I T L S

WORKING PAPER

ITLS-WP-09-15

Constant elasticity of substitution (CES) production function can greatly overestimate the economic costs of climate policies.

By

Truong P. Truong

July 2009

ISSN 1832-570X

INSTITUTE of TRANSPORT and LOGISTICS STUDIES

The Australian Key Centre in Transport and Logistics Management

The University of Sydney

Established under the Australian Research Council's Key Centre Program.

NUMBER: Working Paper ITLS-WP-09-15

TITLE: **Constant elasticity of substitution (CES) production function can greatly overestimate the economic costs of climate policies**

ABSTRACT: In this paper, we look at a modification to the conventional constant-elasticity-of-substitution (CES) production function to arrive at a more general specification which can allow for varying, or 'flexible' elasticity of substitution (FES). The function reduces to the CES as a special case, hence it is more general. We use the new function in a climate policy experiment to test the usefulness and robustness of the new function. It turns out that using the new function in an economic model can give estimates of the economic costs of a climate policy which is about half of the costs as estimated from a conventional CES production function specification. This has significant implication, not only for climate policy, but also for any other economic policy which relies on models which use the CES production function specification as a basic building block.

KEY WORDS: ***JEL C68, D24, D58, Q43, Q54.***
Computable general equilibrium modelling, constant-elasticity-of-substitution production function, climate policy, marginal abatement cost.

AUTHORS: Truong P. Truong

CONTACT: Institute of Transport and Logistics Studies (C37)
The Australian Key Centre in Transport Management
The University of Sydney NSW 2006 Australia

Telephone: +61 9351 0071
Facsimile: +61 9351 0088
E-mail: itlsinfo@itls.usyd.edu.au
Internet: http://www.itls.usyd.edu.au

DATE: July 2009

1. Introduction

Mainstream neoclassical economics often uses a constant-elasticity-of-substitution (CES) production function (Arrow et al., 1961) to describe the process of substitution between energy and other primary inputs such as capital or labour. The CES production function has well-defined economic interpretations but it can have unclear biophysical implications. For example, constant elasticity of substitution implies a pattern of the output elasticity of the various inputs which can be inaccurate and unrealistic from a biophysical or technological viewpoint. To improve on this, a more flexible and realistic specification of the output elasticity function, and hence the production function itself, may be necessary. In this paper, we look at this aspect and consider a second-order approximation to a general output elasticity function as a starting point, and from which derive a more 'flexible' production functional form. The new production function will have varying rather than constant elasticity-of-substitution but can include the latter as a special case. To test this flexible property of the new production function (referred to as 'flexible elasticity of substitution' (FES) production function), we use this new function in a climate policy experiment to estimate the marginal CO2 abatement cost which is often used as an indication of economic cost of climate policy. We found that the results from the FES production function greatly differ from those estimated from a CES production function specification. Importantly, we find the FES tending to give a much lower marginal CO2 abatement costs than the CES production function does. This can be explained in terms of the nature of the underlying 'technological constraints' which are implicitly assumed under these two types of functions, with the FES assuming a more 'flexible' set of technological constraints than does the CES production function, hence the lower marginal CO2 abatement costs.

The outline of the paper is as follows: section 2 describes the conventional neoclassical approach to the specification of input substitution in production activities. Section 3 then considers some biophysical modification to this standard approach which can lead to the specification of a production function which has flexible rather than constant elasticity of substitution between the inputs. Section 4 examines the technological shadow prices of production inputs (especially the energy input) underlying a FES rather than a CES production function (the latter simply assumes these prices to be zero). Section 5 demonstrates the use of the new FES production function in a climate policy experiment to estimate the economic cost of a typical climate policy objective. Section 6 concludes

2. Neoclassical approach to the representation of economic production activities

In a neoclassical economic approach, production activities are often viewed from a rather aggregate perspective as compared to, say, an engineering approach. The latter looks at production activities from the point of view of individual production techniques or technologies[1], and often describes these technologies in terms of fixed production input-output ratios. In an engineering 'model' of production activities such as linear programming (LP), the level of production of each technology is explained in terms of an optimization procedure which selects the least cost combination of technology (or technologies) given the relative input prices

[1] We use the term techniques or technologies in a very loose sense. For example, alternative methods of generating electricity using coal, gas, or enriched uranium, can be considered as different 'techniques' of producing electricity given a particular 'state' of the current 'technology'. Technology, therefore, can be regarded as the combination of all existing techniques. When we refer to a change in 'technology', therefore, this may involve, not just a change in techniques, but also a shift in the 'parameters' of the existing techniques or the creation of a new (and more efficient) technique. In general, however, the distinction between the two words is not very clear because they are closely related.

and the level of demand for total output. When these relative prices change[2], or the total level of demand vary, the least cost combination of technologies will also change resulting in a different combination of the inputs used. A relationship between the level of total output and the (aggregated) inputs used therefore can be referred to as a 'production function' in a similar manner to a neoclassical production function. This is a 'positive' interpretation of a neoclassical production function In contrast, a 'normative' interpretation would seek out to explain the relationship in terms of some underlying economic behaviour of a particular production agent (such as a representative firm in an economy)[3]. In the past, the normative aspect of the neoclassical production function was perhaps the more controversial one which often invited critiques and controversies[4]. With the new emphasis on environmental and energy resources issues, however, the positive aspect of the neoclassical production function now also come under close scrutiny. This is because the crucial question now has become, not just whether wages and capital returns are 'fair', but also whether production and consumption can be maintained in the future in the face of an exhaustible energy resource. To find an answer to this question depends not only on political economy, but also on aspects of thermodynamics and biophysical laws (Georgescu-Roegen, 1966, 1971; Ayres and Warr, 2005; Islam, 1985) and therefore, there are many calls for an 'integration' or 'reintegration' of biophysical laws into the economic laws (Hall et al., 2001; Lindenberger and Kümmel, 2002; Kümmel et al., 2008). One important result of this integration is an explicit rather than implicit consideration of the type of biophysical or 'technological' constraints which can act on production activity, along side with the economic (cost-minimizing) constraints. In a traditional neoclassical approach, often the technological constraints are either assumed away (all points along an isoquant are considered to be technically feasible and equally efficient from a technological viewpoint and therefore, the only type of 'efficiency' left to be considered is 'economic'), or in fact left implicit (the isoquant of a constant elasticity of substitution (CES) production function actually implies a particular form for the 'technological constraints' which govern the relationship between the levels of inputs and output[5]. An important consequence of the neglect of treatment of technological constraints in the traditional approach is that the actual constraints which are actually implied may turn out to be more restrictive than are necessary resulting in a distortion of the calculations relating to economic costs of economic policies.

3. The biophysical modification to the neoclassical approach

In contrast to the neoclassical approach a biophysical approach will seek to explicitly incorporate the type of technological constraints that govern the relationship between inputs and output and consider this alongside with other economic constraints governing economic behaviour. Consider for example, the following simple economic production relationship[6].

---

[2] Non-substitution theorem (Samuelson, 1951) may also allow for the case when relative prices of the inputs may remain quite fixed even in the face of changing total demand for the output. This 'insulates' the relative input prices from 'demand shocks' but in practice, relative input prices may also change, not because of demand shocks but rather of 'supply shocks'. Hence, they are also important sources of stimulations for 'technological change' or technique variation.

[3] This distinction between the positive and normative aspects of a neoclassical production function is perhaps similar to the one used by Shaikh (1974) to distinguish between 'the laws of algebra' and the 'laws of production' in referring to a production function..

[4] When understood from a purely 'positive' perspective, a neoclassical production function can be used simply to predict or forecast the levels of inputs and outputs. When interpreted in a 'normative' sense, however, it can be used as an economic theory to explain and 'justify', for example, the distribution of the total output between wages for labor, interests for capital, and economic returns to natural resources owner tool. Therefore, this aspect can invite more criticism (see, for example, the 'Cambridge controversy' in the theory of capital – Harcourt, 1969, 1972).

[5] This will be considered below in the next section.

[6] Full specification of a production function will have to include other factors like labour, land, natural resources, and other material intermediate inputs. Here, we consider only a 'sub-structure' of the overall production function, i.e. we assume that the production

$$Y = f(K, E) \tag{1}$$

where *Y*, *K*, and *E* are the quantities of output, capital, and energy respectively. In a conventional neoclassical approach, it is assumed that a producer maximizes the output of the activity as specified by the production function (1) and subject only to some economic cost constraint, such as:

$$P_K K + P_E E \leq C \tag{2}$$

Here, P_E and P_K are the market prices of the energy and capital respectively, and *C* is the production cost. Although not explicitly considered, biophysical or technological relationships between the levels of the inputs can be *implied* by a production relationship such as (1). For example, under a Cobb-Douglas or CES production function, the following technological relationship between levels of the inputs are in fact implied:

$$\begin{aligned} \varepsilon_K &= \frac{K}{Y}\frac{\partial Y}{\partial K} = a_K K^{-\rho} / [a_K K^{-\rho} + a_E E^{-\rho}] \\ &= a_K [a_K + a_E (E/K)^{-\rho}]^{-1} \end{aligned} \tag{3}$$

Here, ε_K is the elasticity of output with respect to the capital input; a_K, a_E are constants related to the cost shares of capital and energy inputs respectively, and ρ is a parameter related to the elasticity of substitution[7] between *E* and *K*. To show that technological relationship (3) is in fact implied by a CES production function, first we note that for a constant returns to scale production function such as CES, we have:

$$Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial E} E$$

or:

$$\begin{aligned} 1 &= \frac{K}{Y}\frac{\partial Y}{\partial K} + \frac{E}{Y}\frac{\partial Y}{\partial E} \\ &= \varepsilon_K + \varepsilon_E \end{aligned} \tag{4}$$

From (4) and (3), we can derive the output elasticity for energy input:

$$\begin{aligned} \varepsilon_E &= \frac{K}{Y}\frac{\partial Y}{\partial E} = 1 - \varepsilon_K = [a_E E^{-\rho}]/[a_K K^{-\rho} + a_E E^{-\rho}] \\ &= [a_E (E/K)^{-\rho}][a_K + a_E (E/K)^{-\rho}]^{-1} \end{aligned} \tag{5}$$

function is weakly separable between capital and energy on the one hand, and other factors of production on the other hand. This allows us to focus attention on the question of capital/energy substitutability or complementarity and consider this issue in isolation or 'separately' from other issues of substitution among all factors. The 'output' y in production function (1), therefore, is not the total output, but only the 'sub-output' from the K-E branch, i.e. the level of the "K-E" composite.

[7] If σ is the elasticity of substitution between *K* and *E*, then it can be shown that $\sigma = 1/(1+\rho)$. When $\rho = 0$, this is a Cobb-Douglas production function and ε_K is a constant irrespective of the level of *E* and *K*. When $\rho > (<) 0$, $\sigma < (>) 1$, and when $\rho \to \infty$, $\sigma \to 0$. the production function takes the form of fixed-coefficient or Leontief form and ε_K is undefined.

For any general production function such as (1), we can take the log-differential:

$$\begin{aligned}\frac{dY}{y} &= \frac{K}{Y}\frac{\partial Y}{\partial K}\frac{dK}{K} + \frac{E}{Y}\frac{\partial Y}{\partial E}\frac{dE}{E} \\ &= \varepsilon_K \frac{dK}{K} + \varepsilon_E \frac{dE}{E}\end{aligned} \tag{6}$$

Substituting (3) and (5) into (6) and then integrate, we have:

$$\begin{aligned} d\ln(Y) &= \frac{a_K K^{-\rho}}{[a_K K^{-\rho} + a_E E^{-\rho}]}\frac{dK}{K} + \frac{a_E E^{-\rho}}{[a_K K^{-\rho} + a_E E^{-\rho}]}\frac{dE}{E} \\ &= -\frac{1}{\rho} S_K d\ln(a_K K^{-\rho}) - \frac{1}{\rho} S_E d\ln(a_E E^{-\rho})] \\ &= d\ln([a_K K^{-\rho} + a_E E^{-\rho}]^{-1/\rho}) \end{aligned} \tag{7}$$

or

$$Y = a_0 [a_K K^{-\rho} + a_E E^{-\rho}]^{-1/\rho} \tag{8}$$

with

$$\begin{aligned} S_K &= \frac{a_K K^{-\rho}}{[a_K K^{-\rho} + a_E E^{-\rho}]} = \varepsilon_K; \quad S_E = \frac{a_E E^{-\rho}}{[a_K K^{-\rho} + a_E E^{-\rho}]} = \varepsilon_E; \\ S_K + S_E &= 1 \end{aligned} \tag{9}$$

and a_0 is a normalizing constant.

Technological relationships such as (3) and (5) are not based on any firm economic or technological foundation, and in some cases, they may even violate some biophysical laws. For example, according to equations (3)–(5), output elasticity of capital (energy) will increase (decrease) monotonically with the increase in (E/K) if ρ>0 (substitution elasticity between E and K is less than 1), or remaining constant if ρ=0 (Cobb-Douglas production function). This does not seem to accord well with some empirical evidences. For example, given any state of technology and a fixed level of K, technical efficiency of energy (output per unit of energy) will first increase with increased level of utilization of capital (E/K), but then will reach a maximum level and then decline[8]. This implies equations (3) and (5) are rather unrealistically restrictive, and a modification to these assumptions may be useful. One such modification is to consider a second-order Taylor series approximation to any *general* relationship between output elasticity and the level of capacity utilization evaluated at a particular local point as follows:

$$\varepsilon_K = \frac{K}{Y}\frac{\partial Y}{\partial K} = b_0 + b_1(\frac{E}{K} - \frac{E_0}{K_0}) + b_2(\frac{E}{K} - \frac{E_0}{K_0})^2 \tag{10}$$

Here (E_0/K_0) stands for the initial capital utilization level (local point), and b_0, b_1, and b_2 are coefficients related to the zero-, first-, and second-order derivatives of the underlying general

8 In economics, this can also be described in terms of the ‘law of diminishing returns’ which says the returns to energy input given a fixed level of K in the ‘short run’ will first increase when (E/K) is small, but then will decline when (E/K) gets larger. This implies output elasticity of energy cannot be monotonic with respect to the level of (E/K) except for a limited range. In the ‘long run’ when the level of K is allowed to vary, first with increasing and then eventually decreasing returns to scale (U-shaped long run average cost curve), output elasticity of energy similarly cannot remain monotonic with respect to the level of (E/K) except over a limited range. It may be argued that with perfect competition, it is precisely this limited range that defines the firm’s substitution possibility frontier. However, even if the results of the firm’s behavior may be confined to this limited range, the function describing the firm’s supply behavior (rather than market equilibrium position) should not limit itself only to this limited range. Otherwise market (and technical) inefficiencies cannot be described and detected.

relationship between ε_K and (E_0/K_0) evaluated at this point. Using (10) and continuing to assume (4) (i.e. constant returns to scale), we can derive a technological relationship also for ε_E:

$$\varepsilon_E = \frac{E}{Y}\frac{\partial Y}{\partial E} = 1 - b_0 - b_1(\frac{E}{K} - \frac{E_0}{K_0}) - b_2(\frac{E}{K} - \frac{E_0}{K_0})^2 \tag{11}$$

Substituting (10) and (11) into (6) and then integrate, we have:

$$\begin{aligned}\ln(Y) &= \ln(E) - \int b_0\left[\frac{dE}{E} - \frac{dK}{K}\right] - \int b_1\left[\frac{E}{K} - \frac{E_0}{K_0}\right]\left[\frac{dE}{E} - \frac{dK}{K}\right] - \int b_2\left[\frac{E}{K} - \frac{E_0}{K_0}\right]^2\left[\frac{dE}{E} - \frac{dK}{K}\right] \\ &= \ln(E) + c_1\ln(\frac{E}{K}) + c_2\left[\frac{E}{K}\right] + c_3\left[\frac{E}{K}\right]^2 + \ln(c_0)\end{aligned} \tag{12}$$

or:

$$Y = c_0 E\left[\frac{E}{K}\right]^{c_1}\exp\{c_2\left[\frac{E}{K}\right] + c_3\left[\frac{E}{K}\right]^2\} \tag{13}$$

with $c_1 = [-b_0 + b_1(E_0/K_0) - b_2(E_0/K_0)^2]$, $c_2 = [-b_1 + 2b_2(E_0/K_0)]$, $c_3 = [-(1/2)b_2]$, and c_0 is a normalizing constant. Since $Y = Y_0$ when $E = E_0$ and $K = K_0$, we have:

$$\begin{aligned}Y &= Y_0\left[\frac{E}{E_0}\right]\left[\frac{E/E_0}{K/K_0}\right]^{c_1}\exp\{c_2\left[\frac{E}{K} - \frac{E_0}{K_0}\right] + c_3\left[\left(\frac{E}{K}\right)^2 - \left(\frac{E_0}{K_0}\right)^2\right]\} \\ &= Y_0[e]\left[\frac{e}{k}\right]^{c_1}\exp\{c_2^*\left[\frac{e}{k} - 1\right] + c_3^*\left[\left(\frac{e}{k}\right)^2 - 1\right]\}\end{aligned} \tag{14}$$

where $e = E/E_0$, $k = K/K_0$, $c_2^* = c_2(E_0/K_0)$, $c_3^* = c_3(E_0/K_0)^2$. When $b_1 = b_2 = 0$, $c_2^* = b_2^* = 0$. the function reduces to a Cobb-Douglas form, i.e. $Y = Y_0(E/E_0)^{(1-c_1)}(K/K_0)^{c_1}$. When $b_2 = 0$, the function takes a simpler form[9]:

$$Y = Y_0 e(e/k)^{c_1}\exp\{-2c_2^*[(e/k) - 1]\} \tag{15}$$

We refer to the class of production functions specified by (14)-(15) as *flexible* elasticity of substitution (FES) production function because it can approximate any general production function at a particular point, assuming that the technological constraints can be approximated by a flexible second-order Taylor series expansion of the form (10)-(11). The advantage of using the FES rather than the CES restrictions (3) and (5) is that the assumption of constancy of the elasticity of substitution on a global scale can be relaxed, even though at a local scale, it may still be maintained. This is because equations (10)-(11) can in fact approximate equations (3)

---

[9] A similar form to equations (14)-(15) is often referred to as the "linex" production function (see for example, Kümmel *et al.*, 2008). This is because it has a *lin*ear component in energy input (e) and an *ex*ponential component in the ratio (e/k). This terminology, however, can be confusing because while the variable (e) may appear in the first part of the right hand side as a 'linear' term, it also appears in the second part as an exponential term, and furthermore, if c_1 is not zero, then the relationship also contains a power term $(e/k)^{c_1}$.

and (5) at a particular point (E_0, K_0) if the values of b_0, b_1, and b_2 are given by the zero-, first-, and second-order derivatives of equation (3) evaluated at (E_0, K_0): [10]

$$\begin{aligned} b_0 &= a_K[a_K + a_E(E_0/K_0)^{-\rho}]^{-1} \\ b_1 &= \rho a_K a_E (E_0/K_0)^{-\rho-1}[a_K + a_E(E_0/K_0)^{-\rho}]^{-2} \\ b_2 &= -(1/2)\rho(\rho+1)(a_K a_E)(E_0/K_0)^{-\rho-2}[a_K + a_E(E_0/K_0)^{-\rho}]^{-2} \\ &\quad + a_K[\rho a_E(E_0/K_0)^{-\rho-1}]^2[a_K + a_E(E_0/K_0)^{-\rho}]^{-3} \end{aligned} \tag{16}$$

When the values of b_0, b_1, and b_2 follow equation (16), the FES reduces to the CES at the local point (E_0, K_0). As production moves away from the initial position (E_0, K_0), however, the FES will diverge from the CES. The values of b_0, b_1, and b_2 will also change as the 'initial' position (E_0, K_0) changes and furthermore, the value of the substitution parameter ρ may also change[11].

4. Comparative statics of an FES production function

Assume that a producer maximizes the value of the output from (1) subject to a total cost constraint (2) and also the set of technological constraints as specified by (3) and (5) or (10) and (11) but now taking on a more general form:

$$g_i(K,E) \geq 0; \quad i = 1,...,n. \tag{17}$$

We have the following optimization problem for the producer:

$$\text{Max } L = f(K,E) + \lambda(C - P_E E - P_K K) + \sum_i \mu_i g_i(K,E) \tag{18}$$

where L is the Lagrange function, λ and μ_ι's are the Lagrange multipliers associated with the 'economic' and 'technological' constraints respectively. The first-order conditions for optimality are[12]:

$$\frac{X_j}{Y}\frac{\partial Y}{\partial X_j} = \lambda \frac{X_j}{Y}[P_j + \pi_j]; \quad j = K, E \tag{19}$$

where:

$$\pi_j = -\sum_i \frac{\mu_i}{\lambda}\frac{\partial g_i}{\partial X_j}; \quad j = K, E; \quad i = 1,..,n \tag{20}$$

π_j can be interpreted as the '*technological* shadow price' of input j arising from the existence of various technological constraints acting on j, while P_j is the usual '*economic* price'. The *total* or *effective* price for each input is then equal to ($P_j + \pi_j$). With a CES production function, it is easily seen that constraint (3) in fact implies:

---

10 It can be shown that if ρ >0 (elasticity of substitution between K and E is less than 1), then b_1>0, and b_2<0, i.e. ε_K is monotonically increasing with the level of (E/K). When ρ <0, the opposite will hold true (ε_K is monotonically decreasing with (E/K)). In general, therefore, non-monotonicity of ε_K implies non-constancy of the elasticity of substitution.

11 See the next section on how to estimate this change.

12 For simplicity of notation, we also use the symbol Xj, j=K, E to denote the quantities of the inputs K and E respectively.

$$\varepsilon_K = \frac{K}{Y}\frac{\partial Y}{\partial K} = a_K[a_K + a_E(E/K)^{-\rho}]^{-1} = \frac{a_K K^{-\rho}}{a_K K^{-\rho} + a_E E^{-\rho}} = S_K \qquad (21)$$

Therefore, comparing equation (21) with (19) gives the result: $\lambda^{CES} = 1$, and $\pi_j^{CES} = 0$, since $S_K = (KP_K)/Y$. Thus, by restricting the output elasticity of each input to be equal to just its (economic) cost share (equations (3) and (5)) the CES production function in fact implicitly assumes that all technological constraints are non-binding ($\pi_j^{CES} = 0$). This is consistent with the traditional assumption in neoclassical approach that all points along a production isoquant are technologically feasible and equally efficient (from a technological viewpoint). The only remaining issue to be determined, therefore, is economic efficiency, i.e. how to choose an input combination such that it minimizes the overall *economic* cost of production. The issue of technological efficiency or inefficiency is not explicitly considered or in fact implicitly assumed as being part of the economic efficiency concept. In the new FES production function approach, however, the two concepts of efficiencies are to be considered separately even though still jointly. The 'total price' for each of the production input, therefore, will now consist of two separate components, a resource opportunity cost (represented by the market price P_j), and a technological shadow price (π_j) which stands for the technical efficiency (or inefficiency, depending on the sign of π_j) associated with the use of input j. For example, if $\pi_E < 0$ and $\pi_K > 0$, we can take this to mean technologically it would be more efficient to use more capital and less energy than is currently the case because the marginal productivity of capital is above its market price and the opposite applies to energy. In the traditional approach, this *asymmetry* in the direction of substitution between the inputs will not appear because technological shadow prices are moved to the left hand sides of equations (19) and 'merged' with the (total) output elasticity (or marginal productivity) terms. There are advantages in keeping technological efficiency and economic efficiency separate because when a firm responds to an economic policy (which affects relative input or output prices) it is trying to improve on economic efficiency but cannot always in a position to affect technical efficiency. In fact, in some circumstances, economic efficiency may even require a decrease in technical efficiency rather than an improvement[13], therefore, the two concepts of efficiencies are not always in harmony but can go in apposite directions.

To determine the value of π_j, we note that from equation (19), we can write:

$$\left.\frac{X_j}{Y}\frac{\partial Y}{\partial X_j}\right|_{FES} - \left.\frac{X_j}{Y}\frac{\partial Y}{\partial X_j}\right|_{CES} = (\lambda^{FES} - \lambda^{CES})\frac{P_j X_j}{Y} + \lambda^{FES}\frac{\pi_j X_j}{Y}; \quad j = K, E \qquad (22)$$

where λ^{FES}. and λ^{CES} are the values of λ when CES and FES production functions are used respectively in problem (18). Using equations (3), (5), (10), (11), and after rearranging terms, we have:

$$\begin{aligned} \pi_K &= \frac{Y}{\lambda^{FES}K}\left[b_0 + b_1(\frac{E}{K} - \frac{E_0}{K_0}) + b_2(\frac{E}{K} - \frac{E_0}{K_0})^2 - \frac{a_K}{a_K + a_E(E/K)^{-\rho}}\right] + (\frac{\lambda^{CES}}{\lambda^{FES}} - 1)P_K \\ \pi_E &= \frac{-Y}{\lambda^{FES}E}\left[b_0 + b_1(\frac{E}{K} - \frac{E_0}{K_0}) + b_2(\frac{E}{K} - \frac{E_0}{K_0})^2 - \frac{a_K}{a_K + a_E(E/K)^{-\rho}}\right] + (\frac{\lambda^{CES}}{\lambda^{FES}} - 1)P_E \end{aligned} \qquad (23)$$

[13] For example, when a firm responds to a *carbon* tax on energy use by resorting to a method of production which is less carbon-intensive, the method may result in an improvement on economic efficiency (i.e. reduce the total cost of production) but this can result in, not an increase in technical efficiency but rather a decrease. Zvolinschi *et al.* (2002), for example, showed that the exergy efficiency of a power plant that uses hydrogen (converted from natural gas) as fuel to reduce carbon emissions is around 44.9 % which is actually lower than that of a conventional natural gas fired power plant which is about 55.4 %. The former method of production is therefore less efficient technically, even though from an economic viewpoint it is more efficient.

Equations (22)-(23) can be interpreted as follows: if the production function is truly of the CES form, and therefore output elasticities follow the specifications (3) and (5), then π_j=0 for all *j*'s, and the right hand sides of both equations (22) and (23) disappear. If, however, the underlying production function is closer to an FES rather than a CES form, then the difference in specifications will show up in the values of π_j's which, from equation (23), can be estimated using the observed values of (P_E, P_K , *Y*, *E*, *K*) and the shadow values of λ^{CES} and λ^{FES}. To determine the latter, we note that summing both sides of equation (19), using equation (4), and assuming a CES production function, we have:

$$1 = \lambda^{CES}(\sum_j P_j X_j)/Y = \lambda^{CES}(C/Y) \tag{24}$$

This gives $\lambda^{CES} = Y/C = (1/P_Y)$ where P_Y is the unit price of *Y*. For simplicity and without loss of generality, we can assume P_Y, and hence λ^{CES}, is normalized to 1 for a particular base year . If a FES production function is used instead of CES in problem (18), then equation (24) takes an alternative form:

$$1 = \frac{\lambda^{FES}}{Y}[C + \sum_j \pi_j X_j] = \frac{\lambda^{FES}}{\lambda^{CES}}[1 + (\sum_j \pi_j X_j)/C]$$

or

$$\frac{\lambda^{FES}}{\lambda^{CES}} = \frac{1}{[1 + (\sum_j \pi_j X_j)/C]} \tag{25}$$

Equation (25) says that, if technological constraints on the inputs result in a net positive shadow cost, i.e. $(\Sigma_j \pi_j X_j) > 0$, then $\lambda^{FES} < \lambda^{CES}$. In this case, a CES production function will tend to underestimate the true cost of the output (the true value of the inputs). This is because market prices or resource opportunity costs (as measured by the term $\Sigma_j P_j X_j$) are not sufficient to account for the full productivities of all the inputs. The opposite situation occurs when $(\Sigma_j \pi_j X_j) < 0$, which implies marginal productivities of the inputs are above the market prices. In this case, a CES production function will overestimate the true cost of the output (the true value of the inputs). The 'neutral' position is when $(\Sigma_j \pi_j X_j) = 0$ and in this case, even though some divergences may still exist between a CES and FES production function with respect to individual input prices (π_j's may still be non-zeros), overall, the total cost estimates are consistent between the two specifications. In this special case (when $\Sigma_j \pi_j X_j$=0, and $\lambda^{FES} = \lambda^{CES}$), equation (23) reduces to a simpler form:

$$\pi_K = \frac{C}{K}\left[b_0 + b_1(\frac{E}{K} - \frac{E_0}{K_0}) + b_2(\frac{E}{K} - \frac{E_0}{K_0})^2 - \frac{a_K}{a_K + a_E(E/K)^{-\rho}}\right]$$
$$\pi_E = \frac{-K}{E}\pi_K \tag{26}$$

The shadow prices π_K and π_E in this case always have opposite signs which indicates *relative* rather than *absolute* technical (in)efficiencies between the inputs.[14]

[14]. When π_K and π_E are of opposite signs, we can refer to this as a situation of *relative* technical efficiency (e.g. it is more efficient to substitute energy for capital rather than the reverse when π_K>0 and π_E<0). By nature, technical efficiency is defined and measured only in physical or quantity terms, e.g. the amount of energy (or exergy) inputs which can be converted into useful physical work; both inputs and outputs are measured in physical terms. The shadow prices π_j's, however, are measured in value

5. Application of the FES production function

Empirically, the values of the FES production function parameters b_0, b_1, and b_2 can be estimated directly from (10)-(11), or from (13) and then compared with the values obtained from (16) (which are based on the alternative assumption of a CES function with substitution parameter ρ). This will allow us to estimate empirically how significantly different FES production function is from CES production function. In this paper, however, we follow a simpler approach to demonstrate the significant difference of the two functions. First, we assume a CES production function specification in a typical economic model which is often used to estimate the economic costs of climate policy. Next, we replace the CES with a FES production function specification to see how far the results can differ. Since the FES production function includes the CES production function as a special case (see equation (16)), at the start of each period, in order to give both functions the same starting point, we use equation (16) to ‚calibrate' the parameters of the FES function as though it is a CES function (at the initial position only). As the experiment progresses, however, the decision (on input substitution) in each sector of the economy will alter the energy-to-capital input ratio, and this will change the parameters of the FES function making it different from the CES function (even though at the initial position they are assumed to be the same). As production moves away from the initial point, input substitution will also alter the value of the *elasticity of substitution.* This is captured only by the FES production function specification[15] (whereas the CES production function simply assumes that the value of the elasticity of substiuttion is to remain unchanged).

5.1 Experiment

We use a computable general equilibrium (CGE) model to implement the specification of a FES production function and the economic experiment consists of a simple but typical climate policy simulation exercise[16]. For the purpose of our exercise, we assume that the climate policy objective is to reduce the world CO2 emission from now until the end of the century to restrict the rise of total CO2–equivalent concentration of all greenhouse gases (GHGs)17 in the atmosphere by the year 2100 to a level of around 550 parts per million in volume (ppmv)18. Using a climate model[19], we can estimate the pattern of CO2 emissions for the world as a whole from now until the year 2100 to achieve this particular climate policy objective (see Figure B4, Appendix B). This is then compared to the emissions pattern of a 'reference' or 'Business-as-Usual' (BaU) scenario where there is no climate policy (Figure B3, Appendix B). To reduce CO2 emissions from the levels of the BaU scenario to the level of the climate policy scenario requires the imposition of either a uniform carbon tax (CTAX) or a CO2 emissions permit trading scheme which involves all countries of the world. The unique carbon tax, or equilibrium price of the emissions permits trading scheme, helps to define the (minimum) economic cost of achieving a particular climate policy objective. We use the CGE model to estimate this minimum price, which also stands for marginal CO2 abatement cost (MAC) in all economic activities in the world economy.

(economic) terms. Therefore, there must be a reference point for 'calibrating' or converting the physical concept of technical efficiency into value terms, and a restriction such as $\Sigma_j \pi_j X_j = 0$ (or a constant) provides such a useful reference point..

[15].See Appendix A for a method of estimating this change.

[16].The CGE model is called GTAP-E (see Burniaux and Truong, 2002; Truong *et al.* , 2007) and the climate policy exercise is similar to those reported in Kemfert *et al.* (2006a, 2006b); Kemfert and Truong (2007).

[17] Since we are looking only at CO_2 emissions in this paper, the emissions of other non-CO_2 GHGs are simply assumed to be exogenous and taken from other studies.

[18].This is roughly equivalent to a rise in total radiative forcing of all GHGs to 4.6 W/m^2 , a rise in global annual mean temperature of 2.3 degrees centigrade, and a global mean sea-level rise of about 0.38 metre, according to the ICLIPPS climate model (Tóth *et al.*, 2003; Brückner *et al.*, 2003).

[19] ICLIPPS climate model (see Tóth *et al.*, 2003; Brückner *et al.*, 2003).

5.2 Results

Details regarding the ‚Business-as-Usual' (BaU) and ‚Policy' scenarios in terms of the assumptions regarding per capita GDP growth rates, population growth rates (the main drivers of CO2 emissions)[20] are given in Appendix B (Figures B1 and B2). Figures B3 and B4 shows the levels of CO2 emissions for the BaU and Policy scenarios respectively. Figures B5 and B6 show the changes in energy efficiency index (EEI) (defined as the rate of output growth minus the rate of aggregate energy input growth in production activities) in BaU and Policy scenarios respectively. For the BaU scenario, changes in the EEI can be regarded as ‚autonomous' in the sense that they are not induced by any policy which alters the relative price structure of energy inputs. For the Policy scenario, however, part of the changes in EEI is due to climate policy, hence they cannot be regarded as ‚autonomous', but rather ‚induced' by relative price structure changes. In general, climate policy will tend to induce positive changes (i.e. improvements) in EEI, as can be seen from Figure B6. In Figure B7, we show the different values of EEIs using the FES rather than CES production function specification. In general, the FES will tend to give a slightly smaller value of EEI than the CES specification, especially towards the later periods[21], but this difference is not great. E

Figure B8 gives the estimates of marginal economic cost of climate policy (in terms of the equilibrium price of carbon emissions) for both the CES and FES specifications. It can be seen that in general, the CES can give a much higher estimate of the MAC than the FES, in most cases, nearly twice as much. In measuring the MAC for the FES production function specification, we can make several assumptions. In case (a), we assume the special restriction on technological shadow prices (equation (26)) which implies total shadow costs are zero ($\Sigma_j \pi_j X_j$=0) even though individual shadow prices may be non-zero. This will allow the FES and CES to give exactly the same estimate of the *total* input costs of the inputs – and hence total market costs will also stand for the total opportunity costs. In this case, the discrepancy between CES and FES estimates of the MAC will tend to be the smallest. However, even in this case, as Figure B8 shows, the CES estimate still range about twice the magnitude of the FES estimate. In case (b), we relax this assumption and allow the shadow technological prices to be estimated by the general equation (23) rather than equation (26). However, since we start each period of simulation with the CES and FES functions beginning at the same initial position and with the same value of the elasticity of substitution, this implies the divergence between the CES and FES production functions is still kept to a minimal level. Hence the results of case (b) does not differ much from those of case (a), as can be seen from Figure B8. In case (c), we now assume that the shadow technological prices (π_j's) can be included or 'absorbed' into the actual market prices (P_j's) by assuming that government can levy a tax (or give a subsidy) to the (energy and capital) inputs such that these will equal exactly to the values of the shadow technological prices. This will tend to magnify the difference between the energy and capital input prices further, and hence inducing more changes to the production structure. This will mean the gap between FES and CES results will tend to diverge even further, and this is confirmed in Figure B8. Finally, up to now, we have assumed that the FES production structure is applied only to the electricity generation sector which is responsible for about 40 per cent of the total CO2 emissions of the world in 2005. In case (d), we now assume that the FES production structure is also applied to the transport sector (which is responsible for a further 16 per cent of the world CO2 emissions in 2005). The results, as shown in Figure B8, is that the FES estimate of the MAC is now less than half of the estimate by the CES production function. This shows that if the CES production function is used to estimate the economic cost of climate policy, this can result in more than twice the actual cost, due to the fact that the CES production function assumes a rather restrictive relationship between output elasticity of an input (such as energy)

[20] These assumptions are based on previous studies in this area. See for example, Clarke *et al.* (2007) for a survey.

[21] This can be explained by the smaller price increase for energy input under the FES as compared to the CES specification, hence the smaller ‚inducement'.

and its input level (equation (5)). By using a more general and flexible output elasticity function (equations (10) and (11)), and hence a more general and flexible production function (equation (13) or (14))), we can now give a more accurate estimate of this economic cost.

6. Conclusion

Although mainstream neoclassical economics is often regarded as being more 'optimistic' than ecological or biophysical approaches towards the issues of energy scarcity and environmental degradation, this optimism does not necessarily translate into a more 'optimistic' estimate of the economic cost of climate or environmental policies in general, as has been demonstrated in this paper. Using a conventional neoclassical production function specification such as the CES (constant elasticity of substitution) production function, can result in significant overestimation of the (marginal) economic cost of any economic or environmental policy. The underlying reason for this overestimation can be traced back to the fact that a CES production function ignores – or rather *implicitly* assumes – an underlying *technological* relationship between the inputs, or between the inputs and output, such that this is far more restrictive than the actual relationship, as has been demonstrated in this paper. Starting with a second-order Taylor series approximation to any general output elasticity function (which includes the CES function as a special case), we then derive a general production function form (equation (13) or (14)) which turns out to be more 'flexible' than the CES form. This is because, firstly, the new form can reduce to the CES form as a special case (see equation (16)). Secondly, even if we assume the new form to be identical to the CES function at the beginning, as production moves away from the initial position, there is scope for the parameters of the new function to be 'updated' at the final point with new information obtained from the changed input combination (see equation (A12) in Appendix A). Thus, the assumption of 'constant' elasticity of substitution can be relaxed, so as to allow for this parameter to be flexibly changed as production moves significantly away from the initial position. In other words, the assumption of constant elasticity of substitution can still be maintained 'locally' even if not globally.

Using the new 'flexible elasticity of substitution (FES) production function specification, we conducted some experiments to see the impact of using the new function on the estimation of economic costs of environmental policies. It turned out that the FES can give a much lower cost estimate than the CES function.[22] In a typical climate policy experiment as undertaken in this paper, we observe that the CES can give an estimate (of the marginal abatement cost of CO_2 emissions reduction) which is roughly twice the estimate using the FES production function. This has significant implications for the economic modelling of energy and environmental policy modelling.

Acknowledgements

This paper is a revised version of an earlier paper presented at the Western Economic Association International (WEAI) Conference held at the University of Hawaii in June 2008. Financial assistance from the Department of Economics of the University of Hawaii for the author to attend the Conference is greatly appreciated. Comments from the audience at the Conference and in particular of Jim Roumasset and Chris Wada has resulted in significant improvements to the quality of the paper. Any remaining errors, however, are sole responsibility of the author.

[22] This conclusion may depend on the starting value of the elasticity of substitution used in the simulation. In a sensitivity analysis, we found that the FES continues to give lower estimate of MAC than the CES unless the starting value of the elasticity of substitution is greater than 1.0 (which is regarded as an unrealistic value). The gap between the FES and CES estimates also narrows as the starting value of the elasticity of substitution approaches 1.0.

Appendix A

Microeconomics of CES and FES production functions

(A) Assuming a CES production function of the form:

$$Y = a_0[a_K K^{-\rho} + a_E E^{-\rho}]^{-1/\rho} \tag{A1}$$

The first order condition for producer optimization problem can be written as:

$$\frac{X_j}{Y}\frac{\partial Y}{\partial X_j} = \frac{a_j X_j^{-\rho}}{\sum_i a_i X_i^{-\rho}} = \lambda \frac{X_j P_j}{Y}; \quad j = K, E \tag{A2}$$

with $\lambda = Y/C$. From this, we derive (using the notation $d\ln x = (dx/x)$):

$$-\rho[d\ln X_j - \sum_i S_i d\ln X_i] = d\ln X_j + d\ln P_j - d\ln C; \quad j = K, E \tag{A3}$$

where

$$\begin{aligned} S_i &= \frac{a_i X_i^{-\rho}}{\sum_j a_j X_j^{-\rho}} = \frac{P_i X_i}{\sum_j P_j X_j}, \\ d\ln C &= d\ln(\sum_i P_i X_i) = \sum_i S_i d\ln X_i + \sum_i S_i d\ln P_i. \end{aligned} \tag{A4}$$

Substitute (A4) into (A3) and rearrange terms, we have:

$$d\ln X_j = \sum_i S_i d\ln X_i - \sigma[d\ln P_j - \sum_i S_i d\ln P_i]; \quad j = K, E \tag{A5}$$

where $\sigma = 1/(1+\rho)$ is the elasticity of substitution between K and E.

(B) If production is specified by an FES production function of the form:

$$Y = c_0 E\left[\frac{E}{K}\right]^{c_1} \exp\{c_2\left[\frac{E}{K} - \frac{E_0}{K_0}\right] + c_3\left[\left(\frac{E}{K}\right)^2 - \left(\frac{E_0}{K_0}\right)^2\right]\} \tag{A6}$$

then the following can be derived from (A6):

$$d\ln Y = S_K^* d\ln K + S_E^* d\ln E \tag{A7}$$

where

$$\begin{aligned} S_E^* &= [1 + c_1 + c_2(E/K) + 2c_3(E/K)^2] \\ S_K^* &= (1 - S_E^*) \end{aligned} \tag{A8}$$

The terms S_E^* and S_K^* are cost shares for energy and capital inputs respectively when an FES production function is used. These will not in general be equal to the cost shares S_E and S_K when a CES production function is used, and the difference is due to the presence of the shadow technological prices π_E and π_K. The first-order condition for producer cost minimization when an FES production function is used is given by:

$$\varepsilon_E \equiv \frac{E}{Y}\frac{\partial Y}{\partial E} = [1 + c_1 + c_2(E/K) + 2c_3(E/K)^2] = S_E^* = \lambda\frac{E(P_E + \pi_E)}{Y}, \quad \text{(A9)}$$

$$\varepsilon_K \equiv \frac{K}{Y}\frac{\partial Y}{\partial K} = [-c_1 - c_2(E/K) - 2c_3(E/K)^2] = S_K^* = \lambda\frac{K(P_K + \pi_K)}{Y}. \quad \text{(A10)}$$

From this we can derive:

$$\begin{aligned} A[d\ln E - d\ln K] &= d\ln(\lambda/Y) + d\ln E + d\ln(P_E + \pi_E) \\ B[d\ln E - d\ln K] &= d\ln(\lambda/Y) + d\ln K + d\ln(P_K + \pi_K) \end{aligned} \quad \text{(A11)}$$

or:

$$\begin{aligned} \sigma^* &= d\ln(E/K)/d\ln(P_K^*/P_E^*) \\ &= [(1 - A + B)]^{-1} \end{aligned} \quad \text{(A12)}$$

where σ^* is the elasticity of substitution between *K* and *E* when an FES production function is used, and where:

$$\begin{aligned} A &= [c_2(E/K) + 4c_3(E/K)^2]/S_E^* \\ B &= [-c_2(E/K) - 4c_3(E/K)^2]/S_K^* \\ P_E^* &= P_E + \pi_E; \quad P_K^* = P_K + \pi_K. \end{aligned} \quad \text{(A13)}$$

We note that the substitution elasticity in this case is, in general, not a constant, but varies with the input ratio (*E/K*). In the special case when $c_2=c_3=0$, or when c_2 and c_3 are very large relative to c_1, we have $\sigma^* = 1$ (Cobb-Douglas production function).

From the first-order conditions (A9) and (A10), we also have:

$$\lambda = Y/(EP_E^* + KP_K^*) \quad \text{(A14)}$$

from which we derive:

$$\begin{aligned} d\ln(\lambda/Y) &= -S_K^*(d\ln K + d\ln P_K^*) - S_E^*(d\ln E + d\ln P_E^*) \\ &= -d\ln P - d\ln Y \end{aligned} \quad \text{(A15)}$$

or:

$$d\ln\lambda = -d\ln P \quad \text{(A16)}$$

where:

$$d\ln P = S_K^* d\ln P_K^* + S_E^* d\ln P_E^* \quad \text{(A17)}$$

Substituting (A15) into (A11) and re-arranging terms, we have:

$$
\begin{aligned}
d\ln E &= d\ln Y - \eta_E[d\ln P_E^* - d\ln P_K^*] \\
d\ln K &= d\ln Y - \eta_K[d\ln P_K^* - d\ln P_E^*]
\end{aligned}
\qquad \text{(A18)}
$$

where η_E and η_k can be referred to as the relative price elasticities and are given by:

$$
\begin{aligned}
\eta_E &= (1 - S_E^*) + A\sigma \\
\eta_K &= (1 - S_K^*) - B\sigma
\end{aligned}
\qquad \text{(A19)}
$$

Appendix B

Details on the economic experiments and the results

Table B1: Definitions of countries and regions

Regions	Description
USA	United States of America
EU15	European Union 15
RUS	Russia
JPN	Japan
AUS	Australia
CHN	China
IND	India
RoW	Rest of the World

Table B2: Definitions of sectors

	Sectors
coa	Coal mining
oil	Crude oil
gas	Natural gas, gas manufacture and distribution
p_c	Petroleum and coal products
ely	Electricity
TRN	Transport
AGR	Primary Agriculture and Fishing
CRP	Chemical and rubber product
OMF	Other manufacturing
ROE	Rest of the economy

Figure B1: Per capita GDP growth rates (common to all scenarios)

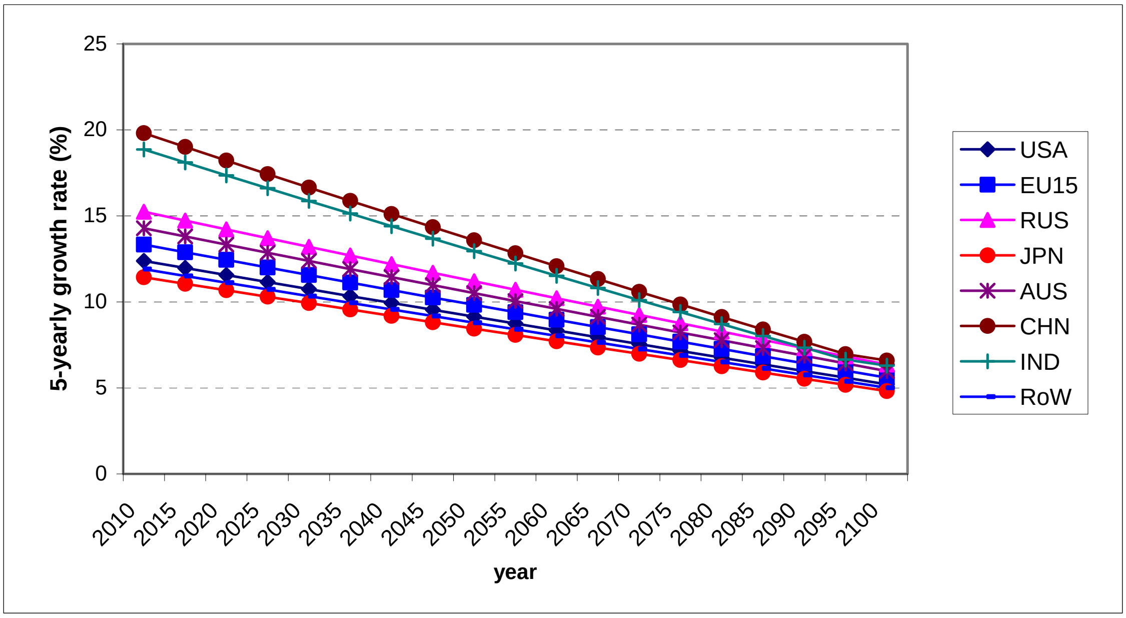


Figure B2: Population growth rates (common to all scenarios)

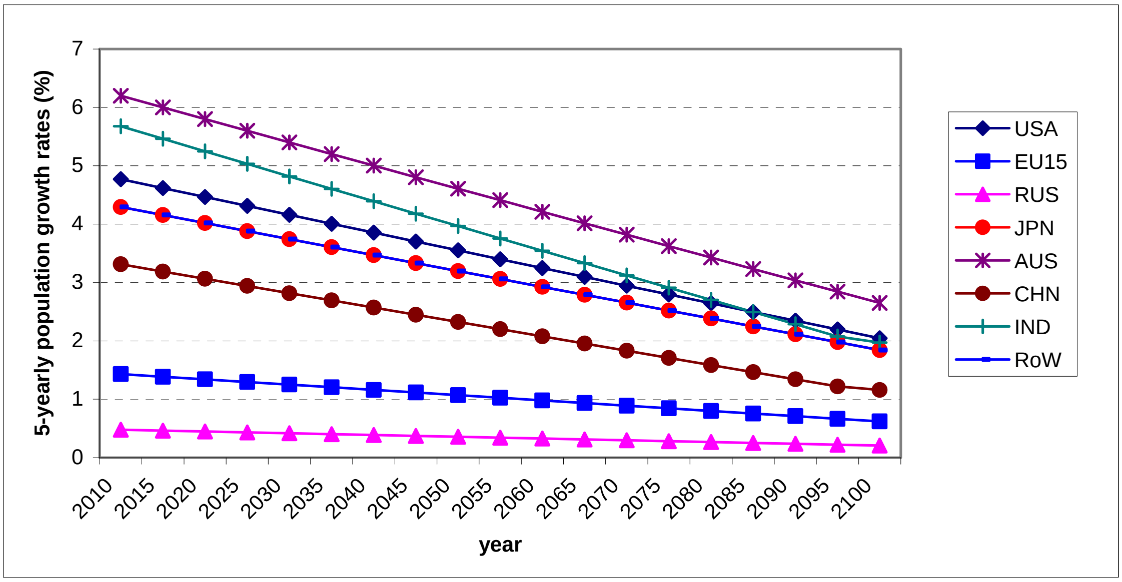


Figure B3: CO_2 emissions (Business-as-usual scenario)

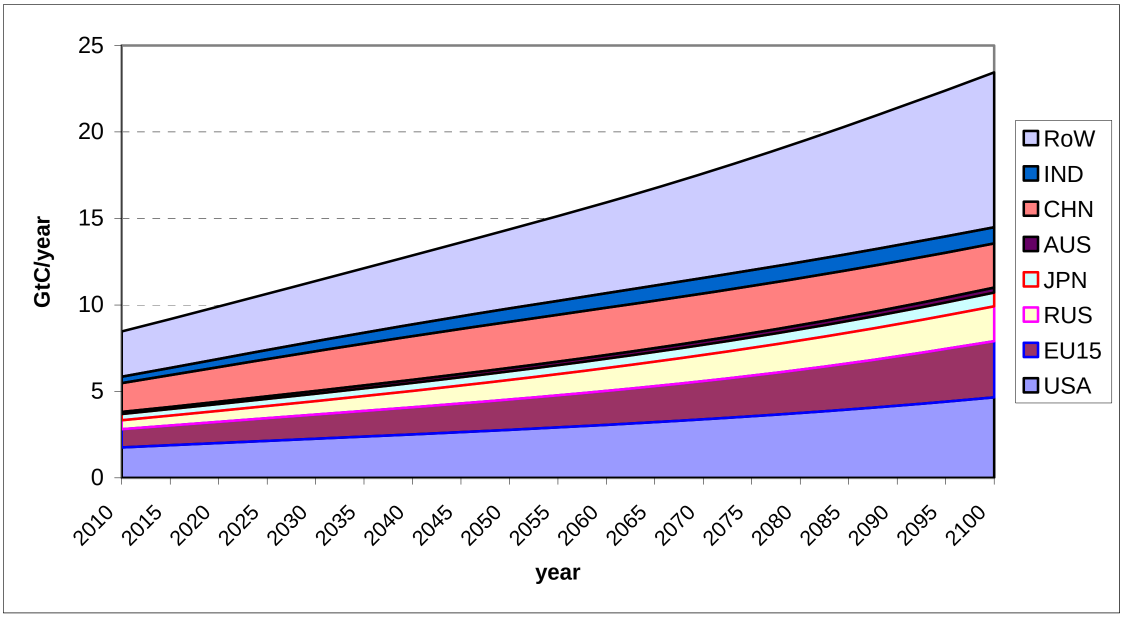


Figure B4: CO_2 emissions (Policy scenario)

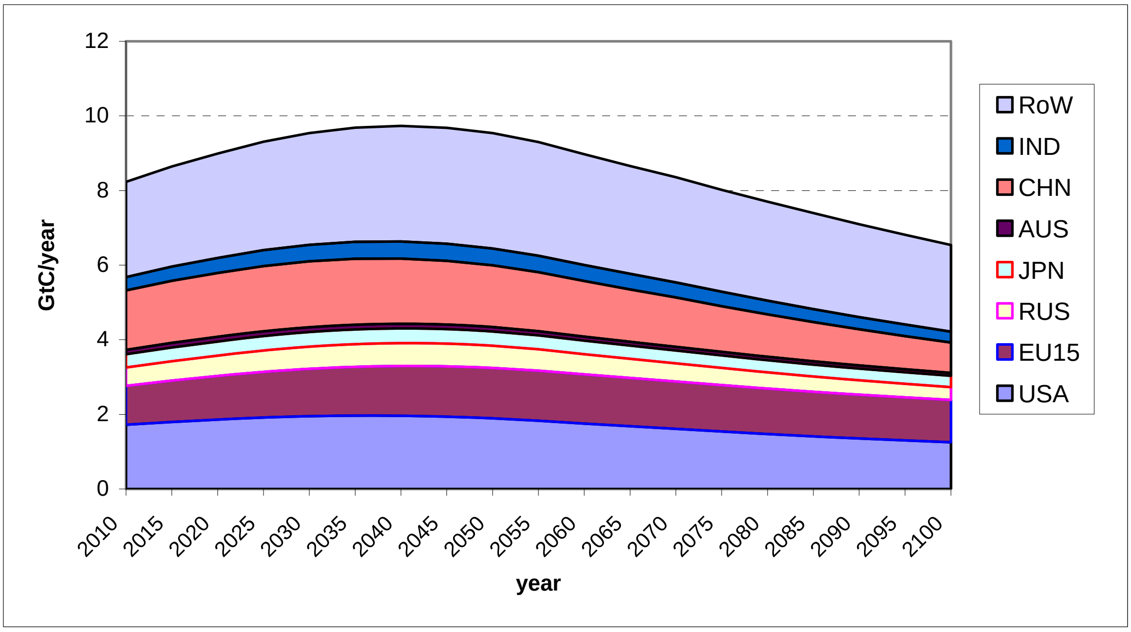


Figure B5: Energy efficiency index (output growth – energy input growth)

(business-as-usual scenario)

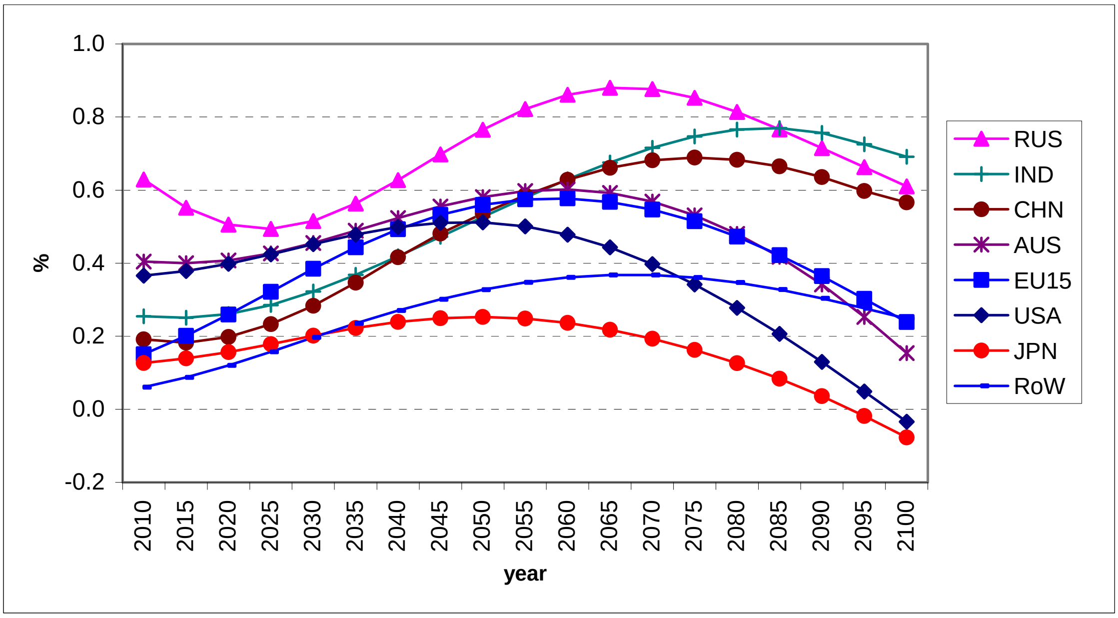


Figure B6: Energy efficiency index (output growth – energy input growth)

(policy scenario – using CES production function specification)

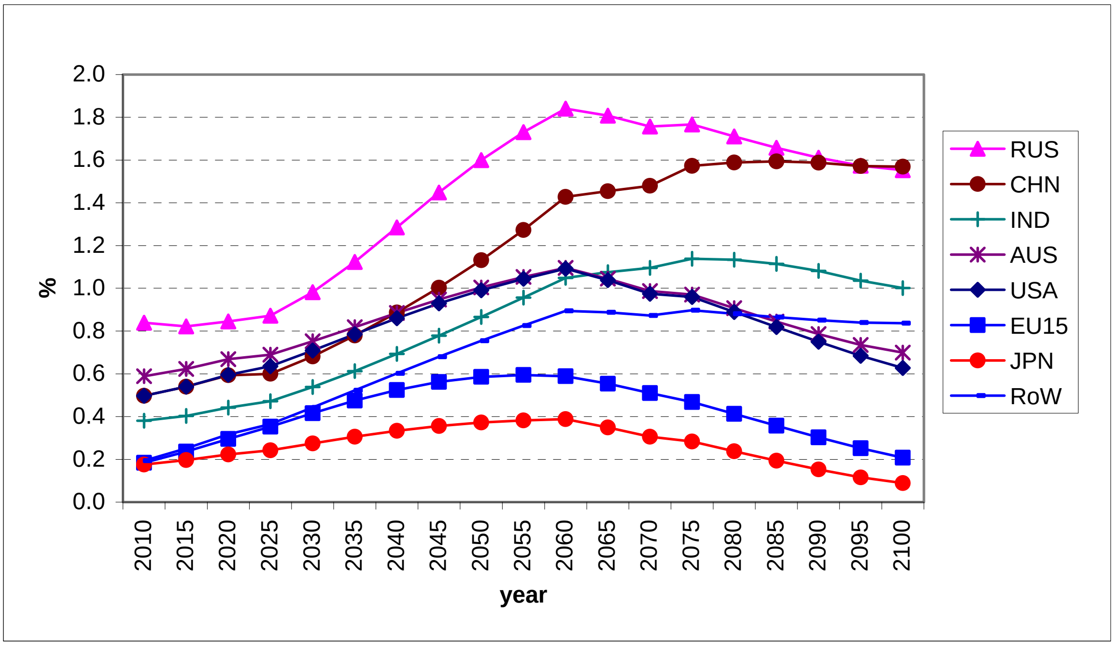


Figure B7: Energy efficiency index (output growth – energy input growth) (policy scenario – using FES production function specification)

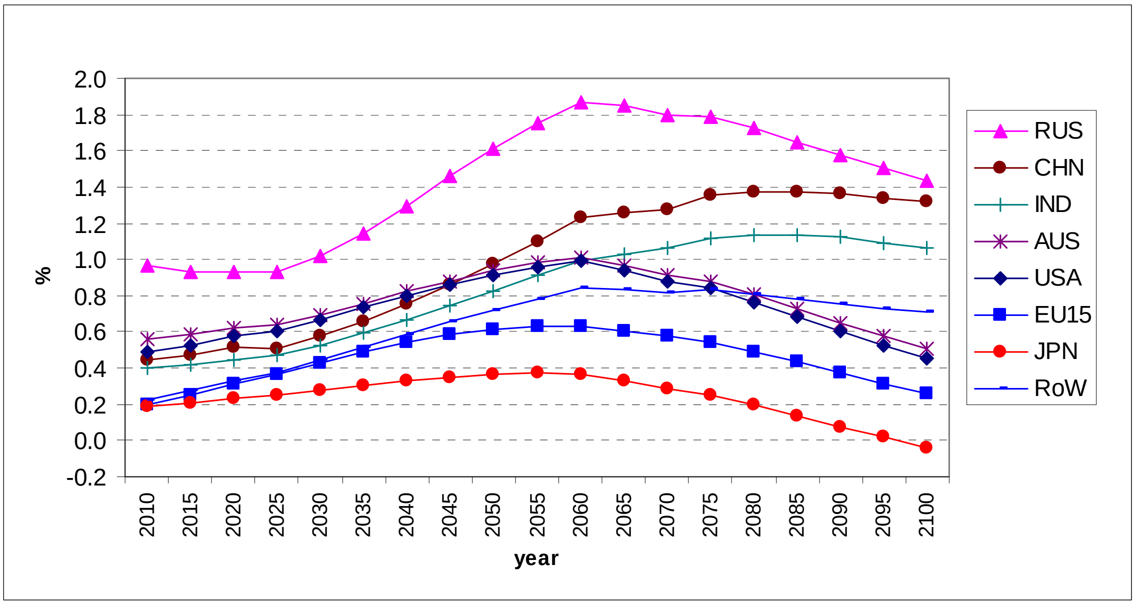
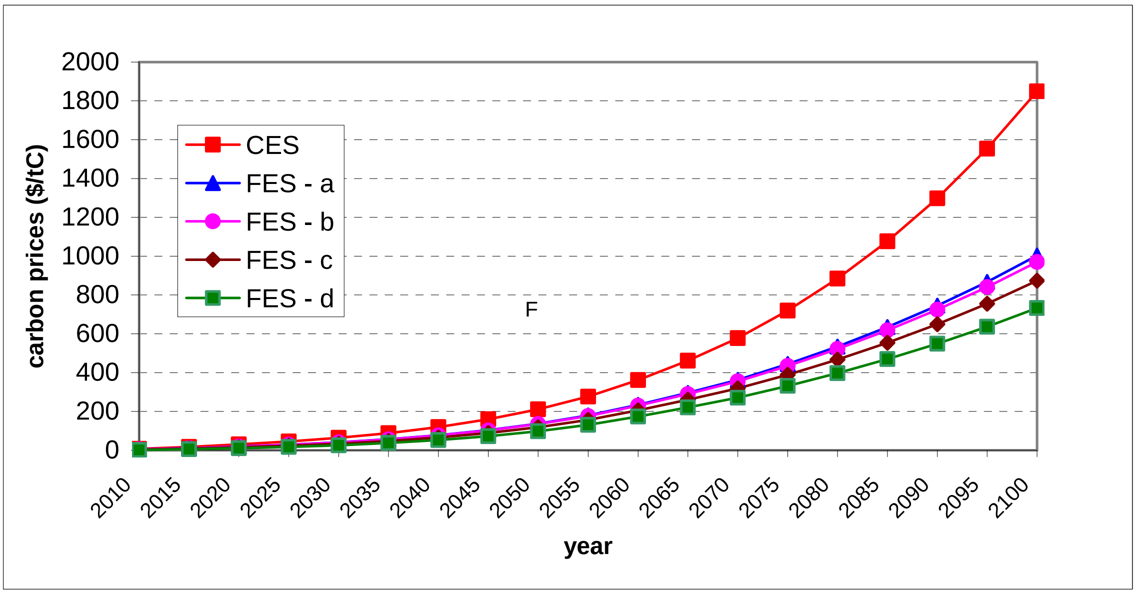


Figure B8: Marginal CO_2 emissions abatement cost or carbon prices (2005 US\$/tonne C) – (policy scenario)



(a) Using the special restriction on technological shadow prices of equation (26).

(b) Using the general specification of technological shadow prices in equation (23).

(c) Assuming shadow prices are ‚absorbed' into market prices via government taxes/subsidies equal to the values of the shadow prices.

(a) to (c): Assuming only the electricity generation sector (ely) to be specified by the FES production structure.

(d) Both the electricity sector and transport sector (TRN) are assumed to be specified by the FES production function structure.

Figure B9: Variation in elasticity of substitution between capital and energy (electricity sector - policy scenario)

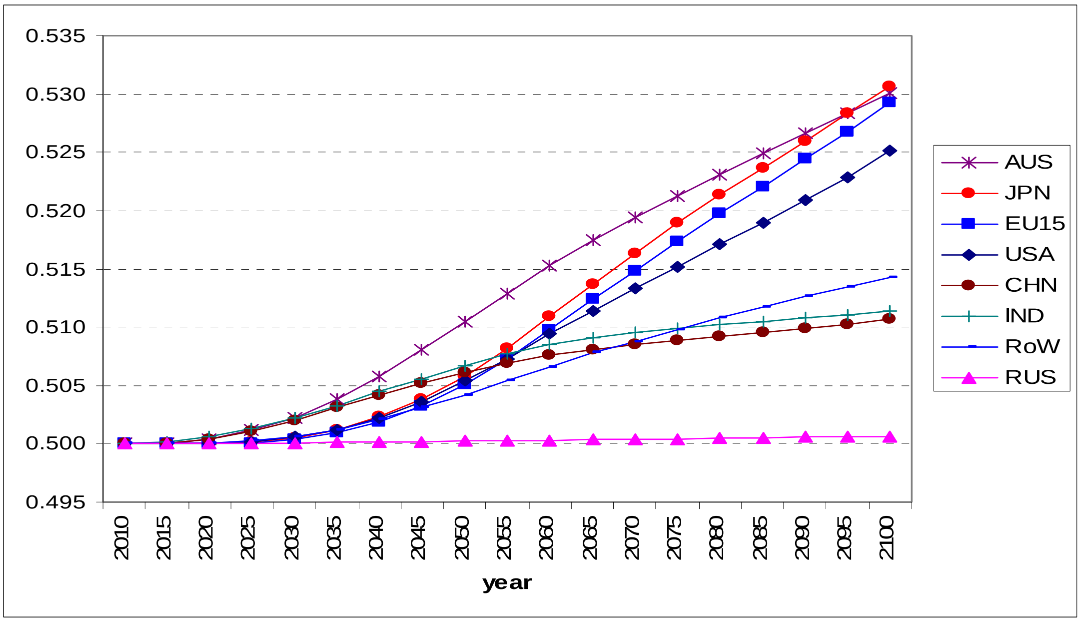
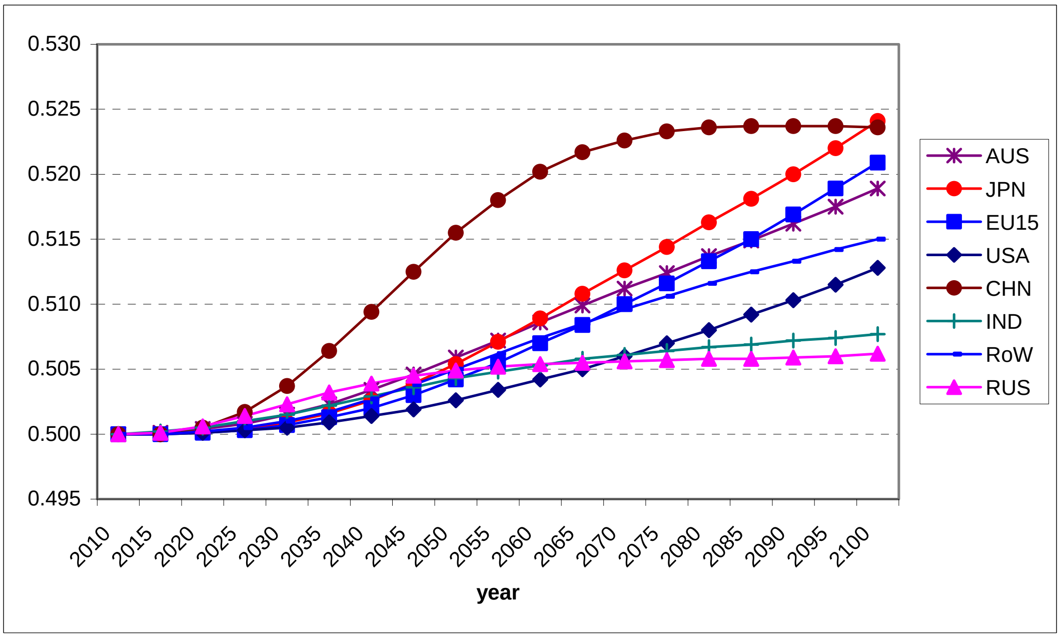


Figure B10: Variation in elasticity of substitution between capital and energy (transport sector - policy scenario)



References

Arrow, K.J., Chenery, H.B, Minhas, ., B.S., and Solow, R.M. (1961), Capital-labor substitution and economic efficiency. *Review of Economics and Statistic*s 43, pp. 225-250.

Ayres, R.U., and Warr, B. (2005).”Accounting for growth; the role of physical work”. *Structural Change and Economic Dynamics* 16 (2):181-209.

Brückner, T., Hooss, G., Fuessel, H. -M., Hasselmann, K. (2003). ‘Climate system modelling in the framework of the tolerable windows approach: The ICLIPS climate model’, *Climatic Change* 56, 119-137.

Burniaux, J. -M., Truong, T. P. (2002). GTAP-E: An Energy-Environmental Version of the GTAP Model, GTAP Technical Paper No. 16 (revised), Center for Global Trade Analysis, Purdue University, West Lafayette, Indiana, USA.

Clarke, L., Edmonds, J., Jacoby, H., Pitcher, H., Reilly, J., and Richels, R. (2007). Scenarios of Greenhouse Gas Emissions and Atmospheric Concentrations. U.S. Climate Change Science Program Synthesis and Assessment Product 2.1a, July.

Georgescu-Roegen, N. (1966). *Analytical Economics: Issues and Problems.* Cambridge, MA.: Harvard University Press.

Georgescu-Roegen, N. (1971). *The entropy law and the economic process*. Cambridge: MA.: Harvard University Press.

Hall, C., Lindenberger D., Kümmel R., Kroeger, T., and Eichhorn, W. (2001), “The need to reintegrate the natural sciences with economics”, *Bioscience* 51, pp. 663-673.

Harcourt, G. C. (1969) “Some Cambridge Controversies in the Theory of Capital”, *Journal of Economic Literature*, 7 (2), pp. 369-405.

Harcourt, G. C. (1972). *Some Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press.

Islam, S. (1985). “Effects of an essential input on isoquants and substitution elasticities.” *Energy Economics* 7, pp. 194-196.

Kemfert, C., Kohlhaas, M., Truong P. T., Protsenko, A. (2006a): “The Environmental and Economic Effects of European Emissions Trading”, *Climate Policy* 6(4), pp. 441-455.

Kemfert, C., Truong, P.T., Brucker, T. (2006b), “Economic Impact Assessment of Climate Change: A Multi-Gas Investigation”, *The Energy Journal*, Special Issue 3 on Multi-Greenhouse Gas Mitigation and Climate Policy, pp. 441-460.

Kemfert, C., Truong P. T. (2007), “Impact assessment of Emission stabilization scenarios with and without induced technological change”, *Energy Policy* 35(11), pp. 5337-5345.

Kümmel, R., Schmid, J., Ayres, R. U., and Lindenberger, D. (2008) “Cost Shares, Output Elasticities, and Substitutability Constraints”, Institute of Energy Economics at the University of Cologne, Working Paper EWI No 08.02.

Lindenberger, D., and Kümmel, R. (2002) “Energy-Dependent Production Functions and the Optimization Model “PRISE” of Price-Induced Sectoral Evolution”, *International Journal of Applied Thermodynamics*, 5(3), pp.101-107.

Samuelson, Paul A. (1951), “Abstract of a Theorem Concerning Substitutability in Open Leontief Models,” in T.C. Koopmans ed., *Activity Analysis of Production and Allocation,* New York: Wiley, 142-46.

Shaikh, A. (1974) "Laws of Production and Laws of Algebra: The Humbug Production Function", *The Review of Economics and Statistics*, 56(1), pp. 115-120.

Tóth, F., Brückner, T., Füssel, H.-M, Leimbach, M., & Petschel- Held, G. (2003). Integrated assessment of long-term climate policies: Part 1 – Model presentation. *Climatic Change* 56, 37–56.

Truong T. P., Kemfert, C., Burniaux, J.-M. (2007). 'GTAP-E: An Energy-Environmental Version of the GTAP Model with Emission Trading', DIW Diskussionspapiere 668, Berlin.

Zvolinschi, A., Kjelstrup, S., Bolland, O., and van der Kooi, H.J., *Including exergy analysis in industrial ecology: The case of combined cycle power plants with CO_2 capture,* Proceedings of ECOS 2002, July 3-5 2002, Berlin, Germany.