A REVIEW OF GAME THEORY MODELS OF LANE CHANGING

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ABSTRACT
Driver lane-changing behaviours have a significant impact on the safety and the capacity of the vehicle-based traffic system. Therefore, modeling lane-changing maneuvers has become an essential component of driving behaviour analysis. Among microscopic LC models, game theory-based lane-changing models highlight the interaction of drivers, which reveal a more realistic image of driving behaviours compared to other classic models. However, the potential of game theory to describe the human driver's lane-changing strategies is currently under-estimated. This paper aims to review the recent development of game-theoretic models that are classified according to their different methodologies and features. They are designed for both human-driven and autonomous vehicles, and we hope they can find applications in future AV industries.

Keywords: Lane-changing models, game theory, driver interaction, driver utility, autonomous vehicle

INTRODUCTION

The lane-changing (LC) maneuver has been one of the most highly analyzed topics in transport modeling. It brings both benefits and drawbacks to the traffic system. On the one hand, empirical evidence shows that LC may help to release the pressure of congestion for vehicles trapped behind a slow-moving vehicle or platoon (*1*). However, it also creates voids to obstruct the following vehicles in the receiving lane, and can cause stop - and - go oscillations or even traffic breakdown, especially near the bottlenecks (*2–5*). In addition, inappropriate LC increases the risk of collision with other drivers and results in about one-tenth of all crashes (*6*). We would like drivers to make 'smarter' lane choices to smooth traffic. However, modeling intelligent drivers' lane-changing interaction becomes complicated since researchers can only observe the consequences but not the motivation of drivers in real trajectory data (*7*) .

A few studies posit this driver LC interaction under certain conditions. For example, reinforcement and inverse reinforcement learning make it possible for autonomous vehicles to imitate successful decisions of human drivers in LC maneuvers by training the controllers (*8–10*). Without any long-period training processes, game theory, as one of the most frequent applications of simulating the process of human natural competitive and cooperative behaviours, enables consideration of the interaction of driver preferences.

Game theory (GT), which was first developed as a mathematical model in the domain of economics, was widely applied to various disciplines to study human decision-making behaviour (*11*). In general, the principle of game theory can be summarized as that decision-makers optimize their respective strategies to achieve better outcomes. That is based on the strategy of their opponents with the assumption that all involved competitors are rational. The game includes three primary elements which are respectively the number of players, the strategies they use, and the revenue (payoff) they get from the corresponding strategies (*12*). The information structure is also considered as an indispensable component of games along with the progress of the theory, because players in games may not get enough information to respond. The incomplete information results from the missing of three fundamental game elements mentioned before, but generally the payoff matrix. Besides, even though players are able to obtain complete information, they may not know the beliefs of the opponents, which is the so-called imperfect game. Harsanyi (13) suggested a transformation from the incomplete game to the imperfect game for computation of solutions.

Over several decades of exploration and discovery, game theory, especially the conventional two-player, non-cooperative game, has been experimentally tested and verified in a variety of cases (*11, 14, 15*). During this period, there have been many optimization methods for best strategy solutions with different assumptions. For games with complete information, the classical Nash Equilibrium optimizes the benefits of individuals with either pure strategy or mixed strategies. When the information in games is imperfect, a Bayesian Nash Equilibrium can be used (*13*). In addition to equilibria with the simultaneous process above, an equilibrium called Subgame Perfect Equilibrium is usually applied into sequential games with perfect information (*16*). Relatively, there is also an optimization method 'Perfect Bayesian Equilibrium' for sequential games with imperfect information (*17*). These progressive refinements of Nash Equilibrium extend conventional theory to adapt to both incomplete information and dynamic scenarios.

In addition to the microscopic level, population games have also been developed to optimize travel behaviours from a macroscopic view. Like early in the transportation field, Wardrop (18) first proposed what came to be known as a Wardrop's User Equilibrium (UE) principle in order to describe the route choice behaviour according to their utilities.

After about 20 years, there was also a prominent discovery in the subject of biology to reveal the nature of animals' conflict and cooperation in the perspective of game theory. Evolutionary Game Theory (EGT) presents the objective of dynamically describing the competition and cooperation between animals in both the same and different species (*19, 20*). It extends Darwin's evolutionary theory of natural selection that organisms are selected for fitness to the environment.

With these optimization methods, we posit that motorists on the roads tend to maximize their 'utility' in terms of various factors, such as safety and travel time, given that others do the same. In other words, we expect that selfish drivers *fear* the risk of collision with others, but they are *greedy* as well to save time and money. In contrast, if they were selfless, they might consider staying in their current lane most of the time not to disturb the crowded traffic, or change lanes to fill long-distance gaps, improving traffic efficiency and capacity.

Instead of adopting the same strategy all the time, motorists may alter their behaviours to obtain the benefit as circumstances vary. We posit the whole traffic system reaches an equilibrium with associated percentages of different strategies, as drivers adopt different rules of behaviour.

This article aims to comprehensively review and summarize the development and achievement of the existing LC models based on both classic microscopic models and GT series, as well as outline and discuss their methodologies and attributes to compare. After that, it will provide some creative ideas and suggestions for further progress of GT-based models. It supplements and extends the GT application content in recent reviews (*21–24*).

DRIVER UTILITY ANALYSIS

Before we introduce how GT works for the decision-making process, as the first challenge of game theory application, we need to identify the utility that drivers may gain from their behaviours. That is, which factors encourage or discourage drivers' LC choices. Sun and Elefteriadou (25) suggested a direct method to figure out actual drivers' characteristics when executing LC behaviours by questionnaires instead. They recruited some participants in focus groups to collect related information that may manifest their aggressiveness towards interactions, however, its small group size of 17 and participants' over-thinking lead to a limited recognition of behaviours, which could be improved by numerous experiments covering different backgrounds of participants but would be quite time-consuming. Similarly, Keyvan-Ekbatani et al. (26) demonstrated that LC strategies would be influenced by driving desired speeds, lane choices and traffic conditions through a driving test and interviews, but the moderately small sample size is still the issue that fails to perform statistically relevant results.

To simplify the complicated utility assessment through assumptions, some models try to indicate the ways that rational individuals achieve their goals when they make choices. As one of the examples, Blomquist (27) presented an economic model which considers accident loss and driver's disutility cost. The expected utility function is shown below:

$$U = U_I - U_d(E,S) - P_r(E,S)F(E,S) \tag{1}$$

where U denotes the driver's expected utility, I and U_d mean incomes and disutilities for drivers, and the product of P_r (possibility of risks) and F (estimated accident loss) represents the expected loss when an accident occurs. Except for incomes, all parameters are influenced by driver effort (E) and exogenous safety (S) factors. (*28*) also presented a similar function that replaces the first two terms in Equation 1 with the gross utility U_0. They both tell how these factors impact on the driver's utility and analyze the solutions to maximize it (with the assumption that there exists a unique Nash Equilibrium). Note that the proposed models are theoretical ones so that parameters need to be calibrated when putting into practice.

Inspired by that, Chatterjee and Davis (29) developed the rational driver utility model and applied it for prediction of rear-end collisions. They claimed that the long reaction time is the main reason for crashes when congested, so they considered reaction time and the collision speed as the accident loss part of the model as illustrated in Equation 2.

$$U = \begin{cases} U_0 - \alpha(r_1 - r_0)^2 & \text{if } r_1 + r_2 \leq 2h + \frac{v}{2}(\frac{1}{a_0} - \frac{1}{a_{max}}) \\ U_0 - \alpha(r_1 - r_0)^2 - \theta \times v_2 & \text{otherwise} \end{cases} \tag{2}$$

where:
U_0 is the estimated gross utility,
α is the weight factor related to disutility from reaction time,
θ is the ratio of the collision speed and accident loss,
r_0, r_1 and r_2 are respectively the reaction time of three different drivers,
v_2 is the speed of driver 2.

However, the assumption of homogeneity of utility functions and parameters above is highly idealized. For example, the aggressive driver may take 'risks', which would be judged as dangerous by other drivers when facing a similar scenario. Addressing heterogeneity across users is a challenge in GT models.

Ahmed et al. (30) noticed the human diversity and the importance of previous driving experience for current choices when modelling the decision-making process of LC and then defined a utility function as:

$$U_{tn} = \gamma^T X_{tn} + \lambda_n + \varepsilon_{tn} \tag{3}$$

where:
U_{tn} denotes the utility of driver n at time t,
X_{tn} is a series of explanatory variables that have affects on utility,
γ is a vector of the weighting or correction numbers for variables,
λ_n varies by individuals, and
ε_{tn} is another random term representing different periods that individuals learn from their experiences.

This expression of utility considers driver heterogeneity and gives an explanation of the reasons for changing lanes. The explanatory variables and corresponding parameters have to be estimated properly, which requires calibration from reliable direct observations and trajectory datasets before application (*31–33*).

Besides, Yu et al. (34) considered the utility constituted by the safety expressed in a function of the time headway as well as the space advantage to avoid from being occupied. When the particular conditions are given, the joint payoff of each strategy could be calculated by the following formula:

$$U_{payoff} = f_w(a, a_0)((1 - F_c(\delta)) * U_{safety}(a) + F_c(\delta) * U_{space}(a) + 1) - 1 \tag{4}$$

where:
a and a_0 denote next acceleration and initial acceleration respectively,
f_w is the penalty due to the change of speed and acceleration,
δ means the driver's aggressiveness that is assumed to follow the Gaussian distribution,
$F_c(\delta)$ is the cumulative distribution of δ, [0-1].

It is noted that δ is a key factor to decide the ratio of safety and space payoffs. In other words, the driver with a large aggressiveness value will prioritize space advantages over safety, which reflects in the large weight of space payoffs in this formula. The aggressiveness method is reasonable to evaluate how greedy drivers are, and it is supported by many findings (*35–37*).

One of the improvements of this method should be that, rather than the assumption that the aggressiveness obeys the Gaussian distribution, a more realistic distribution is supposed to explain overall features of aggressiveness in the whole population of drivers.

Thinking another way of measuring drivers' payoffs, the trade-off method applies economic theories to empirically measure the gain or loss of drivers from their observable choices (*38–40*). The basic objective of this method is to analyze the related externalities in mathematical expressions, and then exploit corresponding solutions to optimize the net utility or deploy policies to regulate. One of the advantages is to put the different factors in a similar magnitude by some adjustments and thereby ensure that the payoffs from different strategies are comparable. Another benefit is that the utility can be facilitated and easy to process. After the concept is well-determined, the main task is that the externalities between LC motorists should be considered, that is to say, which factors change the cost.

Steimetz (41) focused on both traffic delay and crash externalities which link to the traffic flow. In order to consider the joint measurement of travel behaviour, Total Social Costs (C_{TS}) and Marginal External Costs (C_{MX}) are defined with the expression of marginal costs. Besides crash and time external costs, drivers' defensive driving, as well as their effort to avoid the collision, was suggested to be evaluated as another type of crash externality (*42*), which may also influence the costs of delayed travel (*43*). But the cost of the effort itself will be neglected due to its offset between disutility and benefits (*41*). Other factors, such as toll charges, fuel consumption and car insurance are also studied by some of the researchers. But for a brief review here, a classic theory instead will be discussed in the following part. Steimetz (41) stated that the value of density (VOD) was more correlated to reveal the risk and effort towards the risk because it serves as safety margins and then implies the willingness to pay (WTP) for risk avoidance effort as well as traffic delays, where C_{TS} can be computed by the formula below:

$$C_{TS} = \{C_{M_R} R(k, E(k)) + C_{M_T} T(k, E(k)) + C_{M_E} E(k)\} q D_r \tag{5}$$

in which C_{M_R}, C_{M_T} and C_{M_E} are respectively marginal risk loss reduction, value of travel time savings (VTTS) and marginal effort reduction, $R()$ and $T()$ represent the function of risk and travel time, k is the density at a specific time, $E()$ denotes the effort level of drivers, D_r and q mean the duration and the traffic flow respectively. Meanwhile, C_{MX} is determined as the cost per time interval when adding another vehicle on the road. It can be derived from C_{TS} as:

$$C_{MX} = \frac{dC_{TS}}{d(qD_r)} - \frac{C_{TS}}{qD_r} \tag{6}$$

substituting Eq.4 into Eq.5, we get the standard expression of C_{MX}:

$$C_{MX} = C_{M_R}\left(\frac{\partial R}{\partial k} + \frac{\partial R}{\partial E}\frac{\partial E}{\partial k}\right)\frac{\partial k}{\partial q} q + C_{M_T}\left(\frac{\partial T}{\partial k} + \frac{\partial T}{\partial E}\frac{\partial E}{\partial k}\right)\frac{\partial k}{\partial q} q + C_{M_E}\frac{\partial E}{\partial k}\frac{\partial k}{\partial q} q \tag{7}$$

Through empirical measurements of parameters in equations, the C_{MX} value and its confidence interval can be estimated with different percentiles of density and also in different regions. Finally, the results provide a reference to policymakers for congestion pricing and investment.

In terms of analysis for motorists' utility from their behaviours, this methodology presents driver's benefit or loss from their behaviours with a heterogeneity distribution rather than a certain form. Referring to this point of view, it is expected that a microscopic version of the trade-off method will fill the gap of driver's heterogeneity utility theory.

TRADITIONAL MICROSCOPIC LANE-CHANGING MODELS

Early discussions about LC decision models started from the classic rule-based models that studied the driver's activity mostly from the operational (execution) level. One of the most successful studies is Gipps' model that hierarchically determines whether the execution of LC is possible, necessary and desirable (*44*). He defined three zones by different distances from the intended merge and assumed that drivers compare available lanes in terms of various factors such as the speed advantage. This model then describes the decision-making process in a flow chart with both objective and subjective questions and finally outputs a binary answer (change or not change). Moreover, Gipps followed his car-following (CF) structure and then provided a similar updating rule for LC as in Equation 8.

$$v_n(t+\Delta T) = b_n\Delta T + \left\{ b_n^2\Delta T^2 - b_n\left[2\left(x_{n-1}(t) - l_{n-1} - x_n(t)\right) - v_n(t)\Delta T - \frac{v_{n-1}(t)^2}{\hat{b}}\right]\right\}^{1/2} \quad (8)$$

where:
$v_n(t+\Delta T)$ represents the maximum speed of vehicle n in safety in respect of the preceding car at the time interval $t+\Delta T$,
b_n is the most critical deceleration of vehicle n,
ΔT is the time step for updating,
$x_n(t)$ is the location recorded at time t, and
$l_{n-1}(t)$ is the vehicle length of $n-1$.

After the information being updated, the processor decides whether to change lanes or not according to the preset rules. If the conflict occurs, at the execution level, the priority lane selection system will evaluate the choices that are the highest level of the hierarchy, however, it ignores other primary considerations that may also have a significant impact, which should be integrated by trade-offs among considerations (*45*).

Subsequent articles inspired by Gipps' pioneering work set many criteria as well but from different aspects of considerations (*46–50*). As an example, Hidas (46) with his Analysis of Road Traffic and Evaluation by Micro-simulation (ARTEMiS) model applied an autonomous agent technique for simulations of drivers' interactions, so each vehicle in his model operates as a driver-vehicle object (DVO) in some critical scenarios like lane drops and blockages. He classified LC into three types that are respectively free, forced, and cooperative. According to the different types, he designed a detailed algorithm for each one to decide the execution of LC maneuvers. In the controller program, the lead and lag gaps are first measured to check the feasibility of LC. It can be seen that the ARTEMiS model respectively sets the feasibility and essentiality criteria to restrict the plan, which can produce collision-free merging, even in the compulsory LC scenario, the vehicle tends to brake or stop for the appropriate gap. The perfect merging may perform well if all motorists are intelligent and they make no mistakes, while in the real world there should still be some failures due to various factors even with 100% penetration rate of autonomous vehicles. Moreover, the other disadvantages of this model are concluded as incomplete LC reasons, inability to solve conflicts between speed advantages and intended movements, and separate consideration of cooperative and forced LC (*24*).

Learning from the idea of explaining how drivers across the unsignalized intersections or enter T junctions, the gap-acceptance theory is also applied to model LC maneuvers in which drivers consider whether the gaps are large enough to accept. The critical gaps are also called acceptable gaps, which are random variables varying by drivers, and by comparing them with actual gaps drivers can execute their decisions. In general, there are two types of critical gaps, including the lead and the lag gap to be estimated by the formula below:

$$G_n^{cr,j}(t) = exp(X_n^{cr,j}(t)\beta^j + \alpha^j \xi_n + \mu_n^{cr,j}(t)) \tag{9}$$

where:
j represents lead or lag gaps,
$G_n^{cr,j}(t)$ means critical gaps of driver n at t,
$X_n^{cr,j}(t)$ is the vector of explanatory variables,
β^j is the vector of corresponding parameters,
ξ_n is a random variable representing specific drivers,
$\mu_n^{cr,j}(t))$ is also a random variable that is subject to normal distribution $N(0, \sigma^2_{\mu j})$.

And then the possibility of gap acceptance is given as follows:

$$P_n(gap|\xi_n) = P_n(G_n^{lead}(t) > G_n^{cr,lead}(t)|\xi_n) \times P_n(G_n^{lag}(t) > G_n^{cr,lag}(t)|\xi_n) \tag{10}$$

Ahmed (51) applied this theory to model for mandatory lane changing (MLC), discretionary lane changing (DLC) as well as forced merging (FM) decisions with the discrete choice framework that describes the probability of performing three mentioned decisions. A DLC decision is made when the actual lead and lag gaps exceed the critical ones. The driver then calculates the possibility of gap acceptance and finally chooses whether to change lanes or not. Compared to DLC, MLC in heavily congested traffic or forced merging will be executed regardless of the existence of acceptable gaps. This model distinguishes three types of LC and separately describes their features, but the boundary of MLC and DLC is so rigid that drivers cannot make DLC when MLC plans are activated (*24*).

To improve the boundary issues, another gap-acceptance based model was developed by Toledo et al. (52) by considering the trade-offs between MLC and DLC in the utility function. The basic concept of this probabilistic model is similar to Ahmed's model, but it designs a detailed acceleration behaviour for drivers either accepting or rejecting the available gaps after target lane selection. It has two steps to finish the whole LC process: the first one is to choose the destination lane by applying the lane selection model; and the second one is the decision to accept a gap considering four explanatory variables that consist of the gap and speed information, path plan, previous experience, and driving characteristics. Similar to some other microscopic models, the main drawback of this model is that for each choice the driver's utility function is hard to determine. Inspired by their work, there have been many LC models, including either the gap-acceptance or the discrete choice component (*47, 53–55*), which have been widely implemented in microscopic traffic simulators for traffic control and management.

The proposals about both rule-based model and gap-acceptance based model have formed the major part of LC research in decades, but, it is supposed that in the future other adaptable models will realistically explain LC maneuvers with human characteristics consideration instead of being restricted by the same rules.

Moreover, some artificial intelligence simulation models like fuzzy logic-based LC model (*56–58*) and Neutral Network model (*59*) have investigated human drivers' preference of LC to imitate their response towards real-world scenarios, which well describes the human's cognitive thinking towards the uncertainties in LC scenario. However, issues like challenging function expressions and complicated parameters make it much more difficult to calibrate and validate. Models with simple and clear explanations like the game-theoretic model will likely be appreciated in future applications.

TABLE 1 The payoff of each pure strategy

		Player J	
		C	D
Player I	C	$\mathrm{E_I(C,C),E_J(C,C)}$	$\mathrm{E_I(C,D),E_J(C,D)}$
	D	$\mathrm{E_I(D,C),E_J(D,C)}$	$\mathrm{E_I(D,D),E_J(D,D)}$

OVERVIEW OF GAME THEORY

In general, driver activity is a continuous process that can be decomposed into three main levels: strategic, tactical and operational (*60*). Strategic and tactical activities can be modeled with game theory.

To illustrate the model with a toy example, we start with a 2×2 game, which means two competitors in total play two separate strategies, so the number of different combinations is four.

Suppose that there is a game between two players (or agents) I and J with complete information, which means they understand all the strategies that their opponents adopt by observing the mirrors or other advanced technologies. The players are able to choose to either cooperate (hereafter, C) or defect (hereafter, D) to maximize their utilities. Table 1 briefly summarizes the revenues that two players can obtain from their strategies.

In this table, Player I (rows) and Player J (columns) behave simultaneously. The payoffs of each strategy are listed in each corresponding cell. With all revenues displayed in the table, it can be transformed to a 2×2 matrix, called the payoff matrix $\mathbf{M}$. To optimize the individual benefits towards various payoff matrices, Nash Equilibrium is widely used. At least one Nash Equilibrium exists in all finite games when considering mixed strategies. When one of the following requirements is satisfied, the pure strategy exists and will be adopted by players.

- $E_I(C,C) > E_I(D,C)$ and $E_I(C,D) > E_I(D,D)$ or
- $E_J(C,C) > E_J(C,D)$ and $E_J(D,C) > E_J(D,D)$

When the payoffs fail to meet the above conditions, players apply mixed strategies based on the computed possibilities of each strategy. The utility expectation of mixed strategies can be calculated according to different possibilities of strategies. For example, if the probability for Player I to cooperate is p and for Player J to cooperate is q, the overall utility expectation for Player I will be:

$$\begin{aligned} E_I = {} & pqE_I(C,C) + p(1-q)E_I(C,D) + (1-p)qE_I(D,C) \\ & + (1-p)(1-q)E_I(D,D) \end{aligned} \tag{11}$$

Furthermore, when a model holds multiple equilibria, the equilibrium selection process is needed for the scope of actual GT applications with complete and incomplete information (*61, 62*). The possibilities of each equilibrium state (including both pure strategy and mixed strategies) can be empirically estimated by Method of Simulated Moments (MSM) estimator from the specific $\mathbf{M}$ and previous outcomes of repeated plays.

For population games, the studied population is assumed to be well-mixed, and at every time interval, two selected drivers start the game. Note that each player may play with the opponent she has met before, for example, it may happen after a failure to change lanes and the drivers repeat the game in the next trial. The last assumption allows players to learn from their failures and adjust strategies towards a better payoff. With the certain $\mathbf{M}$, Smith and Price (19) stated that there existed 'evolutionary stable strategy' (ESS) for the natural selection from conflicts of the same or different species. Accordingly, to find the ESS for this game, a strategy should meet one of the following

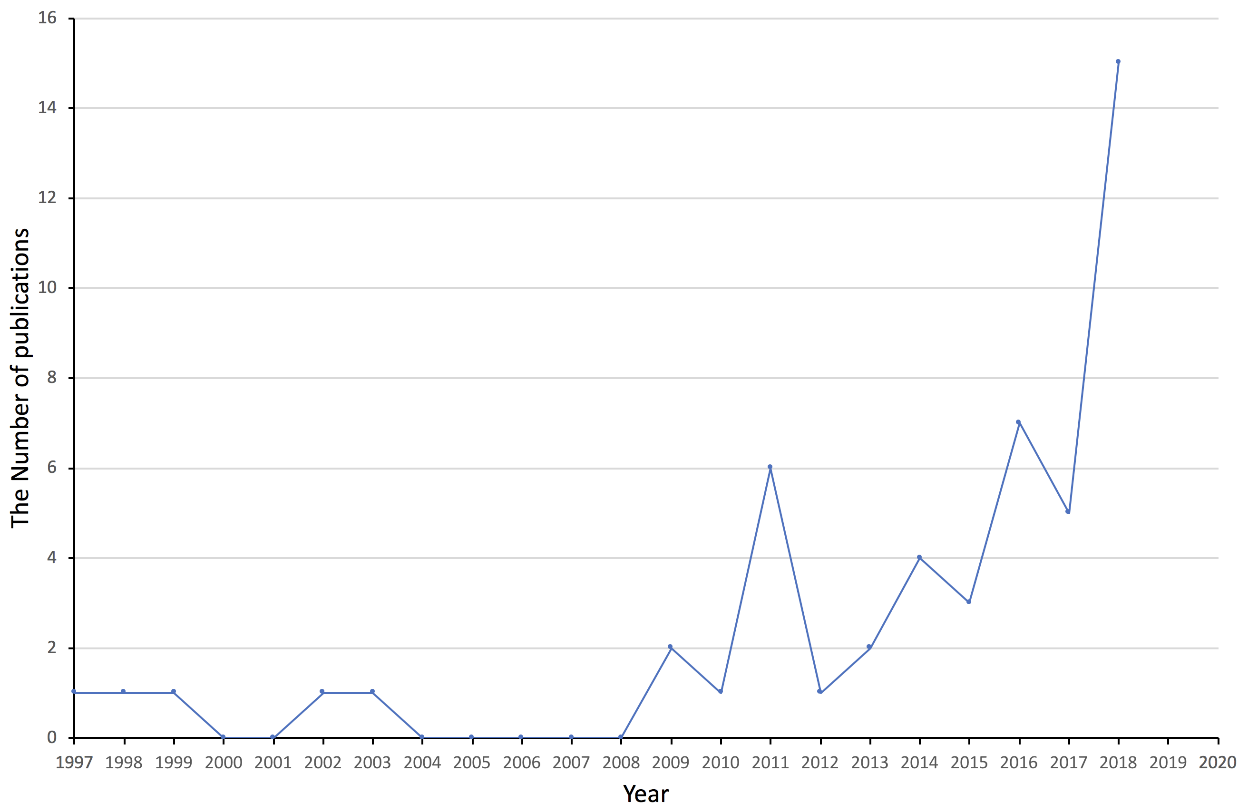

FIGURE 1 Life-cycle of publications in game theory-based lane changing models from 1997 to 2018

requirements:

- $E(C,C) > E(D,C)$ or
- $E(C,C) = E(D,C)$ and $E(C,D) > E(D,D)$

It can be clearly identified that the determination of the ESS depends on the mathematical relationship among R, P, S and T, which can be computed by the driver's utility quantification mentioned before. Accordingly, this relationship will differentiate games into four types, including Prisoner's Dilemma (PD), Hawk and Dove (Chicken), Trivial, and Stag Hunt (SH) games respectively (*63*). They all belong to the types of dilemma games except Trivial, which may cause the benefit conflict between individuals and the group. Therefore, social viscosity mechanisms are proposed to weaken this conflict until the system achieves the equilibrium so that both individuals and the system obtain the expected outcomes.

GAME-THEORETIC LANE-CHANGING MODEL

Game theory better describes driver interactions, so there many LC models have been recently integrated with GT and stand at the forefront of LC research. For a clear and systematic review of the literature, we identified relevant articles with keywords search engines from Google Scholar and Web of Science databases. However, in general, some of the articles in search results may be irrelevant to the study here, meanwhile, some may not include the keywords but the themes are still appropriate. Therefore, we read the abstract of each paper and checked other relevant papers in its reference list to discover articles as possible. The filtered articles are collected with

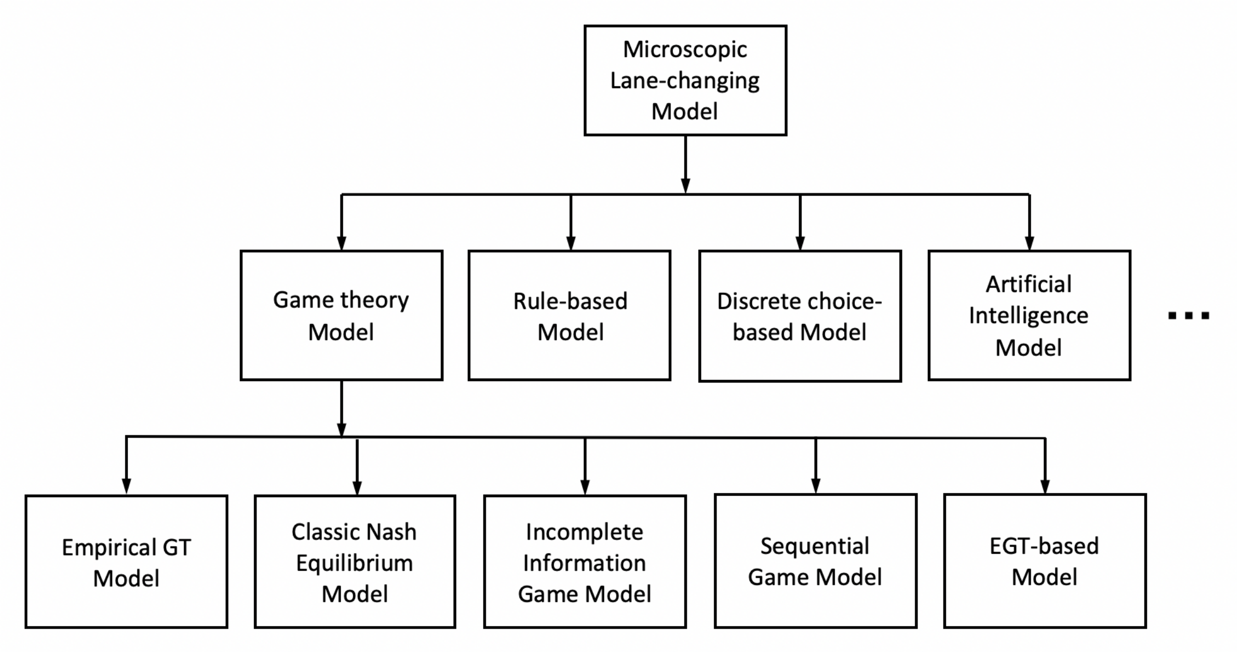

FIGURE 2 Classification of game theoretic lane-changing model

TABLE 2 Features of games with different classifications

Model Classifications	Representative model	Features
Empirical GT Model	Kita (64)	is reliable due to being calibrated from real scenarios
Classic Nash Equilibrium Model	Liu et al. (65)	achieves the best personal choices with complete information
Incomplete Information Game Model	Talebpour et al. (66)	separates LC into two types: mandatory and discretionary
Sequential Game Model	Yu et al. (34)	supposes one reacts first and the other responds later
EGT-based Model	Cortés-Berrueco et al. (67)	promotes the cooperative rate in groups progressively

the criteria, including the research subject is automobiles, the models are microscopic ones, and so on. The life-cycle diagram of total publications per year is presented in Figure 1. After that, we summarized the similar features of published models and categorized them into four types as shown in Figure 2 and Table 2. Note that due to the limited number of articles, some representative GT models will be subsequently introduced in chronological order.

Basic form GT-based models

Kita (64) first formally proposed the concept of the game-theoretical model for merging and give-way interaction to describe how drivers participate in an on-ramp section. He stated that although many previous studies indicated the significance of giveway analysis, few of them investigate the motivation and the desire of giveway actions. For that, he designed a one-directional section and five vehicles involved simulated experiment, where the merging vehicle that affects others will be influenced by others as well. Figure 3 demonstrates the scenario that player 1 has to merge into the major road because of a closure. This interaction process is described by a non-cooperative game with the assumption of complete information, which means drivers understand all the strategies (and pay-off matrices) adopted by each other. To estimate the matrices, the values of payoffs can be computed through the time-to-collision (TTC) measured as well as time headway and their coefficients are estimated by data from an expressway in Japan. The result then shows a high

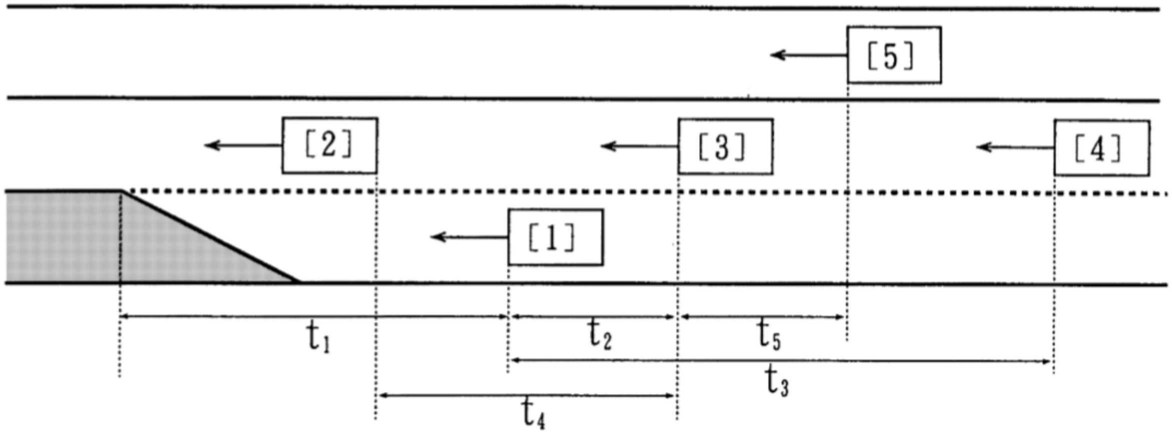

FIGURE 3 Explanatory variables influencing decisions when merging (*64*)

correlation factor, which demonstrates the model's capability of explanation. After several years' development, the payoff estimation part has been enhanced in the direction of interdependence and equilibrium selection (*68, 69*).

Liu et al. (65) claimed the assumption of constant speeds in Kita's model is unrealistic, and in the meanwhile, the giveway strategy may sometimes be performed before merging occurred. Therefore, this conventional game theoretical framework should be improved with multiple equilibrium solutions, but there is no doubt that the study provides an example of empirically simulating LC maneuvers which can be easily replicated. Then, they proposed another GT on-ramp merging model with more realistic behavioural rules. Instead of assuming that the objective of drivers is only to minimize the risk, they supposed that both speed variations and time spent should be considered in the payoff functions, so they applied a series of physical formulae with unknown coefficients to represent the payoffs when two drivers take different strategies. Parameters of the payoff function are estimated by solving an optimal bi-level problem in which Nash equilibrium is at the lower level and a non-linear minimized function of total deviation shown in Equation 12 is at the upper level.

$$min(M,N)\sum_{i=1}^{n}\left[(N_i-\hat{N}_i(M,N))^2+(M_i-\hat{M}_i(M,N))^2)\right] \tag{12}$$

where:
i is the observation index,
N_i is the observed decision of through vehicle (1 means yield, 0 otherwise),
M_i is the observed decision of merging vehicle (1 means merge, 0 otherwise),
$\hat{N}_i$ and $\hat{M}_i$ are respectively the predicted decision of two vehicles.

After solving the formulation by field data from Freeway Data Collection for Studying Vehicle Interactions (DCSVI) project by FHWA, the calibrated parameters then present a high correlation with real LC situations. The payoff functions are able to describe the behavioural characteristics of different drivers on roads.

To develop a GT model based on drivers' feelings about circumstances, Pei and Xu (70) studied drivers' behaviours in a jam condition. They considered the driver's experience from being congested in an over-crowded scenario, and then computed the possibility that the driver is willing to cooperate. The possibility of cooperation depends on the current density of the roads and the maximum tolerant waiting time of drivers when the current velocities do not exceed the maximum velocity of LC execution. Considering both time (U_t) and safety (U_s) payoffs, the expected benefit of each strategy could be expressed by the summation of possible payoffs, which finally gives the Nash Equilibrium point $p = \frac{U_t}{U_t+U_s}$. The model established the theoretical framework of GT description of driver's choices but lacked the specific estimation of payoffs. The future development of this model is to estimate the benefits properly instead of assumptions.

Since then, many articles have emerged to develop the lane-changing mechanism based on a game-theoretic approach, especially in more recent years. Peng et al. (71) also demonstrated a two-competitor but non-cooperative game. Similar to Pei and Xu (70), they considered both the journey time and the safety level as revenues of players, which more realistically account for driver taste. The Nash equilibrium is then applied to explore the optimal solution of the mixed strategy game. Finally, the probability of each strategy is in the form of safety and travel time expressions together with weighting factors α for safety and β for travel time. To estimate importance degree of payoffs, they decided to use questionnaires but it is pretty time-consuming, and it will ignore the variety of objects like genders, ages and driving experiences, which is the main disadvantage of this model or other similar models.

The Autonomous Vehicle (AV) is expected to realize the best strategies when responding to various scenarios to some extent. Therefore, some automatic GT-based LC algorithms have been designed for AVs. For example, by a trade-off of varying factors, cooperative controllers were designed to reduce the total cost caused by merging maneuvers (*72*). The empirical cost function that the driver makes a longitudinal change is then specified as follows:

$$\begin{aligned}\zeta(z(t),u(t),t) = {} & \frac{\beta_{safe}}{s_{i,\sigma_i}}\Delta v_{i,\sigma_i}^2\Theta(-\Delta v_{i,\sigma_i}) + \beta_{eq}(v^e(s_{i,\sigma i}) - v_i)^2 + \beta_{ctrl}a_i^2 \\ & + \beta_{eff}(v^d - v_{\sigma_i}^{\alpha})^2 + \beta_{route}e^{\frac{d^0}{d_{i,\sigma_i}^{end}}} + \beta_{pref}h(\sigma_i) + \beta_{switch}U_{d,switch}(\sigma_i)\end{aligned} \tag{13}$$

where:
α is the weight factor related to disutility from reaction time,
βs are positive weight factors of different travel costs,
σ is the ID of traffic lanes,
s is the distance between two vehicles,
Δv is the speed difference between two vehicles,
v_i is the current speed of the lead vehicle,
v^e is the local equilibrium speed based on density,
v^d is the desired speed,
a_i is the current acceleration of the lead vehicle,
d_0 and d_{end} are the current location of route plan and the distance between the vehicle and the dead end,
h is the cost of lane preference,
$U_{d,switch}$ is the driverâĂŹs unwillingness to change lanes.

The seven components respectively represent safety, equilibrium, control, travel efficiency, route choice, lane preference, and lane switch. All βs in front of each term need to be estimated empirically, while other parameters can be obtained from real-time traffic information.

To approach the solution of optimal LC decision, they defined the Hamiltonian based on Pontryagin's Principle (*73*) and then obtained the necessary conditions for LC execution. Based on this model, the controllers are finally tested by numerical experiment in highway conditions and the results showed that vehicles equipped with the controllers perform well with both AVs and human-driven vehicles with the lowest cost. It is certainly a breakthrough among the microscopic CF and LC models, though due to its complexity and heavy computation, the parameters cannot be efficiently estimated.

Considering drivers may not obtain enough information about beliefs of their opponents, another type of model deals with incomplete information LC games (*66, 74*). It applies Harsanyi transformation (*13*), which introduces 'nature' as one of the players (but without any payoffs) to determine the beliefs of players. After that, the transformed game is presented as an extensive form. To find the solution(s) with uncertain beliefs, Bayesian Nash Equilibrium is commonly utilized by first assuming the possibility of one belief, such as mandatory LC or discretionary LC, and then exploring in which situations drivers can optimize their strategies. The models have been used to predict driver behaviours for both connected environment simulation and real scenarios.

Given that drivers' response could be sequential, Yu et al. (34) implemented an integrated controller with a lower-level controller (responsible for car-following and lane-keeping) and upper-level controller (responsible for LC). The high layer rules are based on GT to achieve the driver's intention of when and how to take actions. The game type is decided as a multiplayer Stackelberg game (*75*), which is a type of sequential game as well. That means in this game one reacts first, and another respond later. Schönauer et al. (76) exploits the same idea to solve the problems in mixed traffic. The estimated total payoffs together with the description of the driver's aggressiveness in this LC game has been introduced in the utility analysis section, and they will be input into the controller for the optimal strategy search. Then by recognizing other drivers' aggressiveness from the estimation algorithm or previous experiences, the optimal acceleration will be computed in every interval as the updating rules whatever the controller chooses to change lane or not. The designed controller can not only recognize its own aggressiveness factor ranged from 0 to 1 but also estimate others' factor and then imitate the response of human drivers.

In summary, the basic form GT-based models have developed from simple static forms with complete information that consider few factors, to complicated dynamic forms with incomplete information that cover multiple factors. They all demonstrate the feasibility of GT to reveal human interaction and decision-making processes. The application of GT models into autonomous algorithmic systems perform well in operating computationally and are expected to be applied with the deployment of AVs.

EGT-based models

Additionally, there are few studies about EGT-based LC models. In general, evolutionary game theory applied in LC models tends to explain drivers' progressive cooperation interactions from the perspective of the whole society. Cortés-Berrueco et al. (67) assumed that all agents in a constant population are able to arrange themselves to achieve the evolution of cooperation with others, and then put them into a traffic simulation controlled by a probabilistic Cellular Automaton (CA) model named GLAI for updating key information needed. In the experiment, the player who decides the cooperative strategies will pay the cost that the other player receives it as a reward. Also, the cooperative probabilities of players will be updated as their behaviours towards ESS (previously mentioned in section 5) according to one of the five protocols (Kin Selection, Direct Reciprocity, Indirect Reciprocity, Network Reciprocity or Group Selection) presented by Nowak (77). The results of the simulation test manifest that the human cooperation behaviour only occurs in certain values of the density, which shows that the traffic condition is the main factor for cooperative LC behaviours.

Similarly, Iwamura and Tanimoto (78) also implied the strategy fractions in a group have the tendency to change because everyone desires to gain the maximum benefits. All drivers applying their (selfishly) optimized strategies brings a serious social dilemma under high traffic densities. They proposed a creative model to reveal the dilemma effect. The model sets the safety and incentive criteria to select the appropriate gaps and then assumes that all vehicles obeying Cellular Automaton (CA) rules and replicator dynamics. After running the simulation, the results indicate that sometimes the solutions of Nash equilibrium and social equilibrium cannot be simultaneously achieved. For direct regulations, the policies should be deployed to adjust either traffic conditions by controlling the traffic density or the payoff drivers may obtain by restricting people from their optimal benefits both aim to encourage cooperative behaviours. Another possible method suggested by Tanimoto (79) is to combine two viscosity mechanisms like indirect reciprocity and network reciprocity to enhance cooperation compared to only one reciprocity, but the finding shows that the integrated mechanisms may negatively affect cooperation. There are also some recent-published studies setting up in simulators in order to motivate cooperation strategies (*80, 81*). Further research about the feasible approach to weaken or eliminate the dilemma effect are supposed to progress.

The models integrated with EGT illustrate the possibility of EGT to describe the decision-making process. They discover the cooperation social dilemma and its strength with different traffic densities and attempt to alleviate it, which emerges as another research interest for GT-based LC model.

CONCLUSIONS

This article summarizes some mainstream microscopic lane changing (LC) models and compares them with the game theoretical LC models to explain the importance of human interaction element. We believe that the potential of Game Thoery (GT)-based LC models is currently under-estimated, and it still needs further exploration including investigation of both realistic analysis of drivers' motivation of LC and the practicality of theoretical models.

GT models focus on the human interaction process under an information structure, presenting an advantage over other models. Rather than directly setting the criteria or rules for controllers, GT logic is contingent on the behavior of other players.

Current GT-based LC models have been well-developed for several decades, but more improvements of models require further study. At first, the payoff function, as the core of GT, should be estimated comprehensively based on factors that impact on drivers' choices. Microscopic utility theory that considers individual heterogeneity with an appropriate specification is preferred. For calibration and validation of LC models, more detailed and sufficient traffic data is required. Future accurate datasets could be developed by innovations like improved GPS and smartphone-collected field data, or better still, AV sensors, which would dramatically promote the development of both existing and future LC models. It is also expected that future GT-based LC models will be modified to cooperate with (or incorporate) some of the well-known car-following models or directly adapt to traffic simulation tools.

Synchronizing selfish and selfless (cooperative) behaviour in the route choice decision, which is the goal of road pricing, also has promise in the lane choice decision. Cooperative behaviour can be embedded in social-aware LC algorithms for AVs, which could be mandated via regulation, or motivated financially with tools like variable prices for lane changing under various conditions (e.g. positive prices when lane changing is discouraged, negative prices when it is encouraged).

Nomenclature

α	the weight factor related to disutility from reaction time
β	weight factors of different travel costs (safety, equilibrium, ...)
ΔT	the time interval
ε	factors varied from experience
γ	weight factors for different explanatory variables
λ	factors varied from individuals
μ	a random variable
σ	the desired lane sequence
θ	the ratio of the collision speed and accident loss
ξ	a random variable
ζ	the estimated total cost
a	the acceleration of vehicles
b	the critical deceleration of vehicles
C	the cooperative behaviour
C_M	the marginal cost
C_{MX}	the marginal external cost
C_{TS}	the total social cost
D	the defective behaviour
d^0	the current location of route plan
d^{end}	the distance between the vehicle and the dead end
D_r	the duration of the travel
E	the driver's effort to avoid collisions
$E(C(D),C(D))$	payoffs with different strategies between player I and J
F	the estimated accident loss
$F_c(\delta)$	the cumulative distribution of δ
f_w	the penalty due to the change of speed and acceleration

G	distance gaps
h	ID of traffic lanes
i	the observation index
j	lead or lag gaps indicators
k	the traffic density
L	the length of vehicles
M	the observed decision of the merging vehicle
N	the observed decision of the through vehicle
n	ID of vehicles
P	possibilities
P_r	the possibility of risk
q	the traffic flow
R	the risk of accidents
r	the reaction time of drivers
S	the exogenous safety factors
T	the travel time
U	the total utility of drivers
u	control vectors
U_0	the gross utility of drivers
U_d	the disutility of drivers
U_I	the income of drivers
$U_{d,switch}$	the driver's unwillingness to change lanes
U_{tn}	the utility of vehicle n at time t
v	the speed of drivers
x	longitudinal location coordinates
X_{tn}	explanatory variables that effect the utility
Y	the distance between the leading and following vehicles
z	the continuous state vector

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