag{18}$$

in which V_X*, V_Y* are the design shears in each leg and V_X, V_Y are the corresponding nominal shear capacities. This linear interaction equation assumes that the excess shear capacity ($\phi V_X + \phi V_Y - V_X^* - V_Y^*$) is available for uniform torsion.

The AISC Specification (AISC, 1993a) limits the sum of the elastic shear stress and the uniform torsion shear stress to ϕ x 0.6 f_y.

6 EXAMPLE

6.1 Problem

A 150 x 100 x 12 unequal angle beam is shown in Fig. 5. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of b = 144 mm, βb = 94 mm, and t = 12 mm are shown in Fig. 5b. The beam is simply supported over a span of L = 6 m, and has a design uniformly distributed vertical load of q^* = 6 kN/m acting parallel to the long leg and with an eccentricity of e = 47 mm from the shear centre at the leg junction, as shown in Fig. 5b. Horizontal deflections of the shear centre are prevented.

The first-order analysis of the beam is summarised in the companion paper (Trahair, 2001) and below, and the checking of the capacities of the beam in the following sub-sections.

6.2 Elastic Analysis

The restraints which prevent horizontal deflections exert a uniformly distributed horizontal force per unit length which is evaluated in the companion paper (Trahair, 2001) as r^* = 2.125 kN/m.

The maximum moments about the rectangular (geometric) X,Y axes are evaluated in the companion paper as

$M_X^* = q^* L^2/8 = 27.0$ kNm

$M_Y^* = -r^* L^2/8 = -9.6$ kNm

The maximum angle leg shears are

$V_X^* = r^* L / 2 = 6.4$ kN

$V_Y^* = q^* L / 2 = 18.0$ kN

The maximum uniform torque is

$M_u^* = q^* e L / 2 = 0.85$ kNm

6.3 Moment Capacity

The maximum moment capacities are evaluated in the companion paper (Trahair, 2001) as

$\phi M_{pX} = 30.5$ kNm > 27.0 kNm = M_X^*, and
- $\phi M_{pY} = 10.8$ kNm > 9.6 kNm = - M_Y^*, OK.

6.4 Shear and Torsion Capacities

Adapting Equation 11, $(b/t)\sqrt{(f_y/250)} = 13.1 < 27$ and the long leg is not slender.

Adapting Equation 8 for shear in the legs and using $\phi = 0.9$,
$\phi V_Y = 233$ kN > 18.0 kN = V_Y^*, OK, and
$\phi V_X = 152$ kN > 6.4 kN = V_X^*, OK.

Adapting Equation 17 for uniform torsion and using $\phi = 0.9$,
$\phi M_u = 2.31$ kNm > 0.85 kNm = M_u^*, OK.

Using Equation 18 for combined shear and torsion,

$$\frac{6.4}{152} + \frac{18.0}{233} + \frac{0.85}{2.31} = 0.485 < 1.0, \quad \text{OK.}$$

6.5 Bearing Capacity

Adapting Fig. 4, $b_{by} = 160$ mm, $b_{bb} = 217.6$ mm.

Adapting Equation 13 for the bearing yield capacity
$\phi R_{by} = 648$ kN > 18.0 kN = R_b^*, OK.

For bearing buckling, the buckling area = 217.6 x 12 = 2611.2 mm^2, and $r = t / \sqrt{12} = 3.46$ mm, so that $(L_e / r)\sqrt{(f_y / 250)} = 100.3$. For this compression member slenderness, the AS 4100 (SA, 1998) design capacity is
$\phi R_{bb} = 342$ kN > 18.0 kN = R_b^*, OK.

7 CONCLUSIONS

This paper develops economical design methods for determining the section capacities of angle section beams under bearing, shear, and uniform torsion, and illustrates their use in a design example.

Proposals are made for checking the shear capacity of each leg of an angle section beam and the uniform torsion capacity of the angle section, and a simple linear interaction equation is developed for combined shear and uniform torsion.

Finally, existing rules for checking the bearing yielding and buckling capacities of an I-section web are adapted for angle section legs.

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9 NOTATION

b	long leg length
b_{bb}, b_{by}	bearing buckling and yielding widths
d	web depth
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_{va}, f_{vm}	average and maximum elastic shear stresses
f_y	yield stress
I_x, I_y	second moments of area about the x, y principal axes
k_s	elastic local buckling coefficient for shear
L	span length
L_e	effective length of web column
M_{pX}, M_{pY}	fully plastic moments about the X, Y axes
M_u	nominal uniform torque capacity
M_u*	design uniform torque
M_{up}	plastic uniform torque
M_{uy}	first yield uniform torque
M_X*, M_Y*	design moments about the X, Y rectangular axes
M_x, M_y	moments about the x, y principal axes
q	intensity of uniformly distributed load
q*	design intensity of uniformly distributed load
R_b*	design bearing load
R_{bb}, R_{by}	nominal bearing buckling and yielding capacities
r	intensity of uniformly distributed reaction, or radius of gyration
r*	design intensity of uniformly distributed reaction
t	leg thickness
V_b	design shear capacity
V_p	plastic shear capacity of a leg
V_X, V_Y	nominal leg shear force capacities
V_X*, V_Y*	design leg shear forces
V_y	first yield shear capacity of a leg
X, Y	rectangular (geometric) axes
x, y	principal axes
X_c, Y_c	X, Y distances from shear centre to centroid
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
β	leg length ratio
δ	central deflection
δ_X, δ_Y	central deflections in the X, Y directions
τ_s	elastic shear buckling stress
τ_y	shear yield stress
ϕ	capacity factor
ν	Poisson's ratio

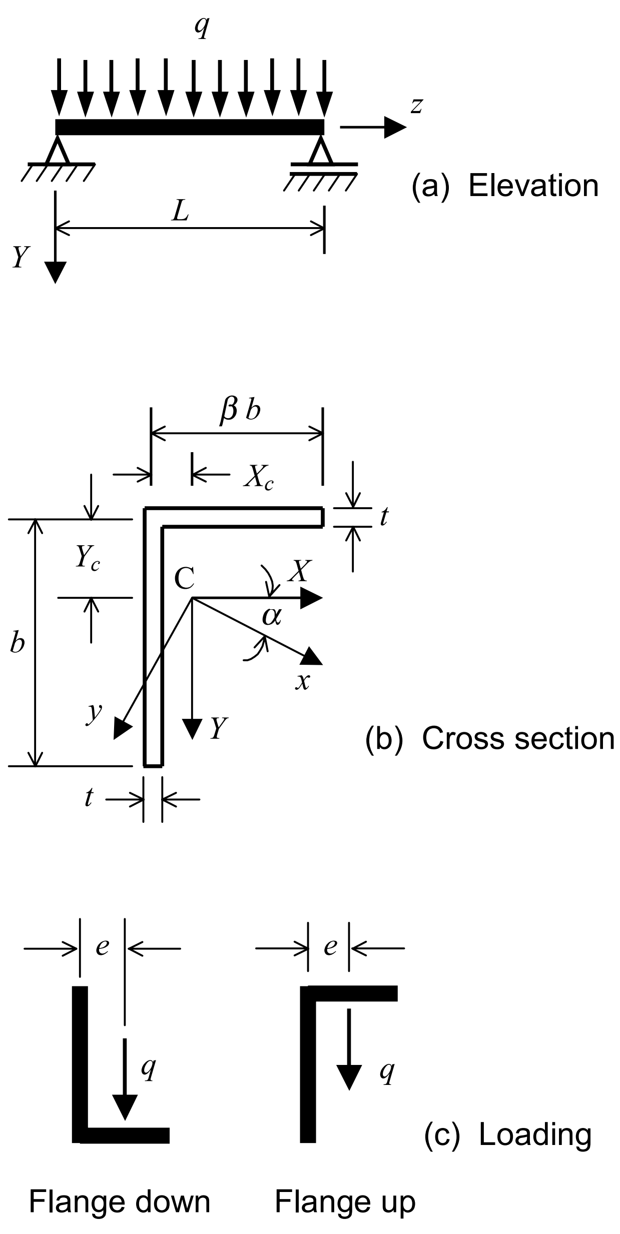


Fig. 1. Simply Supported Angle Section Beam

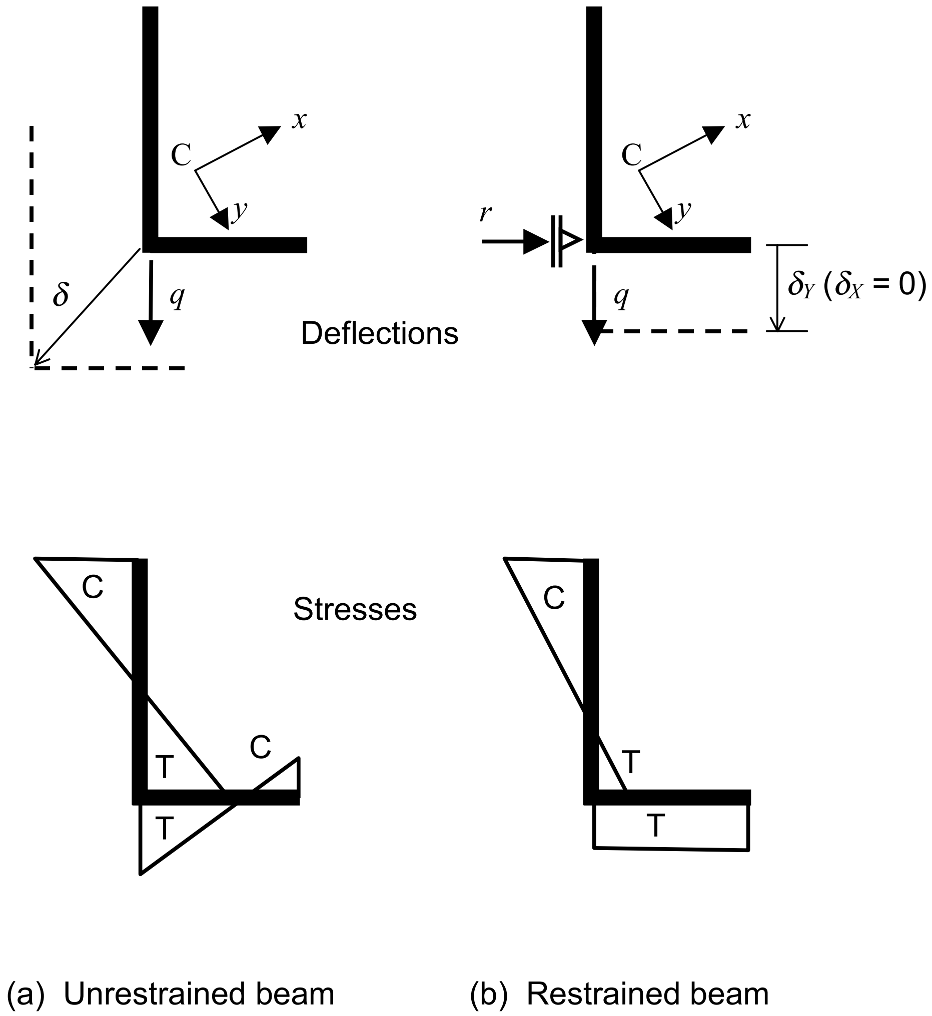


Fig. 2. First-Order Deflections and Stresses

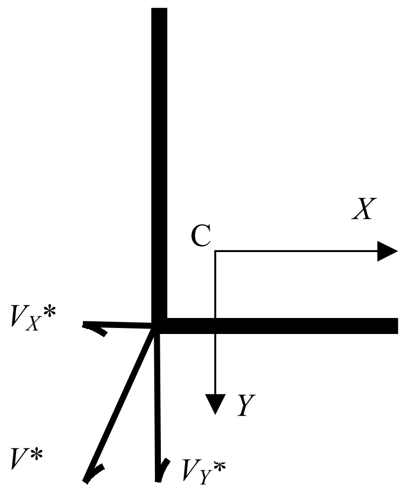


Fig. 3. Design Shear Components

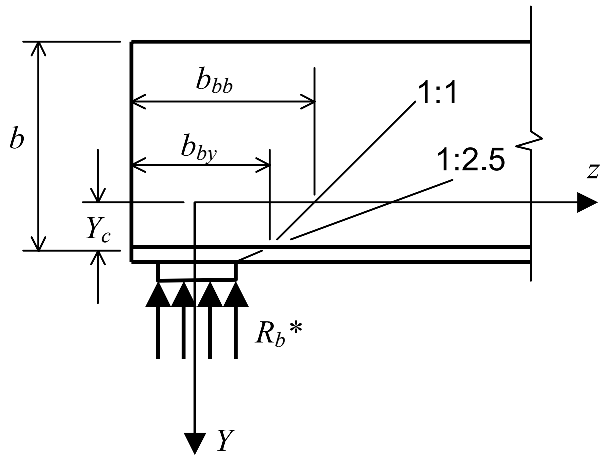
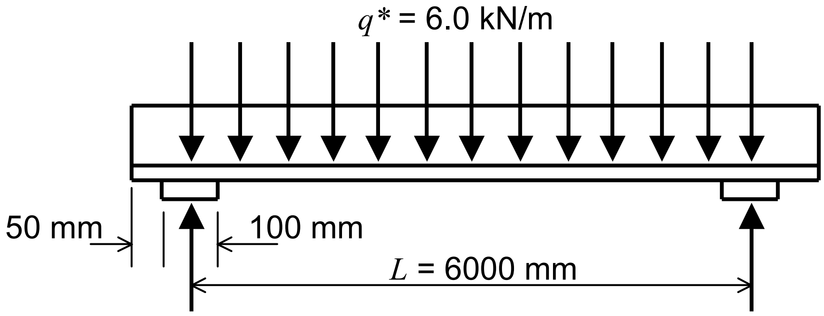
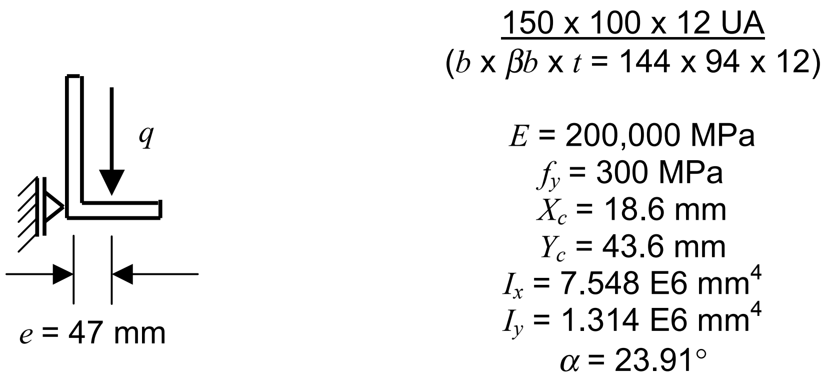


Fig. 4. Dispersion of Bearing Reaction



(a) Elevation



(b) Section

Fig. 5 Example