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# Random regret minimisation and random utility maximisation in the presence of preference heterogeneity: An empirical contrast

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**TITLE:** **Random regret minimisation and random utility maximisation in the presence of preference heterogeneity: An empirical contrast**

**ABSTRACT:** Random regret minimisation (RRM) interpretations of discrete choices are growing in popularity as a complementary modelling paradigm to random utility maximisation (RUM). While behaviourally very appealing in the sense of accommodating the regret of not choosing the 'best' alternative, studies to date suggest that the differences in willingness to pay estimates, choice elasticities and choice probabilities compared to RUM are small. However, the evidence is largely based on a simple multinomial logit form of the RRM model. In this paper we revisit this behavioural contrast and move beyond the multinomial logit model to incorporate random parameters, revealing the presence of preference heterogeneity. The important contribution of this paper is to see if the extension of RRM-MNL to RRM-mixed logit in passenger mode choice widens the behavioural differences between RUM and RRM. The current paper has identified a statistically richer improvement in fit of mixed logit compared to multinomial logit under RRM (and RUM) but found small differences overall between the empirical outputs of RUM and RRM, with no basis of an improved model fit between these two non-nested model forms. The inclusion of both model forms should continue to inform the likely range of behavioural outputs as we investigate a broader range of process heuristics designed to capture real world behavioural response.

**KEY WORDS:** ***random utility, random regret, random parameters, choice probability contrasts, elasticities, model comparisons***

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# 1. Introduction

The great majority of studies using the random regret minimisation (RRM)[1] paradigm have assumed preference homogeneity among decision-makers and estimated a model of the multinomial logit form (without the independence from irrelevant alternatives condition – see below). It is widely accepted that individuals in a population have varying tastes for specific attributes and hence choice outcomes. This recognition of heterogeneity in preferences is something that should matter and hence be investigated whether the underlying behavioural assumption of a model is RRM or random utility (RUM). Extensions to build in preference heterogeneity under RRM through random parameters are rare although there are few studies using non-random parameters and error components to allow for correlated alternatives (see Chorus *et al.* 2014). As far as we are aware, there are three[2]studies that have previously investigated random preference heterogeneity under the random regret minimisation framework. One study (Chorus *et al.* 2008) focussed on a single attribute that was not statistically significant and a random regret threshold that was statistically significant, while the another study (Hess *et al.* 2012) was unable to obtain any evidence to support the presence of preference heterogeneity. Hess *et al.* (2012) estimated models allowing for additional random taste heterogeneity and found that significant variations were only observed in the stand-alone RUM model (MMNL) and in the RUM component of a combined model (MMNL and RRM), but not in the stand-alone RRM model or the RRM component of the combined model. The recent study by Boeri and Maserio (2014) is the most complete on a comparison of MMNL RRM and RUM in the context of freight transport, where they find noticeable differences in mean elasticity estimates and market shares. They suggest that "regret becomes an important choice paradigm when a negative shift in the reference point is introduced" (page 557). Our interest is to see, in the context of passenger transport (or another data set), whether similar findings are obtained, given the somewhat inconclusive evidence from previous passenger based studies by Chorus *et al.* (2008) and Hess *et al.* (2012). Although some readers might see this as yet another study comparing RRM and RUM, it is different in that the focus is on MMNL with random parameters in contrast to the growing number of studies that compare MNL model outcomes.

Given the paucity of evidence on the contribution of RRM compared to RUM under preference heterogeneity assessed through random parameters[3], there remains scope to pursue a further inquiry into the potential gains offered through the main behavioural outputs; namely willingness to pay estimates, choice elasticities and choice probabilities. This is the main motivation and focus of the paper.

[1] Some studies have estimated models in which a subset of attributes is treated as subject to RUM, and a subset subject to RRM. We do not consider this hybrid form in this paper; details are available in Chorus *et al.* (2013) and Hess and Stathopoulos (2013). Caspar Chorus and colleagues have been prominent in the study and promotion of RRM (see Chorus *et al.* 2008, 2013, 2014 and Hensher *et al.* 2013).

[2] Since completing this paper, we have been advised of a recently published paper by Boeri and Maserio (2014) that used freight data to estimate a mixed logit RRM model. We also have found the paper by de Bekker-Grob, and Chorus (2013) in the health setting as well as the review paper published in later 2014 by Chorus et al. (2014). However, of the eight papers cited as MMNL, seven are error components models with fixed parameters, and only Boeri and Maserio (2014) allowed for random parameters.

[3] In a recent paper, Cranenburgh et al. (2015) introduce scale under fixed parameters, and conclude that this noticeably improves the relative fit of RRM compared to RUM models. We are unaware of any RRM modelling that has accommodated both scale and taste heterogeneity.

The paper is organised as follows. We begin with a brief overview of the random regret model given that it is well documented in the literature already, and then set out the context in which we empirically compare RRM and RUM. We then present the model findings, followed by a lengthy assessment of the main behavioural outputs. We compare RRM and RUM within a mixed logit setting as well as within a MNL setting, including a comparison of MNL and mixed logit under RUM and under RRM. The paper concludes with the main findings and suggestions for ongoing research.

# 2. Overview of the Random Regret Mixed Logit Model

Regret (Loomes and Sugden, 1982) is said to occur when a non-chosen alternative leads to a more desirable outcome, for example, when a foregone alternative performs better on a certain attribute compared to the chosen alternative. Under the RRM, respondents are assumed to engage in regret avoidance behaviour by choosing the alternative which minimises regret. The regret for any considered alternative *j*, denoted $Reg(j)$, is the sum of all binary regrets of choosing alternative *j* over the non-considered alternatives $j' \in J$. The random regret minimisation model, proposed in Chorus *et al.* (2008) and subsequently refined by Chorus (2010), has been shown to be able to accommodate the compromise effect.

Specifically, given *K* (*k*=1,…,*K*) attributes:

$$Reg(j) = \sum_{\substack{j' \neq j, \\ j' \in J}} Reg(j, j') = \sum_{\substack{j' \neq j, \\ j' \in J}} \sum_{k} \ln\left(1 + \exp\left[\beta_k (X_{j'k} - X_{jk})\right]\right) \quad (1)$$

In the limit as $\beta_k (X_{j'k} - X_{jk})$ becomes sufficiently negative, $Reg(j, j')$ with respect to attribute *k* falls towards zero. Likewise, if $\beta_k (X_{j'k} - X_{jk})$ becomes sufficiently large, $Reg(j, j')$ with respect to attribute *k* approaches $\beta_k (X_{j'k} - X_{jk})$.

The RRM model is semi-compensatory in the sense that improvements or deteriorations in an alternative depend on the relative attribute level compared to other alternatives. Where an attribute performs well relative to other alternatives, an improvement generates only a small decrease in regret; whereas the same magnitude of improvement generates a larger decrease in regret if the attribute was performing relatively poorly to begin with. Consequently, the RRM model does not exhibit the property of independence from irrelevant alternatives, even with the assumption of *IID* error terms. Moreover, the RRM is just as parsimonious as the standard RUM model, unlike other models of contextual effects which typically require the estimation of additional parameters (Chorus 2010, Hensher *et al.* 2015, Ch 21). Despite these desirable properties, empirical support for the RRM appears mixed. For example, the RRM model under multinomial logit only marginally outperforms its RUM counterpart in three out of the four datasets reported by Chorus (2010). In the remaining dataset, the linear additive RUM turns out to be the better model instead. Similar evidence is given in Hensher *et al.* (2013).

Extending the non-random parameter (preference homogeneity) form to consider preference heterogeneity is relatively straightforward. The choice probabilities of the mixed multinomial logit (MMNL) model under random regret, $P_{nsj}$, (*s*=1,...,S choice scenarios) now depends on the random parameters, with distributions defined by the analyst. The MMNL model is summarised below in (2) for RRM.

$$P_{nsj}=\text{Prob}(choice_{ns}=j\,|\,\mathbf{x}_{nsj},\mathbf{x}_{nsj'},\mathbf{z}_n,\mathbf{v}_n)=\frac{\exp(V_{nsj})}{\sum_{j=1}^{J_{ns}}\exp(V_{nsj})} \tag{2}$$

where

$V_{nsj}$ = $\sum_{\substack{j'\neq j,\\ j'\in s}}\sum_{k}\ln\left(1+\exp\left[\beta_{nk}(X_{nj'sk}-X_{njsk})\right]\right)$

$\boldsymbol{\beta}_n$ = $\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{Z}_n + \boldsymbol{\Gamma}\mathbf{v}_n$

$\mathbf{X}_{nsj},\mathbf{X}_{nsj'}$ = the $k$=1,…,$K$ attributes of alternative $j$ or $j'$ in choice situation $s$ faced by individual $n$,

$\mathbf{Z}_n$ = a set of $M$ characteristics of individual $n$ that influence the mean of the taste parameters,

$\mathbf{v}_n$ = a vector of $K$ random variables with zero means and known (usually unit) variances and zero covariances.

The MMNL-RRM model estimated herein uses stated choice data (six choice scenarios per respondent) and hence a panel form is required. The derivation of the log-likelihood functions of the panel formulation differs to the equivalent cross sectional form, in that the choice observations are no longer assumed to be independent within each respondent (although the independence across respondents assumption is maintained) (as shown in Hensher *et al.* 2015). Mathematically, this means that $E\left(P_1P_2\right)\neq E(P_1)E(P_2)$; hence the log-likelihood function of the panel MMNL model may be represented as Equation (3).

$$\log E(L_N)=\sum_{n=1}^{N}\log E\left(\prod_{s\in S_n}\prod_{j\in J_{ns}}\left(P_{nsj}\right)^{y_{nsj}}\right), \tag{3}$$

The probability that an individual n makes the sequence of choices in $S$ repeated choice situations is:

$$\text{P}_{\text{ni}}=\int\prod_{s=1}^{S}\left[\frac{\exp(R_{nsi})}{\sum_j\exp(R_{nsj})}\right]f(\beta\,|\,\Omega)d\beta \tag{4}$$

$$LL=\sum_{n=1}^{N}\sum_{i}^{J}y_{ni}\ln P_{ni}$$

# 3. The Empirical Setting

The data used in this study of commuter mode choice is drawn from a larger study undertaken in 2009 to investigate the patronage potential of a proposed new metro rail corridor (see Figure 1) in Sydney, designed to offer an alternative alignment and level of service to the existing older rail network. Although the metro plan was abandoned for political reasons soon after the study was completed, the choice data collected remains a rich source of information to investigate alternative choice model forms. Full details of the data and study approach are set out in Hensher *et al.* (2011) including a descriptive overview of the data and the socioeconomic profile. Thus, we only describe the main information relevant to the current paper.

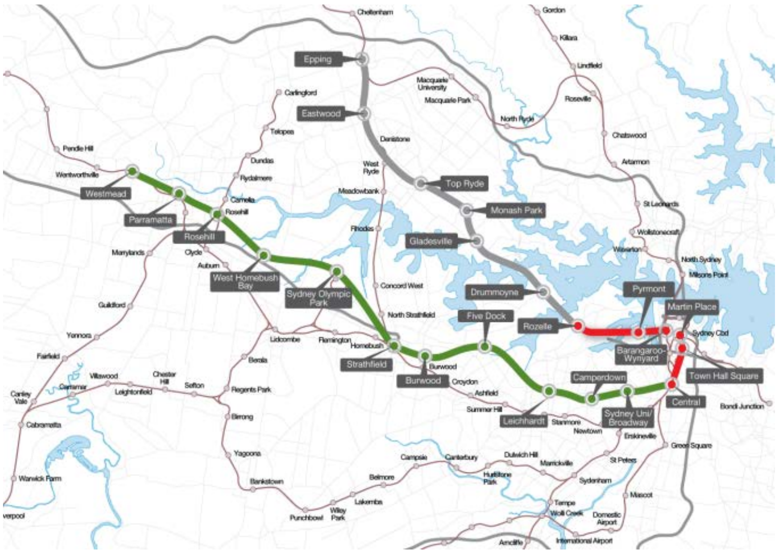



*Figure 1 Proposed Metro Lines in Sydney*

The modal alternatives presented to the respondents vary from one person to another. This variation is determined by the responses given by respondents early in the survey in terms of the availability of the various alternatives for the recent trip. For example, Figure 2 was presented to a respondent who reports not having a car available for the recent trip. As with the modal alternatives, several attributes vary across respondents. These include in-vehicle travel time, travel cost, access and egress times which were pivoted around a reference trip that a sampled individual had recently undertaken. However, for the other attributes such as frequency, crowding, getting a seat and number of transfers, the range was fixed and unlinked from the current trip experience. Table 1 provides all designed levels of crowding on public transport, described by the proportion of seats occupied and the number of people standing, and Figure 2 gives an example of how this information is represented to respondents.

*Table 1 Crowding attribute levels by Mode*

| Level | Seated | Bus | Train | Metro |
|---|---|---|---|---|
| | | Standing | Standing | Standing |
| 1 | 25% | 0 | 0 | 0 |
| 2 | 50% | 0 | 0 | 0 |
| 3 | 60% | 0 | 0 | 0 |
| 4 | 70% | 0 | 0 | 0 |
| 5 | 80% | 0 | 0 | 0 |
| 6 | 80% | 5 | 5 | 5 |
| 7 | 90% | 0 | 0 | 0 |
| 8 | 90% | 5 | 5 | 5 |
| 9 | 100% | 0 | 15 | 16 |
| 10 | 100% | 3 | 30 | 32 |
| 11 | 100% | 7 | 45 | 47 |
| 12 | 100% | 11 | 60 | 63 |
| 13 | 100% | 15 | 75 | 78 |
| 14 | 100% | 19 | 90 | 94 |

| | | | | |
|---|---|---|---|---|
| 15 | 100% | 23 | 105 | 109 |
| 16 | 100% | 27 | 120 | 125 |

The commuter sample of 620 individuals resident in Sydney was selected for model specification.[4] The sampling plan reflects travel across the catchment area that not only includes the central business district, but also areas within the metropolitan area that extend as far west as Westmead (in terms of the rail network), and the north west of Sydney (out to Rouse Hill).

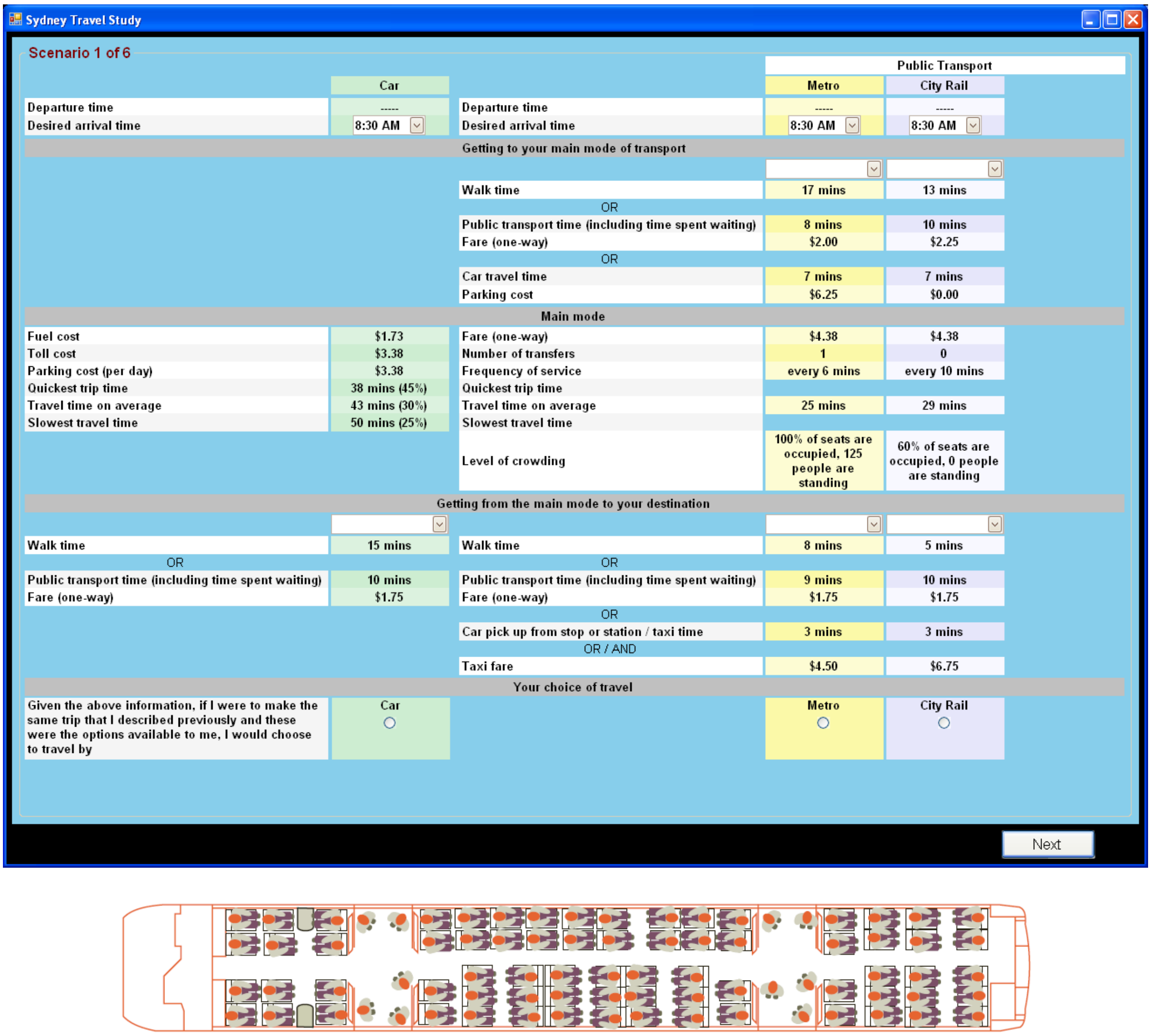



*Figure 2 An Illustrative Choice Scenario*

[4] All trip purposes were studied, but we focus in this paper only on commuting trips.

# 4. Model Results

Four models have been estimated, two under RUM and two under RRM.[5] Given the focus of the majority of previous studies has been on a multinomial logit form, we report the findings under both MNL and mixed multinomial logit. The focus of this section is on assessing the statistical gain in introducing random parameters under random regret in particular, using the Vuong test (Vuong 1989) which is applicable when models are non-nested (such as RUM and RRM). This test aims to establish if there is an improvement in overall statistical fit when migrating from RUM to RRM under both MNL and MMNL. The model results are summarised in Table 2.[6] All random parameters have been estimated under an unconstrained normal distribution, with 500 Halton draws, and allowing for the panel nature of the data. We tested for alternative-specific versus generic parameters and Table 2 reflects the final choice of a mix of generic and alternative-specific parameters.

***Table 2 Summary of estimated RUM and RRM MNL and Mixed Logit models (588 respondents, 3,528 observations) (t-values in parenthesis), 500 Halton draws, and random parameters normally distributed***

| Attribute | Modal Alternative(s) | Multinomial Logit (MNL) | | Mixed Logit (MMNL) | |
|---|---|---|---|---|---|
| | | RUM (model 1) | RRM (model 2) | RUM (model 3) | RRM (model 4) |
| *Mean estimates (for both MNL and MMNL)* | | | | | |
| Travel time (mins) | bus, train, metro | -0.0180 (-3.07) | -0.0117 (-2.94) | -0.0728 (-4.02) | -0.0567 (-4.19) |
| Travel time (mins) | car | -0.0320 (-4.81) | -0.0204 (-4.82) | -0.0762 (-2.95) | -0.0502 (-2.98) |
| Access fare ($) | bus, train, metro | -0.1242 (-5.27) | -0.0976 (-5.22) | -0.2840 (-3.52) | -0.1944 (-2.43) |
| No. people standing | train | -0.088 (-2.50) | -0.0064 (-2.41) | -0.0348 (-2.34) | -0.0283 (-2.40) |
| Egress time | All modes | -0.0186 (-1.50) | -0.0148 (-1.58) | -0.0342 (-0.77) | -0.0235 (-0.63) |
| % seats occupied | bus | -0.8900 (-1.41) | -0.5872 (-1.39) | -1.7418 (-1.69) | -0.9675 (-0.94) |
| *Fixed parameters* | | | | | |
| Fare ($) | bus, train, metro | -0.1260 (-2.66) | -0.1024 (-2.94) | -0.1180 (-1.11) | -0.0956 (-0.90) |
| Access time (mins) | bus, train, metro | -0.0436 (-4.31) | -0.0327 (-4.32) | -0.0856 (-3.90) | -0.0647 (-4.76) |
| Bus constant | bus | 0.7385 (1.33) | 0.5545 (1.28) | 0.9995 (1.27) | 0.6352 (0.81) |
| Metro constant | metro | 0.5076 (2.52) | 0.4862 (3.04) | 0.0058 (0.02) | -0.0664 (-0.30) |
| Car constant | car | 0.0830 (0.18) | 0.038 (0.20) | -0.1152 (-0.10) | -0.2113 (-0.23) |
| Cost (fuel, tolls, parking) | car | -0.0172 (-1.40) | -0.0143 (-1.69) | -0.0675 (-2.27) | -0.0413 (-2.46) |
| *Random parameters – standard deviation* | | | | | |
| Travel time (mins) | bus, train, metro | - | - | 0.0591 (4.08) | 0.0477 (3.52) |
| Travel time (mins) | car | - | - | 0.0589 (3.38) | 0.0329 (2.90) |
| Access fare ($) | bus, train, metro | - | - | 0.3453 (3.04) | 0.2272 (2.24) |
| No. people standing | train | - | - | 0.0357 (2.32) | 0.0302 (2.36) |
| Egress time (mins) | All modes | - | - | 0.2368 (2.90) | 0.1686 (3.02) |
| % seats occupied | bus | - | - | 2.3679 (3.67) | 1.9323 (2.94) |
| *Model fit* | | | | | |
| Log-likelihood (no parameters) | | | -815.14 | | |

[5] The pre-release version of Nlogit6 is used to estimate the mixed logit RRM-model.
[6] We ran models allowing for correlated attributes and correlated alternatives. Both the correlated attributes model (i.e., Cholesky decomposition) and the correlated alternatives (i.e., error components) model for RUM and RRM and did not find statistically significant influences. Model results are available on request from the authors.

| | | | | |
|---|---|---|---|---|
| Log-likelihood at convergence | -452.82 | -453.57 | -360.20 | -358.58 |
| Pseudo $R^2$ | 0.4440 | 0.4436 | 0.549 | 0.551 |
| Information criterion (AIC), sample adjusted | 1.581 | 1.584 | 1.286 | 1.281 |
| Vuong test for non-nested models | 0.3477 | | -0.2168 | |

The overall goodness of fit of the MMNL models is significantly better than the MNL models, of the order of 24 percent higher for both RUM and RRM. With six degrees of freedom difference, this is statistically significant on a Chi-square test at the 95 percent level of confidence. Six attributes were found to be best included as random parameters, although two attributes (i.e., egress time for all modes and proportion of seats occupied for bus) had insignificant mean parameter estimates (with t-value respectively of -0.77 and -1.69 under RUM and -0.63 and -0.94 under RRM). The statistical significance of the fixed mode-specific metro constant is notable (compared to MNL), whereas the main mode fare (although the numerical mean estimates are similar to the MNL models) is not statistically significant under mixed logit in the presence of other random parameters, suggesting that we may be capturing an amount of the unobserved effects associated with the 'new' mode that is otherwise captured in the mean of the constant, by accounting for (random) preference heterogeneity within the sample for six attributes associated with travel time and crowding.

The Vuong non-nested test is based on a comparison of the predicted probabilities of two models that do not nest. This is the appropriate test of differences between RUM and RRM since RRM cannot be specified as a special case of RUM or vice versa. The Vuong closeness test is a likelihood ratio-based test using the Kullback-Leibler (1951) information criterion. The statistic tests the null hypothesis that the two models are equally close to the actual model, against the alternative hypothesis that one model is closer. It cannot make any decision whether the "closer" model is the true model. The Vuong test is based on comparisons of log-likelihoods[7] and works as follows: define $\log L_{0n}$ as the contribution of person $n$ to logL (the overall log-likelihood at convergence) assuming $H_0$; and define $\log L_{An}$ as the contribution of person $n$ to logL assuming $H_A$. The difference is given as $v_n = \log L_{0n} - \log L_{An}$. The Vuong test statistic (VTS) is $\frac{\bar{v}}{sdv(v)/\sqrt{N}}$. A VTS less than -2 favours $H_A$, and a value greater than +2 favours $H_0$. A VTS between -2 and +2 is inconclusive. The VTS is 0.3477 for MNL contrasts and -0.2168 for the MMNL contrasts; hence we can conclude that the evidence is inconclusive and we are not able to suggest that RUM is preferred to RRM or vice versa. For this one data set, RUM and RRM seem to have the same behavioural implications on an overall test of statistical performance. This finding reinforces the evidence from most of the previous studies that compared RUM and RRM (e.g., Chorus *et al.* 2013, Hensher *et al.* 2013).

Despite this finding, there is merit in looking into some specific behavioural outputs of each model form such as the choice probabilities, elasticities and willingness to pay estimates, to see how the models might differ on relevant behavioural metrics. We focus mainly on the mixed multinomial logit model, but before doing so a comparison of the mean, standard deviation and range of choice probability differences between RUM and RRM under MNL and MMNL for each of the alternative modes may add some additional light on possible differences (see Table 3). The full distributions for MNL are summarised in Appendix A, and for MMNL are presented in the following section. The mean differences are negligible, varying from 0.0116 to 0.0241 of a choice probability on the 0-1 scale. The largest difference is for the metro where the highest difference is 0.1474. Although this

[7] The mapping between choice probability and log-likelihood is given in Equation 3.

may seem relatively small, it can aggregate up to a large difference across the population; however there is no case that can be made to suggest that RUM or RRM deliver the better outcome.

***Table 3 Summary of MNL Choice Probability Differences between RUM and RRM by mode (mixed multinomial logit for all alternatives in parentheses, and for chosen only in square brackets)***

| Choice Probability Difference RUM-RRM | Bus | Train | Metro | Car |
|---|---|---|---|---|
| Mean | -0.0241 (0.004) [0.001] | -0.0147(-0.016) [-0.020] | 0.0239 (-0.001) [-0.001] | -0.0116 (0.016) [-0.003] |
| Standard Deviation | 0.0256 (0.029) [0.024] | 0.0289 (0.033) [0.039] | 0.0377 (0.048) [0.047] | 0.0439 (0.056) [0.057] |
| Range | 0.1332 (0.152) [0.097] | 0.1563 (0.205) [0.173] | 0.2245 (0.248) [0.247] | 0.1926 (0.224) [0.206] |
| Minimum | -0.0782 (-0.061) [-0.044] | -0.0729 (-0.145) [-0.113] | -0.0771 (-0.143) [-0.141] | -0.1086 (-0.106) [-0.093] |
| Maximum | 0.0550 (0.091) [0.053] | 0.0835 (-0.060) [0.060] | 0.1474 (0.106) [0.106] | 0.0840 (0.118) [0.113] |

# 5. Behavioural Contrasts between RUM and RRM

This section concentrates on the mixed multinomial logit model results. We set out the absolute choice probability evidence (Table 4 and Figure 3), followed by the distribution of the differences, and then present and discuss the mean elasticity estimates and values of travel time savings. From Table 4 we note that the mean probability of choosing each mode is either slightly lower or virtually the same for RRM compared to RUM, and the standard deviations are remarkably similar, with car being the only moment that is slightly larger under RRM (0.179) than under RUM (0.151). Again there is no stand out case either way.

***Table 4 Summary of Choice Probability Moments for RUM and RRM MMNL by mode***

| Choice Probability | Bus | | Train | | Metro | | Car | |
|---|---|---|---|---|---|---|---|---|
| | RRM | RUM | RRM | RUM | RRM | RUM | RRM | RUM |
| Mean | 0.303 | 0.306 | 0.277 | 0.264 | 0.529 | 0.528 | 0.374 | 0.389 |
| Standard Deviation | 0.104 | 0.101 | 0.165 | 0.155 | 0.196 | 0.197 | 0.179 | 0.151 |

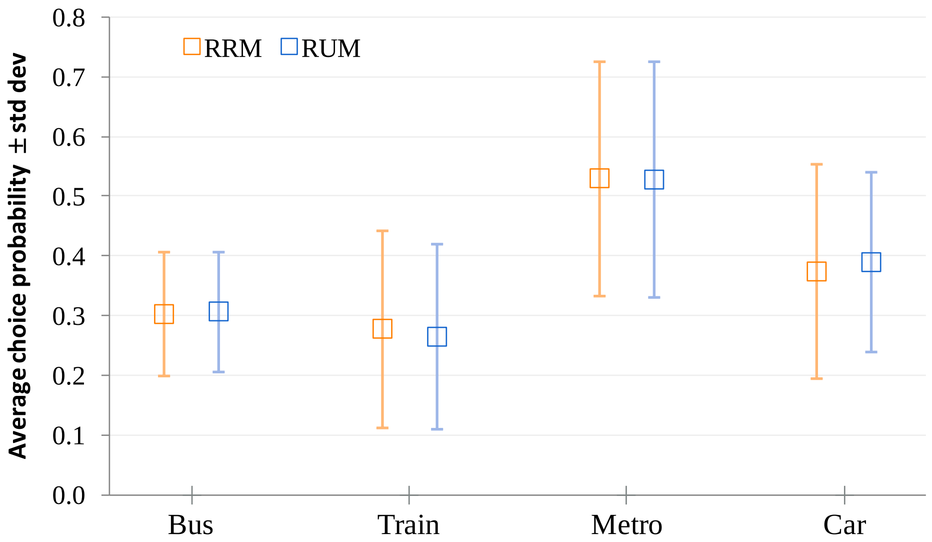



***Figure 3 Choice Probability Moments for RUM and RRM MMNL by mode***

The differences in the choice probabilities between the two model forms by mode is summarised in Figures 4 and 5. Again the differences are extremely small, with the means much closer under mixed logit than under MNL (Table 3), except for car. Figure 5 shows the extent to which RUM choice probabilities are greater than RRM and vice versa. There is asymmetry around zero, but it is interesting to see the sizeable number of choice probability differences in both the positive and negative domains, even though the probability differences are small. Again, however, if we take the extremes of the distributions, we can see some probability differences being as large as -0.14 for train and metro and 0.12 for car. A positive difference indicates that RUM has the higher choice probability and conversely for the negative value. Clearly there are some differences; however they are not significant enough to impact on the overall performance of RRM compared to RUM.

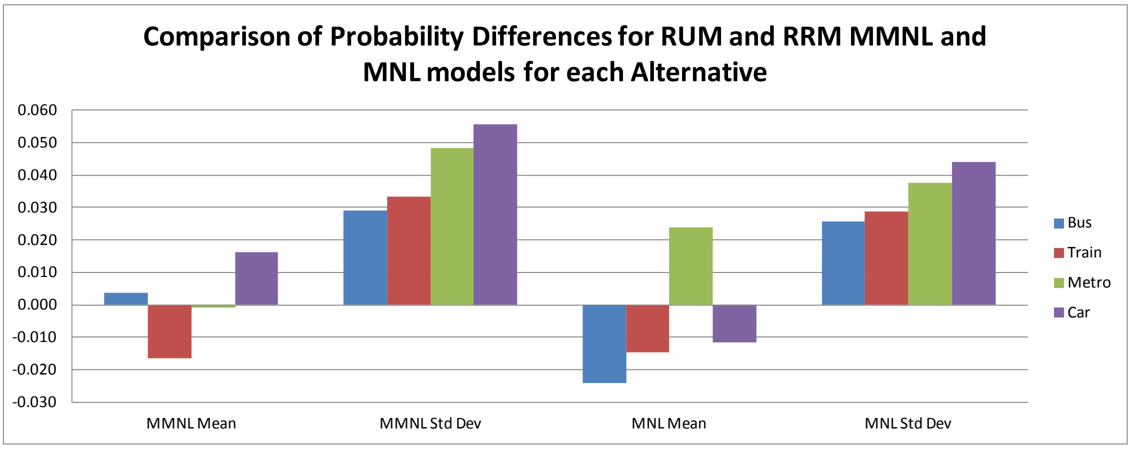
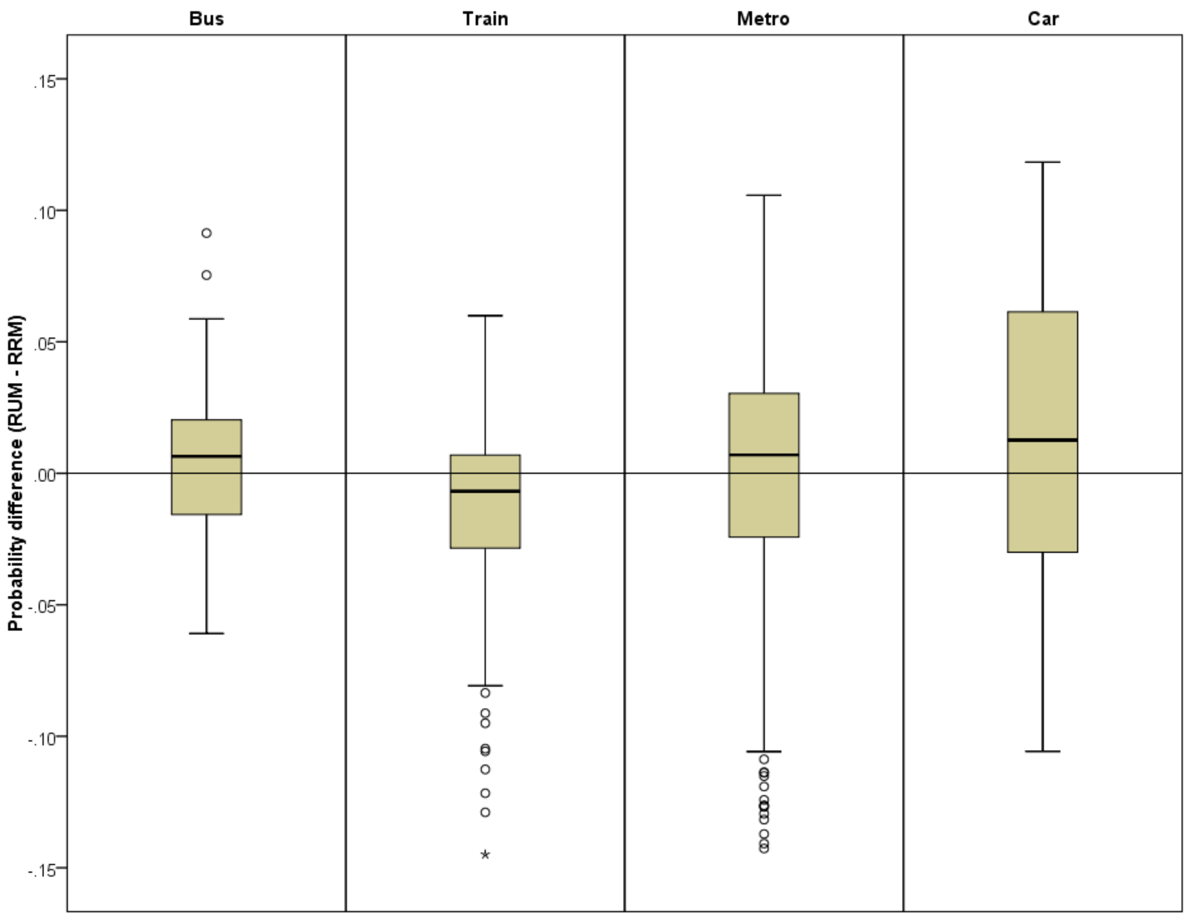



***Figure 4 Differences in estimated probability between RUM and RRM by mode and model specification***




*Note: Extreme values are marked with * (more than 3 Interquartile Range, IQR) and o (> 1.5 IQR)*

***Figure 5 A Box-plot Distribution of difference in estimated probability between RUM and RRM for mixed logit***

Hensher *et al.* (2013) find that in the context of the studied choice-data, the implied mean direct elasticities are quite different for many of the attributes and alternatives between RUM and RRM. Indeed the differences in the elasticities are the strongest hint of behavioural differences between RUM and RRM. The elasticity formula for RRM under mixed logit, requires $\partial R_i/\partial x_{lkn}$, as defined in (4), used to obtain equation (5) – see Hensher et al. (2013) for the full derivation under MNL[8].

$$
\begin{aligned}
&\frac{\partial R_{in}}{\partial x_{lkn}}(\text{where } l \neq i) = \beta_{kn} \frac{\exp[\beta_{kn}(x_{lkn}-x_{ikn})]}{1+\exp[\beta_{kn}(x_{lkn}-x_{ikn})]} = \beta_{kn} q(l,i,k), \\
&\frac{\partial R_{in}}{\partial x_{ikn}}(i.e., \text{where } l = i) = -\beta_{kn}\sum_{j\neq i}^{J} \frac{\exp[\beta_{kn}(x_{jkn}-x_{ikn})]}{1+\exp[\beta_{kn}(x_{jkn}-x_{ikn})]} = -\beta_{kn}\sum_{j=1}^{J} q(j,i,k), \\
&\text{where } q(j,j,k) = 0.
\end{aligned} \tag{4}
$$

$$
\frac{\partial \ln P_{in}}{\partial x_{lkn}} = \beta_{kn}\left[\left(\sum_{j=1}^{J} P_{jn} q(j,i,k)\right) - q(l,i,k)\right] \tag{5}
$$

[8] This the first study, as far as we aware, to obtain mixed logit elasticities for RRM. Boeri and Maerio (2014) only calculated elasticities for MNL.

The elasticity, $\partial lnP_{in}/\partial lnx_{lkn}$ is a simple multiplication of (4) or (5) by $x_{lnk}$. The travel time and car cost elasticities for the MMNL models are summarised in Table 5. The fare elasticities are not reported since the parameter estimate for fare was not statistically significant in both models (see Table 2). There are some very noticeable direct elasticity differences for metro, and especially for car.

***Table 5 Mixed multinomial logit elasticities (negative estimates are direct elasticities, positive estimates are cross elasticities). RUM is the first estimate. Choice probability weighted across the sample.***

| | Bus | Train | Metro | Car |
|---|---|---|---|---|
| Bus travel time | **-0.260, -0.264** | 0.199, 0.057 | 0.113, 0.118 | 0.212, 0.009 |
| Train travel time | 0.458, 0.010 | **-0.495, -0.489** | 0.230, 0.199 | 0.369, -0.027 |
| Metro travel time | 0.631, 0.299 | 0.482, 0.514 | **-0.321, -0.490** | 0.569, 0.259 |
| Car travel time | 0.559, 0.022 | 0.410, 0.031 | 0.205, 0.066 | **-0.302, -0.087** |
| *Car cost* | 0.237, 0.130 | 0.186, 0.093 | 0.114, 0.000 | **-0.201, 0.000** |

To illustrate the way that the evidence is interpreted; in the RUM model, using the bus and car travel times as an example, a ten percent increase in bus travel time results, on average, in a 2.60 percentage reduction in the probability of choosing the bus, given the choice amongst the four modes, holding all other influences constant; however this ten percent increase in bus travel time under RRM takes into account the level of the travel time associated with the other three modes. More specifically, the 2.64 percent reduction in the probability of choosing the bus in RRM explicitly accounts for the levels of travel time in the set of available alternatives, in recognition of the regret that one may have chosen the ‘non-best’ alternative. It is only 1.54 percent higher than the RUM behavioural response, suggesting that accounting for the possibility that the wrong choice may have been made very marginally amplifies the behavioural response that one would normally attribute to a RUM-based elasticity. In contrast, for the car travel time, the difference is 71.9 percent lower for RRM, suggesting that accounting for the possibility that the wrong choice may have been made, significantly suppresses the behavioural response that one would normally attribute to a RUM-based elasticity.

These examples make the very important point that the behavioural responses at an attribute level may well be substantial in the contrast of RRM and RUM, even if one cannot judge one model to be preferred to the other in respect of their overall statistical performance. This is in line with the evidence from previous studies (e.g., Hensher *et al.* 2013). Which one is behaviourally preferred can only be assessed against real market responses, which is something we have no evidence on.

In concluding this section, we contrast the values of travel time savings (VTTS) between RUM and RRM with a recognition that none of the previous comparative studies has done this. The formulae for the RRM model are detailed in Appendix B. The mean estimates are of limited interest; what is of interest is the ratio of VTTS estimates between the two models. The full distribution of such ratios of public transport and car VTTS are presented in Figure 6. The mean (and standard deviation) ratio for public transport (PT) and car are respectively 1.015 (0.152) and 0.922 (0.269), suggesting that the PT estimates are virtually identical on average, whereas the car VTTS is much lower under RUM than under RRM. The range is 1.062 for PT and 2.242 for car; the latter being clearly substantial. A closer look at the full distribution across the sample, given in Figure 6, shows the presence of ratios much closer to zero (below 0.5) for car compared to public transport; hence the variation in the ratio of VTTS across the sample is such that the car VTTS seems to display many more estimates that are higher under RRM than under RUM, even though the ratio of the means is lower.

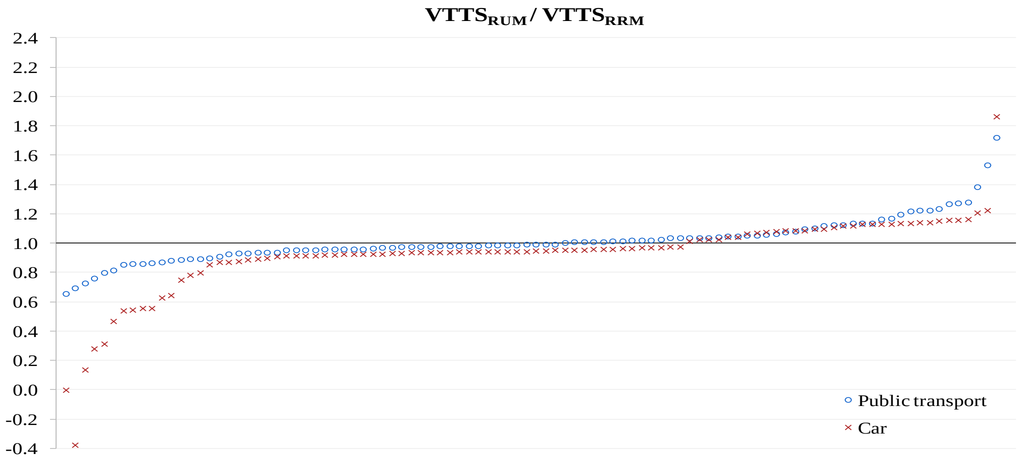



***Figure 6 Sample distributions of public transport and car VTTS ratios for MMNL RUM-RRM***

# 6. Conclusions

This paper set out to explore the difference between RUM and RRM when preference heterogeneity is accounted for through random parameters, in contrast to the predominant assessment in the literature under the non-random parameter MNL form or an error components model form with non-random parameters (as reviewed by Chorus *et al.* 2014). Although the main focus is on comparing the main behavioural outputs from RUM and RRM, we also present the random parameters version of the random regret model together with the formulae for elasticities and willingness to pay estimates, the latter being a simplification of the formulae originally set out by Chorus *et al.* (2013).

The motivation for yet another empirical inquiry on the behavioural contrasts between RRM and RUM was encouraged by the growing support for the semi-compensatory random regret model which, while having considerable behavioural appeal, has not demonstrated conclusively in previous studies its empirical appeal over the RUM form.

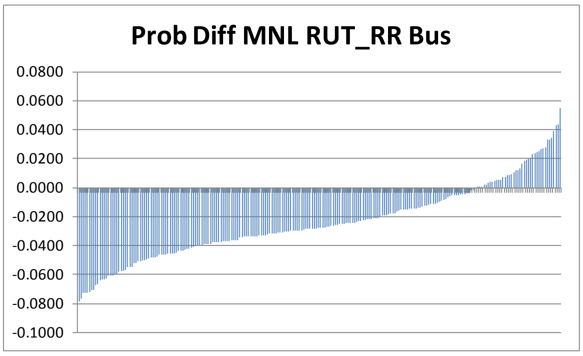
What we find reinforces the accumulating evidence. On the evidence presented in this paper, both models are appealing; however, given that the two models have produced the same (or very similar) empirical results, on a number of occasions including herein, the obvious question to ask is: how can the analyst tell them apart? How can you say which is preferred as the right way to describe the underlying behaviour? We might be guided in answering this question by Chorus (2012a, section 4.2 in Chapter 13) and Chorus *et al.* (2008) who has always suggested that the RRM model has been put forward as an addition to the choice modeller's toolbox, not as a replacement of existing tools such as RUM. The model provides an equally parsimonious but different perspective on decision making, and as such complements, not substitutes, RUM; the idea is that by using different choice models simultaneously, the analyst or policy maker can obtain a broader look at choice behaviour.

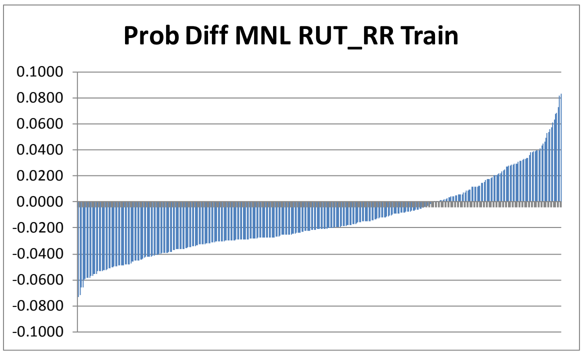
Although the interest in RRM is likely to continue, there may be other more empirically richer candidate attribute and alternative processing strategies worthy of investigation. Finally, ongoing research should investigate the contrasts between various other semi-compensatory RUM forms (i.e., process heuristics including attribute non –attendance and RRM), recognising that in the current paper we have chosen to stay with a model form that is fully compensatory (in respect of attributes). Chorus *et al.* (2014) provide an audit on many studies, the majority being MNL with standard RUM and RRM forms. It is worth noting, however, that Leong and Hensher (2015) compared the Relative Advantage Model (RAM)[9] (based on the smoothed regret function of the RRM model), and the fully compensatory RUM with RRM and found that although model fit differences were small, a comparison shows that the RAM model empirically outperforms the standard random utility maximisation (RUM) model, the RRM model, and a hybrid RUM–RRM model in all eight data sets analysed. The results indicate a need to seriously consider and incorporate context-dependent effects into a literature that has hitherto mainly relied on context-independent models. Leong and Hensher (2012a), however, embed both a linear in parameters and linear in attributes (LPLA) forms under RUM with a non-linear worst level referencing (NLWLR) under RUM into a single model with specific attributes subject one of these behavioural paradigms. Hence utility comprises a context independent effect (in this case LPLA) and a context dependent effect (in this case NLWLR). They conclude that the estimation results show that accounting for some form of referencing and accounting for non-linearity in the utility function are important.

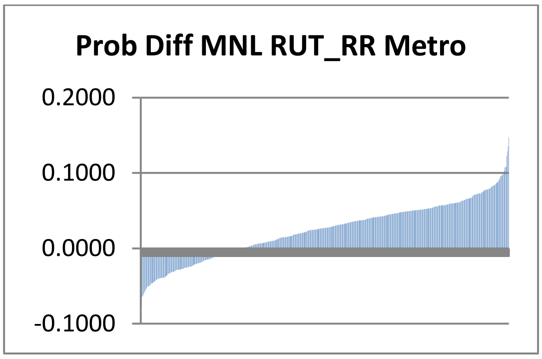
Leong and Hensher (2014) also applied the RAM model to binary choice data on six data sets. For simpler models like the MNL model, the fit of the RAM model is just as good, if not better, than the RUM model, in all six datasets studied, despite the same number of parameters used in both models. However, the improvement in model fit, although significant in some cases, is not very large. Under a random parameters specification that accounts for the pseudo-panel nature of the data, it was found that the RAM model is less successful in explaining the data compared to the RUM model in a larger number of cases. Nevertheless, there are still a few instances where the RAM might still be preferred. There is clearly much scope for ongoing research to next step is to compare a number of model forms under RUM and RRM with random parameters.

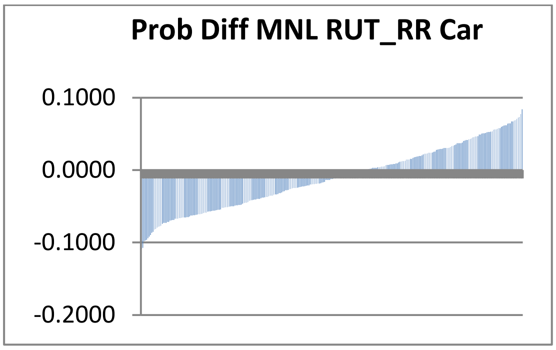
[9] Like the RRM model, the RAM model assumes that each alternative is assessed against all other alternatives in the choice set. However, one key difference between RAM and RRM is that the RAM model explicitly considers the disadvantages and advantages of an alternative, with the advantages of an alternative expressed as a ratio to the sum of advantage and disadvantage.

# 7. Appendix A: MNL model choice probability differences for RUM and RRM by each mode



# 8. Appendix B: Willingness to Pay (WTP) under RRM

We compute willingness to pay using the ratio of marginal utilities, $-MU_{i,t}/MU_{i,c}$ where "i" indexes the alternative in question, t is an attribute, c is the cost (income surrogate), and in what follows, j indexes the set of alternatives. For a random utility specification, as opposed to random regret, the counterpart is

$$WTP_{RUM} = -\frac{\partial U_i / \partial x_{it}}{\partial U_i / \partial x_{ic}} = -\frac{\beta_t}{\beta_c}$$

The following expression for random regret is given in Caspar, Rose and Hensher (2013):

$$WTP_{RRM} = \frac{\partial U_i / \partial x_{it}}{\partial U_i / \partial x_{ic}} = -\frac{\sum_{j\neq i}\left[\dfrac{-\beta_t}{1+\dfrac{1}{\exp\left[\beta_t(x_{jt}-x_{it})\right]}}\right]}{\sum_{j\neq i}\left[\dfrac{-\beta_c}{1+\dfrac{1}{\exp\left[\beta_c(x_{jc}-x_{ic})\right]}}\right]} = -\frac{\beta_t}{\beta_c}\frac{\sum_{j\neq i}\left[\dfrac{1}{1+\dfrac{1}{\exp\left[\beta_t(x_{jt}-x_{it})\right]}}\right]}{\sum_{j\neq i}\left[\dfrac{1}{1+\dfrac{1}{\exp\left[\beta_c(x_{jc}-x_{ic})\right]}}\right]}$$

Note that the willingness to pay measure now depends on the alternative *i* considered as well as the levels of the attributes. It is not a single constant. In a mixed model, the parameters may also vary with the individual. By manipulating the bracketed terms, we obtain

$$WTP_{RRM} = -\frac{\beta_t}{\beta_c}\frac{\sum_{j\neq i}\left[\dfrac{1}{\dfrac{1+\exp\left[\beta_t(x_{jt}-x_{it})\right]}{\exp\left[\beta_t(x_{jt}-x_{it})\right]}}\right]}{\sum_{j\neq i}\left[\dfrac{1}{\dfrac{1+\exp\left[\beta_c(x_{jc}-x_{ic})\right]}{\exp\left[\beta_c(x_{jc}-x_{ic})\right]}}\right]} = -\frac{\beta_t}{\beta_c}\frac{\sum_{j\neq i}\left[\dfrac{\exp\left[\beta_t(x_{jt}-x_{it})\right]}{1+\exp\left[\beta_t(x_{jt}-x_{it})\right]}\right]}{\sum_{j\neq i}\left[\dfrac{\exp\left[\beta_c(x_{jc}-x_{ic})\right]}{1+\exp\left[\beta_c(x_{jc}-x_{ic})\right]}\right]} = -\frac{\beta_t}{\beta_c}\frac{\sum_{j\neq i} p^*_{j,it}}{\sum_{j\neq i} p^*_{j,ic}}$$

where we have defined the notation $P^*_{j|it}$ and $P^*_{j|ic}$ for the terms in the summation. These terms resembles the probability associated with a binary logit model. The omitted term in the summation, when *j* equals *i*, is ½[10]. By adding all pairs of alternatives and then subtracting ½, we obtain the convenient form[11],

$$WTP_{RRM} = -\frac{\beta_t}{\beta_c}\frac{\left[\left(\sum_j p^*_{j|it}\right)-\frac{1}{2}\right]}{\left[\left(\sum_j p^*_{j|ic}\right)-\frac{1}{2}\right]}$$

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[10]For each pair of alternatives the choice probability is one for an alternative associated with itself, so for j=j you get 1/(1+1/1)=1/2.

[11] This formula is included in the pre-release version of Nlogit6.

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