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Capacity Alignment Planning for a Coal Chain: A Case Study

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TITLE: **Capacity Alignment Planning for a Coal Chain: A Case Study**

ABSTRACT: We study a capacity alignment planning problem for a coal chain. Given a set of train operators, a set of train paths, and a terminal comprising of a dump station and a set of routes from the dump station to the stockyard, we seek a feasible assignment of train operators to train paths, to time slots at the dump station, and to routes. The assignment must maximize the number of system paths in the resulting schedule and the schedule should perform well with respect to various performance criteria. We model the problem as a mixed-integer conic programme (MICP) with multiple objectives which we solve using a hierarchical optimization procedure. In each stage of this procedure we solve a single objective MICP. Depending upon whether we evaluate the associated performance criteria under a 2-or 1-norm we reformulate the MICP as either a mixed-integer second-order cone programme or as a mixed-integer linear programme respectively, and can streamline the hierarchical optimization procedure by exploiting properties of the model or observed behaviour on practical instances. We compare the performance of the procedure under the different norms on a real instance of the problem and find that the quality of the solutions found by the faster 1-norm procedure compare well to the solution found under the 2-norm.

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1 Introduction

Aurizon is a large rail freight operator that owns, operates, and manages, the Central Queensland Coal Network, Australia's largest coal rail network. A central planning problem that they face is the alignment of the available capacity of the various components of coal chain infrastructure so as to maximize the opportunity for trains to haul coal from the mines to the ports where the coal is unloaded and stockpiled for export. The available capacity of the rail network is measured in terms of train paths, but a train path is only usable if it can be linked to loading and unloading slots at the mine and port, and the unloaded coal can then be stacked onto a stockpile. These so called system paths are the true measure of the available capacity of a coal chain. We consider the capacity alignment planning problem in which a schedule of feasible system paths are sought that perform well with respect to various performance criteria.

Related literature includes a review by Abril et al. (2008) on maximizing the number of trains which can be scheduled on a single track rail network and the work of Caprara et al. (2002) on the train scheduling problem. Liu and Kozan (2011) and Masoud et al. (2017) also consider the optimization of the capacity of coal rail networks in the Australian setting.

Before formally defining our capacity alignment planning problem we introduce some terminology. The Central Queensland Coal Network is comprised of four main rail corridors, each of which forms the backbone of what is referred to as a coal *system*. Each system has one or more terminals. A *terminal* is a facility where the railed coal is unloaded and stockpiled. A terminal is located at a port and is connected by rail to the rail network.

In our setting, we consider a single terminal that serves two systems. Adjacent to the terminal is a rail yard. The *rail yard* is a facility through which the trains loaded with coal must pass in order to reach the terminal from either rail corridor. Upon arriving at the yard, some trains must undergo *provisioning* operations such as servicing and refuelling before continuing to the terminal.

We define a *train path* along a rail corridor to be the 3-tuple (*system*, *departure time*, *arrival time*). There are two types of train paths: *loaded* and *empty*. Loaded train paths permit travel along the rail corridor in the direction the terminal. The train path originates at the far end of the rail corridor at *departure time* and terminates at the terminal at *arrival time*. Empty train paths serve the opposite direction.

Loaded train paths are typically used by loaded trains hauling coal from a mine to the terminal. Upon arriving at the terminal the loaded train enters a *dump station* and the coal is unloaded into a *pit* where it is then transported by conveyor belt to a pad in the terminal stockyard where it is stockpiled. The path taken by the coal from the pit to the pad is termed a *route*. Typically, a dump station has several pits, not all routes are accessible from every pit, and there are restrictions on using some route combinations simultaneously. An *unload slot* is a 3-tuple (*pit*, *start time*, *finish time*). If an unload slot is assigned to a train, the train must arrive at the pit and begin unloading at *start time* and depart the pit no later than *finish time*.

We define a *system path* to be the 5-tuple (*train path*, *train operator*, *provisioning state*, *unload slot*, *route*). If a system path is used by a train, then the train must follow *train path*, be operated by *train operator*, be provisioned if *provisioning state* is true, use *unload slot*, and the unloaded coal will be transported to a pad via *route*. For a system path, *waiting time* is defined as the time difference between the end of provisioning and start time of the unload slot.

A *schedule* is a set of system paths. The problem is to find one or more schedules that are feasible with respect to a set of constraints and perform well against a set of performance criteria. In the remainder of this section we define the problem input, the constraints, and the performance criteria.

1.1 Problem input

The problem input is the following:

1. A planning horizon of four weeks or equivalently 28 days.
2. A set of train operators.
3. A set of systems.
4. The provisioning times for each train operator.
5. The maximum waiting time of a train following provisioning.
6. The set of routes in the terminal. There are 5 routes A, B, E, G, J.
7. The set of pits at the dump station. The dump station has three pits 1, 2, and 3.
8. The pit-route access mappings which determine the routes each pit has access to. Pits 1 and 2 have access to routes A, B, E, and G. Pit 3 has access to routes B, E, G, and J.
9. A set of available loaded train paths.
10. The unloading times at the dump station. These are fixed at 2 hours and 25 minutes.
11. The total demand for each week in the planning horizon for the system paths corresponding to each (*system*, *operator*) combination. A system path corresponds to the combination (*system*, *operator*) if its train path belongs to *system* and its nominated operator is *operator*.
12. The target route utilizations. These are the desired fraction of system paths corresponding to each (*system*, *operator*, *route*) combination in a schedule.

1.2 Constraints

The constraints or business rules of the problem are the following:

1. For each system path:
   (a) the start time of its unload slot is no earlier than the finish time of its provisioning; and
   (b) the associated pit of its unload slot has access to its route.
2. For each week in the planning horizon, the number of system paths in the schedule for each (*system*, *operator*) combination is at least equal to the loaded train path demand for that system, operator, and week. A system path is counted as a system path of a given week if the the start time of its unload slot occurs in that week.
3. The schedule must respect the pit-route access mappings.
4. Each train path and unload slot can only be used in at most one system path in the schedule.
5. Since there is only single track access to the dump stations at the terminal there must be at least 45 minutes separation between two consecutive arrivals. Therefore, for every two system paths in the schedule, the start times of their unload slots must be at least 45 minutes apart.
6. Two system paths with unload slots that overlap in time cannot have the same pit or the same route.

7. The waiting time of any system path in the schedule cannot be greater than the maximum waiting time.
8. Routing rules:
   (a) Each route can be served by at most one pit at any time.
   (b) Each pit can serve at most one route at any time.
   (c) Pits 1 and 2 cannot serve routes E and G at the same time.

1.3 Performance criteria

The performance criteria for the problem are the following:

1. The number of system paths is maximized.
2. For each week of the planning horizon, the fraction of system paths for each (*system*, *operator*) combination is close to the associated contract share. The contract share corresponding to (*week*, *system*, *operator*) is defined as the ratio of the demand for system paths which belong to *week* and correspond to the (*system, operator*) combination over the total demand. Furthermore, the deviations over (*system*, *operator*) combinations should be balanced, that is, they should be as close to each other as possible.
3. The system paths for each (*system*, *operator*) combination are evenly distributed over each week of the planning horizon. The deviations over (*system*, *operator*) combinations should be balanced.
4. For each week of the planning horizon, the fraction of system paths for each (*system*, *operator*, *route*) combination is close to the desired route utilization. Furthermore, the deviations over (*system*, *operator*, *route*) combinations should be balanced.
5. The total waiting time at the yard is minimized.
6. The system paths corresponding to each (*system*, *operator*) combination allow provisioning with the given frequency. For example, with respect to departure times, every second system path from each system, for each train operator, allows for provisioning. The schedule should be balanced regarding this performance criterion in the sense that the deviations from the ideal frequency over (*system*, *operator*) combinations and all intervals is balanced. Continuing the example, the schedule should not contain two disjoint intervals, each with 4 system paths, in which all the system paths of one interval allow for provisioning, while all of the system paths of the other interval do not.

2 Problem formulation

Without loss of generality, the time horizon is $H = [1, h] = [1, 40320]$, where each time period corresponds to a minute. A system path belongs to the time horizon H if its unload start time is in H. The departure time of some of the system paths is in the day before the planning horizon starts. We represent the day before the planning horizon starts by the interval $[-1439, 0]$. The planning horizon is for scheduling all system paths with unload start times in the planning horizon. Note that all data time periods are in the range $[-1439, 40320]$.

2.1 Parameters and variables

We define the following parameters for the given planning horizon based on the input data:

– Y: Set of systems.

– P: Set of available loaded train paths. Each train path $p \in P$ is described by a departure time $t^d \in H$, a system, an arrival time t^a. The destination of all train paths is the yard and there exists a unique origin per each system.
– O: Set of train operators.
– R: Set of routes.
– D: Set of pits.
– $B = \{0, 1\}$: Set of provisioning states in which the `True` state is indicated by 0 and the `False` state is indicated by 1.
– U: Set of all available unload slots. Each unload slot is described by a pit, and a start time and an end time. The end time depends just on the start time. Therefore, each unload slot can be identified by its pit and its start time. For example unload slot `(1,100)` refers to the unload slot which starts at pit 1 at time 100, and finishes two hours and 25 minutes later at time 245.
– S: Set of all valid system paths which their unload start time is in H. According to the definition, each system path, i, is described by 5-tuple $(p, o, e, u, r) \in S$.

Let H^d, H^a, H^s denote the sets of departure times, arrival times, and unload start times respectively. More formally,

$$H^d = \{t' : \exists y \in Y \text{ and } t^a \in [-1439, 40320]\ (y, t', t^a) \in P\},$$
$$H^a = \{t' : \exists y \in Y \text{ and } t^d \in [-1439, 40320]\ (y, t^d, t') \in P\},$$
$$H^s = \{t' \in H : \exists \ell \in D\ (\ell, t') \in U\}.$$

Let $S_1 = Y,\ S_2 = H^d,\ S_3 = H^a,\ S_4 = O,\ S_5 = B,\ S_6 = D,\ S_7 = H^s$ and $S_8 = R$. Note that $S \subseteq P \times O \times B \times U \times R$, $P \subseteq Y \times H^d \times H^a$, and $U \subseteq D \times H^s$. For a subset $J \subseteq \{1, 2, \ldots, 8\}$, we define map $S_J : \prod_{i \in J} S_i \to 2^S$ as follows:

$$S_J((s_i)_{i \in J}) = \{((s'_1, s'_2, s'_3), s'_4, s'_5, (s'_6, s'_7), s'_8) \in S : s'_j = s_j \text{ for all } j \in J\}$$

Furthermore for $Q \subseteq \prod_{j \in J} S_j$, let $S_J(Q) = \bigcup_{s \in Q} S_J(s)$. We say that system path $i \in S$ has *property* $s \in \prod_{j \in J} S_j$ if $i \in S_J(s)$. Analogously, for $Q \subseteq \prod_{j \in J} S_j$ we say that system path i has property Q, if $i \in S_J(Q)$. For ease of exposition and readability, we define new subscripts for maps S_J. More specifically, let $S_{YOH^s} = S_{\{1,4,7\}}$, $S_P = S_{\{1,2,3\}}$, $S_U = S_{\{6,7\}}$, $S_{H^s} = S_{\{7\}}$, $S_R = S_{\{8\}}$, $S_{YOH^s} = S_{\{1,4,7\}}$, $S_{YOH^sR} = S_{\{1,4,7,8\}}$, $S_{YH^dO} = S_{\{1,2,4\}}$, and $S_{YH^dOB} = S_{\{1,2,4,5\}}$.

We denote the decision of choosing or not choosing system path $i \in S$ in a schedule by binary variable x_i, that is, $x_i = 1$ indicates that system path i is chosen and $x_i = 0$ indicates otherwise. Let $x_J(Q) = \sum_{i \in S_J(Q)} x_i$ denote the summation of the variables corresponding to system paths that have property Q. We similarly define new subscripts for maps X_J.

2.2 Constraints

We formulate the constraints as follows:

Constraint sets 1 and 7: The constraint sets 1 and 7 are implicit in the construction of the set of system paths S.

Constraint set 2: Let the demand for the system paths with property $(y, o) \in Y \times O$ in week j of the planning horizon be denoted by d_{qj}. The system path $i = (p, o, e, u = (\ell, t^s), r) \in S$ belongs to week $j \in W = \{1, 2, 3, 4\}$ if the start time of its unload slot, i.e., t^s, is in the jth week, i.e., $t^s \in I_j = [10080(j-1) + 1, 10080j]$

where 10080 is the number of minutes in a week and discrete interval I_j describes week j. Then

$$x_{YOH^s}(y,o,I_j) \geq d_{yoj} \qquad \text{for all } y \in Y,\ o \in O \text{ and } j \in W. \tag{1}$$

Variable $x_{YOW}(y,o,I_j)$ which denotes $x_{\{1,4,7\}}(\{y\} \times \{o\} \times I_j)$ is the number of system paths in the given schedule $x = (x_i)_{i\in S}$ with property (y,o) where their unload slots $u = (\ell, t^s) \in U$ belong to week j, i.e., $t^s \in I_j$.

Constraint sets 3 and 6: Let $A_t(r), r \in R, t \in H$ be the set of all system paths $i \in S_R(r)$ such that their unload slots overlap time $t \in H$. For each $r \in R$, we denote the collection of all distinct sets $A_t(r)$ over all $t \in H$ by $\mathcal{S}_0(r)$. To define it more formally, we have:

$$A_t(r) = \{(p', o', e', u', r') \in S_R(r) : u' = ((\ell, p), t'), t \in [t', t' + 144]\},$$
$$\mathcal{S}_0(r) = \{A_t(r) : t \in H, \forall t' \in H\ A_t(r) \not\subset A_{t'}(r), |A_t(r)| > 1\}.$$

In a similar way, let $D_t(\ell), \ell \in D$ be the set of all system paths $i \in S_D(\ell)$ such that their unload slots overlap time $t \in H$. For each $\ell \in D$, we denote the collection of all distinct sets $D_t(\ell)$ over all $t \in H$ by $S_1(l)$. Let $B_t, t \in H$ be the set of all system paths $i \in S$ such that

(a) their pits are either pit `1` or `2`;
(b) their routes are either route `E` or `G`; and
(c) their unload slots overlap time $t \in H$.

We denote the collection of all distinct such sets B_t over interval H by $\mathcal{S}_2$. Or more formally,

$$B_t = \{(p', o', e', u', r') \in S : r' \in \{\mathtt{E}, \mathtt{G}\}, u' = ((\ell', p), t'), (\ell', p) \in \{\mathtt{1}, \mathtt{2}\}, t \in [t', t' + 144]\},$$
$$\mathcal{S}_2 = \{B_t : t \in H, \forall t' \in H\ B_t \not\subset B_{t'}, |B_t| > 1\}.$$

Then

$$\sum_{i \in Q} x_i \leq 1 \qquad \text{for all } r \in R, Q \in \mathcal{S}_0(r), \tag{2}$$
$$\sum_{i \in Q} x_i \leq 1 \qquad \text{for all } l \in D, Q \in S_1(l), \tag{3}$$
$$\sum_{i \in Q} x_i \leq 1 \qquad \text{for all } Q \in \mathcal{S}_2. \tag{4}$$

Constraint set 4:

$$\sum_{i \in S_P(p)} x_i \leq 1 \qquad \text{for all } p \in P, \tag{5}$$
$$\sum_{i \in S_U(u)} x_i \leq 1 \qquad \text{for all } u \in U, \tag{6}$$

Constraint set 5: Constraint 5 is equivalent to this constraint that in a feasible schedule in every 45 minute interval in the planning horizon, at most one unload slot starts. There are 40276 45 minute intervals in a planning horizon and the last interval starts at time 40276. More formally, for $t \in [1, 40276]$, let $S^{>45}(t) = S_W([t, t+44])$ and let $\mathcal{S}^{>45} = \{S^{>45}(t) : t \in [1, 40276]\}$, then

$$\sum_{i \in Q} x_i \leq 1 \qquad \text{for all } Q \in \mathcal{S}^{>45}. \tag{7}$$

2.3 Performance criteria

We formulate the performance criteria as follows:

Criterion 1: The number of system paths is equal to $\sum_{i \in S} x_i$. Let $z^{(1)} = -\sum_{i \in S} x_i$. The goal is to maximize this measure.

Criterion 2: Let d_j denote the total demand over week $j \in W$. The associated contract share for (*system*, *operator*) combination $(y, o) \in Y \times O$ in week $j \in W$ is equal to d_{yoj}/d_j. We want the share of system paths with property (y, o) in week j, i.e., $x_{YOH^s}(y, o, I_j)/x_{H^s}(I_j)$ is as close as possible to the associated contract share or equivalently $x_{YOH^s}(y, o, I_j)$ is as close as possible to $x_{H^s}(I_j)d_{qj}/d_j$. Therefore the associated deviation vector is $z^{(2)} = (z^{(2)}_{yoj})_{(y,o,j) \in Y \times O \times W}$ where

$$z^{(2)}_{yoj} = x_{YOH^s}(y, o, I_j) - d_{qj}/d_j x_{H^s}(I_j) \tag{8}$$

for all $(y, o, j) \in Y \times O \times W$.

Criterion 3: Assume we have n system paths in week j. One plausible interpretation of performance criteria 3 is that the number of system paths in each day to be as close as possible to $n/7$. The deviation vector is $z^{(3)} = (z^{(3)}_{yoji})_{(y,o,j,i) \in Y \times O \times W \times [1,7]}$ where

$$z^{(3)}_{yoji} = x_{YOH^s}(y, o, b_{ji}) - x_{YOH^s}(y, o, I_j)/7 \tag{9}$$

for all $(y, o, b_{ji}) \in Y \times O \times W \times [1, 7]$, and $b_{ji} = [10080(j-1) + 1440(i-1) + 1, 10080(j-1) + 1440i]$ describes day $i \in [1, 7]$ of week j.

Criterion 4: Let $f_{yoj}(r)$ be the desired route $r \in R$ utilization by system paths with property $(y, o) \in Y \times O$ (i.e., system paths which are from system y and are operated by operator o) in week $j \in W$ where $f_{yoj}(r)$ is a real number between zero and one. In other word, $100 \times f_{yoj}(r)$ percent of total number of system paths in week j from system y which are operated by operator o, are assigned to route r. The deviation vector is $z^{(4)} = (z^{(4)}_{yojr})_{(y,o,j,r) \in Y \times O \times W \times R}$ where

$$z^{(4)}_{yojr} = x_{YOH^sR}(y, o, I_j, r) - f_{yoj}(r) x_{YOH^s}(y, o, I_j) \tag{10}$$

for all $(y, o, j, r) \in Y \times O \times W \times R$.

Criterion 5: Let w_i denote the waiting time for system path $i \in S$, then the total waiting time is equal to $z^{(5)} = \sum_{i \in S} w_i x_i$. The goal is to minimize this measure.

Criterion 6: Let $g(y, o, I)$ be the desired fraction of system paths with property (y, o) which their departure times are in the interval $I \subseteq H_1 = [-1439, 40320]$ and they are allowed to be provisioned. One suitable choice for $g(y, o, I)$ is to be defined as equal to $1/k$. Let $H_y^d = \{t' \in H_1 : \exists t^a \in [-1439, 40320]\ p = (y, t', t^a) \in P\}$ includes all departure times of system paths from system y. Let $H_y'^d = \{[t_1, t_2] : t_1, t_2 \in H_y^d, t_1 \leq t_2\}$. We define the deviation vector as $z^{(6)} = (z_{yoI}^{(6)})_{(y,o,I) \in Y \times O \times H_y'^d}$ where

$$z_{yoI}^{(6)} = x_{YH^dOB}(y, I, o, 1) - g(q, I) x_{YH^dO}(y, I, o) \tag{11}$$

for all $(y, o, I) \in Y \times O \times H_y'^d$. One of the main improvements one can make to make the above measure less computationally expensive is to reduce the number of values which set I can take. If we know which intervals contain exactly k system paths in the optimal solution, then we just need to consider those intervals. However, since we do not know the optimal schedule before solving the model, we need to consider intervals of all lengths which can contain k consecutive system paths in any possible optimal schedule.

The deviations associated with Criteria 2–4 and 6 are vectors. In order to measure these criteria we need to formalise notions of *deviation* and *unbalancedness*. Let $z = (z_i)_{i \in n}$ denote such a deviation vector. Then each element z_i is the deviation of component i of this criterion from some target value. We define the deviation of z to be $D(z) = \|z\|_p$ and the unbalancedness of z to be $B(z) = \min_{z_0 \in \mathbb{R}}\{\|z - z_0 e\|_p\}$ where $p \in \{1, 2\}$ and $e = (1, \dots, 1)^T$. Thus, the length of the vector z is the measure of deviation and the shortest distance between z and a point on the line $z_1 = z_2 = \dots = z_n$ is the measure of unbalancedness. For each of these criteria we wish to minimize both the deviation and the unbalancedness.

2.4 Model formulation

The aforementioned constraints and performance criteria gives rise to the following mixed-integer conic programme (MICP) with multiple objectives:

$$\begin{aligned} \min &\left(z^{(1)}, z^{(2D)}, z^{(2B)}, z^{(3D)}, z^{(3B)}, z^{(4D)}, z^{(4B)}, z^{(5)}, z^{(6D)}, z^{(6B)}\right) \\ &= \begin{pmatrix} z^{(1)}, D(z^{(2)}), B(z^{(2)}), D(z^{(3)}), B(z^{(3)}), \\ D(z^{(4)}), B(z^{(4)}), z^{(5)}, D(z^{(6)}), B(z^{(6)}) \end{pmatrix} \end{aligned}$$

subject to (1)–(11), $x_i \in \{0, 1\}$ for all $i \in S$.

We now make several observations about this model that we will use to our advantage when solving it. For Criteria 2–4, and 6, we note that for any deviation vector z, the optimal value for z_0 is the average of the components of z under the 2-norm, the median of the components of z under the 1-norm, and that $B(z) \leq D(z)$ for either norm. Furthermore, for a feasible deviation vector, the sum of the components of z is equal to zero. Thus, under the 2-norm, $B(z) = D(z)$ and so a minimum deviation solution also minimizes unbalancedness. Consequently, we can omit the objective functions $z^{(\cdot B)}$ under the 2-norm.

3 Solution methodology

To solve the multi-objective MICP described in the previous section, we employ an hierarchical optimization procedure in which we solve successive single objective

MICPs. The order in which the criteria were presented reflects their relative importance to Aurizon and minimizing deviation is more important than minimizing unbalancedness.

To begin the hierarchical optimization we optimize the MICP with respect to the first objective function $z^{(1)}$. Let $z_*^{(1)}$ denote the value of the best integer solution found and suppose that in a solution to the multiple objective MICP we require that the value of $z^{(1)}$ degrades by a factor of at most $a^{(1)}$ where $a^{(1)} \geq 0$. We refer to $a^{(1)}$ as the degradation factor and add the threshold constraint $z^{(1)} \leq z_*^{(1)}(1 + a^{(1)})$ to the current MICP. In the next stage, we optimize the current MICP with respect to the second objective function $z^{(2D)}$ and then add the corresponding threshold constraint $z^{(2)} \leq z_*^{(2)}(1 + a^{(2)})$ to the current MICP. The above process is repeated until all objective functions $z^{(i)}$ for $i \in \{1, 2D, 2B, 3D, 3B, 4D, 4B, 5, 6D, 6B\}$ have been considered.

Each single objective MICP can be reformulated as a mixed-integer second-order cone programme under the 2-norm, and it can be reformulated as a mixed-integer linear programme under the 1-norm.

4 Computational investigation

In this section, we investigate the performance of the hierarchical optimisation procedure to solving the multi-objective MICP on an instance of realistic size. The investigation is carried out on a machine with dual oct core 3.33GHz Intel Xeon E5-2667 v2 processors and 256 GB of RAM. The number of threads used is 13 out of available 16 threads. We use Gurobi v8.0.0 via the Python API and Python v3.6.4. The time limit for solving each single objective MICP is 600 seconds. The instance has the following characteristics:

- The maximum waiting time is 60 minutes.
- There are two train operators named `op1` and `op2` and two systems named `s1` and `s2`.
- The demand for each week is given in Table 1.
- The unload slots are generated with consideration of constraint set 5 and the implication of constraint set 6 for unload slots. The maximum number of unload slots which can be used in each day is 27. Note that the constraint set 5 is implicit in the construction of unload slots and is therefore not coded.
- The departure times of train paths from systems `s1` and `s2` are 20 minutes and 90 minutes apart respectively. Some of these train paths are cancelled due to maintenance activities.
- The desired provisioning frequency for system paths from system `s2` is one per two and for system paths from system `s1` is one per one (i.e., we prefer all system paths from system `s1` to allow provisioning).
- The route utilization ratios are shown in Table 2. The route utilization ratios for system `s2` and train operator `op2` are zero.

The performance statistics under each norm are shown in Tables 3 and 4. Each row in the table corresponds to a single objective MICP and we use the following notation:

- Relax: the objective function value of the initial MICP relaxation.
- Root: the objective function value of the final MICP relaxation solved at the root node of the branch-and-bound tree.

- BestBnd: the objective function value of the best MICP relaxation found during the branch-and-bound search.
- BestFeas: the objective function value of the best feasible solution found during the branch-and-bound search.
- Gap: the optimality gap of the best feasible solution which we define to be $|\text{BestFeas} - \text{BestBnd}|/\text{BestFeas}$

Under the 2-norm, Gurobi finds an optimal solution to each single objective MICP associated with Criteria 1 and 2, and finds a solution within one percent of the optimal value for 5. However, it cannot find good solutions for the other criteria within 600 seconds. It is not that surprising that the 1-norm leads to much better performance than what was observed under the 2-norm. Gurobi solves the single objective MICPs associated with Criteria 1–3 and 5 to optimality. The quality of the solutions is much better for Criteria 4 and 6, much worse for Criterion 5, and about the same for the other criteria. The reason that the solution for Criterion 5 under the 1-norm is much worse than the solution under the 2-norm is that the threshold constraint associated with Criterion 4 under the 2-norm is not restrictive compared to that under the 1-norm. In Table 3, the value of the best integer solution found under Criterion 4 is 10090 which is likely to be far from optimal.

One interesting observation is that the best integer solution values of the deviation and unbalancedness objectives associated with Criteria 2-4 and 6 are equal. This implies that the median of the components of each deviation vector is zero and that the minimum deviation solution also minimizes unbalancedness. We believe that this is an artefact of the instance rather than the model and so in general we cannot omit the unbalancedness objectives as we did under the 2-norm. However, we have reason to believe that this could be a common occurrence when solving practical instances and so revised the hierarchical optimization procedure to skip minimizing unbalancedness if the median of the deviation solution is zero.

The performance statistics of the revised procedure are shown in Table 5 and Table 6 compares for each criterion, the deviation of the final multiple objective MICP solutions when evaluated under the 2-norm. Overall the solutions found under the 1-norm are better quality and can be found comparatively quickly using the revised hierarchical optimization procedure.

5 Conclusion

We consider a capacity alignment planning problem for a coal chain that is faced by our industry partner Aurizon in which a schedule of system paths are sought that perform well with respect to various performance criteria. For many of these criteria the schedule should not only minimize the deviation from some prescribed targets but also the deviations should be well balanced. We model the problem as a mixed-integer conic programme (MICP) with multiple objectives which we then solve using a hierarchical optimization procedure. In each stage of this procedure a single objective MICP must be solved. Depending upon whether we evaluate the associated performance criteria under a 2- or 1-norm we reformulate the problem as either a mixed-integer second-order cone programme or as a mixed-integer linear programme respectively.

A property of the model is that a minimum deviation solution for a given criteria measured under the 2-norm also minimizes the unbalancedness for that criteria. While this is not a property of the model under the 1-norm we believe

Table 1 Weekly demand

System	Train operator	Demand
s1	op1	98
s1	op2	21
s2	op1	21
s2	op2	0

Table 2 Weekly route utilization

System	Train operator	Route	Route utilization
s1	op1	A	19/98
		B	22/98
		E	15/98
		G	34/98
		J	8/98
s1	op2	A	3/21
		B	2/21
		E	0
		G	2/21
		J	14/21
s2	op1	A	13/21
		B	5/21
		E	0
		G	0
		J	3/21

Table 3 Performance statistics under the 2-norm

Objective	Degrad[%]	Relax	Root	BestBnd	BestFeas	Gap[%]	Time[sec]
1	0	756	-	756	756	0	<1
2D	10	0	0	2.5	2.5	0	49
3D	10	0	0	0	3.4	100	600
4D	10	-	-	0	10090	100	600
5	10	5406	5618	5640	5698	1	600
6D	-	-	-	0	4204	100	600

Table 4 Performance statistics under the 1-norm

Objective	Degrad[%]	Relax	Root	BestBnd	BestFeas	Gap[%]	Time[sec]
1	0	756	-	756	756	0	<1
2D	10	0	0	5.2	5.2	0	26
2B	10	0	0	5.2	5.2	0	30
3D	10	0	-	0	0	0	4
3B	10	0	-	0	0	0	3
4D	10	0	0	12	12	0	162
4B	10	0	0.3	9.8	12	18	600
5	10	5986	6025	6085	6088	0.05	600
6D	10	1153	1303	1399	1516	8	600
6B	-	1153	1165	1170	1516	22	600

Table 5 Revised performance statistics under the 1-norm

Objective	Degrad[%]	Relax	Root	BestBnd	BestFeas	Gap[%]	Time[sec]
1	0	756	-	756	756	0	<1
2D	10	0	0	5.2	5.2	0	18
3D	10	0	0	0	0	0	4
4D	10	0	0	12	12	0	108
5	10	5987	6025	6085	6088	0.05	220
6D	-	1218	1391	1483	1512	2	600

Table 6 Comparison under the 2-norm of the deviations of the final multiple objective MICP solutions

Norm	1	2	3	4	5	6
2-norm	756	2.7	3.4	6441	5698	4204
1-norm	756	2.9	0	4.5	6688	1591
Revised 1-norm	756	2.9	0	5.5	6688	1610

that it will frequently be the case when solving practical instances. Consequently, we revised the hierarchical optimization procedure to omit the unbalancedness objectives under the 2-norm, and check for their redundancy under the 1-norm.

A computational investigation on a real instance of the problem reveals, not unsurprisingly, that the hierarchical optimization procedure under the 1-norm finds good solutions much faster than under the 2-norm. Moreover, the quality of the solutions found by the procedure under the 1-norm compare well to the solution found under the 2-norm when the 1-norm solutions are evaluated for each criterion using the 2-norm.

Future work will include improved modelling of bottleneck performance criteria such as Criteria 6, improved solution procedures such as customised branching for the single objective MICPs within the hierarchical optimization procedure, and extending the problem considered to include additional practical considerations such as dynamic start times for unload slots.

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