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Designing a Dynamic Matching Method for Ride-Sourcing Systems

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Designing a Dynamic Matching Method for Ride-Sourcing Systems

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ABSTRACT
The ubiquity of smart-devices enables the foundation for emerging fast-growing ride-sourcing companies that challenges the traditional taxi services. A crucial aspect of designing ride-sourcing systems is matching mechanism between idle ride-sourcing vehicles and passenger travel requests (i.e. vehicle-passenger matching). Forcing ride-sourcing vehicles to pick up long-distance waiting passengers causes wild goose chase problem that can dramatically increases search time. In this paper, a non-equilibrium based spatio-temporal vehicle-passenger matching method is introduced to determine dynamically and jointly the matching time instances and maximum matching distances to minimize passengers' waiting time (i.e. from the travel request until the pickup) while considering the level of congestion of the network. The performance of the matching method under noticeable variation of traffic congestion and passenger travel requests are investigated with microsimulation.

KEYWORDS
Transportation Network Company (TNC); Taxi; Two-sided Markets; Shared Mobility; Dynamic bilateral matching.

INTRODUCTION
Advancements in information and communication technologies result in growing trends in using ride-sourcing services because of their convenience and affordability. Ride-sourcing service providers like Uber, DiDi, Lyft, Ola and their competitors provide on-demand point-to-point services for passengers through an online platform using a fleet of vehicles owned by self-scheduled drivers. The platform requires the service provider to implement strategies for vacant vehicle-passenger matching to increase profit while ensuring a satisfactory level-of-service such as keeping the expected passenger waiting time below a threshold. In this paper, we develop a method for vehicle-passenger matchings to obtain optimum values for matching time instances and the maximum matching distance to minimize passengers' waiting time.

The research works on ride-sourcing systems exhibit traits of studying cruising taxis. In [17], a stationary model at the equilibrium point is established to formulate movements of cruising taxis. This model is further developed by considering the effect of congestion and passenger demand elasticity in [14]. The steady-state effect of bilateral taxi-passenger searching and meeting behavior at the equilibrium point of cruising taxi systems is studied in [15, 19]. In [16], a two-stage equilibrium-based model is proposed to predict zonal and circulating movements of cruising taxis. In [20, 18], the effect of search friction between vacant taxis and passengers on the equilibrium of the taxi system is scrutinized. A Ride-sourcing market is studied in [21] by proposing a equilibrium-based macroscopic model to capture the taxi-passenger meeting dynamics with external matching function. An equilibrium model in a hybrid market with the coexistence of cruising taxi and ride-sourcing systems is investigated in [7]. [8] initiates non-equilibrium based modeling of cruising taxi systems and proposes a predictive controller to relocate the vacant taxis. A non-equilibrium model for a ride-sourcing market with predictive controller is proposed in [6] to maximize the overall profit by manipulating the fare and wage of the system dynamically.

The effect of inefficient vehicle-passenger matching method on network congestion is more highlighted after imposing harshest-ever set of regulations for ride-sourcing companies by NYC city which necessitates the ride-sourcing companies to carry a passenger at least 69% of the operating time in Manhattan below 96th St from August 2020 [5]. The matching method can be formulated as a bipartite graph structure or stable marriage problem to minimize waiting time, pickup time, idle distance, or the fleet size. The challenges are considering the exogenous arrival rates of travel requests and idle vehicles, congestion of the network, and dynamics of two-sided market. Current works in literature (e.g. [10, 22, 11, 13]) assume that the matching interval is fix and known. Although, there is an intertwine effect between matching intervals (i.e. successive matching instances) and vehicle-passenger matching distance. Long matching intervals result in increasing the possibility of having short matching distances. On the other hand, it inflates the waiting time of the idle vehicles and waiting passengers. To get more insight into the literature of shared mobility, interested readers can refer to [9, 2, 3, 12].

In this paper, a vehicle-passenger matching method is proposed to *dynamically* determine matching intervals and maximum matching distances to optimize expected passengers' waiting time. To fill the gap in designing matching method, the proposed method is developed to consider the level of congestion of the network, anticipation of the future boarding time, and joint effect of optimum matching interval and maximum matching distance. The advantages of the matching method is investigated via a developed microsimulation benchmark.

The remainder of the article is organized as follow. In Section 1, we elaborate different states of ride-sourcing vehicles and passengers in the ride-sourcing system. Also, we illustrate how different subsystems in the proposed model are interact with each other in Section 1. In Section 2, the proposed adaptive spatio-temporal matching algorithm for dispatching idle ride-sourcing vehicles to waiting passengers is presented in detail. Section 3 is devoted to assessing the performance of the proposed adaptive-spatio temporal matching method using microsimulation experiments. Finally, the article is concluded in Section 4.

1 PRELIMINARIES

This section defines terminologies used in this article for representing states of the ride-sourcing vehicles and passengers. Furthermore, we depict the schema of the proposed ride-sourcing system to demonstrate different components of the method and the interactions between them.

1.1 *State Definition*

We design a *ride-sourcing system* as a centralized system for dispatching ride-sourcing vehicles to passengers' origin location and transferring the idle vehicles to the locations with a higher possibility of finding passengers. We determine four states for the ride-sourcing vehicles (i.e. idle, dispatched, transferred, and occupied) and three states for the passengers (i.e. waiting, assigned, and on-board). The four states of ride-sourcing vehicles are as follow:

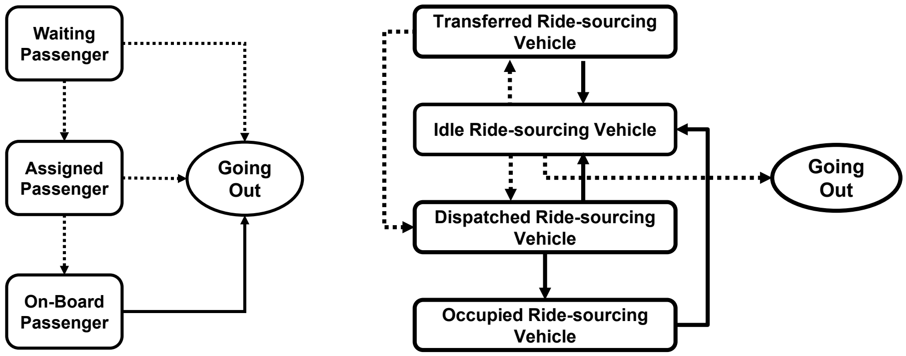
(i) *Idle ride-sourcing vehicle* refers to a vacant vehicle which is not assigned to any passenger's travel request. It waits for receiving a pick-up command from the ride-sourcing system. The number of idle ride-sourcing vehicles in region i at time t is denoted as $c_i^{\mathrm{I}}(t)$.

(ii) *Dispatched ride-sourcing vehicle* is a vacant vehicle assigned to a passenger's travel request. It is sent to the passenger's location through the path recommended by the ride-sourcing system. The dispatched vehicles are not allowed to pick up other passengers along the recommended path.

(iii) *Transferred ride-sourcing vehicle* is a vacant vehicle sent to a location with an excess of passenger's travel request through the path recommended by the ride-sourcing system. Transferred vehicles are not assigned to any passenger's travel request. They are used to balance the vehicle supply and travel request demand in different regions of the network. They can be assigned to passenger's travel request by the ride-sourcing system before reaching the hot-spot locations. The number of transferred vehicles in region i at time t is denoted as $c_i^{\mathrm{T}}(t)$.

(iv) *Occupied ride-sourcing vehicle* is a vehicle servicing a passenger. We assume each vehicle services only one passenger or one group of passengers with the same origin and destination.

The three passenger's states are as:



(a) (b)

Figure 1: State diagram of ride-sourcing systems' agents: (a) passengers and (b) ride-sourcing vehicles.

(i) *Waiting passenger* refers to a passenger who has requested a ride but she/he is not assigned to any ride-sourcing vehicle yet. The number of waiting passengers in region i at time t is denoted as $p_i^{\mathrm{W}}(t)$.

(ii) *Assigned passenger* is a passenger who is not picked up by a vehicle, but a dispatched ride-sourcing vehicle is assigned to her/him. An assigned passenger cannot match with more than one ride-sourcing vehicle at a time.

(iii) *On-board passenger* is a passenger picked up by a vehicle but she/he has not reached her/his destination. The number of on-board passengers in region i at time t is equal to the number of the occupied vehicles in region i at time t.

1.2 *State Transitions*

Consider the movements of an *idle ride-sourcing vehicle* in a network which is divided into a number of regions. Once a *waiting passenger* is matched with an idle vehicle, the vehicle becomes *dispatched* and the passenger becomes *assigned*. Then, the dispatched ride-sourcing vehicle starts its travel towards the passenger's pick-up location with the path recommended by the ride-sourcing system. Once, the dispatched vehicle reaches the location of the assigned passenger, the vehicle and passenger become *occupied* and *on-board*, respectively. Once the occupied vehicle reaches the on-board passenger's destination, the state of the vehicle is changed to idle.

An idle ride-sourcing vehicle might be requested to reposition to other region(s) with an excess number of waiting passengers to balance the vehicle sources and passengers' travel demand. If the ride-sourcing system determines such region(s), a number of the idle vehicles become *transferred* and will be guided to moved to that region(s). The transferred vehicles can be assigned to a passenger if a waiting passenger will appear in a proper-distance of the transferred vehicle. Otherwise, the transferred vehicle will reach to the recommended location and becomes idle again.

To reflect the reality more precisely, we assume if an idle ride-sourcing vehicle remains idle for a long period of time, the driver leaves the ride-sourcing system. In addition, if a waiting or an assigned passenger is not picked up for a long time, the passenger cancels the travel request and quit the ride-sourcing service for other travel choices. By canceling a travel request by an assigned passenger, the ride-sourcing vehicle matched to the passenger becomes idle again.

Figure 1 illustrates the state diagrams of vehicles and passengers in the ride-sourcing system. The dotted lines emphasize the vehicle's/passenger's state changes instantaneously without a physical trip (e.g. the state of an idle ride-sourcing vehicle becomes transferred once the driver receives the transfer command from the ride-sourcing system.). The solid lines reflect a physical trip in the network (e.g. the dispatched vehicle must reach to the location of the assigned passenger to become occupied.).

2 DISPATCHING SUBSYSTEM

The dispatching subsystem which includes perfect and adaptive spatio-temporal matching methods dynamically determines the optimum maximum matching distance and the optimum next matching instance with respect to minimizing passengers waiting time. The dispatching subsystem in each matching instance considers the location of the waiting passengers and idle/transferred ride-sourcing vehicles as well as the aggregated short-term prediction of arrival rate of new passengers and idle/transferred vehicles. In each optimum matching instance that is obtained from the proposed method, firstly, the perfect matching method is applied and then the optimum maximum matching distance that is obtained from the proposed method is used in each matching instance to discard the long-distance matchings of perfect matching method. In Subsection 2.1, the perfect matching method is explained that is built upon maximum matching problem of a bipartite graph. It is a classic problem and has been widely studied in literature [1, 22, 11]. Subsection 2.2 introduces the proposed method for finding the optimum matching intervals and maximum matching distances.

2.1 *Perfect Matching*

The optimum matching between idle/transferred ride-sourcing vehicles and waiting passengers at every matching instance is determined by solving the minimum weighted matching problem for a bipartite graph. It minimizes the total matching distances between idle/transferred vehicles and waiting passengers (or equivalently pickup times). We construct the problem as a bipartite graph by considering, (i) V_1 as the set of idle and transferred vehicles, (ii) V_2 as the set of the waiting passengers, and (iii) E as the edges connecting each element of V_1 to V_2. The sets of V_1 and V_2 are disjoint and independent. The weight of each edge, $w(e)$, is the distance between the idle/transferred ride-sourcing vehicle and the waiting passenger (i.e. the elements of the V_1 and V_2). We obtain the minimum weighted matching for the bipartite graph using integer linear programming method:

$$\begin{aligned} &\text{minimize } \Big(\sum_{e\in E} x_e w(e)\Big), \\ &\text{s.t.} \quad \sum_{e\sim v} x_e \le 1 \quad \forall v \in \{V_1 \cup V_2\} \quad \& \quad x_e \in \{0,1\} \qquad \forall e \in E, \end{aligned} \qquad \textbf{Equation 1.}$$

where, $w(e)$ is the weight of each edge $e \in E$ and $e \sim v$ denotes e is an incident on v. The number of the matching is the minimum of cardinalities of set V_1 and set V_2. Equation 1 is a static optimization problem that minimizes total matching distances. It may suffer from matching vehicles with long-distance passengers that wastes their reserved time. Also, it considers only the *current* location of the idle/transferred ride-sourcing vehicles and waiting passengers at each *fixed* matching interval. We consider the effect of future state of the system, i.e. arrival of idle/transferred ride-sourcing vehicles and waiting passengers, congestion of the network, and the joint relationship of matching intervals and discarding long-distance vehicle-passenger matchings in the proposed method in Subsection 2.2.

2.2 *Adaptive Spatio-Temporal Matching*

The adaptive spatio-temporal matching method which includes perfect matching and adaptive spatio-temporal matching methods dynamically determines the maximum value for the matching distance between idle/transferred ride-sourcing vehicles and waiting passengers as well as occurrence time of the next matching. Increasing the frequency of the matchings in perfect matching methods (i.e. decreasing the time between two successive matching instances) causes assigning an idle/transferred vehicle to a waiting passenger once they become available in the network without considering the effect of new passengers and vehicles that might appear in the network relatively just after the matching instance time. Hence, this might lead to matchings that are unreasonably long. In other words, increasing the matching frequency decreases the *dead-time* but increases the expected *reserved time*. Dead-time for a ride-sourcing vehicle (passenger) is the time between the idle (waiting) state and dispatched (assigned) state. The reserved time for a ride-sourcing vehicle (passenger) is the time takes a dispatched vehicle (assigned passenger) becomes occupied (on-board).

The adaptive spatio-temporal matching method aims at minimizing the expected total passengers' waiting time (i.e. dead time plus the reserved time) to jointly determine the optimum time of the next matching instance and to discard the vehicle-passenger long-distance matchings. To this end, we use the perfect vehicle-passenger matchings (i.e. considering the matching without discarding long-distance matchings) by solving Equation 1. Subsequently, we define an expected passengers' waiting time based on the obtained values of the perfect vehicle-passenger matchings. Then, we determine the optimum value of next matching instance and the maximum distance for vehicle-passenger matchings with respect to minimizing the defined expected total passengers' waiting time. We assume that the average arrival rate of new waiting passengers and idle vehicles are known.

The expected total passengers' waiting time, $\hat{T}_{\mathrm{P}}^{t_{\mathrm{m}}^{i+1}-t_{\mathrm{m}}^{i}}$, between two successive matching time instances, t_{m}^{i} and t_{m}^{i+1}, is sum of four main parts:

(i) Estimated total reserved time of matchings at t_{m}^{i} is:

$$\begin{aligned}\hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i}) &= \frac{\bar{l}_r(t_{\mathrm{m}}^{i})}{\hat{v}(t_{\mathrm{m}}^{i})}\left(m(t_{\mathrm{m}}^{i}) - r(t_{\mathrm{m}}^{i})\right)\\ &= \frac{\bar{l}_r(t_{\mathrm{m}}^{i})}{\hat{v}(t_{\mathrm{m}}^{i})}\Big(\min\left(c^{\mathrm{I}}(t_{\mathrm{m}}^{i}) + c^{\mathrm{T}}(t_{\mathrm{m}}^{i}), p^{\mathrm{W}}(t_{\mathrm{m}}^{i})\right) - r(t_{\mathrm{m}}^{i})\Big),\end{aligned} \qquad \textbf{Equation 2.}$$

where, $\hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i})$ is the estimated total reserved time for the idle/transferred vehicles assigned to the waiting passengers at t_{m}^{i}. $\bar{l}_r(t_{\mathrm{m}}^{i})$ is the average distance of optimum vehicle-passenger matchings after discarding r long-distance matchings from the solution of Equation 1 at time instance t_{m}^{i}. The number of the matchings before discarding at time t_{m}^{i} is denoted by $m(t_{\mathrm{m}}^{i})$. The number of the idle vehicles, transfer vehicles, and waiting passengers at time t_{m}^{i} are indicated by $c^{I}(t_{\mathrm{m}}^{i})$, $c^{\mathrm{T}}(t_{\mathrm{m}}^{i})$, and $p^{\mathrm{W}}(t_{\mathrm{m}}^{i})$. $\hat{v}(t_{\mathrm{m}}^{i})$ denotes the estimated network speed at time instance t_{m}^{i}.

(ii) Estimated total dead-time of the waiting passengers remaining in the network after the vehicle-passenger matching at t_{m}^{i} with discarding r long-distance matching, $\hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i})$, is:

$$\hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i}) = \Big(p^{\mathrm{W}}(t_{\mathrm{m}}^{i}) - m(t_{\mathrm{m}}^{i}) + r(t_{\mathrm{m}}^{i})\Big)\left(t_{\mathrm{m}}^{i+1} - t_{\mathrm{m}}^{i}\right). \qquad \textbf{Equation 3.}$$

(iii) Predicted total reserved time for matchings at t_{m}^{i+1}, $\hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i+1})$ is:

$$\begin{aligned}\hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i+1}) &= \frac{\hat{\bar{l}}_r(t_{\mathrm{m}}^{i+1})}{\hat{v}(t_{\mathrm{m}}^{i+1})}\left(m(t_{\mathrm{m}}^{i+1}) + r(t_{\mathrm{m}}^{i})\right)\\ &= \frac{\hat{\bar{l}}_r(t_{\mathrm{m}}^{i+1})}{\hat{v}(t_{\mathrm{m}}^{i})}\Bigg(\min\Big(c^{\mathrm{I}}(t_{\mathrm{m}}^{i}) + c^{\mathrm{T}}(t_{\mathrm{m}}^{i}) + \rho_{\mathrm{c}}(t_{\mathrm{m}}^{i})\left(t_{\mathrm{m}}^{i+1} - t_{\mathrm{m}}^{i}\right) - m(t_{\mathrm{m}}^{i}),\\ &\qquad p^{\mathrm{W}}(t_{\mathrm{m}}^{i}) + \rho_{\mathrm{p}}(t_{\mathrm{m}}^{i})\left(t_{\mathrm{m}}^{i+1} - t_{\mathrm{m}}^{i}\right) - m(t_{\mathrm{m}}^{i})\Big) + r(t_{\mathrm{m}}^{i})\Bigg),\\ \rho_{\mathrm{c}}(t_{\mathrm{m}}^{i}) &= \rho_{\mathrm{c}}^{\mathrm{en}}(t_{\mathrm{m}}^{i}) + \rho_{\mathrm{c}}^{\mathrm{ex}}(t_{\mathrm{m}}^{i}),\end{aligned} \qquad \textbf{Equation 4.}$$

where, $\hat{\bar{l}}_r(t_{\mathrm{m}}^{i+1})$ is the estimated average optimum matching distance. Because the exact location of idle/transferred vehicles and waiting passengers at time t_{m}^{i+1} is not known, we propose a parsimonious function to estimate the average matching distance of the optimum matching that is only a function of the number of vehicles and passengers. This is described in detail in Subsection 2.3. $m(t_{\mathrm{m}}^{i+1})$ denotes the number of the matchings at time t_{m}^{i+1} without discarding at time t_{m}^{i}. The rates of arrival of idle/transferred vehicles and waiting passengers during interval $[t_{\mathrm{m}}^{i}, t_{\mathrm{m}}^{i+1})$ are denoted by $\rho_{\mathrm{c}}(t_{\mathrm{m}}^{i})$ and $\rho_{\mathrm{p}}(t_{\mathrm{m}}^{i})$, respectively. $\rho_{\mathrm{c}}(t_{\mathrm{m}}^{i})$ has endogenous, $\rho_{\mathrm{c}}^{\mathrm{en}}(t_{\mathrm{m}}^{i})$, and exogenous parts, $\rho_{\mathrm{c}}^{\mathrm{ex}}(t_{\mathrm{m}}^{i})$. $\rho_{\mathrm{c}}^{\mathrm{en}}(t_{\mathrm{m}}^{i})$ captures the rate that occupied or dispatched vehicles become idle. $\rho_{\mathrm{c}}^{\mathrm{ex}}(t_{\mathrm{m}}^{i})$ captures the rate that idle ride-sourcing vehicles leave the network because they are not assigned to any passenger for a long time and the vehicles leaving from or entering to the network due to their working hours.

(iv) Predicted total dead-time for waiting passengers remaining in the network after the vehicle-passenger matching at t_{m}^{i+1}, $\hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i+1})$, is:

$$\begin{aligned}\hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i+1}) = & \\ & \Big(p^{\mathrm{W}}(t_{\mathrm{m}}^{i}) + \rho_{\mathrm{P}}(t_{\mathrm{m}}^{i})\left(t_{\mathrm{m}}^{i+1} - t_{\mathrm{m}}^{i}\right) - \left(m(t_{\mathrm{m}}^{i}) - r(t_{\mathrm{m}}^{i})\right) - \left(m(t_{\mathrm{m}}^{i+1}) + r(t_{\mathrm{m}}^{i})\right)\Big) \\ & \qquad \times \left(t_{\mathrm{m}}^{i+2} - t_{\mathrm{m}}^{i+1}\right) \\ = & \Big(p^{\mathrm{W}}(t_{\mathrm{m}}^{i}) + \rho_{\mathrm{P}}(t_{\mathrm{m}}^{i})\left(t_{\mathrm{m}}^{i+1} - t_{\mathrm{m}}^{i}\right) - m(t_{\mathrm{m}}^{i}) - m(t_{\mathrm{m}}^{i+1})\Big)\left(t_{\mathrm{m}}^{i+2} - t_{\mathrm{m}}^{i+1}\right).\end{aligned} \qquad \text{Equation 5.}$$

By assuming perfect matching at time t_{m}^{i+1}, the expected total passengers' waiting time is:

$$\hat{T}_{\mathrm{P}}^{t_{\mathrm{m}}^{i+1}-t_{\mathrm{m}}^{i}} = \hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i}) + \hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i}) + \hat{T}_{\mathrm{R}}(t_{\mathrm{m}}^{i+1}) + \hat{T}_{\mathrm{D}}(t_{\mathrm{m}}^{i+1}). \qquad \text{Equation 6.}$$

We obtain the optimum value for the next matching time, t_{m}^{i+1}, and the number of discarded long-distance matchings, $r(t_{\mathrm{m}}^{i})$, by minimizing the total passengers' waiting time prediction:

$$\begin{aligned}&\underset{r(t_{\mathrm{m}}^{i}),t_{\mathrm{m}}^{i+1}}{\text{minimize}}\ \left(\hat{T}_{\mathrm{P}}^{t_{\mathrm{m}}^{i+1}-t_{\mathrm{m}}^{i}}\right), \\ &\text{s.t.}\quad 0 < t_{\mathrm{m}}^{i} < t_{\mathrm{m}}^{i+1} \le t_{\mathrm{m}}^{i+2}, \quad t_{\mathrm{m}}^{i+2} \le t_{\mathrm{m}}^{i+1} + t_{\mathrm{m}}^{\max}, \quad 0 \le r(t_{\mathrm{m}}^{i}) \le m(t_{\mathrm{m}}^{i})\end{aligned}, \qquad \text{Equation 7.}$$

where, $t_{\mathrm{m}}^{\max}$ is a predefined upper bound for the next matching time instance. To practically solve the discrete-continues optimization problem of Equation 7, we can discretize the time domain, $[t_{\mathrm{m}}^{i}, t_{\mathrm{m}}^{\max}]$, and iteratively evaluate Equation 6 for any specific t_{m}^{i+1} and $r(t_{\mathrm{m}}^{i})$ that satisfy the criteria of Equation 7 to determine the optimum values. Choosing $t_{\mathrm{m}}^{\max}$ and sampling period for discretization are important factors to reach a solution in meaningful time. To this end, we choose $t_{\mathrm{m}}^{\max} = 120$ [sec] and sampling period of 5 [sec]. The performance of the proposed adaptive spatio-temporal matching for dispatching idle/transferred vehicles to waiting passengers is investigated in Section 3.

2.3 *Estimating a Macro-Function for Average Optimal Matching Distance*

A closed-form macro-function for estimating average optimal matching distance, $\hat{\bar{l}}$, is needed for predicting the total reserved time, see Equation 4. To this end, we propose Algorithm 1 to estimating the average optimal matching distance as a function of the number of the idle/transferred ride-sourcing vehicles and waiting passengers, independent of their location. Algorithm 1 generates the coordinates of idle/transferred ride-sourcing vehicles and waiting passengers randomly with uniform distribution inside a network. Then, a bipartite graph, $G(V_1, V_2, E)$, is built in which the weights of the edges are the Manhattan distance between idle/transferred ride-sourcing vehicles and waiting passengers. The average optimal matching is determined by solving the matching problem on graph G to minimize the sum of matching weights as in Equation 1. To tackle the stochasticity of spatial distribution of ride-sourcing vehicles and passengers, this procedure is repeated N^{itr} times for each number of idle/transferred ride-sourcing vehicles and waiting passengers.

We run the algorithm on a high performance computer (HPC) for N^{itr}=100, N^{P}=50, and N^{T}=50. The result, see Figure 2(b), reveals that the variations of average optimum matching distance with respect to the number of waiting passengers and idle/transferred ride-sourcing vehicles are:

$$\begin{cases}\frac{\partial\hat{\bar{l}}}{\partial(c^{\mathrm{I}}+c^{\mathrm{T}})} > 0,\ \frac{\partial\hat{\bar{l}}}{\partial p^{\mathrm{W}}} < 0 & \text{if}\quad c^{\mathrm{I}} + c^{\mathrm{T}} < p^{\mathrm{W}} \\ \frac{\partial\hat{\bar{l}}}{\partial(c^{\mathrm{I}}+c^{\mathrm{T}})} < 0,\ \frac{\partial\hat{\bar{l}}}{\partial p^{\mathrm{W}}} > 0 & \text{if}\quad p^{\mathrm{W}} < c^{\mathrm{I}} + c^{\mathrm{T}}\end{cases}.$$

Accordingly, the following symmetric form is suggested for the average optimum matching distance:

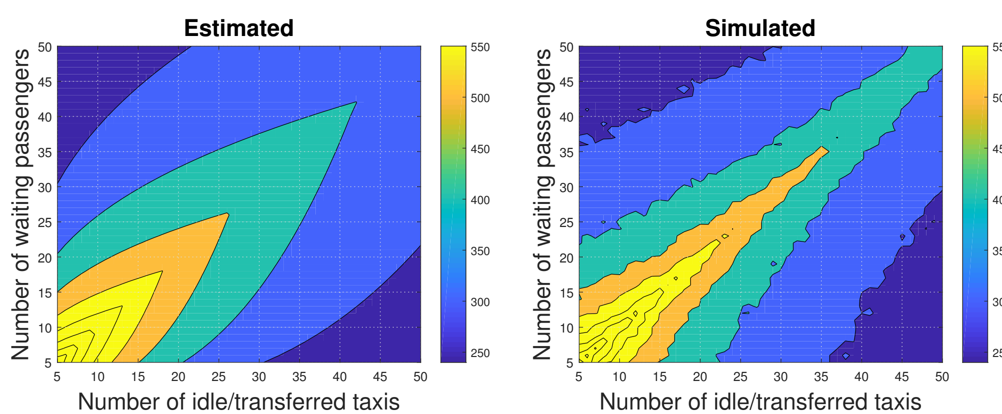
$$\hat{\bar{l}} = \begin{cases}\theta(c^{\mathrm{I}} + c^{\mathrm{T}})^{\zeta_1} p^{\mathrm{W}\zeta_2} & \text{if}\quad c^{\mathrm{I}} + c^{\mathrm{T}} \le p^{\mathrm{W}} \\ \theta(c^{\mathrm{I}} + c^{\mathrm{T}})^{\zeta_2} p^{\mathrm{W}\zeta_1} & \text{if}\quad p^{\mathrm{W}} < c^{\mathrm{I}} + c^{\mathrm{T}}\end{cases} \qquad \text{Equation 8.}$$

Algorithm 1: Pseudocode for estimating the average optimum matching distance

Result: $\{\hat{\bar{l}}_{n^{\mathrm{P}}\times n^{\mathrm{T}}} |\ n^{\mathrm{P}}=1:N^{\mathrm{P}},\ n^{\mathrm{T}}=1:N^{\mathrm{T}}\}$

Spatial initialization;
for $n^{\mathrm{T}}=1:N^{\mathrm{T}}$ **do**
  for $n^{\mathrm{P}}=1:N^{\mathrm{P}}$ **do**
    for $k=1:N^{\mathrm{itr}}$ **do**
      $\{(x_i^{\mathrm{P}},y_i^{\mathrm{P}})|\ i=1:n^{\mathrm{P}}\} \leftarrow$ Generate coordinates of n^{P} passengers randomly;
      $\{(x_j^{\mathrm{T}},y_j^{\mathrm{T}})|\ j=1:n^{\mathrm{T}}\} \leftarrow$ Generate coordinates of n^{T} ride-sourcing vehicles randomly;
      $G(V_1,V_2,E) \leftarrow$ Initialization a bipartite graph with $|V_1|=n^{\mathrm{P}}$ and $|V_2|=n^{\mathrm{T}}$;
      for $i=1:n^{\mathrm{P}}$ **do**
        for $j=1:n^{\mathrm{T}}$ **do**
          $w_{ij} \leftarrow$ Manhattan distance between passenger i and ride-sourcing vehicle j;
          $d_{ij} \leftarrow$ Add an edge to connect vertices $(v_1^{\mathrm{i}},v_2^{\mathrm{j}})$ with weight w_{ij};
      $M_k \leftarrow$ Find optimal matching of G to minimize the sum of matching weights;
      $\hat{l}_k \leftarrow$ Find the average of matched weights;
    $\hat{\bar{l}}_{n^{\mathrm{P}}\times n^{\mathrm{T}}} \leftarrow$ Find the average of $\{\hat{l}_k|\ k=1:N^{\mathrm{itr}}\}$;

where, $\theta>0$, $\zeta_1>0$, and $\zeta_2<0$ are parameters that can be readily estimated using the Least Square method. By utilizing the generated data in algorithm 1, the estimated values of the parameters are: $\hat{\theta}=2394.57$, $\hat{\zeta_1}=0.245$, and $\hat{\zeta_2}=-0.724$, where, $R^2=0.93$. Figure 2 compares the simulated and estimated average optimum matching distances.



(a) (b)

Figure 2: (a) Estimated average of optimum matching distance and (b) simulated average of optimum matching distance.

3 RESULTS

In this section, we investigate the performance of the proposed vehicle-passenger matching using a dynamic ride-sourcing benchmark developed in Aimsun microsimulation. The calibrated Aimsun microsimulation model of the city center of Barcelona [4] is plugged into the benchmark. The studied network approximately covers an area of 8.21 squared kilometers containing 1570 sections and 721 junctions. The Aimsun microscopic model updates the model state (e.g. position of normal and ride-sourcing vehicles, passengers, buses) every half a second. In the following, we scrutinize the effects of the proposed vehicle-passenger matching method in comparison with variants of perfect matching method under noticeable variation of vehicles supply and passengers demand. Figure 3 illustrates the variation of total number of the passengers and ride-sourcing vehicles in 10 replications with different random initialization.

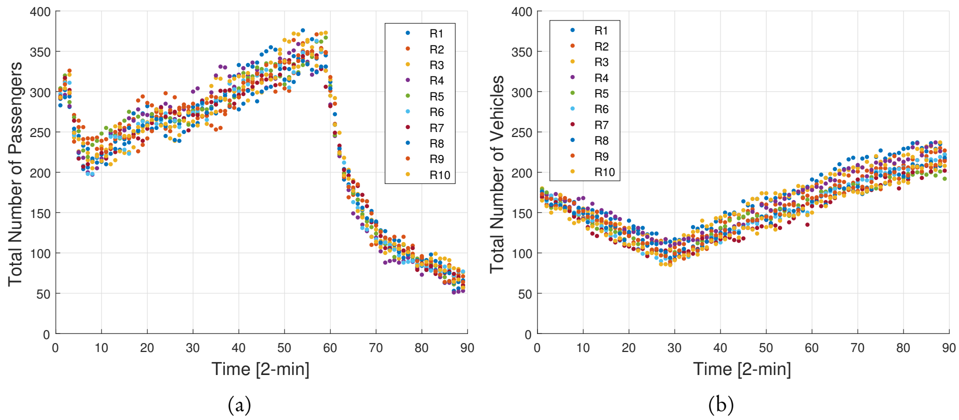


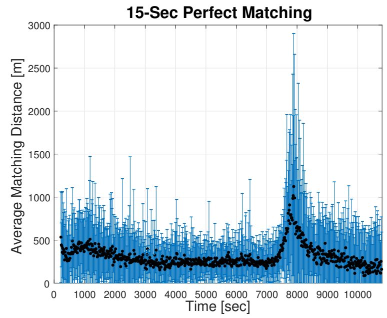
Figure 3: (a) Total number of the passengers including waiting, assigned, and on-board passengers. (b) Total number of the vehicles including idle, dispatched. and occupied vehicles.

Perfect matching method dispatches vehicles to the passengers based on solving Equation 1 in each matching instance without discarding long-distance matchings. This approach considers just the current state of the vehicles and passengers and is sensitive to a predefined matching interval. Moreover, it does not consider the effects of traffic congestion and dynamic of the idle vehicles and waiting passengers. Figure 4 depicts the average of matching distances for different matching intervals. Increasing the matching interval results in lower average and variance of vehicle-passenger matching distances (i.e. shorter vehicle's/passenger's reserved time). However, increasing the matching interval increases the dead time of vehicles and passengers, see Table 1.

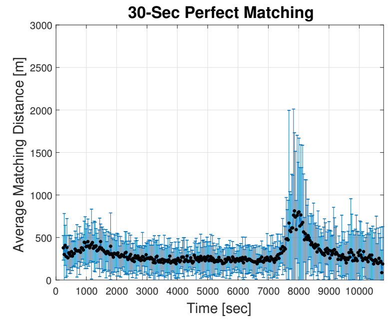
Figure 5 presents the average of matching distances for matching with greedy discarding method. The greedy method prunes the outcomes of the perfect matching by discarding k long-distance vehicle-passenger matching if they are above the predefined distance threshold. This approach only takes into account the current number and location of the passengers and vehicles. The illustrated results in Figure 5 are obtained by choosing $k = 1$ and distance threshold of $900\,[\mathrm{m}]$. Distance threshold is determined using cut off value of 99% for considering potential outlier machings, see Figure 4. Effectiveness of the greedy method is quantified is Table 1.

In the proposed vehicle-passenger matching method, the optimum value of the matching interval and the maximum matching distance are determined by solving Equation 7. These optimum values are time-varying such that at each matching instance, the optimum values of the current distance threshold and the time for the next matching instance are obtained. Figure 6 illustrates the number of waiting passengers and idle vehicles in the network by implementing the adaptive spatio-temporal matching method. Figure 7 elucidates how matching interval and maximum number of the matching intertwined with each other in the proposed method.

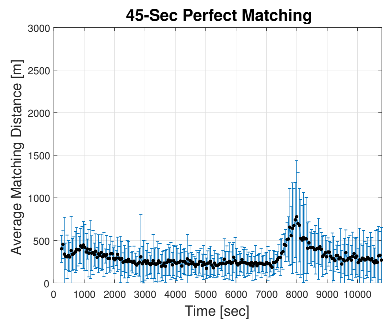
To explore the results of the adaptive spatio-temporal matching method, we discuss about the three-hour simulation results in five time periods: (i) $\Delta t_1 \approx [0 \text{ min}, 12 \text{ min})$, (ii) $\Delta t_2 \approx [12 \text{ min}, 40 \text{ min})$, (iii) $\Delta t_3 \approx [40 \text{ min}, 120 \text{ min})$, (iv) $\Delta t_4 \approx [120 \text{ min}, 140 \text{ min})$, and (v) $\Delta t_5 \approx [140 \text{ min}, 180 \text{ min}]$. In Δt_1, the number of the waiting passengers is decreasing and is greater than the number of the idle vehicles, see Figure 6. Hence, the average optimum matching distance based on Equation 8 must be increasing as shown in Figure 7 (a). The average matching distance has *low-rate* slope because the number of the idle vehicles is *much less* than the number of the waiting passengers. In this time period, the number of the discarded matching is high and increasing, Figure 7 (b) and (c), because the density of idle vehicles and waiting passengers are low and decreasing. When the number of the discarded matchings is increasing, the matching time interval decreases, Figure 7 (d), to compensate the delay caused by discarded matchings in estimated total dead-time (Equation 3) and predicted total reserved time (Equation 4).



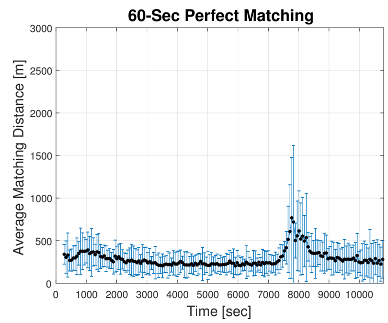
(a)



(b)

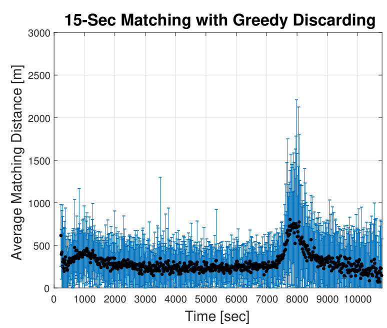


(c)

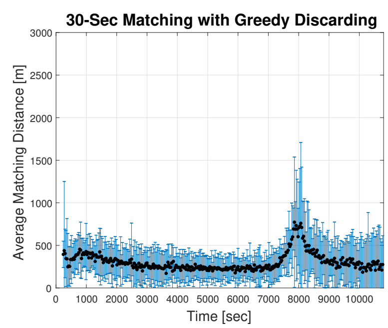


(d)

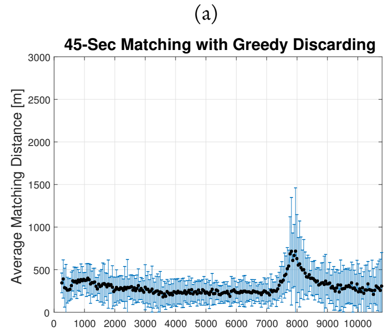
Figure 4: Average of matching distances at each matching time using perfect matching method with different matching interval: (a) 15-second, (b) 30-second, (c) 45-second, and (d) 60-second. The bars represent 95% confidence interval.



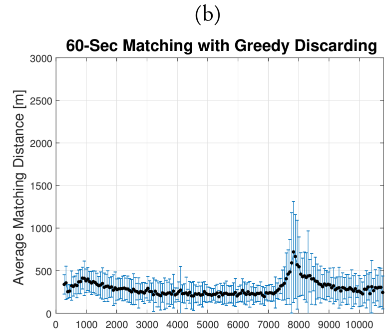
(a)



(b)

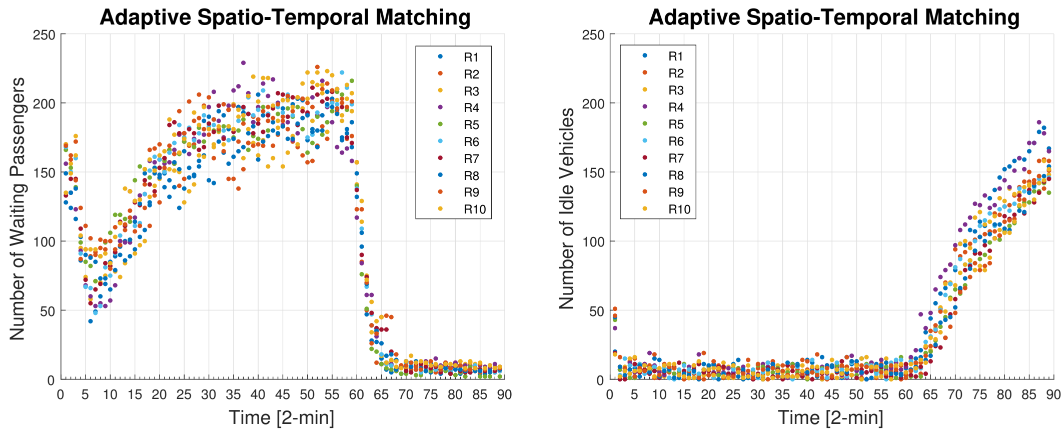


(c)



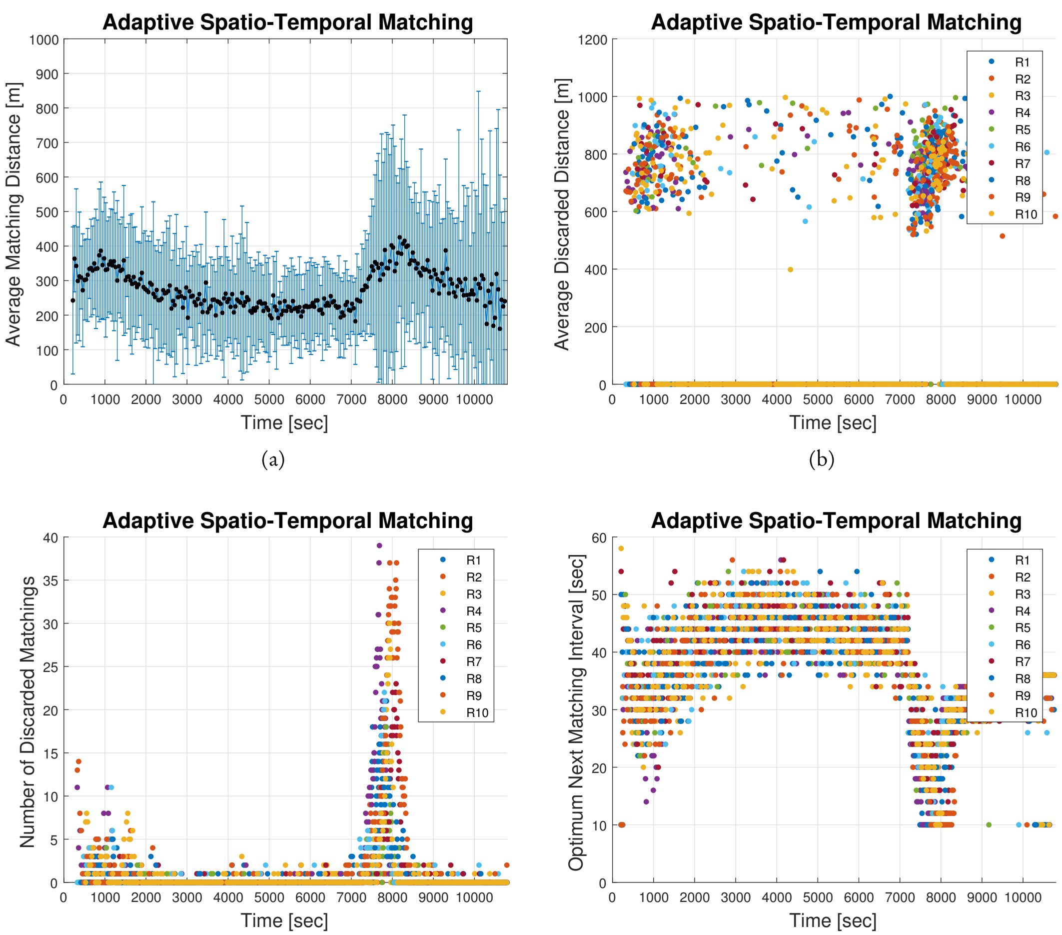
(d)

Figure 5: Average of matching distances in each matching time using greedy discarding method with different matching interval: (a) 15-second, (b) 30-second, (c) 45-second, and (d) 60-second. The bars represent 95% confidence interval.



(a) (b)

Figure 6: (a) Total number of the waiting passengers. (b) Total number of the idle vehicles.



(a) (b) (c) (d)

Figure 7: Output of the adaptive spatio-temporal matching method: (a) average of matching distances after discarding, (b) average distance of discarded matchings, (c) number of discarded matchings, and (d) matching interval. The bars represent 95% confidence interval and different dotted colors show different replications.

In $\Delta t_2 \approx [12 \text{ min}, 40 \text{ min})$, the number of the waiting passengers which is greater than the number of idle vehicles starts to increase while the number of the idle vehicles does not change significantly, see Figure 6. Hence, as expected based on Equation 8, the average optimum matching distance decreases as in Figure 7 (a). The number of the idle vehicles is less than the number of the waiting passengers so the number of the matchings is bounded by the number of the idle vehicles. On the other side, as the number of the waiting passenger increases, the possibility of short-distance matchings are increased that results in less discarded vehicle-passenger matching as illustrated in Figures 7 (b) and (c). Consequently the decrease in the number of discarded vehicle-passenger matchings results in greater value of matching interval, see Figure 7 (d).

In $\Delta t_3 \approx [40 \text{ min}, 120 \text{ min})$, the number of the idle vehicles and the waiting passengers are not changed notably. Hence, the average of matching distances, number of the discarded matching, and matching intervals are not refined significantly. The trend of the idle vehicles and waiting passengers in $\Delta t_4 \approx [120 \text{ min}, 140 \text{ min})$ are almost the same as Δt_1. Hence, the same explanation is valid. The trend of idle vehicles and waiting passengers in $\Delta t_5 \approx [140 \text{ min}, 180 \text{ min}]$ are similar to Δt_2 if we use use idle vehicle and waiting passengers interchangeably. Because of symmetric characteristic of a matching problem, refer to Equation 8 and Figure 2 (b), the explanation for Δt_2 can be applied for Δt_5.

Table 1 presents the quantitative comparison of different matching methods. In this table, total delay is sum of reserved time and dead-time for vehicles and passengers as well as the delay of the impatient waiting/assigned passengers and impatient idle vehicle drivers that cancel their trips or leave the ride-sourcing system. Greater matching intervals decrease the average of matching distance that is equivalent to the reduction in the reserved time of vehicles and passengers. However, this inflates the dead-time of passengers and vehicles. The proposed method shows significant improvement in reducing total delay. It is worth to point out the dead-time of the vehicles in 15-second perfect matching and 15-second matching with greedy discarding is less than the proposed method because vehicles dead-time just considers the vehicles that *successfully* are matched to the waiting passengers. Passengers' dead-time of the proposed method is in a same level of two other methods because the latter methods assign idle vehicles and waiting passengers together once they become available; however, some of these matchings are long-distance such that some assigned passengers leave the network before boarding that are not reflected in passengers' dead-time.

Table 1: Comparison of adaptive spatio-temporal matching method and variants of perfect matching methods with respect to reserved time, dead time, and total delay. The matching interval for adaptive spatio-temporal method is time-varying and the reported value is the average of the matching intervals.

	Matching Interval [sec]	Passengers' Dead Time [sec]		Vehicles' Dead Time [sec]		Vehicles/Passengers Reserved Time [sec]		Total Delay [sec]	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
Perfect Matching (No Discarding)	15	143.7	134.1	38.6	167.9	93.9	85.5	457.3	183.3
Perfect Matching (No Discarding)	30	151.7	137.8	45.5	174.6	91.4	83.1	482.2	196.7
Perfect Matching (No Discarding)	45	157.4	139.7	51.6	159.7	90.0	81.4	500.9	184.7
Perfect Matching (No Discarding)	60	158.9	139.2	55.7	139.8	87.7	81.3	501.0	172.5
Matching with Greedy Discarding	15	143.4	135.2	38.2	157.8	91.8	82.7	458.0	183.5
Matching with Greedy Discarding	30	150.1	136.4	44.8	162.8	91.4	81.7	476.8	182.3
Matching with Greedy Discarding	45	155.3	138.0	51.2	157.1	88.8	79.8	489.9	181.0
Matching with Greedy Discarding	60	158.3	138.8	58.3	171.3	86.8	81.7	503.7	183.8
Adaptive Spatio-Temporal Matching	34.8 (Mean)	143.0	128.1	48.1	159.0	80.6	74.3	371.2	158.4

4 CONCLUSIONS

This article has presented a matching method for ride-sourcing systems. We proposed an algorithm to dynamically determine the optimum matching intervals and maximum matching distance with respect to minimizing passengers' waiting time. The algorithm considered (i) the intertwine effect of matching time interval and maximum matching distance, (ii) level of congestion of the network, and (iii) dynamics of waiting passengers and idle/transferred vehicles to find the optimum values at each matching time instance. The benefits of the matching method in avoiding wild goose chase problem in sending idle/transferred vehicles to long-distance waiting passengers have been demonstrated with microsimulation.

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