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Accounting for travel time variability in the optimal pricing of cars and buses.

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TITLE: **Accounting for travel time variability in the optimal pricing of cars and buses.**

ABSTRACT: This paper addresses the problem of optimal pricing of both cars and buses in a multimodal transport corridor including externalities of congestion, bus crowding and travel time variability. A social welfare maximisation approach is implemented and applied to Sydney, Australia. To characterise travel time variability, a mean-variance model is embedded in the model and a relationship between mean and standard deviation of travel times is empirically estimated. First, we find that as the sensitivity of users to travel time variability increases, the optimal car toll increases approximately linearly, whereas the optimal bus fare remains almost constant, explained by the fact that even though both car and bus users contribute to increased travel time (and bus headway) variability, the contribution of car users is much higher and that is reflected in the socially optimal bimodal pricing structure. Including travel time variability produces substantial increases in toll revenue. Second, if bus headway is variable, the optimal headway is shorter when the users' valuation of savings in travel time variability is large. This result may not hold when headway is reliable but travel time is not, in which case both optimal bus size and headway are adjusted according to travel time variability and crowding costs.

KEY WORDS: ***Optimal pricing, travel time variability, reliability, headway variability, fare, toll.***

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1. Introduction

Travellers do not like wasting time in traffic or waiting at a bus stop. A major element of research in transport economics and traffic engineering has focussed on estimating and monetising the (average) time savings of infrastructure investments and demand management measures targeted at reducing travel times. We are, however, increasingly aware that users are not only willing to pay for a shorter travel time, but also for a more reliable trip. Uncertain and unreliable travel times cause users to arrive earlier or later than expected at their destination, and influences mode choice, route choice and departure time decisions; suggesting that the variability of travel time plays an important role in the generalised cost of travel. For example, in a survey of Dutch drivers, Verhoef *et al.* (1997) reported that 97.4 percent of respondents disliked driving in congested conditions, and when asked about the reasons for disliking congestion, the most important factors were time losses (4.14 points on a five-point scale), *uncertainty* (3.61), and unpleasant driving conditions (3.52). This is one of many pieces of evidence that points to the relevance of certainty and reliability of travel times for users.

Since travel time variability is often related to traffic congestion (e.g., Eliasson, 2007; Tu *et al.*, 2007; Peer *et al.*, 2012), a transport policy aimed at reducing the level of congestion, such as road pricing, has the potential of reducing travel times and increasing trip time reliability. This expected result has been empirically corroborated through the implementation of road pricing in Stockholm and London, which has resulted in reductions in both the mean and the standard deviation of travel times (Transport for London, 2007; Eliasson, 2009). Therefore, there is a case for incorporating the benefits from reducing travel time variability in the pricing of both car and public transport use.

Most analyses on the optimal pricing of urban transport include mean travel times only, whereas the few studies that incorporate travel time variability focus on car tolling only (e.g., Li *et al.*, 2008; Jiang *et al.*, 2011). In contrast, this paper investigates the optimal pricing structure of both cars and public transport (buses), as well as determining the optimal headway and capacity of the public transport mode, with an approach that explicitly accounts for travel time variability as a source of disutility for users.

When analysing car traffic, if there is a positive correlation between the mean (μ) and standard deviation (σ) of travel time, an increase in the number of cars in congested conditions increases both μ and σ. However, the analysis of public transport is not so simple; there are at least three basic differences with the case of cars that can worsen the consequences of unreliability associated with public transport. First, buses and trains have to stop in order to transfer passengers, creating interactions between vehicles and passengers (in the boarding and alighting process), and between vehicles with each other (e.g., queuing delays). The dwell time may also be variable and such variability depends on several factors, including the scheduled headway, the number of passengers getting on and off, and the fare collection system in the case of buses (Dorbritz *et al.*, 2009).

Second, the variability in travel times impacts not only in-vehicle time cost for users, but also waiting time, since unstable travel times yield schedule delays and headway variability, which in turn increases the waiting time of users (Welding, 1957) and influences activity scheduling decisions. Therefore, unlike car traffic, an increase in bus frequency (headway reduction) has counteracting effects that make its impact on total travel time variability hard to predict: increasing bus frequency may increase travel time variability on the road, but reduce dwell time and headway variability.

Third, the unreliability and uncertainty of travel times on public transport operations also represents an extra cost for operators, who need to adjust the scheduling of services with larger slack times in the case of less reliable travel times (Furth, 2000). All of these considerations make the inclusion of buses into a multimodal analysis for the optimal pricing of travel time variability far from trivial.

In this paper, a multimodal social welfare maximisation model is formulated, that accounts for travel time and bus headway variability by using a mean-variance model (Jackson and Jucker, 1982; Senna, 1994). Travellers can choose between travelling by car, bus or walking in order to complete a trip. Demand is spatially disaggregated along a transport corridor. The decision variables are car toll, bus fare, bus headway and bus size. The model is applied to an actual transport corridor in Sydney that is subject to congestion. We find that as the sensitivity of users to travel time variability increases, the optimal car toll increases approximately linearly, whereas the optimal bus fare remains almost constant. Even though both car and bus users contribute to increased travel time (and headway) variability, the contribution of car users is much higher, and is reflected in the socially optimal bimodal pricing structure. Second, if bus headway is variable, the shorter the optimal headway the more sensitive users are to travel time variability. This result may not hold when headway is constant but travel time is not, in which case both optimal bus size and headway are adjusted according to travel time variability and crowding costs.

The remainder of the paper is organised as follows. Section 2 provides a literature review on the determinants of travel time variability and the valuation of travel time variability. Section 3 presents regression models for the relationship between the mean and standard deviation of travel time, estimated with data collected across 423 roads in Sydney. The reliability-sensitive social welfare maximisation approach is introduced in Section 4. In Section 5 the main results of the numerical application are discussed. Conclusions and directions for further research are summarised in Section 6.

2. Literature review

2.1 Determinants of travel time variability: Car traffic

Travel time variability (TTV) is related to random variations in travel time caused by factors that cannot be anticipated or foreseen by a traveller (Fosgerau *et al.*, 2008; Tu, 2008). Tu (2008) divides the sources of TTV in two groups: demand fluctuations and supply fluctuations. Notable sources of variability in traffic demand are temporal effects (e.g., peak/off-peak, weekday/weekend), network effects (effect of traffic in one lane or road over travel times on other parallel or intersecting lanes/roads), and spatial and temporal differences in driving attitude. On the other hand, factors such as volatile or adverse weather conditions, traffic incidents and accidents, and traffic composition influence both demand and road capacity (Tu, 2008).

With the increasing availability of observed travel times, traffic flows and travel speeds on urban and inter-urban networks, analysts have been trying to explain the determinants of TTV based on empirical measurement of these traffic variables. There is no agreement on the dependent variable used as a measure of TTV, and several measures have been proposed to account for the degree of variability of travel time (Pu, 2011), including the standard deviation of travel time (May *et al.*, 1989; Eliasson, 2007; Hellinga *et al.*, 2012; Mahmassani *et al.*, 2012; Peer *et al.*, 2012), the difference between the 90^{th} and 10^{th} percentile of travel time (Eliasson, 2007; Tu *et al.*, 2007), the coefficient of variation of travel time (May *et al.*, 1989; Eliasson, 2006), the standard deviation and the variance of the delay[1] (Mott MacDonald, 2008) and the probability that travel time is below a certain threshold (Asakura, 1998). In some cases, variability is analysed for whole sections or links (May *et al.*, 1989; Eliasson, 2006, 2007; Peer *et al.*, 2012), whereas other authors model variability per unit of road length (per kilometre), as

[1] Delay defined as actual travel time minus free-flow travel time.

a way to have a distance-free measure (Tu *et al.*, 2007; Mott MacDonald, 2008; Mahmassani *et al.*, 2012).

The most common variable used to analyse travel time variability is the mean travel time or the mean delay. A majority of authors have found a positive correlation between travel time variability and mean travel time (May *et al.*, 1989; Eliasson, 2007; Mott MacDonald, 2008; Hellinga *et al.*, 2012; Peer *et al.*, 2012), nevertheless the shape of the relationship varies from case to case. Using travel time data from a set of Dutch highways, Peer *et al.* (2012) show that TTV, measured as the standard deviation of travel time (σ), increases with the mean travel time (μ) and that the relationship is concave, i.e., the rate at which variability grows with the mean travel time decreases with travel time. Hellinga *et al.* (2012) found a similar result, explaining the standard deviation as a function of the mean travel time by using a logarithmic (concave) relationship. On the other hand, Mott MacDonald (2008) analyse TTV for different types of links on English motorways, finding that the shape of the relationship depends on the section or type of highway analysed; in particular, the relationship between μ and σ can be concave or convex, i.e., the coefficient of variation may be an increasing or decreasing function of travel time. In links with extreme congestion, Eliasson (2006) shows that the standard deviation divided by travel time, might be a decreasing function of the travel delay, using data from a number of urban roads in Stockholm containing traffic lights. Eliasson (2007) finds that σ is higher in the “after AM peak” and “after PM peak” periods, which are interpreted as queue dissipation phases, and that a higher speed limit also increases σ.

Instead of analysing the relationship between variability and mean travel time, Tu (2008) relates variability directly to traffic flow. Using highway sections in Beijing, China, and Delft, the Netherlands, it is found that the impact of inflow on TTV depends on the flow itself; there is a low demand range at which travel times are fairly constant and variability is low. However, when flow reaches a ‘critical transition inflow’, an increase in demand is associated with a rapid increase in TTV. This increased variability is maintained until flow reaches a ‘critical capacity inflow’, after which TTV can decrease with demand.

2.2 Determinants of travel time variability: The case of public transport

Research on characterising travel time variability has mainly focused on cars. Nonetheless, public transport modes are also subjected to variations in travel time and headway. The social cost of unreliability in public transport may be substantial; for example, Van Oort (2011) estimates a yearly cost of €12 million in The Hague, The Netherlands, due to unreliable buses and trams. Improving public transport reliability yields multiple benefits, including increased accessibility, additional ticket revenue and reductions in congestion and environmental externalities, if a modal shift from car to public transport is induced (Van Oort, 2011).

There are a number of studies that have analysed bus travel time variability based on empirical data (e.g., Abkowitz and Engelstein, 1983; Strathman and Hopper, 1993; Strathman *et al.*, 1999; El-Geneidy *et al.*, 2008; Mazloumi *et al.*, 2010; Moghaddam *et al.*, 2011). Common indicators proposed to assess the reliability of a public transport service are the standard deviation of travel time, the probability of on-time performance[2], the travel time ratio (observed travel time/scheduled travel time), the average additional travel time per passenger (Van Oort, 2011) and measures to analyse the variability of headways. These studies usually find that travel time variability, however it is measured, increases with factors such as the length of a route, number of stops and signalised intersections, with longer headways and higher passenger activity (boarding and/or alighting), with part-time or unexperienced drivers, and that a deviation in travel time at an early stage on a route (including a late departure from the first stop) propagates further downstream as buses proceed. Mazloumi *et al.* (2010) found an almost linear relationship between the standard deviation and mean travel time for buses on an urban route in Melbourne. Moghaddam *et al.* (2011) also estimated a positive relationship between the

[2] Defined as a bus being between 1 min early to 5 min late at the destination point (Strathman and Hopper, 1993)

standard deviation of travel time and the volume/capacity ratio as an indicator for congestion on the route. On-board fare collections systems, including cash payment, have been found to increase the standard deviation of boarding times (Dorbritz *et al.*, 2009).

2.3 Estimation of users' valuation of travel time variability

In this section we provide a brief summary of the main approaches that have been proposed to examine the users' valuation of travel time variability (for in-depth reviews see Li *et al.*, 2010; and Carrion and Levinson, 2012). The scheduling model and the mean-variance model are the two most common methods to deal with travel time reliability and departure decisions. The scheduling model (Small, 1982; Noland and Small, 1995; Bates *et al.*, 2001) assumes that being early or late at a destination is a source of disutility for travellers. The general form for the utility function U in this model is:

$$U = \delta C + \alpha T + \beta SDE + \gamma SDL + \vartheta D_L \tag{1}$$

where C is the monetary cost of travel, T is travel time, SDE and SDL are the schedule delay early and late, compared to the preferred arrival time, D_L is a dummy variable that is active when arriving late at destination; δ, α, β and γ are the (negative) marginal utilities of cost, travel time, minutes early and minutes late, respectively, and ϑ is a fixed penalty for a late arrival. Parameters for such scheduling models have been estimated by e.g., Small (1982), Bates *et al.* (2001) and Van Amelsfort *et al.* (2008).

The mean-variance approach (Jackson and Jucker, 1982; Senna, 1994; Lam and Small, 2001 among others) suggests that the variability of travel time is a cost by itself, no matter if travellers arrive early or late. Under these assumptions, expected utility can be expressed as:

$$U = \delta C + \alpha \mu + \rho \sigma \tag{2}$$

where μ and σ are the mean and standard deviation of travel time. Analogous to the value of travel time savings (equal to α/δ in equation 1), the *value of reliability* (VOR) is defined as the ratio of the marginal utility of the standard deviation to the marginal utility of cost (i.e., VOR= ρ/δ in equation 2). Another popular outcome of the mean-variance model is the *reliability ratio* (RR), defined as the ratio of the value of saving one minute of the standard deviation of travel time, to the value of reducing one minute of average travel time (RR= ρ/α in equation 2).

Recently, Fosgerau and Karlström (2010) have shown that the scheduling and mean-variance models are equivalent under certain conditions[3]. In this case, the optimal expected utility (after users have chosen an optimal departure time) from the scheduling model can be expressed as a linear function of the mean and standard deviation of travel time, where the factor ρ depends on the scheduling costs β and γ, and the travel time distribution. Empirical evidence suggests that, however, the valuation of travel time variability from a scheduling model may be significantly smaller than that of a mean-variance model (Börjesson *et al.*, 2012).

[3] Namely that the scheduling utility function is linear (such as equation 1), there is no discontinuous penalty for being late (i.e., ϑ =0 in equation 1) and the travel time distribution is independent of the departure time. Fosgerau and Karlström (2010) also analysed the case in which the mean and standard deviation of travel time vary linearly with the departure time, and found that the equivalency between the two approaches (scheduling and mean-variance models) does not hold exactly but can be used as an approximation.

In this paper we use a mean-variance model as the mathematical conceptualisation of travel time variability in the utility function associated with the car and bus travel alternatives. A relationship between the mean and standard deviation of travel time is the simplest construct that can be obtained from empirical data, to be embedded into a microeconomic analysis of optimal pricing and design of a public transport service. The characterisation of travel time variability is described in Section 3.

2.4 Road pricing and travel time reliability

While most researchers looking at optimal road pricing strategies have focused on the minimising travel times in the network (e.g., Yang and Lam, 1996), maximising total toll revenues (e.g., Joksimovic *et al.*, 2005) and minimising emissions (e.g., Johansson, 1997) or externalities in general, some have suggested looking at pricing strategies from a network reliability perspective (e.g., Brownstone and Small, 2005). Chan and Lam (2005) were among the first to look at the impact of road pricing on travel time reliability, and formulated a reliability-based static user equilibrium problem and optimised toll levels to optimise the network travel time reliability based on the probability that the travel time is below a certain threshold.

Setting tolls for the optimisation of network travel time reliability in the context of a dynamic user equilibrium was first investigated by Li *et al.* (2007, 2008) using the standard deviation as a measure of travel time unreliability. Jiang *et al.* (2011) considers a multicriterion dynamic user equilibrium problem in which travel time, travel cost, and travel time reliability are included, and different vehicle types are considered (i.e. low and high occupancy vehicles). As far as we are aware, road pricing in the context of travel time reliability with respect to cars as well as public transport has not yet been considered in the literature.

3. Empirical relationship between mean and standard deviation of travel time: The case of Sydney

In order to estimate a relationship between the mean and standard deviation of travel times, we use a database of floating car data provided by the Roads and Maritime Services (RMS) office of the New South Wales Government in Australia. The data comprises measurements of travel time along several roads in Sydney, in which vehicles are equipped with a GPS device. For each road, a particular trip is repeated ten times over two weeks (from Monday to Friday, the first week in October 2011, the second week in March 2012) at the same time each day. Then, for each trip, a mean and standard deviation of travel time [min/km] is calculated over ten observations. In this paper, only major urban roads are considered (highways are not accounted for). The total number of roads is 423. The scatter plot of mean vs. standard deviation of travel times is shown in Figure 1.

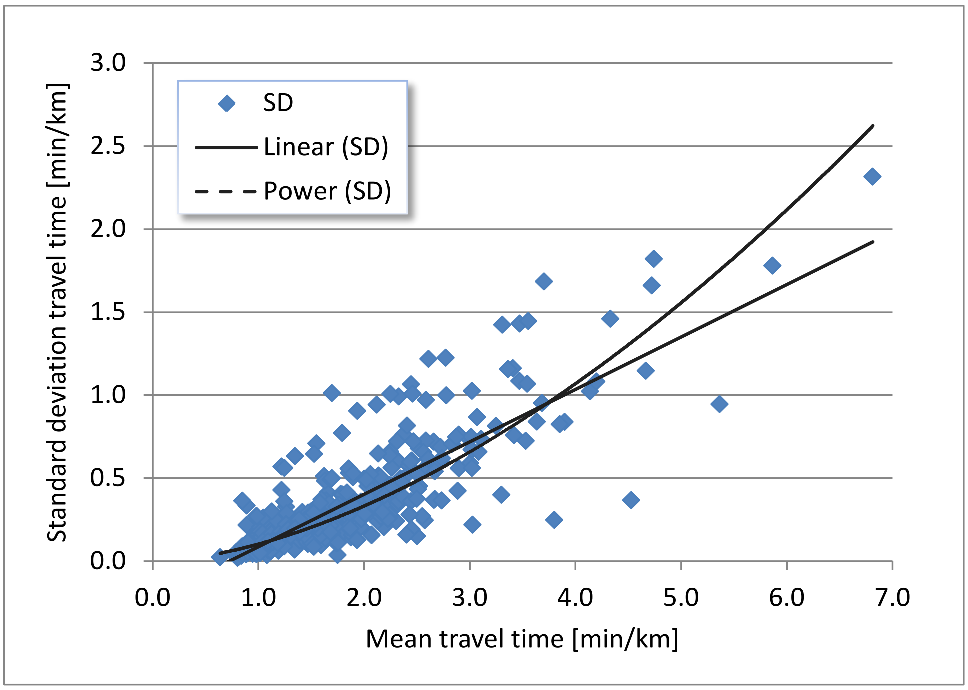

Figure 1: Mean and standard deviation of travel times, Sydney

Linear and power regressions are estimated as follows:

$$\sigma_L = a_0 \mu_L + a_1 \tag{3}$$

$$\sigma_L = a_2 {\mu_L}^{a_3} \tag{4}$$

In (3) and (4), μ_L and σ_L are the mean and standard deviation of travel time per unit of distance (min/km) and coefficients a_i are regression parameters, estimated in Table 1. We observe that the linear model produces a slightly superior model fit relative to the power model, when comparing Adj-R^2. [4]

Table 1: Estimation of regression models

Model	Coefficient	Value	t-ratio	Adj-R2
Linear	a_0	0.316	30.121	0.682
	a_1	-0.229	-10.920	
Power	a_2	0.103	26.557	0.668
	a_3	1.685	29.179	

[4] In a more general power model $\sigma_L = a_2 {\mu_L}^{a_3} + a_4$, constant a_4 is not statistically significant.

The linear model in Table 1 can be compared to Mahmassani *et al.* (2012), who estimate linear and non-linear (square root and quadratic) relationships between μ_L and σ_L for three locations in the U.S. (Irvine, the Baltimore-Washington Corridor and New York City) using the traffic simulation model DYNASMART. They estimate regression models at four aggregation levels: network, O-D pair, path and link. For their path level model (the equivalent to the models of Table 1), coefficient a_0 is between 0.25 and 0.53 for their linear regressions. Our estimate of a_0 =0.316 in Table 1 falls within this range. This figure means that an increase of one minute in the mean travel time per kilometre implies an average increase of 19 seconds in the standard deviation of travel time.

4. Social welfare maximisation approach

The analytical model used in this paper for the study of optimal TTV pricing of cars and public transport is based on the social welfare maximisation model developed by Tirachini (2012) for the analysis of optimal pricing and design of a bus route, including congestion and crowding externalities. In this section we summarise the main elements of the model; further details are provided in Tirachini (2012) and Tirachini *et al.* (2012).

We consider a linear bi-directional road of length L and a single period of operation with directions denoted as 1 and 2. The road is divided into P zones denoted as $i \in \{1,...,P\}$, and the total demand Y^{ij} per origin-destination pair (i, j) is fixed. The distance between zone i and zone $i+1$ is denoted as L_i such that $L = \sum_{i=1}^{P-1} L_i$, as shown in Figure 2. Users can choose to travel by car (a), bus (b) or to walk (e). Let f_{a1}^i be the traffic flow between zone i and zone $i+1$ (direction 1) and f_{a2}^i be the traffic flow between zone $i+1$ and zone i (direction 2). There is only one bus stop per zone and the travel distance between zones is the same for the three modes. The decision variables of the problem are denoted as follows:

f_b : bus frequency [bus/h] (the inverse of bus headway)

s_b : bus length [m]

τ_a : car toll [\$/trip]

τ_b : bus fare [\$/trip]

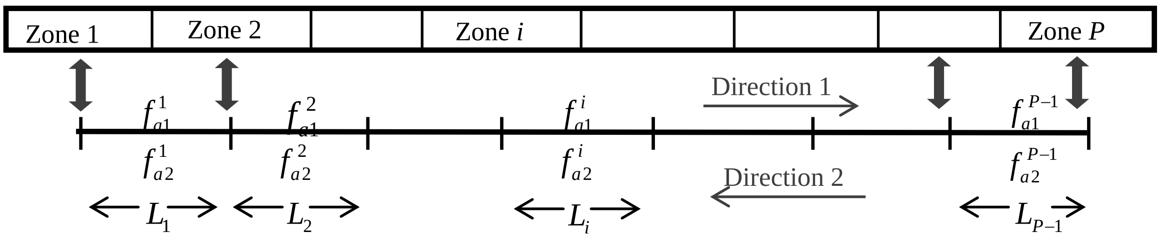

Figure 2: Transport corridor diagram

Bus frequency is assumed to be continuous whereas options on bus lengths are constrained by the size of commercial vehicles (e.g., 8 m, 12 m, 15 m. and 18 m. long buses). We assume that cars and buses share the right-of-way, and that bus stops do not directly affect cars.

Multinomial logit models for modal choice are estimated, including the proportion of available seats and the density of standees as attributes for the bus alternative. Data collected from a stated choice survey conducted in Sydney in 2009 is used to this end[5]. Let U_m^{ij} be the utility associated with travel by mode m in origin-destination (OD) pair (i, j) (direction 1 is used for illustration):

Bus:

$$U_b^{ij} = \alpha_b + \beta_a t_{ab}^i + \beta_h h_b + \beta_c \tau_b + \beta_{vb} t_{vb}^{ij} + \sum_{k=i}^{j-1} \left(\beta_{den} n_{den}^k + \beta_{seat} p_{seat}^k \right) t_{vb}^{k,k+1} + \\ + \theta_b \beta_h \sigma_h + \theta_b \left[\beta_{vb} \sigma_{tb}^{ij} + \sum_{k=i}^{j-1} \left(\beta_{den} n_{den}^k + \beta_{seat} p_{seat}^k \right) \sigma_{tb}^{k,k+1} \right] \qquad (5)$$

Car: $$U_a^{ij} = \beta_{va} t_{va}^{ij} + \beta_c \left(c_r^{ij} + \tau_a \right) / o_r + \theta_a \beta_{va} \sigma_{ta}^{ij} \qquad (6)$$

Walk: $$U_e^{ij} = \alpha_e + \beta_{ve} t_{ve}^{ij} \qquad (7)$$

In (5), the first line is the utility when travel time and headway are not variable, whilst the second line is the utility associated with the standard deviation of headway and travel time. On the first line, t_{ab}^i is the access time at zone *i*, h_b is the mean headway between two consecutive buses, t_{vb}^{ij} is the in-vehicle time between zones *i* and *j*, τ_b is the bus fare, n_{den}^k is the density of standees per square metre between zones (stations) *k* and *k*+1, p_{seat}^k is the proportion of seats occupied between zones *k* and *k*+1, α_b is a modal constant (which will be calibrated to predict an observed modal split) and β_l are the parameters associated with the different attributes. On the second line, θ_b is the bus reliability ratio *RR* (defined as the ratio of the value of reliability to the value of in-vehicle time, see equation 2)[6], σ_h is the standard deviation of headway, and σ_{tb}^{ij} is the standard deviation of bus in-vehicle time. In the application of the model (Section 5), we use the linear specification (3) to characterise travel time variability[7], therefore σ_{tb}^{ij} is given by expression (3) times trip length.

For cars (expression 6), c_r^{ij} is the car running cost to travel between zones *i* and *j*, τ_a is the road charge (decision variable), o_r is the average car occupancy rate, θ_a is the car reliability ratio and σ_{ta}^{ij} is the standard deviation of car travel time. Bus fare and car toll do not vary with trip length. Finally, in (7) walking is assumed neither subject to congestion nor travel time variability (i.e., we ignore the potential influence of factors such as traffic lights on the travel time variability of pedestrians).

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5 The experimental design, study area, sample size and socioeconomic characteristics of respondents are described at length in Hensher et al. (2011).

6 For buses, we assume the same reliability ratio for the variability of both in-vehicle time and headway.

7 The power model (expression 4) does not produce any major difference regarding outcomes on pricing structure and bus design, on the range of car and bus speed obtained in the application of Section 5.

Assuming a multinomial logit model for the estimation of demand, the number of trips by mode m in OD pair (i, j) is given by:

$$y_m^{ij} = Y^{ij} \frac{e^{U_m^{ij}}}{\sum_n e^{U_n^{ij}}} \quad \forall i, j, m \tag{8}$$

where Y^{ij} is the total demand between zones *i* and *j* and y_m^{ij} is the demand between zones *i* and *j* on mode *n*. The parameter estimates for utility functions (5) to (7) are given in Appendix A1.

Consumer surplus B is given by the logsum formula:

$$B = \sum_{ij} \frac{y_m^{ij}}{\mathrm{I}_u} \ln \sum_m e^{U_m^{ij}} + B_0 \tag{9}$$

where I_u is the marginal utility of income (equal to minus the cost parameter β_c in linear utility functions) and B_0 is a constant that has no effect on the solution of the problem, and therefore can be set to zero.

We assume that buses and cars share the right-of-way, which is subject to congestion. Taking direction 1 for illustration, we model the travel time between zone *i* and zone *i*+1 by car (t_{va1}^i) and bus (t_{vb1}^i) as a function of traffic flow (cars/h) and bus frequency (buses/h), by using the well-known Bureau of Public Roads (BPR) formulae:

$$t_{va1}^i\left(f_{a1}^i, f_b\right) = t_{a0}^i\left[1+\gamma_0\left(\frac{f_{a1}^i+\varphi\left(s_b\right)f_b}{K_r}\right)^{\gamma_1}\right] \tag{10}$$

$$t_{vb1}^i\left(f_{a1}^i, f_b\right) = t_{b0}^i\left[1+\gamma_0\left(\frac{f_{a1}^i+\varphi\left(s_b\right)f_b}{K_r}\right)^{\gamma_1}\right] + t_{s1}^i \tag{11}$$

where t_{a0}^i, t_{b0}^i, γ_0 and γ_1 are parameters (t_{a0}^i and t_{b0}^i are the free-flow travel times by car and bus, respectively), $\varphi \geq 1$ is the passenger car equivalency factor of a bus, which depends on the bus length s_b, and K_r is the hourly capacity of the road.

The travel time by bus includes the delay due to bus stops, t_{s1}^i, which consists of the acceleration and deceleration delay t_{ac1}^i and the dwell time t_{d1}^i. The delay in the process of accelerating and decelerating at bus stops is modelled by assuming uniform acceleration and deceleration. We assume that boarding and alighting are allowed at all bus doors, therefore dwell time is estimated as:

$$t_d^i = c_{oc} + p_b \mathrm{B}_b \lambda^{i+} + p_a \mathrm{B}_a \lambda^{i-} \quad (12)$$

where c_{oc} is the time to open and close doors, B_a and B_b are the average alighting and boarding times per passenger, λ^{i+} and λ^{i-} are the number of passengers boarding and alighting a bus at the bus stop, respectively, and factors p_a and p_b are the proportion of passengers boarding and alighting at the busiest door[8].

Next, we formulate bus operator costs. Let operator cost be divided into three components:

$c_1(s_b)$: Station infrastructure cost [$/station-h]

$c_2(s_b)$: Personnel costs (crew) and vehicle capital costs [$/bus-h], and

$c_3(s_b)$: Running costs (fuel consumption, lubricants, tyres, maintenance, etc.) [$/bus-km]

All cost components depend on bus length s_b. The cost per bus-hour $c_2(s_b)$ has two elements: the personnel cost (wages) and the capital cost of a vehicle. The third component of operator cost is the running cost per vehicle-kilometre $c_3(s_b)$, which includes fuel consumption, lubricants, tyres, maintenance, etc. With this, the total operator cost Co can be defined as:

$$C_o(s_b, F) = c_1(s_b)S + c_2(s_b)F + c_3(s_b)VF \quad (13)$$

where S is the number of bus stops, F is the fleet size and V is the operating speed (including running time and stops). The fleet size requirement is given by $F = f_b T_c$, in which T_c is the cycle or round-trip time (given by the summation of bus travel time (11) at all sections and both directions, plus a scheduled slack time at termini if required). Rewriting T_c as $2L/V$, we see that the third term in (13) does not depend on the operating speed and passenger demand. Therefore, the final expression for bus operator cost is given by (14).

$$C_o(f_b, s_b) = c_1(s_b)S + c_2(s_b) f_b T_c(f_b, s_b) + 2c_3(s_b) L f_b \quad (14)$$

Finally, the social welfare maximisation problem is formulated as follows:

$$\underset{\tau_a, \tau_b, f_b, s_b}{Max} \qquad SW = \sum_{ij} \frac{y^{ij}}{\mathrm{I}_u} \ln \sum_m e^{U_m^{ij}} + \sum_{ij} y_a^{ij} \tau_a + \sum_{ij} y_b^{ij} \tau_b - C_o \quad (15)$$

Subject to

$$\max_i \left\{ y_{b1}^i, y_{b2}^i \right\} \leq \kappa f_b S(s_b) \quad (16a)$$

$$f_b^{\min} \leq f_b \leq f_b^{\max} \quad (16b)$$

$$s_b \in \left\{ s_{b1}, \ldots, s_{b4} \right\} \quad (16c)$$

---

[8] We assume p_a=p_b=100 percent for buses with one door, 60 percent for buses with two doors, 43 percent for buses with three doors and 30 percent for buses with four doors. See Tirachini (2012).

$$y_m^{ij} = Y^{ij} \frac{e^{U_m^{ij}}}{\sum_n e^{U_n^{ij}}} \quad \forall i, j, m \tag{16d}$$

Inequality (16a) is a capacity constraint that ensures that the bus transport capacity, $\kappa f_b S\left(s_b\right)$, is large enough to accommodate the maximum bus load; κ is a design factor introduced to have spare capacity to absorb random variations in demand (for example, $\kappa = 0.9$) and $S\left(s_b\right)$ is the bus capacity [pax/h]. Frequencies are constrained by a minimum policy frequency $f_b^{\min}$ (set to have a minimum level of service, if desired) and the maximum feasible frequency $f_b^{\max}$ as given in expression (16b). Expression (16c) establishes that bus size s_b is taken from available (discrete) choices. Finally, in this setting modal choice depends on travel times, which in turn depend on modal choice; this fixed-point problem is solved by iterating between modal choice and travel times until convergence is reached, using the set of equilibrium constraints (16d).

The constrained optimisation (15)-(16) is solved using the optimisation toolbox of Matlab. The solution procedure implemented considers bus frequency as a continuous variable while the bus length, car toll and bus fare are discrete (fare and toll are constrained to be a multiple of 5 cents).

5. Numerical application

5.1 Physical setting, input parameters and assumptions

The social welfare maximisation model is applied with demand and supply data from Military Road in North Sydney, shown in Figure 3. The section modelled comprises 3.44 km of road which is divided into 12 zones (therefore the average zone length is 286 metres). The origin-destination matrix of trips by all modes (car, bus and walking) for the morning peak (7.30 to 8.30am) is shown in Figure 4.

Figure 3: Test corridor, Military Road

O/D	1	2	3	4	5	6	7	8	9	10	11	12
1	0	856	1324	54	23	8	74	99	419	71	16	1405
2	165	0	192	15	4	1	20	19	68	14	3	326
3	829	93	0	0	0	0	0	0	0	0	0	0
4	50	12	0	0	0	0	1	3	13	1	0	91
5	146	0	0	0	0	0	0	0	1	0	0	11
6	235	9	3	0	0	0	0	3	9	0	0	17
7	87	13	4	0	0	0	0	12	48	12	0	187
8	18	1	0	0	0	0	0	0	3	9	0	8
9	396	22	5	1	1	3	24	9	0	27	3	763
10	7	0	0	0	0	0	0	0	0	0	12	1511
11	119	11	1	0	0	0	12	0	3	123	0	1027
12	1780	277	54	21	16	27	151	65	207	3763	1685	0

Figure 4: Origin-destination matrix

The road has two lanes per direction, BPR functions (10) and (11) are assumed to represent travel times with commonly used parameter values $\alpha_0 = 0.15$ and $\alpha_1 = 4$, and a capacity $K_r = 2000 veh/h$ obtained by assuming a 60 percent for effective green time ratio at signalised intersections. Speed at free flow is 50 km/h. With these assumptions plus the calibration parameters of Appendix A1, the average car speed is 26.3 km/h in direction 1 (outbound) and 21.5 km/h in the direction 2 (inbound), similar to the measured average speed of 22 km/h on this road (RTA, 2011, which only reports average speed in the inbound direction in the morning peak).

Four bus sizes are considered in the application of the model, according to the size of commercial vehicles available in the market. These are mini (8 m. long, 2 doors), standard (12 m. long, 3 doors), rigid long (15 m. long, 4 doors) and articulated (18 m. long, 4 doors).The bus equivalency factors $\varphi(s_b)$ are 1.65 for small buses (8 m), 2.19 for standard buses (12 m), 2.60 for rigid long buses (15 m) and 3.00 for articulated buses (18 m), following the linear relationship of Basso and Silva (2010). Fare collection is performed off-board, and hence the average boarding and alighting times are $\mathrm{B}_a = \mathrm{B}_b$=1.46 s/pax-door (Tirachini, 2012). Operator cost parameters are given in Table 2 (Tirachini, 2012).

Table 2: Cost items related to bus size

Bus size [m]	**Bus capital cost [$/bus-h]**	**Driver cost [$/bus-h]**	**Station cost [$/station-h]**	**Operating cost [$/bus-km]**
8	5.1	37.6	4.4	0.9
12	11.9	37.6	6.5	1.3
15	16.9	37.6	8.7	1.4
18	22.0	37.6	10.9	1.6

Users can choose between travelling by car, bus or to walk; other alternatives like switching time period or changing origin and/or destination are not considered. The car operating cost is 14 cents/km (fuel consumption) and the average car occupancy 1.45 pax/car (TDC, 2010), which we assume remains unchanged after pricing reforms (the sensitivity of car occupancy to raising tolls is ignored). Walking speed is 4 km/h.

The parameter estimates for the utility functions (5) to (7) are obtained in the Appendix A1, except for the parameters associated with travel time and headway variability, which cannot be estimated from our Sydney mode choice dataset (see Hensher *et al.* 2011). We make the following assumptions:

- Because the reliability ratio is not known for Sydney, the problem is solved assuming four reliability ratios for the car mode, namely, θ_a = 0.5, 1.0, 1.5 and 2.0, in the range of the values estimated in the literature (two recent reviews are Li *et al.*, 2010; and Carrion and Levinson, 2012).
- The reliability ratio of buses is usually expected to be larger than that of cars because of the discrete nature of bus departures. For example, Bates *et al.* (2001) suggest reliability ratios of around 1.3 for cars and "somewhat higher" for public transport, whereas de Jong *et al.* (2009) suggest reliability ratios of 0.8 for cars and 1.4 for public transport. We assume that the reliability ratio of buses θ_b is 50% larger than that of cars in each case, i.e., θ_b = 0.75, 1.5, 2.25 and 3.0.
- Bus dwell time is unreliable. Dorbritz *et al.* (2009) analysed the mean and standard deviation of bus boarding times for different fare payment scenarios in Zurich, and found that the standard deviation is between 36 and 58 percent of the mean boarding time. We assume that the standard deviation of dwell time (considering boarding and alighting) is 50 percent of the mean dwell time. The standard deviation for dwell time is then added to the standard deviation of travel time for buses, i.e., we assume that travel time and dwell time are not correlated.
- If bus headway is subject to variability, it is assumed that the standard deviation of the headway is equal to its mean. This result stems from assuming that the arrival of buses at bus stops follows a Poisson distribution, as done in several models that consider random bus arrival times at bus stops (e.g., Delle Site and Filippi, 1998; Cominetti and Correa, 2001; Cepeda *et al.*, 2006; Cortés *et al.*, 2011). Implicit in (4) is that we ignore any correlation between headway and travel time in the specification of the bus utility function.

5.2 *Results and discussion*

5.2.1 Base results

Results with the current OD matrix (Figure 4) are shown in Table 3, for reliability ratios $\theta_a \in \{0.0, 0.5, 1.0, 1.5, 2.0\}$ and θ_b such that $\theta_b = 1.5\theta_a$. First, we study the sensitivity of the solution on optimal pricing and bus service design to the increasing values of variability, given by the reliability ratio.

As the sensitivity of users to travel time variability increases, the optimal car toll increases approximately linearly, from \$1.35 when θ_a=0 (i.e., users are assumed insensitive to travel time variability) to \$2.05 when θ_a=2 (the value of reliability doubles the value of travel time savings). On the other hand, the optimal bus fare remains almost constant, on either \$0.35 or \$0.40. That is, even though both car and bus users contribute to increase travel time (and headway) variability, the contribution of car users is much higher and that is reflected in the socially optimal bimodal pricing structure. The MNL demand model was calibrated to predict the current Sydney modal split of trips shorter than 5 kilometres: 62.5 percent car, 31.6 percent walk, and 5.9 percent bus (see Appendix A1); the low bus demand in Sydney explains that the optimal bus size remains constant at a minimum of 8 metres (scenarios with bigger optimal bus sizes are discussed in Section 5.2.2). Comparing to the current situation, Table 3 shows that optimising car toll, bus fare, bus size and bus frequency produces a reduction on the number of car trips and an increase in bus and walking trips. Optimal bus headway reduces from 2.3 to 1.5 minutes (which is attached to an increase in fleet size from 13 to 19 buses), because as the bus reliability ratio θ_b grows, it is optimal to have a greater frequency to reduce not only the mean headway but also its standard deviation. Total social welfare and consumer surplus are lower if users are more sensitive to travel time variability. The optimal public transport subsidy (first best) amounts to between 47 and 56 percent of the total operator cost, but the total loss is more than compensated by the toll revenue (without accounting for toll collection costs). Average car speed is around 22 km/h in the peak direction (D2, towards Zone 1 in Figure 3) whereas bus speed including stops for boarding and alighting is between 17.5 and 18.5 km/h in the same direction.

When compared to the case in which travel time variability is not priced ($\theta_a = \theta_b = 0$), total toll revenue increases by 28 percent if the value of reliability for cars is equal to the value of travel time savings ($\theta_a = 1$) and by 48 percent if the value of reliability doubles the value of travel time savings ($\theta_a = 2$). This is an indication of the substantial effects that including travel time variability into an optimal (first best) transport pricing scheme may have on toll revenue. Note that the increase in revenue happens in spite of the reduction in the total number of cars trips induced by a higher reliability-sensitive toll. Table 3 shows that as the reliability ratio increases, there is a decrease in the modal split of both modes subject to travel time variability (bus and car) and more people walk because walking is assumed uncongestible and reliable.

Table 3: Base results

Reliability ratio car (θ_a)	**0.0**	**0.5**	**1.0**	**1.5**	**2.0**
Optimal bus fare [$]	0.35	0.35	0.40	0.40	0.40
Optimal car toll [$]	1.35	1.55	1.75	1.90	2.05
Optimal bus size [m]	8	8	8	8	8
Optimal bus headway [min]	2.3	1.9	1.8	1.6	1.5
Bus fleet size	13	15	17	18	19
Social welfare [$]	67653	66162	64731	63345	61996
Consumer surplus [$]	57420	54547	51664	49277	46934
Bus operator profit [$]	-437	-542	-557	-629	-692
Subsidy/bus operator cost	0.47	0.53	0.50	0.53	0.56
Toll revenue [$]	10669	12157	13624	14697	15754
Car speed D1 [km/h]	26.3	26.4	26.4	26.5	26.5
Car speed D2 [km/h]	21.6	21.7	21.9	22.0	22.1
Bus speed D1 [km/h]	20.6	20.9	21.2	21.3	21.4
Bus speed D2 [km/h]	17.5	17.8	18.1	18.3	18.5
Modal split bus	7.3%	7.2%	7.2%	7.2%	7.1%
Modal split car	59.6%	59.1%	58.7%	58.3%	57.9%
Modal split walk	33.1%	33.6%	34.1%	34.5%	34.9%

5.2.2 Alternative scenarios on bus reliability and crowding

The previous analysis was carried out assuming that crowding is a source of disutility for users, and that both travel time and bus headways are subject to variability. Next, we compare solutions on optimal pricing for cars and buses, as well as differences in bus service design in the following scenarios, which differ on the assumptions on sources of externalities (results on Table 3 were obtained on scenario S4):

(S1) There are congestion and travel time variability.

(S2) There are congestion, travel time variability and headway variability.

(S3) There are congestion, crowding externalities and travel time variability.

(S4) There are congestion, crowding externalities, travel time variability and headway variability.

The main results on the comparison of scenarios S1 to S4 are summarized next. Figure 5 shows that the optimal bus headway is quite sensitive to the assumptions on the sources of disutility for users. If only congestion and travel time variability (for both cars and buses) are taken into account (S1), the optimal bus headway remains almost constant regardless the bus reliability ratio θ_b assumed. In this case, the optimal bus size is 8 metres for all reliability ratios. Nonetheless, when passenger crowding is also a source of disutility for bus users (S3), the optimal bus headway lies between 2.5 and 3.0 minutes, however the optimal bus size increases to 12 metres (θ_b =0.75 and 1.5 in Figure 5) and 15 metres (θ_b =2.25 and 3.0). This is because, as the reliability ratio increases, the weight of the standard deviation of travel time in the utility function increases, then bigger buses are chosen in order to reduce crowding levels (note the correlation between crowding levels and the valuation of travel time variability introduced in the last term of the bus utility function, equation 5), and therefore reduce the burden of the mean and standard deviation of bus in-vehicle times.

In the two cases with headway variability (S2 and S4), the optimal headway is steadily reduced as θ_b increases, a result previously observed in Table 3. In these cases, increasing frequency (reducing mean headway) has the extra benefit of reducing the cost associated with headway variability; it turns out to be more beneficial to do so rather than increasing bus size. Finally, note that the headway when accounting for crowding externalities (S4) is shorter than when crowding externalities are not considered (S2), an outcome also obtained in models that assume no variability in travel time (Jara-Díaz and Gschwender, 2003; Tirachini *et al.*, 2012). The implications of different optimal headways on fleet size (number of buses required to provide the service) are depicted in Figure 6.

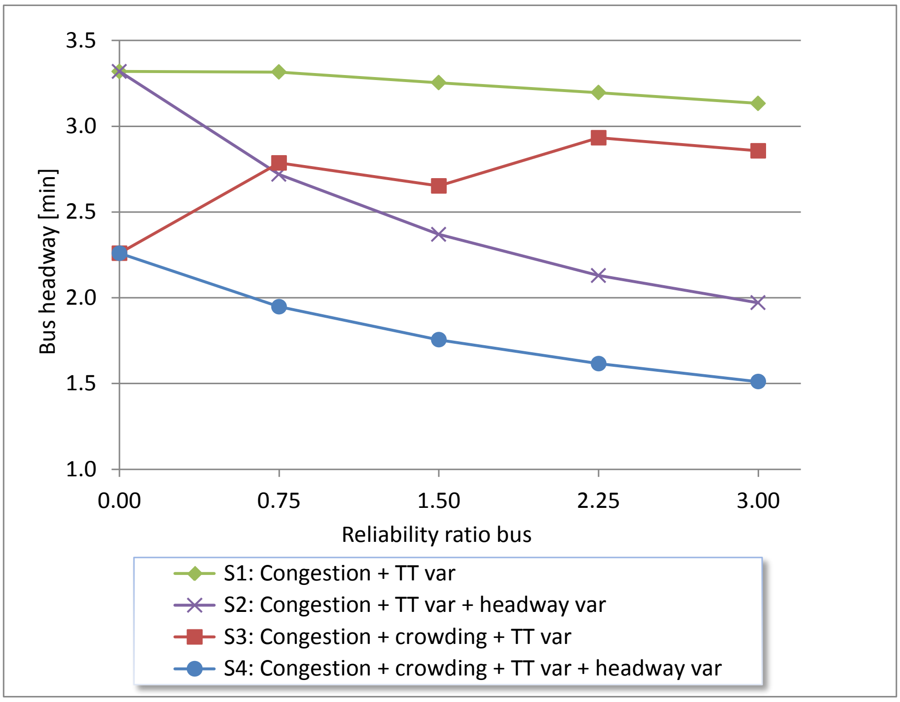

Figure 5: Optimal bus headway

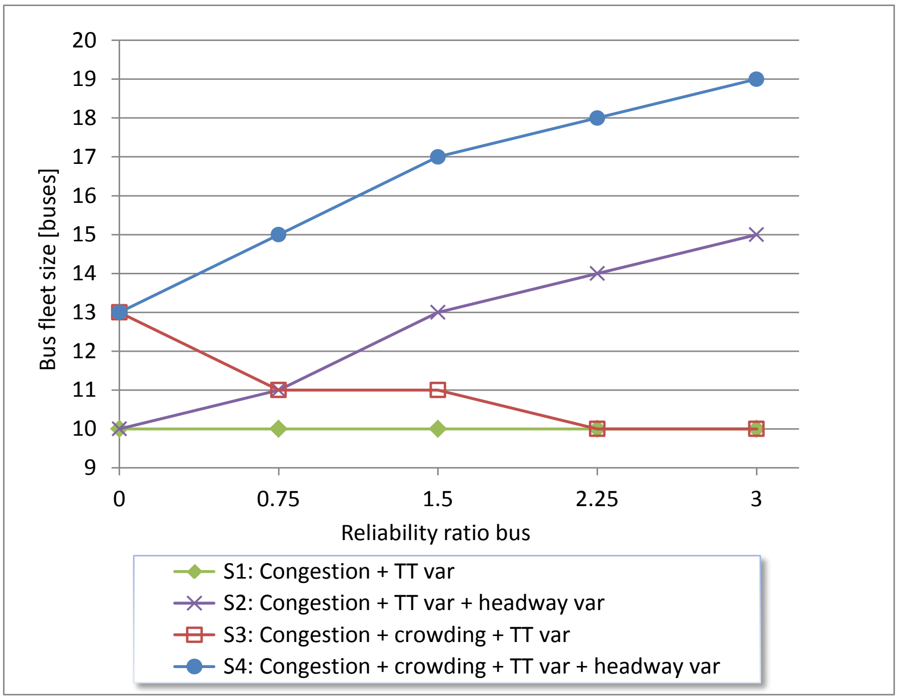

Figure 6: Optimal bus fleet size

The evolution of the optimal pricing structure is depicted in Figure 7. Alternative assumptions regarding sources of disutility influencing bus travel (S1 to S4) have negligible implications on the optimal car toll, so only one case is shown in Figure 7. On the other hand, including headway variability has no noticeable impact on the optimal fare, but including crowding does increase the bus fare for all reliability ratios. Regardless of the scenario considered, the same conclusion as for Table 3 holds, that is, the optimal bus fare remains almost flat as the reliability ratio of buses and cars increases, in contrast to the optimal car toll which steadily increases.

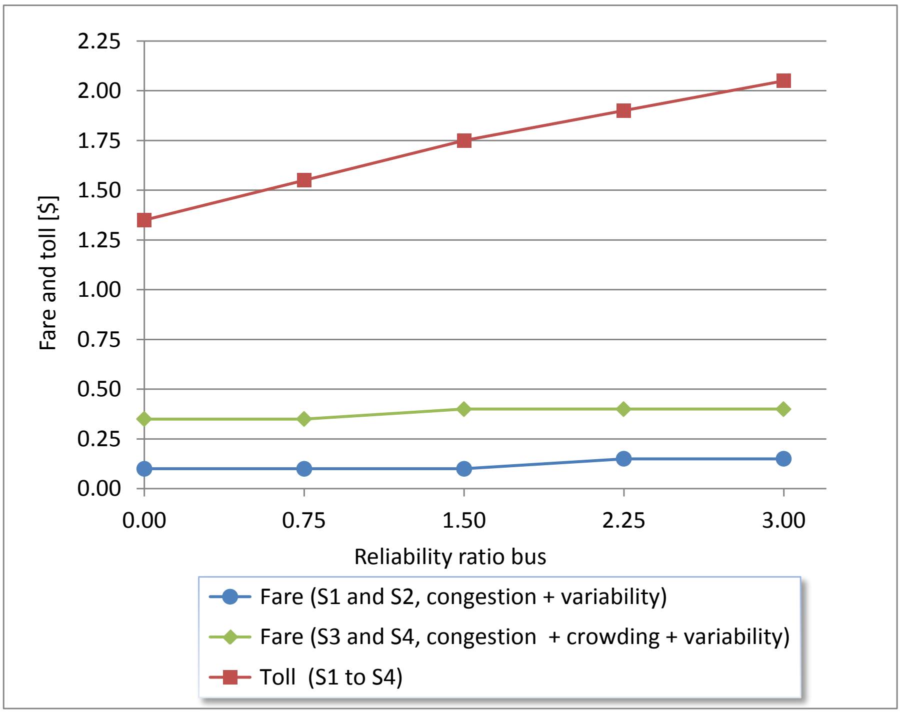

Figure 7: Optimal fare and toll

5.2.3 Increased transport demand

In this section we analyse how the bus service and pricing levels (fare and toll) should be determined when faced with an increase in transport demand (e.g., through a future urban densification around the corridor), assuming that it is not possible to increase road capacity ($K_r = 2000\ veh/h$). The idea is to analyse the evolution of the design variables when the system is stressed and severe congestion arises. The trips by origin and destination of Figure 4 are uniformly scaled in five steps, up to a total demand of 28,850 trips/h (50 percent higher than the current number of trips). Results of the optimal pricing structure and bus headway are shown in Figures 8 and 9. Two cases regarding variability are compared: (i) no variability cost ($\theta_a = \theta_b$ =0), and (ii) scenario S4, with fixed reliability ratios at θ_a =1.0 for cars and θ_b =1.5 for buses. Once again, it is observed that the optimal car toll steadily increases as total demand grows, whereas the optimal bus fare remains almost constant (Figure 8). The optimal bus headway when the variability of travel time and headway is accounted for is always shorter than when no variability is considered (Figure 9).

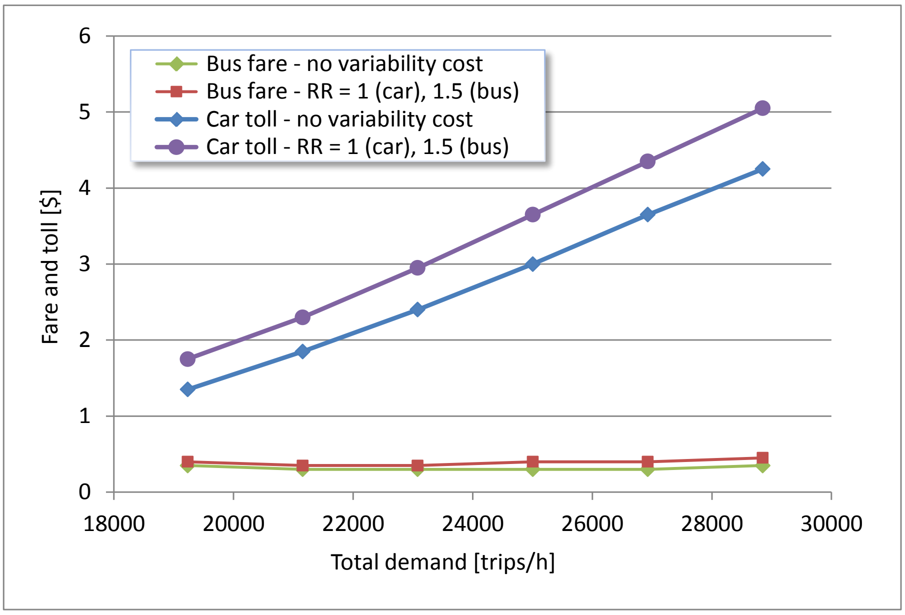

Figure 8: Optimal fare and toll, increased transport demand

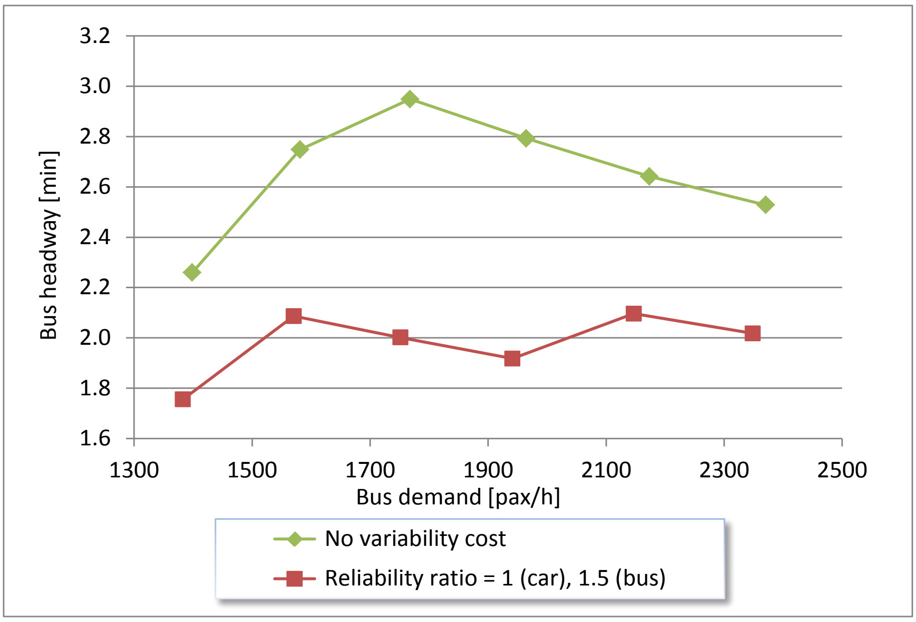

Figure 9: Optimal bus headway, increased transport demand

6. Conclusions

In this paper, we study the optimal pricing structure of both cars and buses when travel times and bus headway are subject to variability, and travellers value reductions in both the mean (μ) and the standard deviation (σ) of travel time. If there is a positive correlation between μ and σ, an additional car on a congested road increases both μ and σ for all drivers. However, when buses are included in the analysis the outcome is not straightforward, because an increase in bus frequency (headway reduction) has counteracting effects on total travel time variability, namely increasing bus frequency may increase travel time variability on the road (for both cars and buses) at the same time that reducing variability of both bus dwell time and headway. Then, the inclusion of buses in a multimodal analysis for the optimal pricing of travel time variability is far from trivial. A multimodal social welfare maximisation model is formulated, that explicitly includes travel time and bus headway variability as sources of disutility for users, through a mean-variance model. Users can choose between travelling by car, bus or walking to complete a trip. Decision variables are car toll, bus fare, bus frequency (inverse of bus headway) and bus size. The relationship between the mean and standard deviation of travel time is empirically obtained using traffic data from Sydney, where the social welfare maximisation model is applied.

We find that as the sensitivity of users to travel time variability increases, the optimal car toll increases approximately linearly, whereas the optimal bus fare remains almost constant. Even though both car and bus users contribute to increase travel time (and headway) variability, the contribution of car users is much higher, and that is reflected in the socially optimal bimodal pricing structure. This result was obtained in a number of different scenarios, including alternative assumptions regarding crowding externalities and travel time and headway variability associated with the bus mode, and for different levels of total transport demand. Second, if bus headway is variable, the more sensitive users are to travel time variability, the shorter is the optimal headway. This result may not hold when the headway is constant and travel time is not, in which case both the optimal bus size and headway are adjusted by the role of travel time variability and crowding costs, which interact with each other.

Future research should investigate further the complex interrelationships that are present in public transport service provision, in particular the possible correlations between headway (which influences waiting time and scheduling delay), dwell time and in-vehicle time, and the correlations between crowding and reliability. The analysis of a full-scale city-wide scenario in which more modal alternatives are in place (e.g., rail, bicycle) is a natural extension of the one-corridor one-period analysis undertaken in this paper, in order uncover the implications on optimal toll system design (location of toll points/gates) and pricing levels of all private and public transport alternatives, when accounting or ignoring travel time variability as a source of disutility for users. The results of this paper suggest that the impact on total toll revenue of including travel time variability is substantial.

References

Abkowitz, M. D. and I. Engelstein (1983). Factors affecting running time on transit routes. Transportation Research Part A 17(2): 107-113.

Asakura, Y. (1998). Reliability measures of an origin and destination pair in a deteriorated road network with variable flow. In: M.G.H. Bell (ed.) Transportation Networks: Recent Methodological Advances, Pergamon Press, Oxford: 273-287.

Basso, L. J. and H. E. Silva (2010). A microeconomic analysis of congestion management policies. 5th Kuhmo Nectar Conference in Transport Economics Valencia, Spain, July 8-9.

Bates, J., J. Polak, P. Jones and A. Cook (2001). The valuation of reliability for personal travel. Transportation Research Part E 37(2-3): 191-229.

Börjesson, M., J. Eliasson and J. P. Franklin (2012). Valuations of travel time variability in scheduling versus mean–variance models. Transportation Research Part B 46(7): 855-873.

Brownstone, D. and K. A. Small (2005). Valuing time and reliability: assessing the evidence from road pricing demonstrations. Transportation Research Part A 39(4): 279-293.

Carrion, C. and D. Levinson (2012). Value of travel time reliability: A review of current evidence. Transportation Research Part A 46(4): 720-741.

Cepeda, M., R. Cominetti and M. Florian (2006). A frequency-based assignment model for congested transit networks with strict capacity constraints: characterization and computation of equilibria. Transportation Research Part B 40(6): 437-459.

Chan, K. S. and W. H. K. Lam (2005). Impact of road pricing on the network reliability. Journal of the Eastern Asia Society for Transportation Studies 6: 2060-2075.

Cominetti, R. and J. Correa (2001). Common-lines and passenger assignment in congested transit networks. Transportation Science 35(3): 250-267.

Cortés, C. E., S. Jara-Díaz and A. Tirachini (2011). Integrating short turning and deadheading in the optimization of transit services. Transportation Research Part A 45(5): 419-434.

Delle Site, P. and F. Filippi (1998). Service optimization for bus corridors with short-turn strategies and variable vehicle size. Transportation Research Part A 32(1): 19-38.

Dorbritz, R., M. Lüthi, U. Weidmann and A. Nash (2009). Effects of onboard ticket sales on public transport reliability. Transportation Research Record 2110: 112-119.

El-Geneidy, A., J. Horning and K. Krizek (2008). Analyzing transit service reliability using detailed data from automatic vehicular locator systems. 87th Annual Meeting of the Transportation Research Board, Washington, D.C.

Eliasson, J. (2006). Forecasting travel time variability. European Transport Conference.

Eliasson, J. (2007). The relationship between travel time variability and road congestion. World Conference on Transport Research, Berkeley.

Eliasson, J. (2009). A cost-benefit analysis of the Stockholm congestion charging system. Transportation Research Part A 43(4): 468-480.

Fosgerau, M., K. Hjorth, C. Brems and D. Fukuda (2008). Travel time variability, definition and valuation. Report, DTU Transport.

Fosgerau, M. and A. Karlström (2010). The value of reliability. Transportation Research Part B 44(1): 38-49.

Furth, P. G. (2000). Data Analysis for Bus Planning and Monitoring. Synthesis of Transit Practice 34, Transit Cooperative Research Program, Transportation Research Board.

Hellinga, B., H. van Lint and F. Hofman (2012). The use of exogenously defined standard deviation versus mean travel time relationships for estimating the impact of policy measures on reliability. Presented at the Annual Meeting of the Transportation Research Board, Washington DC, USA.

Hensher, D. A., J. M. Rose and A. Collins (2011). Identifying commuter preferences for existing modes and a proposed Metro in Sydney, Australia with special reference to crowding. Public Transport 3(2): 109-147.

Jackson, W. B. and J. V. Jucker (1982). An empirical study of travel time variability and travel choice behavior. Transportation Science 16(6): 460-475.

Jara-Díaz, S. R. and A. Gschwender (2003). Towards a general microeconomic model for the operation of public transport. Transport Reviews 23(4): 453 - 469.

Jiang, L., H. S. Mahmassani and K. Zhang (2011). Congestion pricing, heterogeneous users and travel time reliability: multicriterion dynamic user equilibrium model and efficient implementation for large-scale networks. Transportation Research Record 2254: 58-67.

Johansson, O. (1997). Optimal road-pricing: simultaneous treatment of time losses, increased fuel consumption, and emissions. Transportation Research Part D 2(2): 77-87.

Joksimovic, D., M. C. J. Bliemer and P. H. L. Bovy (2005). Optimal toll design problem in dynamic traffic networks - with joint route and departure time choice. Transportation Research Record 1923: 61-72.

Lam, T. C. and K. A. Small (2001). The value of time and reliability: measurement from a value pricing experiment. Transportation Research Part E 37(2-3): 231-251.

Li, H., M. C. J. Bliemer and P. H. L. Bovy (2007). Optimal toll design from reliability perspective. Proceedings of the TRISTAN VI conference, Phuket, Thailand.

Li, H., M. C. J. Bliemer and P. H. L. Bovy (2008). Network reliability-based optimal toll design. Journal of Advanced Transportation 42(3): 311-332.

Li, Z., D. A. Hensher and J. M. Rose (2010). Willingness to pay for travel time reliability in passenger transport: A review and some new empirical evidence. Transportation Research Part E 46(3): 384-403.

Mahmassani, H. S., T. Hou and J. Dong (2012). Characterizing travel time variability in vehicular traffic networks: deriving a robust relation for reliability analysis. Transportation Research Board 91st Annual Meeting, Washington D.C.

May, A. D., P. W. Bonsall and N. W. Marler (1989). Travel time variability of a group of car commuters in North London. Institute of Transport Studies, University of Leeds, Working Paper 277.

Mazloumi, E., G. Currie and G. Rose (2010). Using GPS data to gain insight into public transport travel time variability. Journal of Transportation Engineering 136(7): 623-631.

Moghaddam, S. S., R. Noroozi, J. M. Casello and B. Hellinga (2011). Predicting the mean and variance of transit segments and route travel times. Transportation Research Record 2217: 30-37.

Mott MacDonald (2008). Estimation of DTDV functions for motorways. Report for ITEA division, Department for Transport, London, January 2008.

Noland, R. B. and K. A. Small (1995). Travel-time uncertainty, departure time choice, and the cost of morning commute. Transportation Research Record 1493: 150-158.

Peer, S., C. Koopmans and E. T. Verhoef (2012). Predicting travel time variability for cost-benefit analysis. Transportation Research A 46(1): 79-90.

Pu, W. (2011). Analytic relationships between travel time reliability measures. Transportation Research Record 2254: 122-130.

RTA (2011). Key Roads Performance Report. Roads and Traffic Authority, NSW Government, September 2011.

Senna, L. A. D. S. (1994). The influence of travel time variability on the value of time. Transportation 21: 203-228.

Small, K. A. (1982). The scheduling of consumer activities: work trips. The American Economic Review 72(3): 467-479.

Strathman, J., K. Dueker, T. Kimpel, R. Gerhart, K. Turner, P. Taylor, S. Callas, D. Griffin and J. Hopper (1999). Automated bus dispatching, operations control, and service reliability: baseline analysis. Transportation Research Record 1666: 28-36.

Strathman, J. G. and J. R. Hopper (1993). Empirical analysis of bus transit on-time performance. Transportation Research Part A 27(2): 93-100.

TDC (2010). 2008/09 Household Travel Survey. Summary Report, Transport Data Centre, Transport NSW.

Tirachini, A. (2012). Multimodal pricing and the optimal design of bus services: new elements and extensions. PhD thesis, The University of Sydney. Available at http://ses.library.usyd.edu.au/handle/2123/8584.

Tirachini, A., D. A. Hensher and J. M. Rose (2012). Multimodal pricing and optimal design of public transport services: the interplay between traffic congestion and bus crowding.

Transport for London (2007). Central London congestion charging scheme: ex-post evaluation of the quantified impacts of the original scheme. June 2007.

Tu, H., J. van Lint and H. van Zuylen (2007). Impact of traffic flow on travel time variability of freeway corridors. Transportation Research Record: Journal of the Transportation Research Board 1993: 59-66.

Tu, H. (2008). Monitoring travel time reliability on freeways. PhD thesis, Delft University of Technology, The Netherlands.

Van Amelsfort, D., M. C. J. Bliemer and B. Ubbels (2008). Travellers' responses to road pricing: value of time, schedule delay and uncertainty. In: E. Verhoef, M.C.J. Bliemer, L. Steg, and G.P. Van Wee (eds) Pricing in road transport: a multidisciplinary perspective, Edward Elgar: 64-85.

Van Oort, N. (2011). Service reliability and urban public transport design. PhD Thesis, Delft University of Technology, TRAIL Thesis Series T2011/2, the Netherlands.

Verhoef, E. T., P. Nijkamp and P. Rietveld (1997). The social feasibility of road pricing: a case study for the Randstad area. Journal of Transport Economics and Policy 31: 255-276.

Welding, P. I. (1957). The instability of a close-interval service. OR 8(3): 133-142.

Yang, H. and W. H. K. Lam (1996). Optimal road tolls under the conditions of queueing and congestion. Transportation Research Part A 30: 319-332.

Appendix A1: Estimation of demand models

Multinomial logit models for modal choice are estimated, including the proportion of available seats and the density of standees as attributes for the bus alternative. Data collected from a stated choice survey conducted in Sydney in 2009 is used (Hensher *et al.*, 2011). In order to analyse differences in optimal bus service design and multimodal pricing structure, models with and without crowding variables are estimated (see Tirachini, 2012; Tirachini *et al.*, 2012). The estimation of parameters and specification tests are presented in Table A.1 (n=4155 observations, commuting and non-commuting travel purposes pooled together):

Table A.1: Estimation of parameters, MNL models

Attribute	Model 1 (M1): No crowding variables		Model 2 (M2): Crowding variables	
	Parameter	**t-ratio**	**Parameter**	**t-ratio**
Access time β_a	-0.021	-2.88	-0.019	-2.59
Headway β_h	-0.009	-4.22	-0.010	-4.68
In-veh time β_{vb}	-0.019	-8.29	-0.009	-2.69
Egress time β_e	-0.017	-2.45	-0.019	-2.80
Travel time car β_{va}	-0.019	-5.79	-0.021	-6.30
Cost β_c	-0.110	-4.76	-0.111	-4.79
MSC train α_t	-3.847	-8.38	-4.023	-8.69
MSC bus α_b	-4.751	-9.35	-4.975	-9.68
MSC metro α_m	-3.344	-7.88	-3.362	-7.86
t_{vm} × den stand β_{den}			-0.003	-4.42
t_{vm} × prop seat β_{seat}			-0.012	-2.59
Model Fit				
Log-likelihood	-2946.7		-2917.1	
Adjusted ρ^2 (relative to ASCs)	0.089		0.098	
Specification test				
Likelihood ratio test with respect to Model 1			59.2 (> $\chi_{2,0.001}$ =13.82)	

Note: Time in minutes, cost in $ (AUD).

Focusing on the goodness-of-fit measures, the log-likelihood and adjusted ρ^2 statistics relative to a model with alternative specific constants (ASCs) only, demonstrate that the crowding models (M2) outperform the model with no crowding (M1). A likelihood ratio test indicates that M2 is significantly superior to M1 at the 99.9 percent confidence level.

Parameters for the utility functions (5), (6) and (7) are taken from Table A.1, with the exceptions of the time parameter for walking and the mode specific constants, which are estimated as follows. First, walking as a travel alternative was not considered in the survey of the main stated choice experiment from 2009 in Sydney; therefore a reasonable value for the disutility of travel time while walking has to be supplemented. To this end, a secondary intra-CBD model described in an internal 2009 report by ITLS is used, in which walking was a travel alternative. It was found that the time parameter of walking (β_{ve}) is 1.86 times greater than the in-vehicle time parameter for bus (β_{vb}). Thus, we assume a constant value of β_{ve} across models, equal to 1.86 times β_{vb} on M1 (because the latter is an average value of β_{vb} for all crowding conditions); therefore, $\beta_{ve} = 1.86 \cdot -0.019 = -0.035$.

Second, mode specific constants for demand models M1 to M3 are calibrated to represent the current Sydney modal split of trips shorter than 5 kilometres: 62.5 percent car, 31.6 percent walk, and 5.9 percent bus (TDC, 2010). The current bus frequency of 16 bus/h in the morning peak is used, with a fare of $2.10 and no car toll. With these assumptions, the calibrated alternative specific constants for bus are -2.027 (M1) and -2.071 (M2), and for walking are -0.101 (M1) and -0.109 (M2). The car specific constant is fixed at zero.