System reliabilities of steel frames designed by the inelastic analysis and design method in AISC 360-10

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ABSTRACT

Current methods for designing steel structures, such as the load and resistance factor design (LRFD) method are based on safety checks of individual members using elastic methods. The next generation of steel design methods will move from member-based to system-based design. Recent advances in nonlinear structural modelling make it possible to design a steel frame as a system rather than as a set of independent components, and several steel design specifications worldwide have incorporated provisions for designing for overall system behaviour. However, requirements for minimum system reliability have been implemented in such design-by-inelastic analysis methods rely on existing resistance factors originally developed from member reliability considerations. This paper examines the system reliabilities of a number of simple yet representative structures, including a continuous beam, a portal frame that fails elastically, and three related frames with significant (or limited) capacity of load redistribution. The paper provides an overview of the strengths and system reliabilities of these structures when designed by either second-order inelastic analysis or member-based LRFD provisions in the AISC Specification 360-10 and discusses the reliability implications.

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of these alternative approaches to steel structure design.

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**INTRODUCTION**

Appendix 1 - Inelastic Analysis and Design - of the American Steel Specification *AISC 360-10* (2010) permits, for the first time, the use of inelastic analysis of overall system response to design a steel frame. The inelastic analysis must take into account geometric and material nonlinearity, including the effect of residual stress and geometric imperfections. Appendix 1 states that “Strength limit states detected by an inelastic analysis that incorporates all sources of major nonlinear actions are not subject to the LRFD member-based design checks in the Specification when a comparable or higher level of reliability is provided by the analysis.” Appendix 1 represents a new system-based paradigm for steel frame design, and offers several important advantages over the existing LRFD method. A rigorous inelastic analysis can accurately determine the complex interactions between members of a large structural system, and capture the beneficial system effect of load redistribution after the initial formation of plastic hinges (*Ziemian et al.*, 1991; *Clarke et al.*, 1992; *Chen and Kim*, 1997; *White and Hajjar*, 2000; *Kim et al.*, 2001; *Trahair and Chan*, 2003; *Ngo-Huu et al.*, 2007; *Zhang and Rasmussen*, 2013; *Zhang et al.*, 2014). Thus, inelastic analysis often leads to the design of lighter and more economic structures than LRFD, which is based on a “first-hinge” approach. Inelastic analyses can determine structural performance from initial loading up to collapse, whereby designers are better able to understand the system behavior. This feature is especially important in the new paradigm of performance-based design which is closely coupled to the issue of system behavior. Since inelastic analysis can explicitly indicate the limit states (e.g., onset of structural nonlinearity, incipient system collapse, etc.), it becomes possible to identify different performance limits in design.

The inelastic method is different from the “Direct Analysis Method” (abbreviated as “DM” in the *AISC Specification*) as it appears in Chapter C of *AISC 360-10*. The DM
is based on a rigorous second-order elastic analysis that directly models member imperfections, and uses reduced member stiffnesses to account for the influence of inelasticity (including residual stress) on the stability of structures. While member and system stability are checked/detected by the analysis, the equation based design checks only need to be completed at the cross section level, as stipulated in the remaining Chapters of the Specification. While the DM eliminates the need for calculating effective length factors, it is still a member-based design approach as opposed to the system-level checks in the inelastic method. The inelastic method is an extension of the DM.

From a structural analysis point of view, the technical barriers to the use of inelastic method in practical design have diminished, as significant advances in computerized structural analysis have occurred during the last two decades. Structural analysis software used by structural engineers nowadays often incorporates various levels of inelastic analysis (plastic-hinge analysis or plastic-zone analysis). Unfortunately, even with the advanced nonlinear structural analysis method, the actual performance of a steel frame cannot be predicted with certainty because uncertainties in structural loads, material strength and stiffness will always be present. Because of this, Appendix 1 of AISC 360-10 requires that the inelastic analysis must take into account the uncertainties in system, member, and connection strength and stiffness. According to Section 1.3.1 of Appendix 1, the acceptable method for including uncertainty in system, member, and connection strength and stiffness is to reduce the yield strength and the stiffness of all steel members and connections by a factor of 0.90 for the inelastic analysis. The AISC 360-10 Commentary acknowledges that this reduction factor of 0.90 has its origin in the AISC LRFD resistance factors for tension and flexural members governed by the yield limit states; its use in system-based design, although “deemed acceptable”, is not based on any system reliability calibration. An important question thus has been raised: is the current reduction factor of 0.90 sufficient to fulfill the goal of assuring an “acceptable” level of structural reliability in system-based design?

In the past decade, several research efforts have been made to investigate the system
reliabilities of steel frames and determine the resistance factors for the system-based design by the inelastic method. Buonopane and Shafer (2006) compared the system reliabilities of a group of sixteen, closely related planar low-rise steel frames designed by both LRFD and inelastic analysis methods. The frames were subjected to gravity loads. For a target reliability index of 3.0 on system strength, it was found that the values of system resistance factor range from 0.86 to 0.91. This study only considered the uncertainties in the structural loads and the yield strength. Other random effects, such as the uncertainties in the cross-sectional properties, elastic modulus, and geometric imperfections, were ignored. This study also used a coefficient of variation (COV) of 0.10 for live load, which underestimated the variability of live load as various studies have shown that live load has a COV of about 0.25 (Ellingwood et al., 1982). Accordingly, the system resistance factors derived in Buonopane and Shafer (2006) may be somewhat higher than warranted.

Zhang et al. (2014, 2016a,b) presented a framework for analysing the system reliability of steel frames and determining the system resistance factors for the design-by-inelastic method. The reliability framework is based on the First-Order Reliability Method (FORM). The system resistance factors were calculated for a series of low- to mid-rise planar braced and moment resisting frames for various target reliability levels. Different failure modes and loading conditions (e.g., live-to-dead load ratios) were considered. All important random variables were considered in the reliability assessments, including the uncertainties in yield strength, modulus of elasticity, cross-sectional properties, residual stress, initial geometric imperfections, and structural loads. The study showed that a system resistance factor of 0.80-0.85 would be required for the inelastic method to achieve a target system reliability index of 3.0-3.25.

The steel frames studied in Buonopane and Shafer (2006), Buonopane (2008) and Zhang et al. (2014, 2016a,b) all sustained significant yielding (formation of multiple plastic hinges) when the frames were at the state of incipient collapse. For this category of structures, design by inelastic methods can utilize the frame strength remaining after first yielding to sustain
further loading. The design strengths of such frames predicted by the inelastic analysis can be 10%-30% higher than those estimated from the current elastic-LRFD method. Studies have shown that if such frames are designed by the inelastic method, their system reliability would be comparable to the member reliability implied in current LRFD specifications (Buonopane and Shafer, 2006; Zhang et al., 2014, 2016a,b). On the other hand, there are other types of steel frames in which the system strengths predicted by elastic-LRFD and by inelastic analysis are identical or close, frames in which static redundancies are limited or that are redundant but fail elastically. System reliabilities and resistance factors for this category of structures also must be investigated.

In this paper, we examine the system reliabilities of a number of simple yet representative steel structures, including a continuous beam, a portal frame that fails elastically, a frame with significant capacity for load redistribution, and two frames with limited capacity to redistribute load. We compare the design strengths and system reliabilities of these structures designed by LRFD and the inelastic method. Such a comparison can shed light on the system reliability implications of LRFD and the inelastic method, and the suitability of the current resistance factor stipulated in Appendix 1 of AISC Specification 360-10.

**DESIGN BY INELASTIC METHOD**

Probability-based limit state design criteria have the general format:

\[
\text{Design strength} > \text{Required strength.} \quad (1)
\]

In a system-based safety check, the required strength for a frame is defined by the loads applied to the complete frame, i.e.,

\[
\text{Required strength} = \sum \gamma_i Q_{ni}, \quad (2)
\]

in which \(Q_{ni}\) are nominal loads, and \(\gamma_i\) are the load factors determined from the load combination rules specified in loading standards (e.g., ASCE Standard 7-10 (ASCE, 2010)). We
assume that the current load combination rules, which were developed for member-based safety checks, are equally applicable to system-based safety checks.

When conducting an inelastic analysis for design, the loads are applied incrementally to push down or push over the frame, depending on whether the system capacity under gravity loads or gravity plus lateral (wind) loads is sought. In either case, the actual applied loads are the product of the full load combination and an applied load ratio $\lambda$. The general procedure for checking the integrity of a structural system by the inelastic design method in Appendix 1 of *AISC 360-10* can be summarized as:

1. Develop an inelastic analysis model using the reduced nominal values of yield stress $(0.9F_y)$ and modulus of elasticity $(0.9E)$ for all members;
2. Apply the loads $\lambda \sum q_n i$ by increasing the applied load ratio $\lambda$ incrementally until collapse of the frame;
3. Check if the ultimate load ratio $\lambda_u \geq 1$.

In design by the inelastic method, the system strength limit state is characterized by frame collapse/instability. The design strength of a frame is defined as the peak load in the frame’s load-displacement response. If the load-displacement response does not have a descending branch, it is assumed in this study that the ultimate strength is reached when the slope of the load-displacement curve reduces to 5% of its initial value (Ziemian et al., 1991).

**DESCRIPTIONS OF EXAMPLE FRAMES**

Five planar structures are investigated in this paper. The first, shown in Fig. 1, is a three-span continuous beam. The yield stress and modulus of elasticity are 345 MPa and 200 GPa, respectively. The second is a portal frame, shown in Fig. 2. The two columns are oriented for minor-axis bending and the beam for major-axis bending. The steel has a nominal yield stress of 345 MPa with a modulus of elasticity of 200 GPa. An initial out-of-plumbness of 1/500 is introduced. Cases 3, 4 and 5 are closely related two-bay two-story non-symmetric frames, as shown in Fig. 3. The three frames have the same layout (adopted from Ziemian
et al. (1991)). However, their member sizes and loads are different, as given in Table 1. The nominal yield stress and modulus of elasticity are 320 MPa and 200 GPa, respectively. The three frames all have an initial out-of-plumbness of 1/500. In all cases, residual stresses are assumed to distribute according to the pattern suggested in Galambos and Ketter (1959). All five structures are subjected to gravity load only.

FIG. 1: Case 1: a three-span continuous beam.

FIG. 2: Frame 2: a portal frame.

DESIGN STRENGTHS BY LRFD AND INELASTIC METHODS

For each structure, the design strength is determined using two methods, i.e., the elastic LRFD method and the inelastic method as it appears in Appendix 1 of the AISC Specification 360-10. With the inelastic method, the applied load ratio is increased until the frame collapses. The ultimate load ratio is denoted by $\lambda_{in}$. The inelastic analyses were performed
FIG. 3: Frame 3, 4 and 5: two-bay two-story frame.

TABLE 1: Member sizes and loads for Frame 3, 4 and 5.

<table>
<thead>
<tr>
<th></th>
<th>Frame 3</th>
<th>Frame 4</th>
<th>Frame 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>W12×19</td>
<td>W6×15</td>
<td>W6×15</td>
</tr>
<tr>
<td>C2</td>
<td>W14×159</td>
<td>W14×99</td>
<td>W14×68</td>
</tr>
<tr>
<td>C3</td>
<td>W14×145</td>
<td>W14×68</td>
<td>W14×68</td>
</tr>
<tr>
<td>C4</td>
<td>W6×9</td>
<td>W6×8.5</td>
<td>W6×8.5</td>
</tr>
<tr>
<td>C5</td>
<td>W14×145</td>
<td>W14×145</td>
<td>W14×145</td>
</tr>
<tr>
<td>C6</td>
<td>W14×145</td>
<td>W14×145</td>
<td>W14×145</td>
</tr>
<tr>
<td>B1</td>
<td>W30×116</td>
<td>W30×124</td>
<td>W30×132</td>
</tr>
<tr>
<td>B2</td>
<td>W36×182</td>
<td>W36×182</td>
<td>W36×182</td>
</tr>
<tr>
<td>B3</td>
<td>W24×55</td>
<td>W24×55</td>
<td>W24×55</td>
</tr>
<tr>
<td>B4</td>
<td>W30×116</td>
<td>W30×116</td>
<td>W30×116</td>
</tr>
</tbody>
</table>

Loads \((P_0)\) 146.93 kN/m 142.49 kN/m 107.25 kN/m

using the software package OpenSEES (Mazzoni et al., 2007). In the case of LRFD, the applied load ratio is increased until one of the members reaches its LRFD limit state, i.e., the left-hand side of the beam-column interaction equation of Chapter H in the AISC Specification 360 -10 is equal to 1.0. The applied load ratio corresponding to the LRFD limit is denoted by \(\lambda_e\). All structures considered are code-compliant.

The design strength of the three-span continuous beam in Case 1 is \(\lambda_e = 1.0\) by LRFD and \(\lambda_{in} = 1.29\) for the inelastic method. Fig. 4(a) plots the applied load ratio versus the vertical displacement at point C using the inelastic method. Fig. 4(b) shows the locations
of three plastic hinges developed at the inelastic ultimate limit point. As the inelastic analysis performed in this study is a plastic-zone type of inelastic analysis (as opposed to a plastic hinge type of analysis), it is assumed that a plastic hinge has developed if the cross-sectional yield ratio (percentage of cross-sectional area that has yielded) exceeds 75%. In this example, the design strength given by the inelastic method is 29% higher than LRFD. This is not surprising as the continuous beam has a significant capacity to redistribute loads after first yielding.

A continuous beam can also be designed using the traditional plastic design method, in which a resistance factor of 0.90 is applied to the plastic moment capacities of all members. The design strength given by the plastic design method is \( \lambda = 1.29 \), identical to that of the inelastic method. This is to be expected as the traditional plastic design method is also based on a collapse limit state.

The design strength of the simple portal frame in Case 2 is \( \lambda_c = 1.0 \) by LRFD, and \( \lambda_{in} = 1.02 \) by the inelastic method. Thus, the two design strengths are very close to each other. This is because this frame consists of slender members, and the frame fails essentially elastically. There are significant geometrically nonlinear effects, but no plasticity.

The frame in Case 3 developed significant plasticity at its inelastic ultimate limit state, as shown in Fig. 5(b). The load-roof displacement response from the inelastic analysis is
plotted in Fig. 5(a). The design strength of the frame is $\lambda_e = 1.0$ for LRFD and $\lambda_{in} = 1.19$ for the inelastic method.

![Frame 3: (a) load vs. roof displacement, (b) plastic hinges and section yield ratios at inelastic limit point.](image)

Frame 4 represents a structure in which every beam and column is designed at or close to its LRFD-defined limit state. Fig. 6 shows the LRFD beam-column interaction equation (H1.1) calculations for every member of Frame 4, showing that the interaction equation calculations are all close to unity. The design strength of Frame 4 is found to be $\lambda_e = 1.0$ by LRFD, and $\lambda_{in} = 1.06$ by the inelastic method. The difference between the frame capacities determined by the LRFD and inelastic methods is only minor. The inelastic analysis shows that the frame has developed two plastic hinges at the ultimate limit point, one at the right end of beam B1 and one at the bottom of column C6. The two plastic hinges developed almost simultaneously when the applied load ratio $\lambda$ was about 0.96. This frame has only a limited capability for redistributing forces after first yield. Fig. 7(a) and 7(b) show the load-roof displacement curve and locations of plastic hinges at the inelastic limit point.

Finally, Frame 5 represents a structure in which the strength is governed by one critical member. The design strength is $\lambda_e = 1.0$ for LRFD, and $\lambda_{in} = 1.08$ by the inelastic method. Since the frame has limited capability for load redistribution following the failure of the slender column C2, the system strength given by the inelastic method is only marginally higher than that given by LRFD. The inelastic analysis reveals that at the collapse limit,
FIG. 6: Frame 4: LRFD beam-column interaction equation calculations for every member.

FIG. 7: Frame 4: (a) load vs. roof displacement, (b) plastic hinges and section yield ratios at inelastic limit point.

the slender column C2 is partially yielded with a yield ratio of about 58% (see Fig. 8(b)). Other members are all within their elastic limits at failure. Inelastic yielding only plays a minor role in this example, to a much lesser extent than in Case 1 and Case 3.

SYSTEM RELIABILITY ANALYSIS

Monte Carlo simulation

The collapse probabilities of the five structures designed by LRFD and inelastic methods were determined and compared. As shown in the previous section, the design strengths of a frame given by elastic LRFD and the inelastic method are generally different. The nominal load that a structure can support is $P_0 \cdot \lambda_e$ in the case of LRFD, and $P_0 \cdot \lambda_{in}$ in the case of
FIG. 8: Frame 5: (a) load vs. roof displacement, and (b) section yield ratio at inelastic limit point.

The inelastic method, where $P_0$ is the reference load as shown in Figs. 1 to 3 and in Table 1. The structures are at their design limits in both design methods. Under this assumption we can meaningfully compare the reliabilities associated with both design methods.

The reliability assessments considered the following sources of uncertainties: live load and dead load, yield strength ($F_y$), Young’s modulus ($E$), cross-sectional area ($A$) and moment of inertia ($I$) of each member. Table 2 summarizes the statistical information for these basic random variables. The properties among all columns, and among all beams are assumed to be perfectly correlated. However, the properties between beams and columns are uncorrelated.

The structural loads shown in Figs. 1 to 3 represent the gravity load combination $1.2D_n + 1.6L_n$, and the nominal live-to-dead load ratio is assumed to be $L_n/D_n = 1.5$. As the variability of live load is greater than that of dead load, a structure’s reliability is dependent on the live-to-dead load ratio. Typical nominal load ratios $L_n/D_n$ for steel structures vary from 0.5 to 5.0. The ratio 1.5 adopted in this study is approximately the weighted average of the nominal live-to-dead load ratios for steel structures (Ellingwood et al., 1982). This ratio, being greater than 1.0, reflects the fact that for steel structures, the live load tends to be greater than the dead load.

The system failure probability (for collapse), denoted by $P_f$, for each structure was eval-
TABLE 2: Description of basic random variables.

<table>
<thead>
<tr>
<th>variable</th>
<th>mean nominal</th>
<th>COV</th>
<th>Distribution</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1.05</td>
<td>0.1</td>
<td>Normal</td>
<td>Ellingwood et al. (1982)</td>
</tr>
<tr>
<td>L</td>
<td>1.0</td>
<td>0.25</td>
<td>T1Largest</td>
<td>Ellingwood et al. (1982)</td>
</tr>
<tr>
<td>$F_y$</td>
<td>1.10</td>
<td>0.06</td>
<td>Lognormal</td>
<td>Bartlett et al. (2003)</td>
</tr>
<tr>
<td>E</td>
<td>1.0</td>
<td>0.04</td>
<td>Lognormal</td>
<td>Bartlett et al. (2003)</td>
</tr>
<tr>
<td>A</td>
<td>1.0</td>
<td>0.05</td>
<td>Normal</td>
<td>Ellingwood et al. (1982)</td>
</tr>
<tr>
<td>I</td>
<td>1.0</td>
<td>0.05</td>
<td>Normal</td>
<td>Ellingwood et al. (1982)</td>
</tr>
</tbody>
</table>

COV = coefficient of variation

uated by More Carlo simulation. In each simulation, a random sample of the frame is

generated with randomly generated values for material properties and cross-sectional prop-

ties, according to their statistics in Table 2. Then an inelastic analysis was performed to
determine if the frame would collapse under a random sample of the dead load and live load.

After a sufficient number of simulations (20,000 to $10^6$ simulations in this study depending
on the value of $P_f$), the failure probability $P_f$ is estimated by $P_f \approx n/N$, where $N$ is the
total number of simulations, and $n$ is the number of simulations in which the frame collapsed
(Melchers, 1999).

In applying structural reliability theory to structural code development, the probability
of failure $P_f$ is customarily converted to a reliability index $\beta$, which serves as an alternative
and more familiar measure of reliability (Ellingwood, 1994). The reliability index is related
to the probability of failure by $\beta = \Phi^{-1}(1 - P_f)$, in which $\Phi^{-1}(\cdot)$ represents the inverse
function of the standard normal distribution function $\Phi(\cdot)$. In the following discussions, the
system reliability index is denoted by $\beta_s$, in which the subscript $s$ emphasizes it is a system
reliability.

**System reliability results**

The system reliability indices for the five example structures designed by LRFD and the
inelastic method are summarized in Table 3. Before we discuss the results of system reliability
assessment, it should be noted that in the current LRFD of steel structures, the target
reliability index for compact rolled beams (plastic hinge limit state) and tension members (yielding limit state) is approximately 2.6-2.8 under the ASCE 7 gravity load combination (Ellingwood, 1994, 2000). In the following discussions, the term “member reliability” implies a reliability index about 2.6-2.8.

TABLE 3: System reliability indices and probabilities of failure for LRFD and inelastic method.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>Inelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_s$</td>
<td>$P_f$</td>
</tr>
<tr>
<td>Beam</td>
<td>3.74</td>
<td>$9.201 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Frame 2</td>
<td>2.94</td>
<td>$1.641 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Frame 3</td>
<td>3.44</td>
<td>$2.909 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Frame 4</td>
<td>3.07</td>
<td>$1.070 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Frame 5</td>
<td>3.09</td>
<td>$1.001 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

For the continuous beam, $\beta_s$ is 3.74 for the elastic LRFD, 2.68 for the plastic design method, and 2.68 for the inelastic method. The plastic design method and the inelastic method lead to the same design strength, and thus the same system reliability. The probabilities of failure for elastic LRFD and the inelastic method differ by nearly two orders of magnitude difference (approximately $9.0 \cdot 10^{-5}$ vs $3.7 \cdot 10^{-3}$). A similar observation is made for Frame 3, in which $\beta_s$ for LRFD is 3.44, significantly higher than that of the inelastic method ($\beta_s = 2.71$). Both the continuous beam and Frame 3 have significant capabilities for redistributing force following first yield.

Frame 2 shows a different characteristic in its system reliability than the continuous beam or Frame 3. Its system design strengths given by LRFD and the inelastic method are approximately the same, and thus its system reliability indices also are very similar; $\beta_s$ is 2.94 for LRFD and 2.84 for the inelastic method. This is to be expected, as Frame 2 fails essentially elastically with little yielding and force redistributing is not possible.

The system reliability $\beta_s$ for Frame 4 is 3.07 ($P_f = 1.07 \cdot 10^{-3}$) if designed by LRFD, and 2.79 ($P_f = 2.64 \cdot 10^{-3}$) if designed by the inelastic method. The two failure probabilities are
of the same order of magnitude. The system reliability of Frame 4, if designed elastically, is only marginally higher than the member reliability. This observation is different from Frame 3, although both are redundant frames. However, in Frame 4, all members are sized close to the LRFD limit state, and the frame has only limited capability for redistributing forces following first yield. In this case, the difference between system reliability and member reliability is only minor.

Finally, the reliability result of Frame 5 is similar to that of Frame 4; $\beta_s$ is 3.09 for LRFD and 2.73 for the inelastic method. The two design methods lead to two system failure probabilities of the same order of magnitude. This is because Frame 5, like Frame 4, only has limited capability for load redistribution following the failure of the critical member (column C2).

Based on the system reliability analyses of these five structures, some general observations can be made. If the frames are designed elastically to current practice, the system reliability indices are quite scattered; they vary from 2.94 (Frame 2) to 3.74 (the continuous beam), confirming the conventional wisdom that system reliability is generally higher than individual member reliability. The difference between system reliability and member reliability depends on the extent to which the structural system permits load redistribution.

FIG. 9: System reliability index versus $\lambda_{in} - \lambda_e$. 
following first yield. This difference is greater for the structures with greater capability for inelastic load redistribution, e.g., as seen in the continuous beam and Frame 3. Fig. 9 plots $\beta_s$ versus the difference, $\lambda_{in} - \lambda_e$, for the five structures, this difference represents the structure’s capability for inelastic load redistribution. It can be seen that $\beta_s$ of the elastic LRFD is approximately linearly proportional to $\lambda_{in} - \lambda_e$.

On the other hand, if the design is based on inelastic behavior, the $\beta_s$ for different structural systems are very consistent, ranging from 2.7 to 2.8 for the five frames as shown in Fig. 9, despite their very different failure modes (elastic/inelastic) and structural behaviors (capability for load redistribution). The consistency of the inelastic design method in meeting system reliability goals can also be seen from Fig. 9. Similar observations have been made elsewhere (Zhang et al., 2014). This study confirms that design by inelastic analysis is better able to achieve uniform system reliabilities than the current member-based elastic methods.

Another point that is worthy of note is that the $\beta_s$ for the inelastic method, being around 2.7-2.8, are about the same as the target member reliability index used for many of the LRFD calibrations (Ellingwood, 1994, 2000). The reason that the inelastic method leads to lower system reliabilities than LRFD is because the inelastic analysis takes much of the reserve strength (after first yield) out of the system that elastic LRFD design leaves in.

TARGET SYSTEM RELIABILITY INDEX

By using the inelastic design method, it is possible to achieve a essentially uniform system reliability for steel frames with different failure modes (elastic/inelastic) and behaviors (capability for load redistribution). In particular, the inelastic design procedure in the Appendix 1 of AISC Specification 360 -10 which applies a reduction factor of 0.90 to the yield strength and stiffness of all members for the inelastic structural analysis, appears to achieve a consistent system reliability index of 2.75 under gravity loads. This finding raises an important question: is a system reliability index of 2.75 sufficiently safe (or “acceptable”) for a steel frame? In other words, is the reduction factor of 0.90 currently stipulated in the Appendix 1 of AISC 360 adequate?
The target reliabilities for the current *LRFD Specification* were obtained from a calibration procedure involving the reliability evaluations of a large number of members designed using traditional acceptable practice (*Ellingwood et al.*, 1982; *Ellingwood*, 1994). It would be tempting to establish the target system reliabilities for inelastic design methods using a similar calibration procedure. However, the range of system reliabilities implied by the current *LRFD Specification* is quite wide. As shown in the previous section, the system reliability indices for the four frames (Frames 2 to 5) designed by the current elastic-LRFD practice span the range from 2.94 for Frame 2 which fails with virtually no material nonlinear action to 3.44 for Frame 3 which has a significant capability of load redistribution. All these values of system reliability index are greater than the target member reliability index originally stipulated for LRFD, which is approximately 2.6-2.8. On the other hand, the system reliability index for the continuous beam is 2.68 if designed by the traditional plastic design method, which has been an acceptable practice for many decades. Thus, at least for continuous beams, a system reliability index as low as 2.7 would be acceptable.

The situation is more complex if frames, rather than continuous beams, are considered. The selection of a target system reliability index should consider the mode and consequences of structural failure (*Ellingwood*, 2001). The consequence of system collapse invariably is more severe than the consequence of individual member failure in an indeterminate structure. Furthermore, failures due to instability often occur more suddenly (especially if bifurcation occurs) than failures due to yielding or plastic hinge formation, and may cause the entire structure, or large portions of it, to collapse suddenly. If a distinction for mode/consequence of failure is to be made (as suggested by Commentary Section C1.3.1 to *ASCE Standard 7-10*, which gives target reliabilities for different building Risk Categories), one could argue that a higher target reliability should be stipulated for a system than for a member. On the other hand, if the system target reliability is set too high, there will not be sufficient economic incentives for engineers to adopt the more rigorous and performance-oriented inelastic design method. A suitable target system reliability should be agreed upon by the stakehold-
ers in the building design process - professional engineers, researchers, and the regulatory community (Ellingwood, 2001).

The results of this study suggest that the target system reliability index for steel structures designed by the Inelastic Analysis and Design method should fall in the range 3.0 to 3.25 under gravity loads. Under these conditions, the system failure probability would be less than the member failure probability by a factor of approximately 2 to 5. To achieve this reliability goal for systems designed by the inelastic method, the current resistance factor of 0.90 stipulated in the Appendix 1 of AISC Specification 360-10 may need to be reduced to the range 0.80-0.85.

CONCLUSIONS

Design by second-order inelastic analysis represents a new system-based paradigm for steel design. This paper compares the strengths and system reliabilities of five representative structures designed by LRFD and the inelastic method. The five example structures exhibit different failure modes and structural behaviors.

The continuous beam and Frame 3 both have significant capacity for inelastic load redistribution. The system strengths predicted by the inelastic analysis are significantly higher (more than 18%) than those from elastic LRFD. On the other hand, Frame 2 fails with very little inelastic action, and the system strengths given by LRFD and by inelastic method are almost identical. Frame 4 and Frame 5 have limited capability of load redistribution, and the difference in system strengths given by LRFD and inelastic method is minor (within 8%).

While the current LRFD method was calibrated to achieve a uniform member-reliability, the system reliabilities of frames designed by LRFD are quite variable, varying from 2.94 (Frame 2) to 3.44 (Frame 3). The difference between system reliability and member reliability depends on the extent to which the frame can redistribute load after the initial formation of plastic hinges.

The inelastic method, on the other hand, leads to consistent system reliabilities for dif-
ferent structures; the system reliability index is about 2.7 to 2.8 for all five structures investigated, despite their very different failure modes and structural behaviors. This result demonstrates that system-based design by inelastic analysis is better able to achieve uniform system reliabilities than the current member-based LRFD method. This is to be expected, as the inelastic method is explicitly based on overall system behaviors.

The current inelastic design procedure in Appendix 1 of *AISC Specification 360-10* reduces the yield strength and stiffness of all members and connections by a factor of 0.9 for the inelastic analysis. This procedure leads to a consistent system reliability index of about 2.75, which is comparable to the target member reliability implied by the current LRFD method. Considering that the consequence of system collapse is more severe than that of individual member failure in an indeterminate structure, the authors believe that it might be desirable to adopt a target system reliability index somewhat higher than the member reliability index, e.g., a target system reliability index in the range of 3.0 to 3.25. To achieve this level of system reliability, the reduction factor for the yield strength and stiffness of all members may need to be reduced from the current value of 0.90 to 0.85. However, the determination of target system reliability index ultimately must be agreed upon by the professional, research, and regulatory communities.

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