

I T L S

WORKING PAPER

ITLS-WP-09-02

Ordered choices and heterogeneity in attribute processing

By

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January 2009

ISSN 1832-570X

INSTITUTE of TRANSPORT and LOGISTICS STUDIES

The Australian Key Centre in Transport and Logistics Management

The University of Sydney *Established under the Australian Research Council's Key Centre Program.*

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ABSTRACT: A growing number of empirical studies involve the assessment of influences on a choice amongst ordered discrete alternatives. Ordered logit and probit models are well known, including extensions to accommodate random parameters and heteroscedasticity in unobserved variance. This paper extends the ordered choice random parameter model to permit random parameterization of thresholds and decomposition to establish observed sources of systematic variation in the threshold parameter distribution. We illustrate the empirical gains of this model over the traditional ordered choice model in the context of identifying candidate influences on the role that specific attributes play, in the sense of being ignored or not, in an individual's choice amongst unlabelled attribute packages of alternative tolled and non-tolled routes for the commuting trip. The empirical ordering represents the number of attributes attended to from the full fixed set. The evidence suggests that there is significant heterogeneity associated with the thresholds, that can be connected to systematic sources associated with the respondent (i.e., gender) and the choice experiment, and hence the generalized extension of the ordered choice model is an improvement, behaviourally, over the simpler model.

KEY WORDS: *Ordered choice, heterogeneous thresholds, random parameters, stated choice designs, information processing, ignoring attributes*

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DATE: January 2009

Acknowledgements Research funded under the Australian Research Council Large Grants Scheme, Grant A00103962 and Discover Program grant DP0770618 and the Behavioral Choice Group in the Faculty of Economics and Business at the University of Sydney. Discussions with Chandra Bhat are appreciated as is the support of Ken Train, and especially the detailed advice from a referee and Steven Morrison.

1. Introduction

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A growing number of empirical studies involve the assessment of influences on a choice amongst ordered discrete alternatives. Ordered logit and probit models are well known, including extensions to accommodate random parameters and heteroscedasticity in unobserved variance (see, e.g., Bhat and Pulugurtha 1998, Greene 2007). The ordered choice model allows for non-linear effects of any variable on the probabilities associated with each ordered level (see for example, Eluru et al., 2008). However the traditional ordered choice model is potentially limited, behaviorally, in that it holds the threshold values to be fixed. This can lead to inconsistent (i.e., incorrect) estimates of the effects of variables. Extending the ordered choice random parameter model to account for threshold random heterogeneity, as well as underlying systematic sources of explanation for unobserved heterogeneity, is a logical extension in line with the growing interest in choice analysis in establishing additional candidate sources of observed and unobserved taste heterogeneity¹.

A substantive application herein is used to illustrate the behavioral gains from generalizing the ordered choice model to accommodate random thresholds in the presence of random parameters. It is focused on the influences on the role that a specific attribute processing strategy, of preserving each attribute or ignoring it, plays when choosing amongst unlabelled attribute packages of alternative tolled and non-tolled routes for the commuting trip in a stated choice experiment (see Hensher et al. 2005a, Hensher 2006b, 2008). The ordering represents the number of attributes attended to from the full set. Despite a growing number of studies focusing on these issues (see for example Cantillo *et al.* 2006, Hensher 2006, Swait 2001, Campbell et al. 2008), the entire domain of every attribute is treated as relevant to some degree, and included in the utility expressions for every individual. While acknowledging the extensive study of nonlinearity in attribute specification, which permits varying marginal (dis)utility over an attribute's range, including account for asymmetric preferences under conditions of gain and loss (see Hess at al. 2008), this is not the same as establishing *ex ante* the extent to which a specific attribute might be totally excluded from consideration for all manner of reasons, including the influence of the design of a choice experiment when stated choice data is being used.

The paper is organised as follows. The next section sets out the econometric specification of the generalised ordered choice model, focusing on the derivation of the random threshold structure and its behavioral appeal. We then introduce the empirical context used to test this new model, focusing on the design of the stated choice experiment and associated questions used to define the choice setting and the process used by each respondent in establishing relevance of each attribute. The empirical analysis that follows presents the estimated models – a traditional model and the extended ordered choice model, together with the associated marginal effects that are the basis of behavioral assessment. The paper concludes with some observations on the merits of the extended model form.

 1 A number of authors have introduced random thresholds (e.g., Cameron and Heckman 1998, Cunha et al. 2007, Eluru et al. 2008) but have not integrated this into a generalized model with random parameters and/or decomposition of random thresholds by systematic sources.

2. **Generalizations of the ordered choice model to**

accommodate preference heterogeneity

2.1 The traditional ordered probit model

The ordered probit model was proposed by Zavoina and McElvey (1975) for the analysis of categorical, nonquantitative choices, outcomes and responses. Familiar applications now include bond ratings, discrete opinion surveys such as those on political questions, obesity measures (Greene et al. 2008), preferences in consumption, and satisfaction and health status surveys such as those analyzed by Boes and Winkelmann (2004, 2007).

The model foundation is an underlying random utility or latent regression model,

$$
y_i^* = \beta' \mathbf{x}_i + \varepsilon_i, \tag{1}
$$

in which the continuous latent utility, y^* is observed in discrete form through a censoring mechanism (equation 2).

$$
y_{i} = 0 \text{ if } \mu_{-1} < y_{i}^{*} < \mu_{0},
$$
\n
$$
= 1 \text{ if } \mu_{0} < y_{i}^{*} < \mu_{1},
$$
\n
$$
= 2 \text{ if } \mu_{1} < y_{i}^{*} < \mu_{2}
$$
\n
$$
= ...
$$
\n
$$
= J \text{ if } \mu_{J-1} < y_{i}^{*} < \mu_{J}.
$$
\n(2)

The model contains the unknown marginal utilities, β , as well as J+2 unknown threshold parameters, μ , all to be estimated using a sample of n observations, indexed by $i = 1,...,n$. The data consist of the covariates, xi and the observed discrete outcome, $yi = 0,1,...,J$. The assumption of the properties of the "disturbance," εi, completes the model specification. The conventional assumptions are that ϵi is a continuous disturbance with conventional cdf, $F(\epsilon i|x_i)$ $=$ F(εi) with support equal to the real line, and with density $f(\epsilon i) = F'(\epsilon i)$. The assumption of the distribution of εi includes independence from (or exogeneity of) xi. The probabilities associated with the observed outcomes are given as equation (3).

$$
Prob[y_i = j | \mathbf{x}_i] = Prob[\varepsilon_i < \mu_j - \beta' \mathbf{x}_i] - Prob[\mu_{j-1} - \beta' \mathbf{x}_i], j = 0, 1, \dots, J. \tag{3}
$$

Several normalizations are needed to identify the model parameters. First, given the continuity assumption, in order to preserve the positive signs of the probabilities, we require μ j > μ j-1. Second, if the support is to be the entire real line, then μ -1 = -∞ and $\mu J = +\infty$. Finally, assuming (as we will) that xi contains a constant term, we will require μ 0 = 0. With a constant term present, if this normalization is not imposed, then adding any nonzero constant to μ0 and the same constant to the intercept term in β will leave the probability unchanged. Given the assumption of an overall constant, only J-1 threshold parameters are needed to partition the real line into the J+1 distinct intervals.

Given that data such as ranking data defining the observed ordered choice contain no unconditional information on scaling of the underlying unobserved variable, if yi* is scaled by any positive value, then scaling the unknown μ and β by the same value preserves the observed outcomes; and hence a free unconditional variance parameter, $Var[*εi*] = σε2$, is not identified without further restriction. We thus impose the identifying restriction $\sigma \varepsilon = a$ known constant, σ. The usual approach to this normalization, assuming that ε is independent of x, is to assume that Var[εi|xi] = 1 in the probit model and $\pi/23$ in the logit model – in both cases to eliminate the free structural scaling parameter. The standard treatments in the received literature complete the ordered choice model by assuming either a standard normal distribution for εi, producing the ordered probit model or a standardized logistic distribution (mean zero, variance π2/3), which produces the ordered logit model. Applications appear to be well divided between the two. A compelling case for a particular distribution remains to be put forth.

With the full set of normalizations in place, the likelihood function for estimation of the model parameters is based on the implied probabilities given in equation (4).

$$
Prob[y_i = j | x_i] = F(μ_j - β' x_i) - F(μ_{j-1} - β' x_i) > 0, j = 0, 1, ..., J.
$$
\n(4)

Estimation of the parameters is a straightforward problem in maximum likelihood estimation (see, e.g., Greene 2008 and Pratt 1981). Interpretation of the model parameters is, however, much less so (see, e.g., Daykin and Moffitt 2002). There is no natural conditional mean function, so in order to attach behavioral meaning to the parameters, one typically refers to the probabilities themselves. The partial effects in the ordered choice model are:

$$
\frac{\partial \text{Prob}[y_i = j | \mathbf{x}_i]}{\partial \mathbf{x}_i} = \left[f(\mu_{j-1} - \boldsymbol{\beta}' \mathbf{x}_i) - f(\mu_j - \boldsymbol{\beta}' \mathbf{x}_i) \right] \boldsymbol{\beta} \tag{5}
$$

The result shows that neither the sign nor the magnitude of a coefficient is informative about the corresponding behavioral characteristic in the model, so the direct interpretation of the coefficients (or their "significance") is fundamentally ambiguous. A counterpart result for a dummy variable in the model would be obtained by using a difference of probabilities, rather than a derivative (Boes and Winkelmann 2007 and Greene 2008, Chapter E22). One might also be interested in cumulative values of the partial effects, such as shown in equation (6) (see, e.g., Brewer et al. 2006). The last term in this set is zero by construction.

$$
\frac{\partial \text{Prob}[y_i \leq j \mid \mathbf{x}_i]}{\partial \mathbf{x}_i} = \left(\sum_{m=0}^j \left[f(\mu_{m-1} - \boldsymbol{\beta}' \mathbf{x}_i) - f(\mu_m - \boldsymbol{\beta}' \mathbf{x}_i)\right]\right) \boldsymbol{\beta}
$$
\n(6)

2.2 A generalized ordered choice model

A number of authors, beginning with Terza (1985), have questioned some of the less flexible aspects of the model specification. The partial effects shown above vary with the data and the parameters. It can be shown that for the probit and logit models, this set of partial derivatives will change sign exactly once in the sequence from 0 to J, a property that Boes and Winkelmann (2007) label the "single crossing" characteristic. Boes and Winkelmann (2007) also note that for any two continuous covariates, xik and xi*l*

$$
\frac{\partial \text{Prob}[y_i = j | \mathbf{x}_i] / \partial x_{i,k}}{\partial \text{Prob}[y_i = j | \mathbf{x}_i] / \partial x_{i,l}} = \frac{\beta_k}{\beta_l}
$$
\n(7)

This result in (7) is independent of the outcomes. The ordered choice models above have the property in equation (8); that is, the partial effects are each a multiple of the same β.

$$
\partial \text{Prob}[y_i > j \mid \mathbf{x}_i] / \partial \mathbf{x}_i = K_i \beta \tag{8}
$$

where Kj depends on Xj. This is a feature of the model that has been labeled the "parallel regressions" assumption. Another way to view this feature of the ordered choice model is through the J implied binary choices implied by (8). Let zij denote the binary variable defined by

$$
z_{ij} = 1
$$
 if $y > j$, $j = 0, 1, \ldots, J-1$.

The choice model implies

$$
Prob[z_{ij} = 1 | x_i] = F(\beta' x_i - \mu_j).
$$

The threshold parameter can be absorbed into the constant term. In principle, one can fit these J-1 binary choice models separately. That the same β appears in all of the models is implied by the ordered choice model. However, one need not impose this restriction; the binary choice models can be fit separately and independently. Thus, the null hypothesis of the ordered choice model is that the βs in the binary choice equations are all the same (apart from the constant terms). A standard test of this null hypothesis, due to Brant (1990), is used to detect the condition that the βj vectors are different. The Brant test frequently rejects the null hypothesis of a common slope vector in the ordered choice model. It is unclear what the alternative hypothesis should be in this context. The generalized ordered choice model that might seem to be the natural alternative is, in fact, internally inconsistent – it does not constrain the probabilities of the outcomes to be positive. It would seem that the Brant test is more about functional form or, perhaps, some other specification error. See Greene and Hensher (2009, Chapter 6).

Recent analyses, e.g., Long (1993), Long and Frees (2005) and Williams (2006), have proposed a "generalized ordered choice model.

$$
Prob[y_i = j | \mathbf{x}_i] = Prob[\varepsilon_i < \mu_j - \beta_j' \mathbf{x}_i] - Prob[\mu_{j-1} - \beta_{j-1}' \mathbf{x}_i], j = 0, 1, ..., J \tag{9}
$$

where β -1 = 0 (see e.g., Williams 2006, Long 1997, Long and Frees 2006). The extension provides for a separate vector of marginal utilities for each jth outcome. Bhat and Zhao (2002) introduce heteroscedasticity across observational units, in a spatial ordered response analysis context, along the lines of the generalised ordered logit form.

The generalization of the model suggested above deals with both problems (single crossing and parallel regressions), but it creates new ones. The heterogeneity in the parameter vector is an artifact of the coding of the dependent variable, not a manifestation of underlying heterogeneity in the dependent variable induced by behavioral differences. It is unclear what it means for the marginal utility parameters to

[&]quot; An extended form of the ordered choice model that has attracted much (perhaps most) of the recent attention, is the "Generalized Ordered Logit" (or Probit) model e.g., by Williams (2006). This model is defined in equation (9).

be structured in this way. Consider, for example, that there is no underlying structure that could be written down in such a way as to provide a means of simulating the data generating mechanism. By implication, $y_i^* = \beta_i' \mathbf{x}_i + \varepsilon_i$ if $y_i = j$. That is, the model structure is endogenous – one could not simulate a value of y_i from the data generating mechanism without knowing in advance the value being simulated. There is no reduced form. The more difficult problem of this generalization is that the probabilities in this model need not be positive, and there is no parametric restriction (other than the restrictive model version we started with) that could achieve this. The probability model is internally inconsistent. The restrictions would have to be functions of the data. The problem is noted by Williams (2006), but dismissed as a minor issue. Boes and Winkelmann (2007) suggest that the problem could be handled through a "nonlinear specification." Essentially, this generalized choice model does not treat the outcome as a single choice, even though that is what it is.

To put a more positive view, we might interpret this as a semi-parametric approach to modeling what is underlying heterogeneity. However, it is not clear why this heterogeneity should be manifest in parameter variation across the outcomes instead of across the individuals in the sample. One would assume that the failure of the Brant test to support the model with parameter homogeneity is, indeed, signalling some failure of the model. A shortcoming of the functional form as listed above (compared to a different internally consistent specification) is certainly a possibility. We hypothesize that it might also be picking up unobserved heterogeneity across individuals. The model we develop here accounts for individual heterogeneity in several possible forms.

2.3 Modeling observed and unobserved heterogeneity

Since Terza (1985), with the exception of Pudney and Shields (2000), most of the "generalizations" suggested for the ordered choice models have been about functional form – the single crossing feature and the parallel regressions (see, also, Greene 2008). Our interest in this paper is, rather, in a specification that accommodates both observed and unobserved heterogeneity across individuals. We suggest that the basic model structure, when fully specified, provides for sufficient nonlinearity to capture the important features of choice behavior. The generalization that interests us herein will incorporate both observed and unobserved heterogeneity in the model itself.

The basic model assumes that the thresholds μ are the same for every individual in the sample. Terza (1985), Pudney and Shields (2000), Boes and Winkelmann (2007), Bhat and Pulugurta (1998), and Greene et al. (2008), all present cases that suggest individual variation in the set of thresholds is a degree of heterogeneity that is likely to be present in the data, but is not accommodated in the model. Pudney and Shields discuss a clear example in the context of job promotion, in which the steps on the promotion ladder for nurses are somewhat individual specific.

Greene (2002, 2008) argues that the fixed parameter version of the ordered choice model, and more generally, many microeconometric specifications, do not adequately account for the underlying, unobserved heterogeneity likely to be present in observed data. Further extensions of the ordered choice model presented in Greene (2008) include full random parameters treatments and discrete approximations under the form of latent class, or finite mixture models. These two specific extensions are also listed by Boes and Winkelmann (2004, 2007), who also describe a common effects model for panel data, and Bhat and Pulugurta (1998) as candidates for extending the model.

The model that assumes homogeneity of the preference parameters, β , across individuals, also assumes homogeneity in the scaling of the random term, εi. That is, the homoscedasticity assumption, $Var[\varepsilon x] = 1$ is restrictive in the same way that the homogeneity assumption is. Heteroscedasticity in terms of observables in the ordered choice model is proposed in Greene (1997) and reappears as a theme in Williams (2006).

The model proposed here generalizes the ordered choice model in the directions of accommodating heterogeneity, rather than in the direction of adding nonlinearities to the underlying functional form. The earliest extensions of the ordered choice model focused on the threshold parameters. Terza's (1985) extension suggested

$$
\mu_{ij} = \mu_j + \delta' \mathbf{z}_i. \tag{10}
$$

where zi are individual-specific exogenous variables that represent sources of systematic variation around the mean estimate of a threshold parameter. The analysis of this model continued with Pudney and Shields's (2000) "Generalized Ordered Probit Model," whose motivation, like Terza's was to accommodate observable individual heterogeneity in the threshold parameters as well as in the mean of the regression. We (and Pudney and Shields) note an obvious problem of identification in this specification. Consider the generic probability with this extension,

$$
Prob[y_i \leq j \mid \mathbf{x}_i, \mathbf{z}_i] = F(\mu_j + \delta' \mathbf{z}_i - \beta' \mathbf{x}_i) = F[\mu_j + (\delta^* \mathbf{z}_i + \beta' \mathbf{x}_i)], \delta^* = -\delta.
$$
 (11)

It is less than obvious whether the variables z_i are actually in the threshold or in the mean of the regression. Either interpretation is consistent with the model. Pudney and Shields argue that the distinction is of no substantive consequence for their analysis.

Formal modeling of heterogeneity in the parameters as representing a feature of the underlying data, also appears in Greene (2002) (version 8.0) and Boes and Winkelmann (2004), both of whom suggest a random parameters (RP) approach to the model. In Boes and Winkelmann, it is noted that the nature of an RP specification induces heteroscedasticity, and could be modeled as such. The model would appear as follows:

$$
\beta_i = \beta + \mathbf{u}_i \tag{12}
$$

where $\mathbf{u}_i \sim N[\mathbf{0}, \mathbf{\Omega}]$. Inserting this in the base case model and simplifying, we obtain equation (13).

$$
\text{Prob}[y_i \leq j \mid \mathbf{x}_i] = \text{Prob}[\varepsilon_i + \mathbf{u}_i' \mathbf{x}_i \leq \mu_j - \boldsymbol{\beta}' \mathbf{x}_i] = F\left(\frac{\mu_j - \boldsymbol{\beta}' \mathbf{x}_i}{\sqrt{1 + \mathbf{x}_i' \boldsymbol{\Omega} \mathbf{x}_i}}\right),\tag{13}
$$

Equation (13) could be estimated by ordinary means, albeit with a new source of nonlinearity – the elements of Ω must now be estimated as well². . Boes and Winkelmann (2004, 2007) did not pursue this approach. Greene (2002) analyzes essentially the same model, but proposes to estimate the parameters by maximum simulated likelihood.

Curiously, none of the studies listed above focus on the issue of scaling, although Williams (2006), citing Allison (1999) does mention it. A heteroscedastic ordered probit model with the functional form in (14) appears at length in Greene (1997), and is discussed in some detail in Williams (2006).

 $Var[\varepsilon_i|\mathbf{h}_i] = \exp(\gamma' \mathbf{h}_i)^2$

l

(14)

In microeconomic data, scaling of the underlying preferences is as important a source of heterogeneity as displacement of the mean, perhaps even more so. But, it has received considerably less attention than heterogeneity in location.

In what follows, we will propose a formulation of the ordered choice model that treats heterogeneity in a unified, internally consistent fashion. The model contains three points at which individual heterogeneity can substantively appear: in the random utility model (the marginal utilities), in the threshold parameters, and in the scaling (variance) of the random components. As argued above, this form of treatment seems more likely to capture the salient features of the data generating mechanism than the received "generalized ordered logit model," which is more narrowly focused on functional form.

2.4 Random thresholds and heterogeneity in the ordered choice model

We depart from the base case of the usual ordered choice model,

$$
Prob[y_i = j | \mathbf{x}_i] = F(\mu_j - \beta' \mathbf{x}_i) - F(\mu_{j-1} - \beta' \mathbf{x}_i) > 0, j = 0, 1, ..., J.
$$
 (15)

In order to model heterogeneity in the utility functions across individuals, we construct a hierarchical model in which the coefficients vary with observable variables, z_i (typically such as demographics like age and gender), and randomly due to individual specific unobservables, v_i . The coefficients appear as:

$$
\beta_i = \beta + \Delta z_i + \Gamma v_i \tag{16}
$$

where Γ is a lower triangular matrix and $\mathbf{v}_i \sim N[\mathbf{0}, \mathbf{I}]$. The coefficient vector in the utility function, β*i* is normally distributed across individuals with conditional mean

$$
E[\beta_i|\mathbf{x}_i,\mathbf{z}_i] = \beta + \Delta \mathbf{z}_i \tag{17}
$$

² The authors' suggestion that this could be handled semiparametrically without specifying a distribution for **u***i* is incorrect, because the resulting heteroscedastic probability written above only preserves the standard normal form assumed if **u***i* is normally distributed as well as ε*i*

and conditional variance

$$
Var[\beta_i|x_i, z_i] = \Gamma I \Gamma' = \Omega.
$$
\n(18)

The model is formulated with Γv_i rather than, say just v_i with covariance matrix Ω purely for convenience in setting up the estimation method. This is a random parameters formulation that appears elsewhere, e.g., Greene (2002, 2005). The random effects model is a special case in which only the constant is random. The Mundlak (1978) and Chamberlain (1980) approach to modeling fixed effects is also accommodated by letting $z_i = \bar{x}$ in the equation for the overall constant term.

We are also interested in allowing the thresholds to vary across individuals. See, for example, King et al. (2004) for a striking demonstration of the payoff to this generalisation. The thresholds are modeled randomly and nonlinearly as

$$
\mu_{ij} = \mu_{i,j-1} + \exp(\alpha_j + \delta' \mathbf{r}_i + \sigma_j w_{ij}), \quad w_{ij} \sim \mathcal{N}[0,1] \tag{19}
$$

with normalizations and restrictions $\mu_1 = -\infty$, $\mu_0 = 0$, $\mu_J = +\infty$. For the remaining thresholds, we have (20).

$$
\mu_1 = \exp(\alpha_1 + \delta' \mathbf{r}_i + \sigma_1 w_{j1})
$$
\n
$$
= \exp(\delta' \mathbf{r}_i) \exp(\alpha_1 + \sigma_1 w_{j1})
$$
\n
$$
\mu_2 = \exp(\delta' \mathbf{r}_i) [\exp(\alpha_1 + \sigma_1 w_{j1}) + \exp(\alpha_2 + \sigma_2 w_{j2})],
$$
\n
$$
\mu_j = \exp(\delta' \mathbf{r}_i) (\sum_{m=1}^j \exp(\alpha_m + \sigma_m w_{im})) , j = 1, ..., J-1
$$
\n
$$
\mu_j = +\infty.
$$
\n(20)

Though it is relatively complex, this formulation is necessary for several reasons: (1) It ensures that all of the thresholds are positive. (2) It preserves the ordering of the thresholds. (3) It incorporates the necessary normalizations. Most importantly, it also allows observed variables and unobserved heterogeneity to play a role both in the utility function and in the thresholds. The thresholds, like the regression itself, are shifted by both observable (**r***i*) and unobservable (*wij*) heterogeneity. The model is fully consistent, in that the probabilities are all positive and sum to one by construction. If $\delta = 0$ and $\sigma_i =$ 0, then the original model is returned, with $\mu_1 = \exp(\alpha_1)$, $\mu_2 = \mu_1 + \exp(\alpha_2)$ and so on. Note that if the threshold parameters were specified as linear functions rather than as in (19), then it would not be possible to identify separate parameters in the regression function and in the threshold functions.

Finally, we allow for individual heterogeneity in the variance of the utility function as well as in the mean. This is likely to be an important feature of data on individual behaviour. The disturbance variance is allowed to be heteroscedastic, now specified randomly as well as deterministically. Thus,

$$
Var[\varepsilon_i|\mathbf{h}_i,e_i] = \sigma_i^2 = \exp(\gamma' \mathbf{h}_i + \tau e_i)^2
$$
\n(21)

where $e_i \sim N[0,1]$. Let $\mathbf{v}_i = (v_{i1},...,v_{iK})'$ and $\mathbf{w}_i = (w_{i1},...,w_{iJ-1})'$.

Combining all terms, the conditional probability of outcome *j* is

$$
\text{Prob}[y_i = j \mid \mathbf{x}_i, \mathbf{z}_i, \mathbf{h}_i, \mathbf{r}_i, \mathbf{v}_i, \mathbf{w}_i, \mathbf{e}_i] = \begin{bmatrix} F \left[\frac{\mu_{ij} - \beta_i' \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] - F \left[\frac{\mu_{i,j-1} - \beta_i' \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] \\ \end{bmatrix}, \tag{22}
$$

where it is noted, once again, both μij and βi vary with observed variables and with unobserved random terms. The log likelihood is constructed from the terms in (22). However, the probability in (22) contains the unobserved random terms, vi, wi and ei. The term that enters the log likelihood function for estimation purposes must be unconditioned on the unobservables. Thus, they are integrated out, to obtain the unconditional probabilities,

$$
\text{Prob}[y_i = j \mid \mathbf{x}_i, \mathbf{z}_i, \mathbf{h}_i, \mathbf{r}_i] = \int_{\mathbf{v}_i, \mathbf{w}_i, e_i} \left(F \left[\frac{\mu_{ij} - \beta_i' \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] - F \left[\frac{\mu_{i,j-1} - \beta_i' \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] \right) f(\mathbf{v}_i, \mathbf{w}_i, e_i) d\mathbf{v}_i d\mathbf{w}_i d e_i. \tag{23}
$$

The model is estimated by maximum simulated likelihood. The simulated log likelihood function is given in (24).

$$
\log L_{S}(\boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\alpha}, \boldsymbol{\delta}, \boldsymbol{\gamma}, \boldsymbol{\Gamma}, \boldsymbol{\sigma}, \tau) = \sum_{i=1}^{n} \log \frac{1}{M} \sum_{m=1}^{M} \left(F \left[\frac{\mu_{ij,m} - \boldsymbol{\beta}_{i,m}' \mathbf{x}_{i}}{\exp(\boldsymbol{\gamma}' \mathbf{h}_{i} + \tau e_{i,m})} \right] - F \left[\frac{\mu_{i,j-1,m} - \boldsymbol{\beta}_{i,m}' \mathbf{x}_{i}}{\exp(\boldsymbol{\gamma}' \mathbf{h}_{i} + \tau e_{i,m})} \right] \right)
$$
(24)

 are a set of M multivariate random draws for the simulation3. This is the model in its full generality. Whether a particular data set will be rich enough to support this much parameterization, particularly the elements of the covariances of the unobservables in Γ , is an empirical question that will depend on the application.

One is typically interested in estimation of parameters such as β in (24) to learn about the impact of the observed independent variables on the outcome of interest. This generalized ordered choice model contains four points at which changes in observed variables can induce changes in the probabilities of the outcomes, in the thresholds, μij, in the marginal utilities, $β$ *i*, in the utility function, xi and in the variance, σi2. These could involve different variables or they could have variables in common. Again, demographics such as age, sex, and income, could appear anywhere in the model. In principle, then, if we are interested in all of these, we should compute all the partial effects,

³ We use Halton sequences rather than pseudo-random numbers. See Train (2003) for discussion.

$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{x}_i} = \text{direct of variables in the utility function,}
$$
\n
$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{z}_i} = \text{indirect of variables that affect the parameters } \mathbf{\beta},
$$
\n
$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{h}_i} = \text{indirect of variables that affect the variance of } \varepsilon_i
$$
\n
$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{r}_i} = \text{indirect of variables that affect the thresholds,}
$$

The four terms (in order) are the components of the partial effects (a) due directly to change in x_i , (b) indirectly due to change in the variables z_i that influence β_i , (c) due to change in the variables, h_i in the variance and (d) due to changes in the variables r_i that appear in the threshold parameters, respectively. The, probability of interest is

$$
Prob(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{h}_i, \mathbf{r}_i) = \int_{\mathbf{v}_i, \mathbf{w}_i, e_i} \begin{bmatrix} F \left[\frac{\mu_{ij} - (\beta + \Delta \mathbf{z}_i + \mathbf{L} \mathbf{D} \mathbf{v}_i)' \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] - \\ \frac{\exp(\gamma' \mathbf{h}_i + \tau e_i)}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \end{bmatrix} f(\mathbf{v}_i, \mathbf{w}_i, e_i) d\mathbf{v}_i d\mathbf{w}_i d e_i,
$$

$$
\mu_{ij} = \exp(\delta' \mathbf{r}_i) \left(\sum_{m=1}^j \exp(\alpha_m + \sigma_m w_{im}) \right), j = 1, ..., J-1.
$$
 (25)

The set of partial effects is shown in equation set (26).

$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{h}_i, \mathbf{r}_i)}{\partial \mathbf{x}_i} = \frac{\partial \mathbf{x}_i}{\partial \mathbf{x}_i} \left(\frac{1}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \left[f \left[\frac{\mu_{ij} - \beta'_i \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] - \left[\left(-\beta_i \right) \right] f(\mathbf{v}_i, \mathbf{w}_i, e_i) d\mathbf{v}_i d\mathbf{w}_i d\mathbf{e}_i \right] \right)
$$
\n
$$
(26a)
$$

$$
\frac{\partial \text{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{z}_{i}} = \frac{\partial \mathbf{z}_{i}}{\partial \mathbf{x}_{i}} \left(\frac{1}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \left| f \left[\frac{\mu_{ij} - \beta'_{i} \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right] - \left| \frac{\mathbf{x}_{i} - \beta'_{i} \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right| f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) d \mathbf{v}_{i} d \mathbf{w}_{i} d e_{i} \right)
$$
\n
$$
\left(-\Delta' \mathbf{x}_{i} \right) \left| f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) d \mathbf{v}_{i} d \mathbf{w}_{i} d e_{i} \right|
$$
\n
$$
(26b)
$$

$$
\frac{\partial \text{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{h}_{i}} = \frac{\partial \mathbf{h}_{i}}{\partial \mathbf{x}_{i}} \left\{\int_{\mathbf{exp}(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})}^{\mathbf{E}_{i}} \left[\frac{\mu_{ij} - \beta_{i}' \mathbf{x}_{i}}{\exp(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})} - \frac{\mathbf{h}_{i} \mathbf{x}_{i}}{\exp(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})} \right] - \left[\frac{\mu_{ij} - \beta_{i}' \mathbf{x}_{i}}{\exp(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})} \right] \left[\frac{\mu_{i,j-1} - \beta_{i}' \mathbf{x}_{i}}{\exp(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})} \right] \left[\frac{\mu_{i,j-1} - \beta_{i}' \mathbf{x}_{i}}{\exp(\mathbf{y} \mid \mathbf{h}_{i} + \tau e_{i})} \right] \right\} \left(-\mathbf{y} \right)
$$
\n(26c)

$$
\frac{\partial \text{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{h}_i, \mathbf{r}_i)}{\partial \mathbf{r}_i}
$$
\n
$$
\int_{\mathbf{v}_i, \mathbf{w}_i, e_i} \left\{ \int_{\mathbf{r}}^{\mathbf{r}} \left[\frac{\mu_{ij} - \beta'_i \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] \left(\frac{\mu_{ij}}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right) - \left(\delta \right) \right\} f(\mathbf{v}_i, \mathbf{w}_i, e_i) d\mathbf{v}_i d\mathbf{w}_i d e_i
$$
\n
$$
\left\{ f \left[\frac{\mu_{i,j-1} - \beta'_i \mathbf{x}_i}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right] \left(\frac{\mu_{i,j-1}}{\exp(\gamma' \mathbf{h}_i + \tau e_i)} \right) - \left(\delta \right) \right\} f(\mathbf{v}_i, \mathbf{w}_i, e_i) d\mathbf{v}_i d\mathbf{w}_i d e_i \tag{26d}
$$

Effects for particular variables that appear in more than one part of the model are added from the corresponding parts. Like the log likelihood function, the partial effects must be computed by simulation. If a variable appears only in xi, then this formulation retains both the "parallel regressions" and "single crossing" features of the original model. Nonetheless, the effects are highly nonlinear in any event. However, if a variable appears anywhere else in the specification, then neither of these properties will necessarily remain.

3. Empirical application

 \overline{a}

The context of the application, using stated choice data from a larger study reported in Hensher (2006a,b), is an individual's choice amongst unlabelled attribute packages of alternative tolled and non-tolled routes for the car commuting trip in Sydney (Australia) in 2002. In this paper we are interested in one feature of the way in which individual's process attribute information, namely attribute inclusion or exclusion, given a maximum of five attributes per alternative. The dependent variable in the ordered choice model is the number of ignored attributes, or the number of attributes attended to from the full fixed set associated with each alternative package of route attributes. The utility function is defined over the attribute information processed by each individual, with candidate influences on the each individual's decision heuristic including the dimensions of the choice experiment (e.g., number of alternatives, range of attributes), the framing of the design attribute levels relative to a reference alternative (see below), an individuals socioeconomic characteristics, and attribute accumulation where attributes are in common units (see also Hensher 2006b).

The establishment of attribute inclusion/exclusion in making choices in a stated choice (SC) context is often associated with design dimensionality and the so-called complexity of the SC experiment (Hensher 2006a). It is typically implied that designs with more items to evaluate are more complex than those with less items4 (for example,

⁴ Complexity also includes attributes that are lowly correlated, in contrast to highly correlated, the latter supporting greater ease of assessment in that one attribute represents other attributes.

Arentze et al., 2003, Swait and Adamowicz 2001a, 2001b), impose cognitive burden, and are consequently less reliable, in a behavioral sense, in revealing preference information. This is potentially misleading, since it suggests that complexity is an artefact of the quantity of information, in contrast to the relevance of information (Hensher 2006b). In any setting where an individual has to process information on offer and make a choice, psychologists interested in human judgement theory have studied numerous heuristics that are brought to bear in aiding simplification of the decision task (Gilovich et al. 2002). The accumulating life experiences of individuals are also often brought to bear as reference points to assist in selectively evaluating information placed in front of them. These features of human processing and cognition are not new to the broad literature on judgment and decision making, where heuristics are offered up as deliberative analytic procedures intentionally designed to simplify choice. The presence of a large amount of information, whether requiring active search and consideration or simply assessment when placed in front of an individual (the latter being the case in choice experiments), has elements of cognitive overload (or burden) that results in the adoption of rules to make processing manageable and acceptable (presumably implying that the simplification is worth it in terms of trading off the benefits and costs of a consideration of all information on offer or potentially available with some effort). It is not easy to distinguish between simplified processing because the context is of little interest or the effort is not worth it, versus a genuine interest in the task but with some ex ante biases that translate into heuristics that capture how an individual desires to treat specific pieces of information. Either way, we see gains in investigating attribute processing and in time being able to separate out real behavioral processing from processing for convenience (that lacks behavioral validity in respect to the choice if interest) given the task. Importantly we suggest that the amount of information to process is less important than the relevance of the information, and indeed there are situations where so little information makes processing 'complex' in the sense that the decision maker requires much more detail to define a choice of relevance.

The alternative attribute packages offered to individuals to evaluate are pivoted around the car commuting experiences of sampled respondents. The use of a respondent's experience, embodied in a reference alternative, to derive the attribute levels of the experiment, is supported by a number of theories in behavioural and cognitive psychology, and economics, such as prospect theory, case-based decision theory and minimum-regret theory (see Starmer 2000, Hensher 2006b). Reference alternatives in SC experiments⁵ act to frame the decision context of the choice task within some existing memory schema of the individual respondents, and hence make preferencerevelation more meaningful at the level of the individual.

Four stated choice sub-designs have been embedded in one overall design (Table 1). Each commuter evaluated one randomly assigned sub-design; however, across the full set of stated choice experiments, the designs differed in terms of the range and levels of attributes, the number of alternatives and the number of choice sets. The combination of these dimensions of each design is often seen as the source of design 'complexity', and it is within this setting that we have varied the dimensions of an SC experiment that each respondent is asked to evaluate, and through supplementary questions, established which attributes were 'ignored' in the evaluation and selection of an alternative.

Previous studies were used to identify candidate design dimensions. The five design dimensions are shown in Table 2. Five attributes were selected for each alternative,

 $⁵$ Hensher (2004), Train and Wilson (2008), and Rose et al. (2008) provide details of the design of pivot-based experiments.</sup>

based on previous evidence (see Hensher 2001), to characterise the options: free-flow time, slowed down time, stop/start time, variability of trip time, and total cost. Hensher (2006) explored how varying the number of attributes affects information processing, aggregating attributes according to four patterns, noting that aggregated attributes are combinations of existing attributes⁶. We have selected a generic design (i.e., unlabeled alternatives) to avoid confounding the effect of the number of alternatives with the labeling (e.g., car, train). The sub-design dimensions are shown in Table 1 with the attribute ranges in Table 2.

Choice set of Number of size	alternatives	attributes	Number of Number of levels of attributes	Range of attribute levels
15				Wider than base
9				Base
6		5		Narrower than base
12			3	Narrower than base

Table 1: The sub-designs of the overall design for five attributes

Note: Column 1 refers to the number of choice sets. The four rows represent the set of designs (see Appendix A). The number of alternatives does not include the reference alternative.

As a generic design, each of the alternatives, added as we move from 2 to 3 to 4 alternatives in a choice set (based on Table 1), are exactly the same. That is, for any two alternatives associated with a given design, we should not expect to find the parameter for an attribute (e.g., 'free flow travel time') to be different for the set of non-reference alternatives. Therefore we do not need the attribute 'free flow time alternative one' to be orthogonal to the attribute 'free flow time alternative two' etc up to 'free flow time *J*-1alternatives'. The designs are computer-generated. A preferred choice experiment design is one that maximizes the determinant of the covariance matrix, which is itself a function of the estimated parameters. Knowledge of the parameters, or at least some priors (such as signs) for each attribute, from past studies, provides a useful input. We found that in so doing, the search eliminates dominant alternatives. The method used finds the D-optimality plan very quickly (see Rose and Bliemer 2007).

The *actual* levels of the attributes shown to respondents are calculated relative to those of the experienced reference alternative – a recent car commuter trip. The levels applied to the choice task differ depending on the range of attribute levels and the number of levels for each attribute. The design dimensions are translated into SC screens, illustrated in Figure 1. The range of the attribute levels vary *across* designs. Each sampled commuter is given a varying number of choice sets (or scenarios), but the

 6 This is an important point because we did not want the analysis to be confounded by extra attribute dimensions.

number of alternatives remain fixed. Elicitation questions associated with attribute inclusion and exclusion shown in Figure 2.

a. Transport Study					
Games 1					
	Details of Your Recent Trip	Α	Alternative Road Alternative Road Alternative Road 8	c	
Time in free flow (mins)	15	14	16	16	
Time slowed down by other traffic (mins)	10	12 [°]	8	12	
Time in Stop/Start conditions (mins)	5	4	£.	A	
Uncertainty in travel time (mins)	$+/-10$	$+4.12$	$+4.8$	$+1.8$	
Total Costs	\$2.00	\$2.10	\$2.10	\$1.90	
If you take the same trip again, which road would you choose?	C Current Road	C Road A	C Road B	C Road C	
If you could only choose between the new roads, which would you choose?		C Road A	C Road B	C Road C	
		Go to Game 2 of 6			

Figure1: An example of a stated choice screen

Figure 2: CAPI questions on attribute relevance

4. Empirical analysis

Computer-aided personal interview (CAPI) surveys were completed in the Sydney metropolitan area in 2002^7 . A stratified random sample was applied, based on the residential location of the household. Screening questions established eligibility in

 7 Interviews took between 20 and 35 minutes, with an interviewer present who entered an individual's responses directly into the CAPI instrument on a laptop.

respect of commuting by car. Further details are given in Hensher (2006a). Final models are given in Table 3 for 2,562 observations.

The explanatory variables in the model were guided by the extant literature on heuristics and biases in choice and judgment (see Gilovich et al. 2002), as well as empirical evidence from previous studies on attribute processing by Hensher (2006a,b). We selected candidate influences on the number of attributes actually processed (i.e., deemed relevant) under three broad categories: (i) design dimensions of the choice experiment, (ii) framing around the reference or base alternative, in line with the theoretical argument promoted in prospect theory for reference points, and (iii) the literature on heuristics that suggests that attribute packaging or attribute-accumulation is a legitimate rule for some individuals in stage 1 editing under prospect theory (Gilovich et al. 2002).

The generalized ordered logit model has a preferred goodness of fit over the traditional ordered logit model. With four degrees of freedom difference, the likelihood ratio of 181.92 is statistically significant on any acceptable chi-squared test level. generalized model has included a random parameter form for congestion time framing and has accounted for two systematic sources of variation around the mean of the random threshold parameter (i.e., the accumulation of travel time and gender).

The evidence identifies a number of statistically significant influences on the number of attributes attended to, given the maximum number of attributes provided. The range of the attributes and the number of alternatives⁸ in the choice set condition mean attribute preservation, and the number of levels of an attribute has a systematic influence on the variance of the unobserved effects (or the error term). We framed the level of each attribute relative to that of the experienced car commute as (i) free flow time for reference (or base) minus the level associated with an alternative in the SC design, and (ii) the congested travel time for the base minus the level associated with each SC alternative's attribute level. The parameter estimates are statistically significant and negative suggesting that the more that an SC attribute level ('free flow time' and 'congested time (=slowed down plus stop/start time)) deviates from the reference alternative's level, the more likely that an individual will process an increased number of attributes. The attribute packaging effect for travel time has a negative parameter, suggesting that those individuals who add up components of travel time tend to preserve more attributes; indeed aggregation is a way of simplifying the choice task without ignoring attributes. In the sample, 82 percent of observations undertook some attribute packaging.

The evidence herein cannot establish whether an attribute reduction strategy is *strictly* linked to behavioral relevance, or to a coping strategy for handling cognitive burden, both being legitimate paradigms. It does, however, provide indications on what features of a specific choice experiment have an influence on how many attributes provided within a specific context are processed. It is likely that the evidence is application specific, but extremely useful when analysts compare the different studies and draw inferences about the role of specific attributes.

The threshold parameter has a statistically significant mean and two sources of systematic variation across the sample around the mean threshold parameter estimate.

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⁸ The difference in the number of alternatives (from two to four, excluding the reference alternative) represents a range typically found in SC studies. The actual screens, with the reference alternative is in place, have between three and five alternatives. The number of alternatives is fixed per respondent but it varies across the sample.

Across the sample, there were three levels of the ordered choice observed; level 0 is where all attributes are preserved, level 1 is where 4 of the 5 attributes were preserved, and level 3 is where 3 of the 5 attributes were preserved. No respondent preserved only 1 or 2 attributes. Hence given three levels of the choice variable, there are two threshold parameters, one between levels 0 and 1 and one between levels 1 and 2 (see the explanation in paragraph following equation 3). As indicated in section 2.1, a normalisation is required so that a constant can be identified. We set the threshold parameter for between levels 0 and 1 equal to zero (μ_1) and estimate the parameter between levels 1 and 2 $(\mu_2)^9$.

We investigated an unconstrained random parameter normal distribution; however the standard deviation parameter estimate was not statistically significant from zero. The evidence however justifies the inclusion of a non-fixed threshold parameter, with a higher mean estimate across the sampled population when an individual aggregates the travel time components and when they are male. This is an important finding since it justifies the new formulation of the threshold parameters in ordered choice models as behaviorally meaningful.

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⁹ Estimation of the threshold parameters is not a main object of fitting the ordered choice model per se. The flexibility of the threshold parameters is there to accommodate the variety of ways that individuals will translate their underlying continuous preferences into the discrete outcome. The main objective of the estimation is the prediction of and analysis of the probabilities, e.g., the partial effects. The threshold parameters do not have any interesting interpretation of their numerical values in their own right.

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A direct interpretation of the parameter estimates is not informative, given the logit transformation of the choice dependent variable (see equations 5 and 26). We therefore provide the marginal (or partial) effects in Table 4 which have substantive behavioral meaning, defined as the derivatives of the choice probabilities (equation 25). A marginal effect is the influence a one unit change in an explanatory variable has on the probability of selecting a particular outcome, *ceteris paribus¹⁰*. The marginal effects need not have the same sign as the model parameters. Hence, the statistical significance of an estimated parameter does not imply the same significance for the marginal effect.

We take a closer look at each model, discussing the evidence for design dimensions, framing around the base, attribute packaging, variance decomposition, and other effects. The magnitude and direction of influence is given in Table 4 for the marginal effects which have to be interpreted relative to each of the three levels of the *number of attributes ignored*.

 10 This holds for continuous variables only. For dummy (1,0) variables, the marginal effects are the derivatives of the probabilities given a change in the level of the dummy variable.

Note: the three marginal effects per attribute refer to the levels of the dependent variable.

In commenting on the marginal effects, it should be noted that, for the generalised ordered logit model, some attributes have more than one role; for example the framing of free flow time is both a main effect influence as well as a source of variance decomposition (i.e., systematic source of heterogeneity) for the unobserved variance; and the attribute accumulation for travel time is both a main effect and a systematic source of influence on the distribution of the random threshold parameter. The generalised ordered choice model (GOCM) takes all of these sources into account in identifying the marginal effects for each level of the choice variable. In contrast, where an attribute has multiple roles in the traditional ordered choice model (TOCM), the marginal effects are calculated separately. The marginal effects associated with variance decomposition in GOCM has two unique influences (i.e., number of levels of an attribute and 'who pays for the trip', together with the framing around the base alternative for free flow time which is present elsewhere¹¹).

The dummy variable for 'narrow attribute range' has the highest marginal effect, although its influence is moderated in GOCM compared to TOCM. The probability of considering more (compared to less) attributes from the offered set decreases as an attribute's range narrows, *ceteris paribus*. That is, respondents tend to ignore more attributes when the difference between attribute levels is small. This result is perhaps due to the fact that evaluation of small differences is more difficult or perceptually less relevant than evaluation of large differences. An important implication is that if an analyst continues to include, in model estimation, an attribute across the entire sample that is *ignored by a respondent*, then there is a much greater likelihood of mis-specified parameter estimates in circumstances where the attribute range is narrower than wider.

The marginal effects for the narrow attribute range are positive when one (i.e., 5-1) or two attributes (i.e., 5-2) are ignored. Importantly the positive effect is greater when one attribute is ignored than when two are ignored. This suggests that the probability of considering four or three attributes from the offered set increases as an attribute's range goes from narrow to non-narrow, ceteris paribus, but to a greater extent for four attributes. What we are observing across all three levels of the dependent variable is U- (or inverted U-) shaped response, which appears to be the case for all attributes in GOCM. Thus for the narrow attribute range we have the highest probability of preserving four attributes than of preserving three attributes, given that the probability of preserving all attributes is decreased. Given the observed profile of the sampled respondents preserving five, four and three attributes (Table 2), where there are only 66 observations in the last category (compared to 1415 and 1080 in 5-0 and 5-1), we have

¹¹ For 'Free flow time for base minus SC alternative level' we report this in variance decomposition to show its relatively small effect compared to the overall effect of this variable given in another row in the table.

greater confidence in the relative marginal effects of preserving all (i.e., five) attributes and four attributes.

As we increase the 'number of alternatives' to evaluate (over the range of 2 to 4 plus the reference alternative), ceteris paribus, the importance of considering all attributes increases, as a way of making it easier to differentiate between the alternatives. This finding runs counter to some views, for example, that individuals will tend to ignore increasing amounts of attribute information as the number of alternatives increases. Our evidence suggests that the processing strategy is dependent on the nature of the attribute information, and not strictly on the quantity. The negative marginal effects for ignoring one and two attributes (or preserving four and three attributes) suggest that these rules are less likely to be adopted as the number of alternatives increases.

The theoretical argument promoted in prospect theory for reference points is supported by our empirical evidence. We have framed the level of each attribute relative to that of the experienced car commute trip as (i) free flow time for current (or base) minus the level associated with an attribute and alternative in the SC design, and (ii) the congested travel time for the base minus the level associated with each SC alternative's attribute. The more that an SC attribute level deviates from the reference alternative's level, the more likely that an individual will process an increased number of attributes. This evidence was found for both the 'free flow time' and 'congested time' framing effects. Conversely, as the SC design attribute level moves closer to the reference alternative's level, individuals appear to use some approximation rule, in which closeness suggests similarity, and hence ease of eliminating specific attributes, because their role is limiting in differentiation.

Reference dependency not only has a direct (mean) influence on the number of attributes ignored; it also plays a role via its contribution to explaining heteroscedasticity in the variance of the unobserved effects. This has already been accounted for in the GOCM marginal effects for free flow time framing. It is separated out in the TOCM. The effect of widening the gap between the base and SC 'free flow time' reduces the heteroscedasticty of the unobserved effects across the respondents, increasing the acceptability of the constant variance condition when simpler models are specified.

In GOCM, the congested time framing effect is represented by a distribution across the sample. The random parameter has a statistically significant standard deviation parameter estimate, resulting in a distribution shown in Figure 3. The range is from - 0.857 to 1.257; hence there is a sign change around the mean of 0.70833 and standard deviation of 0.2657. This results in the same mean marginal effect sign in GOCM as free flow time framing; however when we treated congested time framing as having a fixed parameters (in TOCM, where the standard deviation parameter was not statistically significant), the signs are swapped for all levels of the choice variable. The evidence from the GOCM is intuitively more plausible.

Figure 3: Distribution of preference heterogeneity for congested time framing

The attribute-accumulation rule in stage 1 editing under prospect theory is consistently strong for the aggregation of travel time components. The positive marginal effect for the dummy variable 'adding three travel time components' indicates that, on average, respondents who add up the time components, in assessing the alternatives, tend also to ignore more attributes. There is clear evidence that a relevant simplification rule is repackaging of the attribute set, where possible, through addition. This is not a cancellation strategy, but a rational way of processing the information content of component attributes, and then weighting this information (in some unobserved way) in comparing alternatives.

The socio-economic characteristics of respondent's proxy for other excluded contextual influences. A respondent's role in paying the toll was identified, through its influence on variance decomposition of the unobserved effects, as a statistically significant socioeconomic influence on the number of attributes considered. We have no priors on the likely sign of the influence on variance. The positive marginal effect for who pays suggests that those who pay themselves (in contrast to a business paying) tend to resulting in a higher probability of preserving more attributes, although the influence is slightly less in GOCM compared to TOCM. This might mean that males do care more about the time/cost trade-off, in contrast to a situation where only time matters if someone else pays for the travel. Gender was a systematic source of influence on the threshold parameter, increasing its mean estimate for males.

5. Conclusions

The recognition of randomness in the threshold parameters in the presence of random parameters and the identification of systematic sources of heterogeneity in the mean threshold parameter estimate is an important extension of the existing ordered choice model. This paper has brought together all of the key contributions in the literature and extended them, in particular to ensure preservation of the ordering of thresholds in the context of random parameterisation of the thresholds (equations 16 to 20).

The specific application herein, on the role that attributes play in choice making in stated choice experiments, pivoted around a real market experience, has highlighted the role of random thresholds and decomposition, suggesting that the generalized empirical model is a rich behavioral addition to the literature on ordered choice modeling. We need, however, many studies in differing contexts before we can make general conclusions about the specific empirical evidence on sources of influence on the propensity for individuals to invoke specific attribute preservation heuristics.

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Appendix A: Designs for five-attributes

Ordered choices and heterogeneity in attribute processing Greene & Hensher

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