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Moment Capacities of Steel Angle Sections

Research Report No R809

By

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November 2001

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Abstract:
Steel angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion. However, the recommendations of many design codes are unnecessarily conservative when applied to the bending of angle section beams, or are of limited application, or fail to consider some effects which are thought to be important.

In this paper, consideration is given to the first-order elastic analysis of the biaxial bending of angle section beams including the effects of elastic restraints, and proposals are developed for the section moment capacities of angle sections under biaxial bending which approximate the effects of full plasticity in compact sections, first yield in semi-compact sections, and local buckling in slender sections.

Proposals are developed for the bearing, shear, and uniform torsion capacities of angle section beams in a companion paper.

The proposals in this and the companion paper can be used to design steel angle section beams which are laterally restrained so that lateral buckling or second-order effects are unimportant.

Keywords: angles, beams, bending, design, local buckling, moments, plasticity, section capacity, steel, yielding.

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Department of Civil Engineering, Research Report R 809
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Published by:
Department of Civil Engineering
The University of Sydney
Sydney, NSW, 2006
AUSTRALIA

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1 INTRODUCTION

Steel angle sections are commonly used as beams to support distributed loads which cause biaxial bending and torsion, as shown in Fig. 1. However, many design codes (BSI, 2000, SA, 1998) do not have any design rules for torsion, while some recommendations are unnecessarily conservative when applied to the bending of angle section beams, or are of limited application, or fail to consider some effects which are thought to be important. This and a companion paper (Trahair, 2001) develop an economical approximate method of designing restrained angle section beams under biaxial bending and torsion which is consistent with the philosophy of current design codes.

The behaviour of steel angle sections under biaxial bending and torsion is more complex than that of doubly symmetric sections under uniaxial bending and torsion, for which recent research (Pi and Trahair, 1994a,b) has established a better understanding of behaviour and suggested logical methods of design. The complexity arises from the mono-symmetric or asymmetric nature of angle sections, as well as from the common loading condition in which loading parallel to but eccentric from one of the section legs causes biaxial bending about the principal axes and torsion.

The development of a better understanding of the behaviour of steel angle section beams requires special consideration of their loading and restraint, and of the analysis of their elastic behaviour. Firstly, horizontal restraints of beams with vertical loads acting in the plane of one leg induce significant horizontal forces which modify the elastic stress distribution, as indicated in Fig. 2b. These horizontal forces and their effects on the stress distribution need to be accounted for in the elastic analysis of the beam. Secondly, angle section beams are often loaded eccentrically from the shear centre at the intersection of the legs, as shown in Fig. 1c, in which case significant torsion actions may result. These torsion actions need to be accounted for in the analysis.

The strengths of steel angle section beams are related to their section capacities to resist bending, bearing, shear, and torsion actions, and to their member capacities to resist the interactions between biaxial in-plane bending, out-of-plane buckling, and torsion. Very short span beams under distributed loading may fail at the supports, where the shear stresses induced by shear forces and uniform torques are greatest, while long span beams often fail near mid-span, where the normal stresses induced by biaxial bending moments are greatest.

In this paper, consideration is given to the first-order elastic analysis of the biaxial bending of angle section beams including the effects of restraints, and proposals are developed for the moment capacities of angle sections under biaxial bending which approximate the effects of full plasticity in compact sections, first yield in semi-compact sections, and local buckling in slender sections. Proposals are developed for the bearing and combined shear and uniform torsion capacities of angle section beams in a companion paper (Trahair, 2001).

These proposals are generally more economic than those of the AISC Design Specification (AISC, 1993a,b), one of the few design codes with specific rules for angle section beams. This code uses a conservative compromise between the first yield and fully plastic moment capacities for compact beams, even though there is experimental evidence (Madugula et al, 1995, 1996) that the full plastic moment can be reached. In addition, its local buckling rules are based conservatively on those of I-section flange outstands in uniform compression, while the primary recommendation for shear design is based on first yield, rather than full plasticity. Further, the biaxial bending moment capacities are based on a conservative linear interaction equation.

2 FIRST-ORDER ELASTIC ANALYSIS OF BIAXIAL BENDING

A simply supported angle section beam of length L (Fig. 1) has a uniformly distributed vertical load q acting in the plane of one leg of the section, as shown in Fig.2b, and a continuous horizontal restraint which exerts a horizontal uniformly distributed load r acting in the plane of the other leg, and prevents horizontal deflection.

The first-order elastic principal axis deflections u, v can be predicted by resolving the distributed load q and reaction r into principal plane components, whence

$$\frac{u}{\delta_x} = \frac{v}{\delta_y} = \frac{16}{5}\left(\frac{z}{L} - \frac{2z^3}{L} + \frac{z^4}{L^4}\right) \tag{1}$$

in which z is the distance along the beam and δ_x, δ_y are the maximum principal axis deflections which are given by

$$\delta_x = \frac{5L^4}{384EI_y}(-q\sin\alpha + r\cos\alpha) \tag{2a}$$

$$\delta_y = \frac{5L^4}{384EI_x}(q\cos\alpha + r\sin\alpha) \tag{2b}$$

in which E is the Young's modulus of elasticity, I_x, I_y are the principal axis second moments of area of the angle section and α is the inclination of the x principal axis to the horizontal X axis. The maximum horizontal deflection is

$$\delta_X = \delta_x \cos\alpha + \delta_y \sin\alpha \tag{3}$$

and this is zero (as shown in Fig. 2b) when

$$\frac{r}{q} = \frac{(1 - I_y / I_x)\tan\alpha}{1 + (I_y / I_x)\tan^2\alpha} \tag{4}$$

The ratio of the principal axis moments is obtained from

$$\frac{M_y}{M_x} = \frac{-r\cos\alpha + q\sin\alpha}{r\sin\alpha + q\cos\alpha} \tag{5}$$

as

$$M_y / M_x = (I_y / I_x) \tan \alpha \tag{6}$$

and the elastic bending stress distribution caused by these moments is as shown in Fig. 2b.

If the continuous restraint is removed so that $r = 0$ and

$$M_y / M_x = \tan \alpha \tag{7}$$

then the deflections and the elastic stress distribution are as shown in Fig. 2a. The bending stresses vary linearly in both legs of the angle section as shown in Fig. 2a, while the beam deflects horizontally with a maximum value of

$$\delta_X = -\frac{5qL^4 \sin\alpha \cos\alpha}{384EI_y}\left(1 - \frac{I_y}{I_x}\right) \tag{8}$$

3 BIAXIAL BENDING SECTION CAPACITIES

3.1 Full Plastic Moment Capacities

A typical fully plastic stress distribution in an angle section with unequal legs b x t and βb x t is shown in Fig. 3. The plastic neutral axis pp must bisect the area of the cross-section in order to satisfy the condition that the axial force stress resultant for pure bending must be zero. The points of intersection of the neutral axis with the legs are defined by the lengths $\gamma_1 b$ and $\gamma_2 b$ along the legs. If $0 \le \beta \le 1$, then $0 \le \gamma_2 \le \beta$ and

$$\gamma_1 = (1 + \beta) / 2 - \gamma_2 \tag{9}$$

so that

$$(1 - \beta) / 2 \le \gamma_1 \le (1 + \beta) / 2 \tag{10}$$

The full plastic moments M_{pX}, M_{pY} about the rectangular (geometric) axes parallel to the legs of the angle section can be determined by taking moments of the fully plastic stress distribution about axes through the legs of the angle section, whence

$$\frac{M_{pX}}{f_y b^2 t} = \frac{1}{2} - \gamma_1^2 \tag{11a}$$

$$\frac{M_{pY}}{f_y b^2 t} = \frac{\beta^2}{2} - \left\{\frac{1+\beta}{2} - \gamma_1\right\}^2 \tag{11b}$$

when the leg ends are in compression, in which f_y is the yield stress.

When the value of

$$m_r = M_{pY} / M_{pX} \tag{12}$$

is known, the value of γ_1 can be determined as the solution of

$$\gamma_1 = \frac{(1+\beta) \pm \{2m_r^2 - (1-\beta)^2 m_r + 2\beta^2\}^{0.5}}{2(1-m_r)} \tag{13}$$

which lies in the range of Equation 10.

The variations the dimensionless full plastic moment combinations of $M_{pX} / f_y b^2 t$ and $M_{pY} / f_y b^2 t$ for values of the leg length ratio β in the range $0.5 \le \beta \le 1$ are shown by the solid lines in Fig. 4. It can be seen that for each unequal angle, there are two discontinuous lines, indicating that there are ranges of the ratio $m_r = M_{pY} / M_{pX}$ for which the angle section cannot reach full plasticity (because a fully plastic stress distribution for such a range has a non-zero axial force stress resultant). These ranges are defined by

$$\frac{2\beta^2}{2-(1+\beta)^2} \le m_r \le \frac{-2\beta^2}{2-(1-\beta)^2} \tag{14}$$

For these ranges, the maximum moment capacities correspond to elastic-plastic stress distributions, and may be approximated by

$$\frac{M_{pY}}{f_y b^2 t} = -\frac{\beta^2}{2} \tag{15a}$$

$$-\frac{1}{2} + \frac{(1+\beta)^2}{4} \le \frac{M_{pX}}{f_y b^2 t} \le \frac{1}{2} - \frac{(1-\beta)^2}{4} \tag{15b}$$

as shown by the lower dashed lines in Fig. 4.

It may be convenient to replace the rectangular (geometric) axis moments M_{pX}, M_{pY} by their equivalents M_{px}, M_{py} referred to the principal axes x. Thus

$$M_{px} = M_{pX} \cos\alpha - M_{pY} \sin\alpha \tag{16a}$$

$$M_{py} = M_{pX} \sin\alpha + M_{pY} \cos\alpha \tag{16b}$$

The variation of α with the leg length ratio β has been determined by assuming that the angle section is thin-walled and using the computer program THIN-WALL (Papangelis and Hancock, 1997), and is shown in Fig. 5.

The AISC Specification (AISC, 1993a) uses principal axis maximum moment capacities of 1.25 times the first yield moments M_{yx}, M_{yy}, which are equal to 0.833 times the full plastic moments M_{px}, M_{py}. For biaxial bending about both principal axes, the AISC Specification introduces additional conservatism through the use of a linear interaction equation instead of the non-linear relationships shown in Fig. 4.

3.2 First Yield Moment Capacities

Typical elastic stress distributions in an angle section with unequal legs b x t and βb x t are shown in Fig. 2. The first yield moments M_{yX}, M_{yY} about the rectangular (geometric) axes X, Y parallel to the legs of the angle section can be determined by taking moments of the first yield elastic stress distribution about axes through the legs of the angle section. The dimensionless first yield moment combinations of $M_{yX} / f_y b^2 t$ and $M_{yY} / f_y b^2 t$ vary linearly as shown in Fig. 6 between the combinations for which the neutral axis is either at 45° to the X rectangular axis parallel to the horizontal leg or bisects both legs of the angle section.

When the neutral axis is at 45°, the dimensionless first yield moments are given by

$$\frac{M_{yX}}{f_y b^2 t} = \frac{(1+3\beta)}{6(1+\beta)} \tag{17a}$$

$$\frac{M_{yY}}{f_y b^2 t} = \frac{-\beta^2(3+\beta)}{6(1+\beta)} \tag{17b}$$

When the neutral axis bisects both legs, the dimensionless first yield moments are given by

$$\frac{M_{yX}}{f_y b^2 t} = \frac{1}{6} \tag{18a}$$

$$\frac{M_{yY}}{f_y b^2 t} = \frac{\beta^2}{6} \tag{18b}$$

It will often be convenient to replace the rectangular (geometric) axis moments M_{yX}, M_{yY} by their equivalents M_{yx}, M_{yy} referred to the elastic principal axes x, y. This may be done by using a similar transformation to that given by Equations 16.

3.3 Local Buckling Under Elastic Stress Distributions

Although the local buckling resistance of an angle section beam under biaxial bending varies with the ratio of the principal axis moments M_x, M_y, the usual design approach (AISC 1994a, BSI 2000, SA 1998) is to consider the effects of local buckling on the separate principal axis moment capacities, and then to combine the separate capacities in a biaxial bending interaction equation.

Because of this, the elastic buckling of angle sections under the separate uniaxial principal axis elastic moments M_x and M_y has been analysed for this paper using the computer program THIN-WALL (Papangelis and Hancock, 1997). For these analyses, it has been assumed that the Young's modulus is E = 200,000 MPa, the Poisson's ratio is ν = 0.3, and long wavelength flexural buckling effects have been eliminated by preventing lateral displacements of the leg junction. Minimum values of the elastic buckling coefficient k_e were determined from the maximum elastic buckling stresses f_e (tension or compression) using (Timoshenko and Gere, 1961, Bulson, 1970, Trahair et al, 2001)

$$f_e = \frac{\pi^2 E}{12(1-\nu^2)} \frac{k_e}{(b/t)^2} \tag{19}$$

for positive or negative bending about the principal x or y axes. These minimum values are shown in Fig. 7 for angle sections with $0.5 \leq \beta \leq 1.0$ and compared with the corresponding values k_c for angle sections in uniform compression.

For strong axis elastic bending about the x principal axis with the end of the long leg in compression, the buckling mode is one of rotation about the leg junction with distortion of the long compression leg, and k_{exc} varies in the range from 1.25 to 1.30 to 1.18 as β decreases from 1.0 to 0.75 to 0.5. The corresponding values of k_c for uniform compression are significantly lower and vary from 0.44 to 0.59. When the end of the long leg is in tension, the buckling mode is one of rotation of the short compression leg with restraint by flexure of the long tension leg, but k_{ext} increases rapidly from 1.25 to 6.27 as β decreases from 1.0 to 0.5.

For weak axis elastic bending about the y principal axis with both leg ends in compression, the buckling mode is one of rotation about the leg junction, and k_{eyc} increases rapidly from 0.89 to 3.61 as β decreases from 1.0 to 0.5. For both leg ends in tension, the buckling mode is one of rotation about the leg junction with restraint by flexure of both legs, and k_{eyt} is very large and increases from 24.2 to 36.2 as β decreases from 1.0 to 0.5.

3.4 Local Buckling Under Plastic Stress Distributions

The elastic buckling of angle sections under "plastic" stress distributions has also been analysed for this paper using the computer program THIN-WALL (Papangelis and Hancock, 1997), and minimum values of the elastic buckling coefficient k_p were determined from the maximum "plastic" buckling stresses f_p (tension or compression) using an equation similar to Equation 19. Two types of "plastic" stress distribution were considered, one for which the neutral axis intersects only the long leg (at a point defined by $\gamma_1 = (1 + \beta) / 2$) which corresponds to that of an elastic principal axis moment M_x, and one for which the neutral axis bisects both legs (at points defined by $\gamma_1 = 0.5$ and $\gamma_2 = (1 + \beta) / 2$) which corresponds approximately to that of an elastic principal axis moment M_y.

The minimum values k_p are also shown in Fig. 7 and compared with the corresponding values k_e for angle sections with elastic stress distributions. It can be seen that while the values of k_p are always lower that those of k_e, they follow similar trends.

3.5 Section Classification and Capacity

3.5.1 General

Design codes (AISC 1993a, BSI 2000, SA 1998) classify beam sections by comparing their plate slendernesses of a form similar to

$$\lambda = \frac{b}{t}\sqrt{\frac{f_y}{250}} \tag{20}$$

with limiting slenderness values. The British code BS 5950 (BSI, 2000) classifies sections as being either plastic, compact, semi-compact, or slender. The AISC code (AISC, 1993a) and the Australian code AS 4100 (SA, 1998) describe plastic (in the British sense) and compact sections as being compact, and semi-compact beams as being non-compact. For the angle section beams of this paper, the British terminology of plastic, compact, semi-compact, and slender beams is adopted.

A plastic section must have sufficient rotation capacity to maintain a plastic hinge until a plastic collapse mechanism develops. A plastic section satisfies

$$\lambda \le \lambda_p \tag{21}$$

in which λ_p is the plasticity limit. In the Australian code AS 4100, λ_p = 9 for the flange outstands of hot-rolled I-section beams. The plastic classification is only required when a plastic collapse mechanism is to be developed. Some design codes do not allow plastic analysis to be used for angle section beams, in which case a definition of plastic angle section beams is not required.

A compact section must be able to form a plastic hinge. A compact section satisfies

$$\lambda_p < \lambda \le \lambda_c \tag{22}$$

in which λ_c is the compact limit. In the Australian code AS 4100, λ_c = 9 for the flange outstands of hot-rolled I-section beams.

A semi-compact section must be able to reach the first yield moment, but local buckling effects may prevent it from forming a plastic hinge. A semi-compact sections satisfies

$$\lambda_c < \lambda \le \lambda_y \tag{23}$$

in which λ_y is the yield limit. In the Australian code AS 4100, λ_y = 16 for the flange outstands of hot-rolled I-section beams.

A slender section has its moment capacity reduced below the first yield moment by local buckling effects. A slender section satisfies

$$\lambda_y < \lambda \tag{24}$$

3.5.2 Moment Capacities

The nominal section moment capacity M_s of a plastic or compact beam is equal to its fully plastic capacity, so that

$$M_s = M_p \tag{25}$$

Elastic local buckling has a significant effect on the moment capacity of slender beams. In the Australian code AS 4100, the nominal section moment capacity of a slender beam M_s is generally approximated by

$$M_s = M_y \, (\lambda_y / \lambda)^2 \tag{26}$$

in which M_y is the first yield capacity. This approximation is based on the elastic local buckling moment.

For a semi-compact beam, $\lambda_p < \lambda \leq \lambda_y$, and the nominal section moment capacity M_s may be approximated by interpolating linearly between the full plastic and first yield capacities using

$$M_s = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_y - \lambda_p)} \tag{27}$$

3.5.3 Slenderness Limits

In the absence of relevant experimental evidence, the yield and plastic slenderness limits , λ_y, λ_p of angle section beams may be obtained by using local buckling predictions to adapt generally accepted values for the flange outstands of I-section beams, such as those of 16 and 9 of the AS 4100.

3.5.3.1 Yield limits

For angles under strong axis elastic bending about the x axis which causes compression at the end of the long leg, the elastic buckling coefficient k_{exc} varies between 1.18 and 1.30, but the elastic buckling coefficient predicted by THIN-WALL for a flange outstand in uniform compression is 0.44. An appropriate yield slenderness limit may be obtained by multiplying the AS 4100 flange outstand value of 16 by √(1.18 / 0.44) and rounding off. Thus it is suggested that

$$\lambda_{yx} = 26 \tag{28}$$

should be used for the yield limit of angle section beams under strong axis bending which causes compression at the end of the long leg. This yield limit will be conservative for angles under strong axis bending which causes tension at the end of the long leg, which have higher elastic buckling coefficients (Fig. 7).

For angles under weak axis elastic bending about the y axis which causes compression at the leg ends, the minimum value of the elastic buckling coefficient is k_{eyc} = 0.89, which suggests that the yield limit should be reduced from the strong axis value of 26 to

$$\lambda_{yy} = 23 \qquad (29)$$

although a higher value could be used if advantage is taken of the significant increases in k_{eyc} which occur as the leg length ratio β decreases (Fig. 7). This yield limit will be conservative for angles under weak axis bending which causes tension at the leg ends, which have much higher elastic buckling coefficients (Fig. 7).

The AISC Specification (AISC, 1993a) uses a conservative yield limit equivalent to λ_y = 12.6.

3.5.3.2 Plastic limits

The definition of plastic angle section beams has been investigated extensively by Earls and Galambos (1998) and Earls (2001), who in their later papers have advocated classifying plastic angle section beams according to both their leg width-thickness ratio b/t and their member slenderness L/r. However, for this paper, plastic limits are proposed in the form of the traditional width-thickness ratios (in the form of Equation 20) alone.

For angles under strong axis plastic bending about the x axis which causes compression at the end of the long leg, the elastic buckling coefficient k_{pxc} varies between 0.79 and 0.97, but the elastic buckling coefficient predicted by THIN-WALL for a flange outstand in uniform compression is 0.44. An appropriate plastic slenderness limit may be obtained by multiplying the AS 4100 flange outstand value of 9 by $\sqrt{(0.79 / 0.44)}$ and rounding off. Thus it is suggested that

$$\lambda_{px} = 12 \qquad (30)$$

should be used for the plastic limit of angle section beams under strong axis bending which causes compression at the end of the long leg. This plastic limit will be conservative for angles under strong axis plastic bending which causes tension at the end of the long leg, which have higher elastic buckling coefficients (Fig. 7).

For angles under weak axis plastic bending which causes compression at the leg ends, the minimum value of the elastic buckling coefficient is k_{pyc} = 0.59, which suggests that the plastic limit should be reduced from the strong axis value of 12 to

$$\lambda_{py} = 10 \qquad (31)$$

This plastic limit will be conservative for angles under weak axis plastic bending which causes tension at the leg ends, which have much higher elastic buckling coefficients (Fig. 7).

3.5.3.3 Compact limits

It is less easy to suggest compact limits for angle section beams than yield and plastic limits because of a comparative lack of widely accepted limits for I-section beams. In some codes (SA, 1998), plastic limits of I-section members are used conservatively for their compact limits. Thus conservative values of λ_c equal to 12 or 10 could be obtained for the compact limits by using the plastic limits for angle section beams bent about the strong or weak axis.

On the other hand, compact limits which are possibly optimistic could be obtained by using the procedure of Section 3.5.3.1 but with the plastic stress distribution local buckling coefficients k_p instead of the elastic coefficients k_e. This would lead to a value of λ_c equal to 21 or 19 for angles bent about the strong or weak axis.

As a compromise, it is suggested that the compact limits λ_c should be taken as the geometric means of the possibly optimistic values of 21 or 19 and the conservative plastic limits of 12 or 10. Thus the compact limit for angle section beams bent about the strong axis would be given by

$$\lambda_{cx} = 16 \tag{32}$$

and for angle section beams bent about the weak axis by

$$\lambda_{cy} = 14 \tag{33}$$

The AISC Specification (AISC, 1993a) uses a conservative compact limit equivalent to λ_p = 10.8.

4 SHEAR, BEARING, AND TORSION SECTION CAPACITIES

While moment capacity will govern the strength of long span angle section beams, short span beams may have high shears, bearing reactions, and uniform torques near the supports. Proposals for designing angle section legs against shear and bearing are made in a companion paper (Trahair, 2001). Proposals are also made in that paper for design against uniform torsion.

5 EXAMPLE

5.1 Problem

A 150 x 100 x 12 unequal angle beam is shown in Fig. 8. The section properties calculated using THIN-WALL (Papangelis and Hancock, 1997) for the thin-wall assumption of b = 144 mm, βb = 94 mm, and t = 12 mm are shown in Fig. 8b. The beam is simply supported over a span of L = 6 m, and has a design uniformly distributed vertical load of q^* = 6 kN/m acting parallel to the long leg and with an eccentricity of e = 47 mm from the shear centre at the leg junction, as shown in Fig. 8b. Horizontal deflections of the shear centre are prevented.

The first-order analysis of the beam is summarised in Section 5.2 below, and the checking of the moment capacity of the beam in Sections 5.3 and 5.4. The checking of the bearing, shear, and torsion capacities is summarised in a companion paper (Trahair, 2001).

5.2 Elastic Analysis

The restraints which prevent horizontal deflections exert a uniformly distributed horizontal force per unit length r^* which satisfies (Equation 4)

$$\frac{r^*}{q^*} = \frac{(1 - I_y / I_x)\tan\alpha}{1 + (I_y / I_x)\tan^2\alpha} = 0.354$$

so that r^* = 2.125 kN/m.

The maximum vertical deflection is

$$\delta_Y = \frac{5L^4}{384}\left\{\frac{(-q^*\sin\alpha + r^*\cos\alpha)}{EI_y}\sin\alpha + \frac{(q^*\cos\alpha + r^*\sin\alpha)}{EI_x}\cos\alpha\right\} = 77.6 \text{ mm}$$

The maximum principal axis moments are

$$M_x^* = (r^*\sin\alpha + q^*\cos\alpha)L^2/8 = 28.6 \text{ kNm}$$

$$M_y^* = (-r^*\cos\alpha + q^*\sin\alpha)L^2/8 = 2.2 \text{ kNm}$$

The maximum moments about the rectangular (geometric) X,Y axes are

$$M_X^* = q^* L^2/8 = 27.0 \text{ kNm}$$

$$M_Y^* = -r^* L^2/8 = -9.6 \text{ kNm}$$

5.3 Section Classification for Bending

$(b/t)\sqrt{(f_y/250)} = 13.1$

Using Equation 32 for strong axis bending, $\lambda_{cx} = 16 > 13.1$ and the section is compact.
Using Equation 33 for weak axis bending, $\lambda_{cy} = 14 > 13.1$ and the section is compact.

5.4 Moment Capacity

Adapting Equation 12, $m_r = M_Y^* / M_X^* = -0.354$.

Substituting this and $\beta = 0.653$ into Equation 13 leads to $\gamma_1 = 0.215$, so that $0.174 < 0.215 < 0.827$ and γ_1 lies in the range of Equation 10. Adapting Equations 11 and using a capacity factor of $\phi = 0.9$,
$\phi M_{pX} = 30.5$ kNm > 27.0 kNm = M_X^*, OK, and
$-\phi M_{pY} = 10.8$ kNm > 9.6 kNm = $-M_Y^*$, OK.

6 CONCLUSIONS

This paper develops economical design methods for determining the section moment capacities of angle section beams under biaxial bending, and illustrates their use in a design example.

Proposals are made for classifying an angle section as compact, non-compact, or slender, which are based on modifications of existing rules for I-section flange outstands in uniform compression which account for the different local buckling behaviour of angle section legs in compression and tension. Simple expressions are developed for the rectangular (geometric) axis fully plastic moment capacities of compact sections under biaxial bending, and for the first yield capacities which can be used with the fully plastic capacities to approximate the section capacities of non-compact beams. A conservative approximation is proposed for the moment capacities of slender sections.

Proposals are made in a companion paper (Trahair, 2001) for checking the bearing, shear, and torsion capacities of an angle section beam.

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8 NOTATION

b	long leg length
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_e	elastic local buckling stress
f_y	yield stress
I_x, I_y	second moments of area about the x, y principal axes
k_c	elastic local buckling coefficient for compression
k_e	elastic local buckling coefficient for elastic bending
k_p	elastic local buckling coefficient for plastic bending
L	span length
M_{pX}, M_{pY}	fully plastic moments about the X, Y axes
M_{px}, M_{py}	fully plastic moments about the x, y axes
m_r	principal axis moment ratio
M_s	nominal section moment capacity
M_X*, M_Y*	design moments about the X, Y rectangular axes
M_x, M_y	moments about the x, y principal axes
M_x*, M_y*	design moments about the x, y principal axes
M_{yX}, M_{yY}	first yield moments about the X, Y rectangular axes
M_{yx}, M_{yy}	first yield moments about the x, y principal axes
q	intensity of uniformly distributed load
$q*$	design intensity of uniformly distributed load
r	intensity of uniformly distributed reaction.
$r*$	design intensity of uniformly distributed reaction
t	leg thickness
u, v	deflections in the x, y directions
X, Y	rectangular (geometric) axes
x, y	principal axes
X_c, Y_c	X, Y distances from shear centre to centroid
z	distance along beam
α	inclination of x principal axis to X rectangular (geometric) axis
β	leg length ratio
γ_1, γ_2	leg length ratios defining the plastic neutral axis position
δ	central deflection
δ_X, δ_Y	central deflections in the X, Y directions
δ_x, δ_y	central deflections in the x, y directions
λ	long leg slenderness
λ_c, λ_p, λ_y	compact, plasticity, and yield slenderness limits
ϕ	capacity factor

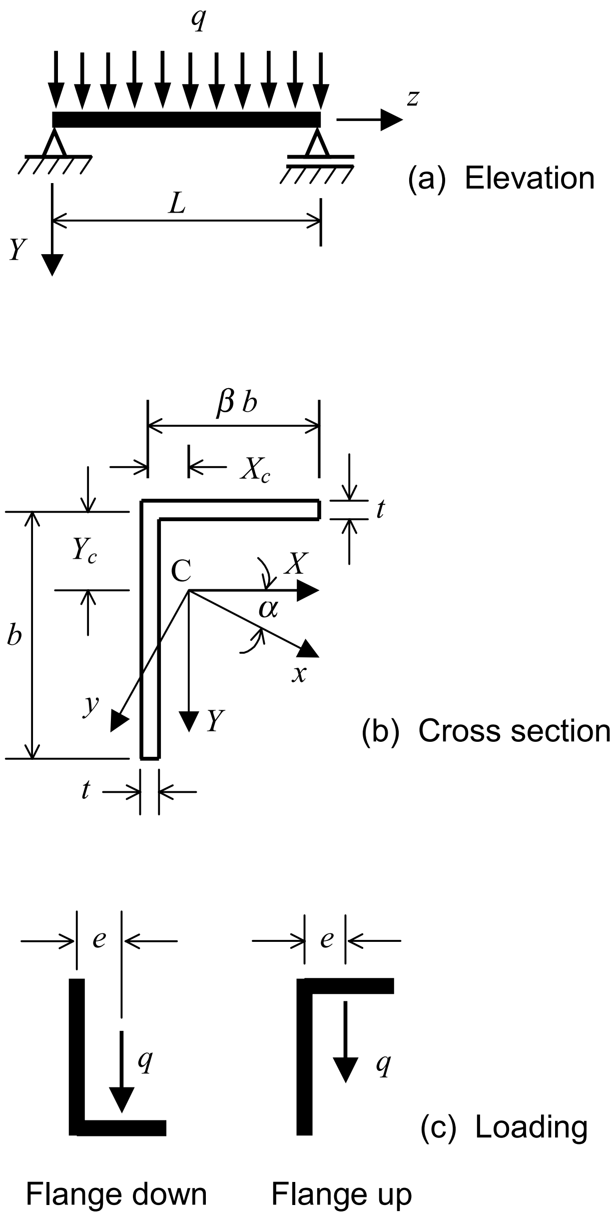


Fig. 1. Simply Supported Angle Section Beam

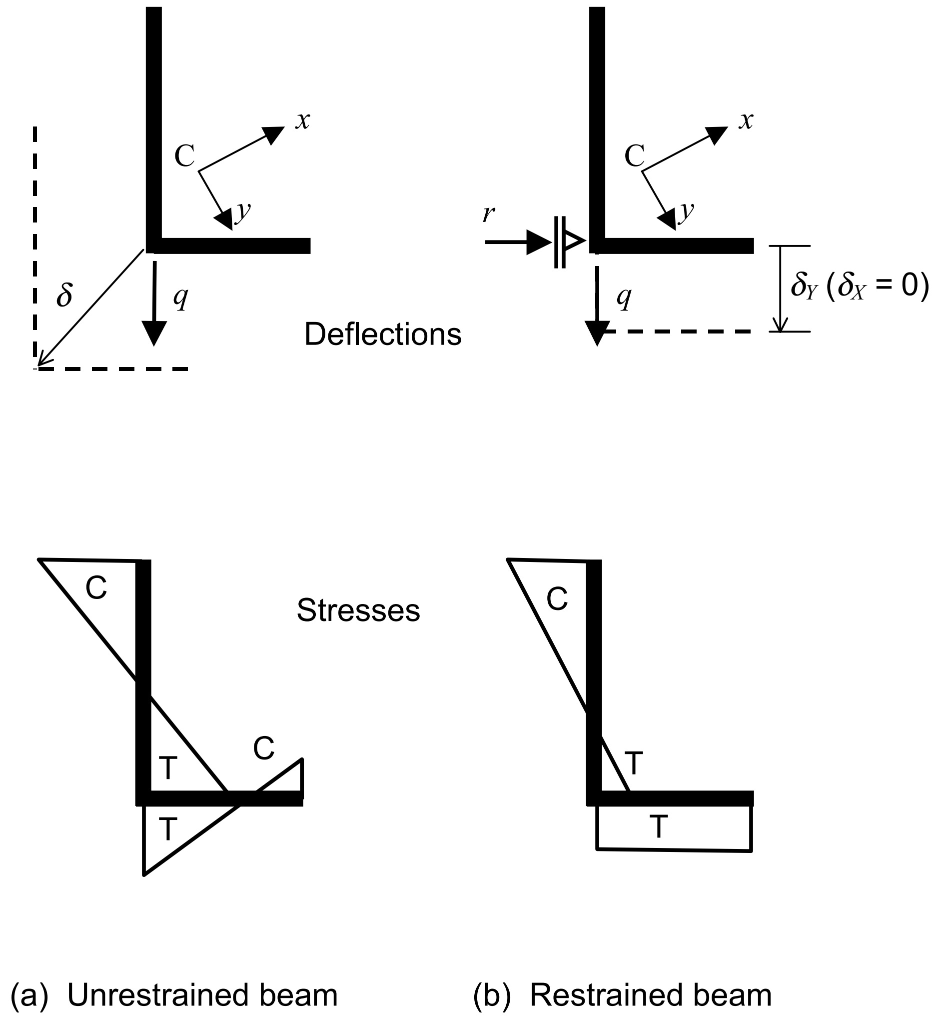
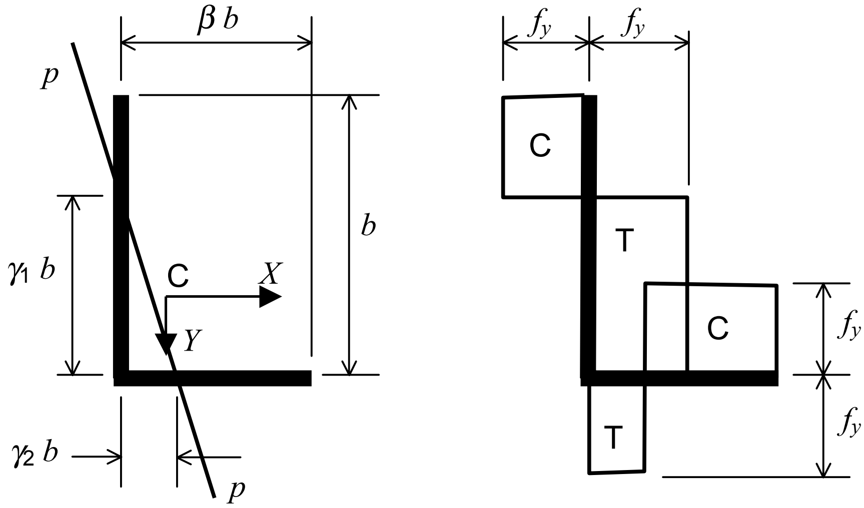


Fig. 2. First-Order Deflections and Stresses



(a) Plastic neutral axis (b) Stress distribution

Fig. 3. Fully Plastic Angle Section

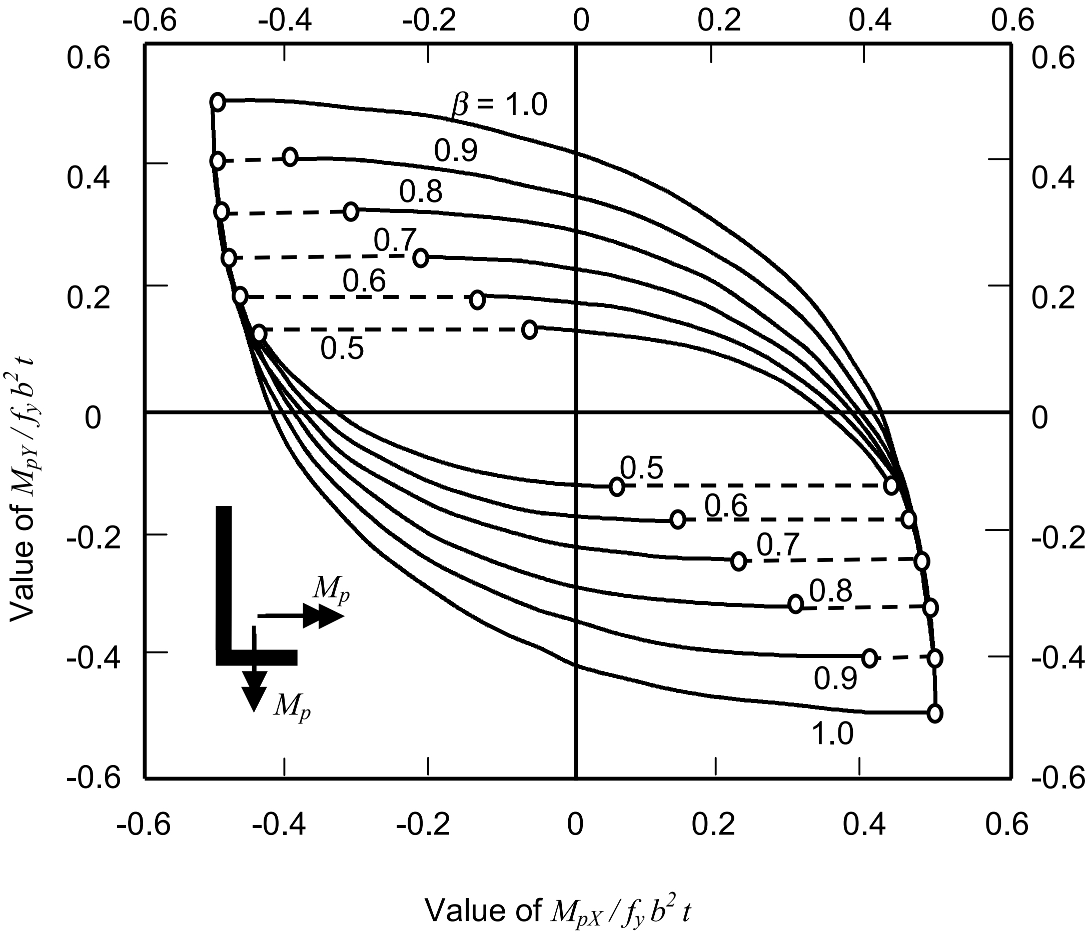


Fig. 4. Plastic Section Moment Capacities

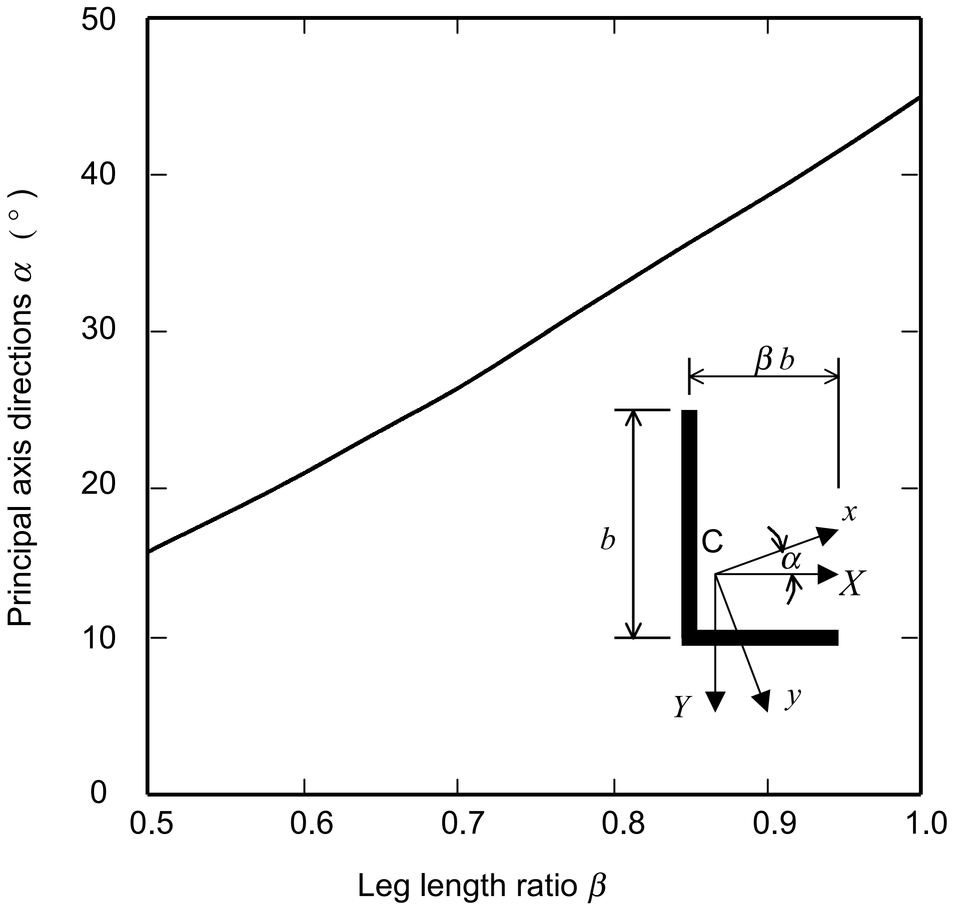


Fig. 5. Principal Axis Directions

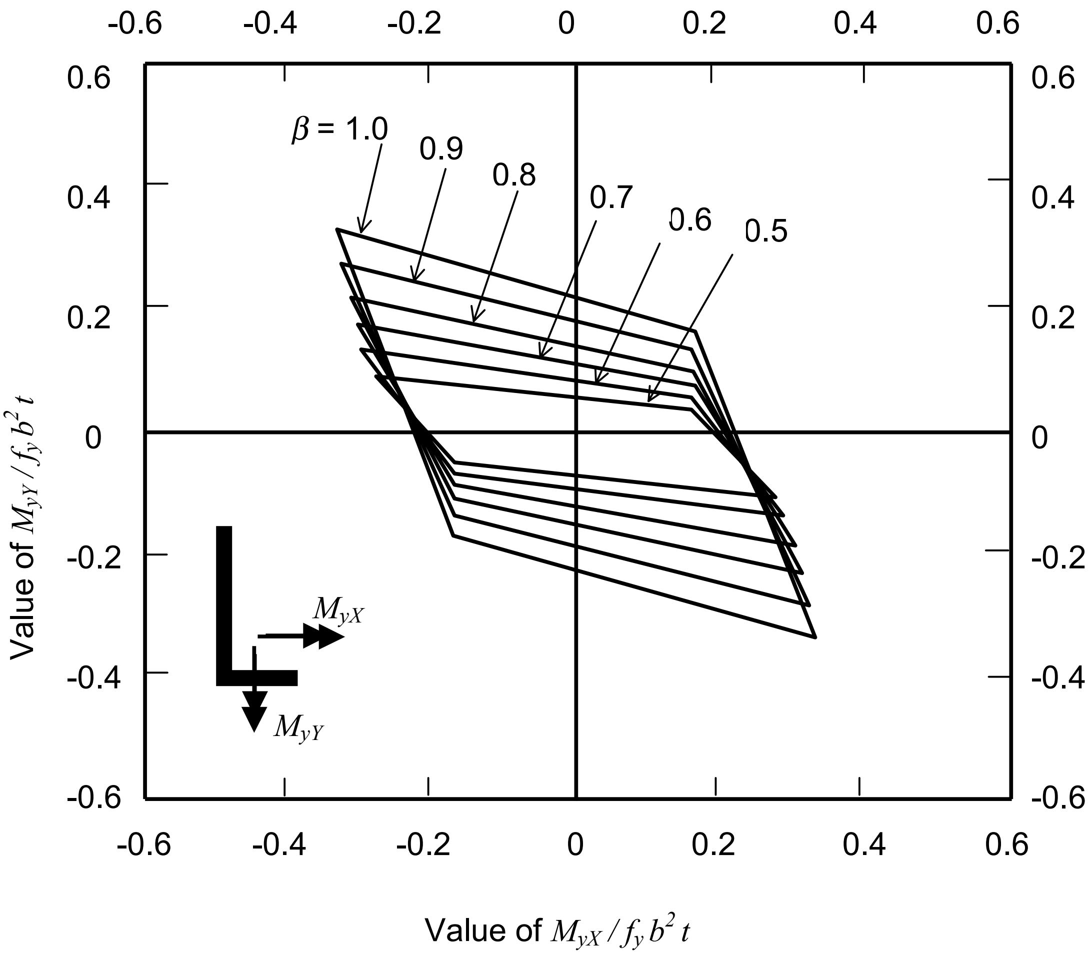


Fig. 6. First Yield Section Moment Capacities

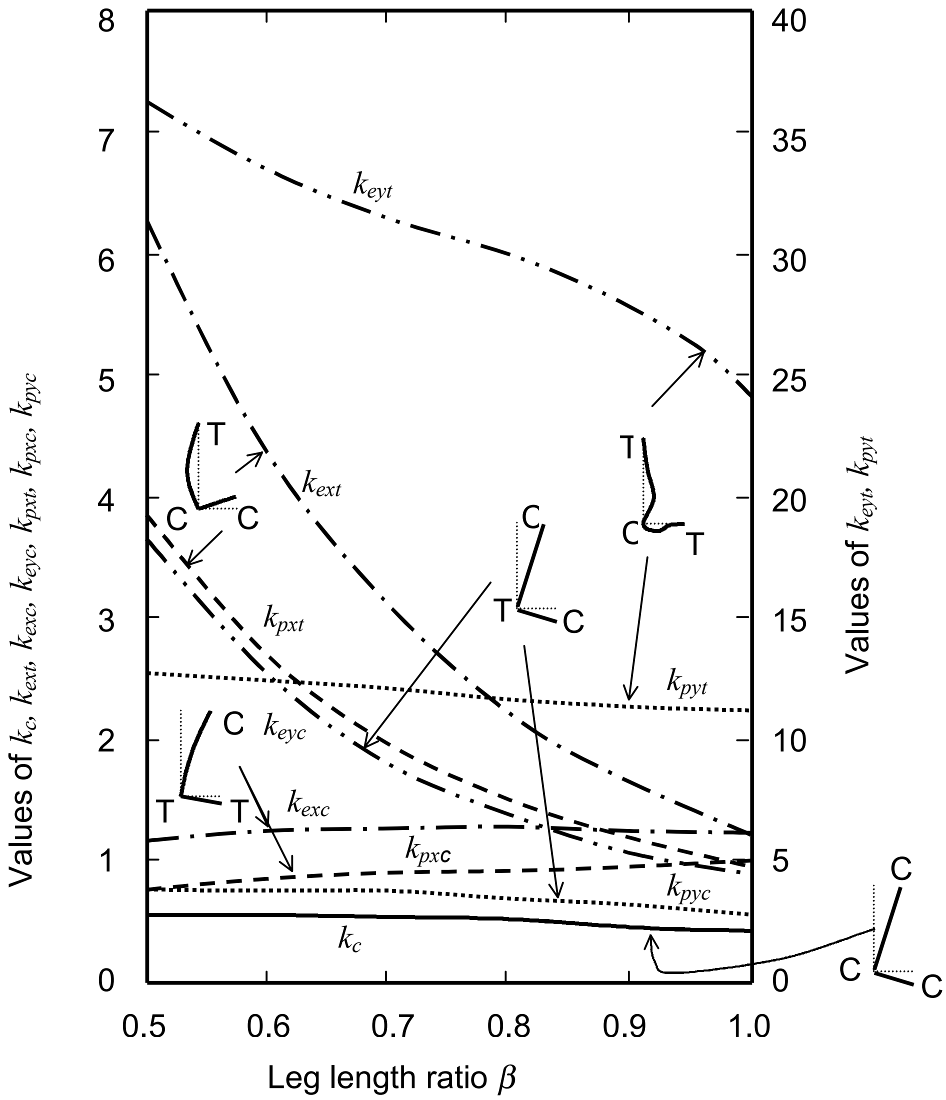
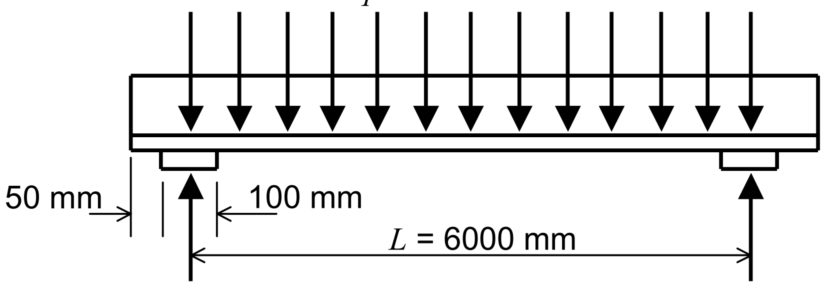
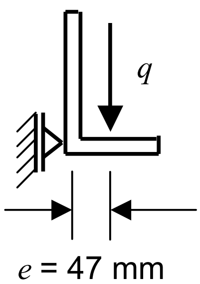


Fig. 7. Local Buckling Coefficients



(a) Elevation



150 x 100 x 12 UA
(b x βb x t = 144 x 94 x 12)

E = 200,000 MPa
f_y = 300 MPa
X_c = 18.6 mm
Y_c = 43.56 mm
I_x = 7.548 E6 mm^4
I_y = 1.314 E6 mm^4
α = 23.91°

(b) Section

Fig. 8 Example