Planning for flexible future grids
under power injection diversity

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A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy
September 2020
Declaration of authorship

I hereby certify, to the best of my knowledge, that all research and writing in this thesis are my own except in cases where proper citation and attribution is made in the text. This work has not been submitted for any other degree or award. Thesis contents that are published or pending publication in journals and conference proceedings are clearly attributed. All intellectual content of this thesis is the product of my own work and all help received have been duly acknowledged.

Sgd.

Adonis Emmanuel DC. Tio

June 2020
Acknowledgement

I was originally hesitant to pursue my Ph.D. degree abroad. But in retrospect, I am grateful and lucky to have experienced what I have experienced here in Sydney, even if it was a struggle at times. I think I have changed.

I would like to thank my supervisors Prof. David Hill and Dr. Jin Ma for their technical guidance and words of encouragement throughout the duration of my candidature; the thesis reviewers for their positive reception of the work; the DOST-ERDT Faculty Development Fund of the Republic of the Philippines for sponsoring my Ph.D. degree, the University of the Philippines, College of Engineering, and the Electrical and Electronics Engineering Institute (EEEI) for the support; my former professors and colleagues at EEEI for pushing me to pursue my PhD abroad and for welcoming me back after thesis submission; the University of Sydney HPC service at The University of Sydney for providing high-performance computing access; the Sydney Informatics Hub, a Core Research Facility of the University of Sydney, for providing technical training and assistance in using the HPC cluster; and my family, friends, and colleagues in Sydney and in the Philippines for being there; special thanks to Jaybie for helping me get settled in Sydney, for taking care of the flat matters, and for the general company.
List of publications

The following are works derived from this thesis that are published or under review.

Journal Papers:


Conference Papers:

Abstract

Power injection scenarios are bound to get more diverse in the future as intermittent renewable sources (RES), energy storage systems (ESS), and aggregated flexible loads become more mainstream, as old and emerging market players gain flexibility in injecting or drawing power from the grid and demand fairer grid access, and as end-users demand more control over their energy footprint. To accommodate these changes, there is emerging interest in (1) exploring ways to reduce a grid’s inherent inability to host diverse power injection conditions via grid expansion and (2) integrating grid flexibility in grid expansion planning to adapt to changing operating conditions, e.g. via line switching. This thesis proposes new tools and frameworks along these research directions that we hope will be useful in future research and practice.

First, we review the evolution of how power injection scenarios are accounted for in the Transmission Network Expansion Planning (TNEP) literature to motivate the need for new tools and frameworks to accommodate more diverse power injection conditions. We then present a new framework for assessing grid inadequacy for power injection diversity using the size of the power flow infeasible set. To characterize the high-dimensional sets involved, we present one approach that evaluates sampled scenarios for infeasibility and two approaches that project the sets in lower dimensions. Then, we propose three sets of metrics based on these approaches. These ideas serve as foundation for finding and evaluating future grid expansion options in the remainder of the thesis.

Next, we explore an idea from the literature that argues that adding new lines that optimize the graph properties of power grids also optimizes the grid’s inherent ability to host diverse power exchanges. Results show that this is not necessarily true and so motivates a more direct approach in reducing grid inadequacy. And so, we develop a linear programming model that directly minimizes grid inadequacy by minimizing the size of the power flow infeasible set. Results show that the model can identify distinct expansion options that are less inadequate, albeit at an added cost.

The last part of the thesis explores how added grid flexibility via line switching can complement new line addition in reducing total grid inadequacy for power injection diversity. We incorporate switch installation and operation in the linear programming model to identify line and switch additions that minimize the size of the infeasible set. Case studies show that a new line can be completely avoided with the addition of switches for about the same level of inadequacy. Finally, we explore the effects of line addition and switch operation on the infeasible set. Using the idea of grid congestion modes, we show that different line and switch expansion options affect the infeasible set differently and thus result in different grid inadequacy profiles.
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<th>Description</th>
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</tr>
<tr>
<td>FACTS</td>
<td>Flexible AC Transmission System</td>
</tr>
<tr>
<td>HVDC</td>
<td>High voltage DC</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed-integer Linear Programming</td>
</tr>
<tr>
<td>RE</td>
<td>Renewable energy</td>
</tr>
<tr>
<td>TCSC</td>
<td>Thyristor-controlled series capacitors</td>
</tr>
<tr>
<td>TNEP</td>
<td>Transmission Network Expansion Planning</td>
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1 Introduction

The future of power grids is uncertain but emerging trends hint at what it could look like. Climate change mitigation and adaptation as well as a transition to a more sustainable way of life will inevitably disrupt the traditional operation and planning of market-based power grids. On the supply side, there is increasing momentum to transition from cheap but dirty and non-renewable fossil fuel resources towards cleaner but intermittent renewable energy (RE) resources such as solar and wind. On the demand side, end-users demand more control over their energy, power, and carbon footprints through initiatives like local energy generation, coordinated demand response, and transition to electric transport. For the grid, increased interconnection and improving grid flexibility gain attention to accommodate the spatiotemporal variations in the availability of and demand for power and energy.

If trends continue, future power grids will need to operate under more diverse power injection conditions. Currently, diversity in power injection operating states come from the intermittency of RE resources, uncertainty in the bid-based dispatch of generators in electricity markets, variability in network loading, and occasional generator outages. But as enabling technologies mature such as bulk energy storage systems (ESS) and aggregated flexible loads, market players will gain greater flexibility in injecting and drawing power from the grid, resulting in greater power injection diversity than before. To enable existing power grids for such a future, new infrastructure, operating protocols, and market regulations will need to be developed.
This thesis develops concepts and methods for grid expansion planning. This area of research is generally concerned with the identification and assessment of necessary grid infrastructure investments that enable secure and reliable grid operation in the long-term future. In particular, this thesis explores the following questions in the context of increased power injection diversity in future grids:

a. Ways to assess grid inadequacy to host diverse steady state power injection conditions,
b. Tools to identify new lines to reduce grid inadequacy for power injection diversity,
c. Tools to co-optimize the allocation of new lines and switches to reduce grid inadequacy for power injection diversity,
d. Ways to characterize the effect of line and switch addition to grid inadequacy.

We elaborate on the contribution of this thesis in these areas in the remainder of this chapter as follows. Section 1.1 gives an overview of the Transmission Network Expansion Planning (TNEP) process and positions this thesis with respect to this process. Section 1.2 reviews how power injection diversity is accounted for in TNEP literature as the power industry evolved. Section 1.3 identifies emerging ideas in addressing power injection diversity in TNEP in a future grid setting. Section 1.4 highlights the contribution of this thesis in relation to the ideas identified in Section 1.3. Section 1.5 outlines the thesis structure.

1.1 The TNEP process

TNEP is an integral part of ensuring a secure and reliable electric power system. It is concerned with putting up the necessary network infrastructure to accommodate the hourly, seasonal,
and annual variations in the power exchanges between generators and end-users in the long-term future, subject to technical and practical constraints.

Transmission line expansion is one of the key network expansion solutions used to enable these power exchanges when generator and load connectivity and overall transmission network capacity are lacking. It is still a significant activity in developing regions as they build capacity to accommodate load growth and connect new generation. The same is true in developed countries, but in a more limited sense, where continuing trends like slowed demand growth, under-utilized transmission assets, public opposition to new transmission corridors, complicated rights-of-way application processing, and uncertain long-term prospects, make a poor case for costlier infrastructure investment in some cases. This motivates the integration of less capital-intensive power flow control options in the planning process as an alternative to line expansion. These alternatives include: (1) power injection control through generation re-dispatch, load islanding, energy storage scheduling, or aggregated demand response, (2) topology control through line switching, (3) line susceptance control through series compensation, (4) voltage angle control through phase shifting transformers, and (5) line flow control through HVDC links.

The combinations of network expansion and power flow control solutions which can be implemented across the network comprise the solution space of TNEP. Choosing which solutions to implement require expertise from various fields of study such as steady-state adequacy analysis, reliability and contingency analysis, transient and stability analysis, short circuit analysis and protection coordination, system loss analysis, reactive power planning, market modeling, cost-benefit analysis, and rate-making among others. Ideally, all aspects should be simultaneously considered to come up with an optimal plan, but this is practically impossible given the computational complexity of these kinds of studies even for small power grids. To make planning manageable, the
process is decomposed into simplified and decoupled subproblems wherein a subset of the required planning objectives is chosen to screen candidate solutions for further detailed analysis later. In traditional literature, for example, TNEP revolves around steady-state network adequacy planning through line expansion. This involves finding least-cost line additions that mitigate network congestion for a projected future load and generation dispatch scenario or scenarios, usually only considering real power flows. Detailed analyses are subsequently performed to the shortlisted solutions, most notably, contingency analysis where the operational performance of the shortlisted plans is tested against component outages. Considering more objectives simultaneously to shortlist solutions is also possible but at the cost of increased computational complexity. Incorporating even marginal additions to a given subproblem however, can render the resulting computations intractable even with current computational technology.

Uncertainty further complicates the cross-disciplinary requirements of TNEP. Uncertainty is naturally embedded in the problem since TNEP deals with the future. It manifests in the operational timescale in terms of customer usage patterns, market bidding behavior, and renewable energy availability to the much more far-out and less predictable strategic timescale in terms of techno-economic and socio-political trends [1]. Different instances of operational and strategic timescale assumptions give rise to the vast scenario space of possible futures that grid operators and planners must be aware of.

Coupling the large scenario and solution spaces together makes TNEP a complex problem that is both theoretically and computationally challenging to solve. Figure 1-1 capture these in a planning framework, adapted with modifications from [2]. This process is repeated every now and then when new information that gives light to some of the uncertainties becomes available. It also
undergoes a cycle of consensus-building with stakeholders so practical and transparent decision-support tools are much needed.

This thesis develops a new framework for assessing the inadequacy of existing grids and proposed expansion options to host diverse power injection conditions. We illustrate how this framework can be used in the following steps of the TNEP process identified in Figure 1-1: (1) as part of Base Case Development in assessing the ability of existing grids to accommodate power injection diversity, (2) as part of Options Shortlisting in identifying new infrastructure additions to enable future grids to host more diverse power injection conditions, and (3) as part of Detailed Analyses in assessing the improvements in grid ability to host power injection diversity brought about by the shortlisted infrastructure investment options.
1.2 Power injection diversity modeling in TNEP research

Over the years, many TNEP decision-support tools and frameworks have been proposed to identify grid infrastructure investments to suit the needs of the power industry as it evolved. The variety of survey papers on the subject is a testament to ongoing attempts to develop models and tools to manage planning in an increasingly more complex power grid [3]–[8]. There are literally thousands of journal and conference papers on TNEP and an exhaustive literature review could prove to be a daunting task.

In this literature review, we take a historical perspective on TNEP research. We explore how the evolution of the electric power industry has shaped TNEP research in the past and how the current technological and socio-political environment continues to shape it. Representative journal and conference papers are chosen to highlight the compounding complexity of TNEP over time. The review focuses on the following aspects of TNEP modeling: (1) how the scenario space is modeled in terms of the power injection scenarios considered in decision-making, and (2) what comprises the solution space in terms of infrastructure investment and power flow control options. This presentation highlights how power injection patterns and infrastructure investment options increasingly diversified as the planning objectives evolved to cater to socio-political demands. Table 1-1 summarizes the key characteristics of TNEP frameworks adapted during key stages of the evolution of electricity grids that the succeeding sub-sections will elaborate.
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1.2.1 Centrally planned power grids

Traditional electric power systems are centrally planned and operated. Generation and transmission infrastructure development are coordinated to meet projected future demand growth
and to ensure supply and network adequacy under normal operation and reliability under contingencies. Generation planning consisted of forecasting future demand and commissioning ample generation and reserve capacity at predetermined points in the grid. Transmission expansion planning was a relatively simple matter of connecting generation and load locations, or reinforcing existing connections, with ample redundancy in case of contingencies. Most necessary information is available to the planner and uncertainty is generally limited to component outage in the short term and load growth in the long term. References [4], [5] provide a survey of key papers during this era. Cost minimization and reliability improvement are common planning objectives although some other metrics and indexes are proposed. Reference [5] provides a review of different mathematical optimization, heuristic, and meta-heuristic methods applied to solve these kinds of models, with varying degrees of modelling detail, including static and dynamic TNEP approaches for single- and multi-year expansion planning respectively.

In terms of the scenario space, only the peak load is often considered in the literature. This is evident in the test systems in [4] for four widely-used test networks. Most papers only consider a unique generation dispatch in the planning process while some allow for flexibility through generation redispatch. By sizing the network infrastructure against this worst-case loading level, sometimes with additional margin for forecast errors, the idea is that the network can essentially operate in any other loading condition. This paradigm worked since the following applied: (1) the loads generally follow known profiles represented by the worst-case peak condition, (2) most of the generating capacity is dispatchable, and hence (3) the resulting power flow patterns across the network are predictable and easily controllable. These assumptions, however, are not applicable in deregulated and future power systems and so, the decision support tools developed around this paradigm become increasingly lacking.
In terms of the solution space, earlier works mostly involve line expansion by building new transmission corridors or reinforcing existing ones. Building network capacity and establishing connectivity was necessary at the time to satisfy a spatially dispersing generation and load growth. Until then, operational flexibility investments, through power flow control options, cannot be widely considered. Regardless, a couple of papers already started to explore other network alternatives, in most cases, through conceptual papers and feasibility studies. Superconducting lines and high order phase lines are explored in [9] and [10] respectively. High voltage DC (HVDC) line planning has been considered in some TNEP decision support tools [11]. Phase-shifters has also been considered in a TNEP context albeit post-optimization [12]. Several researchers and institutions also already contemplated the modeling of energy storage [13], demand-side management [14], Flexible AC Transmission System (FACTS) devices [15], and intermittent renewable generation [16] in the power grid and its impacts in the network planning process. Recent papers integrated fixed series compensation [17], variable series reactors [18], and energy storage planning [19] in TNEP using assumptions for centralized grid planning.

### 1.2.2 Deregulated operation of power grids

Deregulation of the power industry meant that the generation and retail sectors became open for competition while the transmission and distribution networks became regulated monopolies. In some cases, independently-run patches of transmission infrastructure were mandated to connect to each other to facilitate cross-regional power exchanges even outside one’s immediate territorial coverage [20]. These developments drastically changed transmission operations and planning practices because of the shift in planning objectives and the introduction of more uncertainties [6]. In addition to ensuring network adequacy and service reliability, planners must now also ensure that there is adequate infrastructure to foster competition and provide non-
discriminatory open-access. At the same time, the grid should be able to handle deviations from expected short- and long-term variations. This challenge is made more difficult with less available information, with data dispersed across private stakeholders, posing logistic and confidentiality barriers. Transmission expansion planning has become more complicated.

Because of deregulation, power flows, the worst-case of which traditionally dictated the needed network reinforcements, has now become less controllable and predictable. In the short-term, a portion of the generating fleet are now dispatched according to the market clearing that typically varies on a half-hourly basis. In the medium- to long-term, large loads may negotiate directly with generators via bilateral contracts to secure future capacity. The aggregate arrangements between market players can thus vary year-on-year. Because of these new dispatch arrangements, transmission operators have less control on which generators participate in the market as well as which get dispatched and how much. In addition, intermittent wheeling transactions by stakeholders outside one’s immediate jurisdictions add to network loading.

Short-term and long-term variations in power flow patterns, both anticipated and unanticipated, have the potential of overloading weakly-reinforced transmission corridors even outside of peak loading conditions. This necessitated the inclusion of different kinds of dispatch pattern possibilities in the planning process. Failure to prepare the network to handle a variety of power flow patterns could result in the exposure of the network to unanticipated stress and, much more importantly, the curtailment of potential least-cost dispatch for a costlier one, resulting in higher electricity rates and possible conflicts between stakeholders.

To address these challenges, new TNEP planning tools were developed. References [6] and [7] surveyed some of the earlier works on TNEP in a deregulated environment.
In terms of the scenario space, most models now include diversity in generator dispatch. Explicit enumeration identifies specific combinations of generator dispatch using combinatorics or expert information [21], [22]. Market simulation methods use bid models [23]–[28], optimal dispatch, or game theory [29], [30] to simulate market clearing dispatch. The operational scenarios resulting from these methods are then used as input or integrated into an optimization model where single or multiple planning objectives [25], [27], [31] are considered. Some common optimization algorithms include mathematical optimization techniques like mixed-integer linear programming, heuristic or rule-based methods where indices or metrics are iteratively improved via sensitivity analysis or heuristic rules, and meta-heuristic methods like genetic algorithm.

Ideally, multiple peak and off-peak operational power flow patterns should be accounted for, but in most cases, only one or a couple of loading levels and dispatch scenarios are considered. These approaches can work if it is acceptable to forego some savings from least-cost dispatch to defer costly infrastructure investments. Heavy computational requirements inherent in most planning algorithms also limited the scenarios that can be practically considered.

In terms of the solution space, line expansion via line addition is still the dominant network solution of choice to address possible grid congestion. Some papers though start to explore operational flexibility options as alternatives to line expansion planning [32], [33] while some start to integrate grid flexibility solutions like transmission switching, [34] and HVDC lines [35], [36] in the optimization process.

### 1.2.3 Sustainable future grids

The next generation of TNEP developments would have focused on market integration of largely decoupled regional and national electricity markets but the push to realize a sustainable
energy grid gained traction and took center stage [37]. A confluence of factors could have caused this shift. There was a sudden urgency for climate change mitigation and adaptation, recognition for reducing foreign energy imports, and a proliferation of cheap and efficient renewable energy generation technologies [38]. The Fukushima nuclear disaster in Japan in 2011 forced some governments to rethink their dependence on nuclear power [39].

To realize a future sustainable energy system, significant resources are put into developing power generation through wind and solar photovoltaic technologies and supporting technologies such as local generation, bulk energy storage, aggregated demand response, and new generation nuclear technology. Conventional fossil-powered transport technologies are also expected to transition to electric-power technology, ideally powered by renewables, further reducing the dependence on dirty and non-replaceable resources. These developments, together with ubiquitous communications and computing technology, are predicted to usher in a new industrial era [40]. Massive infrastructure investments are anticipated, and the current antiquated power grid will inevitably need to evolve.

As a consequence, next generation TNEP methodologies now include the integration of renewable energy and new technologies as part of the planning objectives. To preserve the current level of grid reliability and living standards on top of these objectives, additional operational and strategic uncertainties need to be incorporated in the planning process. In addition to the market-dependent dispatch of controllable generation, operational uncertainties now include the intermittent non-dispatchable renewable generation profile, dispatchable ancillary resource availability, and dispatchable and non-dispatchable load profiles. In the long term, new sources of uncertainty include new trading patterns and arrangements between emerging market players, and the spatio-temporal diversity in the availability of dispatchable and non-dispatchable generation and loads.
In terms of the scenario space, these additional uncertainties mean that the power flow patterns in future grids are expected to be much more varied than ever before. Some approaches in modeling these patterns as part of TNEP are currently available. A common setup involves drawing from discrete or continuous uncertainty intervals [41]–[43] or from deterministic or probabilistic data-driven models [44]–[56] to identify load and RE generation levels then use economic or rules-based dispatch for controllable generating units. Some papers try to reduce the number of identified power flow patterns through scenario reduction techniques [52], [57], [58]. After identifying the relevant operational scenarios, a variety of approaches can be used to identify the most promising candidate expansion options using a desired objective function. As in the past, common solution selection methods include mathematical optimization [42], [49], [54], [57], rules-based or heuristics approach guided by a metric or index [46], and meta-heuristic techniques [47]. Economic dispatch is commonly integrated in the TNEP optimization, but the necessity of the harder problem of unit commitment has been argued to have little value added in a planning context despite the large computational burden [59].

In terms of the solution space, line expansion is still a common network solution explored to mitigate network congestion as in traditional TNEP models. Recently, however, the importance of considering operational flexibility in TNEP is being recognized as a complement to line expansion [60]–[63]. As a result, co-planning of power flow control schemes with line expansion has become more useful in TNEP. Some of the power flow control options considered include line switching [64], thyristor-controlled series capacitors (TCSC) [65], [66], HVDC [67], ESS [68]–[73], electric vehicle charging stations [74], demand response [75]–[77], phase-shifting transformers [78], microgrids [79], [80] and a mix of these options [81]–[83]. This area of TNEP research will only get more mainstream in the future.
With increased uncertainty and network options come a corresponding explosion of computational complexity. To address this issue, dedicated papers on techniques and algorithms in TNEP for large grids are becoming more available. These include scenario-selection and scenario reduction [53],[84]–[92], network reduction [84], [87], [93]–[96], candidate selection and combinatorial search space reduction [97]–[102], generic optimization frameworks independent of power flow models [103], and efficient algorithms for large test systems [90], [104], [105]. To overcome the limited access to confidential datasets of large grids, some papers work on generating datasets for synthetic grids [106], [107]. Non-conventional methods to complement traditional optimization methods such as graph-based heuristics also emerge [108]–[110].

1.3 Reducing inherent grid inadequacy and integrating grid flexibility in TNEP

The following ideas in TNEP research are emerging to help enable future grids to host more diverse power injection conditions: (1) improving a grid’s inherent ability to host power injection diversity and (2) integrating grid flexibility in TNEP decision-making.

Unlike traditional approaches in literature that find line additions for the peak load dispatch as in [111], a pool of power injection scenarios as in [21], [41], [58], or the worst case scenario within power injection uncertainty intervals as in [112], an emerging idea is to directly improve a grid’s latent or inherent ability to host power injection diversity [108]–[110]. The problem can also be viewed as reducing a grid’s inherent inability or inadequacy to host power injection diversity. The idea is to maximize the diversity of power injection scenarios that the grid can host regardless of the specific mechanisms that shape the diversity of power exchanges in the future. Doing so is potentially useful in identifying alternative grid expansion options that may hedge against a larger set of power injection scenarios that traditional approaches may overlook. This alternative
framework can also pave the way for less computationally intensive methods of identifying meritorious grid expansion options in the future guided by heuristics that use grid graph properties as in [110] or the other metrics that we will present in the next chapter.

Integrating grid flexibility in TNEP is not a new idea and has already been explored in the works reviewed in Sections 1.2.1 to 1.2.3. What is new is the context in which grid flexibility is gaining increasing importance. That is, in traditional TNEP, grid flexibility may be considered to defer more costly infrastructure investments to allow the grid to host a specific peak load dispatch or a pool of pre-defined power injection scenarios. In current and future TNEP research, incorporating grid flexibility in TNEP will be explored to enable grids to host more and more diverse power injection conditions. The idea is as follows: if both the supply- and demand-sides gain flexibility in injecting and drawing power from the grid, then it makes sense for the grid to also be flexible and adaptive to changing power injection states. If there is surplus transmission capacity, adding grid flexibility in an existing grid may be enough by itself to host expected future power injection diversity. Otherwise, co-optimizing grid flexibility with improving inherent grid ability to host power injection diversity could prove to be an interesting and important emerging area of research.

1.4 Contributions of the thesis

This thesis contributes to the two emerging areas of TNEP research that we identified in Section 1.3 namely: (1) reducing inherent grid inadequacy for power injection diversity and (2) integrating grid flexibility planning in TNEP for improved grid adequacy for power injection diversity. In particular, we make the following contributions:
a. We present a new framework for assessing grid inadequacy for power injection diversity. As we mentioned in Section 1.1, this framework is useful in the following steps in the TNEP process: (1) assessing the ability of existing grids to host power injection diversity, (2) designing methods to identify grid expansion options that reduce grid inadequacy, and (3) assessing grid improvements resulting from grid infrastructure investments.

b. The proposed framework uses the infeasible set of the constrained DC power flow model to quantify and characterize grid inadequacy for power injection diversity. This is an approach that has not been done before in this context.

c. We present visualization approaches and metrics that help visualize and quantify the extent of grid inadequacy for power injection diversity. The proposed approaches circumvent the difficulty of characterizing the high-dimensional sets involved in defining the infeasible set.

d. We check whether grid expansion heuristics based on improving grid graph properties as proposed in [110] really reduce inherent grid inadequacy for power injection diversity. Results indicate that improving grid graph properties alone do not necessarily reduce inherent grid inadequacy for power injection diversity, and vice versa.

e. We develop an approach that finds grid expansion options by directly minimizing inherent grid inadequacy for power injection diversity. The approach uses a mixed integer linear programming model to minimize one of our proposed metric, i.e. the model minimizes a scenario-based representation of the power flow infeasible set. Illustrative examples indicate that the method can identify grid expansion options distinct from existing approaches and better in some measures, albeit at an added cost.
f. We explore how grid flexibility is related to total grid inadequacy for power injection diversity and the power flow infeasible set under the proposed framework. We use the scenario-based representation of the infeasible set to quantify the benefits of grid flexibility and to enable its use in a co-optimization model.

g. We develop an approach that identifies new line and switch installation options in a grid that best reduce total grid inadequacy for power injection diversity. The approach uses a mixed integer linear programming model that directly minimizes total grid inadequacy by minimizing a scenario-based representation of the infeasible set. Results indicate that the co-optimizing line and switch additions can have comparable levels of inadequacy as with more costly line-only solutions.

h. We present the idea of grid congestion modes to further characterize the infeasible set. This enables the identification of power exchanges that transition from a congestion mode to feasibility, from one congestion mode to another, or from feasibility to a congestion mode as a result of grid investments. This characterization adds nuance to comparing shortlisted options that we believe will increasingly become important in future TNEP research and practice.

1.5 Structure of the thesis

In Chapter 2, we present a framework and set of metrics that can be used to assess grid inadequacy for power injection diversity. These ideas serve as a common foundational material for the other remaining chapters. This chapter uses materials from the following publication:

In Chapter 3, we explore whether optimized graph properties of power grids result in optimized grid adequacy for power injection diversity. This chapter uses materials from the following publication:


In Chapter 4, we present an optimization model that directly minimizes inherent grid inadequacy for power injection diversity to identify meritorious line addition options. This chapter uses materials from the following publication:


In Chapter 5, we build on the model in Chapter 4 to integrate grid flexibility in the form of line switching. This chapter uses materials from the following publication:

In Chapter 6, we explore how the different expansion options obtained in Chapter 5 affect the power flow infeasible set using what we propose to call grid congestion modes. This chapter uses materials from the following publication submitted for review:

A. E. Tio, D. J. Hill, and J. Ma,, “Integrating grid flexibility in grid expansion planning: Effects on the power flow infeasible set” (submitted for review)

In Chapter 7, we conclude and identify areas for future work.
2 Assessing grid inadequacy for power injection diversity

It is difficult to improve what cannot be measured. To enable future grids to host more diverse power injection conditions, it is useful to devise metrics that quantify the extent of grid ability or inability to do so. Quantifying grid ability to host diverse power exchanges not only enables the study of the extent of grid capability and bottlenecks but also provides a way to design interventions and mechanisms for its improvement.

This chapter presents a new type of grid inadequacy assessment framework relating to a grid’s inherent inability to serve diverse power injection states without intervention. The proposed inadequacy metrics are based on the following idea that has not been explored in existing literature: given all possible power injection scenarios, the proportion of infeasible scenarios that lead to network congestion provide an indicative measure of inherent grid inadequacy against power injection diversity. This chapter provides a framework and set of metrics that explores this characterization of grid inadequacy that researchers and practitioners may find useful in the future. The ideas presented in this chapter also form the foundation of the ideas in the succeeding chapters.

In the proposed framework, we represent the uncertainty in bus power injections from renewables, dynamic loads, and energy storage together with injections from more conventional generation and loads using interval sets that span the whole power injection space. We then use the constrained DC power flow model to identify feasible and infeasible subspaces. Since it is

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generally difficult to analyze the high-dimensional sets involved, we use three approaches to develop numeric quantities or metrics that describe the infeasible set. While single numeric values cannot fully capture the complex relationships between the sets involved, let alone capture the intricacies of defining grid inadequacy, the metrics can still provide valuable insights on network vulnerabilities and bottlenecks that can motivate subsequent detailed analyses and the development of new grid expansion planning tools.

This chapter is organized as follows. Section 2.1 defines grid inadequacy. Section 2.2 reviews related literature on power injection diversity and grid inadequacy assessment, identifies shortcomings, and motivates the need for new metrics in the future grid setting. Section 2.3 presents a framework for measuring inherent grid inadequacy using the power flow infeasible set and defines tractable metrics that embody the proposed framework. Section 2.4 illustrate how the proposed framework and metrics can be used to assess inherent grid inadequacy and reveal existing bottlenecks using two test systems. Section 2.5 discusses the limitations of the proposed framework and identifies potential use-cases. Section 2.6 concludes.

2.1 What is grid inadequacy?

Ref. [113] defines power system adequacy as the existence of enough facilities to meet customer needs and operational constraints at static conditions under a set of probabilistic system states. By extension, grid inadequacy refers to the lack of such facilities or capability. The variety of assessment methods reviewed in the next subsection shows that grid inadequacy assessment is complicated because of complexities in choosing relevant system states and appropriate power system model as illustrated in Figure 2-1.
System states represent probable power injection levels under normal or outage conditions. Uncertainty in generation availability, load consumption, energy storage charge and discharge cycles, and bid-based dispatch in spot markets such as in [114] contribute to the diversity of potential system states [115]. System states are much more predictable in the operational planning setting with future system states more likely to resemble system states from the immediate past. On the other hand, system states in the distant future are much harder to predict because of the dependency on extraneous factors such as climate change, technological development, and legal and market restructuring.

Power system modeling involves the choice of a power flow model and whether power flow control interventions such as power injection control via generation rescheduling or load curtailment, topology control via line switching, or impedance, line power flow, or voltage angle control via FACTS devices are considered. The chosen model determines whether operational constraints or performance targets are not met, which in turn, determine grid inadequacy.

Because of the variety of system states and power system model combinations, measures of grid inadequacy are highly dependent on the assumptions used and often only applicable within the intended use-case.
2.2 Critique of existing methods

2.2.1 Existing metrics

Ref. [113] uses a basic framework that enumerates discrete system states and identifies which states lead to load curtailment. Inadequacy indices or metrics are then used to describe the curtailment states identified. Metrics include the *Probability of Load Curtailments (PLC)*, *Expected Number of Load Curtailments (ENLC)*, and *Expected Energy Not Supplied (EENS)*. *PLC* gives the sum of probabilities of curtailment states while *ENLC* gives an expected value. *EENS* gives the expected energy curtailment within a given period. These metrics are defined broadly enough to accommodate any level of power system modeling fidelity over a range of pre-selected system states. An obvious drawback is that the metrics are only as good as the system states and the assessment models used. As such, proper context and caveats must be established when using these metrics in decision-making.

Other works focus on a specific application. Measures of grid inadequacy have been described in terms of security, flexibility, loadability, or robustness in works related to operations planning [116]–[120]. Despite the different terminologies used, these works assess grid inadequacy for different power injection states under static conditions. Some works characterize inadequacy in terms of sets while others define metrics. In [116]–[118], security regions are defined using allowable variations in bus power injections about an operating point. To compute the allowable adjustments, Ref. [117] uses a linear program to maximize the sum of allowable adjustment range of all generators. A performance index is then defined using the proportion of allowable adjustments relative to the maximum generation in each bus. Ref. [119] also uses allowable generator adjustments to define grid flexibility but solves for the ranges using a different approach, that is, by taking the difference between a reference dispatch and a new dispatch that minimizes
the line loading margins. Flexibility indices are defined using the maximum generation variation in each generator bus and the system in total. Ref. [120] uses the range of allowable load variations to define grid robustness under load uncertainty. A linear program is repeatedly solved to approximate the extent of allowable load changes about a loading condition. A robustness index is then defined in terms of the probability of feasible operation even if the grid operates outside a defined set of operating conditions. And in [121], linear constraints are used to define loadability sets. The paper notes that metrics derived from the regions bounded by such sets, like inner-area approximations and centroid calculations, would provide useful indicators of grid flexibility and are important areas of future research.

In works related to expansion planning, Refs. [41] and [42] use an ad hoc scenario-based approach to compare the robustness of grid expansion options to both load and renewable generation uncertainty. The approach draws \( K \) scenarios from the distribution of uncertain load and generation variables and then, the number of scenarios that do not result in curtailment, \( K_I \), are counted. The larger the ratio of \( K_I \) and \( K \), the more robust a grid is to power injection variability from loads and RE generation. This approach follows the idea behind \( ENLC \) but tailored for a planning problem under load and RE generation uncertainty. Ref. [122] uses the maximum load shedding under uncertainty as a metric for assessing inadequacy and for comparing grid expansion options. Ref. [123] defines a framework to quantify the benefits of grid-side flexibility options provided by line switches and FACTS devices using sets defined from the constrained DC power flow model. The idea is to compare the size of feasible sets with and without flexibility options installed. Instead of defining discrete system states, spaces in the power injection space are used to represent system states in a continuum. Lastly, using a completely different perspective, Ref. [110] uses a graph-theoretic approach to grid inadequacy assessment and design. The work
proposes the use of the volume of feasible bus injections to measure grid flexibility to absorb and deliver power. The authors argue that line additions that maximize the DC power flow susceptance matrix determinant will also maximize the volume of feasible bus injections under special conditions.

2.2.2 Need for new metrics

The inadequacy metrics reviewed, however, are only as good as the assumptions and methods used and each has its limitations and intended use-cases. The basic metrics presented in [113] rely on the enumeration of discrete power injection states. In the future, scenario sampling and scenario reduction techniques can be used to derive a tractable but representative number of power injection states. However, planners must also be able to explore grid vulnerabilities outside of the pre-selected states and quantify risk.

The methods in [116]–[120] are defined for a specific operating point and are limited to modeling either generation variability only or load variability only. While useful in their respective intended applications, overall measures of grid performance that are independent of a specific operating point and can simultaneously capture load and generation variability will also be useful.

To address these issues, the ideas in [41], [42], [110], and [123] provide inspiration, but these works have limitations as well. The scenario-based approach in [41] and [42] suffer the same blindness to unsampled scenarios as with the metrics in [113]. Using the maximum load shedding as indicator of grid inadequacy as in [122] is also useful when used in the proper context but it fails to capture other aspects of network inadequacy. For example, the model assumes flexibility in generation dispatch and as such, ignores grid inadequacy in terms of forfeiture and rescheduling of bid-based dispatch. While [123] provides a useful framework for quantifying inadequacy in
terms of feasible spaces instead of discrete scenarios, it is unclear how to operationalize this framework for many variables. A visual comparison of feasible sets is possible in 2D space for two uncertain variables, as the work illustrates, but is generally difficult for more variables. And while the exploration of the relationship between graph properties and inadequacy of power grids as in [110] is interesting, the work ignores many practical constraints including the contribution of the spatiotemporal distribution of power injections and line capacity limits.

To address the gaps, the ideas presented in this chapter offer the following novel contributions:

a. We present a new framework that characterizes and measures inherent grid inadequacy to accommodate power injection diversity in future grids with more renewables, dynamic loads, and energy storage in a market environment.

b. The framework uses a previously unexplored way of characterizing grid inadequacy that will be useful in future research and practice. That is, given all system states within the whole power injection space, the relative size of the infeasible subspace is indicative of inherent grid inadequacy to accommodate power injection diversity without intervention.

c. Since the sets involved are generally high-dimensional and challenging to analyze, we present three approaches to circumvent this difficulty. One is a scenario-based approach while the other two are novel dimension-reduction approaches.

d. We present three inadequacy metrics obtained from the approaches described in (c).

The advantages of the proposed framework and metrics complement the limitations of the different approaches reviewed as follows. The metrics can capture the spatiotemporal diversity of both positive and negative power injections simultaneously and can be computed using minimal network data. The metrics provide an overall measure of grid inadequacy that is not dependent on
a given operating condition. The first set of metrics generalizes the scenario-based approach in [41] and [42] under the presented framework, and as such, are also dependent on the quality of the system states chosen and can be blind outside of these states. The other two sets of metrics are original and do not have this dependency.

2.3 Proposed framework

2.3.1 Definition and scope

We limit our definition of grid inadequacy to that of what we propose to call inherent grid inadequacy. We use this term to refer to a grid’s inherent inability to allow diverse power exchanges without deploying power flow control intervention including, but not limited to, generation or load redispatch, line switching, or FACTS-based control. Depending on the extent of inherent inadequacy, some generating units may be denied production and some loads may be curtailed in the absence of topology or FACTS-based control. By extension, the extent of inherent grid inadequacy can be used to inform inherent grid adequacy assessment. The term global inherent adequacy can be used to refer to a grid’s ability to accommodate all possible power injection scenarios in the entire power injection space without intervention. On the other hand, the term relative inherent adequacy can be used to refer to inherent adequacy or feasibility relative only to a subspace of the entire power injection space.

2.3.2 Inherent grid inadequacy and the power flow infeasible set

To assess inherent grid inadequacy, we check for the feasibility of power exchanges in the power injection space bounded by bus power injection limits and power balance constraints. The DC power flow model [124] allows a tractable way to do this with sufficient accuracy. This model is ubiquitous in research and practice, in both the operations and expansion planning applications, evident in the literature reviewed in Section 2 and in commercial planning software such as [125].
In the DC power flow model, infeasible operation is characterized by line overloading. That is, a vector of bus power injections $\mathbf{P}^s = [P^s_1, P^s_2, \ldots, P^s_n]^T$ representing scenario $s$ is infeasible in grid $g$ with $l$ lines and $n$ buses if the following relations are satisfied except (2-1):

$$|f^{s,\phi}_{ab}| \leq C^\phi_{ab} \quad \forall ab \in \mathcal{E}^\phi$$  \hspace{1cm} (2-1)

$$f^{s,\phi}_{ab} = B^\phi_{ab}(\theta^{s,\phi}_a - \theta^{s,\phi}_b) \quad \forall ab \in \mathcal{E}^\phi$$  \hspace{1cm} (2-2)

$$P^s_i = \sum_{j \in \mathcal{V}^\phi, j \neq i} f^{s,\phi}_{ij} \quad \forall i \in \mathcal{V}^\phi$$  \hspace{1cm} (2-3)

$$\sum_{i \in \mathcal{V}^\phi} P^s_i = 0$$  \hspace{1cm} (2-4)

$$P^\text{min}_i \leq P^s_i \leq P^\text{max}_i \quad \forall i \in \mathcal{V}^\phi$$  \hspace{1cm} (2-5)

where $f_{ab}$ is the power flow and $C_{ab}$ is the capacity of line $ab$ with end buses $a$ and $b$, $\mathcal{E}$ is the set of lines with cardinality $l$, $B_{ab}$ is the susceptance of line $ab$, $\theta_a$ is the voltage angle at bus $a$, $P_i$ is the power injection at bus $i$, $\mathcal{V}$ is the set of buses labelled from 1 to $n$, $P^\text{min}_i$ and $P^\text{max}_i$ are the power injection limits at bus $i$, and the superscripts indicate dependence in $s$ and $\phi$. Equation (2-1) limits the power flow in a line within capacity. Equations (2-2) and (2-3) are the DC power flow analog of Ohm’s law and Kirchhoff’s Current Law (KCL) respectively. Equation (2-4) is the power balance equation and (2-5) keeps power injection variability within limits.

Any power injection vector satisfying (2-4) and (2-5) represents a valid power exchange. However, some vectors can be infeasible due to network congestion, i.e. when at least one line is overloaded. We call the collection of such vectors as the power flow infeasible set. A large infeasible set means that without intervention, operational constraints will be violated for many power
exchange scenarios and indicates severe inherent grid inadequacy. As such, a grid with a small infeasible set is preferable, especially in a market environment, because it is more likely to be able to accommodate diverse power injection scenarios with minimal changes in the generation dispatch or scheduled load. Owing to this relationship between inherent grid inadequacy and the size of the infeasible set, we propose the following grid inadequacy assessment framework.

To quantify inherent grid inadequacy, we compare the size of the following sets:

- Set $\mathcal{F}$ of valid power injections satisfying (2-4)-(2-5) and
- Set $\mathcal{J}$ of infeasible power injections satisfying (2-2)-(2-5) but fail to satisfy (2-1) for at least one line.

That is, the larger the size of Set $\mathcal{J}$ relative to Set $\mathcal{F}$, the more inadequate a grid inherently is.

Solving for the size of these sets is complicated however, especially for a large number of buses and lines. To overcome this challenge, we present a vector-mapping interpretation of the constrained DC power flow model next. The surfaces are high-dimensional and difficult to analyze in general so we also present novel lower-dimension projection ideas as well.

2.3.3 Visualizing the infeasible set

2.3.3.1 The DC power flow as a vector mapping

The constrained DC power flow model can also be formulated using matrix operations [126]. That is, a power injection scenario is feasible for a given grid if the following relations are met including (2-4):

$$|f^s| \leq C^g$$  \hspace{1cm} (2-6)
\[ f^{s,\phi} = X^{\phi} K^{\phi} \theta^{s,\phi} \]  \hspace{1cm} (2-7)

\[ P^{s} = B^{\phi} \theta^{s,\phi} \]  \hspace{1cm} (2-8)

\[ P^{\text{min}} \leq P^{s} \leq P^{\text{max}} \]  \hspace{1cm} (2-9)

where \( f \) is the vector of line power flows, \( C \) is the vector of line capacities, \( X \) is the diagonal matrix of line susceptances, \( K \) is the edge-to-bus incidence matrix, \( \theta \) is the vector of bus voltage angles, \( P \) is the vector of bus power injections, \( B \) is the full DC power flow susceptance matrix, \( P^{\text{min}} \) and \( P^{\text{max}} \) are the vectors of bus power injection limits, and the superscripts denote dependence in \( s \) and \( \phi \). The elements of diagonal matrix \( X \) correspond to the line susceptances. The elements of matrix \( K \) are obtained by assigning a direction to each line and then defining the elements as follows:

\[ K_{xi} = \begin{cases} 
1 & \text{if line } x \text{ starts at bus } i \\
-1 & \text{if line } x \text{ ends at bus } i \\
0 & \text{otherwise}
\end{cases} \]  \hspace{1cm} (2-10)

The elements of \( B \) are as follows,

\[ B_{ij} = \begin{cases} 
\sum_{k} b_{ik} & \text{if } i = j, k \in \mathcal{V} \\
-b_{ij} & \text{if } i \neq j \\
0 & \text{otherwise}
\end{cases} \]  \hspace{1cm} (2-11)

where \( b_{ik} \) is the line susceptance. Equations (2-6) and (2-9) are the matrix representation of (2-1) and (2-5) respectively while (2-7) and (2-8) are the matrix equivalent of (2-2) and (2-3) respectively. We can combine (2-7) and (2-8) to eliminate \( \theta \) as follows:
\[ f_s^\# = X^\#K^\#(B^\#)^+P^s \]  

(2-12)

where \((B)^+\) is the pseudoinverse of \(B\). Eq. (2-12) highlights the cause-and-effect relationship between \(P\) and \(f\) in terms of the grid topology and network parameters. This also allows a vector mapping interpretation of the constrained DC power flow as follows.

Equation (2-12) can be interpreted as a mapping of vector \(P\) in the \(n\)-dimensional power injection space, \(P\)-space, to the \(l\)-dimensional line power flow space, \(f\)-space, through the linear mapping function \(XKB^+\). This becomes a constrained mapping when combined with Equations (2-4), (2-6), and (2-9). Figure 2-2 visualizes this mapping for a three-bus three-line grid. The susceptances are 1.0 p.u. for Line 1-2 and 0.5 p.u. for Lines 1-3 and 2-3. The power injection limits are 0.0–2.0 p.u. for Bus 1, -1.0–1.0 p.u. for Bus 2, and -1.0–0.0 p.u. for Bus 3. The line capacities are all 1.0 p.u.

![Figure 2-2: Visualization of the constrained DC power flow model as a vector mapping.](image)
Like the three-dimensional objects visualized in Figure 2-2, an \( n \)-dimensional hypercube defined by the bus power injection limits in (2-9) bounds all possible power injection combinations in \( P\text{-space} \). A hypersurface defined by the power balance equation in (2-4) lies within the hypercube and characterizes Set \( \mathcal{F} \) of valid power exchanges. Linear mapping using (2-12) projects this hypersurface into \( f\text{-space} \). Portions of the projection fall inside or outside an \( l \)-dimensional bounding hypercube defined by the line capacity limits in (2-6). Power exchanges projected outside the bounding hypercube have one or more congested lines and hence are infeasible. This characterizes Set \( \mathcal{I} \). The larger the projection outside the \( f\text{-space} \) bounding hypercube, the larger the size of the infeasible power injection set and the more inadequate a grid inherently is to accommodate power injection diversity without intervention.

Traditional assessment for grid inadequacy checks whether the point representing the peak load dispatch or points representing a number of power injection scenarios lie inside the \( f\text{-space} \) bounding hypercube, with or without employing power flow control interventions. However, increased power injection diversity and increased demand for fair grid access in future grids may require a paradigm shift. It will become increasingly important not just to accommodate a number of power injection points but instead accommodate a larger area of the power injection hypersurface as economically as possible. The ideas in this thesis help motivate this paradigm shift that we hope will be useful in future developments in this field.

2.3.3.2 Visualizing in reduced \( P\text{-space} \)

Analysis of feasible spaces is often done in \( P\text{-space} \) as in [116]–[121] and [123]. This involves finding feasible power injection ranges per bus – a process that is generally complicated because the feasible power injection ranges depend on a reference operating point. To remove this dependence, we present an approach that projects instead in a space related to \( P\text{-space} \). That is, we
project along the total network loading vector, $L$, given by the sum of the loads at each bus. The idea is to find thresholds of total network loading that trigger congestion instead of finding allowable power injection ranges or defining feasibility sets as in [116]–[121] and [123].

Under the DC power flow model assumptions, we can find $L_-$ and $L_+$ such that $L_-$ is a network loading level below which congestion is not possible regardless of the generation schedule or load distribution and $L_+$ is the maximum network loading that the grid can accommodate without congestion through at least one generation schedule. We can solve for $L_-$ in two steps: (1) solving for $L_{-,ab}$, the minimum load that triggers congestion of line $ab$ in direction $ab$ and (2) solving for the minimum of the values obtained in Step 1 for all lines in both the forward and reverse power flow directions. That is,

$$L_- = \min \{L_{-,ab} \; \forall ab \in \mathcal{E}^-\}$$  \hspace{1cm} (2-13)

where $\mathcal{E}^-$ is the set of line indices for both forward and reverse power flow directions. Here, each $L_{-,ab}$ is given by

$$L_{-,ab} = \begin{cases} L_{-,ab}^* & \text{if solution exists} \\ L_{\text{max}} & \text{otherwise} \end{cases}$$  \hspace{1cm} (2-14)

where $L_{\text{max}}$ is the maximum load and $L_{-,ab}^*$ is calculated by solving the following linear program:

Objective

$$L_{-,ab}^* = \min L$$  \hspace{1cm} (2-15)

Subject to

$$f_{ab} \geq C_{ab}$$  \hspace{1cm} (2-16)

Eqs. (2-2)-(2-5)  \hspace{1cm} (2-17)
Likewise, we can solve for $L_+$ as follows:

Objective

\[ L_+ = \max L \]  

Subject to

Eqs. (2-1)-(2-5)

Models (2-15)-(2-17) and (2-18)-(2-19) are linear programs solvable using conventional methods [127] and available computer software packages such as [128].

After solving for the relevant points, we can make the one-dimensional visualization like the one illustrated in Figure 2-3 where we can also superimpose $L_{max}$. This diagram visualizes the network loading ranges that are congestion-free wherein $L \leq L_-$ and congestion-prone wherein $L > L_+$. A subset of the latter set is congestion-positive for $L > L_+$ if $L_+ < L_{max}$.

Figure 2-3: 1D Visualization of $P$-space surfaces, 3-bus grid with no generation in Bus 2.
Some useful insights can be obtained from such an exercise. For the three-bus example, operating below \( L_- = 1.5 \) p.u. guarantees no congestion. Since \( L_+ = L_{\text{max}} \), there should be at least one generation dispatch that can serve the maximum load. But since \( L_- < L_{\text{max}} \), operating between \( L_- \) and \( L_{\text{max}} \) requires careful generation scheduling and indicates grid selectivity to some power exchanges. For example, when \( L = L_{\text{max}} = 2 \) p.u, grid congestion does not allow the full dispatch of generators in Bus 1. The load can only be satisfied by curtailing some generation in Bus 1 and dispatching generators in Bus 2, which may be more expensive.

If the generators in Bus 2 are removed, \( L_- \) remains the same but \( L_+ \) is reduced to 1.75 p.u. This means that congestion is guaranteed above this loading level and no feasible generation schedule can serve the maximum load.

Insights such as these can motivate further studies that explore the cause of these thresholds, quantify inherent grid selectivity in accommodating power exchanges, and design appropriate measures.

2.3.3.3 Visualizing in reduced \( f \)-space

Analysis of feasible spaces can also be done in \( f \)-space as follows but this has not been done before. This approach is simpler and provides supplementary insights by identifying lines that are prone to congestion when power injection scenarios are diversified.

In \( f \)-space, the projection of the \( f \)-space hypersurface is spanned by the maximum unconstrained line flows in both directions, \( f_{\text{max,},u,ab} \) and \( f_{\text{max,},u,ba} \). The difference determines a line’s unconstrained loading range. These parameters can be computed with all line capacity constraints relaxed using the following linear program in \( \theta, P, \) and \( f \):
Objective

\[ f_{\text{max},u,ab} = \max f_{ab} \quad (2-20) \]

Subject to

Eqs. (2-2)-(2-5) \quad (2-21)

On the other hand, the projection of the feasible subspace is spanned by the maximum feasible line flows in both directions, \( f_{\text{max},f,ab} \) and \( f_{\text{max},f,ba} \). The difference determines a line’s feasible loading range. These parameters can be computed with all line capacity constraints considered using the following linear program:

Objective

\[ f_{\text{max},f,ab} = \max f_{ab} \quad (2-22) \]

Subject to

Eqs. (2-1)-(2-5) \quad (2-23)

Like the linear programs in Section 2.3.3.2, models (2-20)-(2-21) and (2-22)-(2-23) can be solved using conventional linear programming solution algorithms or available computer software packages. After solving for the unconstrained and feasible line flows, we can superimpose the computed parameters in a line loading range diagram. Figure 2-4 shows this for the three-bus network in Section 2.3.3.1.

Figure 2-4: 1D visualization of the \( f \)-space surfaces, 3-bus grid.
In the figure, red outlines represent the line capacities. White areas represent excess line capacity and remains unused unless the power injection limits change, the network is modified, or non-injection power flow control interventions are used. The gray and black areas represent feasible and infeasible operation respectively and provide useful insights. For example, the figure indicates that there is ample capacity in Lines 1-3 and 2-3 to cover all possible diversity of power injection scenarios within bus power injection limits. However, capacity constraints in Line 1-2 prohibit the dispatch of some power injection scenarios and makes it a bottleneck for diverse power injection operations wherein \( f_{\text{max},f_{12}} = C_{12} < f_{\text{max},u_{12}} \). The prevalence of the infeasible ranges is indicative of inherent network inability to accommodate power injection diversity and can be used to motivate later detailed analyses. For example, the families of power injection scenarios that lead to infeasible states and the likelihood of these states occurring merit further exploration.

2.3.4 Proposed grid inadequacy metrics

2.3.4.1 Scenario Pool Infeasibility Ratio (SPIR)

We can define a representative scenario pool \( S \) of \( |S| \) power injection vectors \( P^1, P^2, ..., P^{|S|} \) to represent the set of valid power exchanges. Each vector represents a power injection scenario that satisfies (2-4) and (2-5). System state selection requires a balance between adequate space representation and computational tractability. It is a challenging problem on its own and is placed outside the scope of this thesis. Some of the common approaches include the following. If data is available and trends are assumed to hold in the planning horizon, probability distribution and correlation models may be used to generate scenarios. Market simulation can also be used if there are reliable datasets and models. Otherwise, combinatoric or worst-case sampling become viable options.
After defining the scenario pool, we can evaluate each scenario using DC power flow for a given network topology $\mathcal{G}$ and solve for the number of line overloading instances per scenario $N_k$, the number of line overloading instances in the scenario pool $N_L$, and the number of scenarios with congestion $N_S$ as follows:

\[
N_{k}^{s,\phi} = \sum_{ab \in \mathcal{E}^{\phi}} \left\{ u\left(f_{ab}^{s,\phi} - C_{ab}^{\phi}\right) + u\left(-f_{ab}^{s,\phi} - C_{ab}^{\phi}\right) \right\} 
\]  

(2-24)

\[
N_{L}^{s,\phi} = \sum_{s} N_{k}^{s,\phi} 
\]  

(2-25)

\[
N_{S}^{s,\phi} = \sum_{s} u\left(N_{k}^{s,\phi}\right) 
\]  

(2-26)

where $s = \{1, 2, \ldots, |S|\}$ indexes the scenarios in $S$, $u(x)$ is a function that returns 1 if $x > 0$ and returns zero otherwise and the superscripts show the dependence on $s$, $\phi$, and $S$. Using $N_S$ and $|S|$, we can then define the Scenario Pool Infeasibility Ratio (SPIR) for a given grid and scenario pool as follows:

\[
SPIR_{S,\phi}^{S} = \frac{N_{S}^{S,\phi}}{|S|} 
\]  

(2-27)

This metric captures the proportion of power exchanges in $S$ that is infeasible. Using the sets defined in Section 2.3.2, we use $N_S$ to measure Set $J$ and $|S|$ to measure Set $\mathcal{F}$. The more representative the scenario pool is of the actual $P$-space hypersurface, the better the approximation given by $SPIR$ of the relative sizes of Sets $\mathcal{F}$ and $J$.

Since $N_L$ is related to $N_S$ as implied in (2-24)-(2-26), $N_L$ can also be useful in some applications. Dividing $N_L$ with the number of line flow assessments given by $|S| \cdot l$ gives the Scenario Pool Line Overloading Ratio (SPLOR) as follows:
\[ SPLOR^{S,\varphi} = \frac{N_{L}^{S,\varphi}}{|S| \cdot l^{\varphi}} \] (2-28)

SPIR is a generalization of the ad hoc approaches in [41] and [42] to the whole power injection space. It shares the same idea with the \textit{ENLC} metric from [113] but counts grid congestion states instead of load curtailment states. As such, it is a flexible metric wherein more detailed models can be used to replace the power system model used here. However, as with all other scenario-based metrics, SPIR is highly dependent on the quality of the selected states and is potentially blind to other states outside this sample. The next two metrics do not require the identification of discrete system states and can be used to complement SPIR.

2.3.4.2 Congestion-prone Network Loading Ratio (CPNLR)

Using the notations in Section 2.3.3.2, we can use \( L_- \) and \( L_{max} \) to define the \textit{Congestion-free Network Loading Ratio (CFNLR)} as follows:

\[ CFNLR^{\varphi} = \frac{L^{\varphi}}{L_{max}} \] (2-29)

This metric gives the proportion of the maximum network loading that is always free from congestion regardless of the generation dispatch, under DC power flow assumptions. Similarly, the ratio of \( L_{max} - L_- \) and \( L_{max} \) gives the \textit{Congestion-prone Network Loading Ratio (CPNLR)} as follows:

\[ CPNLR^{\varphi} = 1 - CFNLR^{\varphi} = \frac{L_{max} - L^{\varphi}}{L_{max}} \] (2-30)
This metric gives the proportion of maximum network loading that may result in congestion if the power injections are not adequately coordinated. Using the sets defined in Section 3.3, we use \( L_{\text{max}} \) to give a measure for Set \( \mathcal{F} \) and \( L_{\text{max}} - L_{\text{}} \) for Set \( \mathcal{J} \) in \( P\)-space.

A limitation of \( CPNLR \) is that it overestimates grid inadequacy while \( CFNLR \) underestimates adequacy. Since operating the grid beyond \( L_{\text{}} \) may or may not result in network congestion depending on the generation schedule, \( L_{\text{max}} - L_{\text{}} \) gives an inflated measure of Set \( \mathcal{J} \). As such, \( CPNLR \) also gives an inflated measure of grid inadequacy. Regardless, \( CPNLR \) is still useful in revealing the extent of grid neediness for careful monitoring and potential intervention. An ideal value of zero means that all loading levels can be served by any combination of generator dispatches without intervention.

### 2.3.4.3 Total Infeasible Line Loading Ratio (TILLR)

From the visualization in Figure 2-4, we can define the Infeasible Line Loading Margin (ILLM), Unconstrained Line Loading Range (ULLR), and Infeasible Line Loading Ratio (ILLR) for each line as follows:

\[
ILLM_{ab}^g = f_{\text{max},u,ab}^g - f_{\text{max},f,ab}^g + f_{\text{max},f,ba}^g - f_{\text{max},u,ba}^g \quad (2-31)
\]

\[
ULLR_{ab}^g = f_{\text{max},u,ab}^g - f_{\text{max},u,ba}^g \quad (2-32)
\]

\[
ILLR_{ab}^g = \frac{ILLM_{ab}^g}{ULLR_{ab}^g} \quad (2-33)
\]

For a given line \( ab \), \( ILLM_{ab} \) gives the difference between the maximum unconstrained line flow and feasible line flow, added for both power flow directions, \( ULLR_{ab} \) gives the difference between the maximum unconstrained line flows, and \( ILLR_{ab} \) gives the ratio of \( ILLM_{ab} \) and \( ULLR_{ab} \).
We can then define the *Total Infeasible Line Loading Ratio (TILLR)* as a measure of overall grid inadequacy. It uses the *ILLM* and *ULLR* of all the lines to compute a metric like *ILLR* for the whole system as follows:

\[
TILLR_{a,b}^\varrho = \frac{\sum_{a \in E} ILLM_{a,b}^\varrho}{\sum_{a \in E} ULLR_{a,b}^\varrho} \quad (2-34)
\]

Using the sets defined in Section 2.3.3.2, we use the total *ULLR* as a measure of Set \( \mathcal{F} \) and the total *ILLM* as a measure of Set \( \mathcal{I} \). As with *CPNLR*, this representation is not exact because of the limitations in projecting in lower-dimensional space and the redundancy in accounting for the projected spaces. Nevertheless, *TILLR* gives a measure of grid inadequacy that can be useful indicators of the extent of network inadequacy.

The individual *ILLRs* give the proportion of the infeasible loading range relative to the total unconstrained loading range of a line. This can be used as an indicator of line adequacy if the value is zero or an indicator for network bottleneck potential otherwise. While a similar physical interpretation cannot be made for *TILLR*, it can be interpreted as the extent of additional power injection diversity that the grid can accommodate only if there are no line capacity constraints. As with *CPNLR*, the ideal value of *TILLR* is zero. If this is the case, no lines in the network will ever get congested even without intervention regardless of the power injection schedule, provided that the injections remain within limits. As with *CPNLR*, the insights provided by *TILLR* are limited by the model assumptions chosen and must be caveated accordingly.

Illustrative examples in the next section showcase the utility of the proposed framework and metrics in revealing inherent grid inadequacy and bottlenecks.
2.4 Illustrative Examples

2.4.1 6-bus test system

The relevant data used for the 6-bus test system is available in [111]. Of the six buses, three have generators and five have loads. There are a total of thirteen installed lines. To construct $\mathcal{S}$ for SPIR evaluation, we use a combinatoric sampling approach since it is tractable for this test system. We sample $T = 3,139,831$ power injection scenarios that represent eleven negative power injection levels at 0%, 10%, …, 100% of maximum and eleven positive power injection levels at 0%, 5%, 15%, …, 95% of maximum, add or subtract up to 5% to balance out the demand. The scenarios represent power injections from traditional loads and generators as well as power injections from renewables, dynamic loads, and energy storage. We specifically chose not to exceed the 100% threshold to illustrate the potential for network congestion even if the power injections remain within limits.

Figure 2-5 visualizes the distribution of the six-dimensional power injection scenario pool in reduced 3D space using principal component analysis (PCA) [129]. The axes are the top three principal components. Unlike the three-dimensional spaces presented as example in Section 2.3.3.1, points corresponding to the scenarios with and without congestion are interspersed with one another, making it difficult to gauge the relative sizes of the infeasible and feasible sets. This challenge reinforces the utility of the proposed inadequacy metrics in quantifying the relative sizes of these sets.
SPIR is equal to 33.1% implying that a third of the sampled scenarios resulted in network congestion. The infeasible scenarios can be analyzed further to determine the family of power injection scenarios that congest the network, explore the likelihood of such scenarios, and then devise appropriate preventive or corrective interventions. CPNLR is equal to 82.1% with $L_\text{min} = 135.7$ MW and $L_\text{max} = L_+ = 760$ MW. This means that operating the grid above $L = 135.7$ MW has potential for congestion. In a way, the grid is inherently “needy” since a large proportion of network loading requires careful monitoring and potential operational intervention. Alternatively, without adequate interventions, the grid is also inherently selective or discriminative from a market perspective because it discriminates against some schedules in serving some loading levels. Despite the inherent selectivity, there is enough capabilities to serve the maximum load through at least one generation dispatch since $L_\text{max} = L_+$. 

Finally, TILLR is equal to 27.1%. Figure 2-6 shows how each line contributes to this value. The diagram shows that only Line 2-4 will never become overloaded regardless of the power injection distribution with an ILLR of 0. The diagram also identifies lines that limit a grid’s ability
to accommodate diverse power injection states wherein \( f_{\text{max}, f, ab} = C_{ab} < f_{\text{max}, u, ab} \) or \( f_{\text{max}, f, ba} = C_{ab} < f_{\text{max}, u, ba} \) as in Lines 1-5, 2-3, 2-6, 3-5, and 4-6. This can serve as basis for later analyses on what kinds of power injection distributions are affected by grid bottlenecks and what remedial action can be made.

![Figure 2-6: Line loading range diagram, 6-bus grid.](image)

### 2.4.2 118-bus test system

We perform a similar analysis using the 118-bus test system using the data in [130]. The grid has 118 buses and 186 lines, with generation at 54 buses and loads at 91 buses. Since defining combinatoric discrete load and generation levels to define the scenario pool is computationally prohibitive, we sample from the line loading space instead to represent low, intermediate, and worst-case loading scenarios. We define one hundred loading levels for each line by dividing each line’s unconstrained loading range into one hundred points. We then solve for the power injection scenarios that result in each line loading level \( F_{ab} \) at minimum network loading as follows:
Objective \quad \min L \quad (2-35)

Subject to \quad f_{ab} = F_{ab} \quad (2-36)

Eqs. (2-2)-(2-5) \quad (2-37)

This process yielded $T = 18,600$ scenarios that represent a spectrum of network loading conditions.

Around forty percent of the sampled scenarios result in congestion with $SPIR = 39.2\%$. $CPNLR$ is equal to $94\%$ with $L_- = 222.9$ MW and $L_{\text{max}} = L_+ = 3,733$ MW. A low value of $L_-$ relative to $L_{\text{max}}$ indicates that congestion is possible even at very low loading levels if there is no coordinated generation scheduling. This is indicative of the grid’s inherent selectivity in choosing generator schedules unless power flow control interventions are available. Since $L = L_+$, there should be at least one feasible generation schedule able to meet the maximum demand. However, this schedule may not necessarily correspond to the winning least cost bid or the most renewable energy utilization rate. $TILLR$ is equal to $31.5\%$. Figure 2-7 shows the top ten lines with the largest $ILLMs$. These results provide indicators of inherent grid bottlenecks wherein $f_{\text{max},f,ab} = C_{ab} < f_{\text{max},u,ab}$ or $f_{\text{max},f,ba} = C_{ab} < f_{\text{max},u,ba}$ such as Lines 8-30, 77-82, 23-24, 69-77, 30-38, and 65-68 that merit further exploration and interventions if necessary.
Figure 2-7: Line loading range diagram showing Top 10 largest ILLM lines, 118-bus grid.

2.5 Discussion

The inadequacy assessment framework and metrics developed here provide indicators of inherent grid inadequacy, vulnerabilities, and bottlenecks to accommodate high power injection diversity without intervention. It complements other inadequacy assessment frameworks and metrics that allow generation rescheduling or load curtailment such as [41], [42] or specifically model contingencies such as [113]. Future work can look at ways to incorporate power flow control interventions in the proposed framework. Contingency modeling may also be incorporated, but accommodating diverse scenarios may not be as important during contingencies compared to securing supply continuity.

Inadequacy assessment using scenario-based metrics such as SPIR are very flexible and can reflect use-case requirements by changing the power system model and the system state selection process. As such, it can easily find potential applications in inadequacy assessment in either operations planning or expansion planning provided that appropriate assumptions are used.
The other two metrics, *CPNLR* and *TILLR*, are more specialized because these metrics test against all possible power injection scenarios which may be impractical in some applications. In the operations setting for example, hour-ahead and day-ahead system states are much more predictable. As such, the level of variability of future states against which inadequacy must be assessed is much more limited. Inadequacy assessment against detailed distribution and correlation models, market simulation models, or tighter interval sets are more appropriate. Testing against potential blind spots is still necessary but not in the scale of exploring the whole space. On the other hand, power injections years or decades ahead in an expansion planning setting are much more uncertain and diverse. In this application, the assumptions underlying the *CPNLR* and *TILLR* metrics become viable preliminary options until more detailed data and simulation models become available. Even if detailed models become available, it will still be useful to scan the whole power injection space for potential vulnerabilities that *CPNLR* and *TILLR* can provide.

While the ideas presented in this chapter focus on transmission grid inadequacy, the idea of using the power flow infeasible set to characterize grid inadequacy for power injection diversity can also be extended to microgrids and distribution grids. However, appropriate power flow models must be used whenever the DC power flow assumptions do not apply like balanced operation, high X/R ratios, near unity voltage magnitudes, and constant-power models. It will be interesting future work to explore how to best model the infeasible set of small and large-scale distribution networks for grid inadequacy assessment for high power injection diversity, given the additional variables and non-linear models often needed to model such grids.

Future work can also look at the development of decision support tools that use the metrics to identify grid structures that are better suited to accommodate power injection diversity in future grids. In Chapters 4 and 5, we present one such approach derived from the *SPIR* metric.
2.6 Conclusion

In this chapter, we presented a new framework and set of metrics that measure inherent grid inadequacy to accommodate increasing levels of power injection diversity. The framework uses the size of the power flow infeasible set to quantify inherent grid inadequacy under power injection diversity without intervention. We present three approaches to give a measure to the high-dimensional sets involved: one uses a scenario-based approach while the other two use novel dimension reduction techniques. We present three inadequacy metrics that describe different but complementary grid inadequacy properties based on these approaches. Illustrative examples show that the metrics can be used as indicators of grid readiness to accommodate diverse operating states as well as indicators of grid vulnerabilities and bottlenecks.

The succeeding chapters build on the framework and metrics presented in this chapter to identify and assess meritorious expansion options that can enable future grids to host more diverse power injection conditions.
3 Inherent grid inadequacy and graph properties of expanded grids

Finding grid expansion options that directly reduce inherent grid inadequacy for power injection diversity, as we defined it in Section 2.3.1, is an emerging problem in TNEP research. The problem diverges from conventional models that minimize cost or maximize social welfare over a pre-defined power injection scenario or scenarios. Finding new tools and approaches to solve this problem may provide distinct grid expansion options that are better in some other measures but are overlooked by conventional methods.

One proposal seeks to optimize grid graph properties as an indirect method of optimizing inherent grid inadequacy. In [110], simple heuristics that improve a chosen grid graph property are proposed. This approach circumvents computationally intensive optimization models but at the expense of power system model fidelity. In this chapter, we explore whether this graph-based framework in finding grid expansion options that aim to reduce inherent grid inadequacy indeed identify solutions that reduce grid inadequacy for power injection diversity.

We structure this chapter as follows. Section 3.1 reviews and critiques the use of graph-based heuristics in power system studies in general and in reducing inherent grid inadequacy in particular. Section 3.2 presents our experimental framework. We first review the relevant grid inadequacy and graph metrics then describe the experiment we used to relate them. Section 3.3 presents the results for two standard test systems. Section 3.4 summarizes the observations and identifies potential use cases. Section 3.5 concludes.

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2 This chapter uses material from the following work: A. E. Tio, D. J. Hill, and J. Ma, “Can graph properties determine future grid adequacy for power injection diversity?,” Physica A, no. 124165, July 2020.
3.1 Graph-based heuristics in power system studies

Commonly proposed TNEP decision-making tools like robust [41], [42], [48], [53] and stochastic [86], [131] programming approaches run detailed power flow and economic models over a pool of identified operating states and over a combinatoric grid expansion solution space. These approaches use an optimization model to select the best lines to add to the grid at least cost, enabling it to accommodate the identified operating states without network overloading. However, the detailed models are computationally intensive to solve and can easily become intractable given the large combinatoric solution and scenario space. Complementary approaches that perform usefully and scale up to large networks need to be explored.

Complex Network Analysis (CNA) provides one such complementary approach in designing future grids. The idea is to identify graph properties of power grids that affect grid performance and then use these properties to inform further study and decision-making. Works already published show increasing interest in this field. Refs. [132] and [133] provide extensive reviews on graph-based indicators of power grid robustness and vulnerability covering hundreds of related works. Ref. [134] identifies applications in several power system assessment, control, and planning problems including micro-grid siting [135], network partitioning [136], network connectivity visualization [137], small-angle disturbance stability assessment [138], line outage and synchronism studies [139], [140], and transmission expansion planning [110], [141]–[144]. Other recent applications include ESS siting [145], topological anomaly detection and algorithm improvement [146], and expansion of distribution grids [147]. There are concerns, however, that the oversimplifying assumptions used in some of these approaches undermines their usefulness in actual power system operation and control [148]. The authors of [133] and [148] emphasize the need to bridge the communication gap between complexity and complex network communities and electrical
engineering communities to better understand the interrelationships between these two bodies of
knowledge.

This chapter tests the simple graph-based grid expansion heuristics in [110] and [141] against the grid adequacy assessment model in Chapter 2 to check whether the heuristics indeed identify grid expansion options that best reduce inherent grid inadequacy for power injection diversity. Refs. [110] and [141] use intrinsic measures of network robustness, derived from the graph properties of power grids, to design more adequate transmission networks. The hypothesis is that improving the intrinsic graph properties of power grids would translate to improved network performance over a wide range of operating states. Ref. [141] uses the condition number while Ref. [110] uses the determinant obtained from the weighted Laplacian matrix of the grid as intrinsic measures of network robustness. Of direct relevance to this thesis, Ref. [110] proposes that under certain assumptions, maximizing the determinant is equivalent to maximizing the ability of grids to absorb and deliver power, i.e. operate under more diverse operating states without line overloading.

The simplicity of the method proposed in [110] and the minimal data requirements is attractive given that the alternative method that uses scenario modeling under a combinatoric optimization model can easily become computationally intractable. Like other works on CNA in power systems however, the method abstracts away practical operational constraints that can potentially limit its usefulness. Some of the constraints overlooked are fundamental in determining the frequency, severity, and distribution of line overloading across the network including the following:

a. the spatial distribution of buses with positive and negative power injections,
b. the relative magnitude of power injections at each bus, and
c. the spatial distribution of low and high-capacity lines.
The line addition heuristic proposed in [110] also ignores the “lumpiness” of actual line addition that comes in discrete increments or “lumps”. That is, the heuristic assumes that the weight of added lines can be continuously varied, the added lines have sufficiently large capacity, and that any pair of buses can be connected. Practically however, standard line configurations limit possible edge weights and line capacities to a set of discrete values. Geographical constraints and the difficulty of acquiring new rights-of-way also make it impossible to directly connect specific bus pairs. Moreover, Ref. [110] failed to demonstrate whether optimizing the determinant indeed translates to optimized network adequacy using metrics that reflect actual grid operation.

The experiment and results that follow provide insight on the efficacy of using some graph-based heuristics as in [110] in finding grid expansion options that reduce inherent grid inadequacy for power injection diversity.

3.2 Experimental setup

3.2.1 Graph representation of power grids

A power grid can be represented by a weighted graph $\mathcal{G}$ with vertex set $\mathcal{V}$ of $n$ buses and edge set $\mathcal{E}$ of $\ell$ transmission lines connecting the buses. The edge weights $w_{ij}$ between buses $i$ and $j$ are the line susceptances. The line susceptances are physical line parameters that determine the distribution of power flows across the network given the spatial distribution of power injections in the buses. Other weighting options are available such as the line capacities or line power flows but these will not be explored in this thesis.

The line-to-bus incidence matrix $K$, weighted adjacency matrix $A$, and network Laplacian $L$ are matrices that represent network connectivity and couplings where $K$ is $\ell \times n$ while $A$ and $L$
are $n \times n$ square symmetric. Matrix $K$ is obtained by first assuming an arbitrary direction for each line then defining its elements as follows:

$$K_{xi} = \begin{cases} 1 & \text{if line } x \text{ starts at bus } i \\ -1 & \text{if line } x \text{ ends at bus } i \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (3-1)

The elements of $A$ are defined as follows:

$$A_{ij} = \begin{cases} w_{ij} & \text{if } ij \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (3-2)

The elements of $L$ are given by $D - A$, where $D$ is the diagonal matrix of weighted node degrees. That is,

$$L_{ij} = \begin{cases} d_i & \text{if } i = j \\ -w_{ij} & \text{if } i \neq j \text{ and } ij \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (3-3)

where $d_i = \sum_j w_{ij}$ is the weighted node degree of $i$. Since the line susceptances are used as edge weights to construct $L$, the network Laplacian is also the DC power flow susceptance matrix $B$ that we defined in Section 2.3.3.1. Many studies cited in [132]-[135] use these matrices to develop heuristics for assessing and reinforcing existing power grids.

### 3.2.2 Graph metrics

From Section 2.3.3.1, it is evident from the vector mapping function from $P$-space to $f$-space (2-12) that matrix $B$, being the network Laplacian, plays a central role in determining the distribution of line power flows in power grids. As such, some of its properties can potentially inform the design of future grids. The following discussion reviews four graph metrics derived
from the network Laplacian and how these metrics have been associated with power system operations and planning problems.

### 3.2.2.1 Algebraic connectivity

The Laplacian spectrum is the set of eigenvalues of $L$ given by $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$, with the corresponding eigenvectors $v_k$, $k = 1, 2, \ldots, n$. The second smallest eigenvalue, $\lambda_2$, is known as the algebraic connectivity or Fiedler value, with its corresponding eigenvector known as the Fiedler vector. The algebraic connectivity increases monotonically with added edges or increased edge-weights, and hence, provides a rough measure for network connectivity [149]. A well-connected graph has a large $\lambda_2$ and can potentially accommodate a more diverse power injection pool. Ref. [150] reviews how networks with large algebraic connectivity have good synchronizability and information distribution properties, albeit in a non-power network application. This metric is related to the other three graph metrics that we discuss next.

### 3.2.2.2 Condition number

The condition number, $\kappa$, is the ratio of the largest and the smallest non-zero eigenvalue of $L$ with $\kappa = \lambda_n / \lambda_2$. If $L$ has a low condition number, it is said to be well-conditioned [141]. Since $\lambda_2 \leq \lambda_n$, the smallest possible value of $\kappa$ is one when $\lambda_2 = \lambda_n$. In the context of power flow analysis, following Equation (2-8), a well-conditioned network Laplacian means that small changes in power injections result in small changes in the bus angles. This property may affect grid adequacy as it is effectively related to line flow sensitivity. Ref. [141] makes a case for using $\kappa$ as an intrinsic measure of network robustness.
3.2.2.3 **Row-reduced Laplacian determinant**

The number of spanning trees in an unweighted graph is given by the determinant of its row-reduced Laplacian obtained by removing any one row and its corresponding column [151]. It is also equal to the product of the non-zero Laplacian eigenvalues divided by \( n \) [152]. Using (2-8) and the graphical interpretation of the matrix determinant, Ref. [110] argues that maximizing the determinant of the row-reduced weighted Laplacian matrix of a power grid, \(|L_R|\), is equivalent to maximizing the volume of feasible power injections, assuming that the added lines have sufficiently large capacity.

3.2.2.4 **Effective resistance**

The effective resistance between buses \( a \) and \( b \) gives the equivalent resistance across \( ab \) if the graph is taken as a resistive circuit. It can be obtained using techniques from circuit theory or from the elements of the generalized inverse of the network Laplacian [143]. Low values indicate strong connectivity and/or the existence of many parallel paths between \( ab \). The total effective resistance, \( R_G \), can be obtained by summing the pairwise effective resistances and can be used as a measure for total network connectivity. It is related to the Laplacian eigenvalues as follows [143]:

\[
R_G = \sum_{i=1}^{n} \sum_{i+1}^{n} R_{ij} = n \sum_{i=2}^{n} \frac{1}{\lambda_i}
\]  

(3-4)

A low value of \( R_G \) implies strong overall graph connectivity and/or the existence of multiple alternative paths between node-pairs. Ref. [143] links a low value of \( R_G \) to a more homogenous distribution of power flows that helps increase grid robustness to line outages. Ref. [144] links the minimization of \( R_G \) to robustness of power grids to disturbances. This metric could also be an indicator of grid robustness to power injection diversity.
It is worth noting that most work that use these four graph metrics in the context of network expansion or optimal topologies do so to achieve robustness against outages as in [143] or to achieve good stability and synchronizability properties as in [144], [150]. Literature review for this chapter reveals that Ref. [110] remains the only work that tries to link graph metrics with power grid adequacy to accommodate diverse operating states.

3.2.3 Methodology

From Section 2.3.3.1, Eq. (2-12) showed that the network connectivity, the edge weights, and the spatial distribution of power injections dictate which lines get more power flows than others while Eq. (2-6) showed that the line capacity determines whether a line gets overloaded or not. Section 2.3 showed how grid inadequacy metrics proposed in Chapter 2 reflect these operational details while Section 3.2.2 showed how the graph metrics focus on the properties of $B$ alone. In this section, we outline the experiment we used to determine the following: (a) whether grids with the best grid inadequacy metrics have distinctive graph properties and (b) whether the four graph properties can be used to predict grid inadequacy despite the simplified assumptions.

We use two well-known standard test systems for the experiment to represent small- and medium-sized power systems, namely the 6-bus and 118-bus test systems with the relevant data available in [111] and [130] respectively. We augment the original grids with new lines taken from a pre-defined candidate line list. To check whether optimizing graph metrics also optimize grid adequacy, we perform two major steps in the experiment: (1) enumerate all possible combinations of line additions to define a population of augmented grids for a given number of line additions and then (2) compute, compare, and rank the inadequacy and graph metrics for each augmented grid in the population. We normalize the graph metric values by dividing with the population maximum for presentation purposes as needed. The small size of the 6-bus test system allowed us to
define a densely-sampled scenario pool for computing SPIR as well as to enumerate all possible line additions for a larger number of lines added. Tests on the 118-bus test system supports the observations on the 6-bus test system for a larger grid but for a smaller scenario pool for computing SPIR and for a smaller number of line additions.

We are particularly interested in the extreme metric values to see whether a grid that is high-ranking in terms of one graph metric is also high-ranking in terms of one inadequacy metric and vice versa. To explore this relationship, we employ two complementary analysis tools as follows:

1. *Analysis using scatter plots.* We use scatter plots to visualize the relationship between one graph metric and one inadequacy metric, using data from the augmented grids. We look at the extreme values and check whether extremes in one graph metric correspond to extremes in one inadequacy metric, and vice versa.

2. *Analysis by comparing ranks.* This method is a quantitative equivalent of the visual check used in the first method. We rank the augmented grids according to each of the seven metrics using the ranking convention in Table 3-1. The grid inadequacy metrics, $\kappa$, and $R_G$ are ranked from smallest to largest while $\lambda_2$ and $|L_R|$ are ranked from largest to smallest. We identify the Top-1 and Top-10 grid configurations in terms of inadequacy metrics to see whether these grids are also top-ranking in terms of the graph metrics. Likewise, we identify the Top-1 and Top-10 grid configurations in terms of the graph metrics to see whether these grids are also top-ranking in terms of the inadequacy metrics.
### Table 3-1: Ranking convention for the seven metrics used

<table>
<thead>
<tr>
<th>Metric</th>
<th>Ranking Convention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph Metrics</td>
<td></td>
</tr>
<tr>
<td>Algebraic connectivity, $\lambda_2$</td>
<td>Population maximum is rank 1.</td>
</tr>
<tr>
<td>Condition number, $\kappa$</td>
<td>Population minimum is rank 1.</td>
</tr>
<tr>
<td>Row-reduced Laplacian determinant, $</td>
<td>L_{\alpha}</td>
</tr>
<tr>
<td>Total effective resistance, $R_G$</td>
<td>Population minimum is rank 1.</td>
</tr>
<tr>
<td>Grid Inadequacy Metrics</td>
<td></td>
</tr>
<tr>
<td>Scenario Pool Infeasibility Ratio, SPIR</td>
<td>Population minimum is rank 1.</td>
</tr>
<tr>
<td>Congestion-prone Network Loading Ratio, CPNLR</td>
<td>Population minimum is rank 1.</td>
</tr>
<tr>
<td>Total Infeasible Line Loading Ratio, TILLR</td>
<td>Population minimum is rank 1.</td>
</tr>
</tbody>
</table>

If graph-based grid expansion, such as those proposed in [110], is really effective in identifying grid designs that best improve grid adequacy, then we must be able to observe the following. In the scatter plots, extreme points in terms of a graph metric should correspond to extreme points in terms of a grid inadequacy metric, and vice versa. In the comparative ranking analysis, grid designs that are optimal (Top-1) in terms of a graph metric should also be optimal (Top-1) in terms of a grid inadequacy metric, and vice versa. Furthermore, Top-10 grid designs in terms of graph metrics should capture high-ranking designs in terms of grid inadequacy metrics, and vice versa.

### 3.3 Results and discussion

#### 3.3.1 6-bus test system

The 6-bus test system has one bus which needs to be connected to the other five buses. Six lines connect the five buses. Fifteen possible lines that can interconnect any given pair of buses comprise the candidate line list. Three buses have generators while five have loads. To generate the scenario pool for SPIR evaluation, we define eleven negative power injection levels from 0%,
10%, 20%, to 100% and eleven positive power injection levels from 0%, 5%, 15%, to 95% per relevant bus to represent power injections from traditional or renewable generation, traditional or dynamic loads, or storage systems. We generate all power injection combinations and vary the positive power injection setpoints by up to ±5% to match the negative power injections. Only the combinations that satisfy the power balance constraint are included in the scenario pool for a total of 3,139,831 power injection scenarios. These parameters are chosen for a reasonable balance between sample fidelity of the power injection space and computational burden in computing SPIR. We note that CPNLR and TILLR do not depend on the scenario pool. All possible one-, two-, three-, and four-line additions that connect all six buses are then taken from the candidate line list for a total of 5, 65, 460, and 2,345 grid configurations respectively.

3.3.1.1 Pairwise metric trends

Figure 3-1 shows the scatter plots between the different graph and grid inadequacy metrics for the 6-bus test system. The black square and diamond dots identify the Top-1 grids for four lines added in terms of a given grid inadequacy and graph metric respectively. Results show that the points corresponding to the lowest values of SPIR and CPNLR do not necessarily correspond to extreme values of graph metrics. The lowest points for TILLR, on the other hand, corresponds better to extreme levels of the graph metric values but the most extreme point in terms of TILLR does not necessarily correspond to the most extreme point in terms of any of the four graph metrics.
Consequently, the largest points for $|L_R|$ and $\lambda_2$ and the lowest points for $\kappa$ and $R_G$ lie within the range of SPIR and CPNLR. This implies that optimizing the grid for $|L_R|$, $\lambda_2$, $\kappa$ and $R_G$ cannot guarantee optimal grid adequacy and can result in getting grids with average to poor SPIR and CPNLR values. On the other hand, the largest points for $|L_R|$ and $\lambda_2$ and the lowest points for $\kappa$ and $R_G$ have better TILLR values close to the population minimum, even if not the best. This means that while optimizing for $|L_R|$, $\lambda_2$, $\kappa$ and $R_G$ may not result in optimal TILLR, good values of TILLR can still be achieved.

**3.3.1.2 Graph characteristics of grids with top-ranking inadequacy metrics**

Table 3-2 identifies the Top-1 grid designs in terms of grid inadequacy metrics and the ranking of its corresponding set of graph metrics. The table also shows how much the graph metric values deviate from the Top-1 graph metric values. Results show that line additions that yield the
optimal SPIR and CPNLR values do not necessarily have optimal graph properties. The graph metrics of the Top-1 SPIR designs rank between 74–408 out of 460 for three lines added and between 320–815 out of 2345 for four lines added. Similarly, the graph metrics of the Top-1 CPNLR designs rank between 40–89 out of 460 for three lines added and between 756–1233 out of 2345 for four lines added. The graph metric values deviate from the best values by 20.1% for $R_G$ to as much as 360.1% for $\kappa$. Similarly, the graph metrics of the Top-1 TILLR line additions are ranked between 64-131 out of 460 for three lines added and 457-706 out of 2345 for four lines added.

Table 3-3 extends Table 3-2 for the Top-10 designs in terms of grid inadequacy metrics. While this range captures some of the Top-1 designs in terms of graph metrics, it also captures grid designs with average- to low-ranking graph metric values. For example, grids in the Top 10 SPIR and CPNLR list have graph metrics ranked up to 191–408 out of 460 for three lines added and up to 320–1,820 out of 2,345 for four lines added. Even grids in the Top 10 TILLR list can have graph metric rankings of up to 143 out of 460 for three lines added and up to 1,294 out of 2345 for four lines added.

Results from Table 3-2 show that a grid does not need to have optimal graph properties to have optimal SPIR, CPNLR, or TILLR values while results from Table 3-3 illustrates that grids can be poorly ranked in terms of graph metrics and still belong to the Top-10 list in terms of grid inadequacy metrics.
Table 3-2: Graph characteristics of the Top-1 grid designs in terms of grid inadequacy metrics, 6-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>Ranking</th>
<th>% Difference from Best</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\lambda_2$</td>
<td>$\kappa$</td>
</tr>
<tr>
<td>Top 1 SPIR</td>
<td>2-3, 3-5, 4-6, -</td>
<td>329</td>
<td>408</td>
</tr>
<tr>
<td></td>
<td>2-6, 2-6, 2-6, 3-5</td>
<td>503</td>
<td>815</td>
</tr>
<tr>
<td>Top 1 CPNLR</td>
<td>1-3, 2-6, 3-6, -</td>
<td>56</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1-3, 2-5, 3-6, 3-6</td>
<td>1,003</td>
<td>756</td>
</tr>
<tr>
<td>Top 1 TILLR</td>
<td>2-6, 3-6, 4-6, -</td>
<td>64</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>2-6, 3-6, 3-6, 4-6</td>
<td>463</td>
<td>457</td>
</tr>
</tbody>
</table>

Table 3-3: Graph characteristics of the Top-10 grid designs in terms of grid inadequacy metrics, 6-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>Ranking Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_2$</td>
<td>$\kappa$</td>
</tr>
<tr>
<td>Top 10 SPIR</td>
<td>3</td>
<td>34-329</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>390-1,587</td>
</tr>
<tr>
<td>Top 10 CPNLR</td>
<td>3</td>
<td>28-260</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>416-1,126</td>
</tr>
<tr>
<td>Top 10 TILLR</td>
<td>3</td>
<td>1-74</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>10-610</td>
</tr>
</tbody>
</table>

3.3.1.3 Grid inadequacy characteristics of grids with top-ranking graph metrics

Table 3-4 shows the Top-1 grid designs in terms of graph metrics and the ranking of its corresponding set of inadequacy metrics. Results show that grids with the top-ranking graph metrics do not necessarily have top-ranking SPIR and CPNLR values. In terms of SPIR, Top-1 grids
in terms of the graph metrics rank from 9–44 out of 460 and deviates by 4.2-10.1% in absolute terms from the best observed SPIR value for three lines added and ranks 216–390 out of 2,345 with a deviation range of 19.1-22.9% for four lines added. Similarly for CPNLR, Top-1 grids in terms of graph metrics rank 26–88 out of 460 with a deviation range of 1.3-1.7% in absolute terms for three lines added and ranks 296–377 out of 2,345 with a deviation range of 3.6-3.8% for four lines added. Top-1 grids in terms of graph metrics have TILLR values ranked 4–41 out of 460 with a deviation range of 1.0-4.9% for three lines added and 11–469 out of 2,345 with a deviation range of 1.1-8.5% for four lines added.

Table 3–5 extends Table 3–4 for the Top-10 designs in terms of the graph metrics. The Top-10 graphs in in terms of $\lambda_2$, $\kappa$, $R_G$, and $|L_R|$ do not necessarily include grids with only the top-ranking SPIR, CPNLR, and TILLR values. This set also fail to include the grid with the Top 1 grid inadequacy metric values. Furthermore, the set includes grids that are average-ranking in terms of SPIR at 111–234 out of 460 for three lines added and 426–866 out of 2,345 for four lines added. For CPNLR, the set include grids that are average-ranking at 131–179 for three lines added and 566–966 for four lines added. Results are better for TILLR with the set including grids that are average-ranking in terms of TILLR at 87–127 for three lines added and 276–469 for four lines added – ranking ranges that are lower than the ones observed for SPIR and CPNLR.

Observations from Table 3–4 show that grids with the optimal graph metrics do not necessarily have optimal inadequacy metrics. Likewise, observations from Table 3–5 show that grids that are high-ranking in terms of graph metrics are not necessarily high-ranking in terms of grid inadequacy metrics.

The observations in Sections 3.3.1.2 and 3.3.1.3 reinforce the observations in Section 3.3.1.1 from the scatter plots that: (1) grids that have poor or average graph metrics can still have
optimal inadequacy metrics and (2) grids that have the optimal graph metrics do not necessarily have optimal inadequacy metrics. Despite these general observations, results from Table 3-5 show that grids that are high-ranking in terms of $\lambda_2$, $\kappa$, $R_G$ can be high ranking as well in terms of $TILLR$. This suggests that optimizing for these graph metrics can give grids with good $TILLR$ values even if not optimal and can find some limited practical applications.

The question arises however, whether these observations arises due to the small size of the system. We present results for a larger grid to see which of these observations still hold.

Table 3-4: Grid inadequacy characteristics of Top-1 grid designs in terms of graph metrics, 6-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>Ranking</th>
<th>Absolute Difference from Best</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SPIR</td>
<td>CPNLR</td>
</tr>
<tr>
<td>Top 1 $\lambda_2$</td>
<td>3-6, 4-6, 5-6, -</td>
<td>44</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>3-6, 4-6, 5-6, 5-6</td>
<td>262</td>
<td>377</td>
</tr>
<tr>
<td>Top 1 $\kappa$</td>
<td>3-6, 4-6, 5-6, -</td>
<td>44</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>3-6, 4-6, 5-6, 5-6</td>
<td>262</td>
<td>377</td>
</tr>
<tr>
<td>Top 1 $R_G$</td>
<td>2-6, 4-6, 5-6, -</td>
<td>20</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>2-6, 3-6, 4-5, 4-6</td>
<td>390</td>
<td>296</td>
</tr>
<tr>
<td>Top 1 $</td>
<td>L_d</td>
<td>$</td>
<td>2-3, 2-6, 4-6, -</td>
</tr>
<tr>
<td></td>
<td>1-5, 2-6, 3-5, 4-6</td>
<td>216</td>
<td>340</td>
</tr>
</tbody>
</table>
Table 3-5: Grid inadequacy characteristics of the Top-10 grid designs in terms of graph metrics, 6-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>SPIR</th>
<th>CPNLR</th>
<th>TILLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10 $z_2$</td>
<td>3</td>
<td>20-235</td>
<td>26-141</td>
<td>4-97</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>86-866</td>
<td>182-566</td>
<td>6-276</td>
</tr>
<tr>
<td>Top 10 $k$</td>
<td>3</td>
<td>20-194</td>
<td>26-131</td>
<td>4-87</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>86-866</td>
<td>182-566</td>
<td>6-276</td>
</tr>
<tr>
<td>Top 10 $R_G$</td>
<td>3</td>
<td>16-132</td>
<td>3-179</td>
<td>4-91</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>86-488</td>
<td>32-865</td>
<td>6-297</td>
</tr>
<tr>
<td>Top 10 $</td>
<td>L_R</td>
<td>$</td>
<td>3</td>
<td>6-111</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4-426</td>
<td>340-966</td>
<td>9-469</td>
</tr>
</tbody>
</table>

### 3.3.2 118-bus test system

The 118-bus test system represents a portion of the U.S. Midwest Interconnect System in 1962 [153]. It has been and is still being used in many power system and network studies in the literature. The test system has 118 buses interconnected by 186 lines. Fifty-four buses have generators and ninety-one have loads. We use the set of existing lines as the candidate line list. We use the method in Section 2.4.2 to construct the scenario pool for evaluating $SPIR$ as follows. We define ten loading levels per line by dividing its unconstrained loading range into equal parts. Then we compute the power injection vector that results in the defined line loading levels at minimum total network loading. This process yields 1,860 power injection scenarios that comprise the scenario pool. We generate all possible one- and two-line additions taken from the candidate line list for a total of 186 and 17,392 grid configurations respectively.
3.3.2.1 Pairwise metric trends

Figure 3-2 shows the resulting scatter plots between the graph and grid inadequacy metrics for one- and two-line additions for the 118-bus test system. The black square and diamond marks indicate Top-1 grid designs in terms of a given grid inadequacy and graph metric respectively. Because the grid is much larger relative to the number of lines added, the observed range of the graph and grid inadequacy metrics is more limited compared to that observed for the 6-bus system. Clusters and extreme points are also more apparent, as is the case in some scatter plots in Figure 3-1 for one- and two-lines added.

Like the observations in Section 3.3.1.1, points corresponding to the lowest SPIR, CPNLR, and TILLR values do not necessarily correspond to optimal graph metric values. Some of these points are scattered within the range of graph metric values. This implies that grids with optimal SPIR, CPNLR, and TILLR can have non-optimal graph properties. Consequently, points corresponding to the optimal graph metric values do not necessarily correspond to points that have optimal grid inadequacy metric values. This is especially true for SPIR and CPNLR where some points corresponding to the optimal graph metrics have high SPIR and CPNLR. A closer look at the Top-1 and Top-10 performing grids in the next two sections verifies these visual observations.
3.3.2.2 Graph characteristics of grids with top-ranking inadequacy metrics

Table 3-6 lists the one- and two-line additions that yield the Top-1 grid inadequacy metrics. As observed for the 6-bus case study, results show that grids can have optimal inadequacy metrics even if the graph metrics are non-optimal. Table 3-7 shows the Top-10 grids in terms of grid inadequacy metric values and their corresponding ranking in terms of graph metrics. Consistent with the observations from the 6-bus case study, the Top-10 designs in terms of inadequacy metrics are not necessarily high-ranking in terms of graph metrics. In fact, some grids that are average- or poorly-ranked in terms of graph metrics made it to the Top-10 with ranks of up to 35–185 out of 186 for three lines added and 353–17,204 out of 17,392 for four lines added. As with the observations for the 6-bus case study, these results support the observation that grids can have non-optimal graph metrics and still be high-ranking in terms of SPIR, CPNL, or TILLR.
Table 3-6: Graph characteristics of the Top-1 grid designs in terms of grid inadequacy metrics, 118-bus grid

| Case Name | Lines Added | λ₂ | κ | R₆ | |L₆| | λ₂ | κ | R₆ | |L₆| |
|-----------|-------------|----|---|----|---|---|----|---|----|---|---|---|
| Top 1 SPIR | 30-38, -   | 4  | 4 | 2  | 64 | 5.9% | 6.3% | 1.7% | 12.3% |
|           | 8-30, 30-38 | 242 | 231 | 191 | 2,828 | 8.9% | 9.7% | 2.5% | 20.3% |
| Top 1 CPNLR | 80-99, - | 7  | 7 | 6  | 141 | 6.7% | 7.2% | 2.3% | 27.2% |
|           | 8-30, 99-100 | 2,025 | 1,638 | 1,087 | 5,105 | 11.3% | 12.6% | 3.7% | 25.9% |
| Top 1 TILLR | 30-38, - | 4  | 4 | 2  | 64 | 5.9% | 6.3% | 1.7% | 12.3% |
|           | 30-38, 65-68 | 205 | 16,856 | 199 | 12,902 | 8.0% | 19.9% | 2.7% | 43.1% |

Table 3-7: Graph characteristics of the Top-10 grid designs in terms of grid inadequacy metrics, 118-bus grid

| Case Name  | Lines Added | λ₂ | κ | R₆ | |L₆| |
|------------|-------------|----|---|----|---|---|
| Top 10 SPIR | 1           | 2-71 | 2-185 | 2-63 | 14-184 |
|            | 2           | 192-657 | 189-16,856 | 189-318 | 1,735-14,584 |
| Top 10 CPNLR | 1          | 5-44 | 5-42 | 5-65 | 54-173 |
|            | 2           | 644-4,107 | 618-3,696 | 259-5,064 | 3,354-16,948 |
| Top 10 TILLR | 1          | 1-35 | 1-185 | 1-53 | 14-184 |
|            | 2           | 3-353 | 3-17,019 | 1-431 | 1,195-14,584 |

3.3.2.3 Grid inadequacy characteristics of grids with top-ranking graph metrics

Table 3-8 shows the grid inadequacy metric rankings of the Top-1 grids in terms of graph metrics. Results are consistent with the observations for the 6-bus test system, that is, grids with the optimal graph metrics are not necessarily optimal in terms of grid inadequacy metrics. Furthermore, relative to CPNLR and SPIR, results for TILLR are better. That is, while Top-1 grids in terms
of $\lambda_2$, $\kappa$, and $R_G$ may not have optimal $TILLR$, these grids can still be high-ranking in terms of $TILLR$ and have values near the minimum.

Table 3-9 extends Table 3-8 for the Top-10 grids in terms of graph metrics. As with observations for the 6-bus test system, the designs in this set can have poor rankings in terms of grid inadequacy metrics, especially for SPIR and CPNLR. And while some of the Top-10 grids in terms of $\lambda_2$, $\kappa$, and $R_G$ are highly-ranked in terms of $TILLR$, the results show that optimizing for $\lambda_2$, $\kappa$, and $R_G$ cannot guarantee optimality in terms of $TILLR$ and may give grids that have poorer adequacy relative to other grids.

Results for the 118-bus test system supports the observations for the 6-bus test system. That is, (a) grids do not need to have distinctive graph properties to have good adequacy and (b) grids with distinctive graph properties do not necessarily have good adequacy. However, while grids that are Top-1 in terms of $\lambda_2$, $\kappa$, and $R_G$ are not necessarily Top-1 in terms of $TILLR$, these grids can still be highly ranked in terms of $TILLR$ and may find some limited practical use.
Table 3-8: Grid inadequacy characteristics of the Top-1 grid designs in terms of graph metrics, 118-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>Ranking</th>
<th>Absolute Difference from Best</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SPIR</td>
<td>CPNLR</td>
</tr>
<tr>
<td>Top 1 λ₂</td>
<td>38-65, -</td>
<td>186</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>38-65, 81-80</td>
<td>17,231</td>
<td>5,498</td>
</tr>
<tr>
<td>Top 1 κ</td>
<td>38-65, -</td>
<td>186</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>38-65, 81-80</td>
<td>17,231</td>
<td>5,498</td>
</tr>
<tr>
<td>Top 1 R₀</td>
<td>38-65, -</td>
<td>186</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>30-38, 38-65</td>
<td>186</td>
<td>8,494</td>
</tr>
<tr>
<td>Top 1</td>
<td>Lₐ</td>
<td>85-86, -</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>85-86, 85-86</td>
<td>10,301</td>
<td>17,139</td>
</tr>
</tbody>
</table>

Table 3-9: Grid inadequacy characteristics of the Top-10 grid designs in terms of graph metrics, 118-bus grid

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Lines Added</th>
<th>Ranking Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SPIR</td>
</tr>
<tr>
<td>Top 10 λ₂</td>
<td>1</td>
<td>1-186</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>186-17,391</td>
</tr>
<tr>
<td>Top 10 κ</td>
<td>1</td>
<td>1-186</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>186-17,391</td>
</tr>
<tr>
<td>Top 10 R₀</td>
<td>1</td>
<td>1-186</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>186-17,391</td>
</tr>
<tr>
<td>Top 10</td>
<td>Lₐ</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3,258-14,160</td>
</tr>
</tbody>
</table>
3.4 Summary and discussions

Experimental results illustrate the following:

a. Grids do not need to have optimal (Top-1) graph metrics to have optimal (Top-1) inadequacy metrics. Furthermore, grids that are poorly-ranked in terms of graph metrics can still be optimal in terms of inadequacy metrics.

b. Grids with the optimal graph metrics do not necessarily have optimal grid inadequacy metrics. Some designs in the Top-10 in terms of graph metrics can still be poorly ranked in terms of inadequacy metrics.

These observations are especially true for SPIR and CPNLR where results showed that grids that are average-ranking in terms of graph metrics can still be top-ranking in terms of SPIR and CPNLR and grids that are high-ranking in terms of graph metrics can be poorly ranked in terms of SPIR and CPNLR. The results are better for TILLR where some grid designs in the Top-10 in terms of $\lambda_2$, $\kappa$, and $R_G$ can be high ranking in terms of TILLR values for both tests systems used.

The results obtained show that using the four graph-based grid expansion heuristics that only use graph topology information may not necessarily lead to optimal grid adequacy, given that important operational constraints are ignored, e.g. the spatial distribution of low- and high-capacity lines, spatial distribution of nodes with positive and negative power injections, and the relative magnitude of power injections at each node, among others. More work is needed to explore (i) how prevalent these observations are to other test systems and actual grids and (ii) whether there exist special classes of grids wherein graph-based grid expansion (using the same four heuristics explored or other heuristics) result in optimal grid adequacy for power injection diversity.
Despite the limitations of the four graph-based grid expansion heuristics explored in giving grid designs with the optimal grid inadequacy characteristics, these methods may still find some limited practical use. Results showed that some grid designs in the Top-10 in terms of $\lambda_2$, $\kappa$, or $R_G$ can be high-ranking in terms of $TILLR$ and still have low values close to the minimum. Even if the solutions given are not optimal in terms of $TILLR$, these designs can still be used in two possible applications:

a. In preliminary planning where engineers need to screen the combinatoric solution space into a handful of interesting solutions that are worth further exploration. Results from graph-based expansion studies can form part of the shortlisted set of solutions. These solutions can then be compared with results from conventional scenario-based optimization models under a multi-criteria decision-making process in later stages of planning.

b. In optimization models as an initial solution or as part of the initial population of solutions. This is especially applicable to metaheuristic methods such as genetic algorithm [41] that rely on iteratively improving an initial set of candidate solutions to arrive at a local optimal solution. A starting solution that has good grid adequacy characteristics may help these types of approaches converge faster or find better solutions.

3.5 Conclusion

Experimental results from this chapter indicate that optimizing either one of the four grid graph properties tested do not necessarily lead to grid expansion options that also have the least grid inadequacy metrics. This motivates a direct approach to reducing grid inadequacy for power injection diversity by minimizing the size of the power flow infeasible set via one of the
inadequacy metrics proposed. In the next chapter, we present one such approach based on the SPIR metric.
4 Reducing inherent grid inadequacy via grid expansion³

As we mentioned in passing in Sections 1.2 and 3.1, most existing TNEP approaches minimize cost or maximize social welfare over a pre-defined power injection scenario or scenarios instead of directly minimizing inherent grid inadequacy for power injection diversity. The latter would appear to be increasingly important in future grids where stakeholders will seem to require so-called ‘plug-and-play’ capability to accommodate distributed energy resources. Having methods of the latter type can identify other promising grid expansion options that methods of the former type may overlook. Literature review indicates that so far, Ref. [110] is the only work that seek to directly reduce inherent grid inadequacy for power injection diversity. However, experimental results from Chapter 3 indicate that some simple graph-based heuristics like the one proposed in [110] do not necessarily give expansion options with the smallest grid inadequacy measures.

In this chapter, we revisit the problem of finding less inherently inadequate grid structures. We formulate the problem using the framework in Chapter 2 then present an optimization model that directly minimizes inherent grid inadequacy based on the SPIR metric. This approach has not been explored before and may give expansion options that are distinct from that of existing approaches and better in accommodating more diverse scenarios. Since the model focuses on minimizing inherent grid inadequacy, we cannot ensure that the solutions found are the best fit to other circumstantial technical and non-technical requirements specific to each grid. However, we believe

that there is inherent value in finding these solutions that planners or other researchers may adjust to their specific needs or find useful on their own. As such, the proposed method is not intended to replace existing methods but rather provide a different way of shortlisting solutions that planners may want to consider in later planning stages.

We structure the chapter as follows. Section 4.1 reviews and critiques existing methods for finding grid expansion options that consider power injection diversity. Section 4.2 formulates the problem of finding grid expansion options under the framework that we presented in Chapter 2 using an MILP model that minimize the size of the infeasible set. Section 4.3 provide examples that illustrate the merits of the proposed approach. Section 4.4 concludes.

4.1 Critique of existing methods

Different approaches have been proposed to find grid expansion options that enable diverse power injection conditions resulting from market interactions, RE intermittency, and/or load uncertainty. In [21], expected market-driven power injection scenarios are first identified using expert judgment or forecasting techniques. The least-cost expansion solution for each scenario is solved and a minimax regret approach is used to choose the best solution. Ref. [22] extended this approach to make a cost-risk curve that allows the planner to choose acceptable investment and risk levels. Since the scenarios are determined a priori, the approaches in [21] and [22] can be extended to a larger scenario pool by updating the scenario selection methodology.

Other approaches integrate scenario identification inside a multi-level optimization model by simulating generation dispatch or market dispatch. Identification of solutions follow different criteria that often involve minimization of expected costs, minimization of maximum cost, or
minimization of costs subject to a robustness criterion. Alternatively, social welfare is considered in some works instead of cost.

Refs. [26] and [28] integrated a bid-clearing model inside a stochastic optimization model that maximizes expected social welfare. Ref. [29] used the worst-case market Nash equilibria to inform planning. These approaches allow for market modeling but the results are sensitive to market model and data assumptions and the models may require large computational resources.

Other approaches use simpler dispatch models to identify power injection scenarios. Ref. [41] used combinatoric sampling to identify RE generation and load coincidence scenarios. Then, a cost minimization model was used to complete the dispatch schedule and find expansion options that are robust against all identified scenarios. Ref. [58] used a clustering approach on historical data to identify RE generation and load scenarios. The model relaxes the strict robustness requirement in [41] by adding chance constraints that allow RE generation curtailment in some scenarios. As with [21] and [22], results in [41] and [58] can be sensitive to the chosen scenario pool. Ref. [112] used uncertainty intervals to constrain RE generation and load variability instead of identifying discrete scenarios. The proposed model finds a solution that is robust against the worst-case curtailment scenario, which is also claimed to be robust against all other scenarios within the uncertainty interval. All three approaches in [41], [58], and [112], assume that other generators are freely dispatchable. This indicates a planning framework that prioritizes service continuity under RE generation and load uncertainty. In the future, service continuity will still be important but some grid operators may want to explore grid expansion solutions that also hedge against the diversity in bid-based market dispatch, RE generation availability, and load and ESS flexibility to foster grid access equity.
Recognizing that expansion planning based on a limited number of scenarios may be inadequate, Ref. [110] proposed a different way that indirectly improves a grid’s intrinsic or inherent ability to absorb and deliver power by improving its graph properties. The work argued that maximizing the determinant of a matrix derived from the DC power flow susceptance matrix also maximizes inherent grid adequacy under special circumstances. This graph-theoretic framework is simple and circumvents computationally intensive optimization modeling but ignores many practical constraints that limits its practical use. As experimental results show in Chapter 3 however, this approach does not necessarily identify grid expansion options that minimizes grid inadequacy. The remainder of this chapter revisits the idea of finding less inherently inadequate grid structures and proposes a more direct approach that capture practical constraints ignored in [110].

4.2 Proposed framework and optimization model

4.2.1 Definition and scope

In Section 2.1, we noted that there are different levels of rigor to grid inadequacy assessment and planning depending on the intended purpose of the study, power system model assumptions, and system states selected. In this chapter, we assume that long-term uncertainties in expansion planning are implicitly embedded in the following given parameters obtained from a separate study: a reference grid topology, location of buses with positive, negative, or mixed power injections, minimum and maximum power injections at each bus, and a list of candidate line additions.

The scope of grid inadequacy assessment and expansion planning then hinges on power system modeling assumptions and state selection preferences. These include the following considerations: the degree of diversity of power injection scenarios assumed to describe system states, whether operational interventions such as load shedding, generation redispatch, or congestion management interventions are modeled, whether contingency conditions are simulated, and
how uncertainties in market bidding and dispatch are considered. In this chapter, we limit our
definition of grid inadequacy to that of what we defined in Section 2.3 as *inherent grid inadequacy*. We use this term to refer to a grid’s inherent inability to allow diverse power exchanges without deploying power flow control interventions including, but not limited to, generation or load redispatch, line switching, or FACTS based control. To assess inherent grid inadequacy, we consider power exchanges in the power injection space bounded by bus power injection limits during normal conditions without line outages.

### 4.2.2 Motivating example

We can expand the grid to reduce inherent grid inadequacy by minimizing the size of the power flow infeasible set. By strategically adding new lines to the grid, changes in line susceptances and capacities change both the mapping function from $P$-space to $f$-space and the $f$-space bounding boxes and hence, may facilitate the feasibility of previously infeasible power exchanges. It is also possible, however, that some previously feasible power exchanges become infeasible. Figure 4-1 illustrates these possibilities by adding new lines to the three-bus three-line grid that we first presented in Section 2.3.3. For ease of reference, recall that the line susceptances are 1.0 p.u. for Line 1-2 and 0.5 p.u. for Lines 1-3 and 2-3. The power injection limits are 0.0–2.0 p.u. for Bus 1, -1.0–1.0 p.u. for Bus 2, and -1.0–0.0 p.u. for Bus 3. The line capacities are all 1.0 p.u. for existing and new lines alike. We use line loading range diagrams presented in Section 2.3.3.3 to visualize the feasible and infeasible sets in $f$-space in lower dimensions and to compare the inherent inadequacy of the different expansion options.

Note that in the base case with no new line added, insufficient capacity in Line 1-2 prevents the dispatch of some power exchanges, e.g. total generation in Bus 1 cannot be at 2 p.u. to supply 2 p.u. of total load. As such, Line 1-2 can be considered as a grid bottleneck. Adding a duplicate
parallel line to either Line 1-2 or Line 1-3 can eliminate the infeasible set but for different reasons: (a) in Figure 4-1b, adding Line 1-2 increases the corridor’s unconstrained line loading but the added capacity more than compensates for this increase while (b) in Figure 4-1c, adding Line 1-3 reduces the unconstrained line loading of Line 1-2 in direction 1-2 to its capacity limit and eliminates the possibility of infeasible power exchanges. This means that adding another Line 1-2 or another Line 1-3 makes the resulting grid globally inherently adequate as we defined the term in Section 2.3.1, i.e. all power exchanges within power injection limits are feasible even without generation rescheduling, load shedding, or other power flow control interventions.

Figure 4-1: Line loading range diagrams for different grid expansion options, 3-bus grid

On the other hand, in Figure 4-1d, adding Line 2-3 increases the unconstrained line loading in Line 1-2 in direction 1-2 without any additional capacity. This worsens inherent grid inadequacy and results in more power exchange combinations becoming infeasible. For instance, the number
of infeasible scenarios increased from 26 in the base case to 31 with Line 2-3 added using a sce-
nario pool with 1,225 power injection scenarios sampled from the power injection space. Based
on these results, and depending on costs and other grid expansion objectives, planners may favor
the addition of Line 1-3 if it is cheaper or Line 1-2 even if in case it is more expensive if additional
capacity margins are desired.

4.2.3 Proposed framework

As the example in Section 4.2.2 illustrates, different grid expansion options result in dif-
ferent inherent grid inadequacy characteristics. The small example allows an exhaustive enumer-
ation and assessment of available options but this is generally intractable for larger problems. In
this section, we present an optimization framework that can help address this problem.

Unlike other works reviewed in 4.1 that minimize cost or maximize social welfare, the
optimization framework that we propose finds grid expansion options by directly minimizing in-
herent grid inadequacy. We do this by minimizing the size of the power flow infeasible set. The
infeasible set, however, is generally high-dimensional and difficult to characterize. In Chapter 2,
we proposed three approaches to measure the size of the infeasible set relative to the power injec-
tion hypersurface: one uses a scenario-based metric and the other two use metrics derived from
lower-dimensional projections of the $P$-space and $f$-space hypersurfaces. Here, we adopt a robust-
like approach that minimizes the number of infeasible scenarios in a scenario-based representation
of the power injection space. We call the approach robust-like to describe infeasibility for a mini-
mal subset and differentiate it from robust approaches that usually connotes feasibility for the en-
tire scenario pool or feasibility under the worst-case scenario.
Since the proposed optimization framework uses a scenario-based representation of the power injection space and infeasible set, we first need to sample a pool of power injection scenarios from the whole power injection space. That is, discrete values of power injections must be assigned to each bus subject to power balance constraint and bus injection limits, see Equations (2-4) and (2-5). The sampled scenarios comprise the scenario pool $\mathcal{S}$ with $|\mathcal{S}|$ scenarios where $|\cdot|$ gives the cardinality of the set. Choosing the appropriate number of scenarios is particularly challenging because it requires a balance between adequate space representation and problem tractability. It is an important and sizeable research problem that we place outside our scope. Some possibilities include the following approaches. Probability distribution models can be used to generate $\mathcal{S}$ if historical data is available and correlation models can be assumed to hold in the future. Market simulation tools can also be used to generate likely scenarios. Otherwise, combinatoric sampling, if tractable, or targeted or worst-case sampling become viable options when the following conditions apply: (a) historical data is not available to develop trend and correlation models, (b) historical trend and correlation assumptions are deemed unlikely to hold in the future, (c) market simulation models are deemed unreliable or intractable, or (d) planners would like to explore solutions that offer some degree of hedging against previously unobserved trends. Even if historical data and market simulation models are available, sampling from previously unobserved scenario subspaces may still be necessary to check for inadequacy against these scenarios, and if so, to inform planners and stakeholders of the existence of such inadequacies. In the case studies, we use combinatoric sampling for a small test system and targeted sampling for a medium-sized test system for illustration purposes. We also use clustering techniques to reduce $|\mathcal{S}|$ for tractability.
4.2.4 Optimization model

After constructing the scenario pool, the following MILP model can be used to find grid expansion options that minimize the size of the infeasible set represented by the number of scenarios in $\mathcal{S}$ that result in congestion:

Objective

$$\min |\mathcal{J}| = \sum_s z^s$$  \hspace{1cm} (4-1)

Subject to:

$$P^s_i - \sum_{j \in \mathcal{V}} \sum_k \{ f^s_{ijk} + \bar{f}^s_{ijk} \} = 0, \forall i \in \mathcal{V}, \forall s$$  \hspace{1cm} (4-2)

$$f^s_{ijk} = B_{ijk}(\theta^s_i - \theta^s_j), \forall ijk \in \mathcal{L}, \forall s$$  \hspace{1cm} (4-3)

$$\bar{f}^s_{ijk} - \bar{B}_{ijk}(\theta^s_i - \theta^s_j) - M_1 x'_{ijk} \leq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-4)

$$\bar{f}^s_{ijk} - \bar{B}_{ijk}(\theta^s_i - \theta^s_j) + M_1 x'_{ijk} \geq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-5)

$$\bar{f}^s_{ijk} - M_1 x_{ijk} \leq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-6)

$$\bar{f}^s_{ijk} + M_1 x_{ijk} \geq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-7)

$$f^s_{ijk} - C_{ijk} - y^s_{ijk} \leq 0, \forall ijk \in \mathcal{L}, \forall s$$  \hspace{1cm} (4-8)

$$-f^s_{ijk} - C_{ijk} - y^s_{ijk} \leq 0, \forall ijk \in \mathcal{L}, \forall s \in \mathcal{S}$$  \hspace{1cm} (4-9)

$$\bar{f}^s_{ijk} - \bar{C}_{ijk} - \bar{y}^s_{ijk} - M_1 x'_{ijk} \leq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-10)

$$-\bar{f}^s_{ijk} - \bar{C}_{ijk} - \bar{y}^s_{ijk} - M_1 x'_{ijk} \leq 0, \forall ijk \in \tilde{\mathcal{L}}, \forall s$$  \hspace{1cm} (4-11)

$$\sum_{ijk \in \mathcal{L}} y^s_{ijk} + \sum_{ijk \in \tilde{\mathcal{L}}} \bar{y}^s_{ijk} - M_2 z^s \leq 0, \forall s$$  \hspace{1cm} (4-12)

$$\theta^s_i = 0, \forall s$$  \hspace{1cm} (4-13)

$$\sum_{ijk \in \tilde{\mathcal{L}}} x_{ijk} \leq x_{max}$$  \hspace{1cm} (4-14)

$$x'_{ijk} = 1 - x_{ijk}, \forall ijk \in \tilde{\mathcal{L}}$$  \hspace{1cm} (4-15)
\[
\begin{align*}
x_{ijk}, z^s & \in \{0,1\} \quad (4-16) \\
y_{ijk}, \bar{y}_{ijk} & \geq 0 \quad (4-17)
\end{align*}
\]

where \( \mathcal{I} \) is the scenario-based representation of the infeasible set, \(|\mathcal{I}|\) is the size of \( \mathcal{I} \), \( z \) indicates whether a scenario is infeasible, \( s = 1, 2, \ldots, |\mathcal{S}| \) indexes the scenarios in \( \mathcal{S} \), \( P_i \) is the power injection in bus \( i \), \( \theta_i \) is the bus angle in bus \( i \), \( \mathcal{V} \) is the set of buses, \( x_{\text{max}} \) is the maximum number of lines that can be installed, and \( M_1 \) and \( M_2 \) are large enough constants. For existing lines, \( f_{ijk} \) is the power flow from bus \( i \) to bus \( j \) in the \( k \)th parallel circuit, \( B_{ijk} \) is the line susceptance, \( C_{ijk} \) is the line capacity, \( y_{ijk} \) indicates the amount of line overload, and \( \mathcal{L} \) is the set of existing lines. For candidate lines, the corresponding variables are \( \bar{f}_{ijk}, \bar{B}_{ijk}, \bar{C}_{ijk}, \bar{y}_{ijk} \), and \( \bar{\mathcal{L}} \) respectively and \( x_{ijk} \) indicates whether a candidate line is installed or not.

Equation (4-2) enforces the power flow balance at each bus, (4-3) models the power flow in an existing line, (4-4)-(4-5) is the counterpart of (4-3) for installed candidate lines, (4-6)-(4-7) forces the power flow to zero for uninstalled candidate lines, (4-8)-(4-9) models the amount of overload in existing lines, (4-10)-(4-11) is the counterpart of (4-8)-(4-9) for installed candidate lines, (4-12) models the existence of line congestion in a scenario, (4-13) sets the bus angle of a chosen reference node \( r \) to zero, (4-14) limits the number of candidate lines that can be installed, (4-15) relates \( x_{ijk} \) to its logic inverse, (4-16) forces \( x_{ijk} \) and \( z^s \) to be binary, and (4-17) forces \( y_{ijk} \) and \( \bar{y}_{ijk} \) to be non-negative.

### 4.2.5 Model characteristics

Like other TNEP models, the number of continuous variables and the number of constraints are dependent on \(|\mathcal{V}|, |\mathcal{S}|, |\mathcal{L}|\), and \(|\bar{\mathcal{L}}|\). But unlike other models, variables representing generator dispatch and load curtailment as well as constraints representing power balance and power
injection limits are no longer needed because these are already captured when constructing the scenario pool. The introduction of line overload variables $y_{ijk}$ and $\bar{y}_{ijk}$ in (4-8) to (4-11) and the formulation of constraint (4-12) that identify congestion scenarios are unique to this model. These make it possible to count the number of scenarios with network congestion and to use it subsequently in the objective function. However, this also increases the number of binary variables. Specifically, there will be $|S|$ additional binary variables and $|S|$ additional constraints representing (4-12). Since each binary variable adds to the chances of more branching during the MILP solution process, ample consideration should be made in choosing $|S|$ to keep the resulting optimization model tractable while maintaining adequate space representation.

One of the key features of the model is that the scenario identification step is fully decoupled from the optimization step as in [21]. The decoupling is only partial in [41] and [58] wherein only RE generation and load variability are sampled and the uncertainty in market dispatch is ignored. Since the dispatch model is fully integrated in the optimization model in [41] and [58], the optimization process favors the dispatch of some generators over others in finding expansion options. Some planners who want to explore less discriminatory grids may thus find partial decoupling of scenarios ill-suited to their needs. Fully decoupling the scenario sampling step from the optimization process gives planners an alternative way to define the diversity of the scenarios represented subject to data availability, forecasting and model accuracy, and planner and stakeholder policy guidelines. Moreover, this can allow for a more transparent process wherein stakeholders know whether their interest for equitable grid access are adequately represented in the scenario identification step, and if not, provide relevant feedback.

The robust-like formulation is similar in idea to the chance-constrained formulation in [58] that allows for infeasibility in a subset of $S$. There are differences however, aside from the full
decoupling of the scenario identification process discussed above. In [58], investment and RE generation curtailment cost is minimized subject to a chance constraint that limits the prevalence of curtailment scenarios. In the proposed approach, we do the opposite case of finding the minimal infeasible subset subject to investment limits. The proposed model can be converted to a similar framework adapted in [58] by converting (4-1) into a constraint and converting (4-14) into the objective. The same conversion is not as straightforward using the chance-constrained model in [58]. While both approaches relaxes the strict robustness requirement in TNEP models such as in [41], the proposed approach provides a different relaxation that allow planners to find other solutions.

In practice, generation and load rescheduling is commonly used to extend grid capability to operate beyond its inherent inadequacy limits. As such, using a strict inherent inadequacy criterion that does not allow for generation or load rescheduling is conservative. However, having solutions from models such as the one we propose is potentially useful. For example, these solutions can provide a benchmark of what is achievable in terms of providing fair access to market participants. More rigorous assessment models in succeeding planning stages can then be used to reveal whether it is desirable to meet this benchmark subject to circumstantial criteria, conditions, and policy targets specific to each grid.

4.3 Case studies

4.3.1 6-bus test system

4.3.1.1 Overview

The six bus test system initially has five interconnected buses and one bus that needs connection. Power can be injected in three and drawn from five buses. These power injections can
represent contributions from either conventional or renewable generation, energy storage, or conventional or dynamic loads. There are fifteen candidate lines and a maximum of four new lines can be installed in a right of way. We adapt the network and loading data in [112] to be able to compare results later where the loads can vary by up to 105% of the original values in [111].

We construct three scenario pools $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$ with $|\mathcal{S}| = 100$, 500, and 1000 respectively for optimization purposes and a much larger pool $\mathcal{S}_4$ with $|\mathcal{S}| = 5,098,771$ for comparing the quality of the solutions obtained. To compare solutions, we take the ratio of the number of infeasible scenarios in a pool with the pool size:

$$SPIR = \frac{|\mathcal{I}|}{|\mathcal{S}|}$$  \hspace{1cm} (4-18)

where $SPIR$ stands for the Scenario Pool Infeasibility Ratio defined in Section 2.3.4.1.

In order to generate a high-fidelity sample of the power injection space, we use combinatoric sampling to construct $\mathcal{S}_4$ by specifying positive power injection levels for the three generating buses from [0, 5, 15, ..., 85, 95, 100%] and negative power injection levels for the five load buses from [0, 5, 15, ..., 85, 95, 105%]. We generate all power injection combinations and allow positive power injection setpoints to vary by up to ±5% to balance the total load. Only the scenarios that satisfy the power balance equation are chosen as part of $\mathcal{S}_4$. The smaller pools are obtained from $\mathcal{S}_4$ using the mini-batch k-means clustering algorithm [154] by specifying the desired number of clusters.

Initial assessment using $\mathcal{S}_4$ shows that $SPIR$ is 38.5% for the solution obtained using a deterministic approach that uses only the peak load dispatch and 43.0% for the solution given in [112] that uses a robust approach that determines the worst-case scenario within the defined uncertainty intervals. This means that in the absence of line switching or FACTS-based control,
market clearing schedules that belong in the infeasible set need to be curtailed using a potentially more expensive generation dispatch or through load shedding. These results highlight the limitation of considering only the peak dispatch or assuming generator dispatch flexibility as in [112] in finding grid expansion options. By ignoring the uncertainty in bid-based dispatch, the solutions may not give the least discriminatory access to market participants. This presents an opportunity to find other solutions that have a lower SPIR that planners may consider to minimize the need for rescheduling or congestion management interventions, to reduce operation costs, or to promote market fairness and competition, albeit at an added investment cost.

4.3.1.2 Optimization results

Table 4-1 shows the optimization results for the three scenario pools used and up to eight new lines added. Figure 4-2 and Figure 4-3 graphs the SPIR and cost values in the table for ease of comparison. Despite the availability of fifteen distinct candidates, only six are commonly chosen as part of the solution. The proposed model identified lines 3-6 and 5-6 in multiple cases which are not part of the solutions given by either deterministic or robust approaches. By ignoring these options, grid planners can miss out on improving grid access fairness. For five lines added for example, Line 3-6 is chosen instead of a third Line 4-6 in the robust solution. This achieves a 21.6% improvement in SPIR for just 13.8% increase in cost as highlighted in the small circle in Figure 4-2 and Figure 4-3. Likewise for seven lines added, installing Line 3-6 or both Lines 3-6 and 5-6 contribute to improving SPIR by 50.8-66.8% for a 13.0-35.0% increase in cost relative to the deterministic solution as highlighted in the large circle. The best solution obtained has a SPIR of 6.7% with eight lines added including all six lines identified in other solutions. This is a reduction of 82.7% compared to the deterministic seven-line solution at just 34.5% added cost.
Table 4-1: Solutions found for different $\mathcal{S}$ and $\Sigma x$, 6-bus grid

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Sigma x$</th>
<th>SPIR</th>
<th>Cost* (in thousands of USD)</th>
<th>Lines Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\mathcal{S}</td>
<td>= 100$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>88.7%</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72.8%</td>
<td>80</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>55.0%</td>
<td>100</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>33.7%</td>
<td>148</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>26.0%</td>
<td>191</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>18.9%</td>
<td>239</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>14.9%</td>
<td>282</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$</td>
<td>\mathcal{S}</td>
<td>= 500$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>88.7%</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>80.3%</td>
<td>50</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>68.7%</td>
<td>70</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>55.0%</td>
<td>110</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>33.7%</td>
<td>148</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>26.0%</td>
<td>191</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>14.1%</td>
<td>226</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$</td>
<td>\mathcal{S}</td>
<td>= 1000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>88.7%</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>80.3%</td>
<td>50</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>67.6%</td>
<td>70</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>55.0%</td>
<td>110</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>33.7%</td>
<td>148</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>21.4%</td>
<td>196</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>12.8%</td>
<td>270</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Deterministic</td>
<td>6.7%</td>
<td>269</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Robust [112]</td>
<td>7</td>
<td>38.5%</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>43.0%</td>
<td>130</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As with other scenario-based approaches such as [21], [22], [41], and [58], the solutions obtained from the proposed approach can be sensitive to the scenario pool chosen. In the case studies tested for the 6-bus test system, increasing $|\mathcal{S}|$ led to solutions with increasingly lower SPIR. This observed solution sensitivity to the scenario pool is inherent to scenario-based approaches. To account for this effect, planners may opt to construct multiple scenario pools then
find the solution for each as in [41]. Planners may then consider for further investigation either high-ranking or all solutions obtained.

![Figure 4-2](image)

Figure 4-2: Optimal SPIR values as a function of lines added and $|S|$

![Figure 4-3](image)

Figure 4-3: Cost values corresponding to the optimal SPIR solutions.

### 4.3.1.3 Solution time

Figure 4-4 shows the solution times in seconds in logarithmic scale for the different optimization case studies in Table 4-1. The models were solved using Gurobi 8.0.1 [128] using the Artemis High Performance Computing system [155] at the University of Sydney with CPU cores that can clock up to 2.60 GHz. Depending on the model size, the test cases used 1-32 cores and
0.1-32.3 GB of memory. Solution times increased exponentially with $x_{max}$ as the number of possible combination of line additions balloon. Solution times also increased by about an order of magnitude each with an increase in $|S|$ from 100 to 500 to 1,000. While some cases took a long time to solve to optimality, solution times are still practicable in a long-term expansion planning setting where there is a more relaxed time constraint. Depending on available computing resources, planners may need to empirically determine suitable values of $x_{max}$ and $|S|$ for tractability, perform the optimization into smaller chunks by incrementally adding a few new lines, or terminate the optimization earlier without waiting for optimality.

![Graph showing solution times for different optimization case studies, 6-bus grid](image)

**Figure 4-4:** Solution times (in log scale) for the different optimization case studies, 6-bus grid

### 4.3.1.4 Comparing alternatives

Showing that a grid expansion option has the least inherent inadequacy level relative to other options for the same investment cap or number of lines added is often not enough to make a case in practice. Other criteria must be considered such as reliability, operational efficiency, economic feasibility, and alignment with policy targets. Finding grid expansion options is only an
initial step and should be followed by a process of evaluating and comparing alternatives using multiple assessment methods and feedback from stakeholders.

One such criteria for comparing alternatives is to compare the infeasible sets spanned by different competing solutions. Table 4-2 shows the proportion of scenarios in $S_4$ uniquely spanned by the robust solution and the unique five-line solution obtained using the robust-like approach. It also shows the proportion of scenarios feasible and infeasible in both.

Table 4-2: Comparison of feasible sets with $x_{max} = 5$, robust-like approach vs. robust solution

<table>
<thead>
<tr>
<th>% of scenarios in $S_4$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.3%</td>
<td>feasible in both</td>
</tr>
<tr>
<td>12.0%</td>
<td>feasible only in the robust-like solution</td>
</tr>
<tr>
<td>2.7%</td>
<td>feasible only in the robust solution [112]</td>
</tr>
<tr>
<td>31.0%</td>
<td>infeasible in both</td>
</tr>
</tbody>
</table>

The robust solution uniquely hedges against 2.7% of the scenarios in $S_4$ while the proposed robust-like solution uniquely hedges against 12.0%. To decide between the two options, the likelihood of future scenarios and the risk of failing to hedge against such scenarios must be assessed. For example, there would be a stronger case for the robust-like solution if future power exchanges are more likely to fall within the space the solution uniquely spans and there are cost-effective risk mitigation measures available for the infeasible set it spans. The converse is true for the robust solution otherwise. Assessing which scenarios are more likely than others, however, is a challenging exercise because of the inherent uncertainty in predicting the future. This will inevitably depend on data availability, stakeholder input, and expert judgment specific to each grid.
4.3.2 118-bus test system

4.3.2.1 Overview

We also perform tests on the 118-bus test system with 54 generating buses, 91 load buses, and 186 lines using the data in [130]. Since it will be intractable to define combinatoric generator and load levels as in the 6-bus case, we used a targeted sampling approach to define the scenario pool as in Section 2.4.2. The process involves solving for the unconstrained loading range of each line, defining 100 loading levels for each line, and solving for the bus power injections that result in a given line loading level. This results in a scenario pool $S_A$ with $|S_A| = 18,600$ that we use for final inadequacy assessment. We then use k-means clustering to generate a smaller pool $S_B$ for optimization purposes with $|S_B| = 100$. We identify 25 candidate lines from the original 186 by identifying bottlenecks comprised of lines with maximum feasible loading ranges at or within 10% of the line’s capacity.

4.3.2.2 Optimization results

Table 4-3 shows the SPIR and solution times for the 118-bus case studies for different values of $x_{max}$. Without additional lines, 39.2% of the scenarios in $S_A$ need either generator rescheduling, load curtailment, or other congestion management intervention. This value can be reduced by almost half to 21.1% if four lines are added. This reduction translates to less rescheduling, load curtailment, or other interventions. It also contributes to less discriminatory grid access. As with the 6-bus case study however, further assessments are needed to determine whether the benefits of reduced inherent grid inadequacy outweigh the costs of adding new infrastructure, among other considerations.
Table 4-3: Solutions found for different $S$ and $\Sigma x$, 118-bus

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Sigma x$</th>
<th>Lines Added</th>
<th>SPIR</th>
<th>Time, (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>--</td>
<td>39.2%</td>
<td>20.97</td>
</tr>
<tr>
<td>1</td>
<td>30-38</td>
<td></td>
<td>26.5%</td>
<td>39.29</td>
</tr>
<tr>
<td>2</td>
<td>30-38, 65-68</td>
<td></td>
<td>24.8%</td>
<td>439.63</td>
</tr>
<tr>
<td>3</td>
<td>8-30, 30-38, 65-68</td>
<td></td>
<td>22.1%</td>
<td>727.99</td>
</tr>
<tr>
<td>4</td>
<td>8-30, 30-38, 65-68, 94-100</td>
<td></td>
<td>21.1%</td>
<td>1,395.63</td>
</tr>
</tbody>
</table>

4.4 Conclusion

In this chapter, we proposed an approach that directly minimizes the size of the power flow infeasible set by minimizing the number of infeasible scenarios in a scenario-based representation of the power injection space. We presented a mixed-integer linear program as used in existing TNEP models but with the capability to count and minimize the number of infeasible scenarios. Test results showed that the model can identify grid expansion options distinct from existing deterministic and worst-case robust approaches. The solutions found have reduced inherent grid inadequacy compared to other solutions, albeit at an extra investment cost. Identifying such options give planners alternatives to consider especially when improving grid access fairness is an important objective.

In the next chapter, we extend the framework and optimization model presented here to integrate grid flexibility from line switching.
5 Integrated grid flexibility and grid expansion planning\textsuperscript{4}

In Chapter 4 we explored how grid expansion via line addition can reduce inherent grid inadequacy for power injection diversity and how this is an emerging research problem that can help enable existing grids for future power injection diversity. Another emerging research problem is the design of flexible and, in a way, dispatchable grids to adapt to changing node power injection circumstances. Integrating grid flexibility planning in TNEP not only may reduce costs and defer infrastructure investments but also improve a grid’s total ability to serve diverse power injection conditions. In this chapter, we extend the ideas in Chapter 4 by integrating grid flexibility in the form of line switching in the framework and optimization model.

We structure the chapter as follows. Section 5.1 reviews existing work on line and switch placement. The review focuses on the type of optimization model used to allocate new lines and switches under power injection uncertainty. Section 5.2 presents the proposed framework and optimization model. The section defines what we propose to call total grid inadequacy for power injection diversity and relates it to the power flow infeasible set, inherent grid inadequacy, and grid flexibility. It then presents an optimization model that directly minimizes total grid inadequacy using the size of the infeasible set via grid expansion and line switching. Section 5.3 provides illustrative examples that motivate the benefits of integrating grid flexibility planning in grid expansion planning. Section 5.4 concludes.

5.1 Related work in integrating grid flexibility in TNEP

Grid expansion via line addition has been explored as a solution to accommodate diverse power injection operation using various optimization models and approaches. We reviewed the related literature in Section 4.1.

Line switching has also been explored as a means for enabling alternative dispatches from cheap or RE generation. Ref. [156] uses a deterministic model for optimal transmission switching (OTS) to enable less costly generator dispatches for one scenario. Ref. [157] uses a chance-constrained OTS model to meet wind utilization targets and minimize generation costs. Ref. [158] presents a robust OTS model that hedges against the worst-case realization of uncertain RE generation. Ref. [159] uses a stochastic model to determine where to install series compensation and line switches to minimize operation costs and use more RE generation across several loading scenarios. Ref. [160] uses a robust model to find series compensation and switching action needed to maximize allowable RE generation uncertainty.

Co-optimization of line addition and line switching has also been explored to accommodate power injection uncertainty. Ref. [34] uses a three-level robust model that ensure feasibility across all identified scenarios while considering generation expansion and contingencies. Ref. [64] uses a stochastic model that minimizes expected costs. Ref. [82] uses a robust model that hedges against the worst-case realization of uncertain variables while also considering energy storage. Ref. [161] uses a stochastic model that also considers energy storage. Ref. [162] uses a robust model that hedges against the worst case realization of uncertain variables but transmission switching is reserved as a corrective action during N-1 contingencies.

Like the literature reviewed in Section 4.1 in the optimal allocation of new lines, non-RE generators are assumed highly dispatchable in the models reviewed in this section that consider
RE generation. As we mentioned in the same section, this reflects a planning objective that prioritizes service availability first and foremost. In a future with more market players, RE generation, bulk ESS, and aggregated flexible loads in a competitive environment, it will be important for the grid not only to ensure service availability but also to accommodate the dispatch of spatially and temporally varying resources as they become available as much as possible. This will allow for least-cost operation and reduced carbon emissions for more scenarios as well as promote market competition and fair grid access and accommodate end-user preferences. Ref. [110], Chapter 3, and Chapter 4 tries to address this emerging need by proposing an alternative framework for finding grid expansion options that improve inherent grid adequacy or reduce inherent grid inadequacy for power injection diversity. The framework and optimization model presented in the succeeding sections builds on the model presented in Chapter 4 by integrating grid flexibility in the form of line switching.

5.2 Proposed framework and model

5.2.1 Motivating example

In Chapter 2, we related the size of the infeasible set to a grid’s inability to host diverse power injection conditions. In Chapter 4, we explored how the infeasible set can be reduced by grid expansion. Another way to reduce the size of the infeasible set, or eliminate it in some cases, is by a combination of line expansion and line switching. Adding a line via grid expansion or removing a line via switching changes the line susceptances and capacities which in turn modifies the mapping from $P$-space to $f$-space as well as the size of the $f$-space bounding box (see the discussion in Section 2.3). These changes may result in the feasibility of previously infeasible scenarios. The union of feasible sets of all possible switching configurations reduces the effective
number of infeasible scenarios and contribute to reduced grid inadequacy to host diverse power injection conditions.

In the three-bus network in Section 4.2.2 for example, network constraints do not allow scenarios that maximize the dispatch of generators in Bus 1 when the total load exceeds 1.5 p.u. In such cases, generators in Bus 2 need to be dispatched. While service availability can still be guaranteed, operating costs or carbon emissions may not necessarily be minimal. By adding a new Line 1-2, a new Line 1-3, or adding a switch in Line 2-3, previously infeasible scenarios become feasible, effectively improving the grid’s ability to accommodate more diverse operating states. Figure 5-1 shows the projection of the infeasible set along the \( f \)-space basis vectors showing the feasible and unconstrained loading range of each line for each of these options (see Chapter 2.3.3.3 to review how the diagrams are constructed).
Infeasible power exchanges in the original grid (such as those that maximize the dispatch of generators in Bus 1) congest Line 1-2 and is captured in the black areas in Figure 5-1a. Scenarios such as these become feasible when either one of the three measures identified are realized to decongest Line 1-2 (note the lack of black areas in Figure 5-1b to Figure 5-1d). The resulting line loading profiles differ however, with limited capacity margins in some lines in some extreme scenarios when a new Line 1-3 or a switch in Line 2-3 is added (see indicated in Figure 5-1c and Figure 5-1d). Depending on the available budget, perceived likelihood of extreme scenarios occurring, need for additional capacity margin, urgency of deployment, and other technical and non-technical barriers to implementation, one of the three options identified may be ultimately chosen.

5.2.2 Measuring grid flexibility using the infeasible set

As the example in Section 5.2.1 illustrates, congestion may be eliminated in some power injection scenarios by switching out some lines, effectively reducing what we propose to call total grid inadequacy for power injection diversity. We define this as grid inability to accommodate power injection diversity even with the use of grid flexibility control interventions that deploy available grid assets like switches, phase-shifting transformers, FACTS devices, and HVDC links.

We can extend the use the infeasible set, mainly used to characterize inherent grid inadequacy in Chapters 2, 3, and 4, to also characterize total grid inadequacy that accounts for grid flexibility as follows. Given a pool of power injection scenarios $\mathcal{S}$ with $|\mathcal{S}|$ scenarios, we can use the number of scenarios with congestion $|\mathcal{I}|$ to represent the infeasible set. We can then use $|\mathcal{I}|$ as an overall measure of total grid inadequacy for power injection diversity. Without any grid flexibility control intervention or power injection control intervention, $|\mathcal{I}|$ also measures inherent grid inadequacy for power injection diversity as is the case in Chapters 2, 3, and 4. With grid flexibility control intervention but no power injection control intervention, the change in $|\mathcal{I}|$ with and without
deploying line switching, for example, can be used as a measure of grid flexibility given by line switching.

5.2.3 Optimization model

We can directly minimize the scenario-based representation of the infeasible set to find line and switch installation options that reduce total grid inadequacy. Like the proposed approach in Chapter 4, directly minimizing total grid inadequacy diverges from conventional approaches that minimize cost under different power exchange scenarios. This different approach allows planners to identify distinct and meritorious expansion options that may be preferable over other shortlisted options after more rigorous analyses in later planning stages.

We can extend the optimization model in Section 4.2.4 to find line and switch additions that reduce total grid inadequacy by directly minimizing $|\mathcal{J}|$ as follows:

Objective

$$\text{min } |\mathcal{J}| = \sum_s z^s$$

Subject to:

$$p_i^s - \sum_{j \in \mathcal{V}} \sum_k \{f_{ijk}^s + \bar{f}_{ijk}^s\} = 0, \forall i \in \mathcal{V}, \forall s$$

$$f_{ijk}^s - B_{ijk}(\theta_i^s - \theta_j^s) - M_1 w_{ijk}^s \leq 0, \forall ijk \in \mathcal{L}, \forall s$$

$$f_{ijk}^s - B_{ijk}(\theta_i^s - \theta_j^s) + M_1 w_{ijk}^s \geq 0, \forall ijk \in \mathcal{L}, \forall s$$

$$\bar{f}_{ijk}^s - \bar{B}_{ijk}(\theta_i^s - \theta_j^s) - M_1 x'_{ijk} - M_1 \bar{w}_{ijk}^s \leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s$$

$$\bar{f}_{ijk}^s - \bar{B}_{ijk}(\theta_i^s - \theta_j^s) + M_1 x'_{ijk} + M_1 \bar{w}_{ijk}^s \geq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s$$

$$f_{ijk}^s - M_1 w_{ijk}^r \leq 0, \forall ijk \in \mathcal{L}, \forall s$$

$$f_{ijk}^s + M_1 w_{ijk}^r \geq 0, \forall ijk \in \mathcal{L}, \forall s$$

$$\bar{f}_{ijk}^s - M_1 x'_{ijk} - M_1 \bar{w}_{ijk}^r \leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s$$
\[
\begin{align*}
\bar{f}_{ijk} + M_1x'_{ijk} + M_1\bar{w}'_{ijk} &\geq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
\bar{f}_{ijk} - M_1x_{ijk} &\leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
\bar{f}_{ijk} + M_1x_{ijk} &\geq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
f_{ijk} - C_{ijk} - y^s_{ijk} &\leq 0, \forall ijk \in \mathcal{L}, \forall s \\
-f_{ijk} - C_{ijk} - y^s_{ijk} &\leq 0, \forall ijk \in \mathcal{L}, \forall s \\
\bar{f}_{ijk} - \bar{c}_{ijk} - \bar{y}^s_{ijk} - M_1x'_{ijk} &\leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
-f_{ijk} - \bar{c}_{ijk} - \bar{y}^s_{ijk} - M_1x'_{ijk} &\leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
\sum_{ijk \in \mathcal{L}} y^s_{ijk} + \sum_{ijk \in \bar{\mathcal{L}}} \bar{y}^s_{ijk} - M_2z^s &\leq 0, \forall s \\
\theta^s_r &= 0, \forall s \\
\sum_{ijk \in \mathcal{L}} x_{ijk} &\leq x_{\text{max}} \\
\sum_{ijk \in \mathcal{L}} u_{ijk} + \sum_{ijk \in \bar{\mathcal{L}}} \bar{u}_{ijk} &\leq u_{\text{max}} \\
x'_{ijk} &= 1 - x_{ijk}, \forall ijk \in \bar{\mathcal{L}} \\
w'^s_{ijk} &= 1 - w^s_{ijk}, \forall ijk \in \mathcal{L}, \forall s \\
\bar{w}'^s_{ijk} &= 1 - \bar{w}^s_{ijk}, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
\bar{u}_{ijk} - x_{ijk} &\leq 0, \forall ijk \in \bar{\mathcal{L}} \\
w^s_{ijk} - u_{ijk} &\leq 0, \forall ijk \in \mathcal{L}, \forall s \\
\bar{w}^s_{ijk} - \bar{u}_{ijk} &\leq 0, \forall ijk \in \bar{\mathcal{L}}, \forall s \\
u_{ijk}, \bar{u}_{ijk}, w^s_{ijk}, \bar{w}_{ijk}, x_{ijk}, z^s &\in \{0, 1\} \\
y^s_{ijk}, \bar{y}^s_{ijk} &\geq 0
\end{align*}
\]
where \( z \) indicates scenario infeasibility, \( s = 1, 2, \ldots, |\mathcal{S}| \) indexes \( \mathcal{S} \), \( P_i \) and \( \theta_i \) are the power injection and bus voltage angle in bus \( i \) respectively, \( x_{max} \) and \( u_{max} \) are the maximum number of lines and switches that can be installed respectively, and \( M_1 \) and \( M_2 \) are large enough constants. For currently existing lines, \( f_{ijk} \) is the line power flow from bus \( i \) to \( j \) in circuit \( k \), \( B_{ijk} \) is the line susceptibility, \( C_{ijk} \) is the line capacity, \( y_{ijk} \) captures the amount of line overload, \( u_{ijk} \) indicates whether a switch is installed, \( w_{ijk} \) indicates whether the line is switched out, and \( \mathcal{L} \) is the set of existing lines. For candidate new lines, the corresponding variables are \( \bar{f}_{ijk}, \bar{B}_{ijk}, \bar{C}_{ijk}, \bar{y}_{ijk}, \bar{u}_{ijk}, \bar{w}_{ijk} \), and \( \bar{\mathcal{L}} \) respectively and \( x_{ijk} \) indicates new line installation.

Equation (5-2) is the bus power balance equation, (5-3) to (5-6) model the line power flow if there is no open switch, (5-7) to (5-10) forces the line power flow to zero if there is an open switch, (5-11) and (5-12) forces the power flow to zero if a candidate line is not installed, (5-13) to (5-16) models the extent of line overload, (5-17) captures line congestion incidence in a scenario, (5-18) sets \( \theta_r \) of reference node \( r \) to zero, (5-19) and (5-20) limits the number of lines and switches that can be installed respectively, (5-21) to (5-23) relates \( x_{ijk}, w_{ijk} \), and \( \bar{w}_{ijk} \) to their logic inverses, (5-24) ensures that a candidate line is installed before a switch can be installed, (5-25) and (5-26) ensure that a switch is installed before it can be opened, (5-27) forces \( u_{ijk}, \bar{u}_{ijk}, w_{ijk}, \bar{w}_{ijk}, x_{ijk}, \) and \( z^s \) to be binary, and (5-28) forces \( y_{ijk}^s \) and \( \bar{y}_{ijk}^s \) to be non-negative. Switch installation and switch status variables can be set to zero for lines not considered for switching.

### 5.2.4 Model characteristics

Model (5-1) to (5-28) is a mixed-integer linear programming (MILP) model that finds the optimal line and switch additions to a grid that minimizes the number of infeasible scenarios in the scenario pool for a given maximum number of lines and switches. It can be solved using classical
methods in solving MILPs [127] or commercial MILP software such as [128]. This model inherits the same properties as the model in Section 4.2.4 such as a fully decoupled scenario selection step and relaxed robustness criterion that allows for congestion in some scenarios. The former property allows greater transparency in defining scenarios with stakeholders while the latter property allows the identification of solutions that are otherwise good if not for infeasibility in some scenarios.

The presented model also inherit limitations inherent to similar MILP models however. The optimization problem can easily become intractable, so computational tractability and space representation must be traded off in defining the scenario pool – an important problem that we place outside the scope of this thesis. Common options include the use of historical data and market models, if available. Otherwise, combinatoric or worst-case sampling may be viable options especially if the following apply: historical data is not available, previous trends are not expected to hold in the future, market models are deemed unreliable, or planners would also like to hedge against previously unobserved patterns. Clustering techniques can be used to further reduce the size of the scenario pool.

The solutions obtained from the model can also be sensitive to the scenario pool chosen. To address this, multiple scenario pools may be used to generate a pool of reinforcement options for later detailed assessment. And like most models that find grid reinforcement options, results of the model should only form part of preliminary expansion planning stages to shortlist a small pool from a large combinatoric solution space. More detailed assessment on the cost, benefits, and risks should follow before deciding which option to eventually implement.
5.3 Case studies

To illustrate the utility of the proposed model and the benefits of integrating line switching in grid expansion planning, we present case studies using the 6- and 118-bus test systems using the data in [111] and [130] respectively. The results are obtained by running the model instances using Gurobi 8.0.1 [128] for at most 24 hours in the Artemis High Performance Computing system [155] using up to 256 GB of memory and up to 24 CPU cores that can clock up to 2.6 GHz each.

5.3.1 6-bus test system

We use the expanded six-bus grid reported in [111] as the base case with positive power injection in three buses and negative injection in five. These represent power injections that may come from traditional or renewable generation, traditional or flexible loads, as well as energy storage. We consider all fifteen candidate lines in [111] for grid expansion and all existing lines for switch installation. We use 50 power injection scenarios taken from the entire power injection space using a similar approach from Chapters 2 to 4 as follows. The unconstrained loading range of each existing line is first solved. We divide each range equally to define 100 line loading levels per line. Power injection scenarios that maximize the total load and result in each line loading level are solved. The total pool is then clustered using the mini-batch k-means algorithm [154] to achieve the desired number of scenarios. After generating the pool, we generate one instance of the optimization model for a given maximum number of lines and switch. We then solve each instance of the model to optimality or for at most 24 hours.

Table 5-1 shows the optimization results for different limits on the number of new lines and switches. Figure 5-2 visualizes the results. Without any new line or switch, 84% of power injection scenarios are inherently infeasible without any operator intervention. This is indicative of inherent grid inadequacy to support dispatch diversity and may result in the need to dispatch
generators with higher operating costs or carbon emissions, curtail available renewable or energy storage capacity, or cut services to some loads. Moreover, allowing for only a limited dispatch diversity may fail to satisfy stakeholders who want equitable grid access.

Table 5-1: Solutions found for different $\Sigma x$ and $\Sigma u$, 6-bus grid

| $\Sigma u$ | $\Sigma x$ | Switch Added | Line Added | $|\mathcal{F}|$ | Opt.Time, s |
|-----------|-----------|--------------|------------|-----------------|-----------|
| 0         | 0         | --           | --         | 42              | --        |
| 0         | 1         | --           | 2-3        | 26              | 1.23      |
| 0         | 2         | --           | 2-3, 3-6   | 16              | 1.78      |
| 0         | 3         | --           | 2-3, 3-5, 3-6 | 7           | 1.66      |
| 0         | 4         | --           | 2-3, 2-5, 3-4, 5-6 | 2   | 2.06      |
| 1         | 0         | 1-5          | --         | 41              | 0.57      |
| 1         | 1         | 1-5          | 2-3        | 23              | 28.79     |
| 1         | 2         | 1-5          | 2-3, 3-6   | 14              | 87.98     |
| 1         | 3         | 1-5          | 2-3, 2-5, 5-6 | 5           | 61.84     |
| 1         | 4         | 4-6          | 2-3, 2-5, 3-4, 5-6 | 1   | 87.51     |
| 2         | 0         | 1-5, 2-4     | --         | 40              | 0.69      |
| 2         | 1         | 1-4, 1-5     | 2-3        | 22              | timeout   |
| 2         | 2         | 1-5, 2-6     | 2-3, 3-6   | 12              | 20,172.76 |
| 2         | 3         | 1-5, 2-6     | 2-3, 3-5, 3-6 | 3           | 188.38    |
| 2         | 4         | 1-5, 4-6     | 2-3, 2-5, 3-4, 5-6 | 0   | 14.46     |
| 3         | 0         | 1-2, 1-5, 2-4 | --         | 39              | 0.34      |
| 3         | 1         | 1-4, 1-5, 2-3 | 2-3        | 21              | timeout   |
| 3         | 2         | 1-2, 1-5, 2-6 | 2-3, 3-6   | 12              | timeout   |
| 3         | 3         | 1-5, 2-6, 4-6 | 2-3, 3-5, 3-6 | 2           | 1,931.10  |
| 3         | 4         | 1-5, 2-3, 4-6 | 1-3, 2-5, 3-4, 3-6 | 0   | 118.22    |
If only switches are added, total grid inadequacy can be reduced to 78% (absolute difference of 6% relative to base case) by adding Switches 1-2, 1-5, and 2-4 (see Figure 5-2 indicated in large rectangle). Switching these lines help relieve congestion in Lines 1-5, 2-3, or 3-5. Line switching alone can only achieve so much in this case study because of the limited excess capacity in other lines to pick up the added power flows. On the other hand, if only lines can be added, total grid inadequacy can be reduced to just 4% (absolute difference of 80% relative to base case) by adding Lines 2-3, 2-5, 3-4, and 5-6. These reinforcements provide additional capacity to frequently congested Line 2-3 as well as three new corridors for some of the power to flow. This solution provides much more significant reduction in total grid inadequacy than switching alone but comes at a much greater cost. Some savings may be achieved for the same level of total inadequacy by instead combining line addition in corridors 2-3, 3-5, and 3-6 with switches in Lines 1-5, 2-6, and 4-6 (see Figure 5-2 indicated in small rectangle). This option provides additional capacity in frequently congested Lines 2-3 and 3-5 as well as provide one new corridor for some of the power to flow. Having the ability to switch Lines 1-5, 2-6, and 4-6 provide additional flexibility in reducing power flows in Lines 1-5, 2-3, or 3-5 by redirecting power flows in other paths with extra capacity.
In later planning stages, these results can be translated into more detailed cost, benefit, and risk measures to further assess the merits of the solutions obtained from the proposed model.

5.3.2 118-bus test system

We also run case studies using the 118-bus test system with positive power injections in 54 buses, negative injections in 91, and with 186 existing lines. We consider 18 out of the 25 candidates in Section 4.3.2.1 for new line and switch installation whose maximum feasible loading is equal to the line capacity. We employ the same scenario generation method as with the 6-bus case study to identify 50 power injection scenarios for optimization purposes. We then generate instances of the optimization model for a given maximum number of lines and switches then solved the resulting model to optimality or for at most 24 hours. Table 5-2 shows the optimization results while Figure 5-3 graphs the resulting total grid inadequacy for the solutions obtained for different values of lines and switches added.

| Σu | Σx | Switches Added | Lines Added | | | Opt.Time, s |
|---|---|---|---|---|---|
| 0 | 0 | -- | -- | 44 | 1.59 |
| 0 | 1 | -- | 94-100 | 39 | 13.59 |
| 0 | 2 | -- | 30-38, 94-100 | 33 | 22.85 |
| 0 | 3 | -- | 30-38, 94-100, 99-100 | 31 | 50.15 |
| 1 | 0 | 94-100 | -- | 39 | 224.11 |
| 1 | 1 | 94-100 | 30-38 | 34 | timeout |
| 1 | 2 | 77-82 | 30-38, 80-99 | 31 | 2,717.42 |
| 1 | 3 | 8-30 | 30-38, 94-100, 99-100 | 30 | timeout |
| 2 | 0 | 8-30, 94-100 | -- | 38 | 862.53 |
| 2 | 1 | 8-30, 94-100 | 30-38 | 33 | timeout |
| 2 | 2 | 8-30, 77-82 | 30-38, 80-99 | 30 | timeout |
| 2 | 3 | 8-30, 99-100 | 30-38, 80-99, 94-100 | 29 | timeout |
Like the 6-bus case study, 88% of power injection scenarios are inherently infeasible without any new line or switch installed nor any grid operator intervention. If only two switches can be installed, total grid inadequacy can be reduced to up to 76% (absolute difference of 12% relative to base case) by adding switches in Lines 8-30 and 94-100 (see Figure 5-3 indicated in large rectangle). Switching out these lines help decongest Lines 30-38, 94-100, and 99-100 in some scenarios. If only up to three lines can be added, total grid inadequacy can be reduced to 62% (absolute difference of 26% relative to base case) by adding Lines 30-38, 94-100, and 99-100. As with the switch-only solution, the line-only solution help address congestion in Lines 30-38, 94-100, and 99-100, not through grid flexibility but through reduced inherent grid inadequacy. A slightly better level of total grid inadequacy at 60% can be achieved if Lines 30-38 and 80-99 and Switches 8-30 and 77-82 are added instead (see Figure 5-3 indicated in small rectangle).

As observed with the 6-bus case study, combining line and switch addition can achieve similar levels of total grid inadequacy as with costlier line-only solutions. Even with both options considered, however, there are still many infeasible scenarios and more investments will be necessary if further reductions in total grid inadequacy are desired. Furthermore, more detailed
assessments need to be performed to determine benefits and risks associated with choosing one option obtained from the model over another.

5.4 Conclusion

In this chapter, we extended the framework presented in previous chapters to relate the power flow infeasible set to inherent grid inadequacy, grid flexibility, and total grid inadequacy. We discussed how combined grid expansion and line switching can help reduce the size of this set. We extended the optimization model in Chapter 4 to find strategic line and switch additions that best reduce the size of the infeasible set for a given equipment budget. Test cases illustrate how new transmission lines can be completely avoided in some cases with the installation of two or three switches for the same reduction in total grid inadequacy.

In the next chapter, we explore how different line and switch expansion options can affect the infeasible set differently. Specifically, we’ll look at how the expansion options affect the feasibility of scenarios relative to the base case or to each other.
6  Effects of line and switch addition on the infeasible set\textsuperscript{5}

There are prior work illustrating the benefits of grid expansion, line switching, and their combination to address different grid operation and planning problems – see Section 6.1 below. However, literature review indicates that none has explored how line and switch addition together affect power flow infeasibility in the context of enabling future grids to host more diverse power injection conditions. In this chapter, we introduce the idea of grid congestion modes not only to classify seemingly unrelated infeasible power exchanges but also to explore how line and switch addition changes the infeasible set by addressing these congestion modes. We build on the ideas presented in Chapters 2, 4, and 5 that relate the DC power flow infeasible set to inherent grid inadequacy, grid flexibility, total grid inadequacy, line addition, and line switching.

Section 6.1 reviews the benefits of combined line and switch addition published in existing work. Section 6.2 introduces the idea of grid congestion modes and outlines the different possibilities on how line and switch addition affect the infeasible set and power exchange membership in grid congestion modes. Section 6.3 provides detailed discussions and visualizations for a 3-bus grid to illustrate the ideas in Section 6.2. Sections 6.4 and 6.5 provide case studies for the 6- and 118-bus grids respectively. Section 6.6 concludes.

6.1  Related work

Several works have illustrated the benefits of line addition in covering diverse operating states. Ref. [21] showed that strategic line additions can be used to hedge against uncertain market-

\textsuperscript{5} This chapter uses material from the following work: A. E. Tio, D. J. Hill, and J. Ma., “Integrating grid flexibility in grid expansion planning: Effects on the power flow infeasible set” (submitted for review)
driven power flows. Ref. [41] showed that specific line additions can host all scenarios in a pre-defined pool of load and RE combinations, assuming that other non-RE generators are freely dispatchable. The work also showed that different scenario pools result in different expansion options that cover varying feasible set percentages. Ref. [58] showed that line addition and RE curtailment can be economical in covering load and RE generation uncertainty. Ref. [112] showed that line additions can be used to hedge against the worst-case combination of RE generation and load within uncertainty intervals. In Chapter 4, we showed that strategic line additions can be used to minimize the set of infeasible power exchanges that a grid can inherently host. We also showed that different expansion options cover different infeasible power exchange sets.

Available work also illustrates the benefits of line switching in enabling alternative power injection schedules. Ref. [156] showed that line switching can enable less costly generator dispatches in known test grids. Using ISO New England data, [163] illustrated that line switching can enable the dispatch of originally infeasible generator schedules hindered by grid congestion. Ref. [157] showed that transmission switching can enable power exchanges with more wind generation. Ref. [158] showed that line switching can be used to hedge against the worst case RE generation scenarios and enable feasible generator schedules when none existed before switching. Ref. [159] illustrated that series compensation and line switching allows scenarios with more RE utilization and lower generation costs across several loading levels. Likewise, [160] showed that series compensation and line switching allows for more uncertainty in RE generation.

Some studies also explore the benefits of combined line and switch addition under diverse conditions. Ref. [64] showed that combined line and switch addition solutions can reduce the need for grid reinforcement, reduce generation costs, and increase RE utilization. Ref. [162] showed that line switching as a contingency response can reduce needed grid expansion investments. Ref.
[161] showed that co-optimizing line and energy storage system (ESS) additions with line switching can help reduce total costs and ESS capacity as well as help in managing load and RE curtailment. Similarly, [82] illustrated that co-optimized line, ESS, and switch addition alter grid expansion plans, reduce needed line or ESS investment costs, reduce generation costs, and help reduce load and RE curtailment. In Chapter 5, we showed that line and switch additions can be used to minimize the set of infeasible power exchanges.

The reviewed literature highlights the benefits of line and/or switch addition in enabling power exchange diversity. These works also indicate that line addition and switching decisions affect the feasibility of power exchanges. Aside from the work presented in Chapters 4 and 5 however, most work fails to consider the larger problem of considering market effects, RE intermittency, and load uncertainty simultaneously. Moreover, modeling the extent of changes to the feasible and infeasible sets when lines and switches are added is not yet fully explored. This chapter addresses these gaps in literature where we explore the mechanisms on how combined line and switch addition affect grid inadequacy for power injection diversity vis-à-vis the power flow infeasible set using the concept of grid congestion modes.

6.2 Theoretical background

6.2.1 Grid congestion modes

We define a grid congestion mode as a set of lines or transmission corridors that will get simultaneously overloaded if a specific power exchange is realized. Infeasible power exchanges that trigger congestion in the same set of lines belong to the same grid congestion mode. If the infeasible set exists for a given grid topology and power injection limits, there is at least one power exchange that triggers overloading in at least one line if realized. It then follows that at least one congestion mode exists and is defined by the set of overloaded lines. Larger infeasible sets may or
may not have more grid congestion modes however, since seemingly different power exchanges may belong to the same congestion mode.

The number of possible grid congestion modes depends in part on the number of lines susceptible to overloading. We can determine which lines are susceptible to overloading by solving for the maximum unconstrained line flows in each line, $f_{\text{max},u,ab}$ or $f_{\text{max},u,ba}$ (see Section 2.3.3.3 and Equations (2-20) and (2-21) for the linear programming model used to solve for these parameters). If either $f_{\text{max},u,ab}$ or $f_{\text{max},u,ba}$ exceed the line capacity, the line is susceptible to overloading, belongs to the set of lines susceptible to overloading $\mathcal{E}^*$, and contributes to the diversity of grid congestion modes. The number of unique 1-, 2-, …, and $|\mathcal{E}^*|$-combinations of lines susceptible to overloading is an upper bound to the number of grid congestion modes, where $|\mathcal{E}^*|$ is the cardinality of $\mathcal{E}^*$. For example, if three out of four lines are susceptible to overloading, the number of grid congestion modes cannot exceed seven, i.e. A, B, C, AB, AC, BC, ABC where A, B, and C identify each line in $\mathcal{E}^*$. Whether all combinations are realizable grid congestion modes depend on factors that affect the simultaneity of line overloading such as the spatial distribution of power injections, line admittances, and line capacities.

Ultimately, grid congestion modes are what hinder grids from hosting more diverse power injection conditions. Grid expansion via line addition or grid flexibility control intervention such as line switching can address some grid congestion modes, in partial or in full, leading to the feasibility or previously infeasible power exchanges. Unfortunately, line addition also redefines the mapping from $P$- to $f$-space that can lead to the infeasibility of previously feasible power exchanges. The relation between changes in grid topology, infeasible set, and grid congestion modes is complex and is best analyzed on a case-to-case basis specific to each grid. Nevertheless, we attempt to enumerate the possibilities below and provide illustrative case studies afterwards.
6.2.2 Effect of line addition

Line addition has the following effects: (i) add capacity along the corridor, (ii) attract flow along the corridor if it is in a loop, and (iii) increase or decrease flow in other lines in the loop depending on the power exchange. It has similar effects as increasing the line susceptance but also comes with an increased line flow limit. Line addition has the following effects to the mapping from \( P \)- to \( f \)-space: (i) dimensionality of the \( f \)-space hypersurface increases with the number of lines added, (ii) change in mapping function from \( P \)- to \( f \)-space distorts the original \( f \)-space hypersurface and projects it to higher dimensions, and (iii) additional capacity extends the \( f \)-space bounding box along new dimensions. The following effects apply for the special case when a line with a similar susceptance and capacity is added in parallel to an existing line: (i) dimensionality of the \( f \)-space hypersurface can be retained if lines in the same corridor are lumped as one, (ii) change in mapping function from \( P \)- to \( f \)-space pivots the original \( f \)-space hypersurface about the origin, and (iii) the \( f \)-space bounding box extends along the corridor proportional to the added capacity.

Changes in the mapping function and dimensionality of the \( f \)-space surfaces interact in complex ways to redefine the infeasible set that may be desirable, undesirable, or a combination of both. In terms of changes in grid congestion modes, there are five possibilities. Case 1 occurs when the added capacity sufficiently accommodates both the original and additional flow levels. Since the corridor is no longer susceptible to overloading, grid congestion modes involving the reinforced corridor can now be eliminated. Case 2 occurs when changes in power flows completely decongest non-reinforced corridors originally susceptible to overloading and eliminate congestion modes involving the decongested corridor. Case 3 occurs when the added capacity is not enough to host the resulting power flows, creating previously non-existent congestion modes involving
the corridor where the line was added. Case 4 occurs when non-reinforced lines originally not susceptible to congestion gets congested, creating new congestion modes involving these lines. Finally, Case 5 occurs when the added capacity and resulting power flow redistribution is enough to eliminate multi-line congestion modes but not enough to eliminate the possibility of congestion in a line, whether reinforced or not.

In terms of scenario feasibility, four scenarios are possible after line addition: (i) previously infeasible power exchanges become feasible, (ii) previously infeasible scenarios remain infeasible in the same congestion mode, (iii) previously infeasible scenarios remain infeasible but in a different congestion mode, and (iv) previously feasible power exchanges become infeasible in a pre-existing or new congestion mode. Case (i) is desirable as inherent grid inadequacy is reduced and as the grid hosts newly feasible power exchanges. Case (ii) is undesirable if the increased power flows in already congested lines make interventions impractical or more costly. Case (iii) is also undesirable if the change in congestion mode forces a different control intervention than what was previously adequate. And finally, Case (iv) is undesirable since the grid can no longer host previously feasible power exchanges without intervention. A combination of Cases (i)-(iv) generally results in a new set of feasible and infeasible scenarios different from that before line addition. Unless the newly added lines can be switched out or other grid flexibility control interventions are available, feasible-turned-infeasible power exchanges can no longer be hosted without power injection rescheduling or curtailment.

6.2.3 Effect of line switching

Like line addition, line switching is like susceptance control but in the extreme case when the line susceptance is reduced to zero. It comes with the following effects: (i) reduce the line power flow to zero in the switched out corridor and (ii) increase or decrease the flow in other lines
in the loop. In terms of the mapping from $P$- to $f$-space, line switching has the opposite effect of adding a line to the grid as follows: (i) dimensionality of the $f$-space hypersurface decreases in proportion to the number of lines switched out, (ii) change in mapping function from $P$- to $f$-space distorts the original $f$-space hypersurface and projects it in lower dimensions, and (iii) dimensionality of the $f$-space bounding box is reduced.

Unlike line addition that applies to all power exchanges, switching action does not need to be deployed in power exchanges where congestion remains after switching. As such, total grid inadequacy and the size of the infeasible set should only decrease or remain the same because of line switching. In terms of scenario feasibility, only previously infeasible scenarios that become feasible need to be accounted for.

6.2.4 Combined effects

If chosen properly, line addition and line switching can complement each other in reducing the size of the infeasible set as follows:

1. As mentioned in Section 6.2.2, line addition can increase power flows in other lines in the loop and trigger congestion in some lines. In cases like these, line switching in specific parts of the grid can be used to redirect power flows to lines with excess capacity, instead of adding new capacity in lines with increased power flows.

2. As mentioned in Section 6.2.3, line switching can increase power flows in other lines in the loop and trigger congestion. In cases where there is limited capacity in lines that see increased power flows, line addition can bring in additional capacity to support line switching.
Choosing the combination of lines and switches to be added that best reduce the infeasible set is a challenging problem however. In Chapter 5, we presented a linear programming model that finds line and switch additions that directly minimize the size of the infeasible set. In the succeeding sections, we explore how some of the solutions obtained in Chapter 5 alter the infeasible set through the various mechanisms enumerated in this section. We start with a 3-bus grid with visualizations and discussions to illustrate some of these mechanisms and possibilities. Then we proceed with the 6- and 118-bus test systems to illustrate how these ideas apply to larger grids.

6.3 Case study: 3-bus grid

6.3.1 Congestion modes

Figure 6-1 shows the line loading range diagram of the modified 3-bus grid with the following grid parameters. The power injection limits are 0.0-2.0, -1.0-1.0, and -1.0-0.0 per unit for Buses 1, 2, and 3 respectively. The line capacities are 0.7, 1.0, and 0.3 per unit while the line susceptances are 1.0, 0.5, and 0.5 per unit respectively for lines 1-2, 1-3, and 2-3. Lines 1-2 and 2-3 are susceptible to overloading since the maximum unconstrained line flows in these corridors exceed the available capacity. All three combinations of one- and two-line overloads are realizable congestion modes after evaluating a scenario pool of 1,225 scenarios sampled from the power injection space.
Figure 6-2 shows the infeasible set subdivided according to the three grid congestion modes. It shows how seemingly unrelated power injection scenarios actually trigger the same congestion mode and form clusters depending on how the $f$-space hypersurface intersect the $f$-space bounding box. For example, the following power exchanges expressed as $(P_1, P_2, P_3)$ seem unrelated but actually trigger the congestion of Line 2-3: $(0.35, 0.35, -0.70)$, $(0, 1, -1)$, and $(1, 0, -1)$. Likewise, the following set of power exchanges seem unrelated but trigger the congestion of Line 1-2: $(1, -1, 0)$ and $(2, -1, -1)$. Interestingly, the latter scenario is the scenario corresponding to the maximum network loading but do not correspond to a congestion mode that overload the greatest number of lines. Instead, two-line overloads are restricted to a small set of specific power exchanges near the following point: $(1.4, -0.4, -1.0)$.

(a) $P$-space  

(b) $f$-space

- No overloaded lines  
- Line 1-2 overloaded  
- Line 2-3 overloaded  
- Lines 1-2 and 2-3 overloaded

Figure 6-2: Infeasible set showing the three grid congestion modes, modified 3-bus grid

6.3.2 Effect of line addition

Figure 6-3 visualizes the changes in the feasible and infeasible sets as well as the grid congestion modes in $f$-space when identical lines are added in parallel to existing lines. Since lines
in the same corridor have the same susceptance, they will share the flow through the corridor equally. Since they also have the same capacity, they will also overload at the same time. Hence, we can lump the flow in one corridor using one line flow variable and preserve the dimensionality of the surfaces in \( f\)-space. This special case of adding identical lines to existing lines results in a simple pivoting of the \( f\)-space hypersurface about the origin as can be observed in Figure 6-3a to Figure 6-3c. Despite this, changes in the infeasible set and congestion modes are not as simple because of the complexity on how the newly pivoted hypersurface intersects the newly expanded bounding box.

All three line addition options share the effect of reducing the size of the infeasible set and eliminating the congestion mode that simultaneously overload Lines 1-2 and 2-3. These are achieved however via different mechanisms and come with other effects depending on the line added. Adding a new Line 1-2 eliminates the possibility of overloading this corridor because the new total capacity exceeds the new maximum unconstrained line flow. Because of this, two out of three of the base case congestion modes are eliminated and only the congestion mode associated with the overloading of Line 2-3 remains. Adding a new Line 2-3 provides additional capacity but is not enough to eliminate the possibility of overloading this corridor. It is enough however to reduce the size of the infeasible set in total and prevent the possibility of simultaneously overloading both Lines 1-2 and 2-3. Hence, two out of three congestion remain. Lastly, adding a new Line 1-3 adds capacity along this corridor. Choosing this option may seem counter-intuitive because the original corridor’s capacity is already enough to cover the new maximum unconstrained line flow. However, this options still helps reduce the size of the infeasible set and eliminate the two-line congestion mode via the diversion of some power flows away from bottleneck Lines 1-2 and 2-3 towards corridor 1-3.
(a) New Line 1-2 added

(b) New Line 2-3 added

(c) New Line 1-3 added

- No overloaded lines  
  
- Line 1-2 overloaded  
  
- Line 2-3 overloaded

Figure 6-3: Effect of line addition on the infeasible set and congestion modes, modified 3-bus grid

Figure 6-4 shows a breakdown of the change in scenario feasibility and congestion modes in the three line addition options. Each node in the figure classifies each scenario in the pool depending on which lines are overloaded. The string of numbers that label each node indicate whether Lines 1-2, 1-3, and 2-3 are overloaded respectively with a ‘1’. That is, node ‘000’ include all
feasible scenarios while ‘101’ include all scenarios that simultaneously overload Lines 1-2 and 2-3. The number inside the parenthesis indicate the number of scenarios belonging to the node while the number between nodes indicate the number of scenarios that changed from one congestion mode to another.

Figure 6-4: Mode transition diagram for three line addition options, modified 3-bus grid
Three transition cases are observable: transition from a congestion mode to feasibility, transition between existing congestion modes, and transition from feasibility to infeasibility. The first transition case is indicative of the efficacy of a given line addition option in reducing inherent grid inadequacy. In Figure 6-4, adding Line 1-3 yields the most change in the number of feasible states with 312 scenarios, followed by Line 2-3 with 288 scenarios and Line 1-2 with 20 scenarios. The second transition case relates to the extent of changes in needed control interventions when planning to add a new line. For example, power exchanges that change congestion mode from ‘100’ to ‘001’ in Figure 6-4a or from ‘001’ to ‘100’ in Figure 6-4b may need different control interventions when the line is added compared to the needed intervention in the reference case. Finally, the third transition case relates to the negative consequence of line addition where other lines gets overloaded after line addition. There are 70 such scenarios when Line 1-2 is added and 25 scenarios when Line 2-3 is added. Unless the newly added lines can be switched out or grid flexibility control interventions are available, these scenarios will either remain infeasible after line addition or will need power injection rescheduling or curtailment.

It is worth noting that reinforcing corridor 1-3 yields the largest feasible set among the three options evaluated – a seemingly counterintuitive result given that the other lines are the ones that lack capacity. Observations like these, those made in Figure 6-3 and Figure 6-4, and other considerations like which scenarios belong to the feasible and infeasible set and the complexity and prevalence of needed control intervention after line addition add nuance to grid expansion decision-making.
6.3.3 Effect of line switching

Figure 6-5 visualizes how portions of the infeasible set in the unswitched state can become feasible when one of the three lines are switched. The black and red points are the 2D projection of the base case $f$-space hypersurface in the plane of the unswitched lines. The black rectangle is the 2D bounding box representing the line capacities of the unswitched lines. The blue and orange dots show how the base case infeasible set (red points) is remapped when one line gets switched out. Because of the change in topology after switching, power flows get redistributed and result in one of the following possibilities: originally infeasible points (red points) become feasible (orange points) or originally infeasible points (red points) remain infeasible (blue points). The visualizations show that of the three switch addition options, adding Switch 1-2 or Switch 2-3 provides useful grid flexibility that help turn originally infeasible power exchanges feasible and contribute to reduced total grid inadequacy, while adding Switch 1-3 does not.

Figure 6-6 shows how power exchanges under the three congestion modes get reclassified when one of the lines get switched out. Unlike the diagrams for line addition, there is only one transition possibility: transition from a congestion mode to feasibility. We do not need to consider cases of transition between congestion modes and transition from feasibility to a congestion mode because switching in these cases do not reduce total grid inadequacy and hence, do not need to be performed.
Figure 6-5: Projection of the base case infeasible set in f-space when one line is switched out, modified 3-bus grid.
Figure 6-6: Mode transition diagram for the three switch addition options, modified 3-bus grid
The case of adding Switch 1-2 or Switch 2-3 illustrate the following mechanisms: (1) eliminating congestion in unswitched lines via power flow redistribution and (2) eliminating congestion in a congested line by switching it out.

The benefits of Switch 1-2 stem from the first mechanism. In Figure 6-6a, this is illustrated in power exchanges that belong in congestion mode ‘001’ (Line 2-3 overloaded) that turn feasible when Line 1-2 is switched. When Line 1-2 is switched, the loop breaks and all generation from Bus 1 can now flow directly to serve the load in Bus 3 through Line 1-3 with no capacity constraints. This eliminates power flows that would have been diverted to Lines 1-2 and 2-3 in the original loop, eliminate congestion in Line 2-3, and turn the power exchanges feasible.

On the other hand, the benefits of Switch 2-3 stem from both mechanisms as illustrated in power exchanges that transition from different congestion modes to feasibility, see Figure 6-6b. Power exchanges that transition from ‘100’ (Line 1-2 overloaded) to ‘000’ (no line overloaded) stem from the following mechanism: switching out Line 2-3 breaks the loop and reduces the loading of Line 1-2. Power exchanges that transition from ‘001’ (Line 2-3 overloaded) to ‘000’ illustrate the second mechanism. Switching out Line 2-3 eliminates congestion in the line itself and redirects the flow to the other lines. However, since there is not enough capacity in the other lines to pick up the slack in all scenarios originally associated with congestion mode ‘001’, this congestion mode remains. Finally, power exchanges that transition from ‘101’ (Lines 1-2 and 2-3 overloaded) to feasibility illustrate both mechanisms as follows: it eliminates congestion in Line 2-3 by switching it out itself and it eliminates congestion in Line 1-2 by eliminating the loop flows as a result of the loop breaking.

Lastly, adding Switch 1-3 illustrates the possibility of no net benefit to switching in terms of providing grid flexibility that reduce total grid inadequacy. No power exchange that was
originally infeasible can be made feasible by switching out Line 1-3. In this case study, this is because switching out Line 1-3 further increases the line flows in Lines 1-2 and 2-3 that are already susceptible to overloading in the first place.

6.3.4 **Combined effects**

Table 6-1 classifies the feasibility of scenarios according to congestion modes when combinations of line and switch additions are used to reinforce the grid. It highlights how the different options result in different infeasible set characteristics in terms of the size of the infeasible set $|J|$ and membership in one of the three congestion modes. Depending on the decision-making criteria such as cost and which power exchange scenarios have priority (e.g. in terms of perceived likelihood), one of these options may be preferable over the others.

If only switch additions are used, the best solution is to add Switch 2-3. Doing so reduces $|J|$ to 94, 20 of which triggers congestion in Line 2-3 and 74 triggers congestion in Line 1-2. If only line additions are used, $|J|$ can be reduced to zero but only through the addition of three lines, one new line across each corridor. Strategically adding one line (Line 1-2) and one switch (Switch 2-3) can also achieve the same effect but uses less equipment. The added capacity in Line 1-2 complements the switching of Line 2-3 by providing additional capacity to cover the additional power flows in Line 1-2 after switching. Cases such as these illustrate how line and switch addition can complement each other well and result in the same level of adequacy as with costlier options that only consider line additions.
Table 6-1: Scenarios per congestion mode per reinforcement case, modified 3-bus grid

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Congestion mode totals</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>‘000’</td>
<td>‘001’</td>
</tr>
<tr>
<td>1-2</td>
<td>1-3</td>
<td>2-3</td>
</tr>
<tr>
<td>+S</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>--</td>
<td>+S</td>
<td>--</td>
</tr>
<tr>
<td>--</td>
<td>--</td>
<td>+S</td>
</tr>
<tr>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>--</td>
<td>+L</td>
<td>--</td>
</tr>
<tr>
<td>+L</td>
<td>--</td>
<td>+L</td>
</tr>
<tr>
<td>+L</td>
<td>+L</td>
<td>+L</td>
</tr>
<tr>
<td>+L</td>
<td>+S</td>
<td>--</td>
</tr>
<tr>
<td>+L</td>
<td>--</td>
<td>+S</td>
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<tr>
<td>+S</td>
<td>+L</td>
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<tr>
<td>--</td>
<td>+L</td>
<td>+S</td>
</tr>
<tr>
<td>+S</td>
<td>--</td>
<td>+L</td>
</tr>
<tr>
<td>--</td>
<td>+S</td>
<td>+L</td>
</tr>
</tbody>
</table>

NOTE: +L means that a line is added, +S a switch; Recall: $|\mathcal{I}|$ is the size of the infeasible set, $|\mathcal{F}|$ is the size of the feasible set.

The line-only and line-and-switch solutions are not equivalent however when the congestion modes are considered as shown in Figure 6-7. The three-line option reduces total grid inadequacy to zero by reducing inherent grid inadequacy to zero. No grid flexibility control intervention is necessary to host the diverse power exchanges. This option also eliminates the possibility of overloading any of the three corridors and eliminates all congestion modes. As such, power
exchanges belonging in the three congestion modes in the base case transition to the feasible mode when the three lines are added (see Figure 6-7a).

(a) Lines 1-2, 1-3, and 2-3 added

(b) Line 1-2 and switch 2-3 added

Figure 6-7: Mode transition diagram, three line vs. the best one-line one-switch option, modified 3-bus grid

On the other hand, adding Line 1-2 and Switch 2-3 reduces total grid inadequacy using a combination of reducing inherent grid inadequacy via line addition and providing grid flexibility via line switching. It is a cheaper option relative to adding three new lines but one of the lines will remain susceptible to overloading. Out of the 1,225 scenarios, 490 scenarios will require switching out Line 2-3 to transition infeasible power exchanges in ‘001’ towards feasibility, see Figure 6-7b.

If N-1 reliability considerations pose an issue after switching, among other concerns, the line-and-
switch option may become less attractive. Other forms of grid flexibility control that reduce the flow in Line 2-3 below its capacity instead of switching it out present possible alternatives and will be interesting areas for future work.

6.4 Case study: 6-bus grid

To illustrate the benefits of combined line and switch addition and to show that different reinforcement options result in different infeasible set profiles and congestion modes in the 6-bus grid, we explore the solutions in Table 6-2 from Section 5.3.1.

Table 6-2: Grid reinforcement cases, 6-bus grid

<table>
<thead>
<tr>
<th>ID</th>
<th>Lines Added</th>
<th>Switches Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>REF</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>S3</td>
<td>--</td>
<td>1-2, 1-5, 2-4</td>
</tr>
<tr>
<td>L4</td>
<td>2-3, 2-5, 3-4, 5-6</td>
<td>--</td>
</tr>
<tr>
<td>L3S3</td>
<td>2-3, 3-5, 3-6</td>
<td>1-5, 2-6, 4-6</td>
</tr>
</tbody>
</table>

Of the eight lines in the original grid, only Line 2-4 is not susceptible to congestion. Lines 1-2, 1-4, and 4-6 are susceptible to minor overloading of up to 1.56%, 6.26%, and 6.77% of rated capacity respectively. Lines 2-6, 3-5, 1-5, and 2-3 are susceptible to moderate to severe overloading of up to 18.4%, 37.7%, 64.8%, and 168.4% respectively. Since seven lines are susceptible to overloading, there are a total of 127 possible 1-, 2-, to 7-combinations of lines susceptible to overloading. Since this is a manageable sum, we were able to check whether power exchanges exist that result in each of the 127 combinations of simultaneously overloaded lines. Results indicate that only 16 are realizable congestion modes. To represent the power injection space with at least one scenario from each congestion mode, we perform the following scenario sampling approach.
similar to those used in preceding chapters. We first divide each line’s unconstrained loading range spanned by the maximum unconstrained line loading values \( f_{\text{max}, u,ab} \) and \( f_{\text{max}, u,ba} \) into 100 line loading levels. Then, we compute for the power exchanges that result in a given line loading level \( F_{ab} \) that maximize the total load. Since this method was only able to capture 15 out the 16 congestion modes, we supplement the pool with power exchanges that result in each of the 16 congestion modes. Each power exchange was found using the following optimization model:

\[
\begin{align*}
\text{Objective} & \quad \max L \\
\text{Subject to} & \quad |f_{mn}| \geq kC_{mn} \quad \forall mn \in \mathcal{E}^+ \\
& \quad |f_{ab}| \leq C_{ab} \quad \forall ab \in \mathcal{E}^g \backslash \mathcal{E}^+ \\
\text{Equation (2-2) to (2-5)} & \quad (6-4)
\end{align*}
\]

where \( k = 1.01 \) is a minimum overloading threshold and \( \mathcal{E}^+ \) is the set of lines that will be simultaneously overloaded. Equation (6-2) forces the simultaneous overload of lines while (6-3) forces no overloading in the other lines.

The overall process yielded 816 scenarios. Table 6-3 shows how many scenarios fall under the different congestion modes per reinforcement case. As indicated in first two columns of Table 6-3, five are one-line congestions coded B-F, seven are two-line congestions coded G-M, and four are three-line congestions coded N-Q indicating that it is not possible to overload four or more lines at a time for any valid power exchange within limits. The last two rows are additional congestion modes associated with L4.

Without any reinforcements and grid flexibility control interventions, 81.5% of the sampled scenarios are infeasible in the reference case (Case REF). If grid reinforcements are limited to switch installations only with a maximum of three switches available, this number can be
reduced to 77.5% by adding switches 1-2, 1-5, and 2-4 (Case S3). Line switching helps remove the congestion in some scenarios in congestion modes B, C, and E that congest Line 1-5, Line 2-3, and Line 3-5 respectively. The number of congestion modes remain the same, however, because of the lack of capacity in the other lines to accommodate increased flows after switching. And as with the switching examples in the 3-bus case study, switching only facilitates transitions from a congestion mode to feasibility. Not all switching combinations are useful except for the following one- and two-line switched states: Line 1-2 switched, Line 1-5 switched, and Lines 1-2 and 2-4 switched. All switched states observed reduce infeasibility via power flow redistribution to decongest lines that are not the switched lines. For example, switching out Line 1-5 help decongest Line 2-3 in 3 scenarios by breaking the loop these lines form with Line 1-2 and 3-5. Likewise, switching out Line 1-2 help decongest Lines 1-5 or 3-5 in 25 scenarios by breaking the loop formed by these lines with Line 2-3. Switching both Lines 1-2 and 2-4 removes all inner loops in the grid to help decongest Line 3-5 in 5 scenarios.
Table 6-3: Scenarios per congestion mode per reinforcement case, 6-bus grid

<table>
<thead>
<tr>
<th>ID</th>
<th>Cong.Lines</th>
<th>REF</th>
<th>S3</th>
<th>L4</th>
<th>L3S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>151</td>
<td>184</td>
<td>787</td>
<td>789</td>
</tr>
<tr>
<td>B</td>
<td>1-5</td>
<td>29</td>
<td>12</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2-3</td>
<td>111</td>
<td>108</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>D</td>
<td>2-6</td>
<td>3</td>
<td>3</td>
<td>--</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>3-5</td>
<td>114</td>
<td>101</td>
<td>4</td>
<td>--</td>
</tr>
<tr>
<td>F</td>
<td>4-6</td>
<td>17</td>
<td>17</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G</td>
<td>1-2, 2-3</td>
<td>1</td>
<td>1</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>H</td>
<td>1-4, 2-3</td>
<td>4</td>
<td>4</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>I</td>
<td>1-5, 2-3</td>
<td>60</td>
<td>60</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>J</td>
<td>2-3, 2-6</td>
<td>41</td>
<td>41</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>K</td>
<td>2-3, 3-5</td>
<td>166</td>
<td>166</td>
<td>--</td>
<td>--</td>
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<tr>
<td>L</td>
<td>2-3, 4-6</td>
<td>21</td>
<td>21</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>M</td>
<td>3-5, 4-6</td>
<td>6</td>
<td>6</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N</td>
<td>1-2, 2-3, 4-6</td>
<td>3</td>
<td>3</td>
<td>--</td>
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<tr>
<td>O</td>
<td>1-4, 2-3, 2-6</td>
<td>9</td>
<td>9</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>P</td>
<td>1-5, 2-3, 2-6</td>
<td>76</td>
<td>76</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Q</td>
<td>1-5, 2-3, 4-6</td>
<td>4</td>
<td>4</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>R</td>
<td>5-6</td>
<td>--</td>
<td>--</td>
<td>7</td>
<td>--</td>
</tr>
<tr>
<td>S</td>
<td>2-6, 5-6</td>
<td>--</td>
<td>--</td>
<td>6</td>
<td>--</td>
</tr>
</tbody>
</table>
Line switching alone can only achieve so much especially since capacity is tight in the other lines. If at most four lines can be installed, the share of infeasible scenarios can be greatly reduced to 0.36% by adding Lines 2-3, 2-5, 3-4, and 5-6 (Case L4). This reduction is much larger compared to the reduction offered by switching alone but comes at a much higher cost. Combining line and switch addition can save costs while offering slightly better reduction in the infeasible set to about 0.33% by instead adding Lines 2-3, 3-5, and 3-6 and Switches 1-5, 2-6, and 4-6 (Case L3S3). Depending on the planning objective, L3S3 might be preferable over L4, or vice versa. Aside from cost, the feasible set spanned by the options as well as the congestion modes remaining need to be considered.

Figure 6-8 compares the congestion modes in L4 and L3S3. In terms of congestion modes, L3S3 only has three while L4 has five including two new congestion modes involving newly added Line 5-6. In the case of L4, 14 scenarios that originally belong to Congestion Modes J (Lines 2-3 and 2-6 overloaded) and O (Lines 1-4, 2-3, and 2-6 overloaded) remain infeasible but now belong to the two new congestion modes involving the newly added Line 5-6, see Figure 6-8a. This presents possible changes in control interventions if the dispatch of these power exchanges is desired after L4 is implemented. In terms of the number of lines susceptible to overloading, L4 has five (Lines 1-5, 2-3, 2-6, 3-5, and 5-6) while L3S3 only has three (Lines 1-5, 2-3 and 2-6). The possibility of congesting Line 3-5 is eliminated in L3S3 because of the added capacity in corridor 3-5 in this option.
While L3S3 seem to be better than L4 in this case study in terms of cost, size of the infeasible set, number of congestion modes, and number of lines susceptible to overloading, there is still merit in considering L4 over L3S3. Table 6-4 shows the number of power exchanges in the scenario pool that are uniquely feasible in each solution. From the table, 8 scenarios are uniquely feasible in L4 including scenarios where loads in Buses 2-5 are served by generators in Buses 1.
and 6. On the other hand, 10 scenarios are uniquely feasible in L3S3 including scenarios where generators in Bus 3 supply loads in Buses 2 and 4. Depending on factors such as the operational costs and emissions reduction associated with these scenarios, as well as the perceived likelihood and frequency of these scenarios occurring within the planning horizon, one reinforcement option may eventually be preferred over the other.

Table 6-4: Comparison of feasible scenarios for cases L4 and L3S3, 6-bus grid

<table>
<thead>
<tr>
<th>No. of scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>779</td>
<td>Feasible in both</td>
</tr>
<tr>
<td>8</td>
<td>Feasible only in L4</td>
</tr>
<tr>
<td>10</td>
<td>Feasible only in L3S3</td>
</tr>
<tr>
<td>19</td>
<td>Infeasible in both</td>
</tr>
</tbody>
</table>

6.5 Case study: 118-bus grid

As with the 6-bus case study, we explore the reinforcement cases in Table 6-5 from Section 5.3.2 to explore how the listed reinforcement cases affect the infeasible set and congestion modes differently.

Table 6-5: Grid reinforcement cases, 118-bus grid

<table>
<thead>
<tr>
<th>ID</th>
<th>Lines Added</th>
<th>Switches Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>REF</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>S2</td>
<td>--</td>
<td>8-30, 94-100</td>
</tr>
<tr>
<td>L3</td>
<td>30-38, 94-100, 99-100</td>
<td>--</td>
</tr>
<tr>
<td>L2S2</td>
<td>30-38, 80-99</td>
<td>8-30, 77-82</td>
</tr>
</tbody>
</table>
Without any reinforcements and grid flexibility control interventions, 58 out of the 186 lines are susceptible to overloading. There are $2.88 \times 10^{17}$ possible combinations of 1-, 2-, to 58-line simultaneous overloads. Since it is intractable to check whether each of these is realizable, we use a pool of 18,600 scenarios to sample representative congestion modes. We generate this pool as follows. We divide each line's unconstrained loading range into 100 line loading levels then find the power exchanges that result in a given line loading level that maximize the total load. We check the scenarios for simultaneous line overloads and found 821 congestion modes comprised of up to 1-40 lines that are simultaneously overloaded. Figure 6-9 shows the number of scenarios according to the number of simultaneously overloaded lines. About 42% of the scenarios result in just one to two simultaneous line overloads. This is indicative of considerable benefits in terms of reducing power exchange infeasibility even if only the one- or two-line congestion modes are addressed.

![Figure 6-9: Scenarios per number of simultaneous line overloads, 118-bus grid](image)

If only two switches can be installed, the size of the infeasible set can be reduced to up to 83.9% (absolute difference of $-11.2\%$ relative to the reference case) by installing switches in Lines...
8-30 and 94-100. Switching out Line 94-100 when it itself is congested help address this congestion mode. Moreover, switching out Line 94-100 also help alleviate congestion in nearby Line 99-100 in some scenarios as a result of power flow redistribution. Likewise, switching out Line 8-30 helps alleviate the congestion in Line 30-38 in some scenarios by diverting flows coming in from Line 8-30. But as with the 6-bus case study, adding switching capability alone can only achieve so much in reducing total grid inadequacy when other lines lack excess capacity.

If only up to three lines can be installed, the number of infeasible scenarios can be reduced to up to 55.9% (absolute difference of f –39.2% relative to reference case) by adding Lines 30-38, 94-100, and 99-100. These reinforcements try to address the same congestion modes as in the switch-only case but employ different means. Instead of switching out Line 8-30 to alleviate congestion in Line 30-38, L3 reinforces Line 30-38 with more capacity. Likewise, instead of switching out Line 94-100 to alleviate congestion in Lines 94-100 or 99-100, L3 reinforces these lines with new capacity each. As with the 3-bus and 6-bus case studies, line addition also resulted in the infeasibility of a couple of scenarios that are originally feasible. This happens when additional power flows congest Line 8-30 in some scenarios after flows increased after reinforcing Line 30-38.

Finally, if both lines and switches are installed, the number of infeasible scenarios can be reduced to up to 62.3% (absolute difference of +6.4% relative to L3) by adding Lines 30-38 and 80-99 and switches 8-30 and 77-82. This solution combines portions of S2 and L3 by adding Line 30-38 and Switch 8-30 to alleviate congestion along these corridors. But instead of reinforcing Line 94-100 as in L3 or switching it out as in S2, L2S2 addresses overloading in Line 94-100 by installing Line 80-99 and Switch 77-82. This combination complements one another in some scenarios as follows. Switching out Line 77-82 makes Bus 80 the only connection point of the
subnetwork containing Line 94-100 to the rest of the grid. This decongests Line 94-100 and loads Line 80-99 that is now aptly reinforced. These results show that in this case study, the benefits of investing in three lines is comparable to the benefits in investing instead in two lines and two switches if only the reduction in infeasible scenarios in the scenario pool is accounted for.

As with the 6-bus case study however, there is more nuance to making the case for either L3 or L2S2. Table 6-6 shows the number of scenarios that are uniquely feasible in these two reinforcement cases. Depending on the perceived likelihood and frequency of the uniquely feasible scenarios in the future as well as perceived benefits and consequences of hosting one set of uniquely feasible scenarios over another, one of the two options may be preferable.

Table 6-6: Comparison of feasible scenarios for cases L3 and L2S2, 118-bus grid

<table>
<thead>
<tr>
<th>No. of scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,654</td>
<td>Feasible in both</td>
</tr>
<tr>
<td>1,555</td>
<td>Feasible only in L3</td>
</tr>
<tr>
<td>359</td>
<td>Feasible only in L2S2</td>
</tr>
<tr>
<td>10,032</td>
<td>Infeasible in both</td>
</tr>
</tbody>
</table>

6.6 Conclusion

In this chapter, we explored how line and switch addition options affect the power flow infeasible set of a given grid. We introduced the idea of congestion modes not only to classify seemingly unrelated power exchanges that actually trigger the same congestion patterns but also to explore how different reinforcement options alter power exchange membership out of a congestion mode towards feasibility, out of one congestion mode towards another, or out of feasibility towards a congestion mode. We presented detailed case studies and visualizations using a 3-bus
grid to reinforce the concepts and to illustrate some possible scenarios. Finally, we presented case studies using the 6- and 118-bus test systems to illustrate how these ideas can be used in small- and medium-sized grids to assess the benefits of combined line and switch addition relative to the base case or to compare between expansion options.
7 Conclusion and future work

In this thesis, we have developed and explored ideas and tools for the infrastructure planning of flexible future grids under diverse power injection operating conditions. The ideas and tools presented are useful across the different stages of Transmission Network Expansion Planning (TNEP) process that we have outlined in Section 1.1 as follows:

1. To assess an existing grid’s ability to host diverse power exchanges, we first developed a framework and set of metrics in Chapter 2. In the framework, we motivated the use of the size of the power flow infeasible set relative to the size of the power injection space as a measure for grid inadequacy for power injection diversity – an idea that has not been proposed nor explored before. We devised three approaches to quantify the size of the infeasible set that circumvents the difficulty of characterizing the high-dimensional sets involved as follows. One uses a scenario-based approach that compares the number of infeasible and feasible power exchanges in a pre-defined scenario pool. The other two approaches project the high-dimensional surfaces in lower dimensions. Case studies illustrated how grid inadequacy metrics derived from these approaches can indicate grid inadequacy for hosting power injection diversity. The ability to quantify the extent of grid inadequacy for power injection diversity is a powerful tool in motivating more rigorous analyses that explore what these limitations are, what are the risks involved, and whether reductions in grid inadequacy can justify the cost of infrastructure investments.
2. To identify infrastructure investments that enable a grid to host more diverse power exchanges, we performed the following studies that build on the framework presented in Chapter 2:

a. In Chapter 3, we tested the validity of four graph-based grid expansion heuristics for identifying good line addition candidates. This study is inspired by existing literature that postulated that optimizing grid graph properties indirectly optimizes a grid’s inherent ability to host diverse power exchanges. A comparison of the ranking of grid expansion options in terms of the graph properties and the grid inadequacy metrics indicate that the postulation may not necessarily hold in all cases. This result is not surprising given that the graph metrics ignore important grid parameters like the spatial distribution of power injection sources and sinks and high- and low-capacity lines.

b. In Chapter 4, we developed a method that directly minimizes inherent grid inadequacy for power injection diversity. We presented a mixed integer linear programming model that finds line additions that minimize the number of infeasible scenarios in a scenario pool representing the power injection space. Case studies show that this approach can identify line addition solutions distinct from that of some existing models and with lower levels of grid inadequacy, albeit at an added cost. And since the generation dispatch is decoupled from the optimization process, the proposed approach also offers flexibility in scenario modeling and transparency in stakeholder consultations that some grid planners may find appropriate for their needs.
c. In Chapter 5, we expanded the model in Chapter 4 to minimize total grid inadequacy for power injection diversity by combining the benefits of line addition that reduce inherent grid inadequacy and line switching that provide grid flexibility. Case studies illustrate how integrating line and switch addition under one optimization framework can help defer the investment in new lines with strategic investments in new lines and switches for the same level of total grid inadequacy.

3. To help compare expansion options against the base case or between themselves, we presented the idea of grid congestion modes to characterize differences in the infeasible sets spanned. Illustrative examples show how different line and switch additions enable not only the transition of previously infeasible solutions belonging to different congestion modes towards feasibility (desirable) but also the transition between congestion modes (may be undesirable) and the transition from feasibility to a congestion mode (undesirable). These dynamics in scenario feasibility add nuance and complexity to TNEP that we believe grid planners and stakeholders will inevitably have to deal with in the future if more equitable grid access is desired.

The framework, ideas, and tools presented in this thesis are initial attempts in the contexts discussed above. There is much to be explored including the following:

1. On assessing grid ability to host power injection diversity:
   a. Can we develop other frameworks and metrics to characterize grid ability to host diverse power exchanges aside from using the power flow infeasible set?
   b. If the power flow infeasible set is used as basis for measuring grid inadequacy for power injection diversity, what other useful metrics can be developed aside from the metrics presented in Section 2.3.4?
c. How sensitive are the proposed metrics to changes in grid parameters?

d. Can we use the AC power flow model instead in the contexts wherein we opted to use the DC power flow model? To what extent will the results agree? To what extent will the results disagree?

e. How useful are the metrics proposed in larger grids and in actual operations and planning practice?

2. On finding grid expansion interventions:

   a. What are other methods to identify grid expansion options that best improve grid ability to host power injection diversity? How will these methods compare to the ones presented in this thesis?

   b. To what extent are the experimental results in Chapter 3 using graph-based grid expansion heuristics applicable? Are there special types of grids wherein the graph-based grid expansion heuristics explored results in optimal inherent grid inadequacy? How will other kinds of graph-based grid expansion heuristics perform in identifying grid expansion options that reduce inherent grid inadequacy? How would graph-based methods perform considering the non-linearities in actual grid operation not considered in this work?

   c. Is it better to use metaheuristic optimization, e.g. genetic algorithm or greedy randomized adaptive search procedures, instead of mixed-integer linear programming in minimizing the scenario-based representation of the infeasible set? Under which condition is one method preferable over the other?

   d. If using a scenario-based approach, how should scenarios be chosen? What are other possible targeted sampling approaches? How critical will scenario sampling
become in TNEP for future grids? How can we make the MILP model tractable for larger grids? Will approaches applied in other problems like relaxation of integrality constraints help?

e. Will using metaheuristic optimization to minimize non-scenario based inadequacy metrics, e.g. TILLR or CPNL in Section 2.3.4, identify better grid expansion options and use less computational resources?

f. In what other ways can we identify co-optimized new line and switch additions?

g. Are there effective graph-based heuristics that can inform the co-allocation of new lines and switches in a grid to improve its ability to host power injection diversity?

h. How will metaheuristic approaches to co-optimizing new line and switch additions compare to a mixed-integer linear programming approach?

i. How will integrating other grid flexibility control interventions such as line susceptance control via series compensation, voltage angle control via phase shifting transformers, or line flow control via HVDC links affect grid ability to host power injection diversity?

j. Would the models in Chapters 4 and 5 be tractable using convex relaxations of the full AC power flow model?

k. How will the tools in finding grid expansion options perform in larger grids and real-world grids?

3. On assessing grid expansion options:

   a. How should planners weigh differences in spanned feasible set against other criteria in multi-objective decision making for designing expansion plans?
b. In case a grid expansion investment is to be justified to promote grid access equity, how should the benefits be shared and the costs divided?

4. On what future grids should be like:

a. If a just and sustainable future grid is desired, are technological solutions, e.g. more renewables and more adequate flexible grids, enough? How necessary is it to consider behavioral solutions such as load adjusting to available generation, e.g. lifestyle changes, instead of the other way around?

b. In such alternative futures, to what extent should existing grids be reinforced and to what aim?
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