Linking Discrete Choice to Continuous Demand in a Spatial Computable General Equilibrium Model

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ABSTRACT:

Discrete choice (DC) models are often used to describe consumer behaviour at a disaggregate level where the choice decision is defined in terms of a set of alternatives (commodities) differentiated mainly by their quality attributes rather than just prices, and individuals making the choice decisions are differentiated by their socio-economic characteristics rather than just income level. DC models therefore are rich in details which are important for policies analysis at a micro or intra-sectoral level (e.g., transport sector, housing sector). In contrast, continuous demand (CD) models are specialized in describing behaviour at an aggregate (inter-sectoral) level (e.g. trade-off between transport and land-use activities). DC and CD models are therefore complements rather than substitutes and increasingly, there is a need to integrate the use of both types of models especially in an economy-wide model to look at the impacts of policies which are implemented at a microeconomic level (e.g. investment in a particular transport network) and yet having impacts which are measured adequately only at an economy-wide level. This paper presents a methodology for integrating the use of DC and CD models in the framework of a computable general equilibrium (economy-wide) model. The paper also illustrates the application of this methodology suggested in an empirical example, taken from a study of the investment in the Northwest Rail network in the Sydney Metropolitan Area (Australia).

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1. Introduction

Discrete choice (DC) models are often used to describe consumer[1] behaviour at a disaggregate level.[2] At this level of observation, a choice decision can be described in terms of a set of alternatives which represent different 'varieties' of a particular product differentiated mainly by their quality or technological attributes, and the individuals or households making the choice decisions are also differentiated by their varied socio-economic characteristics. DC models are thus often rich in details regarding commodity attributes and individual characteristics and therefore can be used to analyse behavioural[3] responses to policies at a microeconomic and intra-sectoral level (e.g., choice decisions within the transport sector, or within the housing sector)). This is in comparison with traditional 'continuous demand' (CD) models[4] which are often lacking in these details but are specialized to look at aggregate choice behaviour at an inter-sectoral level (choice or trade-off decisions between transport and housing activities, or transport and telecommunication, etc). In the past, DC and CD models are often used separately and considered as though substitutes rather than as complements, but there is now an increasing need to consider the use of both types of models within the same the same framework to look at issues which are decided at an individual and microeconomic level (choice of activities, lifestyle and technologies) but have implications at the national and perhaps even global level (international trade and environmental issues such as global warming, etc.). Using both of these types of models within the same framework (such as that of a computable general equilibrium (CGE) model), however, requires some reconciliation and integration of the two types of theoretical approaches and empirical data used by both types of models. For example, DC models are based on the concept of a 'random utility' and describe choice/demand behaviour in terms of a probabilistic distribution rather than as a deterministic outcome. In contrast, CD models are based on the concept of a (deterministic) 'representative' individual with a specific utility or preference structure (e.g. constant-elasticity-of-substitution (CES) function) and the demand outcome from such a model is also deterministic rather than probabilistic. A question thus arise, and that is: under what conditions can the latter type of (aggregate deterministic) behaviour of a CD model be considered as consistent with the aggregation of all the individualistic (and random) behaviour

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[1] In principle, there is no reason why DC model cannot also be used to describe *producer* behvaiour (e.g. choice decision between different technologies for producing a particular commodity such as electricity) although thus far, DC models are employed mainly to describe only consumer behaviour.

[2] by 'disaggregate' it is meant individual or household level.

[3] And/or technological responses, if the choice decision involves the producer and the supply side as well as demand side (e.g. choice of electric cars versus conventional fossil-fuel based cars).

[4] Here the reference is to 'conventional' consumer demand models (e.g. Deaton and Muelbauer, 1980) rather than to the model systems that jointly develop and estimate a discrete choice (e.g., automobile type choice) and an intra-sectoral continuous choice (e.g., vehicle kilometres travelled by the chosen automobile); for examples, Hanemann (1984), Bhat et al. (2009).

of a DC model? This is the issue considered in this paper. The paper presents a methodology for reconciling the two different theoretical frameworks of DC and CD models and suggests a way for integrating the internal structures of the two types of models within the same modeling framework, using a computable general equilibrium (CGE) model as an example.[5] The paper then applies the methodology to an empirical study to illustrate the usefulness of the methodology suggested.

The plan of this paper is as follows. Section 2 discusses the similarities and differences between DC and CD models both from a theoretical as well as empirical viewpoint. Section 3 shows how DC and CD models can be used in an integrated fashion taking into account their similarities and differences. Section 4 illustrates the applicability of the methodology of integration with an empirical example taken from a study on the impacts of transport system improvement on the wider economy.. Section 5 provides some conclusions.

2. DC and CD models – similarities, differences and interrelationships

DC and CD models are similar in the sense that they both are based on the theory of individual utility maximisation subject to a constraint. In the case of DC model, the constraint is described in terms of a discrete choice set. In the case of a CD model, the constraint is expressed in terms of a continuous 'budget set'. From a theoretical viewpoint, the terms 'choice' and 'demand' can be used interchangeably with 'demand' implies 'choice within a budget set'. Empirically, however, the term 'choice' is often used to describe the discrete behaviour of a single individual (or single household) while 'demand' is used to refer to the continuous aggregate behaviour of a group of individuals or household. Individual discrete choice behaviour is also described in terms of a 'random' utility function because the preferences (or utilities) of different individuals are different or 'heterogeneous'. In contrast, aggregate demand behaviour is often described in terms of a deterministic 'representative' utility function which refers only to the aggregate (or 'average') of all the preferences of individuals within the group. 'Demand' therefore is the aggregate of all individual choices (either choices of a single individual over a period of time, or of different individuals at a particular time), and the important question is: under what conditions can the former be said to be a good representation of the latter? For example, under what conditions can the deterministic 'representative' preference or utility function of a CD model be said to be consistent with

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[5] Previous attempts at integrating the use of a DC model within a CGE framework (see for example, Horridge (1994) look only at the use of the probabilistic discrete choice function (such as multinomial logit) to replace the use of a conventional CD function (such as that based on CES utility) in a CGE but not looking thoroughly at the different theoretical and empirical foundations of DC and CD models to see how they can fit together within the framework of a CGE model. This is the issue considered in this paper.

the disaggregate behaviour of all the 'random' individuals in a DC model; can the 'budget set' of the representative individual in a CD model be said to be consistent with the different choice sets of the disaggregate individuals in a DC model? From a theoretical viewpoint, these issues are related to the question of aggregation bias or consistency considered in the traditional (aggregate) economic theory of consumer behaviour, for example, Gorman (1961). The challenge is to relate these traditional discussions to the question of how to link DC to CD models in a consistent manner.

Consider, for example, Gorman (1961) important results. Here is shown that if all the (random, or heterogeneous) individuals/households face with the same set of prices (p={pi}) for different choice commodities/alternatives i's and if the indirect utility function of each individual has an underlying 'polar' utility form (described below) then a consistent aggregate preference structure for the representative individual can be constructed. The 'Gorman polar form' of the indirect utility function can be described as follows:

$$V^h(\boldsymbol{P}, Y^h) = \frac{Y^h - f^h(\boldsymbol{P})}{g^h(\boldsymbol{P})}$$

(1)

where Yh is the budget allocated to the choice activities of individual h; $f^h(.)$ can be regarded as the minimum expenditure level needed to reach a base utility level of 0 for the choice activities; (Yh- $f^h(.)$) therefore can be regarded as the 'excess income' level used to reach a (maximum) positive utility level for the choice activities; $g^h(.)$ is a price index function used to deflate the excess income to reach an equivalent 'real income' level which is represented by Vh(.). The functions $f^h(.)$ and $g^h(.)$ must be homogeneous of degree 1 in prices (so that the expenditure function derivable from the indirect utility function (1) also exhibits this property). The 'real income' function Vh(.) can be regarded as a kind of quantity index6 for the choice activities because it is seen to be given by an expenditure function divided by a price index function. Using Roy's identity, the Marshallian demand for choice/commodity i in the choice set I by individual h can be derived:

$$Q_i^h(\boldsymbol{P}, Y^h) = \frac{\partial V^h / \partial P_i}{\partial V^h / \partial Y^h} = \frac{\partial f^h(\boldsymbol{P})}{\partial P_i} + \frac{\partial g^h(\boldsymbol{P})}{\partial P_i} \cdot \frac{Y^h - f^h(\boldsymbol{P})}{g^h(\boldsymbol{P})}$$

$$= f_i^h(\boldsymbol{P}) + V^h(\boldsymbol{P}, Y^h)\, g_i^h(\boldsymbol{P}); \quad i \in I$$

(2)

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[6] This quantity index is an abstract number and may not be equal exactly to the total 'number of choices' assumed for all choice activities. In some cases, however, if a DC model is used as a 'quantity share' (rather than as an expenditure share) function - see below in the next section - then the quantity index must be related exactly to the total quantity level of the demand/choice activities.

where $f_i^h(.)$ and $g_i^h(.)$ are the partial derivatives of $f^h(.)$ and $g^h(.)$ respectively with respect to the price Pi of choice alternative i. From equation (2), it can be seen that if the coefficient of the income variable Yh is simply $g_i^h(\boldsymbol{P})$ (which does not depend on income), then the individual demand curve for each choice alternative i is linear in income. Furthermore, if $g_i^h(\boldsymbol{P})$ is also independent of the individual index h., i.e. $g_i^h(\boldsymbol{P}) = g_i(\boldsymbol{P}); \forall i$ then all the linear Engel curves are parallel. Under these conditions, an aggregate (representative) demand function for each choice alternative i can be constructed, from and consistent with, the individual demand curves of all disaggregate individuals, once the forms of the functions fh(.) and g(.) are assumed or given.

2.1 *Example: a 'representative' consumer theory of the disaggregate MNL DC model*

Anderson, Palma, and Thisse (1988a,b) have shown that an aggregate or 'representative' theory of the Multinomial Logit (MNL) DC model can be constructed. The theory is based on the assumption of a utility structure for the representative individual either of the form of an 'entropy-type' function, or of a CES form. In the former case, if the representative individual is maximising this entropy-type utility function subject to a total quantity constraint for all the choice decisions, then the results will be a demand quantity share model for each choice alternative which is of a form similar to the MNL DC model. In the latter case, if the representative individual is maximising a CES utility function subject to a total expenditure constraint for all the choice alternatives, then the results will be a demand expenditure share model which is also of a form similar to the MNL DC model. This means an aggregate CD system can be said to exist which is 'equivalent' to the MNL DC model if the MNL DC model is interpreted either as a quantity share or expenditure share demand function. To look at this issue in a general way, consider the following MNL DC model:

$$Prob_i^h = \frac{\exp(V_i^h)}{\sum_{j \in I} \exp(V_j^h)}; \quad i \in I; h \in H.$$

(3)

Here $Prob_i^h$ is the probability of alternative i from a choice set I being chosen by individual h who belongs to a sample H; V_i^h is the deterministic part of a random indirect utility function defined for individual h and choice alternative i. This indirect utility is normally specified as a function of the (observed) attributes of the choice alternative i as well as (observed) characteristics of the individual h:

$$V_i^h = V_i^h(A_i, \boldsymbol{B^h}; \boldsymbol{\alpha}, \boldsymbol{\beta}), \quad i \in I.$$

(4)

Here A_i, B^h stand for the vectors of the observed attribute of alternative i and observed characteristics of individual h respectively and α, β are the corresponding parameter vectors. For a basic MNL, the function (4) can take on a simple form:

$$V_i^h = \sum_{m \in M} \alpha_m A_{im}, \quad i \in I.$$

(4a)

where $m \in M$ refers to the set of attributes describing each choice alternative.7 For a 'mixed' (or random coefficient) MNL model, the indirect utility function can include terms which shows the interactions between choice attributes and individual characteristics:8

$$\begin{aligned}
V_i^h &= \sum_{m \in M} \tilde{\alpha}_m A_{im} \\
&= \sum_{m \in M} [\alpha_m + \sum_{k \in K} \beta_{mk} B_k^h] A_{im} \\
&= \sum_{m \in M} \alpha_m A_{im} + \sum_{m \in M} \sum_{k \in K} \beta_{mk} B_k^h A_{im}, \quad i \in I.
\end{aligned}$$

(4b)

Here $\tilde{\alpha}_m$ stands for the random coefficient of choice attribute m which consists of a deterministic part α_m and a random part which shows the interactions between choice attributes and individual characteristics $k \in K$. Because of the existence of unobserved attributes of the choice alternatives as well as unobserved characteristics of the individual, the empirical indirect utility function must contains a random error term ε_i^h which represents the value of these unobserved variables.

$$U_i^h = V_i^h + \varepsilon_i^h$$

(5)

Given the indirect utility function as specified in (4)-(5), an individual h is said to chose alternative i over all other alternatives $j \neq i$ if and only if $U_i^h > U_j^h$, i.e. $(\varepsilon_i^h - \varepsilon_j^h) > (V_j^h - V_i^h)$ for all $j \neq i$. Depending on the distribution of the random error term, different choice models can be derived. For example, if ε_i^h 's are assumed to be independently and identically distributed (i.i.d.) as a Weibull distribution9, then the probability of condition $(\varepsilon_i^h - \varepsilon_j^h) > (V_j^h - V_i^h)$ being satisfied is given by the choice probability function (3).

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[7] Including the 'alternative-specific constant' as a generic 'attribute' for each alternative where necessary.
[8] See for example, Berry, Levinshon, and Pakes (2004) and also Hensher and Greene (2003), Train (2003), Greene and Hensher (2010).
[9] If the distribution is normal rather than Weibull (also called extreme value type I distribution) then the choice probability function will take on a different form which is referred to as the 'probit' model.

Now, consider the issue of whether the probabilistic choice behavior of the disaggregate individuals h's in a sample H can be said to be equivalent to the typical behaviour of a 'representative' individual called 'H' as described by an aggregate demand model. First, assume that the typical or representative individual faces a price index vector PH for all the choice alternatives and which can be related to the choice attributes, either in level form:

$$P_i^H = (1/\alpha_1)[\sum_{m \in M} \alpha_m A_{im}], \quad i \in I.$$

(6a)

or, alternatively, in log form10:

$$\ln P_i^H = (1/\alpha_1)[\sum_{m \in M} \alpha_m A_{im}], \quad i \in I.$$

(6b)

Noting that α_1 is the parameter of the cost (or price) attribute A_{i1} in the indirect utility function and which can be said to represent the (constant) marginal disutility of the cost attribute (hence it is usually negative).11 The (positive) value of - α_1 therefore represents the marginal utility of money as estimated from a DC model. The values of other parameters $\alpha_m \neq \alpha_1$ are said to represent the (constant) marginal utilities of other 'quality' attributes $A_{im} \neq A_{i1}$. The summation term on the right hand side of equation (6) therefore can be said to represent the total 'quality-adjusted' (indirect) utility of the choice alternative i which is then normalised by the marginal disutility of the money cost attribute to give a quality-adjusted price index for the choice alternative i. In the case of the basic MNL model, this price index is independent of the characteristics of the individuals and depends only on the levels of attributes of the choice alternative. In the case of a mixed or random coefficient MNL model, however, since the random parameters $\tilde{\alpha}_m$ are also dependent on the characteristics of the 'representative' individual ($\boldsymbol{B}^H$):

$$\tilde{\alpha}_m = \alpha_m + \sum_{k \in K} \beta_{mk} B_k^H, \quad m \in M.$$

(7)

the price index therefore is dependent also on these characteristics. In the case of the level form equation (6a), this is now replaced by:

$$P_i^H = (1/\tilde{\alpha}_1)[\sum_{m \in M} \tilde{\alpha}_m A_{im}], \quad i \in I.$$

(8a)

Or, alternatively, for the case of equation (6b), this is now replaced by:

$$\ln P_i^H = (1/\tilde{\alpha}_1)[\sum_{m \in M} \tilde{\alpha}_m A_{im}], \quad i \in I.$$

(8b)

The indirect utility function for each choice alternative can now be expressed in terms of the price index as follows:

$$V_i^H = \alpha_1^H P_i^H, \quad i \in I.$$

(9a)

for the case of the linear price index function (8a); or alternatively

$$V_i^H = \alpha_1^H \ln P_i^H \quad \Rightarrow \quad \exp(V_i^H) = (P_i^H)^{\alpha_1^H}, \quad i \in I.$$

(9b)

for the case of the log-price index function (8b).

2.2   MNL DC model as a conditional quantity share demand model

Given the definition of the price index and the indirect utility function *for each choice alternative i* as shown by equation (9), the next step is to define the indirect utility function for *all* choice activities as a whole. Following from the analysis of Gorman (1961), this (aggregate) indirect utility function for the 'representative individual will be consistent with the (disaggregate) indirect utilities of all the 'random' individuals in the MNL DC model if it is of the Gorman polar form as shown in equation (1), i.e.:

$$V^H(P^H, Y^H) = \frac{Y^H - f^H(P^H)}{g(P^H)}$$

(10)

where P^H is the *aggregate* price index vector for *all* choice alternatives as seen by the representative individual, and Y^H is the representative individual's income level. An important issue is the form which the functions $f^H(.)$ and $g(.)$ can take.

Following from Hausman Leonard and McFadden (1995), define $g(.)$ as a 'logsum' function as follows[12]

$$g(P) = \frac{1}{\gamma} \ln \sum_{i \in I} \exp(\gamma P_i)$$

(11)

Also, using the linear disaggregate price index functions as defined by equation (8a) or (9a), i.e. $P_i = (1/\alpha_1)V_i$, and assuming that $\boxed{} = \alpha_1$, we have:

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[12] Henceforth, the superscript '*H*' will be omitted for simplicity if it is clear that reference is to the 'representative' individual rather than to the disaggregate (random) individuals.

$$g_i(P) = \frac{\partial g(P)}{\partial P_i} = \frac{\exp(\gamma P_i)}{\sum_{i \in I} \exp(\gamma P_i)} = \frac{\exp(V_i)}{\sum_{i \in I} \exp(V_i)} = Prob_i \qquad (12)$$

Substituting this into (2) and also assuming a form for $f(.)$ such that $f_i(.) = 0$ for all i's, this gives (for the representative individual):

$$Q_i(P, Y) = (Prob_i) \left[\frac{Y - f(P)}{g(P),} \right], \quad i \in I \qquad (13)$$

The term in the square bracket on the right hand side can be interpreted as the *quantity index* representing the total aggregate demand (for all choice alternatives), and therefore the probability function $Prob_i$ can be interpreted as a conditional *quantity* share demand function.

2.3   MNL DC model as a conditional expenditure share demand model

The choice of the logsum function (11) to represent the aggregate price index $g(.)$ suffers from one problem as pointed out by Rouwendal and Boter (2009)), and that is, the logsum is not homogeneous of degree 1 in all the prices which is required by the Gorman polar form. Therefore, an alternative to the logsum function (11) is the CES function used by Rouwendal and Boter (2009):

$$g(P) = \left[\sum_{i \in I} (P_i)^\gamma \right]^{1/\gamma} \qquad (14)$$

It can be seen that $g(P)$ is now homogeneous of degree 1 in all the prices P_i's. Furthermore, assuming that the disaggregate price index for each choice alternative is given by the log form of equation (8b) or (9b) rather than (8a) or (9a), we have $(P_i)^{\alpha_1} = \exp(V_i)$. This means that if $\square = \alpha_1$, equation (14) can be re-written as:

$$g(P) = \left[\sum_{i \in I} \exp(V_i) \right]^{1/\alpha_1} \qquad (14a)$$

From equation (14), we can derive:

$$g_i(P) = \frac{\partial g(P)}{\partial P_i} = \frac{1}{P_i} \frac{(P_i)^\gamma}{\sum_{j \in I}(P_j)^\gamma} g(P)$$

$$= \frac{1}{P_i} \frac{\exp(V_i)}{\sum_{j \in I}\exp(V_j)} g(P)$$

$$= \frac{1}{P_i}(Prob_i)g(P) \tag{15}$$

Substituting (15) into (2) and assuming that $f_i(.)$ =0 for all i's, we have

$$Q_i(P,Y) = \frac{\partial g(P)}{\partial P_i} \cdot \frac{Y - f(P)}{g(P)}$$

$$= \frac{1}{P_i}(Prob_i)[Y - f(P)]; \quad i \in I \tag{16}$$

Here, the interpretation of the MNL DC model is different from that given by (13). The term $[Y - f(P)]$ on the right hand side represents the level of 'excess' expenditure (over and above the minimum level $f(P)$ required to reach a zero utility level for choice activities).13 This excess expenditure is to be allocated among the choice alternatives according to the choice probability function Probi, i.e. the DC model is used as a conditional expenditure share model (rather than a conditional quantity share model as in the case of equation (13)). Finally, dividing the expenditure level for each choice alternative i by its specific price index Pi will give the level of quantity demanded of this choice alternative.

From equations (13) and (16), it can now be concluded that a MNL DC model can be interpreted either as a conditional quantity share demand model (as in the case of Hausman et al., (1995)), or as a conditional expenditure share demand model (as in the case of Rouwendal and Boter (2009)). This depends on the assumptions made about the nature of the commodities chosen and the form of the disaggregate and aggregate price indices used to describe the (unit) cost of each choice alternative and of the choice activity as a whole. The forms of these price index functions of course are related to the forms of the underlying utility assumed for the choice activities. For example, a CES-type functional form for the aggregate price index (equation (14)) with the associate 'log form' for the disaggregate price index (equation (8b) or (9b)) are known to to be related to an underlying CES utility function which leads to the interpretation of the DC model as a conditional expenditure share demand model (see Anderson et al. (1988a)). In contrast, a 'logsum' function for the aggregate price index (equation (11)) and the associate

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[13] Often, $f(P)$ can be assumed to be zero, hence $[Y - f(P)]$ is simply equal to Y, the total expenditure level allocated to all the choice alternatives.

'linear form' of the disaggregate price for each choice alternative (equation (8a) or (9a)) can be said to represent an underlying entropy-type utility function and resulting in the interpretation of the DC model as a conditional expenditure share demand model (see Anderson et al. (1988b)). Although both of these types of conditional demand DC models can be used to link to an (unconditional) CD model, it is now clear that only the CES form and the expenditure share DC model is seen to be consistent with a Gorman polar utility form approach.14 In the case of the entropy-type utility and logsum aggregate price function, although the aggregation of all quantity shares may not lead to a consistent aggregation of the expenditures (and prices), the link between DC and CD model is conducted via an aggregate quantity index rather than price index, and the latter (relative prices) are used only to distribute the total quantity to the various alternative choices. The absolute level of the prices may need to be 'calibrated' but this is to be expected because the indirect utility/cost function in a DC model is normally specified only up to an arbitrary origin.15

3. Linking DC models to CD models in a general equilibrium framework

As can be seen from the previous section, one way of linking a DC to a CD model is to assume a two-stage decision process, with the DC decisions representing the 'lower stages' which are concerned primarily with the choices or substitution between *varieties* of a particular product (such as modes of travel, types of cars, modes of dwellings, etc.). These lower-stage decisions are then linked to an 'upper stage' which is concerned primarily with the choices or substitution between aggregate *groups* of commodities (such as between travel and housing, energy and food, work and recreation, etc.) and which are subject to a total expenditure constraint. Crucial in the link between lower stage DC models and upper stage CD models is the formation of the aggregate and disaggregate price/quantity indices which represent the various choice commodities in the lower and upper stages. These price/quantity indices must be consistent with each other at all stages. This would not be a problem if both the lower and upper stages are described in terms of the same theoretical framework but in this case, because there is a mixture of the random utility structure of a DC model and the deterministic utility structure of a CD model, these different structures need to be reconciled so as to ensure consistency in aggregation.

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[14] This result is of no surprise because only the use of a DC model as a conditional expenditure share model that can lead to the obvious result that the aggregation of all individual expenditures of all different choices will always be equal to the total expenditure. The aggregation of quantity shares into an aggregate quantity does not lead to a consistent aggregation of expenditure shares because of the price element.

[15] This is because replacing the value of V_i by $V_i + C$ where C is any constant will not change the choice probabilities in the discrete choice model but this will change the value of the price index (logsum) by a constant.

Consider the following example. Assume that there is an improvement in the transport system such that the (generalised)[16] costs of travel along certain routes are reduced. In an economy where the cost of certain activity ("work") by an individual consists mainly of travel and housing costs, a reduction in transport cost can be passed on to housing expenditure (in the form of higher rent due to increasing demand for housing in a particular area - see, for example, Figure 1, adapted from Venables (2007)).[17] In general, however, such an improvement can also result in a higher level of other activities including transport itself. To consider this issue, an 'upper-stage' demand system may need to be considered where the income effect of the transport improvement (reduction in transport costs is considered as an increase in real income for the workers) as well as the substitution effect between transport and all other activities must be considered (now that the relative price between transport and other commodity prices have changed). The net result of the income and substitution effects would tell us whether an improvement in the transport system would result in more (or less) travel and/or other activities. This ultimately would depend on the structure of the transport system and the pattern of land uses (housing, employment, and other activities) in a particular city. Therefore, an integrated model of DC and CD module linkages (Figure 2) must be considered and put in the context of a spatial general equilibrium (SGEM) model of the city where all the distributions of transport, housing and other economic activities of the city are taken into account.

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[16] The generalized costs of travel may include non-monetary costs such as travel time 'costs', the (hedonic) 'costs' comfort or convenience associated with a particular travel mode, etc. In a DC model of mode choice , the generalized cost of a particular mode can be measured by the value of the indirect utility of the mode divided by the marginal utility of the money cost, i.e. by the (generalized) price index of a mode such as described by equations (8)-(9).

[17] Venables (2007) considered only a monocentric city. Here it is adapted to consider a polycentric city where the 'center of employment' is a particular zone considered in relation to all other zones. The wage gap, rent gap and commuting costs therefore are measured in terms of the average between the zone and all other employment centers.
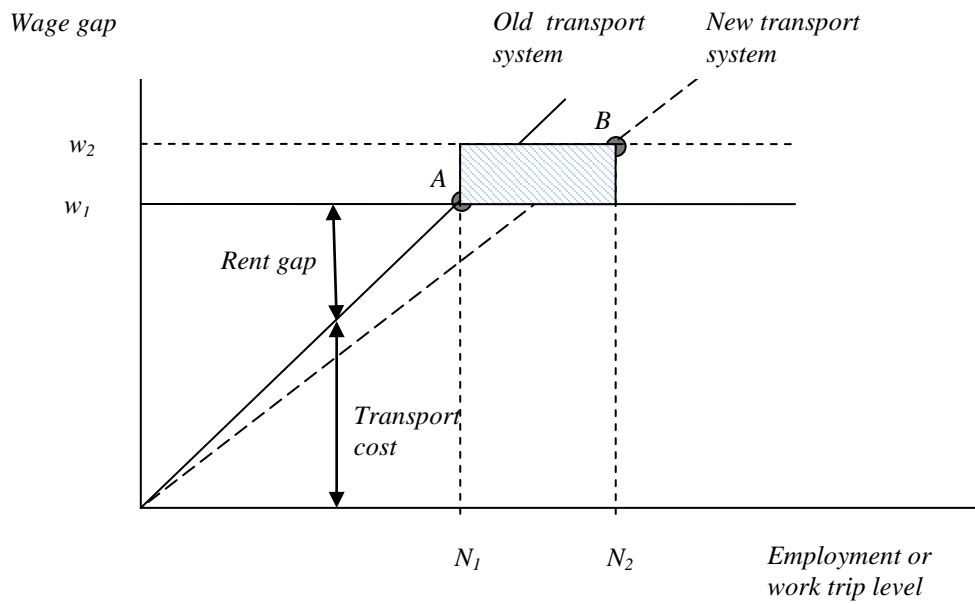
Figure 1 Net Gains from Transport Improvement – polycentric city.
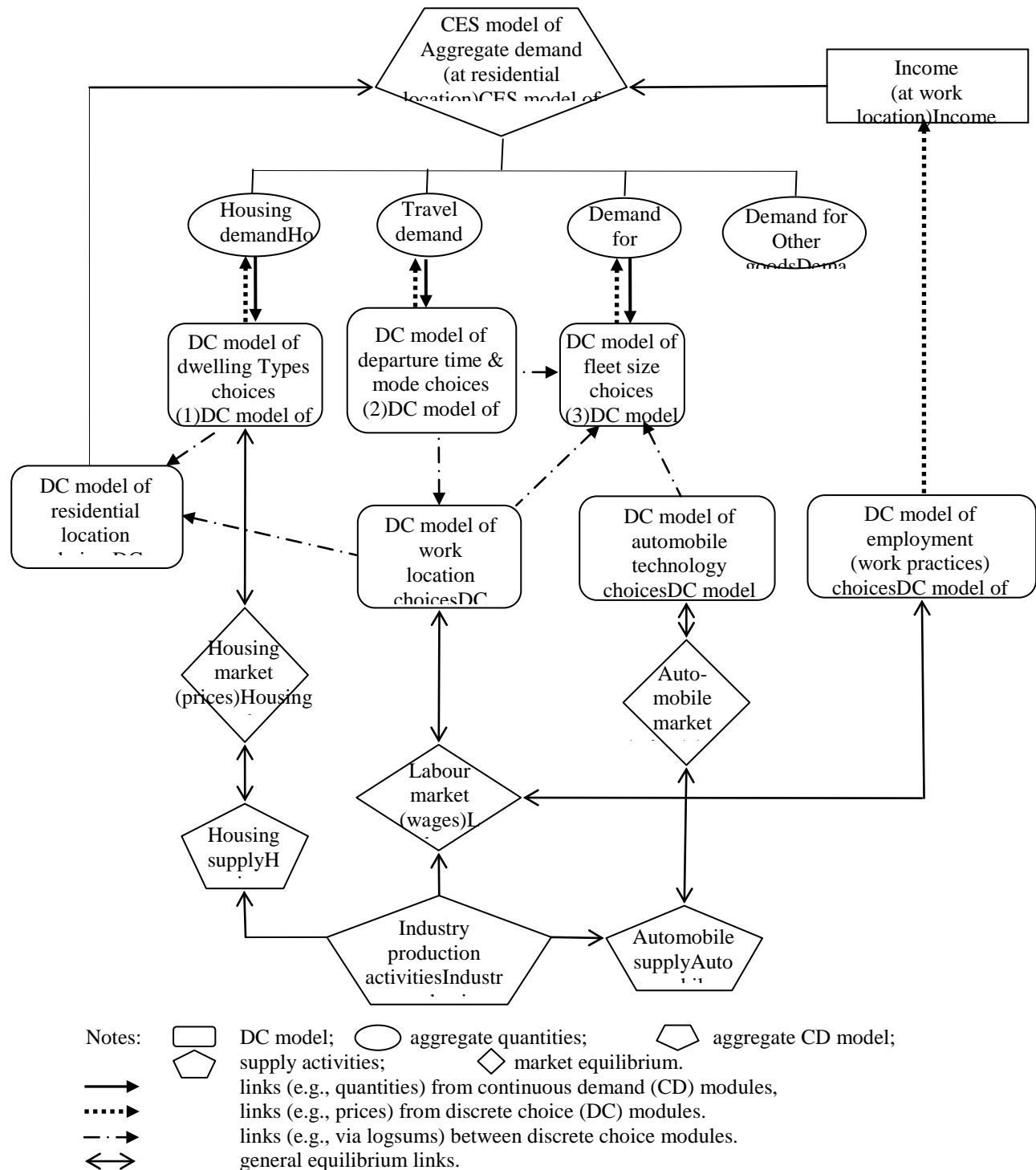
Figure 2: Schematic representation of links from discrete choice (DC) modules to continuous demand (CD) modules within a general equilibrium (GE) framework.

To simplify the analysis, assume that an upper CD demand module consists only of a decision between travel and housing expenditures.[18] Let $P^{(1)}$ and $P^{(2)}$ be the aggregate[19] price indices of 'travel' and 'housing' activities respectively which are estimated from the DC models as explained in the previous sections, and let M be the total budget allocated to travel and housing activities. The aggregate demand for travel and housing activities is then given by the following optimisation problem:

$$\text{Maximise} \quad U = U(Q^{(1)}, Q^{(2)}) = \left[\delta_1(Q^{(1)})^\rho + \delta_2(Q^{(2)})^\rho\right]^{1/\rho}$$

$$\text{s.t.} \quad P^{(1)}Q^{(1)} + P^{(2)}Q^{(2)} \leq M \tag{17}$$

where $Q^{(1)}$ and $Q^{(2)}$ are the *aggregate* levels of demand for travel and housing activities respectively, δ_1 and δ_2 are the distribution parameters of the CES utility function, and $\square = 1/(1\square\square)$ is a parameter representing the elasticity of substitution between these aggregate activities. The solution for the above maximisation problem depends on the form of the utility function $U(.)$, and in the case when this function is assumed to be of a CES form[20] the solution will be given as follows:

$$Q^{(n)} = M(\delta_n / P^{(n)})^\sigma \left[\delta_n(Q^{(n)})^\rho + \delta_n(Q^{(n)})^\rho\right]^{-1}; \quad n = \{1,2\}. \tag{18}$$

To simplify the expression, a 'percentage change' form can be used in place of the absolute level form of equation (19). Using a lower case letter to denote the percentage change (or differential log) form of an upper case variable, we have:

$$\begin{aligned}
q^{(n)} &= m - \sigma p^{(n)} - \sum_{k=\{1,2\}}[S_k(1-\sigma)p^{(k)}] \\
&= (m - \sum_{k=\{1,2\}} S_k p^{(k)}) - \sigma[p^{(n)} - \sum_{k=\{1,2\}} S_k p^{(k)}] \\
&= (\sum_{k=\{1,2\}} S_k q^{(k)}) - \sigma[p^{(n)} - \sum_{k=\{1,2\}} S_k p^{(k)}] \\
&= \bar{q} - \sigma[p^{(n)} - \bar{p}]; \quad n = \{1,2\}
\end{aligned} \tag{19}$$

where:

---

[18] The inclusion of other goods into the CD system will change the details but not the broad conclusions of the analysis.

[19] A variable without a subscript is used to refer to the *aggregate* level (i.e. summarized over all lower-stage choices). This is to be distinguished from the *disaggregate* variable which has a subscript to indicate the specific choice alternative in each lower-stage decision.

[20] Other forms can also be used without affecting the main conclusions of the analysis.

$$p^{(n)} = d \ln P^{(n)}; \quad q^{(n)} = d \ln Q^{(n)}; \quad n = \{1,2\} \tag{20}$$

$$S_n = \frac{P^{(n)} Q^{(n)}}{M} = \frac{(\delta_n)^\sigma (P^{(n)})^{1-\sigma}}{\sum_{k=\{1,2\}} (\delta_k)^\sigma (P^{(k)})^{1-\sigma}}; \quad n = \{1,2\}. \tag{21}$$

$$\bar{p} = \sum_{n=\{1,2\}} S_n p^{(n)} \tag{22}$$

$$\bar{q} = \sum_{n=\{1,2\}} S_n q^{(n)} \tag{23}$$

$$m = d \ln M = \sum_{n=\{1,2\}} S_n (q^{(n)} + p^{(n)}) = \bar{q} + \bar{p} \tag{24}$$

Given the aggregate demand functions (18) (or (19)), the distribution of the aggregate demand into various travel and housing choice alternatives can then be described by the lower-stage DC models.

3.1   *Quantity share DC model linked to a CD model*

Assume that the DC module of travel mode choice is interpreted as a demand *quantity* share (rather than expenditure share) model which describes the distribution of a fixed total *quantity* of travel demand (total number of work trips) from a particular residential location to a particular work location (Origin-Destination pair) by various modes. The reason why a travel mode choice module is regarded as a demand *quantity* share rather than expenditure share model can be explained as follows. Firstly, travel to work is an intermediate rather than final (household production) activity and therefore so long as the same destination is reached, it is immaterial which particular mode has been used. The 'trips' by different modes therefore can be regarded as though perfect substitutes, and their quantities simply be added up. The differences in qualities of travel can be reflected in the 'quality-adjusted price indices' of travel, and because of the 'random' components in these prices (derived from a random utility structure) the aggregation of individual price indices into an aggregate price must be carried out in a consistent fashion, and this has been discussed in the previous sections. Because travel mode choice decision is regarded as a quantity-share demand decision, the appropriate price index structures for travel mode choice decisions are given by equations (9a) and (11) and therefore we can write:

$$
\begin{aligned}
P^{(1)} &= \frac{1}{\gamma^{(1)}} \ln \sum_{i \in I^{(1)}} \exp(\gamma^{(1)} P_i^{(1)}) \\
&= \frac{1}{\alpha_1^{(1)}} \ln \sum_{i \in I^{(1)}} \exp(V_i^{(1)})
\end{aligned}
\tag{25}
$$

where $\boxed{\gamma}^{(1)} = \alpha_1^{(1)}$ is the coefficient of the money cost attribute in the discrete travel mode choice model, $V_i^{(1)}$ is the indirect utility of choice alternative i in the travel mode choice set $I^{(1)}$. Taking the absolute differential[21] of this aggregate price index, we have:

$$
\begin{aligned}
dP^{(1)} &= \frac{1}{\gamma^{(1)}} d\ln \sum_{i \in I^{(1)}} \exp(\gamma^{(1)} P_i^{(1)}) \\
&= \frac{1}{\alpha_1^{(1)}} d\ln \sum_{i \in I^{(1)}} \exp(V_i^{(1)}) \\
&= \frac{1}{\alpha_1^{(1)}} d\vec{V}^{(1)}
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
d\bar{V}^{(1)} &= d\ln \sum_{i \in I^{(1)}} \exp(V_i^{(1)}) \\
&= \sum_{j \in I^{(1)}} \frac{\exp V_j^{(1)})}{\sum_{k \in I^{(1)}} \exp(V_k^{(1)})} dV_j^{(1)} \\
&= \sum_{j \in I^{(1)}} Prob_j^{(1)} dV_j^{(1)}
\end{aligned}
\tag{27}
$$

Since the discrete travel mode choice model is used as a conditional quantity share demand model, this implies:

$$
Q_i^{(1)}(\boldsymbol{P}^{(1)}, Y^{(1)}) \equiv (Prob_i^{(1)})[Q^{(1)}]
\tag{28}
$$

where $Q_i^{(1)}$ stands for the level of demand (number of choices) for travel mode choice alternative i, $Q^{(1)}$ is the aggregate level of demand for all mode choices which is assumed to be given exogenously of a DC model but which can be estimated from an upper-stage CD module using equation (18) or (19), and $(Prob_i^{(n)})$ is the choice probability from the lower-stage DC model. Taking the differential log of both sides of equation (28) and using a lower case letter to denote these differential log (or percentage change) terms, we have:

$$
q_i^{(1)} = q^{(1)} + d\ln(Prob_i^{(1)})
\tag{29}
$$

---

[21] Note that equation (20) requires information on the *percentage* change (or differential log) of the aggregate price indices, therefore, to get this value the absolute differential of equation (26) needs to be divided by the initial value of the aggregate price index which is given by equation (25). As noted from the previous sections, the use of a logsum formula to define the aggregate price index for a DC model requires that its initial value be 'calibrated' so that it is consistent with the initial values of the expenditure level and quantity of all the choices.

where:

$$d \ln(Prob_i^{(1)}) = d \ln(\exp(V_i^{(1)}) - d \ln \sum_{j \in T^{(1)}} \exp(V_j^{(1)})$$

$$= dV_i^{(1)} - d\overline{V}^{(1)}$$

(30)

The first term on the right hand side of equation (29) stands for the *total quantity* effect as estimated from the upper-stage CD model (20), and the last term on the right hand side of equation (29 measures the *quantity substitution* effect (i.e. the percentage change in the quantity share of alternative *i*) which can be estimated from the lower-stage DC model using equation (30).

3.2   *Expenditure share DC model linked to a CD model*

Now consider the case of the aggregate housing location/type choice decision which is also estimated from the upper-stage CD model jointly with the aggregate travel demand decision (as seen from equation (19)) but linked to a lower-stage DC model which is interpreted as a housing *expenditure share* rather than quantity share model. The reason for the interpretation of housing location/type choice decision as a demand expenditure share rather than a demand quantity share decision is given as follows. Firstly, housing quantities in different locations and of different types cannot really be regarded as though 'perfect substitutes'. They are essentially different commodities with significantly different attributes and levels of expenditure. Therefore, their substitution can only be handled via a consideration of the relative price structures and income effects rather than as a matter of quantity substitution.In this case, The aggregate price index for an expenditure share CD module is therefore by equation (9b) and (14) (CES function rather than logsum function):

$$P^{(2)} = \left[\sum_{i \in I^{(2)}} (P_i^{(2)})^{\gamma^{(2)}} \right]^{1/\gamma^{(2)}}$$

$$= \left[\sum_{i \in I^{(2)}} \exp(V_i^{(2)}) \right]^{1/\alpha_1^{(2)}}$$

(31)

where $\gamma^{(2)} = \alpha_1^{(2)}$ is the coefficient of the money cost attribute in the discrete housing type/location choice, and $V_i^{(2)}$ is the indirect utility of the housing type/location choice alternative *i* in this DC model. Taking the differential log of both sides of equation (31) gives:

$$d \ln P^{(2)} = \frac{1}{\gamma^{(2)}} d \ln \sum_{i \in I^{(2)}} \exp(\gamma^{(2)} P_i^{(2)})$$

$$= \frac{1}{\alpha_1^{(2)}} d \ln \sum_{i \in I^{(2)}} \exp(V_i^{(2)}) \tag{32}$$

$$= \frac{1}{\alpha_1^{(2)}} d\vec{V}^{(2)}$$

This is essentially the same as equation (26) except that the left hand side stands for the *percentage change* (or log change) in the price index rather than an *absolute* change. This is explained as follows. The right hand sides (of both equations (26) and (31)) represent the *absolute* change in the logsum (normalised by the marginal utility of money). For a DC model, the logsum is determined only up to an arbitrary origin, but its absolute change is fully determined and therefore can be used to indicate the absolute change in *total* indirect utility associated with *all* choices. When this change in indirect utility is normalised by the marginal utility of money it can be used to represent the change in cost or expenditure level associated with the choice decisions. Now, for a *quantity* share DC model, the total quantity of all choices is assumed to be fixed or given exogeneously of the model and therefore can be normalised to 1 initially (for a DC model). This means the absolute change in the cost or expenditure level (right hand side of equation (26)) can also be used to indicate the absolute change in price index (left hand side of equation (26)). For an *expenditure* share DC model, however, the total level of expenditure (rather than total quantity) is assumed to be fixed or given exogeneously of the DC model therefore the initial total expenditure or (aggregate price level) can be normalised to 1 initially.[22] The absolute change in expenditure or aggregate price level (right hand side of equation (32)) now represents also the percentage (or log) change.[23]

Given that the DC model is used as an expenditure share rather than a quantity share demand model, the absolute demand *quantity* for each housing location/type choice alternative *i* is then given by:

$$Q_i^{(2)}(\boldsymbol{P}^{(2)}, Y^{(2)}) \equiv \frac{1}{P_i^{(2)}} (Prob_i^{(2)})[P^{(2)}Q^{(2)}] \tag{33}$$

where $P_i^{(2)}$ is the disaggregate price index for housing choice alternative *i* (as defined by equation (9b)), $P^{(2)}Q^{(2)}$ is the total expenditure level for housing activities, and $(Prob_i^{(2)})$ is the choice probability for

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[22] This normalisation requires the calibration of the initial indirect utilities in equation (31)such that the aggregate price level is equal to 1 initially. This means the logsum in equation (32) is equal to zero initially.

[23] In fact, from equation (9b), the indirect utility function of an expenditure share DC model is specified as the *log-price* rather than linear price of the choice alternative, hence the absolute change in the 'expected' indirect utility (i.e $d\vec{V}^{(2)}$ in equation 32) must also be said to represent the log change in aggregate price.

the housing choice alternative i as determined by the lower-stage DC model. Taking the differential log of both sides of equation (33) gives:

$$q_i^{(2)} = q^{(2)} - [p_i^{(2)} - p^{(2)}] + d\ln(Prob_i^{(2)}) \tag{34}$$

Compared to the demand for travel mode choice activities as represented by equation (30) of the previous section, the demand for housing choice activities in equation (35) is slightly different. Firstly, apart from the first term which represents the *total quantity* effect as estimated from the upper-stage CD model of equation (20) - which remains the same in both cases, the very last term on the right hand side of equation (34) now represents the percentage change in *expenditure share*rather than quantity share as in the case of equation (30). Since expenditure share involves a relative price component in addition to the quantity components, the middle terms within the square brackets on the right hand side of equation (34) are used to offset this relative price effects. To estimate this relative price effect, firstly, taking the differential log of the disaggregate price index in equation (9b) will give:

$$p_i^{(2)} = d\ln P_i^{(2)} = (1/\alpha_i^{(2)})dV_i^{(2)} \tag{35}$$

Next, from equation (32) this can also be written as :

$$p^{(2)} = (1/\alpha_1^{(2)})d\overline{V}^{(2)} \tag{36}$$

Therefore, the relative price effect is given as:

$$[p_i^{(2)} - p_i^{(2)}] = d\ln(P_i^{(2)}/P_i^{(2)}) = (1/\alpha_i^{(2)})[dV_i^{(2)} - d\overline{V}_i^{(2)}] \tag{37}$$

Substituting this into equation (34) gives:

$$q_i^{(2)} = q^{(2)} + (1 - \frac{1}{\alpha_1^{(2)}})[dV_i^{(2)} - d\overline{V}^{(2)}] \tag{38}$$

Comparing equation (38) to equation (30), the difference is in the extra term representing the relative price effect.

4. An Illustrative experiment

To illustrate the applicability of the approach described in the previous section for linking DC to CD models within a CGE framework, we consider a simple experiment.[24] In this experiment, we investigate the impacts of a transport infrastructure investment project in the Sydney Metropolitan Area (SMA) on transport users and on the local economy. The impacts on transport users are traditionally estimated using a collection of DC modules[25] set up within a 'bottom-up' partial equilibrium (PE) model such as TRESIS.[26] The use of these DC modules requires an implicit assumption that all other activities 'outside' of the DC decisions (for example, total levels of travel and housing demand) are given exogenously of the DC modules. However, these activity levels can in practice be affected by, as well as affecting, the decisions within the DC modules, therefore, to explore the important linkages between the DC decisions and CD decisions, a PE model such as TRESIS needs to be linked to some 'upper-stage' CD modules. For example, to estimate the extent of the substitution or interaction between travel and housing activities, the results from the discrete mode choice and dwelling-type choice modules are used as inputs into the CD module which represents the interaction (at the aggregate level) between travel and housing activities as explained in the previous sections. Both the DC and CD modules are then immersed in a spatial computable general equilibrium model (SCGE) which represents the local economy to determine how much of the improvement in the transport system (in the form of reduced transport costs for certain links in the transport system) can be translated into (i) improved labour productivity at work locations (due to the so-called 'agglomeration effects') which can lead to increased wage levels for certain industry sectors in certain locations, and (ii) increase in rents for certain locations due to improved transport links to these locations (this can be referred to as the 'Venables effect' – see Figure 1). Improved labour productivity due to agglomeration effects in some locations, however, may also lead to 'disagglomeration' effects elsewhere due to spatial competition between locations. Therefore, the 'general equilibrium' effects of a transport improvement may include both winners and losers in terms of changes in wage and rent levels in various locations.

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[24] For more details on this experiment, see Truong and Hensher (2012) and Hensher et al. (2012).

[25] To distinguish between the larger system of interconnected models such as TRESIS or a CGE model and the smaller individual DC (or CD) models within this system, we use the term 'module' to refer to the latter and reserve the term 'model' to refer only to the larger complete system. Thus, a 'model' can consist of several different 'modules' within it.

[26] See Hensher (2002); Hensher and Ton (2002).

4.1 Discrete-choice decisions without linkage to continuous demand decisions

Tables 1 shows the effect of the transport improvement in the rail link between zone 1 (Inner Sydney) and zone 10 (Blacktown Baulkham Hills) on the generalised cost of travel by train between different zones (see Figure A1 for a description of the zones). Table 2 shows the percentage change in the probability of a worker choosing train as the mode of transport for the work trip between these zones. Quite clearly, with the reduction in travel time between zone 10 and other zones, the probability of choosing train as the preferred mode of transport to and from zone 10 will increase, while the probability of choosing other modes will decrease (Table 3). Residential and work location choices are also affected by the transport improvement, and this is shown in Table 4. Here it is seen that zone 10 can become a preferred place of residence even though workers living in zone 10 may now prefer to work in locations other than zone 10 (for example, in zone 1 (Inner_Sydney), 5 (Fairfield_Liverpool), 7 (Inner_West) and 11 (Lower_North_Sydney). The magnitude of these changes, however, are quite small, implying that the *direct* impacts of transport improvement on land uses are small.

4.2 Discrete-choice decisions with linkage to continuous demand decisions in a general equilibrium setting

Assuming that transport and land use decisions by the workers can be translated into housing and employment activities in the local economy,[27] we can now look at the potential impacts of changes in the transport network on the housing and labour market. Firstly, with respect to the housing market, residential location choices and dwelling type choices will affect dwelling demand, which in turn will induce changes in dwelling supply and dwelling prices. Dwelling supply is here assumed to be responding to changes in dwelling prices. The price elasticity of supply of dwellings for various locations in the Sydney metropolitan area is taken from a study by Gitelman and Otto (2012) (see Table 5). Given the potential interactions between supply and demand in various zones, equilibrium results are shown in Table 6 (when DC results are not linked to upper-stage CD decisions) and Table 7 (when DC results are not linked to upper-stage CD decisions). Here it is seen that for a relatively small infrastructure project like the NWRL rail link improvement, the *direct* impacts of the transport improvement on housing activities would be small (Table 6). By 'direct' impacts, this is meant to be attributed only to changes in residential location and work location decisions as estimated from DC modules *without* linkage to an upper-stage continuous demand module which focuses attention on the aggregate trade-off between housing and travel decisions as well as on the 'income' effect of reduced transport costs on travel and

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[27] This requires changes on the *supply* side (i.e. housing supply and employment opportunities) as well as changes on the demand side as induced by changes in household residential and work location choices.

housing activities (the 'Venables' effect). When these indirect impacts are also taken into account, the total impacts of transport improvement on housing activities can be seen to be more substantial (see Table 7). Here, the 'income' effect of transport improvement on housing activities are seen to be most significant for zone 10 (as is expected), but other zones also benefit significantly. Table 8 shows the *indirect* effects as captured by the upper-stage CD module. The indirect effects would consist not only of the aggregate income effects but also aggregate substitution effects between transport and housing. Following an improvement in the transport system, the aggregate price of transport would have been reduced relative to the price of housing which means a substitution away from housing expenditure towards 'cheaper' transport (i.e. workers move places of residence away from the work place towards locations which have cheaper rents (imputed or actual) but requiring greater travel. In a general equilibrium framework, however, this substitution effect would be counter balanced by an opposite (real) income effect where the reduced cost of transport cost would imply an increase in real income and therefore increased level of expenditure on both housing and travel activities. The net impacts of these two opposite effects are shown in Table 8 for the case of housing activities.

For transport activities, the improvement in the rail link between zones 1 and 10 would imply not only a *direct* substitution effect between different *modes* of transport as captured by DC module of mode choice (see Tables 2-3) but also between different *zones* (as places of residence and/or work places – see Table 4). Therefore, the net impacts of these direct effects on train travel could be negative for some zones (for example, zones 4 and 5 - see Table 9) even though they are mostly positive for all other zones. In addition to the direct effects, however, there are also indirect effects from aggregate income and substitution effects between travel and housing (i.e. location or land use) activities. Therefore, these indirect effects may counteract the direct effects to such an extent that it can reverse the original direction of the direct impacts. This is seen for the cases of zones 6 and 8 (Outer_SW_Syd and Centrl_W_Syd), see Table 10. The reduction (rather than increase) in train travel to these locations from all zones (except from zone 10) is seen to arise primarily from a change in place of employment, from zone 6, 8 (and also to some extent, zone 4) to all other zones especially zones 5 (Fairfield_Liverpool), 7 (Inner_West_Sydney) and of course also zone 10 (see Table 12)

.

5. Conclusion

In this paper we have presented a methodology for linking a disaggregate discrete choice (DC) model to an aggregate continuous demand (CD) model and integrate both types of models into the framework of a simple[28] spatial computable general equilibrium (SCGE) which represents the local economy of the Sydney Metropolitan Area (SMA). In this simple model, only a limited number of economic activities are considered: residential location and dwelling type choices, work place location choices, and mode choice activities. These disaggregate choice decisions by individual workers are modelled by discrete-choice (DC) modules contained in a well-known 'bottom-up' transport-land use partial equilibrium model called TRESIS (Hensher and Ton (2002)). The paper presents a methodology for linking these DC modules to a conventional aggregate continuous demand (CD) module which handles the issue of transport – housing expenditure substitution but taking into account the complementaty aspects of these decisions in the context of a spatial general equilibrium model. The methodology is new because up to now, the two types of DC and CD models are often used for different applications with different contexts, relying on different theoretical assumptions and to explain different types of behaviour: individual discrete choices versus continuous aggregate (or 'representative') demand behaviour of a consumer. DC models are used in a 'bottom-up' context because it contains great details on individual attributes and technological or environmental characteristics at a disaggregate level. CD models on the other hand are used in a 'top-down' approach because it can concentrate on 'economy-wide' impacts. Linking these bottom-up to top-down modules is an important challenge not only because of the potential benefits it can bring but also because of the theoretical and empirical difficulties it must overcome due to the differences between the two types of models. This paper has shown a methodology for meeting with this challenge and demonstrated with an empirical example, which shows the potential benefits that can be gained from such a methodology. More specifically, it shows that measuring the impacts of a transport system improvement on a local economy (such as the SMA economy) can be difficult because of the potential interactions between many different types of decisions, not only from the demand side (for example, residential location and dwelling type choice versus travel activities) but also from the supply side (housing supply and wage level associated with a particular work location choice). Using only one particular type of model (e.g. DC models) can concentrate on one type of impacts (e.g. direct impacts of transport improvement on mode choice or locational choice) but neglecting the indirect impacts (or feedbacks) of housing and employment activities on these transport decisions themselves. In a modern city with many 'nodes' of employment and residential locations, the direction of these impacts can be difficult to trace

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[28] In this simple model, only work place, residence location and mode choice activities are considered s

because of the interplay between the economic and spatial elements, therefore the linkage between DC and CD modules need to be put in the context of a *spatial* general equilibrium model (SGEM) to be of effective use. The paper has shown that this approach is feasible, for a simple (but sufficiently comprehensive) SGEM such as that for the SMA. The challenge is to extend this methodology to the case of more complex SGEM, such as that for a state or a nation, and this is left for future contributions

Table 1 Percentage change in the generalised cost of TRAIN travel between Origin-Destination zones following an improvement in the RAIL link between zone 1 and zone 10.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	0	0	0	0	0	0	0	-1.9	0	0	0	0
2	Eastern_Subs	0	0	0	0	0	0	0	0	0	-2.43	0	0	0	0
3	StGge_Suther	0	0	0	0	0	0	0	0	0	-3.76	0	0	0	0
4	Canter_Banks	0	0	0	0	0	0	0	0	0	-0.72	0	0	0	0
5	Fairfd_Livrp	0	0	0	0	0	0	0	0	0	-5.7	0	0	0	0
6	Outer_SW_Syd	0	0	0	0	0	0	0	0	0	-3.59	0	0	0	0
7	Inner_W_Syd	0	0	0	0	0	0	0	0	0	-3.88	0	0	0	0
8	Centrl_W_Syd	0	0	0	0	0	0	0	0	0	-8.87	0	0	0	0
9	Outer_W_Syd	0	0	0	0	0	0	0	0	0	-7.13	0	0	0	0
10	Blck_Baulk_H	-2.18	-2.7	-3.94	-0.81	-6.22	-3.75	-4.24	-10.6	-7.81	-0.56	-2.19	-2.4	-18.13	-3.33
11	Lower_N_Syd	0	0	0	0	0	0	0	0	0	-1.96	0	0	0	0
12	Horns_Kuring	0	0	0	0	0	0	0	0	0	-2.11	0	0	0	0
13	Nth_Beaches	0	0	0	0	0	0	0	0	0	-18.13	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	0	0	0	-3.08	0	0	0	0

Table 2 Percentage change in the probability of choosing TRAIN as the mode of transport between Origin-Destination zones following an improvement in the rail link between zone 1 and zone 10.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	0	0	0	0	0	0	0	4.79	0	0	0	0
2	Eastern_Subs	0	0	0	0	0	0	0	0	0	14.23	0	0	0	0
3	StGge_Suther	0	0	0	0	0	0	0	0	0	22.12	0	0	0	0
4	Canter_Banks	0	0	0	0	0	0	0	0	0	2.32	0	0	0	0
5	Fairfd_Livrp	0	0	0	0	0	0	0	0	0	20.22	0	0	0	0
6	Outer_SW_Syd	0	0	0	0	0	0	0	0	0	22.12	0	0	0	0
7	Inner_W_Syd	0	0	0	0	0	0	0	0	0	17.05	0	0	0	0
8	Centrl_W_Syd	0	0	0	0	0	0	0	0	0	21.61	0	0	0	0
9	Outer_W_Syd	0	0	0	0	0	0	0	0	0	21.71	0	0	0	0
10	Blck_Baulk_H	4.78	14.23	22.12	2.32	19.91	22.12	16.14	20.6	20.8	1.32	6.34	9.88	22.12	18.48
11	Lower_N_Syd	0	0	0	0	0	0	0	0	0	5.86	0	0	0	0
12	Horns_Kuring	0	0	0	0	0	0	0	0	0	9.9	0	0	0	0
13	Nth_Beaches	0	0	0	0	0	0	0	0	0	22.12	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	0	0	0	23.03	0	0	0	0

Table 3 Percentage change in the probability of choosing modes OTHER THAN TRAIN as the mode of transport between Origin-Destination zones following an improvement in the rail link between zone 1 and zone 10.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	0	0	0	0	0	0	0	-3.83	0	0	0	0
2	Eastern_Subs	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	StGge_Suther	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	Canter_Banks	0	0	0	0	0	0	0	0	0	-0.05	0	0	0	0
5	Fairfd_Livrp	0	0	0	0	0	0	0	0	0	-1.56	0	0	0	0
6	Outer_SW_Syd	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	Inner_W_Syd	0	0	0	0	0	0	0	0	0	-4.15	0	0	0	0
8	Centrl_W_Syd	0	0	0	0	0	0	0	0	0	-0.42	0	0	0	0
9	Outer_W_Syd	0	0	0	0	0	0	0	0	0	-0.34	0	0	0	0
10	Blck_Baulk_H	-3.84	0	0	-0.05	-1.81	0	-4.9	-1.25	-1.08	-0.04	-2.98	-0.53	0	-3.7
11	Lower_N_Syd	0	0	0	0	0	0	0	0	0	-3.42	0	0	0	0
12	Horns_Kuring	0	0	0	0	0	0	0	0	0	-0.51	0	0	0	0
13	Nth_Beaches	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4 Percentage change in the probability of choosing a zone as a residential location (origin zone) and given this residential location, the percentage change in the probability of choosing a zone as a work place (destination zone) - following the improvement in the rail link between zone 1 and zone 10.

Origin Zone		% change in probability of choosing zone as residential location	Destination Zone No.													
No.	Name		1	2	3	4	5	6	7	8	9	10	11	12	13	14
			Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	-0.001	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0
2	Eastern_Subs	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	StGge_Suther	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	Canter_Banks	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Fairfd_Livrp	-0.001	0	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.012	-0.001	-0.001	-0.001	-0.001
6	Outer_SW_Syd	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	Inner_W_Syd	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.032	-0.001	-0.001	-0.001	-0.001
8	Centrl_W_Syd	-0.001	0	0	0	0	0	0	0	0	0	0.003	0	0	0	0
9	Outer_W_Syd	-0.001	0	0	0	0	0	0	0	0	0	0.002	0	0	0	0
10	Blck_Baulk_H	0.016	0.017	-0.01	-0.011	-0.009	0.005	-0.009	0.028	0	0	-0.009	0.012	-0.005	-0.01	0.019
11	Lower_N_Syd	-0.001	0	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.026	0	-0.001	-0.001	-0.001
12	Horns_Kuring	-0.001	0	0	0	0	0	0	0	0	0	0.004	0	0	0	0
13	Nth_Beaches	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Gosfrd_Wyong	-0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5 Price elasticity of supply for different types of dwelling in different zones.

Zone		Dwelling type		
No.	Name	1	2	3
		Detached house	Semi-detached house	Unit
1	Inner_Sydney	0.08	0.08	0.35
2	Eastern_Subs	0.08	0.08	0.35
3	StGge_Suther	0.21	0.21	0.655
4	Canter_Banks	0.10	0.10	0.83
5	Fairfd_Livrp	0.32	0.32	0.48
6	Outer_SW_Syd	0.32	0.32	0.48
7	Inner_W_Syd	0.09	0.09	0.59
8	Centrl_W_Syd	0.21	0.21	0.655
9	Outer_W_Syd	0.32	0.32	0.48
10	Blck_Baulk_H	0.32	0.32	0.48
11	Lower_N_Syd	0.09	0.09	0.59
12	Horns_Kuring	0.32	0.32	0.48
13	Nth_Beaches	0.21	0.21	0.655
14	Gosfrd_Wyong	0.32	0.32	0.48

Source: based on Gitelman and Otto (2012)

Table 6 Potential direct impacts of the improvement in the rail link between zone 1 and zone 10 on the housing market in various zones: DC estimates without linkage to CD module.

Zone		Dwelling Expenditure (% changes)			Dwelling Supply Quantity (% changes)			Dwelling Price (% changes)		
No.	Name	1	1	2	3	2	3	1	2	3
		Detached house	Detached house	Semi-detached house	Unit	Semi-detached house	Unit	Detached house	Semi-detached house	Unit
1	Inner_Sydney	-0.03	-0.03	-0.03	0.01	0.03	0.06	0.09	0.09	0.08
2	Eastern_Subs	-0.05	-0.06	-0.06	0.01	0.02	0.05	0.07	0.07	0.06
3	StGge_Suther	0.01	0.01	0.01	0.02	0.05	0.10	0.11	0.11	0.08
4	Canter_Banks	-0.02	-0.02	-0.03	0.01	0.05	0.10	0.09	0.09	0.06
5	Fairfd_Livrp	0.01	0.01	0.01	0.03	0.04	0.08	0.10	0.10	0.09
6	Outer_SW_Syd	0.00	0.00	0.00	0.03	0.04	0.08	0.10	0.10	0.09
7	Inner_W_Syd	0.05	0.06	0.07	0.02	0.07	0.11	0.16	0.17	0.12
8	Centrl_W_Syd	0.04	0.04	0.04	0.03	0.07	0.11	0.13	0.14	0.10
9	Outer_W_Syd	0.01	0.01	0.01	0.03	0.04	0.08	0.10	0.10	0.09
10	Blck_Baulk_H	0.01	0.01	0.01	0.03	0.04	0.08	0.10	0.10	0.09
11	Lower_N_Syd	-0.02	-0.01	-0.02	0.01	0.04	0.09	0.10	0.10	0.07
12	Horns_Kuring	0.01	0.01	0.01	0.03	0.04	0.08	0.10	0.10	0.09
13	Nth_Beaches	0.00	0.01	0.01	0.02	0.05	0.10	0.11	0.11	0.08
14	Gosfrd_Wyong	0.00	0.00	0.00	0.03	0.04	0.08	0.10	0.10	0.09

Table 7 Potential total impacts of the improvement in the rail link between zone 1 and zone 10 on the housing market in various zones: DC estimates with linkage to CD module

Zone		Dwelling Expenditure (% changes)			Dwelling Supply Quantity (% changes)			Dwelling Price (% changes)		
No.	Name	1	1	2	3	2	3	1	2	3
		Detached house	Detached house	Semi-detached house	Unit	Semi-detached house	Unit	Detached house	Semi-detached house	Unit
1	Inner_Sydney	0.12	0.12	0.13	0.01	0.01	0.03	0.11	0.11	0.09
2	Eastern_Subs	0.07	0.06	0.07	0.01	0.01	0.02	0.06	0.06	0.05
3	StGge_Suther	0.17	0.18	0.19	0.03	0.03	0.08	0.14	0.15	0.11
4	Canter_Banks	0.13	0.13	0.14	0.01	0.01	0.06	0.12	0.12	0.08
5	Fairfd_Livrp	0.46	0.48	0.48	0.11	0.12	0.16	0.34	0.36	0.32
6	Outer_SW_Syd	0.26	0.26	0.26	0.06	0.06	0.08	0.19	0.20	0.17
7	Inner_W_Syd	0.52	0.60	0.65	0.04	0.05	0.24	0.48	0.55	0.41
8	Centrl_W_Syd	0.38	0.36	0.36	0.07	0.06	0.14	0.31	0.30	0.22
9	Outer_W_Syd	0.95	0.96	0.96	0.23	0.23	0.31	0.72	0.73	0.65
10	Blck_Baulk_H	2.31	2.33	2.33	0.56	0.56	0.75	1.75	1.76	1.57
11	Lower_N_Syd	0.10	0.10	0.11	0.01	0.01	0.04	0.09	0.10	0.07
12	Horns_Kuring	0.24	0.25	0.25	0.06	0.06	0.08	0.18	0.19	0.17
13	Nth_Beaches	0.17	0.18	0.19	0.03	0.03	0.07	0.14	0.15	0.11
14	Gosfrd_Wyong	0.19	0.19	0.19	0.05	0.05	0.06	0.14	0.15	0.13

Table 8 Potential indirect impacts of the improvement in the rail link between zone 1 and zone 10 on the housing market in various zones: estimates attributed to linkage between DC and CD modules.

Zone		Dwelling Expenditure (% changes)			Dwelling Supply Quantity (% changes)			Dwelling Price (% changes)		
No.	Name	1	1	2	3	2	3	1	2	3
		Detached house	Detached house	Semi-detached house	Unit	Semi-detached house	Unit	Detached house	Semi-detached house	Unit
1	Inner_Sydney	0.14	0.15	0.15	0.00	-0.02	-0.02	0.02	0.02	0.02
2	Eastern_Subs	0.12	0.12	0.12	0.00	-0.01	-0.03	-0.01	-0.01	-0.01
3	StGge_Suther	0.17	0.18	0.18	0.01	-0.02	-0.03	0.03	0.04	0.03
4	Canter_Banks	0.15	0.16	0.16	0.00	-0.03	-0.04	0.03	0.03	0.02
5	Fairfd_Livrp	0.45	0.47	0.47	0.08	0.07	0.08	0.24	0.26	0.23
6	Outer_SW_Syd	0.25	0.26	0.26	0.03	0.02	0.01	0.10	0.10	0.09
7	Inner_W_Syd	0.47	0.54	0.59	0.03	-0.02	0.13	0.32	0.39	0.29
8	Centrl_W_Syd	0.34	0.32	0.32	0.04	-0.01	0.03	0.18	0.16	0.11
9	Outer_W_Syd	0.95	0.95	0.95	0.20	0.19	0.23	0.62	0.63	0.56
10	Blck_Baulk_H	2.30	2.32	2.32	0.52	0.52	0.67	1.64	1.66	1.48
11	Lower_N_Syd	0.11	0.12	0.13	0.00	-0.03	-0.05	-0.01	-0.01	0.00
12	Horns_Kuring	0.24	0.24	0.24	0.03	0.02	0.00	0.08	0.09	0.08
13	Nth_Beaches	0.17	0.18	0.18	0.01	-0.02	-0.03	0.04	0.04	0.03
14	Gosfrd_Wyong	0.19	0.19	0.19	0.02	0.01	-0.02	0.05	0.05	0.04

Table 9 Percentage change in the number of journeys to work by TRAIN between Origin-Destination zones following an improvement in the rail link between zone 1 and zone 10 – DC estimates without linkage to CD module.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	0.9	-0.69	0.47	0.54	2.1	1.35	0.3	5.21	0	0.81	0.72	0.44
2	Eastern_Subs	0	0	0.99	-0.58	0	0.78	2.38	1.3	0.8	0	0	0.85	0	0
3	StGge_Suther	0	0	0.32	-1.26	-0.04	0.32	0	0.93	0	0	0	0.09	0	0
4	Canter_Banks	0	0	0.76	-0.66	0.37	0.56	1.95	1.26	0	2.13	0	0.64	0	0
5	Fairfd_Livrp	0	0	0.44	-1.14	0.1	0.21	1.51	0.95	0	19.72	0	0	0	0
6	Outer_SW_Syd	0	0	0	-1.07	0.03	0.16	1.64	1.04	0	0	0	0	0	0
7	Inner_W_Syd	0	0	0	-1.84	-0.49	-0.31	0.96	0.5	0	16.33	0	-0.3	-0.55	-0.31
8	Centrl_W_Syd	0	0	-0.09	-1.62	-0.32	-0.2	1.31	0.58	-0.32	20.4	0	-0.16	-0.07	0
9	Outer_W_Syd	0	0	0	0	0	0	0	0.84	0.25	21.04	0	0.23	0	0
10	Blck_Baulk_H	4.78	0	0	1.19	20.23	0	18.76	22.01	21.28	0.9	6.34	10.34	0	0
11	Lower_N_Syd	0	0	0.81	-0.63	0.29	0.67	2.29	1.47	0	5.86	0	0.61	0	0.34
12	Horns_Kuring	0	0	0.42	-1.06	0	0	1.68	1.03	0.08	9.36	0	0.17	0	0.1
13	Nth_Beaches	0	0	0	0	0	0	1.63	0	0	0	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	1.74	0	0	0	0	0.28	0	0.12

Table 10 Percentage change in the number of journeys to work by TRAIN between Origin-Destination zones following an improvement in the rail link between zone 1 and zone 10 – DC estimates WITH linkage to CD module.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	1.38	-0.8	5.57	-0.33	15.16	-10.56	0.34	13.27	0	1.91	1.29	1.35
2	Eastern_Subs	0	0	1.2	-1.28	0	0.06	15.57	-7.91	0.27	0	0	1.22	0	0
3	StGge_Suther	0	0	0.43	-2.01	4.6	0.14	0	-9.82	0	0	0	0.17	0	0
4	Canter_Banks	0	0	-0.35	-0.85	5.24	0.59	13.79	-10.36	0	6.75	0	1.64	0	0
5	Fairfd_Livrp	0	0	-1.76	-2.86	2.52	-1.8	13.17	-10.8	0	38.28	0	0	0	0
6	Outer_SW_Syd	0	0	0	-0.89	4.78	-0.14	14.74	-10.02	0	0	0	0	0	0
7	Inner_W_Syd	0	0.02	0	-6.82	2.58	-2.27	8.87	-14.25	0	28.47	0	-3.5	-5.81	-3.03
8	Centrl_W_Syd	0	-0.04	6.7	2.87	8.79	2.75	18.91	-6.48	4.28	49.86	0	3.25	4.52	0
9	Outer_W_Syd	0	-0.01	0	0	0	0	0	-9.08	0.2	40.45	0	0.62	0	0
10	Blck_Baulk_H	8.04	0	0	0.96	37.87	0	44.34	20.99	34.02	4.23	9.93	13.82	0	0
11	Lower_N_Syd	0	0.01	2.43	-0.56	5.8	0.87	15.21	-9.84	0	14.11	0	1.78	0	0.82
12	Horns_Kuring	0	0	-0.25	-1.31	0	0	14.74	-10.87	-0.47	17.17	0	0.33	0	0.29
13	Nth_Beaches	0	0	0	0	0	0	14.32	0	0	0	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	14.14	0	0	0	-0.02	0.63	0	0.39

Table 11 Percentage change in the number of journeys to work by TRAIN between Origin-Destination zones following an improvement in the rail link between zone 1 and zone 10 – effects attributed to CD-CD module linkages.

Origin Zone		Destination Zone No.													
No.	Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Inner_Sydney	Eastern_Subs	StGge_Suther	Canter_Banks	Fairfd_Livrp	Outer_SW_Syd	Inner_W_Syd	Centrl_W_Syd	Outer_W_Syd	Blck_Baulk_H	Lower_N_Syd	Horns_Kuring	Nth_Beaches	Gosfrd_Wyong
1	Inner_Sydney	0	0	0.48	-0.11	5.10	-0.87	13.06	-11.91	0.04	8.06	0	1.10	0.57	0.91
2	Eastern_Subs	0	0	0.21	-0.70	0	-0.72	13.19	-9.21	-0.53	0	0	0.37	0	0
3	StGge_Suther	0	0	0.11	-0.75	4.64	-0.18	0	-10.75	0	0	0	0.08	0	0
4	Canter_Banks	0	0	-1.11	-0.19	4.87	0.03	11.84	-11.62	0	4.62	0	1.00	0	0
5	Fairfd_Livrp	0	0	-2.20	-1.72	2.42	-2.01	11.66	-11.75	0	18.56	0	0	0	0
6	Outer_SW_Syd	0	0	0	0.18	4.75	-0.30	13.10	-11.06	0	0.00	0	0	0	0
7	Inner_W_Syd	0	0.02	0	-4.98	3.07	-1.96	7.91	-14.75	0	12.14	0	-3.20	-5.26	-2.72
8	Centrl_W_Syd	0	-0.04	6.79	4.49	9.11	2.95	17.60	-7.06	4.60	29.46	0	3.41	4.59	0
9	Outer_W_Syd	0	-0.01	0	0	0	0	0	-9.92	-0.05	19.41	0	0.39	0	0
10	Blck_Baulk_H	3.26	0	0	-0.23	17.64	0	25.58	-1.02	12.74	3.33	3.59	3.48	0	0
11	Lower_N_Syd	0	0.01	1.62	0.07	5.51	0.20	12.92	-11.31	0	8.25	0	1.17	0	0.48
12	Horns_Kuring	0	0	-0.67	-0.25	0	0	13.06	-11.90	-0.55	7.81	0	0.16	0	0.19
13	Nth_Beaches	0	0	0	0	0	0	12.69	0	0	0	0	0	0	0
14	Gosfrd_Wyong	0	0	0	0	0	0	12.40	0	0	0.00	-0.02	0.35	0	0.27

Table 12 Potential impacts on residential and work location choices, on employment and wage level (attributed to agglomeration/disagglomeration effects) resulting from the improvement in the rail link between zone 1 and zone 10.

| Zone		Number of residences (% changes)			Employment level (% changes)			Wage level (% changes)		
No.	Name	1	1	2	3	2	3	1	2	3
		DC stand alone estimates	DC estimates WITH linkage to CD module	Effects attributed to DC-CD linkage	DC stand alone estimates	DC estimates WITH linkage to CD module	Effects attributed to DC-CD linkage	DC stand alone estimates	DC estimates WITH linkage to CD module	Effects attributed to DC-CD linkage
1	Inner_Sydney	-0.025	0.121	0.146	0.00	0.00	0.00	0.001	0.001	0.000
2	Eastern_Subs	-0.055	0.065	0.120	0.00	0.00	0.00	0.000	-0.001	-0.001
3	StGge_Suther	0.009	0.184	0.175	0.39	0.61	0.22	0.008	0.031	0.023
4	Canter_Banks	-0.023	0.135	0.158	-1.02	-1.42	-0.40	-0.019	-0.023	-0.004
5	Fairfd_Livrp	0.005	0.473	0.468	0.07	4.51	4.44	-0.003	0.044	0.047
6	Outer_SW_Syd	0.003	0.259	0.256	0.19	-0.37	-0.56	-0.001	-0.012	-0.011
7	Inner_W_Syd	0.058	0.597	0.539	1.60	12.73	11.13	0.019	0.098	0.079
8	Centrl_W_Syd	0.040	0.365	0.325	1.03	-9.58	-10.61	0.013	-0.080	-0.093
9	Outer_W_Syd	0.007	0.964	0.957	0.17	0.04	-0.14	-0.003	0.040	0.043
10	Blck_Baulk_H	0.008	2.336	2.328	-0.54	3.19	3.73	0.020	0.038	0.018
11	Lower_N_Syd	-0.014	0.105	0.119	0.00	0.00	0.00	0.000	-0.001	-0.001
12	Horns_Kuring	0.008	0.250	0.242	0.24	0.70	0.46	0.002	0.016	0.014
13	Nth_Beaches	0.004	0.182	0.178	0.19	0.45	0.25	0.009	0.037	0.028
14	Gosfrd_Wyong	0.003	0.192	0.189	0.13	0.30	0.17	0.000	0.011	0.011

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Appendix
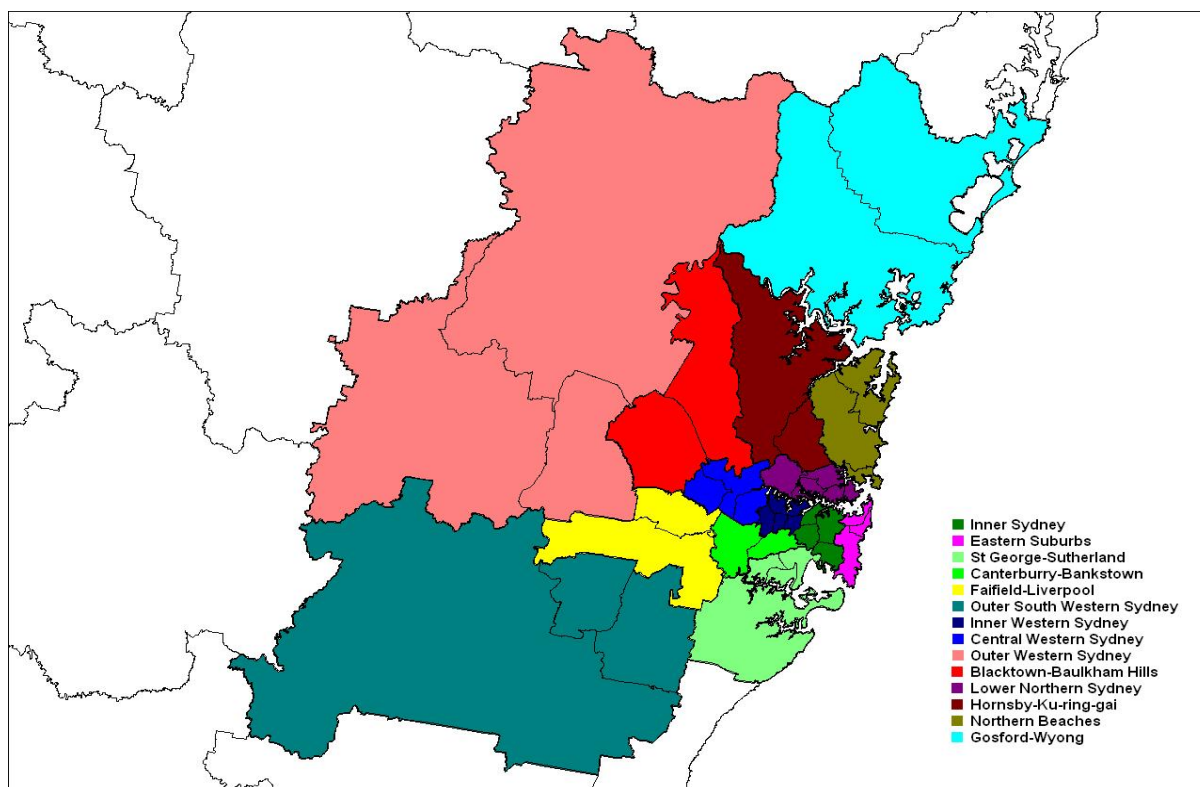
Figure A1 TRESIS-SGEM Zones for the SMA

Table A1 Geographical zones in the Sydney Metropolitan Area with employment levels and journeys to work in 2006

Zone Number	Short Name	Long Name	Employment number in 2006	Journeys to work (daily) in 2006
1	Inner Sydney	Inner Sydney	396498	324963
2	Eastern Suburbs	Eastern Suburbs	63497	84867
3	StGrge Sutherlnd	St George Sutherland	88236	161571
4	Canter. Bankstwn	Canterbury Bankstown	73698	111422
5	Fairfld Liverpl	Fairfield Liverpool	85463	133351
6	Outer SW Syd.	Outer South West Sydney	52938	76538
7	Inner W Syd.	Inner West Sydney	58332	75597
8	Central W Syd.	Central West Sydney	147300	153270
9	Outer W Syd.	Outer West Sydney	81518	119032
10	Blcktwn Blk Hills	Blacktown Baulkham Hills	119490	155674
11	Lower N Shore	Lower North Shore	180611	176070
12	Hornsby Kuringai	Hornsby Kuringai	63148	83344
13	Northern Beaches	Northern Beaches	67010	77238
14	Gosford Wyong	Gosford Wyong	74950	90778
Total	SMA	Sydney Metropolitan Area	1552689	1823716

Source: TRESIS (Hensher (2002); Hensher and Ton (2002)) and ABS (2006)

Table A2 Different labour occupations considered in TRESIS-SGEM

Occupation Number	Short Name	Long Name
1	Managers	Managers
2	Professnals	Professionals
3	TechTrades	Technicians and Trades Workers
4	CommPersServ	Community and Personal Service Workers
5	ClericlAdmin	Clerical and Administrative Workers
6	SalesWorkers	Sales Workers
7	MachOperDriv	Machinery Operators And Drivers
8	Labourers	Labourers
9	Others	Others

Source: ABS (2006)

Table A3 Industries in TRESIS-SGEM

Industry Number	Short Name	Long Name
1	Agr_For_Fish	A,"Agriculture, Forestry and Fishing"
2	Mining	B,"Mining"
3	Manufacturng	C,"Manufacturing"
4	ElyGasWatWst	D,"Electricity, Gas, Water and Waste Services"
5	Construction	E,"Construction"
6	Wholes_Trade	F,"Wholesale Trade"
7	Retail_Trade	G,"Retail Trade"
8	Accom_Food	H,"Accommodation and Food Services"
9	TranPostWare	I,"Transport, Postal and Warehousing"
10	InfoMediaTel	J,"Information Media and Telecommunications"
11	FinanceInsur	K,"Financial and Insurance Services"
12	RentHirRealE	L,"Rental, Hiring and Real Estate Services"
13	ProfSciTech	M,"Professional, Scientific and Technical Services"
14	Admin_Supprt	N,"Administrative and Support Services"
15	PubAd_Safety	O,"Public Administration and Safety"
16	Edu_Training	P,"Education and Training"
17	HlthC_SoAstn	Q,"Health Care and Social Assistance"
18	Arts_Recrtn	R,"Arts and Recreation Services"
19	OthServcs	S,"Other Services"
20	Others	Inadequately described or not stated

Source: ABS (2006)