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TITLE:	**Embedding risk attitudes in a scheduling model: Application to the study of commuting departure time**

ABSTRACT:	Traditionally, the value of travel time savings (VTTS) and the value of reliability (or reduced variability) are estimated within a linear utility functional form, which assumes risk-neutral attitudes for decision makers. In this paper, we develop non-linear scheduling models to address both risk attitude and preference in the context of a stated choice experiment of car commuters facing risky choices where the risk is associated with the trip time. We also investigate unobserved between-individual heterogeneity in time-related parameters and risk attitudes using a mixed multinomial logit (MMNL) model. More importantly, we calculate the willingness to pay values for reducing the mean travel time and variability (earlier/later than the preferred arrival time) within the non-linear scheduling framework. This model is then used to estimate preferred departure times for commuters, assuming that random link capacities are the source of travel time variability. Results show that the more variable travel times are, the earlier commuters depart, and that the non-linear scheduling model predicts earlier optimal departure times than the traditional linear scheduling model. Some important issues related to modelling non-linearity are also discussed.

KEY WORDS:	*Risk attitude; commuter car travel; willingness to pay; travel time variability; non-linear scheduling model; mixed multinomial logit; departure time.*

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1. Introduction

Over the past three decades, there has been a growing number of studies which have investigated the significance of travel time variability in traveller behaviour (e.g., Jackson and Jucker 1982, Small 1982, Bates *et al.* 2001, Senna 1994, Asensio and Matas 2008, see Li *et al.* 2010 for a review). The majority of travel time variability studies have considered day-to-day variations in travel time. Some have also explicitly defined travel time variability as the random variation in travel time (see e.g., Bates *et al.* 2001; Hollander 2006), so as to emphasise the stochastic feature of travel time variability. The scheduling model and the mean-variance models based on the stated choice theoretic framework, are two dominant approaches to empirical measures of the value of travel time savings and the value of time variability, where utility is usually represented by either a linear function of the expected or mean travel time, the expected schedule delay early (ESDE), the expected schedule delay late (ESDL) and other attributes (e.g., travel cost) in the scheduling model, or a linear function of the expected travel time, the standard deviation of travel time (SD) and other attributes in a mean-variance model.

Recent travel time variability studies have included a series of travel times (normally five or 10 levels) for an alternative in their stated preference (SP) experiments to capture time variation (see, e.g., Senna 1994, Small *et al.* 1999, Bates *et al.* 2001, Hollander 2006, Asensio and Matas 2008). Hamer *et al.* (2005) suggested that travel time variability is better presented as a series of travel times associated with each alternative, than it was in early SP experiments often presented as the extent and frequency of delay relative to normal travel time. In previous studies, the expected values of time and time variability are calculated over multiple possible travel times (assumed to be equi-probable) for an alternative within a choice set. Given that the majority of studies specified linear utility functional forms, the value of travel time savings is the ratio of the expected time parameter and the cost parameter (i.e., VTTS $= \beta_{\text{Time}} / \beta_{\text{Cost}}$). Similar to the VTTS, time variability related values are: the value of reliability (VoR $= \beta_{\text{SD}} / \beta_{\text{Cost}}$), the value of expected schedule delay early (VESDE $= \beta_{\text{ESDE}} / \beta_{\text{Cost}}$), and the value of expected schedule delay late (VESDL $= \beta_{\text{ESDL}} / \beta_{\text{Cost}}$).

Linear utility maximisation is still the underlying behavioural framework of travel time variability studies, although some of them reported an association with a theory of Maximum Expected Utility (MEU) (see Noland and Small 1995), which still assumes linear utility specification, unlike full Expected Utility (EU) theoretic models and some other models (e.g., Rank-Dependent Utility Theory) that postulate a non-linear functional form to address attitudes towards risk. Unlike studies in psychology and behavioural economics where the investigation of risk attitudes is a primary focus, transport research has limited evidence on risk attitudes. In this paper, we embed a non-linear utility specification in a scheduling mode to accommodate both risk attitudes and preferences. We also allow for unobserved heterogeneity, using a mixed multinomial logit (MMNL) model, and reveal significant heterogeneity in risk attitudes and time-related parameters between individuals. WTP values for average trip time and reduced variability (earlier/later) are also calculated under the non-linear MNL and non-linear MMNL models. The non-linear scheduling model is then used to estimate preferred departure times for commuters.

2.   Traditional scheduling model for travel time variability

The scheduling model is a dominant approach to modelling travel time variability, which takes into account the consequences of unreliable travel time. Unlike the mean-variance model which assumes that travel time variability leads to the loss of utility by itself, the scheduling model considers that disutility is incurred when not arriving at the preferred arrival time (PAT), either early or late. Let SD be the schedule delay, defined as the difference between the PAT and the actual arrival time:

$$SD = PAT - \left[t_h + \left(T(t_h) \right) \right] \tag{1}$$

The total travel time ($T(t_h)$) is determined by the departure time (t_h). A late arrival (schedule delay late or SDL) relative to the PAT will occur if $(t_h + T(t_h)) - PAT > 0$; otherwise, it will be a schedule delay early (SDE).
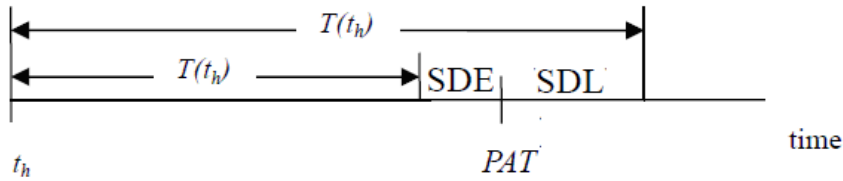

Figure 1: The concept of schedule delay

Given travel time variability, travel time (T) is not deterministic and can be modelled as a random variable (see Section 6); therefore, for some departure times it may be possible to be early or late at the same time. A typical scheduling model is shown in equation (2), where the possible early and late scheduling delays, relative to the preferred arrival time are modelled separately, and their consequences are measured by the estimated parameters.

$$U(t_h) = \eta E[T(t_h)] + \beta E[SDE(t_h)] + \gamma E[SDL(t_h)] + ... \tag{2}$$

$U(t_h)$ is a linear function of the expected travel time ($E[T(t_h)]$), the expected schedule delay early ($E[SDE(t_h)] = P_E * E_{\Delta T}$), and the expected schedule delay late ($E[SDL(t_h)] = P_L * L_{\Delta T}$); P_E is the probability of early arrival; $E_{\Delta T}$ is the amount of time arriving earlier than PAT; P_L is the probability of late arrival shown to respondents; and $L_{\Delta T}$ is the amount of time arriving later than PAT.

3.   The violation of linear utility maximisation

Under Random Utility Maximisation (RUM), the utility function is usually given a linear form as in equation (3).

$$U = \sum_k (\beta_k x_k) + \varepsilon \tag{3}$$

where β_k are the marginal utility parameters and x_k are the attributes that underlie individual preferences; ε is the unobserved component of utility and different assumptions on the

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distribution ε would delivers different models (e.g., an extreme type 1 (EV1) distribution leads to logit models and a standard normal distribution leads to probit models).

To show the violation of linear utility maximisation, a simple time lottery is provided which has two alternatives (Route A and Route B). Under the same departure time, choosing Route A will definitely arrive 10 minutes later than the PAT from home to work, while taking Route B will has a 50 percent chance of arriving 5 minutes later and a 50 percent of chance of arriving 15 minutes later. Under RUM, given the utility function is linear in the attribute parameter, these two alternative will have the same utility $(U_A = \beta_{LaterTime} \times 10 = U_B = \beta_{LaterTime} \times 5 \times 0.5 + \beta_{LaterTime} \times 15 \times 0.5)$. Hence, these two alternatives are indifferent under RUM. However, evidence from psychology and behavioural economics suggests that this indifference exists only if subjects are risk neutral, and the two alternatives with equal expected value will be treated differently according to specific risk attitude (averse or taking).

Given that monetary lottery ticket experiments are widely used in psychological and behavioural economics studies, we use a graph with monetary values to explain risk attitudes. Figure 2 shows a concave utility function, suggesting that the utility of a sure win (U($50)) is higher than the utility of a 50:50 chance of winning $100 or nothing (i.e., $\frac{1}{2}$ U($100)), and hence the sure win is preferred in spite of the same expected value for two alternatives (i.e., $50). This is a typical risk-averse attitude, i.e., a sure alternative is preferred to a risky alternative (i.e., with multiple possible outcomes) of equal or even slightly higher expected value. A convex utility curve suggests risk taking, i.e., a risky alternative is preferred to a sure alternative of equal or even slightly higher expected value. However, almost all existing scheduling models in the transport literature assume that the utility specification is linear, which assumes a risk-neutral attitude in terms of the utility function (see equation 2). This assumption works fine in a deterministic environment (e.g., there is only one travel time with 100 percent chance of occurrence). However, this is often not realistic given that trip time variability is inherent to transport systems and results in multiple possible travel times for a trip. Hence, it may be inappropriate to assume a linear utility specification (risk neutral) in the presence of travel time variability. Compared with a few travel time variability studies where the non-linear scheduling models[1] were estimated, we offer some differences and improvements including the investigation of unobserved heterogeneity, and the calculation of willingness to pay values for travel time savings and reduced variability, which are also influenced by the level of the attributes (see equations 6a-6c)[2].

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[1] Polak *et al.* (2008) estimated a non-linear scheduling model with a constant *absolute* risk aversion (CARA) assumption (i.e., an exponential specification), within a multinomial logit (MNL) framework and a mixed MNL (MMNL) framework. The non-linear utility functional form in this paper is based on constant *relative* risk aversion (CRRA, a power specification), which often delivers "a better fit than alternative families" (Wakker 2008, p.1329). Michea and Polak (2006) estimated a non-linear scheduling model based on the simple power form, using the MNL model.

[2] Michea and Polak (2006) calculated WTP for mean lateness, which is linear in their utility specification. Given that the CARA utility form is used in Polak *et al.* (2008), the WTP values are the ratios between two corresponding parameters (e.g., VTTS = $\beta_{Time} / \beta_{Cost}$; VESDE = $\beta_{ESDE} / \beta_{Cost}$; VESDL = $\beta_{ESDL} / \beta_{Cost}$), same as the traditional RUM or MEU model with a linear utility specification.
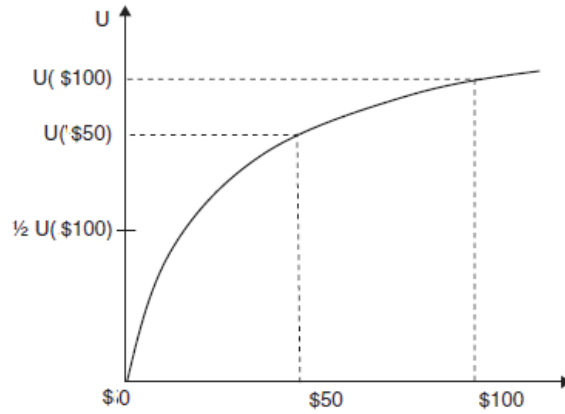
Figure 2: Utility function over states of wealth illustrating risk aversion (Fox and See 2003)

4. Embedding risk attitudes in a scheduling model: Multinomial logit

A non-linear scheduling model is estimated based on data drawn from a study undertaken in Australia in the context of route choice, which utilised a stated choice (SC) experiment involving two SC alternatives (i.e., route A and route B) pivoted around the knowledge base of travellers (i.e., the current trip). Each alternative has three travel scenarios - 'arriving x minutes earlier than expected', 'arriving y minutes later than expected', and 'arriving at the time expected'. Each is associated with a corresponding probability[3] of occurrence, to indicate that travel time is not deterministic but varies from time to time. For a full description of the design and characteristics of the SC experiment, see Li *et al.* (2010, pp. 395-396).

Unlike RUM with a linear utility functional form, some alternative behavioural frameworks allow for non-linearity in utility. For example, Expected Utility Theory (EUT), originally developed by Bernoulli in 1738, is a normative modelling framework for risky choice. EUT models postulate a non-linear functional form, for example, $U = x^{\alpha}$ where α is the risk attitude parameter which explains respondents' attitudes towards risk. EUT assumes that an individual compares the expected utility values associated with particular options. That is, individuals are assumed to compare "the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities" (Mongin 1997, p.342). A basic EUT model is given in equation (4).

$$E(U) = \sum\nolimits_{m} (p_m x_m^{\alpha}) \tag{4}$$

where $E(U)$ is the expected utility; x_m is the mth outcome of an alternative with multiple possible outcomes, and normally there is only one attribute for an alternative (e.g., monetary price) in psychological and experimental economics studies; p_m is the probability associated with the mth outcome.

Risk attitudes play a central role in decision making, especially in a non-deterministic environment. Given a departure time, one specific travel scenario would occur up to a probability with variability in travel time. Hence, travellers' decision making is under risk.

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[3] The probabilities are designed and hence exogenously induced to respondents, similar to other travel time variability studies.

Given this, we embed risk attitudes in a scheduling model, shown in equation (5). We estimate the constant *relative* risk aversion (CRRA) model form as a general power specification (i.e., $U = \dfrac{x^{1-\alpha}}{1-\alpha}$), more widely used than the simple x^{α} form (Holt and Laury 2002).

$$U = \beta_E P_E \frac{E_{\Delta T}^{1-\alpha}}{1-\alpha} + \beta_L P_L \frac{L_{\Delta T}^{1-\alpha}}{1-\alpha} + \beta_{E(T)} \frac{E(T)^{1-\alpha}}{1-\alpha} + \beta_{Cost} Cost + \beta_{Age} Age + \beta_{Tollasc} Tollasc \qquad (5)$$

$E_{\Delta T}$ and $L_{\Delta T}$ are the minutes arriving earlier and later than the preferred arrival time; P_E and P_L are the probabilities of early arrival and late arrival shown to respondents; $E(T)$ is the expected or average travel time; *age* is a person's age in years[4]. *Tollasc* is the dummy variable to indicate whether a specific alternative is a tolled road. $\beta_{E(T)}$, β_E, β_L and βCost, are attribute-associated parameters to be estimated along with βage, and βTollasc,, and α is an additional parameter to be estimated and the value of (1-α) indicates the attitude towards risk. The model is reduced to a linear scheduling model (see equation 2) when $(1-\alpha)=1$. Hence, the traditional scheduling model is a particular case of model (5). This model also offers some innovations relative to the standard EUT Model. Unlike EUT with one attribute only, which by implication sets the attribute-associated parameter equal to 1 (e.g., $U = \dfrac{x^{1-\alpha}}{1-\alpha}$), our model includes a number of attributes (e.g., the travel time and cost), but only if an attribute has multiple possible outcomes (i.e., the travel time in this study), is the risk attitude parameter applied. We also incorporate within-alternative referencing, i.e., minutes of arriving earlier/later relative to the expected or preferred arrival time (PAT), where utility is the highest when arriving at the PAT.5 All existing travel time variability studies specified as non-linear in utility are estimated within a multinomial logit (MNL) framework. Hence to see the empirical gains when unobserved heterogeneity in preference and risk attitudes is introduced, we present empirical estimates of the MNL model first and then compare the evidence with a mixed MNL (MMNL) model in the following section. The MNL modelling results are given in Table 1.

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[4] We investigated a number of socioeconomic effects (e.g., income, gender) but did not find any statistically significant except age.

[5] In the transport literature, another type of referencing (which we refer to as 'between-alternative' referencing) is often used, where typically the reference point is defined as the status quo (the recently experienced preference (RP) alternative, e.g., the recent trip of an driver) and the utility function is defined over gains and losses around the reference alternative (see e.g., Hess *et al.* 2008; Hensher 2008).

Table 1: Scheduling model with embedded risk attitudes (MNL)

(Estimated using Nlogit5)

Variable	Coefficient	t-ratio
Reference constant	0.4864	3.98
Alpha (α)	0.3932	3.53
Average Time (minutes)	-0.3135	-2.44
Earlier (minutes)	-0.1522	-2.54
Later (minutes)	-0.2476	-5.08
Cost ($)	-0.2586	-12.00
Tollasc	-0.3099	-3.27
Age (years)	0.0054	2.14

No. of observations	4480
Information Criterion: AIC	6840.75
Pseudo R-squared	0.31
Log-likelihood	-3412.38

All estimated parameters are significant at or above the 95 percent confidence interval. The estimated parameter for the *Reference* specific constant (i.e., the constant for the current trip) is positive, which suggests, after accounting for the observed influences, that sampled respondents, on average, prefer their current trip relative to the two stated choice alternatives, with this tendency stronger as the age of a respondent increases (0.0054). *Tollasc* is negative, which indicates that, on average after accounting for the time and cost of travel, other factors bundled into a 'toll road quality bonus' are less desirable for a tolled route than a non-tolled route, mainly due to the lack of exposure to tolls for our sampled respondents. *Alpha* is statistically significant from zero with a *t*-ratio of *3.53*. For decision making related to travel time, a risk attitude parameter less than one suggests risk-taking attitudes; and a risk attitude parameter greater than one suggests risk-averse attitudes[6] (see Senna 1994). The calculated risk attitude value ($(1-\alpha)=0.6068<1$) suggests that our sampled commuters tend to be risk-taking when making choices where this risk (probability of occurrence) is associated with a travel time distribution. This finding is in line with Senna (1994) which assumed that his sampled commuters with a fixed arrival time are risk-prone (-taking), where the assumed risk attitude parameter is 0.5(<1), and explained this in the following way: "commuters are frequently travelling in the same route and this situation provides them information about the distribution of travel time" (p.220).

Under a model with a non-linear utility specification and linear probability weighting (i.e., probabilities of occurrence are directly used as the weights), the marginal (dis)utilities associated with arriving earlier/later than the preferred arrival time and the average travel time are not fixed values, but vary according to the minutes and probabilities of arriving earlier and later. The willingness to pay (WTP) formulae for avoiding arriving earlier/later, and reducing the average travel time are given in equations (6a)-(6c).

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[6] This is the opposite to decision making related to monetary outcomes, common to lottery experiments. For example, if a risk attitude parameter is estimated to lie between 0 and 1, the utility function over money would be similar to the curve shown in Figure 2, suggesting decreasing marginal *utility* (i.e., risk averse), given that money is a source of utility. However travel time along with its associated variability is a source of disutility; the corresponding utility function over travel time or variability would show decreasing marginal *disutility* (see e.g., Figure 4) or increasing marginal *utility* (i.e., risk taking).

$$\frac{\partial(U)}{\partial(E_{\Delta T})} \Big/ \frac{\partial(U)}{\partial(Cost)} = (1-\alpha)\beta_E P_E E_{\Delta T}^{(1-\alpha)-1} /(1-\alpha) \,/\, \beta_{Cost} = \beta_E P_E E_{\Delta T}^{-\alpha} \,/\, \beta_{Cost} \qquad (6a)$$

$$\frac{\partial(U)}{\partial(L_{\Delta T})} \Big/ \frac{\partial(U)}{\partial(Cost)} = (1-\alpha)\beta_L P_L L_{\Delta T}^{(1-\alpha)-1} /(1-\alpha) \,/\, \beta_{Cost} = \beta_L P_L L_{\Delta T}^{-\alpha} \,/\, \beta_{Cost} \qquad (6b)$$

$$\frac{\partial(U)}{\partial[E(T)]} \Big/ \frac{\partial(U)}{\partial(Cost)} = (1-\alpha)\beta_{E(T)} E(T)^{(1-\alpha)-1} /(1-\alpha) \,/\, \beta_{Cost} = \beta_{E(T)} E(T)^{-\alpha} \,/\, \beta_{Cost} \qquad (6c)$$

The parameters estimated from the non-linear scheduling model (MNL) are: $\beta_E = -0.1522$, $\beta_L = -0.2476$, $\beta_{E(T)} = -0.3135$ and $\beta_{Cost} = -0.2586$. The calculation of WTP values (values of variability (earlier/later) and VTTS) also requires the assumption of a travel time distribution (e.g., the minutes and probabilities of arriving earlier and later) to account for the specific trip time variability. Suppose the average travel time is 60 minutes, the calculated WTP is Au\$14.54 (\$2008), i.e., each car commuter is willing to pay on average Au\$14.54 to reduce one hour's mean travel time. Suppose the probabilities of arriving earlier/later are 0.1 and the minutes of arriving earlier/later are five minutes, the values of variability are calculated to be Au\$0.16 and Au\$0.25, i.e., a car commuter is willing to pay Au\$0.16 to avoid a 5-minute earlier arrival with a 10 percent chance of occurrence, and Au\$0.25 to avoid a 5-minute later arrival with a 10 percent chance of occurrence. Under the same probabilities (0.1 for both earlier and later), we graph the relationship between the WTP and the minutes of being earlier/later (5-60 minutes) in Figure 1, which shows that the WTP values increase nonlinearly[7] with the increase in the minutes of being earlier/later (from Au\$0.16 to Au\$0.71 for avoiding earlier minutes and from Au\$0.25 to Au\$1.15 for avoiding corresponding later minutes). The gap between the two WTP values grows substantially as the minutes of arriving earlier/later increase, where the difference between a 5-minute earlier and 5-minute later arrival time is Au\$0.10, and increases to Au\$0.44 for a 60-minute earlier/later arrival time. This is mainly attributed to the consequence (penalty) of arriving too late (e.g., after the work starting time).

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[7] Although WTP for total time or schedule delay increases, WTP for reducing one unit of travel time or variability (schedule delay) is lower for longer trips with longer travel time and longer schedule delays, given that the utility curve over travel time and variability ($(1-\alpha)<1$) suggests decreasing marginal disutility. This explains why WTP for one unit of time or schedule delay reduces as average travel time and schedule delay (earlier or later) increase. Empirical studies show that WTP can either decrease or increase with trip length, depending on the context (see Hensher 2010).
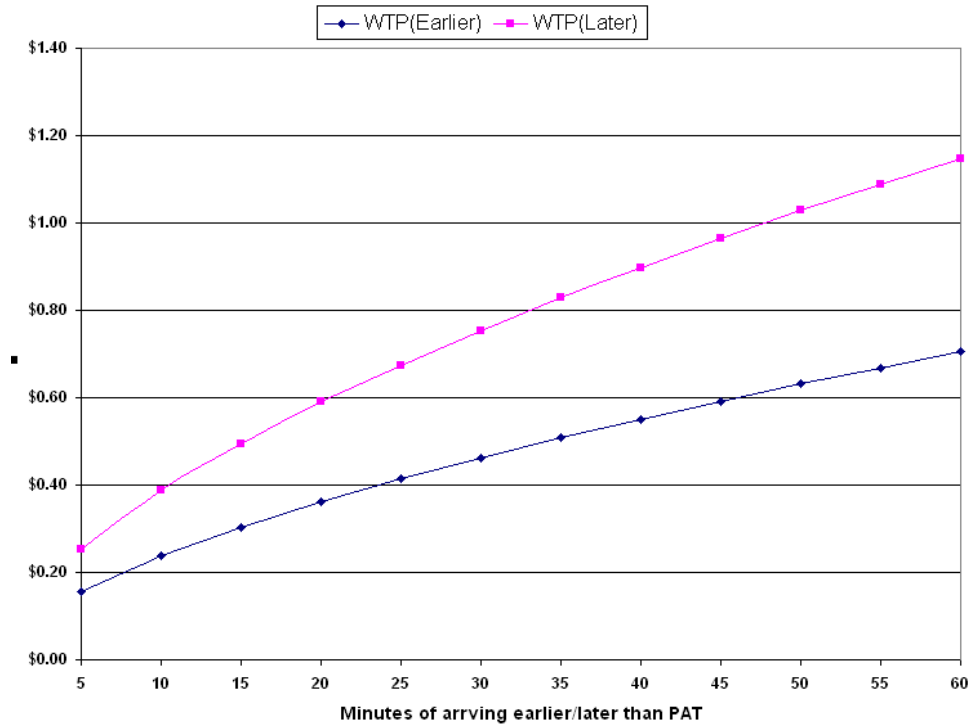
Figure 3: WTP and the minutes of being earlier/later (probability=0.1)

We can also calculate WTP values under different probabilities of occurrence by multiplying corresponding probabilities of arriving earlier and later, given that the probability of occurrence is linear to WTP (see equations 6a-6c). For example, the WTP for avoiding a 10-minute later arrival time with a probability of 0.25 is Au$0.97 per person. Using the above values as an example, if we improve the traffic condition from a 10-minute later arrival time with a probability of 0.25 to a 5-minute later arrival time with a probability of 0.1, the WTP is Au$0.81 per person (=Au$0.97 - Au$0.16). We argue that this is more realistic in the context of a specific transportation project, although it clearly requires knowledge of such a predicted distribution.

The calculated WTP values present another way to explain the identified risk-taking attitude. Suppose there are two scenarios, A: arriving 10 minutes later than the PAT with 100 percent chance of occurrence; B: a 50:50 chance of arriving 5 minutes or 15 minutes later than the PAT. Although both travel scenarios have the same expected later arrival time (10 minutes later), the average WTP for avoiding Scenario A (sure) is Au$3.87, which is slightly higher than the WTP for avoiding Scenario B (risky) which is Au$3.74(= Au$1.27+ Au$2.47). This implies that the sure one incurs higher disutility than the risky one, and hence the risky one is preferred (i.e., risk-taking). The underlying reason is implied decreasing marginal disutility (see Figure 4) due to the curvature of the estimated non-linear utility function where $1-\alpha < 1$ over the expected travel time or variability (minutes of arriving earlier/later than the PAT). We plot, in Figure 4, the utility curve over minutes of arriving later than PAT, all other attributes remaining unchanged. The disutility incurred by a 10-minute later arrival is -1.65, and the disutility incurred by a 50:50 chance of arriving 5 minutes or 15 minutes later is -1.60(=(1.08+2.11)/2). Under this utility shape, a risky scenario is preferred to a sure scenario with the equivalent expected later minutes. This also applies to the average travel time and time arriving earlier than the PAT with the same risk attitude parameter. Many prospect theoretical studies revealed risk-seeking attitudes (less than one) over monetary losses (e.g., 0.88 in Tversky and Kahneman 1992). The increase in travel time would have a similar effect as the increase in monetary losses and hence induce risk-taking attitudes, when the risk attitude parameter is less than one.
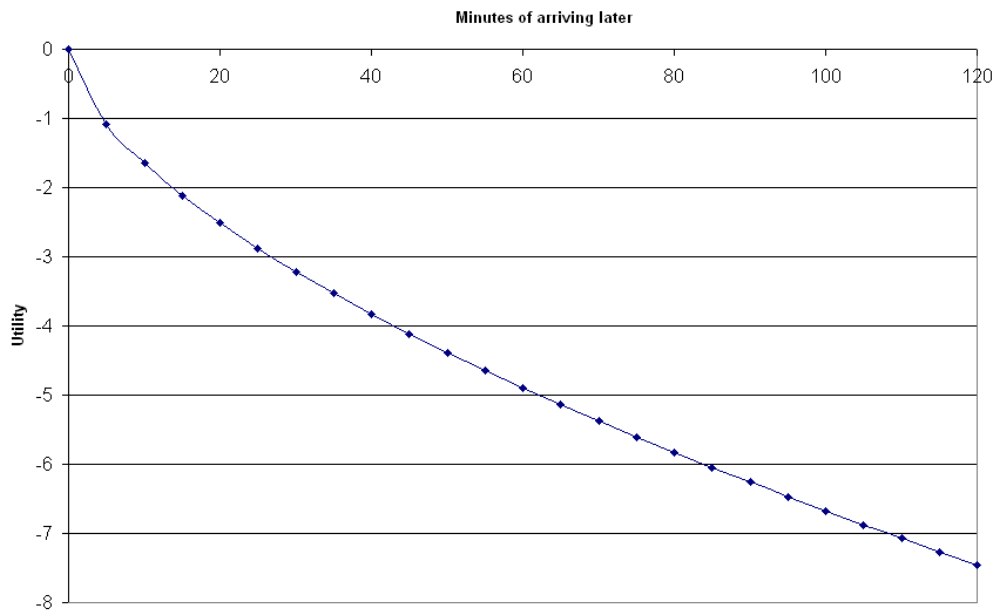
Figure 4: Non-linear utility curve over minuets of arriving later where the risk

attitude parameter (1-α) is less than 1

5. Accounting for unobserved heterogeneity: Mixed multinomial logit

Previous transportation studies that have incorporated a non-linear utility specification under the CRRA assumption have been estimated in a multinomial logit or probit framework (see e.g., Schwanen and Ettema 2009, Michea and Polak 2006).[8] In order to analyse unobserved heterogeneity in time-related parameters and risk attitudes, we extend our MNL non-linear scheduling model to a mixed MNL (MMNL) model. We also compare the behavioural responses and the values of WTP under MMNL and MNL. Constrained triangular distributions are used to ensure all individuals' time parameters are negative[9] and a non-constrained triangular distribution is applied to represent individual risk attitudes. The final MMNL model is given in Table 2.

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[8] The MMNL model of Polak *et al.* (2008) is under the assumption of CARA.

[9] Although a normal distribution can also be constrained and a lognormal distribution can produce all positive or negative individual parameters, they have some serious problems when estimating models (see Cherchi 2009 for a review).

Table 2: Scheduling model with embedded risk attitudes (MMNL)

Variable	Coefficient	*t*-Ratio
(Estimated using Nlogit5)		
Non-random parameters:		
Reference constant	0.6624	4.27
Cost ($)	-0.4307	-9.46
Tollasc	-0.3084	-1.83
Age (years)	0.0244	7.53
Means for random parameters:		
Average Time (minutes)	-1.3337	-18.18
Earlier (minutes)	-0.1938	-2.65
Later (minutes)	-0.3099	-4.52
Alpha (α)	0.7001	42.20
Standard deviations for random parameters:		
Average Time (minutes)	1.3337	18.18
Earlier (minutes)	0.1938	2.65
Later (minutes)	0.3099	4.52
Alpha (α)	0.1081	2.26
No. of observations	4480	
Information Criterion : AIC	5666.87	
Pseudo R-squared	0.43	
Log-likelihood	-2824.44	

Notes: Simulation based on 100 Halton draws

Compared with the MNL model in Table 1, the MMNL model delivers a significant improvement in model fit (AIC (scaled by sample size): 5666.87 vs. 6840.75). Non-random parameters of the MMNL model in Table 2 have the same sign relative to the MNL estimates, suggesting similar behavioural explanations as those presented in Section 4. All random parameters are significant at or above the 95 percent confidence interval, with the exception of the *Tollasc* parameter which is significant at the 90 percent confidence interval. The MMNL model also delivers a stronger mean risk-taking attitude (i.e., 1-0.7001=0.2999) relative to the MNL model (i.e., 1-0.3932=0.6068).

The parameter estimates at the individual level (for 280 respondents)[10] are shown in Figure 5. All individual estimates of *Alpha* are between 0 and 1, hence $1-\alpha<1$, i.e., risk taking. Heterogeneity in risk attitudes is significantly less compared with heterogeneity in the *Average Time* parameter (-2.5546 to -0.5905).

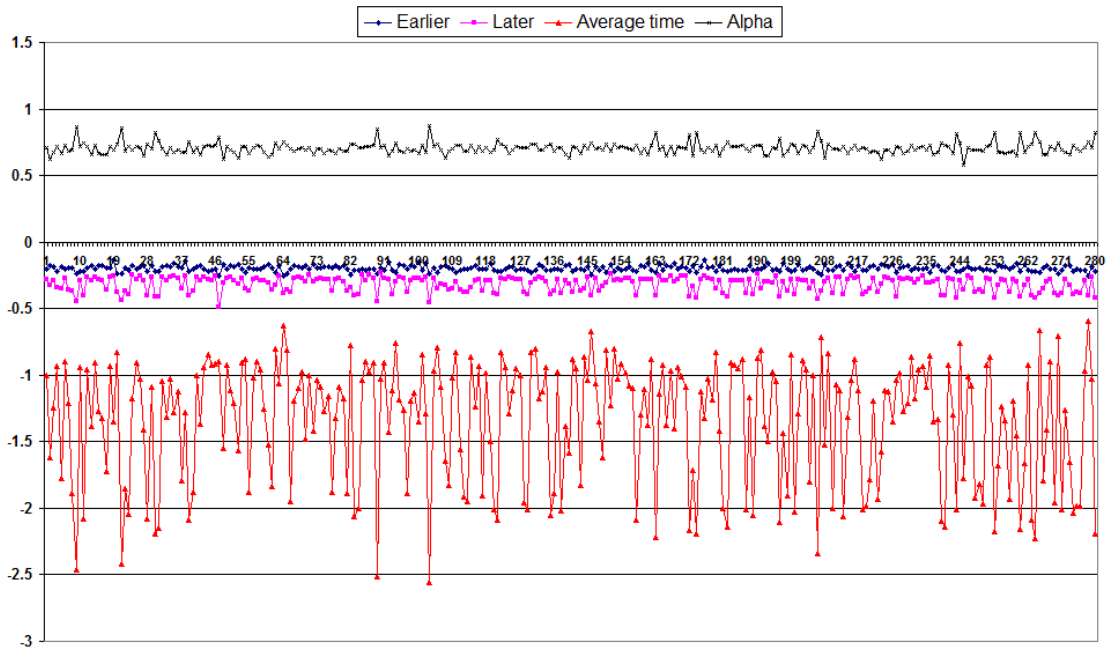[10] One respondent will have a corresponding value for *Alpha, Earlier, later and Average Time.*

Figure 5: Individual-level parameter estimates

Under the MMNL model, on average, each car commuter is willing to pay Au\$10.79 per hour (standard deviation: Au\$1.97) to reduce mean travel time, which is lower than the corresponding value under MNL (i.e., Au\$14.54). The MMNL non-linear model delivers lower mean values of variability than the MNL model. For example, for a 5-minute earlier and a 5-minute later both with a probability of occurrence being 0.1, the WTP values are Au\$0.07 and Au\$0.12 correspondingly (vs. Au\$0.16 and Au\$0.25 under MNL). Based on a linear mean-variance model, Brownstone and Small (2005) estimated lower value of variability and lower value of travel time savings under MMNL where the constant, travel time and variability parameters are treated as random parameters than under MNL.

6. Optimal departure time

In this section we develop a model to estimate the optimal (preferred) departure time (ODT) of commuters, for the MMNL non-linear scheduling model estimated in Section 5. To this end, we construct a utility function with stochastic travel times, which allows us to find expected travel times, probabilities of being early or late and expected early or late schedule delay for every possible departure time, choosing then the ODT as the one that maximises utility.

Travel time variability is mainly explained by inherent fluctuations in traffic demand and road supply (capacity). Notable sources of variability in traffic demand are (Tu 2008): temporal effects (e.g., peak/off-peak, weekday/weekend), network effects (effect of traffic in one lane or road over travel times on other parallel or intersecting lanes/roads), spatial and temporal differences in driving attitude, etc. On the other hand, factors such as volatile or adverse weather conditions, traffic incidents and accidents, and traffic composition influence both the value of instant road capacity. An increasing number of studies have treated capacity as a random variable rather than as a constant factor (Chen *et al.* 2002; Lo and Tung 2003; Li *et. al* 2008; Li 2009 among others). As an illustrative example, Figure 6 shows the variations in capacity measured at different times for a 2-lane freeway section in the Netherlands.
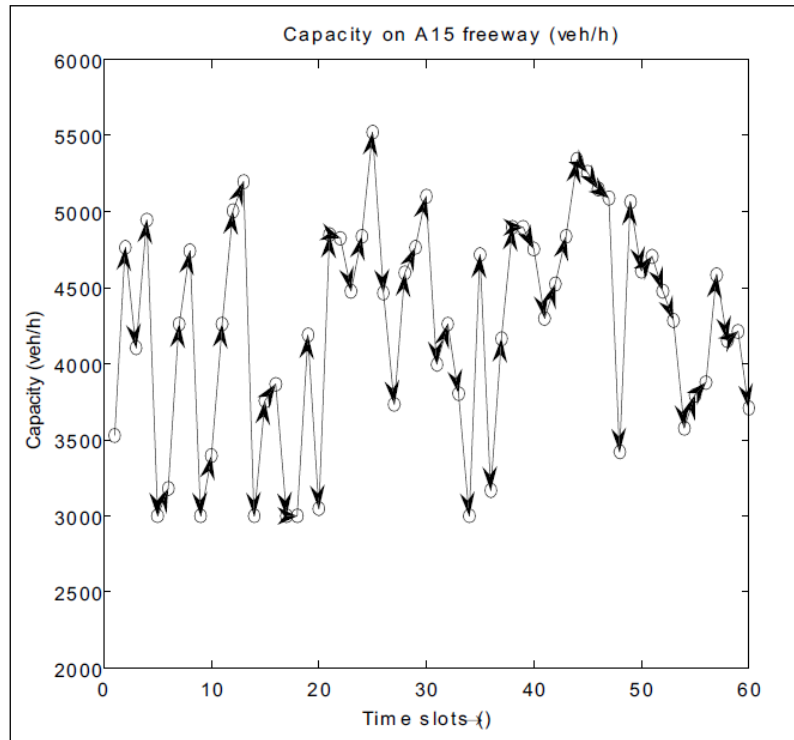
Figure 6: Capacity on a 2-lane freeway section (Source: Tu 2008)

In their model of departure time choice, Noland and Small (1995) disaggregate travel time into three components: free flow travel time, extra delay due to recurrent congestion, and extra delay due to non-recurrent congestion or incidents. The variability in travel times comes from modelling the incident related delay as a random variable that increases travel time over the other two (deterministic) components. In this paper we consider random capacities as the source of travel time variability. Different probability distributions for the random capacity have been proposed in the literature (e.g., uniform, normal, exponential, gamma and weibull). For the sake of simplicity, we will assume a uniform distribution, as in Chen *et al.* (2002) and Li *et al.* (2008).

Let k be the random capacity, with minimum and maximum values K_{min} and K_{max} and a uniform probability density function, $f(k) = 1/(K_{max} - K_{min})$. Consequently, travel time T is also a random variable as a function of k: $y = T(k)$. We use the U.S. Bureau of Public Roads (BPR) travel time function:

$$T(k) = T_f \left[1 + a \left(\frac{q}{k} \right)^b \right] \tag{7}$$

where T_f is the free flow travel time, q is the (deterministic) flow on a link and a and b are parameters (with conventional values a=0.15 and b=4). In order to calculate expected early and late schedule cost, we need to identify the probability density function $g(y)$ for $y = T(k)$, which can be calculated as equation (8).

$$g(y) = f\left(T^{-1}(y)\right)\left|\frac{d}{dy}T^{-1}(y)\right| = \frac{a^{1/b}q}{bT_f\left(K_{max} - K_{min}\right)\left(\dfrac{y}{T_f} - 1\right)^{\frac{b+1}{b}}} \qquad (8)$$

Note that given (7), we are assuming that different departure times *DEP* do not affect travel time itself, but they do affect the probability of being early or late, as in Noland and Small (1995). With this, let $\tilde{U}(DEP)$ be the component of utility (5) that depends on departure time:

$$\tilde{U}(DEP) = \beta_E P_E \frac{\left[PAT - DEP - E_E(T)\right]^{1-\alpha}}{1-\alpha} + \beta_L P_L \frac{\left[E_L(T) - (PAT - DEP)\right]^{1-\alpha}}{1-\alpha} \qquad (9)$$

where P_E and P_L are the probabilities of early and late arrival, and $E_E(T)$ and $E_L(T)$ are the expected travel times, given that the arrival is early and late, respectively. Assuming that both P_E and P_L are positive we have:

$$P_E = \int_{T_{min}}^{PAT - t_{dep}} g(y)\,dy \quad \text{and} \quad P_L = \int_{PAT - t_{dep}}^{T_{max}} g(y)\,dy \qquad (10a)$$

$$E_E(T) = \frac{\displaystyle\int_{T_{min}}^{PAT - t_{dep}} y\,g(y)\,dy}{\displaystyle\int_{T_{min}}^{PAT - t_{dep}} g(y)\,dy} \quad \text{and} \quad E_L(T) = \frac{\displaystyle\int_{PAT - t_{dep}}^{T_{max}} y\,g(y)\,dy}{\displaystyle\int_{PAT - t_{dep}}^{T_{max}} g(y)\,dy} \qquad (10b)$$

where T_{min} and T_{max} are the minimum and maximum travel times:

$$T_{min} = T_f\left[1 + a\left(\frac{q}{K_{max}}\right)^b\right] \qquad \text{and} \qquad T_{max} = T_f\left[1 + a\left(\frac{q}{K_{min}}\right)^b\right] \qquad (11)$$

Thus, the optimal departure time (*ODT*) is obtained as the one that maximises (9).[11]

$$ODT = \arg\max_{DEP} \tilde{U}(DEP) \qquad (12)$$

This must be found numerically. To analyse how the *ODT* varies with the variability of capacity (and consequently, with the variability of travel time), a dispersion factor θ around the mean is defined, such that $\theta K_{mean} = K_{max} - K_{min}$. Then, given θ and K_{mean}, the maximum and minimum capacities are $K_{min} = (1 - \theta/2)K_{mean}$ and $K_{max} = (1 + \theta/2)K_{mean}$. Assuming $K_{mean} = 1500$ veh/h, $T_f = 30$ min, $a=0.15$ and $b=4$, PAT=9 AM, $\beta_E = -0.1938$, $\beta_L = -0.3084$ and $\alpha = 0.7001$, Figure 7 depicts the optimal departure time for the

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[11] Note that maximising this utility function is equivalent to minimising the usual schedule cost function.

deterministic case (with capacity K_{mean}) and three stochastic scenarios with different levels of dispersion in capacity $\theta \in \{0.1, 0.5, 1.0\}$. As expected, after introducing stochasticity in capacity and travel time, optimal or preferred departure time is earlier than in a deterministic environment, and the greater the variability in capacity, the earlier commuters depart from home.
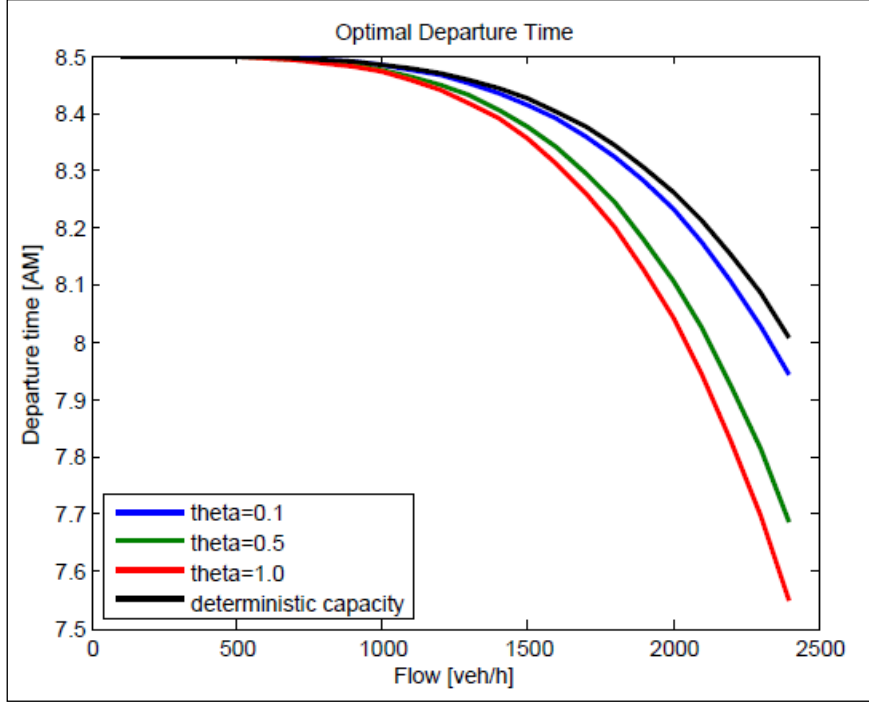

Figure 7: Optimal departure time under the non-linear scheduling model

In the case of a linear scheduling cost ($\alpha = 0$), it can be shown that the probability of late arrival for the *ODT* is $P_L^* = \beta_E / (\beta_E + \beta_L)$ (Bates *et al.* 2001), regardless of the distribution of travel time; nevertheless, when the utility is a non-linear function of schedule delays, P_L^* depends on the variability of travel time. As shown in Figure 8, the probability of being late for the optimal departure time increases with travel time variability, i.e., it is more likely that users will arrive late at destination despite the fact that they depart earlier to adjust to the greater uncertainty in travel times.

We are interested in comparing optimal departure times resulting from this non-linear model with a linear model. If we impose $\alpha = 0$ in utility expression (5), the MMNL parameters estimated for early and late arrival are $\beta_E^{linear} = -0.0782$ and $\beta_L^{linear} = -0.1606$. With these parameter estimates, we can calculate the constant value of optimal probability of late departure as $P_L^* = \beta_E^{linear} / (\beta_E^{linear} + \beta_L^{linear}) = 0.33$, which in Figure 8 intersects P_L^* of the non-linear model at $\theta = 0.83$. This means that for relatively low variability on travel times $(\theta < 0.83)$, the non-linear utility model will predict an earlier departure time than the linear utility model, whilst the opposite result holds if travel time is highly unpredictable $(\theta > 0.83)$. Figure 9 illustrates this for *q*=2000 veh/h.
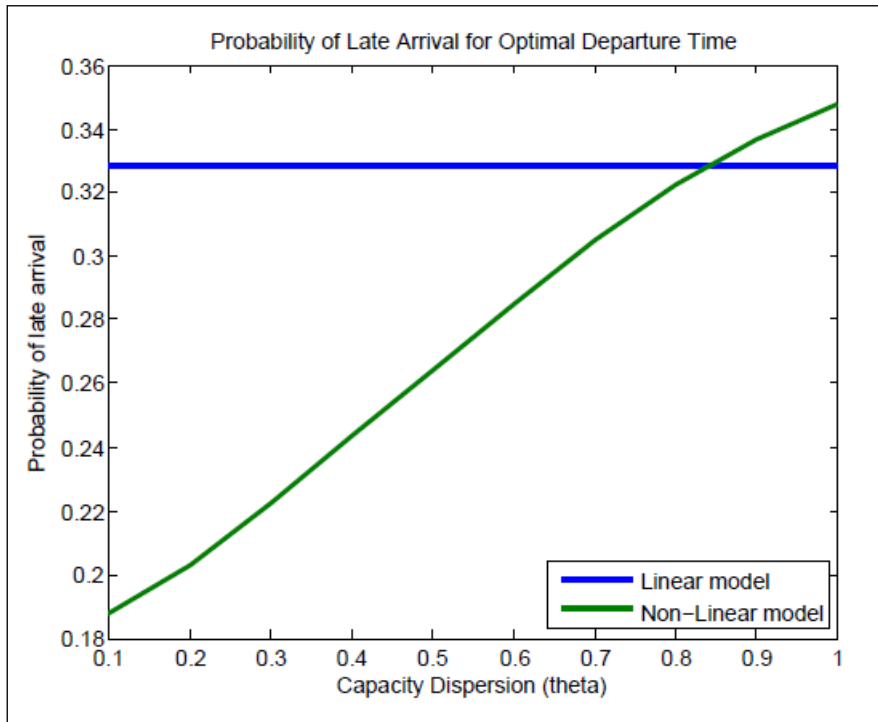
Figure 8: Probability of late arrival for optimal departure time, comparison of
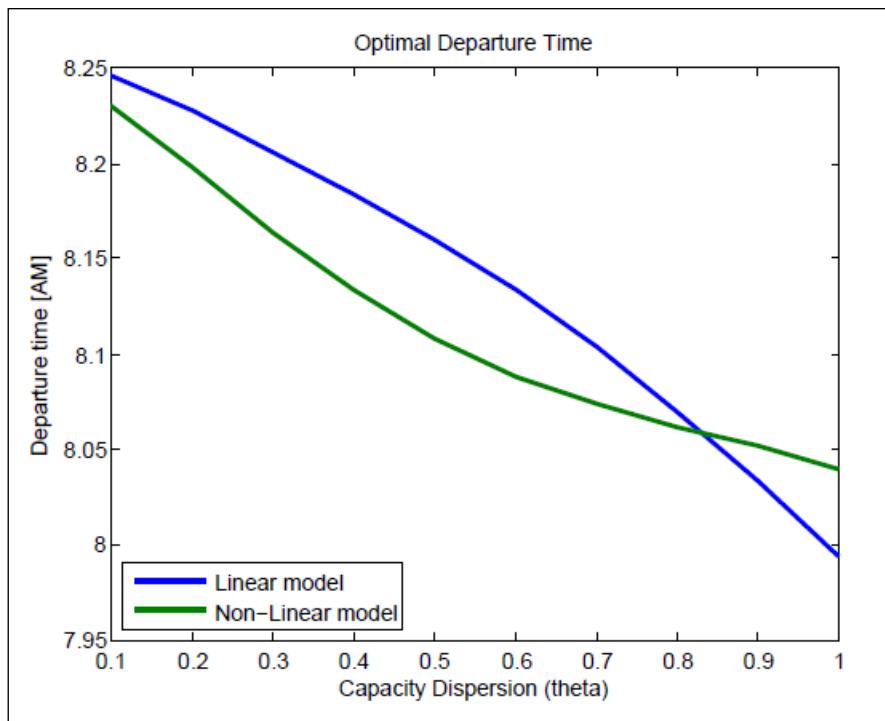
linear and non-linear models

Figure 9: Optimal departure time, comparison of linear and non-linear models

(Traffic flow=2000 veh/h)

In order to provide an educated guess about what model predicts an earlier departure time, we need empirical estimation of random capacity and their dispersions to obtain a range for the parameter θ for real roads. There is very little evidence on the empirical distribution on random capacities. We use results from Brilon *et al*. (2007) that estimate the expected values (*EV*) and standard deviations (*SD*) of the capacity for German highways (they find that the weibull distribution is the one that better suits the estimated values), and also the example of Figure 6 to have rough empirical estimations of θ, even though none of these examples follow a uniform distribution (see Figure 10).

Brilon *et al*. (2007, p. 4) report the *EV* and *SD* for two highways, whose values are $EV_1 = 4367$ veh/h, $SD_1 = 379$ veh/h, $EV_2 = 6874$ veh/h and $SD_2 = 690$ veh/h. If we assume an uniform distribution for these numbers, it yields $\theta_1 = \sqrt{12}SD_1\big/EV_1 = 0.30$ and $\theta_2 = \sqrt{12}SD_2\big/EV_2 = 0.35$. On the other hand, for the example of Figure 6 we would have $\theta_3 = \left(K_{max} - K_{min} \right)\big/K_{mean} = \left(5500 - 3000\right)\big/4246 = 0.59$. These three values fall in the range in which, for our estimated parameters, the non-linear model predicts an earlier departure time than the linear model $\left(\theta < 0.83 \right)$, which in turn suggests that the non-linear scheduling model predicts that our sampled commuters are more conservative when deciding on departure times, compared to the prediction of the traditional linear scheduling model.

Relative to the traditional linear scheduling model with a risk-neutral assumption, the empirical risk attitude estimation (1-α<1) of our non-linear scheduling model suggests that the sampled car commuters tend to be risk taking when making travel time-related decisions. On the other hand, the non-linear scheduling model suggests a more conservative behaviour in terms of departure time, based on empirical estimations of the capacity dispersion parameter ($\theta < 0.83$). This dichotomy can be explained with an example: given a equivalent expected later arrival time of 10 minutes, our sampled commuters would prefer a trip which has a 50 percent chance of arriving 5 minutes later and a 50 percent chance of arriving 15 minutes later, in contrast to a 100 percent (sure) chance of arriving 10 minutes later. As the consequence of this risk-taking attitude, they may have a prospect of a better or worse trip relative the sure one. At the same time, travellers may have conservative beliefs and hence tend to underweight the probability of a good trip. Therefore, they would depart earlier so as to reduce the likelihood of arriving too late relative to the preferred arrival time that may be due to the risk-taking attitude, given that the consequence of arriving late is much more serious than arriving early.

The appropriate interpretation of a behavioural outcome requires both risk attitudes and beliefs of the individual (see e.g., Wakker 2004; Dickinson 2009). For example, Dickinson (2009) found that his subjects are 'risk-averse yet optimistic' using a controlled bargaining experiment. He explained that risk aversion would have an opposite effect relative to optimism. It is expected that conservative beliefs would also offset the effect of risk-taking attitudes, which supports the behavioural implications described earlier.
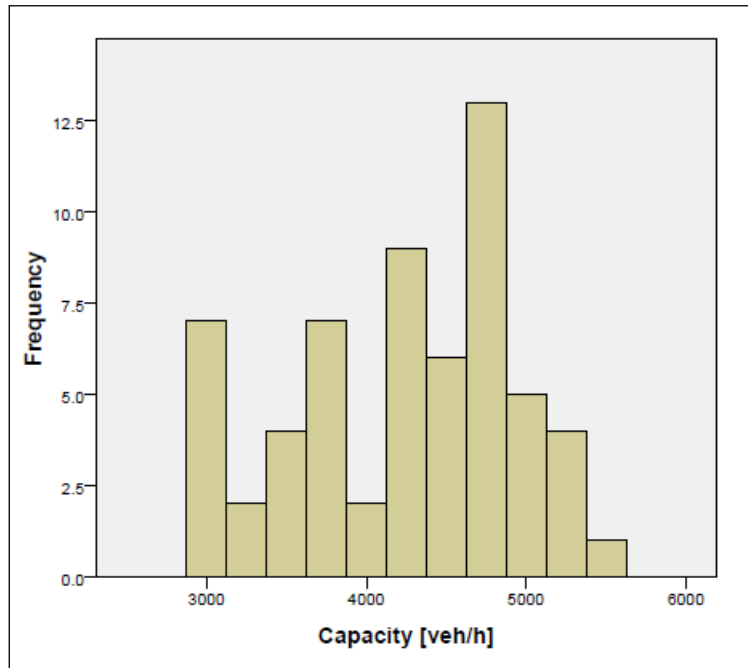
Figure 10: Histogram of frequencies of observed capacities for example of Figure 6

7. Does non-linearity matter?

In the previous section, we showed that non-linear scheduling would predict an earlier departure time, relative to the linear scheduling model. In this section, a comparison of the non-linear and linear models (MNL and MMNL) is carried out in terms of model fit and willingness to pay values, given in Table 3. In order to deliver a meaningful comparison, the same dataset is used and the same utility function is used, with the only difference being that the *Alpha* value is assumed to be 0 for the linear models. For MMNL, the simulation is based on 100 Halton draws for both non-linear and linear models, and constrained triangular distributions are applied to represent time-related random parameters (average travel time and variability).

The comparison suggests some interesting findings. First, the non-linear MNL and MMNL models deliver better model fit than their corresponding linear models. Although the non-linear MMNL model has two additional parameters (i.e., the mean and the standard deviation of *Alpha*), given its much better log-likelihood, the improvement of model fit over the linear MMNL model is substantial (AIC: 5666.87 vs. 6688.21). Secondly, the linear models tend to produce a higher mean WTP than the non-linear models. Given the MMNL model is superior to the MNL model, we focus on the empirical values of MMNL only, where the linear MMNL delivers a much higher mean value of time savings (e.g., Au\$24.39 vs. Au\$10.79) as well as the WTP for reduced variability, compared to the non-linear model's estimates. The better model fit suggests that the WTP values derived from the non-linear scheduling model should be used for calculating time and reliability benefits.

Table 3: Comparison of non-linear and linear models (WTP value: Au$2008)

	Non-linear MNL	Non-linear MMNL	Linear MNL	Linear MMNL
WTPs				
60-min mean travel time	14.54	10.79(1.97)	16.80	24.36(9.94)
5-min earlier (probability=0.1)	0.16	0.07(0.005)	0.18	0.13(0.05)
5-min later (probability=0.1)	0.25	0.12(0.008)	0.25	0.27(0.11)
Model fit				
Information Criterion : AIC	6840.75	5666.87	6850.05	6688.21
Log-likelihood	-3412.38	-2824.44	-3418.02	-3337.10

Notes: 4480 observations (280 respondents);
For MMNL models, the standard deviations are given in parentheses.

The above comparison shows that non-linearity in the utility function form does have a significant impact on model performance, and in this study, the non-linear scheduling model is superior to the linear model. Does this evidence suggest that we should always estimate non-linear utility models? This depends on the content of the stated choice experiment. If the experiment assumes a deterministic or risk-neutral environment (e.g., there is only one travel time (100 percent of occurrence) for an alternative within a choice set), the non-linear utility function seems to be unnecessary, although this assumption may not be realistic, given that travel time variability is embedded in real transport systems. However, if the design itself is embedded with risk (i.e., with multiple possible outcomes for an alternative within a choice set), the non-linear utility model should be estimated to reveal the real attitude towards risk.

In the transportation literature, there are some alternative non-linear utility models. For example, Masiero and Hensher (2010) applied the piecewise linear approximation technique to investigate 'quasi-nonlinearity' which is introduced in the punctuality attribute identifying two decrease and two increase levels with respect to the reference point. However, this approach still maintains the utility function linear in the parameters, and hence assumes that people are risk neutral. Another commonly used approach is to assume a level of non-linearity, e.g., squared attribute (x^2). For example, Small *et al.* (1999) included the squared expected SDE in their scheduling model, using a risky experiment with five equi-probable arrival scenarios (early, late or on time) for each alternative within a choice set, which implicitly assumes their sampled commuters are risk averse. Assuming risk attitude and using a squared time-related attributes may lead to biased findings and conclusions, unless the sampled respondents are indeed risk averse. Even is individuals are assumed to be risk averse, the assumed value (e.g., the risk attitude parameter is two when the squared attribute is used) may not properly address the extent of risk aversion. We suggest that the attitude towards risk should be empirically estimated.

8.   Conclusions

A primary interest of travel behaviour studies is the empirical evaluation of willingness to pay for travel time savings and reduced trip time variability. The majority of studies are established on a linear utility specification to reveal preferences of specific attributes. However, this specification overlooks the attitude towards risk, which also plays an important role in decision making. We develop a non-linear scheduling model to address both preferences and risk attitude. The study context is a stated choice experiment of commuter's choice amongst alternative packages of trip times (average and variability) and costs, where the risk is associated with trip times (i.e., there are more than one possible trip times), like other travel time variability studies.

Compared with a few transportation studies using a non-linear utility specification, a significant improvement of this study is the investigation of unobserved between-individual heterogeneity in time-related parameters and risk attitudes, based on a mixed multinomial (MMNL) model. We estimate taste, risk attitude and some socioeconomic parameters under both MNL and MMNL. Both models produce some similar behavioural explanations, for example, the sampled commuters tend to be risk taking when making time-related choices. In addition to a significant improvement in model fit, the MMNL model delivers much stronger risk-taking attitude than the MNL model. We also calculate willingness to pay (WTP) values under our non-linear scheduling model. With a non-linear utility specification and linear probability weighting, the WTP values for avoiding arriving earlier/later are influenced by the minutes of arriving earlier/later than the preferred time and associated probabilities of occurrence.

Assuming that travel time is a function of stochastic road capacity, we formulated a model to derive preferred or optimal departure times for commuting, given a flow-delay function and random distribution of capacity. It is found that the more variable travel times are, the earlier commuters depart, and that based on empirical estimation of the capacity dispersion, the non-linear scheduling model would predict earlier optimal departure times than the traditional linear scheduling model. The non-linear MMNL model also delivers a much better model fit and lower mean WTP values than the linear MMNL model.

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