Primal and Dual Access

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Abstract

Accessibility, measuring the ease of reaching potential destinations, is increasingly being considered as an effective indicator to evaluate the performance of transport and land use interactions. Primal accessibility, a generalization of the first accessibility formulation proposed by Hansen (1959), has been widely used in many studies and demonstrated to be a reliable tool for project, program, and policy evaluation. The dual of accessibility, measuring the time required to reach a given number of opportunities, is less often considered but can be used for optimization in location-covering type problems. This paper, hence, clarifies the definitions of primal and dual access, and applies both measures to the Minneapolis - St. Paul metropolitan area for auto and transit to demonstrate their practicality as a metropolitan-level measurement. We explore the correlations and differences between the primal and dual access to better understand the relative strengths of the measures. It is found that, as with primal accessibility, dual accessibility is an efficient approach to evaluate accessibility, which is straightforward to calculate and to explain to policy-makers and the public.

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1 Introduction

Accessibility, defined as the potential of reaching valued destinations (Hansen, 1959), can be considered as the joint result of a transport network and land use (Páez et al., 2012), in which the land use describes the spatial distribution of travelers and opportunities, while a transport network provides services for travelers to cover their distance to these opportunities using a specific travel mode. This implies the basic components of access: the speed and directness of travel and the quality and quantity of opportunities (Páez et al., 2012).[1]

A large range of accessibility measures has been applied to various issues and spatial scales deploying the two components in different ways (Harris, 2001). Geurs and Van Wee (2004) reviewed and classified these measures into four categories: infrastructure-based, location-based, person-based, and utility-based measures, by identifying additional components specifying the temporal constraints of activities and the needs of individuals.

The location-based measures evaluate the level of accessibility at a location to spatially distributed activities (Geurs and Van Wee, 2004). Hansen (1959) developed a very basic and widely used location-based method, perhaps the simplest generalization of which counts how many opportunities can be reached in a given time cost, sometimes referred to as the 'cumulative opportunities' measure (Vickerman, 1974, Wachs and Kumagai, 1973, Wickstrom, 1971), 'integral' accessibility (Ingram, 1971, Kwan, 1998), or 'daily' accessibility (Bruinsma and Rietveld, 1998). We call this 'primal' accessibility to contrast with the dual measure below. Primal accessibility is relatively undemanding of data, simple to calculate, and easy to communicate with policy-makers and the public (El-Geneidy and Levinson, 2006, Geurs and Van Wee, 2004, Owen and Levinson, 2012).

Hence, many previous studies applied the primal accessibility measurements for the cases where there is a choice of opportunities, like jobs (Cui and Levinson, 2018*a*,*b*, Deboosere et al., 2018, El-Geneidy and Levinson, 2006, Gutierrez and Gomez, 1999, Hess, 2005, Srour et al., 2002), labor (Cui and Levinson, 2018*a*, Deboosere et al., 2018), population (O'Kelly and Horner, 2003), retail (Apparicio et al., 2007, Guy, 1983, Srour et al., 2002), or health care facilities (Paez et al., 2010). The *Access Across America* study uses primal accessibility for a national job accessibility evaluation across 50 metropolitan areas in the United States, for auto, transit, and walking (Owen et al., 2014, Owen and Murphy, 2017*a*,*c*).

In contrast, we identify the 'dual' accessibility as a measure of the travel costs of accessing a fixed number of opportunities. This term is inspired by the dual in linear programming, which converts a primal minimization problem (the objective function) to its dual maximization problem (bringing constraints into the new objective function), or vice-versa, such that any feasible solutions of the dual problem provides a

[1] Following most of the literature, we use the terms 'access' and 'accessibility' interchangeably.

bound, upper or lower, on the objective of the primal problem (Alevras et al., 2001). It is also inspired by the geometric dual graph, which replaces each face of the primal graph with a vertex and connects the vertices by an edge when two faces are connected (Thulasiraman and Swamy, 1992, Whitney, 1931), increasingly referred to as the 'dual representation of the network' (Masucci et al., 2009, Porta et al., 2006, Rosvall et al., 2005), following the lead of Batty (1981), who first used the term in this way in social network analysis (naming the idea of 'simplicial complexes' that had been developed by Atkin (1974)), and later applied it to physical networks (Batty, 2004) in the domain of Space Syntax.

Compared to primal accessibility, the dual measure is more commonly used for cases where the time to a service rather than quantity of opportunities is critical, but has received less attention. There is a natural analog of primal accessibility with the traditional gravity model of trip distribution, and dual accessibility with the intervening opportunities model indicating that the number of persons going a given distance is inversely proportional to the number of intervening opportunities (Stouffer, 1940).

Similar ideas have been incorporated into several accessibility indices, e.g., the Land Use and Public Transport Accessibility Indexing Model (LUPTAI) (Pitot et al., 2006, Yigitcanlar et al., 2007) and Public Transport and Walking Access Index (PTWAI) (Mavoa et al., 2012) evaluate the accessibility levels, based on walking distance and travel time on vehicles to certain destinations. Some studies solve the travel cost (time or distance) to the nearest facility as well, e.g., hospital (Rosero-Bixby, 2004, Tanser et al., 2006), subway station (Armstrong and Rodriguez, 2006, Chin and Foong, 2006, Shin et al., 2007), and supermarket (Apparicio et al., 2007, Sharkey et al., 2009), which defined a specific dual accessibility evaluation that only considers the ability to reach one opportunity. Travel cost to something like '100,000 jobs' is not widely used.

Breheny (1978) confusingly named this same idea the 'opportunities constant' measure, which was calculated by searching outwards from an origin in cost bands until a certain number of opportunities were reached, and also demonstrated the practicality of this approach for accessibility measurements. This metric has also been used by Echenique et al. (1970), Whitbread (1972) and Breheny (1974). We refer to it as the dual measure, as no other term has been widely adopted and other names are likely to lead to confusion, and in order to logically connect it with the primal measure.

A systematic comparison between primal and dual access is conducted for the first time in the paper. This paper aims to clarify the concepts of primal and dual access and explain how they could be implemented in real-world cases for both auto and transit to demonstrate their practicality, and explore their correlations and differences with multiple composites of the time and opportunity thresholds, as well as the combined time-weighted primal and opportunity-weighted dual access for a better understanding of the measures.

In this study, job access for the Minneapolis - St. Paul (Twin Cities) metropolitan area is measured, to demonstrate proof-of-concept. The definitions of primal and dual access, data collection, primal and dual

access measurements, primal vs. dual access comparison, and the conclusion are in Sections 2 - 6 in turn.

2 Theory

2.1 Hansen's Primal Measure

Primal accessibility as presented here generalizes the first accessibility formulation by Hansen (1959). In the *primal* accessibility (A) problem, we solve for how many opportunities can be reached in t minutes from origin i.

This primal measure of accessibility is foremost *positive*, measuring how many activities can be reached. One could, however, impose a *normative* standard, and insist that it should be above some number (N). It implies the question of whether providing such levels of accessibility is appropriate and sufficient. For some activities, most people probably agree that it is (1 fire station should be within X minutes of anyone who lives in a city, or $X + Y$ minutes in a rural area), and for others it is not. There is no standard for the number of jobs reachable within 30 minutes, but all else equal, more is better than fewer. Discussions of the '30-minute city' (Greater Sydney Commission, 2018), for instance, suggest that 75% of workers should be able to reach their job within 30 minutes using transit. This definition bears on the distribution of jobs and housing as much as on transit service.

$$A_i = \sum_{j=1}^{J} O_j f(C_{ij}) \tag{1}$$

Where:

A_i: primal accessibility of block i;

O_j: number of opportunities in destination j;

$f(C_{ij})$: function of costs, in which C_{ij} is the travel cost between i and j;

J: total number of destinations.

2.2 Cumulative Opportunities, Isochronic, Rectangular

To apply this in practice, the function of costs needs to be specified. We consider time costs in this analysis, though a whole gamut of costs, including monetary as well as time costs (El-Geneidy et al., 2016), and external as well as internal costs could be considered (Cui and Levinson, 2018*c*). First we present the cumulative opportunities formulation. We might think of this as a rectangular distance decay function, as it provides a step function or inclusion or exclusion of an opportunity from the accessibility set.

$$f(C_{ij}) = \begin{cases} 1 & \text{if } C_{ij} < \tau \\ 0 & \text{if } C_{ij} \geq \tau \end{cases} \tag{2}$$

Where:

τ: time threshold.

The time-weighted accessibility score for primal accessibility, proposed by Anderson et al. (2013), combines different time thresholds into a complete measure and scores the accessibility for ranking each block. It applies a decay factor showing that accessibility decreases with a longer time from the origin (Hansen, 1959), which can be written as,

$$A_{i,w_\tau} = \sum_{\tau \in T} (A_{i,\tau_n} - A_{i,\tau_{n-1}}) f(\tau_n) \tag{3}$$

subject to:

$$\tau \text{ is in ascending order.}$$

Where:

A_{i,w_τ}: time-weighted primal accessibility of block i;

τ_n: the nth time threshold in the set T;

$f(\tau_n)$: travel time decay function.

Note that this equation is analogous to the gravity model when $\tau_n - \tau_{n-1}$ approaches zero and the maximum τ in the set T goes to infinity.

The population-weighted primal accessibility is defined as the average accessibility across all blocks with each block's contribution weighted by the population in that block (Cui and Levinson, 2018*b*, Owen and Murphy, 2017*b*), written as,

$$A_{w_p} = \frac{\sum_{i \in I} A_i \times N_i}{\sum_{i \in I} N_i} \tag{4}$$

Where:

A_{w_p}: population-weighted primal accessibility across all blocks;

N_i: population in block i.

2.3 Dual Measure

The *dual* (or *reciprocal*) of the accessibility (A') problem is less commonly seen in urban planning applications, but is widely used in optimization as a location-covering type problem, to determine the optimal location of

facilities to minimize total travel time or ensure a maximum travel time to reach a destination is not exceeded (Church and ReVelle, 1974, ReVelle and Swain, 1970).

In this case we solve for the travel cost required to reach a minimum number of opportunities (for instance, one library, 30 museums, 100,000 jobs), asking: How long does it take to reach Ω opportunities from origin i? In a sense, this is a more *normative* view of accessibility, because it measures the time required to achieve a standardized number of activities.

$$A_i^{'} = \max Q_{ij} C_{ij} \tag{5}$$

that satisfies

$$\min_{Q} \sum_{j=1}^{J} O_j Q_{ij} C_{ij} \tag{6}$$

subject to:

$$\sum_{j=1}^{J} O_j Q_{ij} \geq \Omega \tag{7}$$

$$Q_{ij} \in \{0, 1\} \tag{8}$$

Where:

$A_i^{'}$: dual accessibility of origin i;

Ω: opportunity threshold;

Q_{ij}: cells in incidence matrix Q; '1' if destination j included in the set of destinations, '0' otherwise.

In other words, the dual measure finds the cost of the farthest destination (Equation 5) in the set of 'nearest' (lowest cost) destinations (Equation 6) that contain Ω opportunities (Equation 7).

Significant applications with this measure pertain to questions of *affordability*, especially when combined with more explicit monetary cost measurements. So the question might be how much does it cost (in time and money) to reach Ω jobs?

The application of the dual of isochrone accessibility is less prevalent than the primal measure. Although the dual measure may be more intuitive (since people are more familiar with time than with opportunities), operationally it is more difficult to implement than the primal method. The dual measure can be operationalized using similar methods as in the intervening opportunities method, by ranking all possible destination zones with increasing travel cost (Hayes and Wilson, 1971), then searching outward from an origin location by incrementally increasing travel cost, until the predefined level of accessibility can be obtained.

Stouffer (1940) proposed the law of intervening opportunities that "the number of persons going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunity". On the basis of this theory, we propose an opportunity-weighted accessibility score for dual accessibility, which applies an 'opportunity decay function' to combine different opportunity thresholds of dual accessibility into an integrated measure. The opportunity-weighted dual accessibility ($A^{'}_{i,w_\Omega}$) is written as,

$$A^{'}_{i,w_\Omega} = \sum_{\Omega \in O} (A^{'}_{i,\Omega_n} - A^{'}_{i,\Omega_{n-1}}) f^{'}(\Omega_n) \tag{9}$$

subject to:

$$\Omega \text{ is in ascending order.}$$

Where:

Ω_n: the nth opportunity threshold in the set O;

$f^{'}(\Omega_n)$: opportunity decay function.

The population-weighted dual accessibility can be calculated as with Equation 4.

2.4 The Relationship between Primal and Dual Access

Theoretically, the dual should be the inverse matrix of the primal, if resolutions of the measurements are high enough, for instance, using 1m^2-level of origins with threshold incremented by 1 second (time threshold for primal access) or 1 opportunity (opportunity threshold for dual access). From the case of Figure 1, high resolutions of measurements could give that the primal accessibility of the origin (the white dot) is 3, assuming each destination (the red dots) contains 1 job opportunity, with a time threshold of 23 minutes and its dual accessibility is 23 minutes with a opportunity threshold of 3. However, this calculation is not yet feasible for a metropolitan area due to the expensive computational workload and the lack of data with sufficient detail for either network or land use. Census blocks and 10-minute time threshold bin are commonly used, so, for Figure 1, we only know that the 20-minute primal accessibility is 2 and the 30-minute is 3, without an output for the 23-minute threshold. Comparing the two measures systematically helps us explore where the mismatches between the two matrices happen and to what extent, which provides support for future studies selecting between the primal and dual measures.

3 Data Collection

Several data sources have been applied in this study.

The 2011 TomTom data, which was acquired by the research team from the Metropolitan Council of the Twin Cities metro region, includes both speed data and a road network. TomTom speed data were aggregated based on millions of GPS logging and navigation devices, and provide different speed percentiles for seven time periods: Overnight, Morning Peak Hours (two parts), Mid-Day, Evening Peak Hours (two parts), and Evening, in which the 5th percentile speed refers to the fastest 5 percent of speed records while the 95th percentile speed indicates the slowest 5 percent (TomTom International BV, 2013). This study uses the 50th percentile (median) speed at the morning peak hours (7am - 9am) to represent the auto travel speed on each link.

The TomTom road network is a GIS shapefile containing the spatial and geographical information of the roadways in the Twin Cities. The network can be joined with the TomTom speed data, which generates the network dataset used for searching the shortest travel time path for any given origin and destination pairs and calculating their travel time by auto. Note that the time walking to and from parking is not counted in the auto travel time as we do not have accurate data to measure this.

The General Transit Feed Specification (GTFS) data for the Twin Cities metro area was collected from the Metropolitan Council, which includes the Metro Transit schedule data, transit routes, stops, trips, frequencies, fares, e.g., for both buses and light rail trains (Metropolitan Council, 2018). The GTFS data can be added to the network dataset as well on the basis of the TomTom road network, which allows us to perform the calculations of travel time matrix by transit. Note that the time cost by transit includes the walking time from the origin to the origin transit stop, the waiting time spent on the origin transit stop, on-vehicle travel time, transfer time (if transfers are needed), and the walking time from the destination transit stop to the destination. Note also that if transit services are not available for an origin-destination (OD) pair or the transit travel time is higher than the walking time directly from the origin to the destination, the walking time would be counted as the transit travel time for accessibility calculations. In this study, the selected departure time is 8am for transit travel time estimates.

The 2010 TIGER/Line shapefile was acquired from US Census Bureau (2010), which defines the boundaries of census blocks in Minnesota. The features in the Twin Cities metro area were selected, and the centroid of each block was extracted as the origins and destinations for accessibility measurements. The Population & Housing Unit Counts at the census block level of Minnesota were also collected, which are the product of the 2010 TIGER/Line with Selected Demographic and Economic data (US Census Bureau, 2010). The population counts were applied to calculate the population-weighted average accessibility.

The LEHD Origin-Destination Employment Statistics (LODES), in which LEHD stands for Longitudinal Employment Household Dynamics, was obtained from the US Census Bureau (2013). The Workplace Area Characteristic (WAC) table provides the number of employees working in each census block, and the Origin-

Destination (OD) table tracks the number of home-to-work trips at the census block level. The WAC table can be joined with the TIGER/Line census block shapefile displaying how job opportunities spatially distributed in the Twin Cities region. The OD table is used for the calibration of the decay functions, which is explained later in the primal vs. dual comparison.

This study is a proof-of-concept, so the data described above only meet the basic requirements. In real-world applications such as transport project evaluations, more comprehensive measurements will increase accuracy and precision, though may not much impact broad policy implications. First, an appropriate temporal sampling of departure times is needed to compute the travel time to avoid biased estimates, especially for transit (Owen and Murphy, 2018, Stepniak et al., 2019). We selected a single departure time in this study, which could cause an average error compared to a high temporal resolution. GTFS data, which is based on schedules, have the limitations to catch unscheduled disruptions, e.g., due to severe congestion, vehicle breakdowns, or signal malfunctions (Wessel et al., 2017). Updating GTFS packages with observed trips and arrival times, e.g., by mining smart card system, which collects actual passenger flow data on a real time basis, should help to improve the accuracy of the transit accessibility measurements (Lee et al., 2018). The TomTom road network, what we used to build our accessibility calculation platform, fails to include some pedestrian foot paths, which could result in errors when simulating the travel from/to transit stops or parking lots. A more detailed walking network would work better. Time to find parking spaces should be considered as well, which, however, extends to a problem of parking demand and supply (Hensher and King, 2001, Lam et al., 1999).

4 Primal and Dual Accessibility Measurements

To calculate job accessibility, we used the Network Analyst tool in ArcGIS to search for the shortest travel time path and accumulate the travel time for each OD pair in the Twin Cities based on the TomTom road network, using the TomTom speed data for auto and the GTFS data for transit. We wrote Python scripts to call PostgreSQL for further analysis with the inputs of the travel time and land use data (LEHD WAC table) [2].

Figure 2 summarizes the population-weighted average primal and dual access to jobs by auto and by transit. The time thresholds for the primal measure are set from 1 minute to 600 minutes, in which all job opportunities in the Twin Cities can be reached by either auto or transit. The opportunity thresholds for the dual measure are set from 1,000 jobs to 1,000,000 jobs.

Note that a higher numeric value of primal access represents a higher level of accessibility, as it describes

[2] We have released the ArcGIS model and python scripts that we used for travel time and accessibility measurements on https://github.com/CuiRachel/PrimalAndDualAccess.

the number of reachable opportunities; while a higher numeric value of dual access represents a lower level of accessibility, as it expresses the time cost needed to reach opportunities.

As expected, accessibility by auto is much higher than that by transit from the perspective of both primal and dual access. People driving can reach more job opportunities in a given time threshold or take less time to reach the same number of job opportunities, due to a higher travel speed. The higher thresholds allow more accessibility. Moreover, our primal accessibility measurements are consistent with the *Access Across America* series of study for both auto (Owen and Murphy, 2017*a*) and transit (Owen and Murphy, 2017*c*) with no important differences, after accounting for the different analysis year.

The spatial distributions of the primal and dual access by auto and by transit are discussed in the following subsections.

4.1 Accessibility by Auto

Figure 3 visualizes the primal accessibility to jobs by auto for the Twin Cities metro area with selected time thresholds. Figure 4 gives the dual accessibility to jobs by auto.

The spatial distribution shows clearly that, in general, job accessibility is higher in the downtown area and declines with the increase of distance to the downtown since more jobs are centered on the downtown area. From Figure 3a and Figure 3b, downtown Minneapolis and downtown St. Paul could be easily identified. The blocks between downtown Minneapolis and downtown St. Paul have the highest 10 minutes job accessibility, see Figure 3c, living where travelers can reach both downtown areas in 10 minutes. In addition, blocks along with the primary highways show a higher primal accessibility as well, especially for those to the south of the downtown areas, where several places with a mass of job opportunities, like Minneapolis - St. Paul Airport and the Mall of America, are located.

The time threshold affects primal accessibility, as Figure 2 shows. Comparing the maps, the red area, which represents a higher accessibility, expands with the increase of the time threshold. In 50 minutes, most of the job opportunities could be reached by most residents of the Twin Cities metro area, see Figure 3f.

The spatial distribution of dual accessibility has similar patterns that, in addition to downtown areas, the blocks close to highways have a higher dual accessibility, which reflects the speed advantage of highways. The opportunity threshold affects dual accessibility, which declines with the increase of the opportunity threshold, since travelers need more time to reach a higher number of jobs.

4.2 Accessibility by Transit

Figure 5 displays the spatial distribution of the primal accessibility to jobs by transit for the Twin Cities metro area. The dual accessibility by transit is shown in Figure 6, which again, has similar spatial distribution patterns as the other accessibility maps.

The general patterns are the same as auto that, first, the blocks with higher accessibility are more centered on the downtown; second, time threshold affects the transit accessibility both positively and significantly. Due to a lower speed, transit accessibility is much lower than auto. In 10 minutes, for instance, travelers can only travel within their origin or neighboring blocks by transit, which causes the irregular distribution that the blocks with or adjacent to a larger number of jobs have higher primal accessibility, see Figure 5a. The irregular distribution remains for the dual accessibility, as the red dots shown in Figure 6a, which indicate the blocks (20 in total) with more than 5,000 jobs. It is less likely for travelers to move to an adjacent block in 1 minute by transit, while the intra-zonal travel time (from origin i to the same destination i) is assumed to be 0.

4.3 Auto vs. Transit

Figure 7 shows the population weighted correlations between auto and transit access for both primal and dual access, in which each block in the Twin Cities is an observation in the sample. The correlations are all positive: a higher auto accessibility is associated with a higher transit accessibility. But the auto and transit access are more strongly correlated under the dual measure, for which their correlations vary from 0.6 to 1.0, compared with the primal measure, which vary from 0.1 to 1.0.

Concerning the primal accessibility, it is interesting that the strongest correlations between the modes are not at the same time thresholds. Instead, there are mismatches such that, e.g., 1 min auto and 10 min transit, 2 min auto and 15 min transit, 30 min auto and 250 min transit, are more strongly correlated. This implies the ratios of auto vs. transit travel time to reach similar numbers of destinations. The pairs of dual accessibility with the same opportunity thresholds, however, generally matched up to have the strongest correlations. For instance, the dual access to 1,000 jobs by auto is more correlated with the dual access to 1,000 jobs by transit. This indicates a synchronous increase of needed travel time to reach more opportunities for auto and transit. While, the correlations maintain a high and stable level for the dual access pairs with thresholds above 20,000.

Figures 8 and 9 show the absolute differences of auto and transit access with respect to primal and dual access, respectively, which indicate the benefit of driving in terms of job accessibility due to travel time savings compared with taking transit. For primal accessibility, we measured the auto minus transit results

to explore how many more job opportunities driving can reach than transit in the same time threshold. For dual accessibility, we measured the transit minus auto results to show the additional transit time required by transit to reach the same number of job opportunities as by auto. According to the respective legends, a darker blue refers to a more significant accessibility increase (travel time decrease) by auto.

The spatial distribution patterns of the primal accessibility differences are as expected. With a lower time threshold, the differences are dominated by auto accessibility and basically have the same distribution patterns, see Figure 8a. The blocks between downtown Minneapolis and downtown St. Paul could be clearly identified as well, compared to Figure 3c, which have significant changes on 10 min primal accessibility for using auto or transit. This is mainly because the transit accessibility is too low to affect the differences. Based on the population-weighted average, the transit accessibility is only 0.65% of the auto accessibility with a 10 min time threshold.

A higher time threshold strengthens the relative transit accessibility. In 30 minutes, e.g., see Figure 8b, fewer differences are shown in the downtown areas, which are visualized as a lighter blue, and the lighter blue area expands between the 30 and 80 minute thresholds. This difference mitigation starts from the downtown area where most job opportunities can be reached in a short time by auto. The differences between the modes would be zero when the time threshold is large enough, see Figure 2a.

In contrast, in terms of dual accessibility under any opportunity thresholds, downtown areas are less affected by the auto vs. transit choice. A higher opportunity threshold causes a more severe difference, as it is always easier for auto to reach farther places, compared to transit, see Figure 2b.

Figures 10 and 11 visualize the relative differences of auto and transit accessibility. For the dual accessibility, the relative differences maps share the same spatial patterns as the absolute differences maps (Figure 9). But for the primal accessibility, the relative differences maps look more like the transit primal accessibility maps (Figure 5) rather than the absolute differences ones.

5 Primal vs. Dual Accessibility Comparison

Figure 12 gives the population weighted correlations between the primal and dual access for each of the time and opportunity thresholds. All the correlations shown here are negative, as expected.

For auto, 30 and 40 minutes primal access are more correlated with the dual accessibility, which have the absolute correlation values around 0.6 or higher for all the opportunity thresholds. The absolute correlation values then decline with a higher or lower time threshold that roughly give inverted U-Shape curves. Transit accessibility has a similar pattern that the absolute values of the correlations increase first and then decline with the increase of the time threshold. The turning points here are unrealistic values of 300 and 400 minutes.

Using the LEHD OD table, which does not partition trips by mode of travel, we fit the travel time and opportunity decay functions for auto travel to work. Note, driving accounts for 89.8% of the home-to-work trips in the Twin Cities based on 2017 American Community Survey data (United States Census Bureau, 2017). Future studies should aim to calibrate the decay functions for transit.

To fit the travel time decay function (Equation 3), we measure the percentage of average trip flow in each travel time category, e.g., 0 - 5 minutes or 5 - 10 minutes, where trip flow refers to the number of trips divided by the number of opportunities for an origin in a travel time category. To fit opportunity decay function (Equation 9), we measure the number of intervening opportunities for each OD pair, counting the job opportunities within the isochrone from the origin with the automobile travel time equal to this home-to-work trip. The number of trips is then aggregated based on the intervening opportunities to get, e.g., how many trips with the intervening opportunities of 0 - 1,000 or 1,000 - 2,000.

We tested exponential, power, and natural log functional forms to fit the relationships of travel time vs. percent of average trip flow and intervening opportunities vs. percent of trips. The results are shown in Figures 13 and 14, respectively. The power functions explain both travel time ($R^2 = 0.969$) and opportunity ($R^2 = 0.991$) decay curves better.

Applying the power form decay functions, Figure 15 compares the primal and dual access based on the weighted accessibility scores, in which each dot represents the scores of a census block (Due to network connectivity issues, 33 blocks (out of 54,378) were removed from the analysis.). The figure shows a clear clustered pattern for the time-weighted primal accessibility and the opportunity-weighted dual accessibility, which is well explained by a power format function ($R^2 = 0.955$). We use darker colors to show the dots that are more clustered around the line of the power function.

The exponential travel time decay function is also tested, which is commonly used in previous studies for time-weighted primal accessibility calculations (Anderson et al., 2013, Cui and Levinson, 2018*b*, Owen and Levinson, 2014, Owen and Murphy, 2017*b*,*d*), to compare the weighted primal and dual accessibility scores, replacing the power function. A similar clustered pattern is displayed, which could be explained by a power format function as well (R^2=0.832).

Figure 16 visualizes the spatial distribution of the residuals that measure the differences between the observed and predicted dual accessibility values according to the fitted power function shown in Figure 15. The brown color represents a negative residual where a relatively lower accessibility measurement is offered using the dual measure, compared to the primal one. In contrast, the blue color shows a positive residual, which implies a relatively higher accessibility evaluation when the dual measure is applied.

A spatial clustering can be identified from Figure 16 that the brown areas align with highways. A logit regression model is built on the distance to the nearest highway (D_h, meters) setting the dependent variable

as 1 if the residual is positive and 0 if it is negative. The regression results are shown in Table 1, which finds that a block closer to highways is more likely to have a negative residual, which further demonstrates the spatial clustering around highways. It implies that the dual measure is able to highlight the blocks with lower accessibility near highways and the blocks with higher accessibility away from highways at the same time.

6 Discussion and Conclusions

The concepts of primal and dual access are related and clarified in this study, in which primal accessibility solves for how many opportunities can be reached in a given time or cost threshold, while dual accessibility solves for the travel cost needed to reach a given number of opportunities. We implement both measures for the Minneapolis - St. Paul metropolitan area to demonstrate their practicality for real-world cases and to explore their correlations and differences.

Primal and dual access to jobs are measured for both auto and transit, which have similar spatial distribution patterns. First, job accessibility is higher in the downtown areas, which declines with the increase of distance to the downtown so exurban areas have the lowest job accessibility. Higher access is also found on blocks along highways for auto and the blocks around the light rail stations for transit. Thresholds (time thresholds for primal accessibility; opportunity thresholds for dual accessibility) affect job accessibility significantly.

Based on the weighted accessibility scores (time-weighted for primal accessibility; opportunity-weighted for dual accessibility), the primal and dual accessibility comparison shows a clear clustered pattern, the relationship of which could be well explained by a power function. The spatial distribution of the residuals indicates that the dual measure is able to highlight the blocks with lower accessibility near highways and the blocks with higher accessibility away from highways at the same time. It implies that dual accessibility can be used for, but not limited to, project evaluations associated with changes on highways or near highways to better identify if there are adverse influences on near blocks caused by the network changes, as such changes can cause more localized effects in terms of accessibility (Anderson et al., 2013, Cui et al., 2017).

As a location-based measure, dual accessibility has the same advantages as the primal measure, compared to the other categories of accessibility measures: it is relatively undemanding of data and is straightforward to calculate based on the same travel time matrix for primal accessibility measurements with no need of additional data sources; it is easy to understand by policy-makers or the public, since the travel time required to reach destinations is a question travelers usually face. The measure is commonly used in the objective functions for optimization problems in transport. The dual is more computationally intensive than primal

accessibility for most applications.

The dual access measure can be especially useful in a multi-activity accessibility evaluation, as the value of a location, and thus overall access, should not depend on only one opportunity or activity type or purpose of interest, e.g. jobs, but many different types of opportunities. We might sum over access to individual types of opportunities by applying appropriate weights, e.g., the average share of time per day spent at each activity, to calculate a combined indicator. Using the dual accessibility, it is possible to provide a result showing the weighted average travel time to reach n% of opportunities in a city, for instance, the opportunity threshold equals to the n% of the total for each type of opportunities. It is not as acceptable to set a fixed time threshold for different types of opportunities, as their total numbers, and averge travel times vary significantly. While the weighted-average number of reachable opportunities, using the primal measure, has less meaning.

The dual measure can be used to support the evaluation of transport or land use projects as well. Comparing alternative investments via both accessibility measures may give the same answer (one alternative is better than another by both primal and dual analysis), providing additional confidence in the selection, or different answers (the primal measure suggests one alternative, but the dual suggests another) in which case more evaluation may be required.

Additional research on the communication of alternative access measures may suggest that 'one can reach 123,000 jobs in 30 minutes' is easier or harder to understand, or thought more meaningful or intuitive, by the general public or decision makers than 'it takes 27 minutes to reach 100,000 jobs' by transit.

These reasons make dual accessibility an efficient approach for evaluating accessibility. Hence, dual accessibility deserves more attention.

Future study should be directed to build the travel time and opportunity decay functions for transit, to discover the relationship between primal and dual access for transit according to the weighted accessibility scores. We expect to see a similar clustering pattern as auto.

We would also like to expand the dual measure to consider both the supply of opportunities and the competition among travelers and opportunities. Bunel and Tovar (2014), Cheng and Bertolini (2013), Merlin and Hu (2017), Van Wee et al. (2001), and Allen and Farber (2019) proposed the competitive accessibility measures from the perspective of the primal measure counting the number of reachable and available opportunities. The reciprocal measure showing the travel time to reach available opportunities could allow us to dig deeper into its value of applications for opportunities with competition. In addition, dual accessibility should be used to model travel behavior (Kockelman, 1997, Levinson, 1998, Owen and Levinson, 2015), real estate prices (Ibeas et al., 2012, Srour et al., 2002), and economic productivity (Melo et al., 2017), and compared to the primal measure, to examine its further applicability.

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Table 1: Logit Regression to Identify the Sign of Residuals

	Estimate	Std. Error	Signif.
Intercept	-5.149E-01	1.093E-02	***
D_h	1.185E-04	3.857E-06	***
McFadden R^2		0.0139	

*** p-value<0.001, ** p-value<0.01, * p-value<0.05, . p-value <0.1

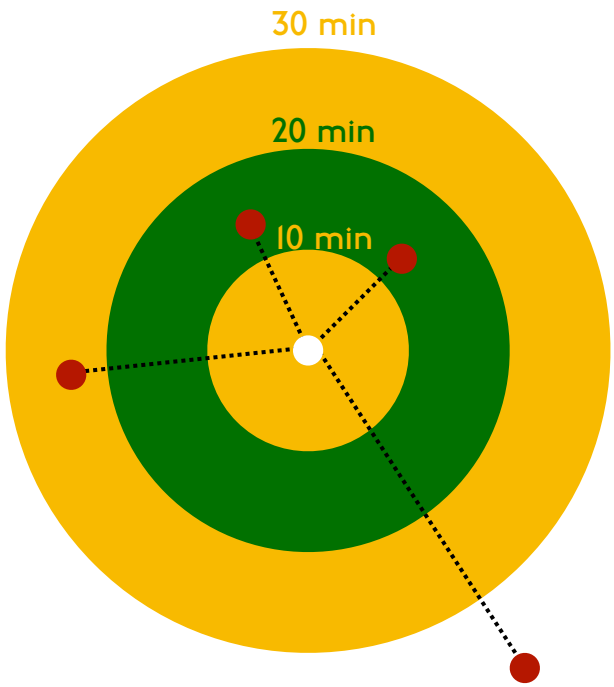
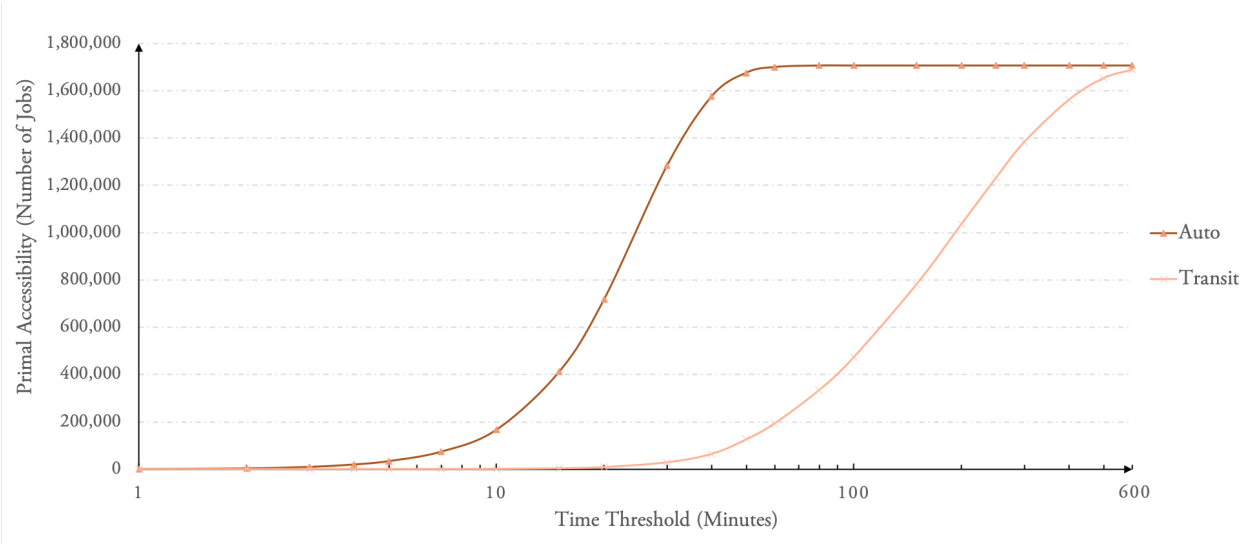
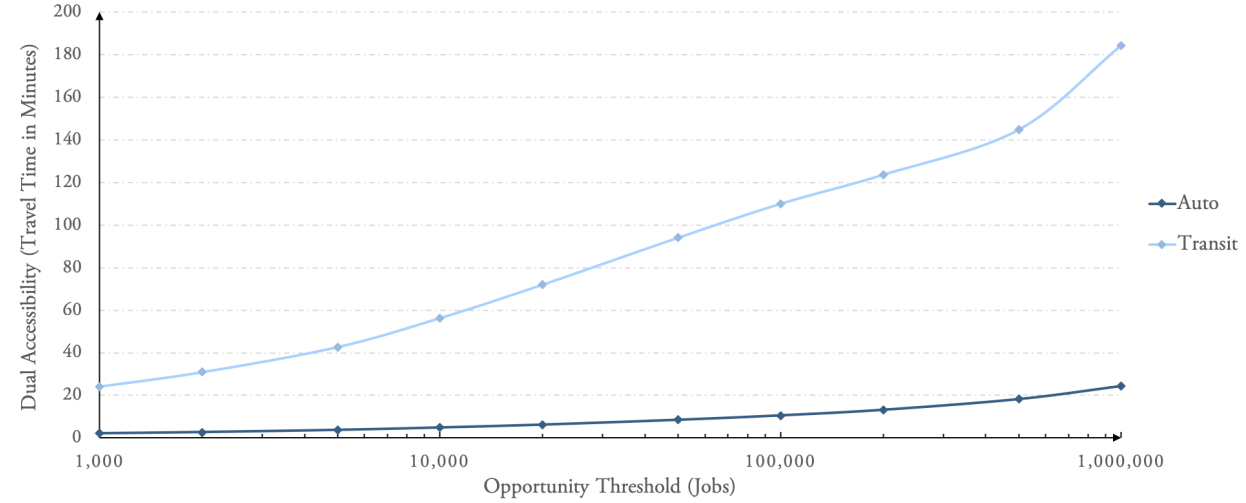


Figure 1: An Illustration of the Primal and Dual Measures. The 20 (30) minute primal accessibility is 2 (3) jobs. The dual accessibility for 2 (3) job opportunities is 14 (23) minutes.

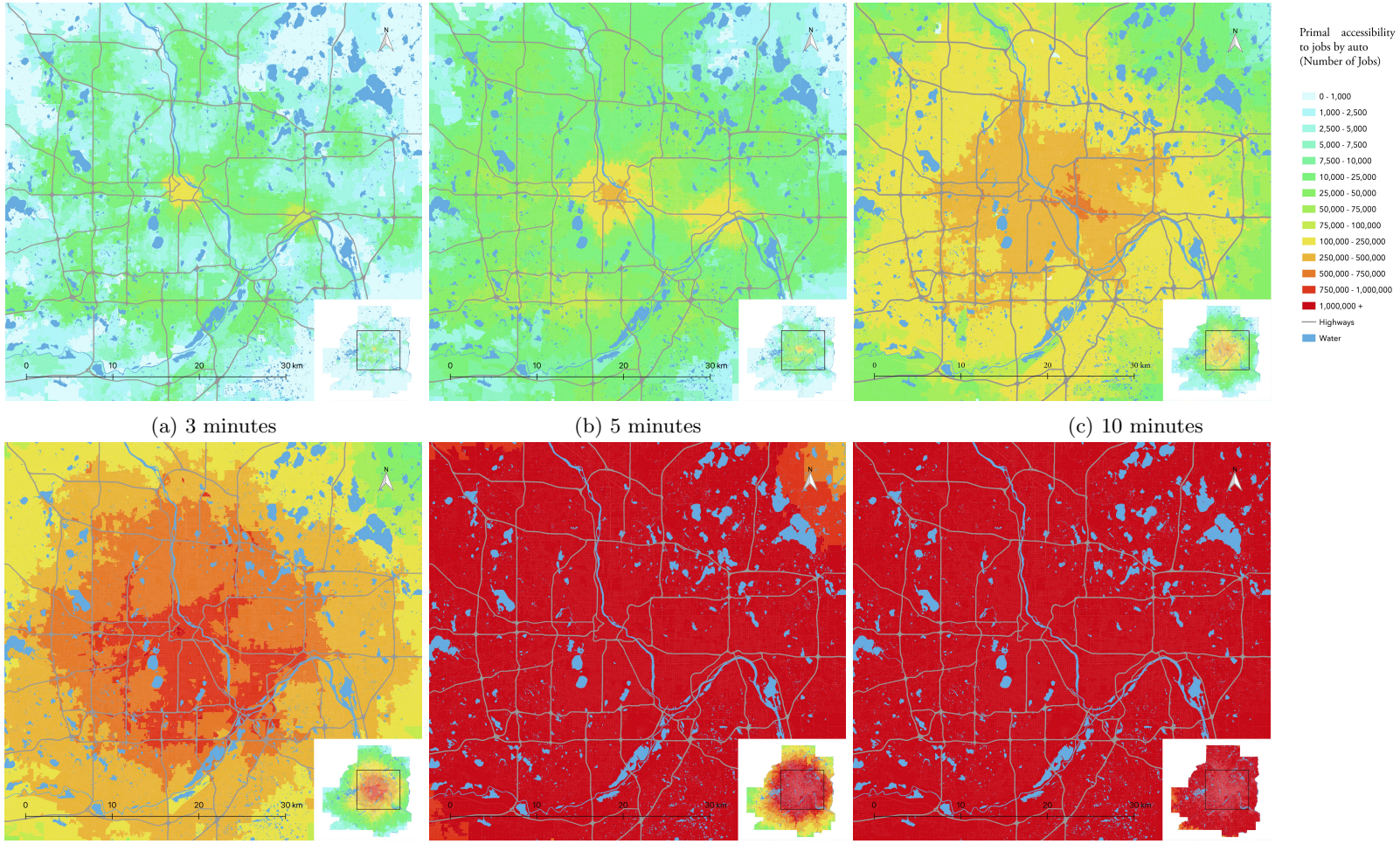


(a) Primal Accessibility (Number of Jobs)



(b) Dual Accessibility (Travel Time in Minutes)

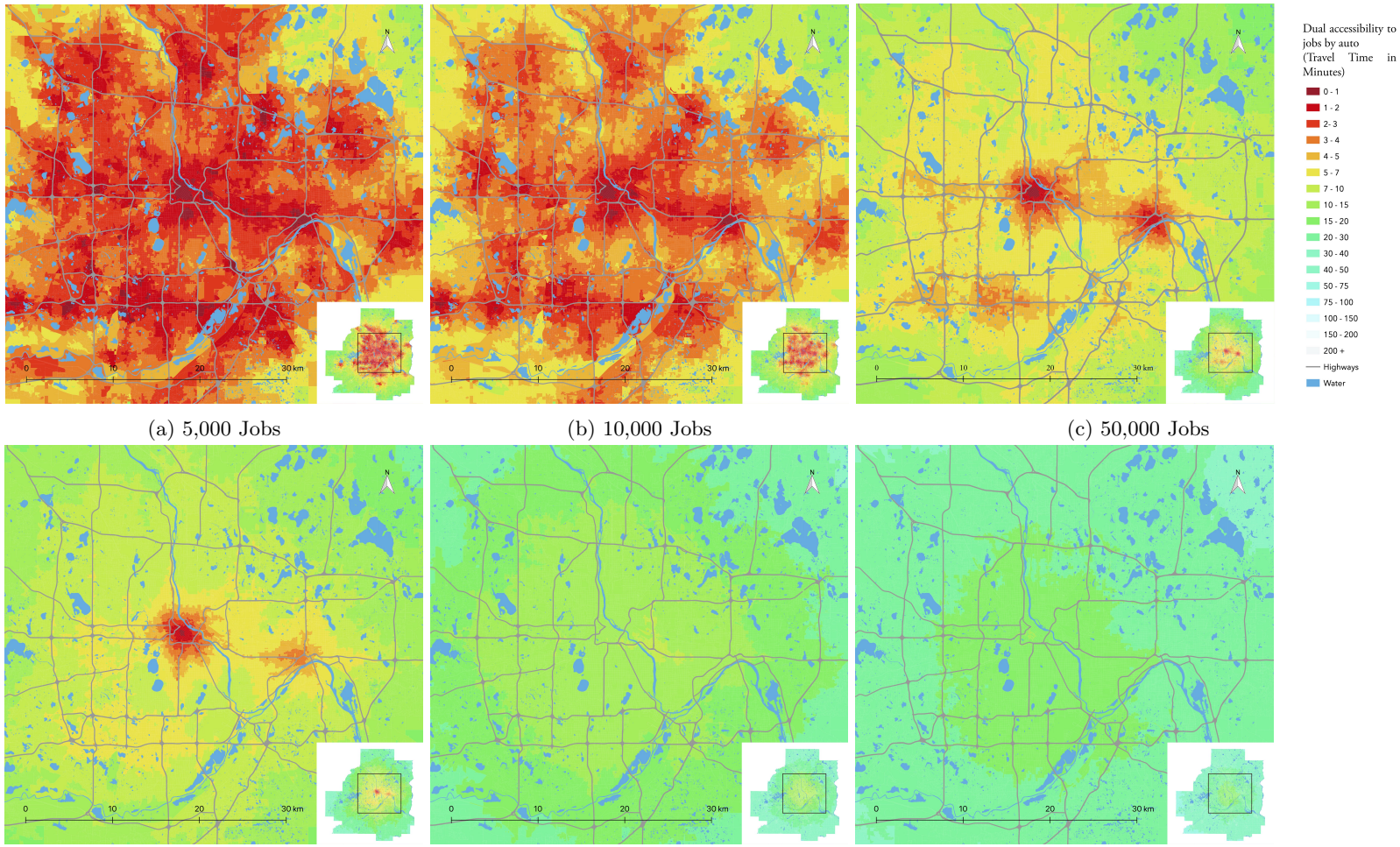
Figure 2: Population-weighted Average Primal and Dual Accessibility by Auto and by Transit



(a) 3 minutes (b) 5 minutes (c) 10 minutes

(d) 15 minutes (e) 30 minutes (f) 50 minutes

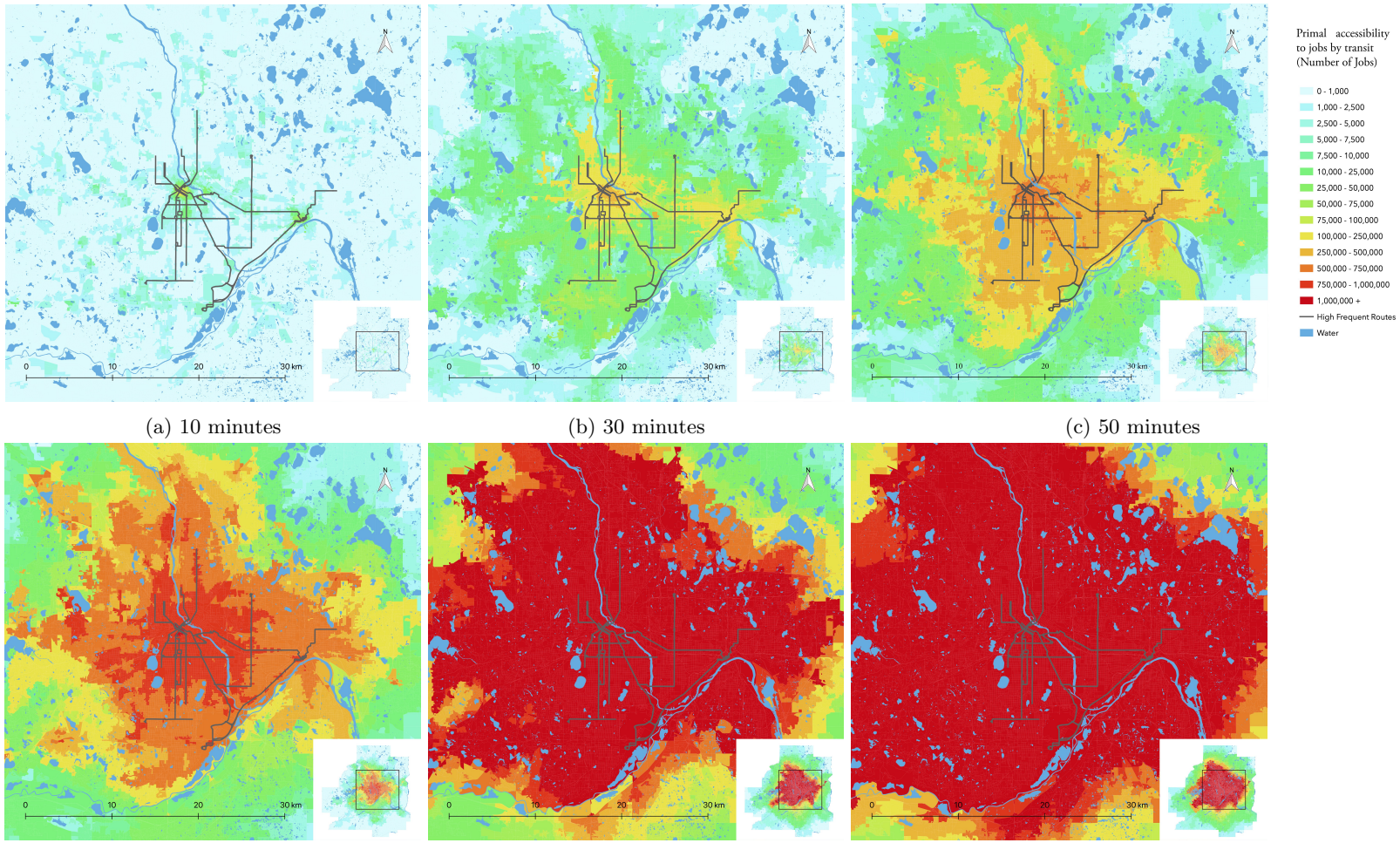
Figure 3: Primal Accessibility to Jobs by Auto (Number of Jobs)



(a) 5,000 Jobs (b) 10,000 Jobs (c) 50,000 Jobs

(d) 100,000 Jobs (e) 500,000 Jobs (f) 1,000,000 Jobs

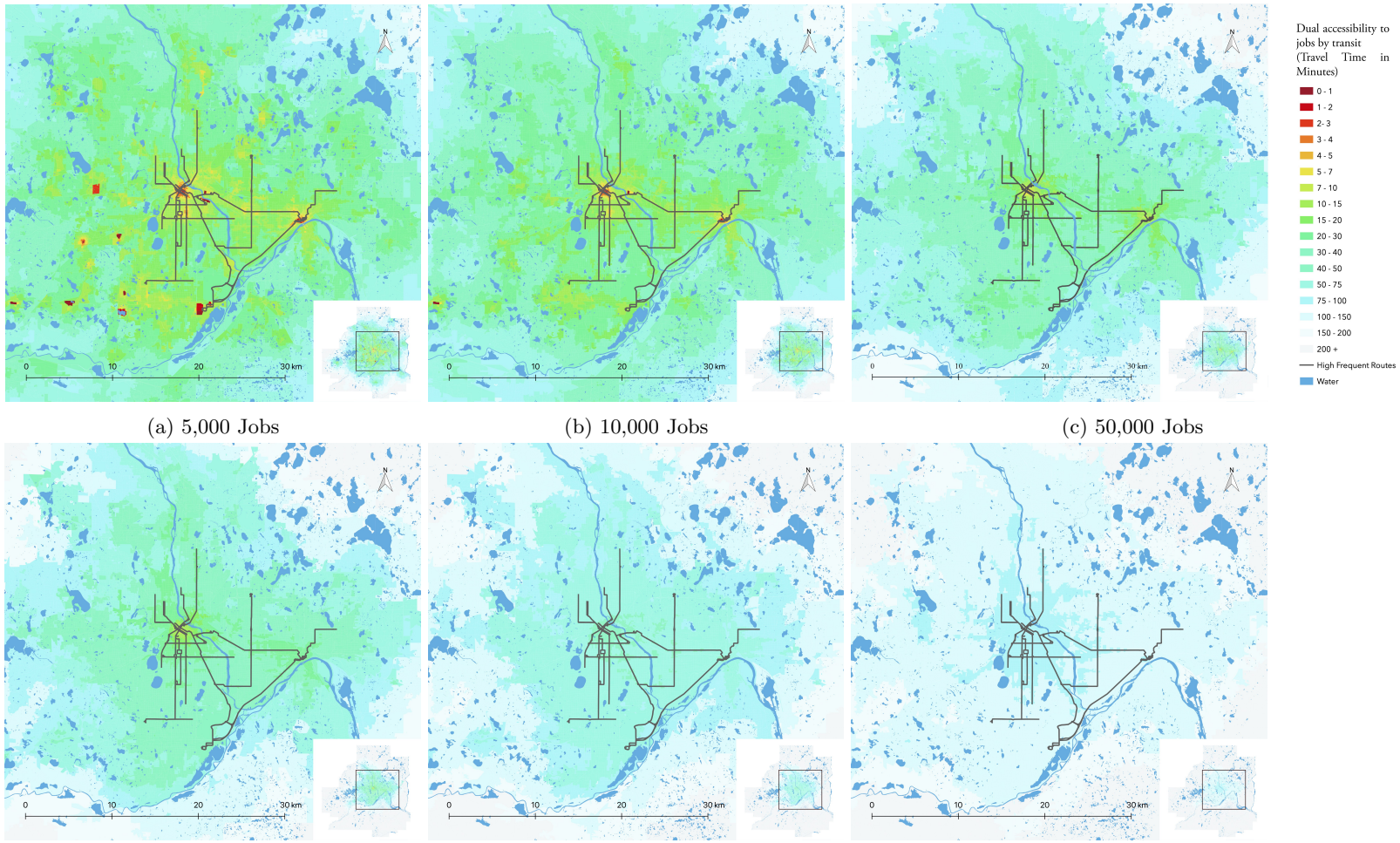
Figure 4: Dual Accessibility to Jobs by Auto (Travel Time in Minutes)



(a) 10 minutes (b) 30 minutes (c) 50 minutes

(d) 80 minutes (e) 150 minutes (f) 200 minutes

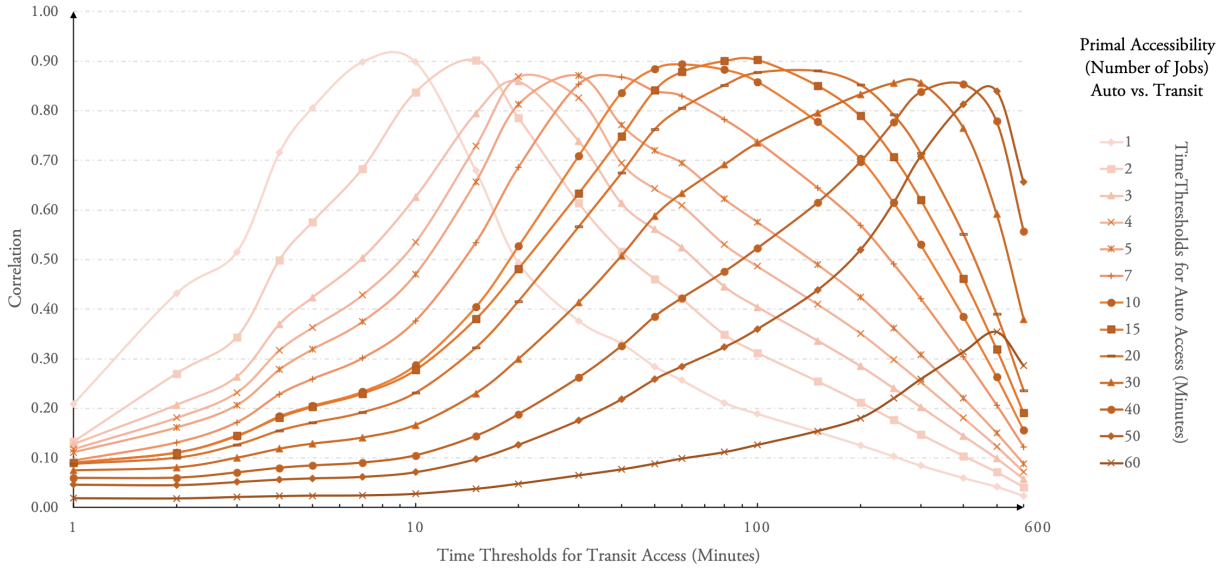
Figure 5: Primal Accessibility to Jobs by Transit (Number of Jobs)



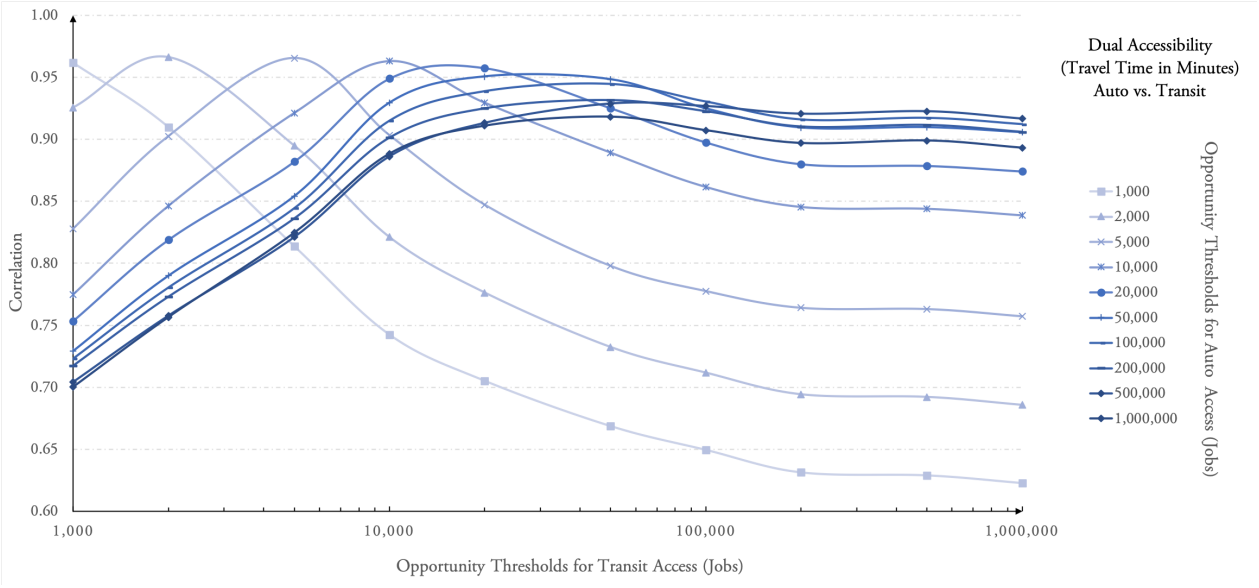
(a) 5,000 Jobs (b) 10,000 Jobs (c) 50,000 Jobs

(d) 100,000 Jobs (e) 500,000 Jobs (f) 1,000,000 Jobs

Figure 6: Dual Accessibility to Jobs by Transit (Travel Time in Minutes)

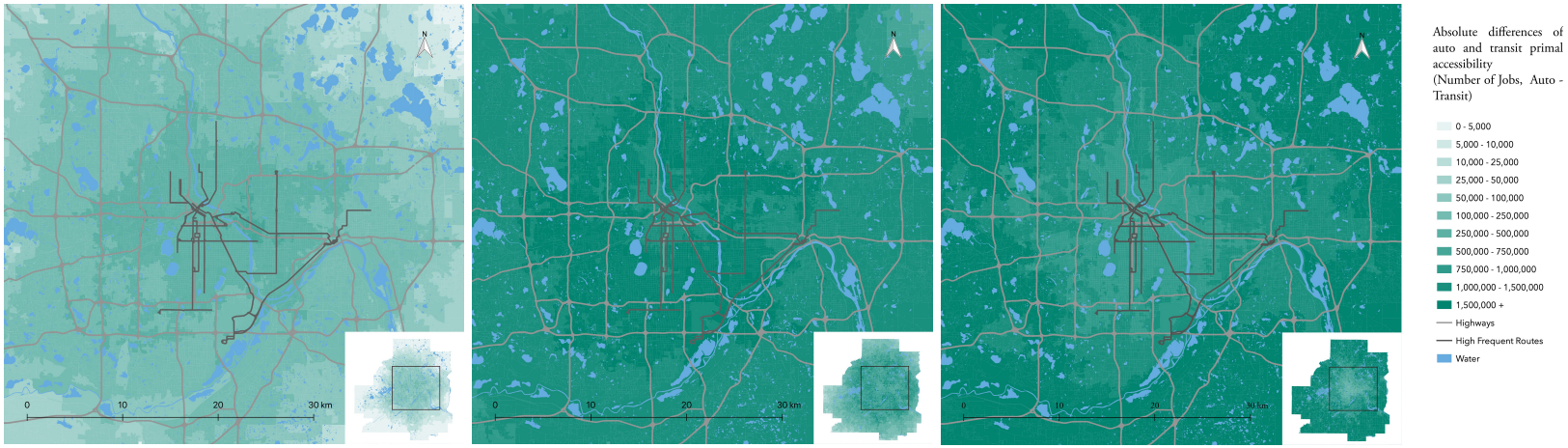


(a) Primal Accessibility (Number of Jobs)



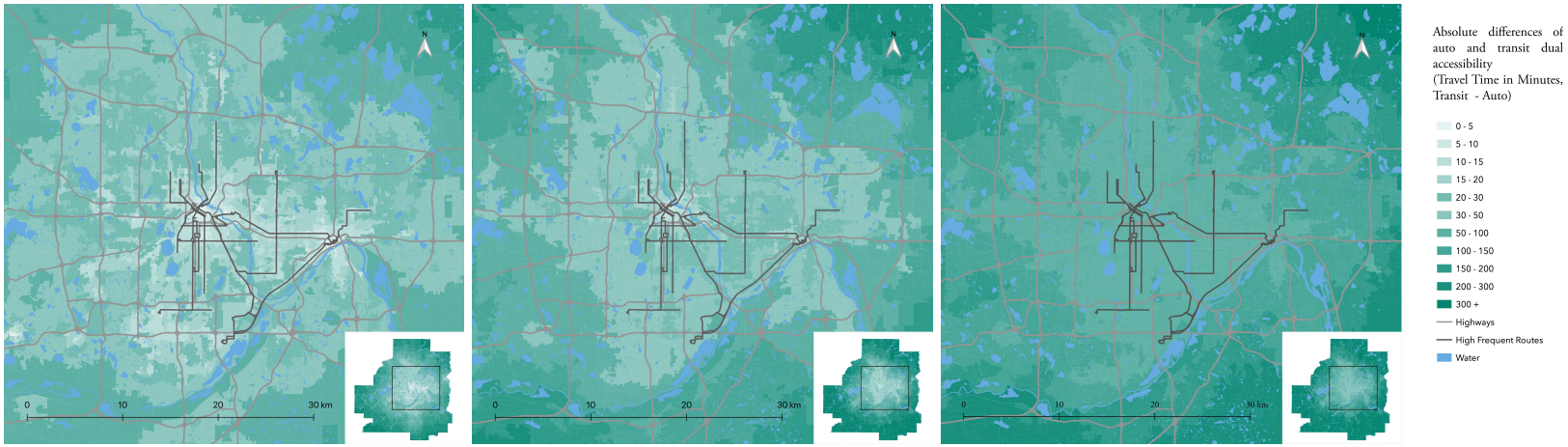
(b) Dual Accessibility (Travel Time in Minutes)

Figure 7: Population Weighted Correlations between Auto and Transit Accessibility



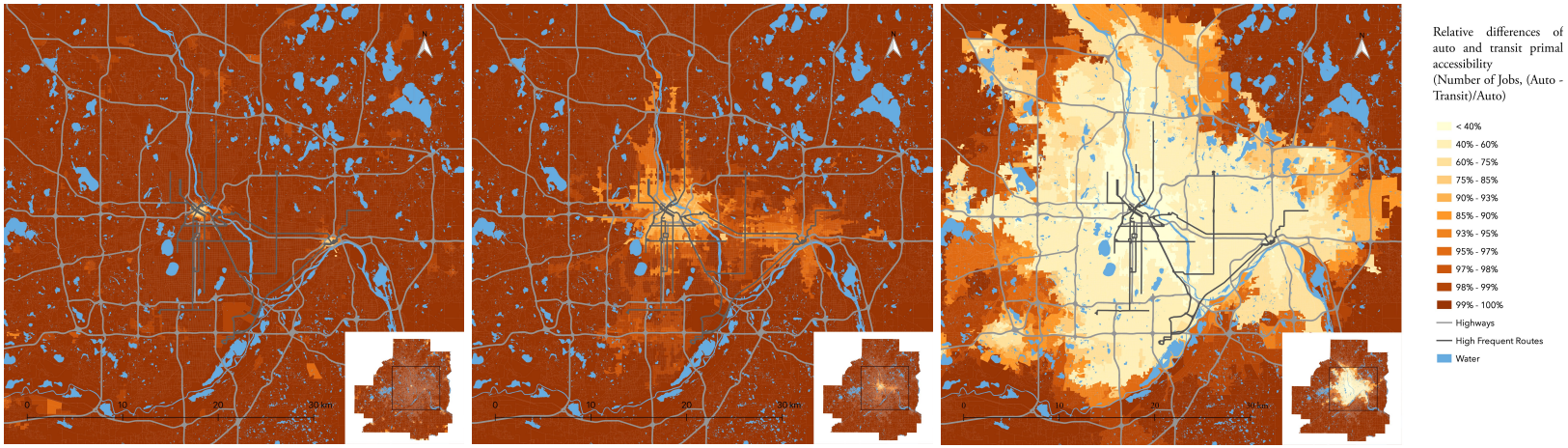
(a) 10 minutes (b) 30 minutes (c) 80 minutes

Figure 8: Absolute Differences of Auto and Transit Primal Accessibility (Number of Jobs) (Auto - Transit)



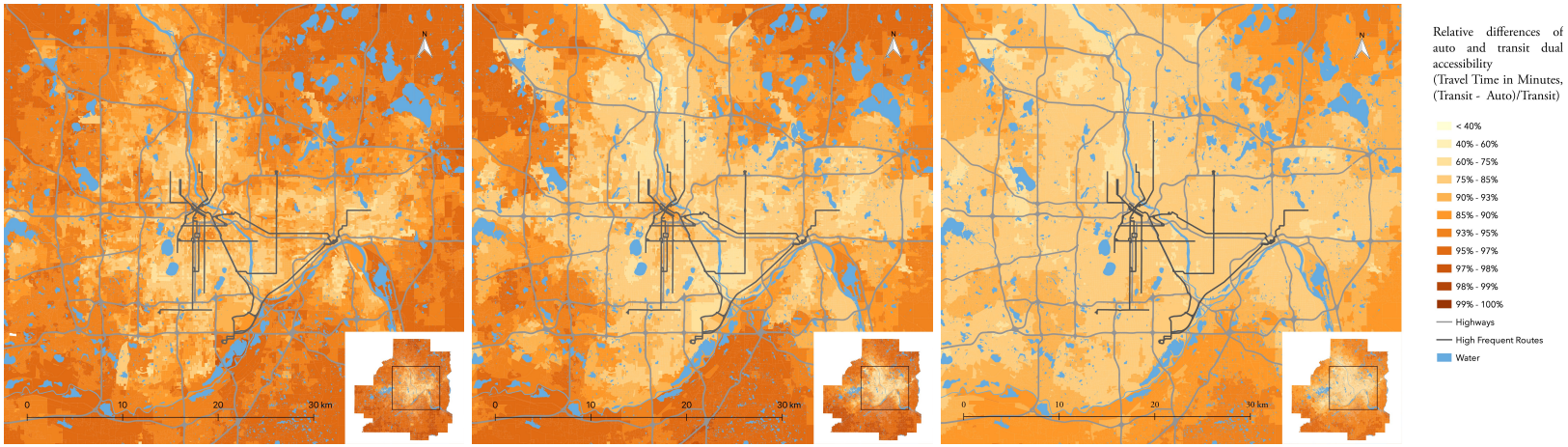
(a) 10,000 Jobs (b) 100,000 Jobs (c) 1,000,000 Jobs

Figure 9: Absolute Differences of Auto and Transit Dual Accessibility (Travel Time in Minutes) (Transit - Auto)



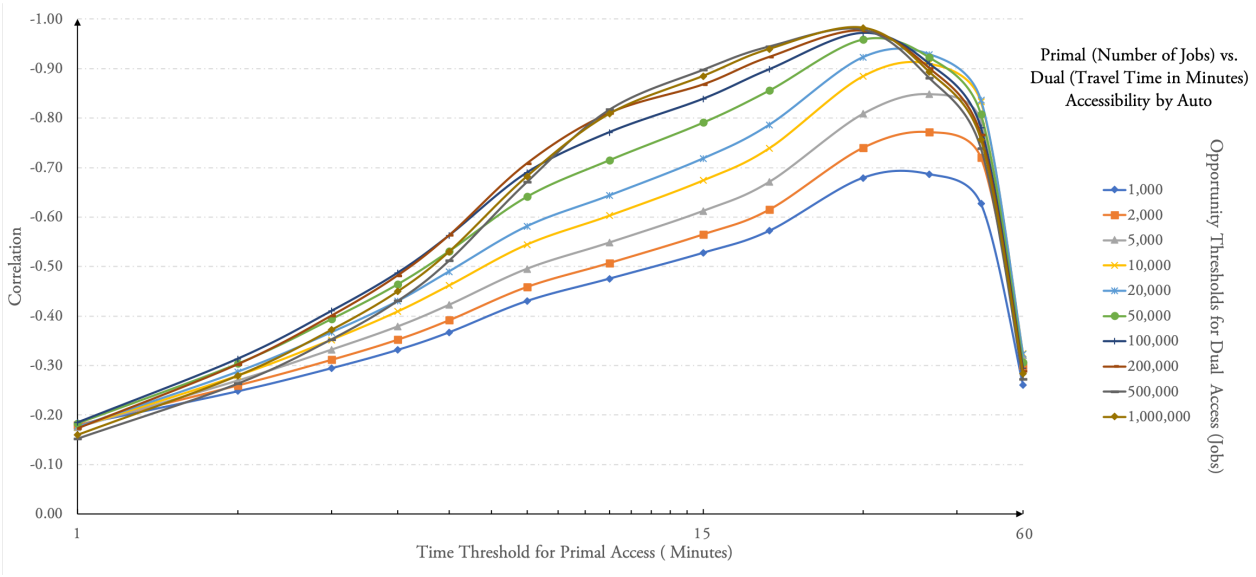
(a) 10 minutes (b) 30 minutes (c) 80 minutes

Figure 10: Relative Differences of Auto and Transit Primal Accessibility (Number of Jobs) ((Auto - Transit)/Auto)

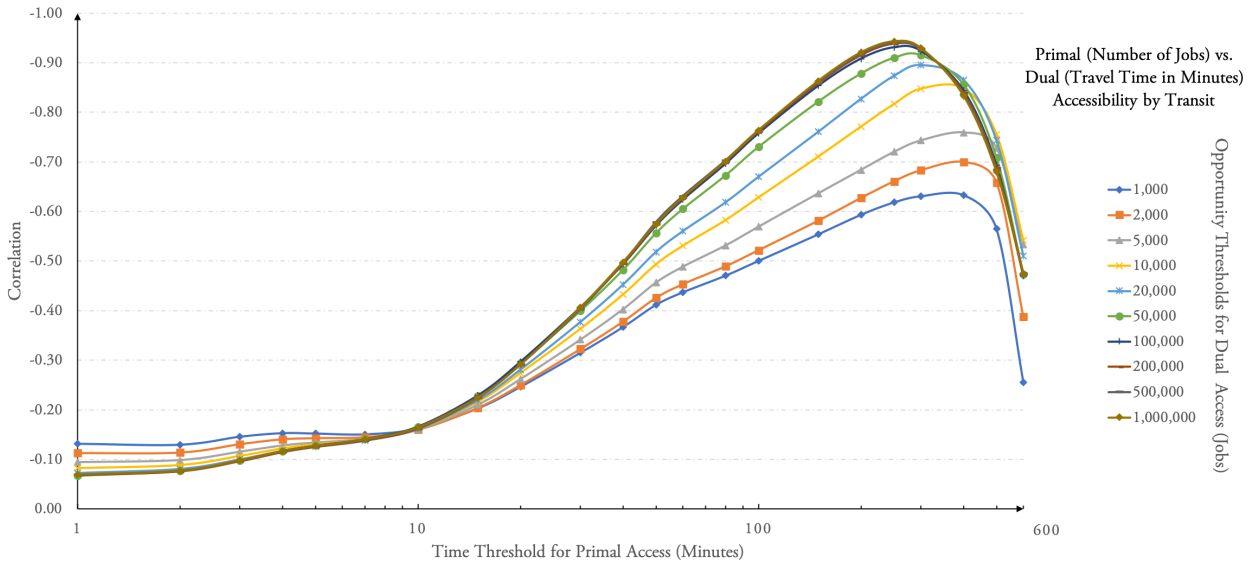


(a) 10,000 Jobs (b) 100,000 Jobs (c) 1,000,000 Jobs

Figure 11: Relative Differences of Auto and Transit Dual Accessibility (Travel Time in Minutes) ((Transit - Auto)/Transit)



(a) Auto



(b) Transit

Figure 12: Population Weighted Correlations between Primal (Number of Jobs) and Dual (Travel Time in Minutes) Accessibility

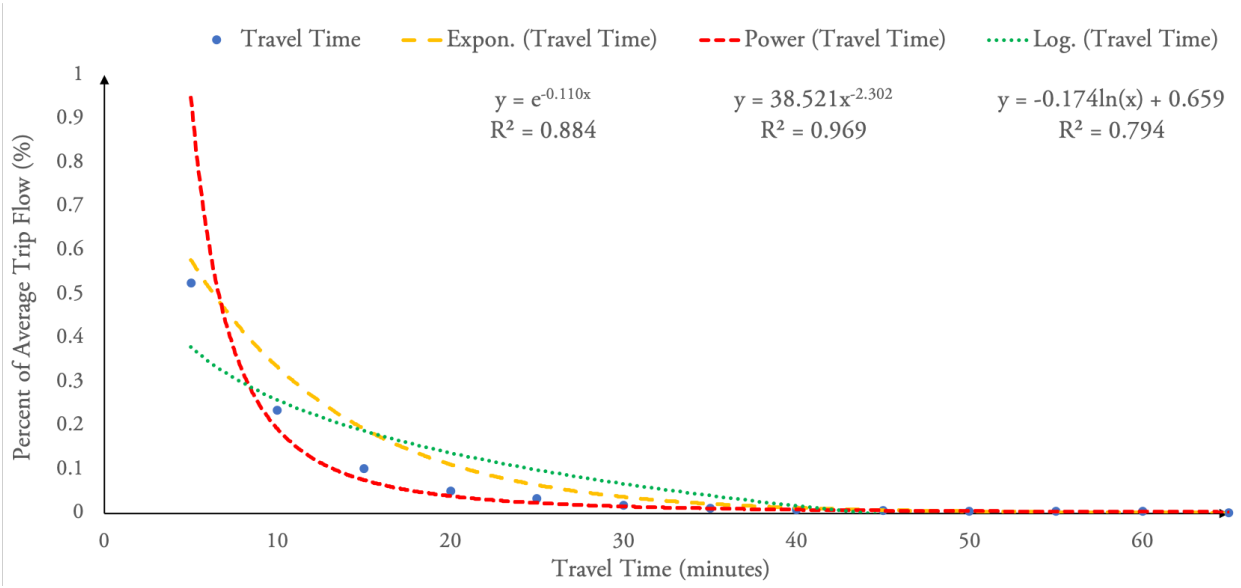


Figure 13: Travel Time Decay Functions for Auto Travel to Work

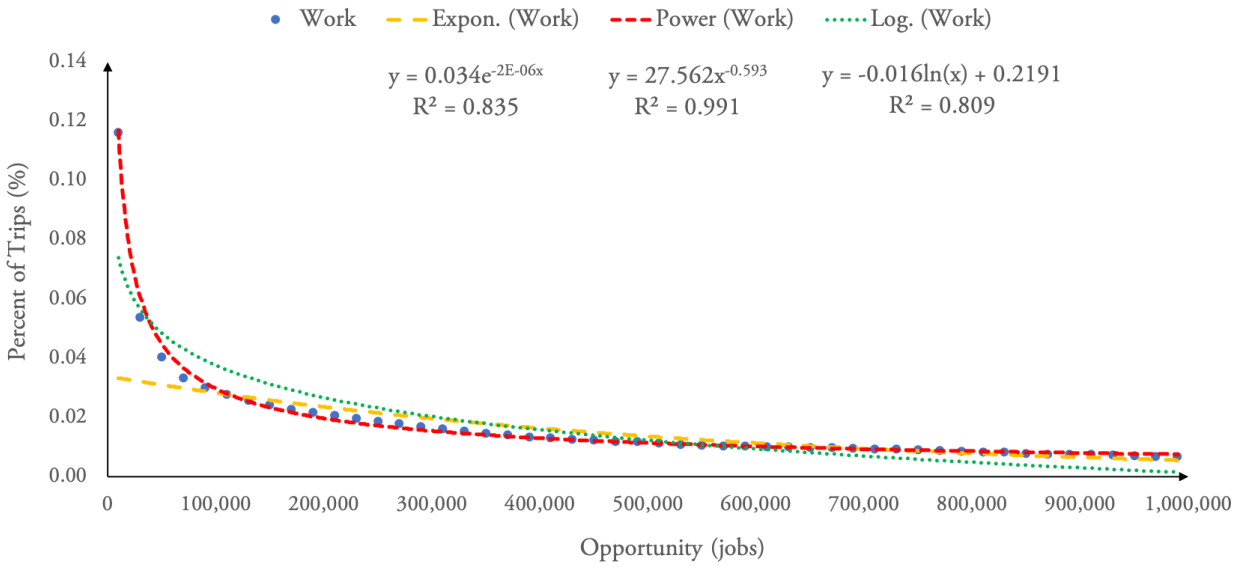


Figure 14: Opportunity Decay Functions for Auto Travel to Work

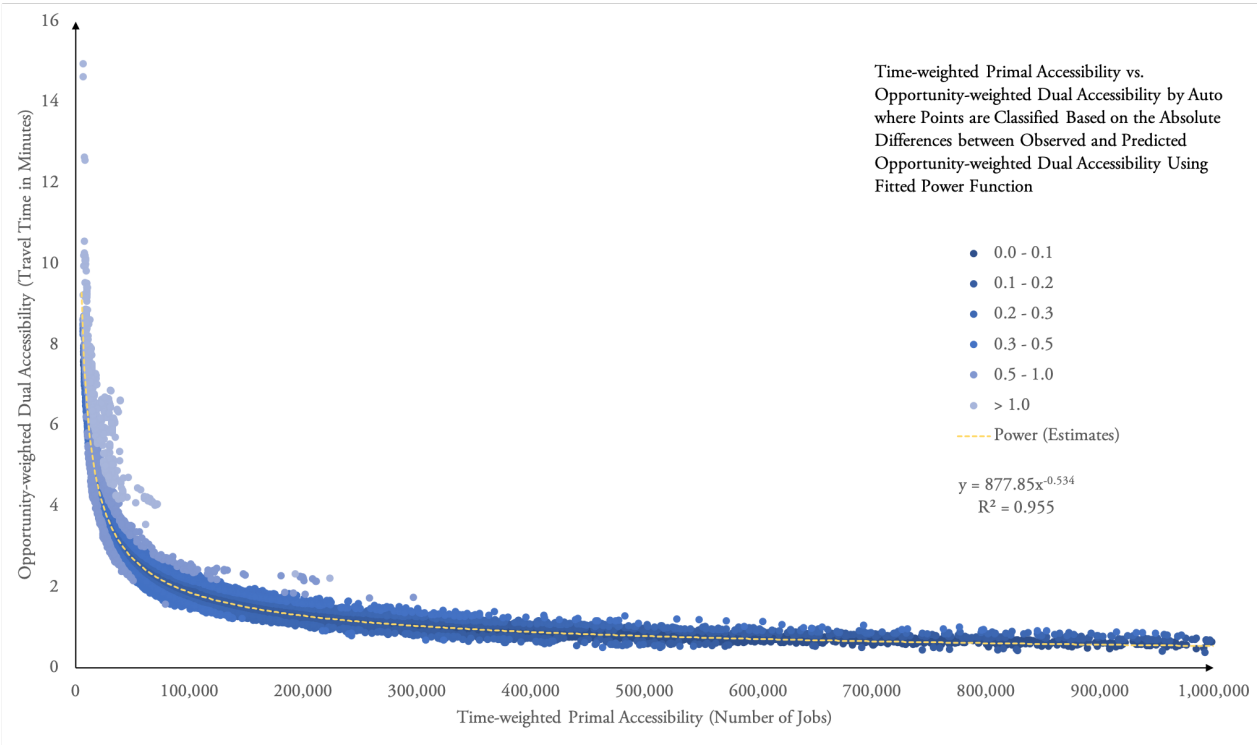


Figure 15: Comparison between Primal (Number of Jobs) vs. Dual (Travel Time in Minutes) Accessibility by Auto Based on the Weighted Accessibility Scores

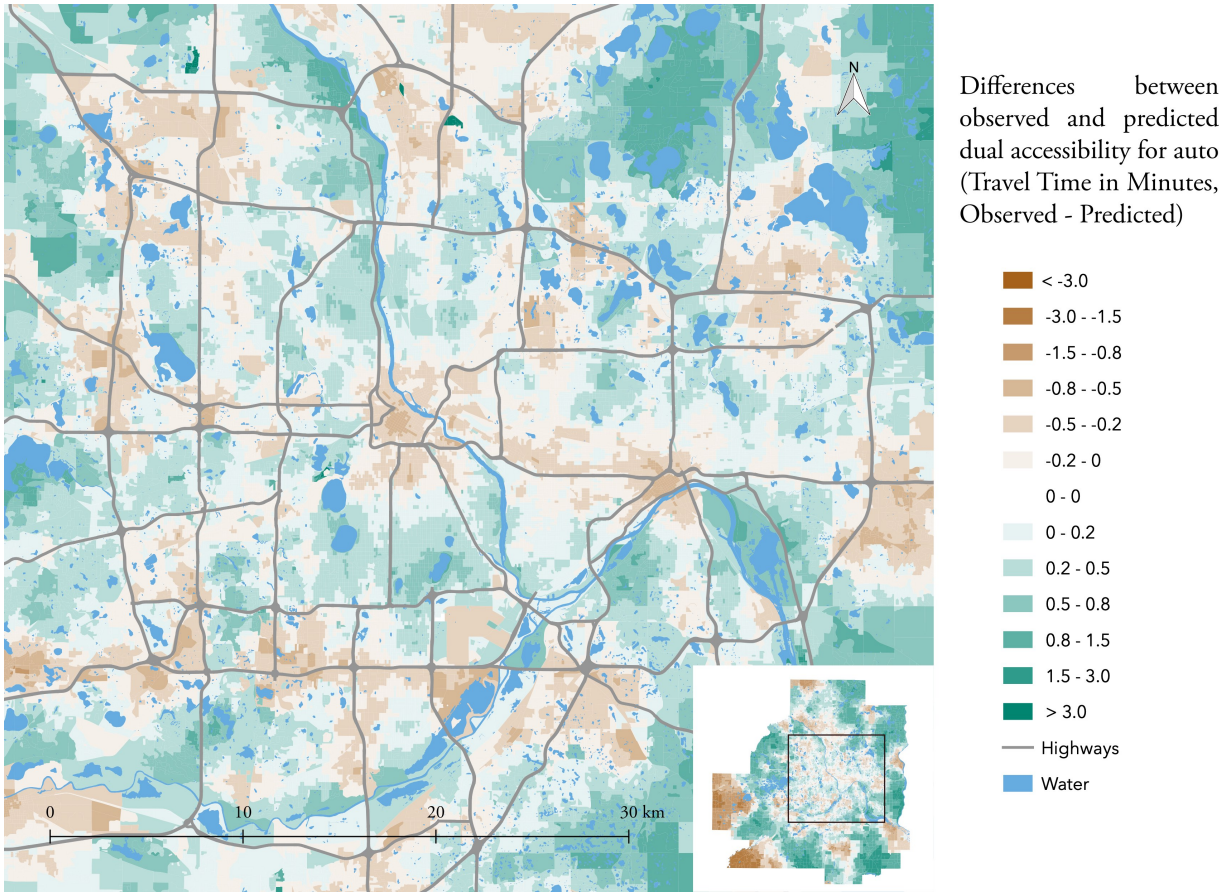


Figure 16: Residuals that Measures the Differences between Observed and Predicted Dual Accessibility (Number of Jobs, Observed - Predicted) for Auto. The Predicted Values Follow the Curve $y = 877.85x^{-0.534}$ Where x Refers to the Observed Primal Accessibility by Auto