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27 February 1979

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Dear Mr Rowe,

I am pleased to advise you that you have been awarded the degree of Doctor of Philosophy. This follows consideration of the reports of the examiners of your thesis entitled "Soil Structure Interaction Analysis and Its Application to the Prediction of Anchor Plate Behaviour".

The next ceremony of conferring of degrees which you could attend will be held at 11.30 am on Saturday 7 April 1979. Alternatively, arrangements can be made for your degree to be conferred "in absentia" at a monthly meeting of the Senate. I should be glad if you would write to me indicating whether you wish your degree to be conferred "in absentia" by the Senate or at the conferring of degrees ceremony.

Yours sincerely,

Kenneth W. Knight,
Registrar.

c.c. Professor E.H. Davis
Dr J.R. Brooker

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SOIL STRUCTURE INTERACTION ANALYSIS AND ITS
APPLICATION
TO THE PREDICTION OF ANCHOR PLATE BEHAVIOUR

VOLUME I

by

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A Thesis submitted for the Degree of Doctor
of Philosophy in the University of Sydney.

July, 1978.

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SYNOPSIS

In this thesis, techniques are developed for the analysis of soil-structure interaction problems. These techniques are used in an extensive study of anchor plate behaviour. The techniques and experience gained from this study are then utilized in the analysis of three complex soil-structure interaction problems.

Consideration is given to the use of classical elasticity and plasticity theory in the analysis of anchor behaviour. In particular, a general method of analysis for horizontal anchors in an elastic soil is developed. Provision is made in this analysis for consideration of anchor depth, soil layer depth, anchor shape, anchor-soil interface conditions, breakaway of the anchor from the underlying soil and interaction between anchors within a large group. This approach is then used to analyse strip, circular and rectangular anchors, and solutions are presented for a wide range of material and geometric conditions.

A technique for performing non-linear soil-structure interaction analysis using the finite element method is developed. This approach retains the well known advantages of sub-structuring and the initial stress method while making provision for failure within the soil and/or structure, as well as for breakaway or shear failure at a cohesive, frictional, dilatant soil-structure interface. Considerable attention is given to numerical errors encountered in non-linear finite element analyses, and in particular to the development of techniques which may be used to provide an accurate prediction of collapse loads.

Using finite element techniques, the general behaviour of both horizontal and vertical anchor plates is studied. This approach permits consideration of many factors previously ignored in anchor analyses, and leads to a better understanding and appreciation of anchor behaviour. The effect of anchor depth, soil cohesion, angle of friction and weight is demonstrated, as well as the consequences of varying the initial stress state and dilatancy of the soil. The circumstances under which anchor roughness will influence the failure load are also investigated. Bearing capacity factors determined from these analyses are presented.

A series of model tests performed by the author, for horizontal anchors resting in overconsolidated clay and normally consolidated sand, are described. The results from these tests compare favourably with the theoretical predictions. Similarly, a comparison between published experimental data for both horizontal and vertical anchors with the theoretical predictions indicates encouraging agreement.

Finally, the techniques and experience developed for the analysis of anchor-soil behaviour are utilised to highlight the effects of the more significant soil and structure parameters upon the behaviour of three complex soil-structure interaction problems; namely

- (1) the interaction between an anchored retaining wall and its anchor;
- (2) the interaction between anchor plates within a group; and
- (3) the usefulness of pile groups in modifying the behaviour of soil slopes.

PREFACE

The candidate carried out the work described in this thesis during the period from Lent term 1975 to Lent term 1978. All of the work was conducted in the School of Civil Engineering, The University of Sydney.

The candidate was supervised by Professor E.H. Davis, Professor of Civil Engineering (Soil Mechanics), except for the period of Professor Davis' sabbatical leave, when he was supervised by Dr. J.R. Booker, Reader in Civil Engineering.

The By-Laws of the University of Sydney require a candidate for the degree of Doctor of Philosophy to indicate which sections of the thesis are original. Although many references have been used during the course of this research program, any information or idea derived from these sources has been acknowledged in the text. In accordance with the above mentioned By-Laws the Author claims originality for the following work:

- (i) the analysis and solutions presented in Chapter 3 for the behaviour of horizontal anchors in an elastic soil;
- (ii) the method of analysing soil-structure interaction problems presented in Chapter 5;
- (iii) the refinement and application of the techniques for the prediction of collapse loads described in Chapter 6;
- (iv) the entire analysis and discussion of horizontal anchor behaviour presented in Chapter 7;
- (v) the planning and conduct of the model anchor experiments

- described in sections 8.2 and 8.3 of Chapter 8;
- (vi) the entire analysis and discussion of vertical anchor behaviour presented in Chapter 9;
 - (vii) all numerical results and the ensuing discussion of the three soil-structure interaction problems considered in Chapter 10.

Two papers were prepared by the author and others during the period of the author's candidature. These papers are submitted in support of his candidature.

1. Rowe, R.K., Booker, J.R. and Balaam, N.P. (1978) "Application of the Initial Stress Method to Soil-Structure Interaction". Accepted for publication, International Journal for Numerical Methods in Engineering.
2. Rowe, R.K. and Booker, J.R. (1978) "A Method of Analysis for Horizontally Embedded Anchors in an Elastic Soil". Accepted for publication, International Journal for Numerical and Analytical Methods in Geomechanics.

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I am indebted to many people for their interest and assistance during the course of this work. In this regard, I would like to thank my fellow research students and the academic staff in the School of Civil Engineering at the University of Sydney for many useful discussions. I would also like to thank those people who assisted in the preparation of the manuscript and diagrams.

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Finally, I would like to thank my parents and my wife Kathy for their support, encouragement and patience throughout both my undergraduate and postgraduate years at University.

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GLOSSARY

ASPECT RATIO: The aspect ratio of a rectangular shaped object (e.g. an anchor or footing) is the ratio of the length 'L' to the width 'B' of the object, and is written symbolically as 'L/B'.

BREAKAWAY: The action of a structure (e.g. an anchor plate) separating from the soil at some point along the soil-structure interface corresponds to 'breakaway' of the soil from the structure. Two limiting cases exist:

No Breakaway is the limiting situation in which there is no separation between soil and structure at any point along the soil-structure interface. This situation may arise where there are very high overburden stresses or where significant adhesion or suction may develop between soil and structure.

Immediate Breakaway is the limiting situation in which complete separation occurs behind an embedded structure upon the application of any tensile load to the structure. This situation may arise where there is no overburden pressure, adhesion or suction acting on the back of the embedded structure.

BONDED ANCHOR: A bonded (or fully bonded) anchor is an anchor which is always in contact with the soil both in front and behind the anchor. Accordingly, no gap may form between anchor and soil. This situation corresponds to one of 'no breakaway' between soil and anchor.

CRITICAL EMBEDMENT RATIO: The embedment ratio at which an anchor exhibits the behaviour of a deep anchor is referred to as the critical em-

bedment ratio. Any increase in embedment ratio beyond the critical value will not lead to any change in the particular aspect of behaviour under consideration.

EMBEDMENT RATIO: The embedment ratio of an anchor is the ratio of the distance 'h' between the bottom edge of the anchor and the soil surface to the width 'B' of the anchor, and is written symbolically as 'h/B'.

FLOW RULE: The flow rule for a material in a state of plastic failure is the relationship between the direction of the vector of plastic strain rate components and the failure surface of the material.

- . Associated Flow Rule: A material is said to have an associated flow rule if the vector of plastic strain rate components is normal to the failure surface for the material when their corresponding axes are superimposed. This is equivalent to saying that the magnitude of the angle of dilatancy ' ψ ' is equal to the magnitude of the angle of internal friction ' ϕ '. That is $\psi = \phi$.

Non-associated Flow Rule: A material is said to have a non-associated flow rule if the vector of plastic strain rate components is not normal to the failure surface for the material when the corresponding axes are superimposed. Thus $\psi < \phi$.

HORIZONTAL ANCHOR: A horizontal anchor is an anchor plate which lies in a horizontal plane and is subjected to a load which is normal to the plane in which the anchor is situated.

DEEP ANCHOR: An anchor is considered to be a deep anchor when the soil surface is so remote as not to effect the particular aspect of anchor behaviour under consideration. Thus the behaviour of a deep anchor is independent of anchor depth for embedment ratios greater than the critical embedment ratio, and hence may be considered to be infinitely deep.

VERTICAL ANCHOR: A vertical anchor is an anchor plate which lies within a vertical plane and is subjected to a load normal to the plane in which the anchor is situated.

NOTATION

All notation and symbols are defined where they first appear in the text. For convenience, the most frequently used symbols and their meanings are listed below.

A	Cross-sectional area of a footing or anchor; matrix relating interface displacements to rigid body motions; Matrix of coefficients (various definitions)
A_j	Area of subregion j
a	Radial distance
B	Full width of a footing or anchor; Displacement- strain transformation matrix; Matrix of coefficients (various definitions)
B_o	Diameter of an anchor shaft
b	Half width of a footing or anchor
C	Matrix of coefficients (various definitions)
c	Material cohesion
c_u	Undrained cohesion
c'	Drained cohesion
c_k	Apparent cohesion at a velocity discontinuity
$\underline{c}$	Vector of constants
D	Depth of soil below an anchor; Matrix of coefficients
D_c	Plastic correction matrix
D_E	Elasticity matrix
D_I	Plasticity matrix
d	Distance between an anchor and a retaining wall
E	Young's Modulus

EA	Axial stiffness of a structural element
EI	Flexural stiffness of a structural element
e	Numerical constant $e = 2.71828 \dots$
F_c, F_γ	Bearing capacity factors
F_s	Correction factor for the effect of superposition
$\dot{\tilde{F}}$	Incremental load vector
$\dot{\tilde{F}}_{iu}$	Vector of unknown incremental nodal forces acting on body i
f	Yield function; functional relationship
G	Shear modulus
g	Plastic potential
H	Depth of soil layer
h	Distance from the soil surface to the lowest point on a buried anchor
I	Matrix of influence coefficients (various definitions)
I_B	Second moment of area for a structural element
I_D	Influence factor for the effect of layer depth upon the apparent stiffness of an anchor with $h/B = \infty$
I_h	Correction factor for the effect of embedment ratio upon the apparent stiffness of an anchor
i	$\sqrt{-1}$; Index parameter
j	Index parameter
K_a	Coefficient of active earth pressure
K_o	Coefficient of lateral earth pressure at rest
K_p	Coefficient of passive earth pressure
K_c	Plastic correction to an elastic stiffness matrix
K_E	Elastic stiffness matrix
K_r	Tangent stiffness matrix for a restrained structure
K_T	Tangent stiffness matrix

k	Index
k ₄	Practical failure load - load at which the footing displacement is four times that predicted from an elastic analysis
M	Matrix of coefficients (various definitions)
N	Matrix of coefficients (various definitions)
P	Applied load
$\tilde{P}$	Fourier transform of $\tilde{p}$
p	Mean compressive stress ie. $p = \frac{1}{2}(\sigma_x + \sigma_y)$
$\tilde{p}$	Vector of field quantities in an elastic soil
Q _u	Ultimate capacity of an anchor
q	Applied traction
q _h	Overburden pressure at the level of an anchor
q _s	Surface surcharge pressure
q _u	Mean bearing capacity
R	Matrix of coefficients (various definitions)
R _K , R _R , R _ψ	Correction factors for the effects of initial stress state, anchor roughness, and dilatancy upon the bearing capacity of an anchor
R _S	Efficiency factor for groups of anchors
r	Radius of the Mohr's Circle
S	Spacing between anchors
s	Internal radius of a cylinder; coefficient for the effect of overburden pressure upon the bearing capacity of an anchor
s _a	Tensile adhesion between the soil and an anchor
s _u	Suction between the soil and an anchor
t	External radius of a cylinder
U	Fourier transform of u

u	Displacement in the x direction
V	Fourier transform of v ; A volume
v	Displacement in the y direction
W	Distance from the centre line of a footing to the lateral boundary; Fourier transform of w
W_a	Weight of an anchor
w	Displacement in the z direction
x, y, z	Cartesian co-ordinates
α, β	Various definitions
γ	Unit weight of the soil
Δ	Finite increment
δ	Displacement of a footing or anchor
δ	Nodal displacement vector
∂	Differential operator
ϵ	Strain
$\dot{\epsilon}$	Vector of incremental strain rate components
$\dot{\theta}$	Incremental rigid body motion vector
Λ	Flow rule proportionality constant
λ	Lame parameter
ν	Poisson's Ratio
π	Numerical constant $\pi = 3.14159\dots$
$\dot{\rho}$	Incremental displacement vector of a soil-structure interface
Σ	Summation
$\Sigma_{zx}, \Sigma_{zy}, \Sigma_{zz}$	Fourier transforms of $\sigma_{zx}, \sigma_{zy}, \sigma_{zz}$
σ	Vector of stress components
σ	Stress tensor
τ	Shear stress
ϕ	Angle of internal friction

ϕ_k	Apparent angle of friction at a velocity discontinuity
χ_i	Vector of known incremental loads acting on a body i
ψ	Angle of dilatancy

CHAPTER ONE
INTRODUCTION

1.1 GENERAL INTRODUCTION

Traditionally, the general problem of soil-structure interaction is divided into two parts; that is,

- (1) the prediction of the behaviour of the foundation-soil system; and
- (2) the determination of the effect of structural stiffness upon the behaviour of the structure-foundation-soil system.

In considering the behaviour of the foundation-soil system, attention has been directed towards what are usually regarded as two distinct aspects of foundation behaviour; namely, the prediction of the deformations, and the stability of the foundation. The success of this approach has varied considerably with the type of foundation in question. For example, the load-deflection characteristics of surface footings, rafts and piles appear to be well understood for situations in which the soil may be idealised as an elastic body (e.g. Egarov et al., 1957; Davis and Poulos, 1963, 1968, 1972; Kerisel and Quatre, 1968; Brown, 1969a, b; Poulos and Mattes, 1971a, b; Gerrard, Davis and Wardle, 1972; Wardle and Fraser, 1974). Similarly, the ultimate behaviour of these foundations has received considerable attention (e.g. Terzaghi, 1943; Meyerhof, 1951; Davis and Booker, 1971; Meyerhof, 1976), and although there is still some doubt as to the bearing capacity of rectangular footings and deep piles, the existing solutions may be adopted with a reasonable degree of confidence.

Current understanding regarding the behaviour of buried foundations, and anchor plates in particular, is somewhat less satisfactory. Anchor plate behaviour has been studied experimentally by

many workers, and a number of approximate theories have been proposed. However, there are significant differences in the literature between the methods of analysis (and the solutions) proposed by various authors (e.g. Meyerhof and Adams, 1968; Vesic, 1971).

In considering the effect of structural stiffness upon the behaviour of structure-foundation-soil systems, conventional structural frame analyses have been linked with Winkler, linear elastic and elastic finite element representations of the soil both within a single analysis and by means of sub-structuring (Lee, 1975). Various extensions of this general approach to incorporate the effect of non-linear soil behaviour have also been attempted. However, attention has been largely restricted to the consideration of an elastic soil with, perhaps, some provision for the effect of local yield within the soil upon the performance of the structure-foundation-soil system.

In addition to the problems of foundation-soil interaction, the general category of soil (or earth)-structure interaction includes problems relating to retaining walls, temporary excavation bracing systems, bulkheads, partially buried continuous walls, cut and cover tunnels or culverts and underground tunnels. In many of these cases, the interaction between soil and structure is particularly complex, frequently involving inelastic-plastic soil behaviour as well as possible plastic failure within the structure itself. Consequently, the general behaviour of the system may be highly dependent upon both the initial stress state within the soil and the construction sequence. Despite this, current practice is still largely based on the pioneering work of Terzaghi (1943; 1954) Tschebotarioff (1949a, b; 1951) and Rowe (1952; 1955a, b, c; 1956; 1957), and although these approaches are, at

best, semi-empirical they have the distinct advantage of having provided acceptable designs over a considerable period of time. However, the growing trend in engineering towards the use of cheaper and less conservative designs has emphasised the need for a better understanding of soil-structure behaviour. Accordingly, finite element techniques have been developed which allow rational consideration of many of the complex factors influencing the performance of geotechnical structures (e.g. see Clough, 1972; Desai, 1975). In particular, the behaviour of both temporary and permanent retaining structures (e.g. Bjerrum, Clausen and Duncan, 1972; Clough and Mana, 1976) and buried culverts (e.g. Duncan, 1977) have been the subjects of considerable research.

Despite recent advances in the analysis of a wide range of geotechnical structures, there remains a large group of soil-structure interaction problems which are not well understood from a theoretical standpoint. These problems include:

- (1) the behaviour of buried foundations subjected to tensile loadings, and in particular, those foundations which may be idealised as an anchor plate or group of anchor plates; (ie. foundations where the size of the shaft or tie rod is small compared to the dimensions of the footing.) This type of foundation would include deadman anchors, underream anchors and buried grillage or mat foundations.
- (2) the interaction between retaining structures and their anchors;
- (3) the interaction between groups of foundations, particularly with regard to the ultimate behaviour; and

- (4) the effect of a large number of piles upon the deformation and stability of a soil mass.

The most basic of these problems is that of predicting the behaviour of an isolated anchor plate; if reliable solutions cannot be obtained for this relatively simple problem then one cannot confidently analyse the more complicated problems. Accordingly, the objectives of this thesis are twofold; firstly, to come to some reasonably complete understanding of the behaviour of anchor plates and to obtain parametric solutions suitable for general application; and secondly, to develop techniques and experience based on the anchor problem which can be extended to more complex problems.

The scope of this study has been limited to the behaviour of soil-structure systems under short term loading and subjected to either 'drained' or 'undrained' conditions. Many phenomena have therefore been omitted from the discussion. Most important amongst these is probably the time-dependent interaction between soil and structure due to consolidation and creep of the soil.

1.2 THE ANALYSIS OF SOIL BEHAVIOUR

The analysis of a soil mass subjected to some form of loading requires the adoption of a suitable mathematical model to describe the response of the soil to applied stress. Because of the complexity of soil composition and behaviour, numerous models have been proposed and adopted in the analysis of soil behaviour.

Clearly, the validity of the results obtained from any ana-

lysis of soil behaviour is dependent upon the accuracy with which the mathematical model approximates real soil. However in order to make realistic problems amenable to analysis, it is necessary to introduce significant idealisation, and it is the oversimplification of reality implicit in this idealisation which is usually the major source of any discrepancy between theoretical predictions and experimental observations. In fact, successful predictions of soil behaviour can be made using quite different and distinct soil models, provided that the soil parameters used in the analysis are appropriate to the particular model (e.g. see Clough and Mana, 1976). Thus, in practice, the reliability of analyses involving any particular soil model depends as much upon the understanding of the model, manifest in the determination of the relevant soil parameters, as it does upon the level of idealisation involved in the model. This is not to suggest that the simplest model is necessarily as useful as a more complete model; on the contrary, available evidence would suggest (e.g. Davis and Booker, 1977) that under certain circumstances, serious errors can arise when incorrect soil models (e.g. Bi-linear models) are adopted in stability analyses. Furthermore, the adoption of the more complete model allows a better understanding of the general behaviour of a soil mass. Accordingly, a number of approaches to the analysis of soil masses, using various soil models, will now be discussed.

Under working load conditions, the performance of many different types of foundations have been successfully predicted using elasticity theory (e.g. Poulos and Davis, 1974), provided that careful attention is given to the selection of soil parameters (Ladd et al., 1977; Burland et al., 1977).

Many different approaches have been adopted in attempting to predict the stability of foundations. These methods may be subdivided into two groups; namely

- (1) semi-empirical engineering analyses, and
- (2) more rigorous theoretical analyses.

Engineering analyses have developed in response to the engineers' need to design and construct services and structures in a period when the necessary analytical tools for a rigorous analysis have not yet been developed. Consequently, engineering analyses lay little claim to being rigorous but rather, rely upon the engineer's intuition, judgement and, most importantly, experience for their successful application. Some of the disadvantages of this approach are that:

- (1) Arbitrary assumptions regarding the shape of the failure surface and distribution of stress within the soil mass are often necessary, and these assumptions may not satisfy equilibrium, the failure criteria or the kinematic requirements for even a simple plastic material. Hence, it is difficult to assess the status and accuracy of solutions obtained from these analyses;
- (2) Only simple theories of failure may be considered, and even then, plastic flow properties are not taken into account;
- (3) The methods implicitly assume that the collapse load is independent of the load path to collapse and hence, is independent of the initial stress state; and
- (4) The methods do not provide any information regarding behaviour and deformations observed prior to collapse.

More rigorous methods generally take the form of a direct analysis using the classical theory of plasticity, or an incremental analysis, usually numerical, following a prescribed load path to failure. The classical plasticity approach has the advantage of providing a clearly defined (exact) collapse load, or bounds to the collapse load, for certain well defined problems in highly idealised materials. This approach has the disadvantages that:

- (1) it is not possible to ascertain collapse loads for problems involving complicated loadings or geometries;
- (2) more complex and realistic soil models cannot be considered;
- (3) the approach implicitly assumes that the collapse load is independent of the load path to collapse and hence, is independent of the initial stress state; and
- (4) the approach does not provide any information regarding the deformation observed prior to collapse.

The adoption of an incremental load path approach to the prediction of failure loads allows rational consideration of complex problems involving complicated geometries, soil models and load paths. In particular, this method avoids the necessity of making arbitrary assumptions regarding the failure mechanism or the statics at failure. Furthermore, the adoption of a load path approach largely removes the necessity for separate deformation and stability analyses since a load path can be followed until collapse occurs; consequently, collapse is only the endpoint of the complete load-deformation behaviour of the soil mass. Thus it is possible to identify situations in which there is a difference between the practical failure load, corresponding to

excessive deformation of the soil mass, and the collapse load which, from a rigorous applied mechanics standpoint, corresponds to infinite displacement.

In a load path approach, soil models of varying complexity may be considered, ranging from non-linear elastic (for example, see Hagman, 1971; Duncan and Chang, 1970; Desai, 1971) to simple elasto-plastic models with provision for both associated and non-associated materials (see Chapters 4 and 5 for details), and to even more realistic and complex models (e.g. Schofield and Wroth, 1968; Wroth and Simpson, 1972; Lade and Duncan, 1975).

In this thesis, a simple elasto-plastic soil model will be adopted in all finite element calculations. It may be expected that the dominant feature of soil-structure interaction problems (in both cohesive and granular materials) can be identified using this model. Furthermore, in the limiting cases, where there is either no plastic failure or complete collapse, the accuracy of the solutions may be assessed by comparison with analytical solutions obtained from direct elasticity and plasticity approaches. The extension of the finite element load path approach to incorporate more sophisticated elasto-plastic soil models is quite straightforward (e.g. Simpson, 1972; Wroth and Simpson, 1972; Ozawa and Duncan, 1976; Duncan et al., 1977) however, it is unlikely that this additional refinement, and complexity, would lead to any better general understanding of soil-structure behaviour within the context of this thesis. The use of these more complete soil models may well be justified in cases where a detailed comparison will be made with the monitored performance of a soil-structure system and where the soil profile and parameters are well

documented.

1.3 OUTLINE OF THE THESIS

In this thesis, a number of approaches to the analysis of soil-structure interaction problems will be considered; namely, the use of elasticity theory, classical plasticity theory and finite element load path methods. These methods will be directed primarily towards an analysis of anchor plate behaviour.

To provide a background to subsequent chapters, the development of research into ground anchor behaviour will be reviewed in Chapter 2.

In Chapter 3, a method of analysis for horizontal anchors in an elastic soil is developed. Provision is made in the analysis for consideration of anchor shape, embedment ratio, (ie. anchor depth/anchor width), layer depth, anchor-soil interface condition, breakaway of the anchor from the underlying soil and interaction between anchors within a large group. The approach is then applied to the case of strip, circular and rectangular anchors and solutions are presented for a wide range of material and geometric parameters.

In Chapter 4, the use of classical plasticity theory in the prediction of collapse loads for buried anchors is discussed.

Chapter 5 outlines the underlying assumptions and the development of the incremental equations for an elasto-plastic material. Consideration is then given to the use of efficient techniques for per-

forming non-linear analyses and a new method for performing soil-structure analysis is presented. This approach retains the well-known advantages of sub-structuring and the initial stress method while making provision for failure within either the soil and/or structure, as well as for breakaway or shear failure at a frictional-dilatant soil-structure interface. The scope of the method will be illustrated by considering a number of examples.

In subsequent chapters, the finite element method, and in particular the soil-structure interaction theory of Chapter 5, will be utilised in the analysis of a number of soil-structure interaction problems. For example, in Chapters 7 and 9, the behaviour of horizontal and vertical anchors will be examined with particular regard to the effect of embedment ratio, soil parameters, anchor roughness and initial stress upon anchor-soil behaviour at or near failure. In this context, it is considered adequate to obtain numerical solutions to within an accuracy of approximately 10%; more accurate solutions could be obtained, although in view of the approximations involved, both in idealising the material and in determining the parameters, the need for additional accuracy would be difficult to justify. However, it is of great importance that the factors affecting numerical accuracy be understood so that one can be assured that the solutions lie within well defined limits, say $\pm 10\%$. Accordingly, in Chapter 6, considerable attention is given to numerical error, and in particular to the development of techniques which may be used to provide an accurate prediction of collapse loads.

In Chapter 8, a series of model tests performed by the author, for horizontal anchors resting in overconsolidated clay and

normally consolidated sand, are described. The results from these tests are then compared with the theoretical predictions of Chapter 7. Comparisons are also made between published experimental results and the theoretical predictions of Chapter 7. Similarly, in Chapter 9, the predicted behaviour of vertical anchors is compared with the results of published field and model tests.

Finally, in Chapter 10, the techniques and experience of the foregoing chapters are utilised in the analysis of three significant soil-structure interaction problems; namely,

- (1) the interaction between an anchored retaining wall and its anchor;
- (2) the interaction between anchor plates within a group; and
- (3) the usefulness of pile groups in modifying the behaviour of soil slopes.

Each of these problems warrants a major study in its own right, and it is not possible in this thesis to investigate all the factors affecting the performance of the various soil-structure systems. Rather, an attempt is made to highlight the effect of a number of the more significant parameters upon the behaviour of the soil-structure system.

To summarise, the aim of this thesis is to develop and apply techniques for the analysis of soil-structure interaction problems. In particular, this study attempts to obtain a better theoretical understanding of the factors affecting the behaviour of anchor plates and it is anticipated that the techniques and experience gained from this examination of anchor behaviour may be utilised in the analysis of more complex soil-structure interaction problems.

CHAPTER TWO
HISTORICAL REVIEW
- GROUND ANCHORS

2.1 INTRODUCTION AND BACKGROUND

2.1.1 Introduction

The purpose of this chapter is to give a general overview of the development of research into ground anchor behaviour. In this context, the range of practical applications for ground anchors will be briefly mentioned. Consideration will then be given to the analysis of plate anchors, with regard to the prediction of both elastic displacements and ultimate bearing capacity. Significant studies relating to the analysis of anchors at various orientations to the soil surface are first considered, and the results of relevant experimental investigations and field studies are then discussed. In addition, some mention is made of experimental observations concerning long term behaviour of anchors and the interaction between anchors within a group. Anchor piles (where the pullout resistance is principally due to shear along the pile) and rock anchors are not considered.

No attempt will be made in this chapter to review the development of either classical plasticity or finite element methods of analysis since both are dealt with extensively in standard texts (e.g. Hill, 1950; Davis, 1968; Chen, 1975; Zienkiewicz, 1971; Desai and Abel, 1972). Where necessary, papers relevant to the development of the theory presented in this thesis will be referred to in the appropriate chapters. Similarly, publications relating to particular types of soil-structure interaction analysis will be referred to in the relevant section of the thesis. Comprehensive 'state-of-the-art' reports concerned with soil-structure interaction problems have been presented by

several authors in recent years (e.g. Clough, 1972; Lee, 1975; Desai, 1975; Burland et al., 1977). These papers review the development of soil-structure interaction analysis and discuss various factors to be considered in applying these techniques to practical problems.

2.1.2 Background

Ground anchors have been used by man since prehistoric times. Over this period, the design of anchors has been largely a matter of empiricism. However, in recent years there has been a growing interest in the use of anchors in supporting larger and more complicated structures, and accordingly, the need for a general understanding regarding the performance of anchors and anchorage systems is greater than at any time in our history.

In a general sense, any foundation subjected to tensile loads may be considered to be an anchor. In particular, buried foundations where the cross-sectional area of the shaft or tie rod (connecting foundation and structure) is small compared to the dimension of the footing may be idealised as an anchor plate. In fact, the types of anchorage used in practice vary considerably. Specifically, they may take the form of;

- (1) 'deadman' anchorages, and grillages which are placed in a pit with the excavated material replaced and compacted above them;
- (2) straight shaft and underream anchors;
- (3) injection anchors; and
- (4) expanding bit, multi-helix or screw plate anchors.

In many cases, the distinction between these various forms of anchorage is not clear and a particular anchor may fall into more than one category. In general, a complete study of these anchors would require a full three dimensional analysis; however, an anchor may often be idealised in terms of a two dimensional situation involving either plane strain or axial symmetry, while groups consisting of a row of similar anchors may also be idealised in terms of a plane strain situation.

The range of applications for ground anchors has been widely reviewed in the literature (e.g. Littlejohn, 1968, 1973; Wycliffe-Jones, 1975; Hanna and West, 1976) and includes footings for transmission towers, tie-backs for retaining walls, bridges, tension roofs and submerged structures subject to uplift pressures. In addition, anchors have been used to preload soil in situations where settlement during, and after construction, would inhibit the performance of a structure. Similarly, anchors may be used as an alternative to providing kentledge for a pile or plate bearing test, provided that the anchors are sufficiently remote from the pile or plate being tested to eliminate any interaction between pile and anchor.

2.2 A REVIEW OF PREVIOUS THEORETICAL ANALYSES

2.2.1 Elastic Behaviour

In the past, elasticity theory has been extensively used for the analysis of surface footings or piles; however only a few investigators have made any attempt to develop techniques for the analysis of buried footings or anchors within an elastic soil. The

methods of analysis proposed by these authors for anchors resting in a homogeneous isotropic infinite or semi-infinite soil mass will now be reviewed.

Fox (1948) integrated Mindlin's equation for the displacement due to a vertical load (Mindlin, 1936), to give the mean displacement of a horizontal, uniformly loaded, rectangular area buried at finite depth in an elastic half-space. This approach was then used to obtain solutions for a range of aspect ratios (ie. anchor length/anchor width) and embedment ratios (ie. anchor depth/anchor width) for the case of undrained loading (ie. Poisson's ratio $\nu = 0.5$).

Douglas and Davis (1964) integrated Mindlin's equation for the displacement due to a horizontal load within an elastic half-space to give the horizontal displacement of a vertical plane subjected to a uniform normal stress over a finite area of the plane. This approach was then used to obtain approximate solutions for the central displacement and rotation of a buried, vertical, rectangular, rigid anchor for a range of embedment and aspect ratios as well as for a range of Poisson's ratios.

Yagolkovskii and Bosakov (1976) adopted an approach similar to that of Douglas and Davis (1964) and obtained a limited range of solutions for a rigid vertical wall extending to the surface of an elastic half-space with a Poisson's ratio of 0.3.

Selvadurai (1976) has derived a general equation specifying the elastic load deflection behaviour of deep, fully bonded, isolated anchors of spheroidal shape (including the limiting case of a circular

anchor plate) embedded within an infinite soil mass.

The foregoing approaches have all considered the limiting case of a fully bonded anchor in which it is assumed that the anchor-soil interface can sustain tension, and/or the compressive stresses due to self weight of the overburden are sufficient to prevent any breakaway (ie. loss of contact) at the interface. It is generally concluded by these authors that this would be a reasonable assumption at working loads for most problems.

Khan'zhov and Khan'zhov (1975) have considered a second limiting case corresponding to the immediate breakaway condition for the axi-symmetric problem of a rigid anchor acting on the upper boundary of a circular slit in an infinite elastic medium, where it was assumed that the lower face of the slit was unloaded. An expression for the load deflection relationship was obtained by using a finite difference representation for the equations of classical elasticity.

In all of these papers, it has been assumed either directly or implicitly that the anchor is perfectly rough and that the elastic layer below the anchor is of infinite extent. Furthermore, no consideration has been given to interaction effects due to neighbouring anchors within an anchor group.

2.2.2 Prediction of Collapse Loads

2.2.2.1 Horizontal Anchors

Numerous approximate or semi-empirical methods for predic-

ting the ultimate uplift capacity of horizontal anchors have been proposed. In the majority of these cases, a failure mechanism is assumed and the uplift capacity is then determined by considering the equilibrium of the soil mass above the anchor and contained by the assumed failure surface. Two of the earliest of these approaches are:

- (1) the 'weight-of-cone' method, in which the failure surface takes the shape of a truncated cone extending above the anchor and meeting the soil surface at an angle of $(45 - \phi/2)^\circ$; and
- (2) the 'friction cylinder' method, in which failure is assumed to occur along the surface of a cylinder of soil above the anchor.

Subsequent variations upon these early approaches have been proposed by Balla (1961), Mariupolskii (1965), and Matsuo and Shinjo (1967). In each case the shape of the assumed failure surface was based upon experimental observations relating to the behaviour of anchors embedded in a particular type of soil. Thus, the usefulness of these approaches is largely dependent upon the similarity between the soil type being considered and the soil for which the approach was developed. In the case of Mariupolskii's approach, the assumptions upon which the uplift load is determined are highly suspect (e.g. see Vesic, 1971).

An approximate general theory for the uplift resistance of both strip and circular anchors has been proposed by Meyerhof and Adams (1968). For a strip anchor, an equation for the ultimate load

(per unit length) was obtained by considering the equilibrium of the block of material directly above the anchor (contained by $b'a'$ or ba as shown in Fig. 2.1 for a shallow and deep anchor). The cohesive force was assumed to act along the planes ab and $a'b'$ while the total passive earth pressure was assumed to act at some angle α to this plane. The angle α was estimated from experimental considerations while the passive earth pressure was evaluated from the results of Caquot and Kerisel (1949). In the case of a shallow anchor, the ultimate load was determined from the equilibrium of the material between the anchor and the soil surface. For a deep anchor the equilibrium of a block of material of height h^* was considered (see Fig. 2.1), where the magnitude of h^* was determined from experimental observations. It was stated that the bearing capacity for a deep anchor should be the lesser of:

- (1) the load calculated as described above for deep anchors; and
- (2) the sum of the bearing capacity of a deep footing and the skin friction on the anchor shaft.

The analysis of circular anchors presented by Meyerhof and Adams parallels that described above for the strip anchor. The major difference between the two cases lies in the fact that the passive earth pressure obtained for the plane strain case was modified by a semi-empirical shape factor for use in the circular case.

Vesic has produced solutions for the expansion of cavities close to the surface of a semi-infinite, homogeneous, rigid plastic solid. These solutions have been applied by Esquivel-Diaz (1967),

FIG. 2.1 SHAPE OF SLIP SURFACE FOR ANCHOR
PULLOUT (MEYERHOF & ADAMS)

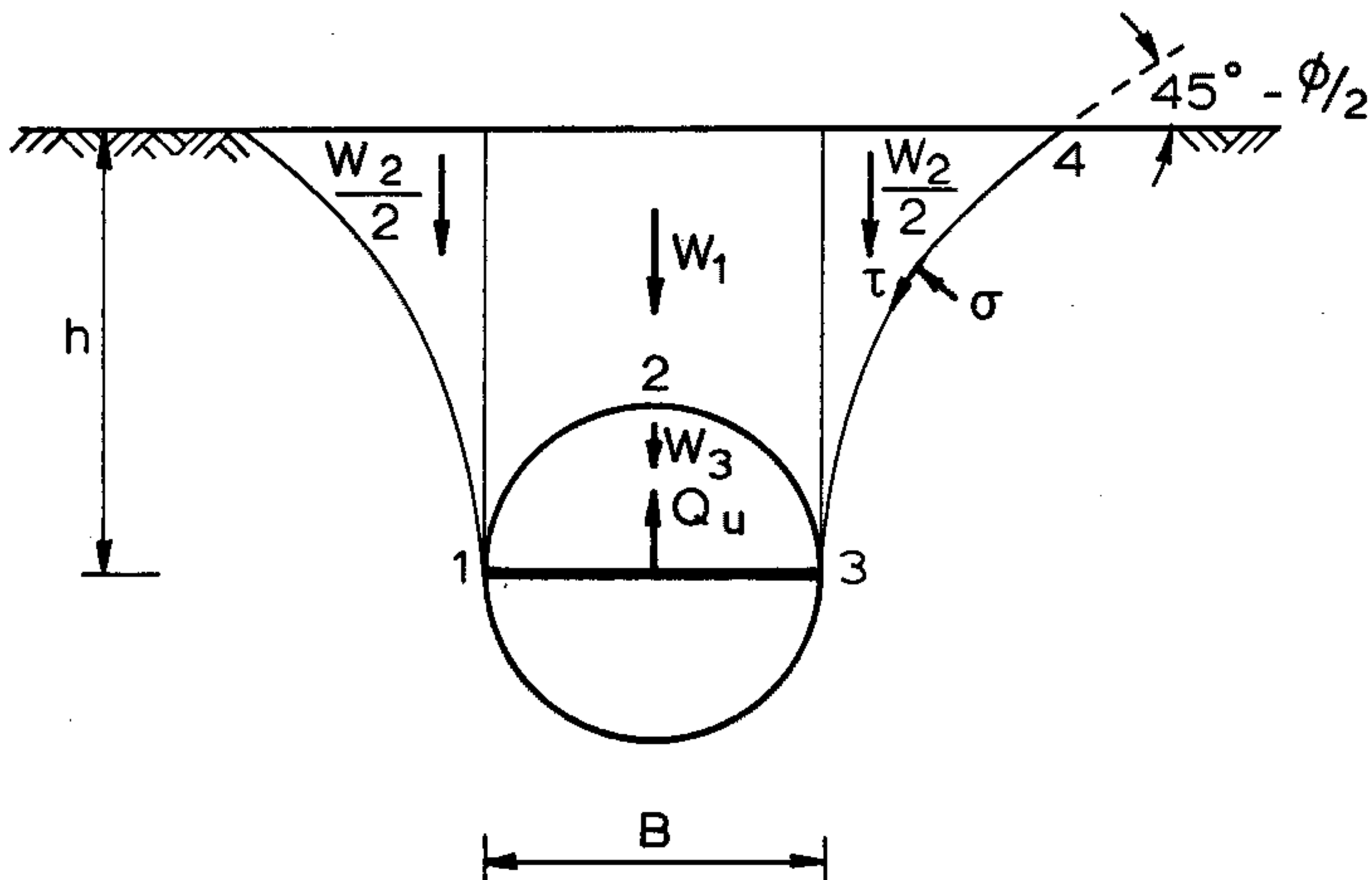


FIG. 2.2 SHAPE OF SLIP SURFACE FOR ANCHOR PULL-
OUT AND EXPANSION OF SPHERICAL CAVITY CLOSE
TO SURFACE (VESIC)

Ali (1968) and Vesic (1971) to the prediction of the bearing capacity of both strip and circular anchor plates.

The solution for a strip anchor was obtained by assuming that the slip surface (as shown in Fig. 2.2) is tangential to the cylinder at points 1 and 3 and meets the soil surface at an angle of $45^\circ - \phi/2$. The ultimate cavity pressure was calculated from an equation of vertical equilibrium involving the weight of soil (W_1 and W_2) and the vertical components of the resultant of internal force along the slip line. The bearing capacity of an anchor plate was then determined by assuming it was equivalent to the ultimate cavity pressure plus the weight, W_3 , of soil within the semi-circular region 1-2-3 above the anchor.

Similarly, the axi-symmetric problem of the circular anchor plate was solved by assuming that the normal and shear stress distributions and the statically correct angles along the slip surface are the same as those found in the corresponding plane strain case. Thus, the slip surface was assumed to be formed by the revolution of the circular slip line 3-4 about the axis of symmetry (Fig. 2.2) and the anchor capacity was calculated in a similar fashion to that outlined for strip anchors.

Meyerhof (1951) proposed the use of the theory for the expansion of a cylindrical or spherical cavity (e.g. Bishop, Hill and Mott, 1945) in determining the bearing capacity of deep foundations. Subsequently, several authors (Mariupolskii, 1965; Vesic, 1971; Ladanyi and Johnston, 1974) have proposed the use of different cavity expansion theories for the prediction of the bearing capacity of deep circular

anchors. The means by which a plate (or wedge) penetration problem is transformed into a cavity expansion problem varies between authors. However, it should be recognised that the deep cavity expansion problem involves symmetric stress and displacement fields which are not consistent with a deep anchor problem; consequently, consideration should be given to this characteristic of the problem in adopting any cavity expansion theories. A more detailed discussion regarding the use of the cavity expansion approach in the analysis of deep anchors (particularly in clay soils) has been given by Swain (1976).

Approaches involving rigid plasticity have been largely restricted to the use of bearing capacity factors for deep footings (e.g. Meyerhof, 1951; Skempton, 1951), assuming that the behaviour of deep foundations is independent of whether the applied load is upward or downward. It should be recognised that these solutions for deep foundations are only approximate; they do not satisfy the requirements of the theory of perfect plasticity for a complete solution. Attempts to develop fields of characteristics for a wedge indenting a purely cohesive, rigid plastic material have been reported by Baligh, 1972. The advantages and limitations of this type of approach will be discussed in Chapter 4.

Two attempts at using the finite element method to determine the behaviour of circular anchor plates have been reported. Ashbee (1969) used a simple non-linear elastic soil model to predict the load-displacement behaviour of an underreamed anchor in a purely cohesive material. Davie and Sutherland (1977) used an elasto-plastic soil model (employing a Von Mises yield criterion) for the analysis of a circular anchor plate in a purely cohesive material. Very few re-

sults were reported in either case. In particular, although it is suggested that theoretical collapse loads could be obtained using the finite element method, no theoretical collapse loads are reported.

2.2.2.2 Vertical Anchors

Historically, the analysis of buried vertical anchors may be considered to be an extension of earth pressure analysis. For example, Terzaghi (1943) has suggested that for anchors with an embedment ratio less than two, the load carrying capacity may be estimated by taking the difference between the passive and active forces acting on walls buried to the same depth and extending to the soil surface.

Casbarian (1967) has proposed a method for determining the ultimate resistance of anchor plates based on Rankine's classical earth pressure theory. It would appear that this 'analysis' consists of a conventional Rankine analysis for a retaining wall, modified for the effect of embedment ratio by using an empirical relationship suggested by Brinch Hansen (1961).

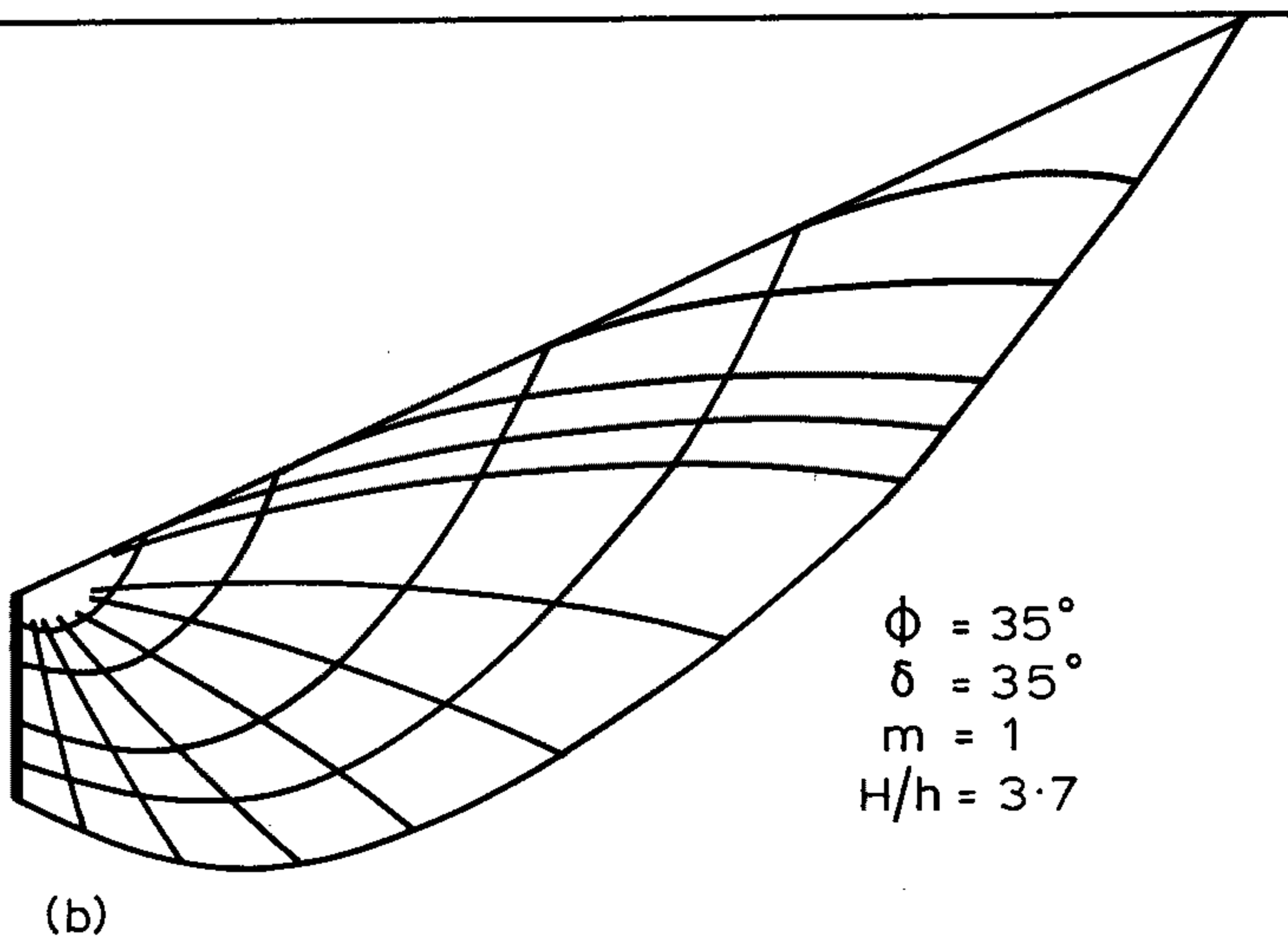
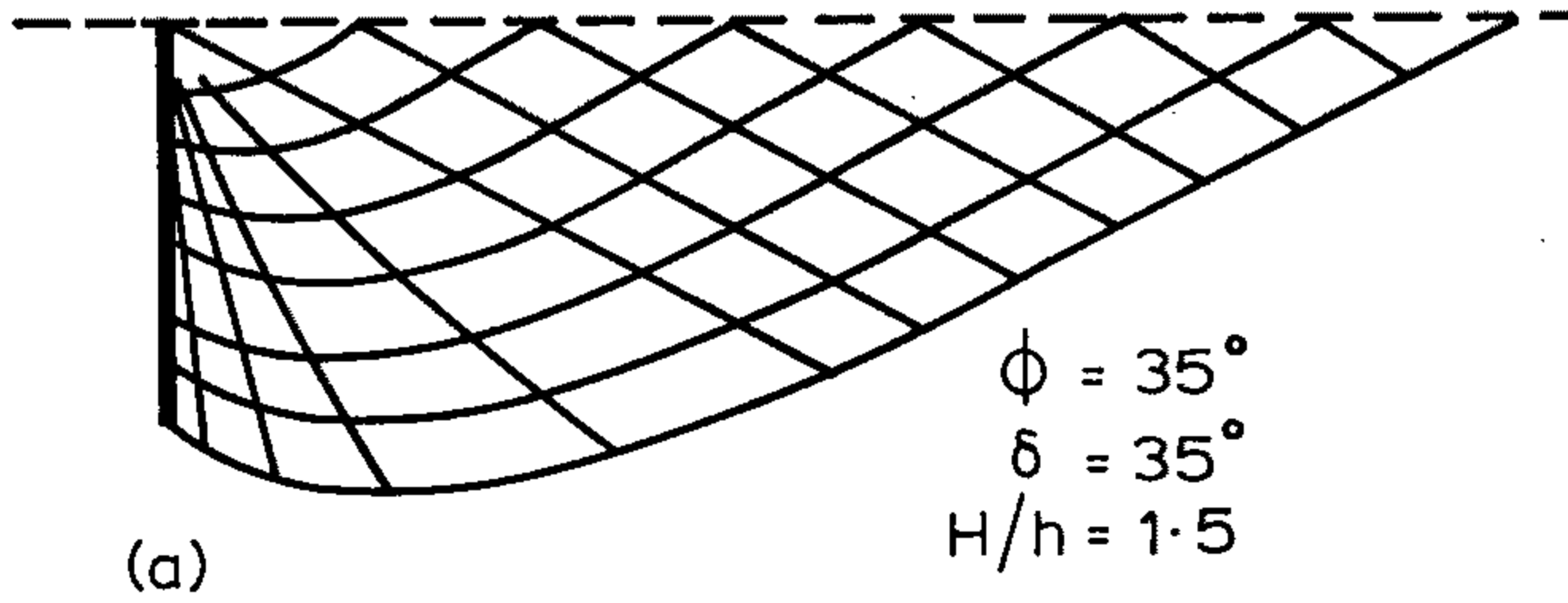
Krebs Ovesen and Stromann (1972) have presented an alternative, but basically similar, approach in which the average bearing capacity, q_o , of a vertical wall (extending to the soil surface) is determined from the difference between the passive and active earth pressures obtained by Brinch Hansen (1953). The bearing capacity, q_u , of a buried anchor was then determined from the bearing capacity, q_o , of a wall extending to the soil surface by multiplying q_o , by an empirical correction factor, f , based on model test results (Krebs Ovesen, 1964).

$$\text{ie. } q_u = fq_o$$

Neely et al., (1973) used both a trial failure surface approach and the method of characteristics (Sokolovski, 1965) to analyse a vertical strip anchor in a purely frictional material. Firstly, a trial failure surface consisting of a combination of logarithmic spirals and straight lines was adopted. It was assumed initially that the soil above the level of the top of the anchor would act as a simple surcharge, as shown in Fig. 2.3a. However, since this approach ignores the shearing resistance of the soil above the anchor, the approach was modified by adopting an 'equivalent free surface', such as OB shown in Fig. 2.3b, along which the normal and shear stresses (p_o and s_o respectively) are calculated by assuming that Rankine active stresses act on the vertical surface OA above the top of the anchor. The stresses p_o and s_o depended on the degree of mobilisation of the internal shearing resistance of the sand along OB which was assumed to be either fully mobilised ($s_o/p_o = \tan \phi$) or zero ($s_o = 0$).

Secondly, the method of characteristics was adopted, as shown in Fig. 2.4, using the surcharge and equivalent free surface assumptions described above for the trial failure surface approach. The solutions obtained using these two methods differed by up to 100% depending upon the assumptions made regarding the degree of mobilisation and the method of analysis adopted.

Although the analysis adopted by Neely et al., (1973) represents a more analytical attempt to predict the ultimate behaviour of vertical anchors than any preceding work, it should be noted that the proposed methods of analysis ignore the active stress distribution behind the anchor, the kinematic behaviour of the material and



(a) Surcharge Method (b) Equivalent Free Surface Method

FIG. 2.4 TYPICAL STRESS - CHARACTERISTIC SOLUTION
(NFFLY et al, 1973)

the initial stress state within the soil mass. Furthermore, while these methods employ some classical plasticity concepts, the solutions obtained from these methods do not satisfy the necessary requirements for classification as either true upper or lower bounds to the collapse load.

2.2.2.3 Anchors at Other Orientations

Approximate methods for the analysis of inclined anchor plates have been presented by Meyerhof (1973) and Harvey and Burley (1973).

Meyerhof's approach is a straightforward extension of the approach proposed by Meyerhof and Adams (1968) and described in Section 2.2.2.1.

Harvey and Burley (1973) developed a theory for shallow inclined circular anchors by adopting a failure mechanism similar to that suggested by Balla (1961) for horizontal anchors.

2.3 A REVIEW OF PREVIOUS EXPERIMENTAL INVESTIGATIONS

Numerous investigators have performed model and field tests with a view firstly to developing empirical rules for anchor design and, secondly, for comparison with theoretical predictions. Most of this work has been devoted to an empirical determination of the pullout capacity of circular anchors.

The purpose of this section is to discuss briefly the aspects

of experimental research of greatest relevance to this thesis. It is intended to be selective, and no attempt is made to present a complete bibliography of experimental research into the very broad subject of ground anchor behaviour.

2.3.1 Horizontal Anchors

Field tests on circular or square anchor plates and anchor piles with enlarged bases in clay have been performed by Clements (1960), Mohan and Jain (1961), Turner (1962), Parr and Vanner (1962), Paterson and Urie (1964), McKenzie (1971), Adams and Hayes (1967), and Adams and Radhakrishna (1971). In each case an attempt has been made to establish the validity of an empirical or semi-theoretical design method. However, the results obtained by the various investigators vary by as much as 600% indicating that the empirical relations produced may be largely restricted to the particular soil and anchor conditions encountered in the individual investigations. Difficulties were experienced in establishing these empirical relations for a number of reasons. Firstly, in many cases, evidence of suction and adhesion between the base of the anchor and the underlying soil was observed but not quantified. Secondly, clearly defined collapse was not always observed and the 'collapse' load was often specified as the load corresponding to a given displacement. These collapse loads are dependent upon the choice of the maximum allowable displacement, [which varied from 13mm (McKenzie) to 50mm (Clements)] and the elastic response of the soil. Thirdly, significantly different material properties were obtained at a given site depending upon the type of test adopted. For example, in the worst case, Adams and Radhakrishna (1971) observed that the shear strength of fissured clay obtained from field

vane tests differed from the undrained residual strength from a tri-axial test by a factor of four. Finally, to some extent the results vary depending upon the techniques adopted for backfilling above the anchor, and the time between placement of the anchor and testing.

Model tests on circular anchors in clay have been performed by Spence (1965), Langley (1967), Adams and Hayes (1967), Ali (1968) and Davie and Sutherland (1977). Although laboratory conditions give greater scope for control of material and test conditions these tests are subject to similar difficulties to those outlined for the field tests. In particular, suction can have a significant effect on the ultimate collapse load. For example, Adams and Hayes obtained quite different dimensionless bearing (ie. pullout) capacity factors from stiff and soft clays if suction was allowed to develop below the anchor. In this case the soft clay was observed to flow around behind the anchor plate whereas in the stiff clay, a gap formed below the anchor. Some tests were performed where the effect of suction was eliminated by using a hollow anchor rod in contact with a piece of filter paper at the lower face of the anchor plate. From these tests it was found that the bearing capacity of anchors in stiff clay was unaffected by suction, however the elimination of suction effects reduced the bearing capacity of anchors in soft clay by a factor of 1.8, giving bearing capacities which were in reasonable agreement with those obtained for anchors in stiff clay.

In the above tests, the transition in behaviour from that of a shallow anchor to that of a deep anchor occurred at embedment ratios ranging between 1.5 and 5.

Once a correction was made for suction, the experimental bearing capacities from both field and model tests were significantly less than those predicted by any of the theories in the previous section. Furthermore, the bearing capacity determined from field tests was considerably lower than that determined from model tests.

Field tests on circular anchor plates or piles in sand have been performed by Mors (1959), Giffels et al., (1960), Balla (1961), Sutherland (1965), Mariupolskii (1965), Kananyan (1966), Baker and Konder (1966) and Adams and Hayes (1967), while Vesic (1971) also reports the results of field tests by Milovic and Martin, and Negre. Most of these tests have been performed for anchors with an embedment ratio less than four.

The results of model tests on circular anchors have been published by Balla (1961), Macdonald (1963), Sutherland (1965), Baker and Konder (1966), Kananyan (1966), Adams and Hayes (1967), Esquivel-Diaz (1967), Matsuo (1968), and Hanna and Carr (1971).

The dimensionless pullout capacities obtained by the various investigators (for similar anchors) were found to differ considerably, while the shape of the failure surface was found to vary with the embedment ratio, angle of internal friction, and the density of the material.

The transition in behaviour from that of a shallow anchor to that of a deep anchor was observed at embedment ratios ranging from 3 to 10.

Comparison of these experimental results for circular anchors in sand with the theoretical predictions of the previous section indicates that, in general, the pullout resistance obtained in dense sand greatly exceeds that predicted by theory while the pullout resistance in loose sand tends to be lower than that predicted by theory, suggesting the need to include the effect of soil dilatancy in any theoretical analysis.

Model tests by Hanna and Carr (Hanna and Carr, 1971; Hanna, 1971; Hanna et al., 1972) indicate that the load carrying capacity of an anchor in sand may vary significantly depending upon the initial stress state. On the basis of these results, Hanna (1971) has suggested that any theory to predict pullout resistance which does not allow for the initial stress state in sand will give misleading results.

Published results from field or laboratory model tests on rectangular (or strip) anchors are relatively scarce.

Swain (1976) has reported the results of two plane strain tests for deep anchors in normally and overconsolidated clay. These tests are reported in considerable detail with regard to both anchor behaviour and soil properties.

Das and Seeley (1975a, b; 1976) have reported the results from a series of model tests on horizontal rectangular anchors in loose sand. In these tests, the embedment ratio at which the transition from shallow to deep anchor behaviour was observed was found to increase with the aspect ratio of the anchor, while the bearing capacity itself decreased with increasing aspect ratio.

2.3.2 Vertical Anchors

The number of model and field tests performed for vertical anchors is extremely limited, particularly with regard to cohesive soils.

Mackenzie (1955; see also Tschebotarioff, 1973) reported the results of a series of model tests for slender, rectangular anchors in clay and these results will be discussed in Chapter 9.

Hueckel and his co-workers (Hueckel, 1957; Hueckel and Szaroniec, 1963) have performed small scale model tests on square anchors in sand. Similar model tests on square and rectangular anchors have been reported by Krebs Ovesen (1964), Krebs Ovesen and Stromann (1972), Neely et al., (1973) and Das and Seeley (1975c, d, e). Field and model tests for deadman anchorages in sand have been reported by Smith (1962) and Smith and Stalcup (1966). In each of these tests attention was directed towards determining the effects of embedment ratio upon the ultimate bearing capacity. In this respect it was found that:

- (1) the bearing capacity increased with embedment ratio over the range considered (ie. for embedment ratios less than or equal to five);
- (2) no transition from shallow to deep anchor behaviour was observed;
- (3) the bearing capacity increased with decreasing aspect ratio of the anchor; and
- (4) the effect of anchor shape (ie. aspect ratio) upon the

bearing capacity increased significantly with embedment ratio.

It is of some interest to note that for anchors with a constant aspect ratio and the same buried depth, h , the ultimate capacity per unit length was found to increase with embedment ratio, reaching a maximum at an embedment ratio of about three and then decreasing for higher embedment ratios as illustrated in Fig. 2.5 for a particular case where $h = 2.1\text{m}$ and $L/B = 1.25$. The empirical curve shown was derived from a statistical analysis of the experimental results obtained for a range of depths, lengths and breadths of the anchor (Smith and Stalcup, 1966).

2.3.3 Other Cases

Experimental investigations into the behaviour of inclined model anchor plates have been reported by Hueckel, (1957); Kananyan, (1966); Larnach, (1973); Harvey and Burley, (1973); Meyerhof, (1973); and Das and Seeley, (1975b, d; 1977). From the results of these investigations it would appear that:

- (1) the load carrying capacity of an anchor plate (loaded perpendicular to the plate) increases with increasing inclination from the horizontal; and
- (2) the load carrying capacity for a particular anchor orientation tends to decrease with decreasing acute angle between the direction of loading and the plane of the anchor plate.

Hueckel (1957), Yilmaz and Hanna (1971), Neely (1972a), Hanna et al.,

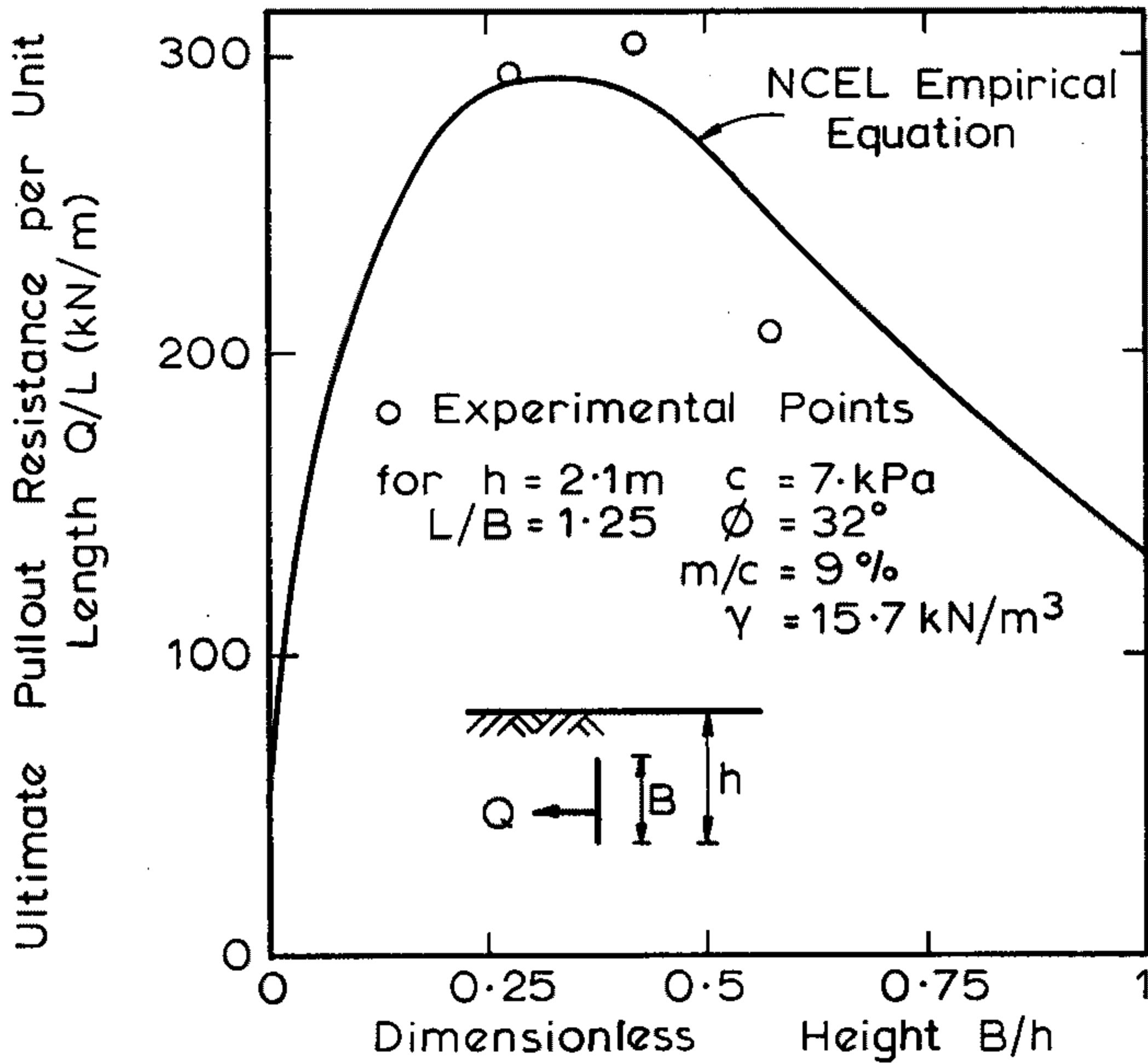


FIG. 2.5 EFFECT OF EMBEDMENT RATIO FOR AN ANCHOR AT A GIVEN DEPTH (From the Results obtained by Smith 1962, 1966)

(1972) and Lanarch (1973) have investigated the behaviour of model anchor groups in sand. As might be expected, it was found that the efficiency of an anchor group depends upon the centre to centre spacing between anchors, the embedment ratio and the number of plates in the group. Unfortunately, there are significant discrepancies between the efficiencies reported by different workers for similar anchor groups tested under relatively similar conditions.

The effect of anchor flexibility upon the ultimate behaviour of anchors has received very little attention. However, on the basis of model tests in sand, Neely (1972b) suggests that the flexibility of an anchor (particularly when sheet piling is used as an anchor) and the location of the tie rod may lead to a significant reduction in the load carrying capacity of the anchor (as compared to a rigid anchor).

In this thesis, attention will be restricted to the behaviour of single anchors, however, it should be mentioned that the success of single underreamed anchors has led to the use of multi-underreamed anchors, where additional underreamed anchors are placed along the anchor shaft. Results of full scale tests on multi-underreamed anchors are recorded by Bassett (1970) and Littlejohn (1970), while Adams and Klyn (1972) and Johnston and Ladanyi (1974) report the results of full scale tests on multi-plate screw anchors. Only Mohan et al., (1969) considered the effect of spacing of the underreamed anchors on the ultimate load carrying capacity of the anchor system. Swain (1976) has performed model tests on multiple underreamed anchors in clay.

The time dependent behaviour of soil, and cohesive soils in particular, may be expected to have an effect on the performance

of earth anchors. Specifically, available data from monitored installations (e.g. Littlejohn, 1973) indicates that there may be a significant loss in anchor preload due to the time dependent displacement of an anchor in clay. Furthermore, model tests (e.g. Adams and Hayes, 1967; Meyerhof and Adams, 1968) indicate that drained or long term uplift capacity in clay can be appreciably less than the undrained or short term capacity, particularly at shallow depths. This reduction with time may be attributed to the dissipation of negative pore water pressures (indicating the need for both drained and undrained analyses) combined with the fact that adhesion or suction between the back of the anchor and the soil cannot be sustained under long term loading.

2.4 SUMMARY

A number of conclusions can be drawn from the foregoing review:

- (1) It appears to be generally agreed (e.g. Vesic, 1971; Hanna, 1971) that existing theories do not adequately describe the behaviour of anchor plates; particularly for shallow anchors.
- (2) The influence of the soil dilatancy and the initial stress state has been neglected in previous analyses; however, experimental data indicates that these factors may have a significant effect upon anchor performance.
- (3) Most methods of analysis are based upon the initial assumption of a particular mode of failure (this in-

cludes the cavity expansion approaches) and the validity of these assumptions has been difficult to establish in view of a lack of accurate experimental methods for monitoring internal soil deformation (Swain, 1976).

- (4) Most experimental programmes, whether at full scale or on models, were concerned with the ultimate load carrying capacity of horizontal circular or square anchors in sand or clay, or with the ultimate load carrying capacity of rectangular vertical anchors in sand. A limited number of tests have been reported for horizontal rectangular anchors and for vertical rectangular anchors in clay. In many cases insufficient data is given to allow a direct comparison between the reported experimental data and theoretical predictions.
- (5) There may be significant differences between the undrained and drained bearing capacity of anchors; in particular, the existence of suction and adhesion between the anchor and soil cannot be relied upon under long term (drained) loading conditions.

CHAPTER THREE

THE ELASTIC LOAD DISPLACEMENT BEHAVIOUR OF HORIZONTALLY EMBEDDED ANCHOR PLATES

3.1 INTRODUCTION

Although the behaviour of anchor plates at working loads is of considerable practical importance, previous investigations into the elastic load-deflection behaviour of horizontally embedded anchors have been restricted to the case of a perfectly flexible or infinitely rigid, fully bonded anchor resting in either an elastic half-space (Fox, 1948) or within an infinite elastic mass (Selvadurai, 1976). In this chapter an analytical technique will be presented for the analysis of an anchor of general shape buried at a depth h below the surface of an isotropic elastic soil layer (with elastic modulus E and Poisson's ratio ν) which extends a further distance D below the anchor plate as shown in Fig. 3.1. The anchor plate itself may be either perfectly flexible or rigid while the anchor-soil interface may be either smooth or rough. In addition, provision is made for slip between anchor and soil as well as breakaway of the anchor from the underlying soil.

In the proposed analysis, the elastic field quantities are assumed to have a Fourier integral representation and to this extent the approach is similar to that used by various other investigators who have assumed either Fourier integral or Fourier series representations in the analysis of surface footings (e.g. Gerrard, 1969; Milovic and Tournier, 1971; Ramsay, 1973). The adoption of this approach allows consideration of a soil layer of finite depth underlain by a smooth or rough rigid base as well as the extreme cases of an infinite or semi-infinite elastic mass; and although no attempt will be made to do so in this chapter, the approach can be easily extended to incorporate the effects of multilayer strata and cross-anisotropic materials

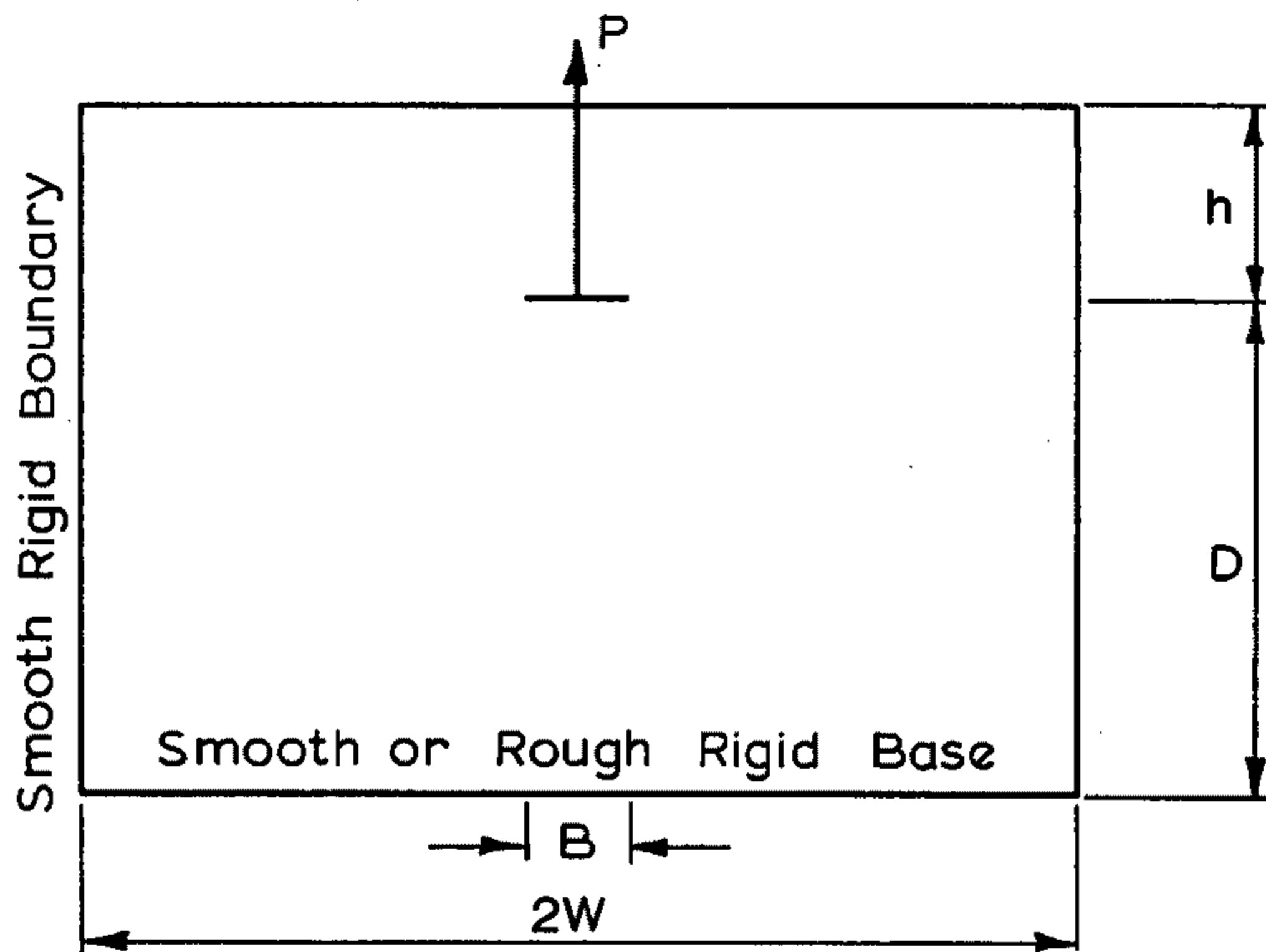


FIG.3.1 BASIC GEOMETRIC PARAMETERS FOR THE CASE OF A STRIP ANCHOR WITHIN AN INFINITE GROUP

(e.g. Gerrard, 1967; Gerrard, 1969; Gerrard and Harrison, 1971).

3.2 THEORY

3.2.1 Development of the Solution Method

Referring to Fig. 3.2, R is a horizontal anchor plate embedded in an elastic layer. The horizontal plane containing R ($z = z_0$) divides the layer into two parts $\Gamma^{(+)} (z_0 < z < z^{(+)})$ above the anchor and $\Gamma^{(-)} (z^{(-)} < z < z_0)$ below the anchor. The stress and displacement fields in $\Gamma^{(+)}$, $\Gamma^{(-)}$ will be denoted by the superscripts (+), (-) respectively. In general these quantities will suffer a 'dislocation' at the interface R due to the presence of the plate. For example, the vertical stresses just above and just below the plate will differ due to the action of the applied load on the plate. Similarly, the horizontal displacements above and below the plate may differ due to slip at the soil-plate interface.

In the most general case the quantity

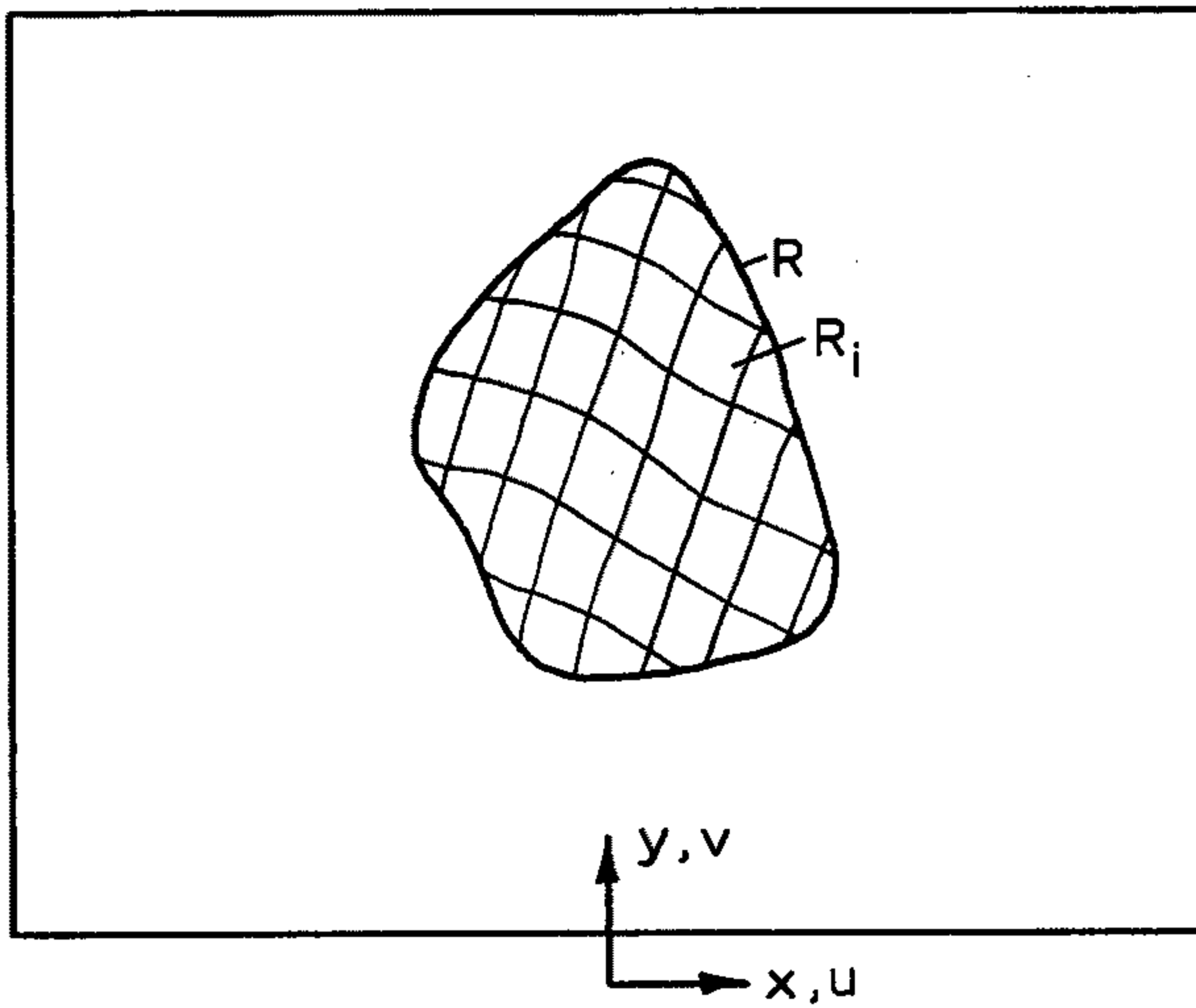
$$\tilde{p}^T = (\sigma_{zx}, \sigma_{zy}, \sigma_{zz}, u, v, w) \quad (3.1a)$$

will undergo the jump

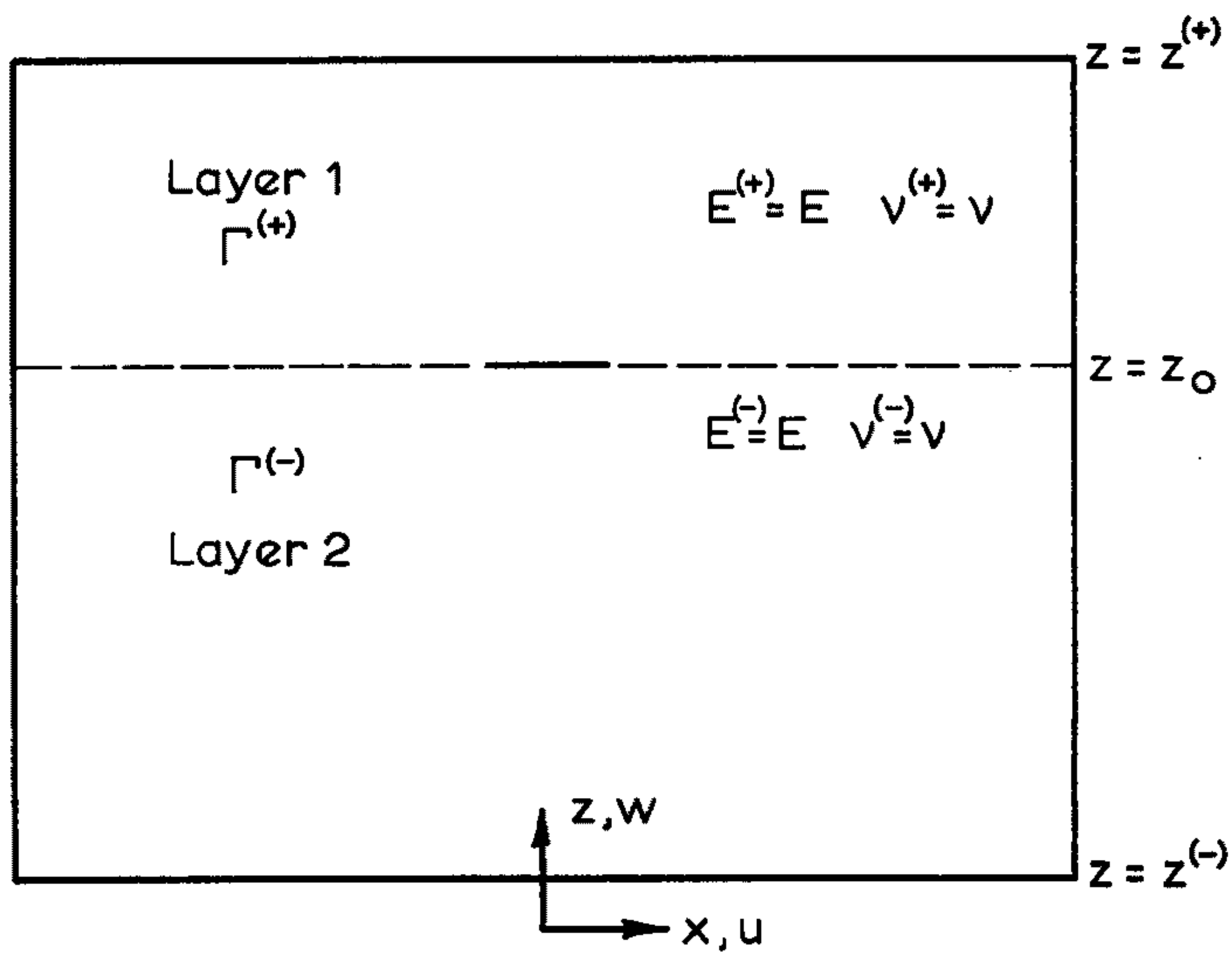
$$\Delta \tilde{p} = \tilde{p}^{(+)} - \tilde{p}^{(-)} \quad (3.1b)$$

at the interface.

In order to analyse the behaviour of the anchor plate, sup-



(a) Plan



(b) Elevation

FIG.3.2 ANALYTICAL CONFIGURATION

pose that the region R is divided into n sub-regions R_i as indicated in Fig. 3.2. Further suppose that, to sufficient accuracy, it can be assumed that Δp takes the constant value Δp_i over each sub-region R_i .

In the following section, it will be shown how it is possible to express the field quantity $\bar{q}^T = (\sigma_{zx}^{(-)}, \sigma_{zy}^{(-)}, \sigma_{zz}^{(-)}, u^{(+)}, v^{(+)}, w^{(+)})$ on the plane $z = z_0$ in terms of the value Δp_i and in particular, that it is possible to establish the relation:

$$\bar{q}_j = \sum_{k=1}^n I_{jk} \Delta p_k \quad (3.2a)$$

where I_{jk} is a 6×6 matrix of influence coefficients and $\bar{q}_j = \frac{1}{A_j} \cdot \int_{R_j} q \, dA$ is the average value of q over the region R_j .

Equation 3.2a may be rewritten in the matrix form

$$\begin{bmatrix} \bar{q}_1 \\ \vdots \\ \bar{q}_n \end{bmatrix} = \begin{bmatrix} I_{11} & \vdots & I_{1n} \\ \vdots & \ddots & \vdots \\ I_{n1} & \vdots & I_{nn} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{bmatrix} \quad (3.2b)$$

The above relation represents a set of $6n$ equations connecting the $6n$ average quantities $(\sigma_{zx}^{(-)}, \sigma_{zy}^{(-)}, \sigma_{zz}^{(-)}, u^{(+)}, v^{(+)}, w^{(+)})_i$ and the $6n$ quantities $(\Delta \sigma_{zx}, \Delta \sigma_{zy}, \Delta \sigma_{zz}, \Delta u, \Delta v, \Delta w)_i$.

This relationship can be used to solve a variety of problems.

1. Rough Bonded Anchor Subjected to a Prescribed Motion

Suppose that the anchor is subjected to the rigid motion

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u_o + yw_z - zw_y \\ v_o + zw_x - xw_z \\ w_o + xw_y - yw_x \end{bmatrix}$$

where $u_o, v_o, w_o, w_x, w_y, w_z$ are all prescribed.

Then the $3n$ quantities $(\bar{u}^{(+)}, \bar{v}^{(+)}, \bar{w}^{(+)})_i$ must take the specified values

$$\begin{bmatrix} \bar{u}^+ \\ \bar{v}^+ \\ \bar{w}^+ \end{bmatrix}_i = \begin{bmatrix} u_o + \bar{y}_i w_z - \bar{z}_i w_y \\ v_o + \bar{z}_i w_x - \bar{x}_i w_z \\ w_o + \bar{x}_i w_y - \bar{y}_i w_x \end{bmatrix} \quad (3.3a)$$

where $\bar{x}_i, \bar{y}_i, \bar{z}_i$ denote the coordinates of the centroid of region R_i .

Also in the absence of slip and breakaway the displacement must be continuous across the plate and thus

$$\begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.3b)$$

Substitution of Eqs. (3a, b) into Eq. (3.2b) leads to a set of $3n$ equations in the $3n$ quantities $(\Delta\sigma_{zx}, \Delta\sigma_{zy}, \Delta\sigma_{zz})_i$

2. Smooth Bonded Anchor Subjected to a Prescribed Motion

In this case the $6n$ quantities $(\bar{w}^{(+)}, \bar{\sigma}_{zx}^{(-)}, \bar{\sigma}_{zy}^{(-)}, \Delta w, \Delta\sigma_{zx}, \Delta\sigma_{zy})_i$ take the specified values $(w_o, 0, 0, 0, 0, 0)$ and thus Eq. 3.2b reduces to a set of $3n$ equations in the $3n$ quantities $(\Delta\sigma_{zz}, \Delta u, \Delta v)_i$

3. Rough Anchor Subjected to a Prescribed Motion and where there is breakaway of the anchor over sub-region R_i from the underlying region in $\Gamma^{(-)}(z = z_0)$.

Irrespective of the nature of the soil-structure interface condition, once breakaway occurs the stresses $(\bar{\sigma}_{zx}^{(-)}, \bar{\sigma}_{zy}^{(-)}, \bar{\sigma}_{zz}^{(-)})_{z=z_0}$ throughout this sub-region must have boundary conditions applicable to a free surface. Thus the 6n quantities $(\bar{u}^{(+)}, \bar{v}^{(+)}, \bar{w}^{(+)}, \bar{\sigma}_{zx}^{(-)}, \bar{\sigma}_{zy}^{(-)}, \bar{\sigma}_{zz}^{(-)})_i$ take the specified values $(u_0, v_0, w_0, 0, 0, 0)$ and Eq. 3.2b represents the full set of 6n equations in 6n quantities $(\Delta\sigma_{zx}, \Delta\sigma_{zy}, \Delta\sigma_{zz}, \Delta u, \Delta v, \Delta w)_i$

4. Smooth Anchor Subjected to a Prescribed vertical displacement and where there is breakaway of the anchor from the underlying region $\Gamma^{(-)}(z = z_0)$.

In this case the free surface boundary conditions must be satisfied over the sub-region R_i however since the anchor itself is smooth the shear stresses σ_{zx}, σ_{zy} are continuous. Thus, the 6n quantities $(\bar{w}^{(+)}, \bar{\sigma}_{zx}^{(-)}, \bar{\sigma}_{zy}^{(-)}, \bar{\sigma}_{zz}^{(-)}, \Delta\sigma_{zx}, \Delta\sigma_{zy})_i$ take the values $(w_0, 0, 0, 0, 0, 0)$ and Eq. 3.2b reduces to (4n) equations in the (4n) quantities $(\Delta\sigma_{zz}, \Delta u, \Delta v, \Delta w)_i$

3.2.2 Determination of Influence Coefficients

In order to determine the influence coefficients of Eqs. 3.2a, b, the equations of elasticity are transformed by the application of a double Fourier transform (Sneddon, 1951), or equivalently the field quantities are assumed to have a Fourier representation. Thus, typically, the displacement in the x direction will have the representation:

$$u(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{+i(\alpha x + \beta y)} U(\alpha, \beta, z) d\alpha d\beta \quad (3.4a)$$

where

$$U(\alpha, \beta, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(\alpha x + \beta y)} u(x, y, z) dx dy \quad (3.4b)$$

In order to be specific in what follows, it will be assumed that the anchor plate is embedded in a single layer of a homogeneous elastic material, although the technique has a much more general application and may be extended to allow consideration of layered cross-anisotropic deposits.

Introduction of the Fourier representation Eq. 3.4a, enables the solution in the upper portion of the layer $\Gamma^{(+)}$ to be expressed in the form

$$\tilde{P}^{(+)} = M(\alpha, \beta, z) \cdot \tilde{c}^{(+)} \quad (3.5a)$$

where $\tilde{P}^{(+)} = (\Sigma_{zx}, \Sigma_{zy}, \Sigma_{zz}, U, V, W)^T$ is the vector of transforms of $(\sigma_{zx}, \sigma_{zy}, \sigma_{zz}, u, v, w)^T$
 $\tilde{c}^{(+)}$ is a vector of 6 arbitrary constants
 and $M(\alpha, \beta, z)$ is the matrix defined in Table 3.1, page 84.

Similarly, the solution in the lower portion of the layer $\Gamma^{(-)}$ may be represented in the form

$$\tilde{P}^{(-)} = M(\alpha, \beta, z) \cdot \tilde{c}^{(-)} \quad (3.5b)$$

The 12 constants $\tilde{c}^{(+)}$, $\tilde{c}^{(-)}$ may be determined from the 3

boundary conditions at the upper surface ($z = z^{(+)}$), the 3 boundary conditions at the lower surface ($z = z^{(-)}$) and the 6 'jump' conditions at the interface ($z = z_0$), by solving

$$A \cdot \tilde{c} = \tilde{b} \quad (3.6)$$

where

$$\tilde{c} = \begin{bmatrix} c^{(+)} \\ c^{(-)} \end{bmatrix}$$

and where A and $\tilde{b}$ are given in Table 3.2, for the specific case of an elastic layer resting on a rough rigid base and with a stress free upper surface.

The solution of Eq. 3.5 leads to the relations

$$\tilde{c}^{(+)} = N^{(+)}(\alpha, \beta) \cdot \Delta \tilde{P} \quad (3.7a)$$

$$\tilde{c}^{(-)} = N^{(-)}(\alpha, \beta) \cdot \Delta \tilde{P} \quad (3.7b)$$

where it will be recognised that the 6×6 matrices $N^{(+)}(\alpha, \beta)$ and $N^{(-)}(\alpha, \beta)$ can be obtained by deleting the columns 1-3, 10-12 from A^{-1} and then deleting rows (7-12) and (1-6) respectively.

It now follows from Eqs. 3.5a and 3.7a and Eqs. 3.5b and 3.7b that

$$\tilde{P}^{(+)} = M(\alpha, \beta, z) \cdot N^{(+)}(\alpha, \beta) \cdot \Delta \tilde{P}$$

and

$$\tilde{P}^{(-)} = M(\alpha, \beta, z) \cdot N^{(-)}(\alpha, \beta) \cdot \Delta \tilde{P}$$

and hence

$$\tilde{Q} = (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) \cdot \Delta \tilde{P} \quad (3.8)$$

where $D^{(-)}$ and $D^{(+)}$ are (6×6) matrices with zero components except for values of unity corresponding to the first three and last three diagonal terms respectively.

It follows from the representation formulae, such as Eq. 3.4a, that

$$\begin{aligned} \underline{q} = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\alpha x + \beta y)} (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) + \\ & D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) \cdot \Delta P \, d\alpha d\beta \end{aligned} \quad (3.9)$$

The average value of q over R_j is given by

$$\bar{q}_j = \frac{1}{A_j} \int_{R_j} \int \underline{q} \, dA$$

so that from Eq. 3.9

$$\begin{aligned} \bar{q} = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_j(\alpha, \beta) (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) \\ & + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) \cdot \Delta P \, d\alpha d\beta \end{aligned} \quad (3.10a)$$

$$\text{where } \phi_j(\alpha, \beta) = \frac{1}{A_j} \int_{R_j} \int e^{i(\alpha x + \beta y)} \, dA \quad (3.10b)$$

If it is assumed that, to sufficient accuracy, the dislocation can be considered to be constant over each sub-region R_k then it follows from Eq. 3.4b that

$$\begin{aligned}
\Delta \tilde{p} &= \sum_{k=1}^n \frac{\Delta p_k}{4\pi^2} \int_{R_k} \int e^{-i(\alpha x + \beta y)} dA \\
&= \sum_{k=1}^n \frac{A_k}{4\pi^2} \phi_k^*(\alpha, \beta) \Delta p_k
\end{aligned} \tag{3.11}$$

where $\phi_k^*(\alpha, \beta)$ is the complex conjugate of $\phi_k(\alpha, \beta)$ defined by Eq. 3.10b and A_k is the area of the sub-region R_k .

Substitution of Eq. 3.11 into Eq. 3.10 leads to the final form

$$q_j = \sum_{k=1}^n I_{jk} \Delta p_k$$

where

$$\begin{aligned}
I_{jk} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{A_k}{4\pi^2} \phi_j(\alpha, \beta) \phi_k^*(\alpha, \beta) (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) \\
&\quad + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) d\alpha d\beta
\end{aligned}$$

The evaluation of the term $\phi(\alpha, \beta)$ for a typical case is illustrated in Appendix 3.A.

3.2.3 Extension to a Fourier Series Representation

The analysis described in the previous section enables the calculation of the influence matrices I_{jk} for an isolated anchor. A case of some practical importance is that of a series of anchors periodically spaced with spacing $2W_x$ in the x direction and $2W_y$ in the

y direction. In this case the solution is best represented as a Fourier Series. Thus, typically, the displacement in the x direction will have the representation:

$$u(x,y,z) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} e^{i(\alpha_n x + \beta_m y)} U(\alpha_n, \beta_m, z)$$

$$\text{where } U(\alpha_n, \beta_m, z) = \frac{1}{4W_x W_y} \int_{-W_y}^{W_y} \int_{-W_x}^{W_x} e^{-i(\alpha_n x + \beta_m y)} u(x,y,z) dx dy$$

$$\begin{aligned} \text{and } \alpha_n &= n\pi/W_x & n &= 0, \pm 1, \pm 2, \dots \\ \beta_m &= m\pi/W_y & m &= 0, \pm 1, \pm 2, \dots \end{aligned}$$

The analysis of this case parallels that described in the previous section, and it is found that:

$$\begin{aligned} I_{jk} = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \frac{A_k}{4W_x W_y} \phi_j(\alpha_n, \beta_m) \phi_k^*(\alpha_n, \beta_m) [D^{(-)} M(\alpha_n, \beta_m, z_0) N^{(-)}(\alpha_n, \beta_m) \\ + D^{(+)} M(\alpha_n, \beta_m, z_0) N^{(+)}(\alpha_n, \beta_m)] \end{aligned} \quad (3.12)$$

3.3 PREDICTION OF THE ELASTIC LOAD DEFLECTION BEHAVIOUR OF HORIZONTAL ANCHOR PLATES

The method of analysis presented in the previous section may be utilised to determine the stresses and displacements anywhere within the elastic soil mass due to a given load P applied to an anchor of general shape (Rowe and Booker, 1978). However, for the purpose of illustration, attention will be restricted to the load displacement relationship for strip, circular and rectangular anchors

expressed in terms of the apparent stiffnesses $P/\delta E$, $P/B\delta E$ and $P/L\delta E$ respectively, where P is the applied load necessary to cause an anchor of length L and width (or diameter) B to displace through a distance δ .

The apparent stiffness depends primarily upon Poisson's ratio ν and four geometric parameters; namely, the anchor shape (ie. aspect ratio, L/B); the embedment ratio h/B ; the depth of the soil layer beneath the anchor D/B ; and for the case of anchor groups, the centre to centre distance to the adjacent anchors $2W_x/B$ and $2W_y/B$ in the x and y directions respectively. However, a number of secondary parameters will also influence the load deflection behaviour of the anchor plate and consequently the following investigation into the effects of the primary parameters upon the anchor behaviour has, unless otherwise stated, been performed for the basic case of a smooth rigid anchor resting in a soil layer underlain by a rough rigid base and with self weight or applied surcharge sufficient to prevent break-away of the anchor from the underlying soil.

The application of the analytical techniques described in section 3.2 to the prediction of the load displacement behaviour of a rigid anchor plate involves an approximation regarding the number of sub-regions which are necessary to simulate rigid anchor behaviour. In this regard, it was found that the load displacement relationship could be obtained to sufficient accuracy by the sub-division of a strip into 8 strip 'elements', a circle into 4 annular rings, and a rectangle into 64 rectangular 'elements'. The actual computational effort may be further reduced by considering symmetry. The adoption of a larger number of 'elements' did not significantly alter the apparent stiffness of the anchor. For example, in a strip analysis, increasing the number of sub-

regions ('elements') from eight to fourteen for a range of parameters (W/B , h/B , D/B , and ν) increased the apparent stiffness by at most 0.8% and generally by less than 0.4%.

In its present form the analytical formulation presented in the previous section involves Lamé's parameter λ (see Table 3.1) which is infinite for Poisson's ratio $\nu = 0.5$. To avoid this difficulty, all the solutions presented for $\nu = 0.5$ were in fact obtained for $\nu = 0.4999999$. Comparison of results obtained for incompressible materials for the limiting case of strip and circular anchors where h/B tended to zero (ie. a surface footing) with classical analytical solutions (e.g. Poulos and Davis, 1974) indicated good agreement and it was considered unnecessary to adopt a special formulation, which would be independent of Lamé's parameters, for incompressible materials.

3.3.1 The Behaviour of Strip Anchors

3.3.1.1 Basic case

The effects of layer depth, embedment ratio and Poisson's ratio upon an isolated, rigid, strip anchor can be conveniently represented by expressing the apparent stiffness $P/\delta E$ in the form

$$\frac{P}{\delta E} = \frac{I_D(h/B=\infty) \cdot I_h}{(1-\nu^2)} \quad (3.13)$$

where P = is the applied load (per unit length) necessary to cause the anchor to displace a distance δ

$I_D(h/B=\infty)$ = is an influence factor for the effect of layer depth D/B upon the apparent stiffness evaluated for an anchor with $h/B=\infty$

I_h = is a correction factor for the effect of embedment ratio h/B upon the apparent stiffness; and

where E & ν = are Young's Modulus and Poisson's Ratio.

The influence factor I_D is shown in Fig. 3.3 as a function of layer depth D/B for the basic case of a rigid anchor with an infinite embedment ratio ($h/B=\infty$). For comparative purposes, the second limiting case of a rigid surface footing is also shown in this figure. It can be seen that the influence factor I_D is relatively insensitive to Poisson's ratio, varying by about 20% at most, whereas the actual stiffness $P/\delta E$ varies by more than 50% in the worst case. It will be observed that for the surface strip, the influence factor increases sensibly with Poisson's ratio; however, in the case of the anchor with an infinite embedment ratio there is no consistent trend for the curves with $\nu = 0.2, 0.3$, and 0.4 , although the maximum difference between these curves is less than 4%. This behaviour is a manifestation of the removal of the factor $1/(1-\nu^2)$ in determining I_D and the actual stiffness itself does increase sensibly with Poisson's ratio. In view of the close bracketing of I_D with Poisson's ratio obtained by removing the factor $1/(1-\nu^2)$, the disadvantage of having influence curves which intersect when plotted against D/B was not considered to be serious.

The correction factor I_h is shown in Figs. 3.4a, b, c and d as a function of embedment ratio h/B for a range of layer depths D/B . This correction factor corresponds to the ratio of the apparent stiffness for a given h/B to the apparent stiffness obtained for a similar case where $h/B=\infty$ and consequently, attains a value of unity at the critical embedment ratio. Three observations can be made from Fig. 3.4.

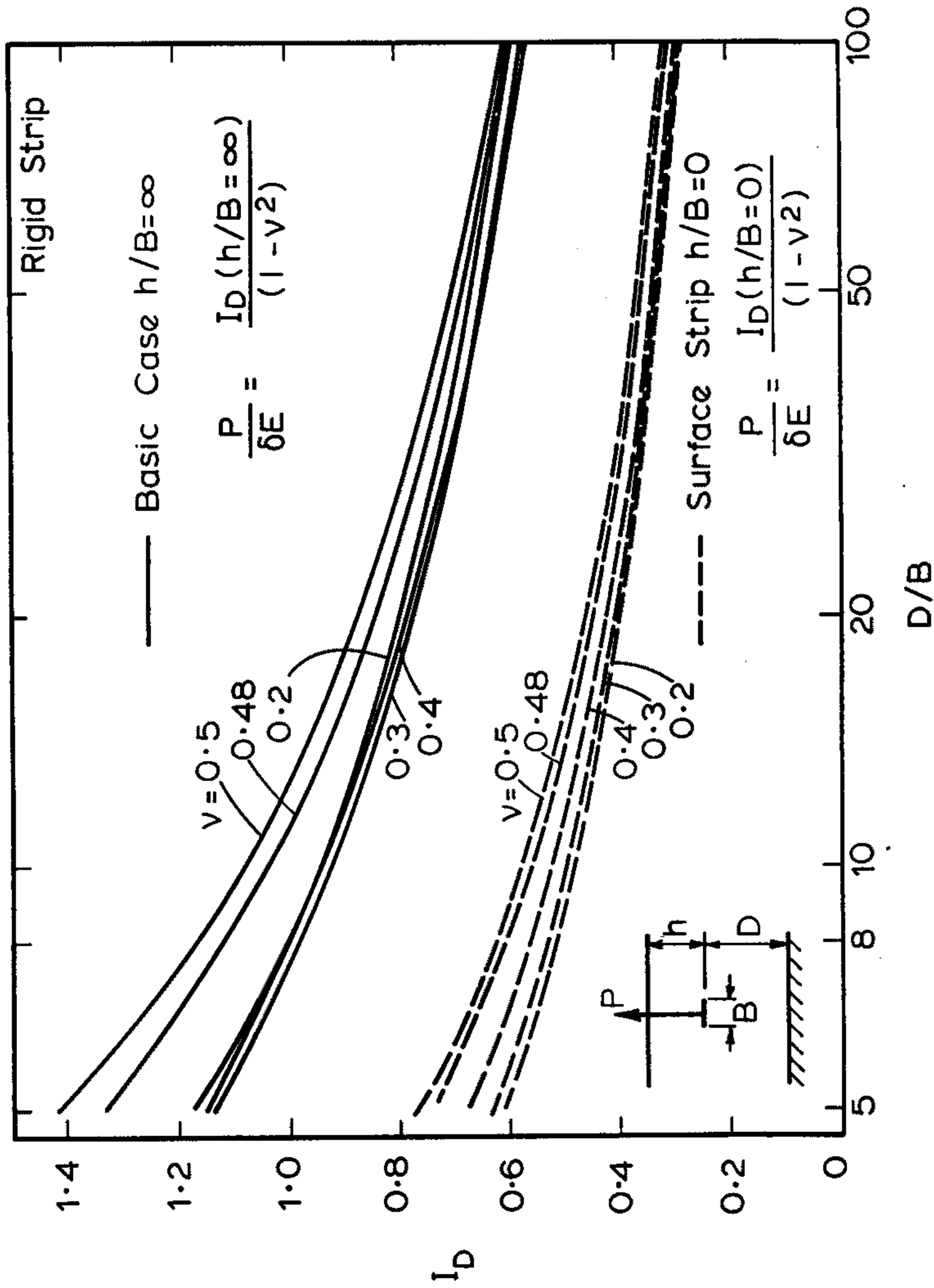


FIG. 3.3 INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH UPON ELASTIC LOAD DEFLECTION BEHAVIOUR OF A RIGID STRIP

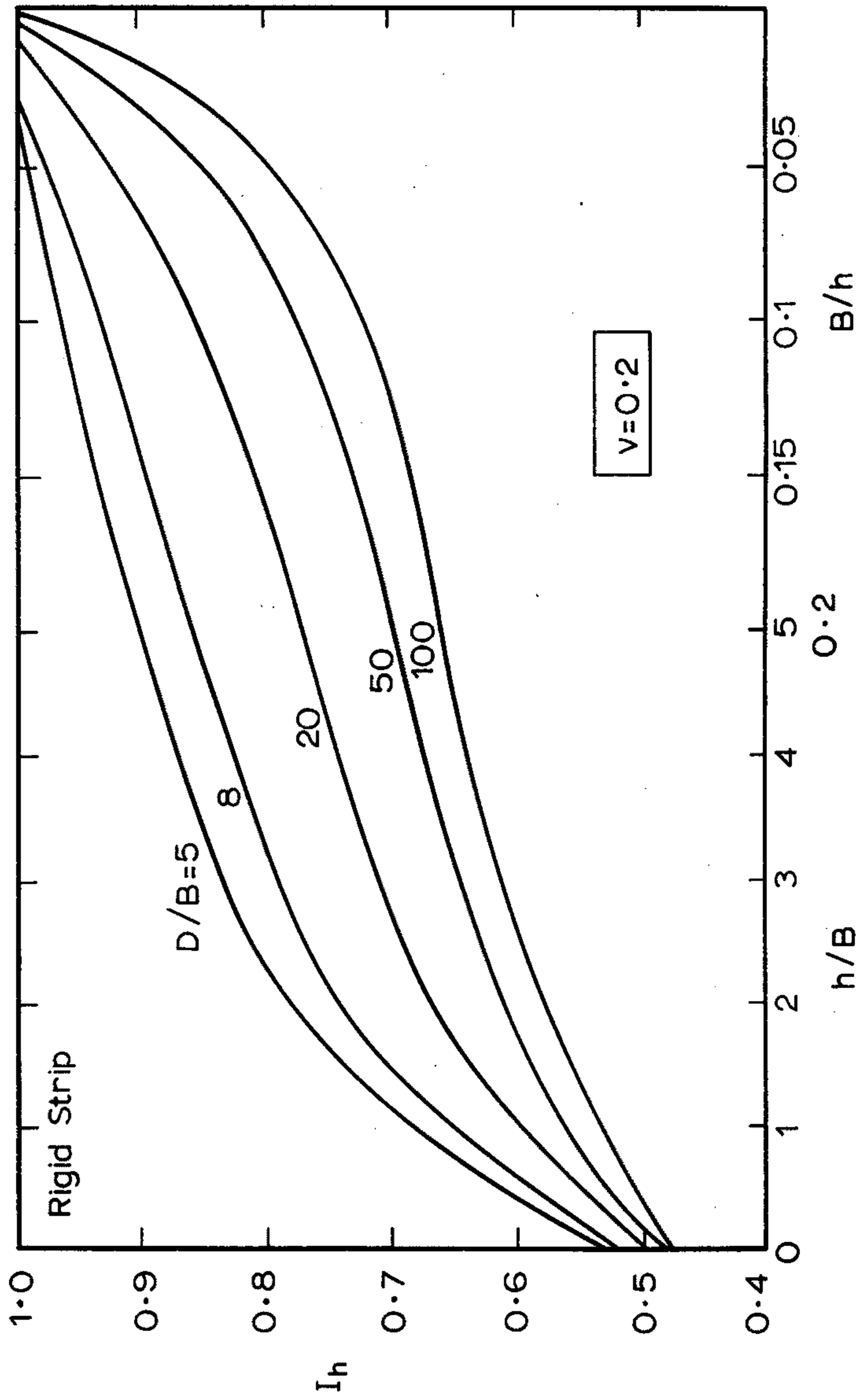


FIG.3.4a CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO- $v=0.2$

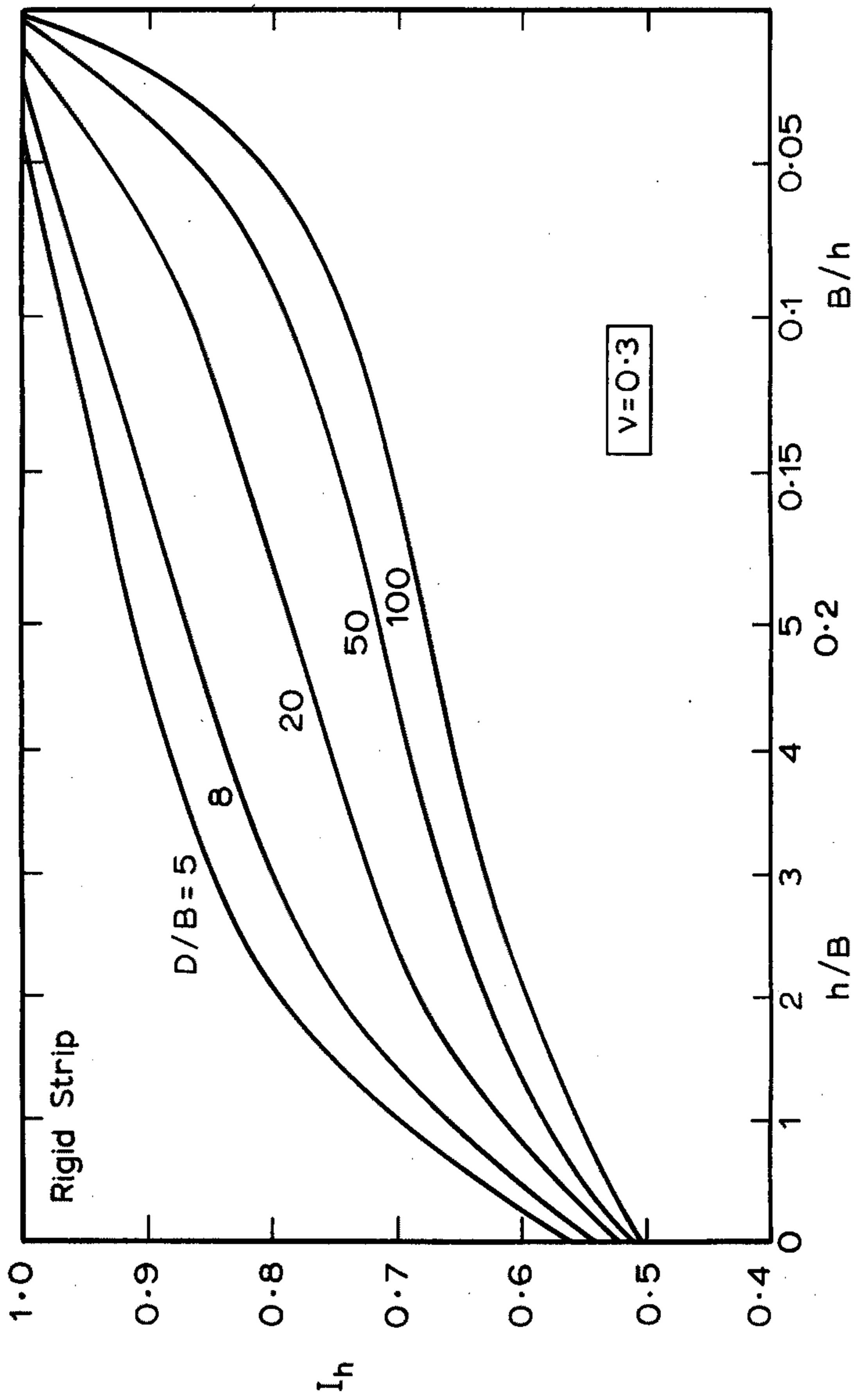


FIG.3.4b CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO - $v = 0.3$

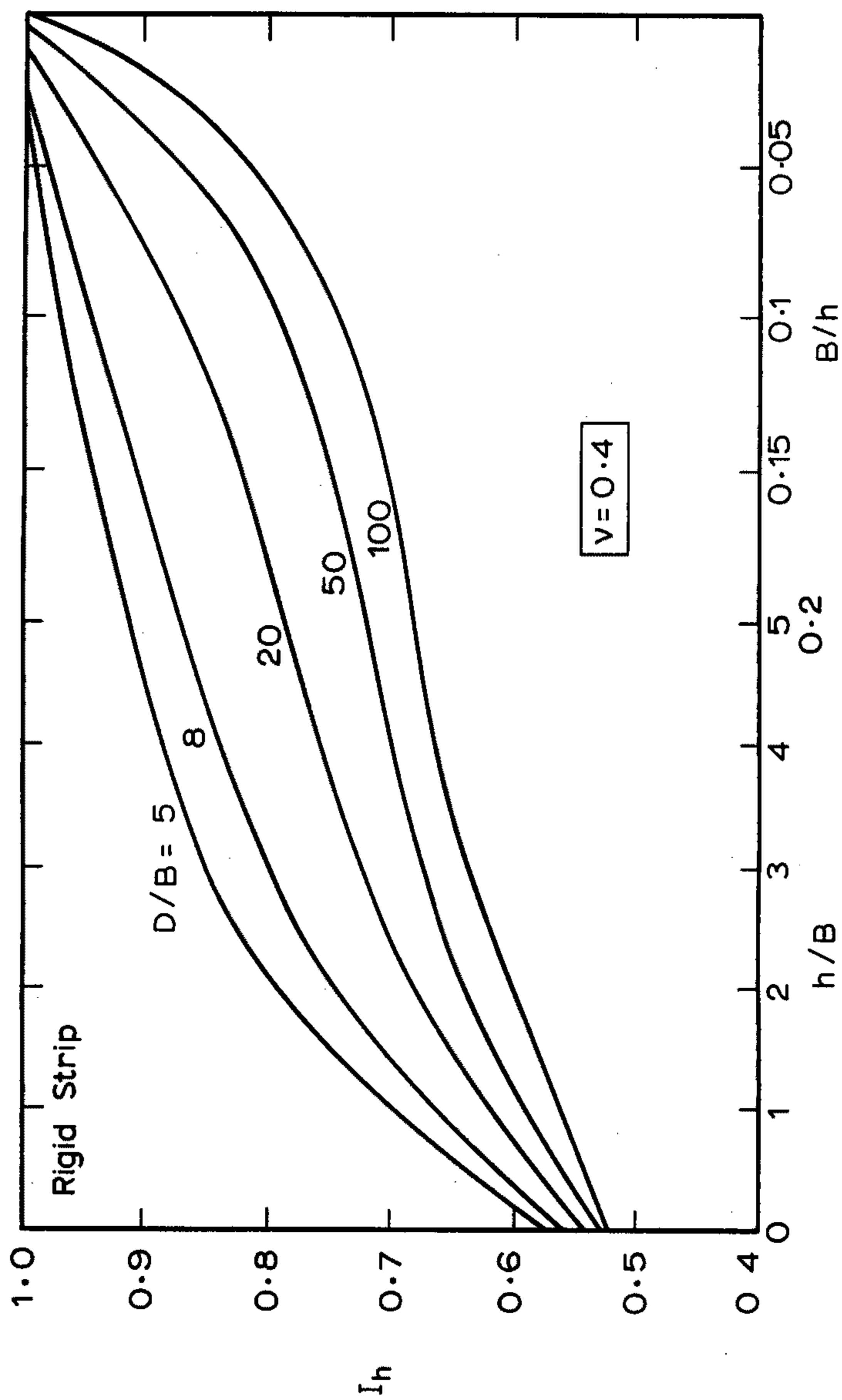


FIG.3.4c CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO - $v = 0.4$

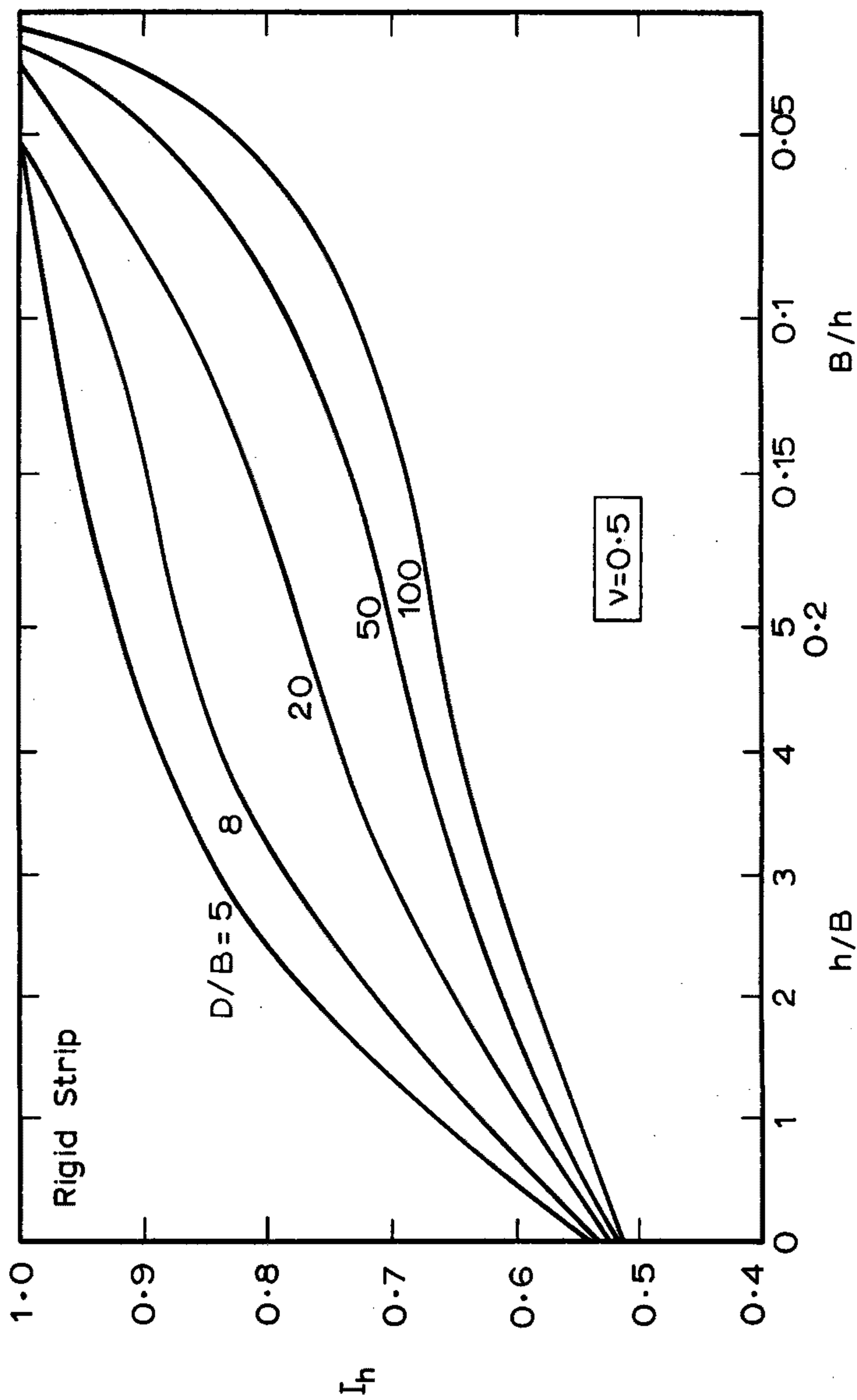


FIG.3.4d CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO- $v=0.5$

Firstly, the critical embedment ratio increases with layer depth. Secondly, the proportion of the maximum possible stiffness that can be obtained for a particular embedment ratio decreases dramatically with layer depth. Finally, it was found that the correction factor I_h is not particularly sensitive to the value of Poisson's ratio, varying by about 10%, (over the range $0.2 \leq \nu \leq 0.5$) in the worst case.

Equation 3.13 may be used in conjunction with Figs. 3.3 and 3.4 to predict the elastic load-deflection response of a horizontal strip anchor for a wide range of embedment ratios, layer depths and values of Poisson's ratio. The use of Equation 3.13 and the associated influence charts is illustrated by a worked example in Appendix 3B.

The foregoing analyses were performed for the basic case of a fully bonded, smooth, rigid anchor resting in a layer underlain by a rough rigid base. The effect of these conditions upon the load deflection behaviour of the anchor will now be considered.

3.3.1.2 Effects of interface between soil layer and rigid base

To investigate the effect of the assumed boundary condition between the soil layer and the rigid base, analyses were performed for an anchor in a layer with $D/B=8$ for the conditions of a rough and smooth rigid base. The apparent stiffnesses obtained from these analyses are shown in Fig. 3.5. The smooth base condition was found to give stiffnesses which were about 4-6% and 14-20% lower than those obtained with a rough base for Poisson's ratio's of $\nu = 0.3$ and $\nu =$

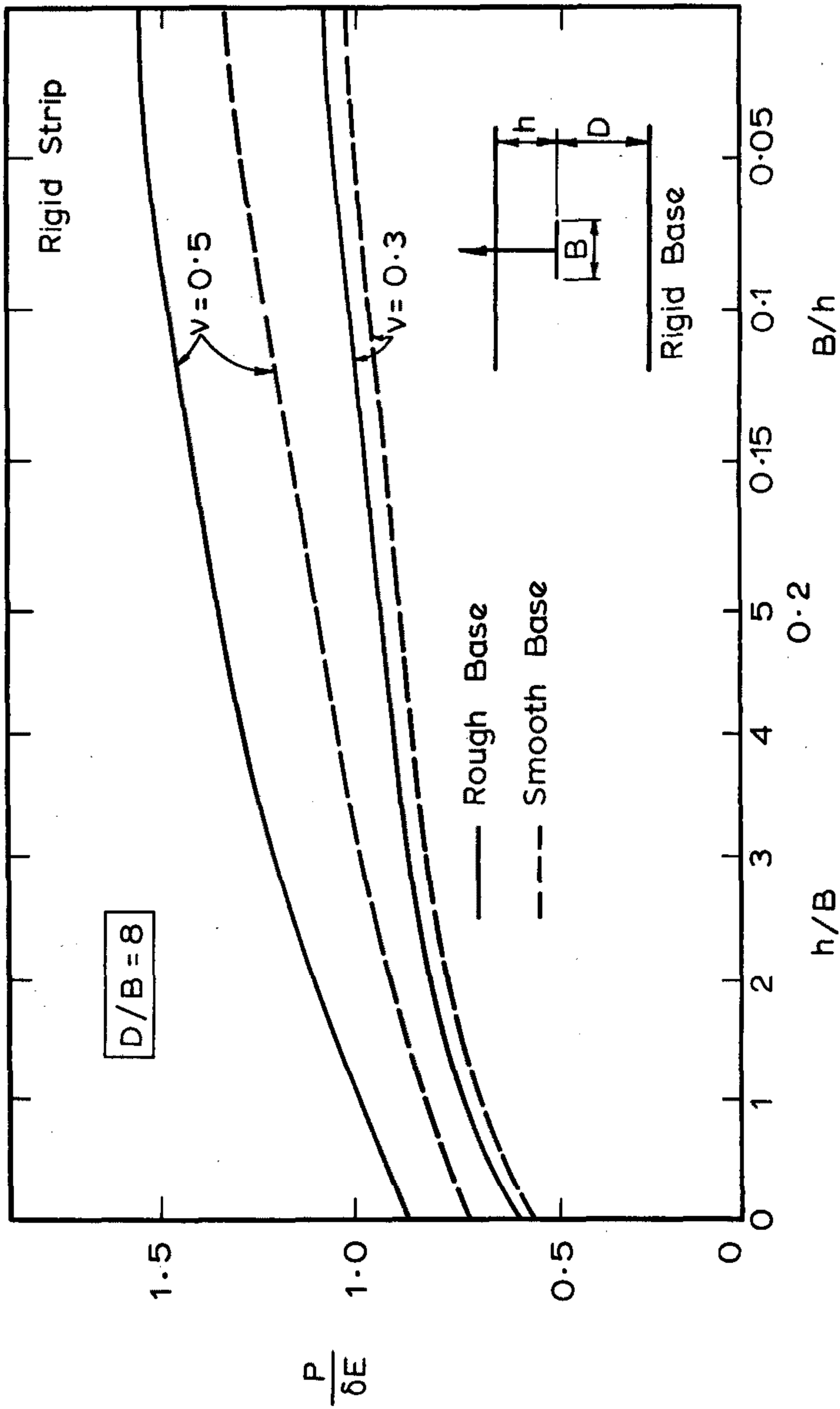


FIG.3.5 EFFECT OF BASE CONDITION UPON APPARENT STIFFNESS.

0.5 respectively. It is of some interest to note that for both values of Poisson's ratio the greatest differences were observed at intermediate depths ($h/B \approx 5$) rather than for the extreme cases of a surface footing ($h/B = 0$) or anchor at infinite depth ($h/B = \infty$), although the variations with h/B were not particularly large in either case.

Since it might be expected that the effect of the base interface condition upon the apparent stiffness would decrease with layer depth D/B , analyses were also performed for an anchor at infinite depth ($h/B = \infty$), for a range of layer depths D/B using the two base conditions. The resulting stiffnesses are shown in Fig. 3.6. From these results, it will be observed that although the significance of the base interface condition does decrease with layer depth D/B , the rate of decrease with depth is relatively slow. For example, a twenty fold increase in layer depth (from $D/B = 5$ to $D/B = 100$) only decreased the discrepancy between the stiffnesses for the smooth and rough bases by a factor of two for both values of Poisson's ratio.

In general it is considered that the rough base condition would be the more practical case, however if the type of base is considered to be significant then the predicted stiffness obtained for a rough base using Equation 3.13 could be approximately modified for a smooth base by multiplying this stiffness by the ratio of smooth to rough stiffness obtained for the particular layer depth D/B from Fig. 3.6 as illustrated by the example in Appendix 3B.

3.3.1.3 Effect of anchor roughness

The type of interface condition between the anchor and soil

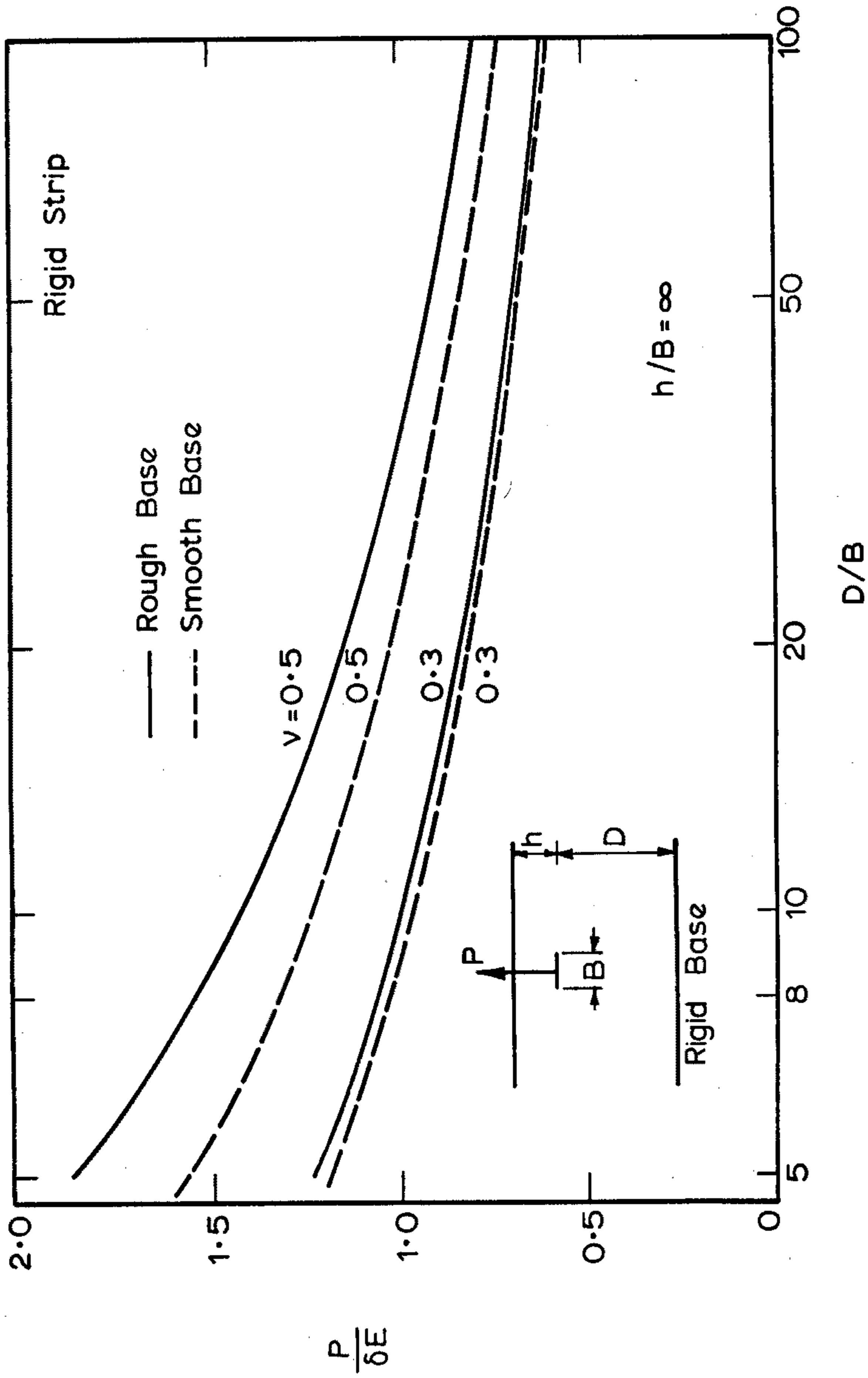


FIG. 3.6 EFFECT OF BASE CONDITION UPON APPARENT STIFFNESS.

was found to be unimportant in the determination of the apparent stiffness, since the difference between the stiffnesses obtained for smooth and rough anchors was always less than 1% and often less than 0.1% for a wide range of layer depths, embedment ratios and values of Poisson's ratio.

3.3.1.4 Effect of anchor rigidity upon apparent stiffness

The apparent stiffness calculated on the basis of the mean displacement of a uniformly loaded region (corresponding to an infinitely flexible anchor) was found to differ from the stiffness determined for a rigid anchor by between 3% and 6% for a wide range of geometric parameters and values of Poisson's ratio. Thus it can be concluded that provided the loading arrangement does not cause structural failure of the anchor plate itself or alter its geometry significantly then the flexibility of the anchor will not greatly alter the load-(mean) deflection behaviour of the anchor.

3.3.1.5 Effect of breakaway

It is generally accepted (e.g. Douglas and Davis, 1964; Selvadurai, 1976) that the fully bonded (no breakaway) condition corresponds to the most practical limiting case for the application of elastic solutions since at working loads the compressive stresses due to the overburden pressure will usually be sufficiently large to prevent the development of tensile stresses between the anchor and the soil. Accordingly, the majority of solutions presented in this chapter are for the case of a fully bonded anchor; however, as outlined in section 3.2, analyses can be performed for the case where

there is breakaway of the anchor from the soil. Assuming that there is no adhesion or suction acting between the anchor plate and the soil, then breakaway will occur when the vertical stress below the anchor is zero. Breakaway corresponds to a separation of the anchor from the underlying soil and results in the formation of a gap below the anchor. Immediate breakaway occurs when the vertical stress below the anchor is initially zero and consequently, any increment in load will result in complete separation of the anchor from the underlying soil. To illustrate this effect, the apparent stiffnesses obtained when there is immediate breakaway are shown in Fig. 3.7 for an anchor resting in a layer with depth ratio $D/B = 8$. Comparison of the apparent stiffnesses obtained for the two cases of immediate breakaway and no breakaway (i.e. Figs. 3.5 and 3.7) indicates that the influence of breakaway decreases with increasing embedment ratio, while it increases with Poisson's ratio.

The situation in which there is progressive breakaway with increasing load can also be analysed, if necessary. For example, Fig. 3.8 shows the elastic load displacement curve obtained for an anchor with embedment ratio $h/B = 3$ resting in a soil with depth given by $D/B = 8$, Poisson's ratio $\nu = 0.3$ and an overburden pressure at the level of the anchor of q_h . Initially, the response of the anchor will correspond to the case of no breakaway. As loading continues, the magnitude of the vertical compressive stress below the anchor will decrease. Breakaway of the anchor from the underlying soil will occur over a particular sub-region once the mean vertical stress below that sub-region reaches zero. Breakaway at each such sub-region leads to a reduction in the apparent stiffness of the anchor with respect to additional increments in load. This procedure continues until complete

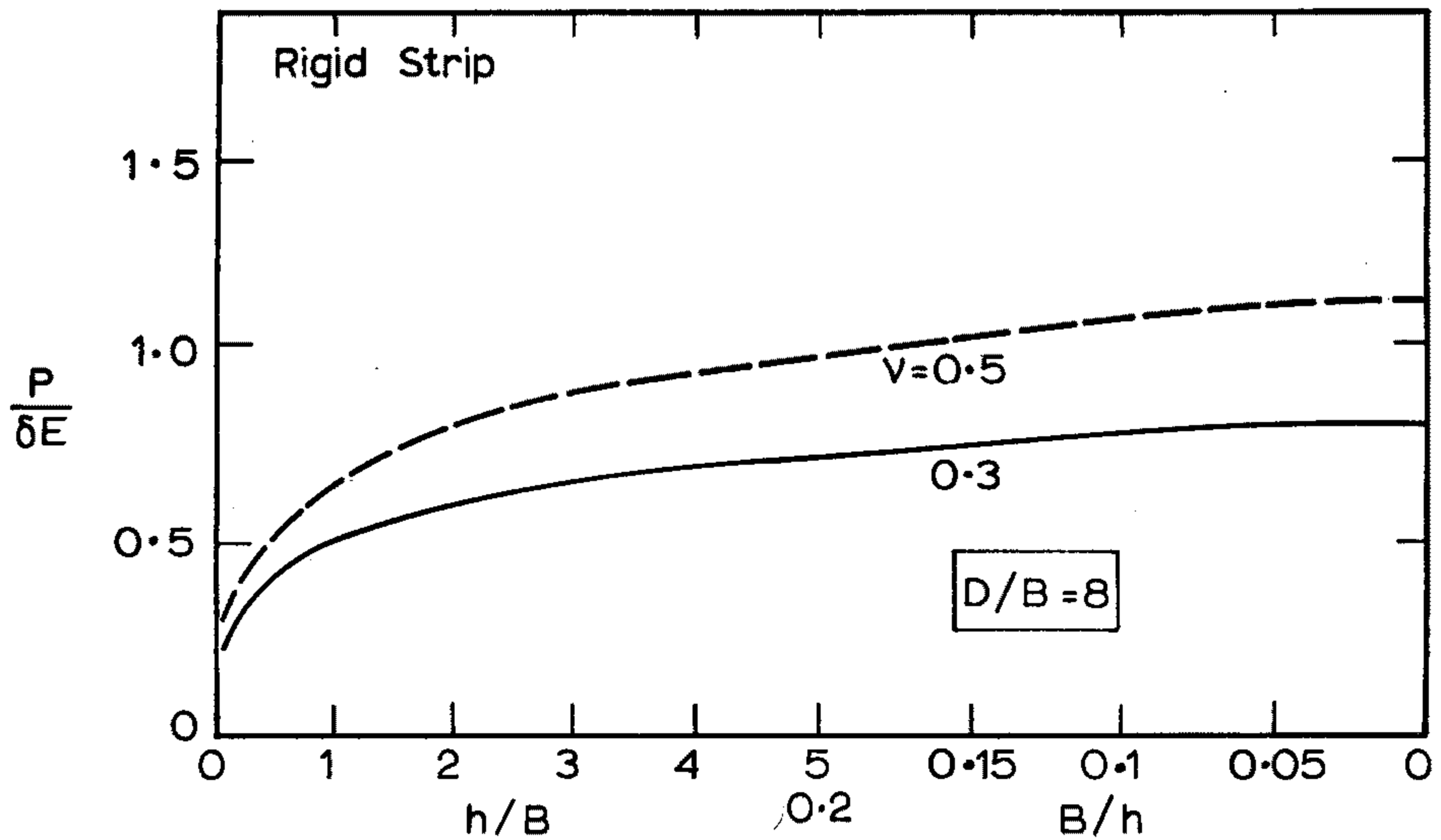


FIG.3.7 EFFECT OF IMMEDIATE BREAKAWAY UPON APPARENT STIFFNESS

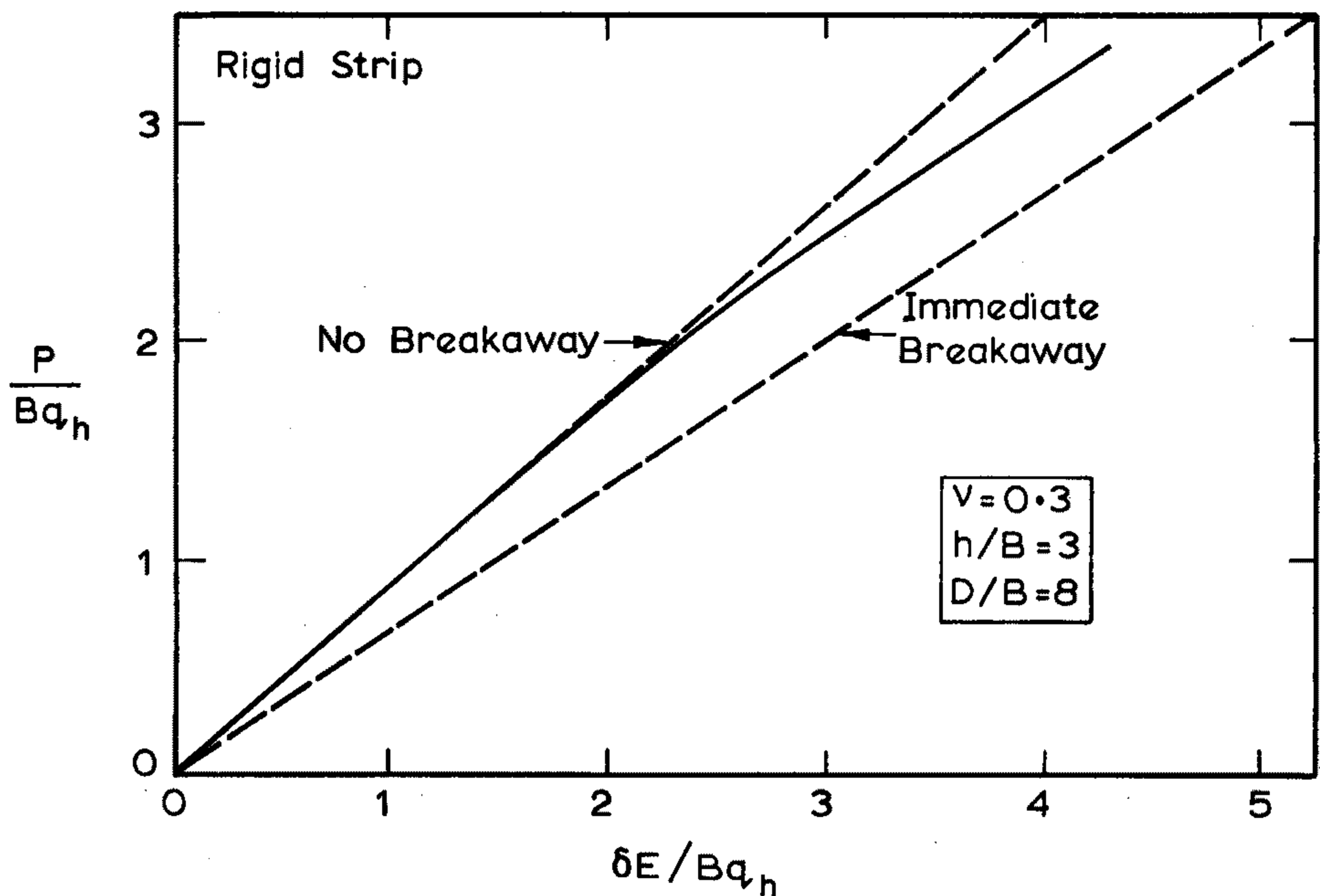


FIG. 3.8 EFFECT OF BREAKAWAY UPON ELASTIC LOAD DEFLECTION BEHAVIOUR

breakaway occurs, whereupon the load displacement curve becomes parallel to that obtained when there is immediate breakaway (i.e. $q_h = 0$).

3.3.1.6 Effect of interaction between anchors within an infinite group

The periodic nature of the Fourier series expansion (Eq. 3.12) corresponds implicitly to an infinite number of anchor plates at a spacing $2W_x$ and $2W_y$ in the x and y directions respectively. Thus, the solution for an individual anchor within an infinite group of parallel strip anchors with centre to centre spacing of $2W$ can be conveniently obtained as a special case. For example, Fig. 3.9 shows the apparent stiffness of a surface anchor ($h/B = 0$) and an anchor at infinite depth ($h/B = \infty$) for a range of anchor spacings W/B . Irrespective of the embedment ratio, it will be observed that for compressible soils ($\nu = 0.3$) the apparent stiffness of the anchor decreases significantly as the spacing between adjacent anchors decreases whereas for an incompressible material the stiffness increases significantly due to the interaction effects. At this layer depth ($D/B = 8$), the interaction between anchors within a group begins to have a significant effect upon the apparent stiffness for values of W/B less than eight and becomes particularly severe for spacings corresponding to W/B less than 5. For values of W/B greater than eight the interaction effect decreases rapidly and it can be concluded that for a spacing $W/B = 40$ the behaviour is essentially that of an isolated anchor.

It will be recognised that the case of an infinite anchor group is completely analogous to the case of an isolated anchor con-

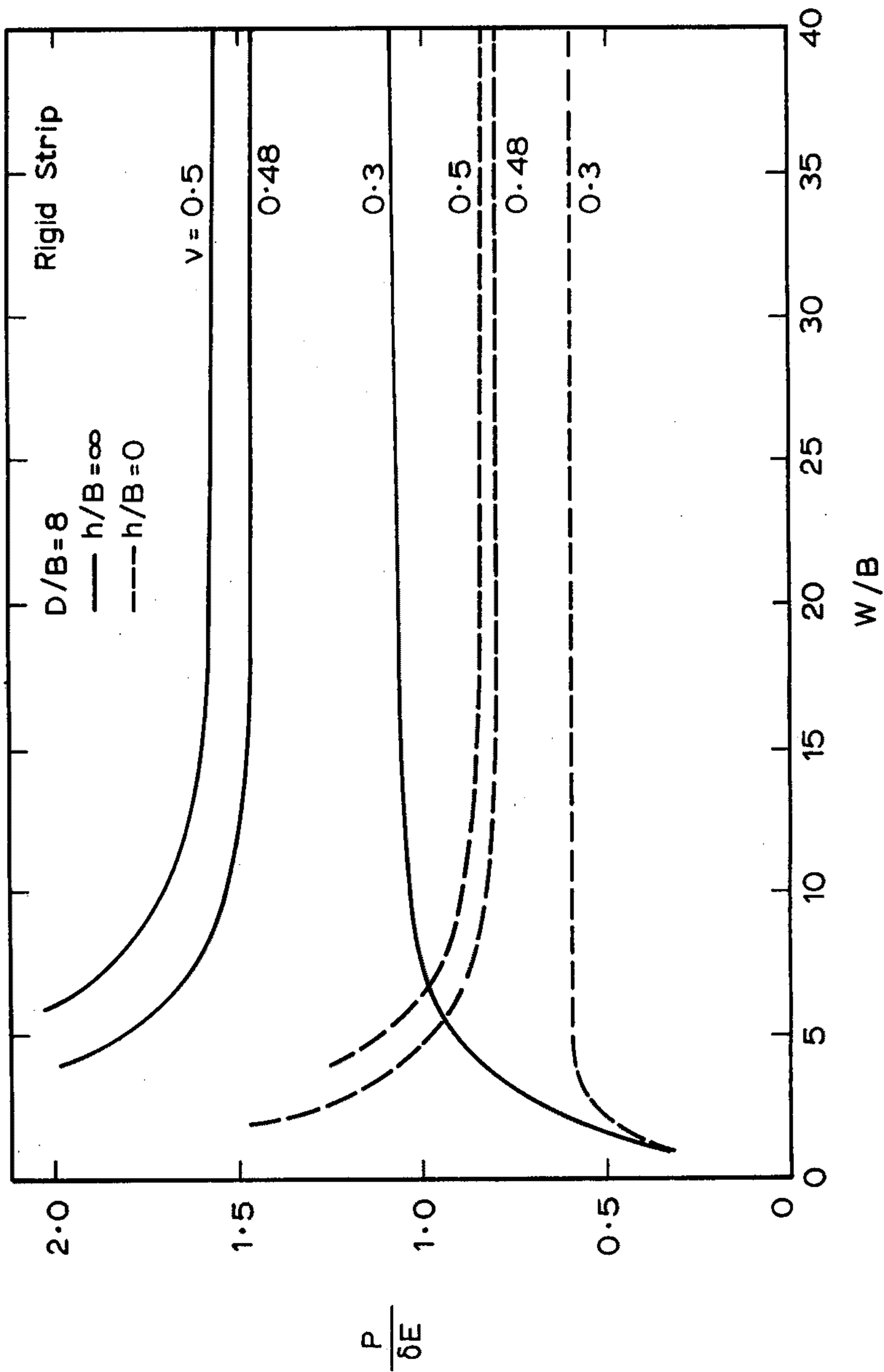


FIG.3.9 EFFECT OF DISTANCE TO LATERAL BOUNDARY UPON APPARENT STIFFNESS $D/B=8$

tained in a box with smooth rigid lateral boundaries at a distance W from the centre of the anchor. Consequently, the solution presented for anchor groups may be used to indicate the distance W to the lateral boundary which would be desirable in a finite element analysis of an isolated anchor.

This distance W to a shear free lateral boundary, (i.e. half the centre to centre spacing between the adjacent anchors) can be expressed in either of two convenient forms namely W/B or W/D , however in general the latter form is of greater significance when considering the spacing necessary to avoid interaction between anchors. To illustrate this, Fig. 3.10 shows the effect of the parameter W/D upon the apparent stiffness for a range of layer depths D/B and it can be seen that for many practical purposes the anchor could be considered to behave as an isolated anchor when $W/D = 3$. This assumption would only lead to an error of 3.5% in the worst case (i.e. $D/B = 1$, $h/B = \infty$, $\nu = 0.5$). Actual isolated anchor behaviour was considered to correspond to a dimensionless spacing $W/D = 5$ since this leads to an error of less than 0.3% in the worst case. Two additional observations can be made from Fig. 3.10. Firstly, the effect of anchor spacing is, as might be expected, greatest for incompressible materials and shallow layers. Secondly, it can be seen that although the embedment ratio alters the magnitude of the apparent stiffness $P/\delta E$ for a given layer depth, it does not greatly alter the general effect of the anchor spacing (in terms of layer depth W/D) upon the stiffness, indicating that even in relatively deep layers (e.g. $D/B = 100$) the behaviour of the anchor is in fact dominated by the depth of soil below the anchor rather than the embedment ratio.

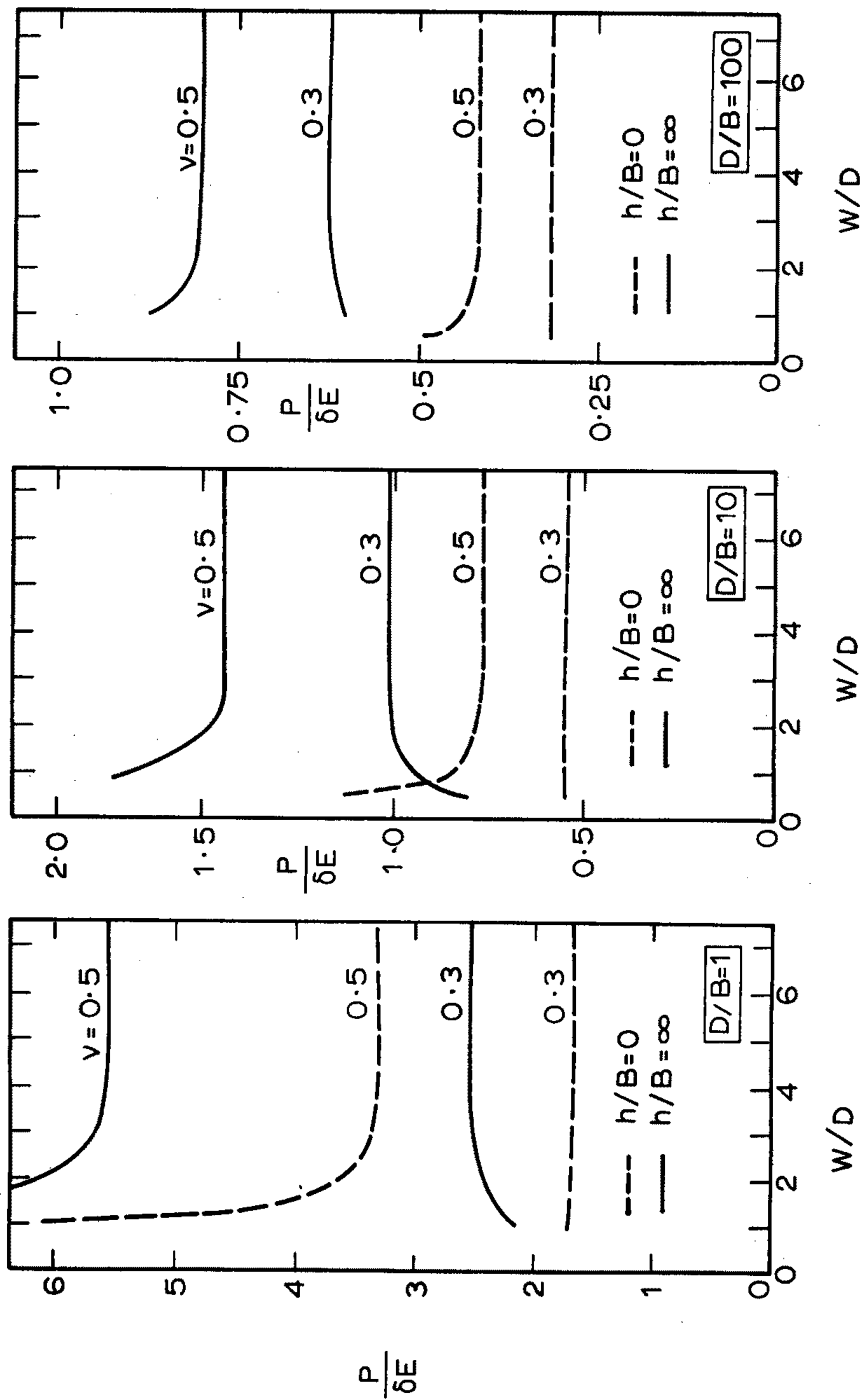


FIG. 3.10 EFFECT OF DISTANCE TO LATERAL BOUNDARY (W/D) UPON APPARENT STIFFNESS OF A RIGID STRIP ANCHOR

3.3.2 The Behaviour of Circular Anchors

3.3.2.1 Basic case

The effects of layer depth D/B , embedment ratio and Poisson's ratio upon the apparent stiffness $P/B\delta E$ of an isolated, rigid, circular anchor can be represented in a similar form to that used for strip anchors viz:

$$\frac{P}{B\delta E} = \frac{I_D (h/B = \infty) \cdot I_h}{1-\nu^2} \quad (3.14)$$

where P is the applied load, and all the other definitions parallel those given for the strip case.

The influence factor I_D is shown in Fig. 3.11 as a function of D/B for the basic case of a rigid circular anchor with an infinite embedment ratio, and for the second limiting case of a rigid surface footing ($h/B = 0$).

As in the case of strip anchors, the influence factor I_D has been defined so that it is relatively insensitive to Poisson's ratio. For a surface anchor (footing) with a depth ratio D/B greater than 30, the influence factor I_D reaches a unique value which is in excellent agreement with the classical elasticity solution for a rigid circular footing resting on a half space (Poulos and Davis, 1974). Similarly, for depth ratios D/B greater than 20, the solution for an anchor with an infinite embedment ratio agrees with the analytical solution for a bonded circular anchor plate within an infinite elastic mass (Selvadurai, 1976) to better than 1%.

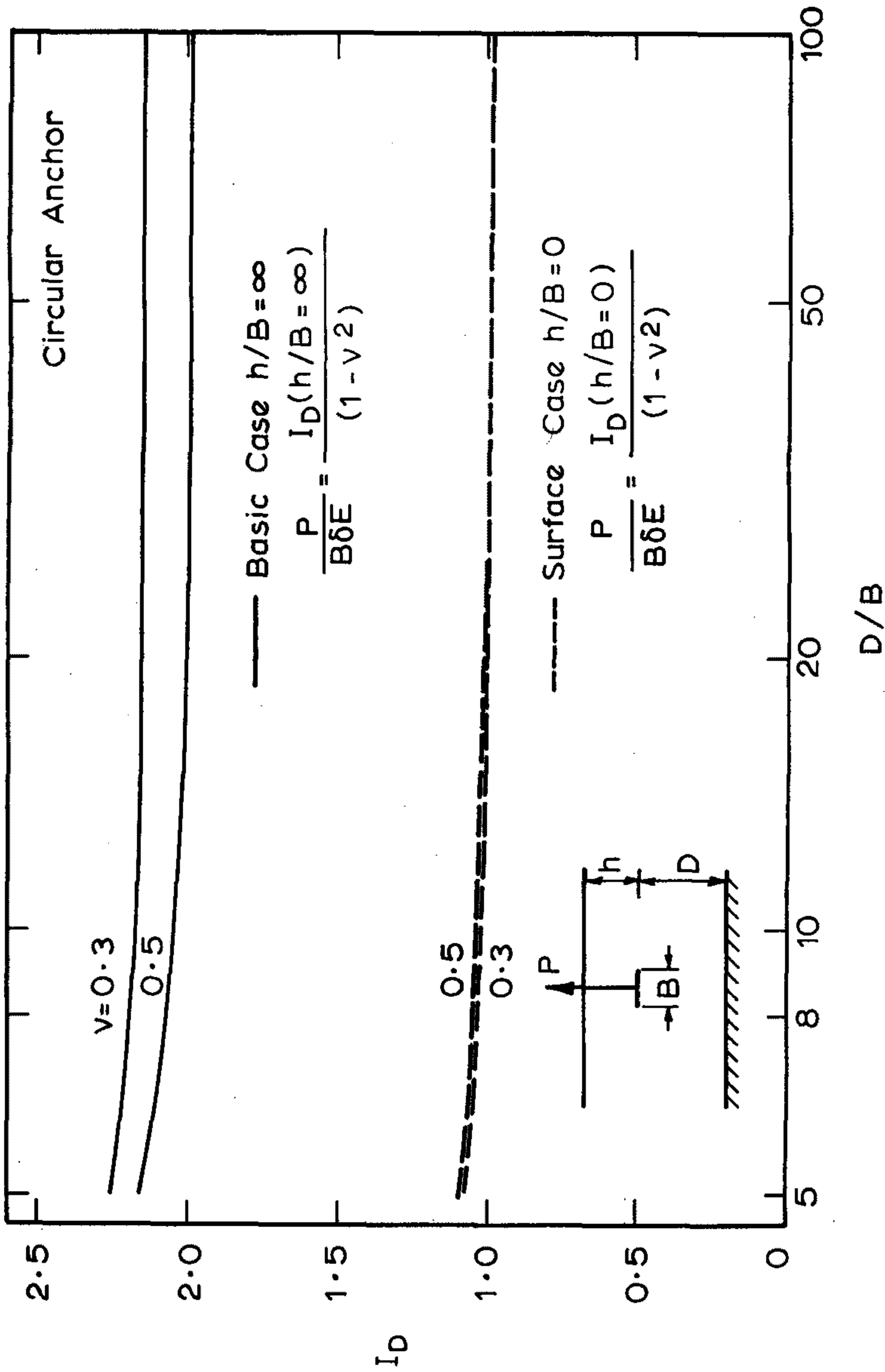


FIG.3.11 INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH UPON ELASTIC LOAD DEFLECTION BEHAVIOUR OF A RIGID CIRCULAR ANCHOR

The correction factor I_h is shown in Figs. 3.12a and 3.12b as a function of embedment ratio h/B for a range of layer depths. Two observations can be made from a comparison of the correction factor I_h obtained for strip and circular anchors (i.e. Figs. 3.4 and 3.12). Firstly, the critical embedment ratio at which a circular anchor behaves as if it was infinitely deep is considerably less than that for a strip anchor. Secondly, the correction factor I_h is relatively insensitive to layer depth for a circular anchor varying by less than 10% for a 10 fold range in D/B (i.e. $5 \leq D/B \leq 50$) whereas the variation of I_h for a strip, over the same range of D/B , may be more than double this value.

Equation 3.14 may be used in conjunction with Figs. 3.11 and 3.12 to predict the elastic load deflection behaviour of circular anchor plates for a wide range of geometric parameters and material properties. The use of these charts is illustrated by a worked example in Appendix 3C.

3.3.2.2 Effect of anchor roughness and flexibility

The effects of anchor roughness and flexibility upon the apparent stiffness of circular anchors are basically the same as for strip anchors. In particular, the difference between the apparent stiffness of a rough and a smooth anchor was less than 1% and the difference between the apparent stiffness determined for a rigid anchor and that determined from the mean displacement of a perfectly flexible anchor was typically about 6%.

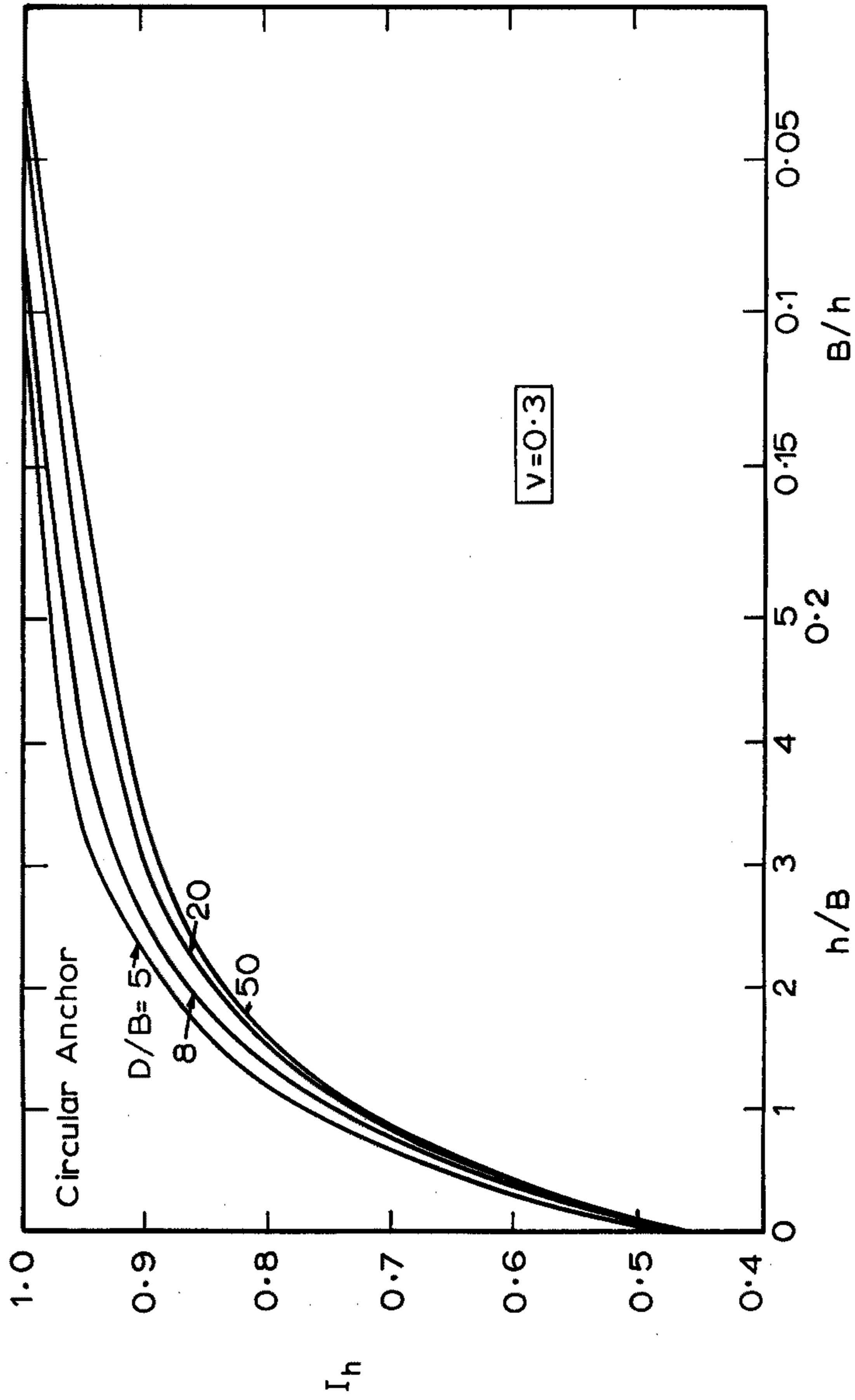


FIG. 3.12a CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT
RATIO $\nu = 0.3$

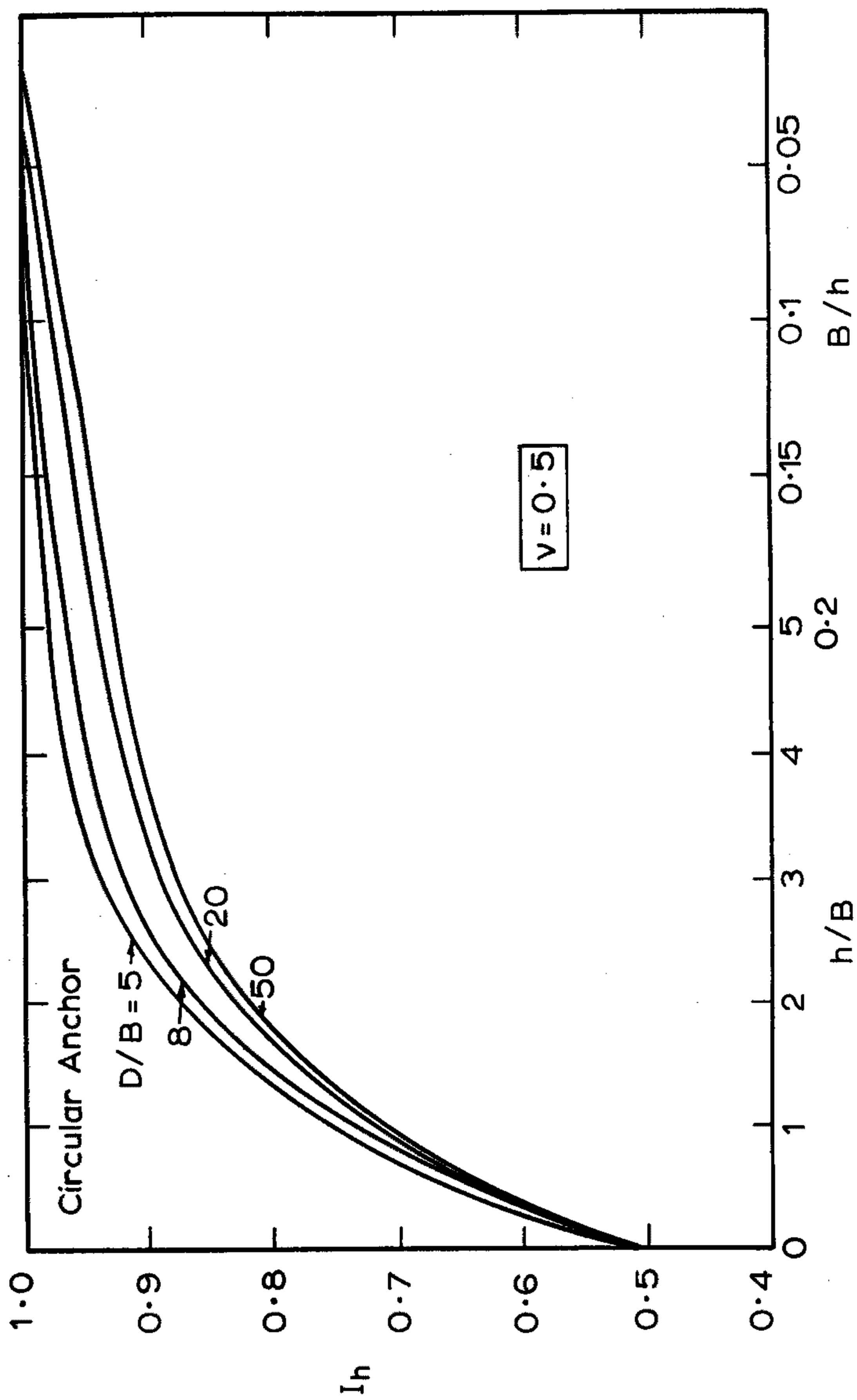


FIG.3.12b CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT
RATIO $\nu = 0.5$

3.3.2.3 Effect of interaction between anchors within an infinite group

For the axi-symmetric problem of a circular anchor plate the Fourier series expressions given in section 3.2.3 reduced to the special case of a Fourier-Bessel series. The periodic nature of this expansion permits the consideration of group effects, or the effect of a smooth lateral boundary, in a similar fashion to that adopted for strip anchors. In this case W is the radial distance from the centre of the anchor to a cylindrical smooth rigid lateral boundary. To a first approximation the analogous case of an infinite group of anchors located on a grid with centre to centre spacing S may be analysed by equating the area of influence for each anchor (i.e. S^2) to the area contained by a smooth cylindrical boundary, and, hence $W = S/\sqrt{\pi}$. To illustrate this effect, the variation in apparent stiffness with spacing ratio W/B for the limiting cases $h/B = 0$ and $h/B = \infty$ where the soil layer has a depth ratio $D/B = 8$ is shown in Fig. 3.13. As in the case of the strip anchor, significant interaction effects are observed for spacing ratios W/B less than eight (i.e. $W/D < 1$) while for spacing ratios W/B greater than 24 (i.e. $W/D > 3$) the behaviour is essentially that of an isolated anchor. The apparent stiffness of circular anchors is in fact slightly less sensitive to the spacing between adjacent anchors than was the case for strip anchors.

3.3.3 The Behaviour of Rectangular Anchors

In the analysis of strip and circular anchors it was found that for these limiting shapes the apparent stiffness of the anchor was virtually unaffected by anchor roughness, while the difference in apparent stiffness based on the displacement of a rigid anchor and the

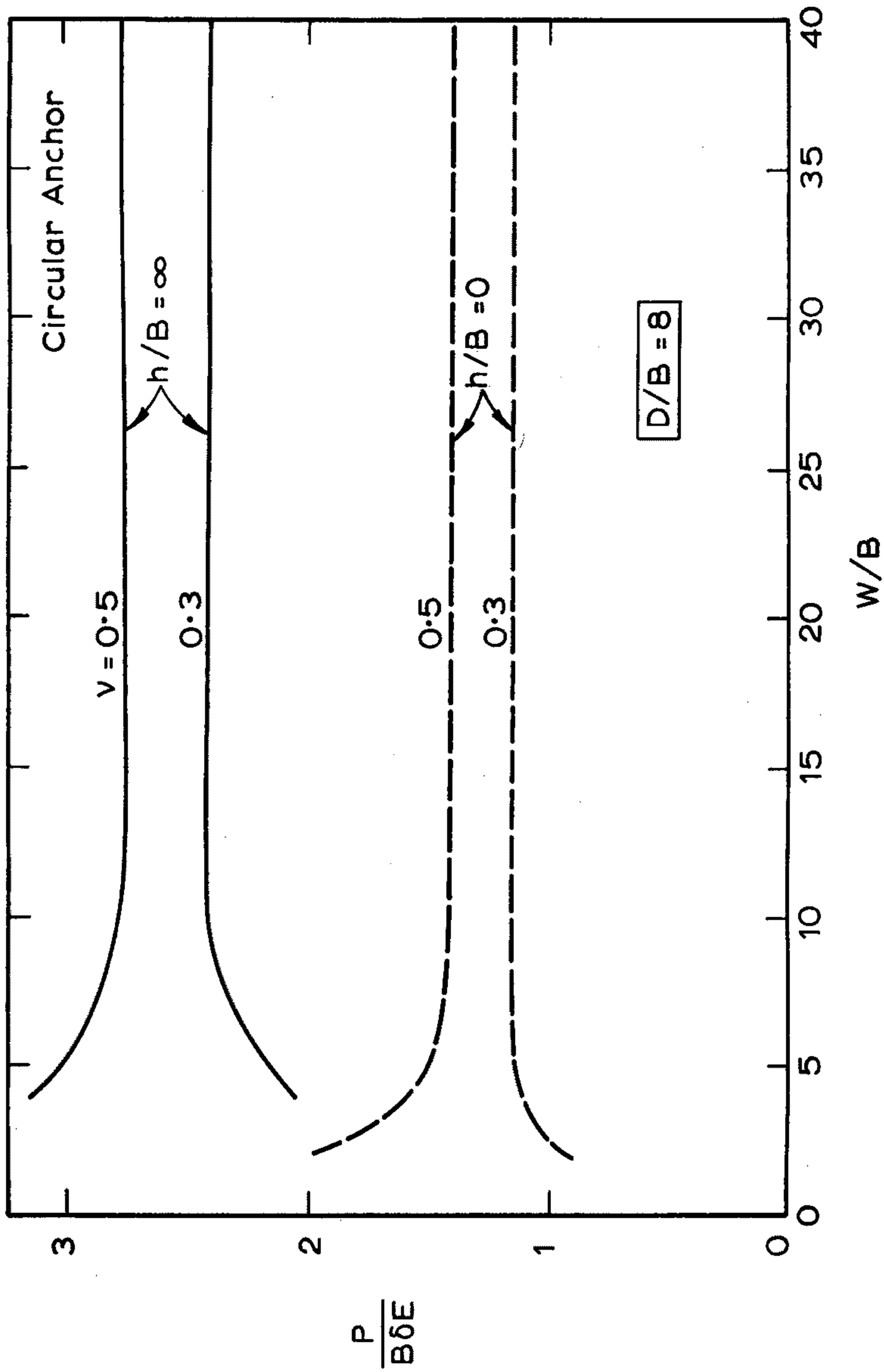


FIG.3.13 EFFECT OF DISTANCE TO LATERAL BOUNDARY UPON APPARENT STIFFNESS OF A RIGID CIRCULAR ANCHOR

mean displacement of a perfectly flexible anchor was typically about 6%. Spot checks indicate that this is also true for rectangular anchors ($1 < L/B \leq \infty$), confirming the assumption made by Fox (1948) when integrating Mindlin's equation to give the mean displacement of rectangular anchors in an elastic half-space.

Accordingly, the load deflection behaviour of rectangular anchors in a soil layer of finite depth can, to sufficient accuracy, be determined from the mean displacement δ_m of a uniformly loaded anchor. This simplifying assumption allows significant computational savings in the determination of the apparent stiffness of the anchor. Further computational savings may be achieved by neglecting the effect of anchor roughness altogether since it was found that this would typically lead to errors in comparison with perfectly smooth and perfectly rough anchors of less than 0.6% and 0.4% respectively.

The effect of layer depth, embedment ratio and Poisson's ratio upon the apparent stiffness $P/L\delta_m$ of a rectangular anchor with aspect ratio L/B may be written in the form

$$\frac{P}{L\delta_m E} = \frac{I_D (h/B) \cdot I_h}{(1-\nu^2)} \quad (3.15)$$

where the definition of all terms parallel those for strip and circular anchors.

The influence factor I_D determined from the mean displacement of rectangular anchors with embedment ratio $h/B = \infty$ is given in Figs. 3.14a and 3.14b for a range of aspect ratios L/B . From these

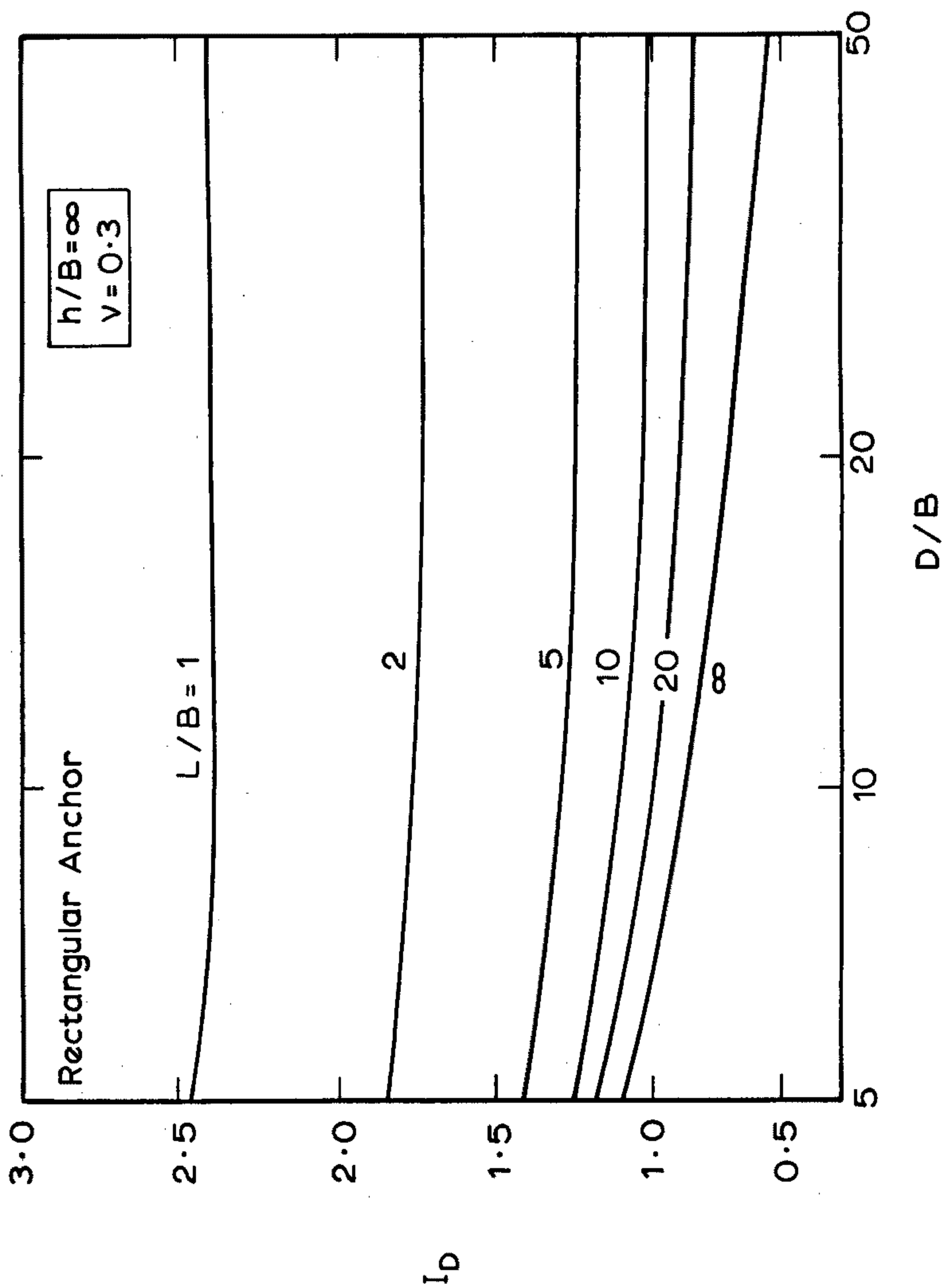


FIG.3.14a INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH DETERMINED FROM THE MEAN DISPLACEMENT OF A RECTANGULAR ANCHOR $v = 0.3$

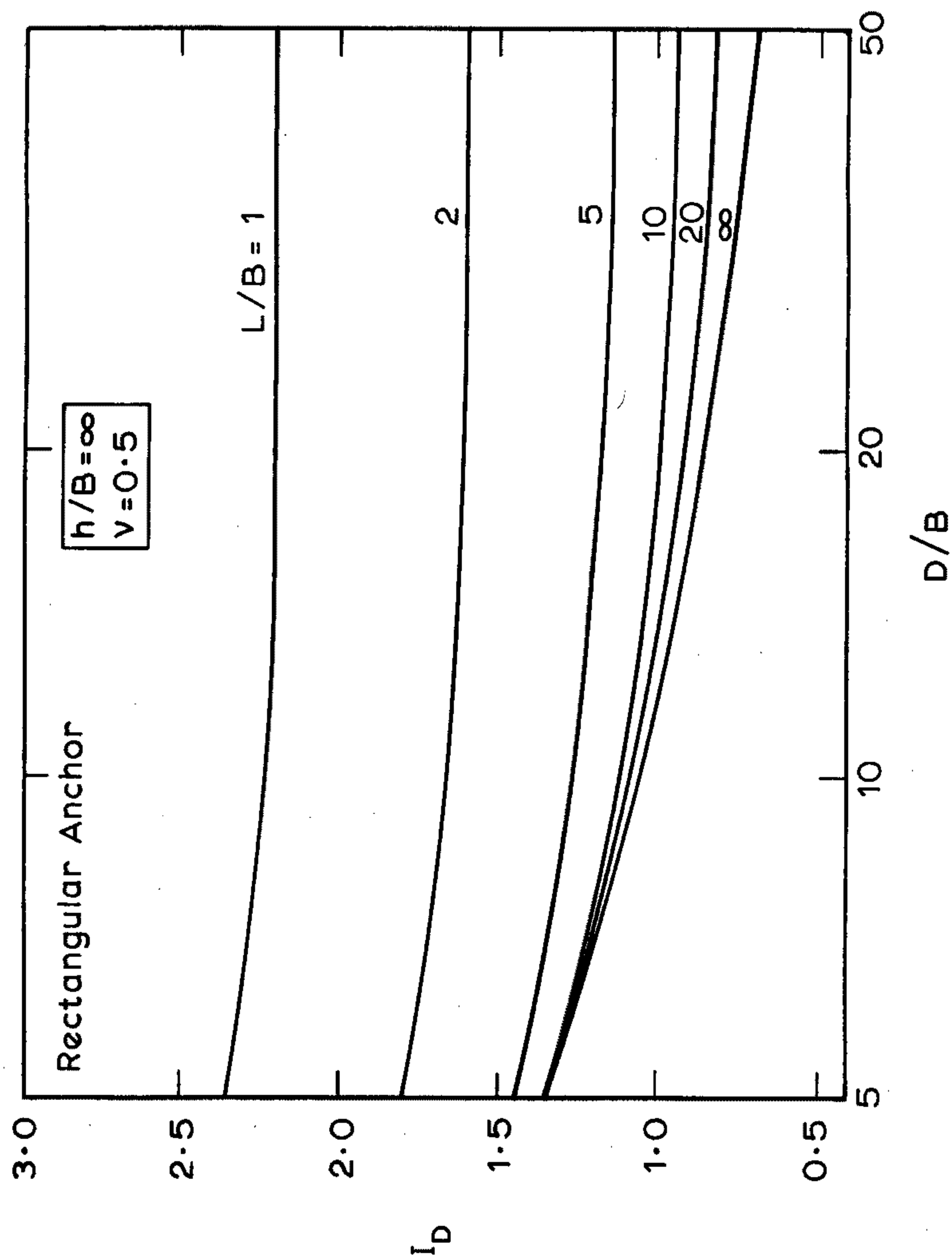


FIG.3.14b INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH
DETERMINED FROM THE MEAN DISPLACEMENT OF A RECTANGULAR
ANCHOR $\nu=0.5$

figures it will be seen that for aspect ratios less than 10 the anchor tends to a limiting apparent stiffness within the range of depth ratios (D/B) considered and, accordingly, the difference between the apparent stiffnesses for different aspect ratios ($L/B \leq 10$) attains a constant value. Furthermore, it will be observed that the difference in the behaviour of strip and rectangular anchors increases with layer depth and that the variation in apparent stiffness with aspect ratio decreases with increasing Poisson's ratio.

The correction factor I_h for a layer with depth $D/B = 20$ is shown in Figs. 3.15a and 3.15b. A number of points emerge from these figures. Firstly, the rate of increase in apparent stiffness with embedment ratio is greatest for small aspect ratios. However, the actual value of the correction factor I_h decreases with aspect ratio for low embedment ratios, and increases with aspect ratio for higher embedment ratios indicating that the relative effect of embedment ratio is greatest for anchors with low aspect ratios.

Secondly, a correction factor similar to I_h may be deduced from Fox's (1948) results for an anchor in an incompressible half space. Comparison of this correction factor with that shown in Fig. 3.15b indicated:

- (1) that for aspect ratio $L/B < 2$, an anchor resting in a layer with $D/B = 20$ behaves almost as if the layer depth were infinite; and
- (2) that the discrepancy between the correction factor I_h obtained by the author for $D/B = 20$ and the similar correction factor deduced from Fox's results for $D/B = \infty$ increases from less than 1% for $L/B = 1$ to a maximum value of 5% for $L/B = \infty$.

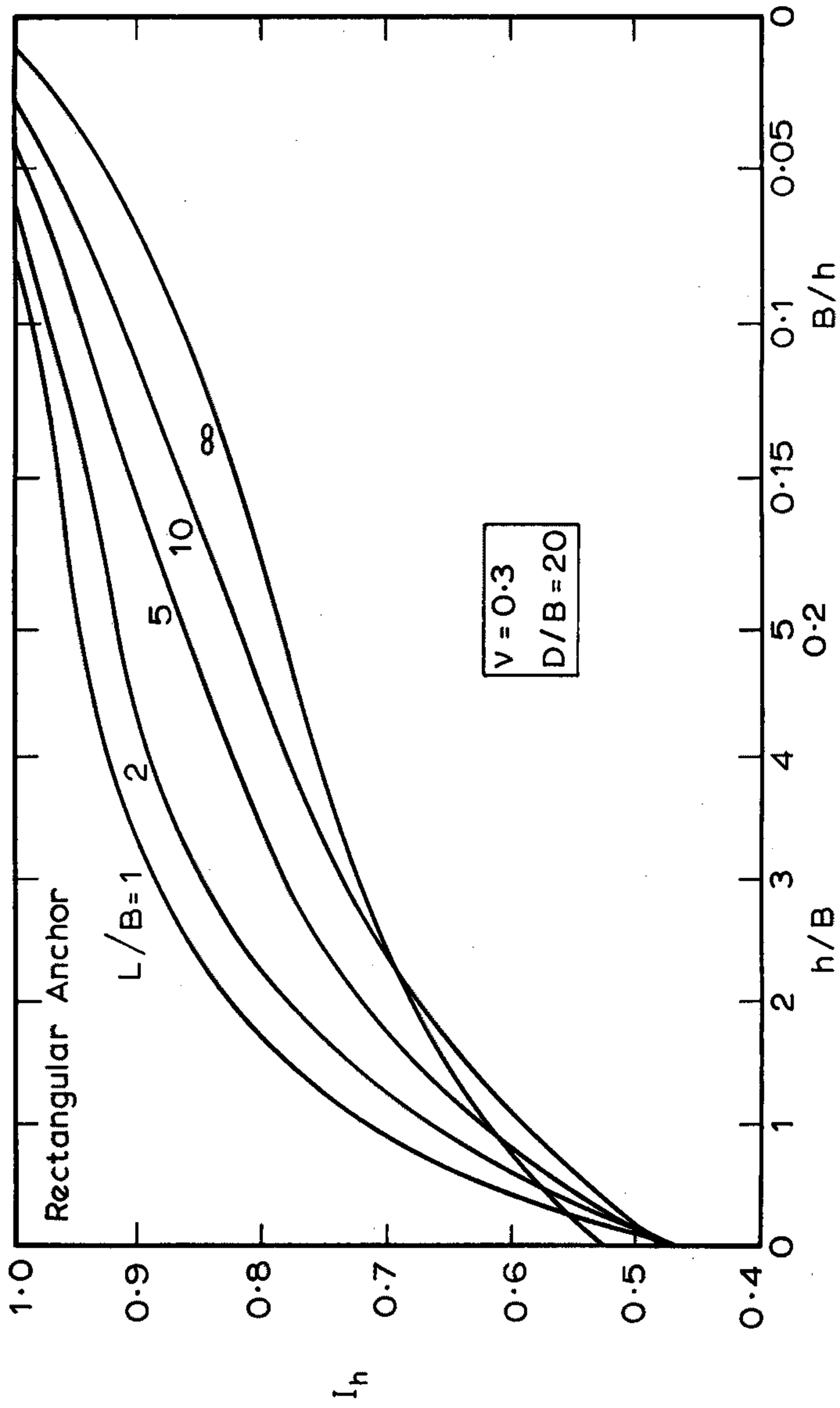


FIG.3.15a CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO

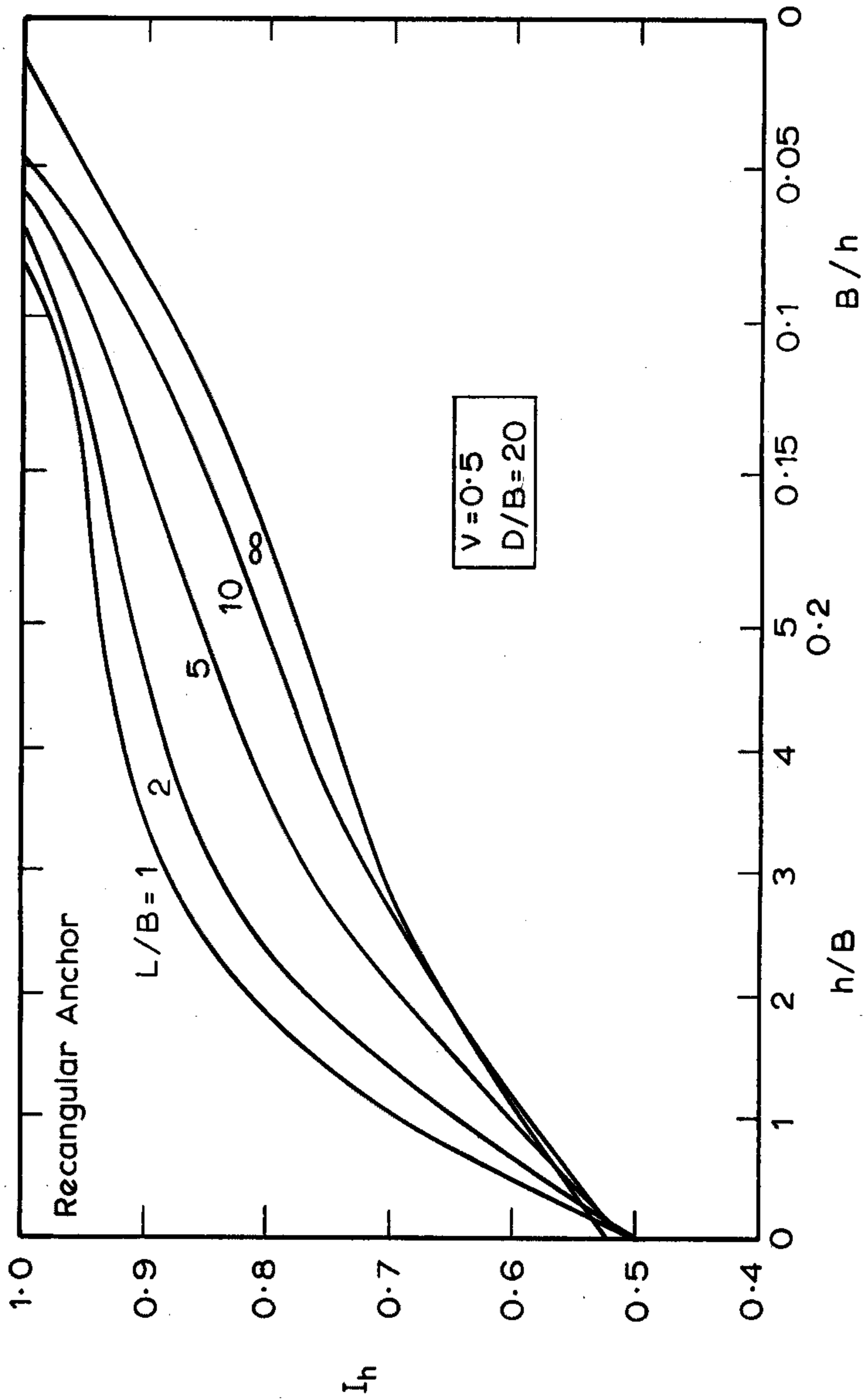


FIG. 3.15b CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO

Finally, it will be observed that the correction factor I_h is relatively insensitive to the value of Poisson's ratio varying by less than 8% between $\nu = 0.3$ and $\nu = 0.5$.

The factors I_D and I_h presented in Figs. 3.14 and 3.15 may be directly applied to predict the apparent stiffness (based on the mean displacement) of anchor plates at infinite embedment ratio for a wide range of aspect ratios and layer depths, while for the particular case where $D/B = 20$ the effect of varying the embedment ratio may also be considered. However, for the limiting cases of circular and strip anchors, it has been found that both the factors I_h and I_D are relatively insensitive to Poisson's ratio. Furthermore, the correction factor I_h is also reasonably insensitive to layer depth D/B . Consequently, the factors I_D and I_h given in Figs. 3.14 and 3.15 may be used to give an approximate prediction of the apparent stiffness of an anchor for a wide range of geometric parameters and values of Poisson's ratio.

To illustrate the application of these influence factors to the prediction of anchor behaviour, a full analysis was performed for a smooth rigid anchor with embedment ratio $h/B = 3$, layer depth $D/B = 8$ and Poisson's ratio $\nu = 0.3$ for a range of aspect ratios and the resulting apparent stiffnesses are compared with the approximate predictions in Fig. 3.16. From this it can be seen that the approximate prediction underestimates the actual apparent stiffness by between 8% and 11% over the range of aspect ratios considered. Direct analyses performed for the otherwise similar case of a flexible anchor indicate that about 6% of this discrepancy is due to the difference between the mean displacement of a flexible footing and the displacement of a rigid

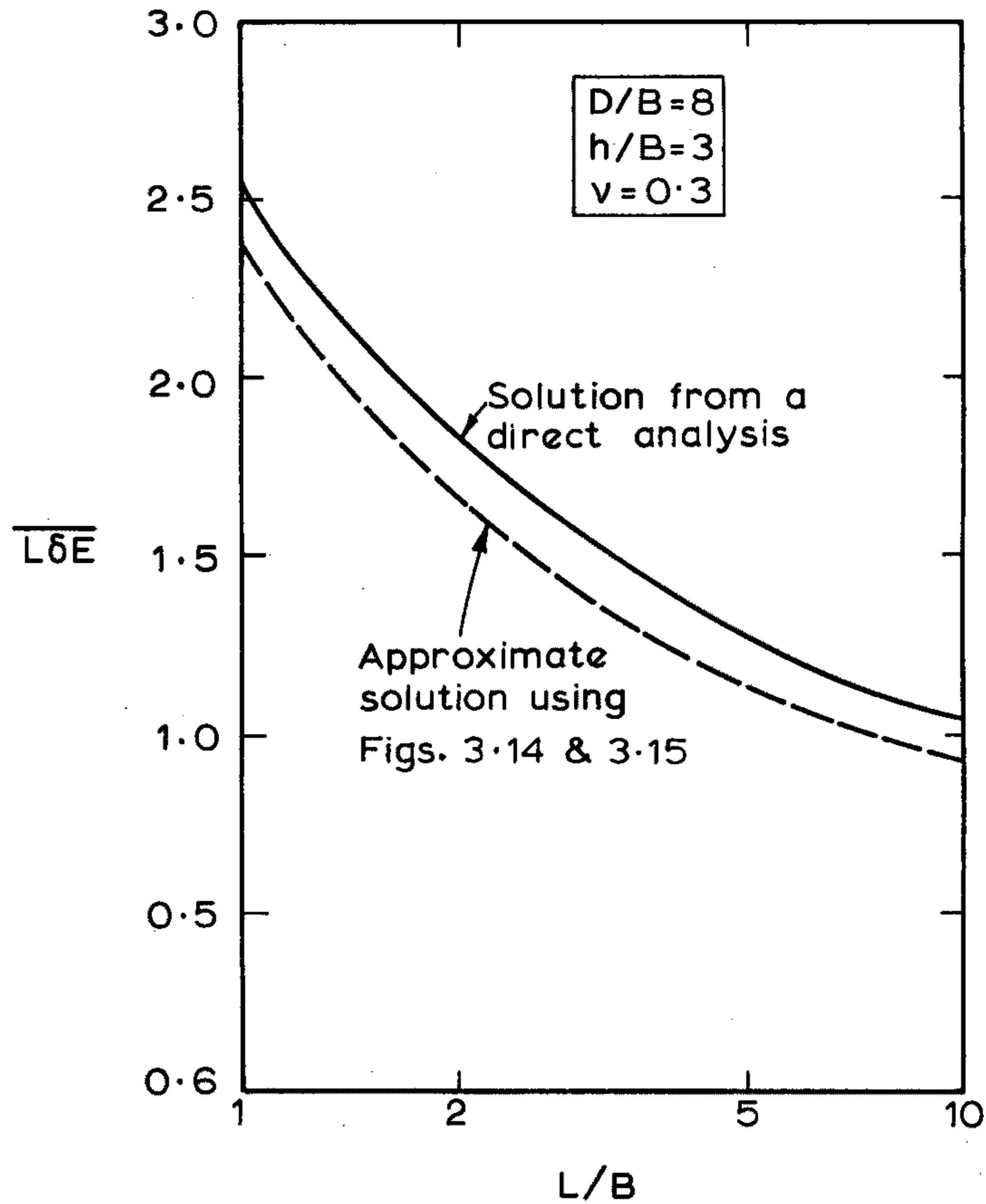


FIG.3.16 APPARENT STIFFNESS FOR RIGID RECTANGULAR ANCHOR.

footing under an equivalent load, while the remaining 2% to 5% discrepancy may be attributed to the effect of layer depth D/B upon the correction factor I_h .

3.4 CONCLUSIONS

In this chapter an analytical technique has been presented for the prediction of the behaviour of horizontal anchor plates embedded in an elastic soil medium. Provision is made in the analysis for the consideration of anchor shape, layer depth, anchor-soil interface condition, breakaway of the anchor from the underlying soil and interaction between groups of anchors.

The approach has been applied to the case of strip, circular and rectangular anchors and solutions have been presented in the form of influence charts which may be used directly in hand calculations to predict the elastic load deflection behaviour of anchor plates for a wide variety of material and geometric conditions.

TABLE 3.1

$M_{11}(z) = \left[\frac{i\alpha}{\gamma} - 2i\alpha\theta z \right] e^{\gamma z}$	$M_{12}(z) = - \left[\frac{i\alpha}{\gamma} + 2i\alpha\theta z \right] e^{-\gamma z}$	$M_{13}(z) = 2i\alpha\gamma e^{\gamma z}$	$M_{14}(z) = -2i\alpha\gamma e^{-\gamma z}$	$M_{15}(z) = \beta\gamma e^{\gamma z}$	$M_{16}(z) = -\beta\gamma e^{-\gamma z}$
$M_{21}(z) = \left[\frac{i\beta}{\gamma} - 2i\beta\theta z \right] e^{\gamma z}$	$M_{22}(z) = - \left[\frac{i\beta}{\gamma} - 2i\beta\theta z \right] e^{-\gamma z}$	$M_{23}(z) = 2i\beta\gamma e^{\gamma z}$	$M_{24}(z) = -2i\beta\gamma e^{-\gamma z}$	$M_{25}(z) = -\alpha\gamma e^{\gamma z}$	$M_{26}(z) = \alpha\gamma e^{-\gamma z}$
$M_{31}(z) = [\lambda + 2G(1 - \gamma\theta z)] e^{\gamma z}$	$M_{32}(z) = [\lambda + 2G(1 + \gamma\theta z)] e^{-\gamma z}$	$M_{33}(z) = 2G\gamma^2 e^{\gamma z}$	$M_{34}(z) = 2G\gamma^2 e^{-\gamma z}$	$M_{35}(z) = 0$	$M_{36}(z) = 0$
$M_{41}(z) = - \frac{i\alpha\theta}{\gamma} e^{\gamma z}$	$M_{42}(z) = i\gamma\theta z e^{-\gamma z} / \gamma$	$M_{43}(z) = i\alpha e^{\gamma z}$	$M_{44}(z) = i\alpha e^{-\gamma z}$	$M_{45}(z) = \beta e^{\gamma z}$	$M_{46}(z) = \beta e^{-\gamma z}$
$M_{51}(z) = - i\beta\theta z e^{\gamma z} / \gamma$	$M_{52}(z) = i\beta\theta z e^{-\gamma z} / \gamma$	$M_{53}(z) = i\beta e^{\gamma z}$	$M_{54}(z) = i\beta e^{-\gamma z}$	$M_{55}(z) = -\alpha e^{\gamma z}$	$M_{56}(z) = -\alpha e^{-\gamma z}$
$M_{61}(z) = (1 + \theta - \gamma\theta z) e^{\gamma z} / \gamma$	$M_{62}(z) = -(1 + \theta + \gamma\theta z) e^{-\gamma z} / \gamma$	$M_{63}(z) = \gamma e^{\gamma z}$	$M_{64}(z) = -\gamma e^{-\gamma z}$	$M_{65}(z) = 0$	$M_{66}(z) = 0$

[M(α, β, z)] =

$$\text{where } \gamma = \sqrt{\alpha^2 + \beta^2} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad G = \frac{E}{2(1+\nu)} \quad \theta = \frac{\lambda+G}{2G} = \frac{1}{2(1-2\nu)}$$

APPENDIX 3A

In this appendix the integral $\phi_j = \frac{1}{A_j} \int_{R_j} e^{i(\alpha x + \beta y)} dA$ will be evaluated for the case of a rectangular sub-region R_j .

Considering a rectangular sub-region with centroidal coordinates $(\bar{x}_j, \bar{y}_j)$ and defined by the limits

$$|x - \bar{x}_j| \leq \frac{1}{2} \Delta x_j ; \quad |y - \bar{y}_j| \leq \frac{1}{2} \Delta y_j , \quad \text{then}$$

$$\begin{aligned} \phi_j &= \frac{1}{\Delta x_j \Delta y_j} \int_{x_j - \frac{\Delta x_j}{2}}^{x_j + \frac{\Delta x_j}{2}} \int_{y_j - \frac{\Delta y_j}{2}}^{y_j + \frac{\Delta y_j}{2}} e^{i(\alpha x + \beta y)} dy dx \\ &= \frac{4e^{i(\alpha \bar{x}_j + \beta \bar{y}_j)} \sin(\alpha \Delta x_j / 2) \sin(\beta \Delta y_j / 2)}{\Delta x_j \Delta y_j \alpha \beta} \end{aligned}$$

APPENDIX 3BWorked Example - Strip Anchor

A) Predict the elastic load-displacement behaviour of a strip anchor of width $B = 1\text{m}$ buried to a depth of 4m in an elastic soil with a total depth 16m and Poisson's ratio $\nu = 0.35$. Assume the base of the soil layer is rough. Basic parameters: $\frac{h}{B} = 4$, $\frac{D}{B} = \frac{16-4}{1} = 12$, $\nu = 0.35$. From Eq. 3.13 the apparent stiffness is given by

$$\frac{P}{\delta E} = \frac{I_D (h/B = \infty) \cdot I_h}{1-\nu^2}$$

Step 1

From Fig. 3.3 for $h/B = \infty$, $D/B = 12$

ν	I_D
0.3	.880
0.4	.892

By interpolation for $\nu = 0.35$ $I_D = .886$

Step 2

Also from Fig. 3.3 the rate of change of stiffness with D/B is given by

$$\frac{I_D (D/B = 12) - I_D (D/B = 8)}{I_D (D/B = 20) - I_D (D/B = 8)} = \frac{.892 - 1.0}{.785 - 1.0} = 0.502$$

for $\nu = 0.3$, $h/B = \infty$.

Step 3

For $h/B = 4$

From Figs. 3.4b, 3.4c the correction factor I_h can be determined for $D/B = 8$ and 20, for $\nu = 0.3$ and 0.4

$\frac{D}{B}$	ν	
	0.3	0.4
8	.835	.838
20	.754	.758

$\therefore$ with $\nu = 0.35$, I_h is estimated by linear interpolation for each layer depth ratio viz:

$\frac{D}{B}$	I_h
8	.836
20	.756

Step 4

Since the variation in stiffness with layer depth D/B is relatively insensitive to both the embedment ratio h/B and Poisson's ratio the influence factor I_h can be estimated for $D/B = 12$ using the rate of change in stiffness with depth ratio D/B obtained in step 2 ie.

$$\frac{I_h(D/B=12) - I_h(D/B=8)}{I_h(D/B=20) - I_h(D/B=8)} = 0.502$$

$$\frac{I_h(D/B=12) - .836}{.756 - .836} \doteq 0.502$$

giving $\therefore I_h = .796$

Step 5

Substitution of I_D , I_h and ν into Eq. 3.13 gives $\frac{P}{\delta E} = \frac{0.886 \times 0.796}{1 - 0.35^2} = 0.804$ which agrees with the analytical value ($\frac{P}{\delta E} = .8038$) obtained from a direct analysis to better than plotting accuracy.

Note:

A more straight-forward, but slightly less accurate approach would be to use a linear interpolation to obtain the value of I_h in step 4 rather than determine the rate of change in stiffness with D/B as in step 2.

$$\text{Thus } \frac{I_h(D/B = 12) - I_h(D/B = 8)}{I_h(D/B = 20) - I_h(D/B = 8)} \doteq \frac{12 - 8}{20 - 8} = 0.3$$

$$\text{i.e. } \therefore \frac{I_h(D/B = 12) - .836}{.756 - .836} = .3$$

and hence $\therefore I_h(D/B) = .809$

$$\text{giving } \frac{P}{\delta E} = \frac{.886 \times .809}{1 - .35^2} = .817$$

which is in error by 1.6%

B) Predict the elastic load deflection behaviour of a strip anchor of width $B = 1\text{m}$ buried to a depth of 4m in an elastic soil with a total depth of 16m . Assume the soil layer has a Poisson's ratio $\nu = 0.35$ and rests on a smooth rigid base.

Step 1

Following the procedure given in part A, the apparent stiffness of the anchor in a soil layer resting on a rough rigid stratum can be determined and is given by

$$\left(\frac{P}{\delta E}\right)_{\text{rough}} = 0.804$$

Step 2

From Fig. 3.6 the apparent stiffness of an anchor with $h/B = \infty$ and embedded in a soil layer with $D/B = 12$, resting on either a rough or smooth base can be determined for $\nu = 0.3$ and $\nu = 0.5$. Hence by interpolation the apparent stiffness can be determined for $\nu = 0.35$

Case	ν		
	0.3	0.5	0.35
Rough	.953	1.344	1.051
Smooth	.922	1.175	0.985

$$\text{Thus } \frac{\left(\frac{P}{\delta E}\right)_{\text{smooth}}}{\left(\frac{P}{\delta E}\right)_{\text{rough}}} \doteq \frac{0.985}{1.051} = 0.937 \text{ for } D/B = 12 \quad \nu = 0.35$$

Step 3

The approximate apparent stiffness of the anchor resting on a smooth base layer is obtained by multiplying the known value for a rough layer by the approximate ratio of stiffness obtained in Step 2

$$\text{viz } \left(\frac{P}{\delta E}\right)_{\text{smooth}} \approx \left(\frac{P}{\delta E}\right)_{\text{rough}} \times 0.937$$

$$\approx 0.804 \times 0.937$$

$$= 0.753$$

which differs from the result $\left(\frac{P}{\delta E} = 0.748\right)$ obtained from a direct analysis by less than 1%.

APPENDIX 3CWorked example - Circular Anchor

Predict the elastic load displacement behaviour of a circular footing of diameter 1m buried to a depth 4m in an elastic soil with a total depth 16m and Poisson's ratio $\nu = 0.4$. Basic parameters $h/B = 4$, $D/B = \frac{16-4}{1} = 12$, $\nu = 0.4$. From Eq.3.14, the apparent stiffness is given by

$$\frac{P}{B\delta E} = \frac{I_D (h/B = \infty) \cdot I_h}{(1 - \nu^2)}$$

Step 1

I_D may be obtained from Fig. 3.11 for $h/B = \infty$, $D/B = 12$

ν	I_D
.3	2.16
.5	2.02

Thus interpolating for $\nu = 0.4$ gives $I_D = 2.09$.

Step 2

I_h may be determined from Fig. 3.12 for $h/B = 4$, thus

v	D/B	
	8	20
.3	.948	.925
.5	.945	.918

Interpolated for $v = 0.4$ for each value of D/B gives

D/B	I_h
8	0.946
20	0.922

and then linear interpolation for $D/B = 12$ gives $I_h = 0.938$.

Step 3

Substitute I_D , I_h , v into Eq. 3.14 to obtain

$$\frac{P}{B\delta E} = \frac{2.09 \times 0.938}{1 - .4^2} = 2.334$$

which can be compared with the directly computed value of $\frac{P}{B\delta E} = 2.307$ indicating an error of 1.1%.

CHAPTER FOUR
PLASTICITY THEORY AND
ITS APPLICATION

4.1 INTRODUCTION

Classical plasticity theory (e.g. see Hill, 1950; Prager and Hodge, 1951; Davis, 1968) provides a mathematically rigorous means of performing stability analyses subject to the assumptions that the resulting collapse load is independent of the load path to collapse, and that the material behaviour can be approximated by a rigid - plastic model. However, it is extremely difficult (or impossible) to apply classical plasticity theory to many engineering problems, and simplifying assumptions have frequently been introduced to make the analysis tractable. These simplifying assumptions often violate some condition of the problem; consequently, the analysis will be invalid from the standpoint of the mathematical theory of plasticity, although in many cases the solutions obtained may have been quite successfully adopted in practice. Accordingly, in this chapter the fundamental features of the theory of plasticity will be outlined, and consideration will be given to the difficulties involved in utilising the theory to predict collapse loads for all but the most straightforward of problems. In particular, attention will be given to the role of plasticity theory in the prediction of the ultimate behaviour of anchor plates.

The plasticity equations for a rigid plastic material are given in standard texts (e.g. Hill, 1950; Davis, 1968; Chen, 1975) and will not be restated here.

4.2 PLASTICITY THEORY

4.2.1 Background to the Mathematical Theory of Plasticity

The earliest attempts to analyse the behaviour of plastic materials were made by Coulomb (1773) and Rankine (1857) in regard to earth pressure calculations. However, the development of the theory of plasticity was relatively slow and although some quite significant advances were made prior to 1930 (e.g. Kötter, 1888, 1903; Harr and von Karman, 1908; von Mises, 1913; Prandtl, 1920, 1924; Hencky, 1924) it was not until the interdependence between the stress and velocity fields was recognised (Geiringer, 1930) that plasticity theory began to take its present form. Subsequent theoretical developments in the mathematical theory of plasticity were primarily directed at the behaviour of 'purely cohesive' materials (e.g. metals) and culminated in the publication of Hill's book (1950).

The proof of the extremum and variational principles by Hill was followed by the development of the limit theorems by Drucker et al., (1951, 1952). These theorems are applicable to the general class of materials with an associated flow rule, and enable upper and lower bounds to be placed on the load at which collapse occurs (even if the true collapse load cannot be obtained). The proof of these theorems greatly enhanced the usefulness of classical plasticity theory, particularly with regard to soil mechanics.

4.2.2 Basic Requirements of Plasticity Theory

In the terminology of classical plasticity theory, a stati-

cally admissible stress field is a stress field which:

- . Is in equilibrium with the applied tractions;
- . Everywhere satisfies the equation of equilibrium; and
- . Nowhere exceeds the strength:

while a kinematically admissible velocity field (or plastic strain rate field) is a velocity field which:

- . Satisfies the boundary velocity conditions;
- . Satisfies the plastic strain rate equations which describe the volumetric behaviour of the material during plastic deformation; and
- . Gives rise to boundary velocities such that the rate of plastic work is always positive.

Accordingly, from the standpoint of plasticity theory, a complete solution to a problem must satisfy the following requirements:

- (1) A statically admissible stress field must be found for any plastic regions within the loaded mass;
- (2) The stress field must be capable of extension into the region outside the plastic region without violating the yield criterion; and
- (3) A kinematically admissible velocity field must be found.

The limit theorems of Drucker et al., (1952) apply to both a cohesive or cohesive-frictional material, with an associated flow rule (i.e. where the dilation of the material during plastic deformation is a unique function of the angle of internal friction ϕ).

The theorems may be briefly restated by observing that:

- . a stress field which satisfies conditions (1) and (2) above gives a lower bound to the true collapse load; and
- . a kinematically admissible velocity field (condition (3) above) gives an upper bound to the true collapse load.

Thus for an associated material, the collapse load obtained from any complete solution (ie. a solution which satisfies the requirements of both an upper and lower bound) is unique and does not depend upon the initial stress state or load path to failure.

Exact solutions to some soil mechanics problems may be obtained by developing stress and velocity fields which satisfy the necessary conditions for completeness. However, in some cases it may be more convenient to derive a simple statically admissible solution and a kinematically admissible solution as a lower and upper bound respectively.

The development of statically admissible lower bounds may be facilitated by the postulation of a stress discontinuity either between a plastic and non-plastic stress field, or between two plastic fields. The normal and shear stress across a stress discontinuity must be in equilibrium, however there may be a change in principal stress direction (and magnitude).

A simple, but often useful, way of obtaining statically admissible solutions for problems in which the unit weight $\gamma = 0$ and the angle of friction $\phi = 0$ is to subdivide the mass into fields of constant stress separated by straight line stress discontinuities. The geometric

properties of stress discontinuities between fields of constant stress have been investigated by Winzer and Carrier (1948).

Similarly, the development of kinematically admissible upper bounds may be facilitated by the introduction of velocity discontinuities and the relevant equations are given by Davis (1968).

Many investigators have assumed an associated flow rule in the analysis of soil stability or bearing capacity problems. However, experimental evidence indicates that for a frictional material an associated flow rule significantly overestimates the rate of volumetric plastic strain. In an attempt to overcome this difficulty, a number of investigators (e.g. Hansen, 1958; Mroz, 1963; Davis, 1968) have independently postulated a class of material which obeys a non-associated flow rule. In many cases, this idealisation provides a more realistic representation of soil behaviour, although this advantage is partially offset by the fact that the normal limit theorems do not hold for a non-associated material (Davis, 1968). However, a very much weaker upper bound can be invoked for a non-associated material; that is, it can be shown that a kinematically admissible collapse load (and in particular, the true solution) for an associated material with an angle of internal friction ϕ , is an upper bound to the collapse load for a non-associated material with the same angle of friction.

4.2.3 Approximate Approaches

Closed-form analytical solutions to soil mechanics problems have been obtained for a number of situations where the soil may be assumed to be weightless (e.g. Prandtl, 1920; Reissner, 1924; Cox et

al., 1961). However, the important inclusion of soil weight considerably complicates the mathematical solution. Consequently, many approximate methods have been developed for the analysis of the bearing capacity or stability of a ponderable soil. The level of approximation involved may vary quite significantly between methods. For example, Davis and Booker (1971) and Booker (1970) used a numerical technique for the integration of the equation of plasticity to obtain an otherwise rigorous analysis of the bearing capacity of footings. However, many other approaches, such as the method of characteristics (often called the slip line method) and limit-equilibrium methods, are far less rigorous. These two approaches will now be considered briefly.

The method of characteristics (ie. slip line method) has received considerable attention in recent years (e.g. Sokolovskii, 1965). In this approach fields of characteristic lines are developed within a plastic region from the known boundary conditions. The resulting stress field is statically admissible and hence the first condition for a complete solution is satisfied. However, in general no attempt is made to show that the stress field can be extended without violating the yield criteria. In many cases this omission arises because it is simply too difficult to show that the field is extensible. The solution obtained in this approach is often quite adequate for engineering purposes, although it should be recognised that strictly speaking the solution is not necessarily a true lower bound. Similarly, no consideration is given to the kinematic behaviour of the soil mass, and hence the solution obtained from the stress characteristics is not an upper bound.

So called 'limit equilibrium methods' have been used to obtain approximate solutions for many stability problems in soil mechanics (Terzaghi, 1943; Taylor, 1937; Bishop, 1955). In these analyses, it is usual to make an assumption about the shape of the failure surface in the soil mass and then analyse the statics of the surface. (To do so usually requires further arbitrary assumptions regarding the stresses or forces involved.) In general, these solutions do not satisfy the requirements of a true lower bound solution. Furthermore, the solutions may not even satisfy the requirements of a true upper bound, although experience would indicate that this approach tends to give an unconservative estimate of the collapse load. (For example see Davis and Booker, 1971).

Recent advances in computer technology have been accompanied by the development of a number of new methods of performing stability analyses using plasticity theory. In each case, the continuum is subdivided into a number of discrete regions or elements to facilitate the analysis, however the actual methods of analysis are quite distinct.

The first of these methods was developed from an extension of 'The Finite Element Method' (Clough, 1960; Zienkiewicz and Chung, 1967; Zienkiewicz, 1971; Desai and Abel, 1972) to incorporate the effects of failure within the continuum. Initially, provision for failure was incorporated by reducing the elastic (or shear) modulus of the soil once the stress within an element touched the failure surface. However, this bi-linear approach is not rigorous from a plasticity standpoint and under certain circumstances may lead to significant error (Davis and Booker, 1977). Accordingly, attention has been directed to the incorporation of a more rigorous idealisation of soil behaviour into the finite element

approach. In particular, incremental equations have been developed from the constitutive laws for a simple elasto-plastic material, and an approximate solution to these equations is obtained using finite elements. Thus stability analyses may be performed for an elasto-plastic material by following an incremental procedure whereby the entire history of loading is examined. The principal advocates of this type of approach include Argyris (1965), Pope (1965), Marcel and King (1967), Yamada and his co-workers (1968) and Zienkiewicz and his co-workers (1969).

An elasto-plastic finite element analysis may be considered to be rigorous in the sense that equilibrium, failure criteria and the stress-strain behaviour of the soil are all incorporated into the general incremental equations and hence one might expect that the solutions obtained from these analyses would satisfy all three requirements for a complete solution. Unfortunately, because of the approximation involved in idealising the continuum as a number of discrete elements, the solution itself is only an approximate solution and even for a material with an associated flow rule, may not be regarded as either a true upper or lower bound.

For problems with known analytical solutions, the finite element approach often (although not necessarily!) gives collapse loads above the true solution. Despite this disadvantage, the finite element approach to stability analysis is becoming increasingly popular, because of its versatility in terms of both geometrical and material considerations. The approach has major advantages over approximate techniques such as stress characteristic and equilibrium methods in that both the static and kinematic behaviour are considered without the need for making

arbitrary assumptions about the extent of plastic failure or the shape of the failure surface. Details regarding the development and application of the finite element method for predicting non-linear behaviour of soil will be given in subsequent chapters.

In this thesis, it will be shown that in many circumstances the load deflection behaviour prior to collapse may be of considerable significance; however if only the collapse load itself is required then the adoption of a load path technique may involve too much computation. Alternative approaches have been suggested which avoid the load path procedure. In these analyses, the problems of direct limit analysis are reduced to linear or non-linear programming problems. A number of approaches have been developed along these lines (e.g. Ranaweeran and Leckie, 1970; Faccioli and Vitiello, 1973; De Donato and Franchi, 1973) however, in this section attention will be restricted to the approaches of Lysmer (1970) and Hung (1976).

Lysmer (1970) has suggested a method of obtaining lower bounds in which linear programming is used to optimise the choice of statically admissible stress-fields. This approach is attractive in that it provides a convenient means of obtaining a statically admissible stress field, however to show that this solution is a true lower bound requires proof that the material is not overstressed; it is often extremely difficult to obtain this proof. Furthermore, because the Mohr-Coulomb failure criteria must be approximated by a set of simultaneous linear equations for each region (ie. element), the method is expensive (in terms of computational costs) when compared to the cost of a non-linear finite element analysis of similar accuracy.

Finally, Hung (1976) has presented an approach for direct limit analysis using rigid plastic finite elements. In this approach, the theoretical non-linear problem is converted to an equivalent linear elastic problem and an iterative procedure is followed until the collapse load is found. To date, application of the approach has been restricted to the analysis of plates and shells composed of a material with a von Mises yield criteria, although it is claimed that the approach may be extended to Mohr-Coulomb materials.

4.3 PREDICTION OF COLLAPSE LOADS FOR ANCHOR PLATES

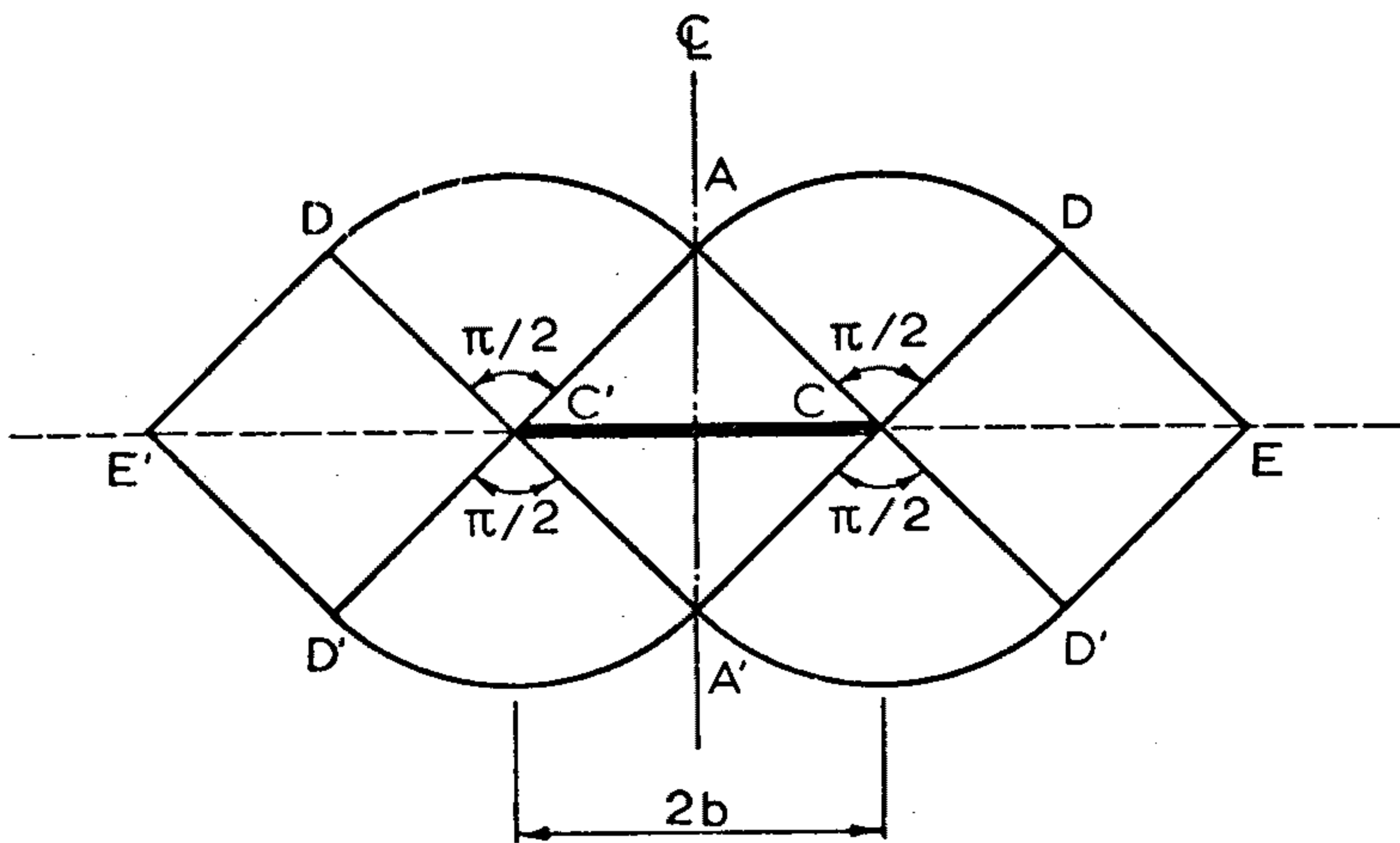
All of the existing theoretical predictions for the bearing capacities of anchor plates (see Chapter 2 for details) involve some form of plasticity; however, none of the proposed solutions can be regarded as complete, and in most cases they do not even constitute a true upper or lower bound. This situation raises the question as to why rigorous solutions (or even a bracketing of the collapse loads) have not been obtained using classical plasticity theory. Hence in this section, consideration will be given to the use of classical plasticity in the prediction of collapse loads for the special case of a strip anchor resting in a purely cohesive material.

The development of stress and velocity fields using the method of characteristics to solve the relevant plasticity equations is generally not possible because of the lack of boundary conditions from which the stress field can be developed. Consequently, with the possible exception of an anchor at infinite depth, it does not appear to be possible to obtain an exact solution for an anchor in even the simplest of materials (ie. purely cohesive). The application of clas-

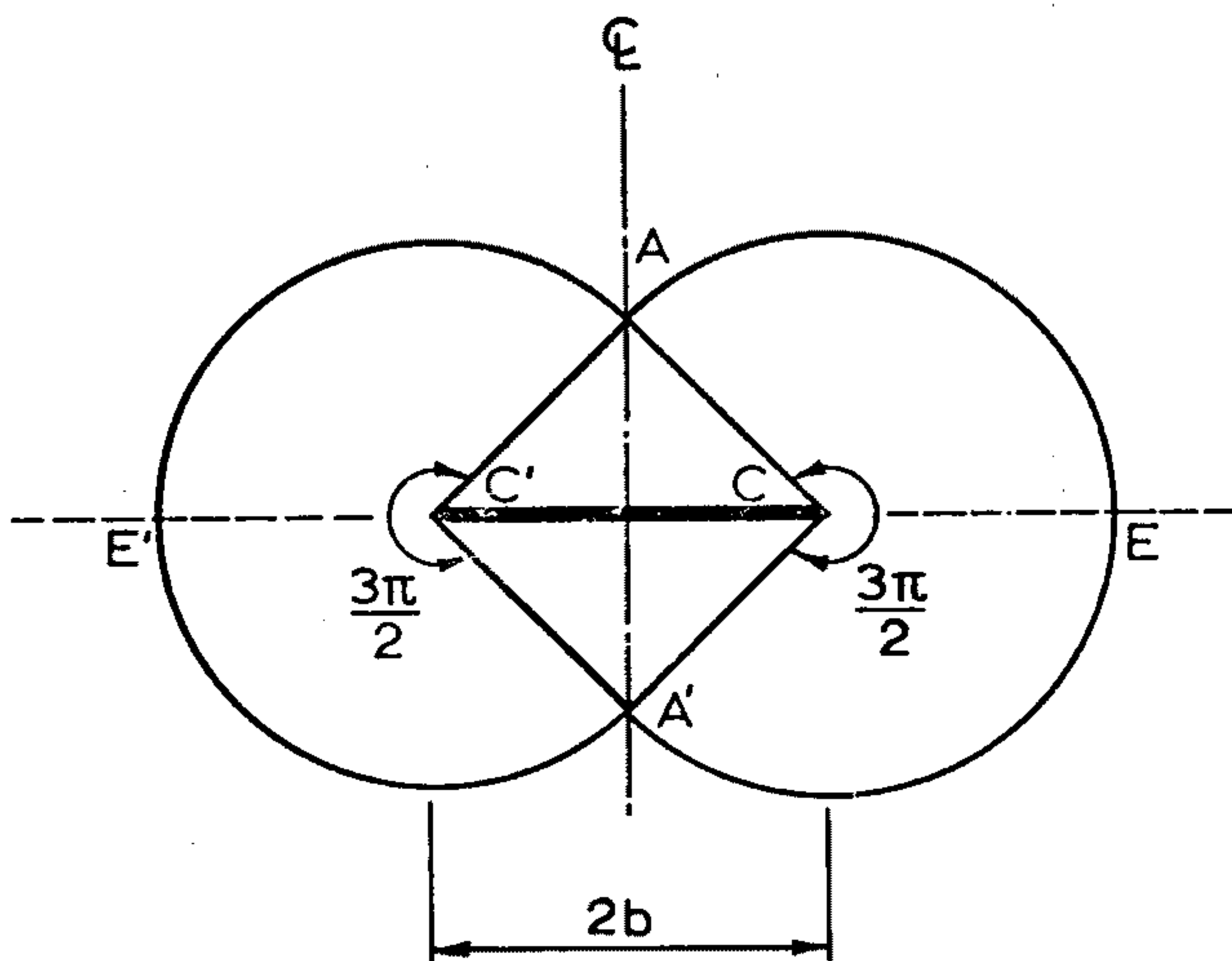
sical plasticity theory is therefore restricted to obtaining upper and lower bound solutions to the collapse loads for anchors in a material with an associated flow rule.

Considering a strip anchor of width B at infinite depth in an ideal rigid plastic material with a cohesion c , then upper and lower bounds may be obtained as outlined below;

- (1) In Fig. 4.1(i) the well known Prandtl (1920) solution for a footing is used to obtain a lower bound. The solution is anti-symmetric about the discontinuity EE' , being tensile on one side and compressive on the other. Thus, the net difference in vertical stress between the top and bottom of the anchor yields a statically admissible limit load equal to $2(2 + \pi) Bc = 10.28 Bc$.
- (2) Fig. 4.1(ii) shows an upper bound solution in which the material in regions $AC'C$ and $A'CC'$ moves vertically with uniform velocity while the material in the regions AEA' and $AE'A'$ distorts and rotates about C and C' respectively. The corresponding stress distribution is also anti-symmetric about the line EE' and can easily be shown to give a kinematically admissible limit load equal to $(2 + 3\pi) Bc = 11.42 Bc$. The stress distribution associated with this velocity field is statically admissible and so this solution satisfies requirements (1) and (3) for a complete solution (given in section 4.2.2). This upperbound is similar to the solution proposed by Meyerhof (1951) for deep footings. If it could be shown that this stress field can be extended into the rigid material without exceeding the material



(i) Lower Bound LB1



(ii) Upper Bound UB1

FIG.4.1 UPPER AND LOWER BOUNDS FOR ANCHOR AT INFINITE DEPTH IN A PURELY COHESIVE SOIL

strength then this would be an exact solution. As yet, no such extension field has been devised.

It can be shown that provided the soil can sustain tension ternately, provided the overburden pressure is sufficient to prevent the occurrence of tension) both these upper and lower bounds are independent of the overburden pressure. This result follows directly from the Hencky and Geiringer equations (see Hill, 1950; Prager and Hodge, 1951) and means that for a rigid plastic material the occurrence of 'breakaway' (ie. where the vertical stress below or behind the anchor reaches zero and consequently, separation between the soil and anchor may occur) does not alter either the upper or lower bound, and by inference would not alter the actual collapse load.

For anchors at finite depth it may be anticipated that the boundary condition behind the anchor would influence the bearing capacity. In particular, two limiting cases may be considered, namely;

- (1) that where there is immediate breakaway (ie. the normal and tangential tractions on the soil surface behind the anchor are both zero and hence a gap may form between the anchor and soil. This situation corresponds to a weightless material)
- (2) that where there is no breakaway (ie. the soil below the anchor is always in contact with the anchor plate. This situation corresponds to the case where the overburden pressure is very large with respect to the soil cohesion).

Considering firstly the case where the vertical stress below

a horizontal anchor is zero (ie. immediate breakaway is possible) then lower bounds may be developed along the lines proposed by Davis (1968) for the solution to Terzaghi's trapdoor problem. For example, Figs. 4.2(a) and 4.2(b) show two types of lower bound solutions involving straight line stress discontinuities and fields of constant stress. In Fig. 4.2(a) field 1 is non-plastic while fields 2, 3 and 4 are all plastic. From consideration of equilibrium and geometry for this particular arrangement of discontinuities it can be easily shown that the corresponding average pressure q applied to the anchor plate is given by

$$q/c = 1 - 2 \cos 2\alpha + \cos (2\beta - 4\alpha) + \sin (2\beta - 4\alpha) \tan \beta \quad (4.1)$$

This bearing pressure, q/c , may be maximised with respect to α for a given $\beta = \tan^{-1} (2h/B)$ giving an optimum relationship between the two parameters viz;

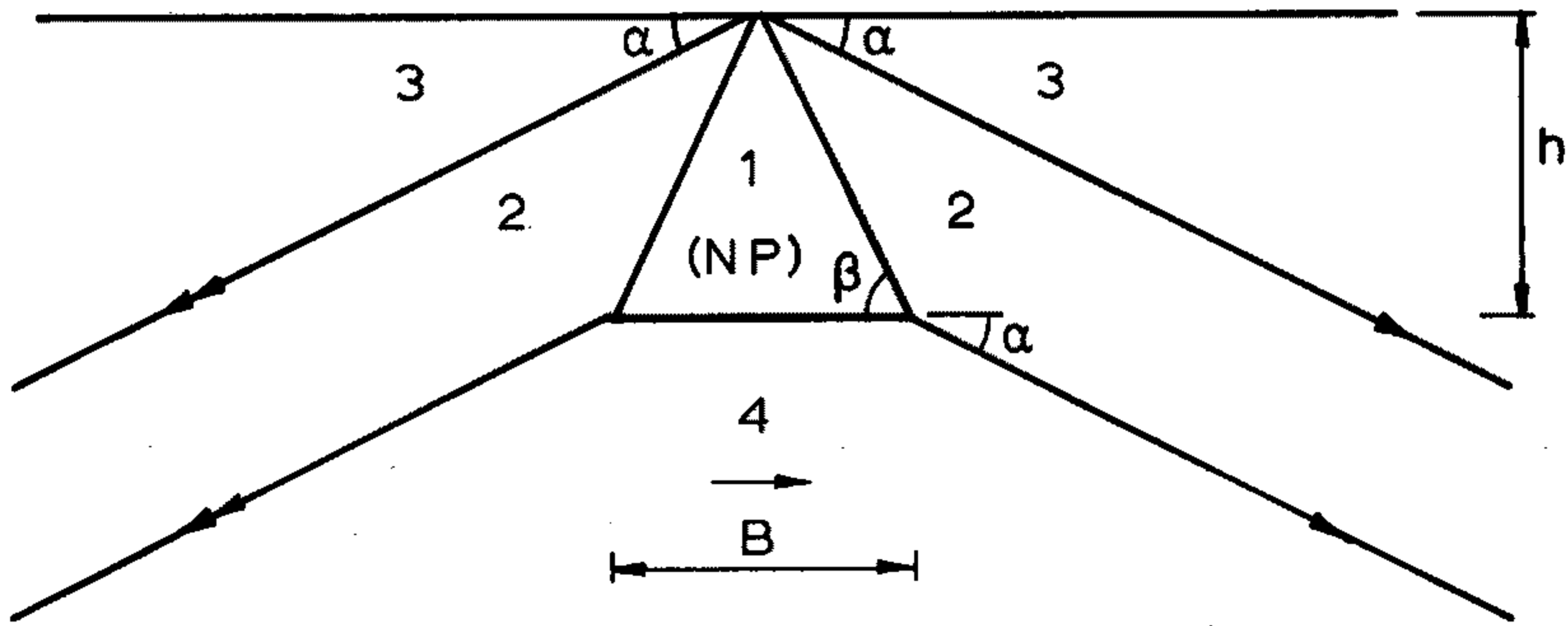
$$\tan \beta = (\sin 4\alpha - \sin 2\alpha) \sec 4\alpha \quad (4.2)$$

Similarly, in Fig. 4.2(b) fields 1, 2, 3, 4, 6 are all plastic while field 5 is not plastic. Again from the consideration of geometry, equilibrium and the allowable change in principal stress direction across a discontinuity it can be shown that the average pressure q applied to the anchor plate is given by

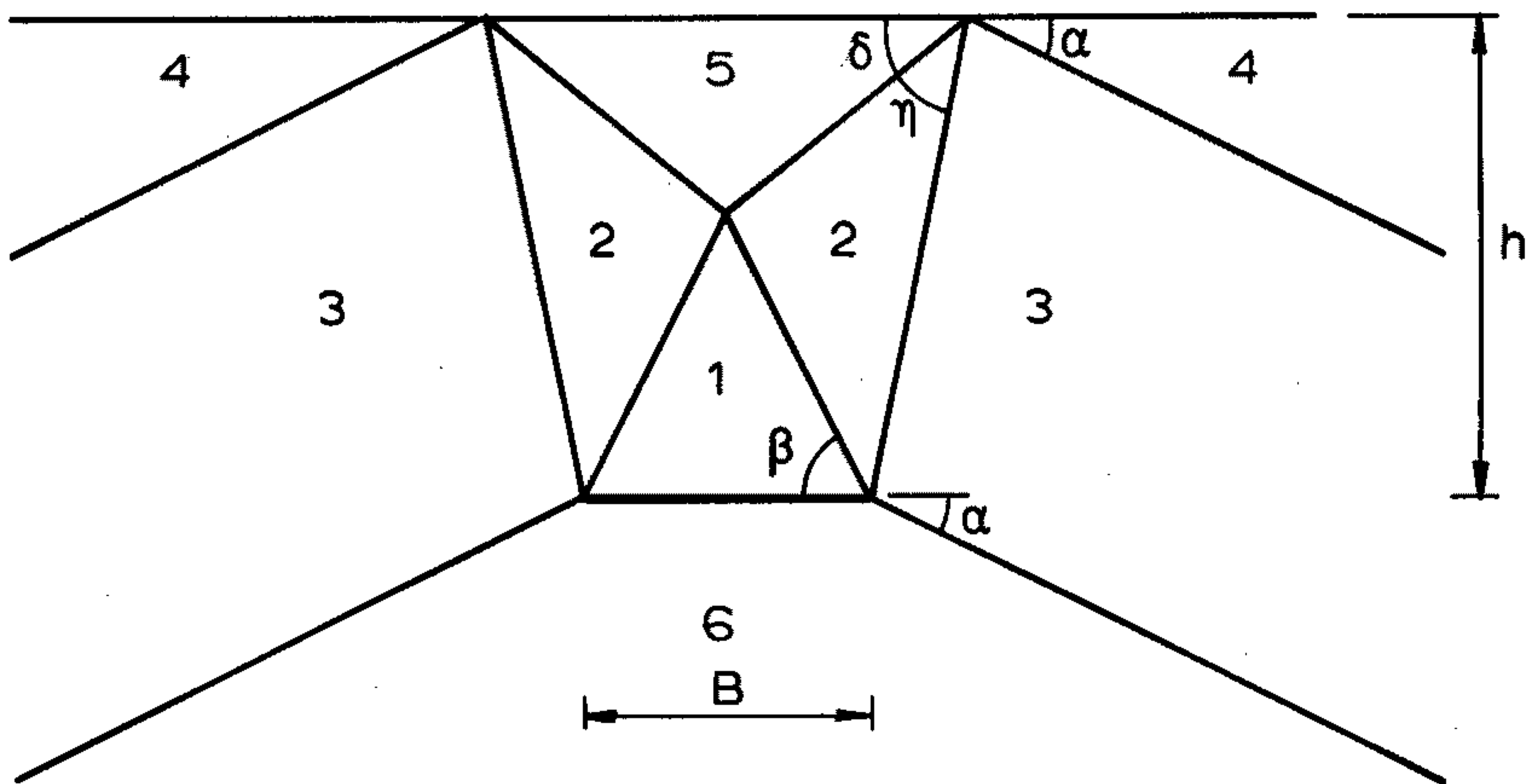
$$q/c = 2 - 2 \cos 2\alpha + 2 \cos (2\alpha - 2\beta) - 2 \cos 2\beta \quad (4.3)$$

where

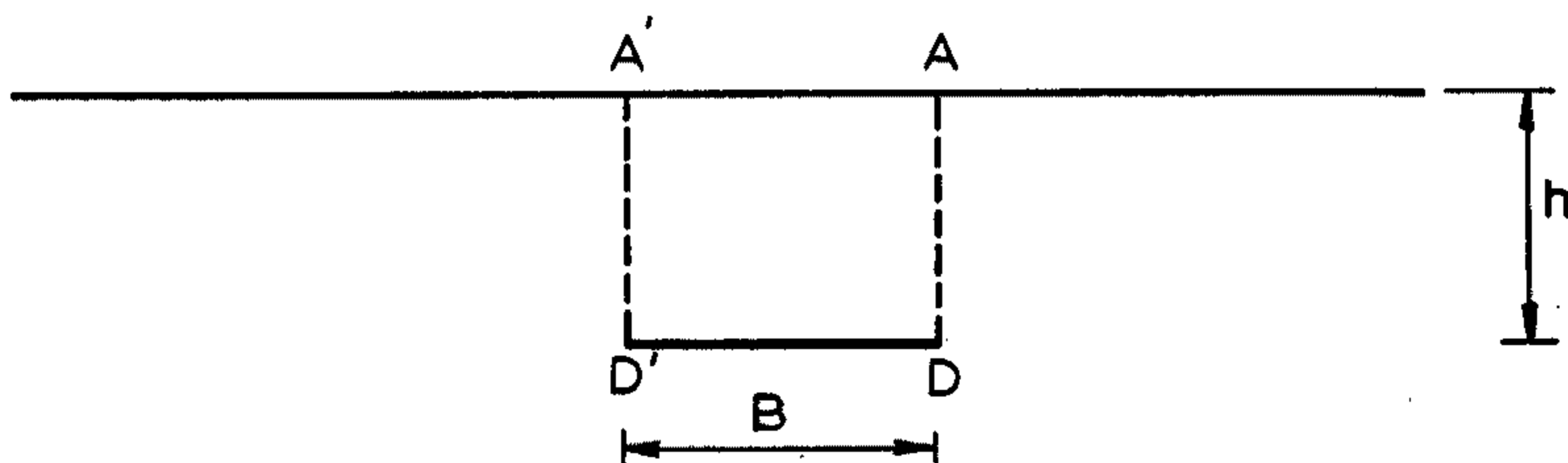
$$h/B = \frac{1}{2}(\sin \alpha \cos \alpha + \cos^2 \alpha \tan \beta + \frac{\sin^2 \alpha}{\tan \eta} + \frac{\sin \alpha \cos \alpha \tan \beta}{\tan \eta}) \quad (4.4)$$



(a) Lower Bound LB2



(b) Lower Bound LB3



(c) Lower Bound UB2

FIG. 4.2 LOWER & UPPER BOUNDS FOR A SHALLOW ANCHOR (IMMEDIATE BREAKAWAY)

and where the condition of zero vertical stress and a mobilised shear stress less than c in region 5 are enforced by the conditions that

$$\tan \delta = \frac{-1 + 2 \cos 2\alpha - 2 \cos (2\alpha - 2\beta) + \cos 4\beta}{\sin 4\beta} \quad (4.5)$$

and $|\sin 4\beta \cot 2\delta + \cos 4\beta| \leq 1 \quad (4.6)$

Both of these solutions can be shown to be statically admissible and clearly extend to infinity without overstressing the materials.

The simple upper bound solution shown in Fig. 4.2(c) which involves rigid upward movement of the material in the region ADD'A', is kinematically admissible and it can be shown that this corresponds to a limiting pressure of

$$q/c = 2 h/B \quad (4.7)$$

The lower bound solutions given by Eqs. 4.1 and 4.3 as well as the upper bound solution of Eq. 4.7 are plotted against embedment ratio h/B in Fig. 4.3. The two lower bound solutions and the upper bound solution bracket the true answer quite well over the range $0 \leq h/B \leq 2.5$, however for higher embedment ratios the best upper and lower bound diverge rapidly.

For the case where the anchor is fully bonded to the underlying soil, the lower bounds shown in Figs. 4.2(a) and 4.2(b) are still applicable. The upper bound solution shown in Fig. 4.2(c) is not kinematically admissible for the fully bonded case. Fig. 4.4(b) shows an

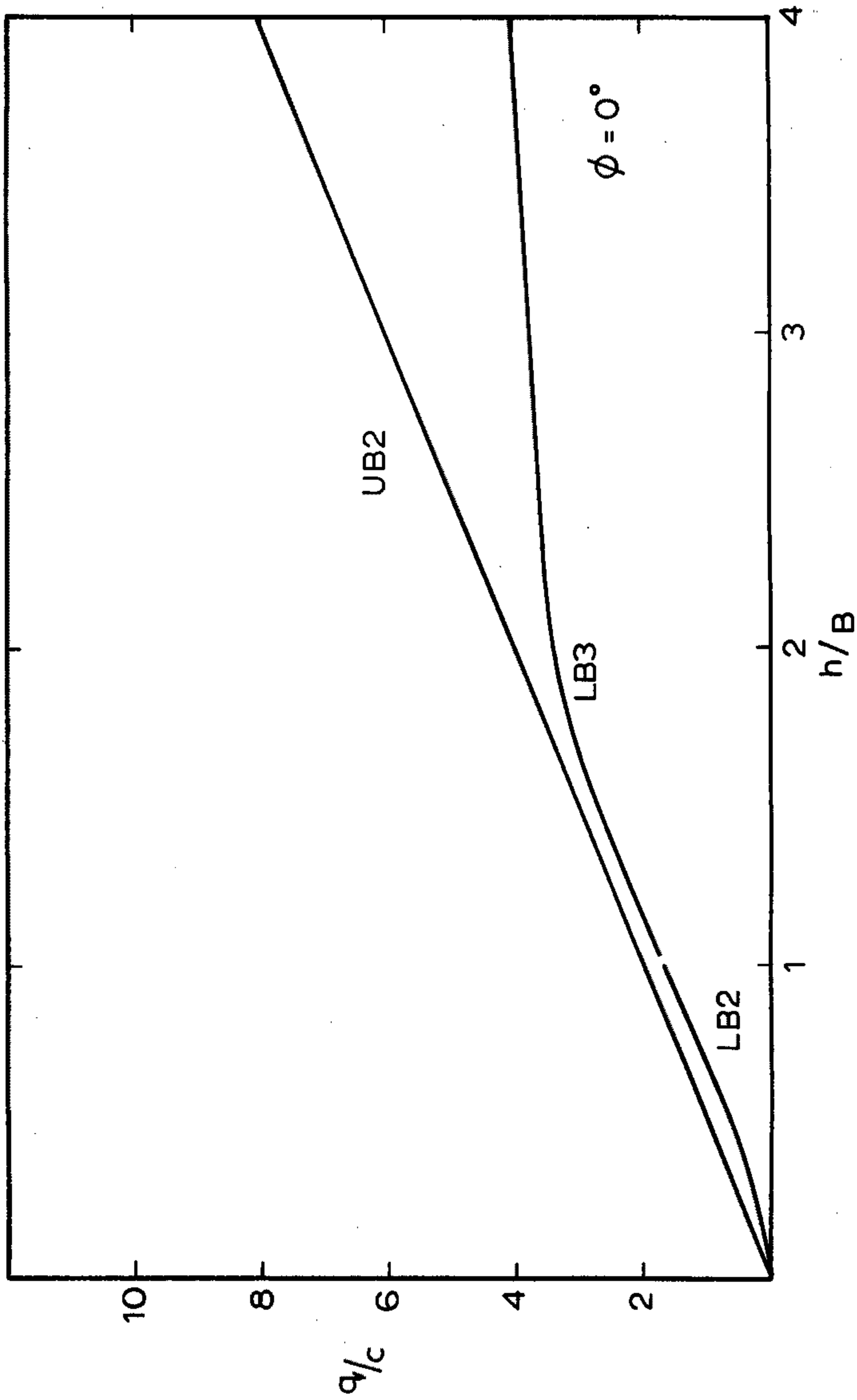
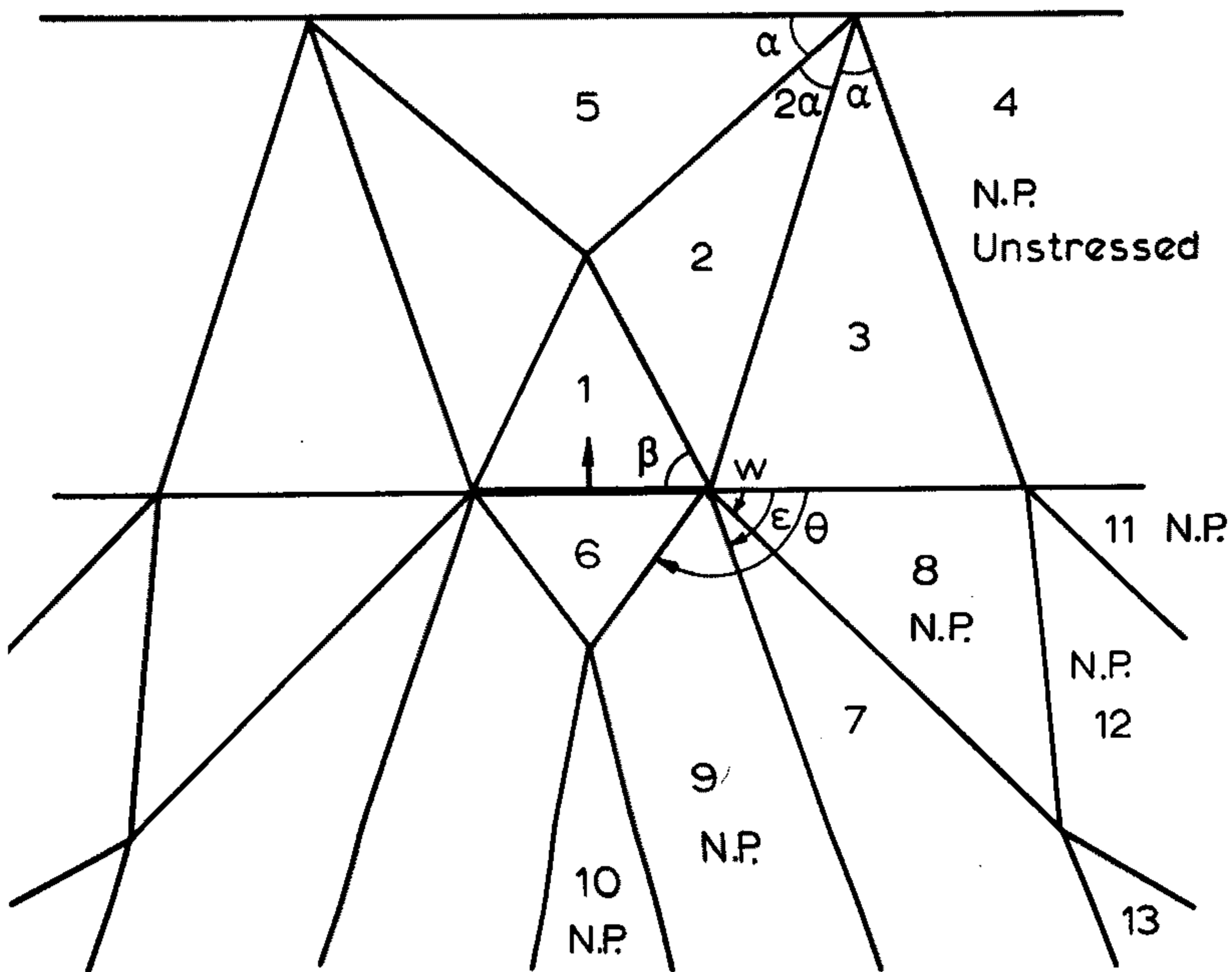
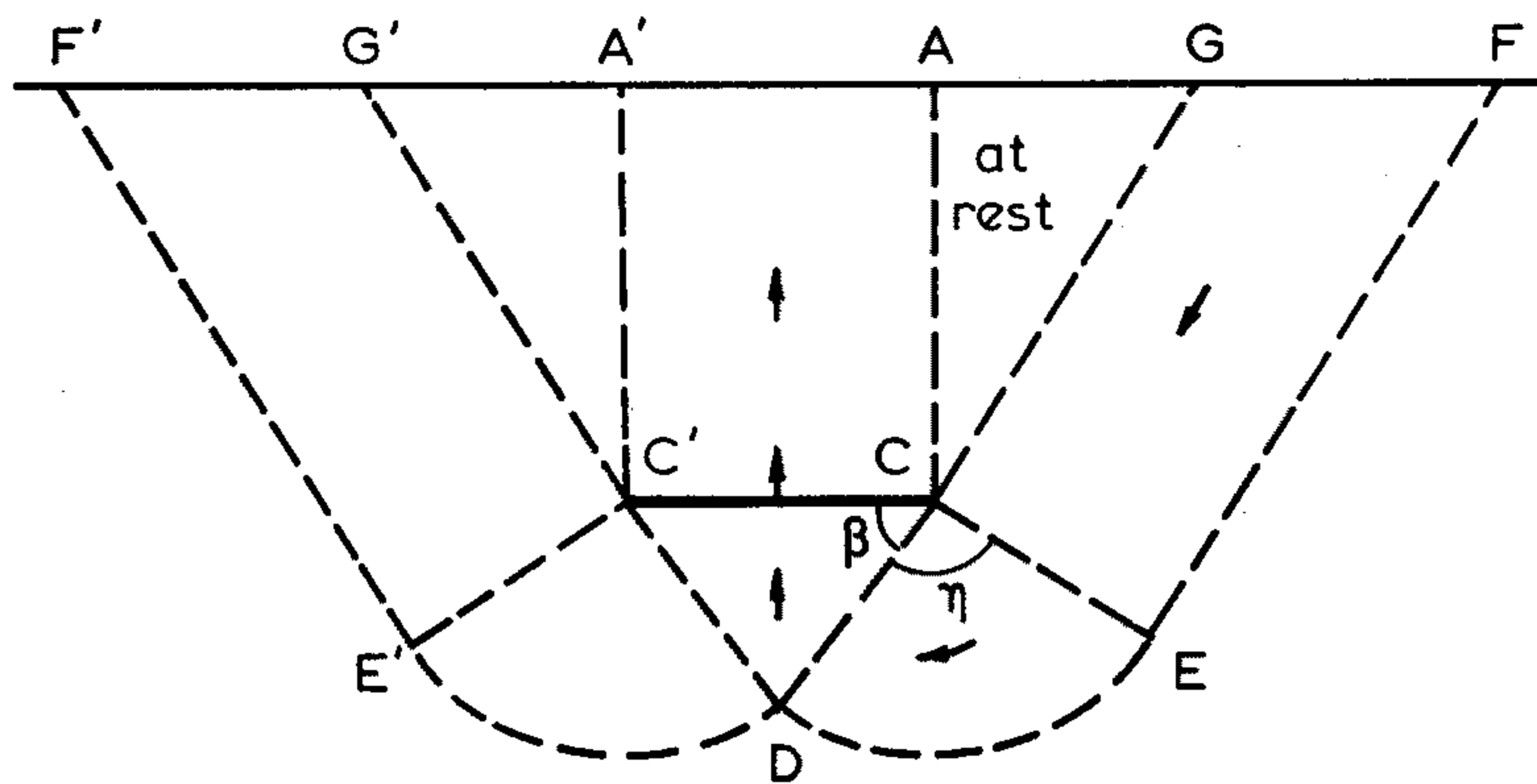


FIG. 4.3 LIMIT SOLUTIONS FOR A SHALLOW ANCHOR - IMMEDIATE
BREAKAWAY



(a) Lower Bound LB4



(b) Upper Bound UB3

FIG. 4.4 LOWER AND UPPER BOUNDS FOR A SHALLOW ANCHOR (FULLY BONDED)

upper bound which is kinematically admissible for a fully bonded anchor. This solution involves rigid upward movement of the blocks ACC'A' and CDC', and a rigid downward movement of regions CEF'G and C'E'F'G'. The region CDE and C'D'E' distort and rotate about C and C' respectively. The regions ACG and A'C'G' are both at rest. It can be shown that the limit pressure corresponding to this arrangement is given by

$$q/c = 2 h/B [1 - \sec (\beta + \eta) \cos \beta] + \tan \beta + 2\eta - \tan (\beta + \eta) \quad (4.8)$$

This solution is plotted against embedment ratio in Fig. 4.5 and can be seen to be a better upper bound than that for infinitely deep anchors (UB1 of Fig. 4.1) up to an embedment ratio of 1.5. It will be noticed that for the limiting case as h tends to zero, this solution will give the Prandtl solution.

Although the solutions obtained for the immediate breakaway case are also valid lower bounds for the no breakaway situation, they are not good lower bounds, and it is necessary to devise a statically admissible field which will give a higher estimate of the limit load. Clearly, one such field is given by the Prandtl solution for a footing and hence the Prandtl load $q/c = (2 + \pi)$ is a lower bound for all values of h/B (while giving the exact collapse load for $h/B = 0$). However, it is still necessary to obtain a solution which will allow a transition between the lower bounds generated from the Prandtl solution for the limiting cases of a very shallow and very deep anchor. One such solution is shown in Fig. 4.4(a). Here it is assumed that regions of constant stress (nos 1 to 13) are separated by straight line discontinuities. Regions 1, 2, 3, 5, 6, 7 are all plastic while

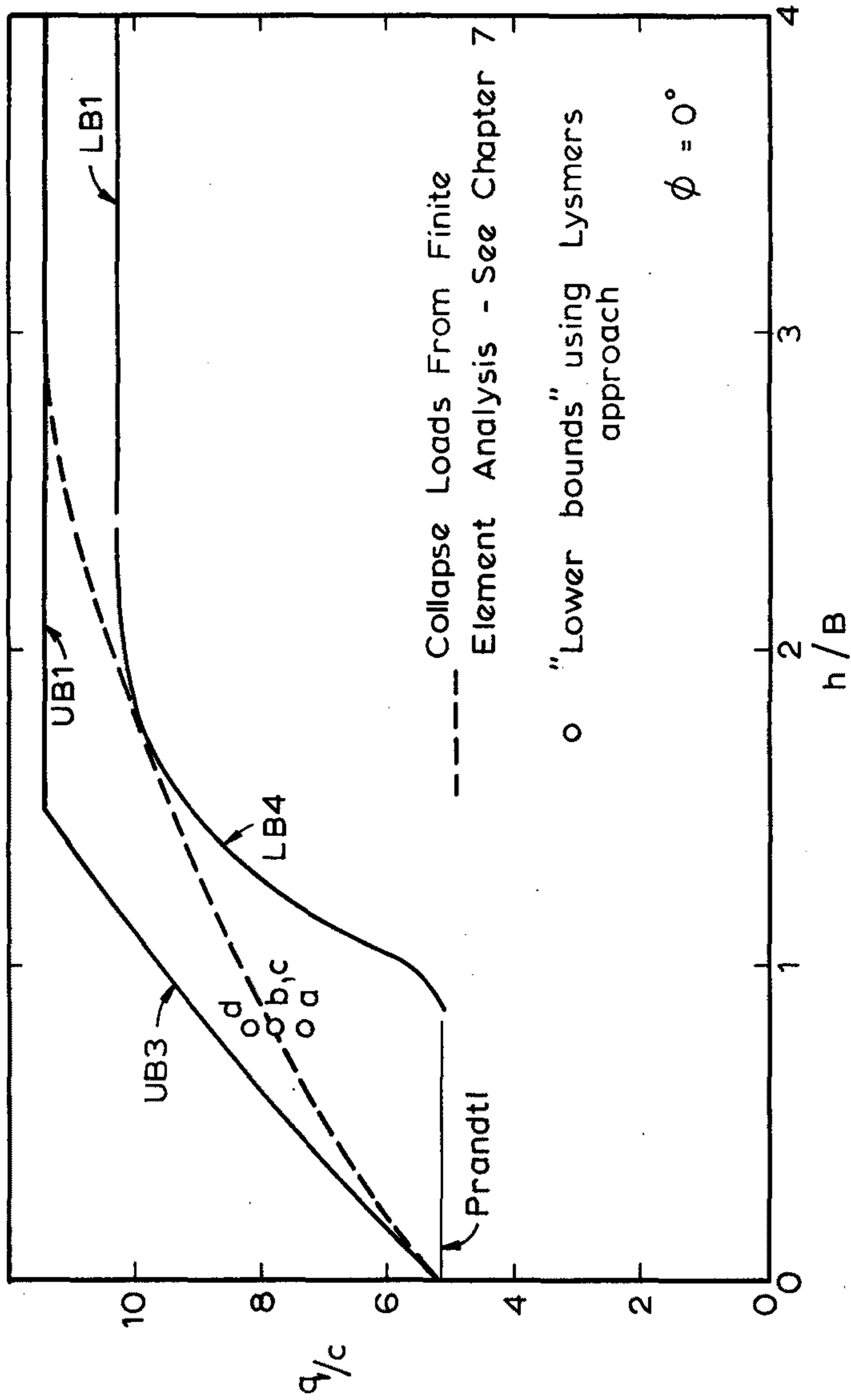


FIG. 4.5 LIMIT SOLUTIONS FOR A SHALLOW ANCHOR - FULLY BONDED

regions 4, 8, 9, 10, 11, 12, 13 are non-plastic. It is also assumed that region 4 is either unstressed, or else subjected to a hydrostatic stress state, and that the major principal stress is vertical and horizontal in regions 1 and 6 respectively. The limit loads obtained for this arrangement of stress continuities are given in Appendix 4A and the results are plotted against embedment ratio (h/B) in Fig. 4.5.

The limit load itself is determined initially from consideration of equilibrium across the discontinuities between regions 1 to 9, and although this is a statically admissible stress field, the establishment of a true lower bound also requires proof that the field can be extended to infinity without exceeding the material strength. This cannot be done for the general problem, and even for a particular selection of h/B , α , β , ω , ϵ , and θ it is extremely difficult to obtain an extension field because the discontinuities frequently intersect generating a new set of discontinuities at each such point. Consequently, the solution shown in Fig. 4.5 for this arrangement (LB3) may be a lower bound but, strictly speaking, it has not been proved to be a true lower bound.

It will be appreciated that there are infinitely many stress fields which are statically admissible and that difficulties arise firstly in trying to select stress fields which will give good lower bounds, and secondly, in showing that the proposed field is extensible (and therefore leads to a true lower bound). Accordingly, Lysmer (1970) has proposed a method which at least makes it easier to select good, statically admissible fields.

Lysmer's approach considers the family of stress fields having the property that all stresses vary linearly within each triangular element of some mesh which spans the soil mass under study. For this type of stress field it is possible to express all equilibrium and failure conditions as a set of linear constraints; the optimal lower bound may then be isolated by the method of linear programming.

Since all stresses vary linearly within each element, the condition that the stress field does not violate the Mohr-Coulomb failure criteria will be satisfied provided that the yield condition is not violated at the element corners. In anticipation of the use of linear programming, it is desirable to linearise the no-yield condition and hence, the Mohr-Coulomb failure surface is approximated by an inscribed regular polygon with m sides. Thus if there are p elements, the no-yield condition gives rise to $3pm$ linear equations. Experience has shown that the computer time required to solve a given problem increases with at least the third power of the number m . Consequently, Lysmer proposed an iterative procedure for approximating the no-yield surface and this procedure (which is fully described by Lysmer, 1970) was adopted by the author.

Lysmer's paper gives examples in which the method was applied to simple earth pressure and bearing capacity problems. In each case favourable agreement with known solutions was obtained using only six triangular elements.

The author has written a computer program based on the method

proposed by Lysmer. The program was checked by performing standard bearing capacity and earth pressure analyses and these results were then compared with classical solutions as well as those obtained by Lysmer. The method of analysis was then applied to the case of a fully bonded anchor at an embedment ratio $h/B = 0.8$. Four mesh arrangements were considered, and these are shown in Fig. 4.6. The lower bounds obtained from these mesh arrangements are given in Table 4.1. The highest limit load for each arrangement of discontinuities is plotted in Fig. 4.5. A number of observations may be drawn from these analyses.

Firstly, the analysis of buried structures requires a much larger number of 'elements' than was found necessary for surface footings or earth pressure analyses. Secondly, the limit load was reasonably sensitive to the number of sides to the inscribed polygon approximating the Mohr-Coulomb failure criteria and it would appear that at least twelve sides should be adopted. Finally, the limit load obtained from this analysis is automatically a true lower bound if the surrounding rigid material is of infinite strength, however, in general, the rigid material may be assumed to have the same strength as that surrounding the buried structure; consequently, the limit load is only a true lower bound if it can be shown that the statically admissible field can be extended into the rigid material without exceeding the soil strength.

In many cases it is extremely difficult to devise a suitable statically admissible field which extends to infinity (alternatively, it is even more difficult to prove that no such extension field exists). For engineering purposes, the limit load obtained from this type of

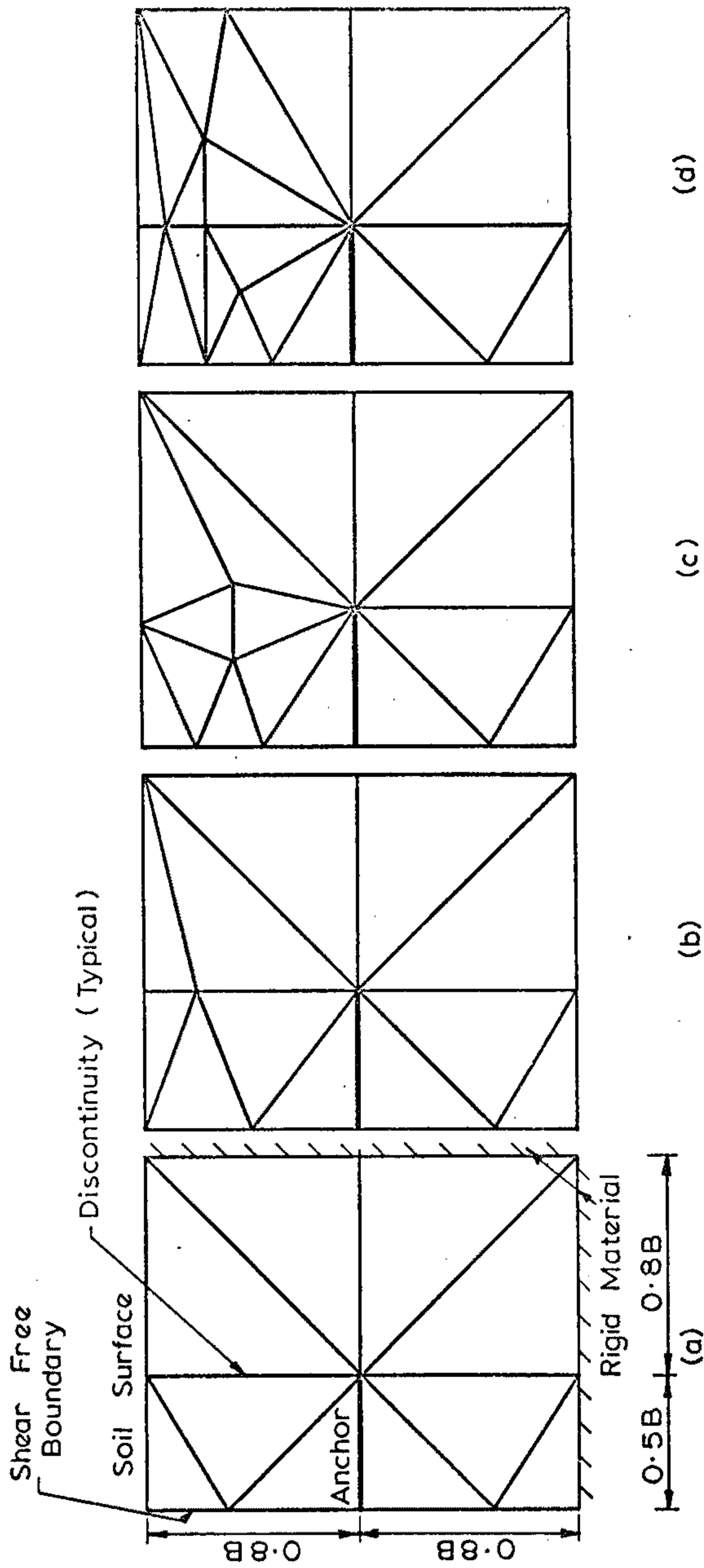


FIG. 4.6 MESH ARRANGEMENT FOR LOWER BOUND ANALYSIS

analysis may be adopted with reasonable confidence if the rigid boundary is remote from the buried structure, however, in the authors experience the cost of analyses involving sufficient 'elements' to allow a suitably remote rigid boundary is prohibitive.

TABLE 4.1

Limit Loads from Lower Bound Analysis Using Lysmer's Approach

$$\underline{h/B = 0.8 \quad \phi = 0}$$

Mesh Code	No. of Elements	No. of sides to yield condition m	Limiting pressure acting on the anchor plate q/c
a	10	3	5.55
		6	6.64
		12	7.22
		18	7.22
		24	7.26
b	12	3	5.69
		6	7.04
		12	7.78
		18	7.82
		24	7.87
c	15	3	5.78
		6	7.12
		12	7.74
		18	7.80
		24	7.85
d	20	3	6.08
		6	7.37
		12	7.97
		18	8.11
		24	8.11
		30	8.14

4.4 CONCLUSIONS

The application of classical plasticity to the prediction of the bearing capacity of anchor plates (and other complicated stability problems) is largely restricted to the use of limit analysis for materials with an associated flow rule. However, even for this restricted class of problem it is extremely difficult to obtain true upper, and more particularly, lower bounds which will bracket the collapse load to sufficient accuracy. To some extent the difficulty of obtaining good statically admissible fields for lower bound solutions may be reduced by automatic procedures such as that proposed by Lysmer; however, these analyses may be expensive in terms of computer costs (ie. as expensive, and often more expensive, than a full elasto-plastic finite element analysis of comparable accuracy) and when a good statically admissible field is obtained, it is still necessary to prove that the field is extensible.

Consequently, in this thesis a finite element approach to the prediction of collapse loads will be adopted. In so doing, numerical collapse loads may be obtained for a wide range of geometric and material properties. In particular, both associated and non-associated materials may be investigated and the effects of initial stress condition may be considered. Furthermore, the adoption of a load path technique will allow consideration of the general behaviour of the anchor both prior to and at collapse; thereby reducing the traditional distinction between stability analysis and deformation analysis.

By adopting a numerical technique such as the finite element method to solve the incremental equations describing the response of

the elasto-plastic soil mass to a loading condition, it is possible to avoid many of the arbitrary assumptions used in previous analyses involving anchor plates; however it is recognised that the solutions obtained using this approach are only approximate. Accordingly, the solution obtained from finite element analyses will be compared with solutions obtained from classical plasticity as well as with experimental results wherever possible. In this regard, it is of some interest to compare the bearing capacity of a fully bonded anchor in a purely cohesive material deduced from a finite element analysis (details of which are given in Chapter 7) with the upper and lower limit loads obtained from classical plasticity as shown in Fig. 4.5. It will be observed that the finite element collapse loads generally lie between the upper and lower bounds and consequently might be expected to give a reasonable estimate of the collapse load.

APPENDIX 4ALimit Load for LB3

It can be shown that for the solution shown in Fig. 4.4(a) the limit load is given by

$$\begin{aligned}
 q/c = & -4 \cos 2\beta + 2 - \cos 2\theta - s/c \cos (2d + 2\omega - 2\theta) \\
 & + s/c \cos 2d - \cos (2b + 2\epsilon - 2\omega) + \cos 2b - r/c \cos (2a - 2\epsilon) \\
 & - \cos 8\alpha + r/c \cos 2a
 \end{aligned} \tag{4.9}$$

where β, a, b, d are given by

$$\begin{aligned}
 2h/B &= (3 \tan^2 \beta - 1)/(2 \tan \beta) \\
 \sin 2a &= -c/r \sin 8\alpha \\
 \sin 2b &= r/c \sin (2a - 2\epsilon) \\
 \sin 2d &= c/s \sin (2b + 2\epsilon - 2\omega)
 \end{aligned}$$

and subject to the restriction that

$$\sin 2\theta = s/c \sin (2d + 2\omega - 2\theta)$$

where r and s (which are the radii of the Mohr circles corresponding to the stress state in regions 8 and 9) must be less than cohesion c .

CHAPTER FIVE
FINITE ELEMENT THEORY

5.1 INTRODUCTION

The preceding chapters have been concerned with two limiting models of soil behaviour; namely, that of a linear elastic and a rigid plastic material. However, real soils are neither elastic nor rigid plastic but rather, they exhibit a non-linear stress-strain behaviour. A better approximation of the soil behaviour may be obtained by assuming that the soil is an elasto-plastic material.

In general, problems involving elasto-plastic materials are not amenable to analytical treatment and it is necessary to adopt a numerical technique such as the Finite Element Method. The basic formulation of the Finite Element Method for an elastic material has been extensively described in standard texts (e.g. Zienkiewicz, 1971; Desai and Abel, 1972) and will not be restated here. The formulation for non-linear materials is also presented in both of these texts along with numerous other publications referred to therein. However, because of the number of variations in the approach to non-linear behaviour which have received attention in the literature, as well as the fundamental importance of the formulation (and its implicit assumptions) to the validity of the results presented in the ensuing chapters, the general formulation adopted for elasto-plastic material will be described in this chapter.

A sub-structure approach will then be presented for the analysis of soil-structure interaction problems involving non-linear behaviour of either soil or structure as well as slip or breakaway at the soil-structure interface. The scope of this approach along with

some of its features will be illustrated by consideration of a number of examples.

Previous workers have shown that phenomena such as consolidation (Small et al., 1976), viscous effects such as creep (Zienkiewicz and Cormeau, 1974) and large strain effects (Carter et al., 1977) may be included in an elasto-plastic finite element analysis. Accordingly, modification of the soil-structure interaction approach of the following sections to incorporate these effects is quite straightforward.

5.2 PLASTICITY THEORY AND THE FINITE ELEMENT METHOD

5.2.1 Soil Model and Failure Criteria

For an elasto-plastic material, the failure criterion determines whether the material is in an elastic or plastic state while defining the behaviour of the material once it is plastic. The two most commonly adopted failure criteria are the Extended von Mises [proposed for metals by von Mises (1913) and extended for a frictional material by Drucker and Prager (1952)], and Mohr-Coulomb [proposed by Coulomb (1773) and Mohr (1900) and extended to the three dimensional situation by Shield (1955)]. Experimental investigation by Kjellerman (1936), Kirkpatrick (1957), Wu et al., (1963), and Bishop (1966) indicate that the failure criterion for sands and clays are experimentally very close to the Mohr-Coulomb criterion.

In recent years, more complicated failure criteria have been developed in an attempt to more accurately describe soil behaviour.

For example, Roscoe and Burland (1968), Di Maggio and Sandler (1971) and Lade and Duncan (1975) have proposed various models involving modified or alternative failure criteria.

Under certain circumstances (e.g. Roscoe and Burland (1968) for clays; Di Maggio and Sandler (1971) and Lade and Duncan (1975) for granular materials) these more complicated models give a closer approximation to real soil behaviour. The incorporation of these elaborate theories into the analysis is quite straightforward; however in this thesis a Mohr-Coulomb criterion will be adopted throughout since broad trends in the behaviour of both cohesive and granular materials may be readily understood in terms of this single failure criterion.

The general Mohr-Coulomb failure criterion may be expressed in the form

$$f = \sigma_{\max} - N_{\phi} \sigma_{\min} - 2c\sqrt{N_{\phi}} \quad (5.1)$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum of the three principal stresses, and

$$N_{\phi} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

where ϕ is the angle of friction of the soil and c is the cohesion.

For a material in a plastic state the value of f in Eq. 5.1 is zero while for an elastic material f is negative. In this general

case the Mohr-Coulomb failure surface consists of six planes each intersecting two other planes to form a six sided pyramid when plotted in principal stress space. For stress states on this failure surface, numerical difficulties may arise when any two of the principal stresses are equal. Geometrically this corresponds to a point in stress space lying on the intersection of two planes and the problem which arises is that of deciding how the stresses behave on further incrementation. In axi-symmetric analyses performed by the author, this difficulty was overcome by adopting the procedure proposed by Small (1977) wherein a transition surface was inserted between the Mohr-Coulomb planes at each corner, rounding them off so that the failure surface resembles the Lade-Duncan model.

For plane strain analysis the Mohr-Coulomb failure criterion reduces to the form

$$f = (\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 - \sin^2 \phi (\sigma_{xx} + \sigma_{yy} + 2c \cot \phi)^2 \quad (5.2)$$

where σ_{xx} , σ_{yy} and σ_{xy} are components of stress referred to the cartesian co-ordinate system.

Once an elasto-plastic material becomes plastic the strain at a given stress state is no longer determinate. However, it has been experimentally observed that the strain rate ratios are determinate and may be related by a flow rule. Many workers have in fact assumed that the volumetric behaviour of a material is a unique function of ϕ . In this case the plastic potential of the material coincides with the failure surface f and the plastic strain rates are given by

$$\frac{\dot{\epsilon}_{vp}}{\dot{\epsilon}_{1p}} = 1 - N_{\phi} \quad (5.3)$$

$$\frac{\dot{\epsilon}_{2p}}{\dot{\epsilon}_{1p}} = 0$$

where $\dot{\epsilon}_{vp}$ is the rate of plastic volume strain, and $\dot{\epsilon}_{1p}$, $\dot{\epsilon}_{2p}$ and $\dot{\epsilon}_{3p}$ are the principal plastic strain rates.

A material whose volumetric behaviour during plastic deformation is a unique function of ϕ (as given in Eq. 5.3) is said to have an associated flow rule.

In regard to classical plasticity the assumption of an associated flow rule involves several computational advantages and consequently has received considerable attention. The significance of an associated flow rule has been discussed in detail by Davis (1968). In particular, Davis has observed that although real soils do initially dilate*, they do not continue to dilate indefinitely. Furthermore, the rate of dilation is a function of the density of the soil rather than ϕ , and may even be zero although ϕ is not. Accordingly, Hansen (1958), Mroz (1963) and Davis (1968) have independently postulated a class of ideal material which obeys a non-associated flow rule. In these materials, the rate of dilation may be selected to suit experimental evidence; however, to make the analysis of such material tractable, in this thesis it will be assumed that the material does dilate indefinitely at a constant rate given by the expression proposed by Davis

* An important exception is in the undrained deformation of saturated clays.

(1968) viz.

$$\frac{\dot{\epsilon}_{vp}}{\dot{\epsilon}_{1p}} = 1 - N_{\psi}$$

(5.4)

$$\frac{\dot{\epsilon}_{2p}}{\dot{\epsilon}_{1p}} = 0$$

$$\text{where } N_{\psi} = \frac{1 + \sin \psi}{1 - \sin \psi}$$

ψ is an angle of dilatancy which fulfils the geometric role in relation to strain rates that ϕ does with regard to stress. It will be observed that the special case where $\psi = \phi$ corresponds to an associated flow rule and Eq. 5.4 is identical to Eq. 5.3.

A second limiting case arises when the angle of dilatancy $\psi = 0^\circ$ and corresponds to plastic deformation at constant volume. This behaviour is observed in the undrained deformation of saturated clays, is fairly closely approximated in the deformation of very loose sands, and is generally observed for most soils once the residual or ultimate strength is reached.

In the most general case, the flow rule of Eq. 5.4 may be written in the form

$$\dot{\epsilon}_{\sim p} = \Lambda \dot{\epsilon}_{\sim r} \quad (5.5)$$

where $\dot{\epsilon}_{\sim p}$ is the vector of plastic strain increment

Λ is a single signed scalar multiplier

$\tilde{r}$ is a vector determined from the flow rule.

In fact, if g is the plastic potential of the material then

$$\tilde{r} = \partial g / \partial \sigma \quad (5.6)$$

In this case, the plastic strain increment will be normal to the plastic potential surface g and will be inclined at an angle to the failure surface f . The form of the vector $\tilde{r}$ to be used is given in Appendix 5A.

To avoid undue numerical complications and to reduce the number of parameters to be studied, in the following chapters the soil was assumed to be homogeneous and isotropic. In particular, it was assumed that for cohesive materials no cracking would occur when subjected to tensile stresses. In addition, for purely cohesive materials the yield strength in tension and compression were assumed to be equal.

5.2.2 Incremental Equations for an Elasto-plastic Material

The general formulation of incremental equations governing the behaviour of an elasto-plastic material has received considerable attention in the past (see for example Zienkiewicz, 1971; Desai and Abel, 1972; Ring, 1975; and Small, 1977) however for completeness of presentation, the basic steps in the formulation will be restated here.

Consider an elasto-plastic material with a yield criterion given by

$$f(\sigma) \leq 0 \quad (5.7)$$

where the inequality indicates that the material is elastic while an equality indicates that the material is in a plastic state. It is assumed that the failure criteria is dependent only on the stress state σ .

While the material is elastic it will obey the laws of elasticity, and the stress-strain relationship can be simply expressed as

$$\underline{\dot{\sigma}} = -D_E \underline{\dot{\epsilon}} \quad (5.8)$$

where $\underline{\sigma}$ is the vector of stress components
 $\underline{\epsilon}$ is the vector of strain components
 D_E is a matrix of elastic constants and is given in standard texts (e.g. Zienkiewicz, 1971).

Once the stress state is such that the criterion of Eq. 5.7 indicates that the material is plastic, the total strain rate $\underline{\dot{\epsilon}}$ can be thought of as being composed of an elastic component $\underline{\dot{\epsilon}}_E$ and a plastic component $\underline{\dot{\epsilon}}_p$ e.g.

$$\underline{\dot{\epsilon}} = \underline{\dot{\epsilon}}_E + \underline{\dot{\epsilon}}_p \quad (5.9)$$

where $\underline{\dot{\epsilon}}_E = -D_E^{-1} \underline{\dot{\sigma}}$ (See Eq. 5.8)

As well the stresses must lie on the yield surface $f(\underline{\sigma}) = 0$ so that any change in the plastic stress state must be such that

$$df = 0 \quad (5.10)$$

Since

$$df = \frac{\partial f}{\partial \sigma_{xx}} d\sigma_{xx} + \frac{\partial f}{\partial \sigma_{yy}} d\sigma_{yy} + \dots$$

we can write Eq. 5.10 as

$$\underline{s}^T \dot{\underline{\sigma}} = 0 \quad (5.11)$$

where $\underline{s}^T = \left(\frac{\partial f}{\partial \sigma_{xx}}, \frac{\partial f}{\partial \sigma_{yy}}, \dots \right)$

and $\dot{\underline{\sigma}} = (\dot{\sigma}_{xx}, \dot{\sigma}_{yy}, \dots)$

is the vector of stress components. $\dot{\underline{\sigma}}$ indicates a change in the stress vector over some time interval.

Equation 5.11 is the dot product of two vectors and since the product is zero the vector $\underline{s}^T$ must be perpendicular to $\dot{\underline{\sigma}}$. Since the stress increment $\dot{\underline{\sigma}}$ takes place along the failure surface, then the vector $\underline{s}$ must be normal to the failure surface.

The plastic material may be supposed to have a flow rule such as that described in Eq. 5.5 e.g.

$$\dot{\underline{\epsilon}}_p = \Lambda \underline{r}$$

so that Eq. 5.9 becomes, on substituting for $\dot{\underline{\epsilon}}_E$ and $\dot{\underline{\epsilon}}_p$

$$\dot{\underline{\epsilon}} = -D_E^{-1} \dot{\underline{\sigma}} + \Lambda \underline{r} \quad (5.12)$$

Rearranging gives

$$\dot{\underline{\sigma}} = -D_E \dot{\underline{\epsilon}} + \Lambda D_E \underline{r} \quad (5.13)$$

The resulting equation may be multiplied by $\tilde{s}^T$ and noting that $\tilde{s}^T \dot{\tilde{\sigma}} = 0$ (Eq. 5.11) it may be seen that Eq. 5.13 becomes

$$\dot{\tilde{\sigma}} = -\tilde{s}^T D_E \dot{\tilde{\epsilon}} + \Lambda \tilde{s}^T D_E \tilde{r}$$

and solving for Λ we get

$$\Lambda = \frac{\tilde{s}^T D_E \dot{\tilde{\epsilon}}}{\tilde{s}^T D_E \tilde{r}} \quad (5.14)$$

By substituting this value of Λ into Eq. 5.13 the final incremental stress-strain law results

$$\dot{\tilde{\sigma}} = -D_I \dot{\tilde{\epsilon}} \quad (5.15)$$

where

$$D_I = D_E - \frac{\alpha \beta^T}{\alpha^T \tilde{s}}$$

and

$$\alpha = D_E \tilde{r}$$

$$\beta = D_E \tilde{s}$$

It may be seen that the incremental matrix D_I is merely the matrix of elastic constants diminished by the dyadic matrix $\alpha \beta^T / \alpha^T \tilde{s}$ which is dependent on the current stress state. It is also worth noting, that in general the matrix D_I unlike D_E is not symmetric and consequently the incremental stiffness matrix K_T will not be symmetric. Symmetry will only occur for the case of a material with an associated flow rule.

Equation 5.15 is the general form of the incremental stress-strain law for a perfectly plastic material. The specific form of the vectors $\tilde{r}$ and $\tilde{s}$ are given in Appendix 5A for the case of plane strain.

Equations similar to the virtual work equations may be obtained by considering the equilibrium of a material subjected to known applied tractions and body forces during a load increment. Substitution of the incremental stress strain law into these equations leads to equations describing the general incremental behaviour of the material. An approximate solution to these equations can then be obtained using finite elements and may be written in the form

$$K_T \dot{\tilde{\delta}} = \dot{\tilde{F}} \quad (5.16)$$

where $\dot{\tilde{F}}$ is the incremental load vector; and

K_T is the incremental stiffness matrix defined by

$$K_T = \int_V B^T D_I B \, dv \quad (5.17)$$

The matrix B defines the relationship between the change in nodal displacements $\dot{\tilde{\delta}}$ of an element and the elemental strain $\dot{\tilde{\epsilon}}$ during a load increment ie.

$$\dot{\tilde{\epsilon}} = B \dot{\tilde{\delta}}$$

The form of the matrix B depends upon the choice of element and is given in standard texts (e.g. Zienkiewicz, 1971).

5.2.3 Solution Method

Application of Eq. 5.16 to problems involving elasto-plastic materials is complicated by the fact that K_T depends upon the current average stress over any particular load increment. The average stress state is not known when an increment is first applied and so special methods must be used to obtain a solution to the equations.

A straightforward application of the finite element method would be to adopt an incremental approach wherein the tangent stiffness equations are solved at the i th increment by using the value of stress at the end of the $(i - 1)$ th increment in the formation of the i th tangent stiffness matrix K_T .

For each load increment Eq. 5.16 may be applied iteratively by obtaining an improved estimate of the change in stress over the increment and then using this in the formation of the tangent stiffness matrix for the following iteration. This solution procedure may be applied quite satisfactorily by iterating over each load increment until the desired accuracy is achieved and then proceeding to the subsequent load increment. However, such an approach proves to be extremely costly in computing time since

- (1) the tangent stiffness matrix must be assembled, factorized and solved for each iteration within each load step; and
- (2) for non-associated materials the tangent stiffness matrix is non-symmetric involving a considerable increase in both storage and computer time.

A more efficient approach to the solution of the non-linear equation given by Eq. 5.16 is the initial stress approach developed by Zienkiewicz et al., (1969). Because of its computational efficiency, this approach was adopted for all the finite element analyses performed by the author and it will be briefly described below.

The incremental plastic matrix D_I defined in Eq. 5.15 may be split into two parts; namely the elastic matrix of constants D_E and a dyadic matrix D_C where

$$D_I = D_E - D_C$$

$$D_C = \frac{\tilde{\alpha} \tilde{\beta}^T}{\tilde{\alpha}^T \tilde{s}}$$

where D_C may be considered to be a correction to the elastic matrix of constants and will be dependent upon the plastic stress state.

Since the matrix D_I may be separated into two parts it follows that the tangential stiffness matrix K_T may be similarly split and thus

$$K_T = K_E - K_C \quad (5.18)$$

where $K_E = \int_V B^T D_E B dV$

and $K_C = \int_V B^T D_C B dV$

where K_E is the elastic component of the soil stiffness matrix and K_C is the reduction in stiffness due to plastic yielding. For each load

increment, the solution to the equations governing the behaviour of an elasto-plastic material may now be obtained by application of the algorithm

$$K_E \dot{\delta}_i^{(n+1)} = K_C^{(n)} \dot{\delta}_i^{(n)} + \dot{F}_i \quad (5.19)$$

where the superscript n denotes the n th estimate of that variable.

For the first iteration in each load step the correction vector $K_C^{(1)} \dot{\delta}_i^{(1)}$ may be estimated from the values obtained at the end of the previous increment. The iterative procedure specified by Eq. 5.19 is then repeated until the solution has converged to sufficient accuracy. Details regarding the incremental procedure and convergence will be considered in Chapter 6.

A number of features of this initial stress approach should be mentioned; namely,

- (1) the elastic stiffness K_E need only be set up and reduced to triangular form once, so that the cost of solving for the load sets $K_C^{(n)} \dot{\delta}_i^{(n)} + \dot{F}_i$ is relatively small;
- (2) the tangential stiffness matrix K_T is never directly evaluated since the effect of plasticity is manifest as an initial stress correction to the load vector formed from the 'elemental reductions' in stiffness $K_C^{(n)}$ and the incremental displacement $\dot{\delta}_i^{(n)}$ from the previous iteration.
- (3) the non-symmetric nature of the tangential stiffness matrix does not affect the storage and computation time since the

tangential stiffness matrix is never assembled.

5.2.4 Correction to the Yield Surface and Negative Plastic Work

In following an incremental procedure it is assumed that elastic elements remain elastic, and similarly plastic elements remain plastic, until the physical conditions compatible with this assumed state are violated. Thus, an elastic element will not become plastic until the stress state reaches, or moves outside, the yield surface due to an elastic increment in stress. If the stress state moves outside the yield surface, it violates the yield criterion which implicitly states that such a stress state cannot exist. There are two commonly adopted methods of allowing for this difficulty. Ideally, the load increment size may be selected so that only one element yields in each load increment, and consequently the increment size may be adjusted so that the stress state for that element lies on the yield surface. Unfortunately, for problems involving large numbers of elements and extensive plastic failure, the procedure may be extremely time consuming. An alternative approach is to correct the stress state of elements which have yielded back on to the yield surface. Several methods of performing this correction have been proposed (e.g. Christian and Marr, 1972) and there appears to be no real justification for preferring one method of correction to another (Ring, 1975). In the author's computer programs, any stress state outside the yield surface was corrected back to the yield surface along a path normal to that surface. This corrected stress state now satisfies the yield criterion, however it may no longer be in equilibrium. Thus further iterations may be required in the solution procedure so that both equilibrium and the yield criterion are adequately satisfied. The development

of equations to determine the correction normal to the yield surface is relatively straightforward and has been described by various authors (e.g. Ring, 1975; Carter, 1977; Small, 1977).

Once an element becomes plastic, the governing incremental equation (Eq. 5.16) constrains the stress state such that it may only move along the yield surface, while its volumetric deformation must be compatible with the selected flow rule. However, in some problems (e.g. beneath a buried anchor) plastic material may unload elastically thereby returning to an elastic state. It is therefore necessary to check that the direction of each increment in plastic strain is along the outward normal to the plastic potential. This may be achieved by checking the sign of the scalar multiplier Λ given in Eqs. 5.5 and 5.14 to ensure that it has the correct sign (ie. positive for the sign convention used in this thesis). An incorrect sign would imply that negative plastic work is being performed and hence the basic constitutive laws of the ideal material are being contravened. This indicates that unloading is taking place in the plastic material and consequently the element should return to the elastic state. Thus within each iteration of the numerical procedure provision is made for plastic elements to become elastic as well as the usual possibility of elastic elements becoming plastic. The importance of negative plastic work has been more fully discussed by Davis and Booker (1975), and Davis, Ring and Booker (1974).

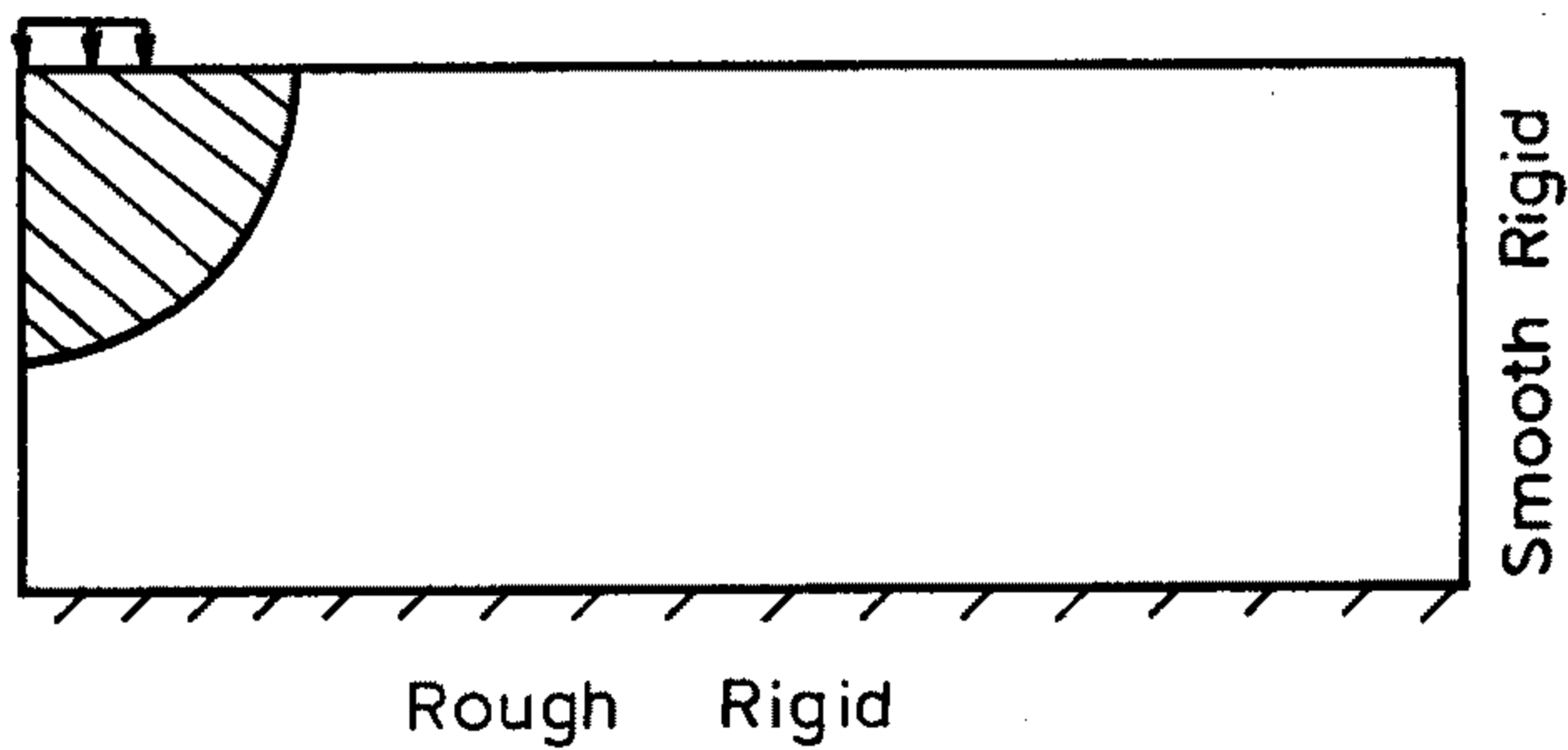
5.3 SUB-STRUCTURING AND THE CONCEPT OF A SUPER ELEMENT

The concept of subdividing a body into a number of sub-structures to allow more detailed analysis of the composite structure has

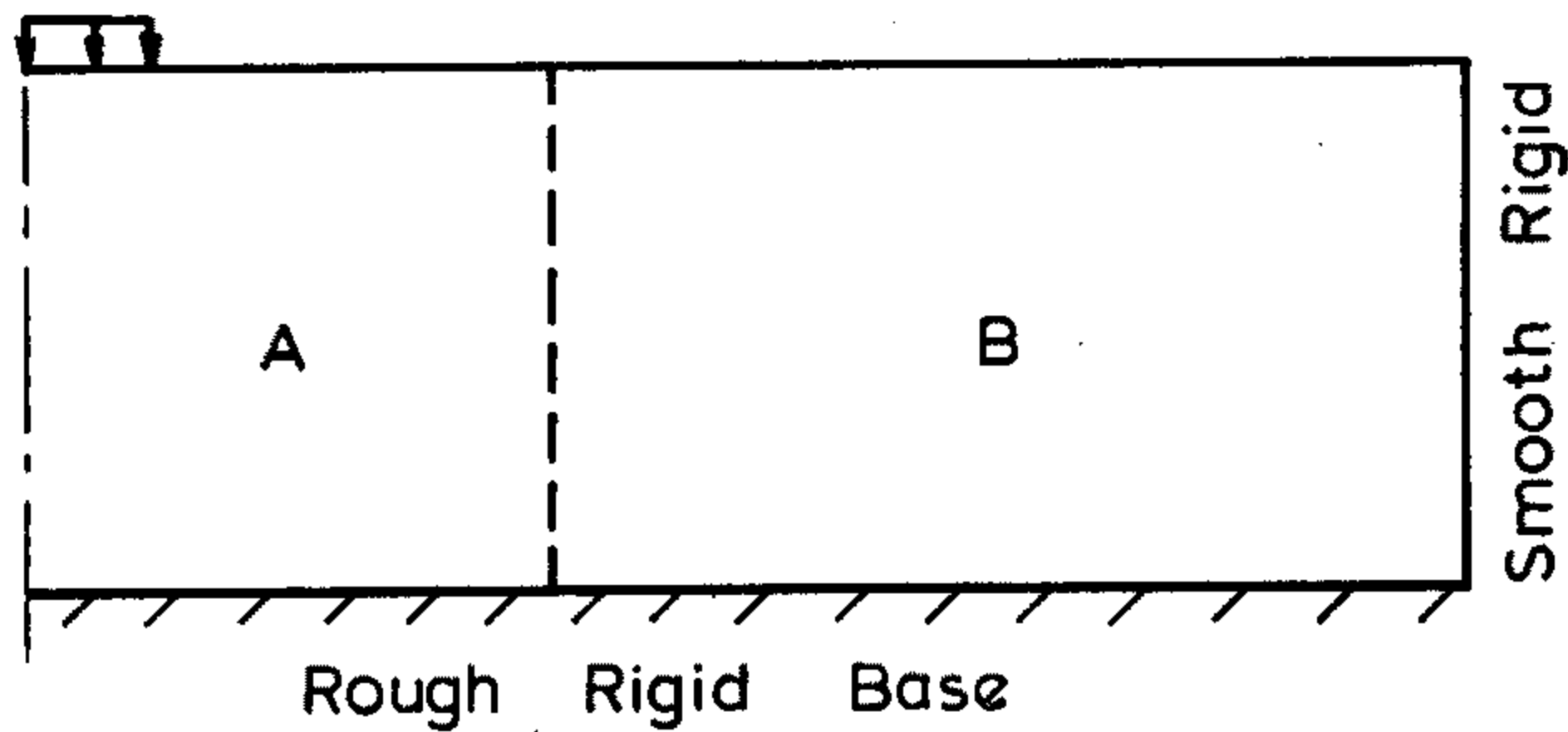
been widely recognised, particularly with regard to aerospace research. However, it does not appear to have been generally recognised that this concept of sub-structuring can be of significant benefit in the analysis of soil mechanics problems involving non-linear behaviour of a soil mass. Consequently, in this section the procedure involved in sub-structuring a soil mass will be discussed while in the following section the concept will be extended to the more general analysis of soil structure interaction.

It is often found that in the elasto-plastic analysis of problems involving plastic failure near a foundation or soil structure (be it a footing, retaining wall, anchor, soil slope etc.) there are extensive portions of the soil mass which remain elastic for the entire analysis. The prediction of the behaviour of the elastic portion of the soil mass is relatively straightforward; consequently, the direct inclusion of the contribution of elements within this elastic mass into the stiffness matrix K_E used in the application of the initial stress algorithm (Eq. 5.19) is wasteful in terms of both computer storage and computational effort.

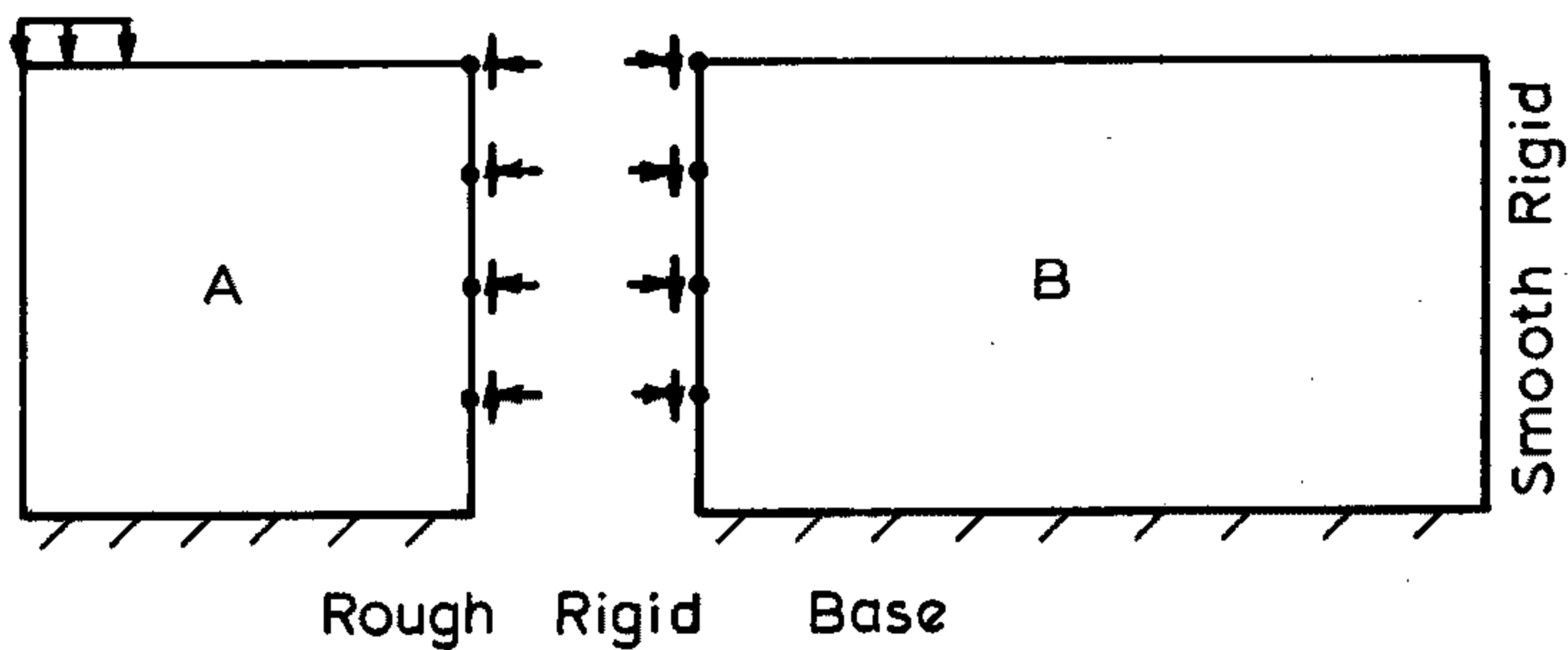
In most cases, the maximum likely extent of the plastic region for a given problem can be reliably estimated. Suppose that such an estimate is made and that the general soil mass is subdivided into two bodies, one which may exhibit elasto-plastic behaviour (Body 1) and one which is assumed will remain elastic for the entire analysis (Body 2) as shown in Fig. 5.1. Now considering each body as a separate entity, the stiffness matrix may (in each case) be partitioned such that the incremental nodal forces and displacements within the body $(\dot{F}_m, \dot{\delta}_m)$ and at the boundary between adjacent bodies $(\dot{F}_b, \dot{\delta}_b)$ may be



Typical Plot of Yielded Zones at Ultimate Load



Non-Linear Block A and Elastic Block B



(c) Blocks A & B Analysed Separately

FIG. 5.1 DIVISION OF CONTINUUM INTO AN ELASTIC BLOCK (B) AND A NON-LINEAR BLOCK (A)

separated, thus for Body 1

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \begin{bmatrix} \dot{\delta}_{\sim m} \\ \dot{\delta}_{\sim b} \end{bmatrix} = \begin{bmatrix} \dot{F}_{\sim m} \\ \dot{F}_{\sim b} \end{bmatrix} \quad (5.20)$$

and for Body 2

$$\begin{bmatrix} M & N \\ N^T & R \end{bmatrix} \begin{bmatrix} \dot{\delta}'_{\sim m} \\ \dot{\delta}'_{\sim b} \end{bmatrix} = \begin{bmatrix} \dot{F}'_{\sim m} \\ \dot{F}'_{\sim b} \end{bmatrix} \quad (5.21)$$

Now from Eq. 5.21 it can be shown that if there are no applied nodal forces within this Body (ie. $\dot{F}'_{\sim m} = 0$), then

$$[R - N^T M^{-1} N] \dot{\delta}'_{\sim b} = \dot{F}'_{\sim b} \quad (5.22)$$

From the consideration of equilibrium between the two bodies, it is

$$\text{known that } \dot{F}'_{\sim b} + \dot{F}_{\sim b} = 0 \quad (5.23)$$

Thus, the combination of Eqs. 5.20, 5.21, 5.22 and 5.23 with the com-

patability requirement that $\dot{\delta}_{\sim b} = \dot{\delta}'_{\sim b}$ gives

$$\begin{bmatrix} A & B \\ B^T & C+I^{-1} \end{bmatrix} \begin{bmatrix} \dot{\delta}_{\sim m} \\ \dot{\delta}_{\sim b} \end{bmatrix} = \begin{bmatrix} \dot{F}_{\sim m} \\ 0 \end{bmatrix} \quad (5.24)$$

where $I^{-1} = [R - N^T M^{-1} N]$ and $F_{\sim m}$ is the vector of applied nodal forces acting on Body 1.

In practice the influence matrix I may be obtained by determining the displacement of Body 2 at the interface between the two bodies, due to a unit nodal force applied in turn to each degree of freedom at the interface. This influence matrix is positive definite and may be inverted to give what may be regarded as the elemental stiffness matrix of a 'Super Element' which represents the apparent stiffness of Body 2. The elemental stiffness may be directly added into the total stiffness matrix of Body 1 as indicated in Eq. 5.24. It has been shown by Balaam (1978) that this adoption of the 'Super Element' as an alternative to including the entire contribution of the elements within Body 2 into the stiffness matrix for the entire mass can lead to very considerable savings in both storage and computation time.

Clearly the general soil mass may be sub-divided into as many elastic sub-regions as desired and by repeated application of this process the finite element boundaries may be made as remote as considered necessary without the heavy computational cost this usually entails in a non-linear analysis.

Eq. 5.24 is directly applicable to the case where Body 2 remains elastic and is not subjected to applied tractions (other than at the interface with Body 1). However, the procedure may be extended to the case where the entire mass is subjected to incremental body forces and where Body 2 is visco-elastic (Brown and Redman, Pers. Comm.)

In the case where the entire mass is subjected to an increment in body force Eq. 5.24 is replaced by

$$\begin{bmatrix} A & B \\ B^T & C+I^{-1} \end{bmatrix} \begin{bmatrix} \delta_m \\ \delta_b \end{bmatrix} = \begin{bmatrix} F_m \\ \gamma I^{-1} k \end{bmatrix} + \begin{bmatrix} F_{\gamma m} \\ F_{\gamma b} \end{bmatrix} \quad (5.25)$$

where I^{-1} is obtained as previously described;

k is a vector of nodal displacements at the interface boundary of Body 2 due to a unit value of applied density in Body 2;

F_m is the vector of nodal forces due to the applied external traction acting on Body 1; and

$F_{\gamma m}$, $F_{\gamma b}$ are vectors of nodal force due to the increment γ in body force within Body 1.

5.4 SOIL-STRUCTURE INTERACTION ANALYSIS

The general problem of soil-structure interaction is one of considerable interest to engineers and has, consequently, received a great deal of attention. The types of geotechnical structure involved include retaining walls, temporary excavation bracing systems, soil anchors, lined tunnels, piles, rafts and strip footings.

Historically the methods of analysis adopted for the many different soil-structure interaction problems are as varied as the problems themselves. More recently, the finite element method has been applied to the entire spectrum of soil-structure interaction analysis, (for example, Chang and Duncan, 1970; Wong, 1971; Clough and Duncan, 1971; Desai and Holloway, 1972; Jaworski, 1973; Desai, 1974; Murphy, Clough and Woolworth, 1975; Romstad, Hermann and Shen, 1974; and Clough, 1972). These analyses have represented both the soil and structure

using finite elements which in some cases have been linked together by interface elements to allow for shear failure at the soil structure interface.

As would be expected, analysis of such complicated behaviour may lead to a variety of numerical problems.

Firstly, the difference in stiffness between soil and structural elements may lead to serious numerical errors in the solution process (Roy, 1971).

Secondly, the presence of the structure (for example a retaining wall tied back to an anchor plate) may establish a link between nodes which are a long distance apart and lead to a very large increase in bandwidth of the equations being solved. Finally, it has been the authors experience that straightforward application of the initial stress method to non-linear problems involving stiff link or joint elements is often accompanied by poor convergence of the method. Some attempt has been made by Ghaboussi et al., (1973) and Wilson (1975) to reduce the numerical error due to stiff joint elements however, difficulties arise in an initial stress approach since the incremental stiffness correction terms may still be quite large.

In this section an alternative approach, in which the soil and the structure are analysed as separate bodies, related only by compatibility of nodal displacements and nodal forces at their interface, is presented.

The concept of separating an elastic body into sub-structures has been recognised by various authors [e.g. Brown and Wiesner (1975), Poulos and Davis (1968)], in the analysis of problems such as pile and

pile strip behaviour. When applied to non-linear finite element methods this substructure approach avoids the difficulties outlined above while retaining the computational advantages of the initial stress technique.

5.4.1 An Initial Stress - Sub-Structure Approach

Consider a structural body embedded in an elasto-plastic soil mass. The composite body can be separated into two components, namely the soil and the structure.

Suppose that incremental loads $\dot{\chi}_1$ and $\dot{\chi}_2$ are applied respectively to the soil mass and the structure, then the two substructures will be governed by the equations

$$K_{1T} \dot{\delta}_1 = \dot{\chi}_1 + \dot{F}_{1u} \quad (5.26)$$

$$K_{2T} \dot{\delta}_2 = \dot{\chi}_2 - \dot{F}_{2u} \quad (5.27)$$

where K_{1T} , K_{2T} are the tangent stiffness matrices for soil and structure respectively;
 $\dot{\delta}_1$, $\dot{\delta}_2$ are the vectors of incremental nodal displacement for soil and structure;
 $+ \dot{F}_{1u}$ is the vector of incremental nodal forces due to the structure acting on the soil; and
 $- \dot{F}_{2u}$ is the vector of incremental nodal forces due to the soil acting on the structure.

The vectors $\dot{\delta}_1$, $\dot{\chi}_1$, $\dot{F}_{1u}$ are of length k while the vectors $\dot{\delta}_2$,

$\dot{\chi}_2$, $\dot{F}_{2u}$ are of length ℓ where k and ℓ are the number of degrees of freedom of soil and structure respectively. Thus, if there are m degrees of freedom at the soil-structure interface, then the vector $\dot{F}_u$, of length m , which corresponds to the forces exerted by the structure onto the soil can be augmented by the addition of zero terms for each degree of freedom which does not correspond to an interfacial node to give the vectors $\dot{F}_{1u}$ and $\dot{F}_{2u}$ for soil and structure.

Analysis of the Soil

Suppose for the moment that the nodal force vectors $\dot{F}_u$ were known, then the initial stress approach could be used to analyse the soil using the following algorithm.

$$K_{1E} \dot{\delta}_{1l}^{(n+1)} = K_{1C}^{(n)} \dot{\delta}_{1l}^{(n)} + \dot{\chi}_1 + \dot{F}_{1u}^{(n+1)} \quad (5.28)$$

where K_{1E} is the elastic component of the soil stiffness matrix

$K_{1C}^{(n)} = K_{1E} - K_{1T}^{(n)}$ is the reduction in stiffness due to plastic yielding; and the superscript n denotes the 'nth' estimate of that variable.

It is convenient to rewrite Eq. 5.28 in the form

$$K_{1E} \dot{\delta}_{1K}^{(n+1)} = \dot{F}_{1K}^{(n)} \quad (5.29a)$$

$$K_{1E} \dot{\delta}_{1u}^{(n+1)} = \dot{F}_{1u}^{(n+1)} \quad (5.29b)$$

where $\dot{F}_{1K}^{(n)} = K_{1C}^{(n)} \dot{\delta}_{1l}^{(n)} + \dot{\chi}_1$

so that $\dot{\delta}_{\sim 1}^{(n+1)} = \dot{\delta}_{\sim 1K}^{(n+1)} + \dot{\delta}_{\sim 1u}^{(n+1)}$

Notice that it is only necessary to reduce $K_{\sim 1E}$ to triangular form once and so the cost of solving for the load sets $\dot{F}_{\sim 1K}^{(n)}$ is relatively small.

Now suppose that $\rho_{\sim}$ denotes the vector of nodal displacements at the soil structure interface, then clearly $\rho_{\sim}$ can be written in the form

$$\dot{\rho}_{\sim 1}^{(n+1)} = \dot{\rho}_{\sim 1K}^{(n+1)} + \dot{\rho}_{\sim 1u}^{(n+1)} \quad (5.30)$$

where the vectors $\rho_{\sim 1}$ are obtained by deleting all the non-interface components from the vectors $\dot{\delta}_{\sim 1}$, and will be of length m .

Since the matrix $K_{\sim 1E}$ in Eq. 5.29b is constant, successive estimates of $\dot{\rho}_{\sim 1u}$ can always be written in the form

$$\dot{\rho}_{\sim 1u}^{(n+1)} = I_{\sim 1E} \dot{F}_{\sim u}^{(n+1)} \quad (5.31)$$

where $I_{\sim 1E}$ is a $(m \times m)$ matrix of influence coefficients, which can be determined initially and will remain constant for each iteration and loadstep while the geometry of the problem remains unchanged. Eqs. 5.30 and 5.31 can now be combined to give

$$\dot{\rho}_{\sim 1}^{(n+1)} = \dot{\rho}_{\sim 1K}^{(n+1)} + I_{\sim 1E} \dot{F}_{\sim u}^{(n+1)} \quad (5.32)$$

Analysis of the Structure

In general, if the structure is separated from the soil it will be a mechanism, so that the solution for deflection would contain a number of arbitrary rigid body motions. This difficulty can be overcome by applying sufficient artificial constraints to remove the indeterminacy.

This leads to a set of equations of the form

$$K_{2r} \dot{\delta}_{2r} = \dot{\chi}_2 - \dot{F}_{2u} \quad (5.33)$$

where K_{2r} is the tangent stiffness matrix of the restrained structure and $\dot{\delta}_{2r}$ are the incremental deflections of the restrained structure. For solution purposes this may be replaced by the scheme

$$\begin{aligned} K_{2E} \dot{\delta}_{2r}^{(n+1)} &= \dot{\chi}_2 - \dot{F}_{2u}^{(n+1)} \\ &+ (K_{2E} - K_{2r}^{(n)}) \dot{\delta}_{2r}^{(n)} \end{aligned} \quad (5.34)$$

where K_{2E} is the elastic stiffness matrix of the restrained structure. For simple structures the necessary restraints are obvious from inspection while for more complicated structures the necessary restraints, and the matrix A defined below, can be isolated by performing Gaussian elimination on the full structural stiffness matrix.

When this is done the actual incremental displacements $\dot{\rho}_2$ of the structural interface nodes can be obtained from the restrained

structure plus the arbitrary rigid body motions as follows:

$$\dot{\rho}_2 = \dot{\rho}_{2r} + A \dot{\theta} \quad (5.35)$$

where $\dot{\theta}$ is the vector of incremental rigid body motions,
and has length p ;

A is a $(p \times m)$ matrix relating the interface deflections
to rigid body motions; and

p is the number of artificial restraints applied to
the structure.

The vector $\dot{\rho}_{2r}$ can be derived by a method similar to that
described in the previous subsection and may be written

$$\dot{\rho}_{2r}^{(n+1)} = \dot{\rho}_{2K}^{(n+1)} - I_{2E} \dot{F}_u^{(n+1)} \quad (5.36)$$

where the definitions $\dot{\rho}_{2K}$, I_{2E} parallel those of $\dot{\rho}_{1K}$, I_{1E} .

When Eqs. 5.32, 5.35, 5.36 are combined it is found that

$$\begin{aligned} \dot{\rho}_1^{(n+1)} - \dot{\rho}_2^{(n+1)} &= (I_{1E} + I_{2E}) \dot{F}_u^{(n+1)} \\ &\quad - A \dot{\theta}^{(n+1)} + (\dot{\rho}_{1K}^{(n+1)} - \dot{\rho}_{2K}^{(n+1)}) \end{aligned} \quad (5.37)$$

If there are m unknown nodal forces and p rigid body motions,
then Eq. 5.37 represents m equations in $(m+p)$ unknowns. The remaining
equations can be derived from the observations that the structure must
be in a state of equilibrium. The equations of equilibrium correspon-
ding to the artificial restraints can be written in the form

$$A^T (\dot{\chi}_2 - \dot{F}_u^{(n+1)}) = 0 \quad (5.38)$$

where for convenience, it has been assumed that the applied loads $\{\dot{\chi}_2\}$ are located at the soil structure interface. The more general case may be obtained by a simple modification of the product $A^T \dot{\chi}_2$.

Eqs. 5.37 and 5.38 may be written in the simple interaction equation

$$\begin{bmatrix} I_E & -A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} \dot{F}_u^{(n+1)} \\ \dot{\theta}^{(n+1)} \end{bmatrix} = \begin{bmatrix} \dot{a}^{(n+1)} \\ \dot{b}^{(n+1)} \end{bmatrix} \quad (5.39)$$

where $\dot{a}^{(n+1)} = (\dot{\rho}_{2K}^{(n+1)} - \dot{\rho}_{1K}^{(n+1)}) - (\dot{\rho}_2^{(n+1)} - \dot{\rho}_1^{(n+1)})$

$$\dot{b}^{(n+1)} = -A^T \dot{\chi}_2$$

and $I_E = I_{1E} + I_{2E}$

For a soil-structure interface at which no slip occurs, the vectors $\dot{\rho}_1^{(n+1)}$ and $\dot{\rho}_2^{(n+1)}$ may be taken as equal and thus Eq. 5.39 may be solved to determine $\dot{F}_u^{(n+1)}$ and $\dot{\theta}^{(n+1)}$. Once these quantities are known the soil deflections can be determined from Eq. 5.28.

Similarly the deflections of the structure can be calculated by solving Eq. 5.34 and adding the rigid body contributions due to $\dot{\theta}^{(n+1)}$. The process is then continued until convergence is achieved. This approach derives its power from the fact that it is only necessary to triangulise the matrices K_{1E} , K_{2E} once for all iterations and load-steps while the problem configuration remains constant.

Problems in which slip or breakaway (e.g. behind an anchor plate) occurs at the interface can also be solved using this method. The most common criterion for slip (or breakaway) is that once the nodal force reaches a specified value it remains constant. (More complicated criteria are considered in Appendix 5B). Suppose that this nodal force is F'_u and the corresponding nodal deflection is ρ' . Once the critical value is reached F'_u will remain constant so that $\dot{F}'_u$ will be zero for all future increments unless unloading occurs. Thus one unknown is lost from Eq. 5.39 however, it is replaced by $(\dot{\rho}_2 - \dot{\rho}_1)$, the relative displacement between structure and soil. So Eq. 5.37 may still be solved to obtain $\dot{F}_u^{(n+1)}$ and $\dot{\theta}^{(n+1)}$. Problems involving a smooth interface can be treated as above by assuming the shear forces F'_u and $\dot{F}'_u$ are always zero.

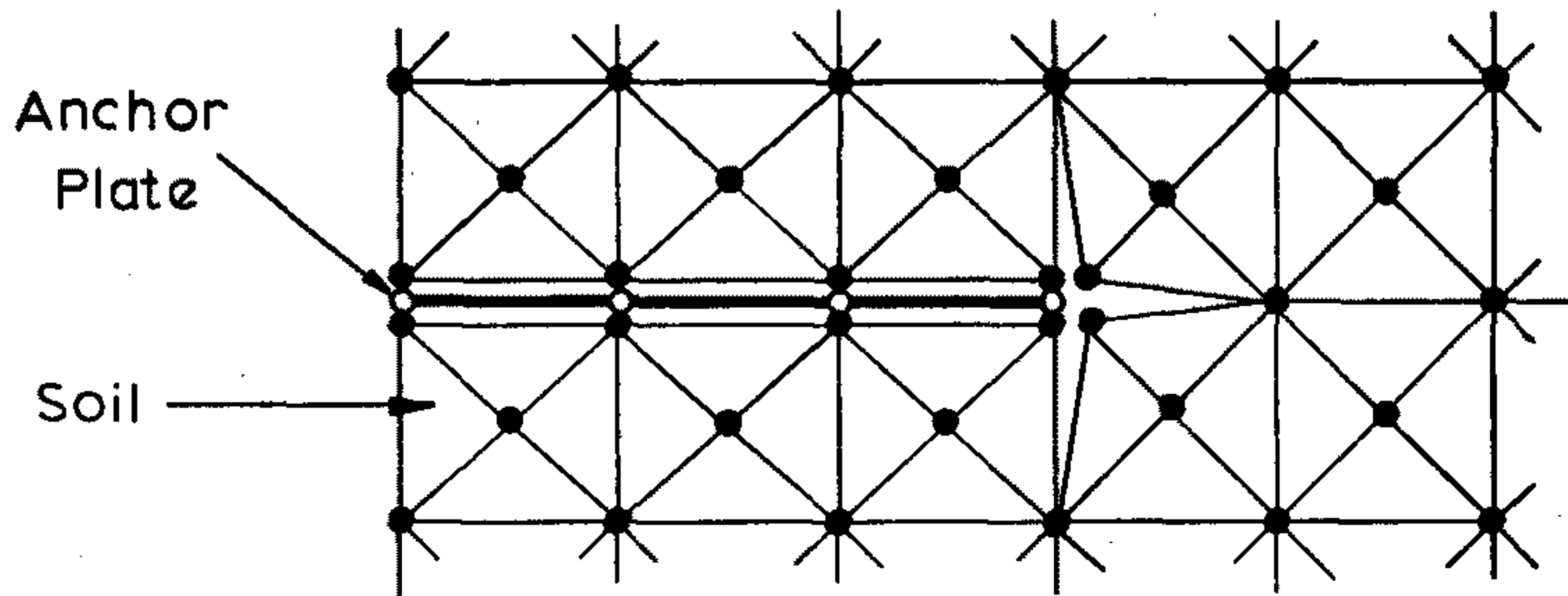
The solution algorithm presented above has several advantages. Firstly the widely differing magnitudes of soil stiffness and structural stiffness are separated and only come together in Eq. 5.39. This set of equations, which for most soil-structure problems is far smaller than the stiffness equations for the entire body, may be solved using multi-precision arithmetic if this is thought necessary. Secondly, the inclusion of interface behaviour is quite straightforward and does not involve correction to the load vector. Thus, the initial stress method when applied in this way seems far more stable than when applied in a conventional manner to the entire body.

5.4.2 Special Considerations for Anchor Plates

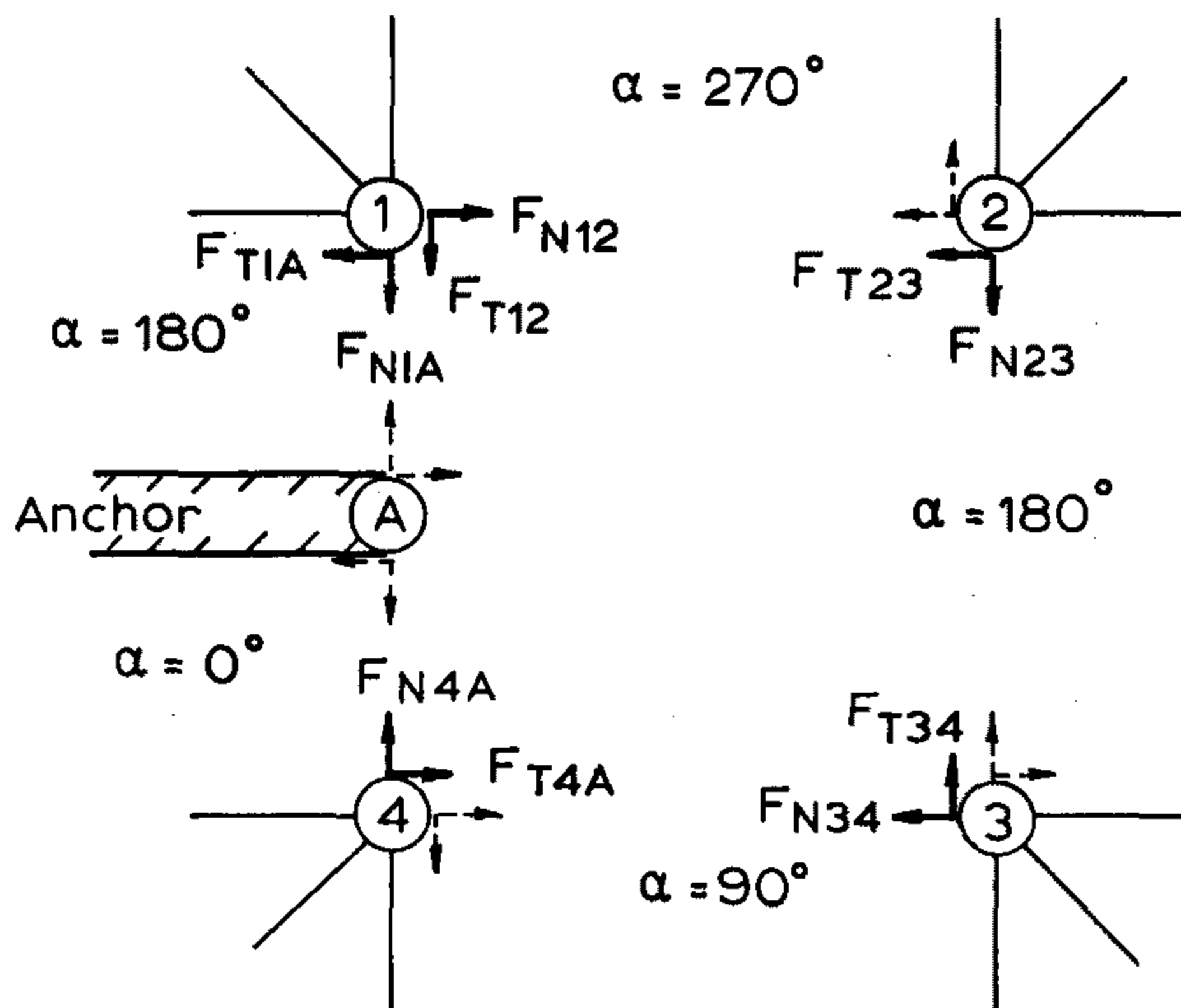
Two possible conditions may exist at any point beneath (or behind) an anchor plate during a loading sequence; that is, either the anchor plate is in contact with the underlying soil or it is not. In general, provision must be made for the case where the soil (which is initially in contact with the underside of the anchor) breaks away from the anchor plate.

Except at the edge of the anchor, provision for breakaway may be simply achieved by devising a finite element mesh such that there is a dual node pair in the soil corresponding to each node on the anchor plate as shown in Fig. 5.2a. Below (or behind) the anchor plate conditions of compatability in normal displacement between anchor and soil are enforced until the compressive normal force has been reduced to zero at a point. Once this occurs the compatability requirement is no longer enforced, resulting in the formation of a gap between the anchor and the underlying soil. This procedure may take place at each point behind the anchor plate until the gap spans the entire width of the anchor.

It will be observed that the provision of dual soil nodes for each anchor node allows independent slip in front and behind the anchor as would be expected to occur in reality. It will also be appreciated that breakaway at a point must be associated with a reduction in both the normal and tangential nodal forces at that point to zero, since the normal and shear stresses on a free surface must be zero.



(a) Mesh Arrangement at Soil - Anchor Interface



(b) Details of Mesh at Anchor Tip

FIG. 5.2 SOIL - ANCHOR DETAILS

To allow for the complex deformation that might be expected at the edge of the anchor a mesh arrangement involving a minimum of four soil nodes at the edges was adopted in the anchor analyses performed for subsequent chapters. Arrangements involving less than four nodes at the edge of the anchor were tried but were found to be unsatisfactory in that they were quite sensitive to the mesh arrangement for the case where breakaway occurred, and did not provide adequate freedom for deformation to occur around the edge.

The most commonly adopted arrangement of nodes at the edge of the anchor is shown in Fig. 5.2. It will be observed that when there is no breakaway or slip, soil nodes 1 and 4 are connected to the anchor node by the requirements of normal and tangential compatibility. Once the limiting tangential nodal force was attained slip could occur independently between node 1 and the anchor, and between node 4 and the anchor. Once the limiting normal force was reached the compatibility requirements between node 4 and the anchor node were no longer enforced (they are in fact replaced by the requirement that the normal and tangential forces N_{4A} and T_{4A} must be zero). It now remains to discuss the role played by nodes 2 and 3. In fact nodes 2 and 3 were linked together, and to nodes 1 and 4 respectively, by requirements of normal and tangential compatibility. This may be achieved by application of the same techniques outlined for the two body case in the previous section at the cost of a little additional bookkeeping to ensure that the nodal forces and displacements are obtained for the relevant body when considering the effect of the second node in each nodal pair. It follows that provision can be made for slip to occur between the soil node pairs 1-2, 2-3 and 3-4 when the

limiting shear strength of the soil is reached between each of the nodal pairs. It is this provision for slip in these locations that allows freedom of deformation at the anchor tip. Since the normal compatability requirements were always enforced between the soil nodes, no separation of material occurred although localised rupture (e.g. between nodes 1 and 2) could occur adjacent to the anchor as has been observed experimentally both by the author and by other workers (e.g. Adams and Hayes, 1967). The significance of this provision for rupture within the soil mass will be discussed in more detail in Chapter 6.

5.4.3 Other Considerations

1) Breakaway

Once breakaway occurs, checks must be made to ensure that the soil nodes do not 'overtake' the anchor nodes (ie. the soil cannot pass through the anchor, or vice versa). If numerical overlapping of nodes is detected within a load increment then the compatability requirement must be reintroduced thereby closing the gap initially formed at breakaway. In this situation a compressive force would develop between the soil and anchor preventing the physically impossible situation of soil passing through the anchor.

2) Slip

As in the case of plastic elements, care must be taken to ensure that the rate of plastic work along a slip line is always positive. This may be simply achieved by checking that the increment

in relative displacement between nodal pairs is in the same sense as the relevant shear force. If negative work is detected then the compatability requirements must be reintroduced to allow unloading at that point on the slip line.

Neither of the above situations arose in the analyses performed for this thesis; however, cases exist (e.g. if loading is followed by unloading) where failure to detect these effects could lead to significant errors.

3) Dilatancy of the Soil-Structure Interface

Provision can be made for a dilating interface (or rupture line) as described in Appendix 5C. However, in general the most realistic situation is considered to be that of no dilation (ie. $\psi = 0^\circ$ at the interface) since this corresponds to the limiting situation where the narrow zone of plastic material at the soil-structure interface has reached its residual strength. In all the analyses performed for this thesis it was assumed that the angle of dilatancy at the soil structure interface (or along a rupture line) was zero.

From the standpoint of classical plasticity, slip can only occur along a velocity discontinuity and hence the apparent adhesion c_k and angle of friction ϕ_k at a rough soil structure interface are given by the expressions (Davis, 1968)

$$c_k = \frac{c \cos \psi \sin \phi}{1 - \sin \phi \sin \psi} \quad (5.40)$$

and

$$\tan \phi_k = \frac{\cos \psi \sin \phi}{1 - \sin \phi \sin \psi}$$

where c and ϕ are the real strength properties of the soil at the interface and ψ is the angle of dilatancy.

In the analysis of rough anchors, c and ϕ were taken to be the same as the values for the general soil mass while, as stated above, ψ was taken to be zero. The values of c_k and ϕ_k at the soil structure interface were then deduced from Eq. 5.40.

In the analysis of footings and horizontal anchors (Chapters 6 and 7) the apparent strength parameters c_k and ϕ_k along potential rupture lines were assumed to have the same values as the strength parameters in the rest of the soil mass (ie. $c_k = c$, $\phi_k = \phi$). For $\psi = 0^\circ$ along the rupture lines this implies that (for $\phi > 0^\circ$) the actual strength along the rupture lines was slightly greater than in the surrounding material. This assumption was made to avoid any possible suggestion that the introduction of potential rupture lines could in any way weaken the material. In the analysis of vertical anchors (Chapter 9) the theoretically correct values of c_k and ϕ_k obtained from Eq. 5.40 were adopted.

5.4.4 Examples

The range of possible applications of the foregoing theory includes footing, tunnel, anchor, retaining wall, pile and slope stability analyses (Rowe, Booker and Balaam, 1976, 1978). Consequently,

in the ensuing chapters the approach will be used to predict the behaviour of footings, anchors, retaining walls and soil slopes; and where possible, the numerical predictions will be compared with relevant analytical or experimental results. However, for the remainder of this chapter, attention will be restricted to an illustration of the major features of the approach by a consideration of four highly idealised examples. In each case, constant strain triangles were used to obtain elastic or elasto-plastic solutions for a homogeneous isotropic soil, with elastic modulus E_s and Poisson's ratio $\nu = 0.48$. The soil in each case was assumed to be purely cohesive with a cohesion c , unit weight γ and coefficient of earth pressure at rest K_0 .

In the second, third and fourth examples the 'structure' is modelled by a number of beam elements.

5.4.4.1 Rigid Footing

In this example the author's approach is compared with various other techniques used in the analysis of a rigid strip footing. Roundoff error was determined by comparison of the results obtained in the elastic range using single and multiprecision arithmetic. Clearly, the significance of any roundoff error present in the elastic solution is compounded for each load step in the full elasto-plastic analysis.

All calculations in this example were performed on a Prime 300 computer, using 32 bit words for single precision and 64 bits for multiprecision floating point arithmetic.

The four methods of simulating rigid footing behaviour considered were:

Method (a): Substructure approach

The displacement and stresses within the soil mass due to a loaded rigid footing were determined using the substructure interaction theory of the previous sections. The analysis is in fact simplified by the observation that a rigid beam can only undergo rigid body motion and hence its contribution to the interaction matrix $[I_{2E}]$ of Eq. 5.39 is known to be zero and need not be calculated.

Method (b): Displacement Defined

Loading of the soil by a rigid footing was simulated without actually modelling the footing, by forcing the vertical displacement along the base of the loaded area to adopt the same, specified, displacement. This was achieved by adding a large 'blanketing' number, X_B , to the appropriate diagonal terms of the stiffness matrix while adopting a corresponding right hand side equal to the blanketing number multiplied by the specified displacement. Rigid behaviour was observed provided the blanketing number was several orders of magnitude larger than the other terms in the stiffness matrix. These equations were then solved for the general displacement vector. Once the displacement vector was known the equilibrating nodal forces at the displacement defined nodes could be calculated.

Method (c): Stiff Elements

In this approach the rigid footing was approximated by applying a uniform load to a layer of elements whose modulus of elasticity, E_B , was considerably greater than the modulus of the underlying soil. Results were then obtained for various stiffness ratios E_B/E_S . This approach can only be used to simulate loading by a rough rigid footing.

Method (d): Link Elements

A convenient method of analysing the rigid footing problem is to introduce dual nodes into the finite element mesh along the soil structure interface. Using beam elements to model the footing [alternatively stiff soil elements could have been used as in method (c)], the nodal forces applied by the structure to the soil at a dual node can be evaluated by incorporating 'springs' between the nodes in both the vertical and horizontal directions. This is analogous to the introduction of special joint elements (Goodman et al., 1968) along the interface. These springs are assigned vertical and horizontal stiffness K_V and K_H respectively. Displacement compatibility at the interface is achieved by assigning the springs very large stiffnesses.

For this example a rigid footing, of width B , was analysed by adopting a high beam stiffness ($E_B I_B$), a high vertical spring stiffness K_V and zero horizontal spring stiffness K_H . Calculations were performed for various ratios of K_V/E_B and $E_B I_B/E_S B^3$ using single and multiprecision arithmetic and the central displacements of

the footing were compared as shown in Fig. 5.3.

Comparisons of Methods

The relative computation times required to set up and factorize the stiffness matrix along with ten load steps for each of the five methods are shown in Table 5.1, (page 172).

It can be seen that substructure and displacement defined methods involve the same number of equations and bandwidth although the substructure approach requires slightly greater computation time.

The introduction of stiff soil or beam elements to model the footing as in methods (c) and (d) increases both the number of equations and the bandwidth with a corresponding increase in computation time.

A comparison between the displacements and stresses obtained from single and multiprecision arithmetic for each approach is given in Table 5.2. The substructure approach [method (a)] gave identical results for the two analyses. The use of a blanketing number on the diagonal in method (b) was seen to cause very little roundoff error. Using multiprecision arithmetic, rigid behaviour was obtained for the stiff soil elements of method (c), however, the use of single-precision arithmetic resulted in unacceptable roundoff errors. Similarly, the introduction of a high stiffness for the quasi-rigid beam element and the corresponding higher link element stiffnesses into the soil stiffness matrix [method (d)] caused serious roundoff errors as can be seen from Fig. 5.3.

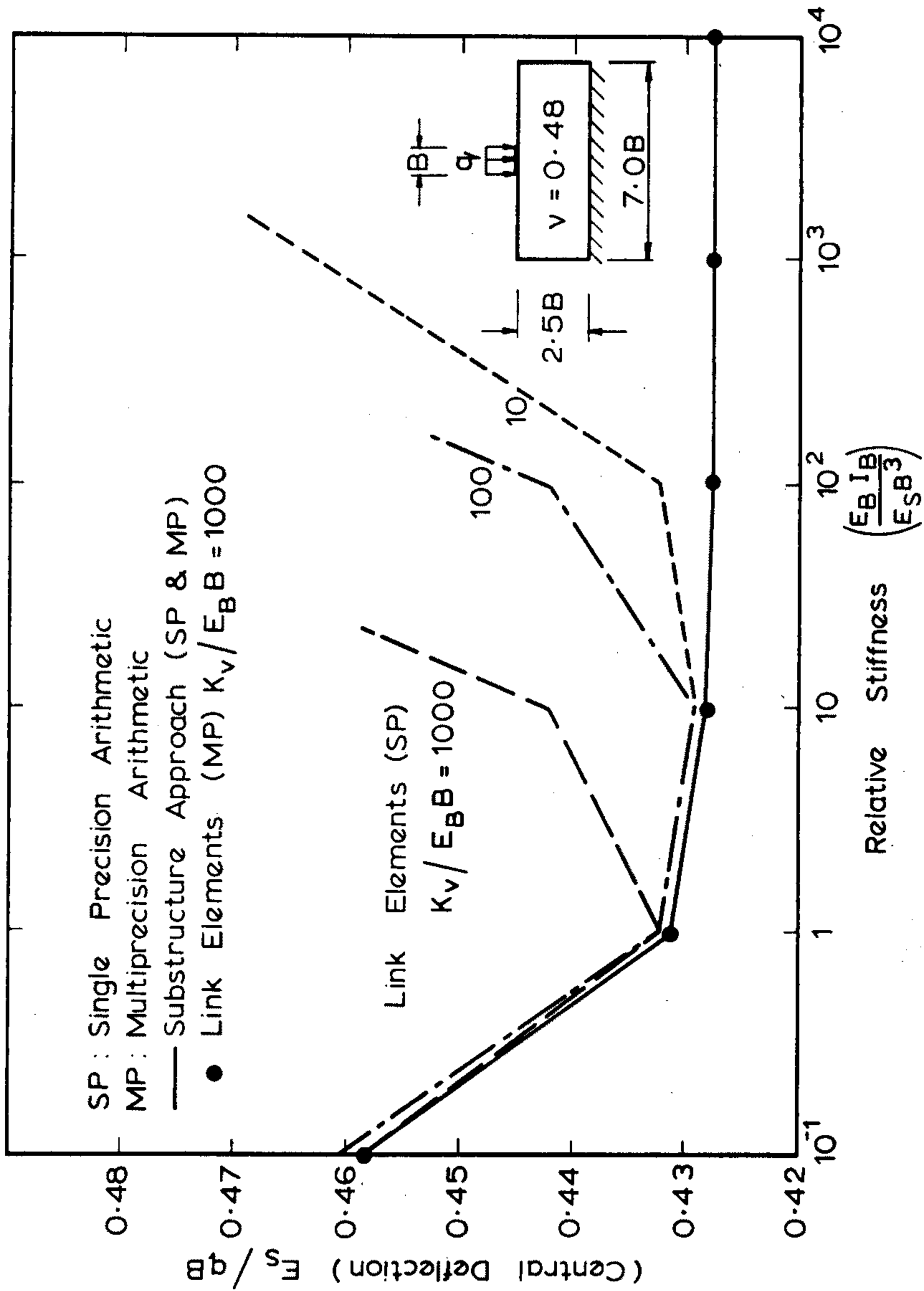


FIG. 5.3 ACCURACY OF SOLUTION METHODS

Except for method (c) which is restricted to the analysis of rough footings, all other methods can be used to analyse either perfectly rough, or perfectly smooth footings. Furthermore, shear failure at the soil-structure interface can be accommodated in the link element and sub-structure approaches.

In the link element approach a rough footing can be analysed by adopting a large horizontal spring stiffness K_H . The shear forces can be calculated for each load increment using a spring analogy. Now considering an interface which exhibits a purely adhesive behaviour, once the tangential nodal force reaches the allowable value, horizontal compatibility at the dual node can be released for future load increments. When using the initial stress approach this modification takes the form of a correction added to the load vector. However, the magnitude of the correction terms are such that it is extremely difficult to obtain a convergent solution to these equations for suitable spring stiffness ratios K_H/E_B and this severely inhibits the advantages of adopting the initial stress method.

The sub-structure approach [method (a)] can be seen to have computational advantages over the alternative methods in that no alteration is made to the soil stiffness matrix. Thus the roundoff error for a rigid or flexible footing analysis would be the same as that for the analysis of a uniform load on the soil. Furthermore, despite the additional computation time required to set up and solve the interaction equation the computation time is less than for all but one of the alternative approaches.

5.4.4.2 A Flexible Strip Footing

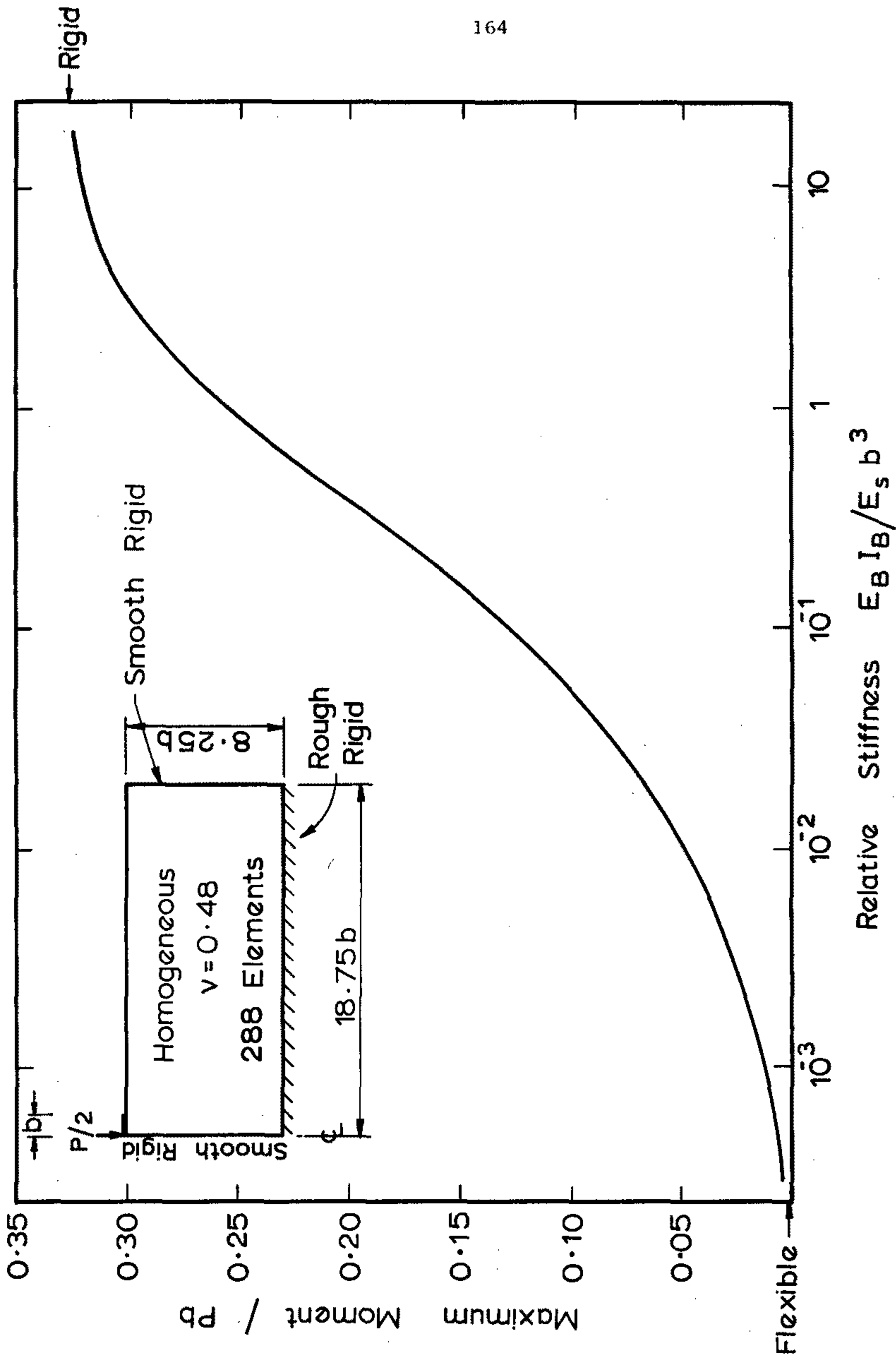
Although the initial stress sub-structure approach was developed with a view to elasto-plastic analyses, it can also be useful in problems where the elastic soil-structure response is required for a number of different structural systems. For example, the variation in maximum transverse bending moment flexural rigidity ($E_B I_B$) was determined for a strip footing subjected to a centrally applied line load P as shown in Fig. 5.4. Similar curves were also obtained showing the effect of flexural rigidity upon both central deflection and differential deflection.

The bending moment distribution within a rigid strip obtained from this analysis is compared with the numerical results obtained by Gorbunov-Possadov and Malikova (1973) for a rigid strip on an elastic half space, as shown in Fig. 5.5.

The major advantage of this approach is that since the soil stiffness matrix remains constant it need only be determined and factorized once. The response for the entire range of flexural rigidities can then be determined simply by modifying the much smaller structural stiffness matrix and the structural contribution to the interaction matrix.

5.4.4.3 Tunnel

In this example the interdependent behaviour of soil and structure is illustrated by considering a smooth rectangular service tunnel of height h resting on a rigid base in a shallow soil layer



Relative Stiffness $E_B I_B / E_s b^3$

FIG. 5.4 MAXIMUM MOMENT IN FOOTING

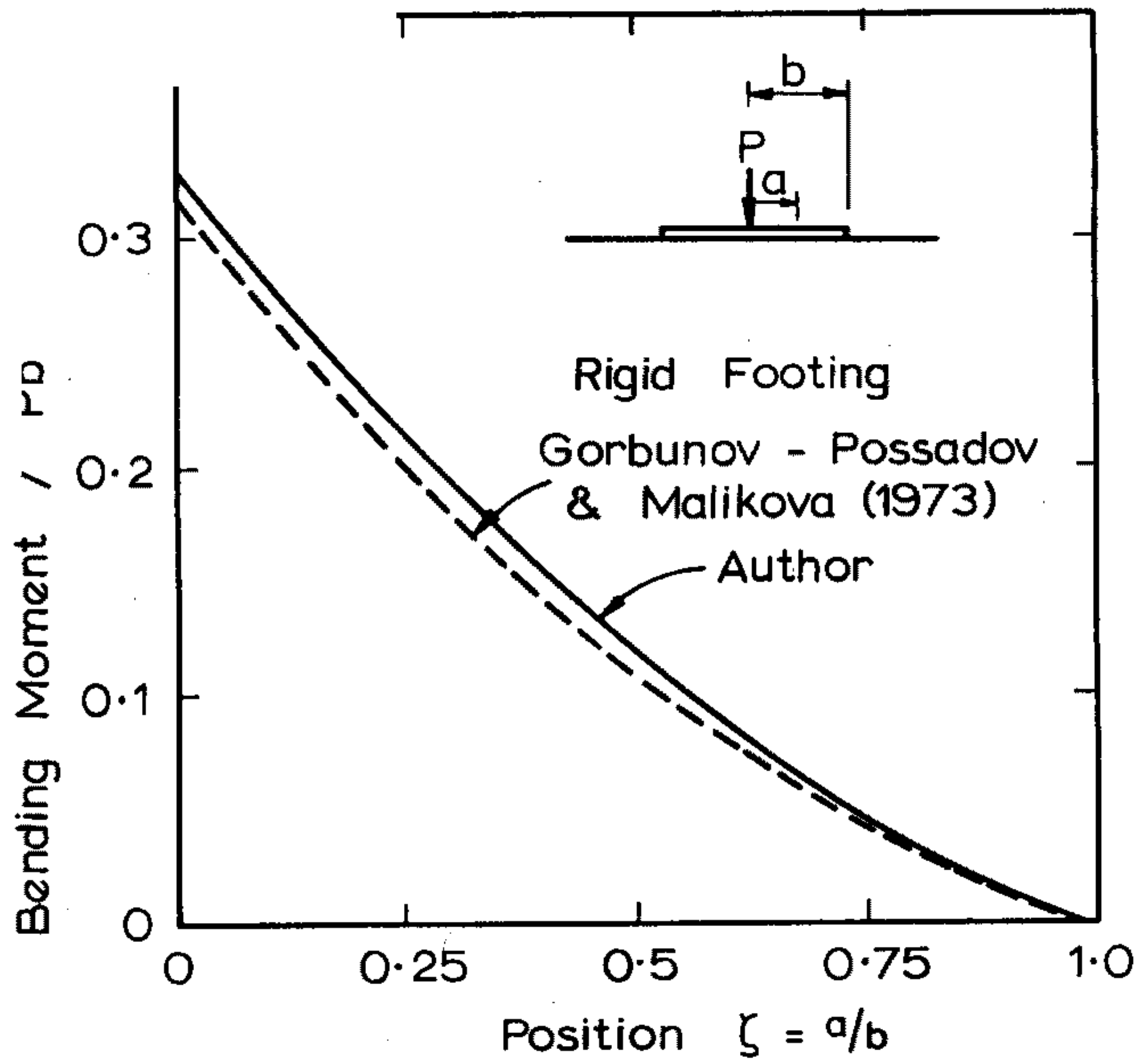


FIG.5.5 TRANSVERSE MOMENT IN RIGID FOOTING

of depth $3.5h$. It is assumed that the tunnel lining remains elastic and has an elastic modulus E_B and uniform thickness t . The soil is assumed to be purely cohesive with dimensionless properties $E_s/c = 200$, $\frac{\gamma h}{c} = 2.0$ and a coefficient of earth pressure at rest $K_0 = 0.93$.

The construction sequence was simulated by applying general K_0 conditions followed by progressive removal of boundary loads. Thus the excavation loads are 'externally supported' during the process of lining and any stresses within the lining are due to the subsequent removal of the support.

The two cases considered correspond to a concrete lining 150 mm thick and 230 mm thick respectively. Fig. 5.6 shows the distribution of bending moment around the moderately flexible tunnel lining with dimensionless properties $E_B t^3/E_s h^3 = 0.131$ and $E_B t/E_s h = 93.8$. It can be seen that the results obtained assuming an elastic soil model are significantly different from those obtained when a full elasto-plastic analysis is performed.

The distribution of bending moment around the semi-rigid lining with properties $E_B t^3/E_s h^3 = 0.469$ and $E_B t/E_s h = 125.0$ obtained from elastic and elasto-plastic analyses are given in Fig. 5.6(ii).

These results indicated that the significance of plastic yield within the soil mass is largely dependent upon the relative stiffness of the tunnel lining. The fairly flexible lining in case (i) permitted considerable plastic yield to occur resulting in a large redistribution of soil stress around the tunnel. The maximum bending moment produced from this soil stress distribution is 37% higher than

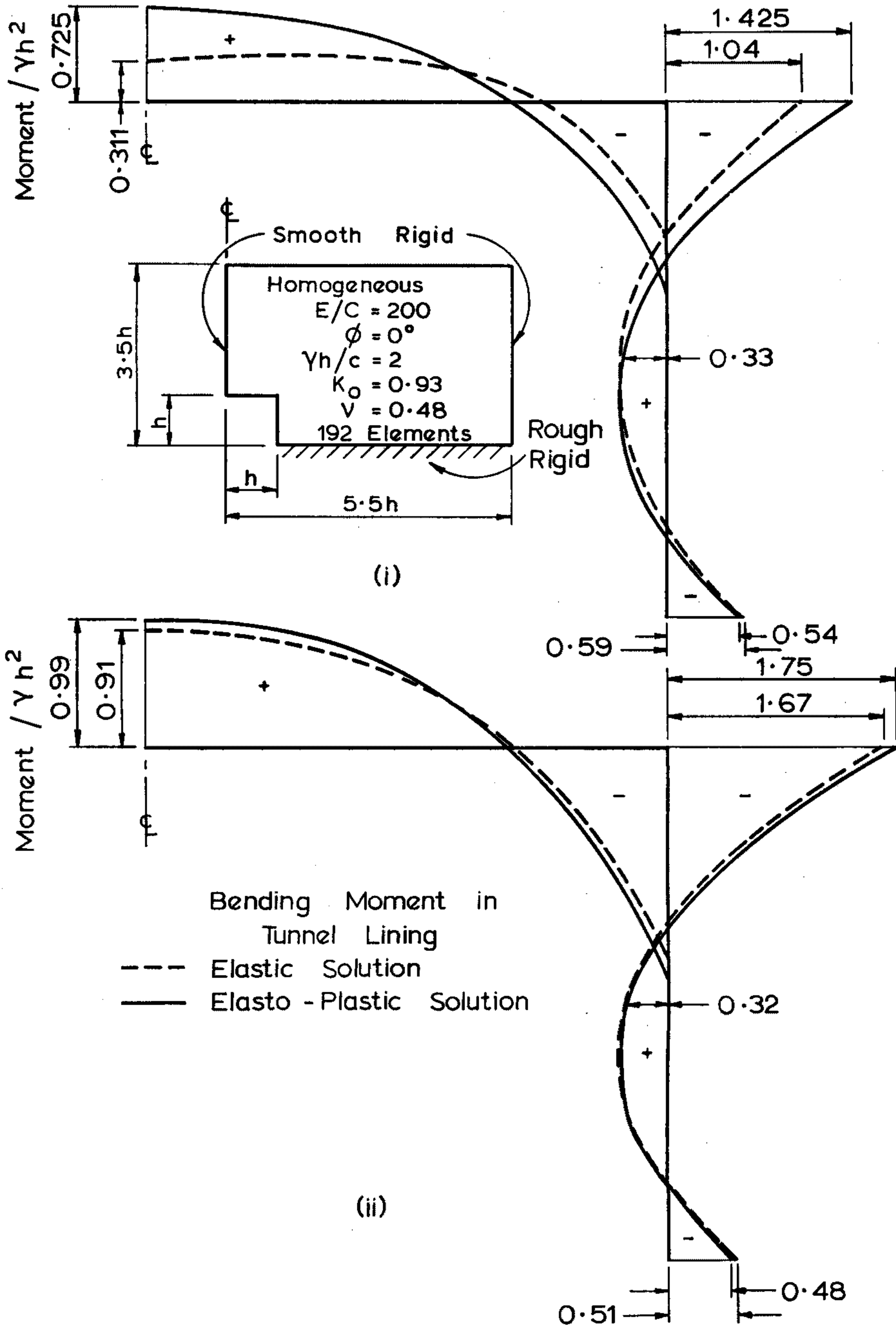


FIG. 5.6 BENDING MOMENT IN TUNNEL LINING

that obtained assuming no soil failure. The semi-rigid lining of case (ii), however, provides much greater support to the soil mass thereby reducing the general spread of plastic yield. Correspondingly the maximum bending moment from the elasto-plastic stress distribution is only 5% greater than that obtained from an elastic distribution.

5.4.4.4 Soil Anchor

Plastic yield within both soil and structure as well as complicated interface conditions can be easily incorporated into the analysis and may be seen to significantly alter the load carrying behaviour of the structure. Consider, for example, a smooth horizontal strip anchor of width B and transverse flexural rigidity $E_B I_B / E_S B^3 = 5.7$ which is buried to a depth $h = 3B$, in a purely cohesive soil mass with coefficient of earth pressure at rest $K_0 = 1.0$, and dimensionless properties $E_S / c = 200$ and $\gamma h / c = 1.13$. The anchor is subjected to a centrally applied line load P .

Three cases are considered as shown in Fig. 5.7. For shallow anchors the assumption made regarding the tension allowed to develop behind the anchor has a significant effect on the load deflection behaviour. In case (i) it is assumed that the soil adheres to the base of the anchor and hence the load is directly transferred to the soil both above and below the anchor. This may be compared with case (iii) where it is assumed that no tension can develop between anchor and soil. Once the stress at the base of the anchor ceases to be compressive the anchor 'breaks away' from the soil behind the anchor.

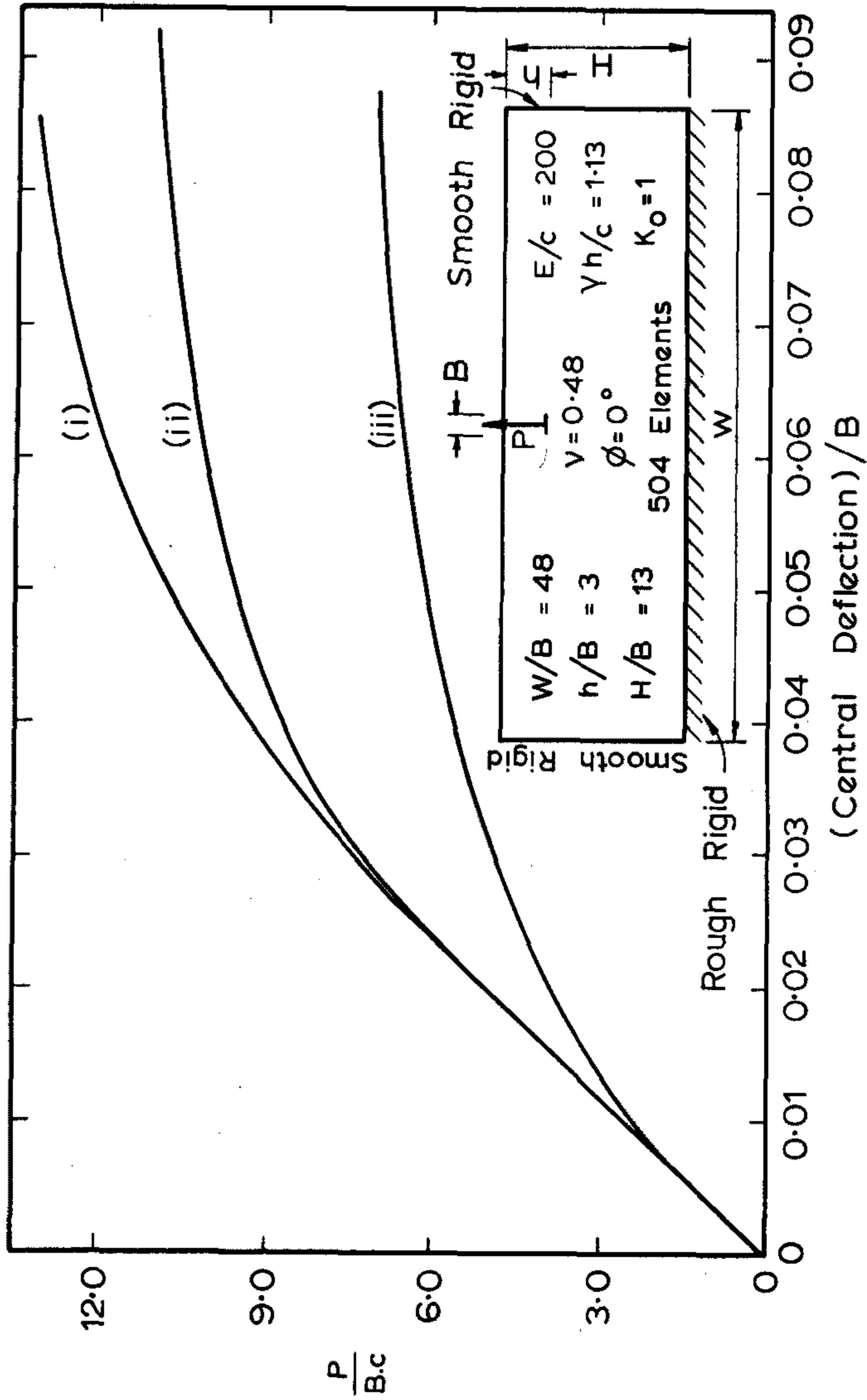


FIG.5.7 LOAD DEFLECTION CURVE FOR A STRIP ANCHOR

In case (ii) it is assumed that the soil adheres to the anchor, and that the anchor has a plastic moment, M_p , where $M_p B/E_B I_B = 2.4 \times 10^{-5}$. A plastic hinge developed at the centre of the anchor for an applied load $P = 6.5 Bc$. The effect of yield within the anchor on its load carrying behaviour can be judged by comparison of curves (i) and (ii) on Fig. 5.7.

This example serves to illustrate the general effects that provision for breakaway, and for the formation of a plastic hinge within the structure, may have upon anchor behaviour. Other possible applications include the provision for the development of tension cracks behind a retaining wall in cohesive material, and provision for the formation of plastic hinges within a braced wall.

5.5 CHOICE OF ELEMENT TYPE

The theoretical expressions presented in the preceeding sections are quite general in that they may be applied to an elastoplastic soil-structure interaction analysis irrespective of the choice of finite element adopted for either soil or structure.

In the context of this thesis, constant strain elements were used in the analysis of soil behaviour while, in general, the structure was modelled using beam elements. For the special case of an infinitely rigid structure (e.g. rigid anchor or footing) a direct analysis of the structure was not necessary since it could only undergo rigid body motion and hence its contribution to the interaction matrix (I_{2E} of Eq. 5.39) is known to be zero and need not be calculated.

It is recognised that the soil and structural elements are not entirely compatible (ie. compatibility at the node does not ensure compatibility between nodes); however, it was found that this limitation is of no practical importance provided that the soil structure interface is adequately subdivided.

Finally, it is recognised that there is an increasing trend towards the use of isoparametric elements and many examples of their successful application have been cited in the literature. However, although there is strong evidence regarding the superior computational efficiency obtained with these elements (as compared to lower order elements) for elastic analyses, the available evidence (for example, Nagtegaal et al., 1974; and more recently Balaam, 1978) indicates that provided both types of element are used intelligently then similar results are obtained for similar computational effort in non-linear analyses; under these circumstances the choice of elements becomes largely a matter of taste.

TABLE 5.1

RELATIVE COMPUTATION TIME FOR SET UP AND
SOLUTION FOR TEN LOAD STEPS

Method	Relative Time	No. of Equations	Bandwidth
a) Substructure	1.07	190	20
b) Displacement Defined	1.00	190	20
c) Stiff Soil Elements	1.17	200	24
d) Link Elements	1.15	199	24

TABLE 5.2

COMPARISON OF ROUNDOFF ERROR SINGLE PRECISION

ARITHMETIC PRIME 300

Method	Percentage Error*	
	Worst error in displacement	Worst error in stress
a) Substructure	<0.05	<0.01
b) Displacement Defined		
$X_B/E_S = 10^4$	0.1	0.13
$X_B/E_S = 10^5$	0.1	0.13
c) Stiff Soil Elements		
$E_B/E_S = 500$	10.0	21.0
$E_B/E_S = 1000$	30.0	53.0
$E_B/E_S = 2000$	83.0	170.0
d) Link Elements	(See Figure 5.3)	

* Relative to results from the same type of analysis using multi-precision arithmetic.

APPENDIX 5AForm of Vectors $\underline{r}$ and $\underline{s}$

In this appendix the form of vectors $\underline{r}$ and $\underline{s}$ used by the author will be presented for the case of plane strain. The actual derivation of these vectors has previously been described by various authors (see for example Small, 1977) and will not be reproduced here. The form of the vectors $\underline{r}$ and $\underline{s}$ for the axi-symmetric case have been given by Small (loc cit).

(i) Form of vector $\underline{r}$

Consider a plastic material with an angle of dilatancy ψ .

If it is assumed that the material has a flow rule specified by Eq. 5.4 and has principal strain rates $\dot{\epsilon}_{1p} > \dot{\epsilon}_{2p} > \dot{\epsilon}_{3p}$, then Eq. 5.4 may be rewritten

$$\begin{aligned}\dot{\epsilon}_{1p} &= \Lambda \\ \dot{\epsilon}_{2p} &= 0 \\ \dot{\epsilon}_{3p} &= -N_{\psi} \Lambda\end{aligned}\tag{5.41}$$

now assuming that the plastic strain increments are in the same directions as the principal stresses then the flow rule is given by Eq. 5.42.

$$\text{ie. } \dot{\underline{\epsilon}}_p = \Lambda \underline{r}$$

where $\dot{\underline{\epsilon}}_p^T = (\dot{\epsilon}_{xp}, \dot{\epsilon}_{yp}, \dot{\gamma}_{xyp})$

$$\underline{r} = \begin{bmatrix} a - b \\ -a - b \\ c \end{bmatrix} \quad (5.42)$$

$$a = 2(\sigma_{xx} - \sigma_{yy})$$

$$b = 2 \sin \phi \sin \psi (\sigma_{xx} + \sigma_{yy} + 2c \cot \phi)$$

$$c = 8\sigma_{xy}$$

(ii) Form of the vector $\underline{s}$ for the Mohr-Coulomb failure criterion

In equation 5.11 the vector $\underline{s}$ was defined as

$$\underline{s} = \frac{\partial f}{\partial \underline{\sigma}}$$

where f was the failure criterion. For the particular case where the failure criterion is the Mohr-Coulomb criterion, $f = \sigma_{\max} - N_{\phi} \sigma_{\min} - 2c\sqrt{N_{\phi}}$ we can write in general

$$\underline{s} = \frac{\partial f}{\partial \sigma_{\max}} \left\{ \frac{\partial \sigma_{\max}}{\partial \underline{\sigma}} \right\} + \frac{\partial f}{\partial \sigma_{\min}} \left\{ \frac{\partial \sigma_{\min}}{\partial \underline{\sigma}} \right\}$$

or

$$\underline{s} = \left\{ \frac{\partial \sigma_{\max}}{\partial \underline{\sigma}} \right\} - N_{\phi} \left\{ \frac{\partial \sigma_{\min}}{\partial \underline{\sigma}} \right\} \quad (5.43)$$

where $\underline{\sigma}$ is the vector of stress components

$$\underline{\sigma}^T = (\sigma_{xx}, \sigma_{yy}, \dots)$$

For a Mohr-Coulomb material under conditions of plane strain

we can write the failure criterion more simply as given in Eq. 5.2

ie.

$$f = (\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 - \sin^2\phi(\sigma_{xx} + \sigma_{yy} + 2c \cot \phi)^2$$

where σ_{xx} , σ_{yy} are the normal stresses (compression considered positive) referred to an x-y Cartesian coordinate system and σ_{xy} is the shear stress.

The vector $\underline{s}$ now becomes simply

$$\underline{s} = \begin{bmatrix} 2(\sigma_{xx} - \sigma_{yy}) - 2 \sin^2\phi(\sigma_{xx} + \sigma_{yy} + 2c \cot \phi) \\ 2(\sigma_{yy} - \sigma_{xx}) - 2 \sin^2\phi(\sigma_{xx} + \sigma_{yy} + 2c \cot \phi) \\ 8\sigma_{xy} \end{bmatrix} \quad (5.44)$$

APPENDIX 5BEquations for $c - \phi$ Slip at the Soil-Structure Interface

In this appendix the analysis of section 5.4 will be extended so that the soil structure interface strength can be either adhesive, adhesive-frictional or purely frictional. This may be achieved in a number of ways. One such method (Rowe et al., 1978; Balaam, 1978) involves transformation of the nodal force and deflection axes at the interface by the angles of dilatancy and friction respectively. This approach has some advantages for the case of frictional dilatant slip, particularly where the interface has an associated flow rule. However, this approach may be slightly cumbersome when the soil structure interface is inclined at variable angles to the x-y axis system. Consequently in this and the following appendix a more straightforward approach will be presented for consideration of $c - \phi$ slip and a dilatant interface. The difference between the approach presented here and that presented by Rowe et al., (1978) is one of form rather than of essence.

Consider the general case of a soil structure interface rotated at some angle to the x coordinate direction, then the normal nodal force is assumed to be negative when acting from the first soil node towards the second node (be it a structural or soil node) within the pair. Accordingly, a negative normal force corresponds to a tensile stress acting over the relevant portion of the interface.

Assuming that slip between the soil and structure is governed by a Mohr-Coulomb law (where there is an apparent interface adhesion c_k and angle of friction ϕ_k) then:

- (1) The soil and structure adhere completely if

$$|F_T| < c_k \cdot l + F_N \tan \phi_k \quad (5.45)$$

where l is the length of influence of the nodal pair
and F_T , F_N are the tangential and normal nodal forces
respectively; and

- (2) slip occurs at the soil structure interface when

$$|F_T| = c_k \cdot l + F_N \tan \phi_k \quad (5.46)$$

Assume for simplicity that $F_T > 0$ then if during a load
increment the nodal forces move from a valid state specified by F_T
and F_N to a new state on the yield surface specified by $(F_T + \Delta F_T)$,
 $(F_N + \Delta F_N)$ then

$$\Delta F_T - \Delta F_N \tan \phi_k = c_k \cdot l - F_T + F_N \tan \phi_k \quad (5.47)$$

If F_T and F_N already lie on the failure surface this reduces
to

$$\Delta F_T - \Delta F_N \tan \phi_k = 0 \quad (5.48)$$

Thus, once the occurrence of slip is detected (by violation of Eq. 5.45)
the relevant tangential compatibility equation in Eq. 5.39 is replaced
by Eq. 5.47 (which degenerates to Eq. 5.48 for future load increments).

In applying this numerical routine, checks must be made to
ensure that Eq. 5.46 is not violated; that is to ensure that

$$F_N \geq -c_k \cdot l \cot \phi_k \quad (5.49)$$

APPENDIX 5CEquations for Dilatancy at a Soil-Structure Interface

As described in the previous appendix, once slip occurs Eq. 5.39 is modified such that the relevant tangential compatibility requirement between the soil and structural nodes is replaced by the failure condition (Eq. 5.48) for further increments in applied load. If the interface does not dilate then the requirement for compatibility in the normal direction is maintained. To incorporate a simple dilatancy behaviour into the analysis, the appropriate compatibility equation in the normal direction is replaced by

$$\tan \psi = \frac{\Delta u_{N2} - \Delta u_{N1}}{\Delta u_{T2} - \Delta u_{T1}} \quad (5.50)$$

for all additional increments in applied load, where

Δu_{N1} , Δu_{T1} and Δu_{N2} , Δu_{T2} are the increments in normal and tangential displacement at nodes 1 and 2 (e.g. soil and structural nodes) respectively.

CHAPTER SIX
APPLICATION OF THE FINITE
ELEMENT METHOD TO THE
PREDICTION OF COLLAPSE LOADS

6.1 INTRODUCTION

The suitability of the elasto-plastic finite element method, not only for the prediction of the load-deformation behaviour after initial yielding, but also for the prediction of the load at which complete collapse occurs, has been widely recognised in recent years (e.g. Yamada et al., 1968; Zienkiewicz et al., 1969; Desai and Reese, 1970; Chung and Lee, 1972; Zienkiewicz et al., 1975). This has been accompanied by ever increasing demands to apply the method to more and more complicated and varied problems; problems for which the use of classical plasticity approaches are too difficult or impossible. Practical application of these techniques, however, requires confidence in the numerical collapse loads obtained, and this can only be achieved by consideration of less complicated problems with known solutions.

Much of the early work (Yamada et al., 1968; Zienkiewicz et al., 1969) was concerned with the application of the method to certain well defined problems such as the loading of a 'V' notched tensile specimen or expansion of a cylinder. These problems involve relatively simple stress states at failure and the finite element method gives good estimates of the collapse load even when a fairly coarse mesh and loading procedure is adopted. However, recent studies (e.g. Nagtegaal et al., 1974; Davidson and Chen, 1974) have found that for more complex problems the finite element method often fails to give a clearly defined collapse load and even with the use of fine mesh arrangements may give numerical collapse loads well above the analytical values obtained from plasticity theory.

A considerable amount of research has been published on the

effects of mesh refinement and convergence of iterative procedures (e.g. Yamamoto and Tokuda, 1971; Nayak and Zienkiewicz, 1972a, b; Yamamoto, 1973; Nagtegaal et al., 1974 and Carey, 1976) and it appears to be generally accepted that a numerical estimate of the collapse load for a given problem can be improved by refining the finite element mesh, although Nagtegaal points out that this will only be true for certain types and/or arrangements of elements. Refinement of the mesh often increases convergence difficulties and is usually accompanied by a decrease in the incremental step size used. Unfortunately, the computational costs involved in adopting a finite element mesh and load path which would give a numerical collapse load within a few percent of the true failure load is prohibitive for many practical problems.

In this chapter, the general features of an elasto-plastic analysis will be examined with particular reference to a rigid footing resting on a deep homogeneous soil layer, and a rigid anchor buried at infinite depth in a homogeneous soil mass. Based on this experience, two techniques are presented for the prediction of collapse loads. The first is a simple extrapolation technique and the second attempts, at least in an approximate way, to allow the development of free plastic flow (which tends to be inhibited in the usual stiffness formulation of the finite element method), by permitting the formation of velocity discontinuities in regions with high stress and velocity gradients. Both techniques give a better estimate of the collapse load for a particular problem than would otherwise have been obtained with the same degree of numerical sophistication. The first has the advantage that it may be adopted without changing existing programs, while the second requires some modification to the general

finite element approach but is applicable to a wider range of problems.

It will be seen that the adoption of these special techniques can lead to an improved estimate of collapse load for a number of straightforward problems where the theoretical collapse load is known. However, the advantages of adopting these techniques are of greatest significance in the analysis of buried structures, such as an anchor plate, where the stress and velocity fields at collapse are considerably more complex than that encountered in rigid footing analyses. In fact, a buried anchor generally requires four times the number of finite elements necessary for an analysis of similar accuracy involving a surface footing.

6.2 APPLICATION OF ELASTO-PLASTIC TECHNIQUE TO WELL BEHAVED PROBLEMS

Although the use of elasto-plastic techniques in the analysis of simple problems is well documented (e.g. Yamada et al., 1968; Zienkiewicz et al., 1969), it is useful to consider some further examples here, firstly to demonstrate that the finite element formulation adopted gives reasonable and believable results, and secondly to indicate the effect increasing complexity has upon the accuracy of the numerical collapse load.

Unless otherwise stated, all the finite element analyses presented in this chapter were obtained for a smooth rigid footing or anchor with half width b resting on a homogeneous soil layer of depth H . The soil has a modulus of elasticity E , Poisson's ratio ν , cohesion c and angle of internal friction and dilatancy of ϕ and ψ res-

pectively. A Mohr-Coulomb failure criterion was assumed.

The soil mass was modelled using constant strain triangles, while rigid loading was simulated by solving a set of interaction equations as described in Chapter 5. Typically, the finite element mesh was divided into a rectangular grid in which each quadrilateral shown was further divided into four constant strain triangles. In general, these triangular elements are not shown for clarity.

As a check on the numerical methods adopted, and the computer program itself, a thick wall cylinder with internal radius 'a', external radius '2a' was analysed as shown in Fig. 6.1. The cylinder was subjected to an internal pressure q and zero external pressure. This problem has been previously analysed by Small (1977) and was selected because an analytical solution can be obtained for the entire elastic/elasto-plastic/plastic range (Chapman, unp.). The cylinder was analysed for both the drained and the related undrained parameters, where

$$\begin{array}{lll}
 E'/c' = 200 & \text{and} & E_u/c' = 300 \\
 \nu' = 0.0 & & \nu_u = 0.48 \\
 \phi' = 30^\circ & & c_u/c' = 0.866 \\
 \psi = 0 & & \phi_u = 0
 \end{array}$$

The results of the elasto-plastic finite element analysis are shown in Fig. 6.1 and can be seen to agree quite closely with the analytical solution for the entire range of the behaviour.

As a second example, consider a smooth rigid punch indenting

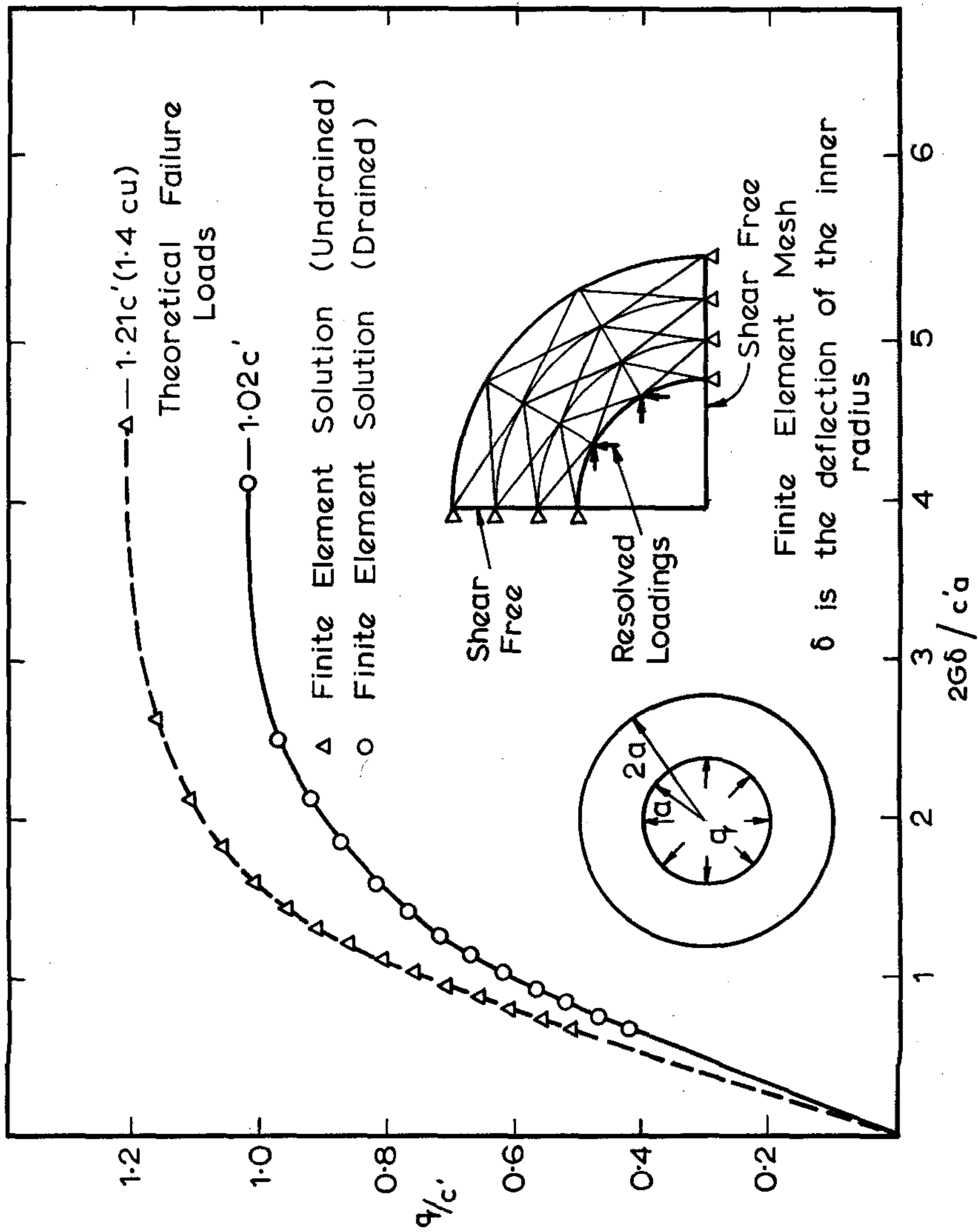


FIG. 6-1 EXPANSION OF A CYLINDER

a soil block of depth $H = b$ and width $W/b = 3$ as shown in Fig. 6.2. The block is underlain by a smooth rigid base, and the lateral boundaries are stress free. The soil was assumed to be purely cohesive with properties $E/c = 266$ and $\nu = 0.48$.

Although some local yielding takes place at the edge of the punch for low loads, the response of the punch is essentially elastic until the load level reaches 95% of the collapse load. The zone of yielded material then rapidly spreads from the edge of the footing towards the base of the soil layer. For a given increment in footing displacement no additional load can be carried at a load level equivalent to 99.6% of the theoretical collapse load of $4bc$.

A slightly more complex problem can be devised by increasing the depth of the soil layer to a value of $H/b = 6$ as shown in Fig. 6.3. In terms of classical plasticity this problem is analogous to the 'V' notched tension specimen analysed by Yamada et al., (1968) and Zienkiewicz et al., (1969). The load deflection curve obtained from the finite element analysis indicates a clearly defined collapse load which lies between the upper and lower bounds obtained by Davis and Booker (1975) and was 1.7% above the best lower bound solution.

The mode of failure observed in the previous problem can be significantly altered by increasing the distance to the stress free boundary to give a dimensionless width $W/b = 11$ as shown in Fig. 6.4. The collapse load indicated by the finite element analysis of this problem was found to be 2% higher than the theoretical failure load obtained by Hill (1950).

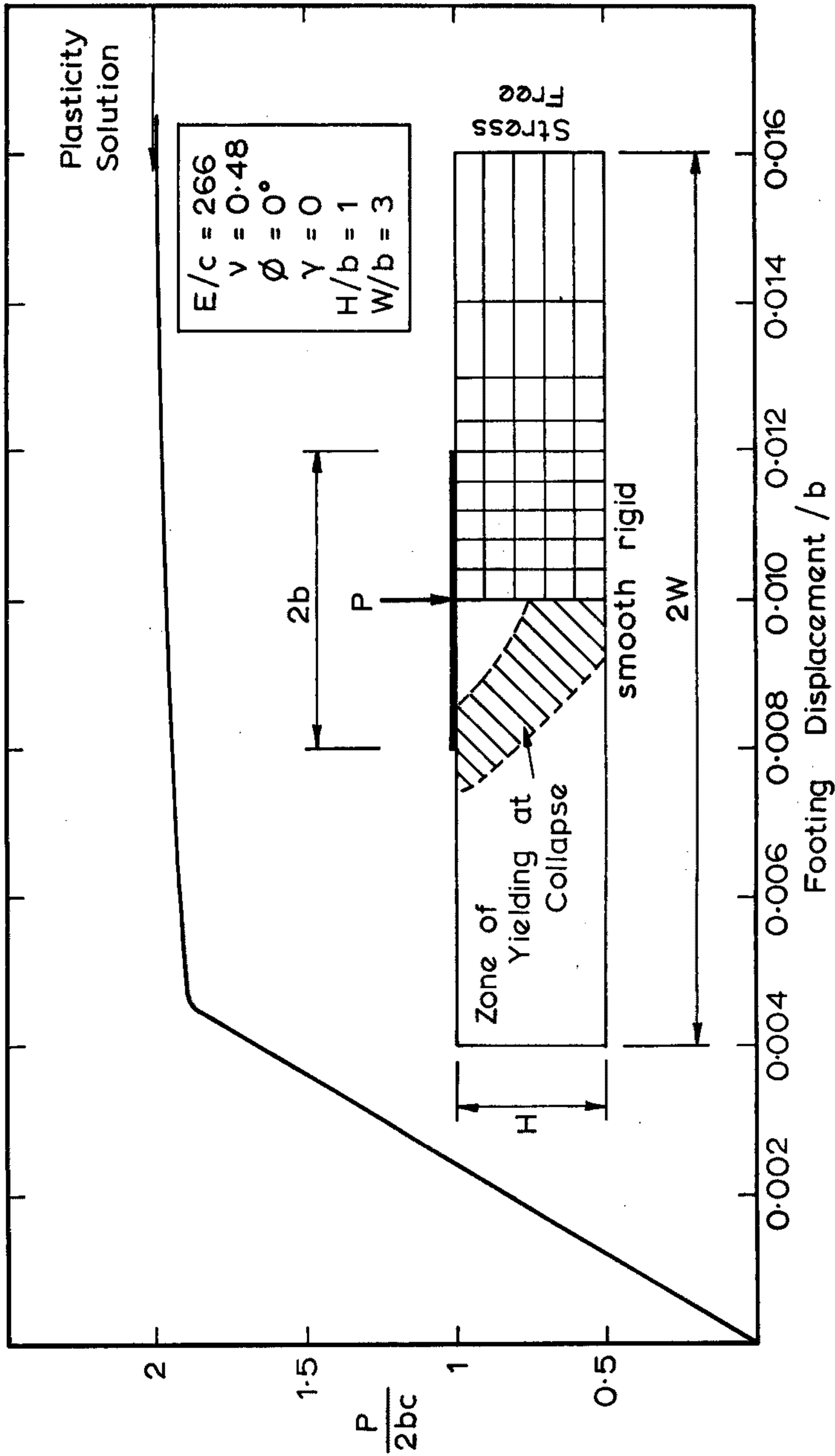


FIG.6.2 LOAD DEFLECTION CURVE FOR RIGID FOOTING ON SHALLOW SOIL LAYER

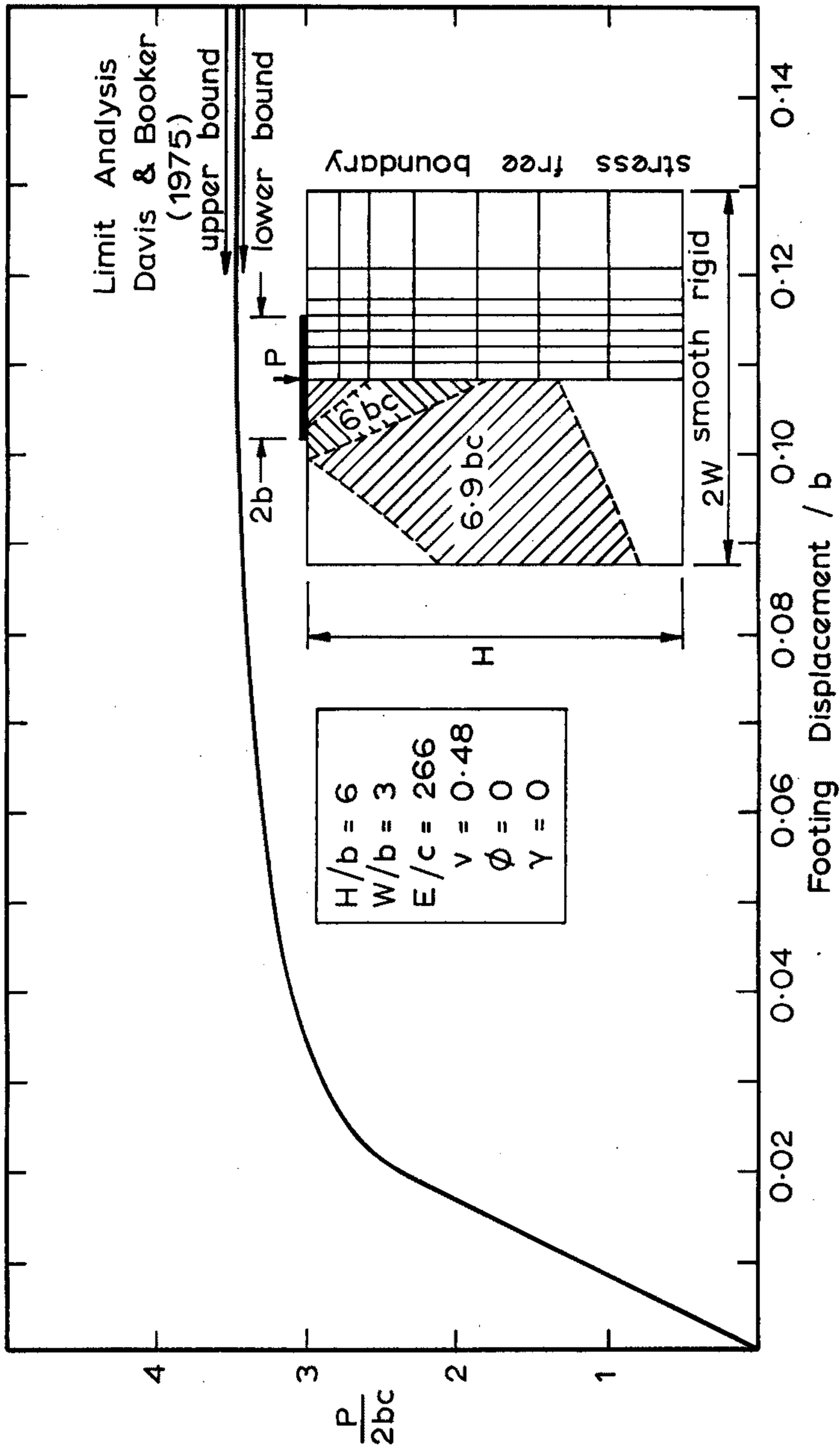


FIG. 6.3 LOAD DEFLECTION CURVE FOR A RIGID FOOTING NEAR A
FREE EDGE

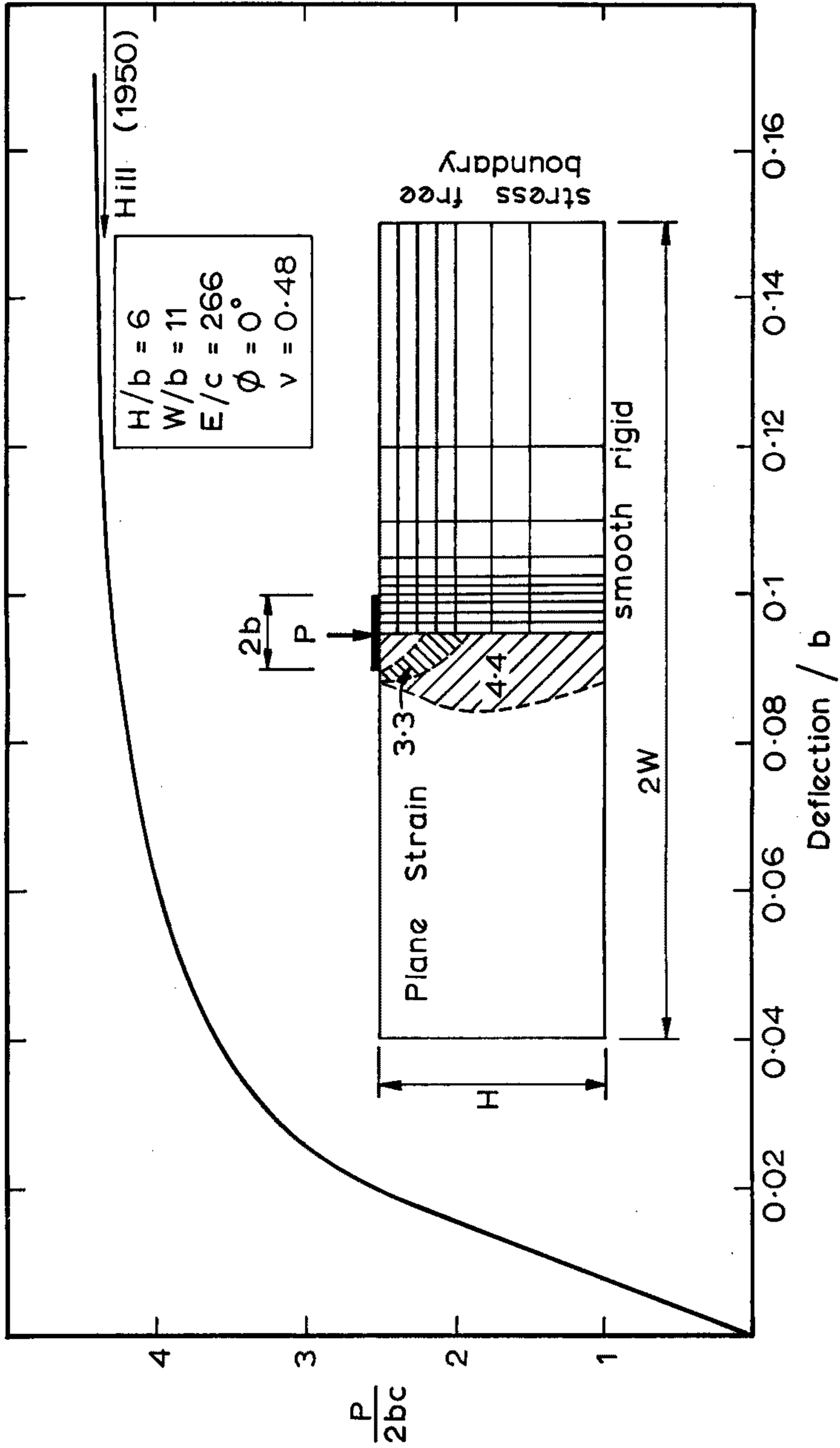
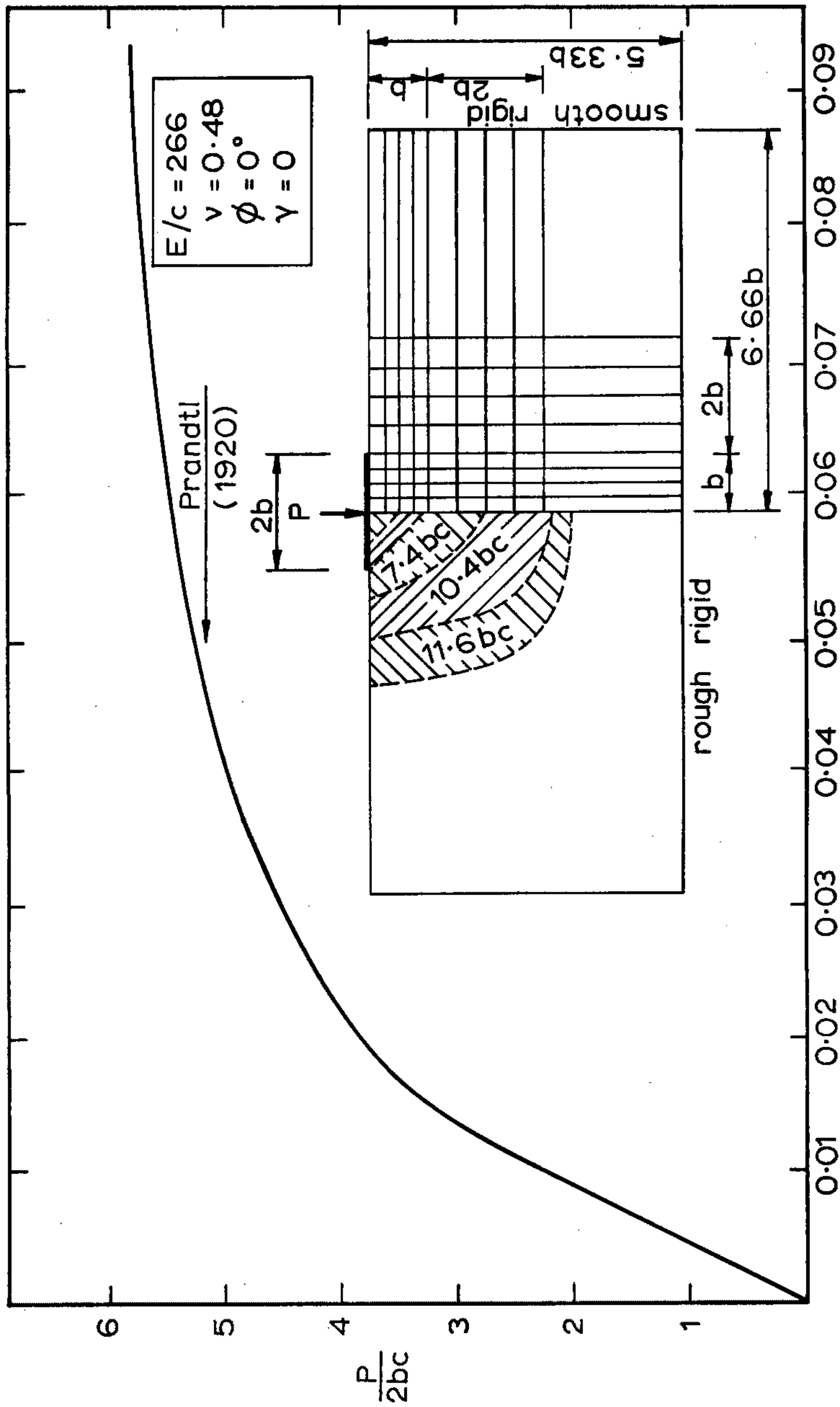


FIG.6.4 LOAD DEFLECTION CURVE FOR PUNCH INDENTATION

Finally, consider a rigid footing resting on a soil layer of depth $H = 5.33b$ underlain by a rough rigid base and with a smooth rigid boundary at a distance $W/b = 6.66$ from the centre line of the footing, as shown in Fig. 6.5. In this problem failure was not clearly defined when loading was stopped at an applied load 14% above the analytical collapse load (Prandtl, 1920). The point of termination of this run was somewhat arbitrary since the iterative procedure had converged satisfactorily at each load step, and the load displacement curve was still rising despite the existence of an extensive region of plastic material surrounding the footing, as shown in the insert to Fig. 6.5. The fact that uncontained plastic flow, corresponding to collapse of the footing, had not occurred indicates that while the plastic deformation corresponding to collapse could be adequately modelled for problems with fairly simple failure mechanisms, such as in examples one to four, difficulties are encountered in modelling the intense plastic deformation which must occur in problems with more complex failure mechanisms.

In examples two to five, which involve indentation of a rigid die or footing into a soil mass, the difference in the complexity of the stress state and velocity field at collapse can be quantified in terms of the significance of the centred fan which occurs at the footing tip in the classical plasticity solution. Thus, in the second example, the depth of the soil-layer and boundary conditions are such that failure is associated with a fairly simple stress state which does not involve a centred fan and the discrepancy between the finite element collapse load and the analytical values is less than 0.4%. The difficulty in accurately predicting the collapse load for the remaining three problems increases with the complexity of the stress state and the associated ve-



Footing Displacement / b

FIG. 6.5 LOAD DEFLECTION CURVE FOR A RIGID FOOTING

locity field at collapse. Thus the lower bound solution for the third problem can be associated with a 41° centred fan at the edge of the footing while the fourth and fifth problems involve a 67° and 90° centred fan respectively. Correspondingly, the discrepancy between the best statically admissible solution and the finite element collapse load increases from 1.7% for the third problem to 2% for the fourth and 14% for the fifth. Thus the accuracy of the finite element collapse load for a given arrangement of finite elements is seen to decrease significantly as the stress state becomes more complex.

6.3 APPLICATION TO DIFFICULT PROBLEMS

The application of elasto-plastic finite element techniques to the prediction of collapse loads for complex problems requires careful consideration of the many factors involved in obtaining the finite element solution. In particular it is necessary to ensure that the behaviour at the singularity is modelled as accurately as is economically possible and that the secondary considerations, such as the step size and convergence criteria, do not lead to a false solution. These requirements will now be considered in more detail.

6.3.1 Mesh Refinement and Element Distribution

Three related finite element meshes corresponding respectively to 100, 196 and 324 constant strain triangular elements are shown in Fig. 6.6. Rigid footing analyses were performed as previously described using a displacement increment of 0.0026b. The load deflection curves obtained for the three mesh arrangements

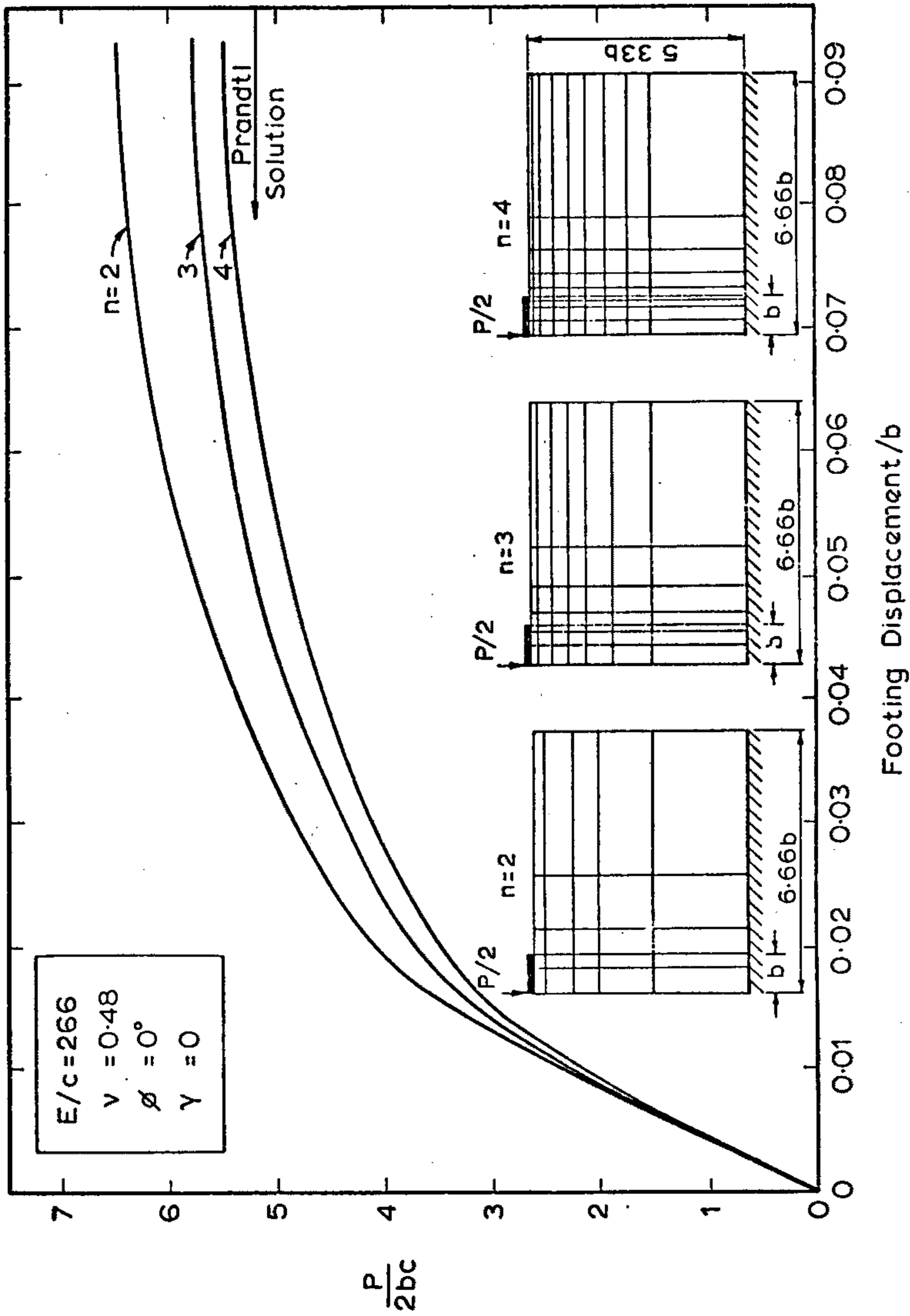


FIG.6.6 LOAD DEFLECTION CURVES FOR RIGID FOOTING-SERIES B

(Fig. 6.6) can be seen to vary significantly with the different numbers of elements. As might be expected, the finest mesh gave the best estimate of the collapse load, however, even in this case, the collapse load was not clearly defined when the run was terminated at an applied load 7% above the Prandtl solution.

To investigate the influence of element distribution upon the collapse load for an approximately constant number of elements, load deflection curves were obtained for a rigid footing using the three meshes shown in Fig. 6.7. From Fig. 6.8 it can be seen that the accuracy of the numerical collapse load increased as the proportion of elements clustered around the footing tip increased. It should be observed, however, that while there was a 6% difference in the collapse load obtained using meshes (i) and (ii), there was only a 3% difference between the collapse loads of meshes (ii) and (iii) indicating that the rate of improvement in predicted collapse load falls off rapidly as the number of elements around the footing tip increases.

Two points emerge from these numerical experiments.

Firstly, the accuracy to which the complex behaviour at the edge of the rigid footing can be modelled significantly alters the numerically predicted collapse load for the footing, and secondly, even with a fairly large number of elements, the numerical collapse load was not clearly defined at load levels well above the analytical collapse load. Furthermore, as will be seen later, these observations are even more apparent in the analysis of anchor behaviour where the stress state of collapse may involve up to a 270° centred fan at the edge of the anchor as compared to the 90° fan for the rigid footing.

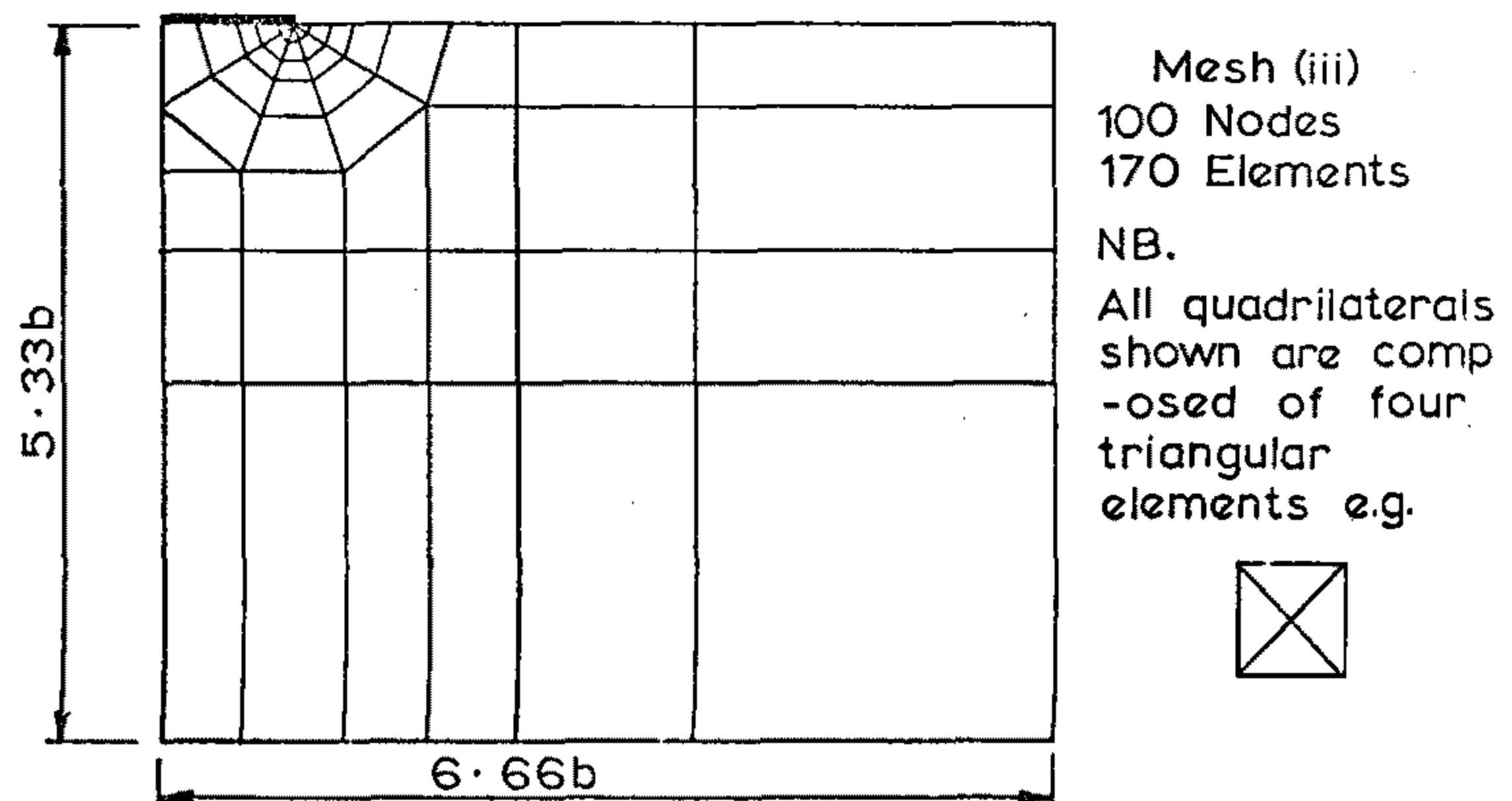
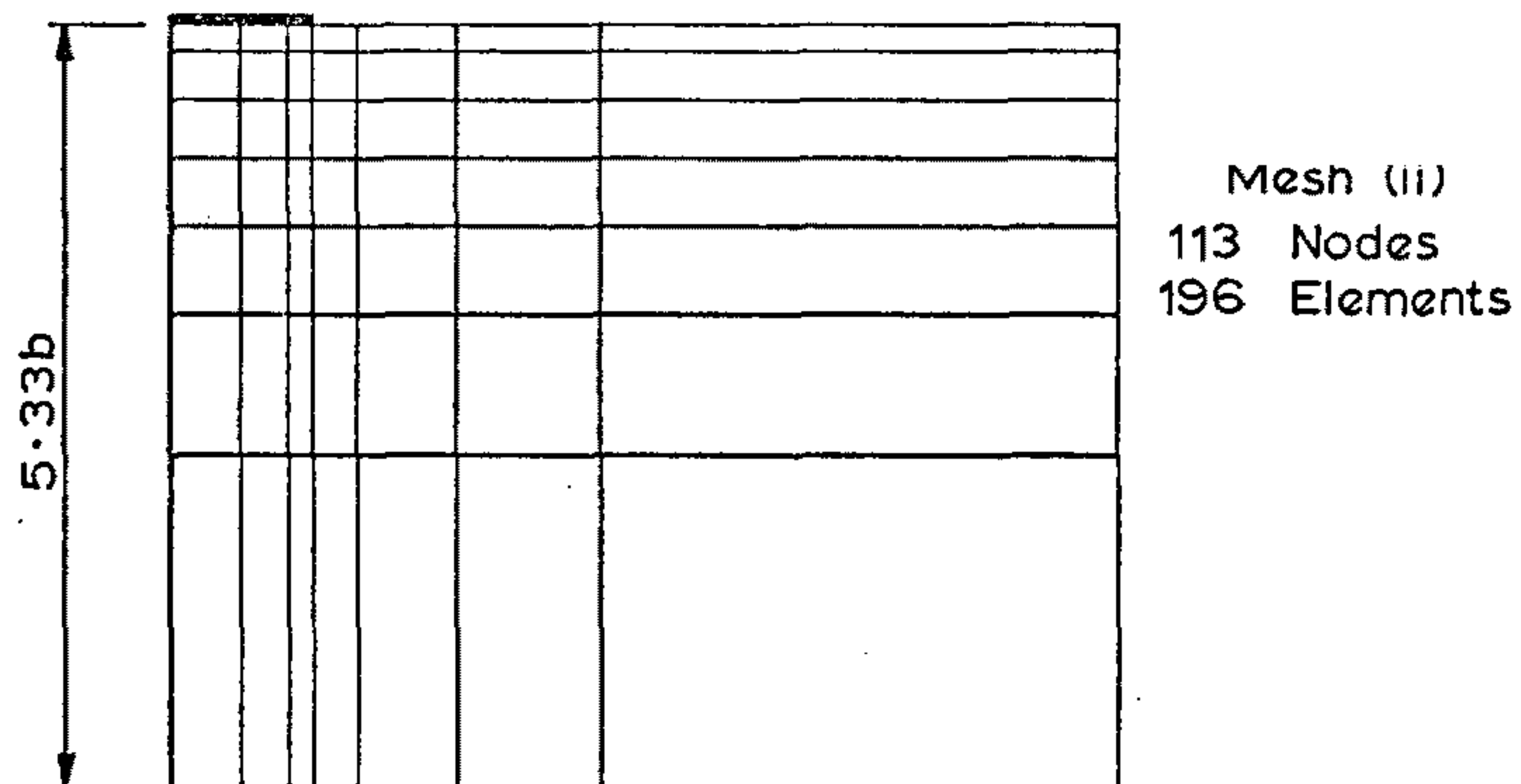
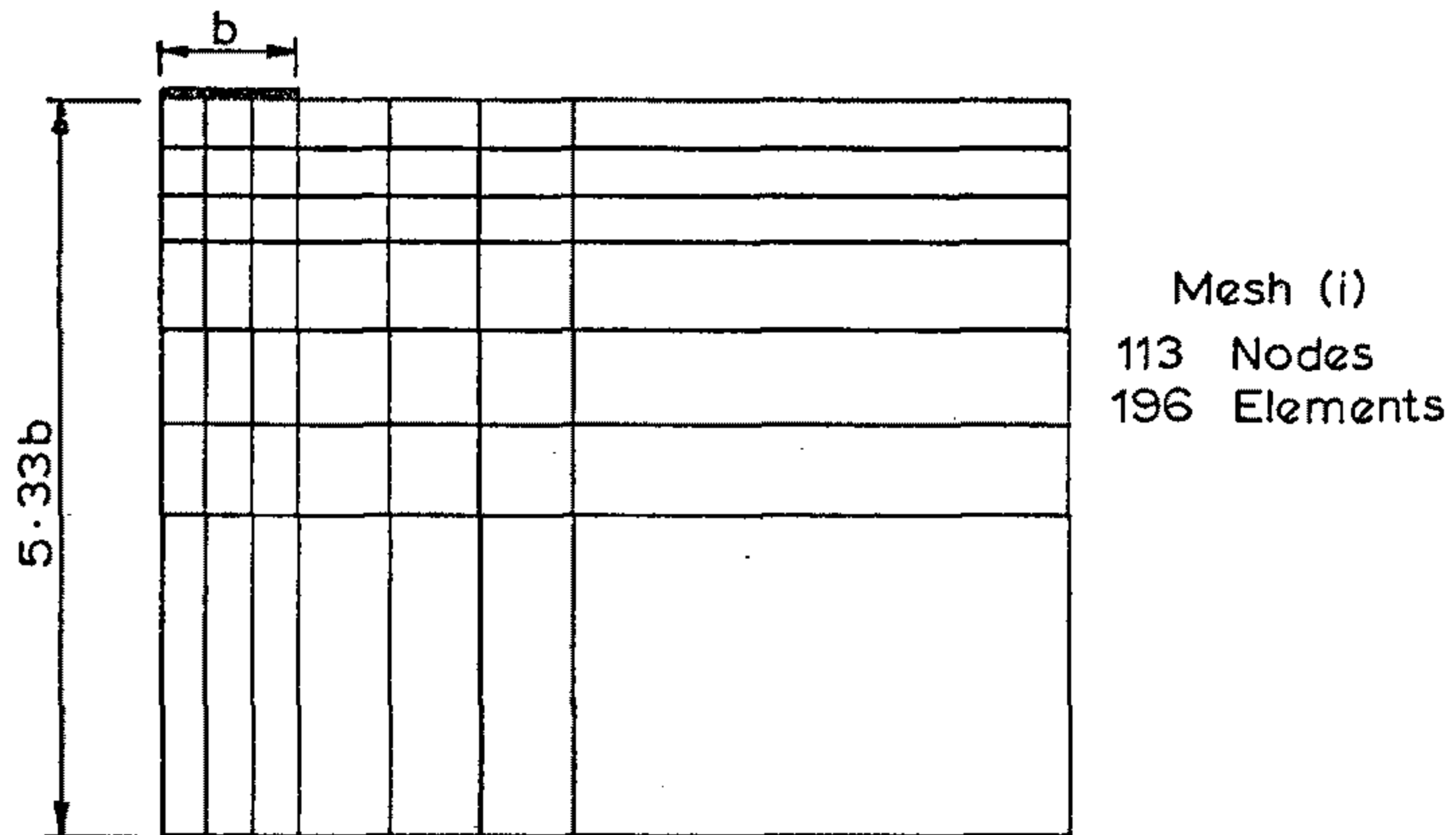


FIG.6.7 VARIOUS MESH ARRANGEMENTS

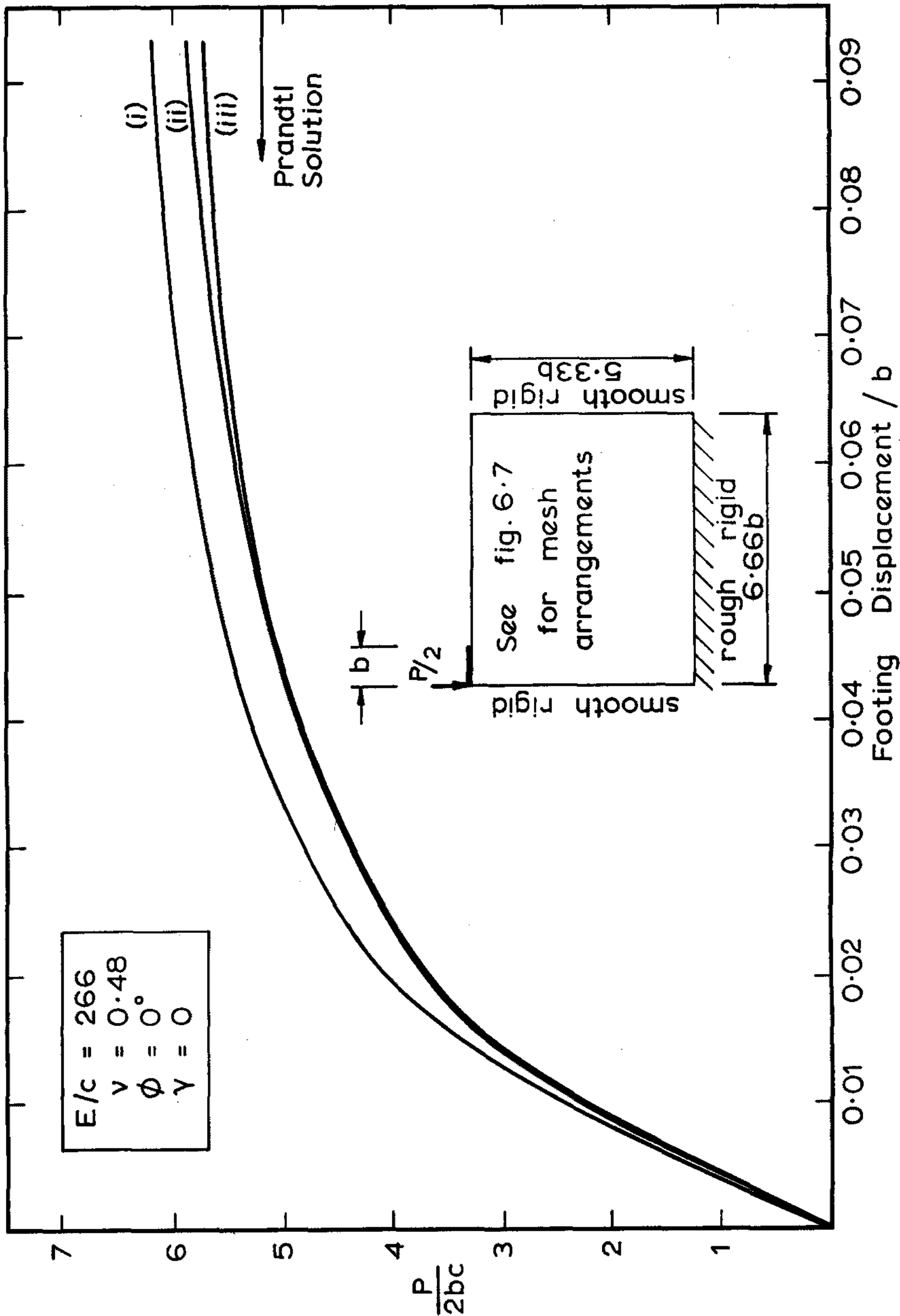


FIG.6.8 EFFECT OF MESH ARRANGEMENT ON THE LOAD - DEFLECTION CURVE OF A RIGID FOOTING

6.3.2 Convergence Criteria and Increment Size

The adoption of an incremental plasticity theory and an iterative solution technique such as used in the initial stress method leads to errors due to increment size and convergence. These errors are related, with the effect of increment size upon the numerical results (and the load deflection curve in particular) depending upon the type of loading, the nature of the problem, and convergence.

Convergence

Convergence criteria may take many forms, however they are all intended to indicate whether the non-linear equations describing the behaviour of the material when subjected to an increment in load have been solved to sufficient accuracy. This is often achieved by comparing the change in some aspect of the solution between iterations, and terminating the procedure once this change has become sufficiently small. Clearly, therefore, the usefulness of convergence criteria depends both on the aspect being considered and the interpretation of what constitutes a 'sufficiently small' change.

The computer programs used to obtain the results presented in this thesis incorporate seven convergence criteria which may be used separately or in any selected combination. Some of these convergence criteria have been previously described in the literature (e.g. Nayak and Zienkiewicz, 1972b) and have undoubtedly been adopted

by many other workers in the field. However, for completeness, all convergence criteria considered by the author will be described.

These convergence criteria were:

1. Stress

For each element, the change in the mean stress $p = (\sigma_1 + \sigma_3)/2$ and the radius of the Mohr circle $r = (\sigma_1 - \sigma_3)/2$ between iteration $i-1$ and iteration i are compared as a percentage of the shear strength of the element, viz.

$$\frac{(|p_i - p_{i-1}| + |r_i - r_{i-1}|)}{(p_i \sin \phi + c_i \cos \phi)} \times 100$$

2. Displacement

The change in each incremental displacement $\Delta\delta$ is compared as a percentage of the mean incremental displacement, viz.

$$\frac{(\Delta\delta_i - \Delta\delta_{i-1})}{\frac{n}{(\sum_{j=1}^n \Delta\delta_j)_i / n}} \times 100$$

3. Norm of Displacement

The norm $||\Delta\delta||_i = \sqrt{\{\Delta\delta\}_i^T \{\Delta\delta\}_i}$ is calculated and then the change in norm between iterations is compared as a percentage of the norm, viz.

$$\frac{||\Delta\delta||_i - ||\Delta\delta||_{i-1}}{||\Delta\delta||_i} \times 100$$

4. Correction Vector

The change in each term Δf of the incremental correction vector $\{\Delta f\}_i = [K_c]_i \cdot \{\Delta \delta\}_{i-1}$ is compared as a percentage of the mean correction term, viz.

$$\frac{\Delta f_i - \Delta f_{i-1}}{\frac{1}{n} \sum_{j=1}^n (\Delta f_j)_i} \times 100$$

5. Norm of the Correction Vector

The change in the norm of the incremental correction vector $||\Delta f||_i = \sqrt{\{\Delta f\}_i^T \cdot \{\Delta f\}_i}$ is compared as a percentage of the norm, viz.

$$\frac{||\Delta f||_i - ||\Delta f||_{i-1}}{||\Delta f||_i} \times 100$$

6. Equilibrium

The incremental equilibrating nodal forces

$$\{\Delta F'\} = ([K_E] - [K_c]_i) \cdot \{\Delta \delta\}_i$$

are calculated and each term is compared as a percentage of the incremental applied load ΔF_A , viz.

$$\frac{\Delta F' - \Delta F_A}{\Delta F_A} \times 100$$

7. Change in Nodal Forces

In a displacement defined analysis, the nodal force ΔF_i required to produce the specified displacements will change between increments. This change in nodal force is compared as a percentage of the nodal force, viz.

$$\frac{\Delta F_i - \Delta F_{i-1}}{\Delta F_i} \times 100$$

The effectiveness of various convergence criteria can be illustrated by considering the problem of a block of cohesive frictional material subjected to uniform compression as shown in Fig. 6.9(a).

In this problem, all elements have the same stress state and consequently all reach failure at the same load, producing an elastic/fully plastic behaviour. Once the load required to cause collapse has been reached, the displacement will be largely independent of any load in excess of the collapse load and although, fundamentally, the displacement is indeterminate at collapse, in fact the numerical formulation is such that the displacement appears to be a function of the number of iterations as shown in Fig. 6.9(b).

At failure, the stresses which are identical for each element and must, therefore, remain constant for each iteration, satisfy vertical equilibrium and the yield condition but cannot satisfy horizontal equilibrium for applied loads exceeding the collapse load.

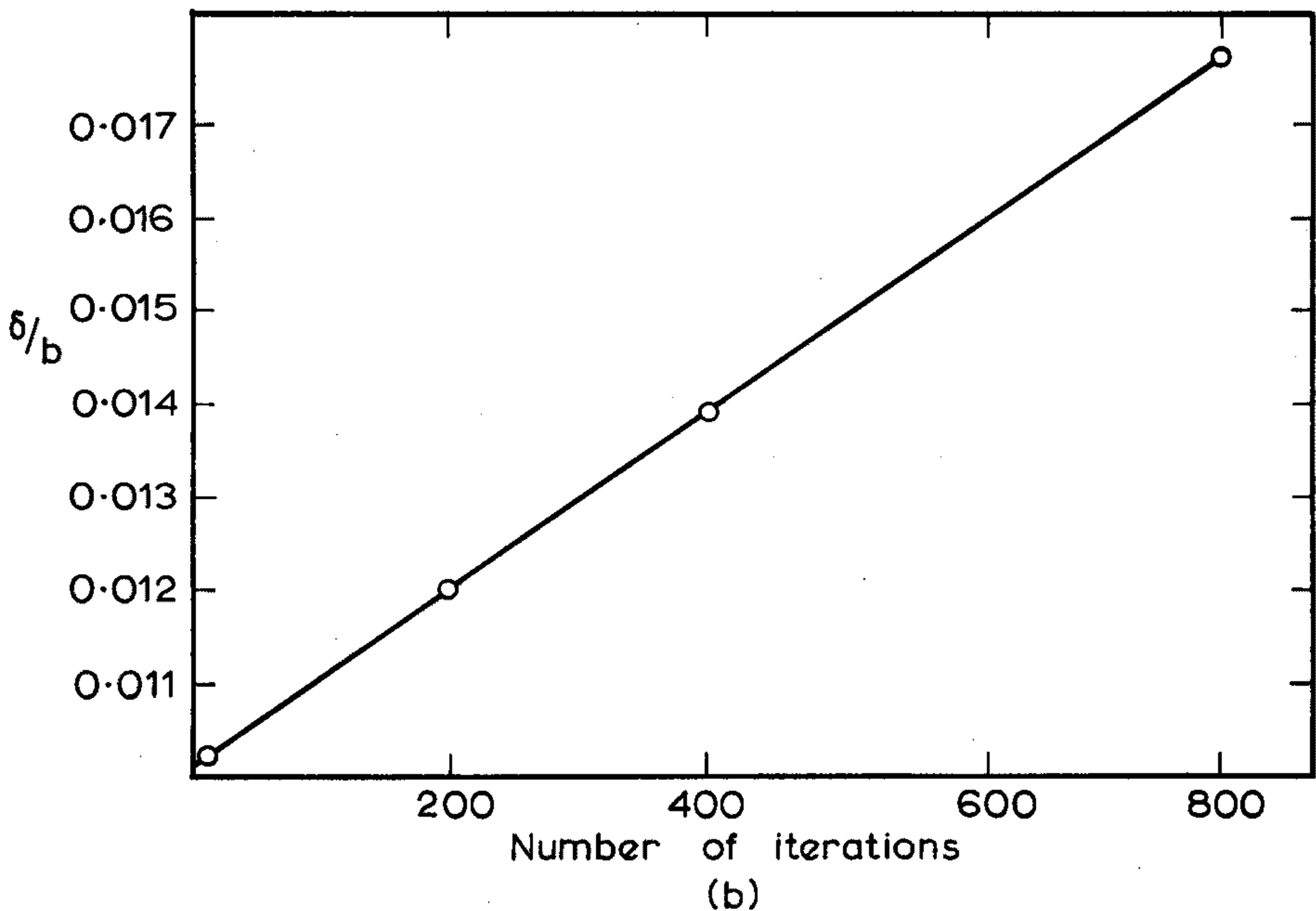
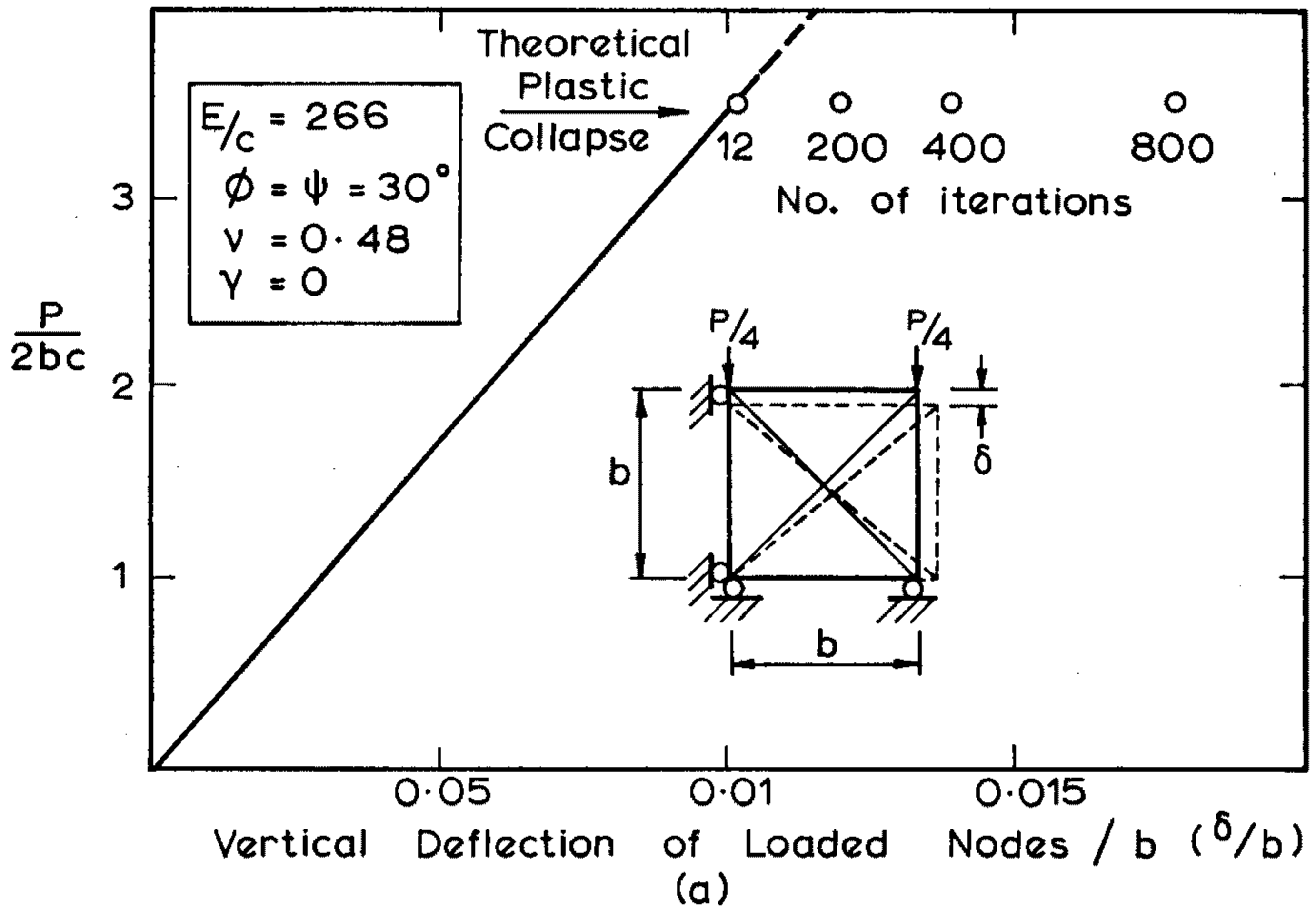


FIG.6.9 EFFECT OF CONVERGENCE CRITERIA ON THE PREDICTED LOAD DEFLECTION CURVE OF A BLOCK UNDER UNIFORM COMPRESSION

Thus any convergence criteria based on the change in stress brought about by stress redistribution between iterations will indicate a converged solution after the second iteration and is of little value in indicating collapse.

This example is an especially severe test of the stress convergence criteria; however, it has been found that even for problems with a significant elasto-plastic region, stress convergence becomes progressively less sensitive as failure approaches. Hence the major advantage of a stress convergence criteria lies in its usefulness for indicating when a solution is not well behaved due to either a poor mesh or too large an increment size, rather than as a means of identifying collapse.

All but one of the other convergence criteria used in the problem (viz nos. 2 to 5) are either directly or indirectly affected by the changing displacements and hence, providing the convergence tolerance is sufficiently small, they will all indicate that the solution has not converged. The various criteria do, however, have significantly different sensitivities since the divisor in each case is a function of the increasing displacement. Consequently these convergence criteria will appear to converge as the deflection terms become very large. The number of iterations required to satisfy the various convergence criteria are shown in Table 6.1. It will be seen that the equilibrium convergence criteria is never satisfied, since equilibrium cannot be satisfied for an applied load exceeding the collapse load. Of the remaining criteria, checks on the norm of the displacement and maximum change in the correction vector are the most sensitive while the maximum change in displacement is the least sen-

sitive.

TABLE 6.1

NO. OF ITERATIONS REQUIRED TO SATISFY CONVERGENCE CRITERIA

Criteria	No. of iterations required		
	2%	1%	0.5%
1 Stress	2	2	2
2 Displacement	48	135	385
3 Norm of displacement	115	252	558
4 Norm of correction vector	110	211	404
5 Correction vector	98	191	375
6 Equilibrium	∞	∞	∞

The relative merits of the different criteria vary from problem to problem and while the equilibrium and maximum change in correction vector are generally the most useful, inspection of all the various criteria can give insight into the numerical behaviour of the problem.

The load deflection curves obtained for a rigid footing analysis using the mesh shown in Fig. 6.7(iii) and adopting a displacement step size of .0026b are shown in Fig. 6.10 for the two cases where all the previously described convergence conditions were satisfied to better than 2% and 0.03%. The number of iterations required to satisfy the two convergence conditions is shown in the insert to Fig. 6.10. It will be seen that the 2% convergence condition requires at most 10 iterations per load step and usually only 4 to 6 iterations.

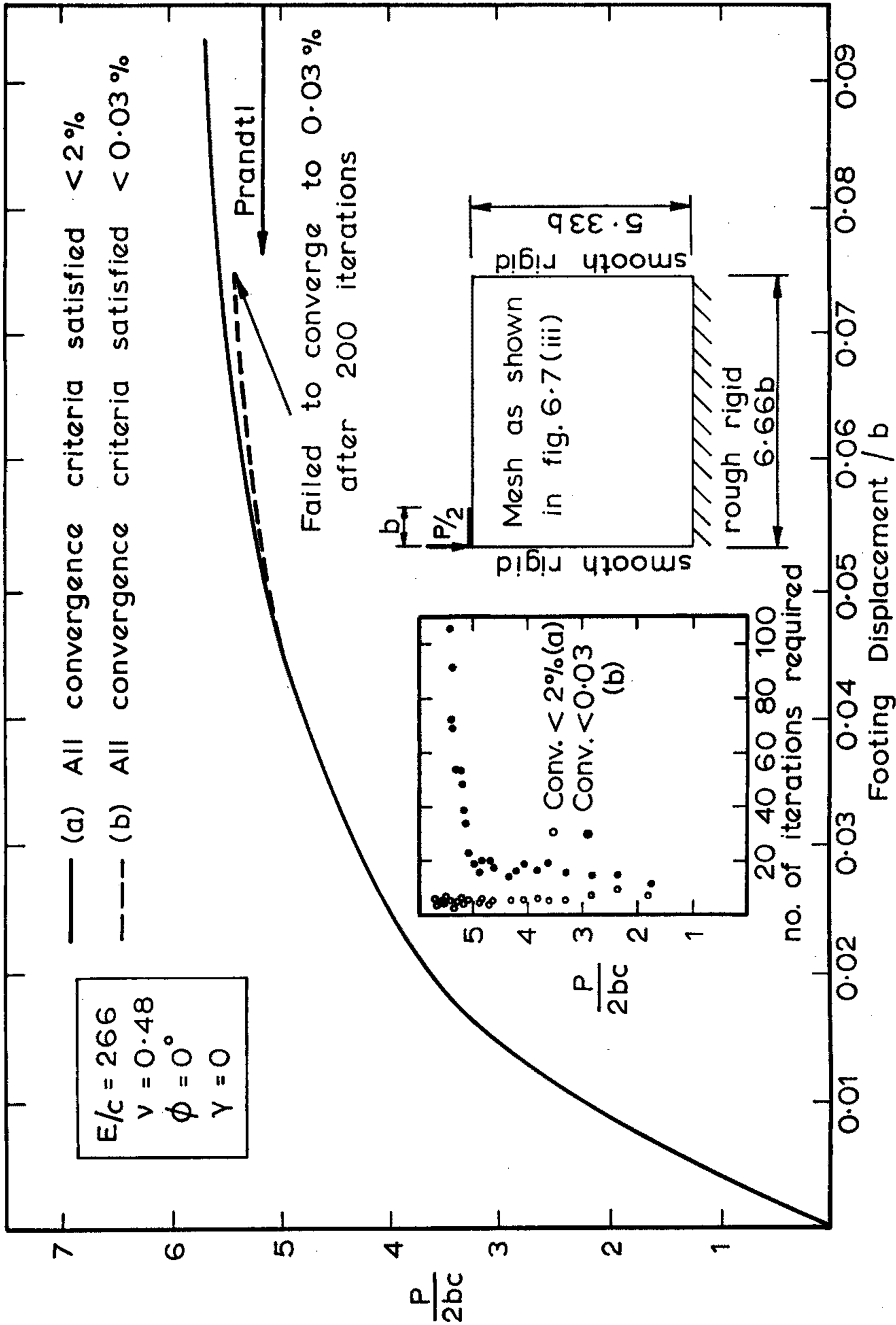


FIG.6.10 EFFECT OF CONVERGENCE CRITERIA ON THE LOAD - DEFLECTION CURVE FOR A RIGID STRIP

In comparison, the .03% criteria required approximately 15 iterations per load step up to a load level of $5 \times (2bc)$ with the number of iterations increasing dramatically beyond this point. The 0.03% criteria had not been satisfied after 200 iterations at a load of $5.4 \times (2bc)$.

The load displacement curves for the two cases are identical up to a load of about $5 \times (2bc)$. Beyond this, the higher convergence criteria lowered the load required to produce a given displacement, and at the point where solution (b) failed to converge the predicted load was 3% lower than that obtained using the coarser convergence criteria. This improved result was, however, only obtained at a cost of a 930% increase in computational effort, indicating that for most practical applications of these techniques consideration must be given to the trade off between desirable accuracy and the computational cost involved in obtaining that accuracy.

Type of Loading

Analyses involving rigid body movement of a structure in relation to the soil mass can take either of two forms; namely a displacement defined or load defined analysis. In so doing, the same problem (e.g. a rigid footing) is being analysed and the only difference lies in the way in which increments of load are added to the footing. Thus, in principle, the same result will be obtained irrespective of the type of loading adopted. However, in practice it is generally more convenient to adopt a displacement defined analysis, since for a constant increment size, failure can be identified by a very small change in load without the need for a large number of iterations. In a load defined

analysis, failure can only be identified when the displacement becomes indeterminate (such as in the block problem shown in Fig. 6.9) and the displacement continues to increase with each additional iteration. Thus the definition of failure is at least partly dependent upon the increment size and choice of convergence criteria. Furthermore, lack of convergence may also be due to the erratic behaviour of a few elements caused by either a poor mesh or too coarse an increment size. This could lead to an erroneous estimate of the 'collapse load', with different values being obtained by either altering the mesh or the increment size.

Provided these potential sources of error are recognised, identical results can be obtained using either a load defined or displacement defined analysis as shown in Fig. 6.11.

With some modifications, the concept of a displacement defined analysis can also be applied to structures which are not rigid by specifying a particular nodal displacement or selecting loads to give an approximately consistent increment in average displacement.

For uniform loading over n nodes the incremental displacement $\Delta\delta_j$ of a particular node j can be written in the form

$$\Delta\delta_j = \sum_{i=1}^n I_i \Delta F_i + \Delta\delta_{k_j} \quad (6.1)$$

where I_i is the elastic influence coefficient obtained by applying a unit load at the i th node under the footing; ΔF_i is the applied nodal force at the i th node under the footing; and $\Delta\delta_{k_j}$ is the equivalent

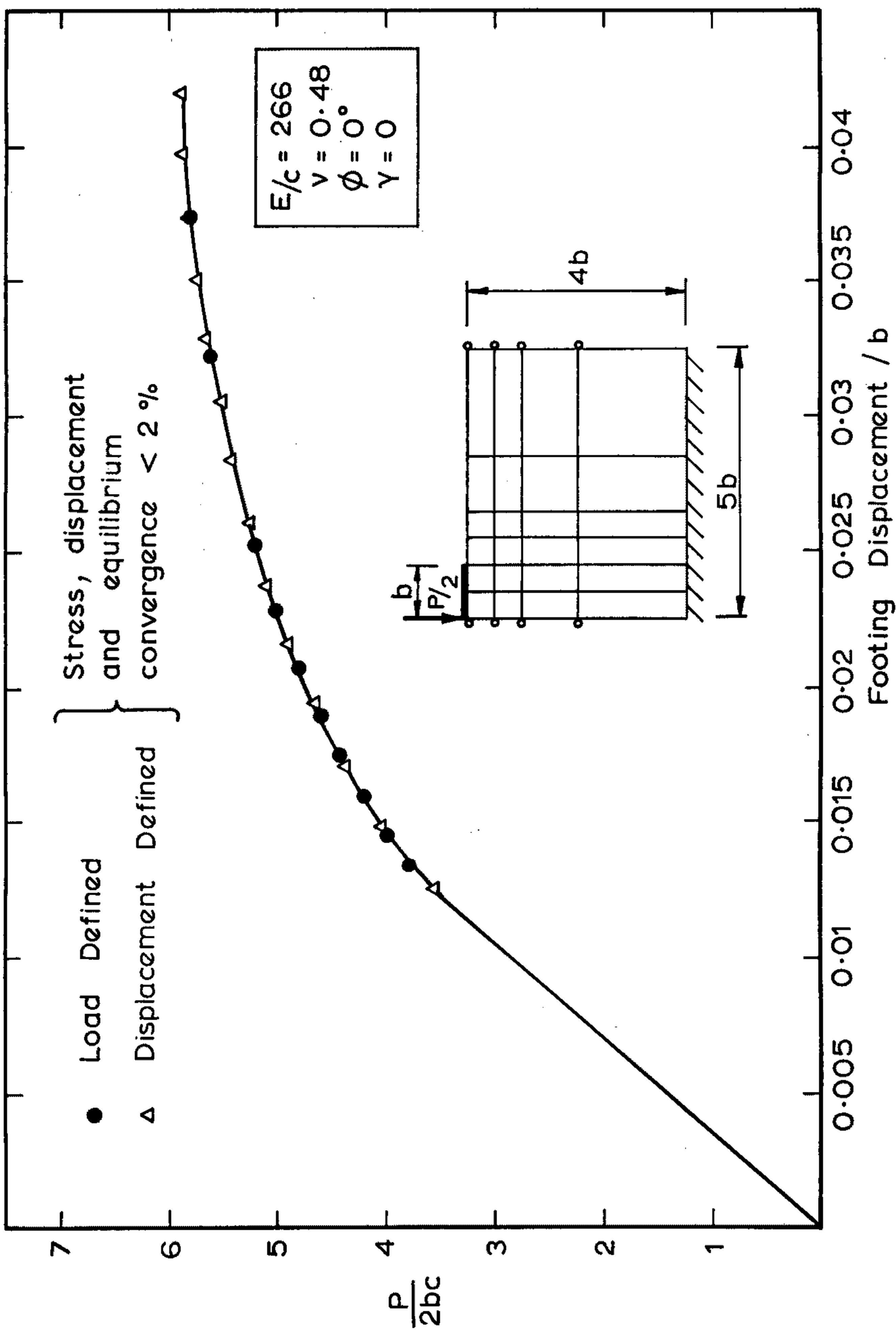


FIG.6.11 EFFECT OF CONVERGENCE CRITERIA ON THE LOAD - DEFLECTION CURVE FOR A RIGID FOOTING

nodal displacement at the i th node due to the plastic correction which can be estimated by solving the equations

$$[K_E] \{\Delta\delta_k\} = [K_C] \{\Delta\delta\}$$

and selecting the appropriate n terms from the vector $\{\Delta\delta_k\}$, where $\{\Delta\delta_k\}$ is a vector of nodal displacement due to the apparent plastic correction forces; $[K_E]$ is the elastic stiffness matrix; $[K_C]$ is the plastic correction matrix; and $\{\Delta\delta\}$ is the incremental nodal displacement from the prior iteration. Since each nodal force is a function of the increment in applied pressure Δq they may all be expressed in the form

$$\Delta F_i = k_i \Delta q \quad (6.2)$$

where k_i is a known constant. Thus combining equation (6.1) and (6.2) the required increment in pressure for a specified nodal displacement $\Delta\delta_j = \delta$ is given by

$$\Delta q = \frac{\delta - \Delta\delta_{k_j}}{\sum_{i=1}^n I_i k_i} \quad (6.3)$$

This 'displacement probe' technique leads to a logical change in load step size as failure approaches, and although it will lead to an increase in the number of load steps, the fact that the load level is selected to give a fairly constant change in displacement significantly reduces the number of iterations required to obtain a converged solution. Furthermore, generalisation of this approach to give an approximately constant increase in any selected subset of nodal dis-

placements for any arbitrary choice of structural flexibility is quite straightforward.

The load deflection curve obtained for a uniformly loaded strip using both a conventional analysis and a displacement probe is shown in Fig. 6.12. It can be seen that the two approaches yielded identical results. Despite the increase in the number of load steps required in the displacement probe analysis, the decrease in the number of iterations necessary to ensure a converged solution resulted in a 14% saving in computer time.

Increment Size

In general, the errors in the numerical value of collapse loads due to discretization error, error in the incremental procedure and round off error, are not independent and an attempt to reduce one source can increase that due to another source. Refinement of the mesh in critical regions, for example, may make convergence more difficult (Yamamoto, 1973) and increase roundoff error (Shah, 1966). In particular it has been found that while mesh refinement schemes involving a large number of elements in regions of high stress gradient, reduce the discretization error compared with other arrangements involving a similar number of elements, they considerably increase difficulties in the incremental procedure and require smaller step sizes to avoid numerical instability of small elements in highly stressed regions.

Accepting that the increment size is small enough to avoid these difficulties, then the sensitivity of the numerical collapse

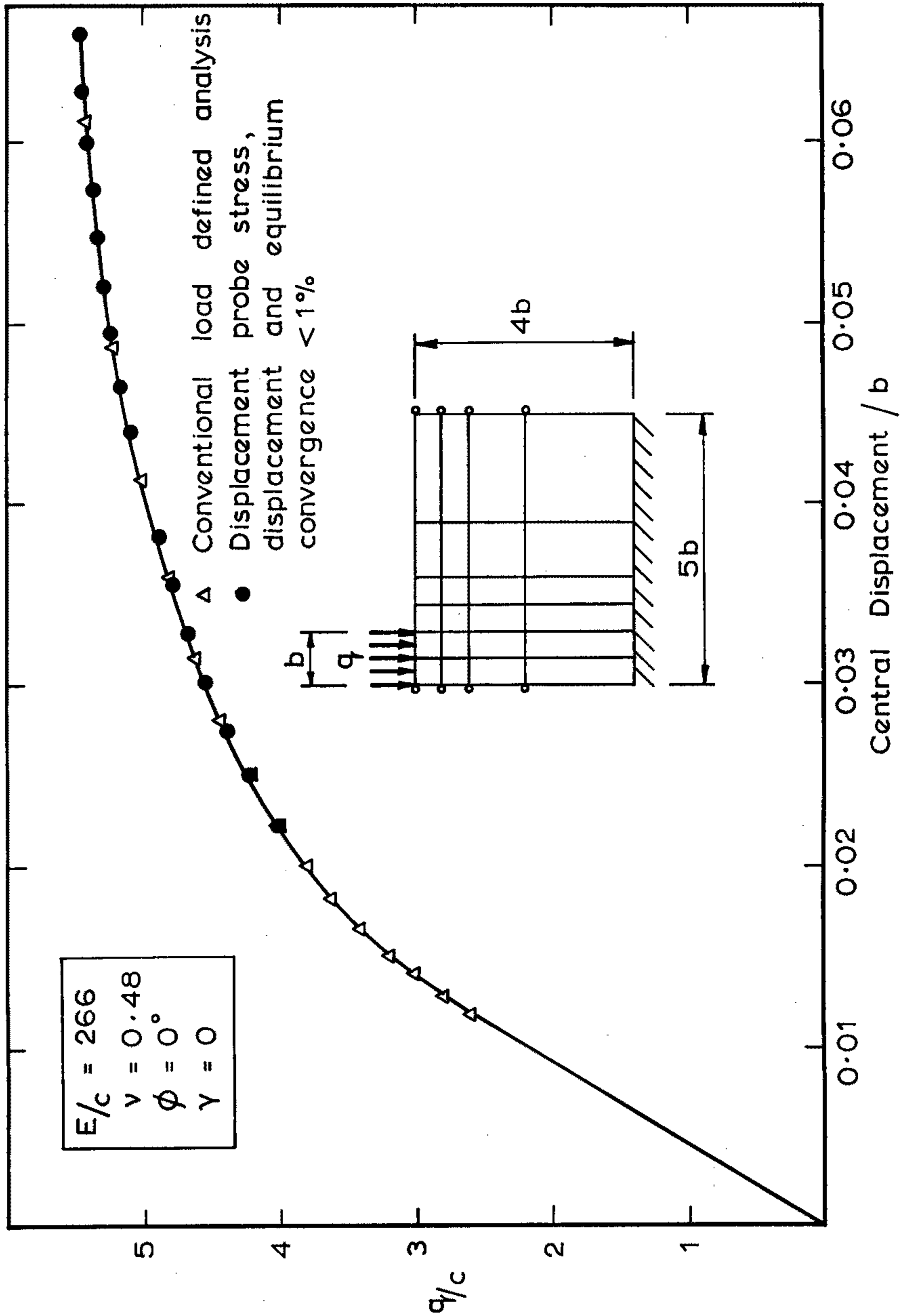


FIG.6.12 LOAD DEFLECTION CURVE FOR A UNIFORMLY LOADED STRIP

load to the choice of increment size is a good indication of the adequacy of the convergence criteria. Thus, if the convergence tolerance is sufficiently small, then a reduction in increment size of, say, 50% will not significantly alter the collapse load.

6.4 TECHNIQUES FOR PREDICTING COLLAPSE LOADS

As indicated in the previous section, quite different collapse loads can be obtained for different mesh arrangements, corresponding to a decrease in discretization error as the number of elements within the critical region increases. A straightforward approach to obtaining accurate collapse loads would be to adopt a very fine finite element mesh along with a small increment size and tight convergence criteria. Unfortunately, this approach, involves very considerable computational costs and is, therefore, particularly unsuitable for problems where the collapse load is of primary importance, indicating a need for a computationally economical technique which will give collapse loads sufficiently accurate for engineering application. Two such techniques will now be presented.

6.4.1 Extrapolation Technique

It is generally recognised (e.g. Yamamoto, 1973; Yamamoto and Tokuda, 1971; Carey, 1976; Davidson and Chen, 1974) that discretization error may be reduced and an improved finite element solution obtained by refining the mesh. This observation suggests the possibility of extrapolation to an accurate solution from a series of solutions none of which employ a very fine mesh.

Essentially, the proposed extrapolation method consists of plotting the solutions obtained from a number of mesh arrangements against some measure of the mesh refinement while attempting to keep the errors due to the incremental procedure constant. The method is best illustrated by an example. Consider a smooth rigid footing with half width b resting on a purely cohesive homogeneous soil mass with cohesion c , elastic modulus $E = 266c$ and Poisson's ratio $\nu = 0.48$. For the purposes of mesh refinement each mesh was divided into nine regions as shown in Fig. 6.13. Three meshes were considered as shown in Fig. 6.14. In the first mesh ($n = 4$) regions one to four were divided into four sub-regions. Similarly the first four regions in the other two meshes ($n = 9$, $n = 16$) were divided respectively into 9 and 16 sub-regions. All other regions were then subdivided as required to be consistent with the first four and each quadrilateral sub-region was further divided into four constant strain triangular elements.

A displacement defined analysis was performed for each mesh using an increment in displacement of $0.0027b$ and the same convergence criteria in an attempt to keep the errors due to the incremental procedure small and constant. Each run was terminated after about 35 increments although all convergence criteria were still satisfied. The load displacement curves obtained for the footing are shown in Fig. 6.14. While none of these solutions individually gives anything but a crude overestimate of the collapse load, they can be used together to obtain a good estimate of the collapse load by plotting the load obtained from the three meshes for a given displacement against $1/\sqrt{n}$ where n is the number of sub-regions into which the regions under

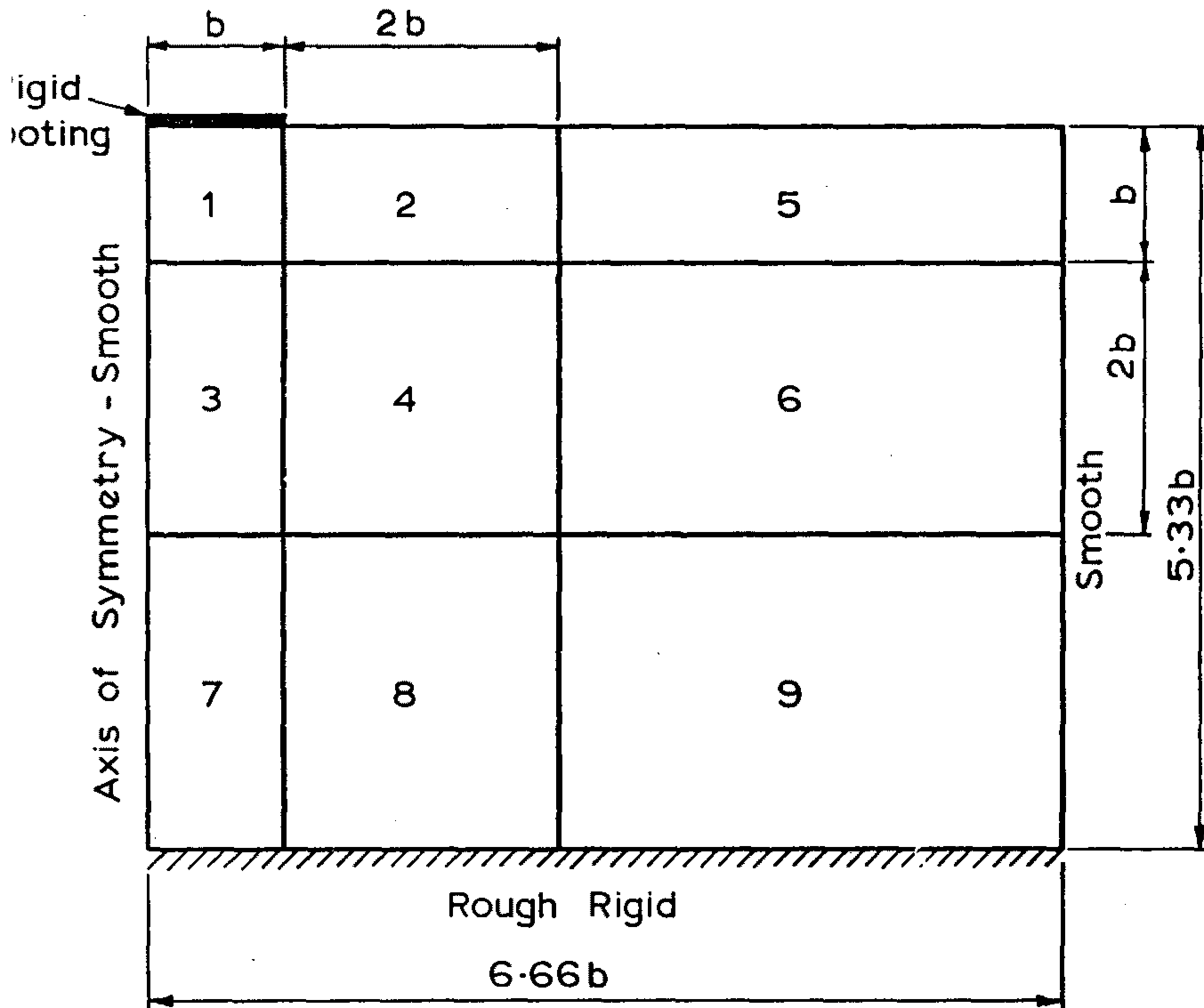


FIG. 6.13 RIGID FOOTING-MESH REFINEMENT REGIONS

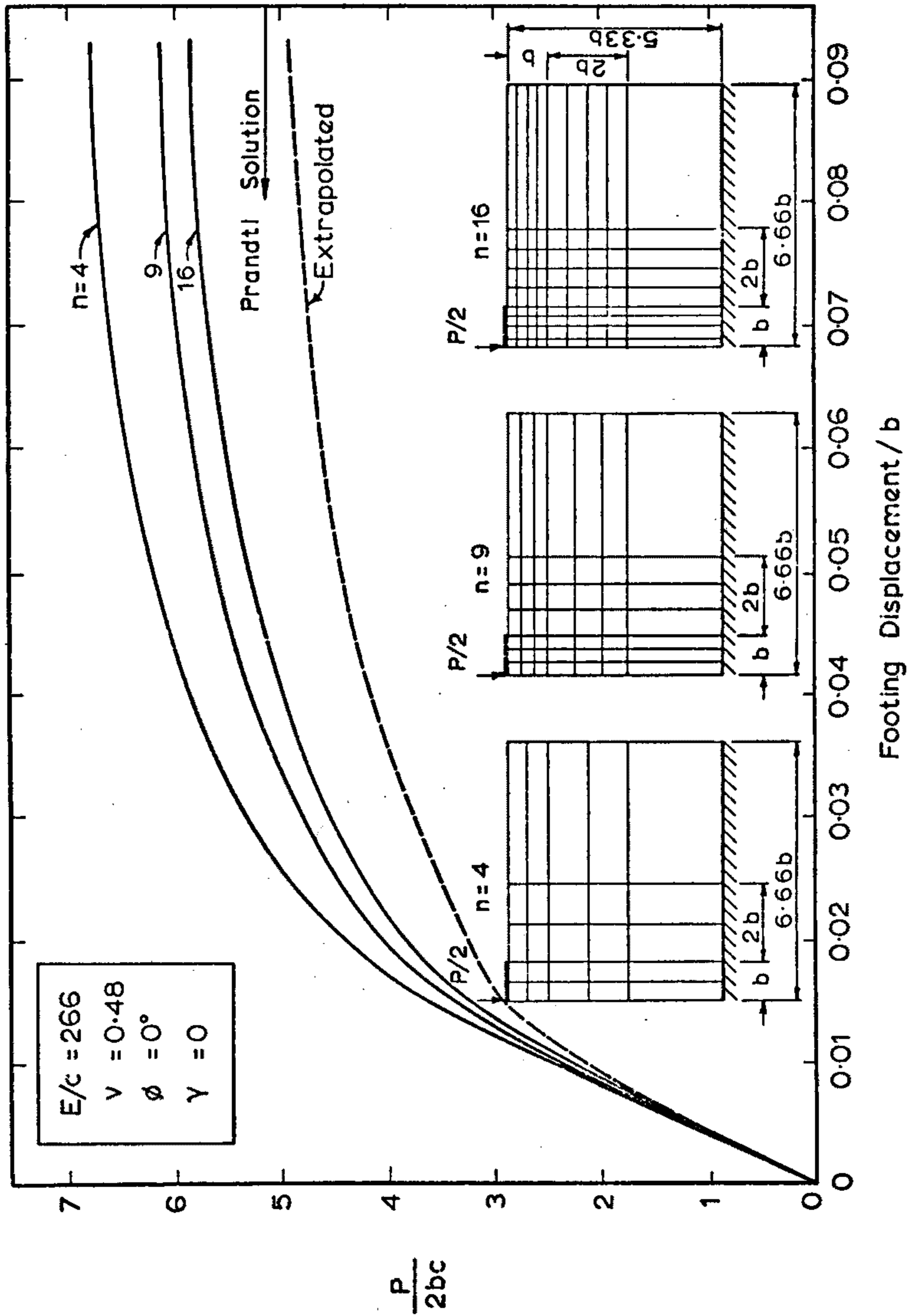


FIG.6-14 EFFECT OF MESH REFINEMENT ON THE LOAD-DEFLECTION BEHAVIOUR OF A RIGID FOOTING - SERIES A

and adjacent to the footing were subdivided. From Fig. 6.16 it can be seen that the loads corresponding to the same displacement for the three mesh arrangements form a series of lines which can be extrapolated to the case where the number of sub-regions n approaches infinity. (i.e. $1/\sqrt{n}$ approaches zero).

The load displacement curve constructed from the extrapolated load for each displacement contour (See Fig. 6.16) indicates a collapse load 2.7% below the Prandtl value. This compares favourably with the best direct estimate of the collapse load which exceeds the Prandtl value by approximately 14%.

In general, there is no reason to believe that the variation of collapse load with the function of mesh refinement $1/\sqrt{n}$ should be linear. To illustrate this, consider the series of meshes, shown in Fig. 6.15. Recognising that the most critical stress region for a rigid footing problem is at the edge of the footing, each of the regions was subdivided such that the length of the sides of the sub-region within each region were in the ratios 1:2, 1:2:3 and 1:2:3:4 for the three cases $n = 4$, $n = 9$ and $n = 16$ respectively, thereby increasing the number of elements near the footing edge. In each case the load displacement curve indicates a collapse load closer to the Prandtl value than that obtained using the corresponding mesh shown in Fig. 6.14. However, the extrapolation is less satisfactory than in the previous example, indicating a collapse load (see Figs. 6.15 and 6.17) 10% lower than the Prandtl solution, compared with the best finite element estimate which is approximately 7% above the Prandtl value.

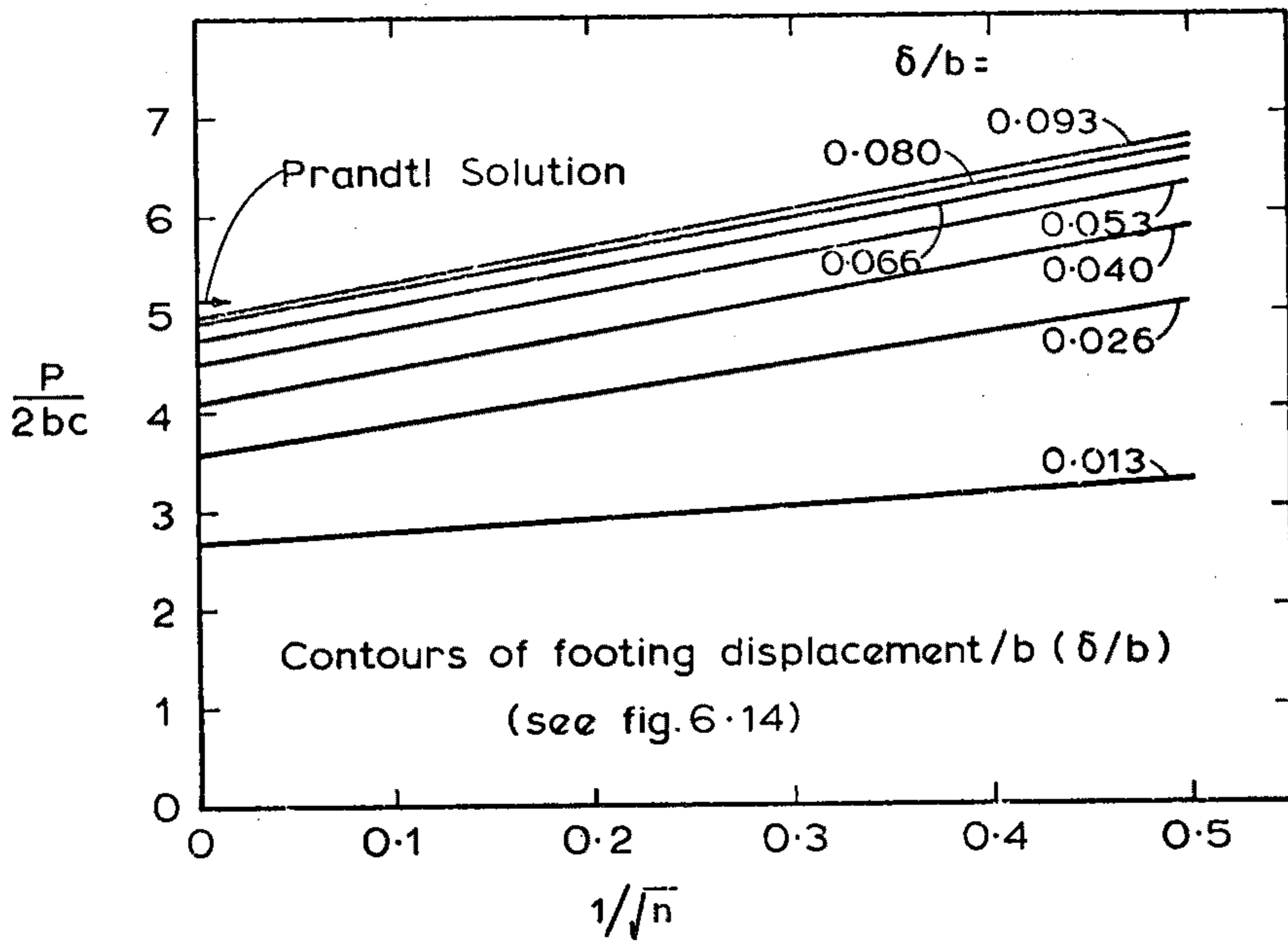


FIG. 6.16 EXTRAPOLATED LOADS - SERIES A - RIGID FOOTING

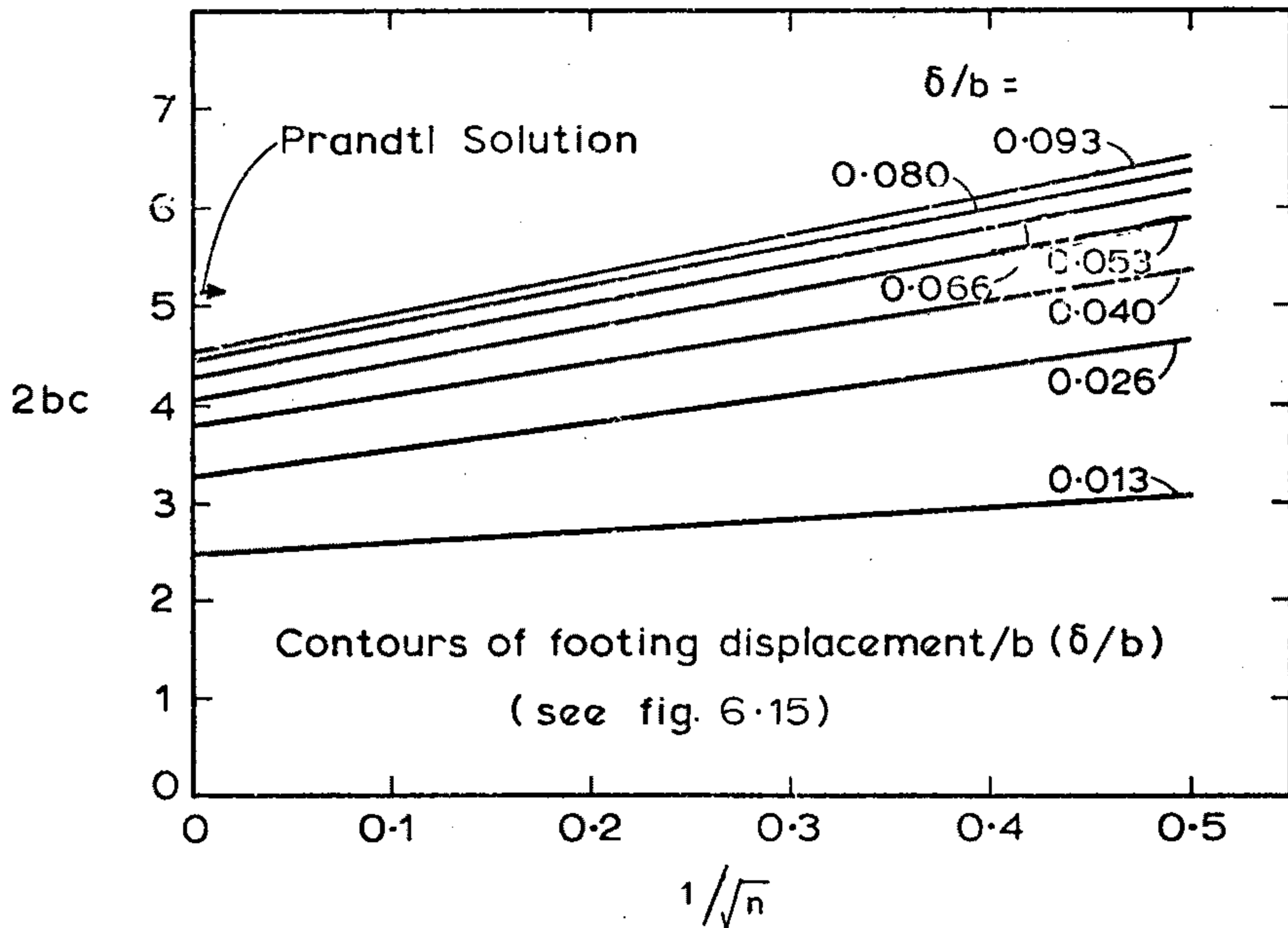


FIG.6.17 EXTRAPOLATED LOADS-SERIES B-RIGID FOOTING

The extrapolation procedure has been used successfully for a number of different problems. For example, a smooth rigid footing on a cohesive frictional soil with an angle of internal friction $\phi = 30^\circ$ and angle of dilatancy during plastic deformation $\psi = 0^\circ$ was analysed using the same elastic soil properties and mesh arrangement as shown in Fig. 6.14. The displacement contours and extrapolated load displacement curve obtained from these solutions were found (see Figs. 6.18 and 6.19) to indicate a clearly defined collapse load only fractionally below the Prandtl value, whereas the best finite element estimate of the collapse load exceeded the Prandtl value by more than 6%.

Possible applications of the extrapolation technique are not restricted to rigid footing problems but, rather, extend to the whole class of problems where errors due to the incremental procedure and round off can be kept small and constant for the different mesh arrangements considered. To illustrate this, consider a rough rigid anchor with half width b embedded at infinite depth in a purely cohesive soil. If it is assumed that the soil adheres to the back of the anchor then the problem is antisymmetric about a line through the anchor and parallel to it. Thus, it is only necessary to analyse a quarter of the problem as shown in Fig. 6.20. Although the exact analytical collapse load for this problem is not known, good upper and lower bounds can be obtained from limit analysis as outlined in Chapter 4. The load displacement curves obtained for this problem from the finite element analysis are shown in Fig. 6.20. From the extrapolated load displacement curve (see Figs. 6.20 and 6.21), the extrapolated collapse load can be seen to be between the upper and lower bound, and is only 1.5% below the upper bound whereas the lowest numerical collapse

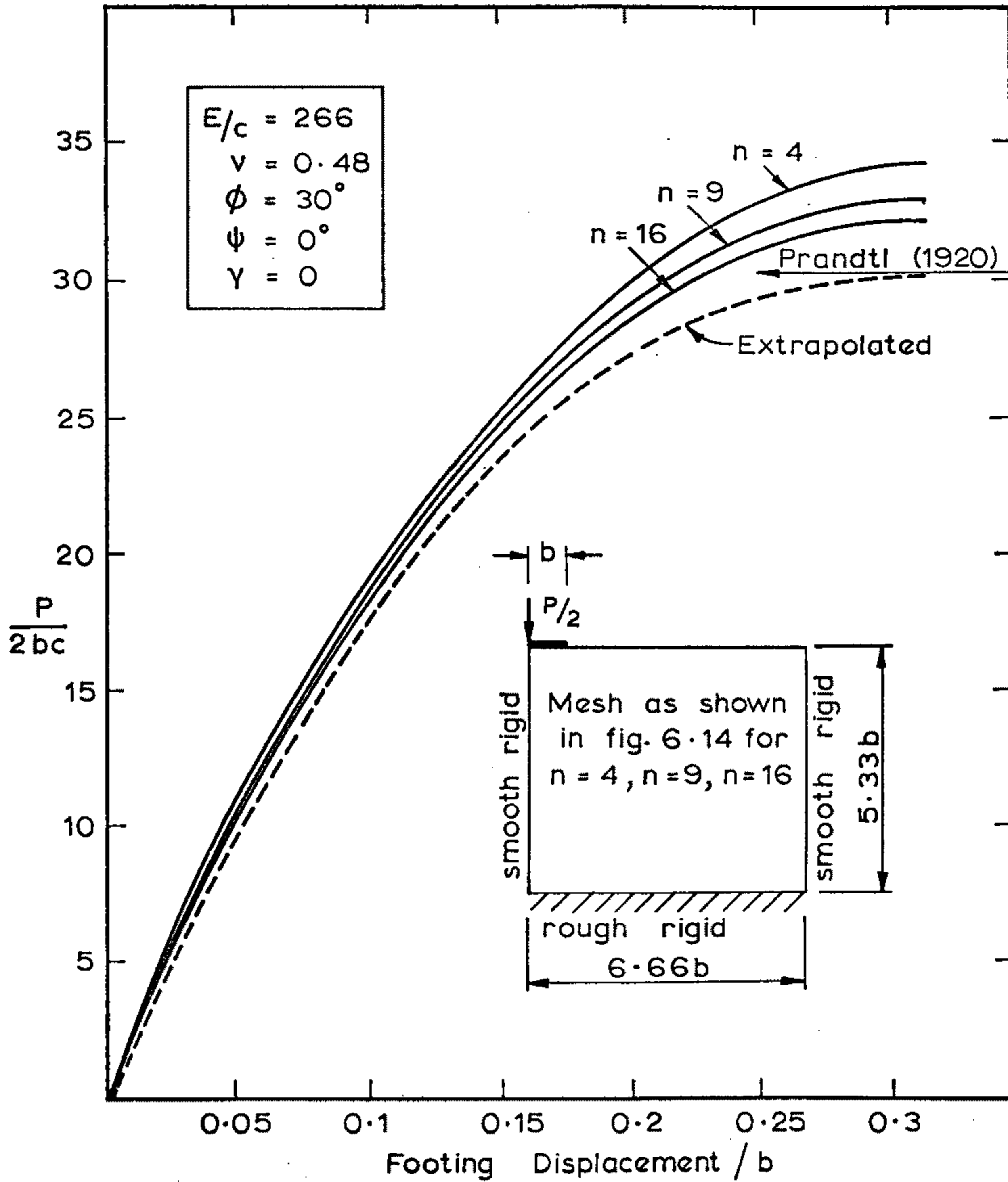


FIG. 6.18 LOAD DEFLECTION CURVES - RIGID FOOTING

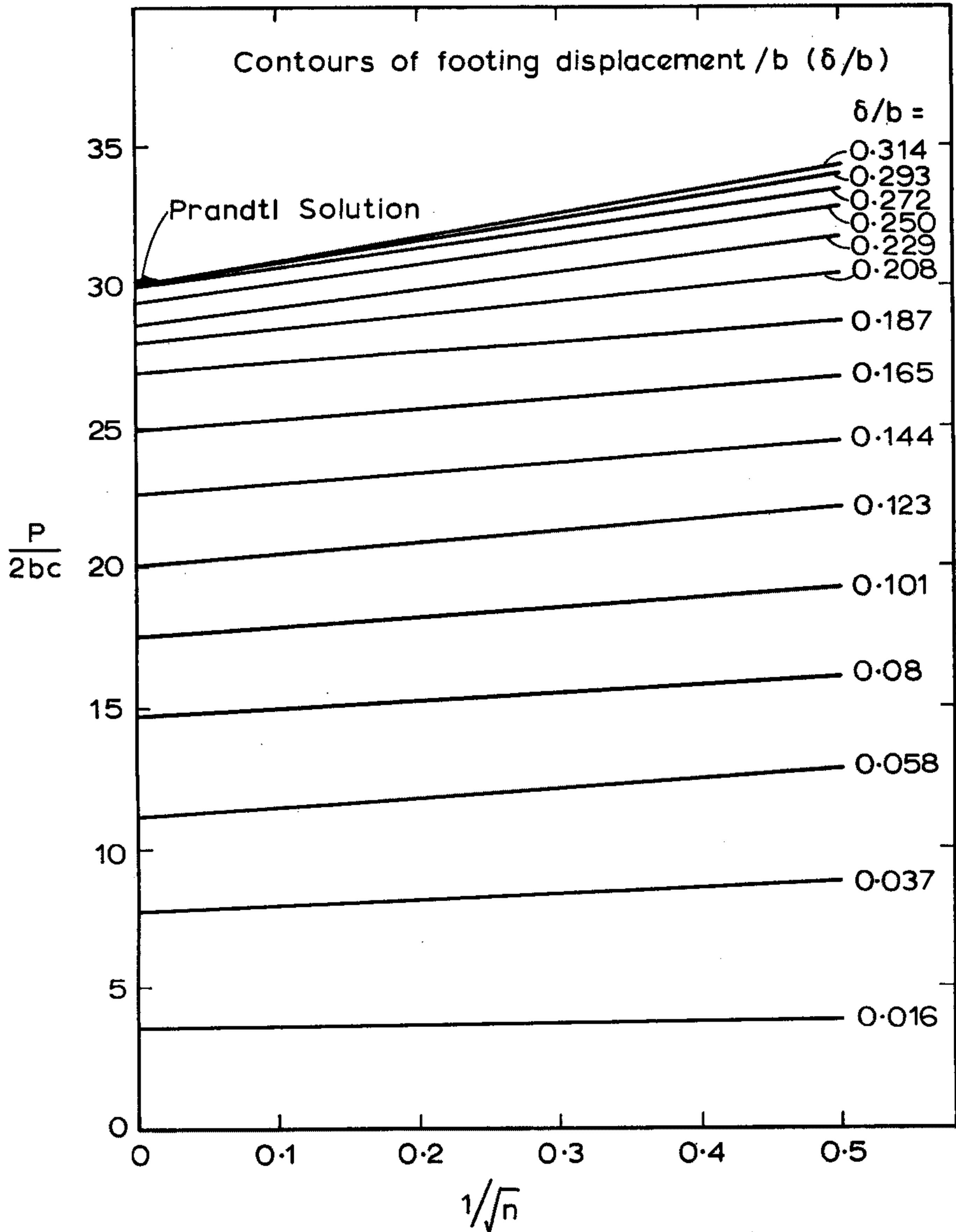


FIG. 6-19 EXTRAPOLATED LOADS - RIGID FOOTING
ON COHESIVE - FRICTIONAL SOIL MASS

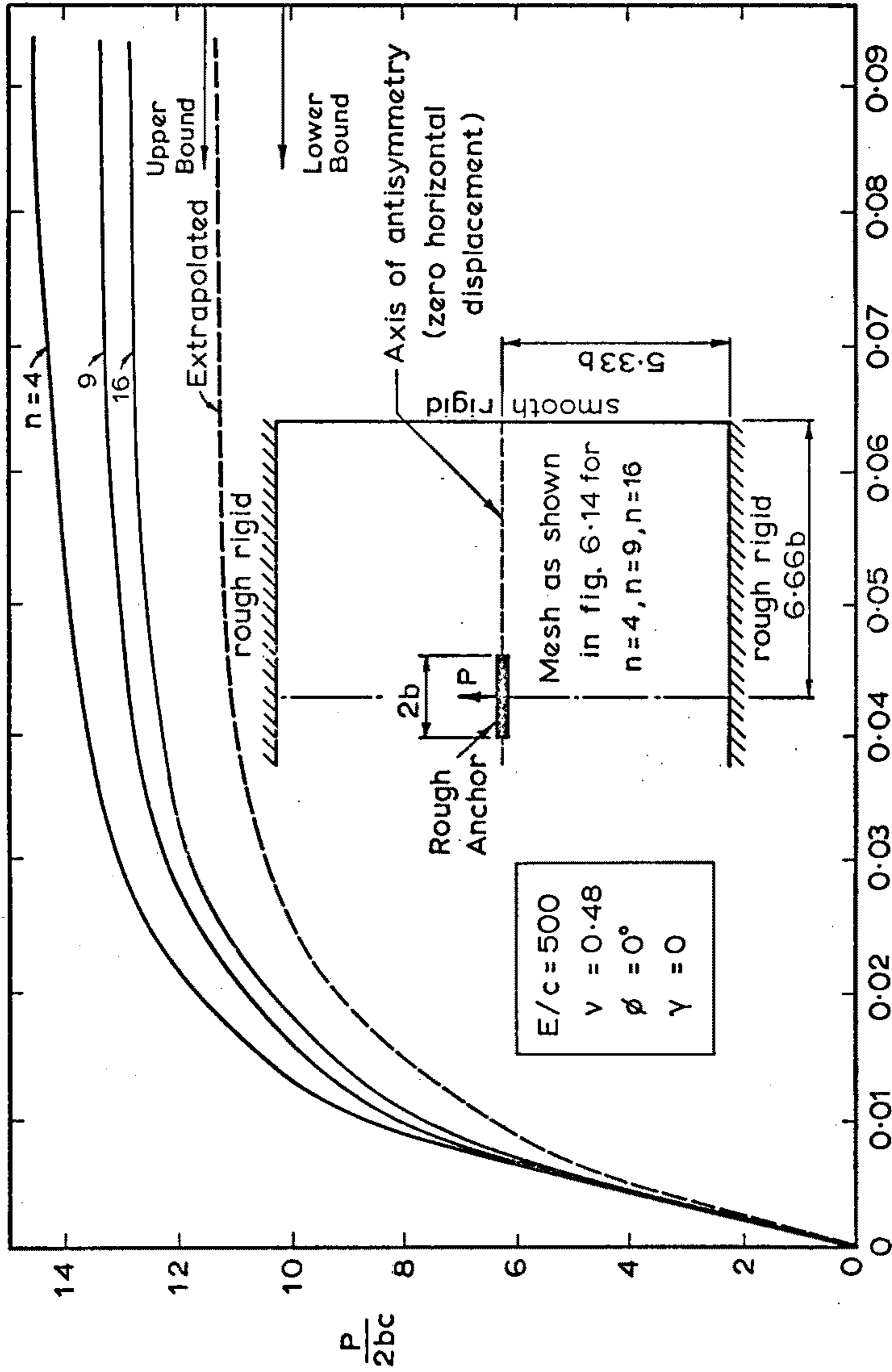


FIG.6-20 LOAD DEFLECTION CURVES FOR A ROUGH RIGID ANCHOR AT INFINITE DEPTH

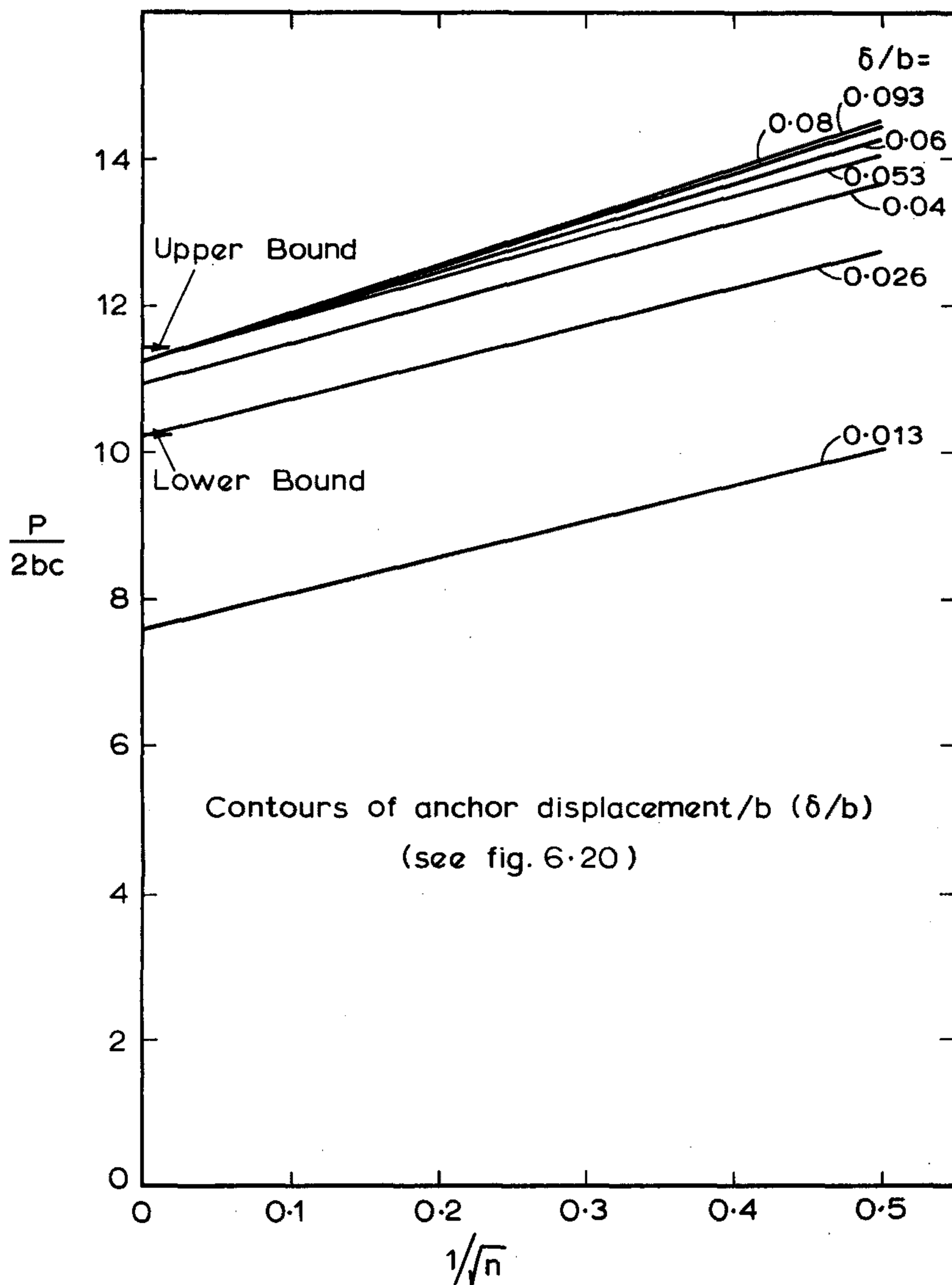


FIG. 6.21 EXTRAPOLATED LOADS - ROUGH RIGID ANCHOR

load obtained directly from the finite element solutions exceeds the upper bound value by more than 14%.

These examples illustrate a number of features of the extrapolation technique. Firstly, the best extrapolated solution is obtained if a straightforward mesh refinement scheme is adopted. Secondly, in all cases studied by the author, the extrapolated load has been less than or equal to the collapse load obtained from classical plasticity theory. Thus, with care in selecting the mesh refinement scheme, a good estimate of the collapse load can be obtained. However, for any systematic refinement scheme the extrapolated load will be a conservative estimate of the collapse load and hence more suitable for engineering purposes than the normal finite element solution which usually overestimates the failure load. Finally, it should be emphasised that the extrapolation technique only deals with discretization error and, hence, it is necessary that the other sources of error are held at low and fairly constant values for the different finite element solutions.

6.4.2 Potential Rupture Lines

Difficulties in obtaining good numerical collapse loads are generally encountered in problems which involve high stress and velocity gradients prior to and at collapse. Typically, these problems are associated with a stress singularity such as at the edge of a rigid footing. The conventional approach to the analysis of such a problem would be to refine the finite element mesh in this region, in an attempt to model the rapid variation in stress, and to provide for the intense plastic distortion which must occur in these regions at

collapse. An alternative approach is to introduce potential rupture lines into the finite mesh. In many respects these potential rupture lines are analogous to joint or spring elements which have been extensively used in geomechanics to allow for shear slip along zones of weakness such as along rock seams or at a soil-structure interface (Goodman et al., 1968; Ellison et al., 1971; Chung and Lee, 1972; Ghaboussi et al., 1973; Wilson, 1975; Sharma et al., 1976). In this paper, provision is made for rupture to occur within the elasto-plastic continuum to allow additional freedom for the development of plastic velocity fields and to reduce the influence of elastic tensions on the elasto-plastic solution. As with joint elements, dual nodes are placed along a selected potential rupture line such as MM' shown in Fig. 6.22, however, the numerical approach differs from that adopted with joint elements in that the dual nodes are not connected by joint stiffness terms inserted into the global stiffness matrix. Rather, nodal forces are applied to the dual nodes to ensure compatibility until the shear strength of the material is reached. Once this occurs, the compatibility equations are replaced by the equation describing the failure condition and the volumetric behaviour of the material during plastic deformation. Typically, a Mohr-Coulomb relationship was adopted between the normal and tangential nodal forces and it was assumed that rupture corresponded to the ultimate condition of zero volume change during plastic deformation. Provision for dilation of the rupture surface is easily incorporated using the simple relationship suggested by Davis (1968), and described in Appendix 5C. Details of the numerical formulation have been described with regard to the more general problem of soil-structure interface behaviour by Rowe et al., (1978), and are given in Chapter 5.

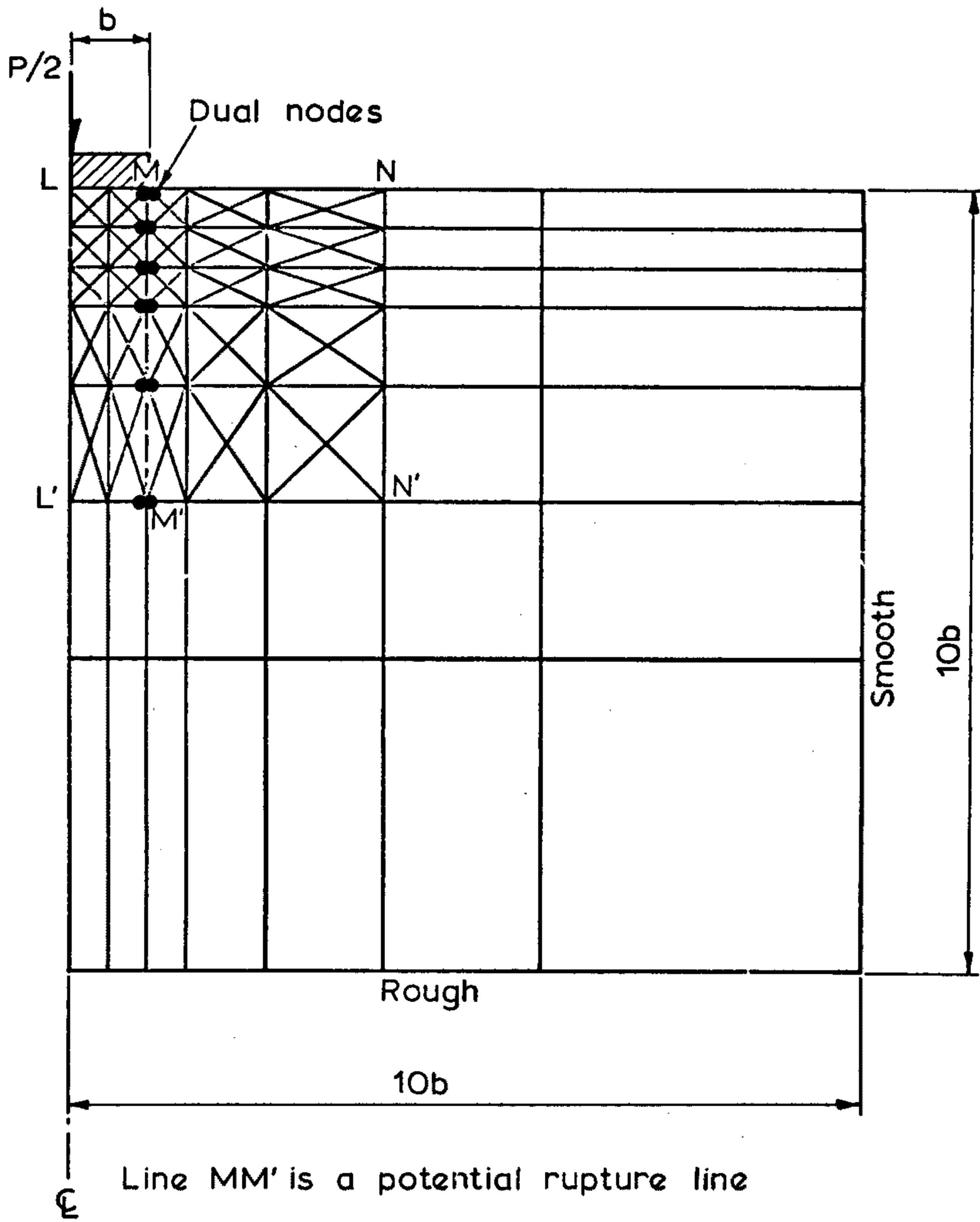
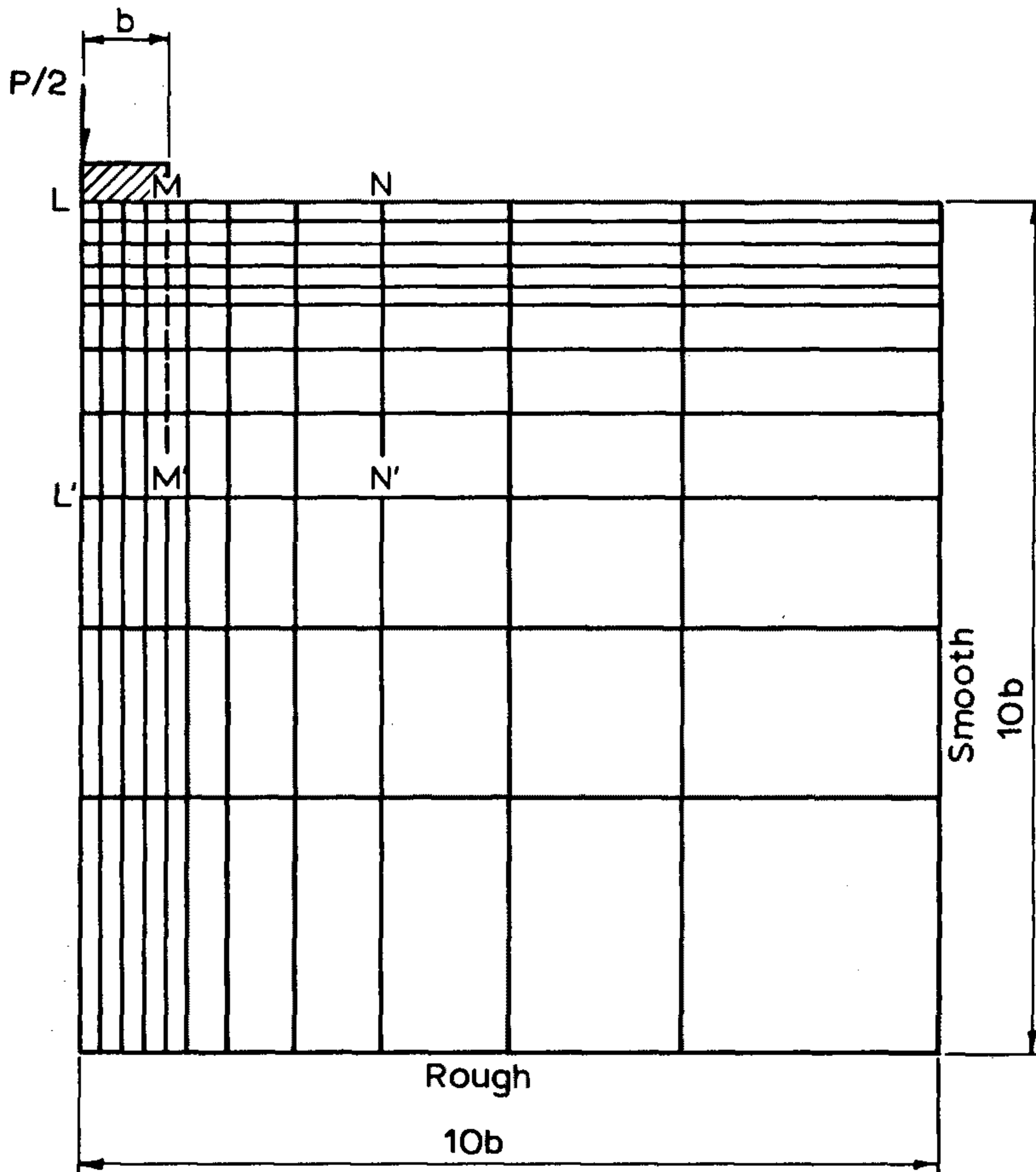


FIG.6-22 MESH 1

It should be emphasised that these potential rupture lines are not zones of weakness in the material since they obey the same material laws and have the same strength properties as the rest of the soil mass. Furthermore, irrespective of the number, length and orientation of the potential rupture lines, actual rupture will only occur at those nodes where the shear strength of the material is reached.

The advantages of introducing these potential rupture lines can best be assessed by consideration of a number of examples. Figs. 6.22 and 6.23 show two finite element meshes used in the analysis of a smooth rigid footing with half width b resting on a homogeneous, weightless purely cohesive soil mass with cohesion c , elastic modulus $E = 100c$, and Poisson's ratio $\nu = 0.48$. The soil was assumed to obey a Mohr-Coulomb failure criteria. Two cases were considered for each mesh. In the first instance, an elasto-plastic analysis was performed using a normal mesh without rupture lines by defining the displacements, in increments of $.005b$, along the base of the footing, and then back figuring the applied load P . The very fine increment size was adopted to ensure that the dominant error in the solution would be discretization error. In the second case, a potential rupture line was introduced along the line MM' as shown in Figs. 6.22 and 6.23 and an identical analysis performed. The displacements and stresses in the elastic range were identical both with and without the rupture line and were in reasonable agreement with the analytical elastic solutions, as shown for mesh 2 in Fig. 6.24.

Fig. 6.25 shows the load deflection curves for the four analyses. Using mesh 1 without rupture lines, no clearly defined collapse

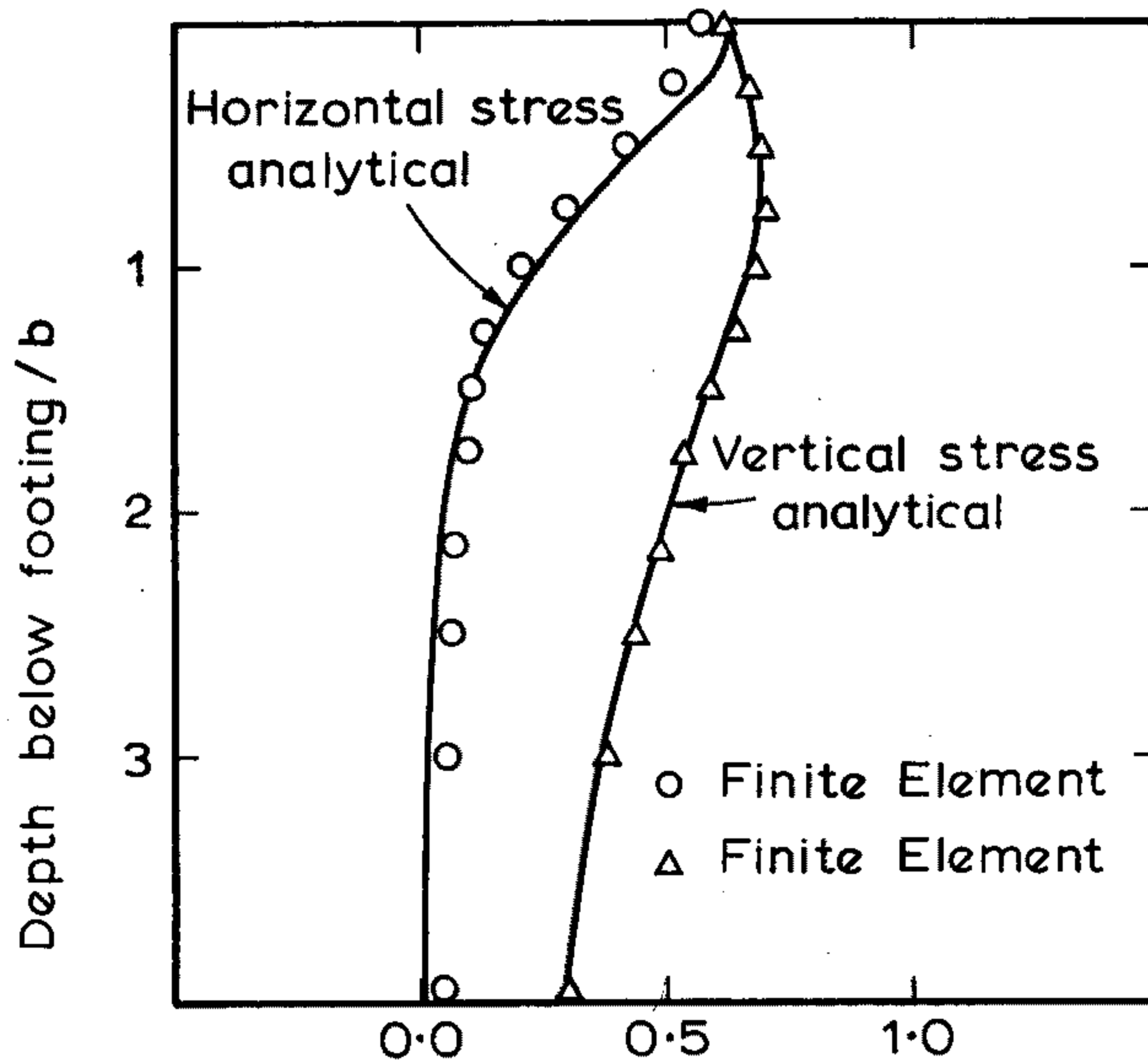


Elements within LMNN'M'L' are

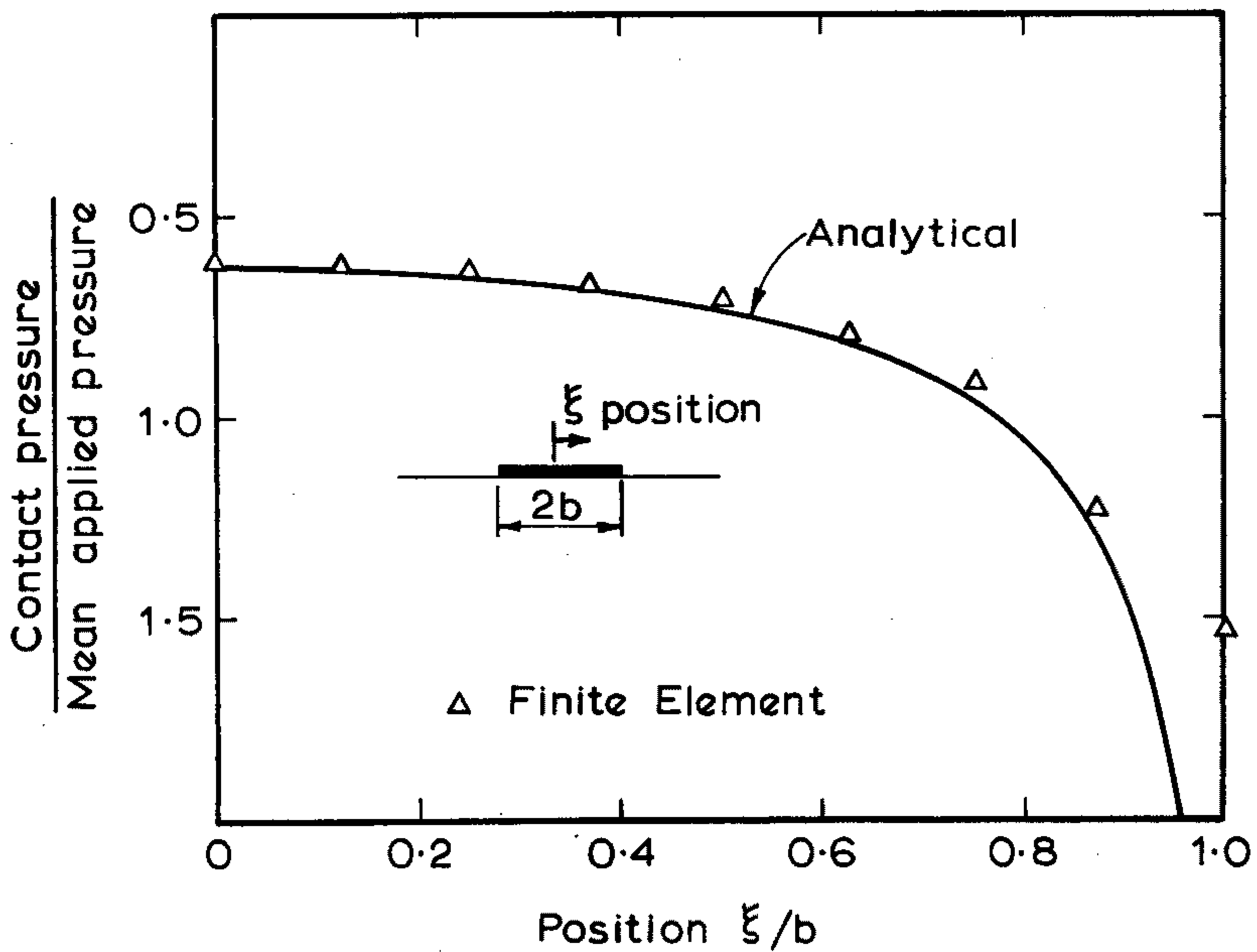


Line MM' is a potential rupture line

FIG.6-23 MESH 2



(Stress below centre of footing) / (Mean applied pressure)



Variation of contact pressure with position

FIG.6.24 ELASTIC STRESS DISTRIBUTION BENEATH A SMOOTH RIGID FOOTING-MESH 2

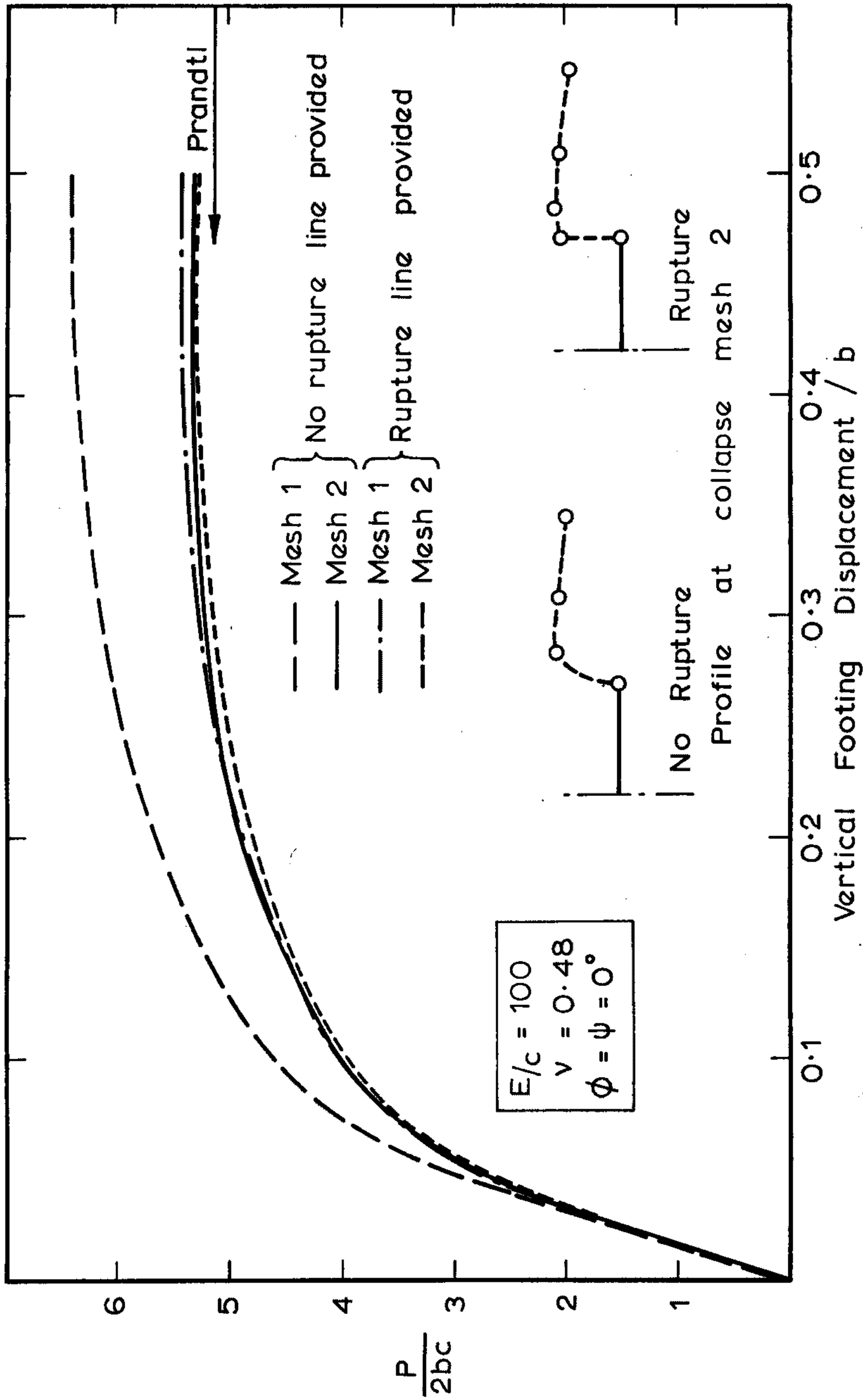


FIG.6-25 SMOOTH RIGID FOOTING ON HOMOGENEOUS MATERIAL

load had been observed when the analysis was discontinued at an applied load 24% above the Prandtl value. In comparison, the introduction of the rupture line into this mesh can be seen to have had a significant effect on the general load displacement behaviour of the footing, giving a clearly defined numerical collapse load only 5% above the Prandtl solution.

The variations in mean compressive stress and principal stress direction along a stress characteristic, at collapse, are shown in Fig. 6.26 for the two cases. From this, it can be seen that in addition to lowering the numerical collapse load the introduction of potential rupture lines leads to a significantly more accurate representation of the stress state throughout the plastic region. In particular, the stress state near the edge of the footing is poorly modelled in the conventional analysis. For example, the vertical and horizontal stresses at the point X shown in Fig. 6.26a were respectively 2.7c and 2.5c for the no rupture case, as compared to 0.6c and 2.4c when rupture was permitted. Comparison of these values with the theoretical vertical and horizontal stresses of zero and 2c show that the introduction of rupture lines leads to a much better representation of the stress state at collapse. The same conclusion can be drawn from the comparison of the values of mean compressive stress and principal stress direction at the point X as shown in Fig. 6.26.

From Fig. 6.25 it can be seen that the load displacement curves obtained using mesh 2 both with and without a rupture line are fairly similar to that obtained using the much coarser mesh 1 with a rupture line introduced. The numerical collapse load obtained for

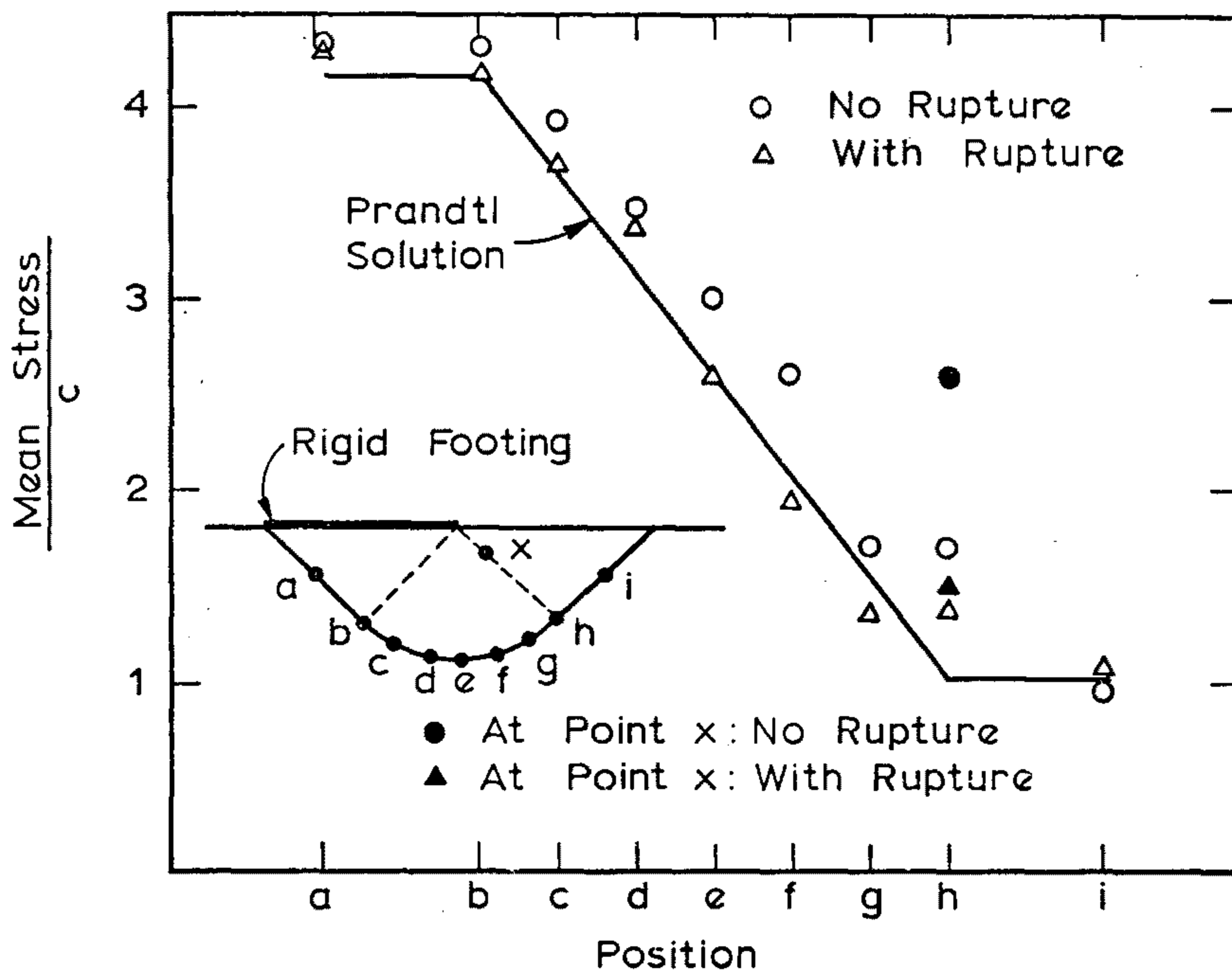


FIG.6-26(a) VARIATION IN MEAN COMPRESSIVE STRESS
ALONG A STRESS CHARACTERISTIC

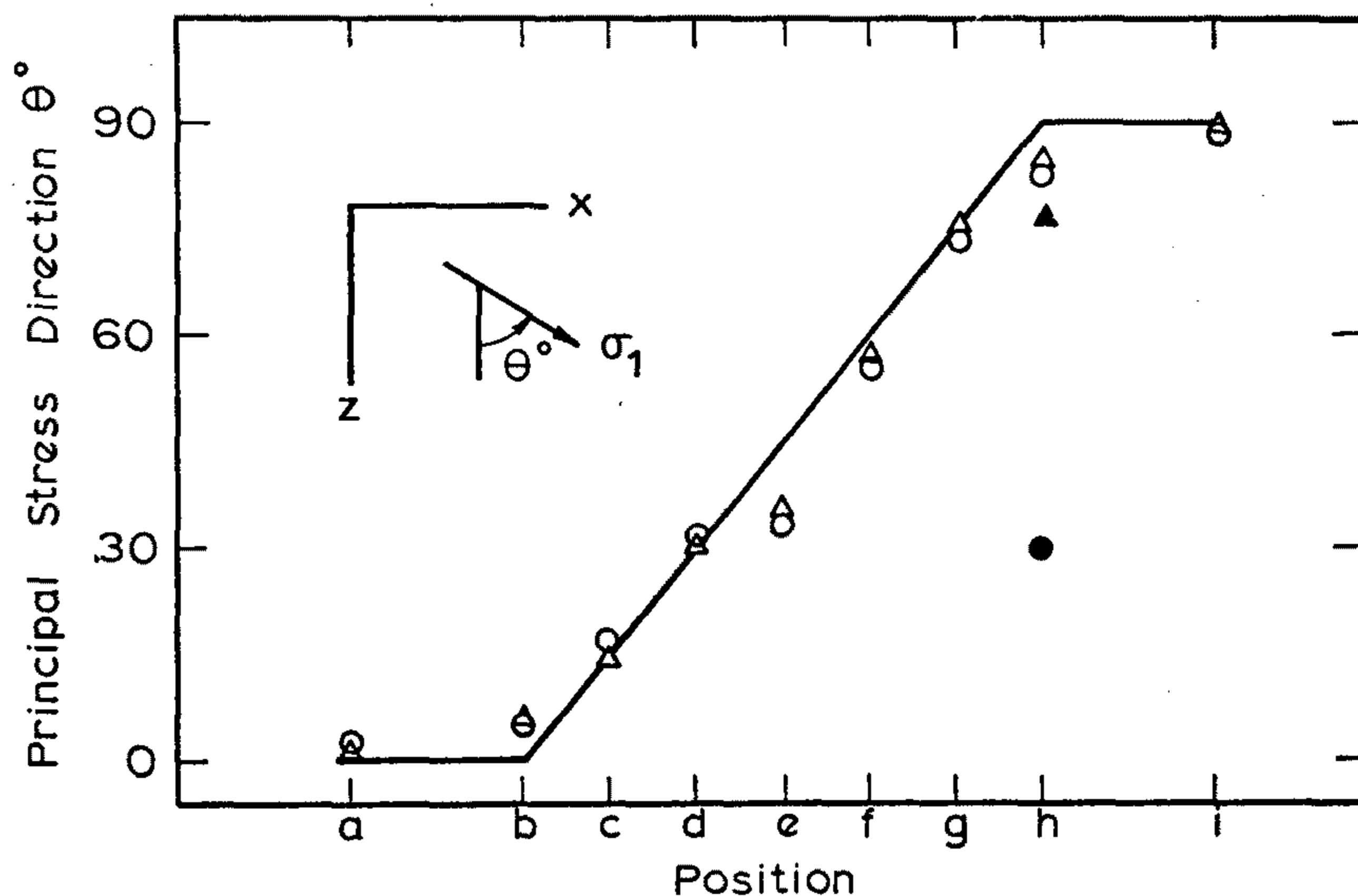


FIG.6-26(b) VARIATION IN PRINCIPAL STRESS DIRECTION
ALONG A STRESS CHARACTERISTIC

MESH 1

the more refined mesh was 3% above the Prandtl value in the no rupture case and 2.6% above when rupture was permitted. The variations in mean compressive stress and principal stress direction at collapse are shown in Fig. 6.27 for the two cases using mesh 2 and it can be seen that while the collapse loads were only slightly different, the corresponding stresses within the plastic region obtained when rupture was permitted were somewhat better than those obtained without rupture. The effect of the rupture line upon the soil profile at collapse is indicated in the insert to Fig. 6.25.

This example illustrates a number of features of the analysis. Firstly, the introduction of rupture lines into a coarse mesh will have the same effect on the collapse load as significant refinement of the mesh, while requiring considerably less computational effort. Secondly, the introduction of rupture lines into a mesh which is already sufficiently refined to give a good, well defined collapse load, does not alter that collapse load. This emphasises a point made earlier, that the introduction of potential rupture lines does not weaken the material, but rather, introduces additional freedom for plastic deformation in cases where the original finite element mesh is inadequate.

The technique of introducing rupture lines to facilitate the determination of collapse loads has been used for a number of different problems to illustrate the range of applications for the technique. Fig. 6.28 shows two finite element meshes used in the analysis of a smooth rigid footing resting on a homogeneous, cohesive frictional material with cohesion c , angle of internal friction $\phi = 30^\circ$ and angle of dilatancy $\psi = 0^\circ$. The soil was assumed to be

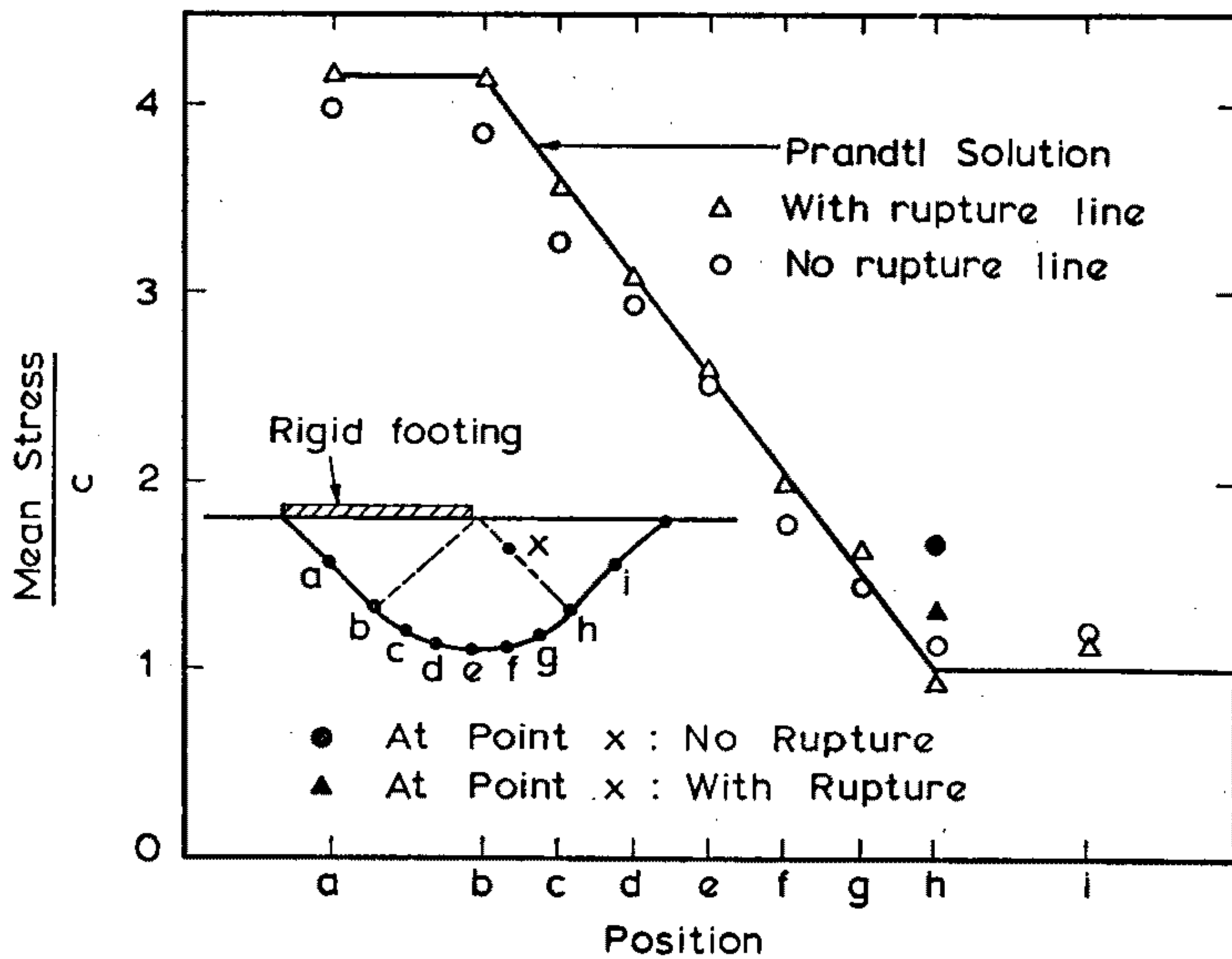


FIG. 6.27a VARIATION IN MEAN COMPRESSIVE STRESS ALONG
A STRESS CHARACTERISTIC - MESH 2

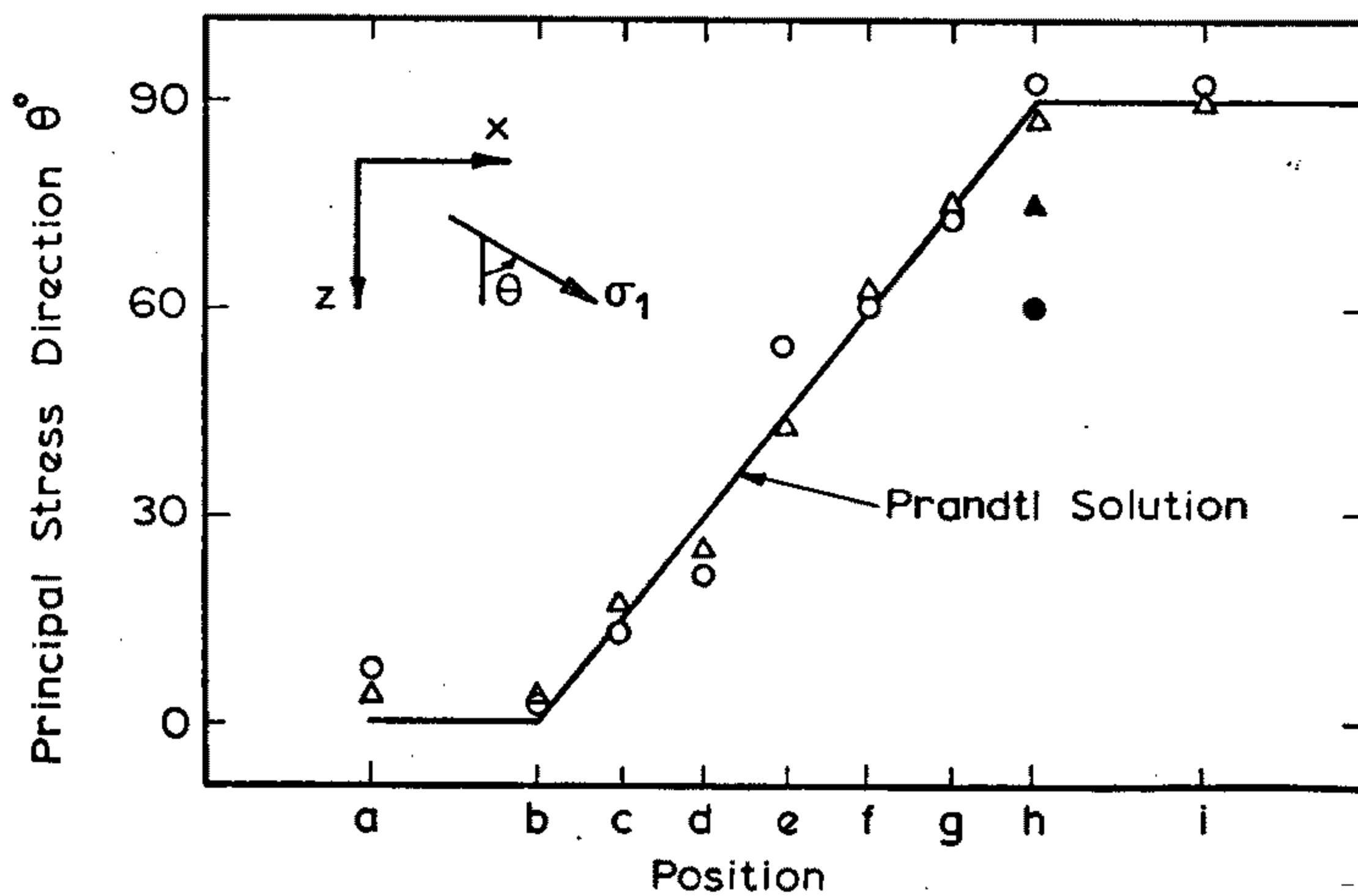


FIG. 6.27b VARIATION IN PRINCIPAL STRESS DIRECTION ALONG A
STRESS CHARACTERISTIC - MESH 2

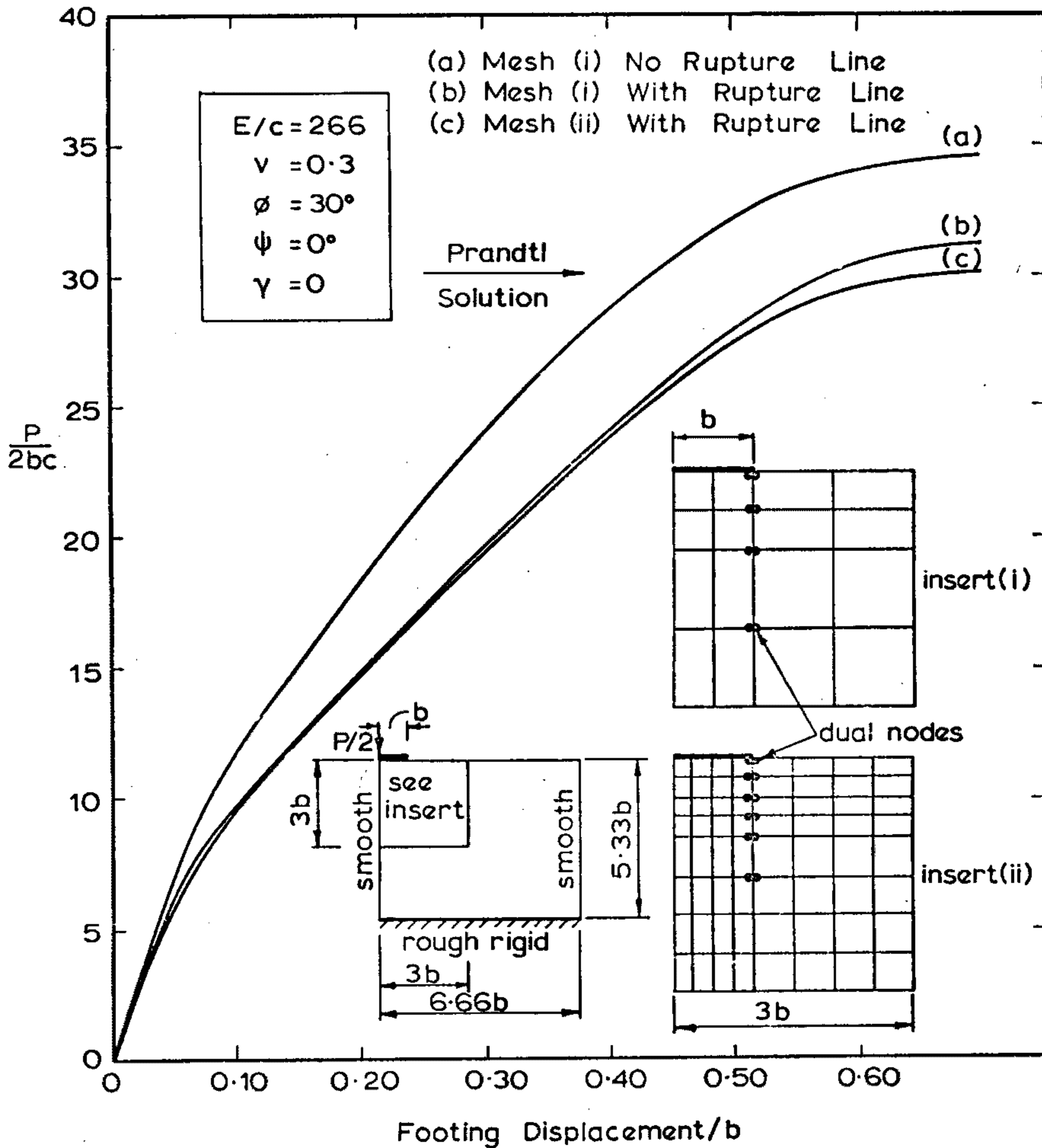


FIG.6.28 EFFECT OF RUPTURE LINE ON LOAD DEFLECTION
 CURVES FOR A RIGID FOOTING ON A COHESIVE FRICTIONAL SOIL

weightless and to have an elastic modulus $E = 266c$ and a Poisson's ratio $\nu = 0.3$. The load displacement curve obtained using mesh (i) without a rupture line [curve (a)] was 16% higher than the Prandtl solution when the computation was stopped. This may be compared with curve (b) obtained using the same mesh but with a potential rupture line included. Curve (c) shows the load displacement relationship obtained using the finer mesh [mesh (ii)] with a potential rupture line included. The coarser mesh with a rupture line gave a collapse load which would be adequate for most engineering applications, while the solution using the finer mesh is only just above the analytical Prandtl collapse load.

Application of this technique is not restricted to plane strain problems and, indeed, the method is of considerable benefit in the analysis of axi-symmetric problems because of the greater difficulties often encountered in obtaining good collapse solutions, even with considerable mesh refinement (Nagtegaal et al., 1974). To illustrate this, a smooth circular rigid footing resting on a purely cohesive soil with cohesion c , modulus of elasticity $E = 500c$ and Poisson's ratio $\nu = 0.48$ was analysed using the mesh shown in Fig. 6.29. The collapse load obtained without a potential rupture line was found to be 27% above the plasticity solution obtained by Cox et al., (1961), however, with a rupture line, the collapse load was only 2% above the Cox solution.

In the previous examples it was found that only one potential rupture line was needed to effect a sufficient improvement in evaluation of the collapse load. There is, however, no restriction on the number of potential rupture lines which can be included and,

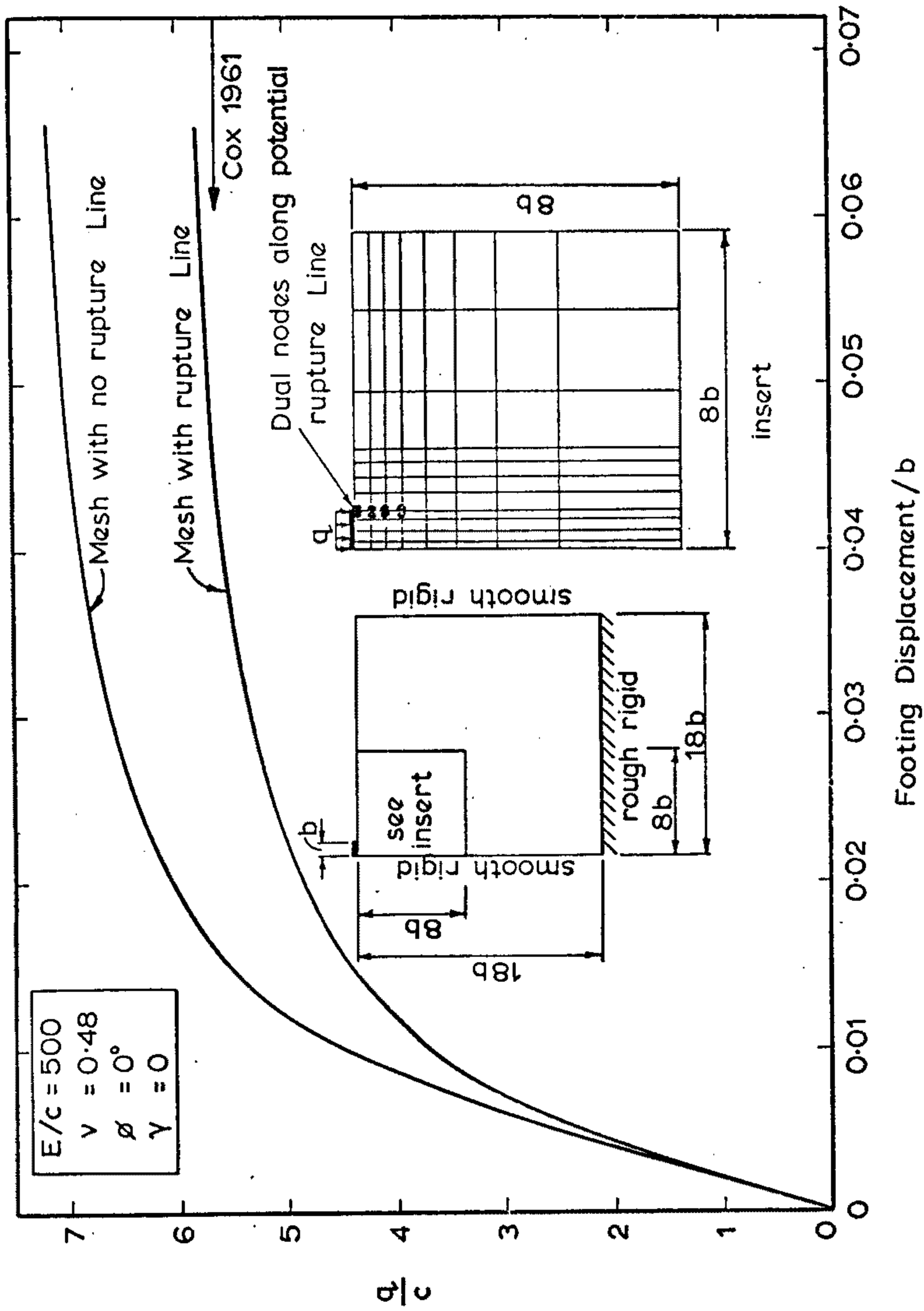


FIG.6.29 LOAD DEFLECTION CURVE FOR A SMOOTH RIGID CIRCULAR FOOTING

in problems where a singularity plays an even more dominant role than for a surface footing, more than one potential rupture line may be needed. Consider, for example, a horizontal smooth anchor at very great depth in a purely cohesive soil with cohesion c , modulus of elasticity $E = 500c$ and Poisson's ratio $\nu = 0.48$. It is assumed that the soil adheres to the back of the anchor and hence only a quarter of the problem need be analysed. Two potential rupture lines were incorporated in the mesh, as shown in Fig. 6.30. The load displacement curves with and without the rupture lines are also shown in Fig. 6.30. In the no rupture case, the load displacement curve rises well above the best upper bound available from limit analysis, while the curve obtained with rupture permitted lies between the best upper and lower bounds, and was in fact just below the upper bound solution. Fig. 6.31 shows the velocity field, for the rupture case, at collapse. Superimposed on this is the limiting velocity characteristic from the upper bound solution. Two points emerge from this example. Firstly, the finite element velocity field can be seen to correspond well with the upper bound. In particular, it will be noticed that all the large deformation in the finite element field takes place within the region defined by the upper bound solution, while the material is practically at rest outside this region. Secondly, rupture can be seen to have only affected the two dual node pairs closest to the anchor tip on each potential rupture surface. Thus, although provision was made for possible rupture along the entire length of these lines, actual rupture only occurs close to the stress singularity.

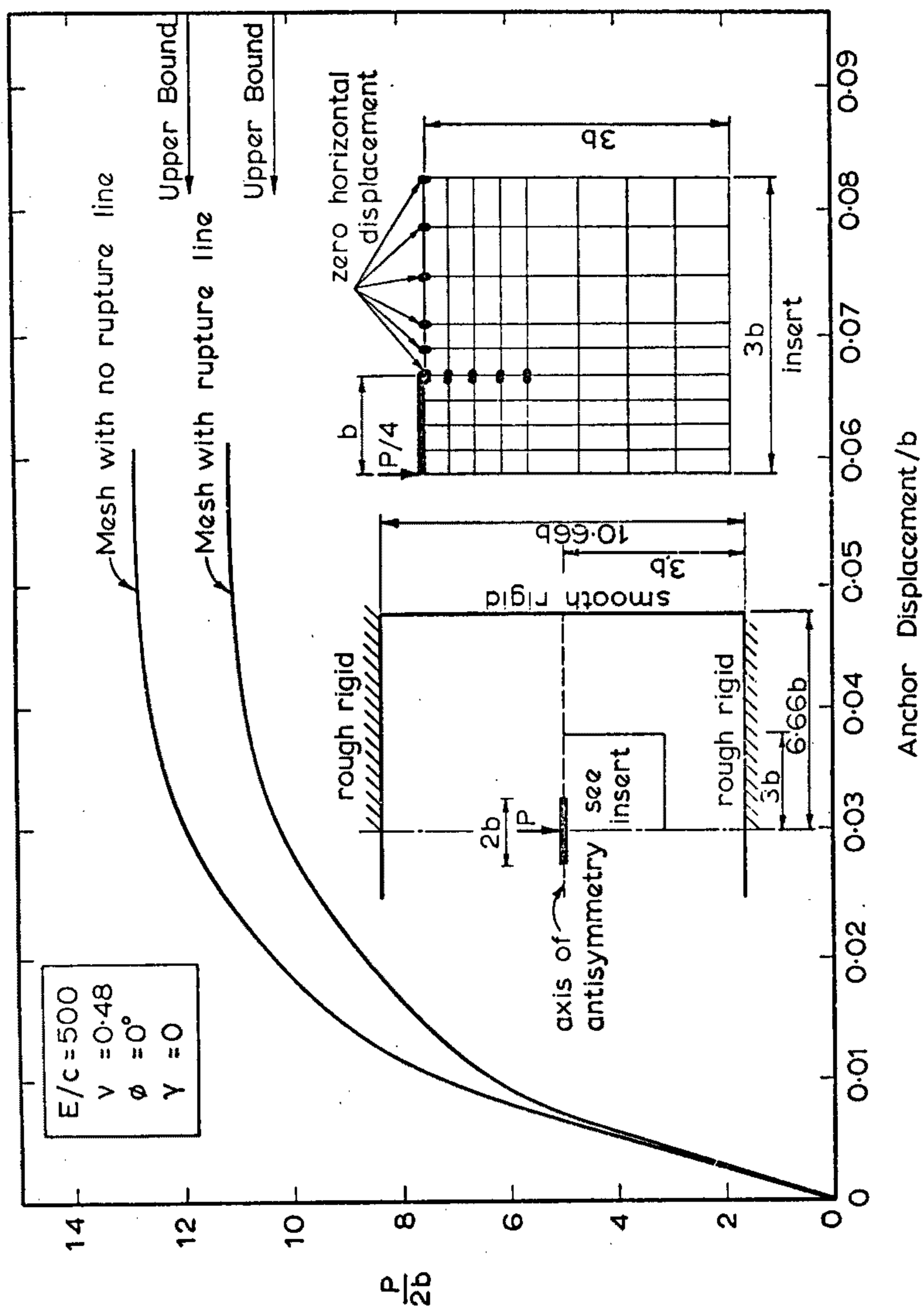


FIG.6.30 LOAD DEFLECTION CURVE FOR A SMOOTH STRIP ANCHOR AT INFINITE DEPTH

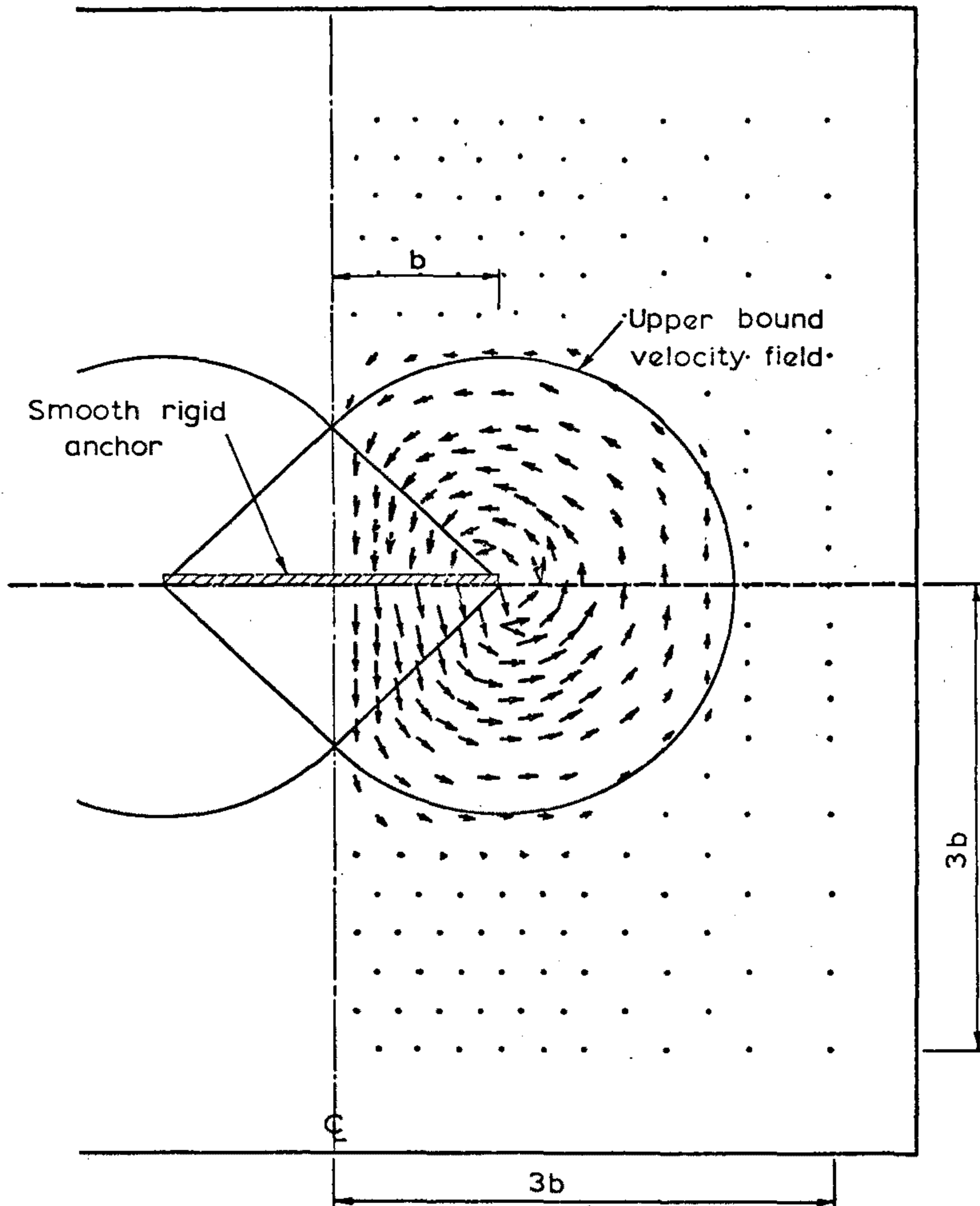


FIG.6.31 VELOCITY FIELD AT COLLAPSE FOR RIGID ANCHOR

6.4.3 Conclusions

Two finite element techniques for the prediction of numerical collapse loads have been presented, and it is concluded that for a particular problem, these techniques give a better estimate of the collapse load than would otherwise have been obtained from a conventional finite element analysis involving similar computational effort.

CHAPTER SEVEN

THE BEHAVIOUR OF HORIZONTALLY EMBEDDED ANCHORS

7.1 INTRODUCTION

In this chapter, the ultimate bearing capacity of horizontally embedded anchor plates subjected to vertical pullout forces will be investigated.

The general configuration of the problem to be analysed and its finite element idealisation are shown in Figs 7.1(a) and 7.1(b) respectively.

The ultimate uplift capacity, Q_u , of an anchor plate can be written in the form

$$Q_u = Aq_u + W_a$$

where A is the area of the anchor plate;

q_u is the average bearing capacity of the anchor;

and W_a is the effective weight of the anchor plate.

The contribution of shearing resistance along the stem of the anchor, if any, will be neglected in this analysis.

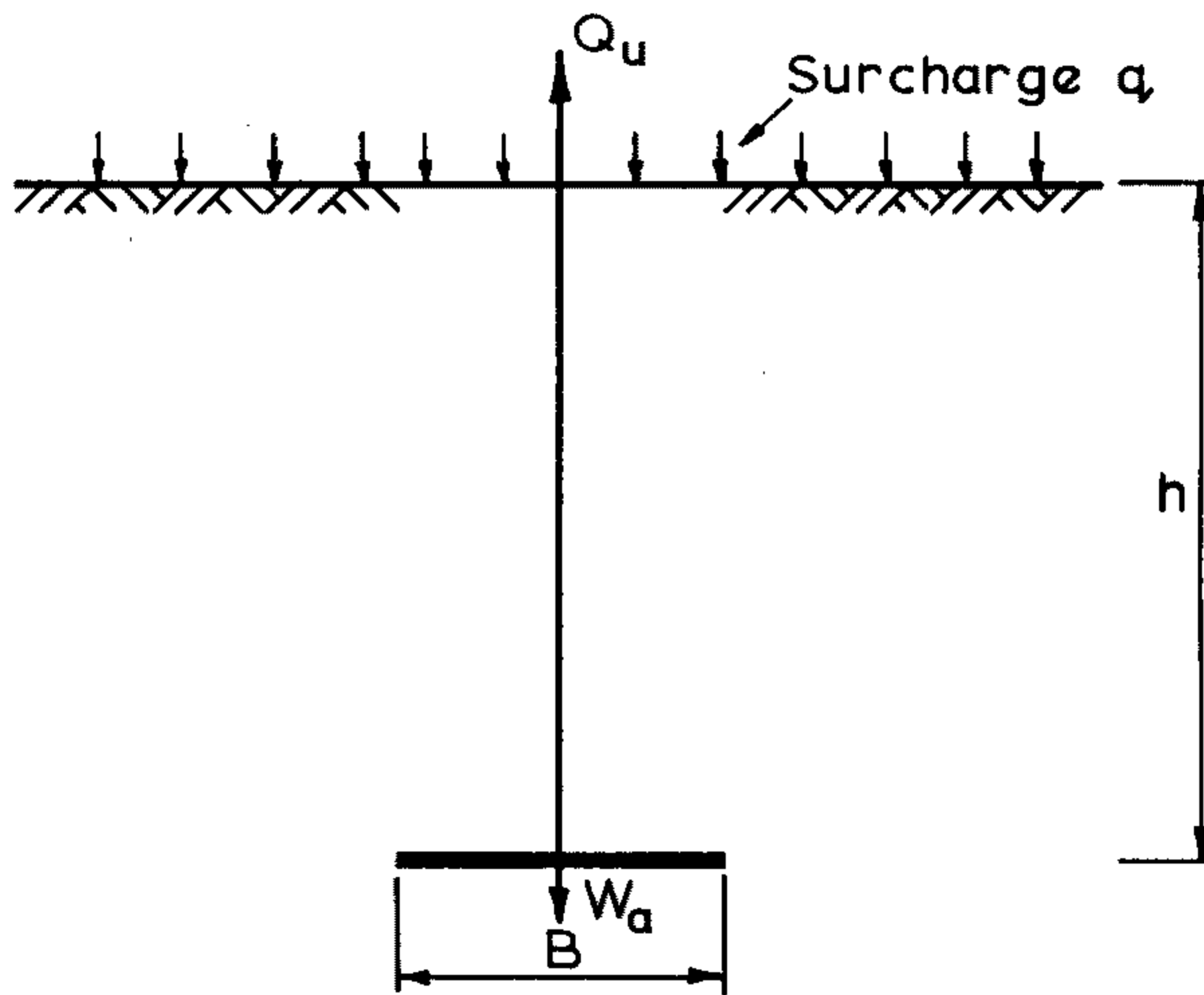
The anchor capacity, q_u , will be presented in a form similar to that used in the analysis of surface footings, namely

$$q_u = F_s (cF_c + \gamma h F_\gamma) \quad (7.1)$$

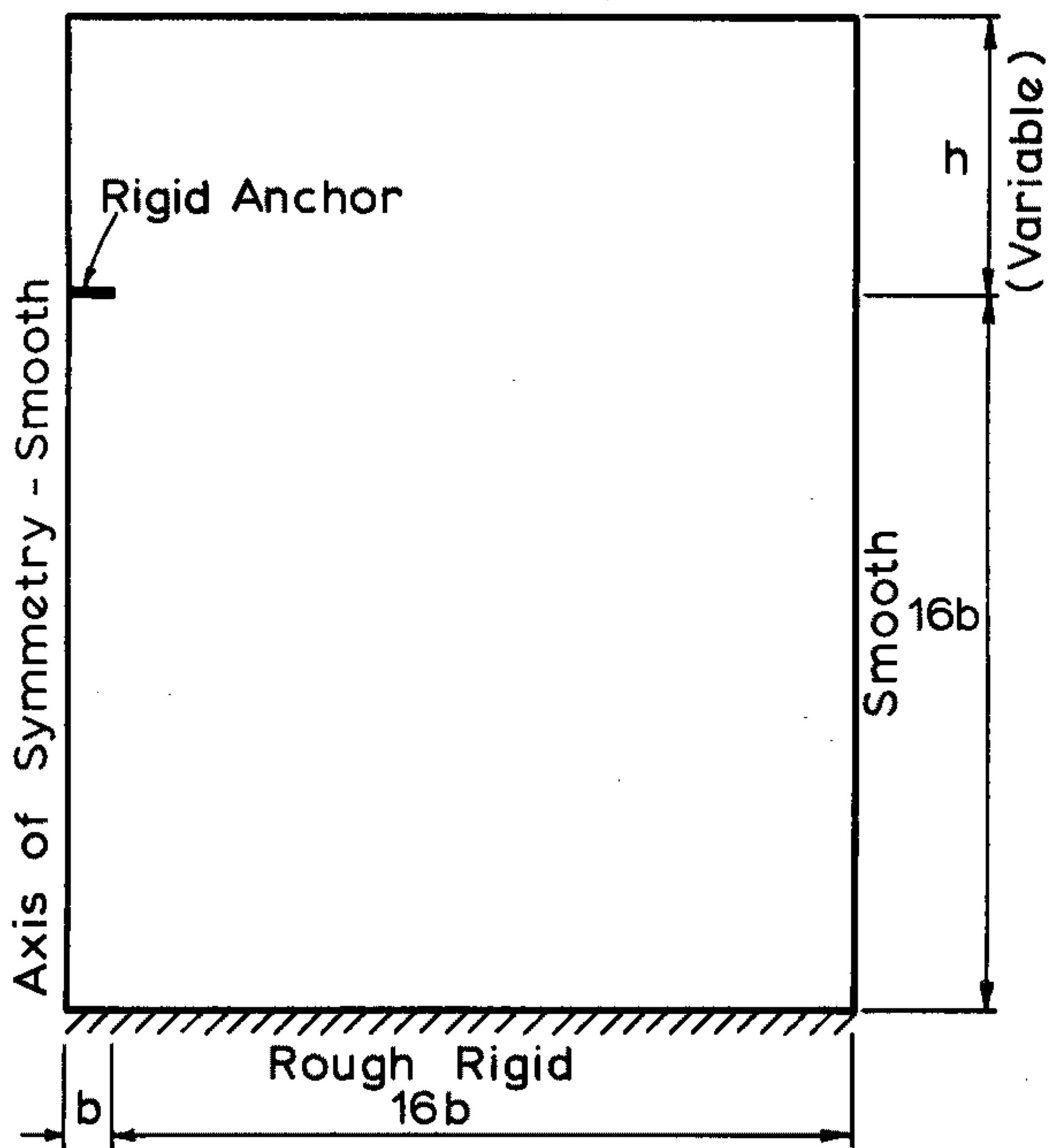
where c and γ are the cohesion and unit weight of the soil respectively;

h is the depth to which the anchor is buried;

$F_c = \left(\frac{q_u}{c}\right)_{\gamma=0}$ and $F_\gamma = \left(\frac{q_u}{\gamma h}\right)_{c=0}$ are bearing capacity factors which



(a) Basic Problem



(b) Finite Element Idealisation

are primarily functions of the angle of internal friction, ϕ , and the embedment ratio h/B , (although the secondary effects of dilatancy, ψ , initial stress state and anchor roughness will also be considered); and

F_s is a correction factor to allow for the fact that the bearing capacity is not simply the sum of the contributions of cohesion and overburden pressure. Thus, strictly speaking, the bearing capacity factors may not be 'superimposed', however experience indicates (e.g. see Davis and Booker, 1971) that this is often a good approximation.

The effect of a surcharge q_s above the soil surface is not included in Eq. 7.1 since the bearing capacity of an anchor in a soil mass with cohesion c^* , subjected to a surcharge q_s , can be determined by considering a soil mass with cohesion $c = c^* + q_s \tan \phi$ and no surcharge, provided that the influence of the surcharge upon the occurrence of breakaway of the anchor plate from the underlying soil is considered in the determination of F_c . Thus in addition to the parameters mentioned above, F_c is also a function of the dimensionless surcharge q_s/c .

In general, foundation design involves consideration of both bearing capacity and settlement. Thus the maximum safe foundation load is obtained by dividing the bearing capacity by a factor of safety. Settlement of the foundation is then checked independently, usually on the basis of an elastic analysis with, perhaps, some modification for local yield (Poulos, 1977).

Although this approach to foundation design is well established in the profession, it is worth emphasising that the clear distinction

between bearing capacity and settlement considerations is quite artificial for all but the simplest problems. Most soil-structure interaction problems, both in reality and in the finite element analysis, exhibit three modes of behaviour; namely,

- elastic (which may involve minor local yield);
- elasto-plastic (which involves significant contained plastic flow); and
- plastic collapse (which involves uncontained plastic flow).

From the standpoint of the rigorous theory of plasticity, the ultimate bearing capacity is the maximum load the soil can sustain (general shear failure), and as such corresponds to complete collapse of the foundation. However, in many cases this collapse load is only attained after extensive contained plastic deformation. In these cases the settlement of a footing which is subjected to the 'safe foundation load' (obtained by dividing the ultimate bearing capacity by a factor of safety of 2.5 or 3) would be in excess of that predicted from an elastic settlement analysis. Thus, while there is an 'adequate' factor of safety against complete collapse, the foundation may be rendered unserviceable due to unsatisfactorily high deflections. Consequently, if the conventional (two part) engineering approach to foundation design is to be adopted, then it is necessary to define the bearing capacity (which may be less than or equal to the ultimate bearing capacity) such that reasonable settlement predictions can be made at the 'safe working load' determined from bearing capacity considerations.

The definition of failure in these cases where there is significant contained plastic flow must, to some extent, be an arbitrary one. One commonly adopted approach is to determine the bearing capacity

(for local shear failure) from the load at which the load-deflection curve passes into a straight and fairly steep tangent. (Terzaghi, 1943; Meyerhof, 1951). In this chapter, an alternative approach will be proposed in which the bearing capacity is determined from the 'practical failure' load (to be denoted as the 'k4' failure load) which is the load level at which the actual displacement is four times the displacement predicted from an elastic analysis.

This investigation into the behaviour of horizontal anchor plates will be arranged as follows:

Firstly, the numerical procedure and idealization will be discussed.

Secondly, the effects of embedment ratio, angle of internal friction and surcharge upon the behaviour of an anchor resting in a cohesive material will be considered and the bearing capacity factor F_c determined for this case.

Thirdly, the effects of embedment ratio and angle of internal friction upon a purely frictional material with self weight will be investigated and the basic bearing capacity factor F_γ determined.

Fourthly, the likely effects of the coefficient of lateral earth pressure at rest (K_0), angle of dilatancy ψ , anchor roughness and layer depth upon anchor behaviour will be considered.

Fifthly, the assumption of superposition implicit in Eq. 7.1 will be examined for a number of cases. The suitability of the bearing

capacity factors for hand calculations will then be illustrated by comparison of the failure loads predicted on this basis with the failure loads obtained directly from the finite element calculations.

Finally, the likely effect of shape upon bearing capacity will be illustrated by the consideration of a circular anchor resting in a purely cohesive soil.

7.2 NUMERICAL DETAILS

The finite element results presented in this chapter were obtained using the soil-structure interaction theory described in Chapter 5, and the numerical procedure described in Chapter 6. The behaviour at the edge of the anchor was modelled as described in Chapter 5. It should be emphasised that the parametric study presented in this chapter was only possible because of the computational savings obtained by using potential rupture lines (see Chapter 6) as an alternative to a much more refined finite element mesh.

As discussed in Chapter 3, the elastic load deformation behaviour of an anchor buried within a soil mass is dependent on three geometric parameters; namely, the embedment ratio of the anchor (h/B), the depth of soil below the anchor (D/B) and the distance to the lateral boundary (W/B). The combination of these geometric parameters with the soil parameters affecting both the elastic and plastic behaviour (e.g. c , γ , ϕ , ψ , K_0 , E , ν) leads to a somewhat unmanageable number of variables. Accordingly, the geometric parameters least likely to affect the ultimate behaviour (ie. D/B and W/B) were held

at constant values for the majority of the analyses.

The finite element boundary conditions generally adopted are shown in Fig. 7.1(b). The distance to the lateral boundary (W) was chosen to be 8.5 times the footing width (B), since it was found in Chapter 3 that for relatively shallow layers ($D/B \leq 10$), the elastic displacement of an anchor was virtually independent of this distance for values of $W/B > 8$. The depth of the soil layer below the anchor (D) was taken to be eight times the footing width for most of the analyses. This depth was sufficient to ensure that plastic failure never occurred close to the base of the layer and, consequently, it was anticipated that the collapse load would not be significantly altered by increasing the depth of the layer. On the other hand, the apparent stiffness of an anchor does vary with the depth of the soil layer (see Chapter 3), and hence, strictly speaking, the finite element solutions only apply to a layer of depth $8B$. However, in section 7.4.4 it will be shown by means of an example that the entire elasto-plastic behaviour to collapse can be normalised so that it is largely invariant for a reasonably wide range of layer depths. Clearly, this approach will be subject to the limitations that: firstly, the base of the layer should be sufficiently deep so as not to interfere with the growth of plastic regions; and secondly, the layer must be finite, otherwise the elastic displacement of the strip would be infinite. This latter restriction is not considered to be of practical significance since soil layers are not infinite, and furthermore, the strip itself would not be infinite but, rather, would have a finite length L , and hence would have a finite displacement.

In the context of this chapter, an anchor is considered to be

at infinite depth ($h/B = \infty$) when the free surface is so remote as not to effect the anchor behaviour. However, difficulties arise in analysing very deep anchors because of the requirement that the distance, h , from the anchor to the soil surface, should be sufficiently small to allow adequate finite element idealisation. This distance h was usually limited to eight times the anchor width, although one case with an embedment ratio of fifteen was investigated. In a number of instances, deep anchor behaviour (ie. $h/B = \infty$) was observed at embedment ratios less than eight; but in general, the transition from shallow to deep anchor behaviour was not detected over this range of embedment ratios. Consequently, the anchor at 'infinite' depth was analysed as a special case by adopting identical boundary conditions above and below the anchor. Thus, the anchor at 'infinite depth' was in fact contained in a box, with a rough rigid boundary at distance $8B$ above and below the anchor.

The basic finite element mesh adopted for the majority of the analyses in this chapter is shown in Fig. 7.2. The distribution of elements below the anchor was the same for all embedment ratios, while the details of the element distribution above the anchor for the various embedment ratios are given in the inset to Fig. 7.2. The anchor itself and the potential rupture lines are identified by dual nodes.

As far as possible, the effect of mesh variation upon the results for different embedment ratios was avoided by maintaining fairly constant finite element bay dimensions, with different embedment ratios being simulated by removing unnecessary bays of elements.

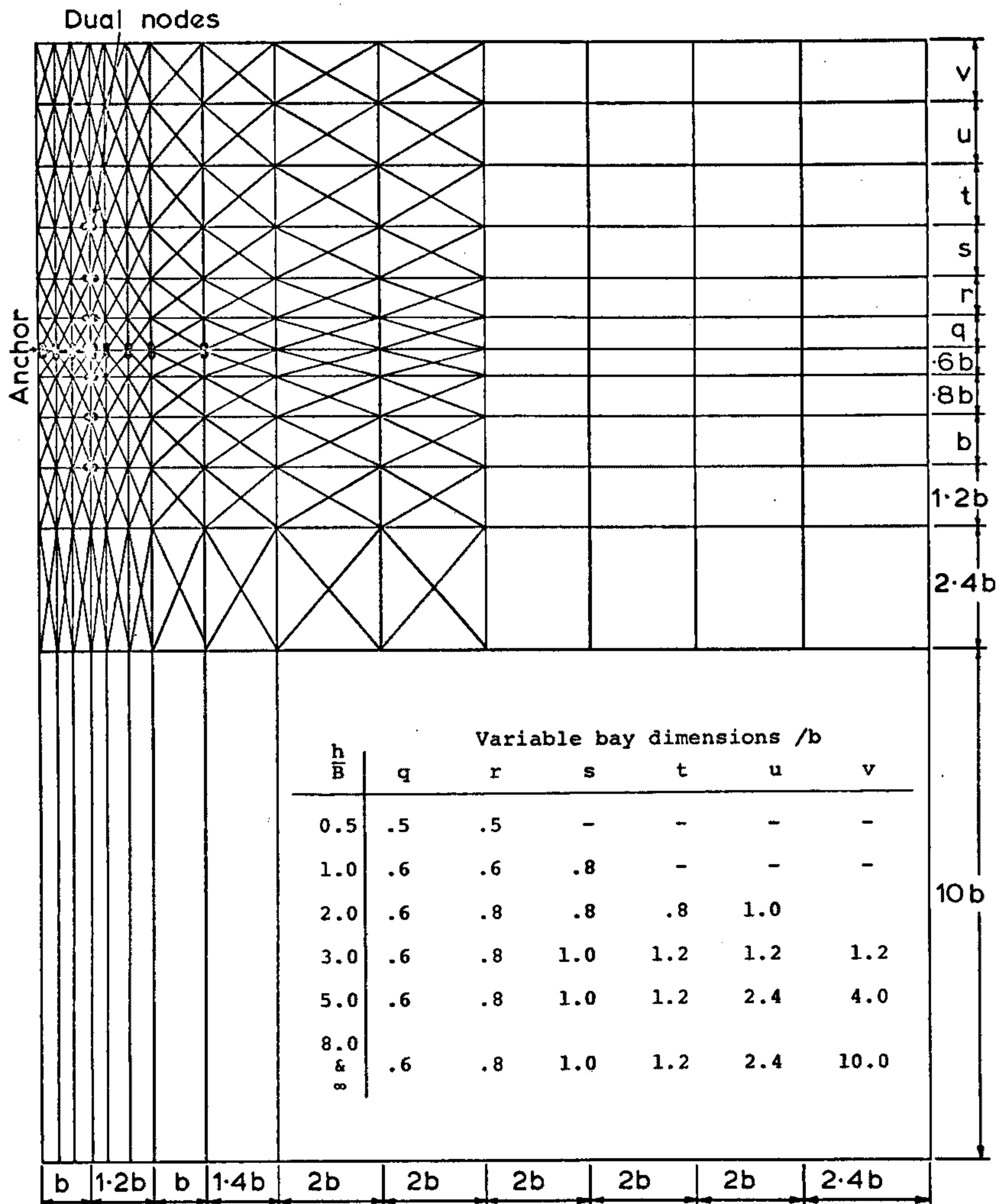


FIG.7.2 GENERAL FINITE ELEMENT MESH ARRANGEMENT TO SCALE FOR $h/B = 3$

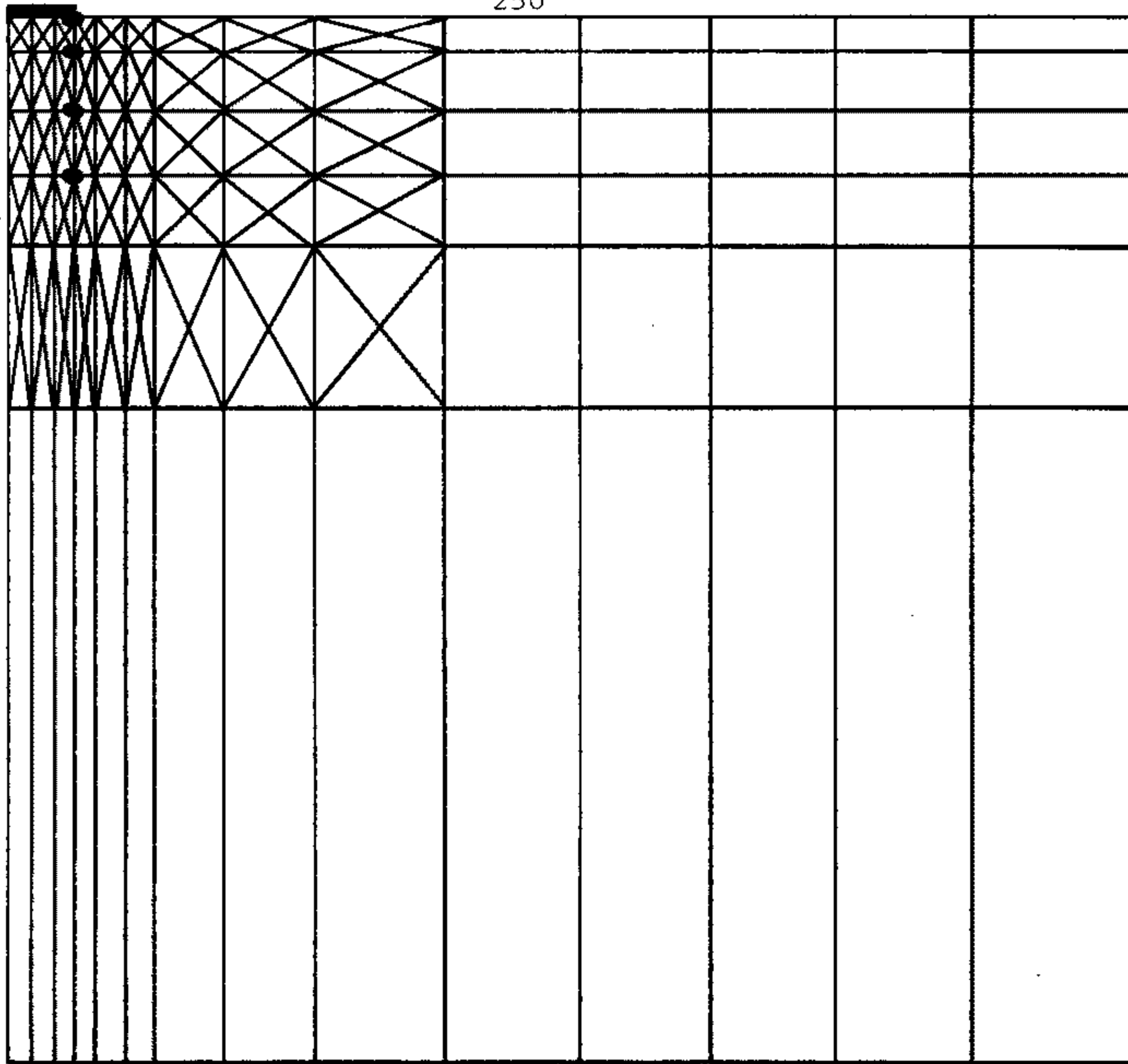
Two test problems with a known solution were analysed using the basic mesh so as to determine the order of accuracy which could be expected.

Fig. 7.3(a) shows the finite element mesh used in the analysis of an anchor resting on the surface of a purely cohesive material. It is assumed that the soil surface is subjected to a surcharge, q_s , sufficient to prevent breakaway of the anchor from the soil. This problem is analogous to that of a rigid footing and the ultimate bearing capacity corresponds to the Prandtl value $q_u = (2 + \pi)c$.

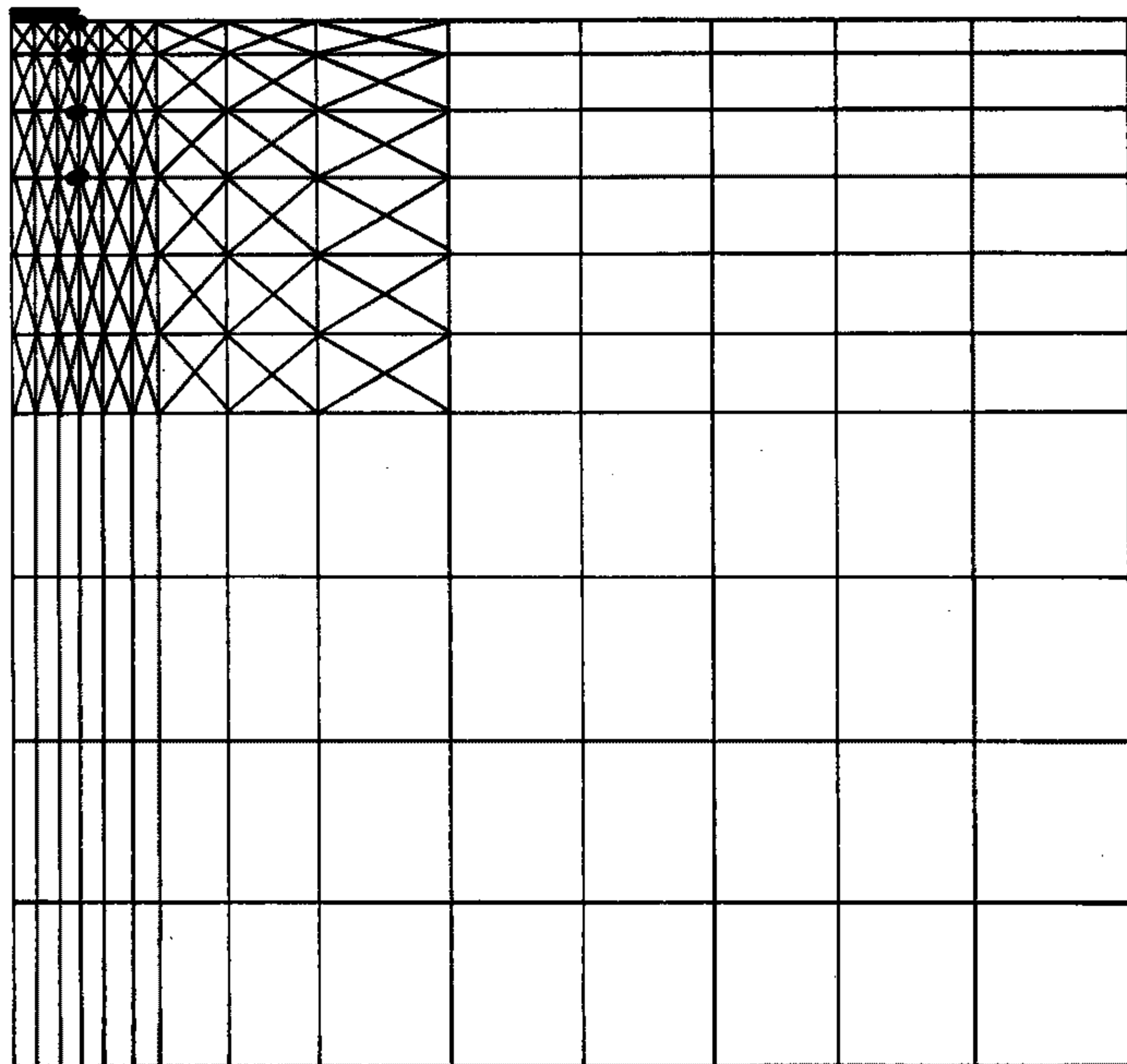
The elastic stresses below the centre and edge of the anchor obtained from the finite element analysis are shown in Fig. 7.4 and can be seen to be in reasonable agreement with the analytical solution.

The load displacement curve obtained for the surface anchor resting on a soil mass with $E/c = 500$ and $\nu = 0.48$ is shown in Fig. 7.5. The numerical collapse load can be seen to exceed the Prandtl value by 4.3%.

To simulate the ultimate behaviour of an anchor at infinite depth, a plate resting in the centre of a mass of purely cohesive material ($E/c = 500$, $\nu = 0.48$) and contained in a box with a rough rigid top and bottom and smooth rigid sides was analysed using the appropriate mesh shown in Fig. 7.2. It was assumed that there was an initial hydrostatic stress state sufficient to prevent breakaway of the anchor from the soil (alternatively it could be assumed that the soil-anchor interface could sustain sufficient tension to ensure full adhesion of the soil and anchor). The load displacement curve obtained from this



(a) Basic Mesh



(b) Basic mesh with aspect ratio of slender elements reduced

FIG. 7.3 FINITE ELEMENT MESH FOR ANCHOR WITH ADHESION

$$\underline{h/B = 0}$$

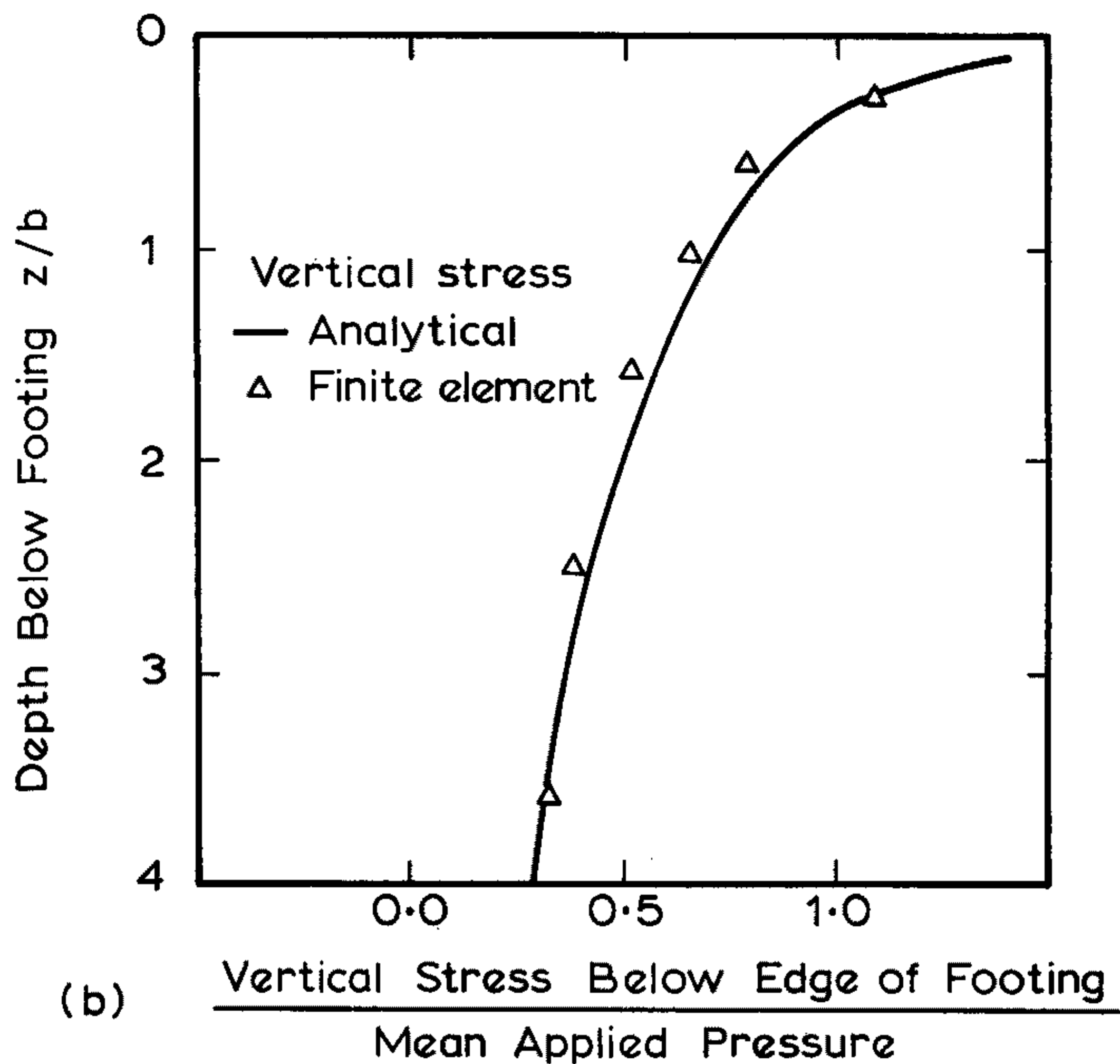
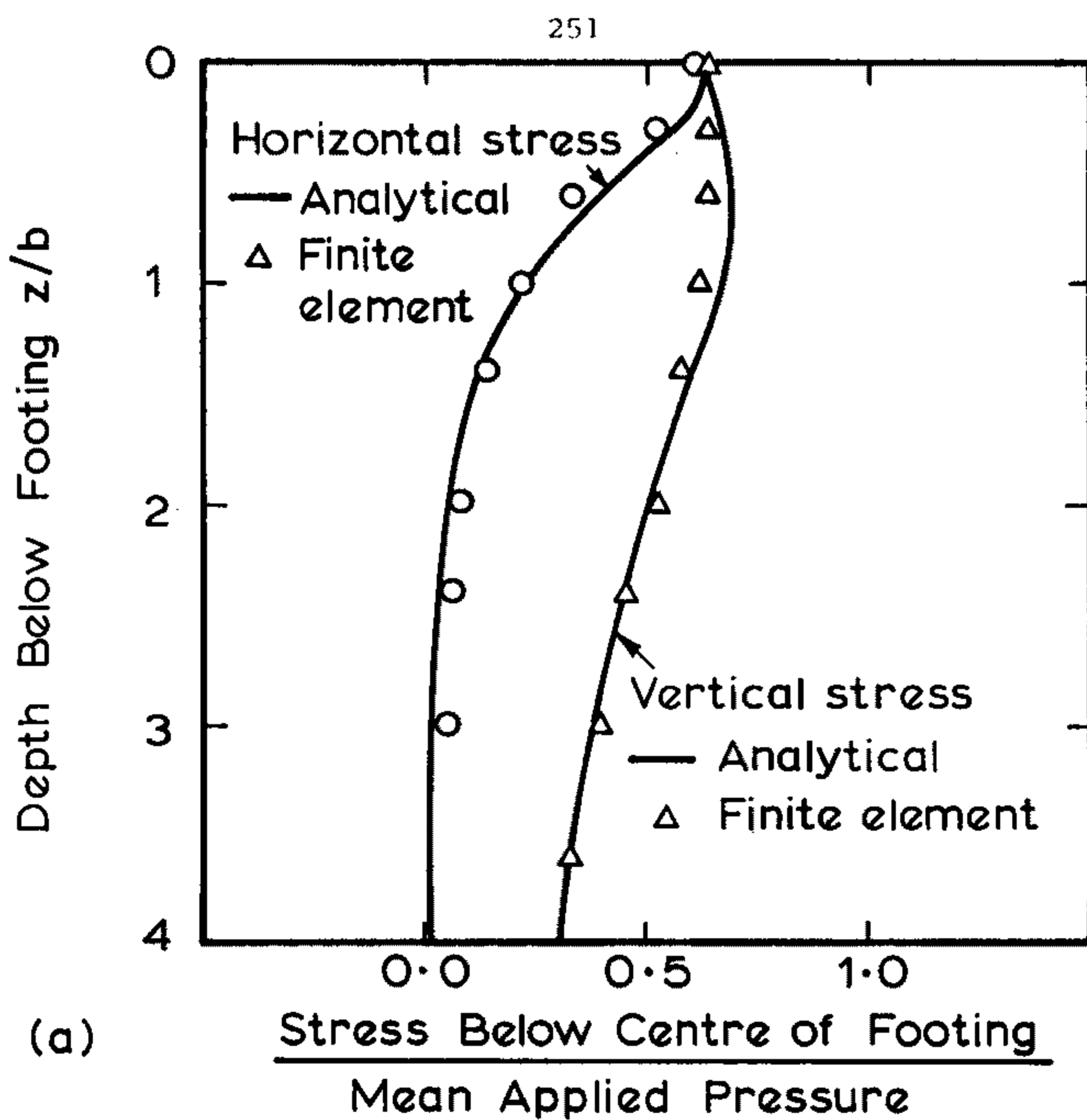
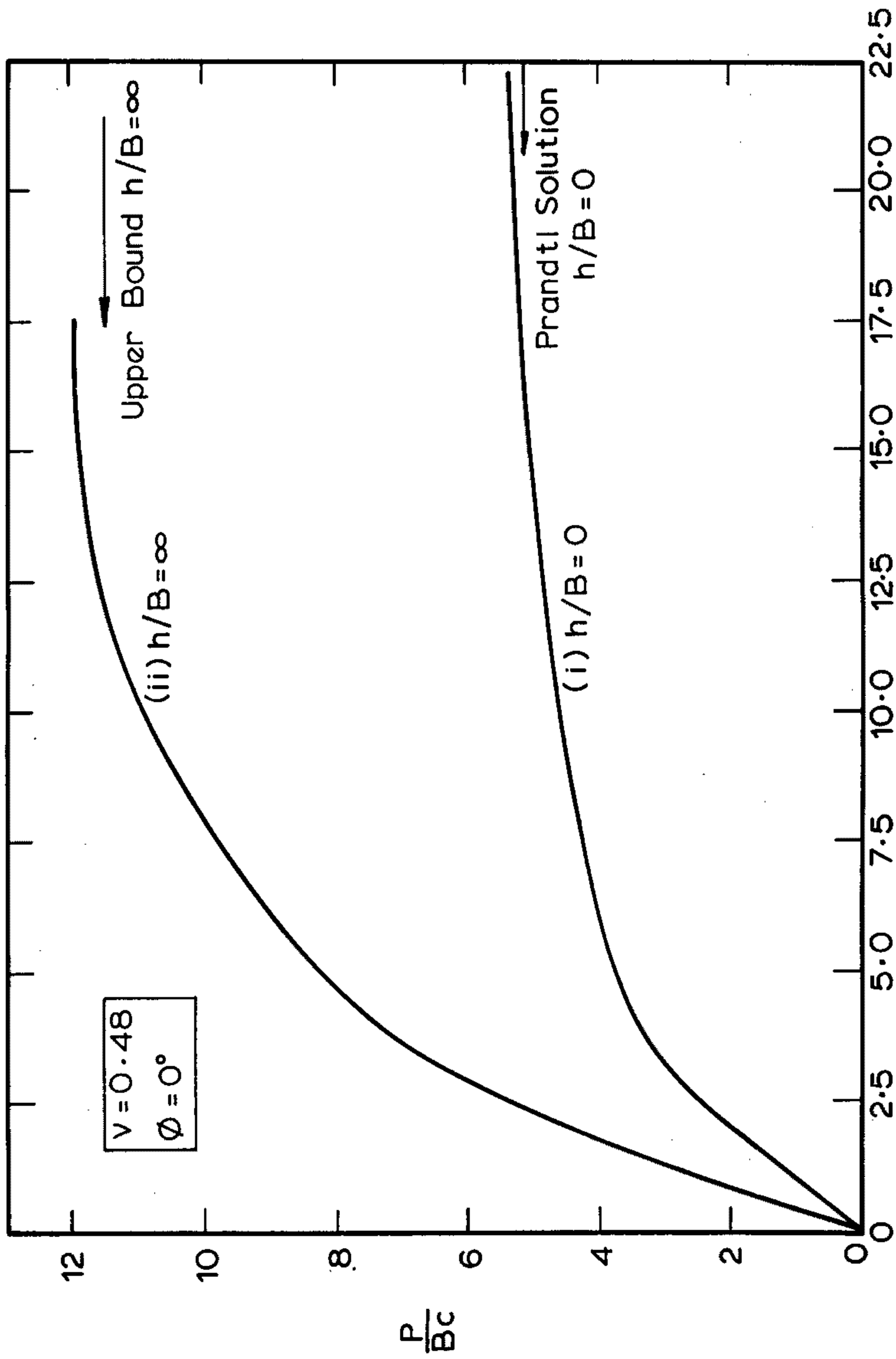


FIG.7.4 VARIATION IN STRESS WITH DEPTH BELOW A RIGID FOOTING



Anchor Displacement E/Bc

FIG.7.5 LOAD DISPLACEMENT CURVES FOR TWO RIGID ANCHORS - NO BREAKAWAY

analysis is shown in Fig. 7.5. In this case, the numerical collapse load exceeded the upperbound load (see Chapter 4) $q_u = (2 + 3\pi)c$ by 4.6%.

The fairly close agreement between the numerical and analytical loads for these extreme cases of $h/B = 0$ and $h/B = \infty$ suggests that the numerical collapse loads obtained for intermediate embedment ratios could be expected to be within approximately 5% of the actual value. Strictly speaking, this inferred order of accuracy only applies to cohesive materials, however, experience suggests that the mesh would also be suitable for cohesive-frictional and purely frictional materials which will be seen to generally involve simpler failure mechanisms than those observed in the purely cohesive case.

The elastic load-deflection behaviour (expressed in terms of the apparent stiffness $P/\delta E$) predicted from the finite element analysis was compared with the behaviour predicted from the classical elasticity approach (see Chapter 3) for a range of embedment ratios. It was found that the stiffness predicted by the finite element analysis was, typically, about 18% greater than the Fourier integral solution. This discrepancy in elastic stiffness arises because computer limitations on the number of finite elements which could be adopted in the analysis of deep anchors necessitated the use of slender elements near the base of the soil layer. To determine the likely effect of these slender elements on the prediction of collapse loads and upon the load displacement curve itself, a rigid footing analysis was performed using the mesh shown in Fig. 7.3(b), in which the aspect ratio of the slender elements in Fig. 7.3(a) was reduced by a factor of four. The load dis-

placement curves from this analysis and the prior analysis using mesh (a) are shown in Fig. 7.6. It will be seen that there is a 14.6% difference in the apparent stiffness obtained from the two analyses, and compared to the analytical solution, the stiffnesses obtained using meshes (a) and (b) are in error by 17% and 2% respectively.

Thus, it may be anticipated that the adoption of the basic finite element mesh would lead to an underestimate of the displacement at a given load level, but would not give rise to serious errors regarding either the stress distribution or general behaviour of the anchor. Indeed, the collapse loads obtained using the two meshes shown in Fig. 7.3 were in error (when compared to the Prandtl solution) by only 4.3% and 4% respectively; hence, the slender elements in the basic mesh resulted in an additional error of less than 0.4%. Furthermore, the load displacement curves can be 'normalised' by comparing loads obtained at displacements corresponding to given multiples of what would be the elastic displacement at this load level if plastic deformation had not occurred. Consider, for example, the load displacement curve (see Fig. 7.6) obtained using mesh 7.3(a). If the apparent elastic stiffness obtained from this mesh is ' k ', then the load at which the total displacement is twice, three times etc. the elastic displacement is obtained by constructing lines with slope $k/2$, $k/3$ etc. and then finding the intersection of these lines with the load displacement curve as shown in Fig. 7.6. The loads corresponding to a displacement of twice, three times, the elastic displacement are termed the ' k_2 ', ' k_3 '.. values. Similar; ' k_2 ', ' k_3 ' etc. loads may be obtained from an analysis using mesh 7.3b as shown in Fig. 7.6. Comparing the results of the two analyses, it was found that although the apparent elastic stiffnesses

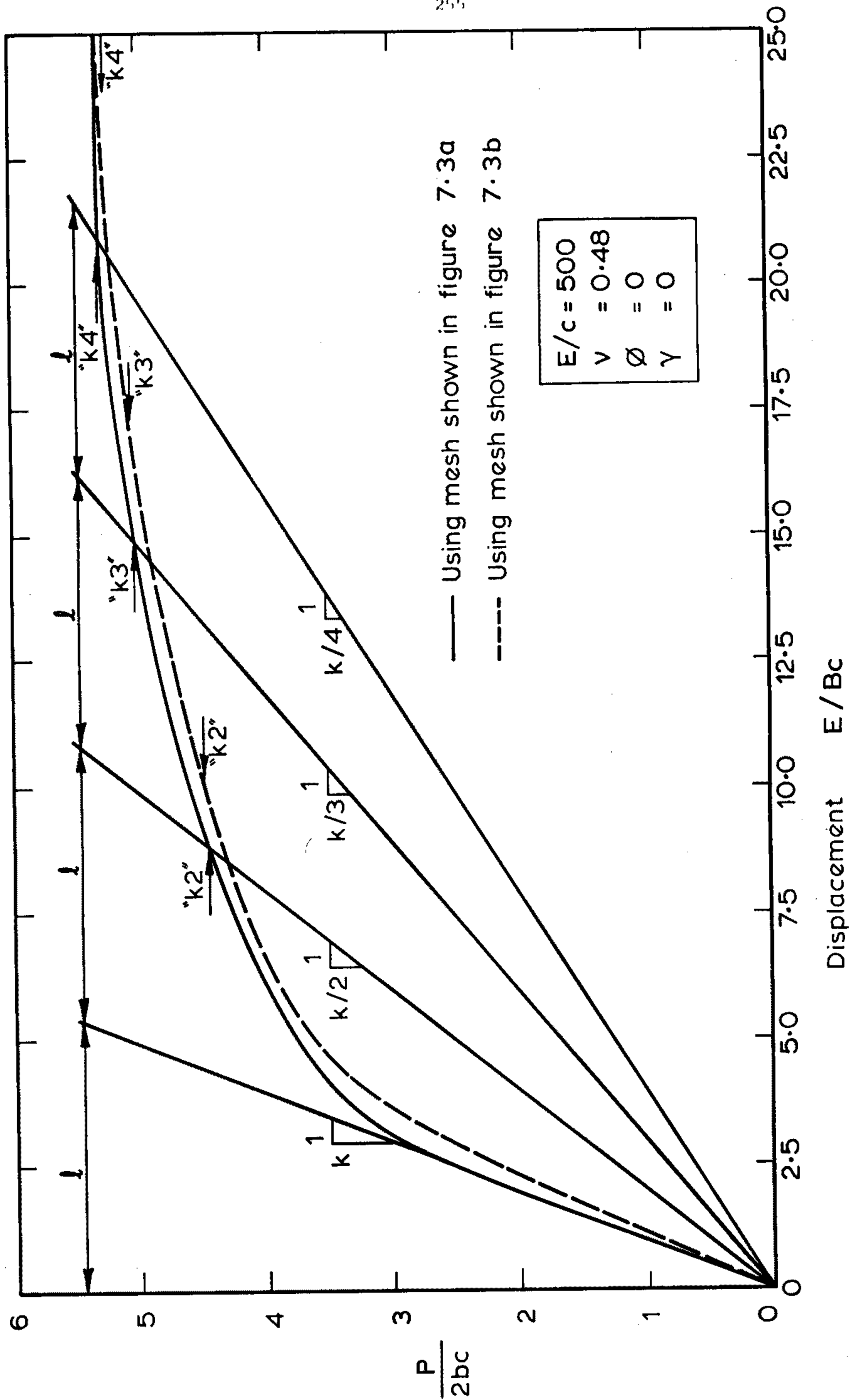


FIG.7.6 LOAD DEFLECTION CURVE FOR RIGID FOOTING-EFFECT OF SLENDER ELEMENTS

differed by more than 14%, the load at the normalised displacements differed by only 1.3% at the 'k2' displacement, 0.9% for 'k3' and 0.4% for 'k4', indicating that the effect of errors in apparent stiffness upon the load displacement curve can be largely eliminated by normalising in this way. Consequently, the entire load displacement curve to failure could be easily reconstructed using these 'k2', 'k3' etc. loads and an elastic stiffness 'k' which is determined from a more accurate elastic analysis. In an approximate way, this also makes it possible to include the effects of differing layer depths and the finite length of the strip by virtue of their influence upon the elastic stiffness; although care is obviously needed in determining the compatibility of any assumptions regarding the finite length of the anchor in the elastic analysis with the assumption of plane strain implicit in the determination of the 'k2', 'k3' loads.

Displacement defined analyses were performed for all rigid anchor analyses. The increment size adopted for cohesive and cohesive frictional materials was $0.5 Bc/E$, while an incremental size of $0.125 \gamma hB/E$ was adopted for purely frictional materials. The convergence criteria of Chapter 6 applicable to displacement defined problems were satisfied to better than 2%. As a check on the effectiveness of this iterative procedure, a second analysis of a fully bonded surface anchor (footing) was performed using an increment size $0.25 Bc/E$. In addition, an anchor with an embedment ratio $h/B = 3$, resting in a cohesive frictional material with $E/c = 500$, $\phi = 45^\circ$, $\psi = 0^\circ$, $\gamma = 0$ and $\nu = 0.3$ was analysed using increment sizes of $0.5 Bc/E$ and $0.25 Bc/E$. The 50% reduction in increment size altered the collapse load for the surface anchor on the purely cohesive material by 1.3%, and for the buried anchor in cohesive frictional material by 0.8%.

The numerical procedure for a purely frictional material will be discussed in section 7.3.3.1.

7.3 BASIC PARAMETERS - THE EFFECTS OF h/B , c , ϕ , γ

In this section, the effect of embedment ratio h/B , cohesion c , angle of internal friction ϕ , and unit weight γ upon the bearing capacity of a smooth, rigid strip anchor will be investigated, assuming that the soil has a coefficient of lateral earth pressure at rest, K_0 , equal to unity, and an angle of dilatancy $\psi = 0^\circ$ (corresponding to zero volume change during plastic deformation).

Unless otherwise stated, the analyses were performed for the case where $q_s = 0$, $\gamma = 0$ and where there is no adhesion between anchor and soil.

The results for cohesive materials were obtained for a ratio of elastic modulus to cohesion (E/c) equal to 500, however, they are equally valid for all values of E/c and are presented in a dimensionless form which is independent of the E/c ratio. In the case of purely frictional materials the analyses were performed for a ratio of elastic modulus to overburden pressure $E/\gamma h$ equal to 333, although the results were presented in a dimensionless form which was independent of the $E/\gamma h$ ratio.

Finally, all undrained ($\phi = 0^\circ$) and drained ($\phi > 0^\circ$) analyses were performed for values of Poisson's ratio ν equal to 0.48 and 0.3 respectively.

The three cases of a purely cohesive material (undrained loading), a cohesive-frictional material (drained loading) and a frictional material with weight (drained) will be considered in section 7.3.1, 7.3.2 and 7.3.3 respectively.

7.3.1 Purely Cohesive Material (Undrained Loading)

From a rigorous applied mechanics viewpoint, there are two possible 'basic cases' in the analysis of anchor behaviour. In the first of these cases, the soil both above and below the anchor is always in contact with the anchor plate and the situation can be classified as one in which there is no breakaway of the anchor from the underlying soil. In the second case, there is no contact between the anchor and the underlying soil once the anchor is loaded. This situation will be classified as one of immediate breakaway. If the soil-structure interface cannot sustain tension, then breakaway of an anchor plate from the underlying soil will occur when the stress redistribution caused by loading the anchor reduces the vertical stress at the underside of the anchor to zero. Initially, the vertical stress (q_h) at the level of an anchor plate will be equal to the sum of the overburden pressure due to the soils' self weight, γh , and any surcharge q_s above the soil surface; thus

$$q_h = \gamma h + q_s$$

For a purely cohesive material, the magnitude of the overburden pressure will not alter the mobilised strength of the material, however, it will influence the occurrence of breakaway. Thus the body force effects will be identical to the effect of a surface surcharge and it is only necessary to determine a single bearing capacity factor

$F_c \left(\frac{h}{B}, q_h/c, \phi = 0 \right)$, which is a function of embedment ratio and overburden pressure.

If the dimensionless overburden pressure q_h/c is sufficiently large to ensure that the vertical stress below the anchor is compressive for all applied anchor loads up to and including the failure load, then the soil will always be in contact with the anchor and no breakaway can occur. Clearly, any overburden pressure higher than the limiting value $(q_h/c)_1$ required to prevent breakaway will not increase the bearing capacity. It may be anticipated that there would be a range of bearing capacities and types of behaviour for overburden pressures in the range $0 \leq q_h/c \leq (q_h/c)_1$ corresponding to the transition from the immediate breakaway to the no breakaway condition.

It is recognised that the development of adhesion and/or suction between the anchor plate and the soil will reduce the likelihood of breakaway, and will have a similar short term effect on the bearing capacity to that of the overburden pressure q_h . However, in general, any contribution to the bearing capacity due to either adhesion or suction cannot be relied upon and will be neglected in the following discussion.

The two extreme cases (ie. no breakaway and immediate breakaway) will now be investigated. Consideration will then be given to the case where breakaway occurs during the loading sequence.

7.3.1.1 Effect of h/B - No Breakaway

The behaviour of a fully bonded strip anchor is significantly

influenced by its embedment ratio (h/B), as illustrated by the load deflection curves shown in Fig. 7.7. Specifically, the load carrying capacity of an anchor increases rapidly with embedment ratio for values of h/B less than three, however, an increase in h/B above three did not lead to any further increase in collapse load. In contrast, the general load deflection behaviour varied with embedment ratio over the entire range of h/B considered. Thus, the displacement observed prior to collapse decreased with increasing embedment ratio, while the collapse load appeared to be more clearly defined for higher values of h/B .

The influence of embedment ratio upon the general behaviour of an anchor plate can also be appreciated from consideration of the plastic region and velocity fields at failure (as shown in Figs. 7.8, 7.9, 7.10). In particular, it can be seen that both the velocity field and the plastic region above the anchor were significantly altered by the embedment ratio. From consideration of the behaviour exhibited in the four figures (ie. Figs. 7.7, 7.8, 7.9 and 7.10) it is seen that the embedment ratio of $h/B = 3$ marks the changeover from shallow anchor to deep anchor behaviour. At depths greater than this, collapse occurs when there is uncontained plastic flow around the anchor and although the extent of the plastic region for $h/B = 5$ is influenced by the presence of the free surface, the velocity field and collapse load are almost identical to the infinite case. For embedment ratios less than three, failure can be associated with plastic flow to the surface. At embedment ratios less than or equal to one, the plastic failure above the anchor is restricted to a narrow shear zone above the edge of the anchor, which corresponds to an upward block movement of the material directly above the anchor towards the soil surface as shown for $h/B = 1$

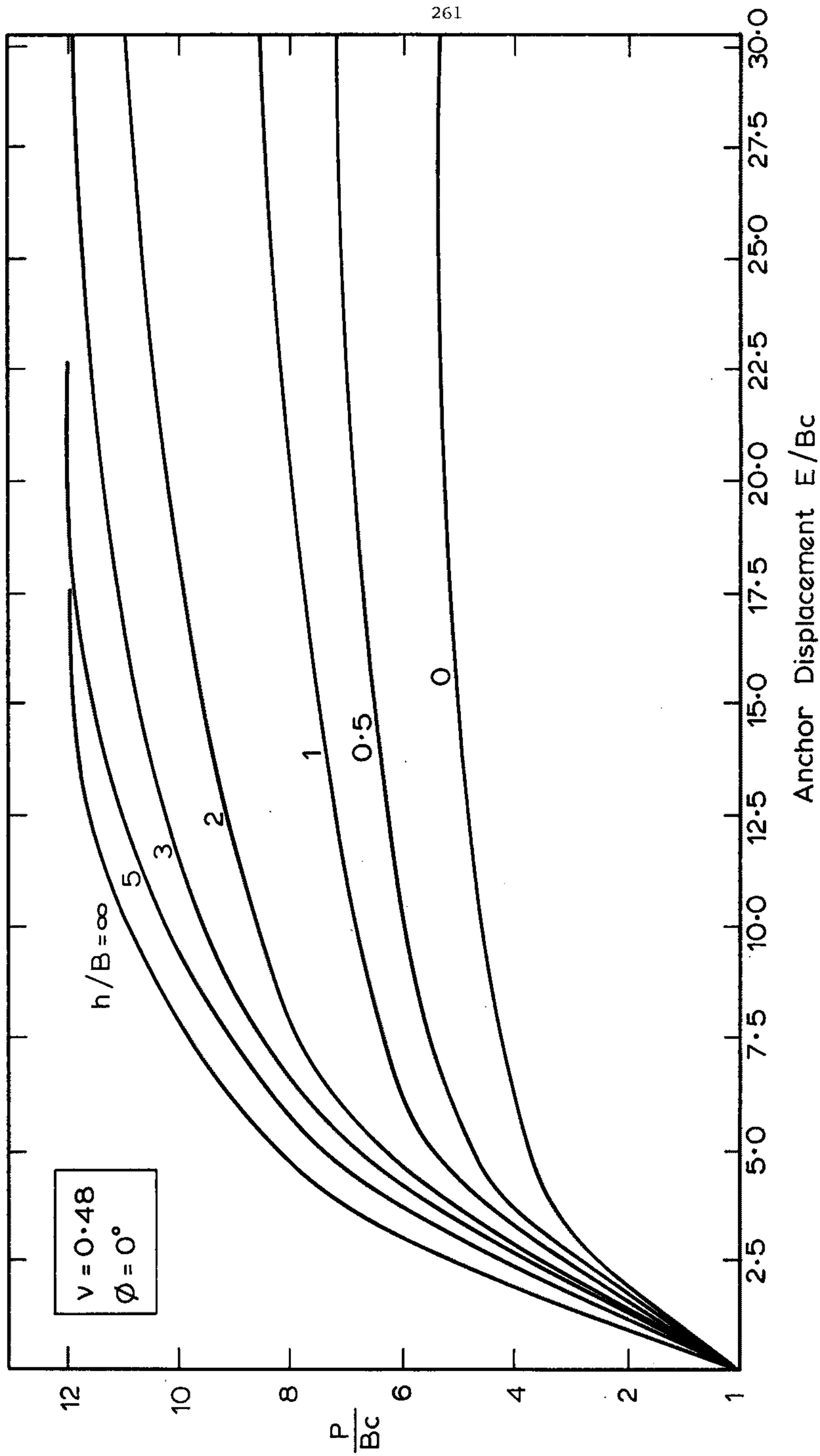


FIG.7.7 LOAD DISPLACEMENT CURVES FOR HORIZONTAL ANCHORS - NO BREAKAWAY

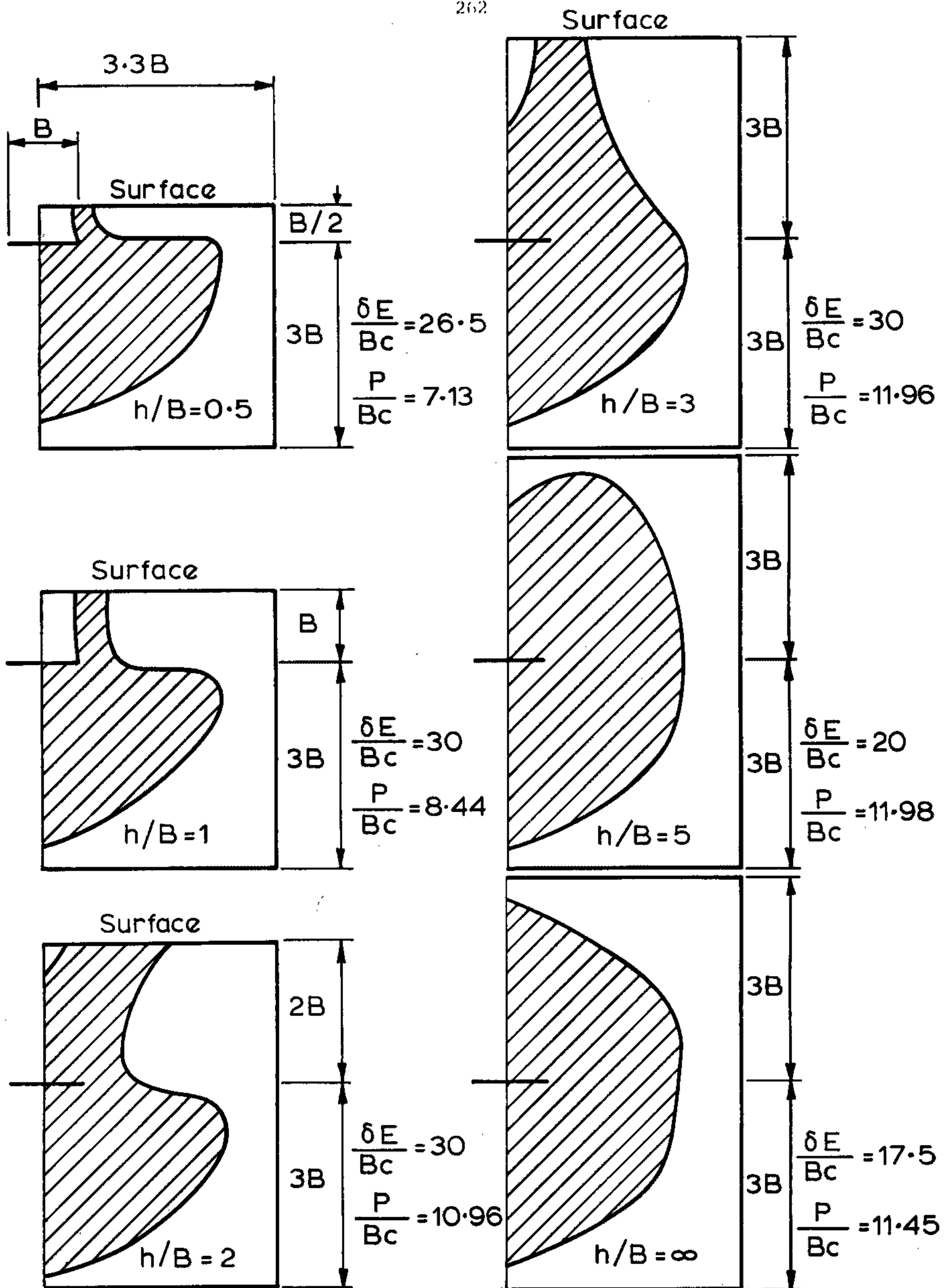
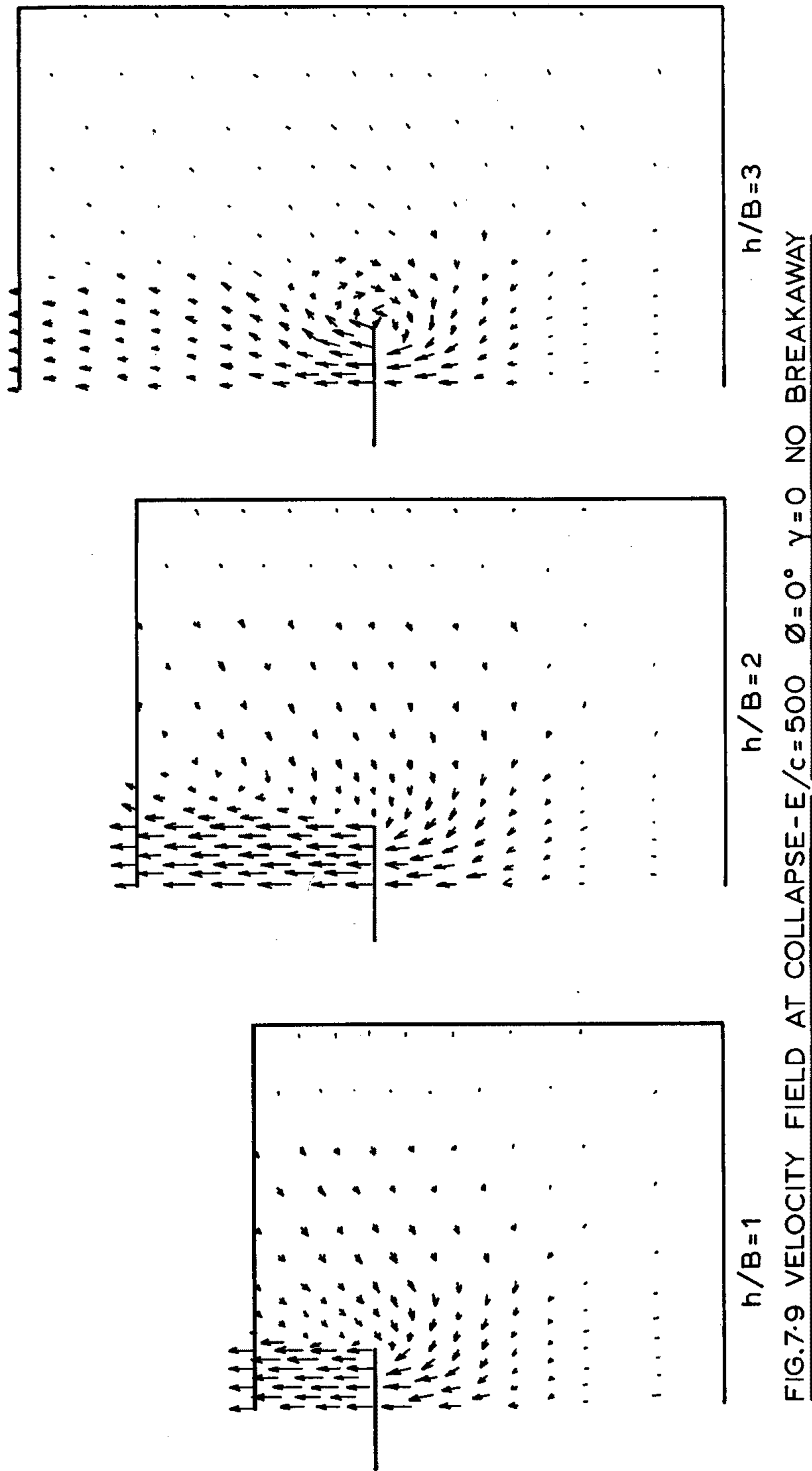
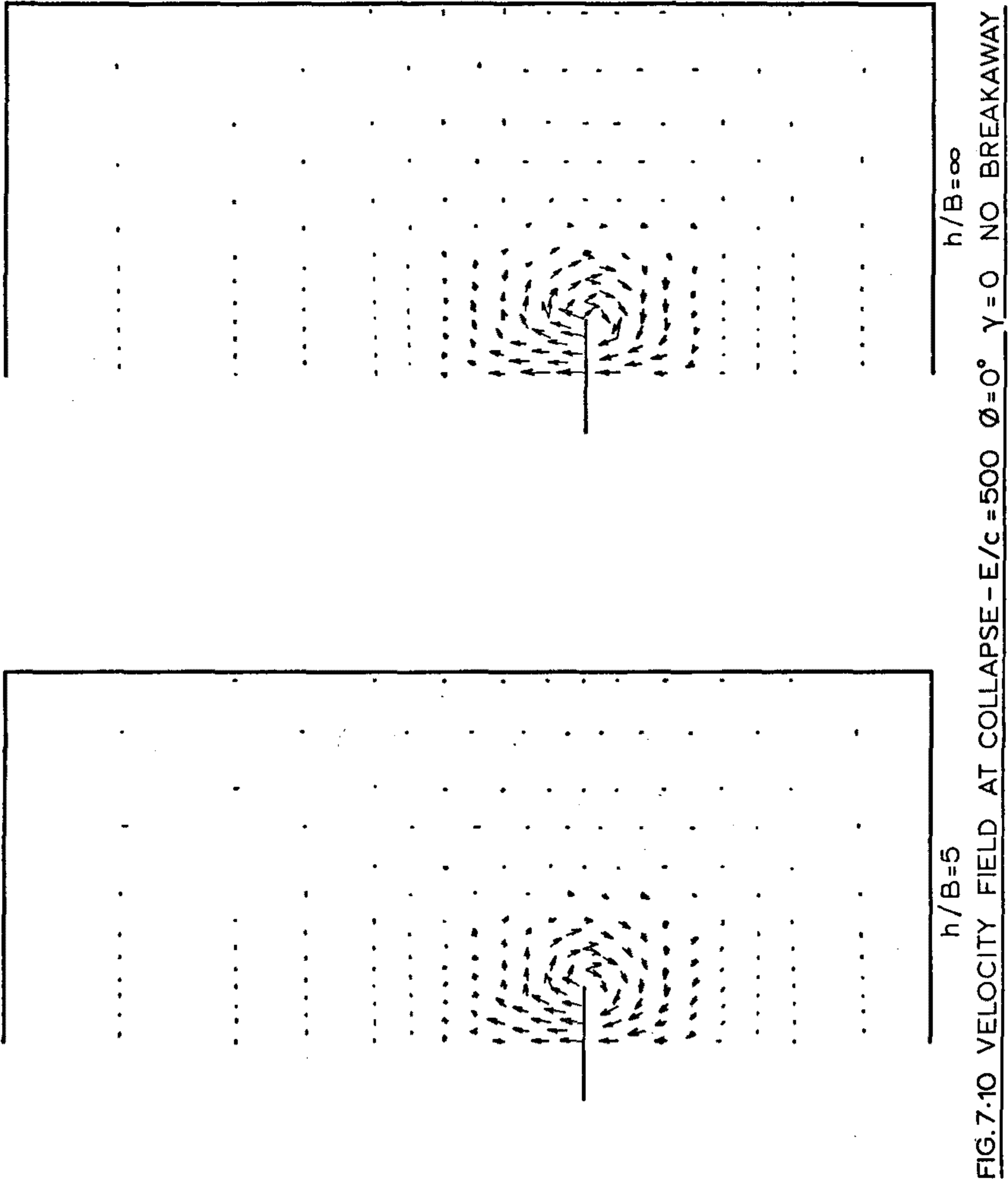


FIG. 7.8 PLASTIC REGIONS AT LAST LOAD INCREMENT
 $E/c = 500$ $\phi = 0$ $\gamma = 0$ NO BREAKAWAY





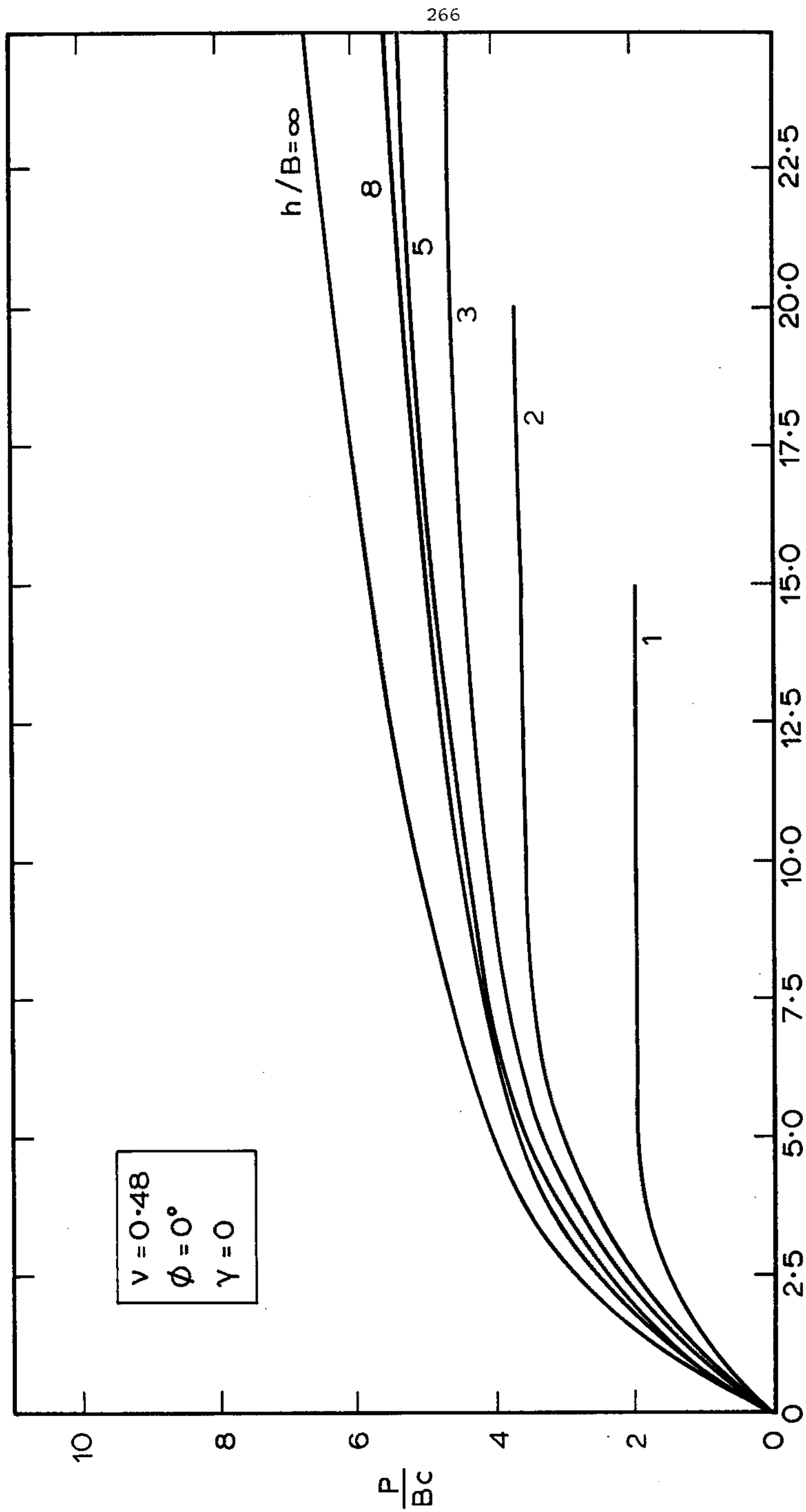
in Fig. 7.9.

The zones of plastic failure for embedment ratios $h/B = 2$ and 3 both reach the surface. However, in the former case, plastic flow is still predominantly toward the surface, while in the latter case plastic flow is predominantly around the anchor with some residual flow towards the surface.

Thus, it would appear that for soft clays, where the dimensionless overburden pressure q_h/c is sufficiently high to prevent breakaway, there is a changeover from shallow anchor to deep anchor behaviour at an embedment ratio of three. Furthermore, for embedment ratios greater than zero but less than three, collapse (in the rigorous applied mechanics sense) had not occurred when loading was ceased. In these cases, the load displacement curve was still rising, although the displacement was more than four times the elastic prediction for that load level and hence, for all practical purposes, failure may be considered to have occurred.

7.3.1.2 Effect of h/B - Initial Breakaway

The load displacement curves obtained at various embedment ratios for the case where there is no adhesion, suction or initial stress state (ie. where breakaway of anchor from the soil occurs immediately) are shown in Fig. 7.11. Here, the collapse load is clearly defined at low displacements for shallow anchors. From consideration of the load displacement curves (Fig. 7.11), the plastic regions at the last load increment (Fig. 7.12) and the corresponding velocity fields [Fig. 7.13(a) and 7.13(b)], it can be seen that shallow anchor



Anchor Displacement E/Bc

FIG.7.11 LOAD DISPLACEMENT CURVES FOR HORIZONTAL ANCHORS - IMMEDIATE

BREAKAWAY ($q_h/c=0$)

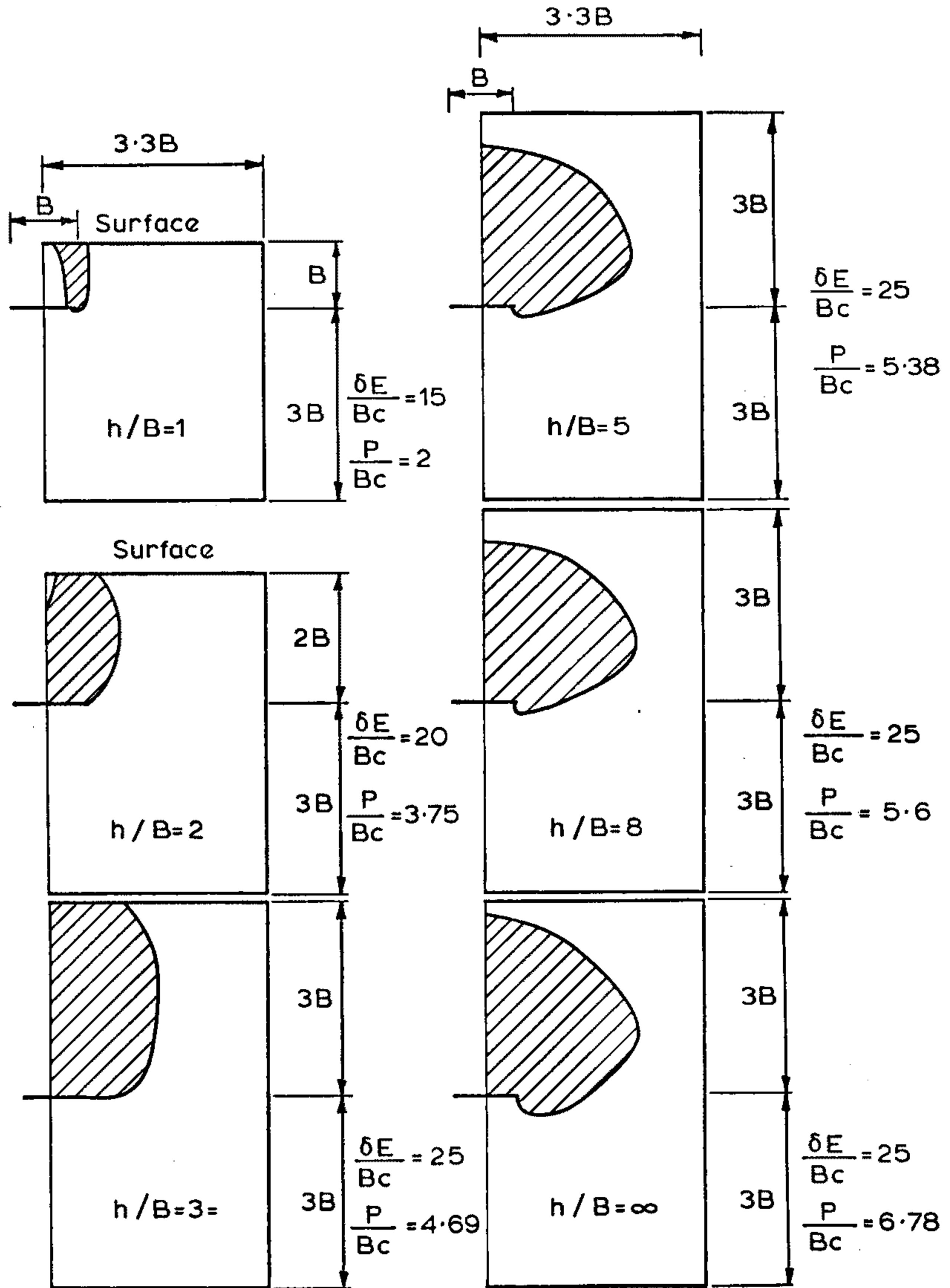


FIG.7.12 PLASTIC REGIONS AT LAST LOAD INCREMENT
 $E/c = 500$ $\phi = 0^\circ$ $\gamma = 0$ IMMEDIATE BREAKAWAY
 $(q_h / c = 0)$

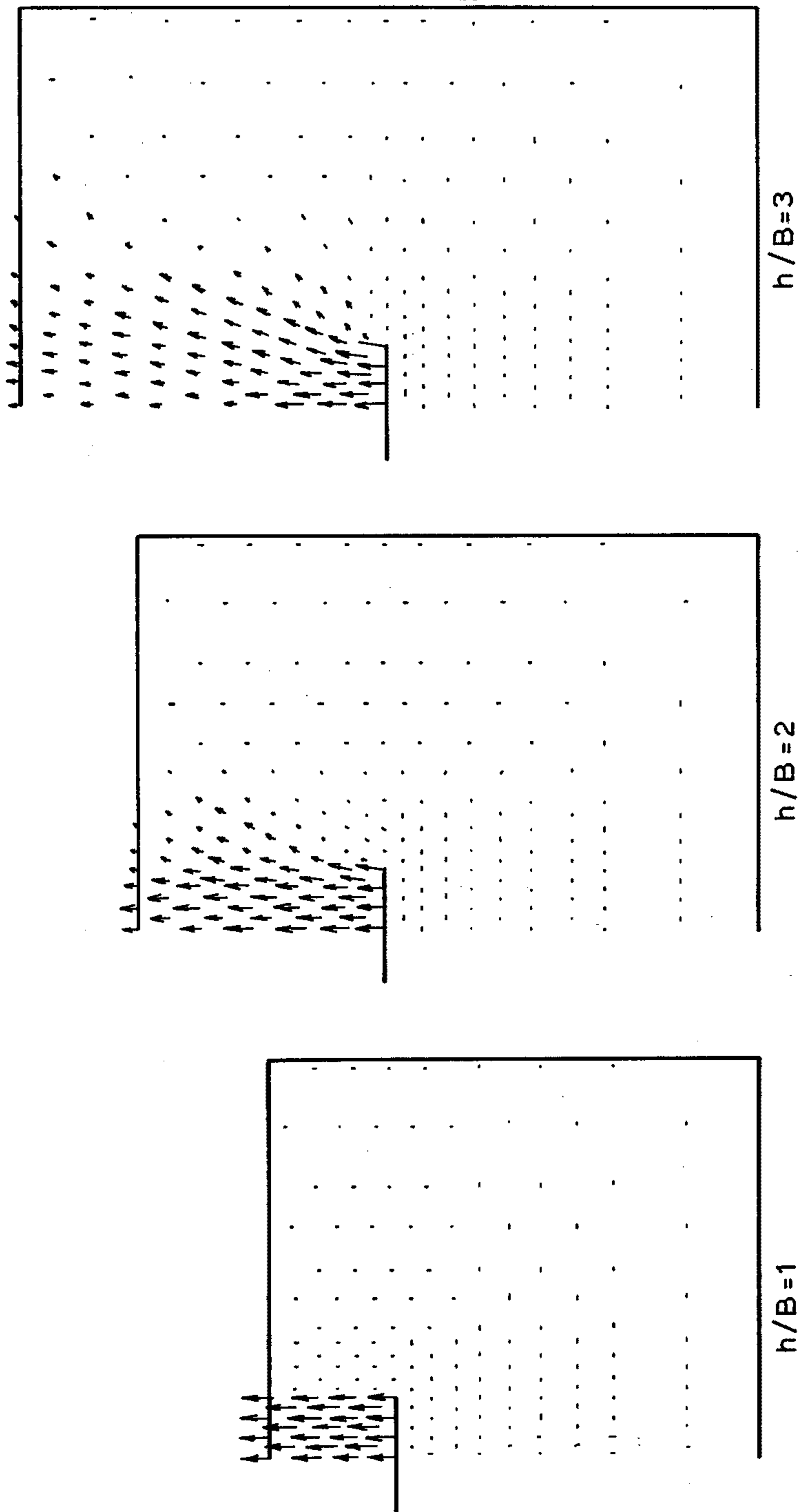


FIG.7.13a VELOCITY FIELD AFTER LAST LOAD INCREMENT $E/c=500$ $\phi=0^\circ$ $\gamma=0$ IMMEDIATE
BREAKAWAY

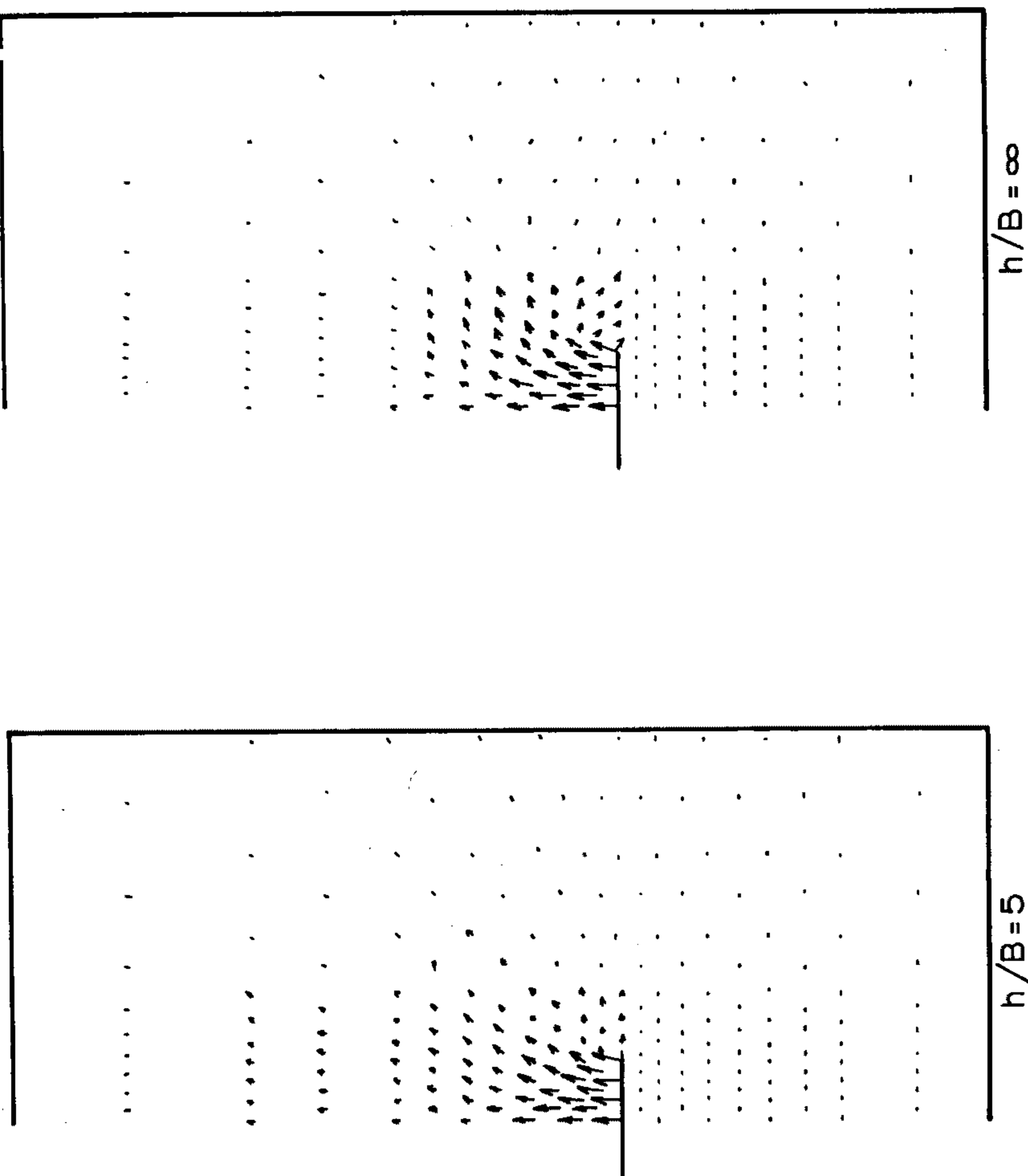


FIG. 7.13b VELOCITY FIELD AFTER LAST LOAD INCREMENT - $E/c = 500$ $\phi = 0^\circ$ $\gamma = 0$
IMMEDIATE BREAKAWAY

behaviour is exhibited for embedment ratios less than or equal to three. In these cases, failure occurs due to shearing of the soil above the anchor. For an embedment ratio of one, the plastic zone is restricted to a narrow band above the edge of the anchor, and the velocity field shows that the material above the anchor moves as an almost rigid block, while the collapse load corresponds to the simple rigid plastic upper bound solution of $2 Bc$.

At an embedment ratio of two, the plastic region extends beyond the zone above the anchor and the velocity field can be seen to spread as it approaches the surface. The collapse load in this case is less than the simple rigid plastic upper bound calculated on the basis of a velocity discontinuity rising from the edge of the anchor up to the soil surface.

In the no breakaway case, an embedment ratio of three was seen to correspond to deep anchor behaviour. However, with immediate breakaway, the behaviour at this depth is clearly that of a shallow anchor. Both the plastic region and the velocity field are primarily directed towards the surface, although some tendency for the material to flow around the anchor is observed.

For embedment ratios greater than three, deep anchor behaviour is observed. Here, the velocity fields and the growth of the plastic region indicate that plastic flow will be predominantly around the anchor rather than towards the surface. In view of the discussion in Chapter 4 of the effect of breakaway upon the collapse load of an anchor at infinite depth, it seems likely that, in the applied mechanics sense, the ultimate collapse load for all deep anchors would be the same irres-

pective of the breakaway condition. This definition of failure is, however, rather academic since when loading ceased, the displacement in each case was at least six times that predicted from an elastic analysis at the same load level, although the load itself was more than 40% below the estimated ultimate collapse load of 11.4 Bc.

The definition of failure in these cases, where there is significant contained plastic flow, must to some extent be an arbitrary one. Terzaghi (1943) for example, suggests that where there is no peak to the load settlement curve, the bearing capacity should be taken as the load at which the curve passes into a steep and fairly straight tangent. In the case of the anchor plate considered here there is little doubt that a peak or ultimate collapse load does exist. Rather, problems arise because of the very large contained plastic deformation which is observed before reaching the ultimate collapse load. If the existence of this peak (or ultimate) load is neglected in favour of some definition of failure related to deformation behaviour, as suggested by Terzaghi, then difficulties arise in specifying just when the curve passes into the 'straight tangent' portion.

In this thesis, failure will be deemed to have occurred at a load level where the actual displacement is four times the displacement predicted from the elastic stiffness, as shown in Fig. 7.14. While this definition is somewhat arbitrary, it is also simple to apply and will make the modification of elastic settlement prediction to incorporate some degree of local shear failure quite straightforward. Furthermore, for any case where failure is fairly clearly defined, then the selection of the factor four as opposed to three or five will not

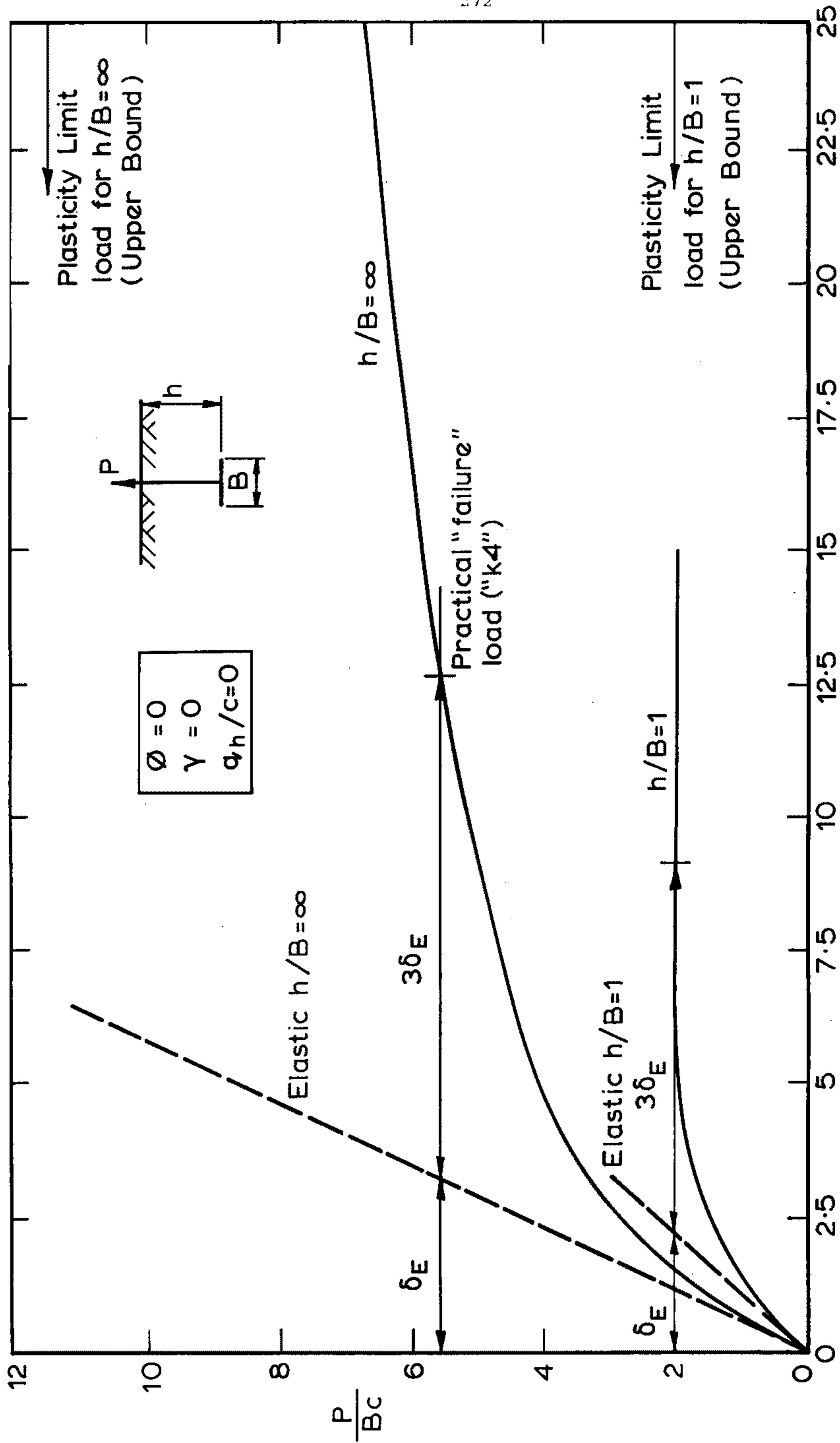
Anchor Displacement E/Bc

FIG. 7.14 DEFINITION OF FAILURE

significantly alter the estimate of the collapse load. However, for cases where the elasto-plastic tail is reasonably steep, this estimate of the failure load gives working loads in the elastic or early elasto-plastic range, and hence providing the settlement (calculated with the appropriate local yield modification) is satisfactory, one is assured of adequate resistance against ultimate collapse without adopting an unnecessarily conservative estimate of the failure load.

In regard to any discussion of failure loads which are related to deformation, it should be remembered that the analysis presented here makes no allowances for finite strain effects and changes in geometry. This fact may be safely ignored in many foundation engineering problems; such as in footing analyses, where the effect of geometry change is to increase the collapse load, (e.g. Carter 1977). However, in the case of anchors being pulled out of the soil, the geometry changes may be expected to decrease the embedment ratio. Here, the critical parameter is the displacement, divided by the buried depth h , which is a function of both the embedment ratio (h/B) and the modulus to strength ratio E/c . Thus, while the significance of changes in geometry cannot be predetermined, it seems likely that the loads corresponding to displacement much in excess of four or five times the elastic value would, in practice, be altered by finite changes in geometry, and hence should be viewed with some degree of caution. This is particularly true when considering the results obtained from pullout tests for model anchors.

7.3.1.3 Effect of Body Force and Surcharge q_h/c

The bearing capacity factor for a purely cohesive material will depend upon both the embedment ratio and the dimensionless overburden pressure q_h/c . Thus, from the previous two sections, bearing capacity factors can be obtained corresponding to the two limiting cases $F_c = \left(\frac{P}{Bc}\right) q_h/c = 0$ and $F_c = \left(\frac{P}{Bc}\right) q_h/c = \infty$. For intermediate values of overburden pressure $0 < q_h/c < (q_h/c)_1$, the behaviour of the anchor will be the same as in the no breakaway case until breakaway occurs, after which the behaviour will tend to approximate that observed with immediate breakaway. To illustrate this, consider an anchor at 'infinite depth' where there is a finite overburden pressure $q_h = q_s + \gamma h$. The load displacement curves obtained for a range of values of overburden pressure (q_h/c) are shown in Fig. 7.15. In the case of no breakaway, ($q_h/c \geq 6$) the collapse load of $11.95 Bc$ can, as previously mentioned, be compared with the upperbound load of $11.42 Bc$ obtained from classical plasticity and the discrepancy of 4.6% attributed to discretization error, due to the finite element mesh. From Fig. 7.16(a) it can be seen that the finite element velocity field at collapse corresponds well with the upperbound solution given in Chapter 4. (The twin velocities at certain points are due to the behaviour of dual nodes once rupture has occurred along a potential rupture line). Fig. 7.16(b) shows the extent of the plastic region at various load and displacement levels. It will be observed that the growth of the plastic region is symmetric about a horizontal plane through the anchor plate. This is to be expected since the reduction in vertical stress below the anchor and the increase in vertical stress above the anchor will have an equivalent magnitude in any increment because of the anti-symmetric nature of the problem. Con-

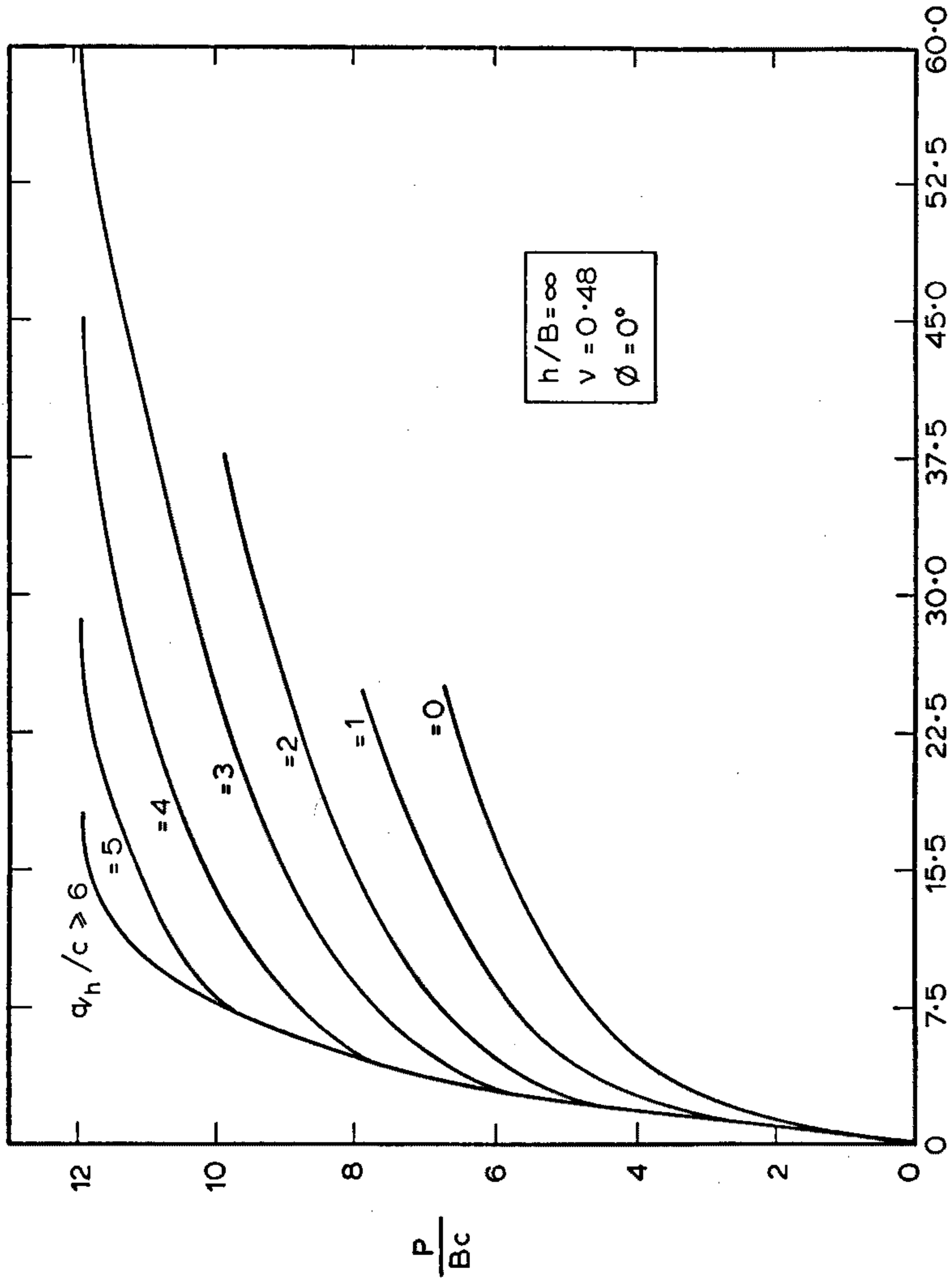
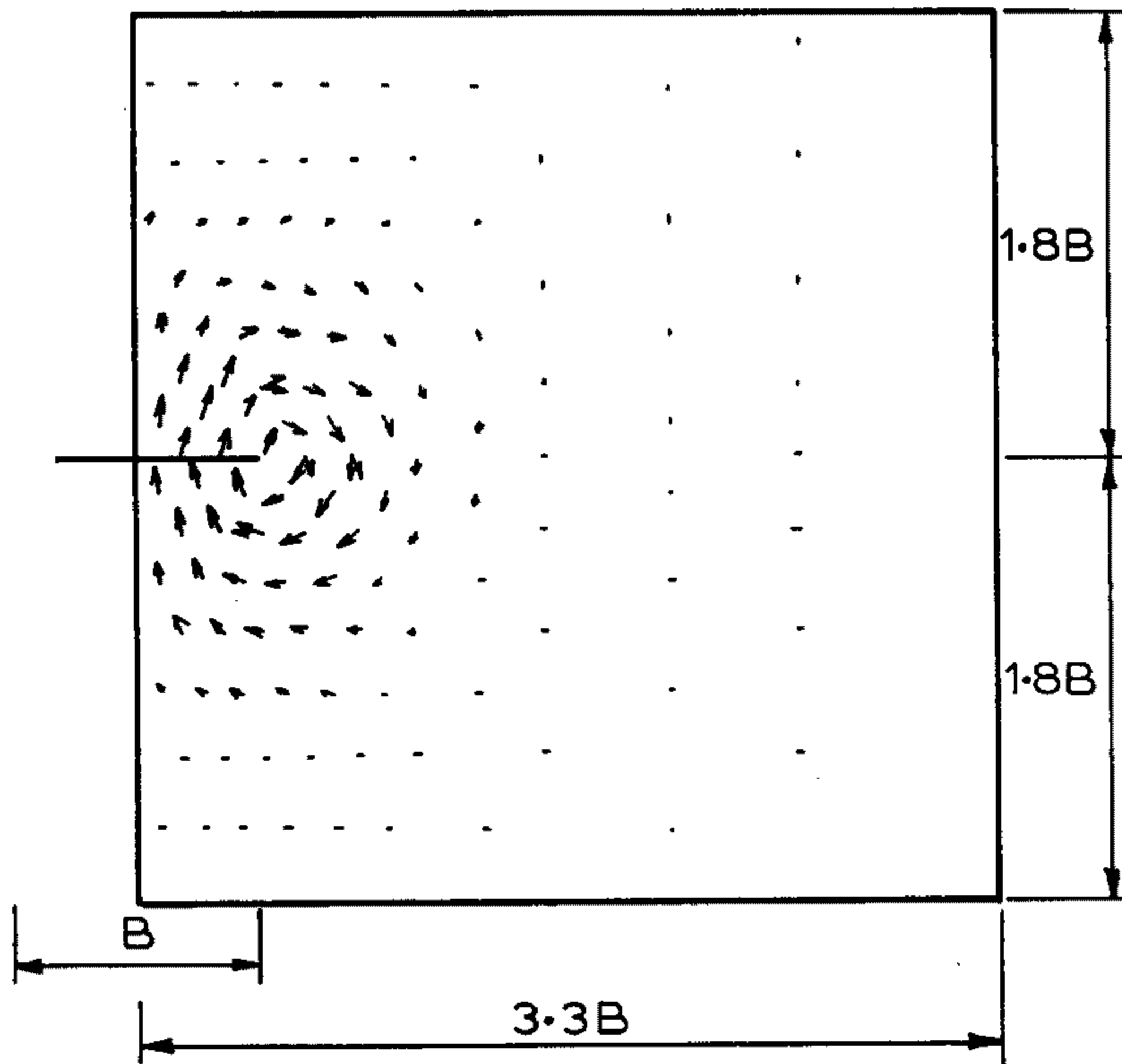
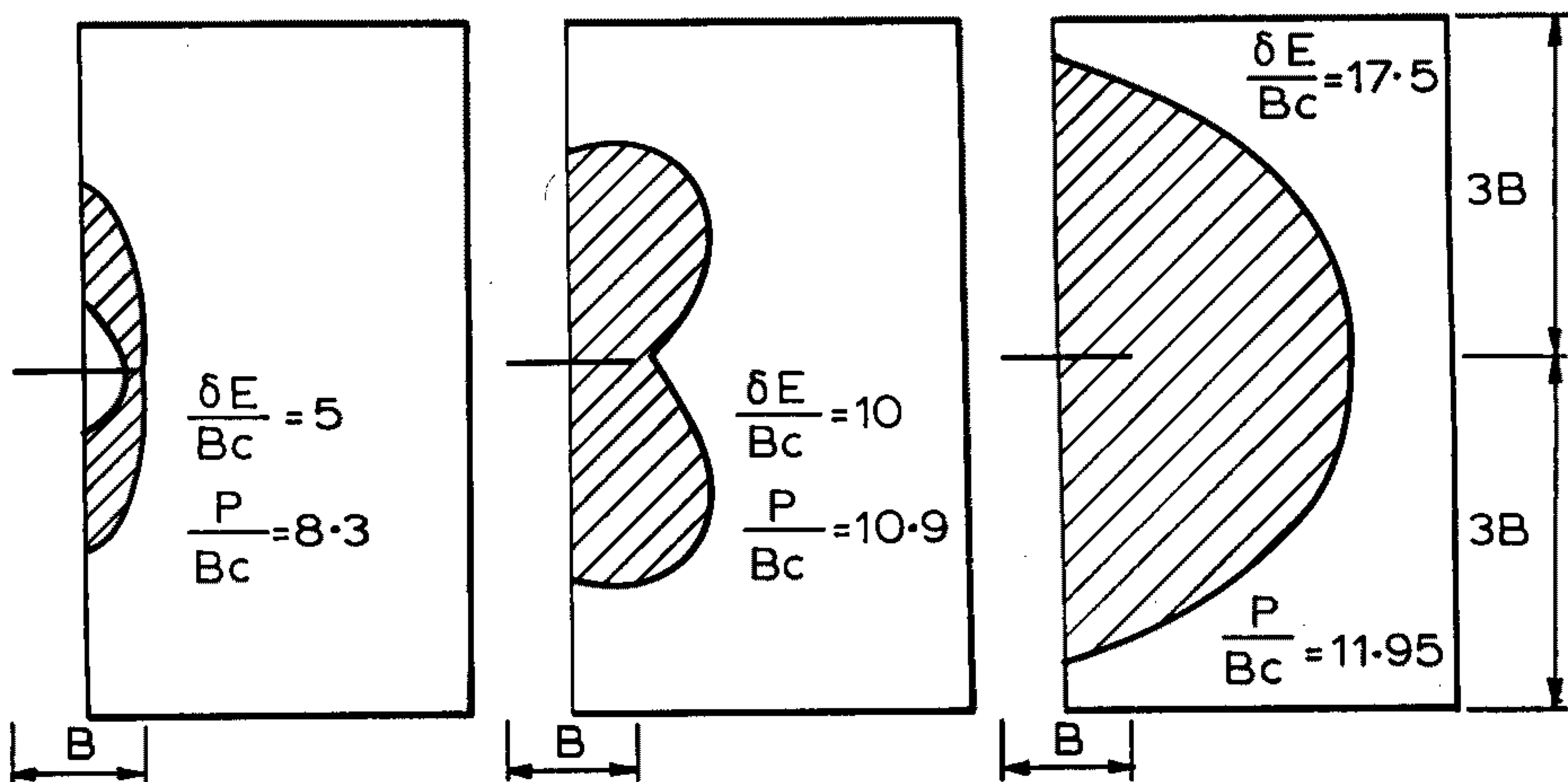


FIG.7.15 EFFECT OF OVERBURDEN PRESSURE ON BEARING CAPACITY OF A DEEP ANCHOR



(a) Velocity Field at Collapse



(b) Growth of Plastic Region

FIG.7-16 ANCHOR AT INFINITE DEPTH NO BREAKAWAY $q_h/c \geq 6$

sequently, the growth of the plastic region is fairly rapid and uncontained plastic flow commences at a displacement of $17.5 Bc/E$.

As observed in Chapter 4, the collapse load determined from classical plasticity for an anchor at infinite depth is independent of the initial stress state, and hence the collapse load for the immediate breakaway ($q_h/c = 0$) condition should be the same as that for the no breakaway ($q_h/c \geq 6$) condition (ie. $11.42 Bc$). However, classical plasticity does not indicate the deformation required to obtain the collapse load.

In an elasto-plastic analysis, it is found that the initial stress state (ie. q_h/c) significantly alters the development of the plastic region, as well as the displacements which occur prior to collapse. In fact, once breakaway occurs the growth of plastic zones ceases to be symmetric about the anchor plate and instead tends to grow from above the anchor until it eventually encompasses the entire anchor allowing for uncontained plastic flow and collapse. For example Fig. 7.17, shows the growth of plastic zones for the case where $q_h/c = 3$. From this, and the load displacement curves shown in Fig. 7.15, it can be seen that there is very considerable elasto-plastic deformation prior to collapse, and although the same collapse load is obtained as in the no breakaway case, the total displacement prior to collapse was $60 Bc/E$, which is 3.4 times greater than that in the no breakaway case.

The load displacement curves for the cases where $q_h/c = 5$ and $q_h/c = 4$ are also shown in Fig. 7.15, and the same trend towards significantly increased displacement prior to collapse with decreasing overburden pressure q_h/c can be observed. Indeed, in the early elasto-

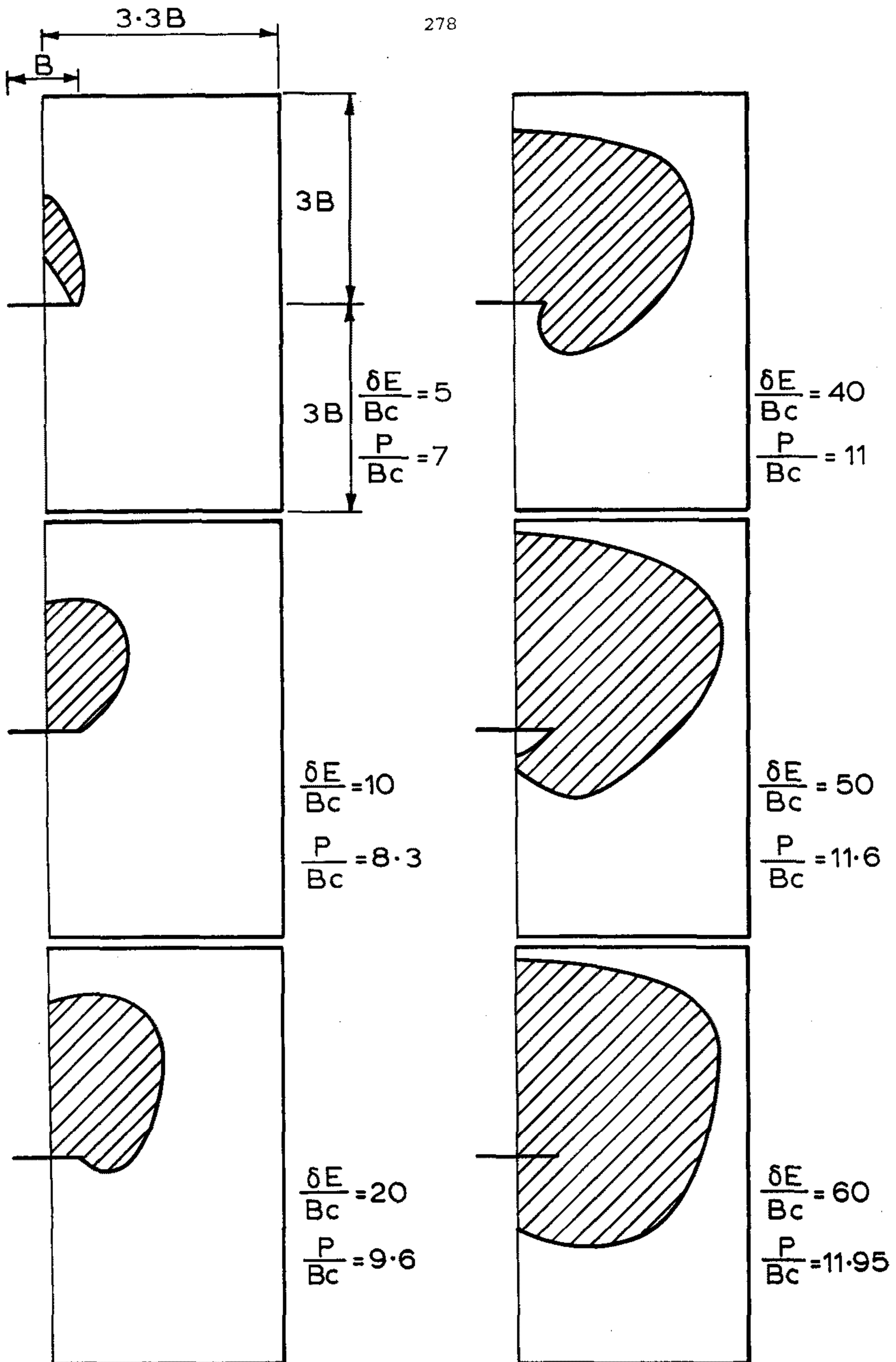


FIG.7-17 GROWTH OF PLASTIC REGIONS FOR ANCHOR AT INFINITE DEPTH $E/c = 500$ $\phi = 0$ $q_h/c = 3$

plastic portion of the load deflection curves, the difference in applied load (at a given displacement) between the curves for various overburden pressures is approximately equal to the difference in q_h/c between the curves; although it should be noted that the value of q_h/c did not alter the ultimate collapse load.

In the case where $q_h/c = 0, 1$ and 2 , the analysis was not continued until collapse occurred because of the cost involved in following the very long elasto-plastic portion of the curve. However, from a knowledge of the rigid plastic solution and the behaviour of the finite element solutions for $q_h/c = 3, 4, 5$, it is reasonable to infer that the collapse load for $q_h/c < 3$ would be the same as that of the no breakaway case.

The final velocity fields for the cases where $q_h/c = 0$ and $q_h/c = 3$ are shown in Fig. 7.18. In the immediate breakaway ($q_h/c = 0$) case, only local shear failure had occurred in the region above the anchor when loading ceased. Consequently, the plastic deformation [see Fig. 7.18(a)] is also restricted to this region. As the loading procedure is continued, the plastic region can be expected to grow in the same general fashion as observed for $q_h/c = 3$ (Fig. 7.17) which, at collapse, exhibited a general shear failure involving flow of the material around the anchor as shown in Fig. 7.18(b).

Two points emerge from this study of the deep anchor in purely cohesive material. Firstly, the ultimate collapse load has been seen to be independent of the overburden pressure q_h/c . Secondly, the load displacement behaviour was seen to be significantly altered by the overburden pressure. Thus, because of the very large elasto-

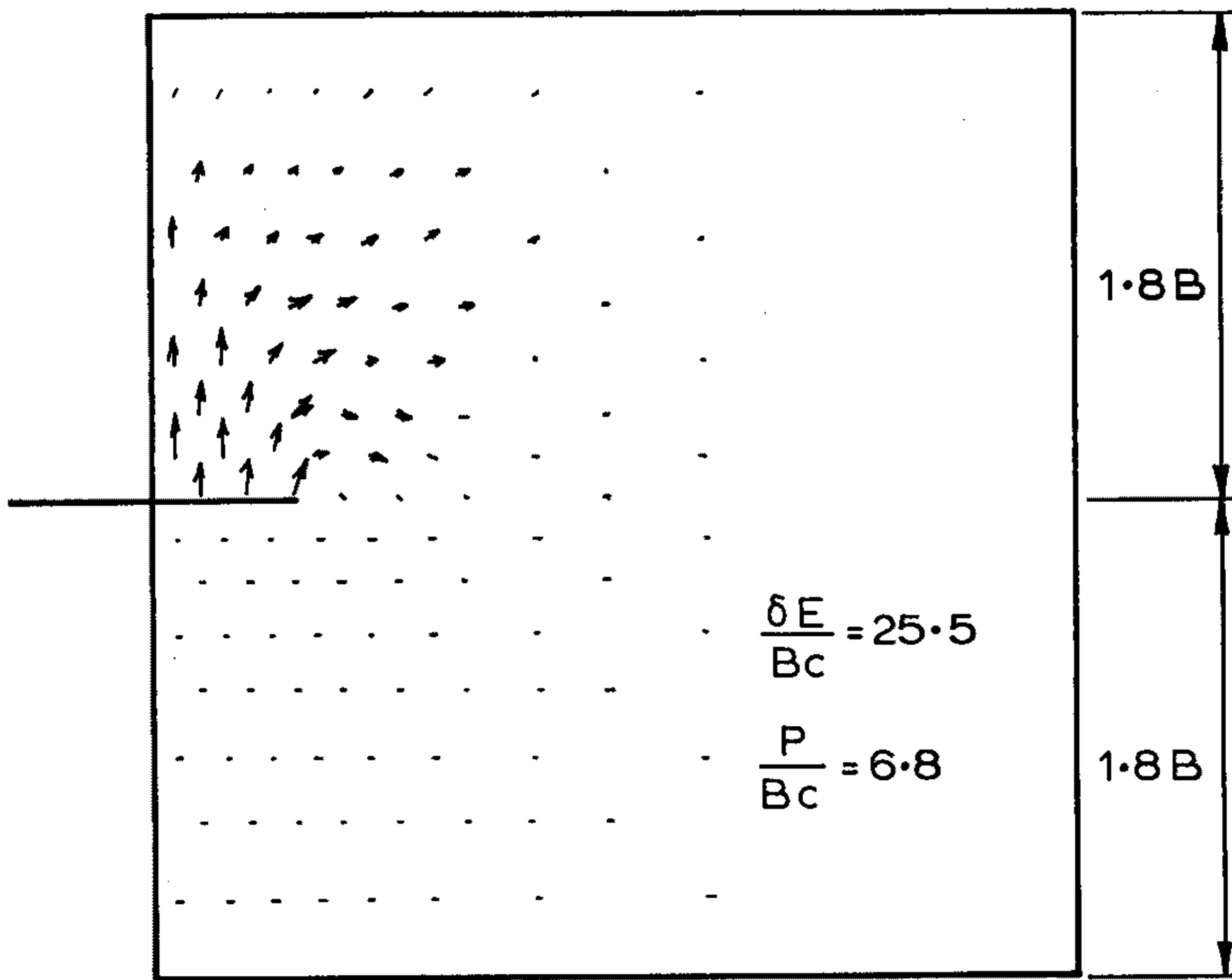
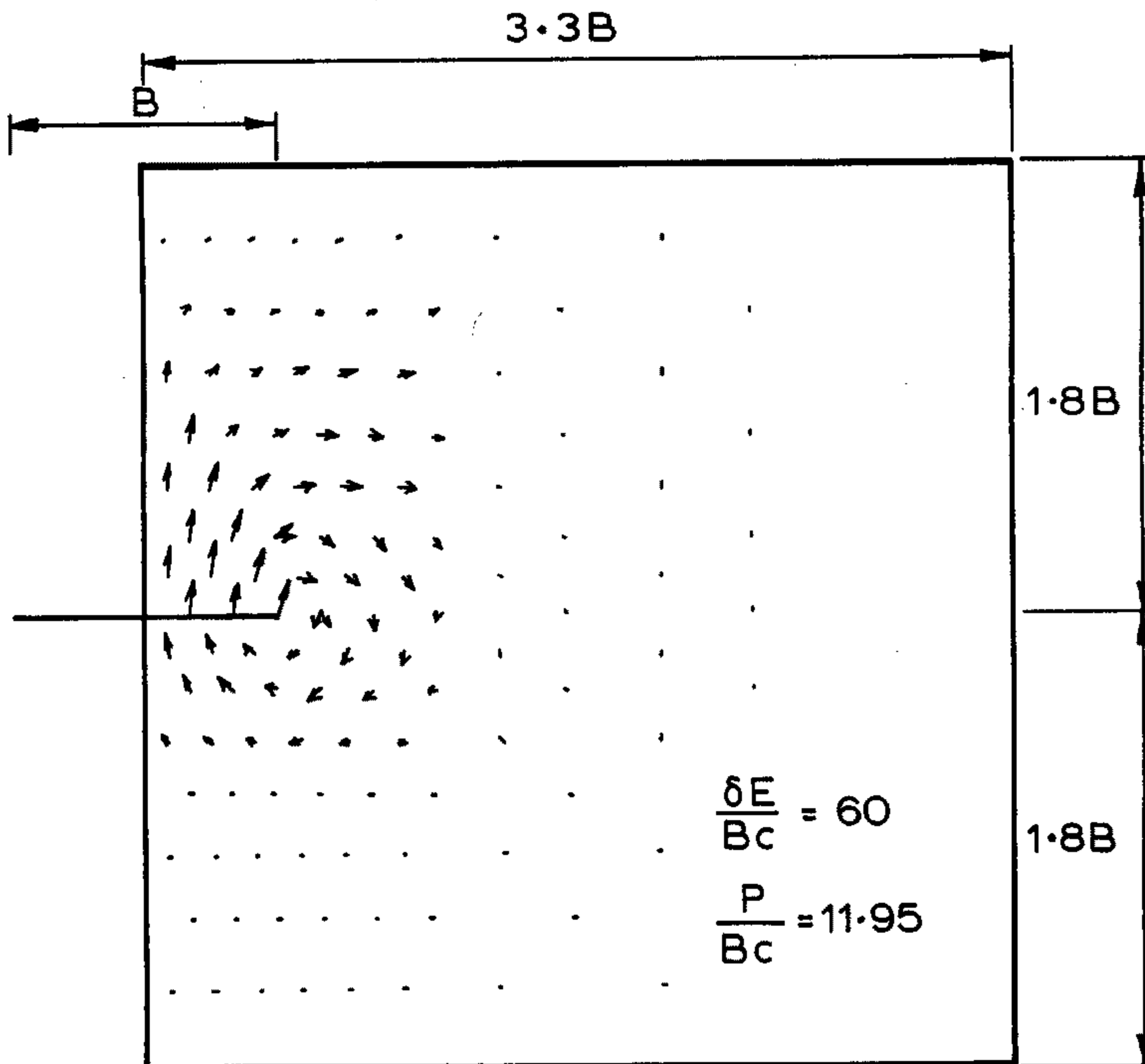
(a) $q_h/c=0$ Immediate Breakaway(b) $q_h/c=3$ Breakaway

FIG.7.18 VELOCITY FIELDS FOR ANCHOR AT INFINITE DEPTH

$$E/c = 500 \quad \phi = 0$$

plastic deformation which occurs prior to complete collapse, it may be considered that (to all practical purposes) failure had occurred at the 'k4' load. This practical failure load is well below the ultimate collapse load for low values of q_h/c .

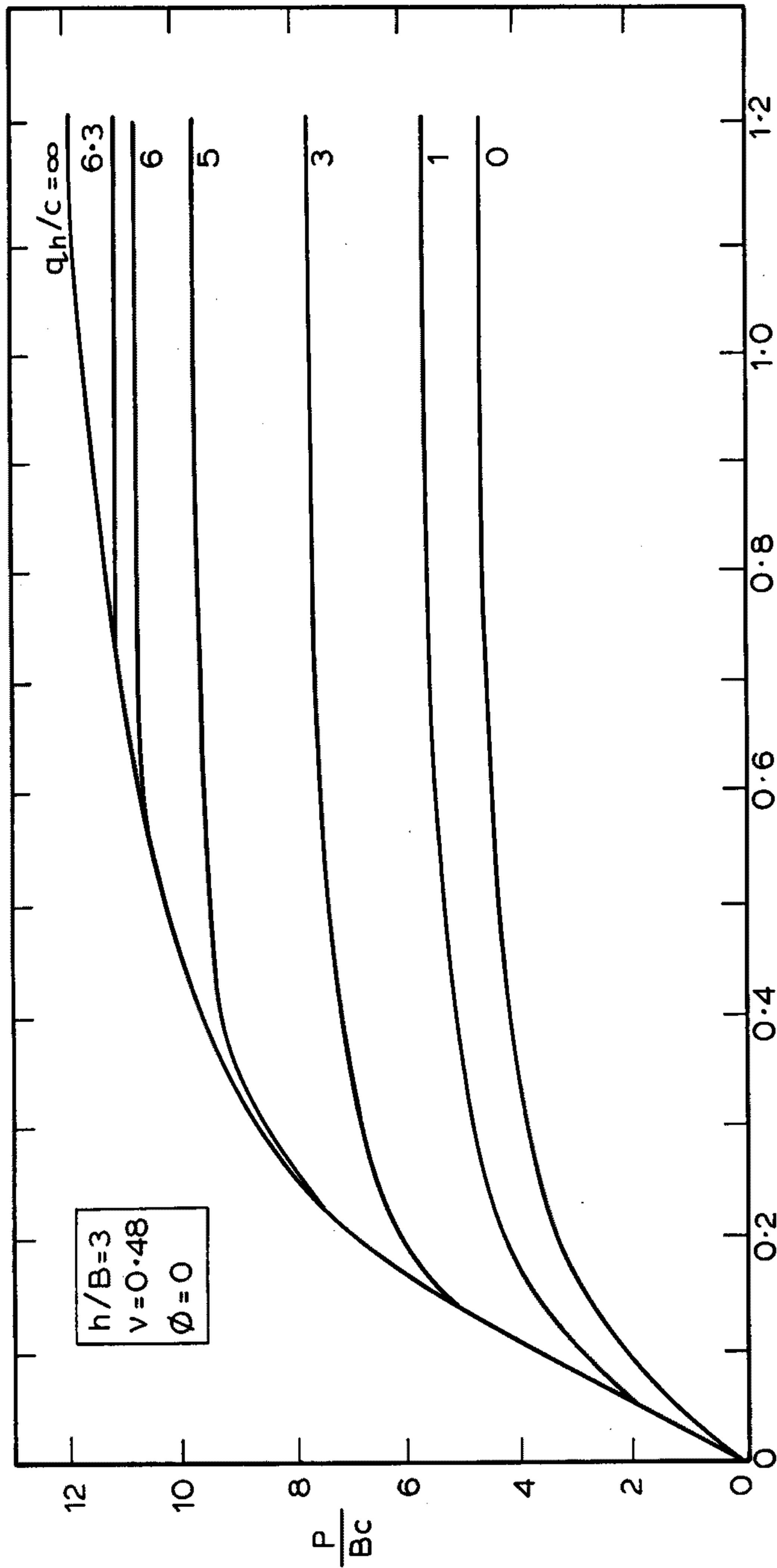
The load displacement curves for a range of overburden pressures q_h/c are shown in Fig. 7.19 for an anchor with an embedment ratio $\frac{h}{B} = 3$. From this, it can be seen that for a shallow anchor, the difference in collapse load is generally equal to the difference in the overburden pressure (q_h/c) between the curves, however, there exists a theoretical limit to the increase in bearing capacity due to overburden pressure equal to approximately 6 c.

These analyses of shallow and deep anchors indicate that the bearing capacity factor F_c is a complicated function of both the embedment ratio and the overburden pressure. Furthermore, the nature of this function depends upon the definition of failure adopted in determining F_c , since for deep anchors the ultimate collapse load is independent of the overburden pressure whereas the practical ('k4') failure load varies significantly with q_h/c .

In Fig. 7.20, the practical 'k4' failure load is plotted against the overburden pressure for a number of embedment ratios, and it can be seen that for an isotropic, homogeneous soil, the bearing capacity factor may be approximately expressed in the form

$$F_c = F_{c_0} + \frac{q_h}{c} \quad (7.2a)$$

$$\text{provided that } F_c \leq F_c^* \quad (7.2b)$$



Anchor Displacement E/Bc

FIG. 7.19 EFFECT OF SURCHARGE q_h/c UPON LOAD DISPLACEMENT BEHAVIOUR

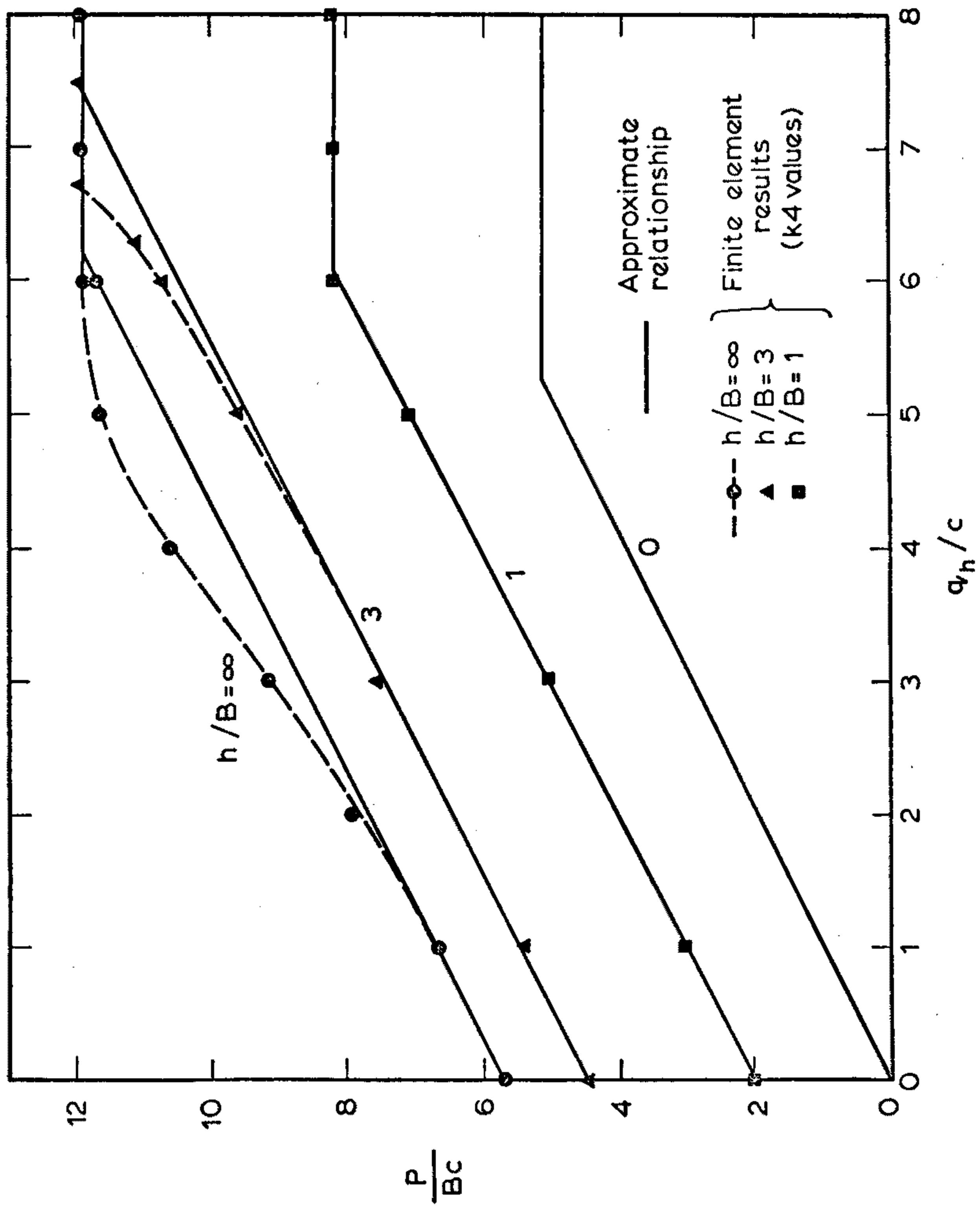


FIG.7-20 VARIATION OF BEARING CAPACITY WITH SURCHARGE q_h/c , $\phi = 0^\circ$

otherwise $F_c = F_c^*$

where $F_c = F_c \left(\frac{h}{B}, \frac{q_h}{c}, \phi = 0 \right)$

$$F_{c_o} = F_c \left(\frac{h}{B}, \frac{q_h}{c} = 0, \phi = 0 \right)$$

and $F_c^* = F_c \left(\frac{h}{B}, \frac{q_h}{c} = \infty, \phi = 0 \right)$

Note that F_{c_o} and F_c^* are the practical failure loads corresponding to the immediate breakaway and no breakaway cases respectively. This approximate relationship is reasonably accurate for shallow anchors and in the worst case it leads to a conservative estimate of the failure load at very great depth ($\frac{h}{B} = \infty$).

7.3.1.4 Summary of Results for Undrained Loading ($\phi = 0^\circ$)

In the foregoing investigation of undrained anchor behaviour it was found that, although the collapse load is often easily identified, there are cases where significant deformation occurs before the ultimate collapse load is reached. In these instances the failure load may be determined by finding the intersection of the load settlement curve with a straight line passing through the origin and having a stiffness corresponding to one quarter of the elastic value for that anchor. This load is referred to as the 'k4' load. Similarly, loads corresponding to a second stiffness of one half and one third the elastic stiffness are referred to as 'k2' and 'k3' loads.

The bearing capacity factor, F_c , determined on this basis is shown in Fig. 7.21 for embedment ratios ranging from zero to infinity.

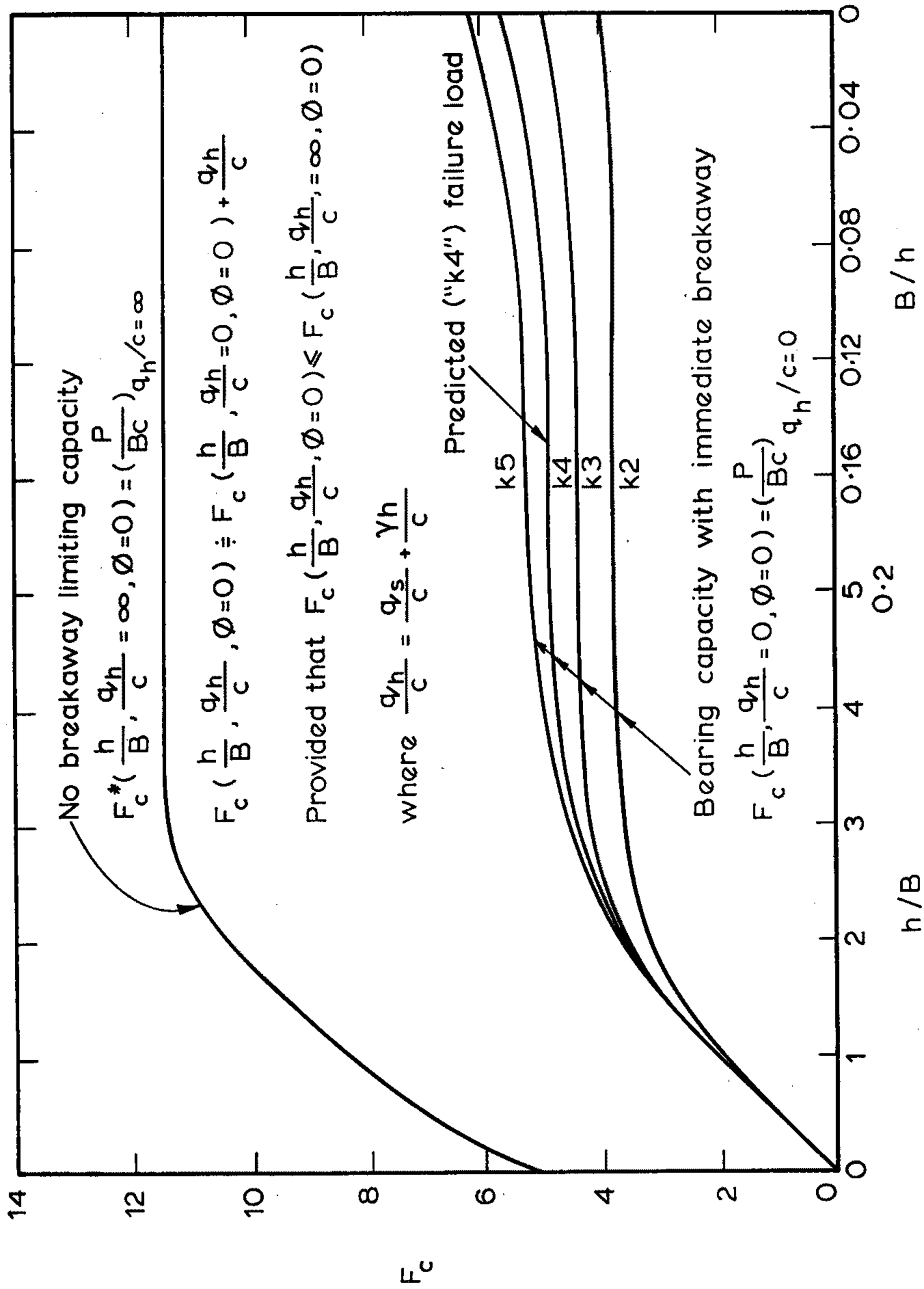


FIG. 7.21 VARIATION OF BEARING CAPACITY FACTOR F_c WITH EMBEDMENT RATIO, $\phi = 0^\circ$

F_c is plotted against a linear function of h/B in the range of zero to five and against the inverse, B/h , for embedment ratios greater than five. To provide a measure of the sensitivity of the practical failure loads to the definition of failure, the loads corresponding to 'k2', 'k3' and 'k5' displacements are also shown in Fig. 7.21.

All the loads for the no breakaway condition have been reduced by approximately 4.5% so as to agree with the rigid plastic ultimate collapse loads for $h/B = 0$ and $h/B = \infty$.

From Fig. 7.21, it can be seen that if there is no breakaway, then an anchor can be classified as a deep anchor for embedment ratios greater than or equal to three. In the case of immediate breakaway, the transition from shallow to deep anchor behaviour can be seen to occur at an embedment ratio of approximately four, since there is very little increase in bearing capacity for embedment ratios greater than this value until the infinite case is approached. In fact, it seems likely that this increase in load obtained for the various 'k' ratios in the infinite case arises from the finite element idealisation which approximated the infinite boundary by rough boundaries at a distance $8B$ above and below the anchor.

Finally, it has been found that for a homogeneous, isotropic material, the bearing capacity factor F_c for intermediate values of overburden pressure $0 < q_h/c < (q_h/c)_1$ may be approximately expressed in the form

$$F_c = F_{c_0} + \frac{q_h}{c}$$

provided that $F_c \leq F_c^*$

otherwise $F_c = F_c^*$

where $F_c = F_c \left(\frac{h}{B}, \frac{q_h}{c}, \phi = 0 \right)$

$$F_{c_o} = F_c \left(\frac{h}{B}, \frac{q_h}{c} = 0, \phi = 0 \right)$$

$$F_c^* = F_c \left(\frac{h}{B}, \frac{q_h}{c} = \infty, \phi = 0 \right)$$

and where $q_h = q_s + \gamma h$ is the overburden pressure due to a surface surcharge q_s and body forces γh within the soil mass.

7.3.2 Weightless Cohesive - Frictional Material - (Drained Loading)

7.3.2.1 Effect of Poisson's Ratio

In the analysis of anchor behaviour in a purely cohesive material (Section 7.3.1), the value of Poisson's ratio was assumed to be 0.48 to correspond with undrained conditions. However, for cohesive-frictional materials a typical drained value of Poisson's ratio equal to 0.3 will be adopted.

In general the ultimate collapse load is relatively insensitive to the value of Poisson's ratio. Indeed it follows from the limit theorems of classical plasticity that the collapse load for a material with an associated flow rule must be completely independent of Poisson's ratio. However, as might be expected, the displacement at a given load level (prior to collapse) does vary with Poisson's ratio. To illustrate this, consider an anchor, with

embedment ratio $h/B = 3$, which is resting in a purely cohesive material. The load displacement curves obtained for the two cases where $\nu = 0.48$ and $\nu = 0.3$ are shown in Fig. 7.22. It will be observed that, while the elastic stiffness is quite different in the two cases, the collapse loads agree to within 0.5%, and this discrepancy may be attributed to numerical error.

The above argument regarding the insensitivity of collapse load to choice of Poisson's ratio is only strictly correct where collapse is clearly defined. However, it has been found that the load corresponding to a given number of elastic displacements (e.g. the 'k3', 'k4' etc. loads previously defined) are also relatively insensitive to the choice of Poisson's ratio since its effect is taken into account by the different elastic stiffnesses. Consequently, failure loads defined in this way will not be seriously affected by Poisson's ratio.

7.3.2.2 Effect of h/B on Collapse Load - Various ϕ , $\psi = 0^\circ$

The load displacement curves obtained for a range of embedment ratios for values of ϕ equal to 15° , 30° and 45° are shown in Figs. 7.23, 7.24 and 7.25. As in the case where $\phi = 0^\circ$, collapse is clearly defined for embedment ratios less than or equal to three. For embedment ratios greater than three, the load displacement curve increases steadily over the loading range studied and the slope of this curve can be seen to increase significantly with friction angle. It will also be observed that the difference between the load displacement curves obtained for $h/B = 5$, 8, and ∞ increases with increasing friction angle. The velocity fields obtained for all of

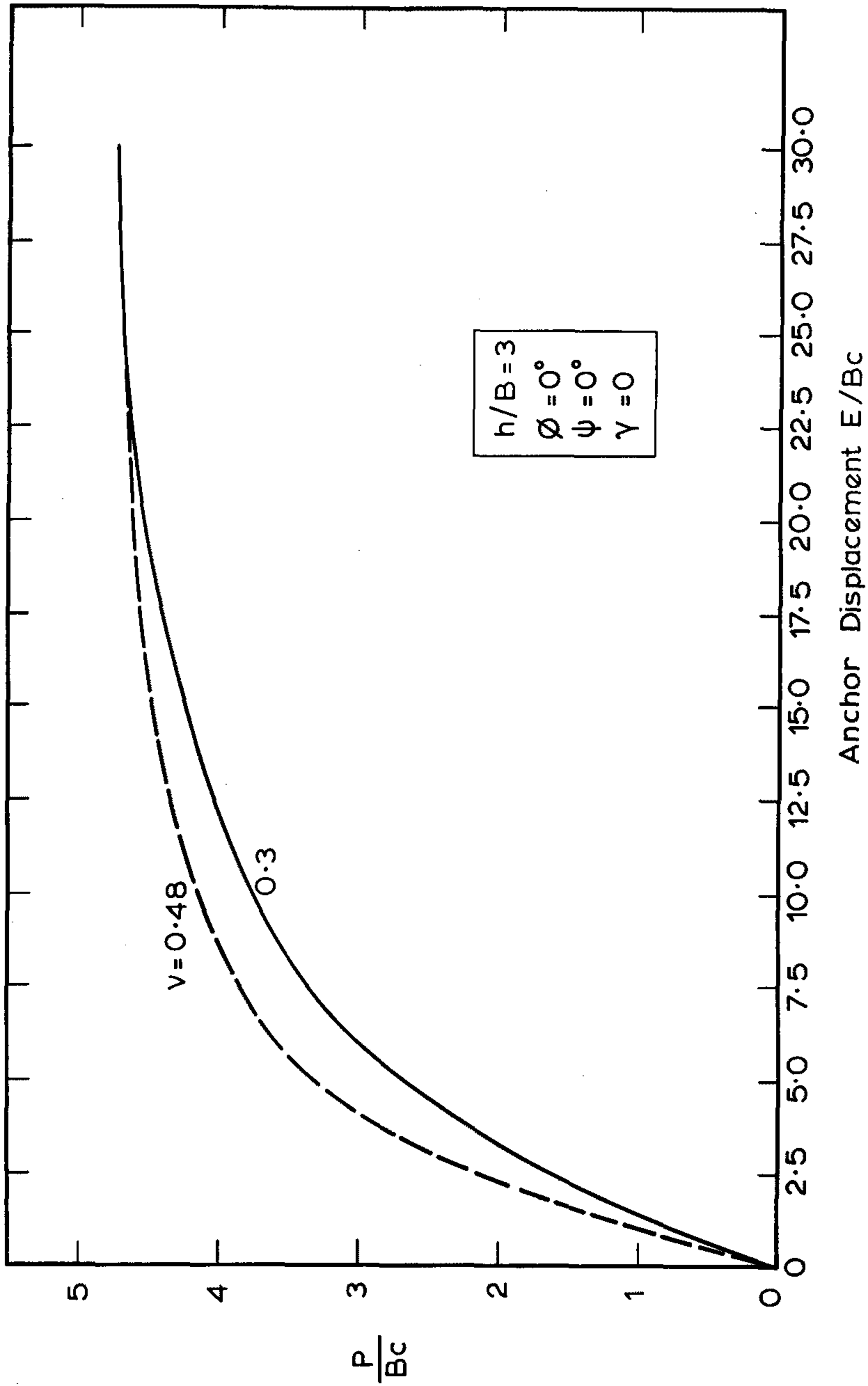
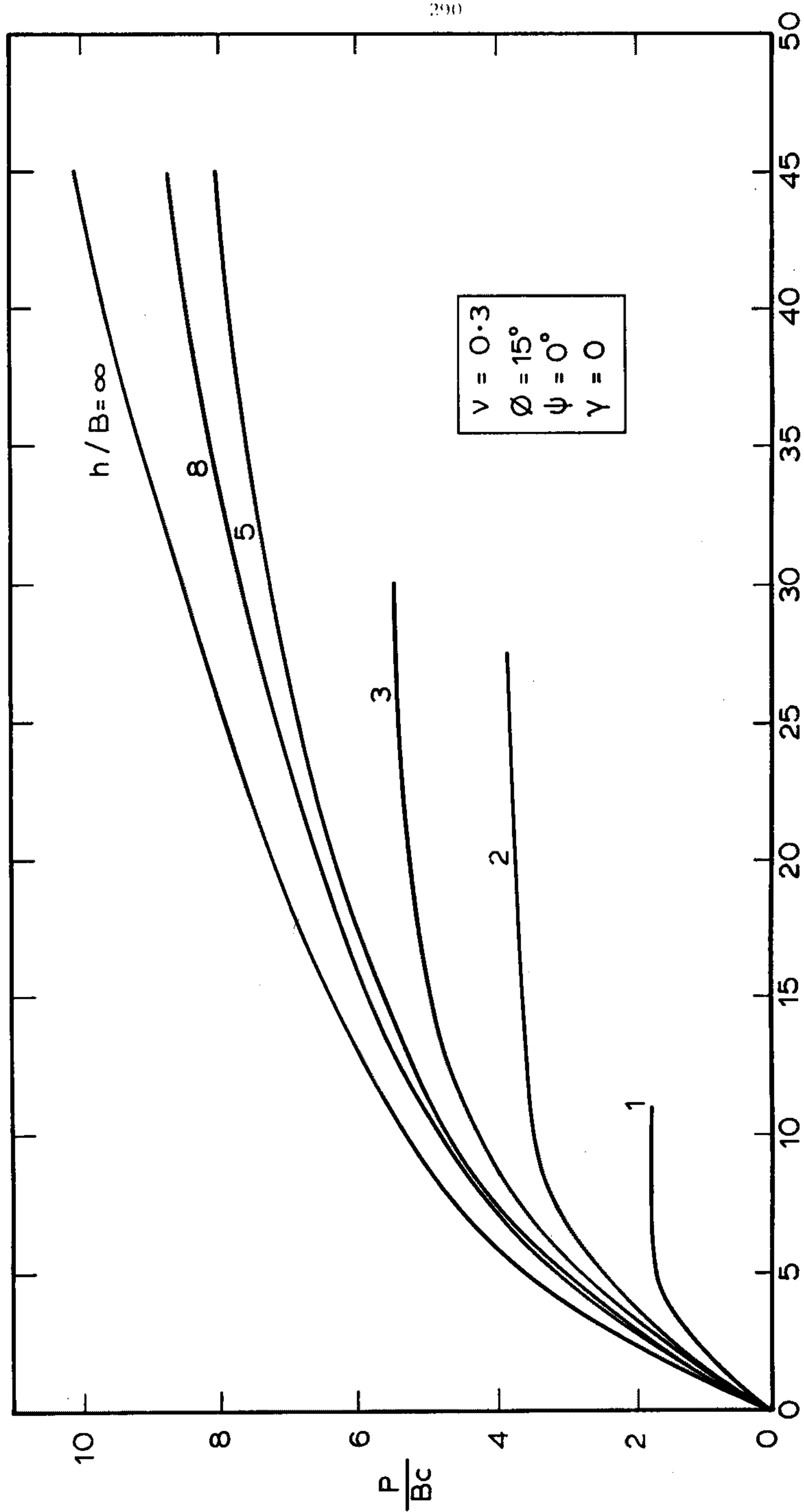


FIG.7.22 EFFECT OF POISSONS RATIO ON LOAD DISPLACEMENT CURVE



Anchor Displacement E/Bc

FIG. 7.23 LOAD DISPLACEMENT CURVES $\phi = 15^\circ$ $\gamma = 0$ VARIOUS h/B

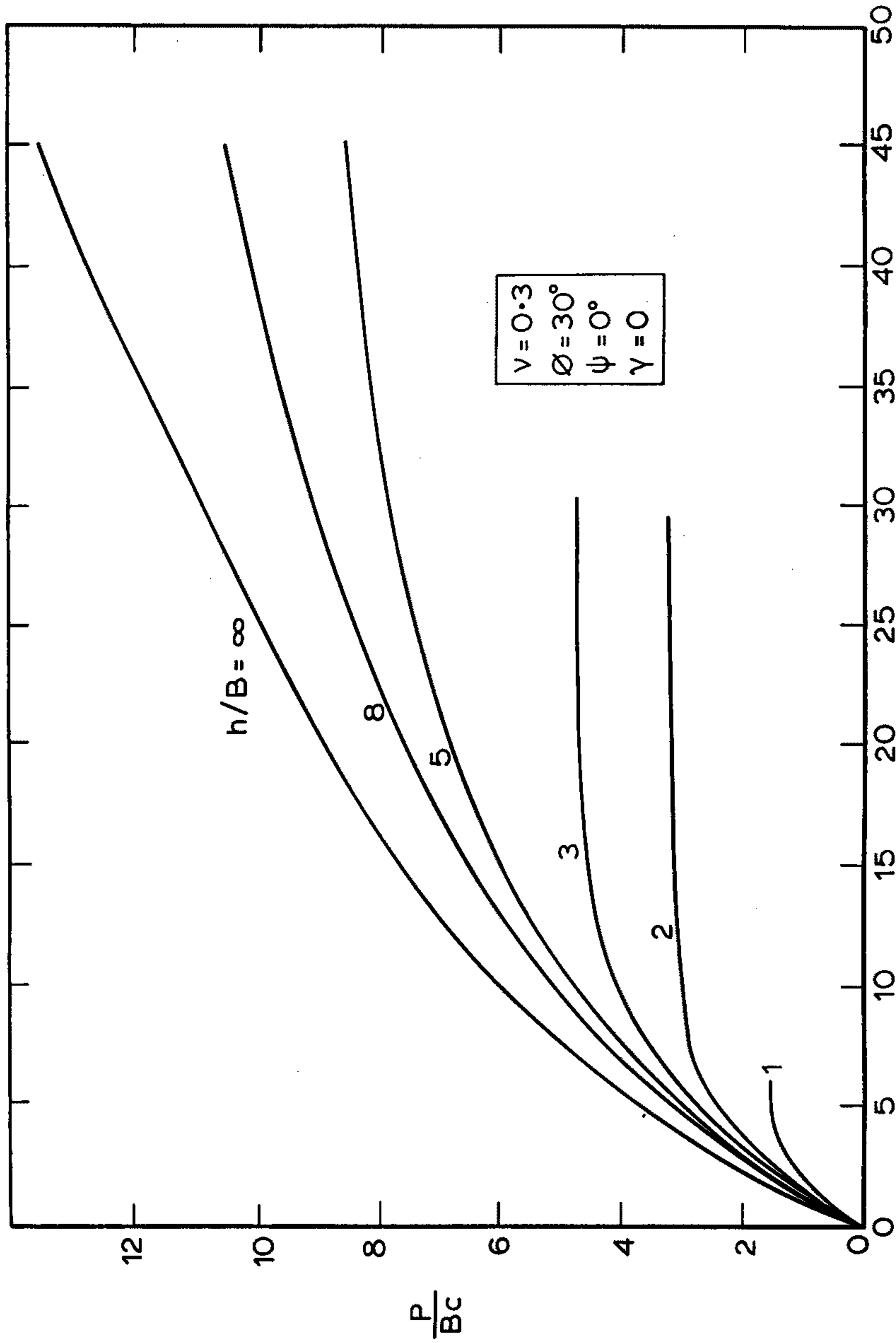
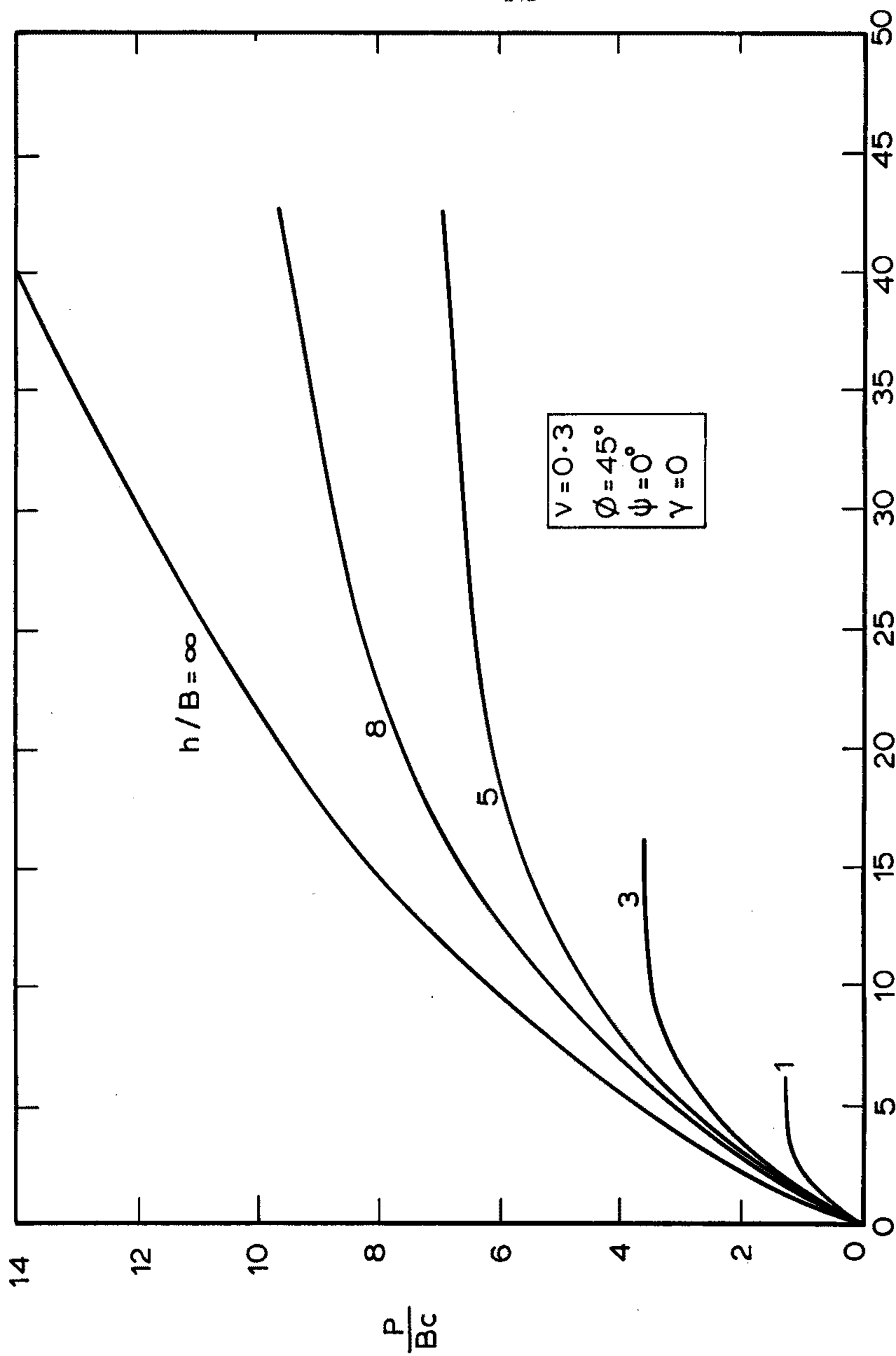


FIG.7.24 LOAD DISPLACEMENT CURVES $\phi = 30^\circ$ $\gamma = 0$ VARIOUS h/B



these cases are essentially the same as those shown for $\phi = 0^\circ$ in Fig. 7.13.

The effect of the angle of internal friction upon the collapse load will now be considered in more detail for the cases where $h/B = 3$ and $h/B = \infty$.

7.3.2.3 Effect of ϕ on Collapse Loads - $h/B = 3$

In many bearing capacity analyses, the collapse load $(\frac{P}{Bc})_{\gamma=0}$ is found to increase with increasing angle of internal friction, ϕ (e.g. Davis and Booker, 1971). However, this is not generally true of anchor plates. For example, Fig. 7.26 shows the load displacement curves obtained for an anchor with $h/B = 3$ for angles of internal friction, ϕ , in the range 0° to 45° . In every case, failure was clearly defined with the highest collapse load, for $\phi = 15^\circ$, being 18% above the value obtained for $\phi = 0^\circ$. The collapse load can be seen to decrease for values of ϕ both greater and less than 15° and approximately the same collapse load was obtained for $\phi = 0^\circ$ and $\phi = 30^\circ$. The collapse load for $\phi = 45^\circ$ was 23% lower than that obtained for $\phi = 0^\circ$.

This type of behaviour can be attributed to the counter-acting effects of an increase in the strength of soil in compressive zones with a decrease in the strength of soil in flexural regions where incremental tensile stresses are encountered. The extent of the flexural region can be estimated from Fig. 7.27 which shows the contours of mean stress $p = (\frac{\sigma_{yy} + \sigma_{xx}}{2})$ for the case where $\phi = 0^\circ$. Fig. 7.28 shows the plastic regions at collapse for the four values

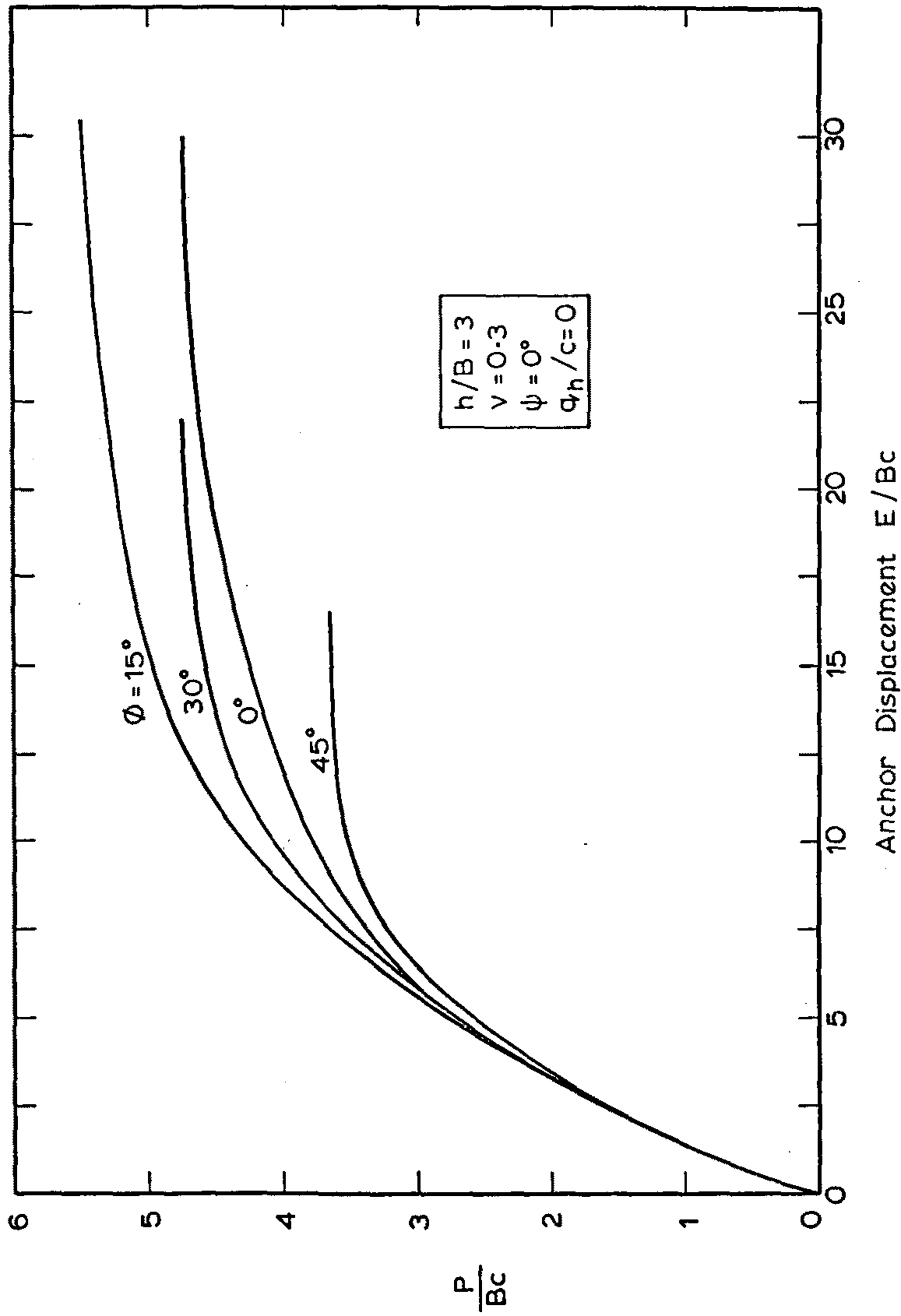


FIG.7.26 EFFECT OF ϕ ON LOAD DISPLACEMENT BEHAVIOUR- $h/B=3$
(CONSTANT COHESION)

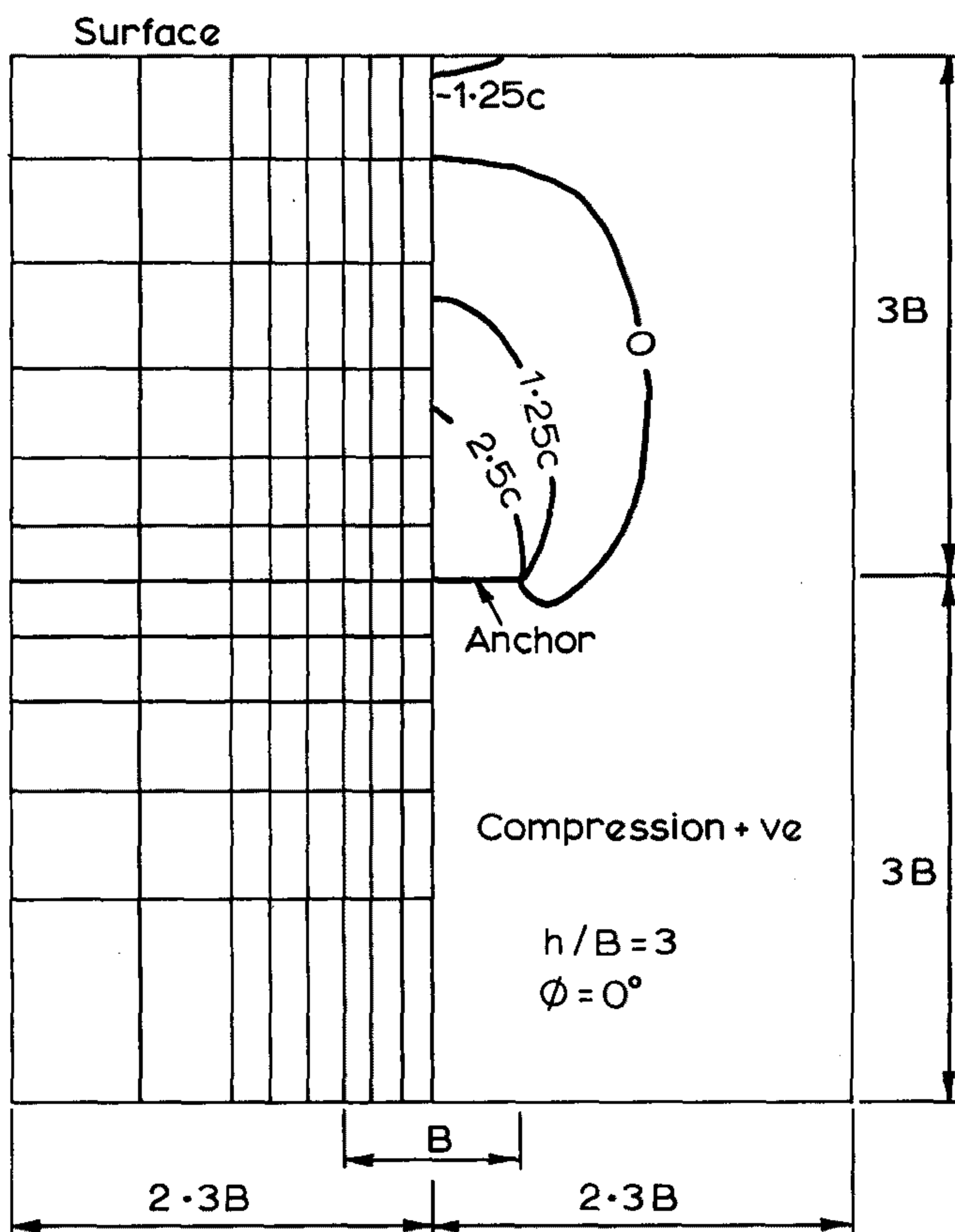


FIG.7.27 CONTOURS OF MEAN COMPRESSIVE STRESS
AT COLLAPSE - $h/B = 3$ IMMEDIATE BREAKAWAY $\phi_h/c = 0$

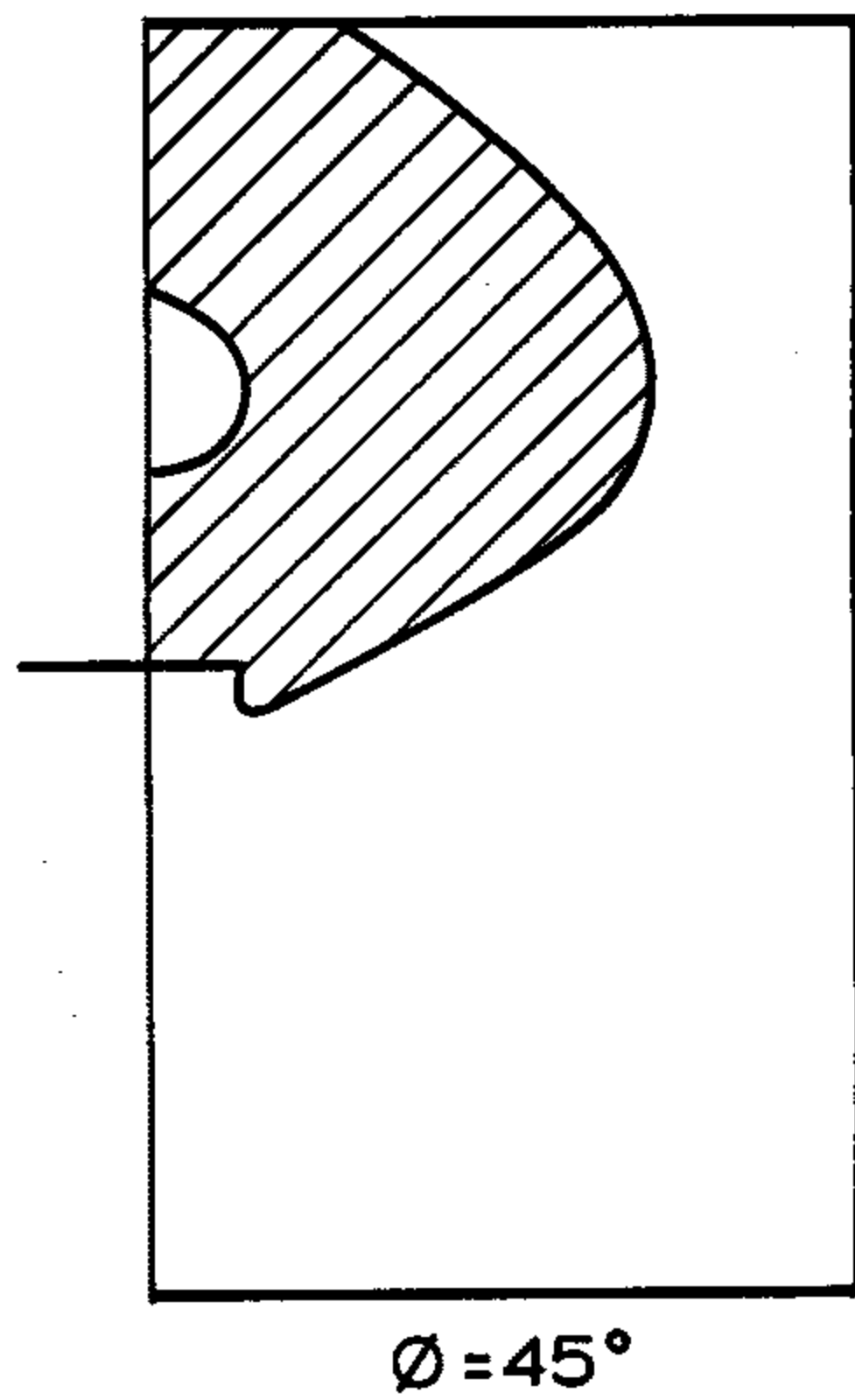
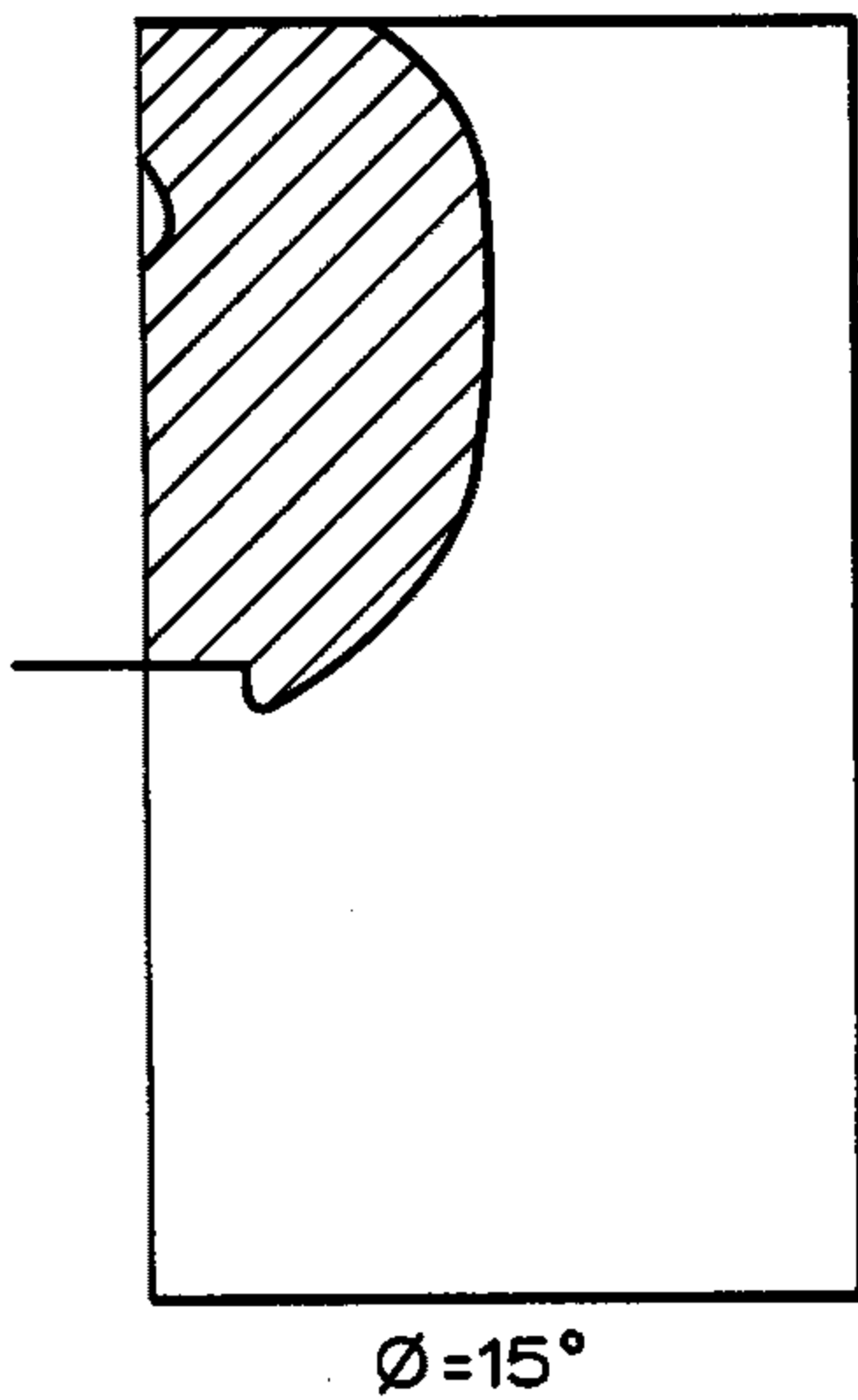
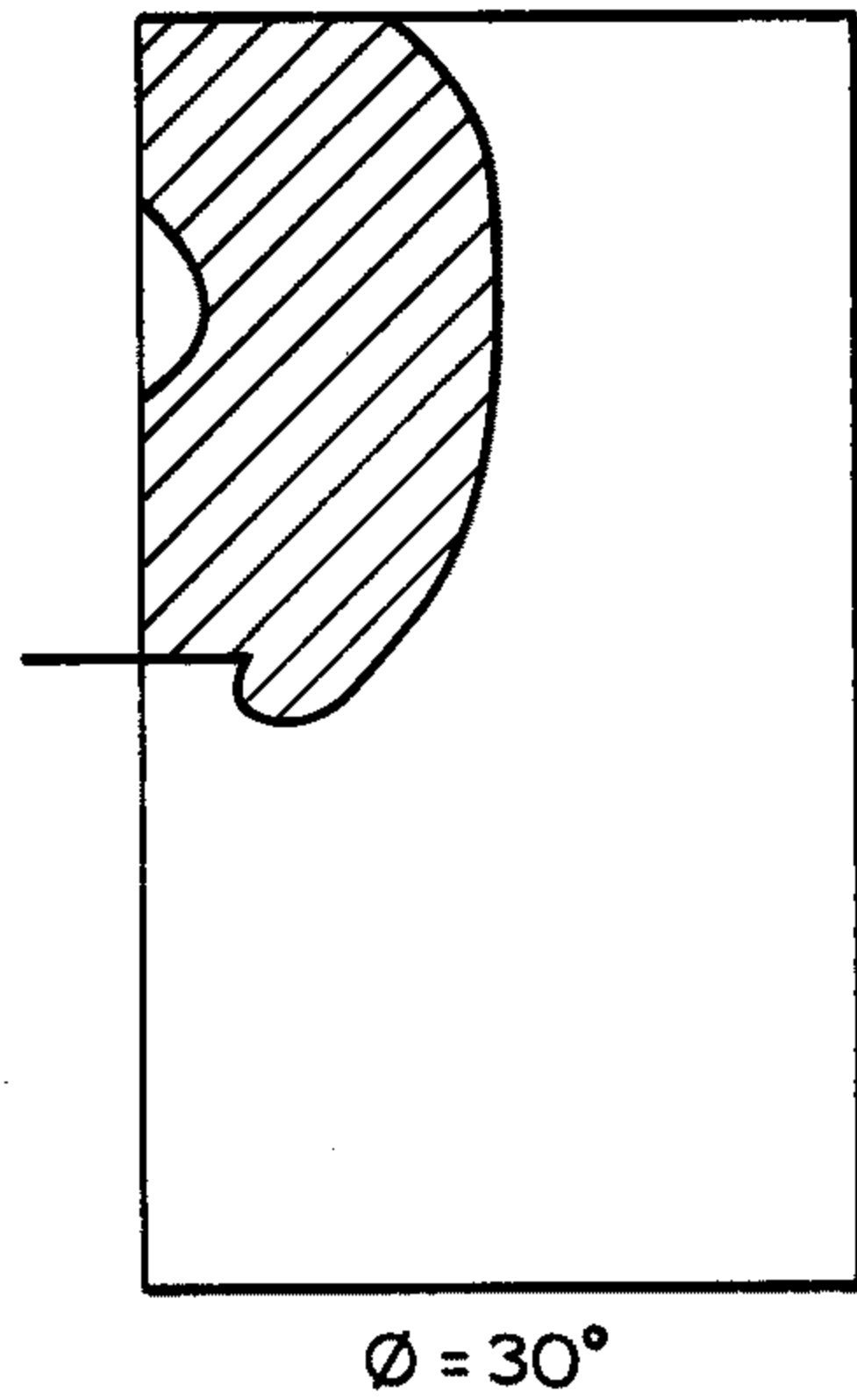
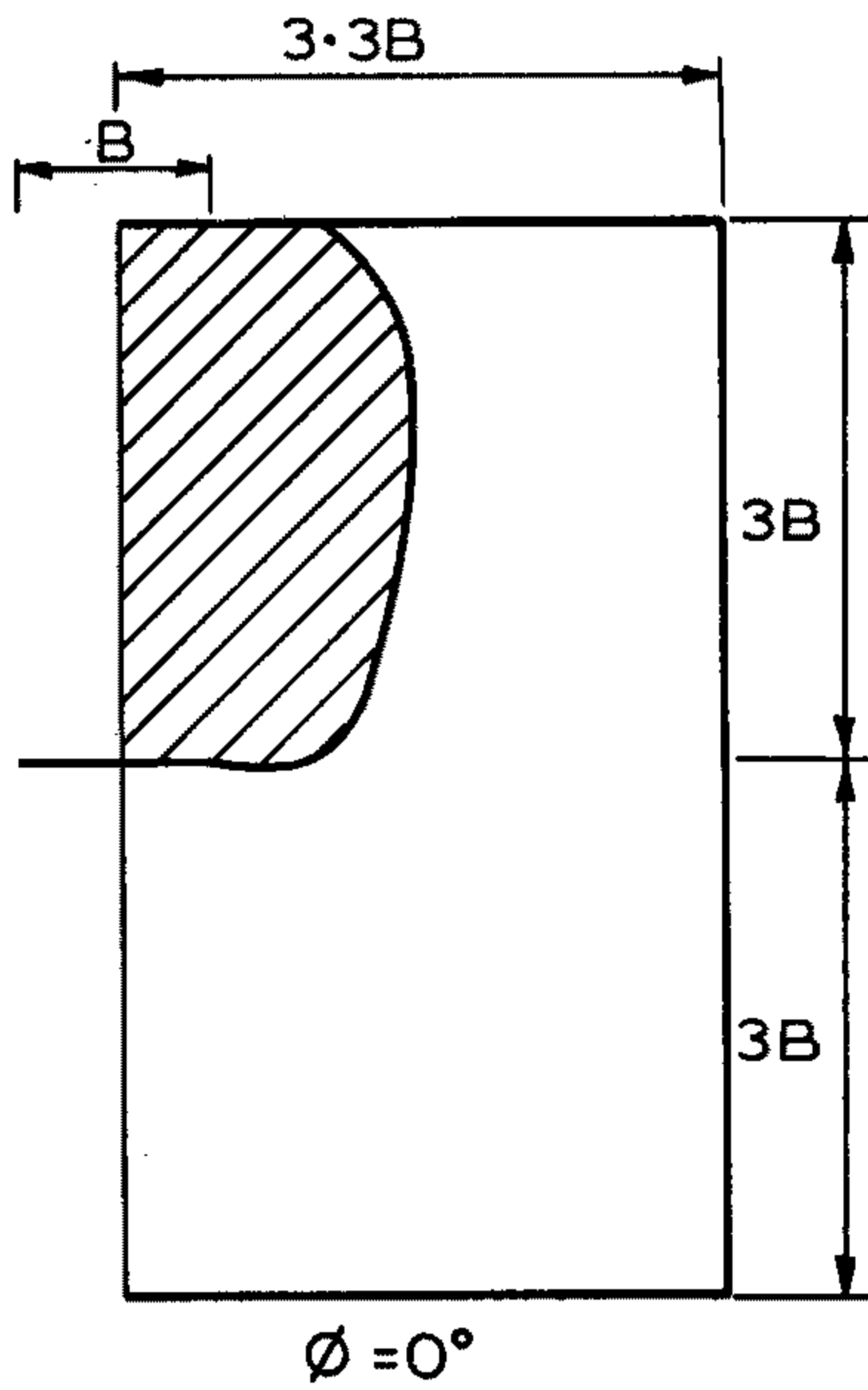


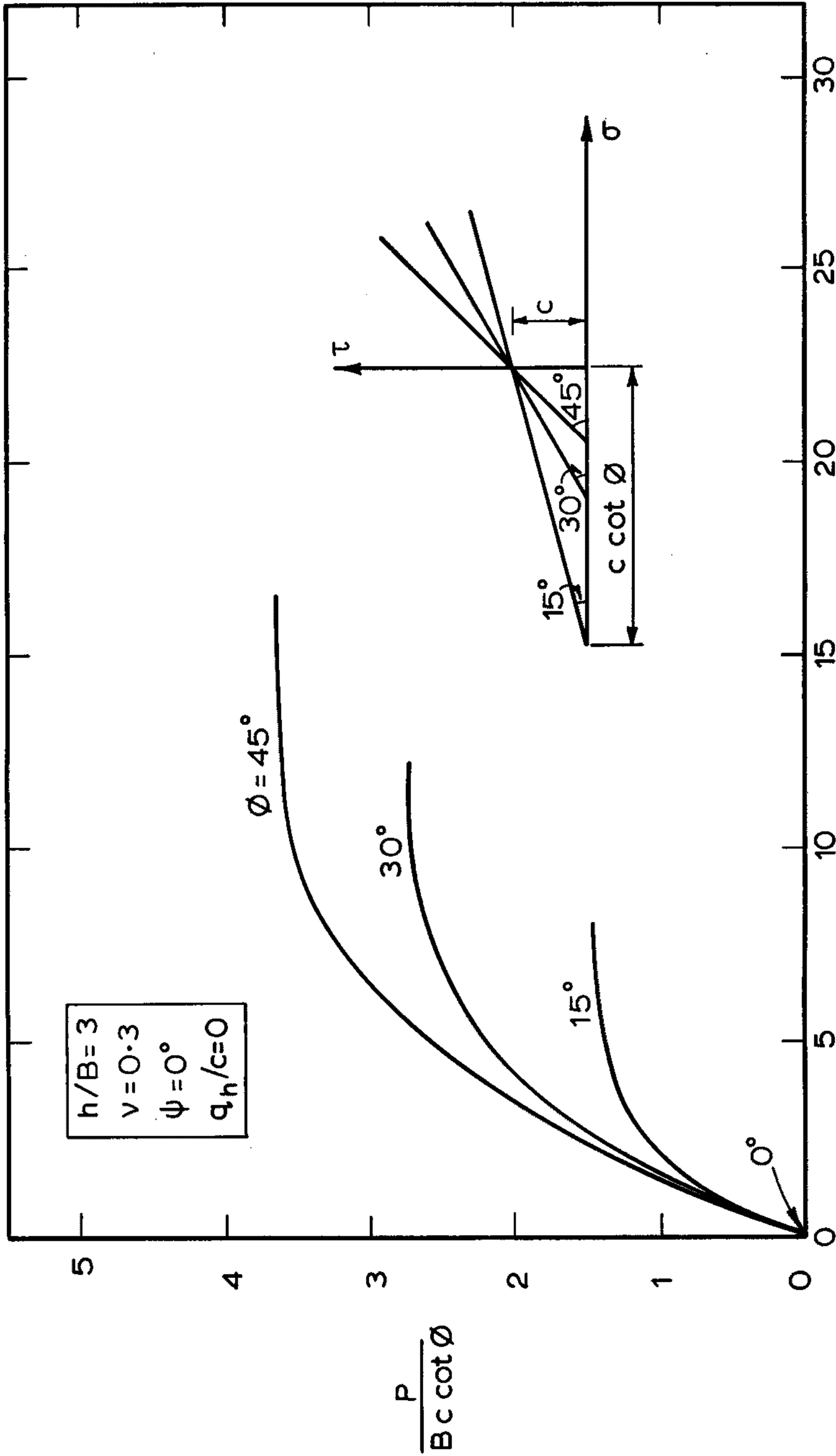
FIG. 7.28 PLASTIC REGIONS AT COLLAPSE - VARIOUS $\emptyset$
 $h/B = 3$ $E/c = 500$ $\gamma = 0$ $\psi = 0^\circ$ $q_h/c = 0$

of $\phi = 0^\circ, 15^\circ, 30^\circ$ and 45° . As ϕ increases, it can be seen that the plastic region extends out into the regions associated with lower values of mean stress and shrinks in the regions of high mean compressive stress leaving a region of elastic material contained within the plastic zone above the anchor.

This dependence of the bearing capacity of a shallow anchor in a cohesive frictional material upon the material strength in flexure is emphasised by the fact that the results in Fig. 7.26 are plotted with respect to a constant cohesion c , and hence the maximum allowable tensile stress $\sigma_3 = -c \cot \phi$ decreases with increasing angle of internal friction ϕ . If the same results are plotted for a constant maximum tensile stress $\sigma_{3\max} = -c \cot \phi$, then the trend of increasing strength with friction angle is again observed as shown in Fig. 7.29. However, this method of presentation has the disadvantage that the case where $\phi = 0^\circ$ must be considered as a special case and so the more general method of presentation, where the load is expressed in terms of the cohesion as shown in Fig. 7.26, will be adopted for the remainder of the chapter.

7.3.2.4 Effect of ϕ on Collapse Loads $h/B = \infty$

From the load displacement curves shown in Fig. 7.30, it can be seen that the bearing capacity increases with the angle of internal friction for a very deep anchor. Indeed this is to be expected since, at great depth, flexure of the soil above the anchor ceases to be of significance (for example, see Fig. 7.31), and the behaviour is largely dominated by the growth of the plastic zones in compression. The load displacement curve in each case is associated with a long



Anchor Displacement $E/Bc \cot \phi$

FIG.7-29 EFFECT OF ϕ ON LOAD DISPLACEMENT BEHAVIOUR - $h/B = 3$

(CONSTANT $\sigma_{3\max} = c \cot \phi$)

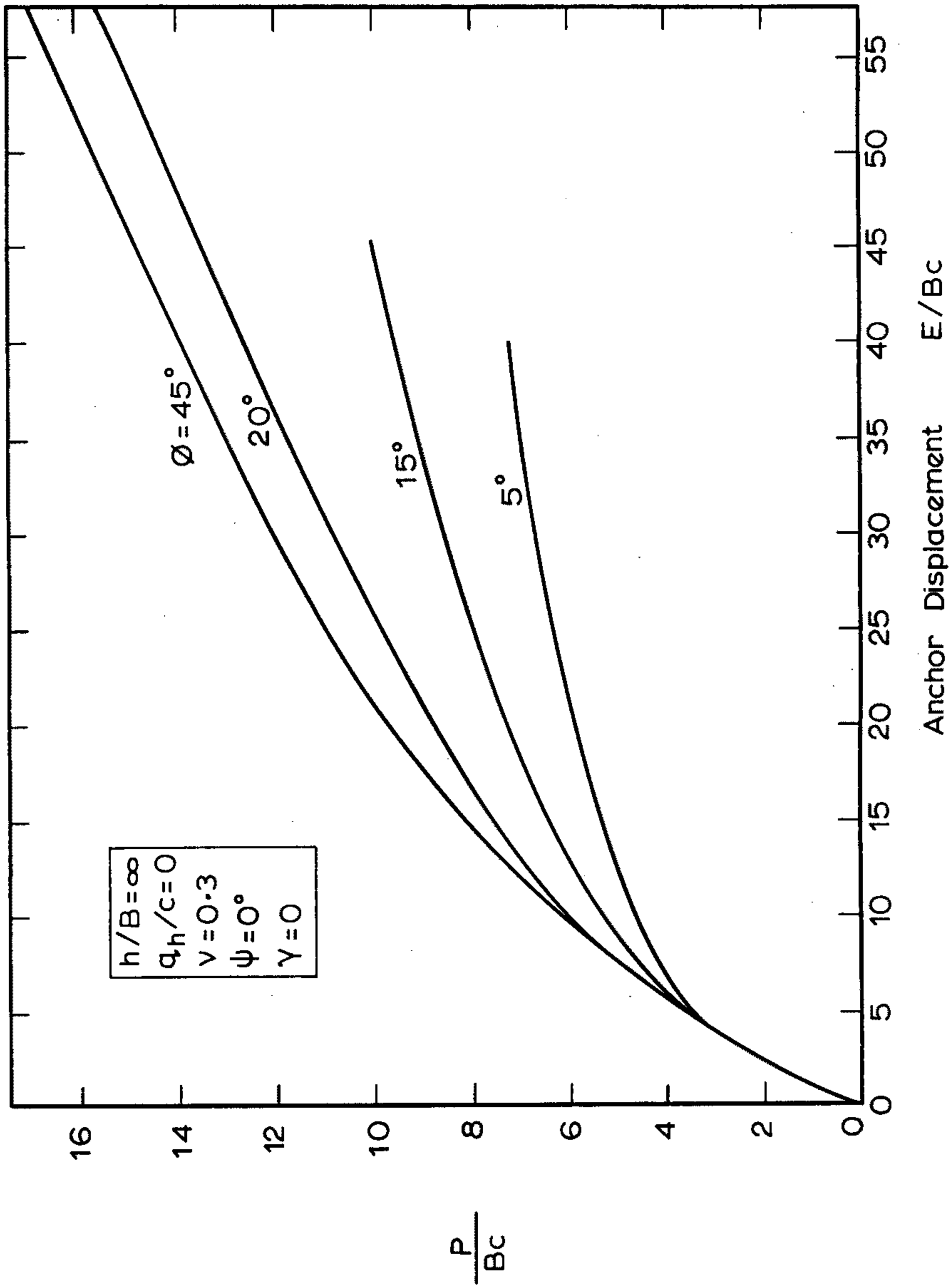


FIG. 7.30 LOAD DISPLACEMENT CURVES - ANCHOR AT INFINITE DEPTH

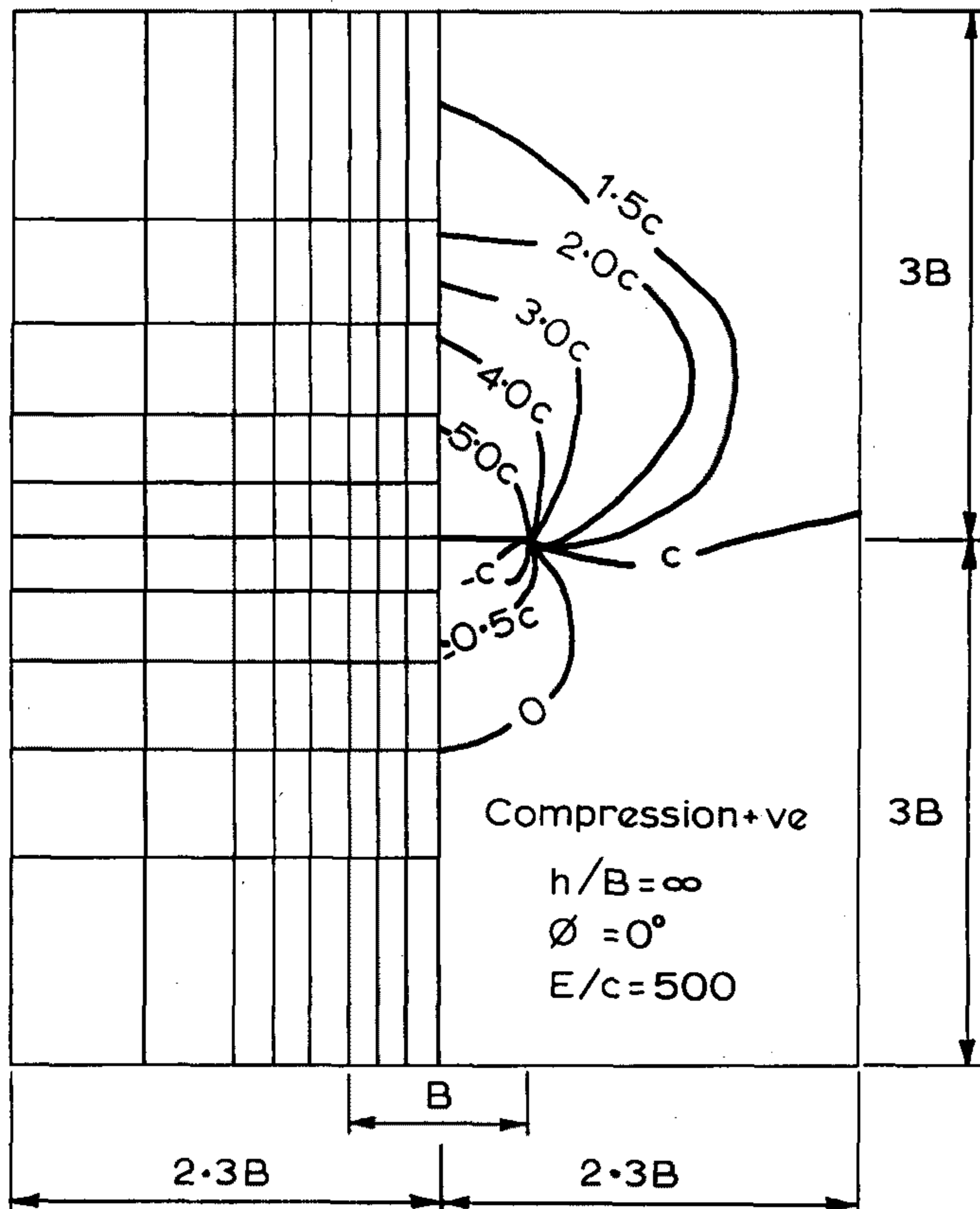


FIG. 7.31 CONTOURS OF MEAN COMPRESSIVE STRESS AT
LAST LOAD STEP - $h/B = \infty$ IMMEDIATE BREAKAWAY

$(q_h/c = 0)$

elasto-plastic tail, and it is necessary to adopt a practical definition of failure related to anchor-displacement, such as the 'k4' definition previously discussed. The ultimate collapse load for an anchor at infinite depth in a cohesive frictional material is unknown, however it is certain that the collapse load must be at least as great as that for the surface footing. To illustrate this, Fig. 7.32 shows the load displacement curve obtained for the case where $\phi = 30^\circ$. It will be observed that when loading was discontinued, after several hundred load steps and a displacement of $525 Bc/E$, there was no indication of likely collapse, although the applied load was almost twice the Prandtl value (ie. $P = 30.1 Bc$) for a surface footing.

7.3.2.5 Effect of Surface Surcharge q_s/c

Unlike the purely cohesive case, for soil in which $\phi > 0^\circ$ the effects of a surface surcharge (q_s) and of body force within the soil (γh) must be considered separately.

The analysis of anchor problems in which there is a surface surcharge $q_s > 0$ is quite straightforward and follows the same lines as discussed in section 7.3.1. Thus the bearing capacity factor $F_c \left(\frac{h}{B}, \frac{q_s}{c}, \phi \right)$ may be expressed in terms of an approximate equation similar to that adopted for purely cohesive materials, viz.

$$F_c = F_{c_o} + q_s/c \quad (7.3)$$

provided that $F_c \leq F_c^*$

where $F_c = F_c \left(\frac{h}{B}, \frac{q_s}{c}, \phi \right)$

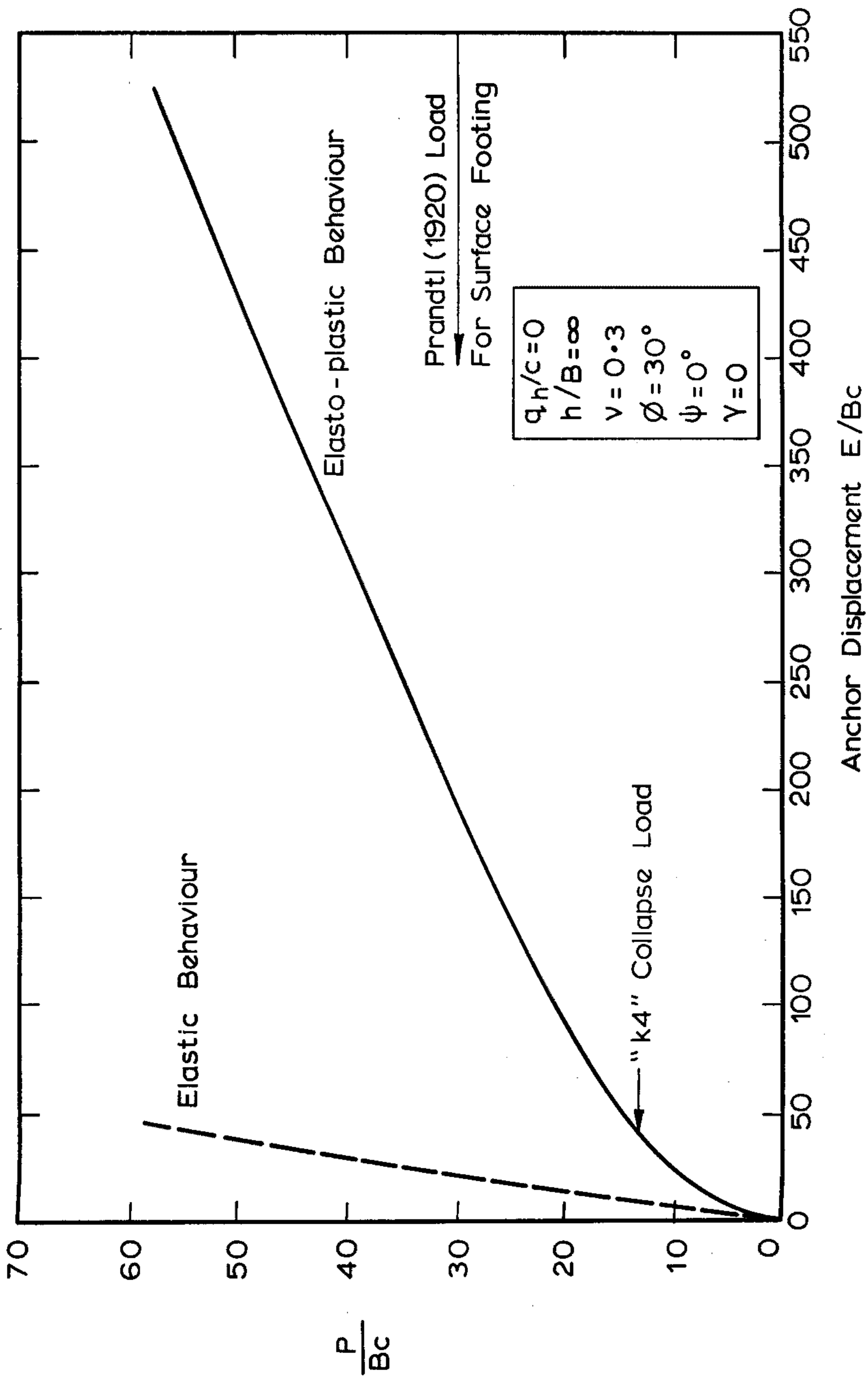


FIG.7.32 ANCHOR AT INFINITE DEPTH - IMMEDIATE BREAKAWAY

$$F_{c_o} = F_c \left(\frac{h}{B}, \frac{q_s}{c} = 0, \phi \right)$$

and
$$F_c^* = F_c \left(\frac{h}{B}, \frac{q_s}{c} = \infty, \phi \right)$$

The determination of bearing capacity factors for the limiting no breakaway case will not be considered in detail for frictional materials since, in general, the body force will dominate the overburden pressure (ie. $\gamma h \gg q_s$) and this will be discussed separately in the following section. The effect of the breakaway condition upon the bearing capacity of a shallow anchor is, however, illustrated for an anchor with an embedment ratio of three, resting in a cohesive frictional soil with $\phi = 30^\circ$, $\gamma h = 0$. From Fig. 7.33 it can be seen that the breakaway condition gives rise to a 72% increase in bearing capacity and this may be compared to the 260% increase in bearing capacity due to the no breakaway condition which is obtained for a similar anchor in a purely cohesive material. The reduced influence of the no breakaway condition upon the bearing capacity for $\phi = 15^\circ$ is another effect of the reduced maximum tensile strength of the material (ie. $c \cot \phi$) with increasing frictional angle for a constant cohesion. This indicates that caution should be adopted in applying Eq. 7.3 for large values of the dimensionless surcharge q_s/c and for high angles of internal friction ϕ . This problem is not, however, considered to be of great practical importance since in these cases the dimensionless body force $\gamma h/c$ will usually dominate the overall behaviour of the anchor.

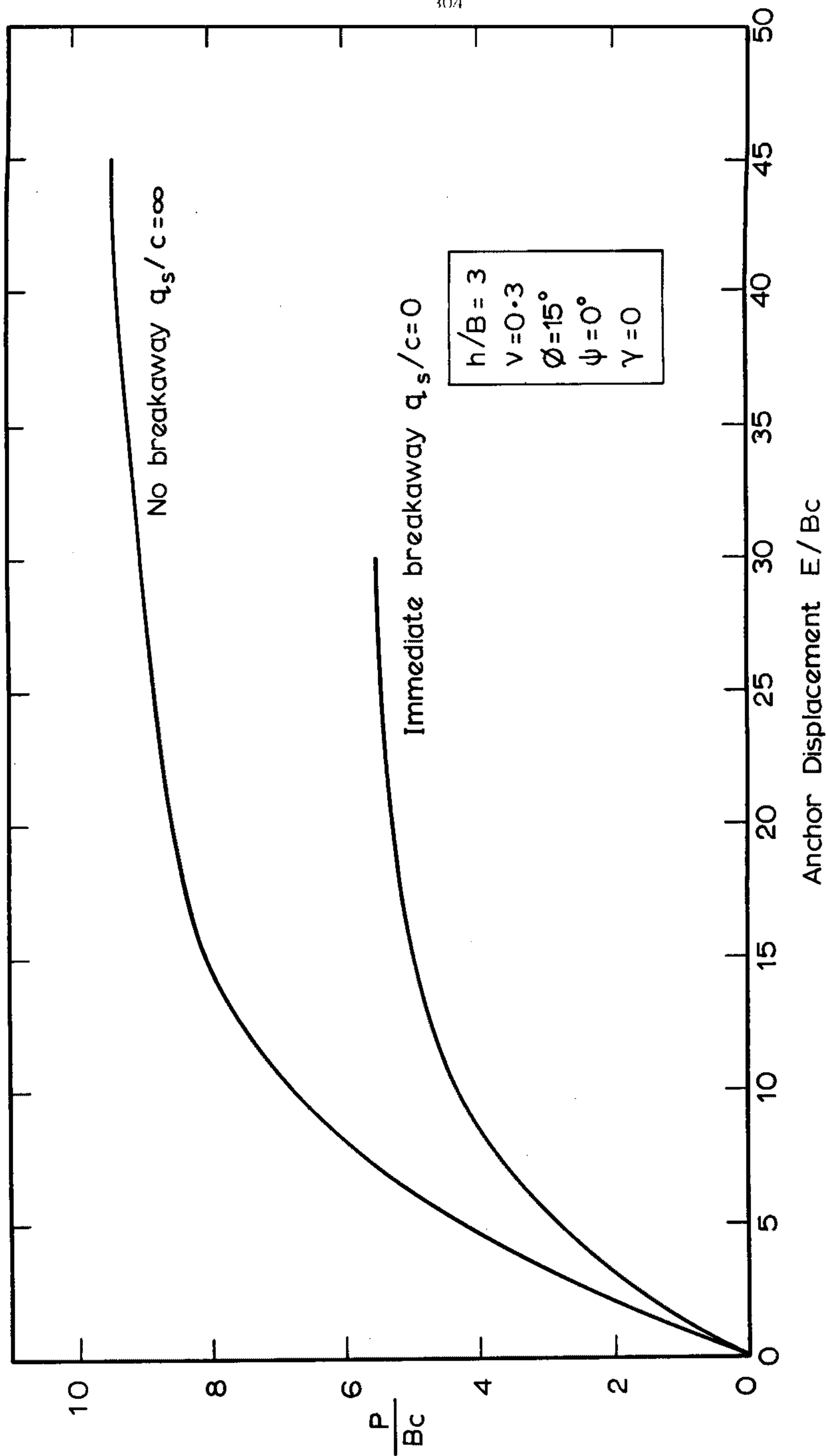


FIG. 7.33 EFFECT OF SURCHARGE UPON LOAD DEFLECTION CURVE $h/B=3$

7.3.2.6 Summary of Results for a Cohesive-Frictional Material
(Drained Loading)

As was previously the case for undrained behaviour, it is necessary here to define failure for anchors with embedment ratios greater than three in terms of a practical ('k4') failure load. The bearing capacity factors F_c determined on this basis for the entire range of embedment ratios are shown in Figs. 7.34, 7.35 and 7.36 for $\phi = 15^\circ$, 30° and 45° respectively. Also shown are the 'k2' and 'k3' loads corresponding to a secant stiffness of one half and one third the elastic value.

When $\phi = 15^\circ$, it would appear that deep anchor behaviour occurs at an embedment ratio, h/B , between five and eight. In this case there is still a difference between the 'k4' values obtained for $h/B = 8$ and $h/B = \infty$, however as explained in section 7.3.1.4, this is a result of the displacement being underestimated due to the adoption of the same distance to the 'infinite boundary' as to the surface in the $h/B = 8$ case.

For ϕ greater than 15° , deep anchor behaviour cannot be identified within the range of embedment ratios considered, and hence must occur at an embedment ratio significantly greater than eight.

The effect of ϕ upon the collapse load for various embedment ratios is illustrated in Fig. 7.37. In the case of very shallow anchors (e.g. $h/B = 1$), the tension due to flexure of the soil above the anchor dominates the solution for all ϕ greater than zero and

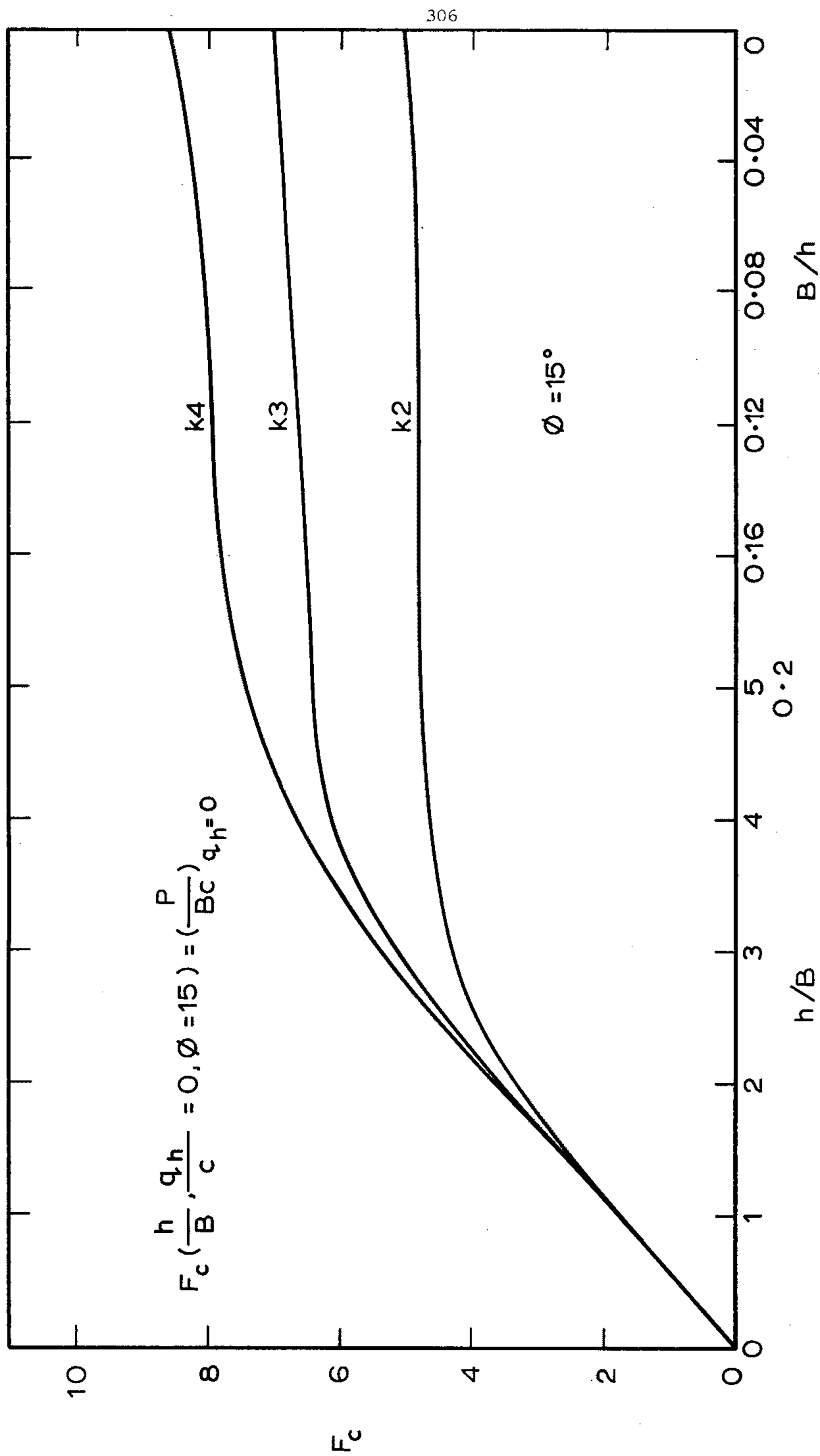


FIG.7.34 VARIATION OF F_c WITH EMBEDMENT RATIO $q_h/c=0$ $\phi=15^\circ$

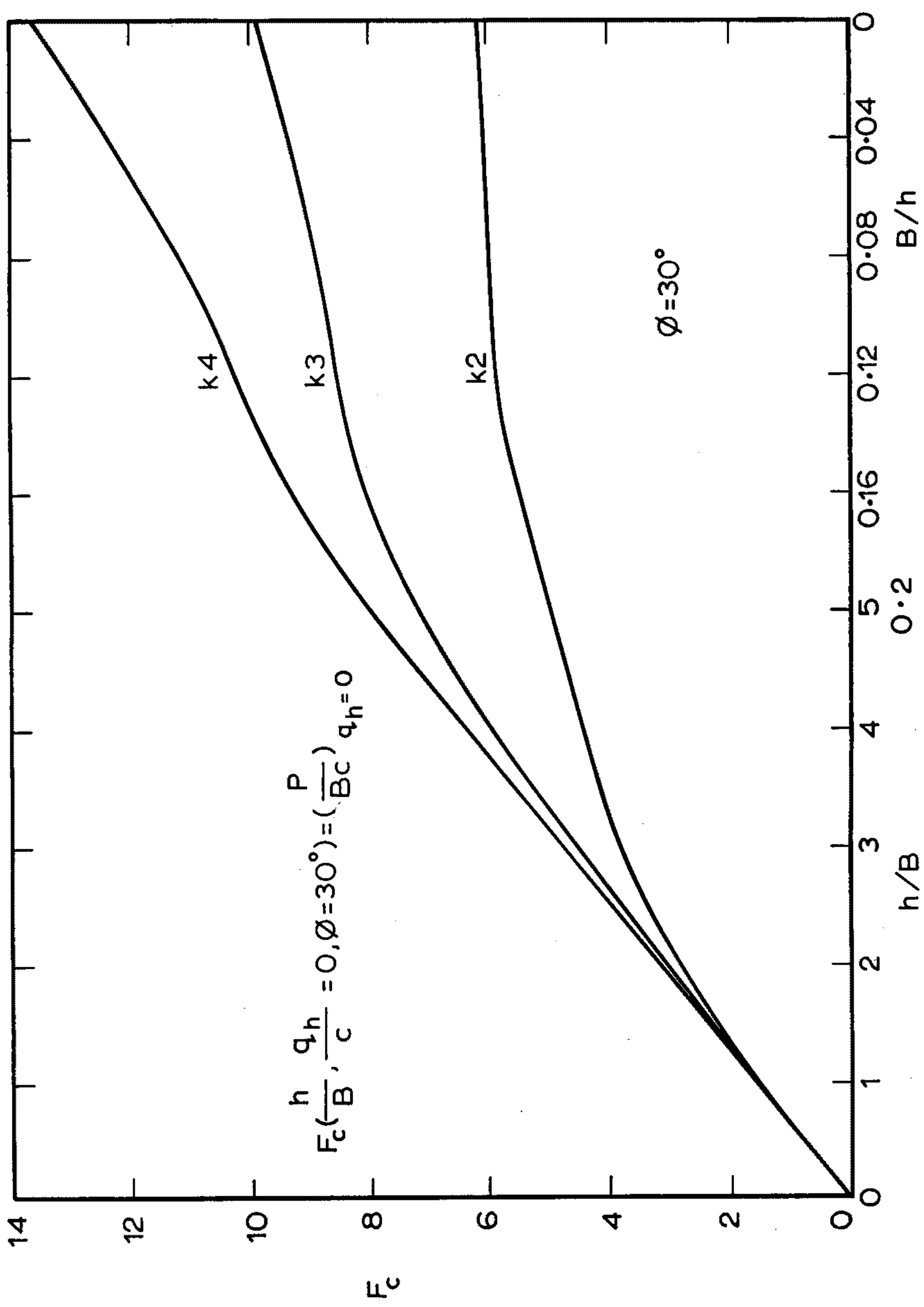


FIG. 7.35 VARIATION OF F_c WITH EMBEDMENT RATIO $q_h/c=0$ $\phi=30^\circ$

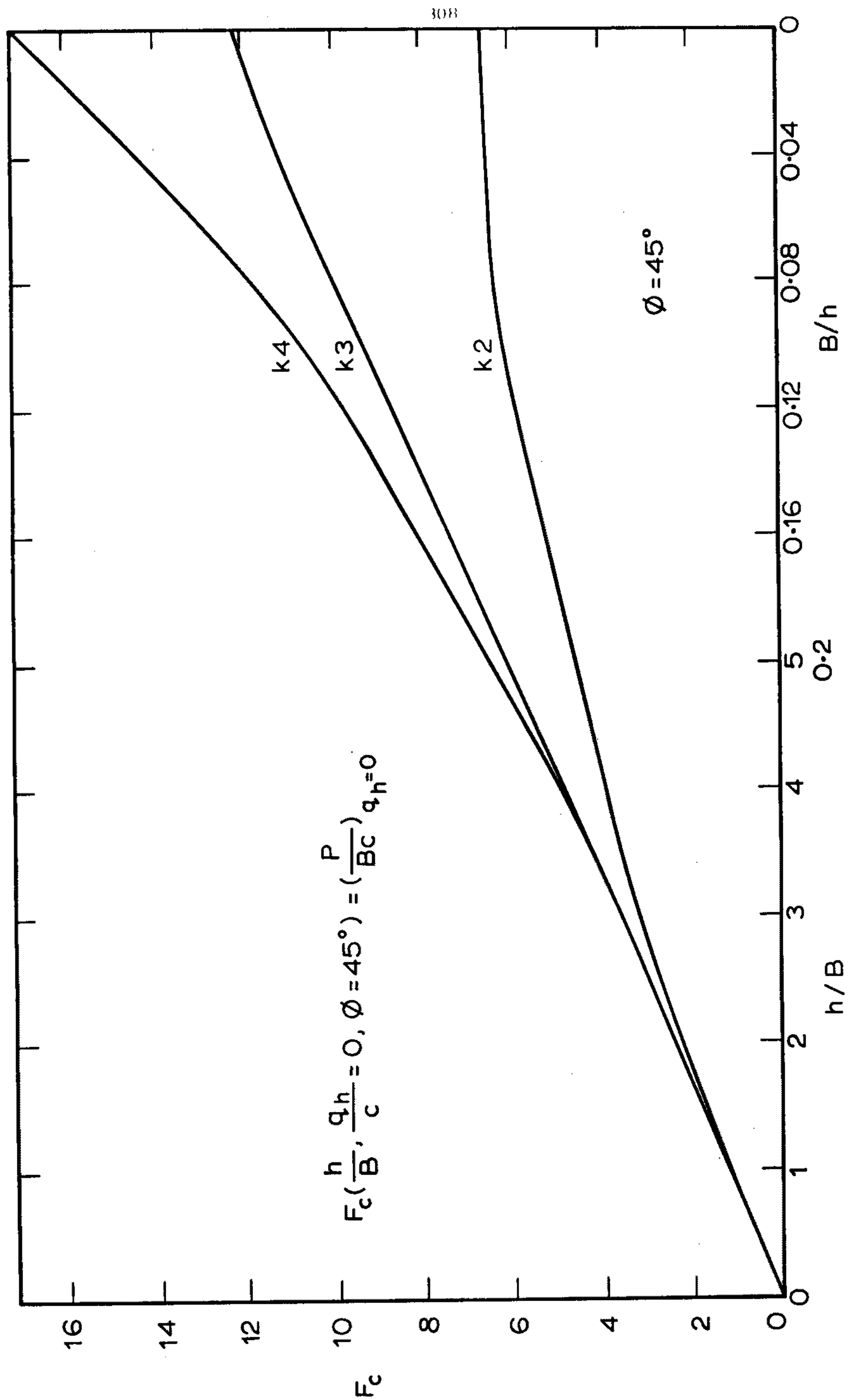


FIG. 7.36 VARIATION OF F_c WITH EMBEDMENT RATIO $q_h/c=0$ $\phi=45^\circ$

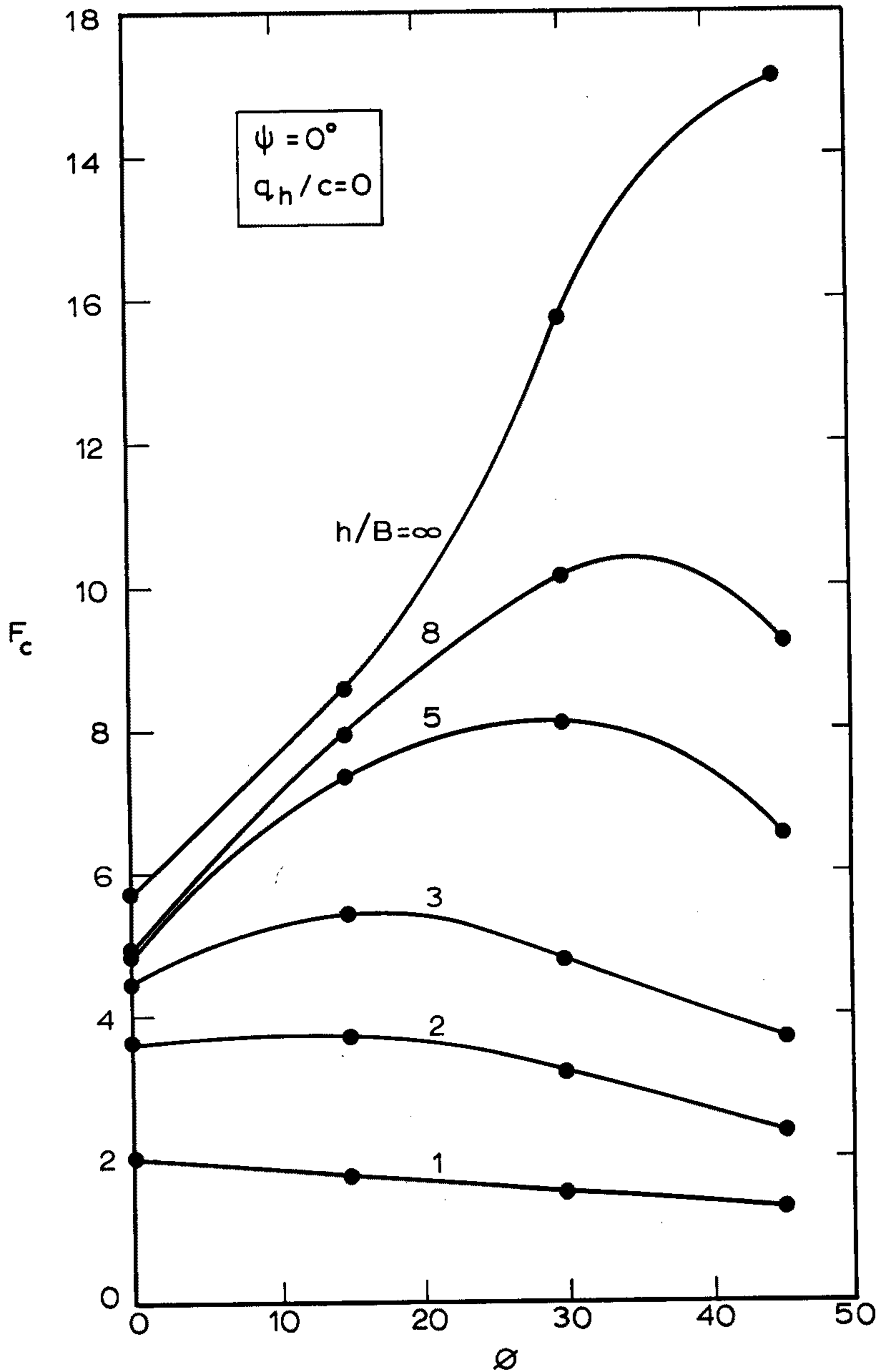


FIG.7.37 VARIATION OF F_c WITH $\emptyset$ FOR HORIZONTAL ANCHOR - IMMEDIATE BREAKAWAY "k4" VALUES

hence the bearing capacity with respect to the cohesion decreases with friction angle. As the embedment ratio increases, the significance of these flexural tensions tends to decrease. Thus the maximum bearing capacity factor corresponds to $\phi = 0^\circ$ for $h/B = 1$; to $\phi = 15^\circ$ for $h/B = 3$; to $\phi = 30^\circ$ for $h/B = 5$; and in excess of 45° for $h/B = \infty$.

The general effect of ϕ upon the collapse load shown for $h/B = 3$ has been previously recognised by Vesic (1971) however, his paper suggests that the peak bearing capacity factor F_c will occur for $\phi = 15^\circ$ irrespective of embedment ratio.

7.3.3 Frictional Material With Weight (Drained Loading)

7.3.3.1 Numerical Approximations

The analysis of purely frictional materials with weight is complicated by the fact that, since the geostatic stresses are very low near the soil surface, the shear strength of the material is correspondingly low and, consequently, the material fails at very low load levels. This implies the need either for extremely small increment size or a variable increment size procedure. Neither of these alternatives is particularly desirable since, in the first instance, the cost is unacceptably high, while in the second instance the increasing increment size unnecessarily complicates the numerical procedure. A more acceptable alternative is to analyse a material with a very small cohesion, and in fact for $K_0 = 1$, this corresponds to the analysis of a purely frictional material where the shear strength of the material to an equivalent depth $\Delta h = c/\gamma \tan \phi$

is neglected. In all these analyses, a cohesion given by $\frac{\gamma h}{c} = 120$ was adopted. This leads to two methods of determining F_γ from an analysis: namely, either $F_\gamma = \frac{q_u}{\gamma h} - \frac{c}{\gamma h} F_c$, where it is assumed that the superposition factor $F_s = 1$; or else the cohesion is treated as a surcharge due to the additional height $\Delta h = c \cot \phi / \gamma$ and hence $F_\gamma = \frac{q_u}{\gamma h^*}$, where $h^* = h + c \cot \phi / \gamma$. The difference in the value of F_γ determined by the two methods was found to be insignificant. In fact, even if all corrections are neglected and F_γ is calculated from $F_\gamma = \frac{q_u}{\gamma h}$ then the error is $\frac{c}{\gamma h} \cot \phi$, or typically about 1.5% for $\phi = 30^\circ$. The appropriate corrections have been made in determining the bearing capacity factor F_γ , however the load scales on the load-displacement curves have not been modified.

The insensitivity of the solution to the shear strength of the top layer of material can be illustrated further by removing the top 10% of the material and replacing it by a surcharge which does not contribute any strength. For example, an anchor with embedment ratio $h/B = 3$ was analysed for the case where $\phi = 30^\circ$, $\gamma h/c = 120$ and surface surcharge $q_s = 0$. The load displacement curve was then compared with that obtained for an anchor with embedment ratio $h/B = 2.7$, surface surcharge $q_s = 0.3 \gamma B$ and the same cohesion as before. From Fig. 7.38 it can be seen that the difference between the collapse load obtained in the two cases is only about 2.8%.

In all the analyses of 'purely frictional' material, an increment size of $0.125 B\gamma h/E$ was adopted. As a check on the increment size and convergence criteria, an anchor with embedment ratio $h/B = 3$ resting in a frictional material with $\phi = 30^\circ$, $\psi = 0^\circ$, $E/\gamma h = 333$ and $\nu = 0.3$ was analysed using displacement increments of $0.125 B\gamma h/E$

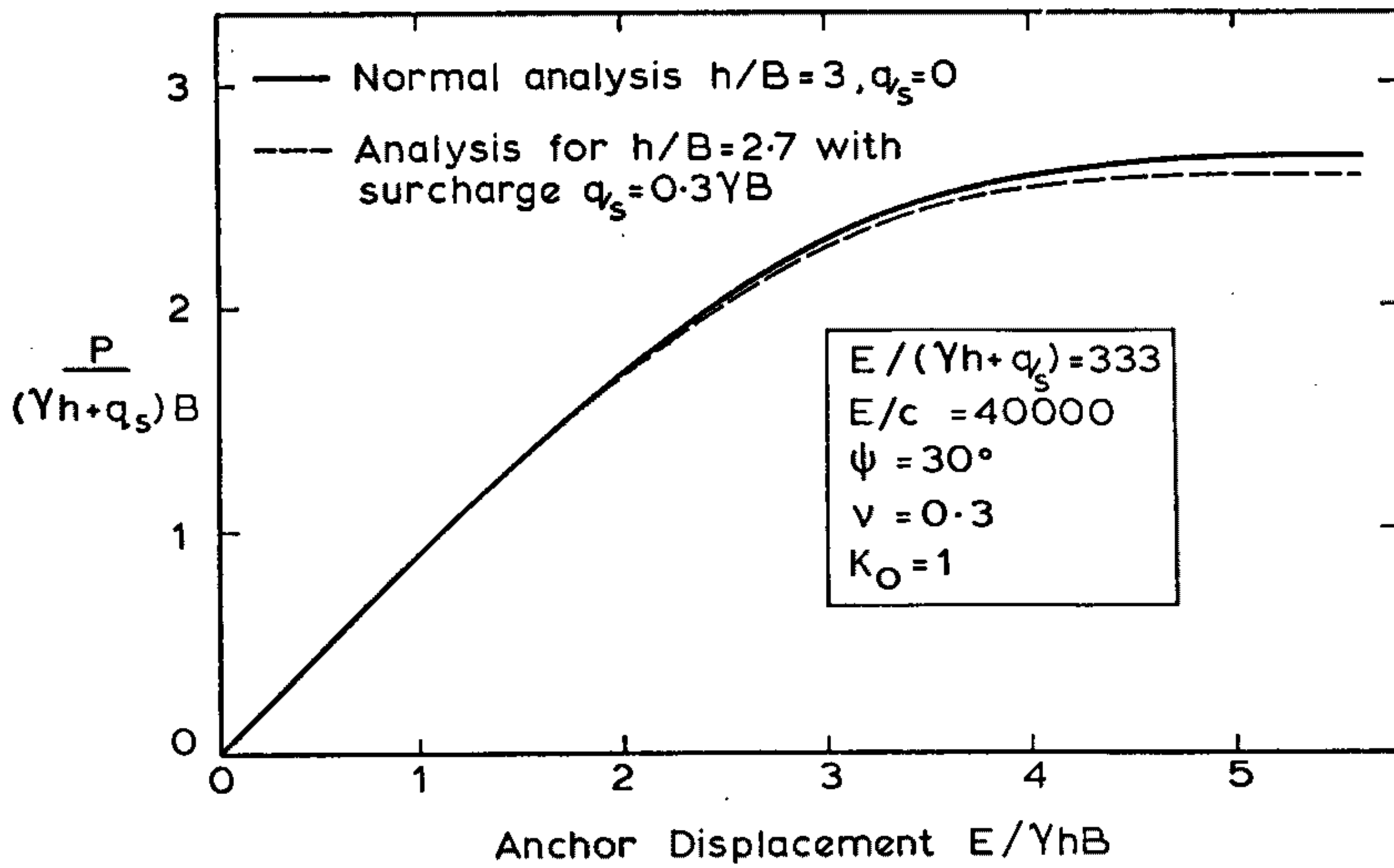


FIG.7.38 EFFECT OF SURCHARGE ON LOAD DISPLACEMENT CURVE

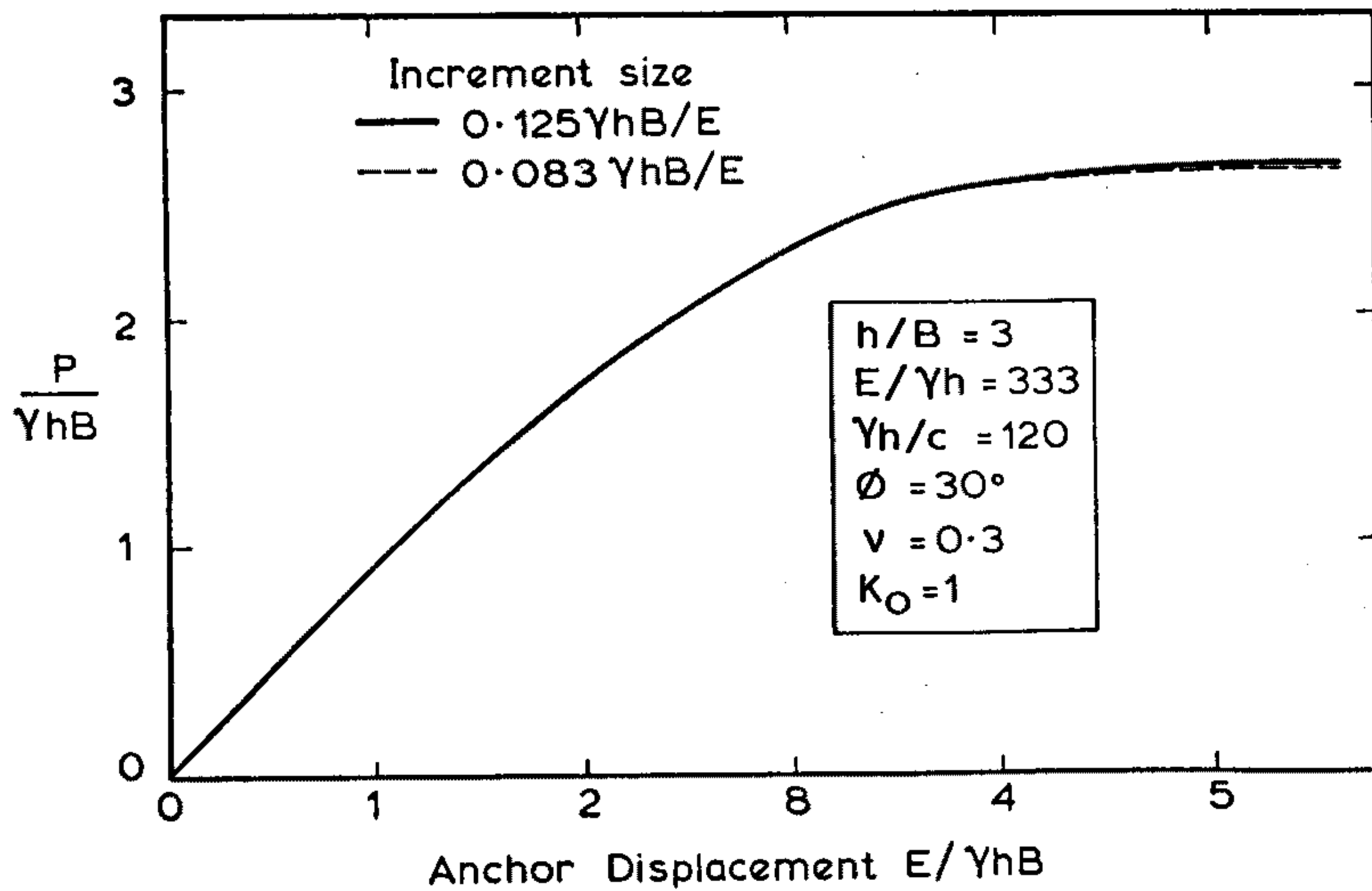


FIG. 7.39 EFFECT OF INCREMENT SIZE ON LOAD DISPLACEMENT CURVE

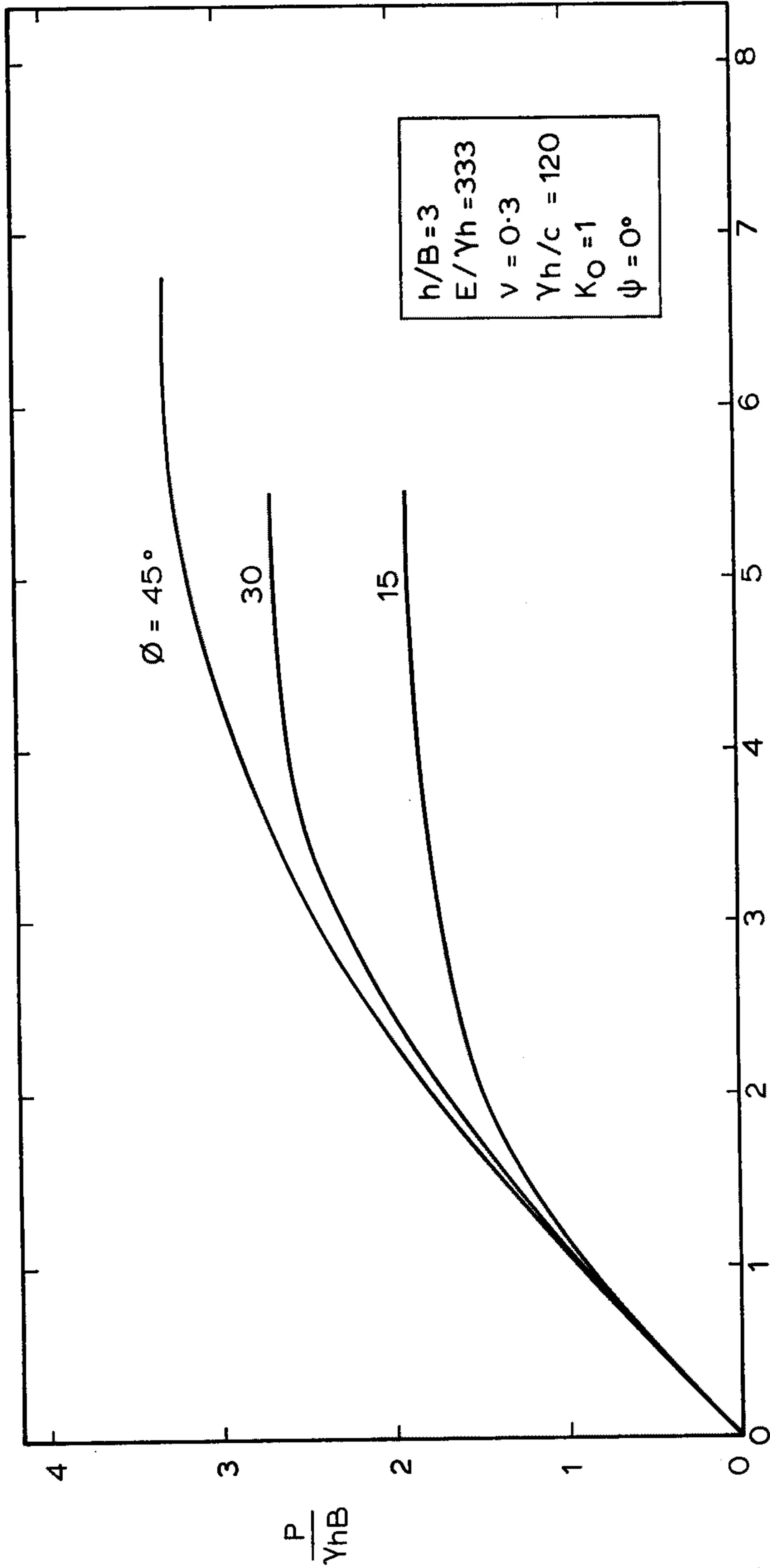
and $0.083 B\gamma h/E$. From the load displacement curves shown in Fig. 7.39 it can be seen that this 33% reduction in step size altered the collapse load by less than 1% and hence the selected increment of $.125 B\gamma h/E$ appears to be adequate.

7.3.3.2 Effect of ϕ for $h/B = 3$

Fig. 7.40 shows the load displacement curves obtained for an anchor with embedment ratio $h/B = 3$ for the three cases $\phi = 15^\circ$, $\phi = 30^\circ$ and $\phi = 45^\circ$. The extent of the plastic region and the velocity field at collapse are shown in Figs. 7.41 and 7.42 respectively. For comparison, the plastic region at collapse for $\phi = 0^\circ$ is also shown in Fig. 7.41.

From the load displacement curves, it will be observed that collapse is clearly defined in each case and is preceded by an elasto-plastic region of very limited extent. Correspondingly, the plastic regions and velocity fields can be seen to extend to the surface indicating shallow anchor behaviour.

The extent of the elasto-plastic region at collapse changes quite markedly with increasing friction angle. Thus, in a purely cohesive material ($\phi = 0^\circ$), failure is associated mainly with material flow around the anchor, (see for example Fig. 7.9) however, this tendency decreases with increasing friction angle. For $\phi = 15^\circ$ there is quite extensive plastic failure both above and below the anchor, while the velocity field shows significant flow behind the anchor. With increasing ϕ the plastic region shrinks until, for, $\phi = 45^\circ$, it is restricted to a fairly narrow band of material above and below the edge



Anchor Displacement $E/\gamma h B$

FIG. 7.40 LOAD DISPLACEMENT CURVES FOR ANCHOR IN A FRICTIONAL MATERIAL
WITH WEIGHT

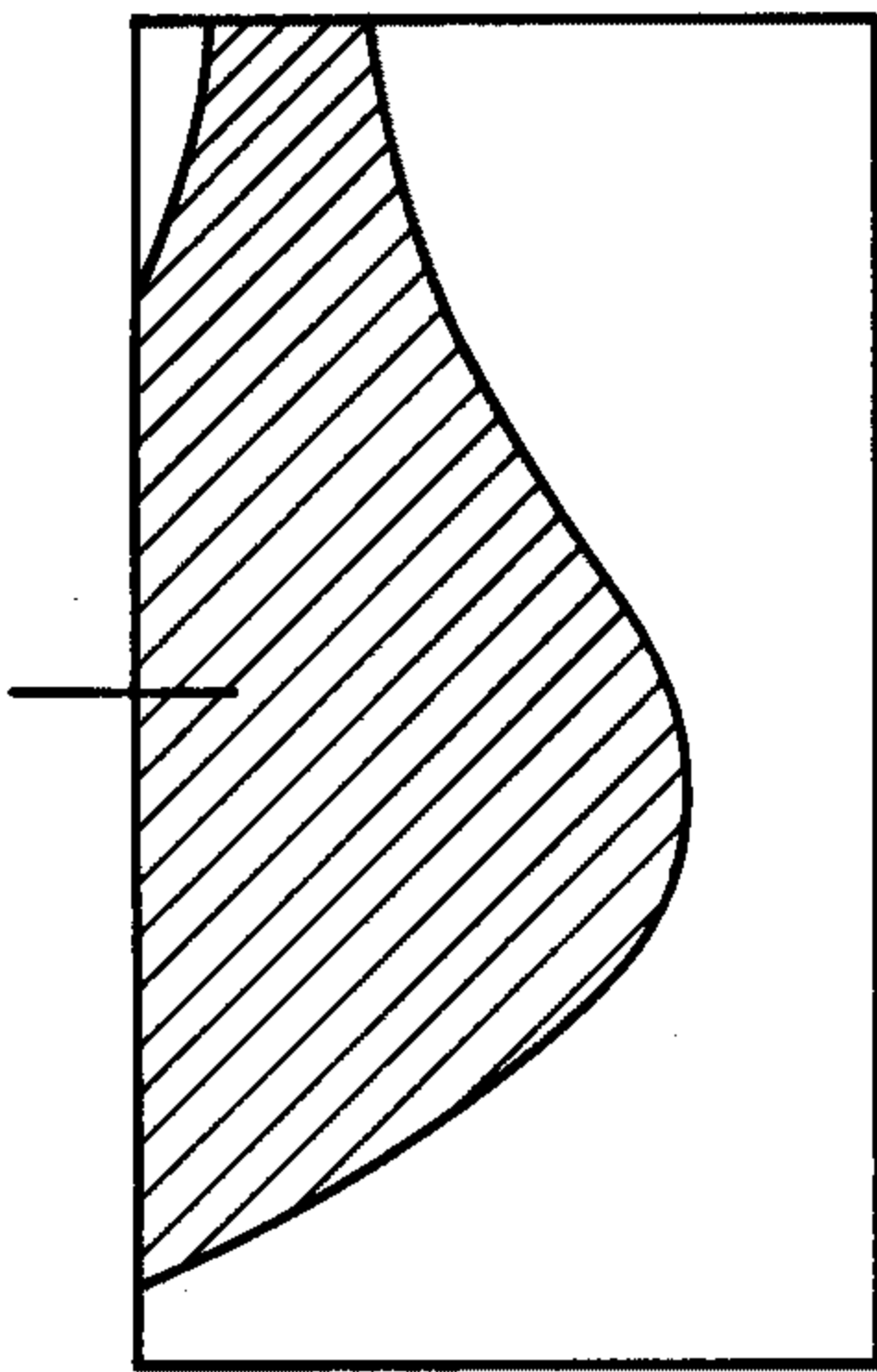
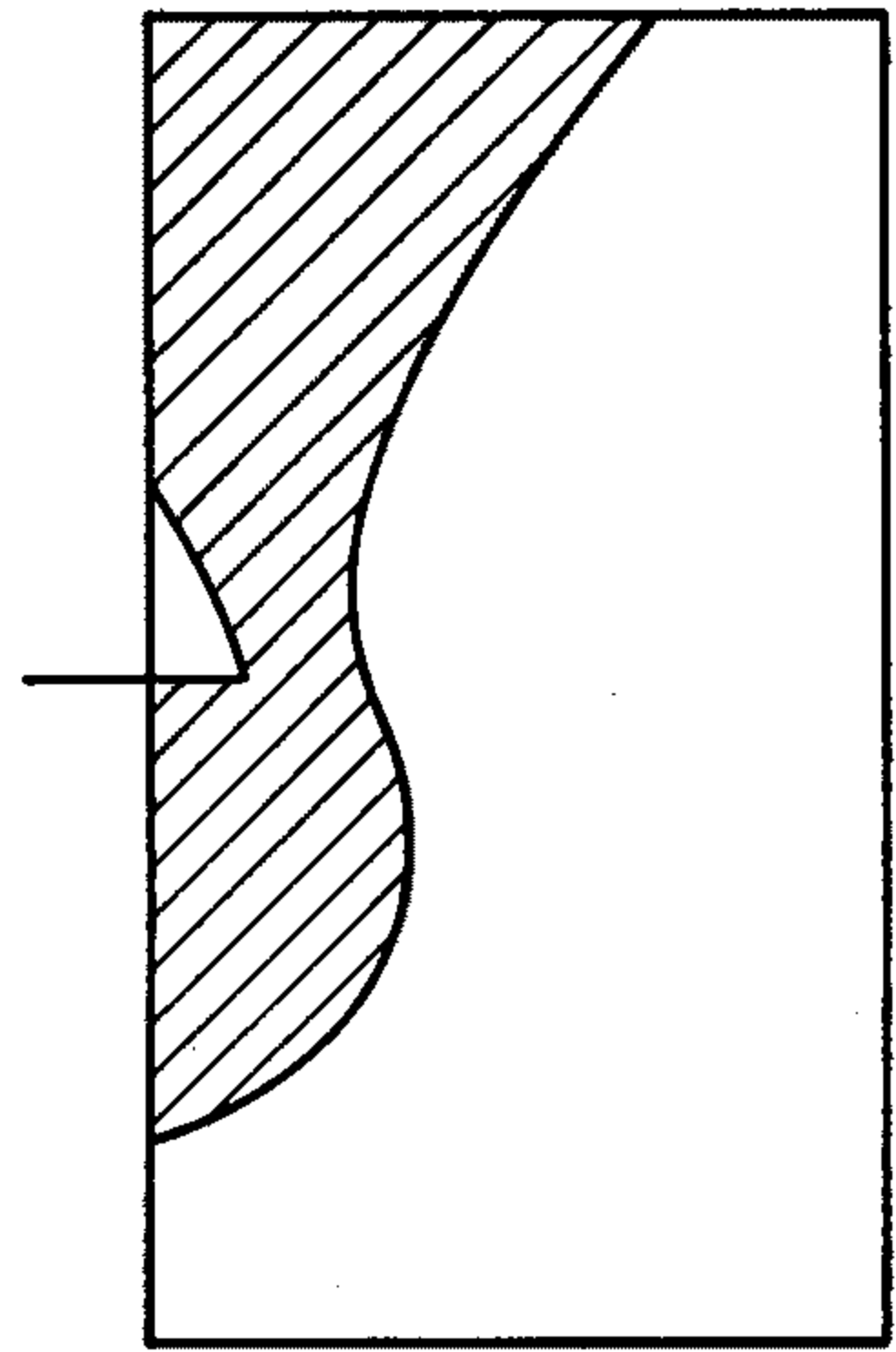
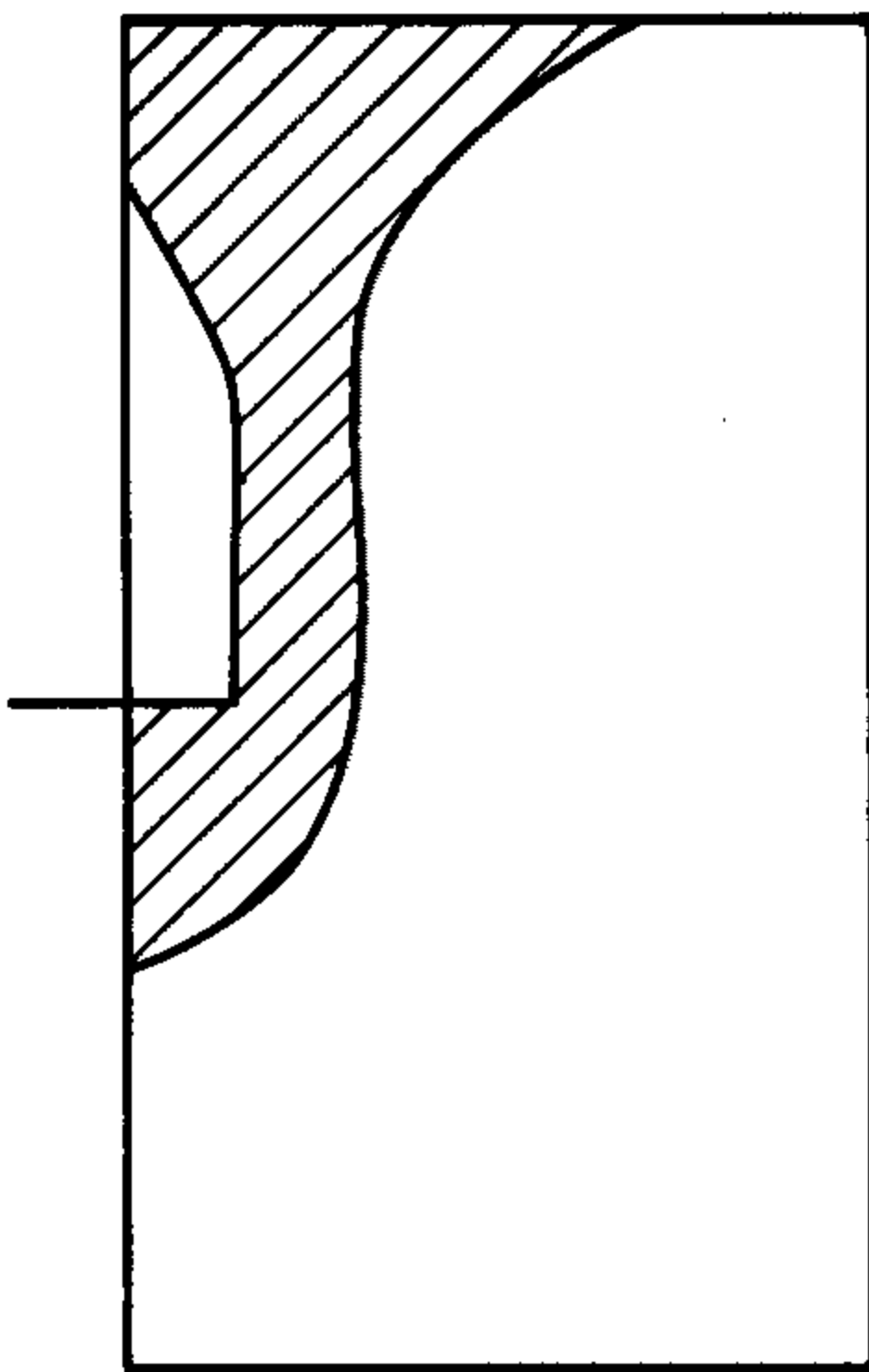
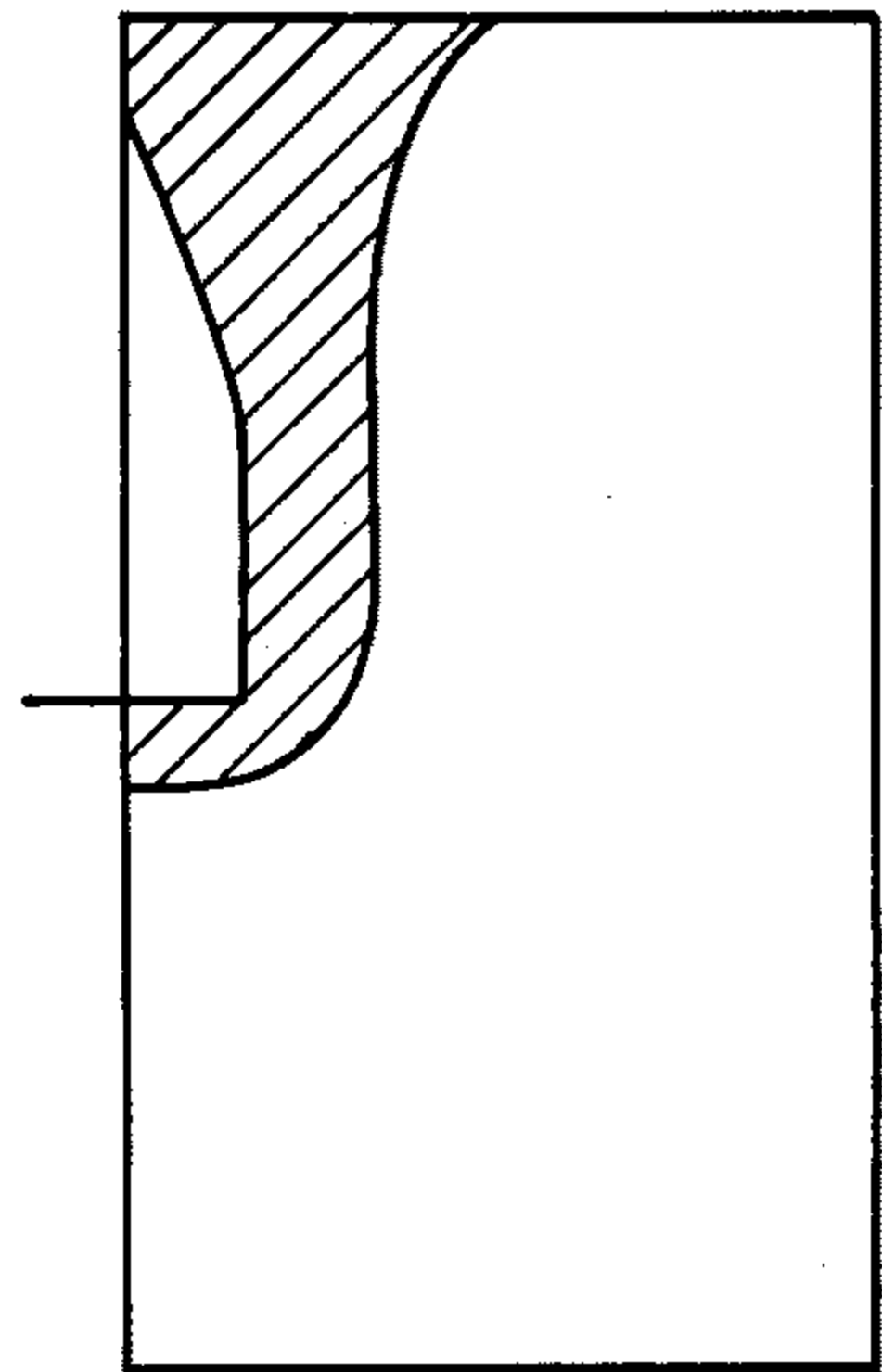

 $\emptyset = 0^\circ$

 $\emptyset = 15^\circ$

 $\emptyset = 30^\circ$

 $\emptyset = 45^\circ$

FIG.7·41 PLASTIC REGIONS AT COLLAPSE - VARIOUS $\emptyset$
 $h/B = 3, E/c = 40000, \gamma h/c = 120, \psi = 0^\circ$

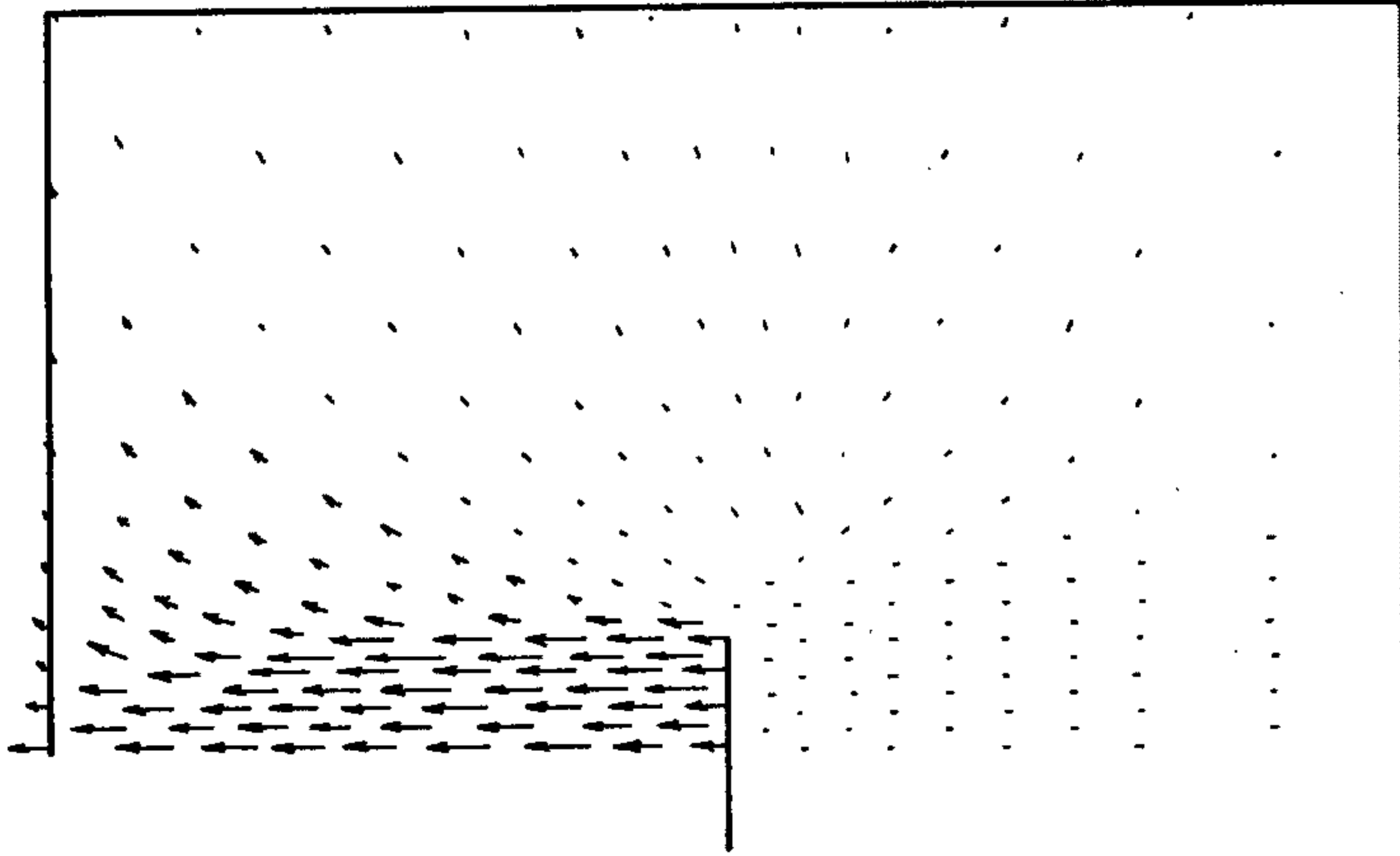
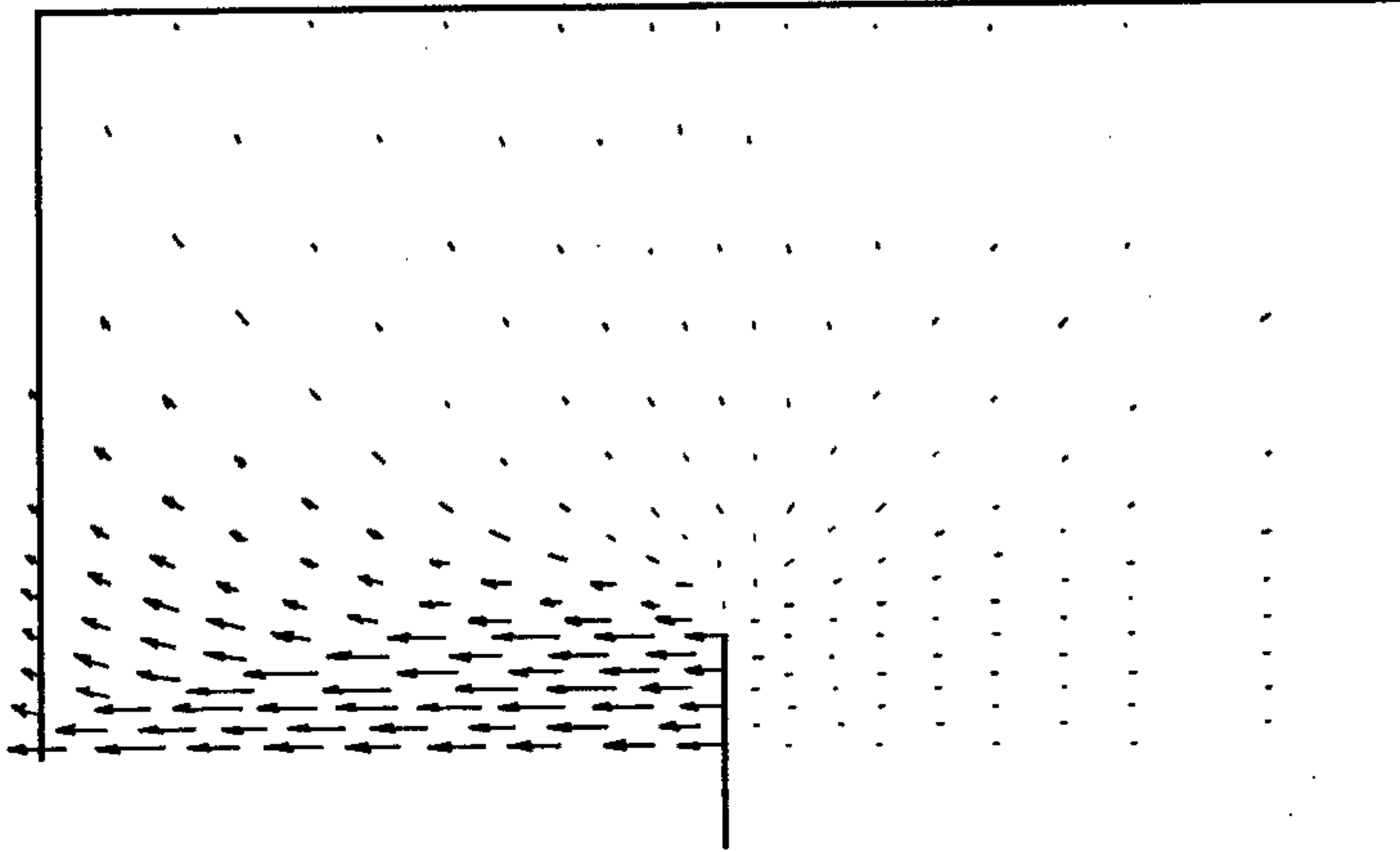
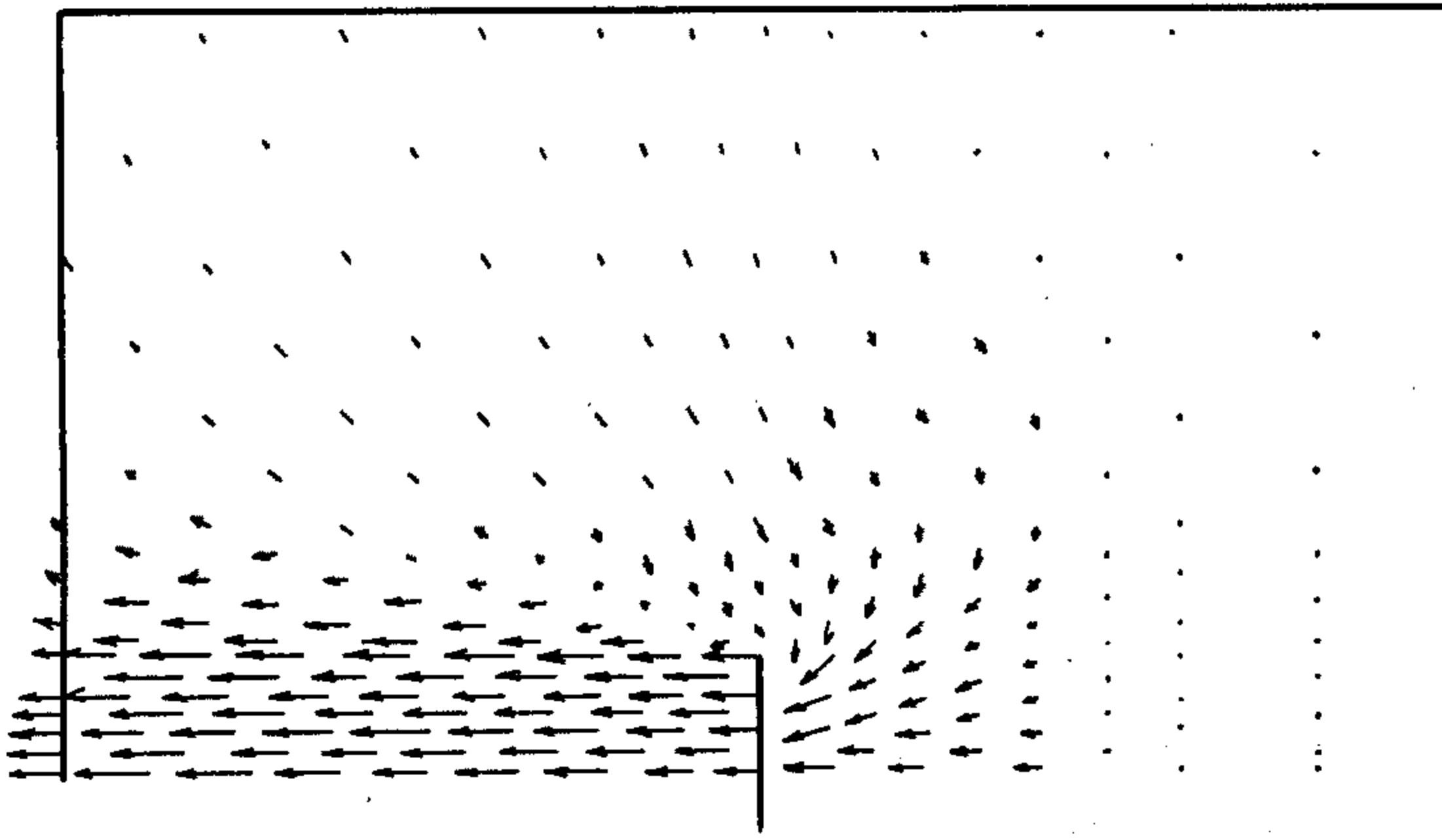

 $\varnothing = 45^\circ$

 $\varnothing = 30^\circ$

 $\varnothing = 15^\circ$

FIG.7.42 VELOCITY FIELD AT COLLAPSE $h/B=3$, VARIOUS $\varnothing$, $\psi=0$, $\gamma h/c=120$

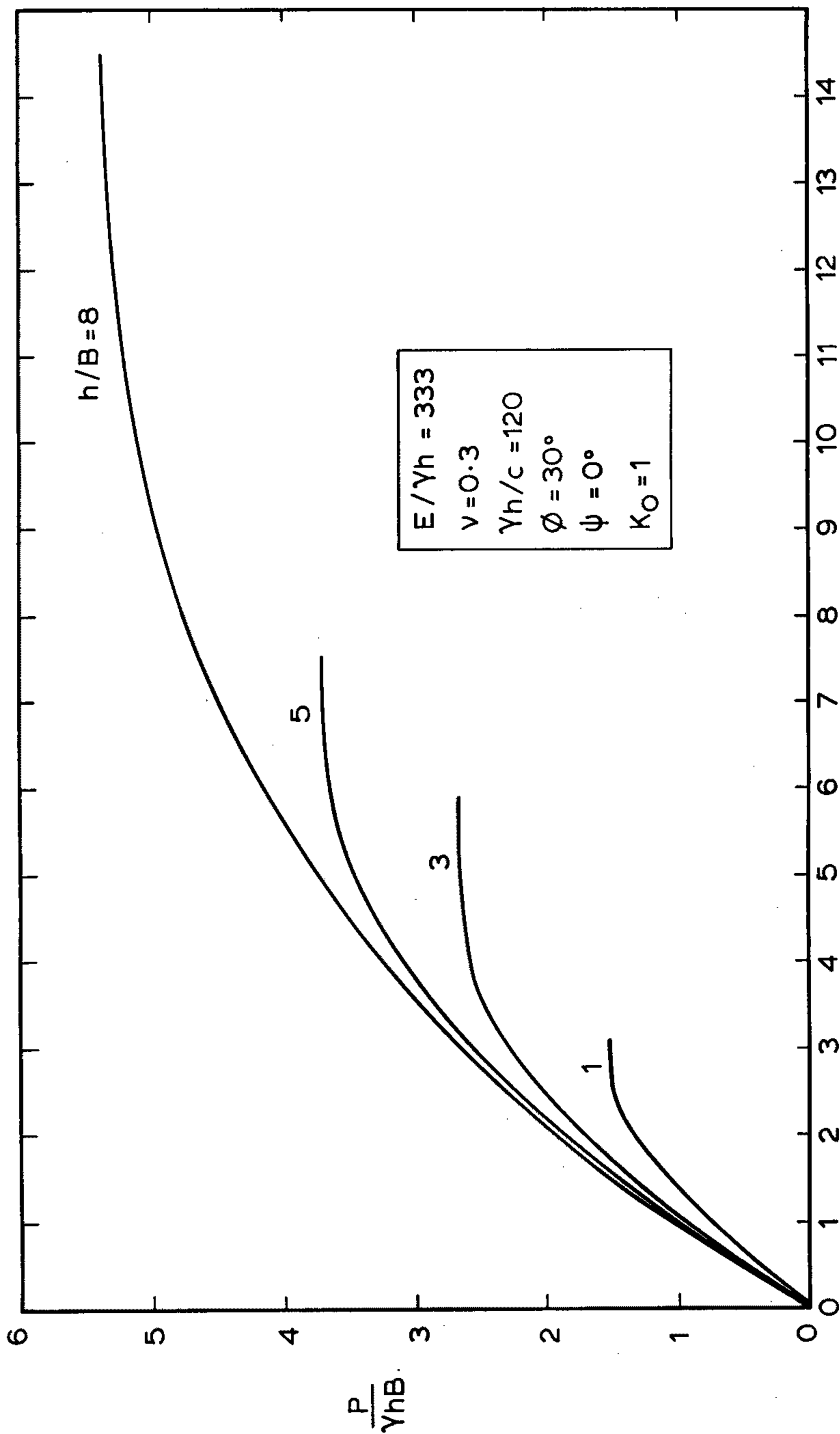
of the anchor. Similarly, the flow around the anchor decreases with increasing ϕ and is non-existent for $\phi = 45^\circ$.

7.3.3.3 Effect of the Embedment Ratio ($\frac{h}{B}$)

The load-displacement curves for an anchor with various embedment ratios resting in a soil with properties $E/\gamma h = 333$, $\nu = 0.3$, $\phi = 30^\circ$ and $\gamma h/c = 120$ are shown in Fig. 7.43. The extent of the plastic region and the velocity fields at failure are shown in Figs. 7.44 and 7.45 respectively.

It will be observed that in each case collapse is clearly defined and is associated with plastic failure within a band of material extending from the edge of the anchor to the surface. As the embedment ratio increases the extent of the plastic zone, particularly below the anchor, increases and very early signs of flow around the anchor were noticeable for embedment ratios $h/B = 5$ and 8. However, in both cases failure is still quite clearly that of a shallow anchor.

The bearing capacity factor F_γ is shown in Fig. 7.46 as a function of embedment ratio h/B and the angle of internal friction ϕ . It can be seen that the bearing capacity factor is very nearly a linear function of the embedment ratio for the entire range of embedment ratios considered. The existence of a linear relationship has been previously predicted by Meyerhof and Adams (1968) and Vesic (1971) although, in general, the actual values of F_γ shown in Fig. 7.46 differ from those predicted by either of these authors.



Anchor Displacement $E/B\gamma h$

FIG.7.43 LOAD DISPLACEMENT CURVES FOR ANCHOR IN A FRICTIONAL MATERIAL WITH
WEIGHT - $\phi = 30^\circ$

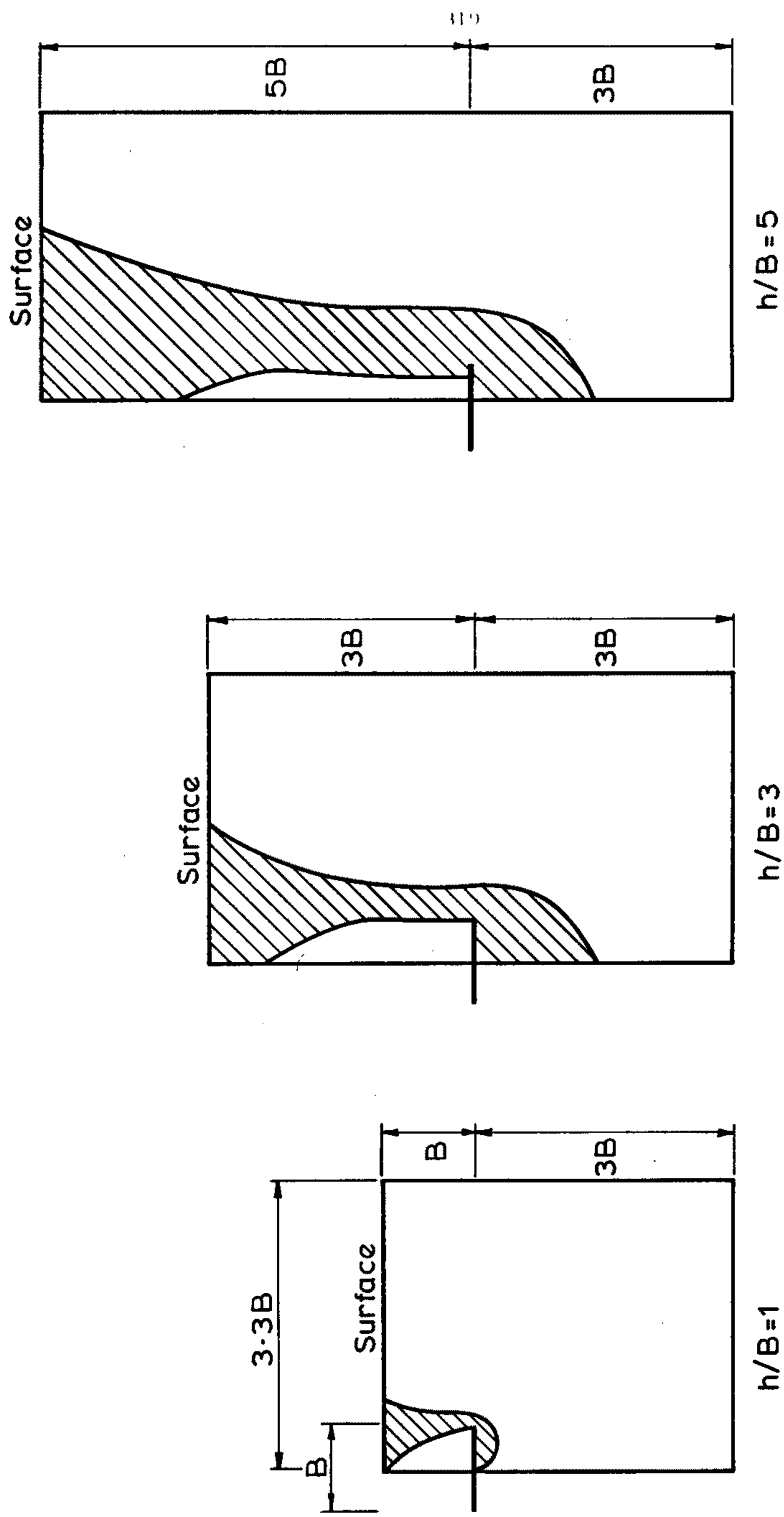


FIG.7.44 PLASTIC REGIONS AT COLLAPSE - VARIOUS h/B , $\phi = 30^\circ$, $\psi = 0^\circ$, $\gamma_h = 120$

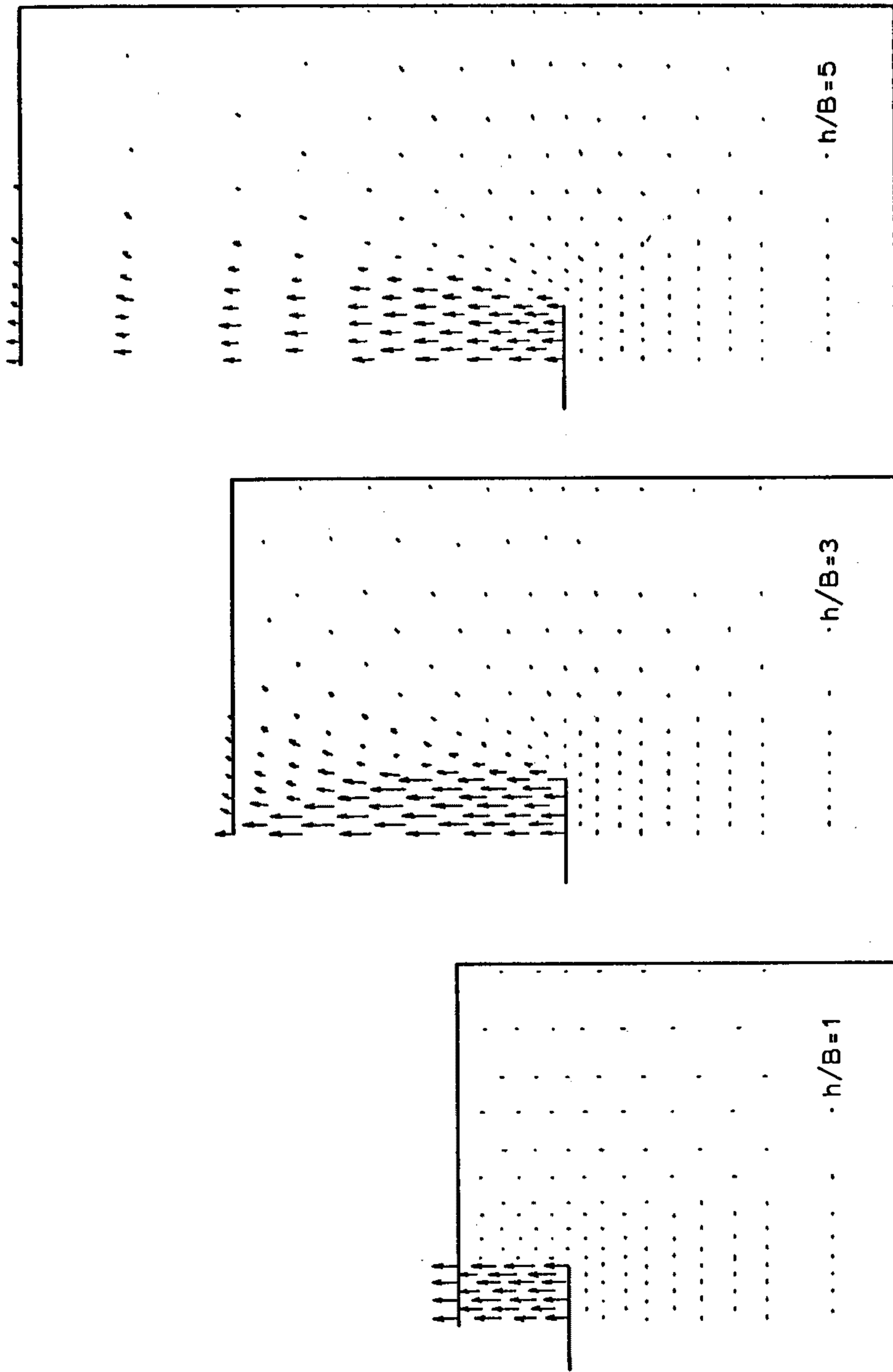


FIG.7.45 VELOCITY FIELD AT COLLAPSE - VARIOUS h/B , $\phi = 30^\circ$, $\psi = 0^\circ$, $\gamma h = 120$, $E = 400000$

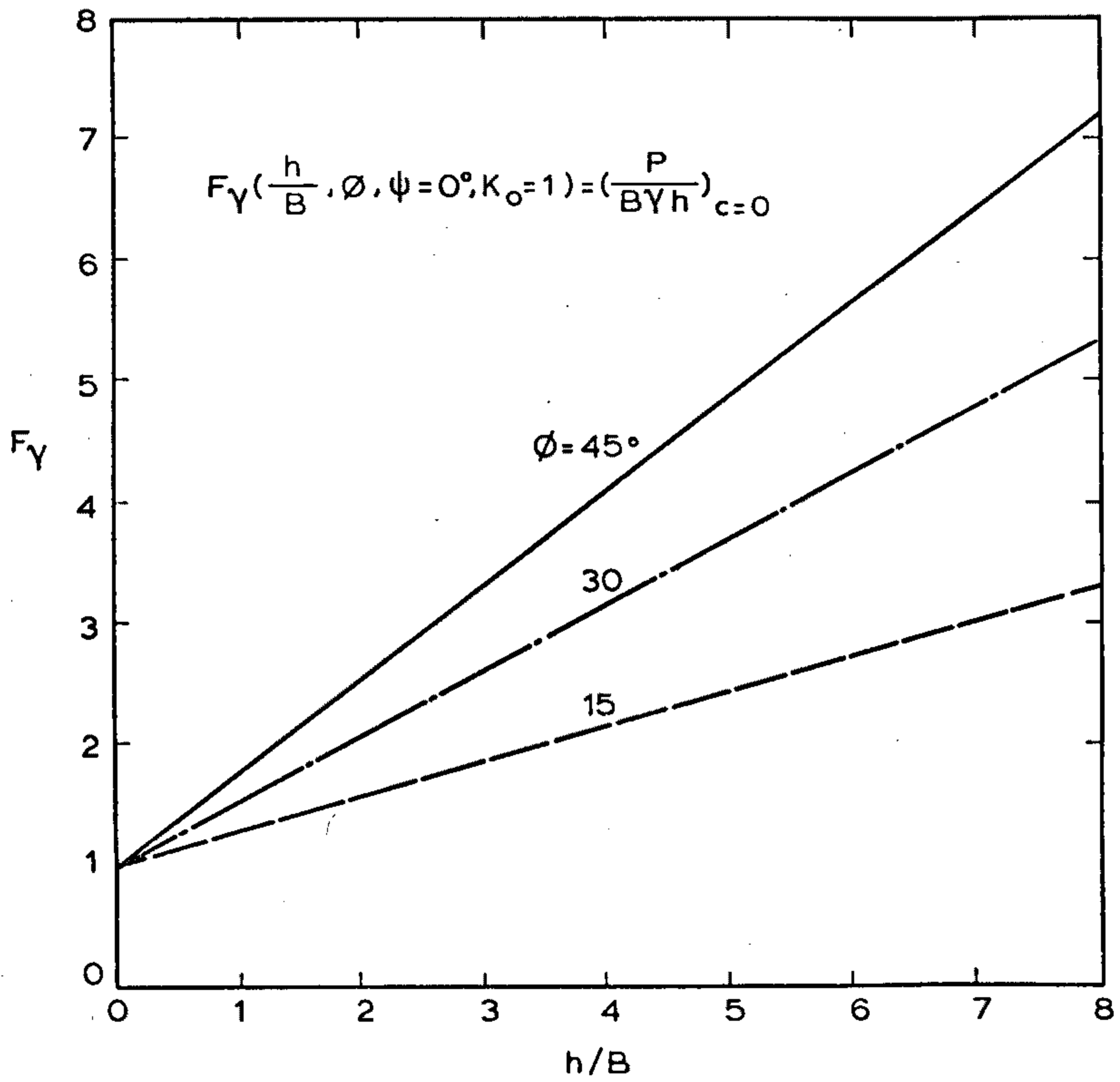


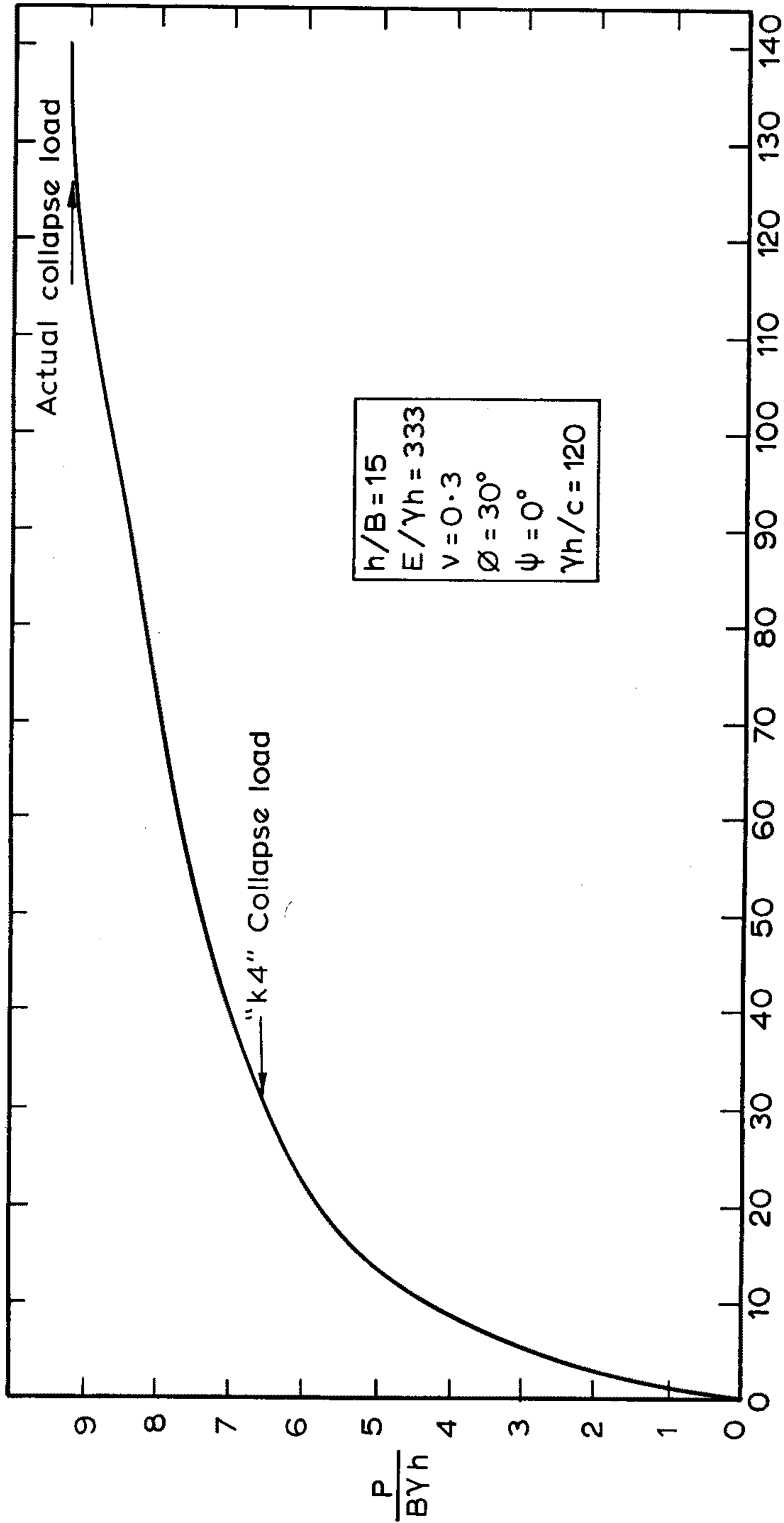
FIG. 7-46 BEARING CAPACITY FACTOR F_Y .

The load displacement curve obtained from an analysis of an anchor with embedment ratio $h/B = 15$ and $\phi = 30^\circ$ is shown in Fig. 7.47. The value of F_γ calculated from this analysis is found to be consistent with the linear relationship shown in Fig. 7.46, indicating that (for $\phi = 30^\circ$) deep anchor behaviour is not apparent for embedment ratios less than fifteen. The velocity fields and growth of the plastic region were also found to indicate shallow anchor behaviour. Very large deformations were, however, observed and the displacement at collapse was almost thirteen times that predicted by an elastic analysis for the same load. The practical 'k4' failure load was found to be $F_\gamma = 6.49$, which is considerably lower than the actual collapse value of $F_\gamma = 9.07$.

7.3.3.4 Summary of Results for a Frictional Material with Weight

The variations in bearing capacity factor F_γ with angle of internal friction ϕ and embedment ratio h/B are summarised in Fig. 7.48 for embedment ratios less than or equal to eight. It has been found that for a given ϕ , the bearing capacity increases almost linearly with embedment ratio over the range of values considered.

It is of interest to note that, for a weightless, cohesive, frictional material, the embedment ratio at which deep anchor behaviour is observed increases with increasing friction angle, while in a purely frictional material with weight, deep anchor behaviour is not observed at all for embedment ratios less than or equal to eight. Thus it may be concluded that for frictional materials the



Anchor Displacement E/BYh

FIG.7-47 LOAD DISPLACEMENT CURVE FOR ANCHOR WITH $h/B=15, \phi=30^\circ$

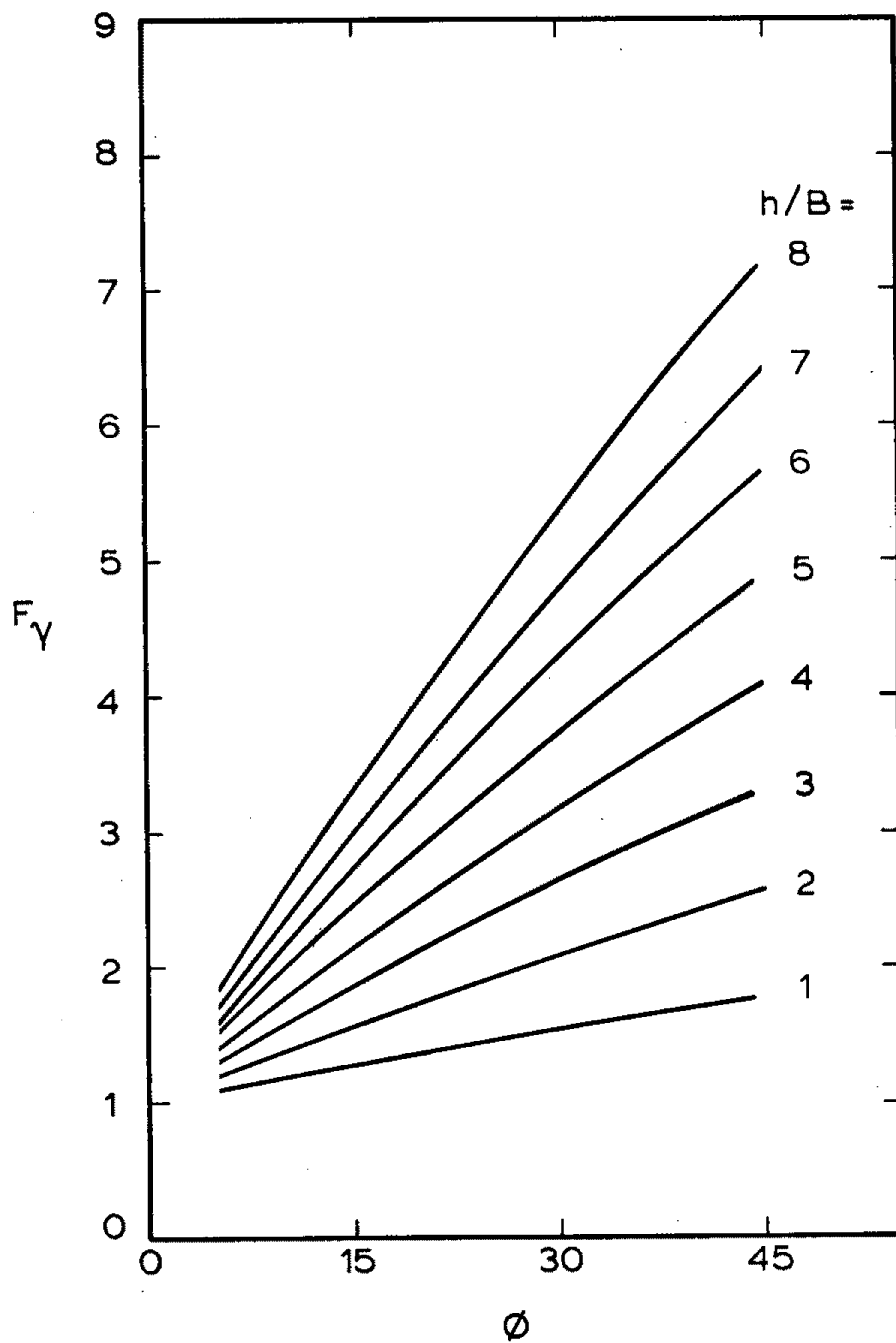


FIG. 7-48 VARIATION OF F_γ WITH ϕ

embedment ratio required to ensure deep anchor behaviour increases with both ϕ and $\gamma h/c$. Despite this, however, the very large displacement observed prior to collapse for an anchor with a high embedment ratio ($h/B = 15$, $\phi = 30^\circ$) indicates the need for caution in extrapolating the bearing capacity factors beyond the limits shown. In particular, the result for $h/B = 15$ suggests that it may be necessary to adopt a 'k4' definition of failure for embedment ratios greater than eight since the practical collapse load within this range may be well below that predicted by extrapolation of the results from Fig. 7.46. (For example, for $h/B = 15$, $\phi = 30^\circ$ the 'k4' collapse load is 28% below the actual collapse load.)

7.4 EFFECT OF SECONDARY PARAMETERS

In the previous section, consideration was given to the effects of the parameters h/B , ϕ and $\gamma h/c$ upon the bearing capacity of a smooth rigid strip anchor buried in a soil mass with a coefficient of lateral earth pressure $K_0 = 1$ and angle of dilatancy $\psi = 0^\circ$. In this section, the likely effects of those conditions which are constant in the previous section will be investigated to indicate the extent to which the bearing capacity factors of the previous section can be directly applied.

7.4.1 Effect of Dilatancy

The assumed angle of dilatancy $\psi = 0^\circ$ adopted in the previous section corresponds to a material which deforms plastically with zero volume change. In regard to the surface footing analyses, it has been often found that the flow rule (ie. value of ψ) does not

significantly alter the collapse load (Davis and Booker, 1973). However, this is not the case for buried anchors. Fig. 7.49 shows the bearing capacity factor F_γ determined for a range of embedment ratios h/B for the cases of a non-associated flow rule, where $\psi = 0^\circ$, and for an associated flow rule where $\psi = \phi = 30^\circ$. The coefficient of lateral earth pressure at rest, K_0 , was taken to be one in both cases. It will be observed that the effect of ψ upon the collapse load increases significantly with the embedment ratio. Figs. 7.50 and 7.51 show the extent of the plastic region and the velocity fields at collapse for the cases $\psi = 0^\circ$ and $\psi = \phi = 30^\circ$ for an anchor with embedment ratio $h/B = 5$. For the associated flow rule, the zone of plastic material spreads to a distance more than twice that obtained with $\psi = 0^\circ$, although the zone of plastic material directly above the anchor decreases with increasing ψ . From the velocity fields, it can be seen that, for an associated flow rule, the material above the anchor moves almost as a rigid block towards the surface and involves significant plastic deformation in regions which were essentially elastic for the non-associated flow rule.

Thus it can be seen that, for an associated flow rule, the material above the anchor tends to 'lock up' as it attempts to dilate during plastic deformation and it is necessary for the plastic zones to extend well out into the material before there is sufficient freedom for the material above the anchor to move as a rigid body.

The extent of the plastic region necessary to allow plastic collapse increases dramatically with embedment ratio and leads to a correspondingly significant increase in the bearing capacity factor as the embedment ratio increases.

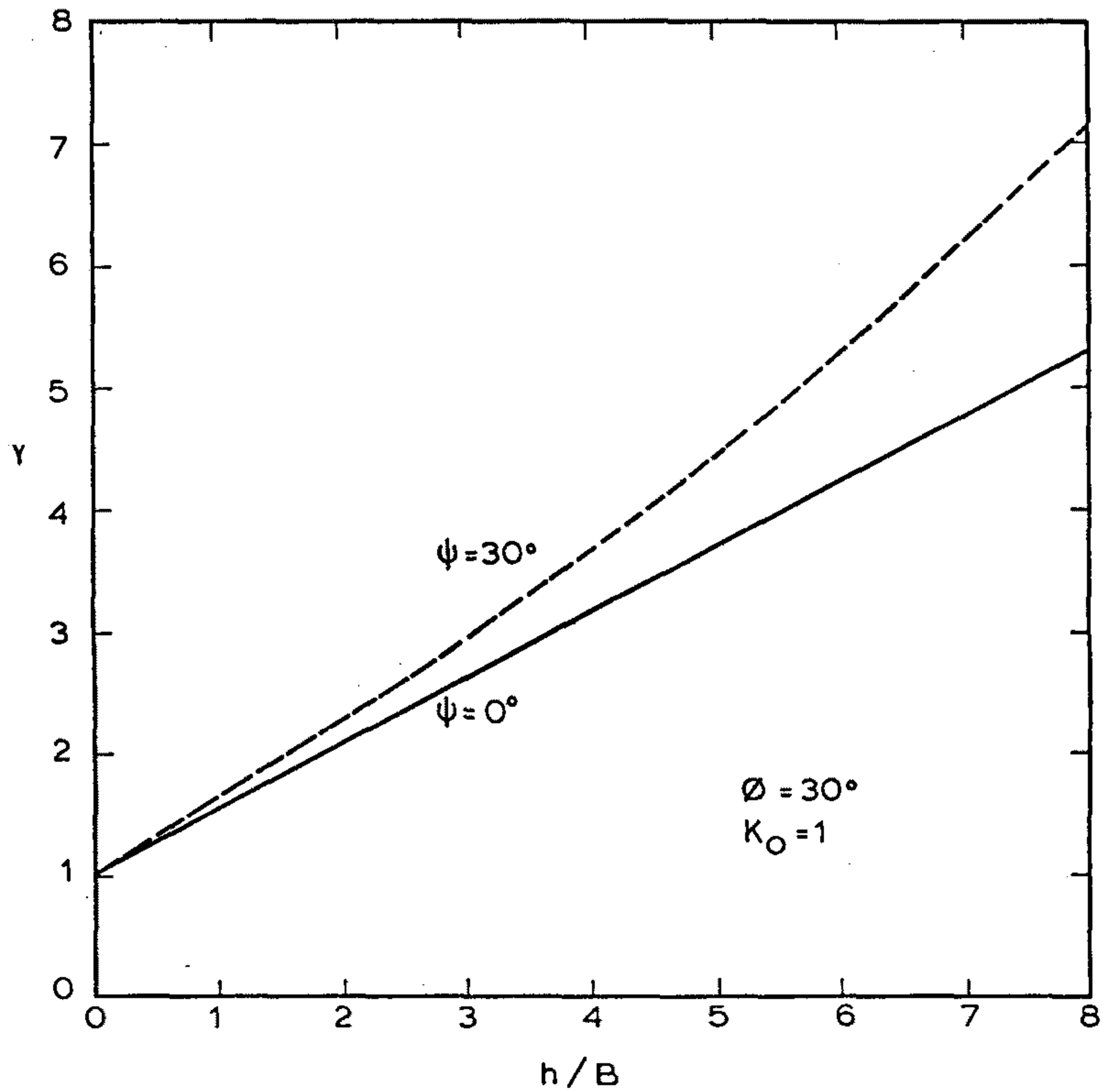


FIG.7.49 EFFECT OF ψ UPON BEARING CAPACITY FACTOR F_γ .

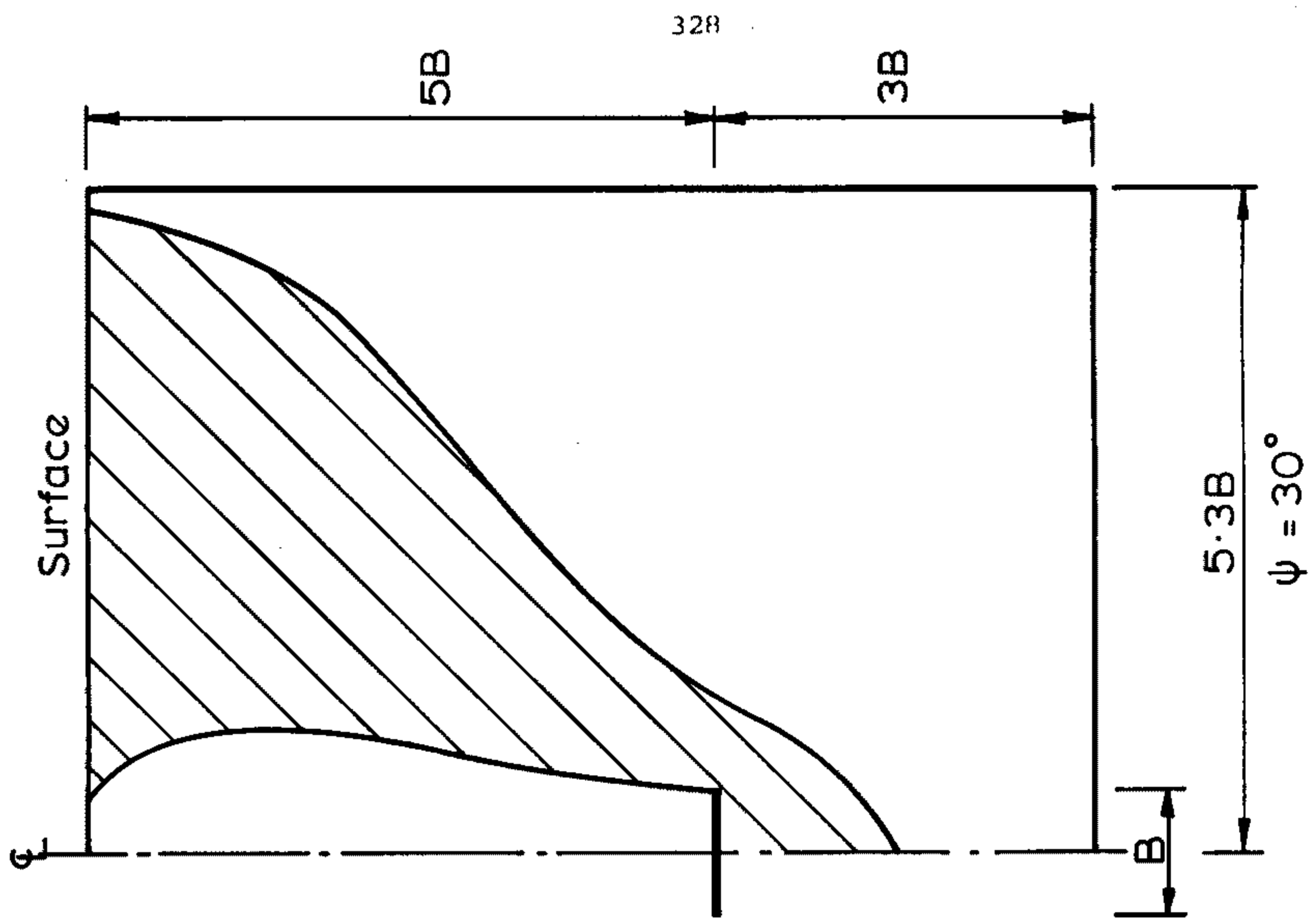
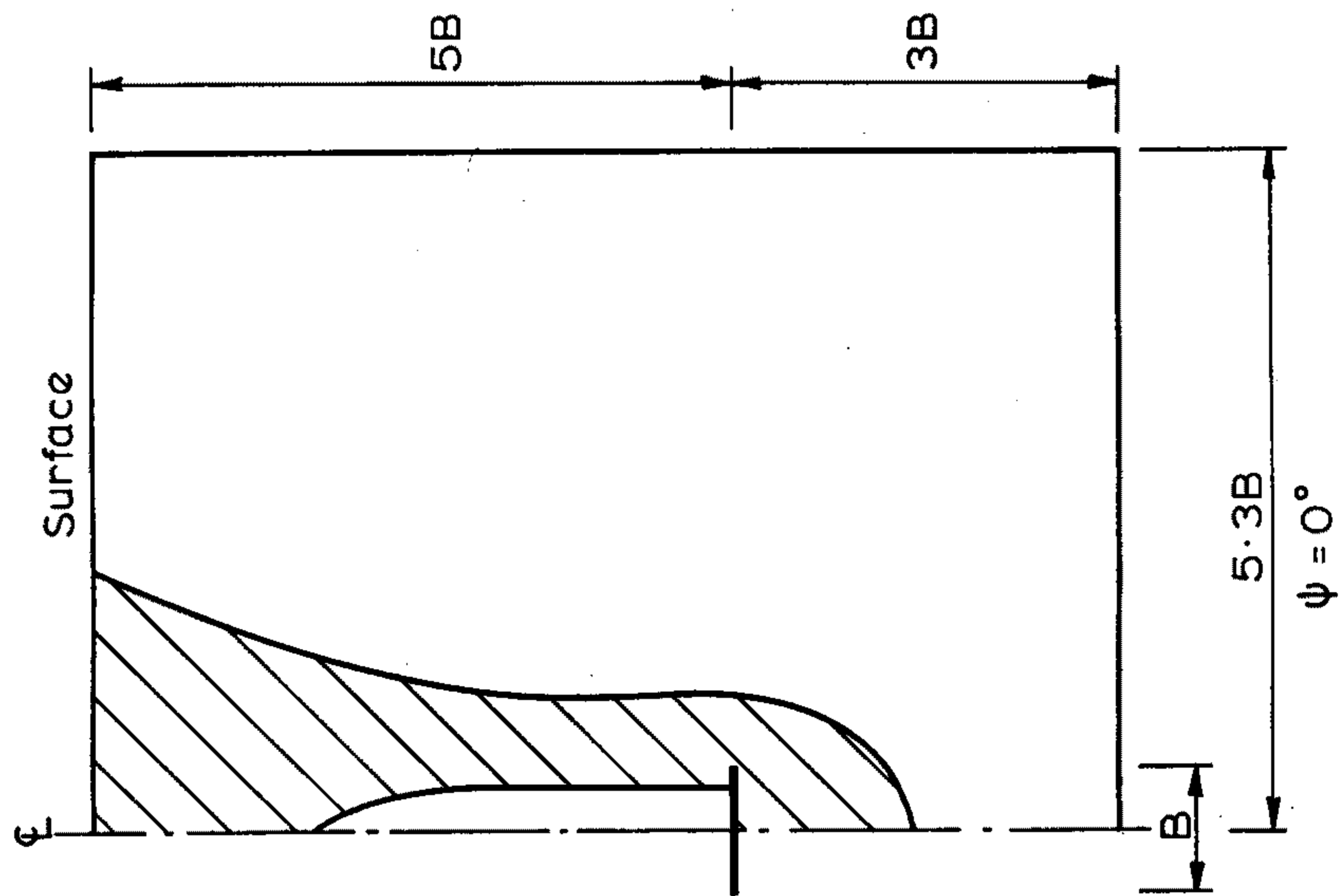


FIG.7.50 PLASTIC REGIONS AT COLLAPSE $h/B = 5$, $\phi = 30^\circ$, $E/\gamma h = 333$ EFFECT OF ψ

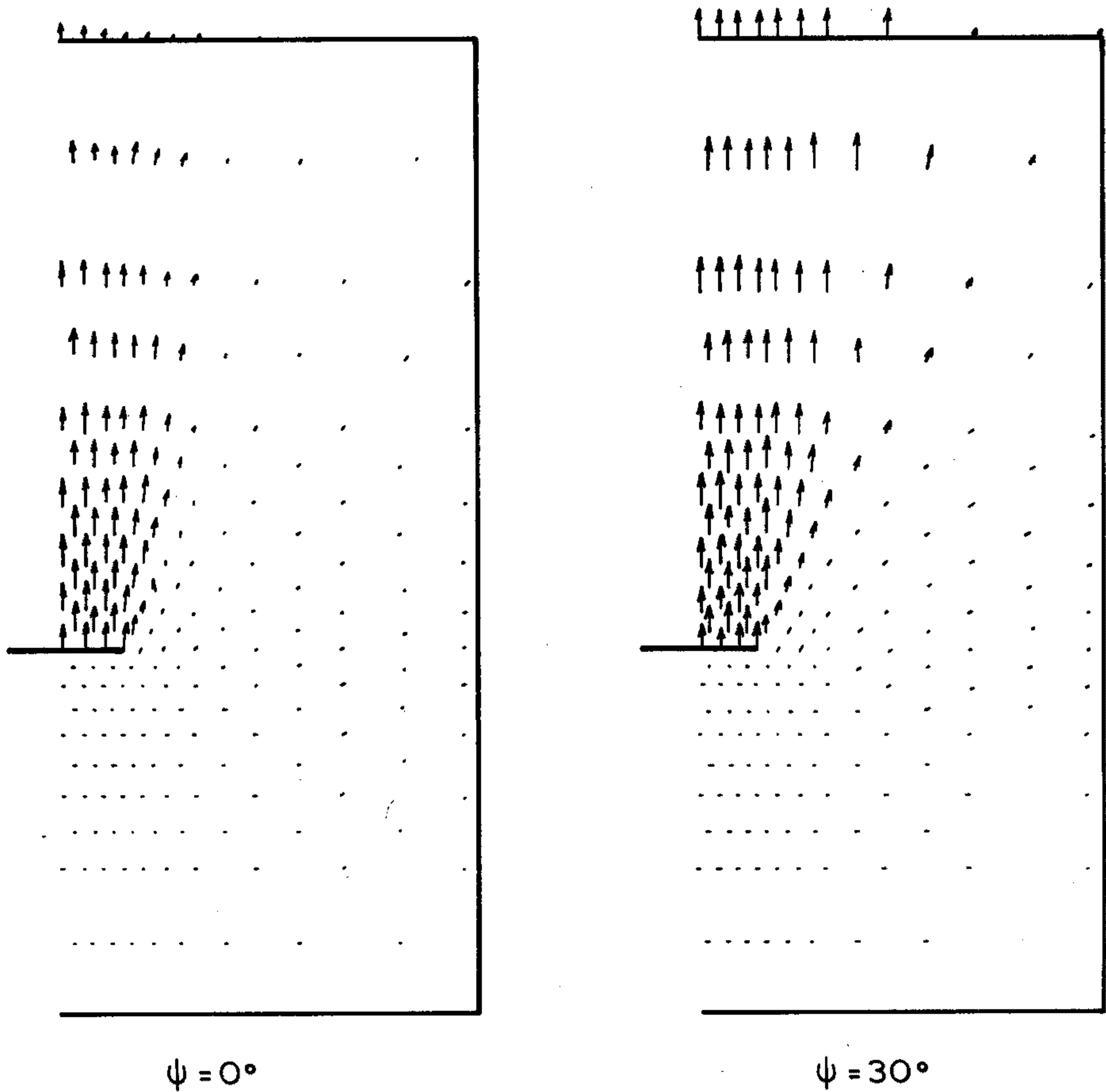


FIG.7.51 VELOCITY FIELD AT COLLAPSE $h/B=5$, $\emptyset=30^\circ$,
 $\gamma_h/c=120$, $E/\gamma_h=333$ EFFECT OF ψ

The increase in anchor capacity due to the effect of $\psi > 0$ can be conveniently expressed in terms of a correction to the basic bearing capacity factor F_γ obtained for $\psi = 0$. Thus the adjusted value F'_γ can be written in the form

$$F'_\gamma = F_\gamma \cdot R_\psi \quad (7.4)$$

where F_γ is the bearing capacity factor previously determined for $\psi = 0^\circ$; and

R_ψ is a correction factor which makes allowance for the effect of ψ .

The value of the correction factor R_ψ is plotted against embedment ratio in Fig. 7.52. It will be observed that when the results for $\psi = \phi$ are expressed in the form given by Eq. 7.4, each of the factors F_γ and R_ψ is a linear function of the embedment ratio. The variations in the correction factor R_ψ with various values of ψ for an anchor with $h/B = 3$ are shown for the case of an associated flow rule, i.e. $\phi = \psi$ and for the case where $\phi = 30^\circ$, in Fig. 7.53(a) and 7.53(b) respectively.

These results indicate that the bearing capacity factor given in section 7.3 for $\psi = 0^\circ$ can be expected to be slightly conservative for sands or silts with an angle of friction less than or equal to 30° and values of ψ less than about 15° . For densely packed gravels and similar materials with $\phi > 30^\circ$ and relatively high angles of dilatancy it would appear that the basic bearing capacity factor F_γ would be quite conservative and at least an approximate estimate of the actual bearing capacity could be obtained by using the cor-

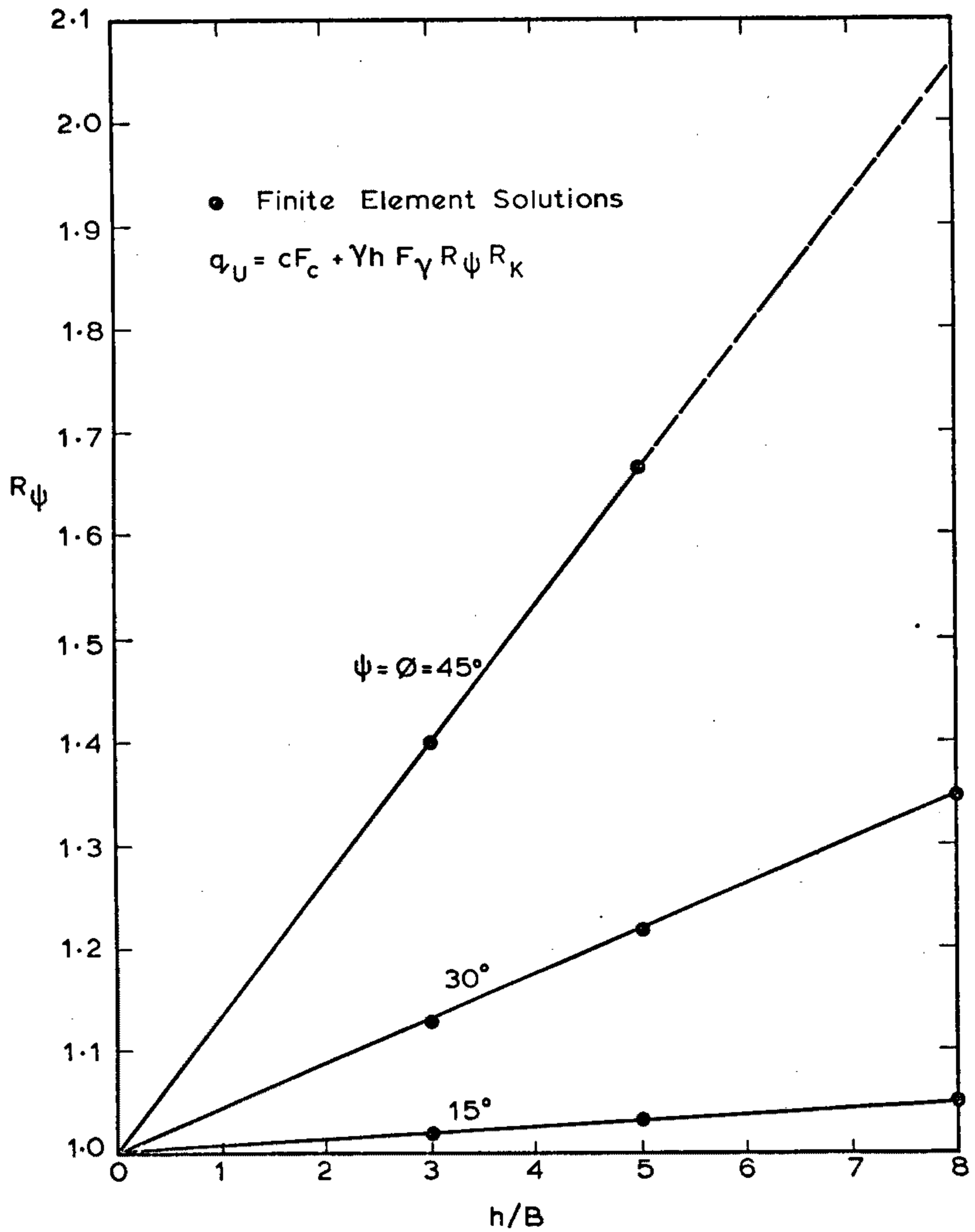
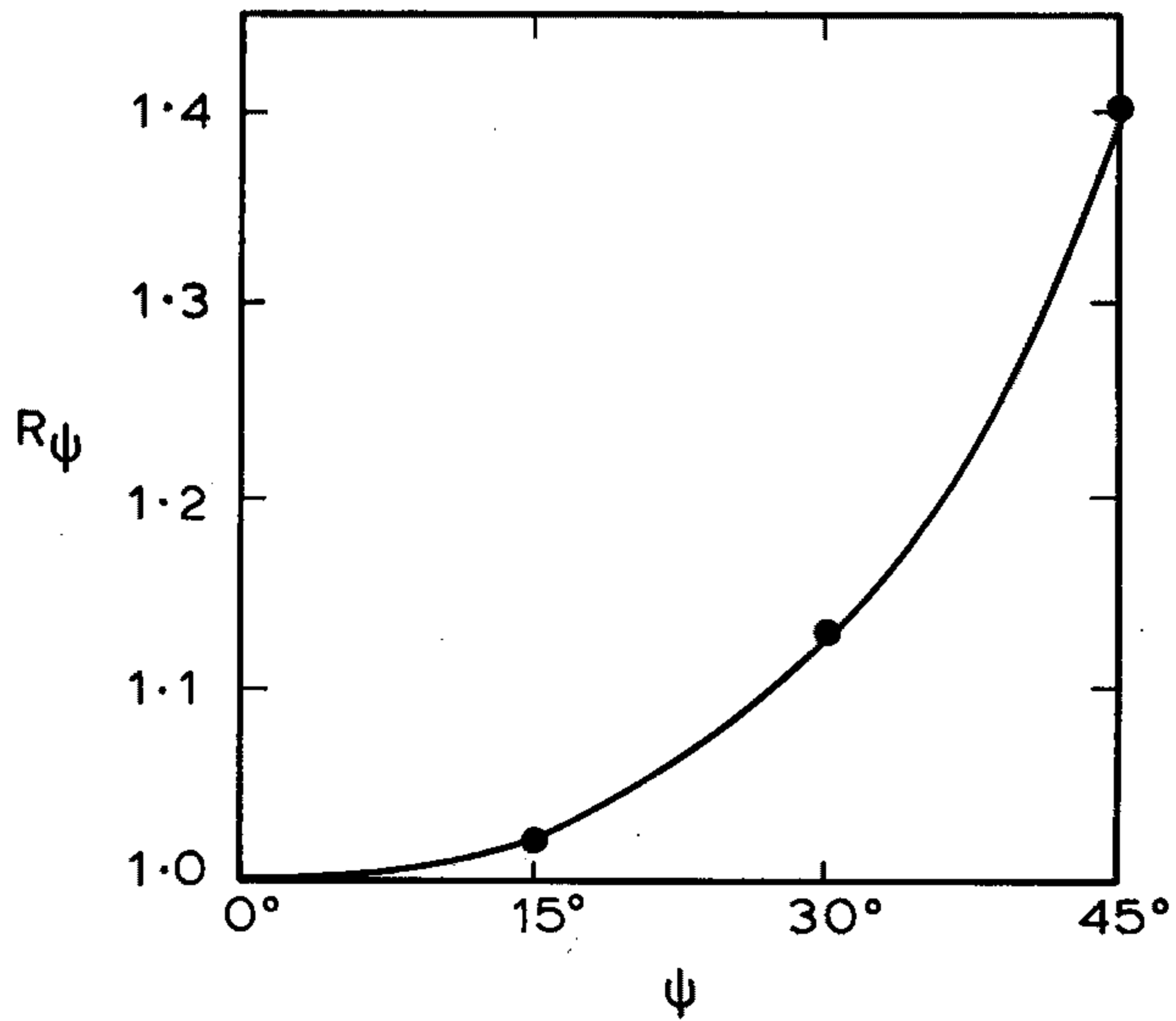
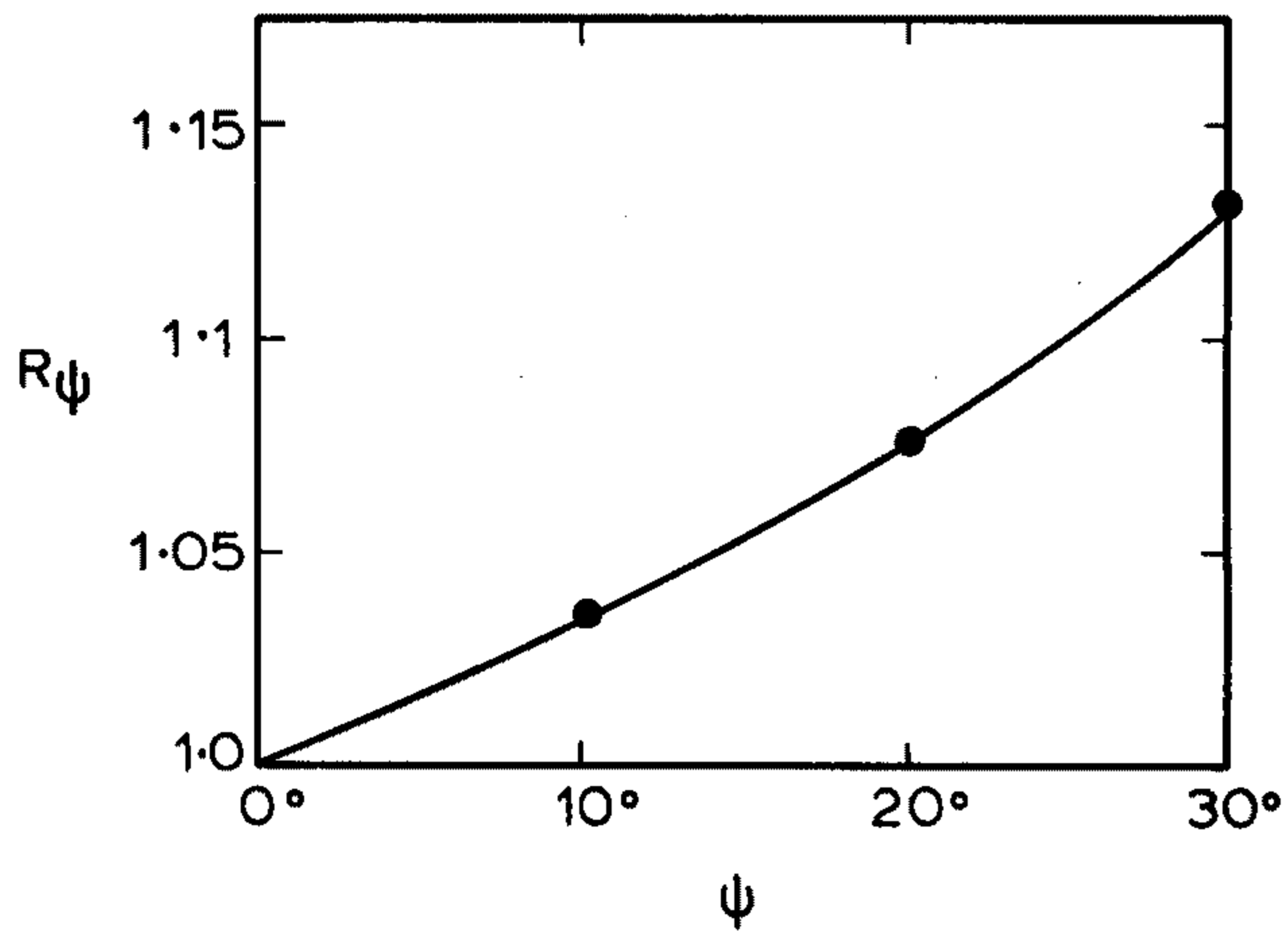


FIG.7.52 CORRECTION FACTOR FOR EFFECT OF DILATANCY
UPON F_γ



(a) Associated Flow Rule $\phi = \psi$



(b) Constant $\phi = 30^\circ$

FIG. 7.53 EFFECT OF ψ UPON R_ψ FOR $h/B = 3$

rection factor R_ψ and Eq. 7.4.

7.4.2 Effect of Initial Stress State

Fig. 7.54 shows a number of load displacement curves for an anchor with embedment ratio $h/B = 3$ and material properties $\phi = 30^\circ$, $E/\gamma h = 333$, $\gamma h/c = 120$, and $\nu = 0.3$. The cases considered fall into two categories; namely, the effect of initial stress state expressed in terms of the coefficient of lateral earth pressures at rest, K_0 , for the values $K_0 = 0.5$, $K_0 = 1$, $K_0 = 2$, and the effect of a non-associated flow rule ($\psi = 0^\circ$) and an associated flow rule ($\psi = \phi = 30^\circ$).

The extent of the plastic regions at collapse at various levels of dimensionless load $P/B\gamma h$ are shown in Fig. 7.55, while the velocity fields at collapse are shown in Figs. 7.56, and 7.57.

Considering, firstly, the case of a material with an associated flow rule, it will be observed that although the load displacement curve is significantly altered by the initial stress condition, the collapse load is the same in each case. This is consistent with the limit theorems of classical plasticity, which imply that the collapse load must be independent of the load path and the initial stress state.

For the case of a material with a non-associated flow rule, there is no unique collapse load, and in fact the collapse load obtained for $K_0 = 0.5$ is less than that obtained for $K_0 = 1$ by approximately 6% while the collapse load obtained for $K_0 = 2$ exceeded that

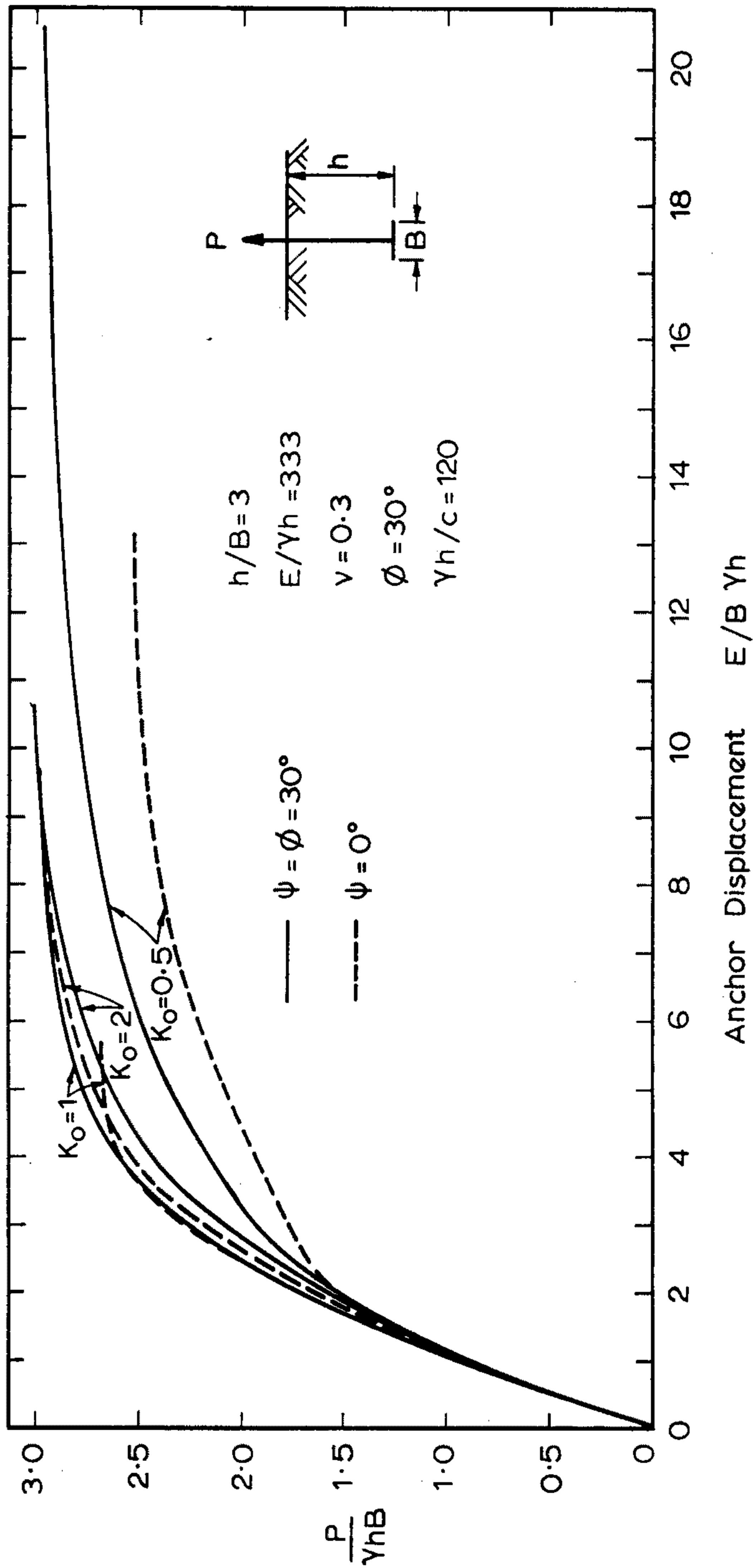
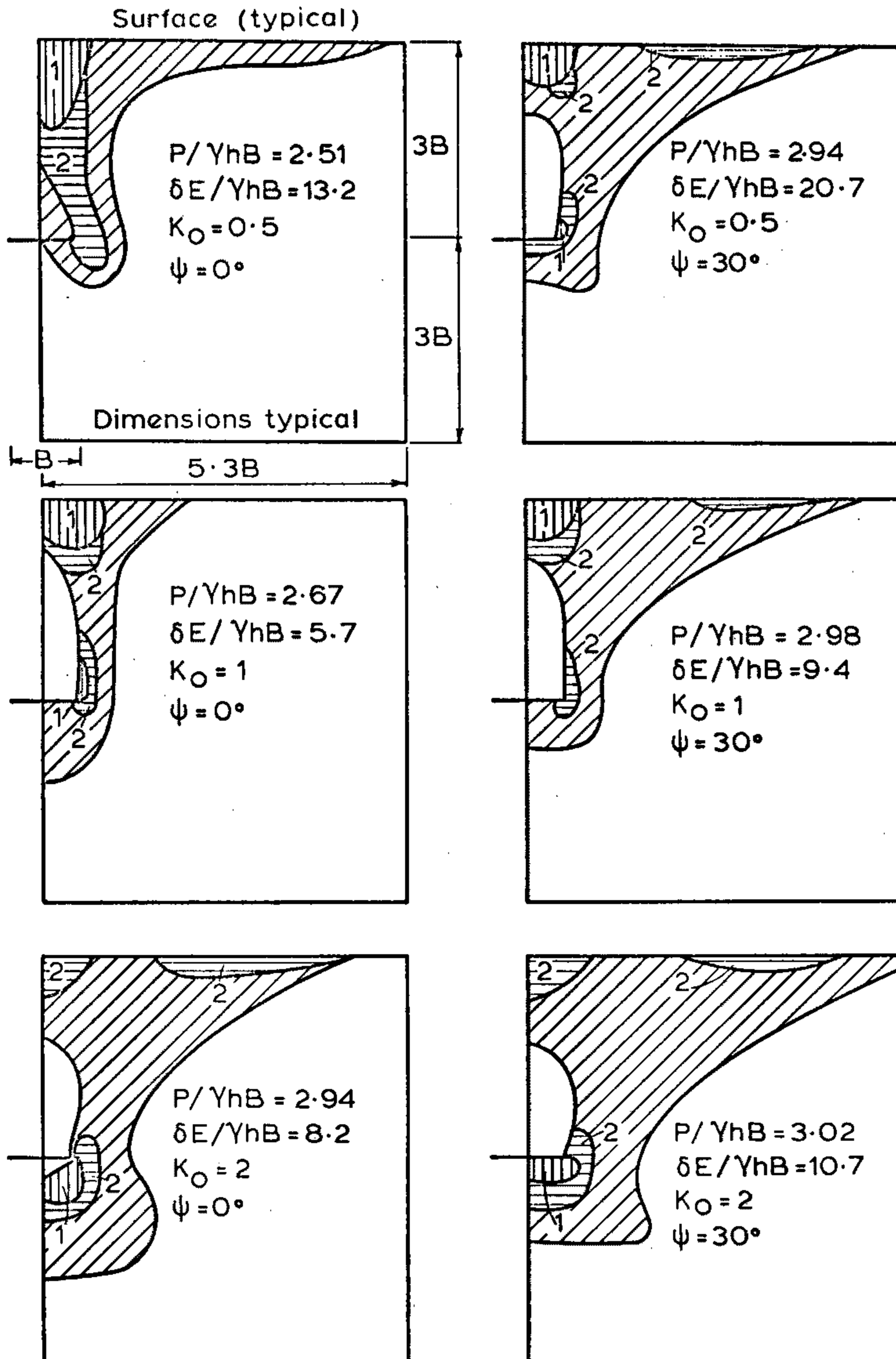


FIG.7.54 EFFECT OF K_o UPON COLLAPSE LOAD $h/B = 3$



N.B. Shaded regions $\parallel$ and $\equiv$ indicate extent of plastic region at load levels $P/YhB=1$ and $P/YhB=2$ respectively

FIG.7.55 EXTENT OF PLASTIC ZONES AT COLLAPSE $h/B=3$,
 $E/Yh=333$, $\phi=30^\circ$

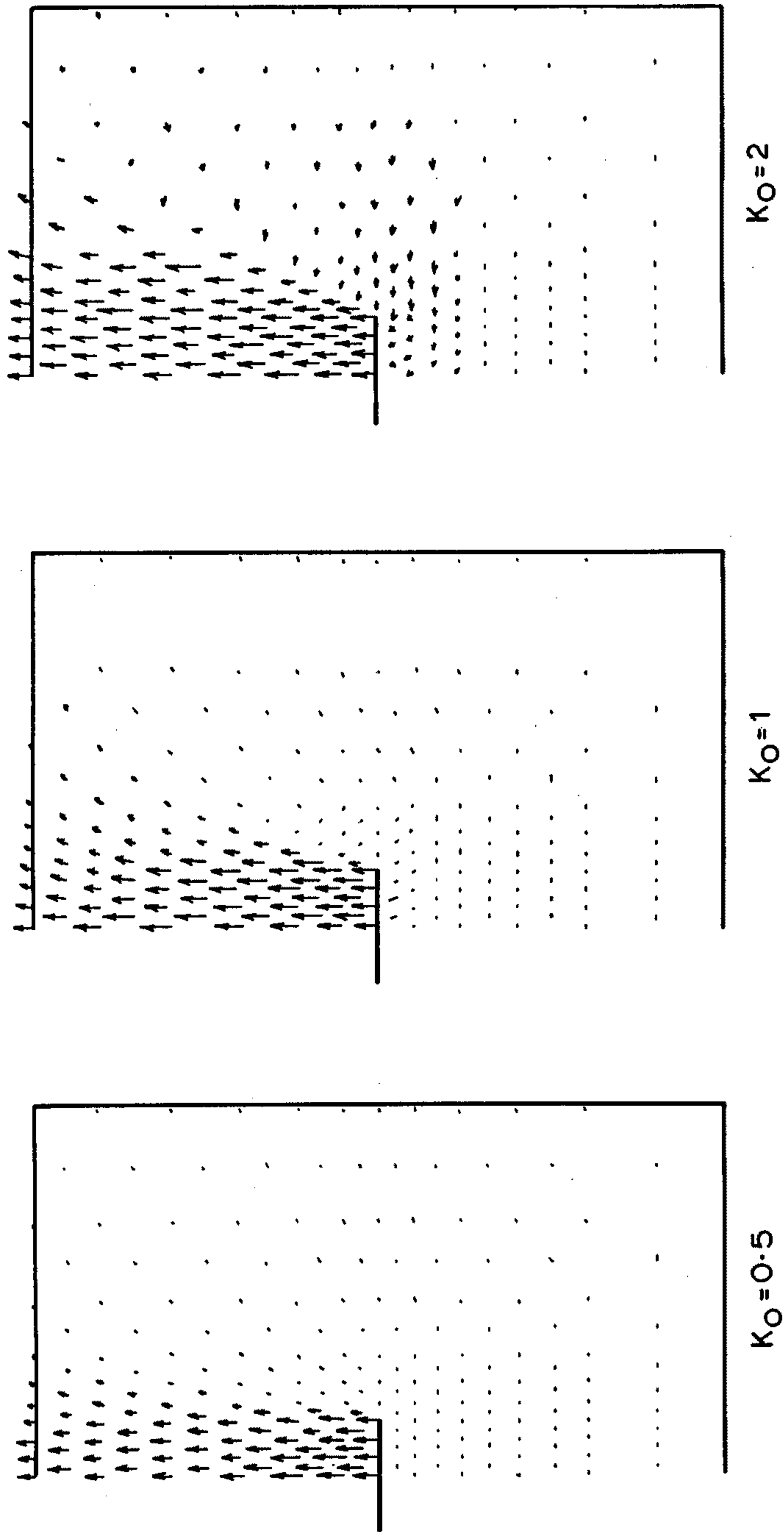


FIG. 7.56 VELOCITY FIELDS AT COLLAPSE - ANCHOR $h/B=3$, $\phi=30^\circ$, $\psi=0^\circ$, $E/Y_h=333$, $Y_h/c=120$

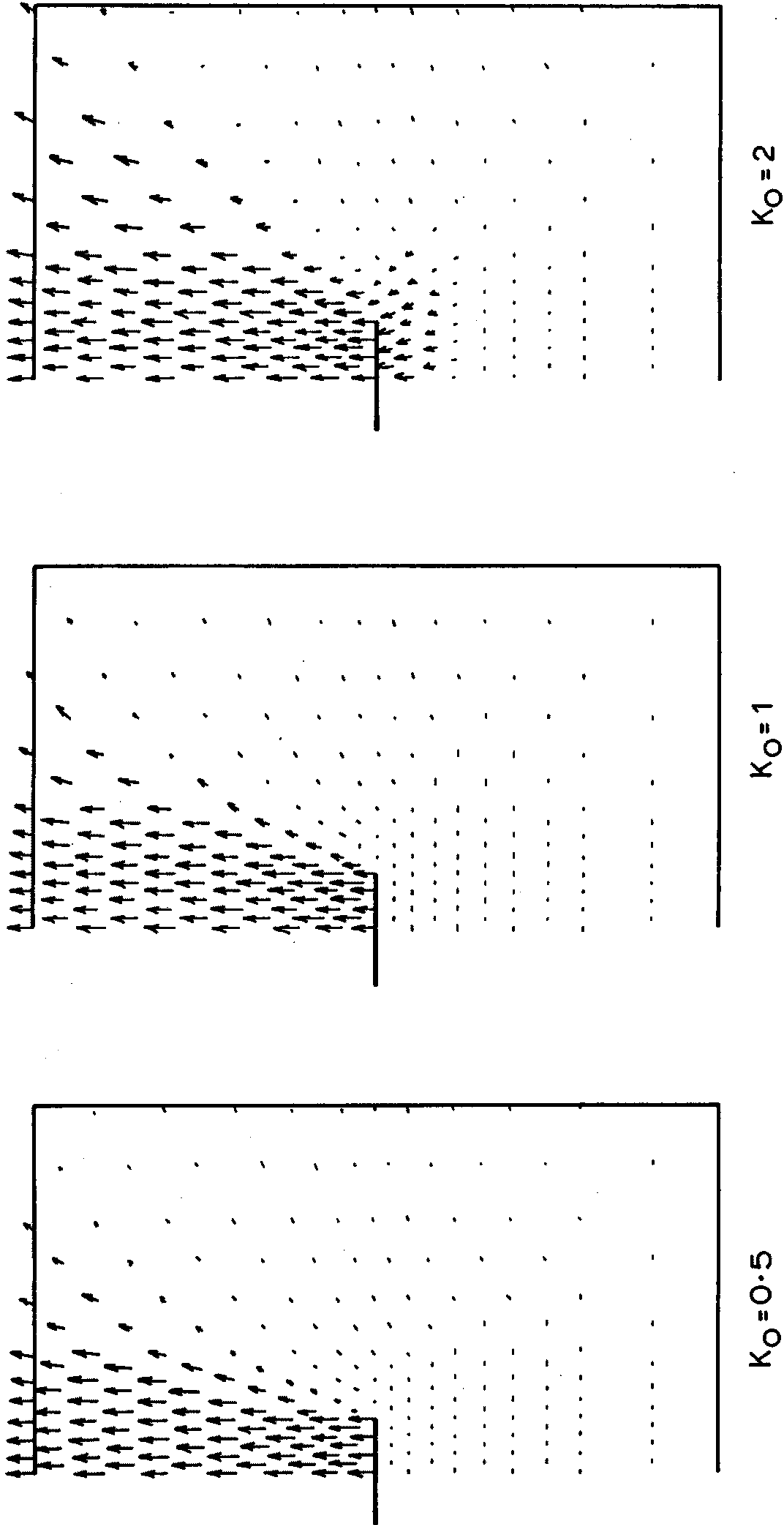


FIG.7.57 VELOCITY FIELDS AT COLLAPSE -ANCHOR $h/B=3$, $\phi = 30^\circ$, $\psi = 30^\circ$, $E/\gamma h = 333$, $\gamma h/c = 120$

obtained for $K_0 = 1$ by 10% and was almost equal to the collapse load obtained for the associated flow rule.

The range of initial stress states considered corresponds to a range of collapse loads varying by up to 16% of the base value obtained for $K_0 = 1$, and although the unique collapse load for the associated material marks the uppermost limit of the range, the value obtained for $K_0 = 0.5$ does not necessarily correspond to the lowest possible collapse load. Thus it should be recognised that the bearing capacity factors presented for $K_0 = 1$ may be significantly conservative for some initial stress states and unconservative for others.

The significant differences in the behaviour of the anchor in the various cases is emphasised by the plastic regions and the velocity fields at collapse (see Figs. 7.55, 7.56 and 7.57). In particular, it should be observed that although the order in which regions become plastic depends upon the initial stress state, the extent of the plastic region at collapse for a material with an associated flow rule is fairly similar in each case. In addition, the velocity fields above the anchor at collapse are largely independent of the initial stress state when $\psi = \phi$, although for $K_0 = 2$ there is far more plastic deformation of material below the anchor than in the other two cases.

For the non-associated material ($\psi = 0^\circ$), the extent of plastic region and velocity field at collapse varies more markedly with initial stress conditions than is the case for associated materials.

The extent of the plastic region and the velocity field for $K_o = 2$ are fairly similar for the cases where $\psi = 0^\circ$ and $\psi = 30^\circ$, in that they both involve extensive plastic failure and rigid movement of a large mass of material above the anchor towards the surface. However, the velocity fields below the anchor are quite different, and it can be seen that flow around the anchor is far more easily accommodated when $\psi = 0^\circ$. This suggests that the difference in the collapse load obtained for $K_o = 2$, $\psi = 0^\circ$ and $K_o = 2$, $\psi = 30^\circ$ would increase as the embedment ratio increases since the containment of the dilating material will dominate the overall behaviour for deeper anchors. This was in fact found to be the case for an anchor with an embedment ratio $h/B = 8$, $\phi = 30^\circ$, where the ultimate collapse load obtained for $K_o = 2$, $\psi = 0^\circ$ was only 17% above the basic value obtained with $K_o = 1$, $\psi = 0^\circ$, whereas the bearing capacity for the associated flow rule ($\phi = \psi = 30^\circ$) was 35% above the basic value.

In the case of deep anchors with low values of K_o , the deformations which occur due to contained plastic flow prior to collapse are quite significant and it may be necessary to invoke a 'k4' definition of failure. For example, an anchor with an embedment ratio $h/B = 8$, resting in a soil mass with properties $\phi = 30^\circ$, $\psi = 0^\circ$ and $K_o = 0.5$ was found to have a practical 'k4' failure load 25% below the ultimate collapse load. The ultimate collapse load for $K_o = 0.5$ was found to be almost identical to that obtained in the basic case where $K_o = 1$, however, the anchor displacement prior to collapse (for $K_o = 0.5$) was over three times the displacement observed for the basic case (ie. $K_o = 1$).

7.4.3 Effect of Anchor Roughness

For horizontal anchors subjected to vertical loading, it has been found that neither the load-displacement curve nor the collapse load is noticeably affected by anchor roughness. This situation arises because the problem is symmetric about the axis of the anchor and (as can be appreciated from the velocity fields obtained for the smooth case) does not involve any significant horizontal velocity component for soil particles adjacent to the anchor. Consequently, no significant shear stresses develop between the rough anchor plate and the soil; hence roughness cannot alter the anchor behaviour. Clearly this would not be the case for either an inclined anchor or an inclined loading.

7.4.4 Effect of Layer Depth

To determine the likely effect of layer depth upon the load deflection behaviour of an anchor, four analyses were performed for an anchor with embedment ratio $h/B = 5$ buried in a purely cohesive material ($E/c = 500$, $\phi = 0^\circ$, $\nu = 0.48$, $q_h/c = 0$) for layer depths $D = 5, 8, 13$ and 18 times the anchor width B . The same basic mesh was used as shown in Fig. 7.2 for $D/B = 5$ and $D/B = 8$, except that in the former case the depth of the bottom bay of elements was $2B$. For the cases where $D/B = 13$ and 18 , an extra one and two bays of depth $5B$ were added respectively.

The load deflection curves obtained for $D/B = 5, 8$ and 18 are shown in Fig. 7.58. The apparent stiffnesses obtained for $D/B = 5$, $D/B = 13$ and $D/B = 18$ were found to differ from the basic case ($D/B = 8$)

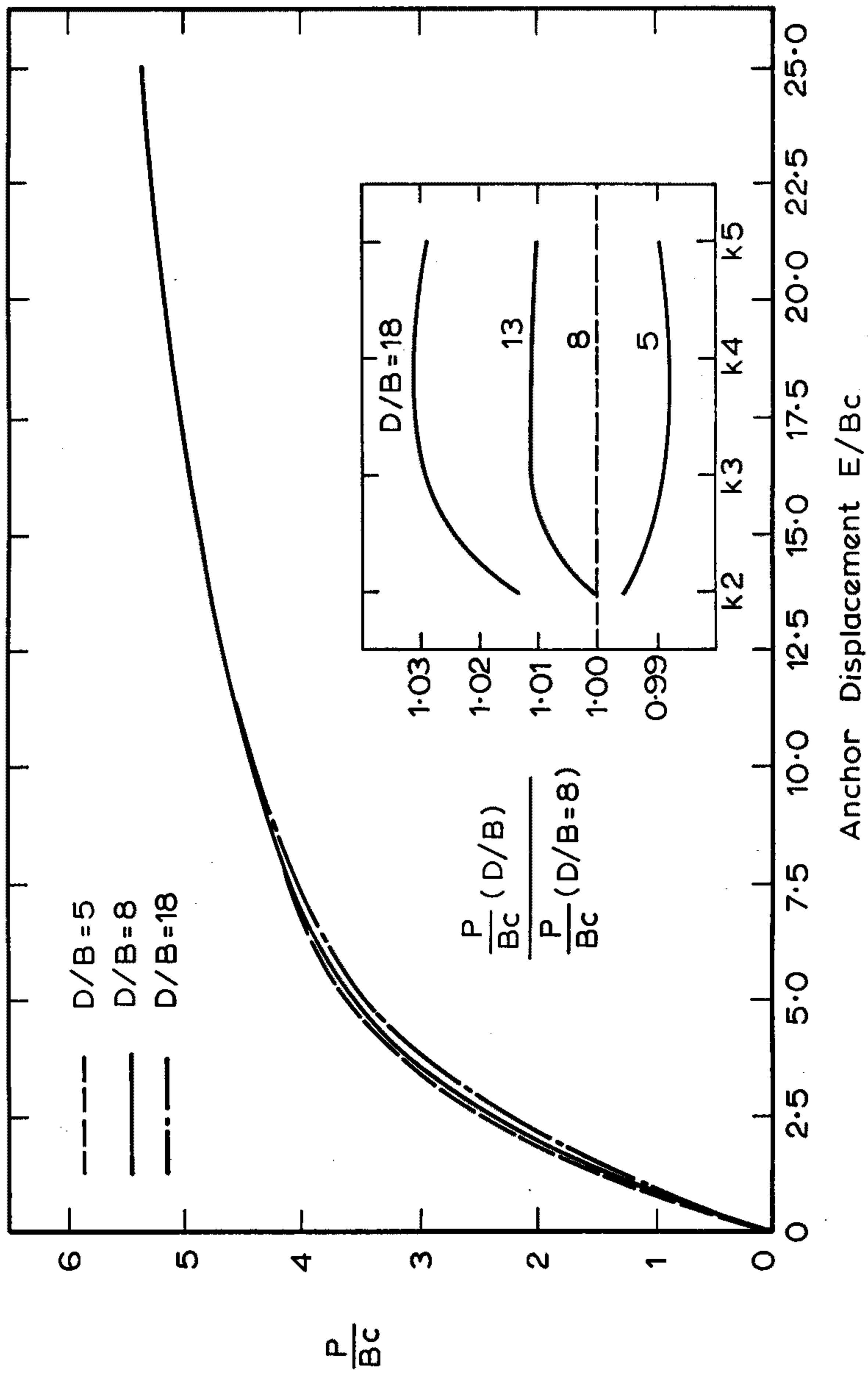


FIG.7.58 EFFECT OF LAYER DEPTH D UPON LOAD DISPLACEMENT BEHAVIOUR $h/B=5, \varnothing=0^\circ$

by 13%, 7% and 11%, respectively and these ratios can be compared with the ratios obtained from a more exact analysis (Chapter 3) of 14%, 7%, and 11%.

The 'k2', 'k3', 'k4', 'k5', loads were obtained for each load deflection curve and the ratio of these values to the basic values obtained for $D/B = 8$ are shown in the insert to Fig. 7.58. From this, it can be seen that even though increasing the layer depth by twice the embedment ratio decreases the apparent stiffness by 11%, the normalised loads (ie. 'k2', 'k3' etc.) are relatively insensitive to the change in layer depth with an error, in the worst case, of about 3%.

As might be expected, once the plastic deformation begins to dominate the behaviour, the load displacement curves are independent of the layer depth, and hence clearly defined collapse loads will be completely independent of the layer depth.

7.5 APPLICATION OF BEARING CAPACITY FACTORS FOR HORIZONTAL ANCHORS

7.5.1 Superposition Error

The separation of the bearing capacity expression (Eq. 7.1) into two parts, depending upon cohesion and body force respectively, assumes that the two types of behaviour are totally independent of each other and hence the bearing capacity results can be superimposed. The error due to superposition can be expressed in the form $F_s = \frac{q_{\text{actual}}}{q_{\text{predicted}}}$ where q_{actual} is the collapse load obtained from a direct analysis; and $q_{\text{predicted}}$ is the predicted collapse load obtained from the bearing ca-

capacity factors F_c and F_γ , hence $q_{\text{predicted}} = cF_c + \gamma hF_\gamma$.

To determine the likely magnitude of superposition error, an anchor with $h/B = 3$, $\phi = 30^\circ$, $E = 40000$, $\nu = 0.3$, $K_o = 1$ and $\psi = 0^\circ$ was analysed for a range of values of $\gamma h/c$ as shown in Fig. 7.59. In each case considered, the superposition error was less than 5.6% and in all but one case the predicted load was conservative.

The velocity fields obtained were fairly similar in each case although the extent of the plastic region changed quite significantly with $\gamma h/c$ as can be seen from Fig. 7.60. The major difference between these plastic regions is restricted to the areas below the anchor and along the centre line above the anchor. However, the majority of soil resistance to shallow anchor pullout is due to shearing of the material above the edge of the anchor and since the mode of failure is fairly similar in each case, it is not particularly surprising that the superposition assumption gives quite good results.

The anchor with embedment ratio $h/B = 3$ was also analysed for a range of $\gamma h/c$ for $\phi = 15^\circ$ and 45° and the error between actual and predicted collapse load was found to be less than 6% in every case.

The error in superposition might be expected to be greatest for problems where there is a transition from deep anchor to shallow anchor behaviour within the range of $\gamma h/c$ considered. Thus, an anchor with embedment ratio $h/B = 8$ was analysed for the case where $\phi = 15^\circ$, $\psi = 0^\circ$, $E/c = 333$, $\nu = 0.3$ and $\gamma h/c = 1$. At this embedment ratio,

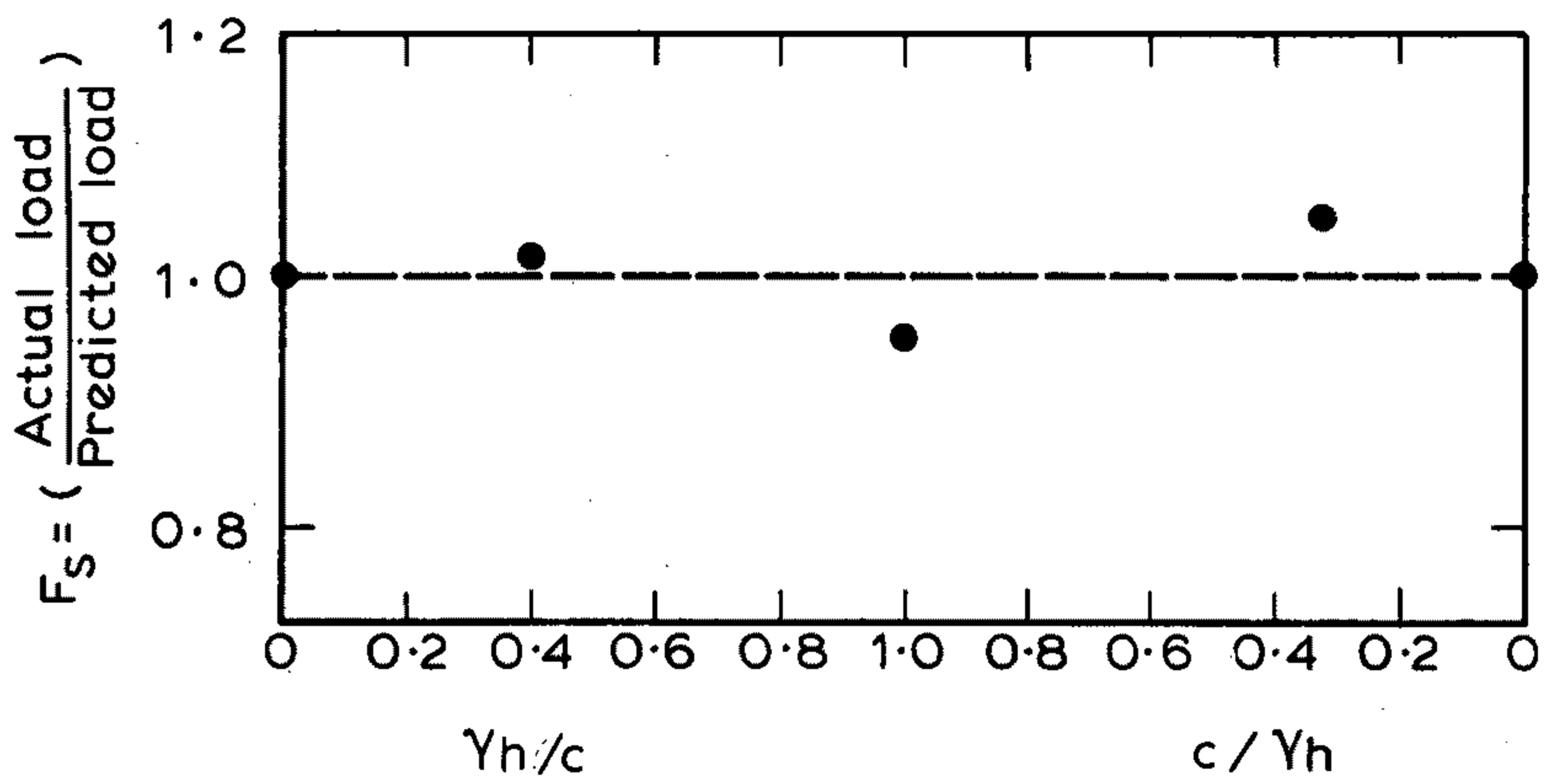


FIG. 7.59 SUPERPOSITION FACTOR $h/B=3$ $\phi=30^\circ$

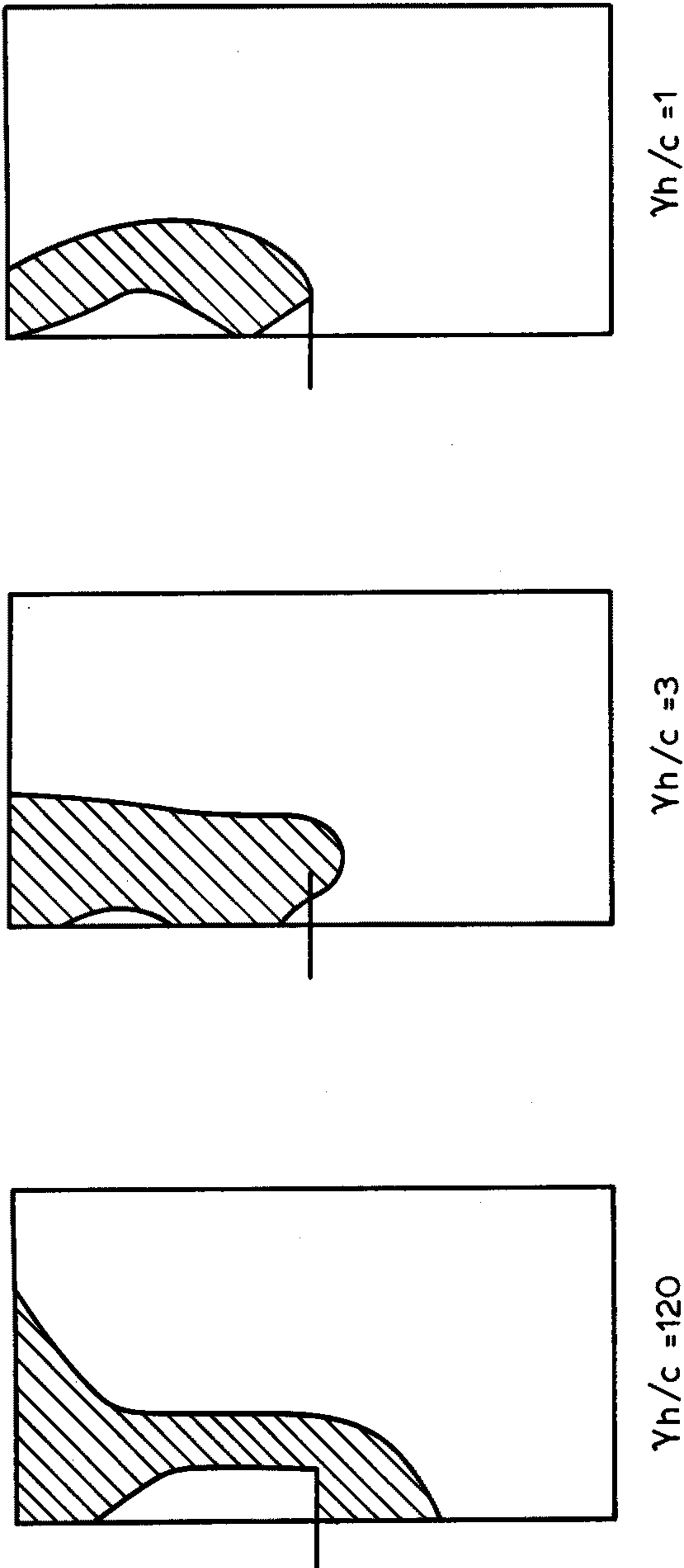


FIG.7.60 PLASTIC REGIONS AT COLLAPSE - $h/B=3$, $E/c = 40000$, $\varnothing = 30^\circ$, $\psi = 0^\circ$

the anchor behaves as a deep anchor for the limiting case where $\gamma h/c$ approaches zero and it is necessary to determine the value of F_c using a practical 'k4' definition of failure. In the second limiting case as $(\frac{c}{\gamma h}) \rightarrow 0$, this anchor behaves as a shallow anchor and failure is clearly defined at a displacement less than the 'k4' displacement. The load displacement curve for the case where $\gamma h/c = 1$ is shown in Fig. 7.61 and it can be seen that failure was not clearly defined when loading ceased. The 'k4' collapse can, however, be seen to exceed the predicted collapse load (using a 'k4' value for F_c) by less than 2%. This indicates that the superposition error is not likely to be any larger for anchors where there is a transition from deep to shallow behaviour within the possible range of values of $\gamma h/c$ than for shallow anchors.

In every case considered, the superposition errors were less than 6% and hence, were of the same order as the anticipated numerical error in the solutions. Thus it would seem reasonable to neglect the effect of superposition when applying Eq. 7.1, by assuming that the superposition factor F_s is equal to unity.

7.5.2 Application of Solutions to Hand Calculations

The application of the solution obtained in section 7.3 and 7.4 to the prediction of anchor capacity will be illustrated by the consideration of two examples.

Firstly, consider an anchor of width $B = 2\text{m}$ buried to a depth of 5.4m in a loose granular material with material properties $c' = 1\text{ kPa}$, $\phi' = 30^\circ$, $\psi = 0^\circ$, $K_o = 1$ and $\gamma = 20\text{ kN/m}^3$. It is assumed that there is

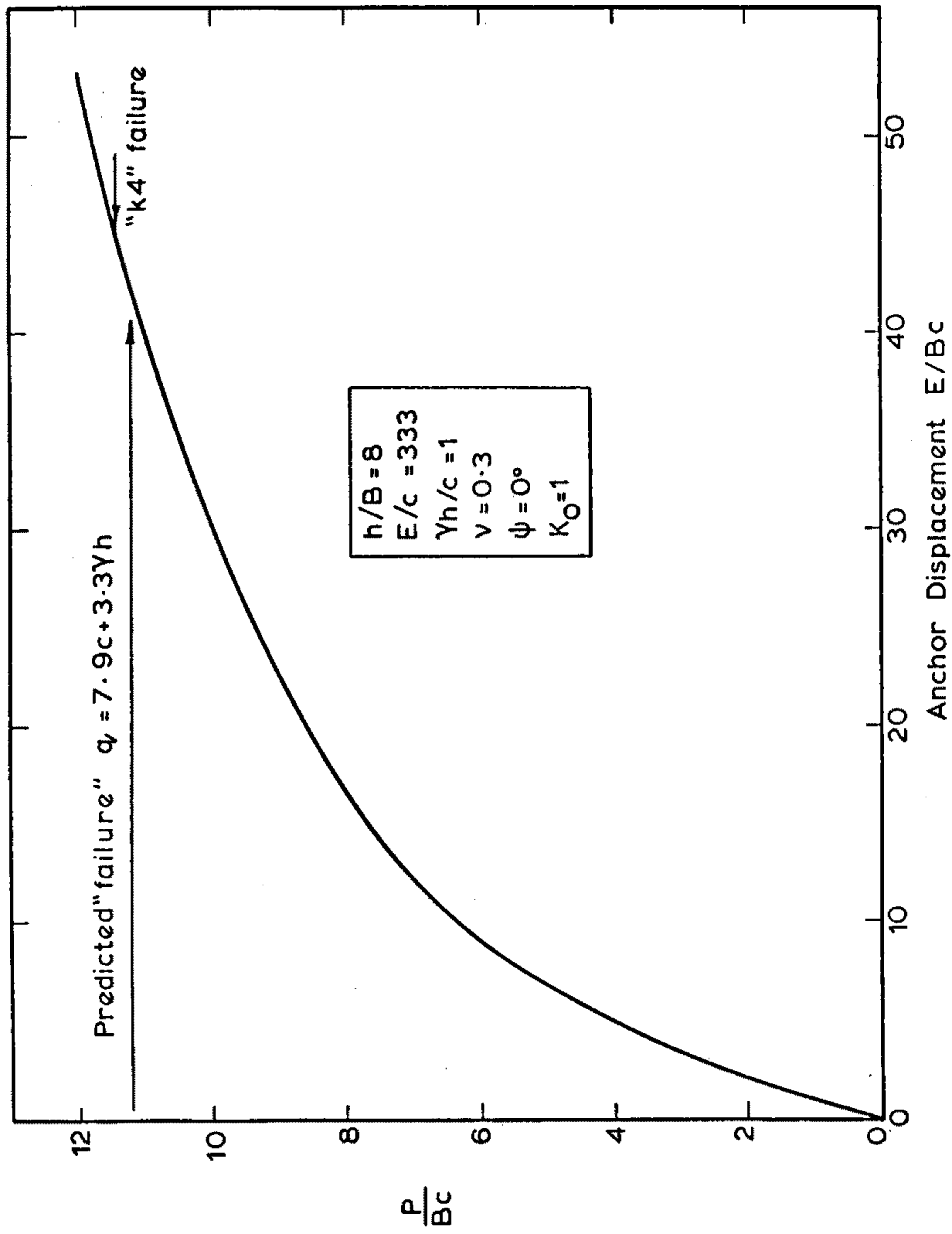


FIG.7.61 LOAD DISPLACEMENT CURVE FOR ANCHOR IN COHESIVE-FRICTIONAL SOIL WITH WEIGHT

a surface surcharge q_s equal to 12 kPa.

From Eq. 7.1 the anchor capacity is given by

$$q_u = cF_c + \gamma h F_\gamma$$

where $F_c = F_{c_o} + q_s/c$ from Eq. 7.3

$$F_{c_o} = F_c \left(\frac{h}{B} = 2.7, \frac{q_s}{c} = 0, \phi = 30^\circ \right) = 4.3 \text{ from Fig. 7.35}$$

$$\begin{aligned} c &= c' + q_s \tan \phi' && \text{from section 7.1} \\ &= 1 + 12 \tan 30^\circ \\ &= 7.93 \text{ kPa} \end{aligned}$$

and $q_s/c = 1.51$

Thus $F_c = 4.3 + 1.51 = 5.81$ from Eq. 7.3

$$F_\gamma = F_\gamma \left(\frac{h}{B} = 2.7, \phi = 30^\circ, \psi = 0^\circ, K_o = 1 \right) = 2.46$$

from Fig. 7.46

and the average anchor capacity

$$\begin{aligned} q_u &= 7.93 \times 5.81 + 20 \times 5.4 \times 2.46 \\ &= 46.1 + 265.7 \\ &= 311 \text{ kPa} \end{aligned}$$

which is almost identical to the collapse load $q_u = 312$ kPa obtained from a direct finite element analysis of this case.

Secondly, consider an anchor of width $B = 2\text{m}$ buried at a depth of 6m in a granular material with properties $c' = 1$ kPa, $\phi = 35^\circ$, $\psi = 12^\circ$, $K_o = 0.5$ and $\gamma = 20$ kN/m³. It is assumed that there is no

surface surcharge (ie. $q_s = 0$).

From Eq. 7.1, the anchor capacity is given by

$$q_u = cF_c + \gamma h F_\gamma$$

where

$$F_c \left(\frac{h}{B} = 3, \frac{q_s}{c} = 0, \phi = 35^\circ \right) = 4.4 \quad \text{from Fig. 7.37}$$

$$F_\gamma \left(\frac{h}{B} = 3, \phi = 35^\circ, \psi = 0^\circ, K_o = 1 \right) = 2.88 \quad \text{from Fig. 7.48}$$

$$R_\psi = 1.04 \quad \text{for } h/B = 3, \phi = 30^\circ, \psi = 12^\circ$$

$$R_\psi = 1.13 \quad \text{for } h/B = 3, \phi = \psi = 30^\circ \quad \text{from Fig. 7.53}$$

$$R_\psi = 1.18 \quad \text{for } h/B = 3, \phi = \psi = 35^\circ$$

and by interpolation $R_\psi = 1.06$ for $\phi = 35^\circ, \psi = 12^\circ$

$$R_K = 0.94 \quad \text{for } \phi = 30^\circ, K_o = 0.5 \quad \text{from section 7.4.2}$$

$$\text{hence } F_\gamma \left(\frac{h}{B} = 3, \phi = 35^\circ, \psi = 12^\circ, K_o = 0.5 \right) = 2.88 \times 1.06 \times 0.94 = 2.87$$

$$\text{where } q_u = 1 \times 4.4 + 120 \times 2.87 = 349 \text{ kPa}$$

which is 1% above the collapse load $q_u = 346$ obtained from a direct analysis of this case, as shown in Fig. 7.62.

7.6 SHAPE EFFECTS - CIRCULAR ANCHOR (UNDRAINED LOADING) $\phi = 0^\circ$

Although the application of the elastoplastic finite element techniques to the prediction of anchor behaviour has so far been res-

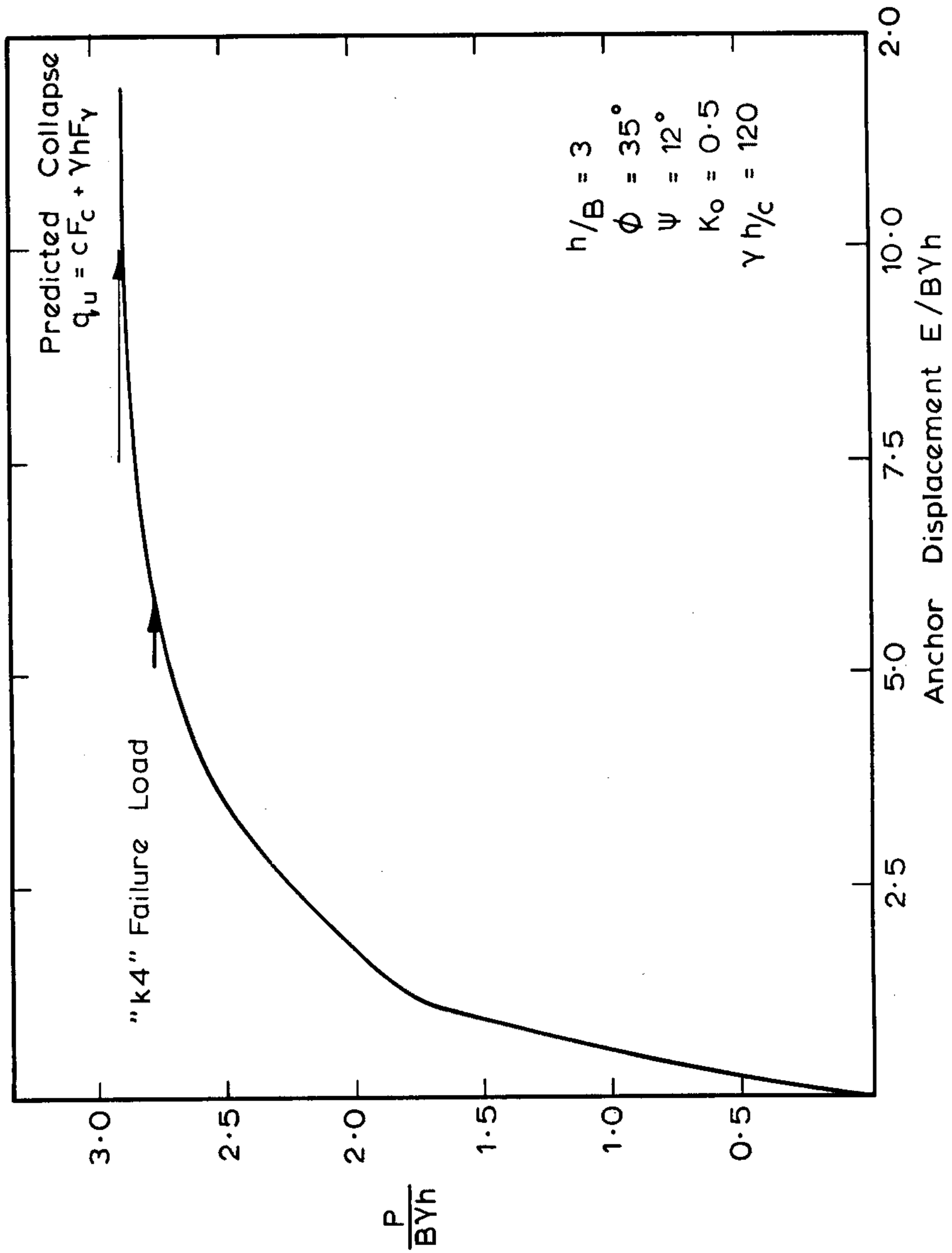


FIG. 7.62 LOAD DISPLACEMENT CURVE FOR HORIZONTAL ANCHOR

stricted to the analysis of strip anchors subjected to normal loading, there is no fundamental reason why the approach could not be extended to analyse the behaviour of either rectangular or circular anchors with either normal or inclined loading. Unfortunately, the cost involved in performing such a general analysis is prohibitive. The determination of the effect of anchor shape upon bearing capacity will, therefore, be limited to the special case of a horizontal circular anchor subjected to a centrally applied vertical load and resting in a purely cohesive soil mass.

Various authors (e.g. Balaam, 1978, Nagtegaal et al., 1974) have recognised that the difficulties involved in obtaining accurate elastic and elasto-plastic solutions in an axi-symmetric analysis are greater than in the corresponding plane strain case, irrespective of whether triangular or 8 node quadrilateral (iso-parametric) elements are adopted. However, despite this, the technique of introducing potential rupture zones into the finite element mesh can, as indicated in Chapter 6, still lead to an improved estimate of the collapse load.

The finite element mesh and boundary conditions adopted for the axi-symmetric analysis are shown in Figs. 7.1(b) and 7.2 respectively, except that for this analysis all the quadrilateral bays shown were subdivided into four triangular elements. Solutions were obtained for a rigid anchor resting in a soil mass which obeyed the Mohr-Coulomb failure criterion and with properties $E/c = 500$, $\nu = 0.48$, $\phi = 0^\circ$ and $\gamma = 0$.

A rigid footing was analysed using the basic mesh shown in Fig. 7.3(a) (where each quadrilateral bay was divided into four trian-

gular elements) and the collapse load was found to exceed the value obtained by Cox et al., (1961) by 4.7%, indicating that with the inclusion of potential rupture zones, the numerical error obtained from the axi-symmetric analysis is not significantly greater than that obtained in the corresponding plane strain analysis.

The load displacement curves obtained from analyses performed for anchors with embedment ratios of $\frac{h}{B} = 1, 2, 3$ and ∞ are shown in Fig. 7.63 and the bearing capacity factors $F_c = \left(\frac{P}{\pi B^2}\right) c$ corresponding to different values of apparent stiffness (ie. 'k2', 'k3' etc. values) are shown in Fig. 7.64.

Deep anchor behaviour was observed at an embedment ratio of about 2.5 in the case of circular anchors, as compared to 4 in the strip analysis. Furthermore, it can be seen from Fig. 7.64 that if the load displacement curve for the anchor at the 'infinite' depth ($h/B = \infty$) is non-dimensionalised and expressed in term of the 'k' values, then there is no apparent difference between these results and those obtained for $h/B = 3$. In fact, this might be expected since the difficulties of infinite elastic displacement for an anchor at infinite depth encountered in the plane strain case do not arise for circular anchors, and hence the finite distance to the base boundary will have far less effect on the elastic and elasto-plastic behaviour.

As in the plane strain analyses, extensive deformation due to contained plastic flow was observed necessitating the adoption of a 'practical' 'k4' failure load. In the case of deep anchors, this load is considerably less than the anchor collapse load as can be seen from Figs. 7.63 and 7.64 where in the case of an anchor at 'in-

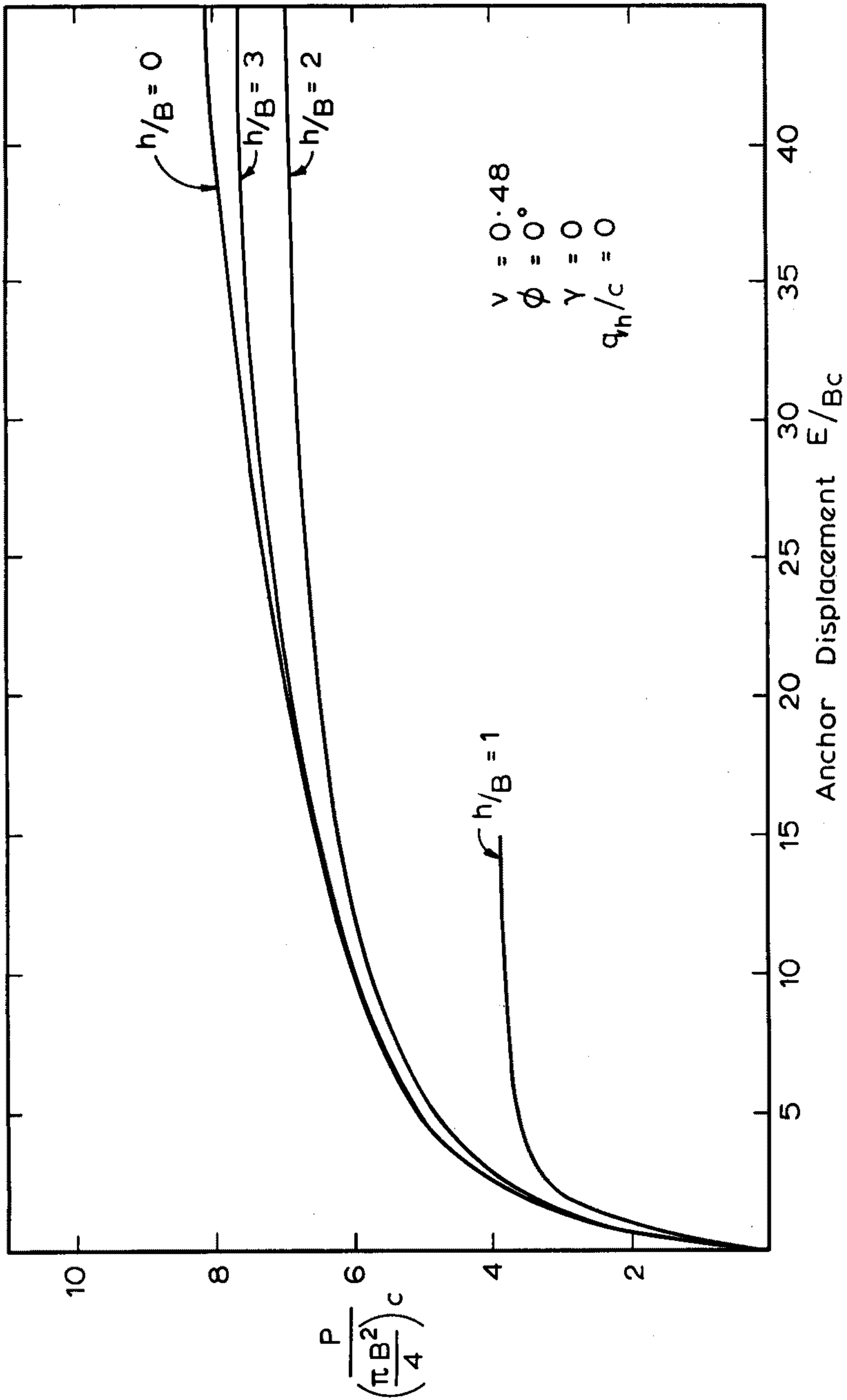


FIG. 7.63 LOAD DISPLACEMENT CURVES FOR CIRCULAR HORIZONTAL ANCHOR

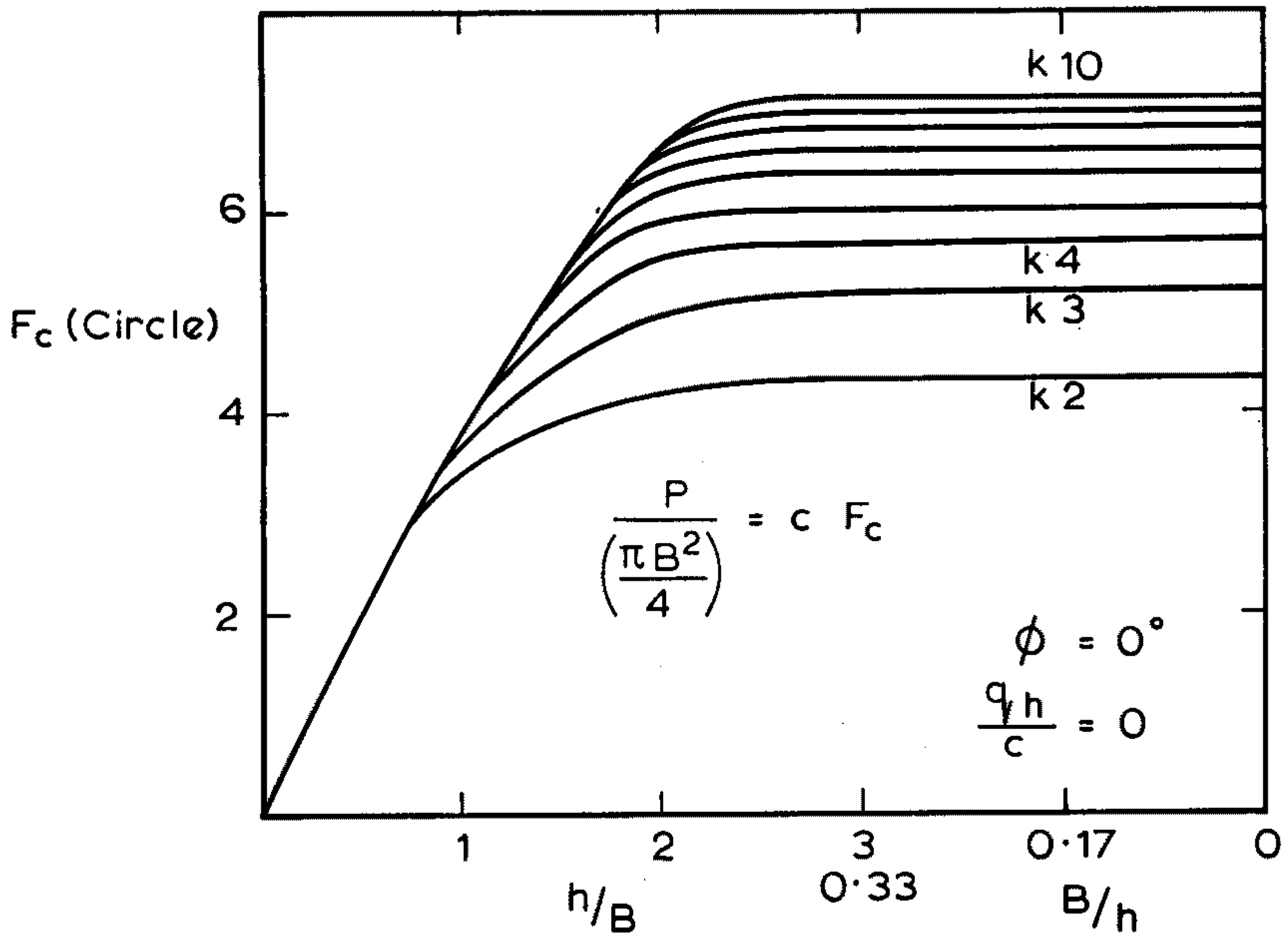


FIG. 7.64 BEARING CAPACITY FACTORS FOR A CIRCULAR ANCHOR

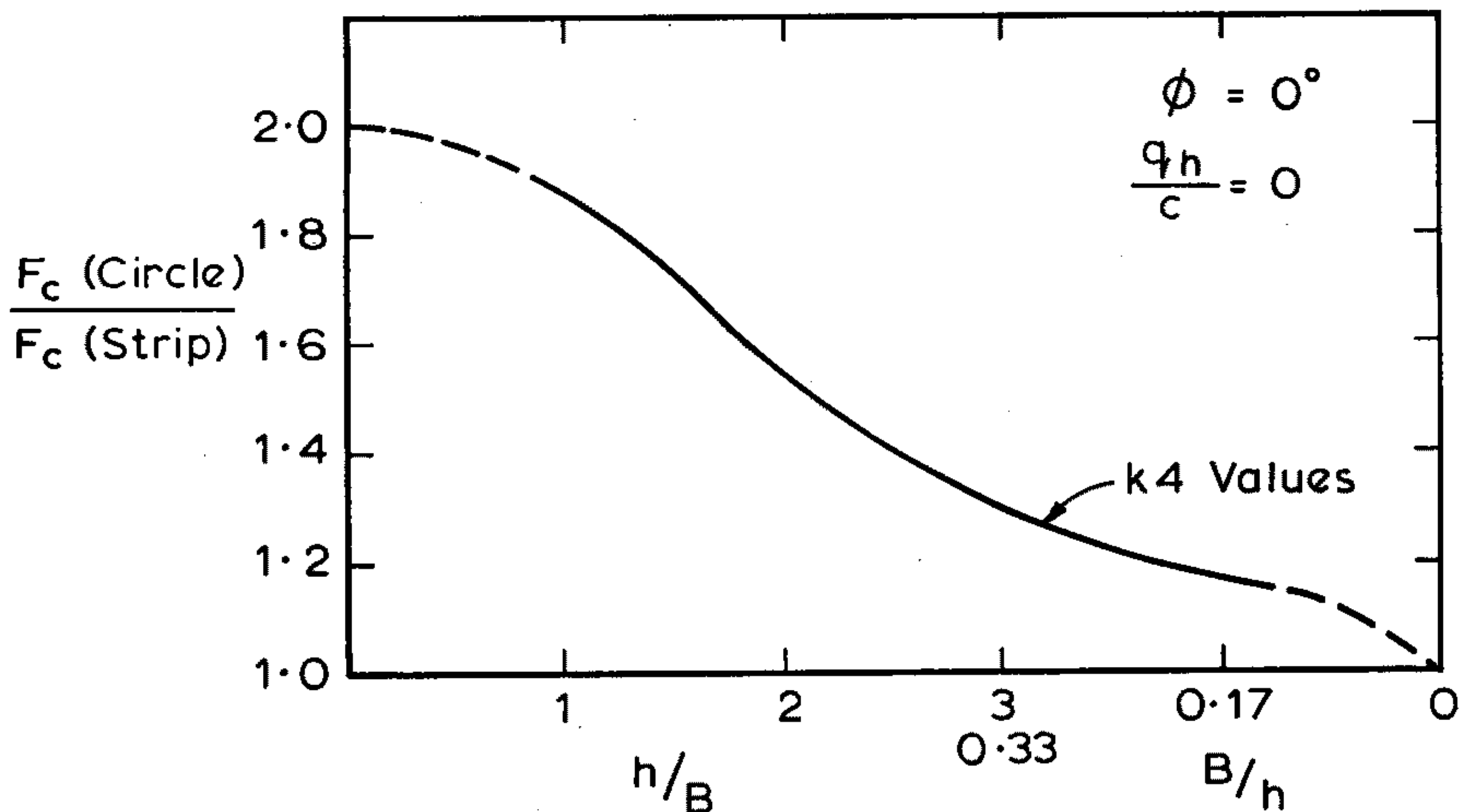


FIG. 7.65 RATIO OF BEARING CAPACITY FACTORS F_c FOR CIRCLE AND STRIP

finite depth' the 'k4' bearing pressure of 5.72c may be compared to the 'k10' bearing pressure of 7.18c which in turn is well below the bearing pressure of 8.1c at which loading was terminated

The effect of shape upon the practical 'k4' failure load is illustrated in Fig. 7.65, where the bearing pressure for the circular anchor is divided by the corresponding bearing pressure for a strip. It will be noticed that the effect of shape is greatest for shallow anchors and decreases rapidly with depth. In the case of shallow anchors ($h/B = 1$) the increased bearing capacity would to some extent be offset in practice due to the considerably higher flexural tensions which were observed above the anchor. In these analyses, tension within the soil mass was permitted. This assumption may be justified provided that the sum of the overburden pressure and the increment in tension due to anchor loading remains compressive; however, for very shallow anchors where this is not the case and where the material cannot sustain tension, the bearing capacity factors for both strips and circles will overestimate the actual bearing capacity.

7.7 CONCLUSIONS

In this chapter, the effects of cohesion, angle of internal friction, body force, and embedment ratio upon the behaviour of a strip anchor have been investigated and basic bearing capacity factors $F_c = \left(\frac{P}{Bc}\right)_{\gamma=0}$ and $F_\gamma = \left(\frac{P}{\gamma h}\right)_{c=0}$ have been determined.

Particularly in the case of anchors in cohesive and cohesive frictional materials, it has been found necessary to adopt a practical

definition of failure corresponding to the load at which the deformation is four times the elastic prediction for that load level. In many cases this practical failure load is well below the ultimate collapse load and is necessary because in these cases the anchor deformations at the working load (determined by dividing the ultimate bearing capacity by a factor of safety of 2.5 or 3) would considerably exceed that predicted from an elastic analysis.

The effects of secondary parameters such as the soil dilatancy angle, the coefficient of lateral earth pressure at rest, anchor roughness and layer depth have also been considered. Except in the case of material with high angles of friction and dilatancy the basic bearing capacity factors were found to be within approximately 10% of the actual bearing capacity for a wide range of these parameters. For loose, normally consolidated sands ($K_0 < 1$, $\psi \approx 0$) the basic bearing capacity factor F_γ may be unconservative for a range of embedment ratios although it is anticipated that the effects would be less than 10% and would be largely nullified for angles of dilatancy greater than zero.

The use of the bearing capacity factors for the prediction of failure loads in cases other than those for which direct calculations were made was illustrated by means of two examples, and it was found that the predicted values agreed well with those obtained from a direct finite element analysis for the particular choice of parameters.

Finally, the likely effect of shape upon the bearing capacity

of an anchor was illustrated by consideration of a circular anchor in a purely cohesive material.

APPLICATION OF THE INITIAL STRESS METHOD TO SOIL-STRUCTURE INTERACTION

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SUMMARY

A general method is presented for the analysis of soil-structure interaction problems using the initial stress finite element approach. The soil and structure will be treated as separate bodies related only by nodal forces at the soil-structure interface. Provision is made for plastic failure of both soil and structure, shear failure at the interface, and breakaway of the structure from the soil.

INTRODUCTION

A number of numerical difficulties may arise in the analysis of problems involving soil-structure interaction. Firstly, the difference in stiffness between soil and structural elements may lead to serious numerical errors in the solution process.¹ Secondly, the presence of the structure (for example, a retaining wall tied back to an anchor plate) may establish a link between nodes which are a great distance apart and lead to a very large increase in bandwidth of the equations being solved. Finally, it has been the authors' experience that straightforward application of the initial stress method² to non-linear problems involving stiff link or joint elements is often accompanied by poor convergence of the method. Some attempt has been made by Ghaboussi and Wilson^{3,4} to reduce the numerical error due to stiff joint elements, however, difficulties arise in an initial stress approach since the incremental stiffness correction terms may still be quite large.

In this paper an alternative approach, in which the soil and the structure are analyzed as separate bodies related only by compatibility of nodal displacements and nodal forces at their interface, is presented.

The concept of separating the general body into substructures related by compatibility and equilibrium conditions has been recognized by various authors^{5,6} in the elastic analyses of problems such as pile and pile strip behaviour. When applied to non-linear finite element methods this substructure approach avoids the difficulties outlined above while retaining the computational advantages of the initial stress technique.

AN INITIAL STRESS—SUBSTRUCTURE APPROACH

Consider a structural body embedded in an elasto-plastic soil mass. The composite body can be separated into two components, namely the soil and the structure.

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Suppose that incremental loads $\{\dot{\phi}_1\}$ and $\{\dot{\phi}_2\}$ are applied respectively to the soil mass and the structure, then the two substructures will be governed by the equations

$$[K_{1T}]\{\dot{\delta}_1\} = \{\dot{\phi}_1\} + \{\dot{F}_{1u}\} \quad (1)$$

$$[K_{2T}]\{\dot{\delta}_2\} = \{\dot{\phi}_2\} - \{\dot{F}_{2u}\} \quad (2)$$

where $[K_{1T}], [K_{2T}]$ are the tangent stiffness matrices for soil and structure respectively;
 $\{\dot{\delta}_1\}, \{\dot{\delta}_2\}$ are the vectors of incremental nodal displacement for soil and structure;
 $-\{\dot{F}_{1u}\}$ is the vector of incremental nodal forces due to the structure acting on the soil; and
 $-\{\dot{F}_{2u}\}$ is the vector of incremental nodal forces due to the soil acting on the structure.

The vectors $\{\dot{\delta}_1\}, \{\dot{\phi}_1\}, \{\dot{F}_{1u}\}$ are of length k while the vectors $\{\dot{\delta}_2\}, \{\dot{\phi}_2\}, \{\dot{F}_{2u}\}$ are of length l where k and l are the number of degrees-of-freedom of soil and structure respectively. Thus, if there are m degrees-of-freedom at the soil-structure interface, then the vector $\{\dot{F}_{1u}\}$, of length m , which corresponds to the forces exerted by the structure onto the soil can be augmented by the addition of zero terms for each degree-of-freedom which does not correspond to an interfacial node to give the vectors $\{\dot{F}_{1u}\}$ and $\{\dot{F}_{2u}\}$ for soil and structure.

Analysis of the soil

Suppose for the moment that nodal force vectors $\{\dot{F}_u\}$ were known, then the initial stress approach could be used to analyse the soil using the following algorithm.

$$[K_{1E}]\{\dot{\delta}_1^{(n+1)}\} = [K_{1E}^{(n)}]\{\dot{\delta}_1^{(n)}\} + \{\dot{\phi}_1\} + \{\dot{F}_{1u}^{(n+1)}\} \quad (3)$$

where $[K_{1E}]$ is the elastic component of the soil stiffness matrix

$$[K_{1E}^{(n)}] = [K_{1E}] - [K_{1T}^{(n)}]$$

and the superscript n denotes the n th estimate of that variable.

It is convenient to rewrite equation (3) in the form—

$$[K_{1E}]\{\dot{\delta}_{1K}^{(n+1)}\} = \{\dot{F}_{1K}^{(n)}\} \quad (4a)$$

$$[K_{1E}]\{\dot{\delta}_{1u}^{(n+1)}\} = \{\dot{F}_{1u}^{(n+1)}\} \quad (4b)$$

where

$$\{\dot{F}_{1K}^{(n)}\} = [K_{1E}^{(n)}]\{\dot{\delta}_1^{(n)}\} + \{\dot{\phi}_1\}$$

so that

$$\{\dot{\delta}_1^{(n+1)}\} = \{\dot{\delta}_{1K}^{(n+1)}\} + \{\dot{\delta}_{1u}^{(n+1)}\}.$$

Notice that it is only necessary to reduce $[K_{1E}]$ to triangular form once and so the cost of solving for the load sets $\{\dot{F}_{1K}^{(n)}\}$ is relatively small.

Now suppose that $\{\rho\}$ denotes the vector of nodal displacements at the soil-structure interface, then clearly $\{\rho\}$ can be written in the form

$$\{\rho_1^{(n+1)}\} = \{\rho_{1K}^{(n+1)}\} + \{\rho_{1u}^{(n+1)}\} \quad (5)$$

where the vectors $\{\rho\}$ are obtained by deleting all the non-interface components from the vectors $\{\dot{\delta}_1\}$, and will be of length m .

Since the matrix $[K_{1E}]$ in equation (4b) is constant, successive estimates of $\{\dot{\rho}_{1u}\}$ can always be written in the form—

$$\{\dot{\rho}_{1u}^{(n+1)}\} = [I_{1E}]\{\dot{F}_u^{(n+1)}\} \quad (6)$$

where $[I_{1E}]$ is a $(m \times m)$ matrix of influence coefficients, which can be determined initially and will remain constant for each iteration and loadstep while the geometry of the problem remains unchanged. Equations (5) and (6) can now be combined to give—

$$\{\dot{\rho}_1^{(n+1)}\} = \{\dot{\rho}_{1K}^{(n+1)}\} + [I_{1E}]\{\dot{F}_u^{(n+1)}\} \quad (7)$$

Analysis of the structure

In general if the structure is separated from the soil it will be a mechanism so that the solution for deflection would contain a number of arbitrary rigid body motions. This difficulty can be overcome by applying sufficient artificial constraints to remove the indeterminacy.

This leads to a set of equations of the form

$$[K_{2r}]\{\dot{\delta}_{2r}\} = \{\dot{\phi}_2\} - \{\dot{F}_{2u}\} \quad (8)$$

where $[K_{2r}]$ is the tangent stiffness matrix of the restrained structure and $\{\dot{\delta}_{2r}\}$ are the incremental deflections of the restrained structure. For solution purposes this may be replaced by the scheme—

$$[K_{2E}]\{\dot{\delta}_{2r}^{(n+1)}\} = \{\dot{\phi}_2\} - \{\dot{F}_{2u}^{(n+1)}\} + ([K_{2E}] - [K_{2r}^{(n)}])\{\dot{\delta}_{2r}^{(n)}\} \quad (9)$$

where $[K_{2E}]$ is the elastic stiffness matrix of the restrained structure. For simple structures the necessary restraints are obvious from inspection while for more complicated structures the necessary restraints, and the matrix $[A]$ defined below, can be isolated by performing Gaussian elimination on the full structural stiffness matrix.

When this is done the actual incremental displacements $\{\dot{\rho}_2\}$ of the structural interface nodes can be obtained from the restrained structure plus the arbitrary rigid body motions as follows:

$$\{\dot{\rho}_2\} = \{\dot{\rho}_{2r}\} + [A]\{\dot{\theta}\} \quad (10)$$

where $\{\dot{\theta}\}$ is the vector of incremental rigid body motions, and has length p ;

$[A]$ is a $(p \times m)$ matrix relating the interface deflections to rigid body motions; and

p is the number of artificial restraints applied to the structure.

The vector $\{\dot{\rho}_{2r}\}$ can be derived by a method similar to that described in the previous subsection and may be written—

$$\{\dot{\rho}_{2r}^{(n+1)}\} = \{\dot{\rho}_{2K}^{(n+1)}\} - [I_{2E}]\{\dot{F}_u^{(n+1)}\} \quad (11)$$

where the definitions $\{\dot{\rho}_{2K}\}$, $[I_{2E}]$ parallel those of $\{\dot{\rho}_{1K}\}$, $[I_{1E}]$.

The equations of equilibrium corresponding to the artificial restraints can be written in the form—

$$[A]^T(\{\dot{\phi}_2\} - \{\dot{F}_u^{(n+1)}\}) = 0 \quad (12)$$

where for convenience, it has been assumed that the applied loads $\{\dot{\phi}_2\}$ are located at the soil-structure interface. The more general case may be obtained by a simple modification of the product $[A]^T\{\dot{\phi}_2\}$.

Equations (7)–(12) may now be written in the simple interaction equation—

$$\begin{bmatrix} [I_E] & -[A] \\ -[A]^T & [0] \end{bmatrix} \begin{bmatrix} \{\dot{F}_u^{(n+1)}\} \\ \{\dot{\theta}^{(n+1)}\} \end{bmatrix} = \begin{bmatrix} \{a^{(n+1)}\} \\ \{b^{(n+1)}\} \end{bmatrix} \quad (13)$$

where

$$\begin{aligned}\{a^{(n+1)}\} &= (\{\dot{\rho}_{2K}^{(n+1)}\} - \{\dot{\rho}_{1K}^{(n+1)}\}) - (\{\dot{\rho}_2^{(n+1)}\} - \{\dot{\rho}_1^{(n+1)}\}) \\ \{b^{(n+1)}\} &= -[A]^T\{\dot{\phi}_2\}\end{aligned}$$

and

$$[I_E] = [I_{1E}] + [I_{2E}]$$

For a soil-structure interface at which no slip occurs the vectors $\{\dot{\rho}_1^{(n+1)}\}$ and $\{\dot{\rho}_2^{(n+1)}\}$ may be taken as equal and thus equation (13) may be solved to determine $\{\dot{F}_u^{(n+1)}\}$ and $\{\dot{\theta}^{(n+1)}\}$. Once these quantities are known the soil deflections can be determined from equation (3). Similarly, the deflections of the structure can be calculated by solving equations (9) and adding the rigid body contributions due to $\{\dot{\theta}^{(n+1)}\}$. The process is then continued until convergence is achieved. This approach derives its power from the fact that it is only necessary to triangularize the matrices $[K_{1E}]$, $[K_{2E}]$ once for all iterations and load-steps while the problem configuration remains constant.

Problems in which slip or breakaway (e.g. behind an anchor plate) occurs at the interface can also be solved by using this method. The most common criterion for slip (or breakaway) is that once the nodal force reaches a specified value it remains constant. (More complicated criteria are considered in the Appendix). Suppose that this nodal force is F'_u and the corresponding nodal deflection is ρ' . Once the critical value is reached F'_u will remain constant so that $\dot{F}'_u$ will be zero for all future increments unless unloading occurs. Thus one unknown is lost from equation (13) however, it is replaced by $(\dot{\rho}_2 - \dot{\rho}_1)$, the relative displacement between structure and soil. So equation (13) may still be solved to obtain $\{\dot{F}_u^{(n+1)}\}$ and $\{\dot{\theta}^{(n+1)}\}$. Problems involving a smooth interface can be treated as above by assuming the shear forces F'_u and $\dot{F}'_u$ are always zero.

The solution algorithm presented above has several advantages. Firstly the widely differing magnitudes of soil stiffness and structural stiffness are separated and only come together in equation (13). This set of equations, which for most soil-structure problems is far smaller than the stiffness equations for the entire body, may be solved using multi-precision arithmetic if this is thought necessary. Secondly, the inclusion of interface behaviour is quite straightforward and does not involve correction to the load vector. Thus, the initial stress method when applied in this way seems far more stable than when applied in a conventional manner to the entire body.

EXAMPLE

The foregoing theory has been successfully applied to the analysis of footing, tunnel, anchor and pile problems;^{7,8} however for brevity, its application here will be restricted to a comparison of the authors' approach with various other techniques used in the analysis of a rigid strip footing.

Constant strain triangles were used to obtain elasto-plastic solutions for a homogeneous isotropic, purely cohesive soil, with elastic modulus E_s and Poisson's ratio $\nu_s = 0.48$, which obeys the Mohr-Coulomb failure criteria. Roundoff error was determined by comparison of the results obtained in the elastic range using single and multiprecision arithmetic. Clearly, the significance of any roundoff error present in the elastic solution is compounded for each load step in the full elasto-plastic analysis.

All calculations in the example were performed on a PRIME 300 computer, using 32 bit words for single precision and 64 bits for multiprecision floating point arithmetic.

The four methods of simulating rigid footing behaviour considered, were:

Method (a): substructure approach

The rigid footing was modelled using the theory of the previous section along with the observation that for a rigid structure $[I_{21}]$ of equation (13) is a zero matrix and need not be calculated.

Method (b): displacement defined

Loading of the soil by a rigid footing was simulated by adding a large 'blanketing' number, X_B , to the appropriate diagonal terms of the stiffness matrix while adopting a corresponding right hand side equal to the blanketing number multiplied by the specified displacement. Rigid behaviour was observed provided the blanketing number was several orders of magnitude larger than the other terms in the stiffness matrix.

Method (c): stiff elements

The rigid footing was approximated by applying a uniform load to a layer of elements whose modulus of elasticity, E_B , was considerably greater than the modulus of the underlying soil.

Method (d): link elements

Beam elements used to model the rigid footing, were linked to the soil by incorporating 'springs' between corresponding beam and soil nodes. These springs were assigned vertical and horizontal stiffness K_v and K_H respectively. Displacement compatibility at the interface was achieved by assigning the springs very large stiffnesses. This is analogous to the introduction of special joint elements (9).

For this example a smooth rigid footing was analysed by adopting a high beam stiffness ($E_B I_B$), a high vertical spring stiffness K_v and zero horizontal spring stiffness K_H . Calculations were performed for various ratios of $K_v/E_B B$ and $E_B I_B/E_S B^3$ using single and multiprecision arithmetic and the central displacements of the footing compared as shown in Figure 1.

Comparisons of methods

The relative computation times required to set up and factorize the stiffness matrix along with ten load steps for each of the five methods is shown in Table I.

It can be seen that substructure and displacement defined methods involve the same number of equations and bandwidth although the substructure approach requires slightly greater computation time.

The introduction of stiff soil or beam elements to model the footing as in methods (c) and (d) increases both the number of equations and the bandwidth with a corresponding increase in computation time.

Table I. Relative computation time for set up and solution for ten load steps

Method	Relative time	No. of equations	Bandwidth
(a) Substructure	1.07	190	20
(b) Displacement defined	1.00	190	20
(c) Stiff soil elements	1.17	200	24
(d) Link elements	1.15	199	24

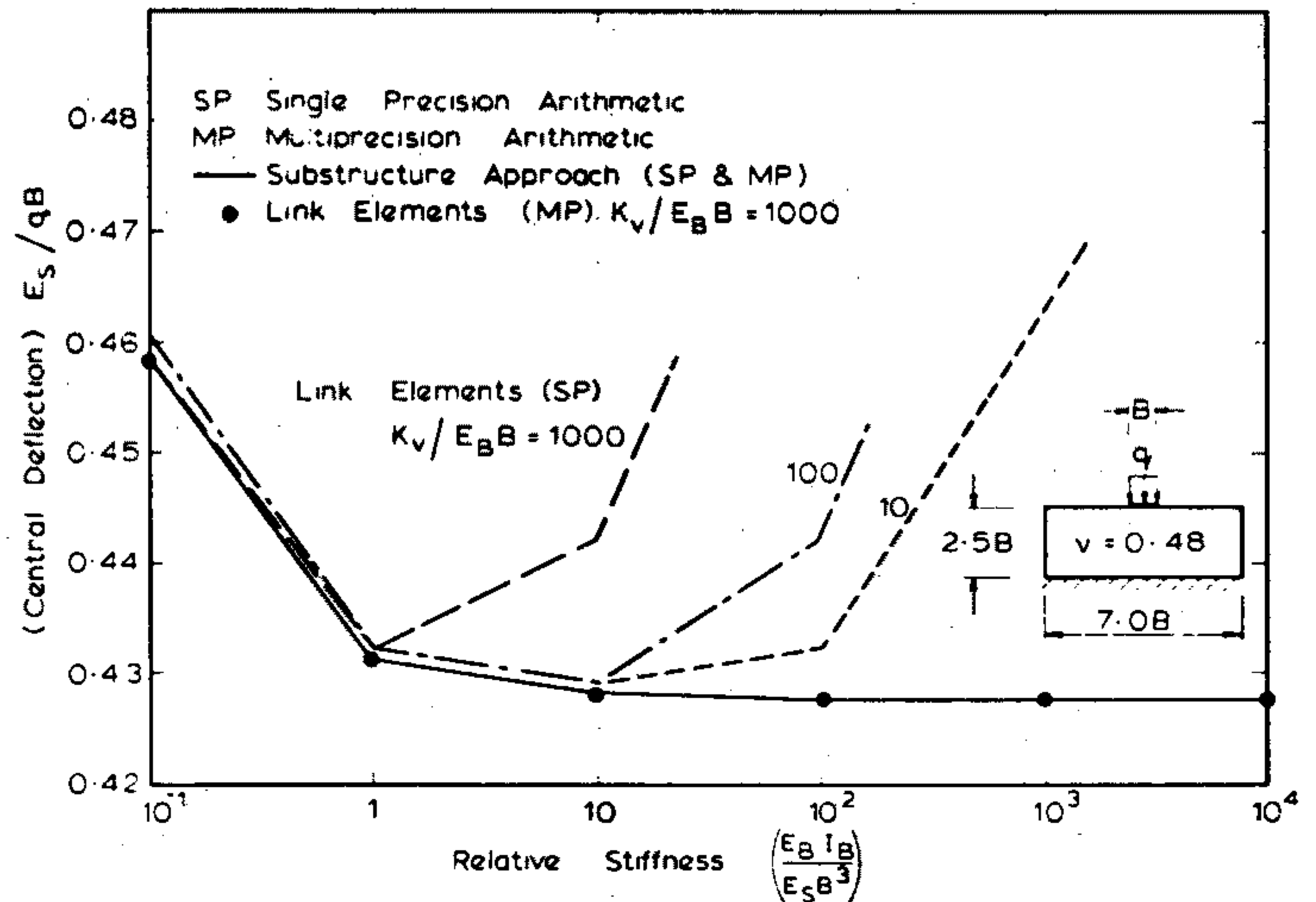


Figure 1. Accuracy of solution methods

A comparison between the displacement and stresses obtained from single and multiprecision arithmetic for each approach is given in Table II. The substructure approach (method (a)) gave identical results for the two analyses. The use of a blanketing number on the diagonal in method (b) was seen to cause very little roundoff error. In a single precision analysis, the stiff soil element of method (c) caused unacceptable roundoff errors for stiffness ratios sufficiently high to give a rigid behaviour when using multiprecision arithmetic. Similarly, the introduction of high stiffness for the quasi-rigid beam element and the corresponding higher link element

Table II. Comparison of roundoff error single precision arithmetic prime 300

Method	Percentage error*	
	Worst error in displacement	Worst error in stress
(a) Substructure	<0.05	<0.01
(b) Displacement defined		
$N_B/E_s = 10^4$	0.1	0.13
$N_B/E_s = 10^5$	0.1	0.13
(c) Stiff soil elements		
$E_B/E_s = 500$	10.0	21.0
$E_B/E_s = 1000$	30.0	53.0
$E_B/E_s = 2000$	83.0	170.0
(d) Link elements	(see Figure 1)	

* Relative to results from the same type of analysis using multiprecision arithmetic.

stiffnesses ~~into the soil~~ stiffness into the soil stiffness matrix (method (d)) caused serious roundoff errors as can be seen from Figure 1.

Except for method (c) which is restricted to the analysis of rough footings, all other methods can be used to analyse either perfectly rough, or perfectly smooth footings. Furthermore, shear failure at the soil-structure interface can be accommodated in the link element and sub-structure approaches.

Using the link element approach, rough footings were analysed by adopting a large horizontal spring stiffness K_H . Considering a purely adhesive interface, once the tangential nodal force reaches the allowable value, horizontal compatibility at that dual node can be released for future load increments. When using the initial stress approach this modification takes the form of a correction added to the load vector. However, the magnitude of the correction terms is such that it is extremely difficult to obtain a convergent solution to these equations for suitable spring stiffness ratios $K_H/E_B B$ and this severely inhibits the advantages of adopting the initial stress method.

CONCLUSION

A general method has been presented for the analysis of soil-structure interaction problems which may involve plastic failure of both soil and structure, breakaway, or shear failure of a frictional, dilatant soil-structure interface. This approach avoids the computational difficulties which may arise from largely differing soil and structural stiffnesses, and the convergence problems which may arise when using conventional joint elements with the initial stress technique.

Although no attempt is made to do so in this paper, clearly, the approach can be extended to problems involving interdependent behaviour of several structures embedded in the soil mass.

APPENDIX

Frictional and dilatant behaviour of the interface

Consider an interface between soil and structure which is governed by a Mohr-Coulomb law and has an angle of friction ϕ_a and an angle of dilatancy¹⁰ ψ_a .

If no slip has occurred the increments in normal and tangential displacements at a dual node $(\dot{U}_{N1}, \dot{U}_{T1}), (\dot{U}_{N2}, \dot{U}_{T2})$ must be equal. Once slip occurs ~~this condition~~ can be replaced by the yield condition and the condition that

$$\tan \psi_a = (\dot{U}_{N2} - \dot{U}_{N1}) / (\dot{U}_{N2} - \dot{U}_{T1}) \quad (14)$$

Thus, if $\psi_a = 0$ the interface is non-dilatant while $\psi_a = \phi_a$ corresponds to the limiting case of a material with an associated flow rule.

The same solution procedure as used in section 2, 'An initial stress-substructure approach', for a purely adhesive interface can be retained by rotating the incremental nodal forces $(\dot{F}_{T1}, \dot{F}_{N1})$ and nodal displacements $(\dot{U}_{T1}, \dot{U}_{N1})$ at each dual node pair through the angles ϕ_a and ψ_a respectively. Thus

$$\begin{Bmatrix} \dot{F}_{\epsilon 1} \\ \dot{F}_{\eta 1} \end{Bmatrix} = \begin{bmatrix} \cos \phi_a & -\sin \phi_a \\ \sin \phi_a & \cos \phi_a \end{bmatrix} \begin{Bmatrix} \dot{F}_{T1} \\ \dot{F}_{N1} \end{Bmatrix} \quad (15a)$$

$$\begin{Bmatrix} \dot{U}_{\epsilon 1} \\ \dot{U}_{\eta 1} \end{Bmatrix} = \begin{bmatrix} \cos \psi_a & -\sin \psi_a \\ \sin \psi_a & \cos \psi_a \end{bmatrix} \begin{Bmatrix} \dot{U}_{T1} \\ \dot{U}_{N1} \end{Bmatrix} \quad (15b)$$

where $(\dot{U}_{\epsilon 1}, \dot{U}_{\eta 1})$ and $(\dot{F}_{\epsilon 1}, \dot{F}_{\eta 1})$ are the rotated incremental nodal displacement and forces.

The nodal forces in the ξ direction are now analogous to the nodal forces in the tangential direction in the previous analysis of a purely adhesive interface with adhesive strength F_{ca} , and the failure condition can now be written in the form—

$$F_{\xi i} = F_{ca} \cos \phi_a \quad (16)$$

By assembling global rotation matrices $[M]$ and $[N]$ corresponding to (15a) and (15b) for the vectors of nodal forces and deflections at the interface, it can be shown that equation (13) can now be replaced by—

$$\begin{bmatrix} [N][I_k][M]^T & -[N][A] \\ -[A]^T[M]^T & [0] \end{bmatrix} \begin{Bmatrix} \dot{F}_{u_{\eta\xi}} \\ \dot{\theta} \end{Bmatrix} = \begin{Bmatrix} [N]\{\dot{\rho}_{2k}\} - \{\dot{\rho}_{1k}\} \\ -[A]^T\{\dot{\phi}_2\} \end{Bmatrix} \quad (17)$$

Once the calculated forces $\{\dot{F}_{u_{\eta\xi}}\}$ are transformed back to the normal and tangential directions using the inverse of equations (15a) the solution procedure is identical to that outlined previously.

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SOIL-STRUCTURE INTERACTION
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A METHOD OF ANALYSIS
FOR HORIZONTALLY EMBEDDED ANCHORS
IN AN ELASTIC SOIL

by

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SUMMARY

A general method is presented for the analysis of horizontally embedded anchors in an elastic soil. Provision is made in the analysis for the consideration of anchor shape, layer depth anchor-soil interface condition, breakaway of the anchor from the underlying soil and interaction between groups of anchors.

Application of the analytical technique is illustrated for strip and circular anchors and these solutions are presented in the form of influence charts which may be used directly in hand calculations to predict the elastic load deflection behaviour of anchor plates for a wide variety of material and geometric conditions.

1. INTRODUCTION

Although the behaviour of anchor plates at working loads is of considerable practical importance, previous investigations into the elastic load-deflection behaviour of horizontal anchors have been restricted to the case of a fully bonded, perfectly flexible or infinitely rigid anchor resting in either an elastic half-space (Fox, 1948) or within an infinite elastic mass (Selvadurai, 1976). In this paper, an analytical technique will be presented for the analysis of an anchor of general shape buried at a depth h below the surface of an isotropic elastic soil layer (with elastic modulus E and Poisson's ratio ν) which extends a further distance D below the anchor plate. The anchor plate itself may be either perfectly flexible or rigid while the anchor-soil interface may be either smooth or rough. In addition, provision is made for slip between anchor and soil as well as breakaway of the anchor from the underlying soil.

2. THEORY

2.1 Development of the Solution Method

Referring to Fig.1, R is a horizontal anchor plate embedded in an elastic layer. The horizontal plane containing the anchor $R (z = z_0)$ divides the layer into two parts $\Gamma^{(+)} (z_0 < z < z^{(+)})$ above the anchor and $\Gamma^{(-)} (z^{(-)} < z < z_0)$ below the anchor. The stress and displacement fields in $\Gamma^{(+)}$, $\Gamma^{(-)}$ will be denoted by the superscripts $(+)$, $(-)$ respectively. In general, these

quantities will suffer a "dislocation" at the interface R due to the presence of the plate. Thus, for example, the vertical stress just above and just below the plate, will in general differ, due to the action of the applied load on the plate and the horizontal displacements above and below the plate may differ due to slip at the soil-plate interface.

In the most general case, the quantity

$$\underline{p}^T = (\sigma_{zx}, \sigma_{zy}, \sigma_{zz}, u, v, w) \quad (1a)$$

will undergo the jump

$$\Delta \underline{p} = \underline{p}^{(+)} - \underline{p}^{(-)} \quad (1b)$$

at the interface.

In order to analyse the behaviour of the anchor plate, suppose that the region R is divided into n sub-regions R_i as indicated in Fig.1. Further suppose that, to sufficient accuracy, it can be assumed that $\Delta \underline{p}$ takes the constant value $\Delta \underline{p}_i$ over each sub-region R_i .

In the following section, it will be shown how it is possible to express the field quantity $\underline{q}^T = (\sigma_{zx}^{(-)}, \sigma_{zy}^{(-)}, \sigma_{zz}^{(-)}, u^{(+)}, v^{(+)}, w^{(+)})$ on the plane $z = z_0$ in terms of the value $\Delta \underline{p}_i$ and in particular, that it is possible to establish the relation.

$$\bar{\underline{q}}_j = \sum_{k=1}^n I_{jk} \underline{p}_k \quad (2)$$

where I_{jk} is a 6 x 6 matrix of influence coefficients and $\bar{\underline{q}}_j = \frac{1}{A_j} \cdot \int_{R_j} \underline{q} \, dA$ is the average value of $\underline{q}$ over the region R_j .

Equation (2) may be rewritten in the partitioned matrix form:

$$\begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} I_{11} & \vdots & I_{1n} \\ \vdots & \ddots & \vdots \\ I_{n1} & \vdots & I_{nn} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{bmatrix} \quad (3)$$

The above relation represents a set of $6n$ equations connecting the $6n$ average quantities $(\bar{\sigma}_{zx}^{(-)}, \bar{\sigma}_{zy}^{(-)}, \bar{\sigma}_{zz}^{(-)}, \bar{u}^{(+)}, \bar{v}^{(+)}, \bar{w}^{(+)})_i$ and the $6n$ quantities $(\Delta\sigma_{zx}, \Delta\sigma_{zy}, \Delta\sigma_{zz}, \Delta u, \Delta v, \Delta w)_i$.

This relationship can be used to solve a variety of problems by specifying the appropriate stress, displacement and dislocation conditions at the interface. In particular, provision can be made for either a rough or smooth anchor with or without breakaway of the anchor from the underlying region in $\Gamma^{(-)}$. For example, consider a smooth anchor subjected to a prescribed vertical displacement w_0 . Provided it is assumed that the anchor-soil interface cannot sustain tensile tractions, then breakaway of the anchor from the underlying region in $\Gamma^{(-)}$ ($z = z_0$) will occur over the sub-region R_i , if, and only if, the normal traction acting on that portion of the interface has reached zero. Consequently, the general case where there is progressive breakaway with increasing load can be analysed by following an incremental procedure in which breakaway occurs over a

sub-region R_i when the vertical stress at the interface of R_i with $\Gamma^{(-)}(z=z_0)$ reaches zero. Prior to breakaway the quantities $(w^{(+)}, \sigma_{zx}^{(-)}, \sigma_{zy}^{(-)}, \Delta w, \Delta \sigma_{zx}, \Delta \sigma_{zy})_i$ take the specified values $(w_0, 0, 0, 0, 0, 0)$ whereas after breakaway the 6 quantities $(w^{(+)}, \sigma_{zx}^{(-)}, \sigma_{zy}^{(-)}, \sigma_{zz}^{(-)}, \Delta \sigma_{zx}, \Delta \sigma_{zy})_i$ take the specified values $(w_0, 0, 0, 0, 0, 0)$. Equation (3) then represents a set of $6n$ equations in $6n$ unknowns for each increment of load.

2.2 Determination of Influence Coefficients

In order to determine the influence coefficients of equations (2), (3), the equations of elasticity are transformed by the application of a double Fourier transform (Sneddon, 1951), or equivalently the field quantities are assumed to have a Fourier representation. Thus, typically, the displacement in the x direction will have the representation:

$$u(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{+i(\alpha x + \beta y)} U(\alpha, \beta, z) d\alpha d\beta \quad (4a)$$

where

$$U(\alpha, \beta, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(\alpha x + \beta y)} u(x, y, z) dx dy \quad (4b)$$

In order to be specific in what follows, it will be assumed that the anchor plate is embedded in a single layer of a homogeneous elastic material, although the technique has a much more general application and may be extended to allow consideration of layered cross-anisotropic deposits. (e.g. Gerrard & Harrison, 1971).

Introduction of the Fourier representation equation (4a), enables the solution in the upper portion of the layer (Γ^+) to be expressed in the form

$$\tilde{P}^{(+)} = M(\alpha, \beta, z) \cdot \tilde{c}^{(+)} \quad (5a)$$

where $\tilde{P}^{(+)} = (\tilde{\Sigma}_{zx}, \tilde{\Sigma}_{zy}, \tilde{\Sigma}_{zz}, U, V, W)^T$ is the vector of transforms of $(\sigma_{zx}, \sigma_{zy}, \sigma_{zz}, u, v, w)^T$
 $\tilde{c}^{(+)}$ is a vector of 6 arbitrary constants
and $M(\alpha, \beta, z)$ is the matrix defined in Table 1.

Similarly, the solution in the lower portion of the layer (Γ^-) may be represented in the form

$$\tilde{P}^{(-)} = M(\alpha, \beta, z) \tilde{c}^{(-)} \quad (5b)$$

The 12 constants $\tilde{c}^{(+)}$, $\tilde{c}^{(-)}$ may be determined from the 3 boundary conditions at the upper surface ($z = z^{(+)}$), the 3 boundary conditions at the lower surface ($z = z^{(-)}$) and the 6 "jump" conditions at the interface ($z = z_0$), by solving

$$A \cdot \tilde{c} = \tilde{b} \quad (6)$$

where

$$\tilde{c} = \begin{bmatrix} \tilde{c}^{(+)} \\ \tilde{c}^{(-)} \end{bmatrix}$$

and where A and $\tilde{b}$ are given in Table 2, for the specific case of an elastic layer resting on a rough rigid base and with a stress free upper surface.

The solution of equation (5) leads to the relations

$$\underline{c}^{(+)} = N^{(+)}(\alpha, \beta) \cdot \Delta \underline{P} \quad (7a)$$

$$\underline{c}^{(-)} = N^{(-)}(\alpha, \beta) \cdot \Delta \underline{P} \quad (7b)$$

where it will be recognised that the 6x6 matrices $N^{(+)}(\alpha, \beta)$ and $N^{(-)}(\alpha, \beta)$ can be obtained by deleting columns 1-3, 10-12 from A^{-1} and then deleting rows (7-12) and (1-6) respectively.

It now follows from equations (5a) and (7a) and equations (5b) and (7b) that

$$\underline{p}^{(+)} = M(\alpha, \beta, z) \cdot N^{(+)}(\alpha, \beta) \cdot \Delta \underline{P}$$

and
$$\underline{p}^{(-)} = M(\alpha, \beta, z) \cdot N^{(-)}(\alpha, \beta) \cdot \Delta \underline{P}$$

and hence

$$\underline{Q} = (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) \cdot \Delta \underline{P} \quad (8)$$

where $D^{(-)}$ and $D^{(+)}$ are (6x6) matrices with zero components except for values of unity corresponding to the first three and last three diagonal terms respectively.

It follows from the representation formulae, such as equation (4a), that

$$\underline{q} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\alpha x + \beta y)} (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) \cdot \Delta \underline{P} \, d\alpha d\beta \quad (9)$$

The average value of q over R_j is given by

$$\bar{q}_j = \frac{1}{A_j} \int_{R_j} q \, dA$$

so that from equation (9)

$$\begin{aligned} \bar{q} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_j(\alpha, \beta) & (D^{(-)} M(\alpha, \beta, z_0) N^{(-)}(\alpha, \beta) \\ & + D^{(+)} M(\alpha, \beta, z_0) N^{(+)}(\alpha, \beta)) \cdot \Delta P \, d\alpha d\beta \end{aligned} \quad (10a)$$

$$\text{where } \phi_j(\alpha, \beta) = \frac{1}{A_j} \int_{R_j} e^{i(\alpha x + \beta y)} \, dA \quad (10b)$$

If it is assumed that, to sufficient accuracy, the dislocation can be considered to be constant over each sub-region R_k then it follows from equation (4b) that

$$\begin{aligned} \Delta P &= \sum_{k=1}^n \frac{\Delta p_k}{4\pi^2} \int \int_{R_k} e^{-i(\alpha x + \beta y)} \, dA \\ &= \sum_{k=1}^n \frac{A_k}{4\pi^2} \phi_k^*(\alpha, \beta) \Delta p_k \end{aligned} \quad (11)$$

where $\phi_k^*(\alpha, \beta)$ is the complex conjugate of $\phi_k(\alpha, \beta)$ defined by equation (10b) and A_k is the area of the sub-region R_k .

Substitution of equation (11) into equation (10) leads to the final form

$$\bar{q}_j = \sum_{k=1}^n I_{jk} \Delta p_k$$

where

$$I_{jk} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{A_k}{4\pi^2} \phi_j(\alpha, \beta) \phi_k^*(\alpha, \beta) (D^{(-)} \cdot M(\alpha, \beta, z_0) \cdot N^{(-)}(\alpha, \beta) + D^{(+)} \cdot M(\alpha, \beta, z_0) \cdot N^{(+)}(\alpha, \beta)) d\alpha d\beta$$

The evaluation of the term $\phi_j(\alpha, \beta)$ is quite straightforward. For example, it can be easily shown that for a rectangular sub-region R_j defined by the limits

$$|x - \bar{x}_j| \leq \frac{1}{2} \Delta x_j \quad |y - \bar{y}_j| \leq \frac{1}{2} \Delta y_j$$

then

$$\phi_j(\alpha, \beta) = \frac{4e^{i(\alpha \bar{x}_j + \beta \bar{y}_j)} \sin(\alpha \Delta x_j / 2) \sin(\beta \Delta y_j / 2)}{\Delta x_j \Delta y_j \alpha \beta}$$

2.3 Extension to a Fourier Series Representation

The analysis described in the previous section enables the calculation of the influence matrices I_{jk} for an isolated anchor. A case of some practical importance is that of a series of periodically spaced anchors with spacing $2W_x$ in the x direction and $2W_y$ in the y direction. In this case the solution is best represented as a Fourier Series. Thus typically the displacement in the x direction will have the representation:

$$u(x, y, z) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{i(\alpha_n x + \beta_m y)} U(\alpha_n, \beta_m, z)$$

$$\text{where } U(\alpha_n, \beta_m, z) = \frac{1}{4W_x W_y} \int_{-W_y}^{W_y} \int_{-W_x}^{W_x} e^{-i(\alpha_n x + \beta_m y)} u(x, y, z) dx dy$$

$$\begin{aligned} \text{and } \alpha_n &= n\pi/W_x & n &= 0, \pm 1, \pm 2, \dots \\ \beta_m &= m\pi/W_y & m &= 0, \pm 1, \pm 2, \dots \end{aligned}$$

The analysis of this case parallels that described in the previous section, and it is found that for this case

$$I_{jk} = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \frac{A_k}{4W_x W_y} \phi_j(\alpha_n, \beta_m) \phi_k^*(\alpha_n, \beta_m) \left[D^{(-)} \cdot M(\alpha_n, \beta_m, z_0) \cdot N^{(-)}(\alpha_n, \beta_m) + D^{(+)} \cdot M(\alpha_n, \beta_m, z_0) \cdot N^{(+)}(\alpha_n, \beta_m) \right] \quad (12)$$

3. PREDICTION OF THE ELASTIC LOAD DEFLECTION BEHAVIOUR OF ANCHOR PLATES

The method of analysis presented in the previous section may be utilised to determine the stresses and displacements anywhere within the elastic soil mass due to a given load applied to an anchor of general shape. However, for the purpose of illustration, attention will be restricted here to the load displacement relationship for strip and circular anchors. A more detailed investigation, which includes consideration of rectangular anchors, has been presented by Rowe and Booker (1978).

The apparent stiffness of an isolated anchor depends primarily upon Poisson's ratio ν and three geometric parameters; namely, the anchor shape; the embedment ratio h/B ; and the depth of the soil layer beneath the anchor D/B . Unless otherwise stated, the analysis has been performed for the basic case of a smooth rigid anchor resting in a soil layer underlain by a rough rigid base and with self-weight or applied surcharge sufficient to prevent breakaway of the anchor from the underlying

soil, although the effect of certain secondary parameters has also been investigated.

Application of the analytical techniques described in Section 2 to the prediction of the load displacement behaviour of a rigid anchor involves a numerical approximation regarding the number of sub-regions which are necessary to simulate rigid anchor behaviour. In this regard it was found that the load displacement relationship could be obtained to sufficient accuracy by sub-division of a strip into eight strip "elements" and a circle into four annular rings. The adoption of a larger number of "elements" did not significantly alter the apparent stiffness of the anchor. For example, in a strip analysis increasing the number of sub-regions ("elements") from eight to fourteen for a range of parameters (W/B , h/B , D/B , and ν) increased the apparent stiffness by at most 0.8% and generally by less than 0.4%.

The analysis of anchor behaviour within an elastic soil using this method of analysis only requires the solution of a small number of equations whereas alternative approaches which involve sub-division of the soil into numerous finite elements require the solution of quite large systems of equations. Thus for an analysis of comparable accuracy, the proposed method of analysis leads to significant computational savings. Furthermore, these computational savings increase dramatically when considering deep anchors, deep soil layers and when considering the general three dimensional case of a rectangular anchor. A detailed investigation into the application of finite element techniques

for the prediction of anchor behaviour up to and including collapse has been described by Rowe (1978).

3.1 The Behaviour of Strip Anchors

3.1.1 Basic case

The effects of layer depth, embedment ratio and Poisson's ratio upon an isolated, rigid, strip anchor can be conveniently represented by expressing the apparent stiffness $P/\delta E$ in the form

$$\frac{P}{\delta E} = \frac{I_D (h/B = \infty) \cdot I_h}{(1-\nu^2)} \quad (13)$$

where P is the applied load(per unit length) necessary to cause the anchor to displace a distance δ

$I_D (h/B = \infty)$ is an influence factor for the effect of layer depth D/B upon the apparent stiffness evaluated for an anchor with $h/B = \infty$;

I_h is a correction factor for the effect of embedment ratio h/B upon the apparent stiffness; and

where E and ν are Young's Modulus and Poisson's Ratio.

The influence factor I_D is shown in Fig.2 as a function of layer depth D/B for the basic case of a rigid anchor with an infinite embedment ratio ($h/B = \infty$). For comparison purposes, the second limiting case of a rigid surface footing is also shown in this figure. It can be seen that the influence factor

I_D is relatively insensitive to Poisson's ratio, varying by about 20% at most, whereas the actual stiffness $P/\delta E$ varies by more than 50% in the worst case. It will be observed that for the surface strip, the influence factor increases sensibly with Poisson's ratio; however, in the case of the anchor with an infinite embedment ratio there is no consistent trend for the curves with $\nu = 0.2, 0.3$, and 0.4 , although the maximum difference between these curves is less than 4%. This behaviour is a manifestation of the removal of the factor $1/(1-\nu^2)$ in determining I_D and the actual stiffness itself does increase sensibly with Poisson's ratio. In view of the close bracketing of I_D with Poisson's ratio obtained by removing the factor $1/(1-\nu^2)$, the disadvantage of having influence curves which intersect when plotted against D/B was not considered to be serious.

The correction factor I_h is shown in Figs. 3a, b, c and d as a function of embedment ratio h/B for a range of layer depths D/B . This correction factor I_h corresponds to the ratio of the apparent stiffness for a given h/B to the apparent stiffness obtained for a similar case where $h/B = \infty$ and consequently, attains a value of unity at the limiting embedment ratio. Three observations can be made from Fig. 3. Firstly, the limiting embedment ratio increases with layer depth. Secondly, the proportion of the maximum possible stiffness that can be obtained for a particular embedment ratio decreases dramatically with layer depth. Finally, it was found that the correction factor I_h is not particularly sensitive to the value of Poisson's ratio, varying by about 10%, (over the range $0.2 \leq \nu \leq 0.5$) in the worst case.

The foregoing analyses were performed for the basic case of a fully bonded, smooth, rigid anchor resting in a layer underlain by a rough rigid base. The effect of varying these conditions upon the load deflection behaviour of the anchor will now be considered.

3.1.2 Effects of interface between soil layer and rigid base

To investigate the effect of the assumed boundary condition between the soil layer and the rigid base, analyses were performed for an anchor in a layer with $D/B = 8$ for the conditions of a rough and smooth rigid base. The apparent stiffnesses obtained from these analyses are shown in Fig.4. The smooth base condition was found to give stiffnesses which were about 4-6% and 14-20% lower than those obtained with a rough base for Poisson's ratio's of $\nu = 0.3$ and $\nu = 0.5$ respectively.

Since it might be expected that the effect of the base interface condition upon the apparent stiffness would decrease with layer depth D/B , analyses were also performed for an anchor at infinite depth ($h/B = \infty$), for a range of layer depths D/B using the two base conditions. The resulting stiffnesses are shown in Fig.5. From these results, it will be observed that although the significance of the base interface condition does decrease with layer depth D/B , the rate of decrease with depth is relatively slow.

In general, it is considered that the rough base condition would be the more practical case; however if the type of base is considered to be significant then the

predicted stiffness obtained for a rough base using equation (13) can be approximately modified for a smooth base by multiplying this stiffness by the ratio of smooth to rough stiffness obtained for the particular layer depth D/B from Fig.5.

3.1.3 Effect of anchor roughness

The type of interface condition between the anchor and soil was found to be unimportant in the determination of the apparent stiffness, since the difference between the stiffnesses obtained for smooth and rough anchors was always less than 1% and often less than 0.1% for a wide range of layer depths, embedment ratios and values of Poisson's ratio.

3.1.4 Effect of anchor rigidity upon apparent stiffness

The apparent stiffness calculated on the basis of the mean displacement of a uniformly loaded region (corresponding to an infinitely flexible anchor) was found to differ from the stiffness determined for a rigid anchor by between 3% and 6% for a wide range of geometric parameters and values of Poisson's ratio. Thus it can be concluded that provided the loading arrangement does not cause structural failure of the anchor plate itself or alter its geometry significantly then the flexibility of the anchor will not greatly alter the load-(mean)deflection behaviour of the anchor.

3.1.5 Effect of breakaway

It is generally accepted (e.g. Douglas and Davis 1964; Selvadurai, 1976) that the fully bonded (no breakaway) condition corresponds to the most practical limiting case for the application of elastic solutions since at working loads the compressive stresses due to overburden pressure will usually be sufficiently large to prevent the development of tensile stresses between anchor and soil. Accordingly, the majority of solutions presented in this paper are for the case of a fully bonded anchor; however, as outlined in section 2, analyses can be performed for the case where there is breakaway of the anchor from the soil. Assuming that there is no adhesion or suction acting between the anchor plate and the soil, then breakaway will occur when the vertical stress below the anchor is zero. Breakaway corresponds to a separation of anchor from the underlying soil and results in the formation of a gap below the anchor. Immediate breakaway occurs when the vertical stress below the anchor is initially zero and consequently, any increment in load will result in complete separation of the anchor from the underlying soil. To illustrate this effect, the apparent stiffnesses obtained when there is immediate breakaway are shown in Fig.6 for an anchor resting in a layer with depth ratio $D/B = 8$. Comparison of the apparent stiffnesses obtained for the two cases of immediate breakaway and no breakaway (i.e. Figs. 4 and 6) indicates that the influence of breakaway decreases with increasing embedment ratio, while it increases with Poisson's ratio.

The situation in which there is progressive breakaway with increasing load can also be analysed by adopting an incremental procedure (Rowe and Booker, loc. cit.).

3.1.6 Effect of interaction between anchors with an infinite group

By replacing the Fourier integral representation (equation (4)) by a Fourier series expansion (e.g. see equation (12)), it is possible to analyse an individual anchor within an infinite group of parallel anchors with a centre to centre spacing $2W$. To illustrate this, Fig.7 shows the effect of the parameter W/D upon the apparent stiffness for a range of layer depths D/B and it can be seen that for many practical purposes the anchor could be considered to behave as an isolated anchor when $W/D = 3$. This assumption would only lead to an error of 3.5% in the worst case (i.e. $D/B = 1$, $h/B = \infty$, $\nu = 0.5$). Actual isolated anchor behaviour was considered to correspond to a dimensionless spacing $W/D = 5$ since this leads to an error of less than 0.3% in the worst case. Two additional observations can be made from Fig.7. Firstly, the effect of anchor spacing is, as might be expected, greatest for incompressible materials and shallow layers. Secondly, it can be seen that although the embedment ratio alters the magnitude of the relative stiffness $P/\delta E$ for a given layer depth, it does not greatly alter the general effect of the anchor spacing (in terms of layer depth W/D) upon the stiffness, indicating that even in relatively deep layers (e.g. $D/B = 100$) the behaviour of the anchor is in fact dominated by the depth of soil below the anchor rather than the embedment ratio.

3.2 The Behaviour of Circular Anchors

3.2.1 Basic case

The effects of layer depth D/B , embedment ratio and Poisson's ratio upon the apparent stiffness $P/B\delta E$ of an isolated rigid circular anchor can be represented in a similar form to that used for strip anchors viz:

$$\frac{P}{B\delta E} = \frac{I_D (h/B = \infty) \cdot I_h}{1-\nu^2} \quad (14)$$

where P is the applied load, and all the other definitions parallel those given for the strip case.

The influence factor I_D is shown in Fig.8 as a function of D/B for the basic case of a rigid circular anchor with an infinite embedment ratio, and for the second limiting case of a rigid surface footing ($h/B = 0$).

As in the case of strip anchors, the influence factor I_D has been defined so that it is relatively insensitive to Poisson's ratio. For a surface anchor (footing) with a depth ratio D/B greater than 30, the influence factor I_D reaches a unique value which is in excellent agreement with the classical elasticity solution for a rigid circular footing resting on a half space (Poulos & Davis, 1974). Similarly, for depth ratios D/B greater than 20, the solution for an anchor with an infinite embedment ratio agrees with the analytical solution for a bonded circular anchor plate within an infinite elastic mass (Selvadurai, 1976) to better than 1%.

The correction factor I_h is shown in Figs. 9(a) and 9(b) as a function of embedment ratio h/B for a range

of layer depths. Two observations can be made from a comparison of the correction factor I_h obtained for strip and circular anchors (i.e. Figs. 3 and 9). Firstly, the limiting embedment ratio at which a circular anchor behaves as if it was infinitely deep is considerably less than that for a strip anchor. Secondly, the correction factor I_h is relatively insensitive to layer depth for a circular anchor varying by less than 10% for a 10 fold range in D/B (i.e. $5 \leq D/B \leq 50$) whereas the variation of I_h for a strip, over the same range of D/B , may be more than double this value.

3.2.2 Effect of anchor roughness and flexibility

The effects of anchor roughness and flexibility upon the apparent stiffness of circular anchors are basically the same as for strip anchors. In particular, the difference between the apparent stiffness of rough and smooth anchors was less than 1% and the difference between the apparent stiffness determined for a rigid anchor and that determined from the mean displacement of a perfectly flexible anchor was typically about 6%.

3.2.3 Effect of interaction between anchors within an infinite group

For the axi-symmetric problem of a circular anchor plate, the Fourier series expressions given in section 2.3 reduces to the special case of a Fourier-Bessel series. The periodic nature of this expansion permits the consideration of group effects, or the effect of a smooth lateral boundary,

in a similar fashion to that adopted for strip anchors. In this case W is the radial distance from the centre of the anchor to a cylindrical smooth rigid lateral boundary. To a first approximation the analogous case of an infinite group of anchors located on a rectangular grid with centre to centre spacing S may be analysed by equating the area of influence for each anchor (i.e. S^2) to the area contained by a smooth cylindrical boundary, and, hence $W = S/\sqrt{\pi}$. As in the case of the strip anchor, significant interaction effects were observed for spacing ratios W/D less than one, while for spacing ratios W/D greater than three the behaviour is essentially that of an isolated anchor. The apparent stiffness of circular anchors is, in fact, slightly less sensitive to the spacing between adjacent anchors than was the case for strip anchors.

CONCLUSIONS

In this paper an analytical technique has been presented for the prediction of the behaviour of horizontal anchor plates embedded in an elastic soil medium. Provision is made in the analysis for the consideration of anchor shape, layer depth, anchor-soil interface condition, breakaway of the anchor from the underlying soil and interaction between groups of anchors.

The approach has been applied to the case of strip and circular anchors and solutions are presented in the form of influence charts which may be used directly in hand

calculations to predict the elastic load deflection behaviour of anchor plates for a wide variety of material and geometric conditions.

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$M_{11}(z) = \left[\frac{i\alpha}{\gamma} - 2i\alpha\theta z \right] Ge^{\gamma z}$	$M_{12}(z) = - \left[\frac{i\alpha}{\gamma} + 2i\alpha\theta z \right] Ge^{-\gamma z}$	$M_{13}(z) = 2i\alpha\gamma Ge^{\gamma z}$	$M_{14}(z) = -2i\alpha\gamma Ge^{-\gamma z}$	$M_{15}(z) = \beta\gamma Ge^{\gamma z}$	$M_{16}(z) = -\beta\gamma Ge^{-\gamma z}$
$M_{21}(z) = \left[\frac{i\beta}{\gamma} - 2i\beta\theta z \right] Ge^{\gamma z}$	$M_{22}(z) = - \left[\frac{i\beta}{\gamma} - 2i\beta\theta z \right] Ge^{-\gamma z}$	$M_{23}(z) = 2i\beta\gamma Ge^{\gamma z}$	$M_{24}(z) = -2i\beta\gamma Ge^{-\gamma z}$	$M_{25}(z) = -\alpha\gamma Ge^{\gamma z}$	$M_{26}(z) = \alpha\gamma Ge^{-\gamma z}$
$M_{31}(z) = \left[\lambda + 2G(1-\gamma\theta z) \right] e^{\gamma z}$	$M_{32}(z) = \left[\lambda + 2G(1+\gamma\theta z) \right] e^{-\gamma z}$	$M_{33}(z) = 2G\gamma^2 e^{\gamma z}$	$M_{34}(z) = 2G\gamma^2 e^{-\gamma z}$	$M_{35}(z) = 0$	$M_{36}(z) = 0$
$M_{41}(z) = - \frac{i\alpha\theta}{\gamma} ze^{\gamma z}$	$M_{42}(z) = \frac{i\alpha\theta}{\gamma} ze^{-\gamma z}$	$M_{43}(z) = i\alpha e^{\gamma z}$	$M_{44}(z) = i\alpha e^{-\gamma z}$	$M_{45}(z) = \beta e^{\gamma z}$	$M_{46}(z) = \beta e^{-\gamma z}$
$M_{51}(z) = - \frac{i\beta\theta}{\gamma} ze^{\gamma z}$	$M_{52}(z) = \frac{i\beta\theta}{\gamma} ze^{-\gamma z}$	$M_{53}(z) = i\beta e^{\gamma z}$	$M_{54}(z) = i\beta e^{-\gamma z}$	$M_{55}(z) = -\alpha e^{\gamma z}$	$M_{56}(z) = -\alpha e^{-\gamma z}$
$M_{61}(z) = \frac{(1+\theta-\theta z)}{\gamma} e^{\gamma z}$	$M_{62}(z) = - \frac{(1+\theta+\theta z)}{\gamma} e^{-\gamma z}$	$M_{63}(z) = \gamma e^{\gamma z}$	$M_{64}(z) = -\gamma e^{-\gamma z}$	$M_{65}(z) = 0$	$M_{66}(z) = 0$

$$[M(\alpha, \beta, z)] =$$

$$\text{where } \gamma = \sqrt{\alpha^2 + \beta^2}$$

$$\lambda = \frac{vE}{(1+v)(1-2v)}$$

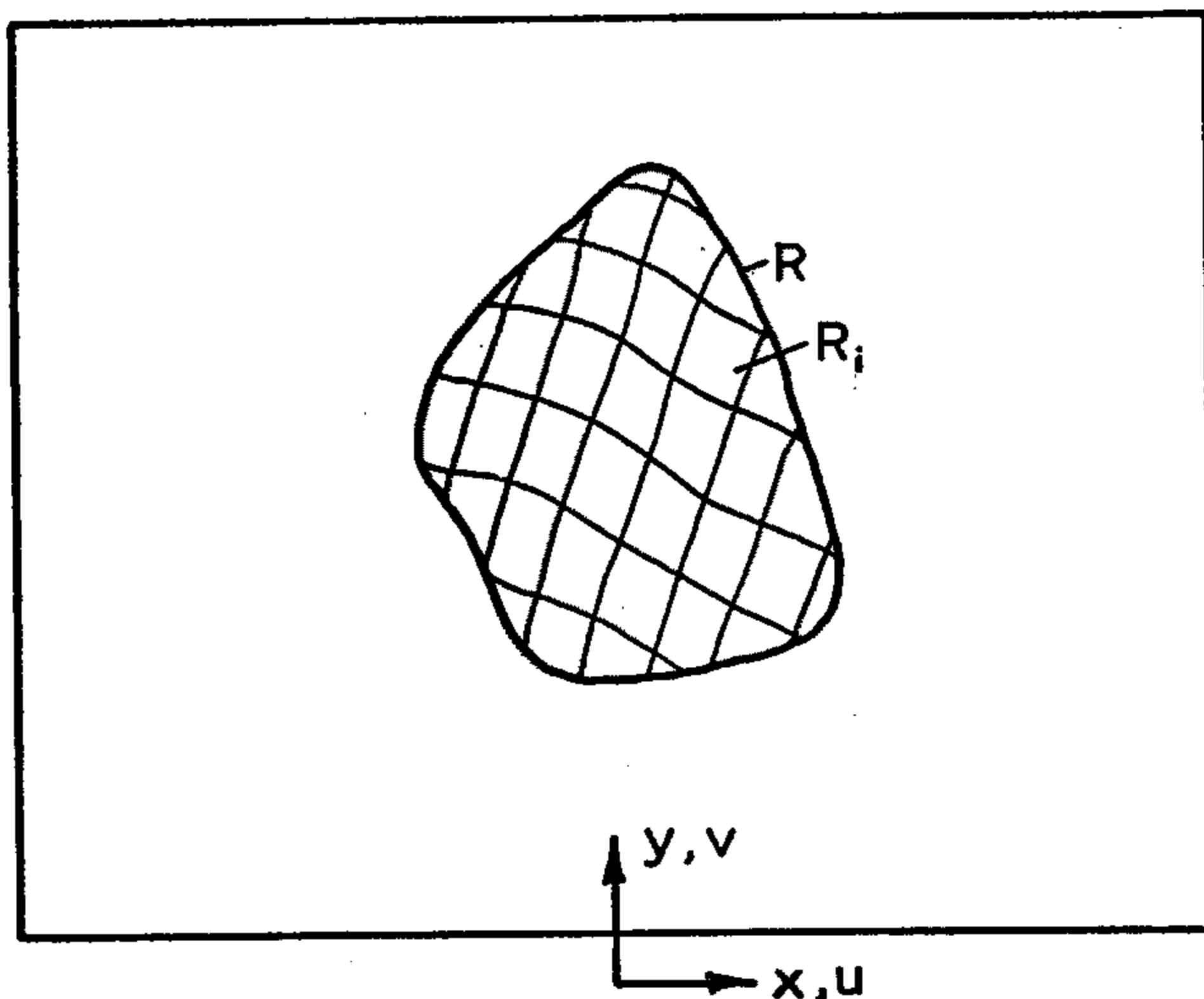
$$G = \frac{E}{2(1+v)}$$

$$\theta = \frac{\lambda+G}{2G} = \frac{1}{2(1-2v)}$$

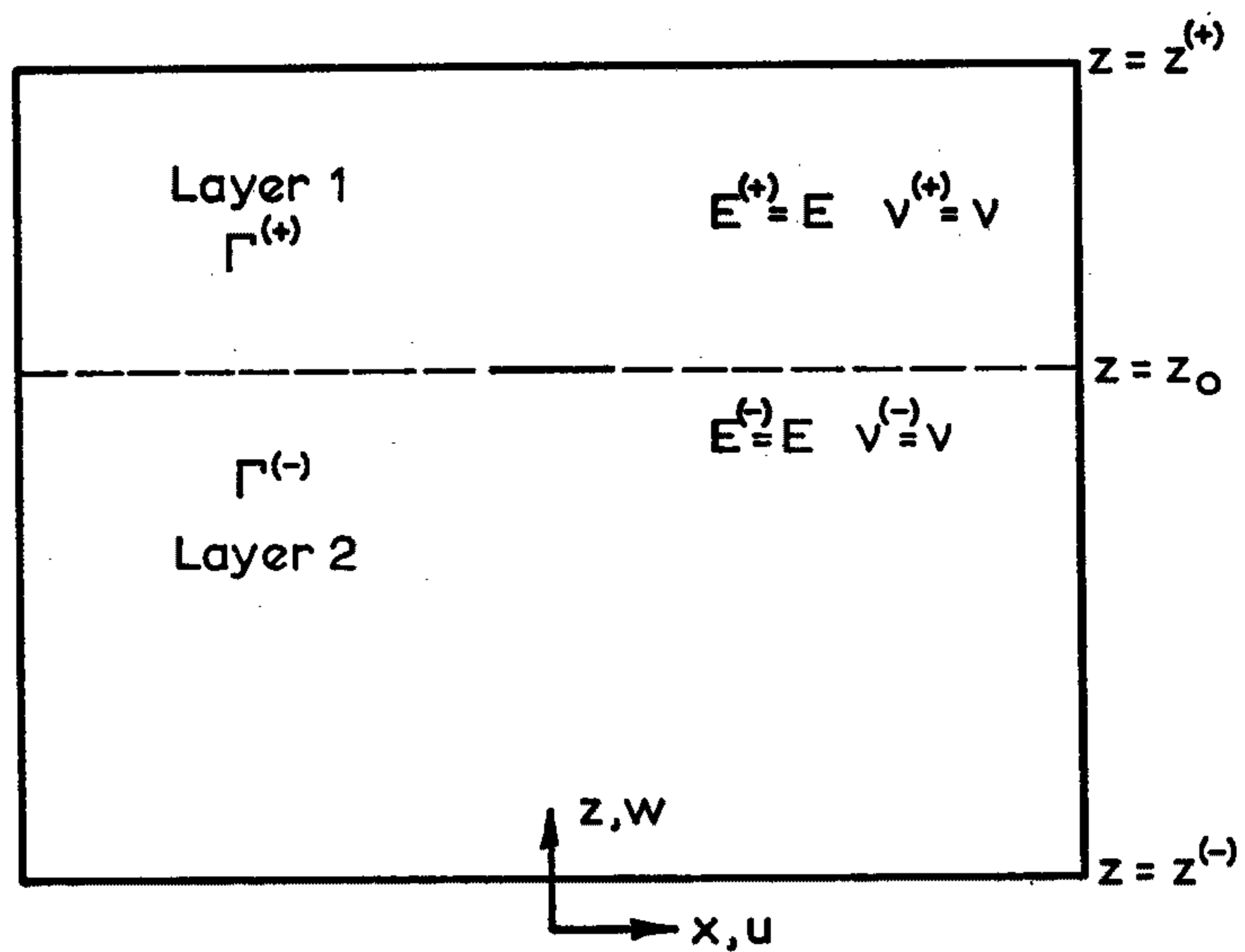
Table 1

[A] =	$M_{11}(z+)$	$M_{12}(z+)$	$M_{13}(z+)$	$M_{14}(z+)$	$M_{15}(z+)$	$M_{16}(z+)$		$-M_{11}(z_0)$	$-M_{12}(z_0)$	$-M_{13}(z_0)$	$-M_{14}(z_0)$	$-M_{15}(z_0)$	$-M_{16}(z_0)$	c_1	(+)	$\Delta\Sigma_{zx}$	0
	$M_{21}(z+)$	$M_{22}(z+)$	$M_{23}(z+)$	$M_{24}(z+)$	$M_{25}(z+)$	$M_{26}(z+)$		$-M_{21}(z_0)$	$-M_{22}(z_0)$	$-M_{23}(z_0)$	$-M_{24}(z_0)$	$-M_{25}(z_0)$	$-M_{26}(z_0)$	c_2	(+)	$\Delta\Sigma_{zy}$	0
	$M_{31}(z+)$	$M_{32}(z+)$	$M_{33}(z+)$	$M_{34}(z+)$	$M_{35}(z+)$	$M_{36}(z+)$	0	$-M_{31}(z_0)$	$-M_{32}(z_0)$	$-M_{33}(z_0)$	$-M_{34}(z_0)$	$-M_{35}(z_0)$	$-M_{36}(z_0)$	c_3	(+)	$\Delta\Sigma_{zz}$	0
	$M_{41}(z_0)$	$M_{42}(z_0)$	$M_{43}(z_0)$	$M_{44}(z_0)$	$M_{45}(z_0)$	$M_{46}(z_0)$		$-M_{41}(z_0)$	$-M_{42}(z_0)$	$-M_{43}(z_0)$	$-M_{44}(z_0)$	$-M_{45}(z_0)$	$-M_{46}(z_0)$	c_4	(-)	$\Delta\Sigma_{zy}$	0
	$M_{51}(z_0)$	$M_{52}(z_0)$	$M_{53}(z_0)$	$M_{54}(z_0)$	$M_{55}(z_0)$	$M_{56}(z_0)$		$-M_{51}(z_0)$	$-M_{52}(z_0)$	$-M_{53}(z_0)$	$-M_{54}(z_0)$	$-M_{55}(z_0)$	$-M_{56}(z_0)$	c_5	(-)	$\Delta\Sigma_{zz}$	0
	$M_{61}(z_0)$	$M_{62}(z_0)$	$M_{63}(z_0)$	$M_{64}(z_0)$	$M_{65}(z_0)$	$M_{66}(z_0)$		$-M_{61}(z_0)$	$-M_{62}(z_0)$	$-M_{63}(z_0)$	$-M_{64}(z_0)$	$-M_{65}(z_0)$	$-M_{66}(z_0)$	c_6	(-)	$\Delta\Sigma_{zz}$	0
0																	
=	$M_{41}(z-)$	$M_{42}(z-)$	$M_{43}(z-)$	$M_{44}(z-)$	$M_{45}(z-)$	$M_{46}(z-)$		$M_{41}(z-)$	$M_{42}(z-)$	$M_{43}(z-)$	$M_{44}(z-)$	$M_{45}(z-)$	$M_{46}(z-)$	c_4	(-)	$\Delta\Sigma_{zy}$	0
	$M_{51}(z-)$	$M_{52}(z-)$	$M_{53}(z-)$	$M_{54}(z-)$	$M_{55}(z-)$	$M_{56}(z-)$		$M_{51}(z-)$	$M_{52}(z-)$	$M_{53}(z-)$	$M_{54}(z-)$	$M_{55}(z-)$	$M_{56}(z-)$	c_5	(-)	$\Delta\Sigma_{zz}$	0
	$M_{61}(z-)$	$M_{62}(z-)$	$M_{63}(z-)$	$M_{64}(z-)$	$M_{65}(z-)$	$M_{66}(z-)$		$M_{61}(z-)$	$M_{62}(z-)$	$M_{63}(z-)$	$M_{64}(z-)$	$M_{65}(z-)$	$M_{66}(z-)$	c_6	(-)	$\Delta\Sigma_{zz}$	0

Table 2



(a) Plan



(b) Elevation

FIG. 1 ANALYTICAL CONFIGURATION

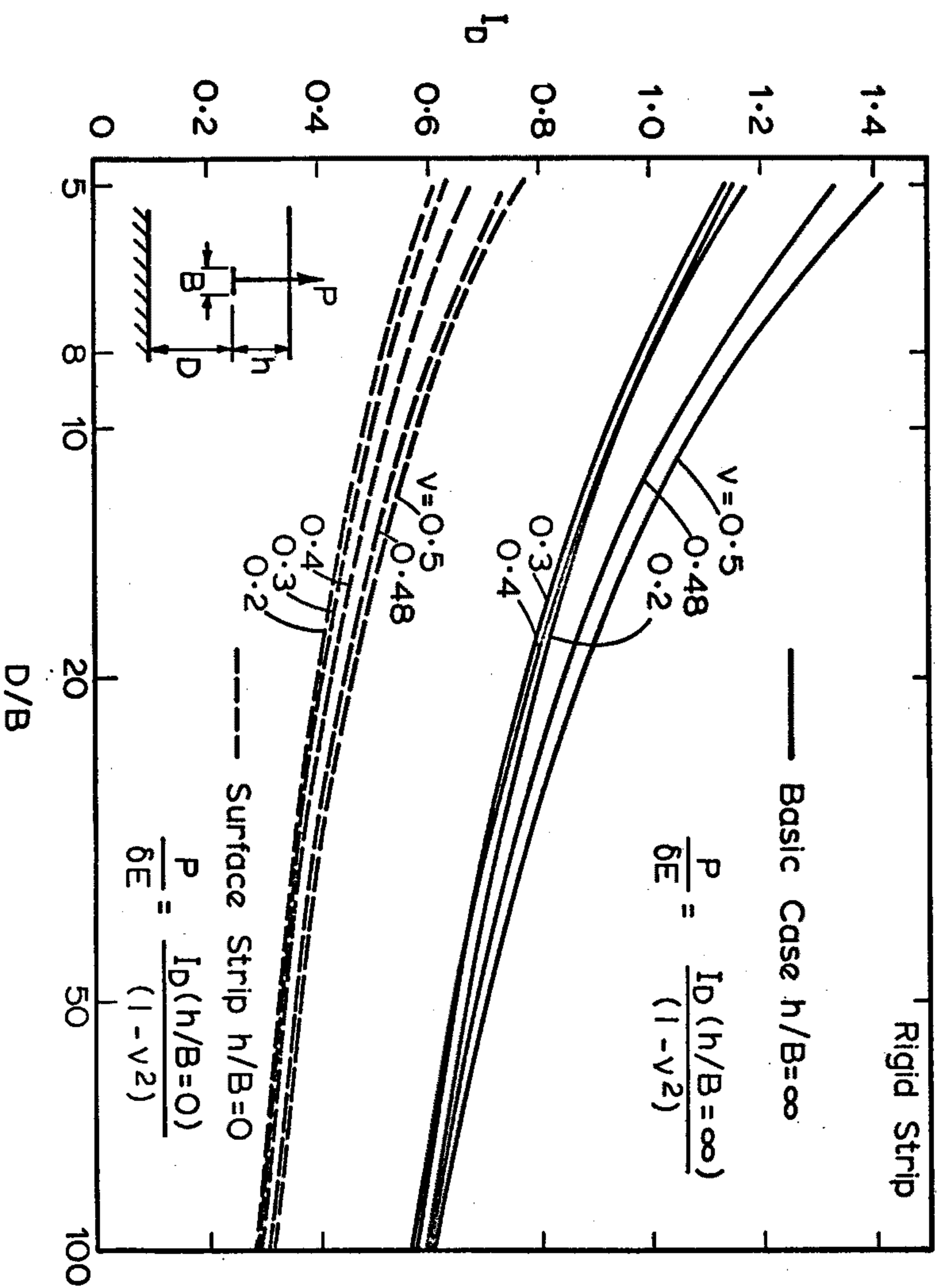


**FIG.2 INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH
UPON ELASTIC LOAD DEFLECTION BEHAVIOUR OF A RIGID STRIP**

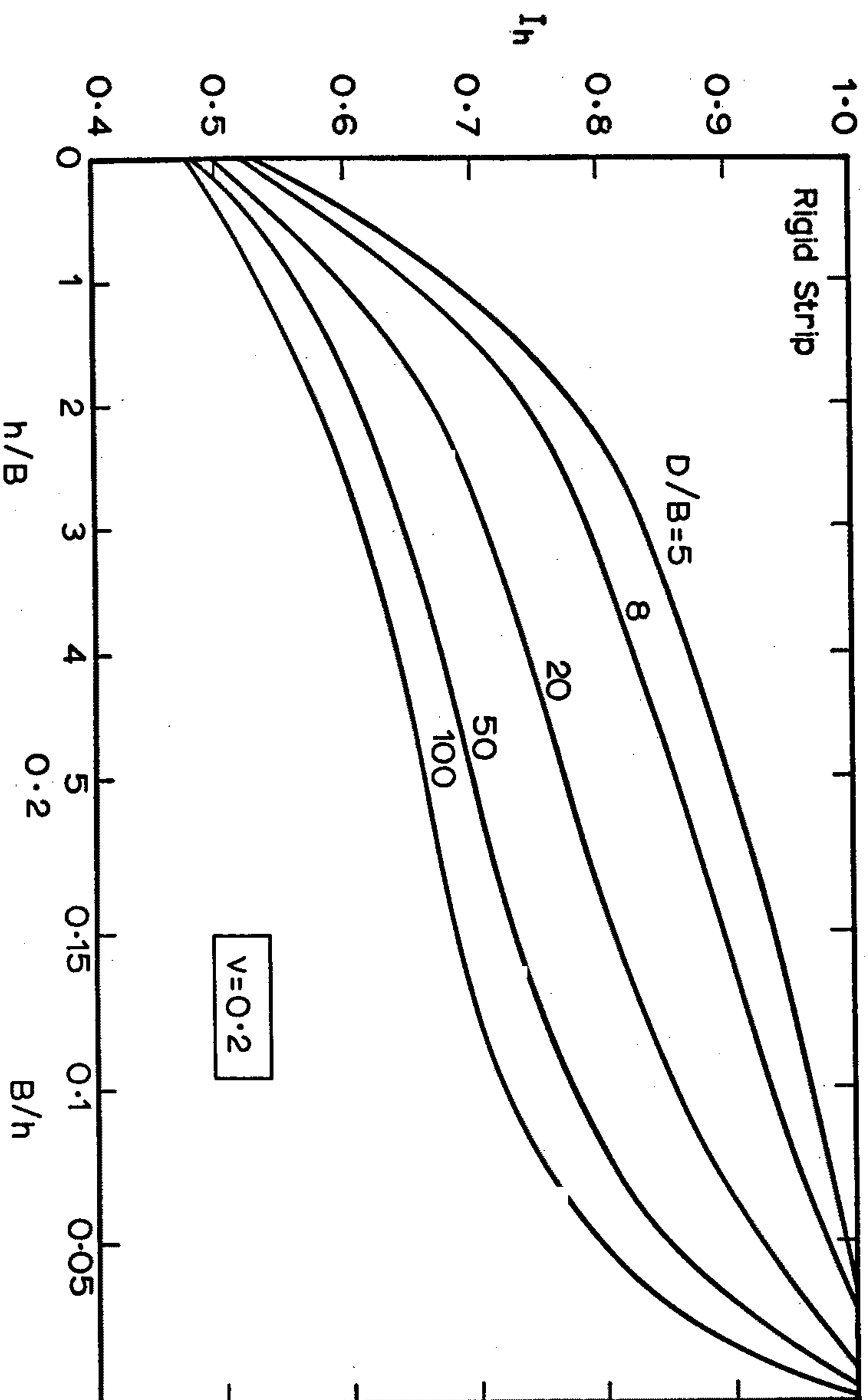


FIG. 3d CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO- $\nu=0.2$

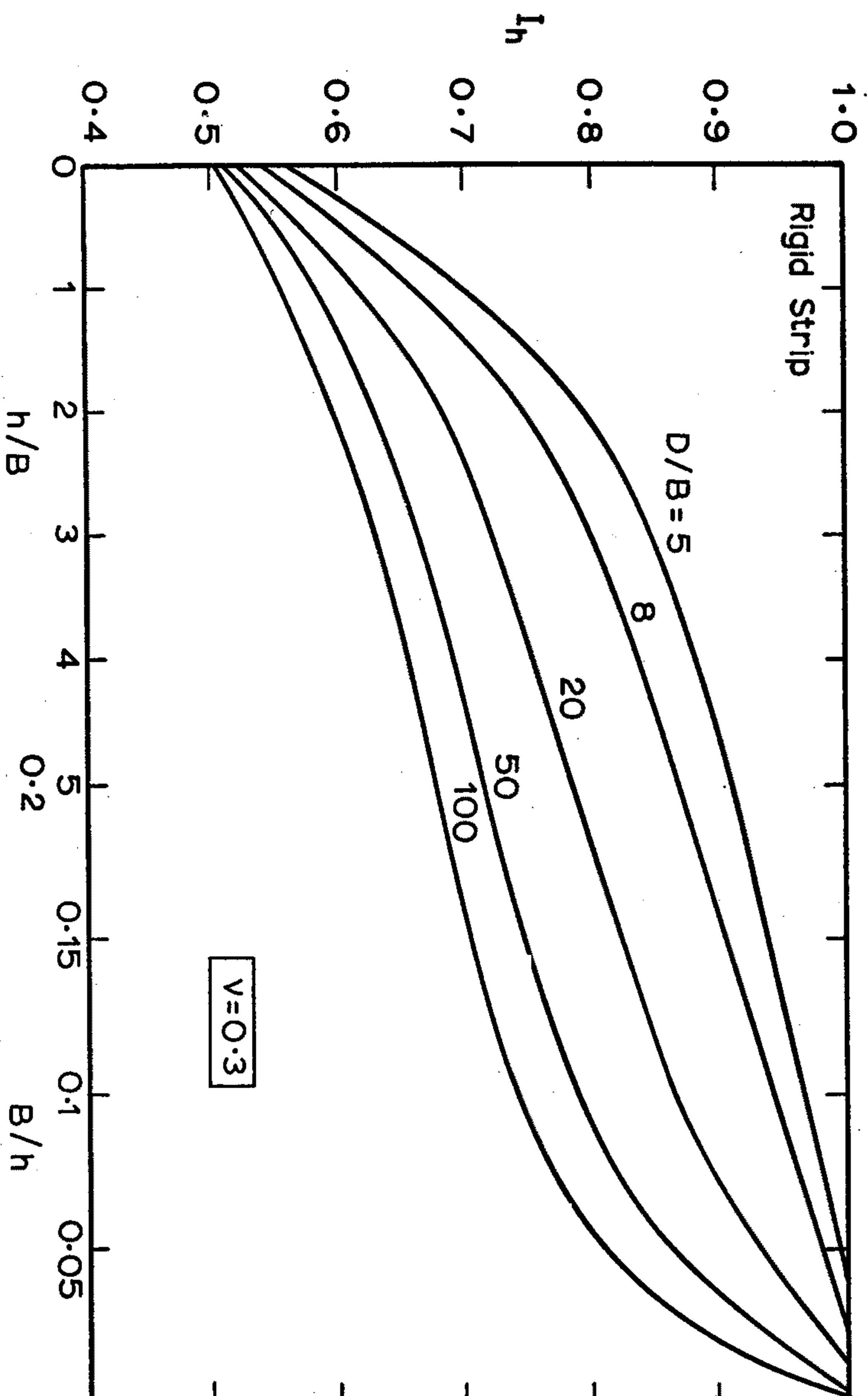


FIG. 3b CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO - $\nu = 0.3$

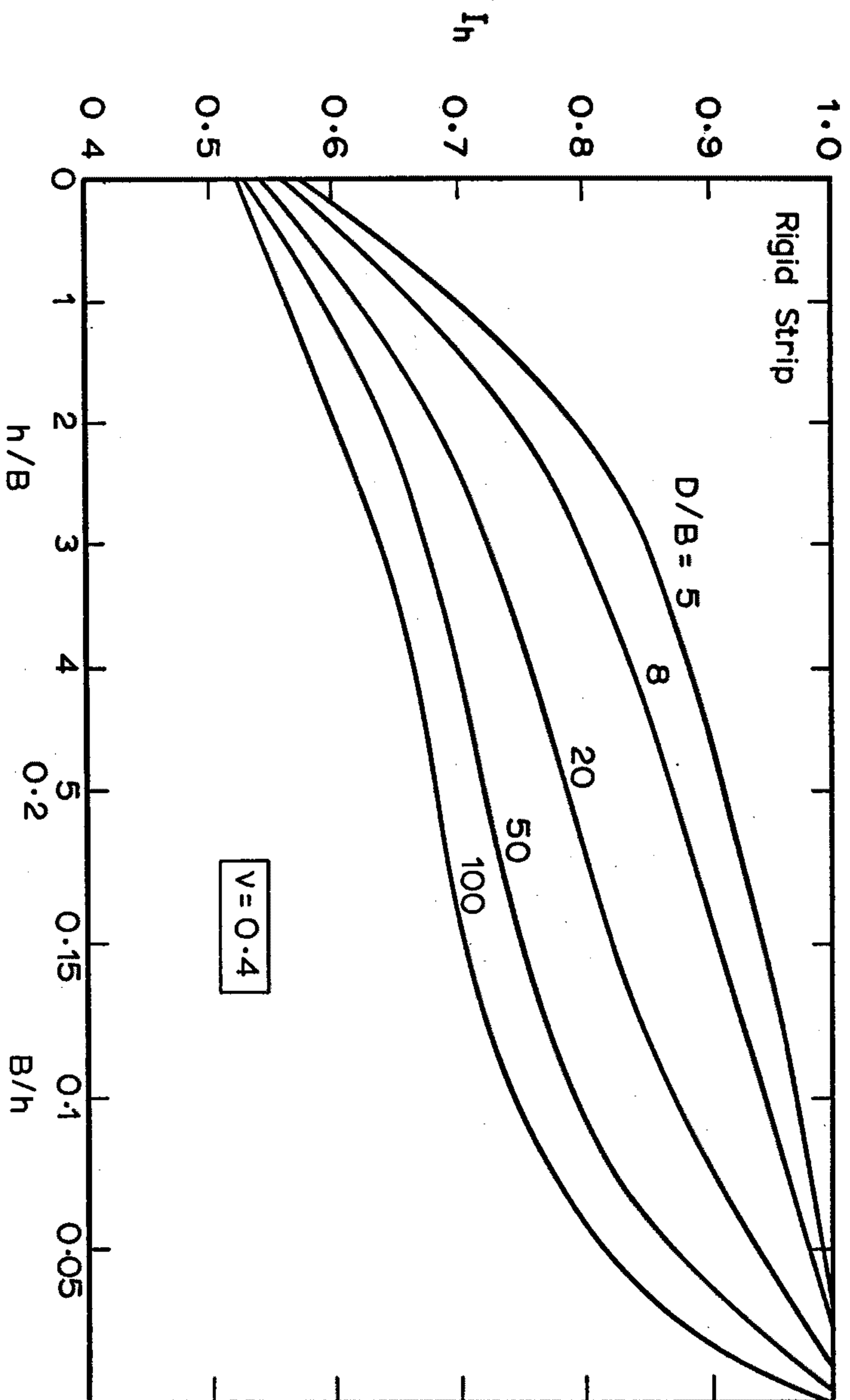


FIG.3c CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO - $\nu = 0.4$

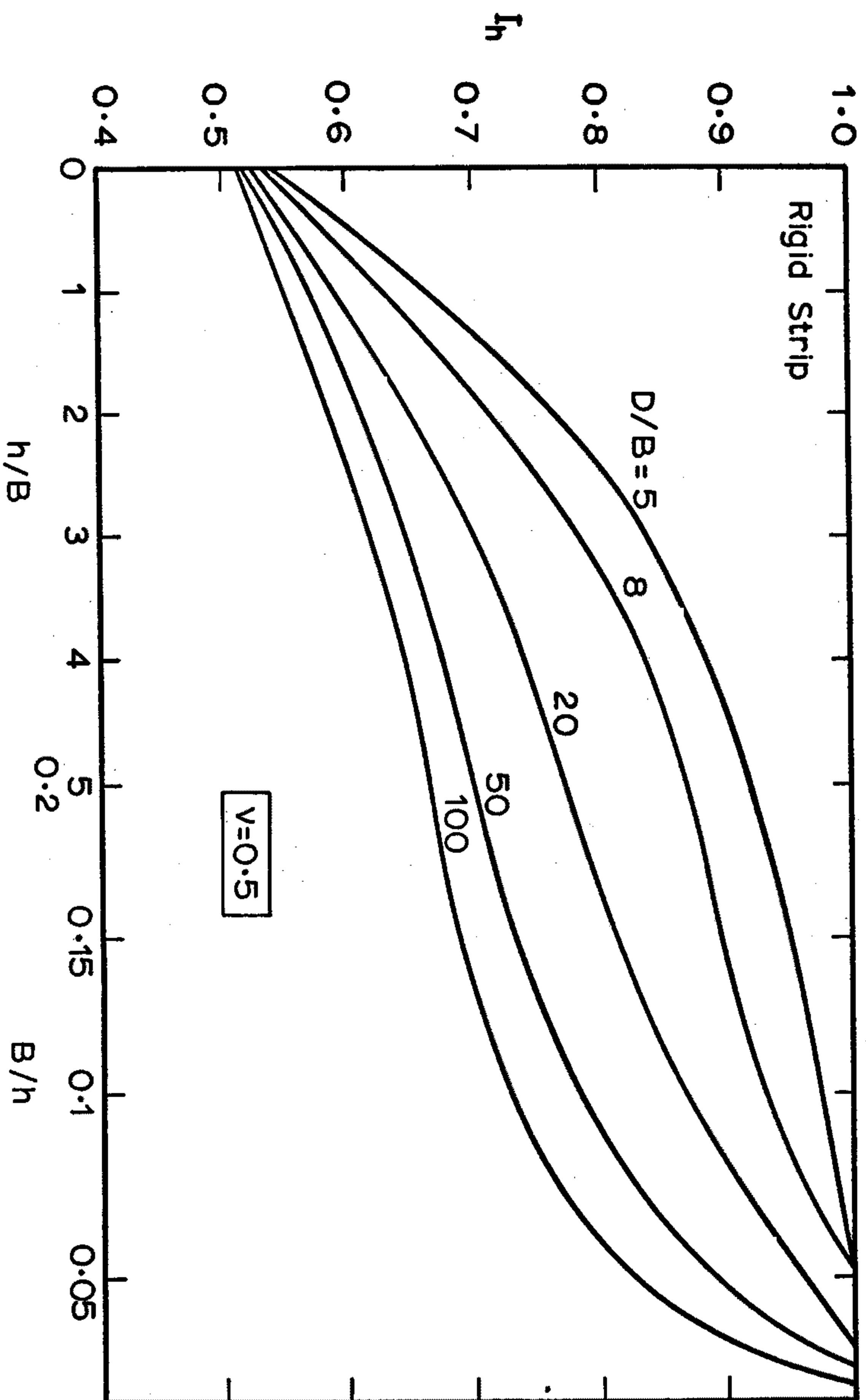


FIG. 3d CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT RATIO- $\nu=0.5$

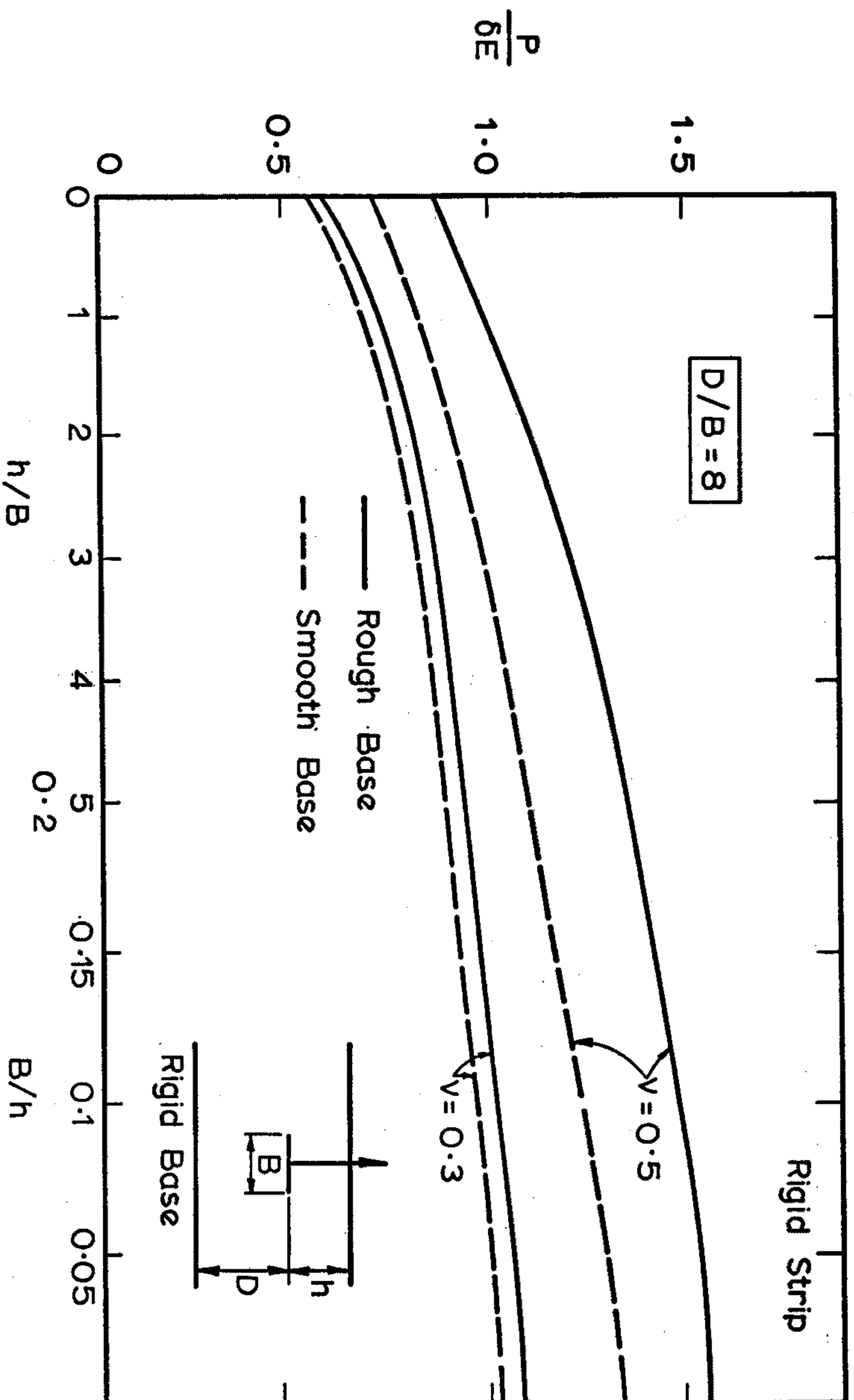


FIG. 4 EFFECT OF BASE CONDITION UPON APPARENT STIFFNESS.

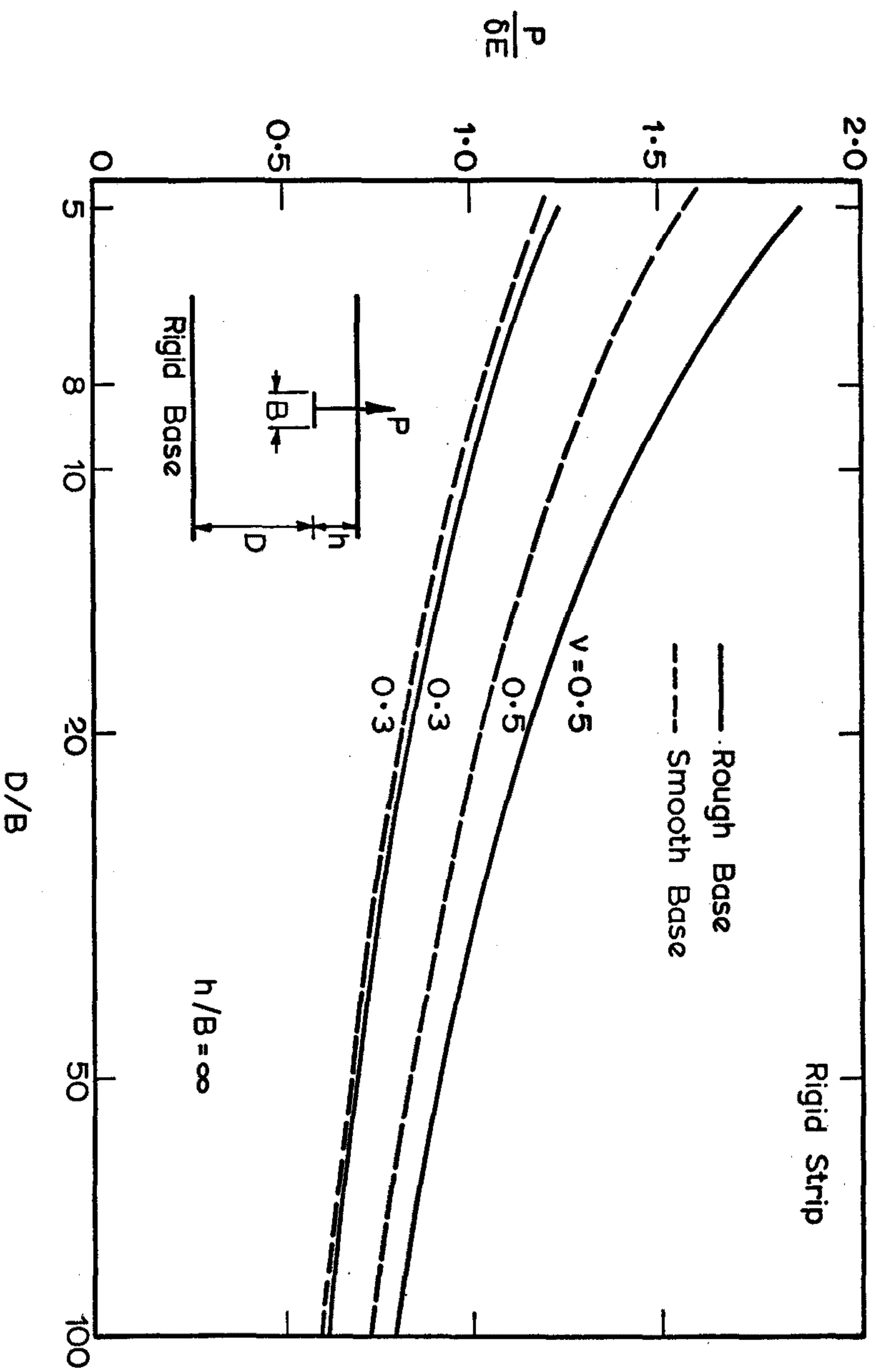


FIG. 5 EFFECT OF BASE CONDITION UPON APPARENT STIFFNESS.

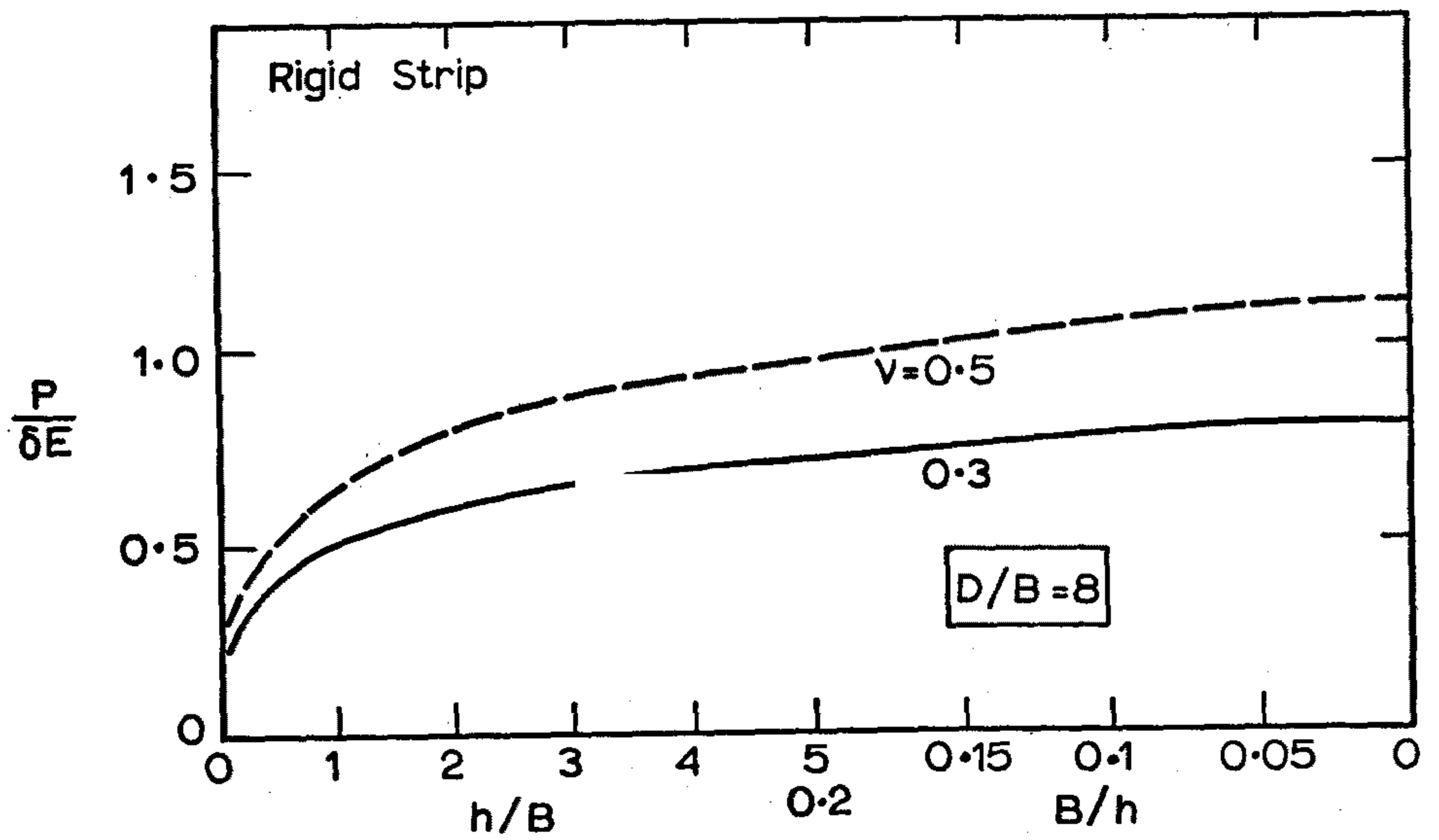


FIG.6 EFFECT OF IMMEDIATE BREAKAWAY UPON APPARENT STIFFNESS

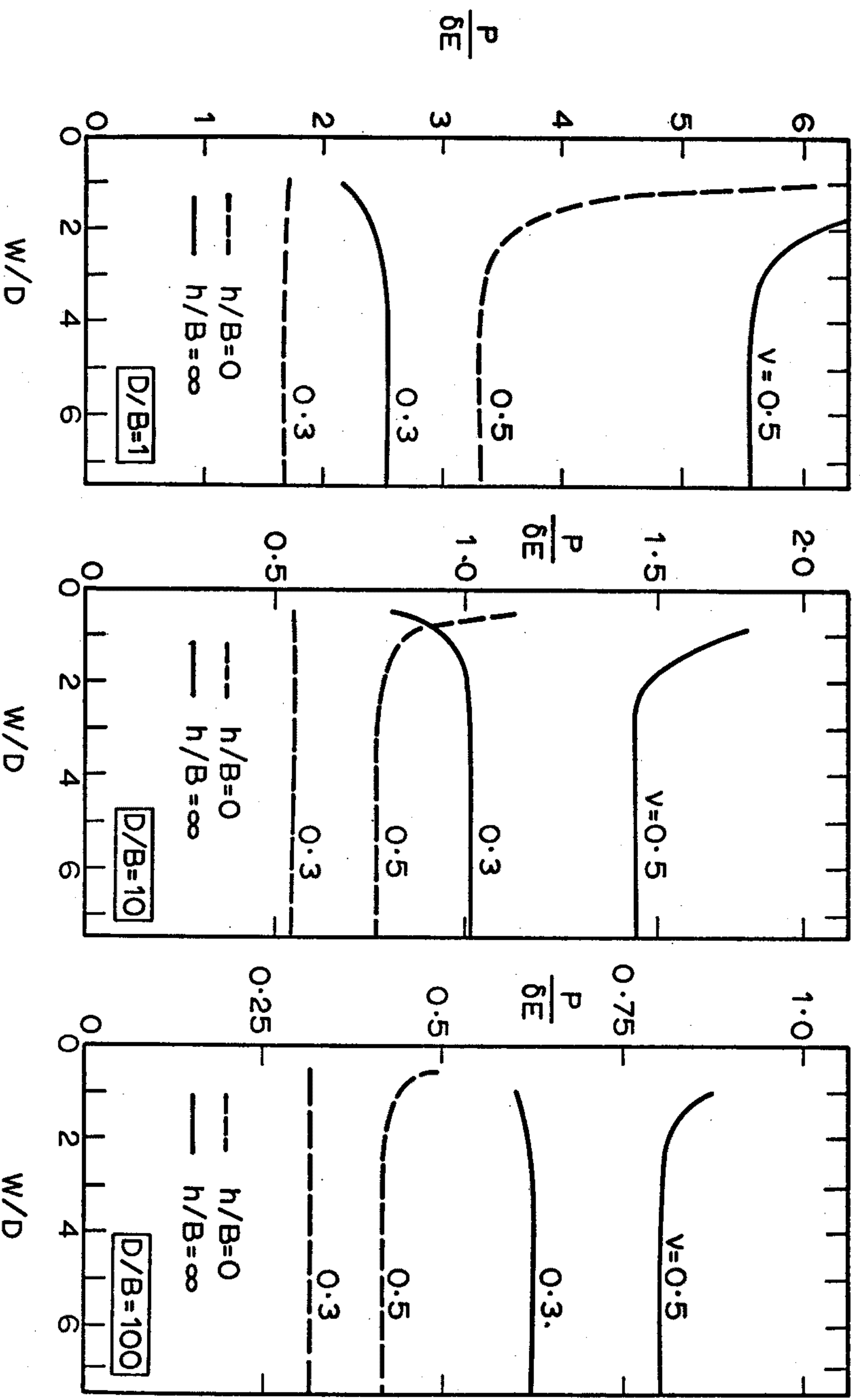


FIG.7 EFFECT OF DISTANCE TO LATERAL BOUNDARY (W/D) UPON APPARENT STIFFNESS OF A RIGID STRIP ANCHOR

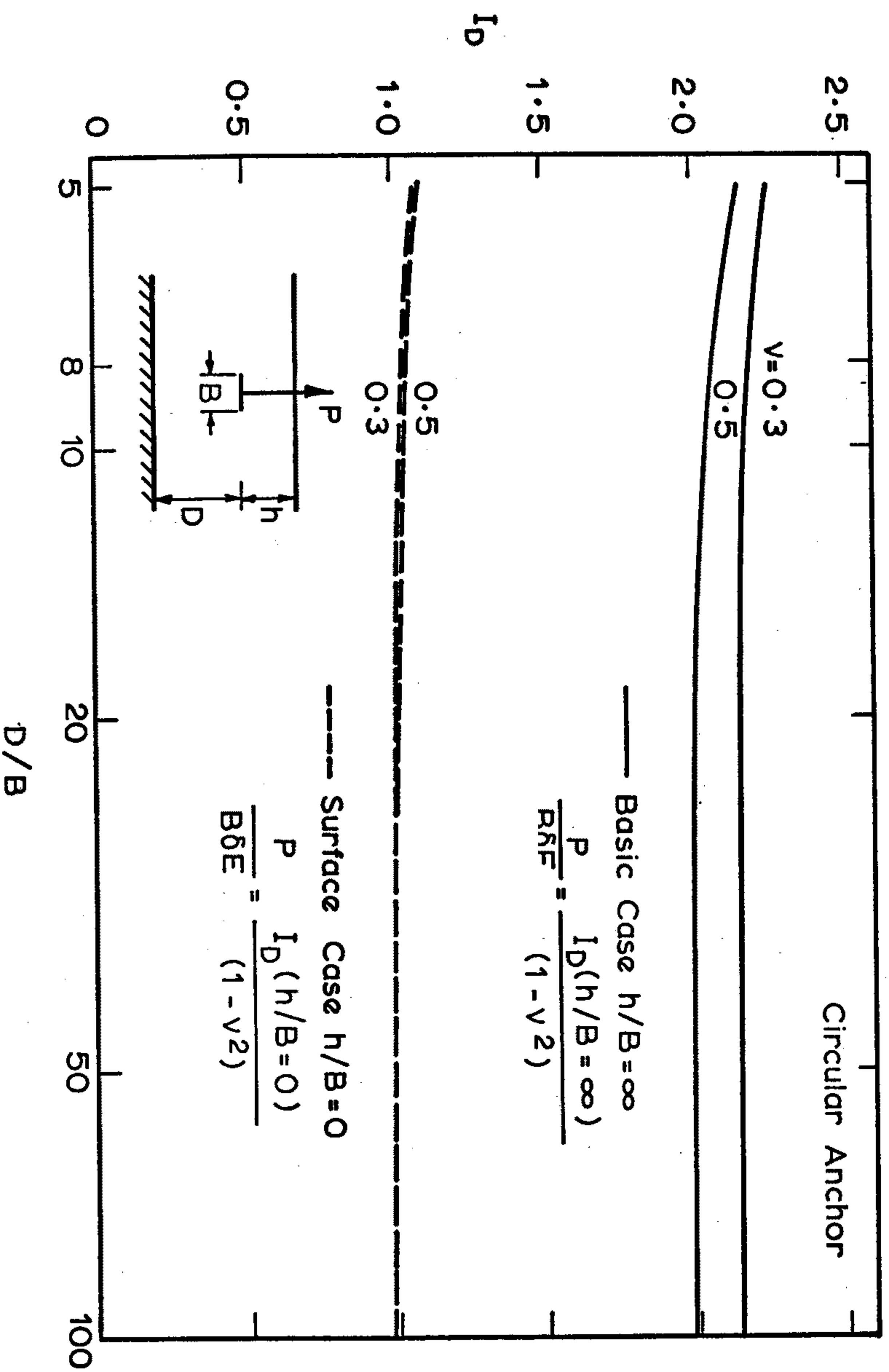


FIG. 8 INFLUENCE FACTOR I_D FOR THE EFFECT OF LAYER DEPTH UPON ELASTIC LOAD DEFLECTION BEHAVIOUR OF A RIGID CIRCULAR ANCHOR

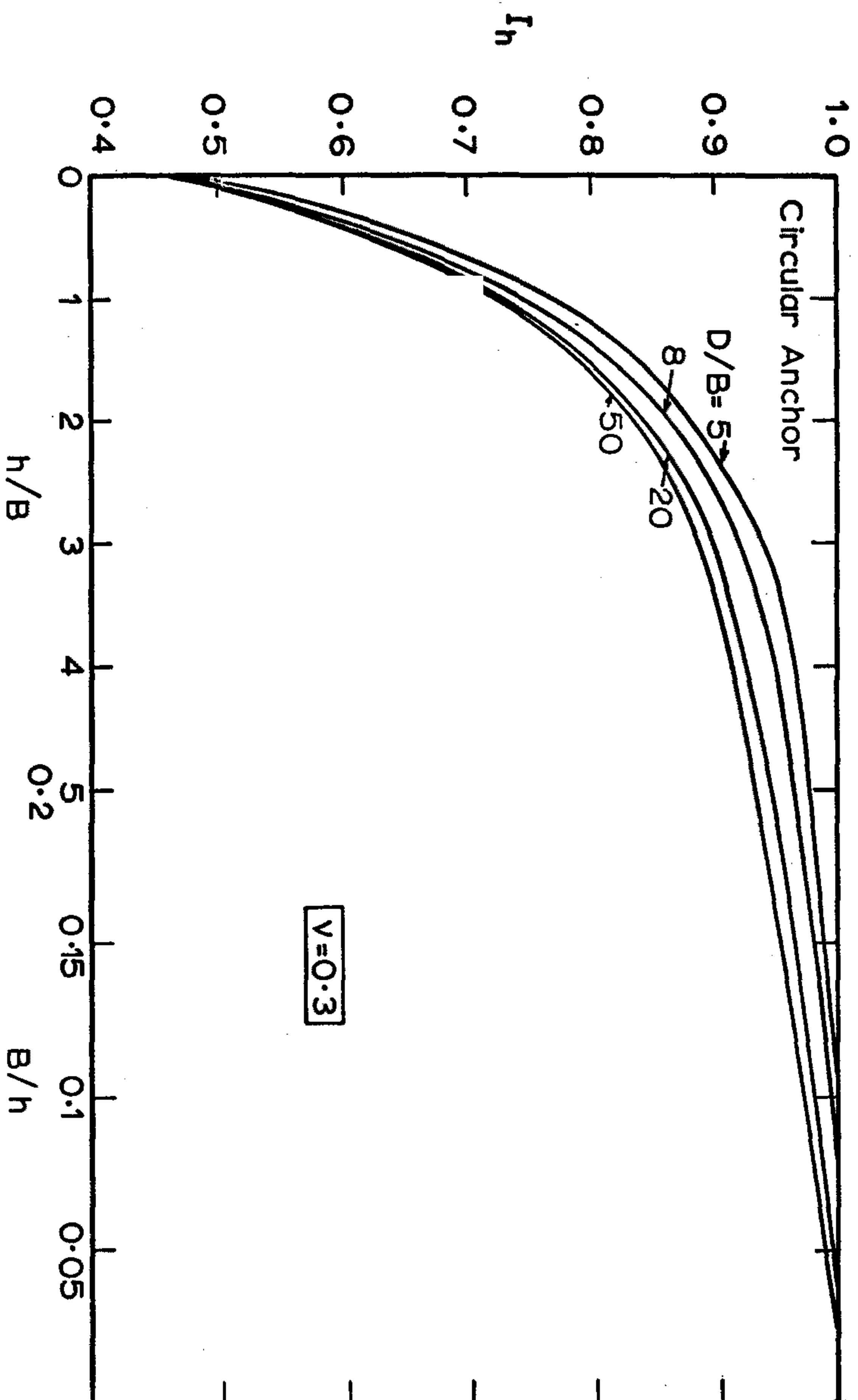


FIG. 9a CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT

ATIO $\nu = 0.3$

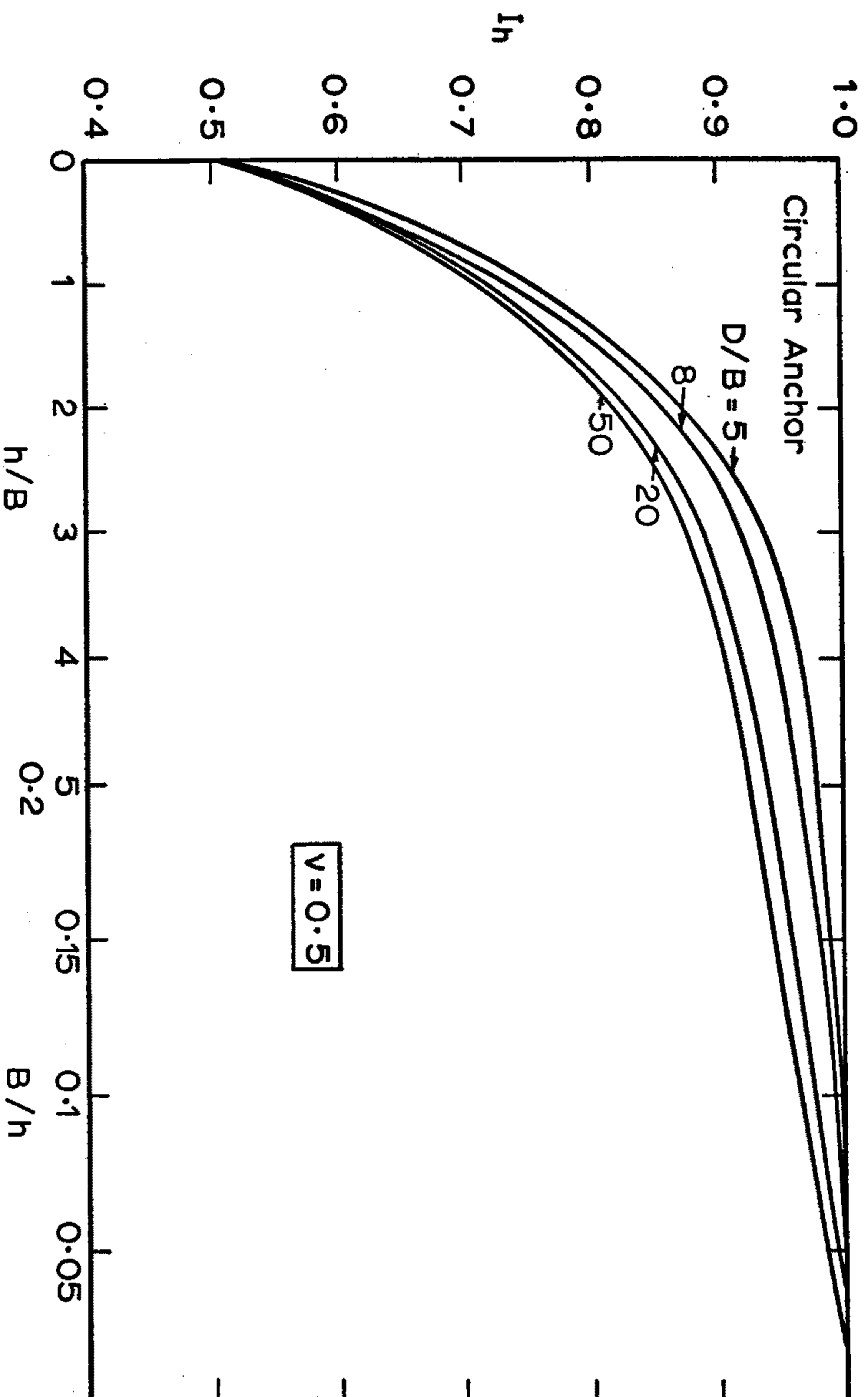


FIG.9b CORRECTION FACTOR I_h FOR THE EFFECT OF EMBEDMENT

RATIO $\nu = 0.5$

SOIL STRUCTURE INTERACTION ANALYSIS
AND ITS APPLICATION TO THE PREDICTION
OF ANCHOR PLATE BEHAVIOUR

VOL. 2

SOIL STRUCTURE INTERACTION ANALYSIS AND ITS
APPLICATION
TO THE PREDICTION OF ANCHOR PLATE BEHAVIOUR

VOLUME II

by

Ronald Kerry Rowe, B.Sc., B.E.

A Thesis submitted for the Degree of Doctor
of Philosophy in the University of Sydney.

July, 1978.

VOLUME II

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CHAPTER EIGHT
LABORATORY TESTS ON
HORIZONTALLY EMBEDDED
ANCHOR PLATES

8.1 INTRODUCTION

In this chapter, the results of two series of model anchor pullout tests are presented and comparisons are made with the theoretical predictions of Chapter 7. As mentioned in Chapter 2, a considerable amount of literature has been published on experimental investigations into the general behaviour of both isolated anchors and groups of anchors; however, as there appears to be very little experimental data regarding horizontally embedded rectangular anchors subjected to vertical pullout loads, it was considered desirable to perform a number of simple tests for this case.

In the first series of tests the anchors were embedded in overconsolidated clay, while in the second series the anchors were embedded in loose, dry, normally consolidated sand. In both cases, attention was directed towards the load displacement behaviour of the anchor with particular regard to the determination of the collapse load for a range of depth-to-width (ie. embedment) ratios. No attempt was made to determine the strains or deformations within the soil mass, as this was considered to be beyond the scope of this thesis in regard to limitations on both financial and technical assistance.

Sections 8.2 and 8.3 describe the tests carried out in clay and sand respectively, and include a comparison of the experimental results with the theoretical predictions of Chapter 7. A comparison between published results from model and field tests will be made in section 8.4.

8.2 MODEL TESTS IN CLAY

8.2.1 Properties and Preparation of Clay

The series of tests reported in this section were performed using a refined, type C/10 Kaolin. This material was selected because it consolidates quickly and does not creep excessively. It has been used successfully in model footing tests by Davis and Poulos (1968), Mattes and Poulos (1971), Brown (1972) and Wiesner (1977). The grading curve obtained from a particle size analysis using the hydrometer method (Wiesner, loc cit), indicated that in terms of M.I.T. grain size classification, the material contains 70% silt and 30% clay. Liquid and plastic limits for C/10 Kaolin were 45 and 33 respectively and hence by Casagrande's system it may be classified as a silty clay.

The large pressure vessel in which the clay was consolidated and tested is pictured in Fig. 8.1 and is shown diagrammatically in Fig. 8.2. The internal diameter of 590mm provided sufficient surface area for the installation of four or five anchor plates, depending upon the selection of sizes. The depth of 480mm was more than sufficient to allow the embedment of anchors up to the maximum required depth of 180mm. Drains on either side at the top of the pot, and in the centre of the base plate were protected with gauze and covered with filter paper to permit drainage of water from the clay under pressure. The covering of the pot during consolidation of the clay is shown in Fig. 8.2 and consisted of an O ring (to provide the seal), a rubber membrane, spacing ring, domed lid and stiffening ring. This top assembly was secured to the base of the pot with twenty Allen screws. The design of the pressure vessel has been described by Wiesner (1977).

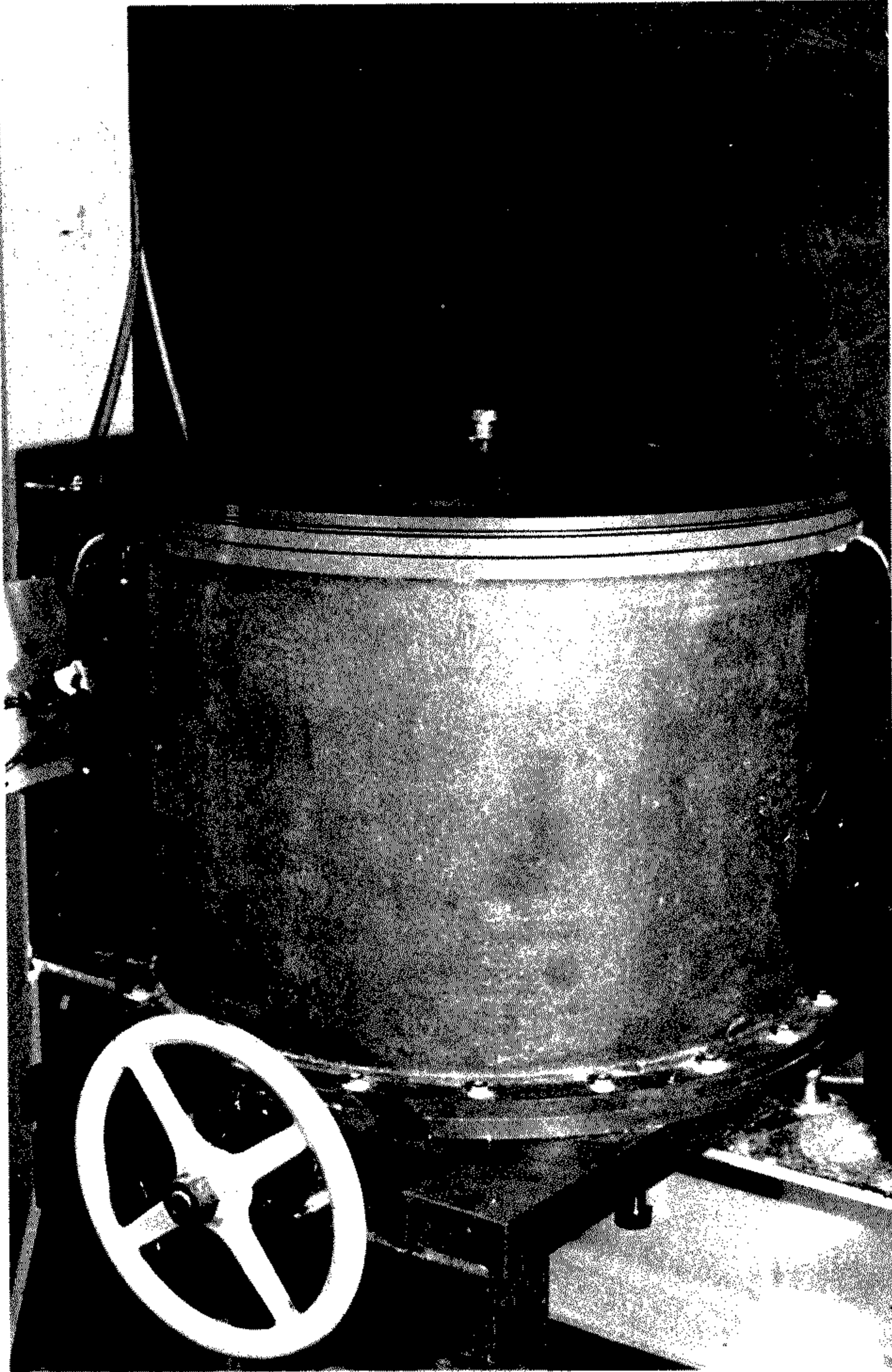


FIG.8.1 POT FOR CONSOLIDATION OF CLAY

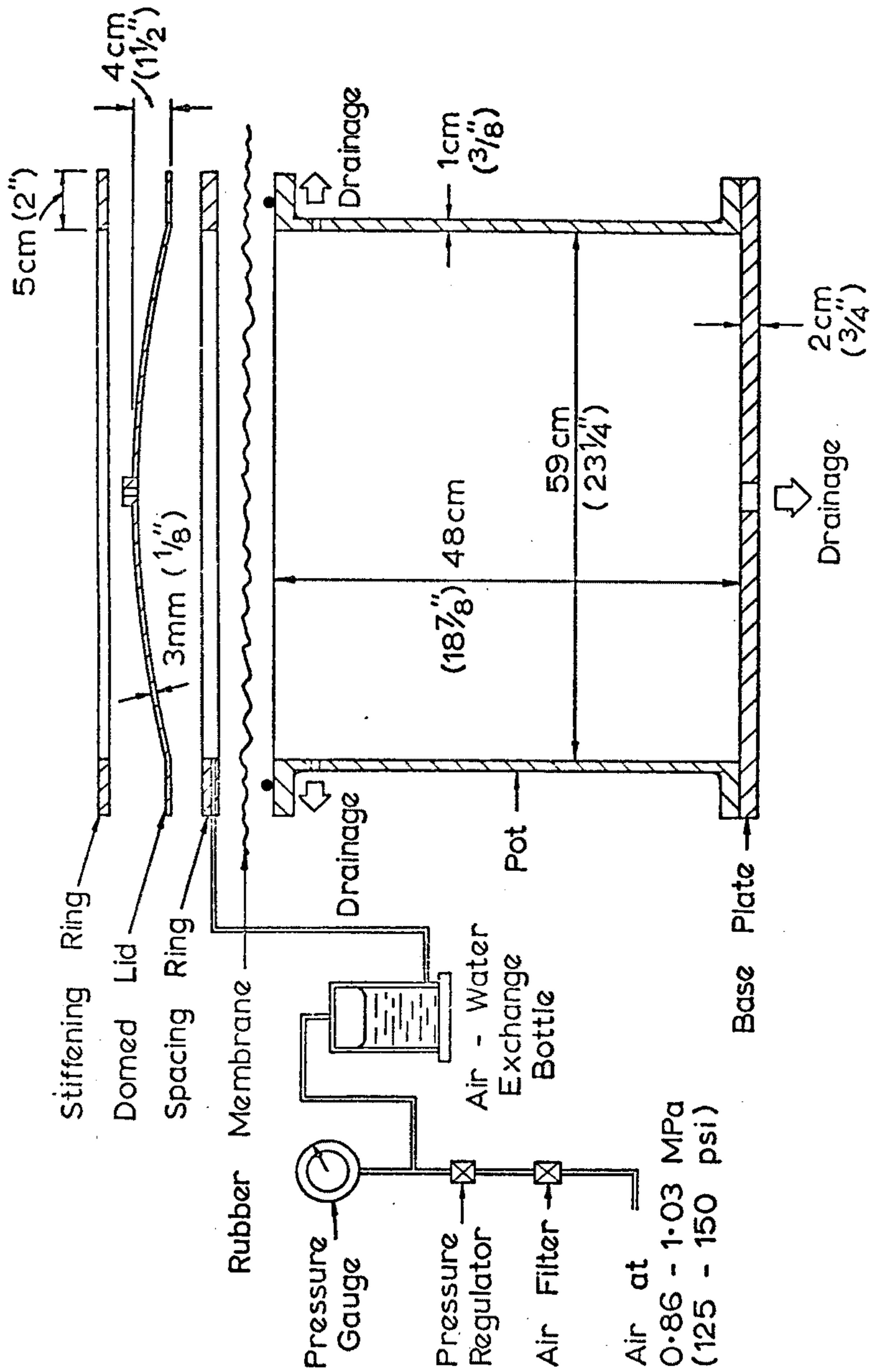


FIG. 8.2 POT FOR CONSOLIDATION OF CLAY
(after Wiesner 1977)

The clay was initially prepared by adding 51% of water (by weight) to 180 kg of powdered Kaolin in a number of galvanised steel containers. This was mixed mechanically, using a power drill fitted with a propeller until a reasonably uniform paste was formed. The containers were sealed and allowed to stand for 24 hours, after which the clay was again thoroughly mixed. It was then poured into the pressure vessel and vibrated with an immersion vibrator to remove as much of the entrapped air as possible. When the pot was full of clay, the top assembly (ie. O-ring, rubber membrane, spacing ring, domed lid and stiffening ring) was screwed onto the pot. Water under pressure was injected between the domed lid and rubber membrane to consolidate the clay. The pressure was applied via an air-water exchange bottle using a Norgren (0-413 kPa) pressure regulator coupled with a Budenberg Standard Test Gauge. As the clay consolidated, water flowed out of the drains at the top and bottom of the pot and the total volume of expelled water was continuously measured in a measuring cylinder. To simplify the apparatus and test procedure, no backpressure was applied to the pore water.

The actual depth of the pot (ie. 480mm) was much greater than the maximum depth to which it was intended to place anchors (ie. 180mm). Accordingly, it was considered unnecessary to remove the entire mass of clay after each test; and in fact, the bottom 240mm of clay was never removed during the series of tests. So as to establish a uniform procedure for subsequent tests, a pilot test was performed. The object of this test was:

- (1) to investigate the difficulties involved in placing anchors in soil and testing them; and

- (2) to establish the preconsolidated lower layer.

For this pilot test, initial consolidation of the clay under a pressure of 100 kPa led to a settlement at the top surface of about 100mm. Additional clay was added to the pot and vibrated with the existing clay. A number of shallow anchors were then inserted and the clay mass was consolidated under 200 kPa until primary consolidation was complete. After the anchor pullout tests had been performed, the clay was removed to a depth of about 240mm and its moisture content was determined.

The clay which had been removed was reconstituted by the addition of sufficient water to give a moisture content of 51%, mixed mechanically to break up the consolidated clay and then vibrated. Each of the containers was sealed and left for two days, after which it was again mixed and vibrated. After a further two days, the clay was mixed and then poured into the pressure vessel, where it was vibrated to remove any remaining air voids. The surface was smoothed off and the anchor plates inserted.

Each anchor plate and rod was smeared with a thin layer of petroleum jelly. A piece of filter paper, overlain by a piece of gauze, which in turn was overlain by a further piece of filter paper, all cut to the dimension of the appropriate anchor, was placed on the surface of the clay. The anchor plates were placed on top of this filter-gauze-filter combination and were pushed down to approximately half the desired depth. The clay was then vibrated until the air voids caused by the introduction of the anchor had been removed. The surface was smoothed off and the anchor rods were pushed in until only the top 6mm of each rod protruded. The final surface was levelled off and covered with a layer

of filter paper. Holes were cut in the filter paper to allow the ends of the anchor rods to pass through. The anchor rods were prevented from piercing the membrane by covering the filter paper with a 25mm thick layer of damp sand.

The top assembly was screwed onto the pot and the clay was consolidated under a pressure of 200 kPa for six days. After this time, the lid, sand and filter paper were removed and undrained pullout tests were performed. The clay surface and model anchors prior to testing are pictured in Fig. 8.3.

At this stage, the pot generally contained 230 kg of clay at approximately 35% moisture content and with a bulk density of 1.85 t/m^3 . The average cohesion, determined from undrained triaxial tests on samples removed from the pot, was 50 kPa.

The technique of underlaying the anchor with filter paper and gauze was adopted to prevent adhesion between the underside of the anchor plate and the soil. Hollow anchor rods were used to prevent the development of suction below the anchor. This procedure is similar to that used by Adams and Hayes (1967).

8.2.2 Anchor Plate Details

The model anchor plates were made from rectangular brass bars with a Young's Modulus of $9.2 \times 10^4 \text{ MPa}$. Each anchor was 6mm thick and had a width B and a length L as shown in Table 8.1. The majority of tests were performed using anchors with a length to breadth ratio (L/B) of five although a few tests were also carried

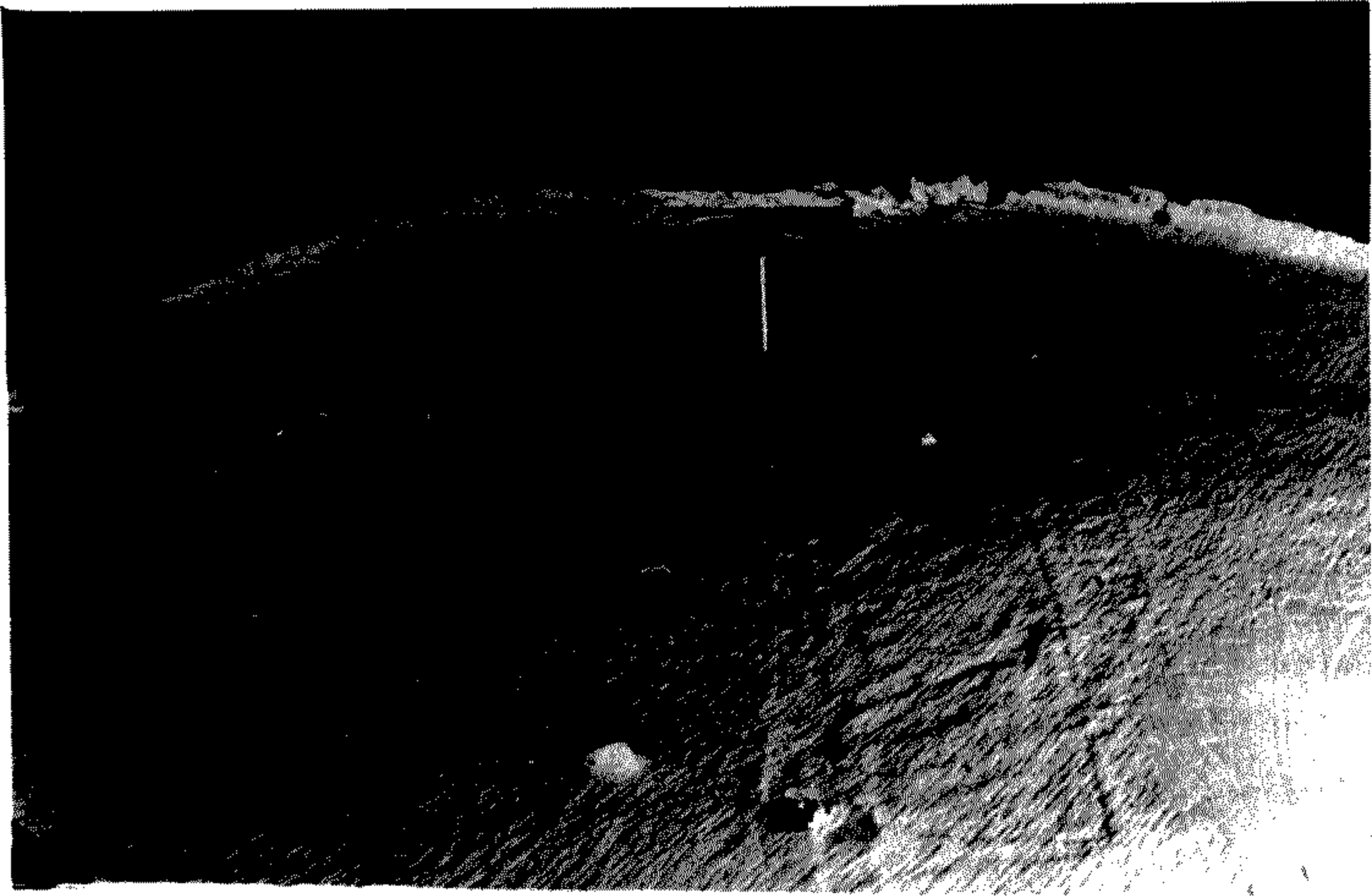


FIG. 8-3 CLAY SURFACE AND PROTRUDING ANCHOR
RODS PRIOR TO TESTING

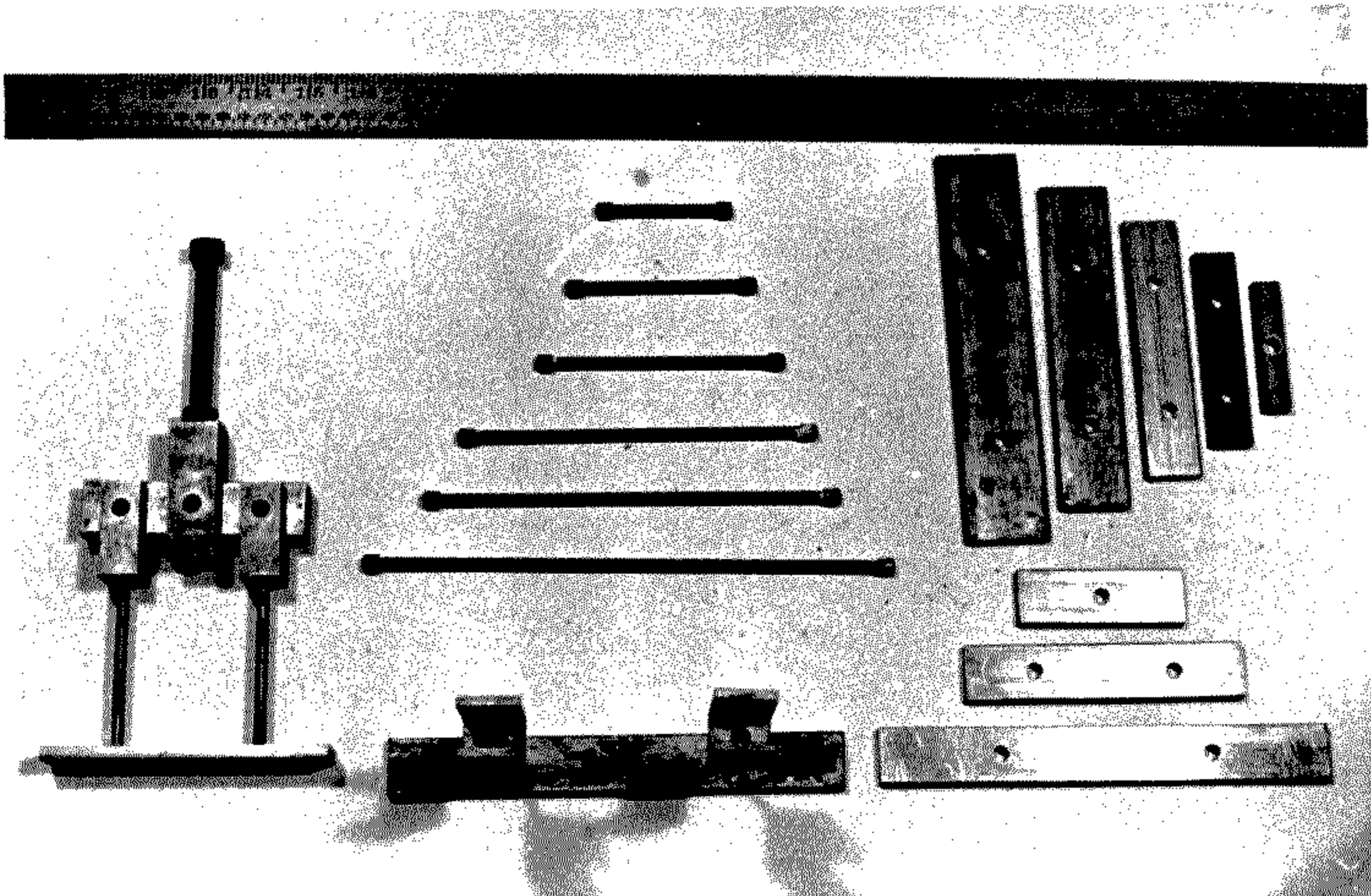


FIG. 8-4 ANCHORS, ANCHOR RODS AND LOADING
DEVICE FOR CLAY TESTS

out using L/B ratios of 3 and 8 to determine the effect of shape upon the collapse load.

Load was applied by means of hollow stainless steel anchor rods which had an outside diameter of approximately 3mm and an inside diameter of 2mm. At each end of the anchor rods, a 6mm long and 6mm diameter brass screw thread was silver soldered onto the anchor rod to facilitate easy connection of the rods to both the anchor and the loading device. The connections are illustrated in Fig. 8.4. Tests were performed at different embedment (h/B) ratios by using a number of different rod lengths to prevent the rods from extending too far above the soil surface during consolidation. In general, each anchor was connected to two anchor rods located along the axis of the anchor at a distance of L/4 from each end; however, anchors A and F were only connected to one rod located at the centre of the anchor. The anchor rods extended to the bottom of the anchor plate so that during testing the underside of the anchor was at atmospheric pressure, and hence there was no suction at the bottom of the anchor plate.

TABLE 8.1

DIMENSIONS OF ANCHOR PLATES IN CLAY

Code	Width (mm)	Length (mm)	L/B	No. of rods
A	12.7	63.5	5	1
B	19.1	95.3	5	2
C	25.4	127.0	5	2
D	31.8	158.8	5	2
E	38.1	190.5	5	2
F	25.4	76.2	3	1
G	25.4	203.2	8	2

8.2.3 Loading Apparatus Details

The apparatus used for loading the buried anchors is pictured in Figs. 8.4, 8.5, 8.6 and 8.7. Two brass 'U'-shaped sections were screwed onto the top end of each anchor rod as shown in Fig. 8.4. These sections were then connected to a brass distribution bar by means of 3mm diameter steel bolts passing through slightly oversize holes to allow free rotation. A different distribution bar was used for each anchor and the centre to centre spacing of the connection to the U sections was $L/2$ in each case where two rods were used. The distribution bar was pinned to an additional U shaped brass section which in turn was connected by means of a 6mm diameter steel rod to the loading arm as shown in Figs. 8.5 and 8.6. In the case where only one anchor rod was used, the distribution bar was pinned vertically between the U shaped section attached to the anchor rod and the U section attached to the loading arm.

Uplift loads were applied to the anchor by means of a one-to-one lever system, as shown in Figs. 8.6 and 8.7. The lever arm was constructed from a 63mm square hollow steel section and pivoted from a hanger using a 12mm diameter steel bolt in an oversized hole. The distance from the fulcrum to the point of application of both the uplift load and the applied load was 711mm and the moment of inertia of the section was $0.82 \times 10^6 \text{ mm}^4$.

Load was applied to the lever arm (and hence to the anchor) by a piston which formed part of a constant rate of penetration machine (see Fig. 8.7). The load was measured using a Soilcrete proving ring and a 7DCDT-100 displacement transducer. The displacement of the

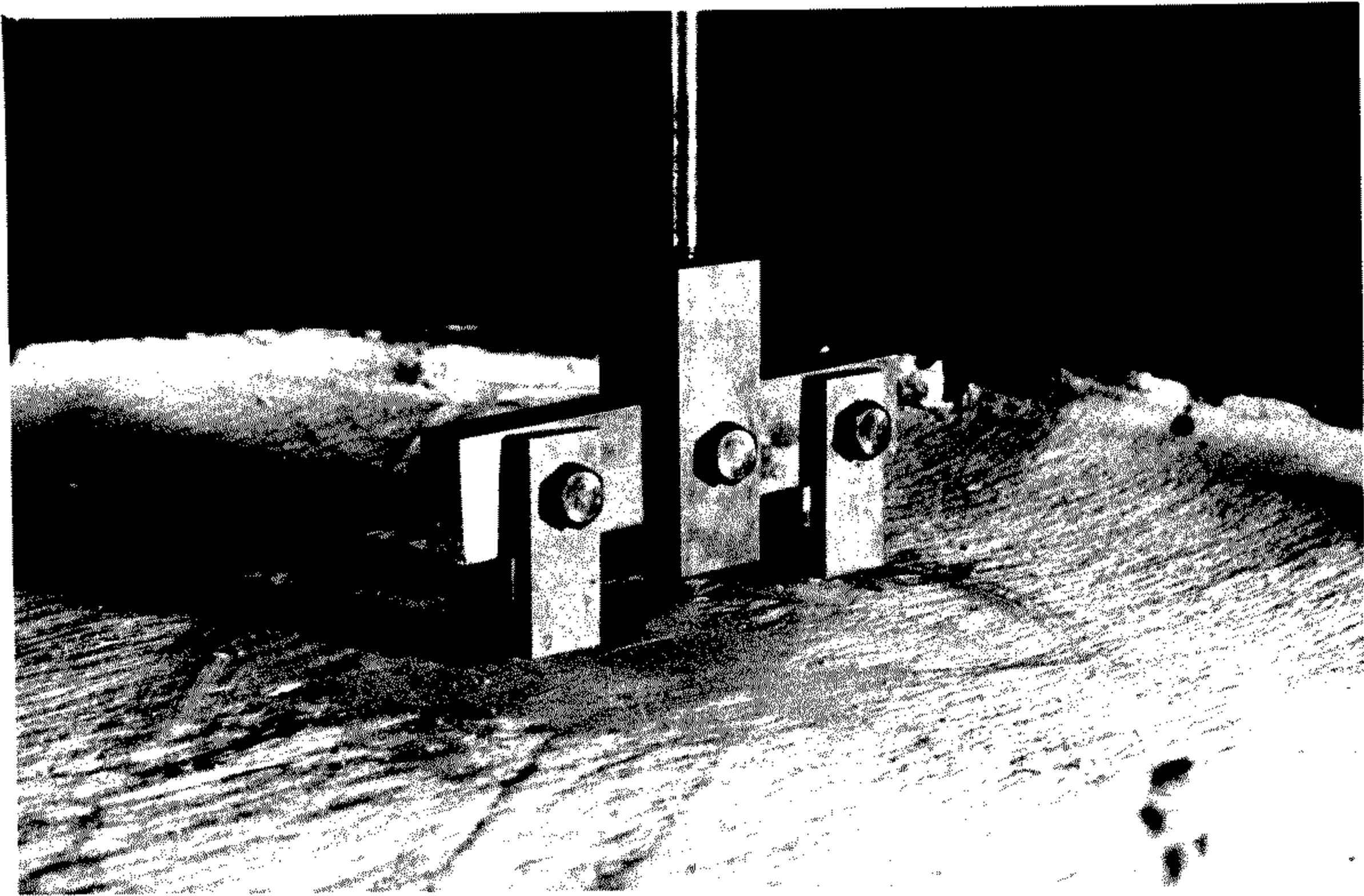


FIG. 8.5 LOADING DEVICE FOR CLAY TEST



FIG. 8.6 LOADING ARM AND PROVING RING
FOR CLAY TEST

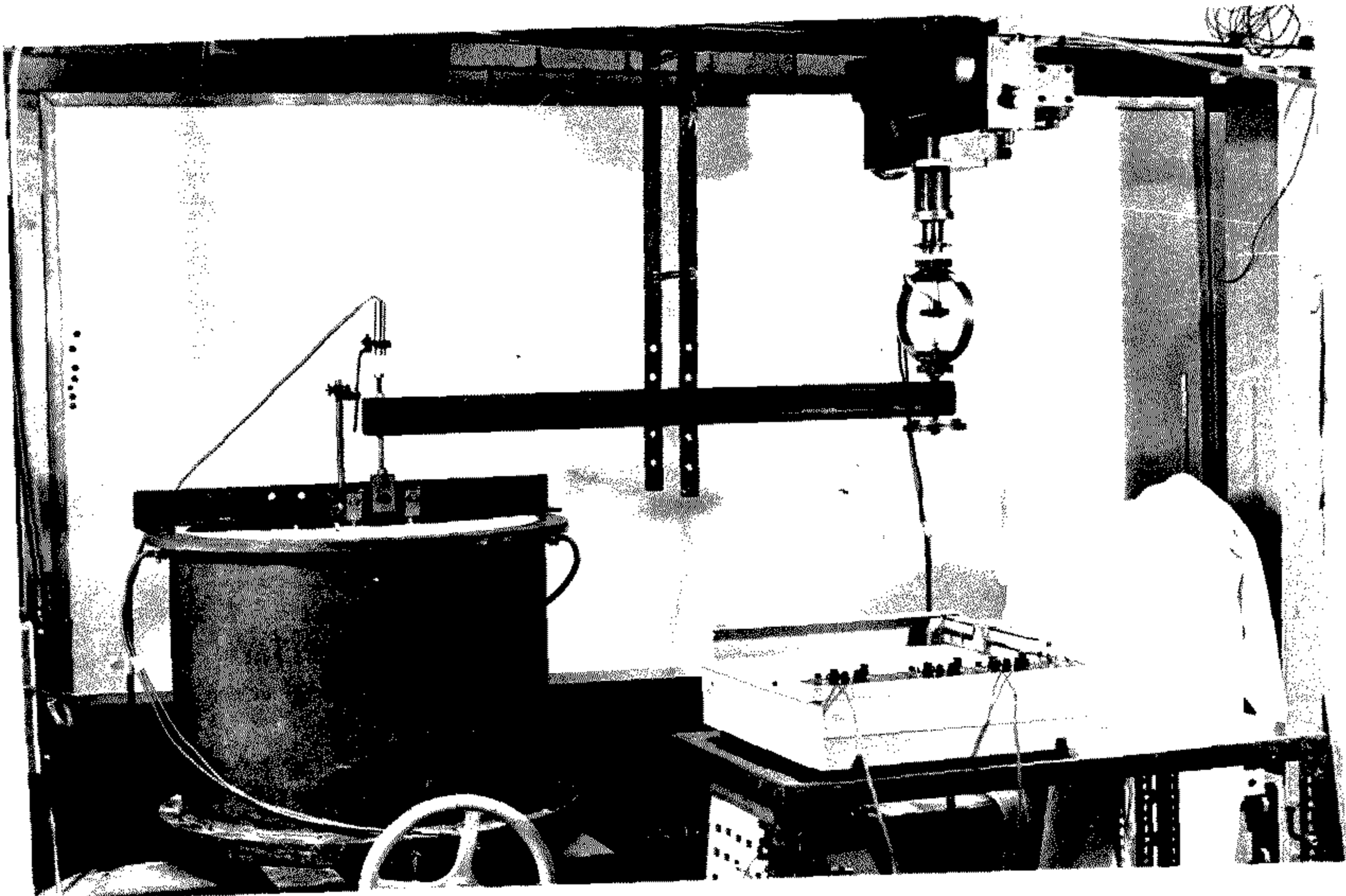


FIG. 8-7 COMPLETE TEST RIG FOR CLAY
PULLOUT TESTS

anchor plate was measured using a 7DCDT-500 displacement transducer located directly above the loading device and connected to the anchor as shown in Fig. 8.7. A 5 volt Hewlett Packard 62005A power supply provided a stable input voltage to the two transducers. The outputs from the transducers were fed into the X (load) and Y (displacement) axes of a Hewlett Packard 7046A X-Y recorder.

The proving ring and associated transducer were calibrated by applying dead weights to the anchor loading device, thereby removing the influence of any minor lack of symmetry within the loading system. The displacement transducer was calibrated using a micrometer.

The effect of the dead weight of the anchor and its loading device was removed by applying a counterweight directly below the applied load, as shown in Fig. 8.6.

The loading apparatus was mounted on a 2.3m high loading frame and could be positioned over any anchor in the pressure vessel (pot) by a combination of movement of the apparatus along the frame and movement of the pot on its trolley across the frame (see Fig. 8.7).

8.2.4 Experimental Procedure

Once consolidation of the pot had been completed, the lid, membrane, sand and filter paper were removed, revealing a fairly smooth surface which was usually 18 to 25mm below the rim of the pot. Typically, the anchor rods protruded a distance of about 12mm above the soil surface. The amount of anchor rod above the surface was

measured in each case and the inside of each hollow rod was cleaned with a long needle to ensure a clear passage of air to the base of the anchor. The loading device was then screwed onto the selected anchor and attached to the lever arm. The lever arm was adjusted to ensure that it was horizontal by means of a bolted nut connection between the loading device and the lever arm. The predetermined counterweight was then added to the applied load end of the lever, after which the proving ring and displacement transducer were moved into position, and the initial readings taken.

In an attempt to achieve undrained conditions, it was considered necessary to load the anchor as quickly as possible; thus the anchor was loaded by hand winding the rapid feed mechanism of the constant-rate-of-penetration machine. Each test took between 10 and 30 seconds from the commencement to the completion of loading. In the case of shallow anchors, collapse was clearly defined and loading ceased once the load displacement curve began to drop.

Deep anchors usually did not exhibit a clearly defined collapse load and so loading was ceased after an anchor movement of between 30 and 40mm.

8.2.5 Strength and Deformation Parameters

Because of the difficulties involved in performing stress path tests which give parameters applicable to overconsolidated clay under open pot conditions (Brown, 1972), the undrained cohesion, c , and elastic modulus, E , of the clay at the end of each series of tests was obtained from conventional undrained triaxial tests (Bishop and Henkel, 1957). The tests were performed on from four to

eight 38mm x 76mm core samples taken at anchor level from each pot. The mean cohesion and modulus obtained from these tests are given in Table 8.2. Despite the small number of tests from each pot, the standard deviation of the results is also given in Table 8.2 to indicate the variability of the basic parameters. The variation in both cohesion and elastic modulus between the different pots of clay is to be expected because of the difficulties of ensuring that the clay was uniformly reconstituted to the same (51%) moisture content prior to each new series of tests.

Of greater significance is the variability of these parameters within a pot of clay. Typically, approximately 70% of the results for undrained modulus were within 25% of the mean value (although larger scatters were observed for series 6 and 7). Admittedly, much of the variability of these results may be attributed to sampling error due

- (1) to any effects that the tests on adjacent anchors may have had on the sample; and
- (2) to disturbance involved in removing the core sample;

however, the results also suggested that there was a consistent variability within the pot. It would appear that this variation is due to non-uniform drainage of the clay, despite the care taken with the provision of drainage at the top, bottom, and sides of the soil mass.

Both the cohesion and the modulus given in Table 8.2 represent the material behaviour with respect to vertical loading; however, there is some evidence to suggest (Poulos, Pers. Comm.) that

TABLE 8.2

UNDRAINED MODULUS AND COHESION

Series No.	^{1,2} $\bar{c}$	³ s.d	$\frac{s.d}{\bar{c}}$	$\bar{E}$	s.d	$\frac{s.d}{\bar{E}}$	$\frac{\bar{E}}{c}$
	kPa	kPa		kPa	kPa		
1	53.8	2.8	.05	2410	610	.25	45
2	46.9	3.0	.06	1920	230	.12	41
3	55.5	12.3	.22	2100	480	.23	38
4	43.6	4.1	.09	1940	390	.20	44
5	54.5	9.0	.16	2290	540	.24	42
6	47.6	4.1	.09	1850	550	.30	39
7	43.7	4.1	.09	1880	520	.28	43

Notes: 1. $\phi = 0^\circ$ and $\nu = 0.5$

2. $\bar{c}$, $\bar{E}$ = mean cohesion and elastic modulus from core samples

3. s.d = standard deviation of test results for each series.

there is no significant variation in the deformation or strength parameters with orientation in this type of overconsolidated Kaolin.

8.2.6 Experimental Results

The results obtained from 31 pullout tests are summarised in Table 8.3, while a number of typical load deflection curves are shown in Figs. 8.8 and 8.9. For convenience in the ensuing discussion, the load deflection curves obtained from these pullout tests have been tabulated by presenting the elastic stiffness ($P/\delta L$) and the mean pressure corresponding to displacements 2, 3, 4 and 5 times those that would be obtained at the same load level if the behaviour had remained elastic.

TABLE 8.3a

EXPERIMENTAL RESULTS - CLAY TESTS

Series No.	Test No.	B (mm)	h (mm)	h/B	L/B	$\frac{P}{\delta L}$ (kPa)	k2 (kPa)	k3 (kPa)	k4 (kPa)	k5 (kPa)	Max Pressure (kPa)	Max Displacement (mm)	Comment
1	1	12.7	70	5.5	5	4250	203.7	245.5	268.9	300.6	372.3	25	C
	2	19.1	86	4.5	5	3300	171.7	224.8	264.1	295.8	352.3	27	C
	3	25.4	76	3.0	5	2120	177.2	216.5	237.9	241.3	241.3	15	C
	4	31.8	78	2.45	5	3030	126.9	158.6	175.1	182.0	182.0	11	C
2	5	19.1	175	9.16	5	3670	208.4	233.0	254.4	267.5	351.0	27	R
	6	25.4	178	7.0	5	3250	191.0	215.1	230.3	242.0	293.7	34	R
	7	31.8	176	5.55	5	3810	182.7	206.2	220.0	231.7	277.9	44	R
	8	19.1	178	9.3	5	2360	242.0	263.4	280.9	293.7	337.2	42	R
3	9	19.1	178	9.3	5	6070	226.8	259.3	275.8	286.1	335.1	36	R
	10	25.4	173	6.82	5	4820	227.5	254.4	269.6	284.7	347.5	37	R
	11	25.4	178	7.0	5	3020	224.1	251.0	269.3	281.1	317.3	26	R
	12*	25.4	178	7.0	5	3440	353.0	383.6	401.9	419.2	421.3	28	R
4	13	19.1	92	4.8	5	7750	229.6	251.6	264.8	275.8	333.0	19	C
	14	19.1	68	3.6	5	2780	152.2	192.0	214.0	227.3	254.7	22	C
	15	25.4	76	3.0	5	2610	140.4	168.2	178.6	184.8	205.5	22	C
	16	25.4	67	2.6	5	1810	150.3	180.6	190.9	195.8	195.8	18	C
	17	38.1	64	1.67	5	3200	74.5	102.4	114.5	114.5	114.5	13	C

* Adhesion/suction not prevented.

TABLE 8.3b

EXPERIMENTAL RESULTS - CLAY TESTS

Series No.	Test No.	B (mm)	h (mm)	h/B	L/B	$\frac{P}{\delta L}$ (kPa)	k2 (kPa)	k3 (kPa)	k4 (kPa)	k5 (kPa)	Max Pressure (kPa)	Max Displacement (mm)	Comment
5	18	25.4	73	2.87	3	9340	223.4	254.4	279.9	294.4	350.9	24	C
	19	25.4	70	2.75	5	3260	188.0	224.0	245.5	257.9	265.5	22	C
	20	25.4	73	2.87	8	4125	203.4	236.5	252.4	257.2	257.2	12	C
	21	25.4	98	3.87	8	3860	210.6	243.4	257.2	270.3	297.2	28	C
6	22	25.4	42	1.65	5	1400	99.3	112.5	112.5	112.5	112.5	20	C
	23	25.4	80	3.15	8	2450	173.7	200.0	209.6	212.4	212.4	16	C
	24	25.4	83	3.25	5	2540	189.7	225.0	240.5	247.1	253.7	22	C
	25	25.4	88	3.45	5	2380	180.0	211.8	228.2	240.6	256.0	19	C
7	26	25.4	111	4.38	3	5310	202.0	231.7	252.4	267.5	342.7	27	R
	27	25.4	118	4.63	3	9240	173.7	195.8	211.6	223.4	300.6	27	R
	28	25.4	118	4.63	5	4230	166.8	193.7	213.7	229.6	271.6	26	R
	29	25.4	114	4.5	5	5580	180.6	199.3	213.7	225.5	279.3	35	R
	30	25.4	127	5.0	5	2070	162.0	194.4	219.3	237.9	253.0	33	R
	31	25.4	114	4.5	8	2510	166.8	193.7	209.9	219.9	228.2	27	R

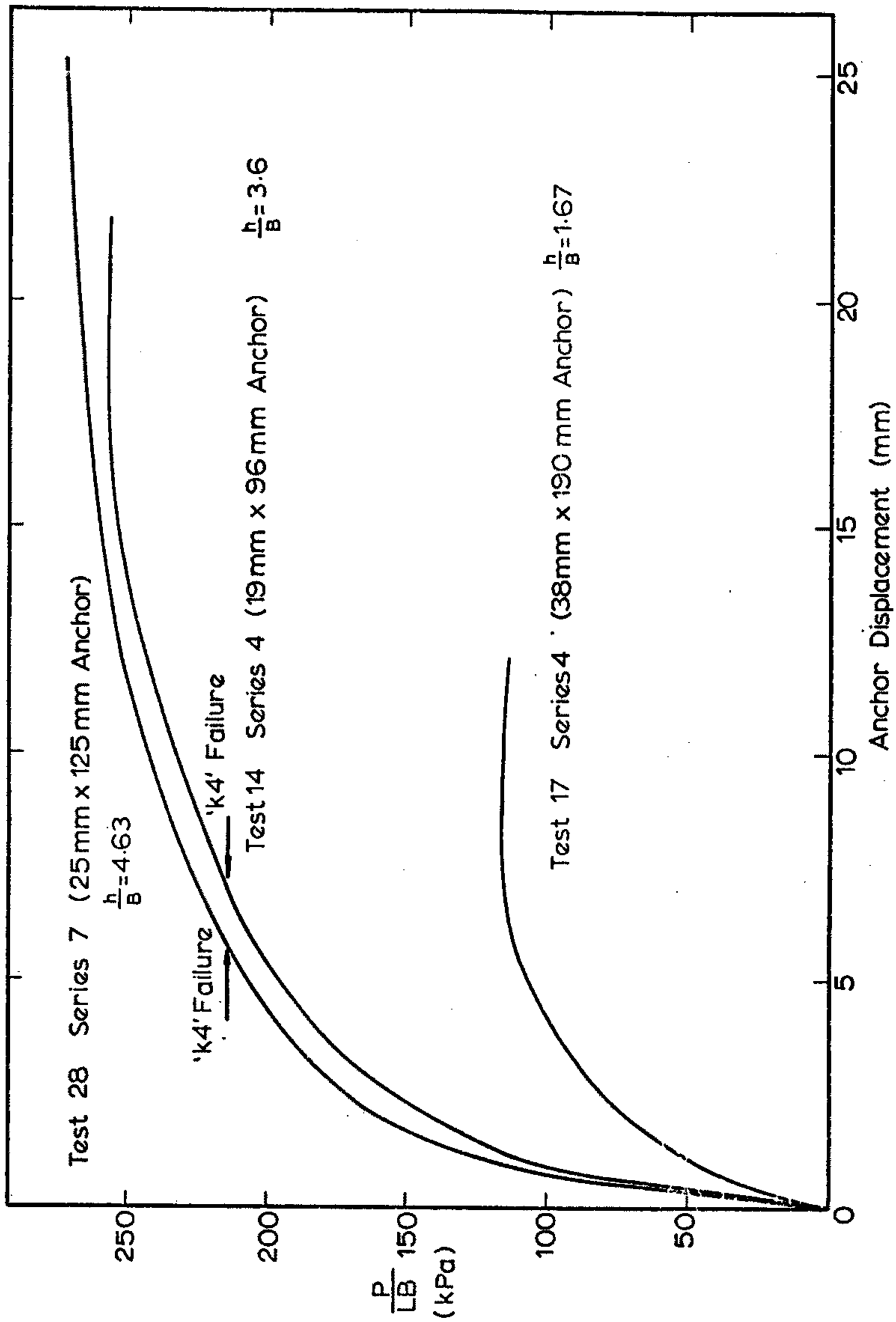


FIG. 8.8 LOAD DISPLACEMENT CURVES FOR ANCHOR PLATES
IN CLAY $\frac{L}{B} = 5$.

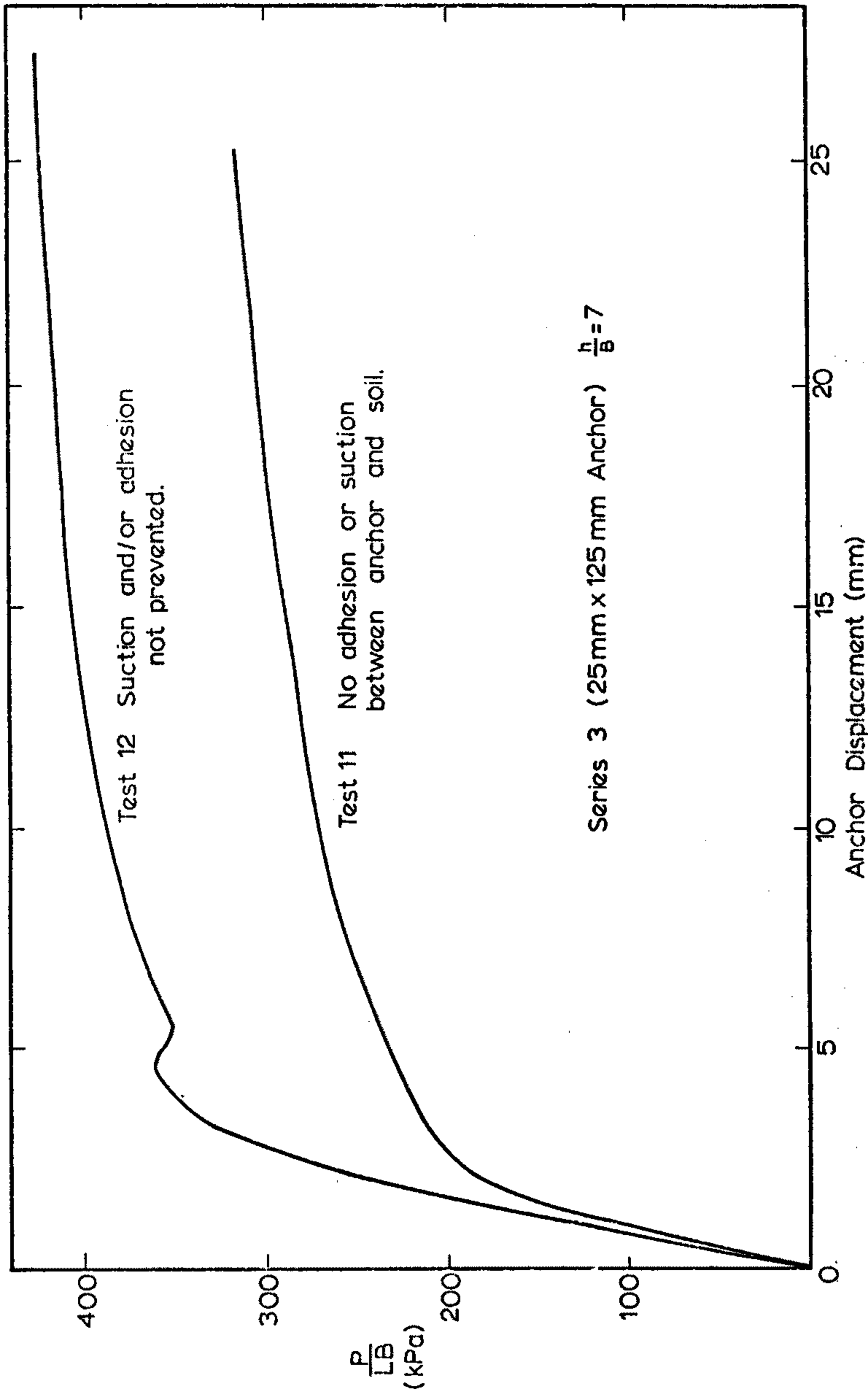


FIG. 8.9 EFFECT OF PARTIAL ADHESION OR SUCTION ON EXPERIMENTAL BEHAVIOUR.

These pressures correspond to the 'k2', 'k3', 'k4' and 'k5' load levels discussed in Chapter 7. From this information, the relevant portion of each load deflection curve may be reconstructed if desired. The maximum applied pressure corresponding to the collapse load in cases where collapse was clearly defined (denoted by 'C'), or the load at which the test was terminated in cases where the load deflection curve was still rising (denoted by 'R') are included in Table 8.3, as is the maximum displacement at which each test was terminated.

It was found that presentation of the experimental results in terms of 'k2', 'k3', etc. load levels had advantages similar to those observed with regard to the theoretical analyses; that is, presentation of the results in this form tended to reduce the sensitivity of the results to experimental variations. In particular, load deflection curves for similar anchors which appeared to be different (because of a difference in the elastic stiffness) were found to be quite similar when expressed, for example, in terms of the 'k2', 'k3' load/displacement levels. Thus, the adoption of this approach allows a more meaningful comparison of load deflection curves since it effectively reduces the influence of variations in elastic modulus, as well as other tangible effects (such as proximity to the edge of the pot and other, previously tested, anchors). It is anticipated that this approach may also have useful practical applications in that the load deflection behaviour of an installed anchor may be estimated from a non-destructive test in which the elastic stiffness of the anchor was determined. For this purpose, the 'k2', 'k3' etc. load levels could be obtained theoretically or from a field test conducted under geometrically similar conditions.

The results presented in Table 8.3 may be broadly divided

into three categories of behaviour, viz.

- (1) very shallow anchor behaviour, for $h/B \leq 2.5$
- (2) shallow anchor behaviour, for $2.5 < h/B < 4.5$
- (3) deep anchor behaviour, for $h/B \geq 4.5$

All anchors with embedment ratios less than 2.5 (at the beginning of the test) were found to exhibit very shallow anchor behaviour characterised by a clearly defined collapse load (such as shown for Test 17 in Fig. 8.8) and the formation of a flexural tension crack (as shown in Fig. 8.10). The formation of the tension crack was seen to dominate the behaviour of the anchor. The crack began to develop at relatively low displacements and resulted in an abrupt collapse of the anchor. In fact, for these displacement controlled tests, an actual drop in the load carrying capacity could be measured after the formation of the tension crack.

Shallow anchor behaviour, characterised by clearly defined collapse loads without the development of tension cracks, was observed for anchors with embedment ratios in the range $2.5 < h/B < 4.5$. A typical load deflection curve, obtained from Test 14 ($h/B = 3.6$), is shown in Fig. 8.8. From this and from the results tabulated in Table 8.3 it can be seen that although collapse was clearly defined in these cases, considerable deformation had occurred prior to collapse and the adoption of a 'k4' practical failure load could be justified. In most cases, the geometry changes just prior to collapse are quite significant with the anchor generally having displaced through almost one footing width. For anchors at the shallow end of the range, the formation of small tension cracks was observed after collapse had occurred, as shown in Fig. 8.11. In these cases the development of tension cracks appeared

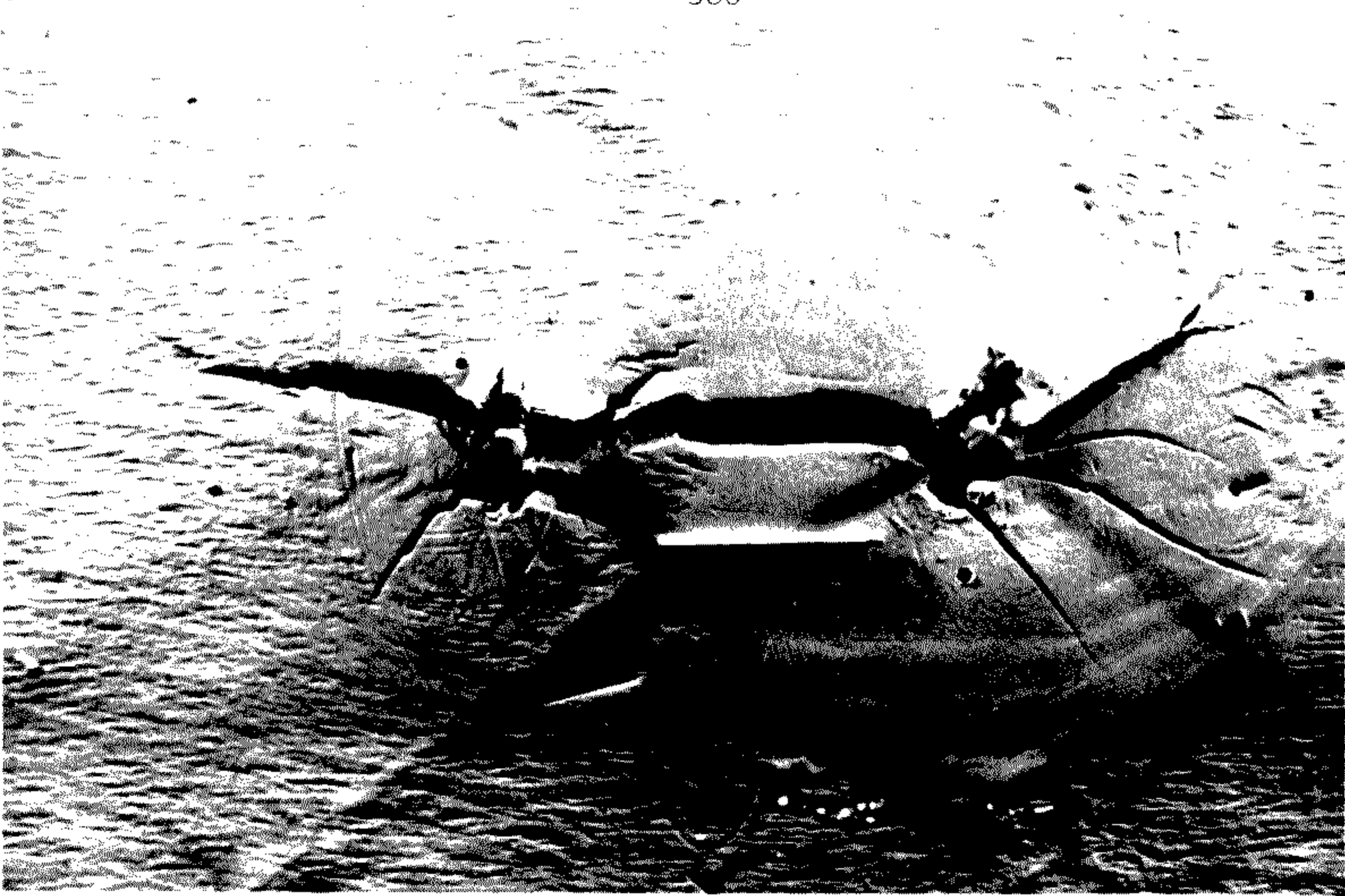


FIG. 8.10 TENSION CRACKS FORMED FOR VERY SHALLOW ANCHORS

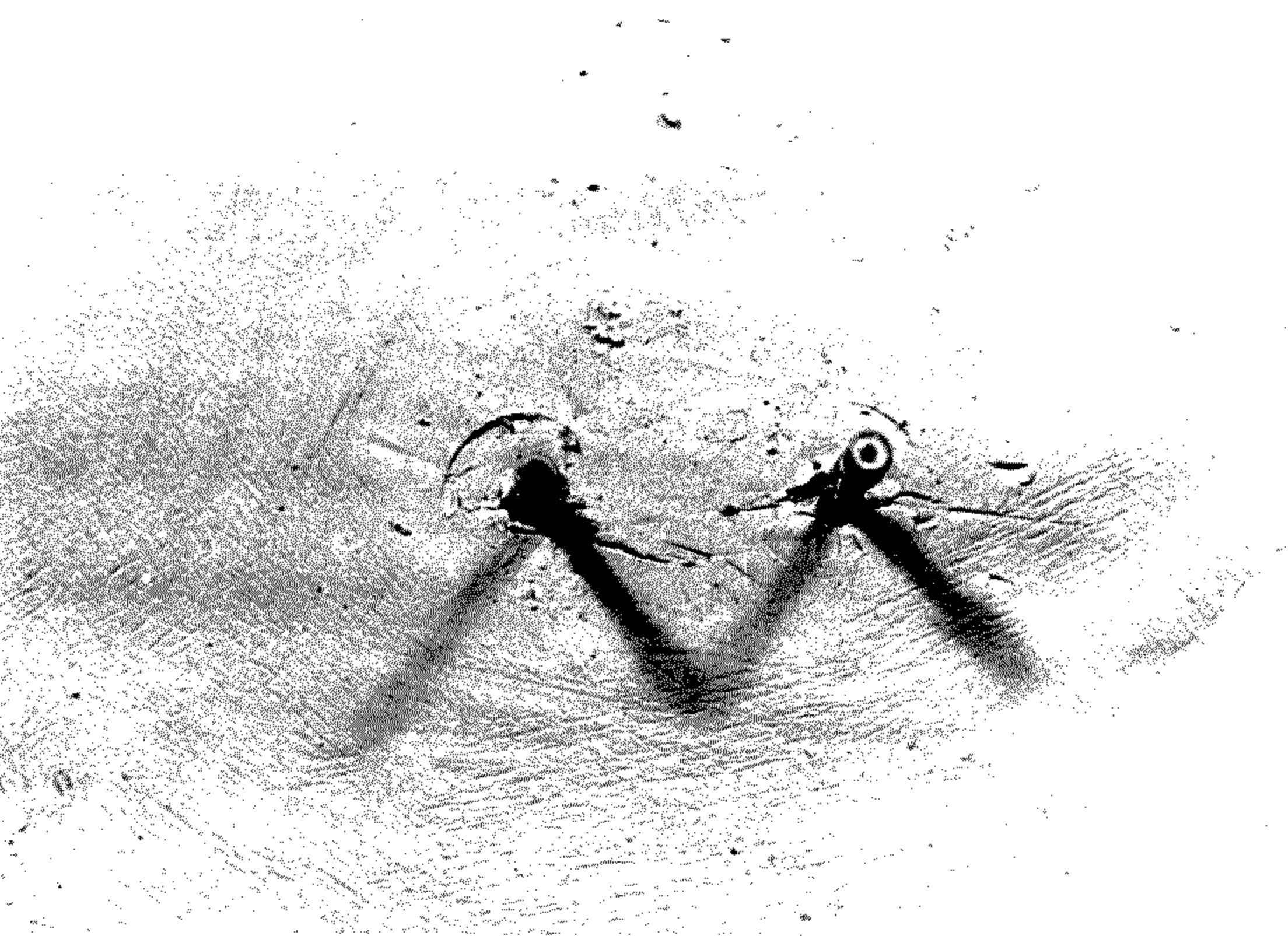


FIG. 8.11 FORMATION OF SMALL TENSION CRACKS AFTER COLLAPSE FOR SHALLOW ANCHORS

to be the result of post-collapse geometry changes and as such are not considered to be an intrinsic part of shallow anchor behaviour. At the deeper end of the range, no tension cracks were observed even after quite significant deformation of the anchor; however, in these cases the occurrence of clearly defined collapse may, in itself, be partly attributable to the effect of geometry changes. For example, in Test 14 the initial embedment ratio was 3.6; at the 'k4' failure load the effective embedment ratio had reduced to 3.3; however, at collapse the effective embedment ratio was only 2.5. Thus although the initial embedment ratio is also reasonably applicable at the 'k4' failure load, there is some doubt as to what embedment ratio (ie. initial, final or somewhere in between) should be associated with actual collapse. This obvious and important effect of geometry change for model tests (particularly in soft clay) does not appear to have been recognized in the majority of publications on model anchor behaviour.

Deep anchor behaviour was generally observed for anchors with embedment ratios greater than 4.5. In these cases, failure was not clearly defined and the load deflection curve was still rising when the test was terminated after a displacement of between one and two anchor widths. There was no indication of tension crack formation in any of these tests, and in most cases there was no apparent surface indication that the anchor had even been loaded despite the fact that it had displaced through a distance of between 25 and 45mm. Any consideration of bearing capacity in these cases would necessitate the adoption of an arbitrary definition of failure such as the Terzaghi/Meyerhof approach, the intersection of tangents approach, percentage displacement approach, or the 'k4' type definition suggested by the

author.

In two cases, clearly-defined collapse was observed for anchors with embedment ratios greater than 4.5; namely in Tests 1 and 13. In both these cases, the anchor was relatively slender (12mm and 19mm respectively) and the deformation at collapse corresponded to a change in embedment ratio of from 5.5 to 3.5 and from 4.8 to 3.8 respectively. Consequently, these anchors, despite their initial embedment ratio, could reasonably be considered to be shallow anchors and thus the clearly defined collapse is a result of geometry change rather than an exception to deep anchor behaviour.

Although there is insufficient evidence to make any definitive statement, it would appear that a reduction in aspect ratio also decreases the depth at which deep anchor behaviour is observed. For example, in Test 26 the initial embedment ratio of an anchor with $L/B = 3$ was 4.38 and the embedment ratio at termination of the test was 3.3, however, the load deflection curve was still rising quite steeply at termination and there was no indication of impending collapse. By contrast, for $L/B = 5$ in the previously cited Tests 1 and 13 as well as in Test 2 where $(h/B)_{\text{initial}} = 4.5$ and $(h/B)_{\text{final}} = 3.1$, collapse was quite clearly defined at similar embedment ratios.

Because of the difficulties involved in assessing the magnitude of suction and adhesion between anchor and soil, care was taken in these tests to ensure that no such suction or adhesion developed between the anchor and the soil. However, to indicate the likely effect suction and adhesion could have on the behaviour of anchors, one test was performed where the anchor was placed in direct contact

with the underlying soil and where the hollow anchor rods were replaced by solid rods. The load displacement curves obtained for the standard case without adhesion (Test 11) and the special case where adhesion was permitted (Test 12) are shown in Fig. 8.9. As previously observed for deep anchors, in the case of no adhesion the behaviour is characterised by a steadily rising load displacement curve corresponding to extensive plastic deformation even after a displacement of one anchor width, and there was no indication of impending collapse when the test was terminated. In contrast, the load displacement curve from Test 12 (suction and adhesion not prevented), which is slightly stiffer elastically, suggests that there was virtually no plastic deformation prior to the 'kink' in behaviour at a displacement of about 6mm. It is believed that this 'kink' corresponds to breakaway of the anchor from the underlying soil. After breakaway, the load deflection curve indicates that extensive plastic deformation is taking place and in fact the curve is almost parallel to the no adhesion curve. Clearly defined collapse had not been observed in either case when loading was terminated. It will be recognised that while there is insufficient evidence to make any conclusive statements, the behaviour exhibited in these two cases is very similar to that predicted in Chapter 7 for the effect of $\gamma h/c$, where breakaway occurs during the loading procedure. On this basis, suction s_u and adhesion s_a may be deduced to be $(s_u + s_a)/c \doteq 1.4$ for this particular case. The fact that not only can soil sustain this additional load, but that the load deflection curve continues to rise after breakaway occurs, would suggest that the actual collapse load in both the breakaway and no breakaway condition is well in excess of 420 kPa and hence in the no breakaway case an enormous deformation would be required to obtain this collapse load. (By extrapolation of the load deflection

curve, it was found that a minimum displacement of 75mm would have been required to attain even the 420 kPa load in Test 11).

After each series of tests, the soil was removed in such a way as to allow some assessment to be made of the failure mode for each anchor. It was found that in each case, failure was associated with indentation of the anchor into the clay, and involved rupture between the soil above the anchor and the soil at the sides of the anchor. In the case of very shallow anchors, the tension crack was located along the longitudinal axis of the anchor plate and extended from the soil surface to the top of the anchor. In these cases, the crack was up to 5mm wide at the soil surface and reduced to a hair-line crack at the anchor level. It appeared that crack propagation had been encouraged by the presence of the anchor rods.

For all shallow anchor tests (including very shallow anchors), there was no significant indication of plastic flow around the anchor and the void below the anchor had almost the same cross-sectional area as the anchor plate itself. However, in the case of deeper anchors, there was noticeable flow around the anchor, and the cross-sectional area of the void below the anchor varied from approximately 60% to 80% of the plate area.

8.2.7 Discussion and Comparison with Theoretical Predictions

It will be appreciated that many of the observations of the previous section regarding the general behaviour of model anchors is in accordance with the predictions made in Chapter 7. In particular, the distinction between shallow and deep anchor behaviour and the need

for a practical definition of failure accord with the finite element findings. On the other hand, although the significant role played by tension failure for very shallow anchors was anticipated in Chapter 7, no such allowance was made in the analysis; consequently, it may be anticipated that the theoretical prediction for very shallow anchors will be somewhat unconservative.

The load displacement curves obtained both experimentally and theoretically for a shallow anchor ($h/B = 3$) are shown in Fig. 8.12. The experimental curve shown is that obtained from Test 15 (Series 4) and although this is classified as a shallow anchor, failure did not occur until a displacement of $0.9B$ and the load deflection curve was still rising quite sharply at the practical 'k4' failure load. The theoretical curve shown in Fig. 8.12 was obtained for the case where $\phi_u = 0^\circ$, $\nu_u = 0.48$, $E_u = 1940$ kPa and $c_u = 43.4$ kPa. These parameters correspond to the mean values obtained from triaxial tests. An alternative comparison between the experimental and theoretical curves can be obtained by collocation of the two curves at two points and then plotting in a non-dimensional form as shown in Fig. 8.13. Consideration of Figs. 8.12 and 8.13 indicates that, subject to uncertainty as to the actual soil parameters, the experimental and theoretical curves are in acceptable agreement. However, it will be noted that the finite element prediction tends to overestimate the deflection in the early elasto-plastic portion of the curve. This may be attributed to the differences between the nature of physical rupture and the numerical provision for rupture at the anchor tip. The discrete nature of the rupture lines in the finite element analysis implies that rupture of a nodal pair corresponds to instantaneous rupture along a segment of the rupture line determined by the spacing to adjacent nodal

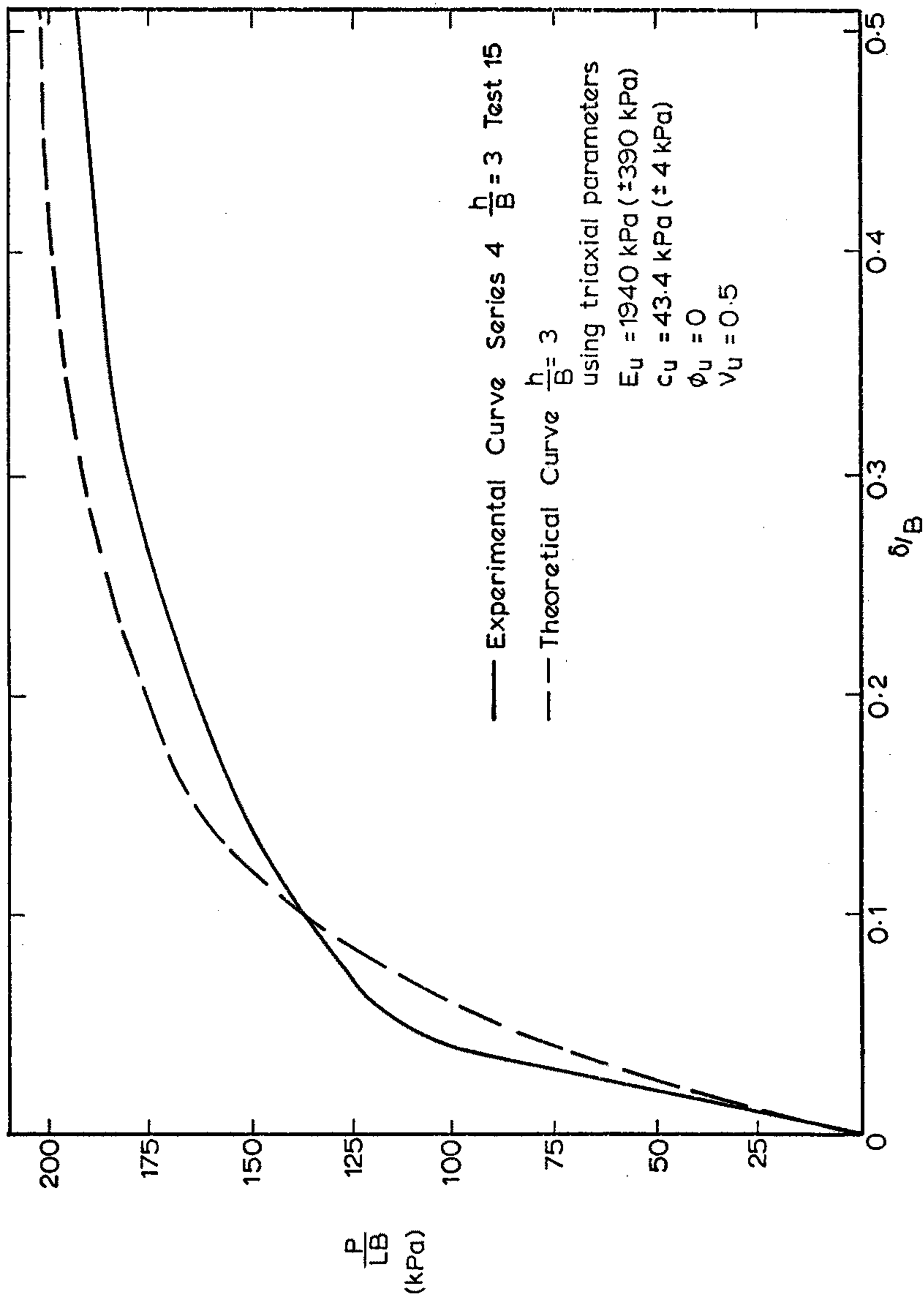


FIG. 8.12 COMPARISON OF THEORETICAL AND EXPERIMENTAL LOAD DEFLECTION CURVES.

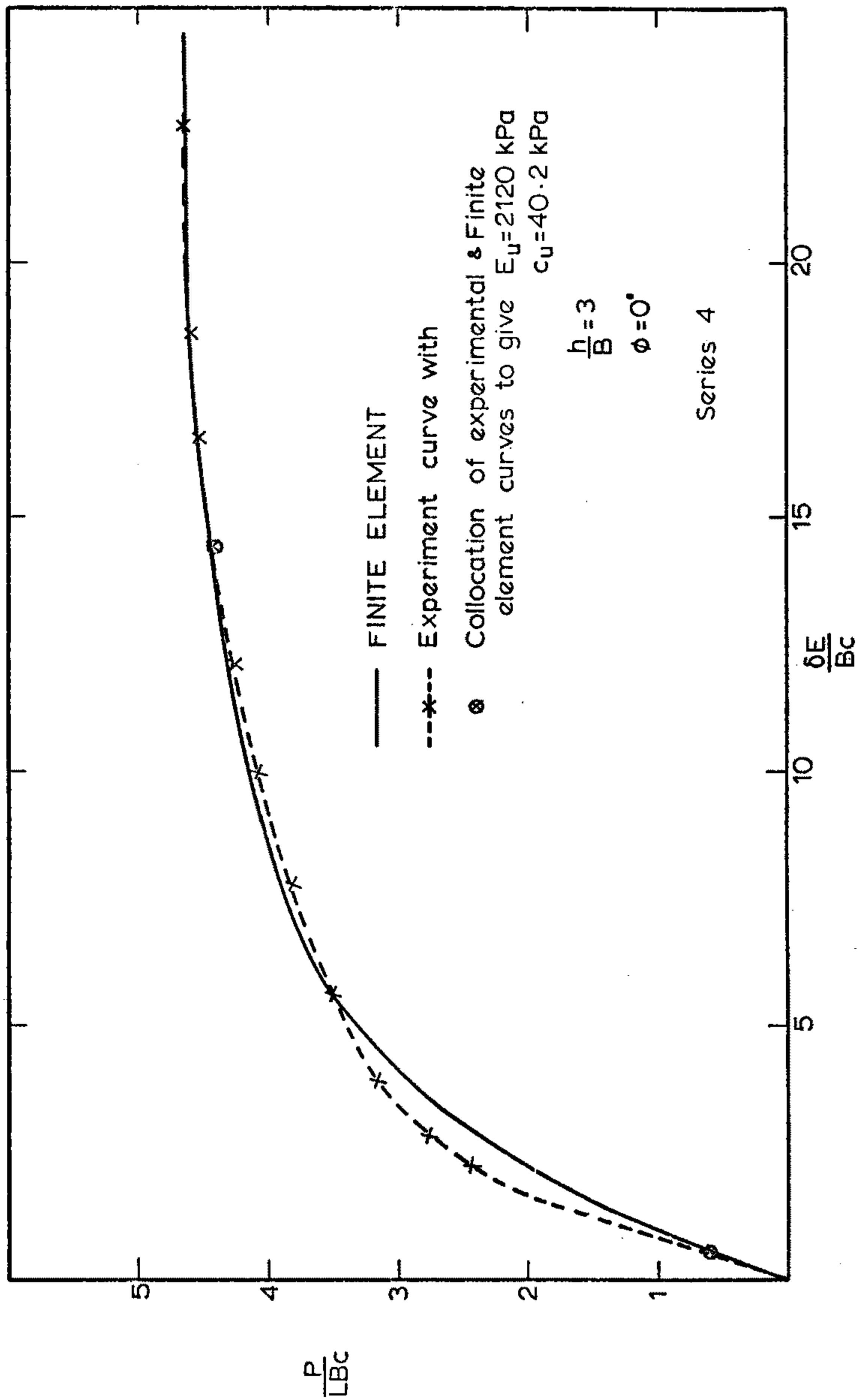


FIG. 8.13 COMPARISON OF THEORETICAL AND EXPERIMENTAL LOAD DEFLECTION CURVES.

pairs. Accordingly, unless an extremely fine finite element mesh is adopted, rupture will occur more suddenly than would be the case in reality (where rupture can occur gradually). Thus the discrepancy of the load deflection curves for deflections less than twice the elastic deflection may be attributed to this numerical artifice and it will be observed that this limitation is offset by the relatively comparable form of the load deflection curves for larger displacements.

Adopting the mean cohesion given in Table 8.2 and the 'k2', 'k3', 'k4' and 'k5' pressures given in Table 8.3, the experimental and theoretical dimensionless loads may be compared as shown in Figs. 8.14, 8.15, 8.16 and 8.17. From these figures it will be appreciated that for embedment ratios greater than 2.5, there is encouraging agreement between the experimental and theoretical load factors. In particular, it will be observed that the dimensionless factors are virtually independent of embedment ratios for $h/B \geq 3$ and that the majority of experimental results lie within $\pm 15\%$ of the theoretical prediction.

The direct comparison of the plane strain finite element results with the experimental results for rectangular anchors is only valid if the rectangular anchor behaves as a strip. To determine the likely effect of aspect ratio, L/B , upon the behaviour of rectangular anchors, the results obtained from comparable tests for aspect ratios $L/B = 3, 5$ and 8 are summarised in Fig. 8.18. In each case, the experimental dimensionless load factor has to be normalised by dividing throughout by the theoretical bearing capacity factor F_c for each embedment ratio. From this figure, it will be seen that although there is a 10% scatter of results for embedment ratios of 5 and 8, there is

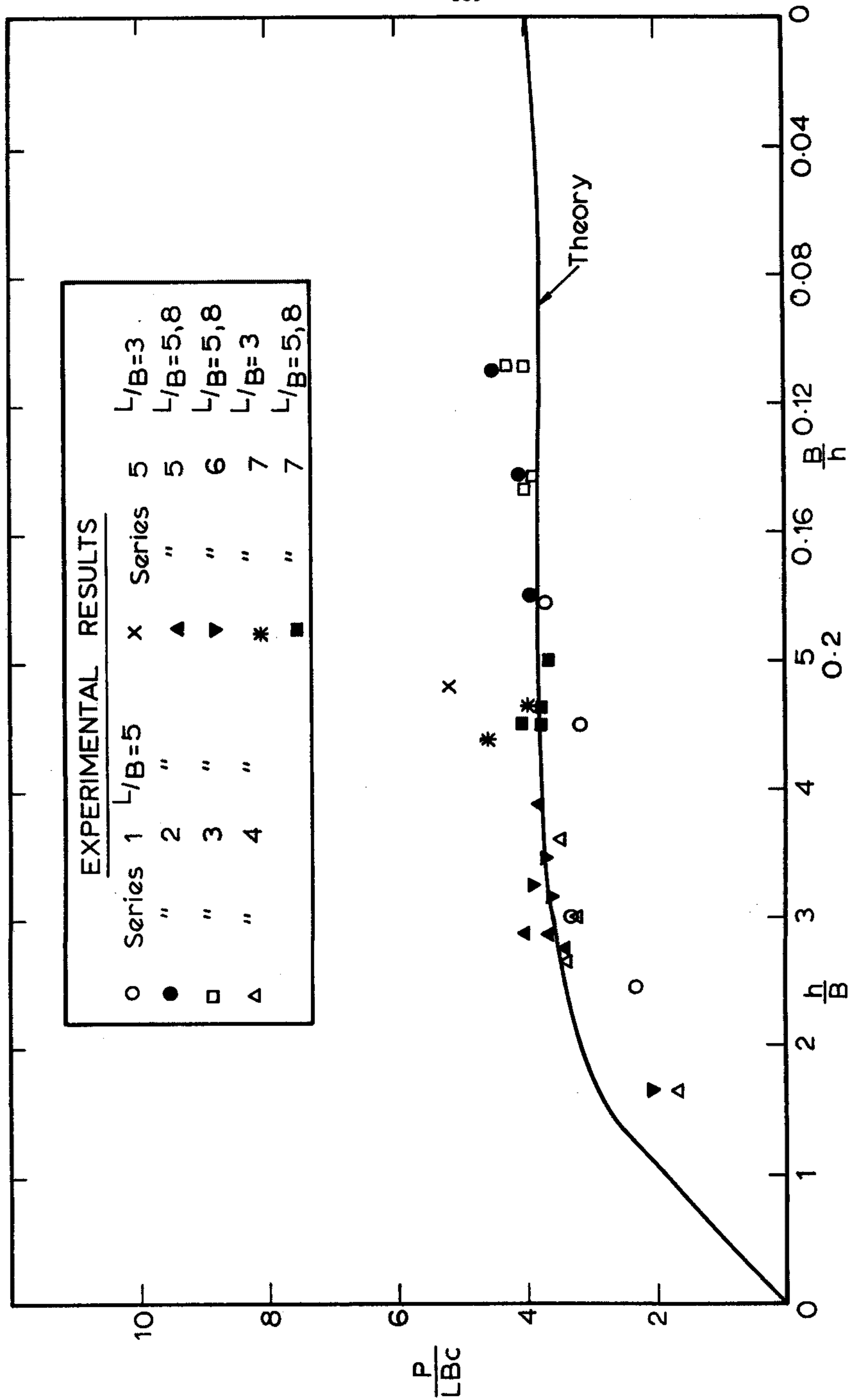


FIG. 8.14 COMPARISON OF EXPERIMENTAL AND THEORETICAL 'k2' VALUES.

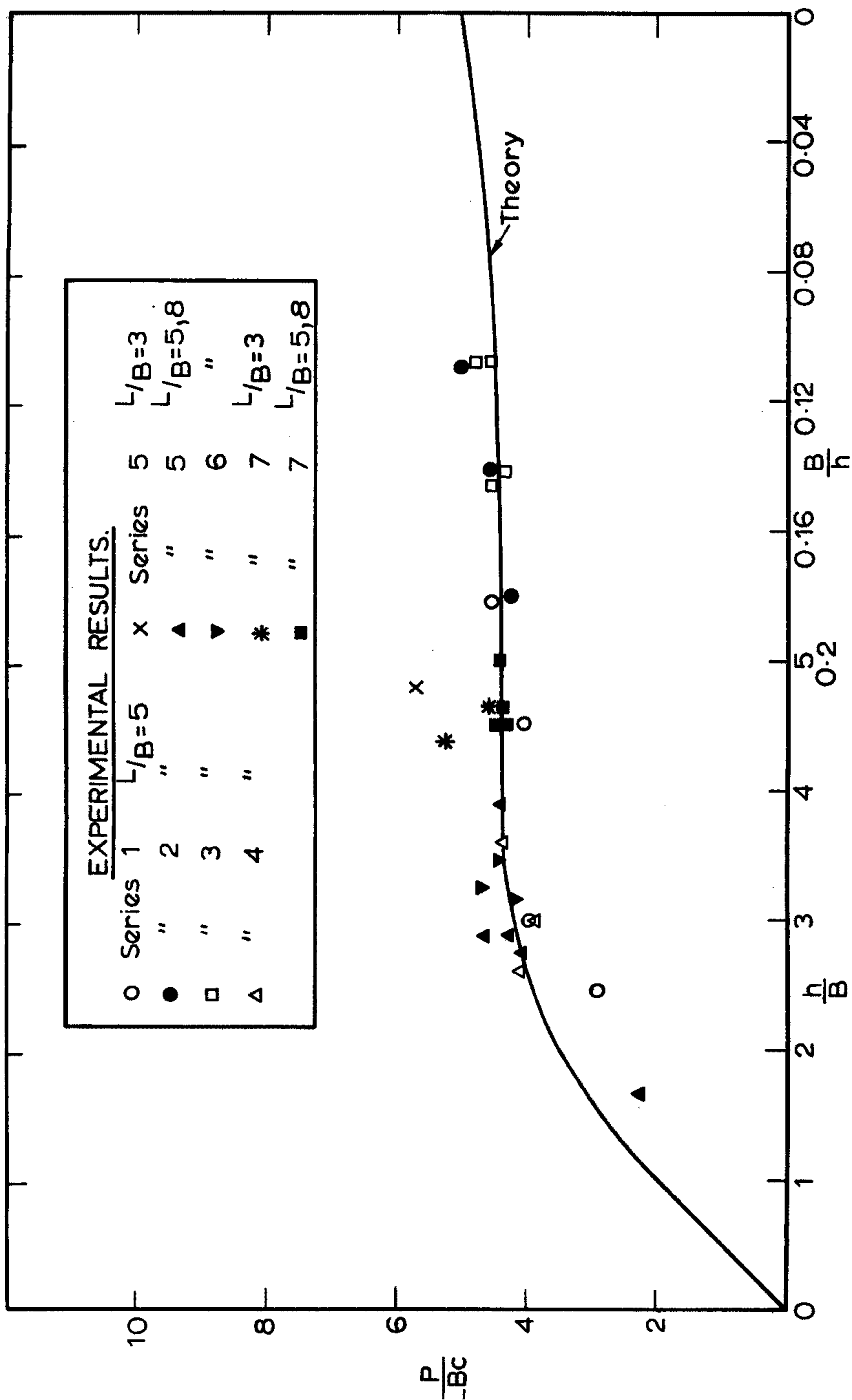


FIG. 8.15 COMPARISON OF EXPERIMENTAL AND THEORETICAL 'k3' VALUES.

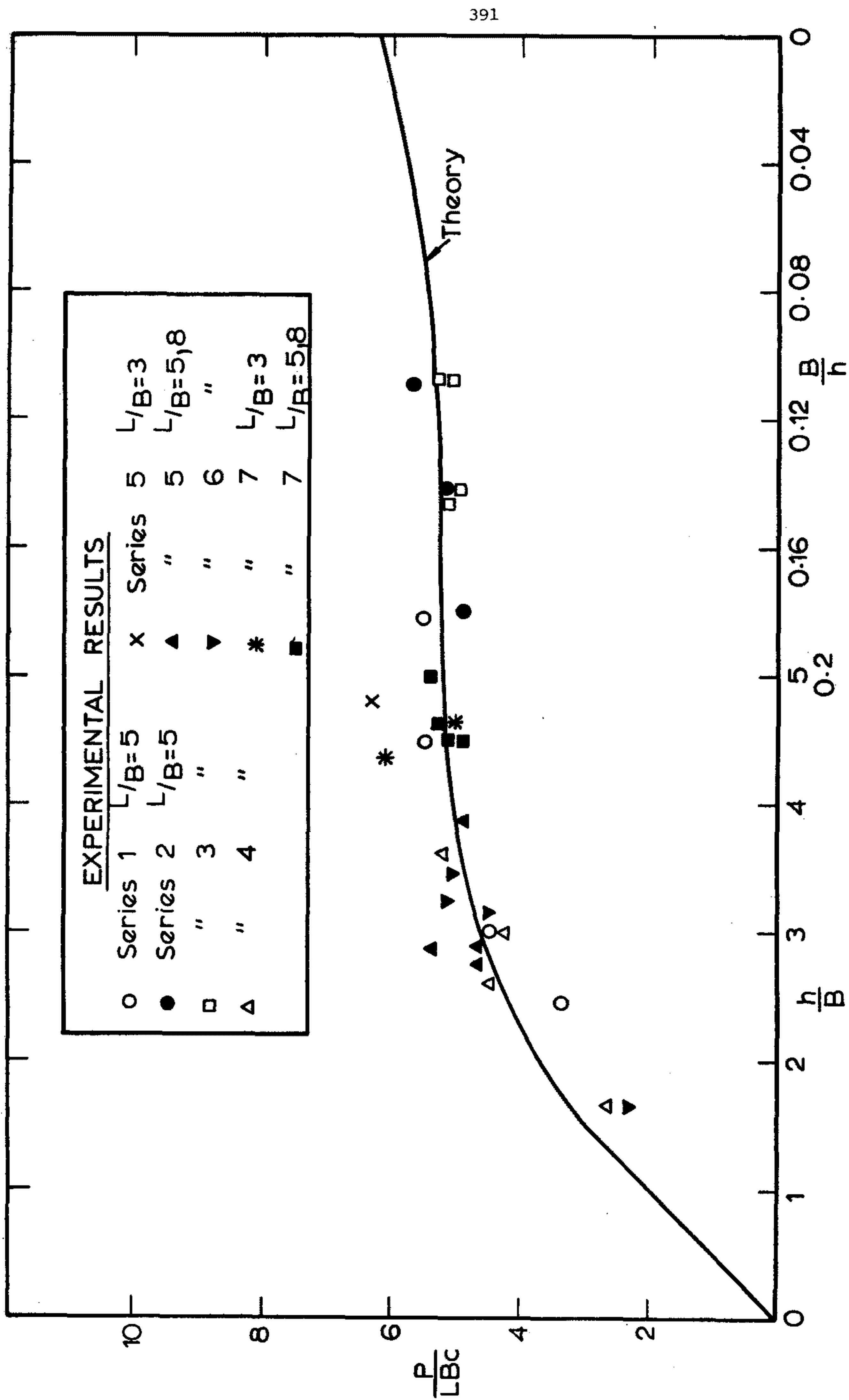


FIG. 8.16 COMPARISON OF EXPERIMENTAL AND THEORETICAL 'k5' VALUES.

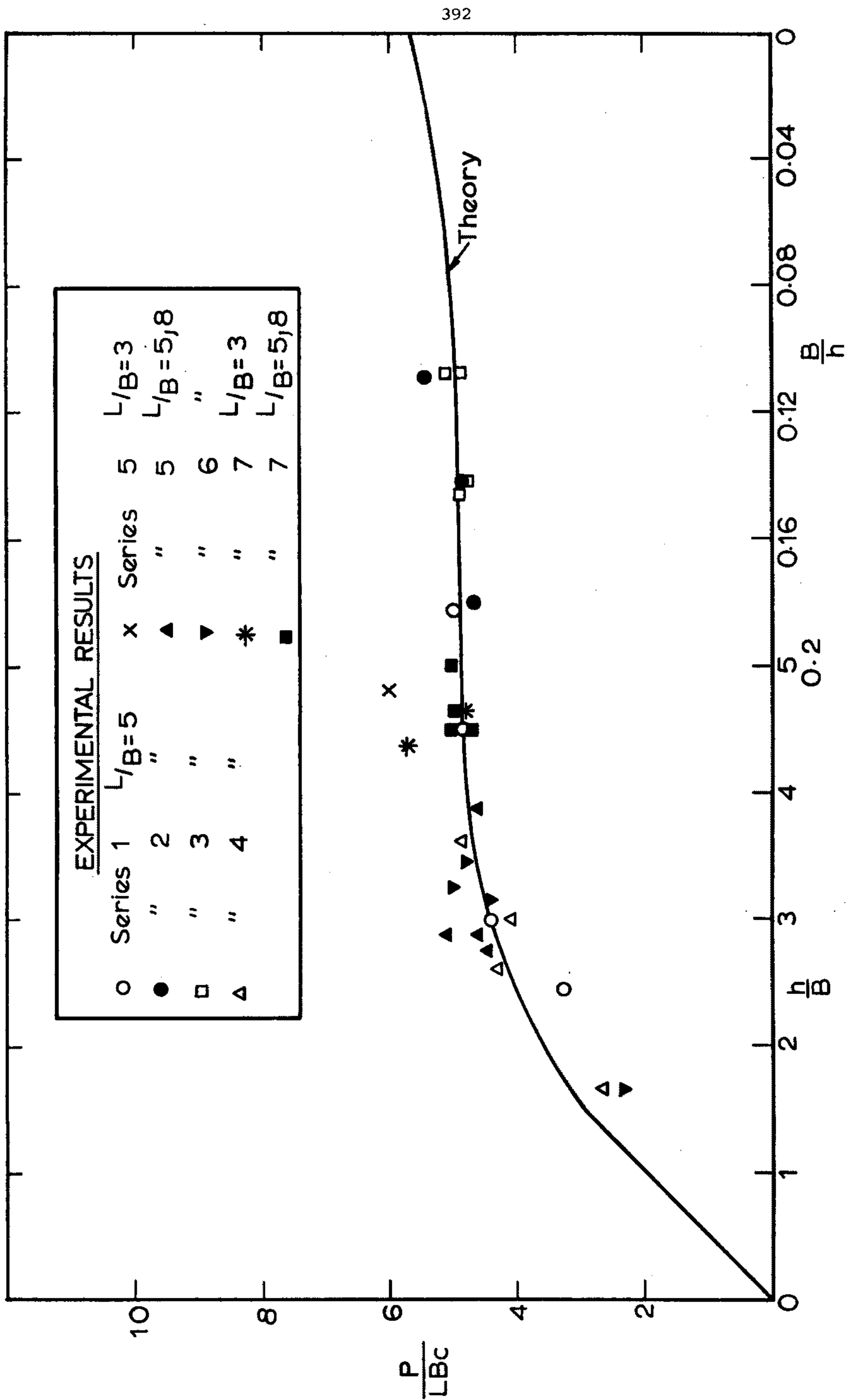


FIG. 8.17 COMPARISON OF EXPERIMENTAL AND THEORETICAL 'k4' VALUES.

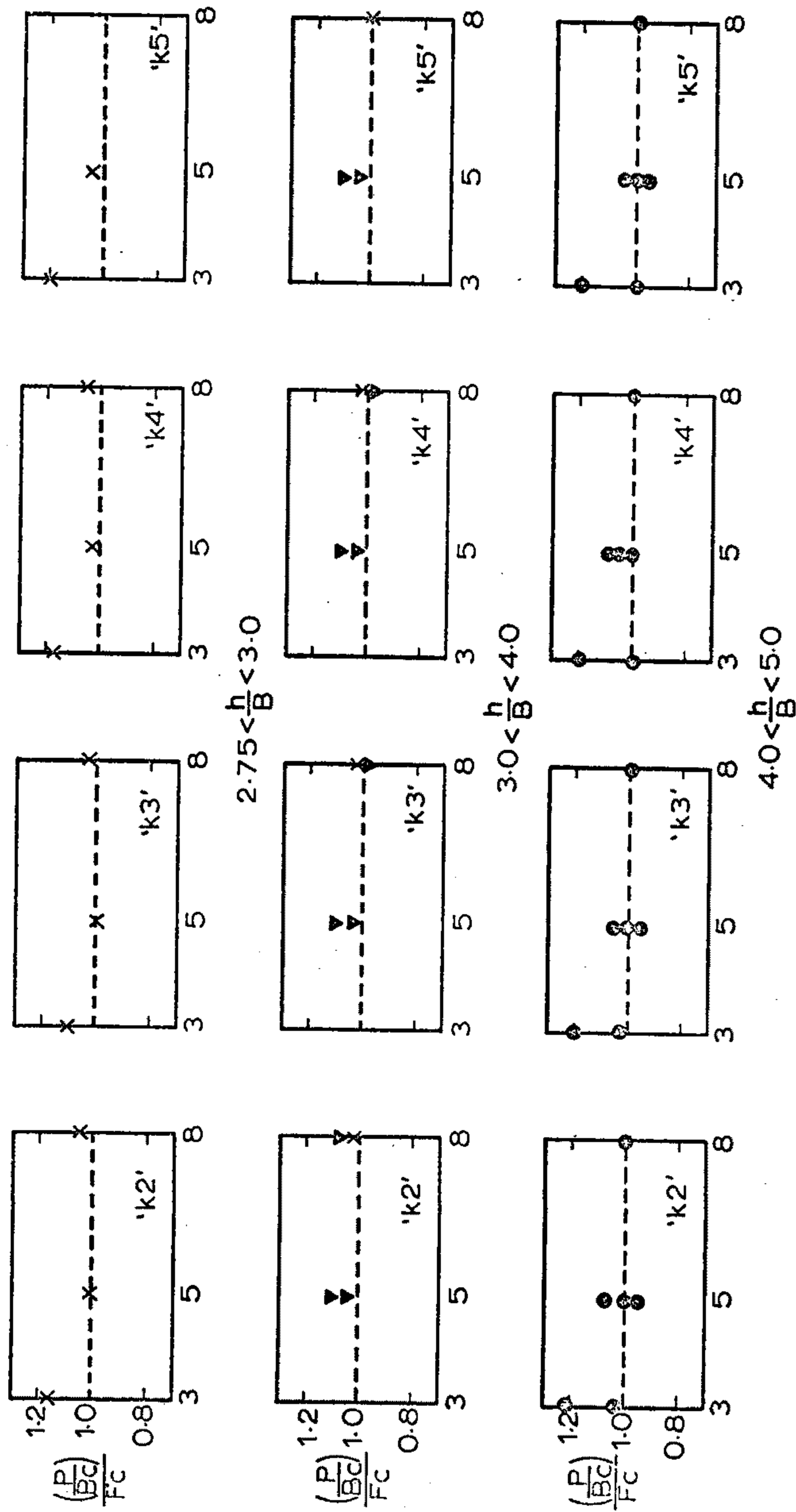


FIG. 8-18 COMPARISON OF EXPERIMENTAL AND THEORETICAL BEARING CAPACITY FACTOR FOR VARIOUS $\frac{L}{B}$

no constant difference between the two cases and it would appear that both anchors behave as a strip anchor. With regard to the smallest aspect ratio, $L/B = 3$, there is insufficient evidence to make a definite statement; however, there is some evidence to suggest that the reduction in aspect ratio does correspond to an increase in the bearing capacity of the anchor and hence the strip results obtained from the finite element analysis should give a conservative estimate of the bearing capacity for rectangular anchors.

8.3 MODEL TESTS IN SAND

8.3.1 Details of Sand Properties and Preparation

The tests reported in this section were performed using a dry medium grained quartz sand from Sydney: $\gamma_{\min} = 14.3 \text{ kN/m}^3$; $\gamma_{\max} = 17.3 \text{ kN/m}^3$. The grading curve is shown in Fig. 8.19.

The tests were performed in a steel box with dimensions 450mm x 606mm and height 381mm. The box was designed so that an additional section could be bolted on to the top flange giving a total height, for the modified box, of 692mm.

In this series of tests, attention was directed towards the bearing capacity of anchors in loose, normally consolidated sand. Loose sand was used so that the effect of embedment ratio and angle of internal friction, ϕ , upon the bearing capacity could, as far as possible, be determined without the additional complications due to dilation during plastic deformation. In addition, the consideration of loose sands reduces the likely scale effects which are frequently

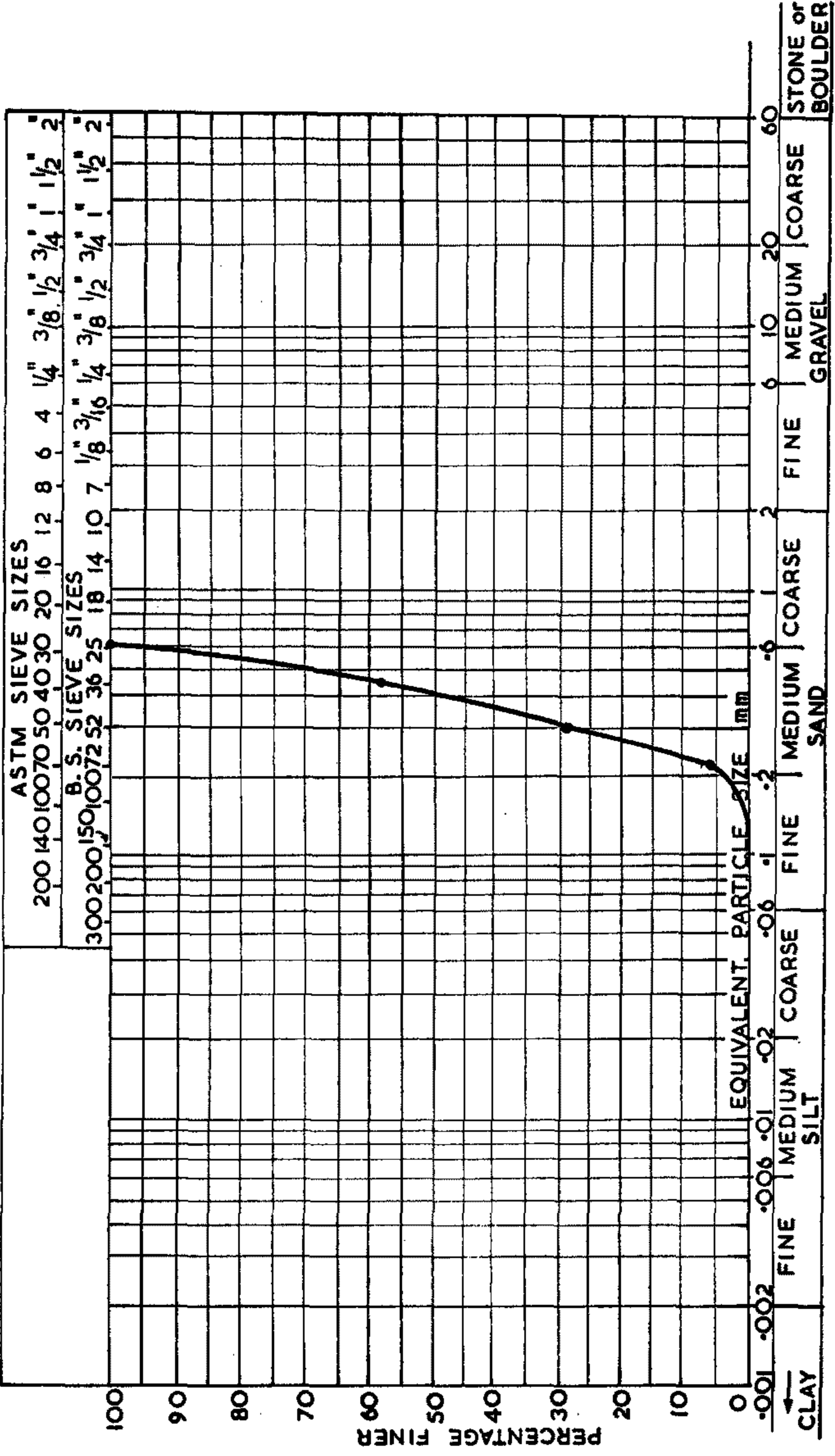


FIG. 8.19 PARTICLE SIZE FOR SYDNEY SAND.

observed with sands (e.g. Graham and Stuart, 1971; and Vesic, 1973), since for loose sands the angle of friction (ϕ) is far less affected by confining pressure than is the case for denser sands (Ladd et al., 1977).

At the beginning of each test, a 140mm thick layer of sand was placed in the bottom of the box by lightly sprinkling the sand from a height of approximately 25mm above the soil surface. This layer was gently levelled off, and the anchor plate placed in position as shown in Fig. 8.20. Additional sand was then sprinkled into the box from a height of 25mm above the surface until the plate was buried to the required depth. The density of the sand was determined directly from the known volume occupied by the sand and the weight of sand within the box. Over a total of 41 tests, the mean density was found to be $\gamma = 14.9 \text{ kN/m}^3$ with a variation of less than 1% between individual tests. Conventional triaxial tests performed on 76mm diameter samples for cell pressures in the range 30 - 100 kPa indicated that over the relevant range of density (viz: $14.76 < \gamma < 15.04$) the angle of internal friction ϕ was 32° . The dilatancy of the material was determined from the volumetric and axial strain measured in triaxial tests on saturated sand. At the relevant density, the maximum angle of dilatancy was found to be approximately 4° . No direct attempt was made to measure the coefficient of lateral earth pressure (K_0) of sand within the test box. However, K_0 may be estimated from Jaky's relationship (1944) to be 0.47 for $\phi = 32^\circ$.

To ascertain the sensitivity of the anchor bearing capacity to the method of sand placement, six tests were performed in which the sand was poured from a height of 250mm above the soil surface. In this

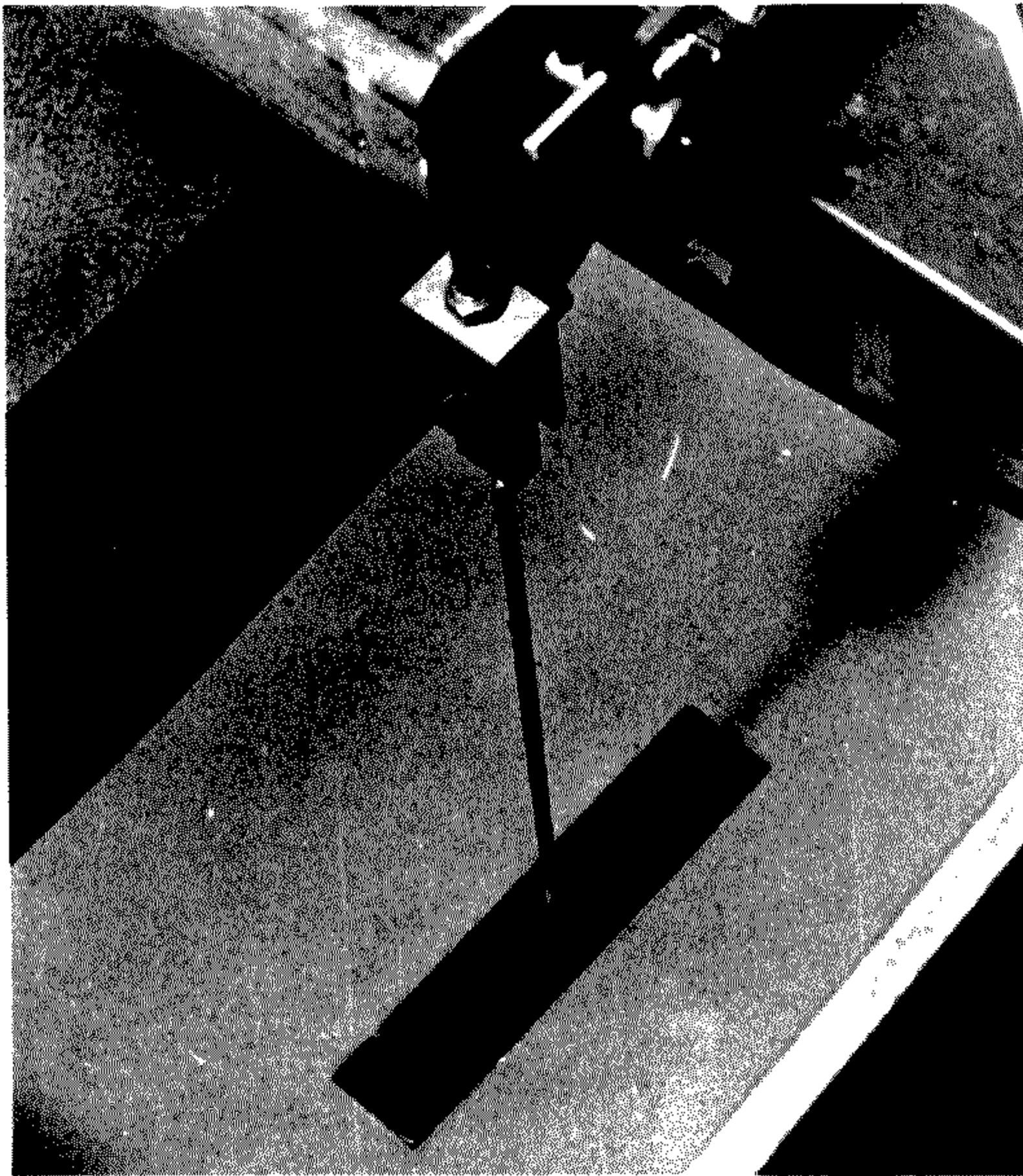


FIG. 8.20 ANCHOR RESTING ON BASE LAYER
OF SAND

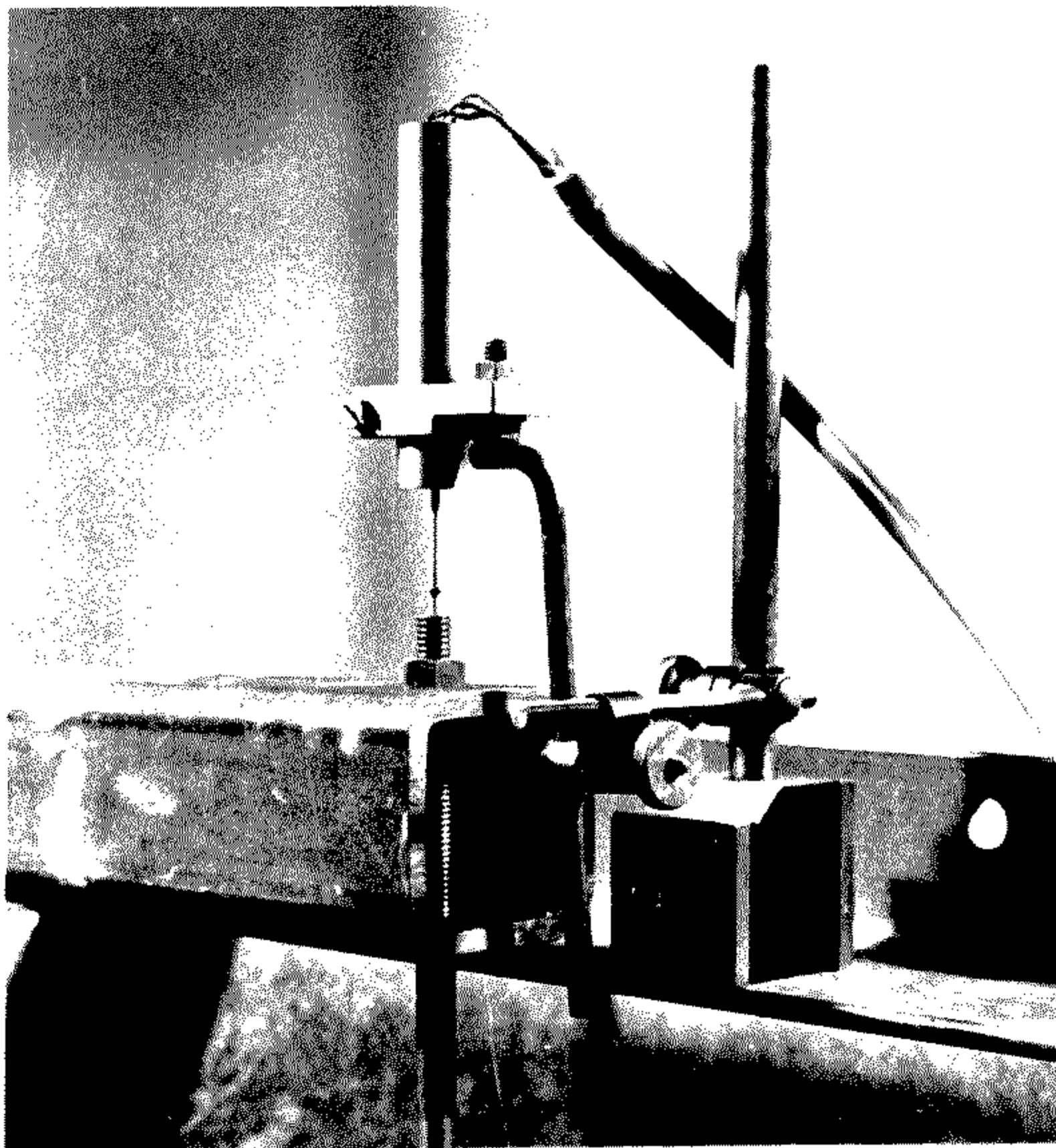


FIG. 8.21 CONNECTION TO LOADING ARM

case, the relevant soil parameters were $\gamma = 15.27 \text{ kN/m}^3$, $\phi = 33.3^\circ$, $\psi_{\text{max}} \approx 10^\circ$ and the estimated coefficient of lateral earth pressure was in the range $0.4 \leq K_0 \leq 0.45$.

The modulus of elasticity, E , of the sand was obtained from triaxial tests performed at cell pressures of 50, 70 and 100 kPa. The modulus was then normalised with respect to the initial vertical stress σ_{vo} and may be expressed in the form

$$E/p_a = A(\sigma_{\text{vo}}/p_a)^n \quad (8.1)$$

where p_a is standard atmospheric pressure.

The best description of the experimental modulus was obtained when the parameters n and A in Eq. 8.1 were 0.88 and 440 respectively. If it is assumed that the parameter n is equal to unity, then the best description of the experimental modulus is given for $A = 450$.

8.3.2 Anchor Plate and Loading Apparatus Details

The model anchor plates were all 51mm wide, with lengths of 51, 102, 152, 203, 254 and 445mm corresponding to length to breadth ratios of 1, 2, 3, 4, 5 and 8.75. Each anchor was cut from 8mm thick mild steel. The anchor surfaces were polished and the maximum angle of interface friction $\phi_k = 16.7^\circ$ was determined using the largest anchor plate resting on the sand surface, and subjecting it to both vertical and horizontal loading.

The anchor plates were connected to the loading device (see

Fig. 8.21) by a 440mm long and 8mm diameter steel rod which screwed into the mid-point of the anchor at one end, and into a brass connecting bar at the other end. The brass connecting bar was pinned to a 'U'-shaped brass section (see Fig. 8.22) by a 6mm diameter steel bolt passing through oversized holes. The 'U'-section was in turn connected to the steel lever arm, using a threaded 6mm steel bar and nut arrangement. This allowed adjustment of the level of the loading beam to ensure it was horizontal prior to testing. The remainder of the loading apparatus was the same as previously described for the model tests in clay (section 8.2.3).

8.3.3 Experimental Procedure

Once the sand and anchor had been placed as described in section 8.3.1, a counter weight equivalent to the weight of the anchor and loading device was placed on the applied load end of the lever. The proving ring and displacement transducers were then moved into position and the initial readings taken.

The anchor was loaded using the constant rate of penetration machine running at a speed of 1.5mm/min. The displacement and load were measured and recorded using transducers and the Hewlett Packard X-Y recorder in a similar fashion to that described for the clay tests.

In a number of cases, once failure had occurred the anchor was unloaded and then reloaded to determine the effects of prior collapse on anchor behaviour.

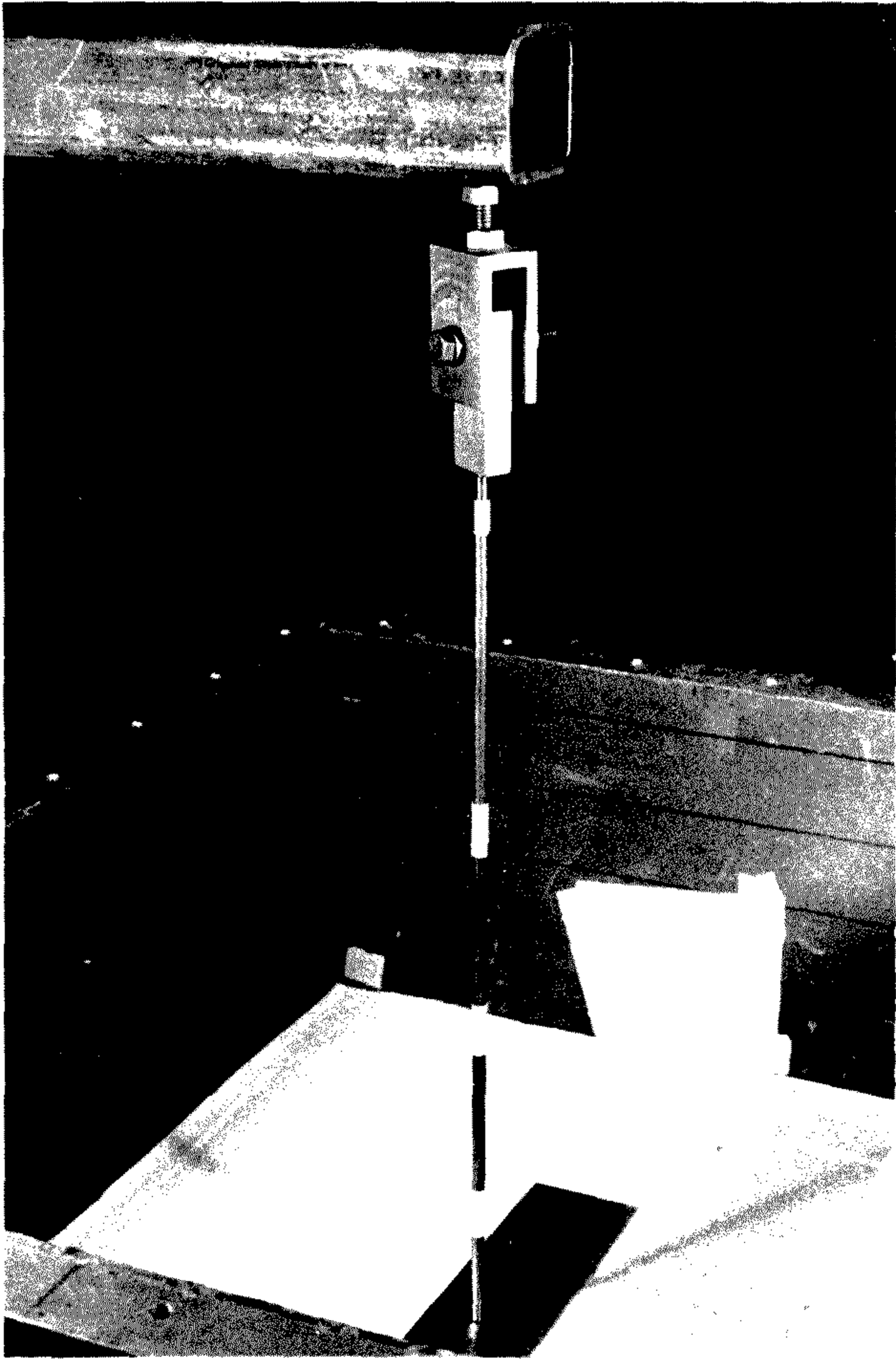


FIG. 8-22 CONNECTION TO ANCHOR ROD

8.3.4 Experimental Results

The ultimate bearing pressure determined from 47 pullout tests on horizontal anchors resting in dry sand are summarised in Table 8.4. The geometric parameters L , B , and h , as well as the unit weight γ and dimensionless bearing capacity $P/LB\gamma h$, are also shown in Table 8.4.

The majority of the pullout tests (ie. numbers 1 - 36) were performed for anchors with an aspect ratio $L/B = 8.75$ and a number of typical load deflection curves obtained for this case are shown in Figs. 8.23 and 8.24. In each of these tests, the collapse load was clearly defined from the load deflection curve, however in general, there was no evidence of anchor failure apparent at the soil surface. From Fig. 8.23, it will be seen that for quite shallow anchors ($h/B = 3$), collapse of the anchor was associated with plastic deformation without any change in the applied load after a displacement of about 2mm (ie. $\delta/h \doteq 0.013$). For higher embedment ratios, collapse is associated with oscillation in the load deflection behaviour which increases in severity with embedment ratio. It seems likely that this behaviour is due to the collapse of sand from the front of the anchor into the gap which will form behind the anchor once breakaway occurs. This type of behaviour was observed for deeper anchors (ie. $h/B \geq 5$) after a displacement of approximately 8mm, irrespective of embedment ratio, although the magnitude of the oscillations did increase with embedment ratio.

It was found that anchor collapse was usually preceded by extensive inelastic (probably largely contained plastic) deformation

TABLE 8.4

EXPERIMENTAL RESULTS - SAND TESTS

Test No.	L (mm)	h (mm)	h/B	L/B	γ kN/m ³	$\left(\frac{P}{BL}\right)$ collapse kPa	$\frac{P}{BL\gamma h}$
1	445	51	1	8.75	14.86	1.15	1.52
2	445	51	1	8.75	14.84	1.12	1.48
3	445	51	1	8.75	14.82	1.17	1.55
4	445	51	1	8.75	14.93	1.00	1.31
5	445	102	2	8.75	14.76	3.03	2.01
6	445	102	2	8.75	14.82	2.96	1.96
7	445	102	2	8.75	14.91	3.03	1.99
8	445	102	2	8.75	14.77	2.94	1.95
9	445	152	3	8.75	15.00	5.75	2.52
10	445	152	3	8.75	15.00	5.87	2.57
11	445	152	3	8.75	14.90	5.66	2.50
12	445	152	3	8.75	14.86	5.66	2.50
13*	445	152	3	8.75	15.29	6.13	2.64
14*	445	152	3	8.75	15.27	5.93	2.55
15	445	203	4	8.75	15.13	9.65	3.14
16	445	203	4	8.75	15.02	9.10	2.98
17	445	203	4	8.75	14.86	8.69	2.88
18	445	203	4	8.75	14.86	8.93	2.96
19	445	254	5	8.75	14.91	14.5	3.83
20	445	254	5	8.75	14.85	14.2	3.76
21	445	254	5	8.75	14.79	14.3	3.81
22	445	254	5	8.75	14.83	14.8	3.93
23*	445	254	5	8.75	15.24	15.5	4.00
24*	445	254	5	8.75	15.29	15.8	4.07
25	445	305	6	8.75	14.85	21.0	4.64
26	445	305	6	8.75	14.75	19.9	4.42
27	445	305	6	8.75	14.88	21.0	4.63
28	445	305	6	8.75	14.88	19.9	4.38

Table 8.4 continued.

Test No.	L (mm)	h (mm)	h/B	L/B	γ kN/m ³	$\left(\frac{P}{BL}\right)$ collapse kPa	$\frac{P}{BL\gamma h}$
29	445	356	7	8.75	14.87	27.1	5.12
30	445	356	7	8.75	14.77	25.8	4.91
31	445	406	8	8.75	14.90	34.2	5.65
32	445	406	8	8.75	15.02	33.1	5.43
33	445	406	8	8.75	15.04	33.8	5.54
34	445	406	8	8.75	14.81	32.2	5.36
35*	445	406	8	8.75	15.29	37.0	5.96
36*	445	406	8	8.75	15.26	38.2	6.16
37	254	153	3	5	15.00	6.4	2.79
38	254	153	3	5	14.99	6.3	2.75
39	153	153	3	3	15.15	7.4	3.19
40	153	153	3	3	14.99	7.0	3.05
41	102	153	3	2	14.80	7.3	3.22
42	102	153	3	2	15.04	8.0	3.48
43	51	153	3	1	15.15	12.7	5.48
44	51	153	3	1	14.99	12.7	5.54
45	254	305	6	5	14.77	23.5	5.22
46	254	406	8	5	14.79	37.3	6.21
47	254	406	8	5	14.80	37.7	6.27

Notes:

(1) B = 51mm for all tests.

* Sand placed by dropping from a height of 250mm.

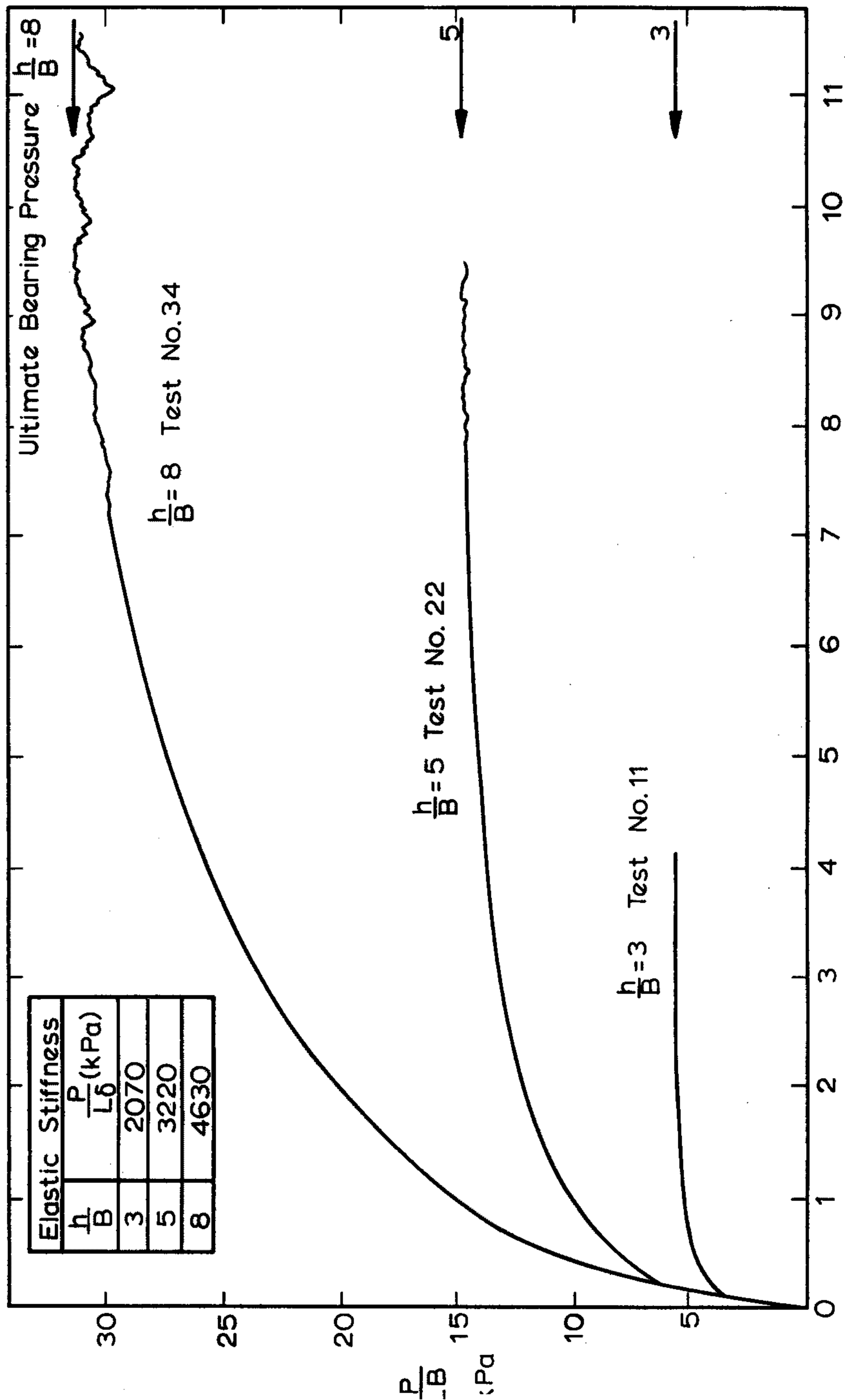


FIG. 8.23 TYPICAL LOAD DISPLACEMENT CURVES FOR ANCHORS IN DRY SAND.

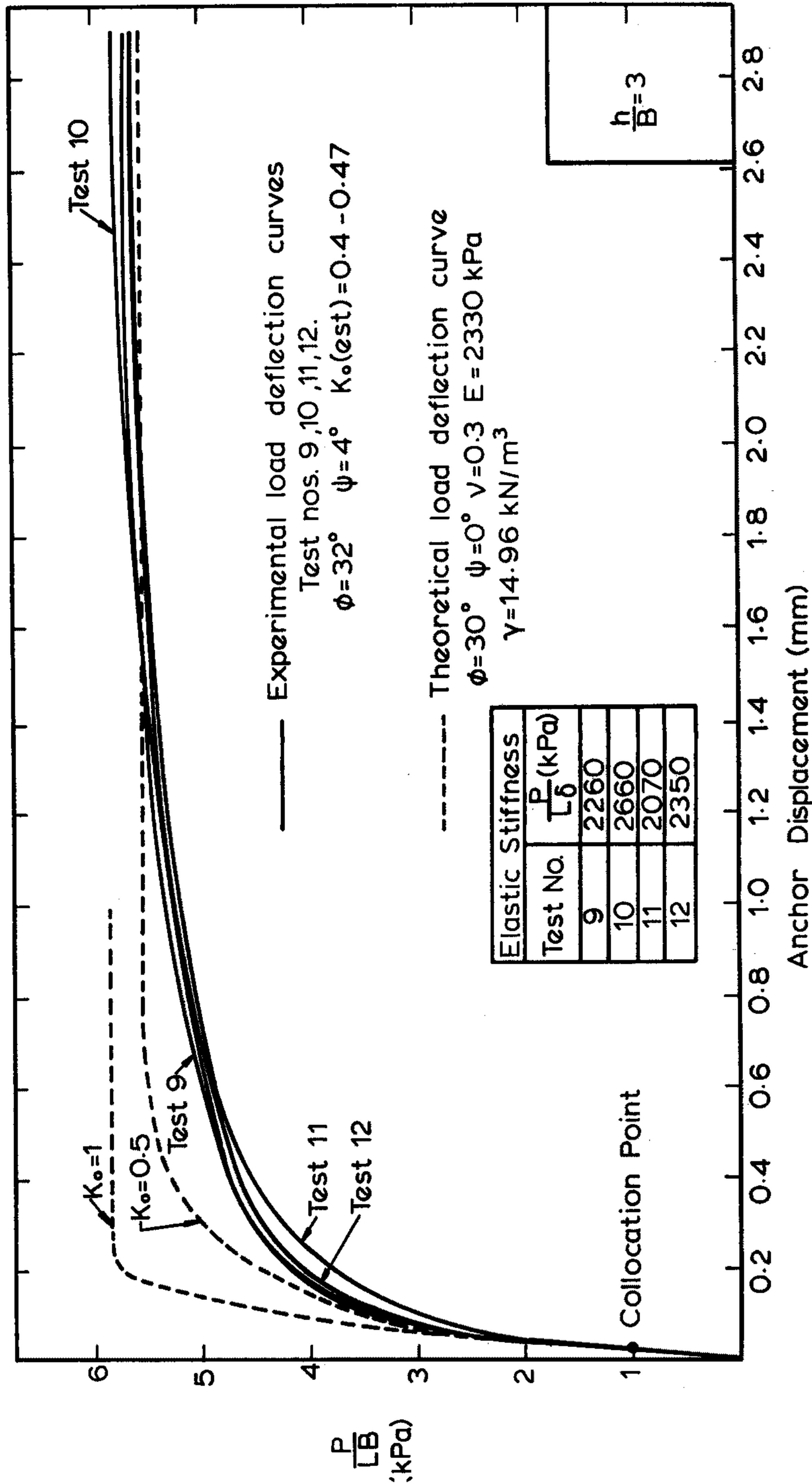


FIG. 8.24 COMPARISON OF EXPERIMENTAL AND THEORETICAL LOAD DEFLECTION CURVES.

and in fact genuine elastic behaviour was only apparent for displacements less than 0.1mm. Thus the elastic deformation constitutes an extremely small portion of the total load deflection curve (less than 1% for $h/B = 8$) and in tests primarily directed at the determination of the ultimate bearing pressure, the elastic stiffness could not be accurately resolved from the plot of test results, and similarly, cannot be accurately represented on the reduced copies of the load deflection curves shown in Figs. 8.23 and 8.24. For the cases shown in Figs. 8.23 and 8.24, special attention was given to the determination of the elastic stiffness as summarised in the respective figures. From Fig. 8.24, it will be seen that despite the differences in measured elastic stiffness, the load deflection curves are fairly similar in each case, while the ultimate bearing pressures vary by less than 4%. This observation was generally true for all embedment ratios. The dimensionless ultimate bearing pressures $P/LByh$ obtained for $L/B = 8.75$ are plotted against embedment ratio in Fig. 8.25, and it can be seen that these increase approximately linearly with embedment ratio.

The six tests performed with the sand deposited by spilling from a height of 250mm are summarised in Table 8.4., (ie. Tests 13, 14, 23, 24, 35, 36) and the dimensionless bearing capacities are plotted in Fig. 8.25. Comparison of these results with those obtained from similar tests in sand placed using the normal (25mm drop) procedure indicate that the increase in bearing pressure is considerably greater than that which would be expected due to the slightly higher density alone. Insufficient tests were performed for any definite statement to be made about the effects of deposition and dilatancy upon the collapse load; however, it would appear that the increase in dilatancy does significantly effect the collapse and, fur-

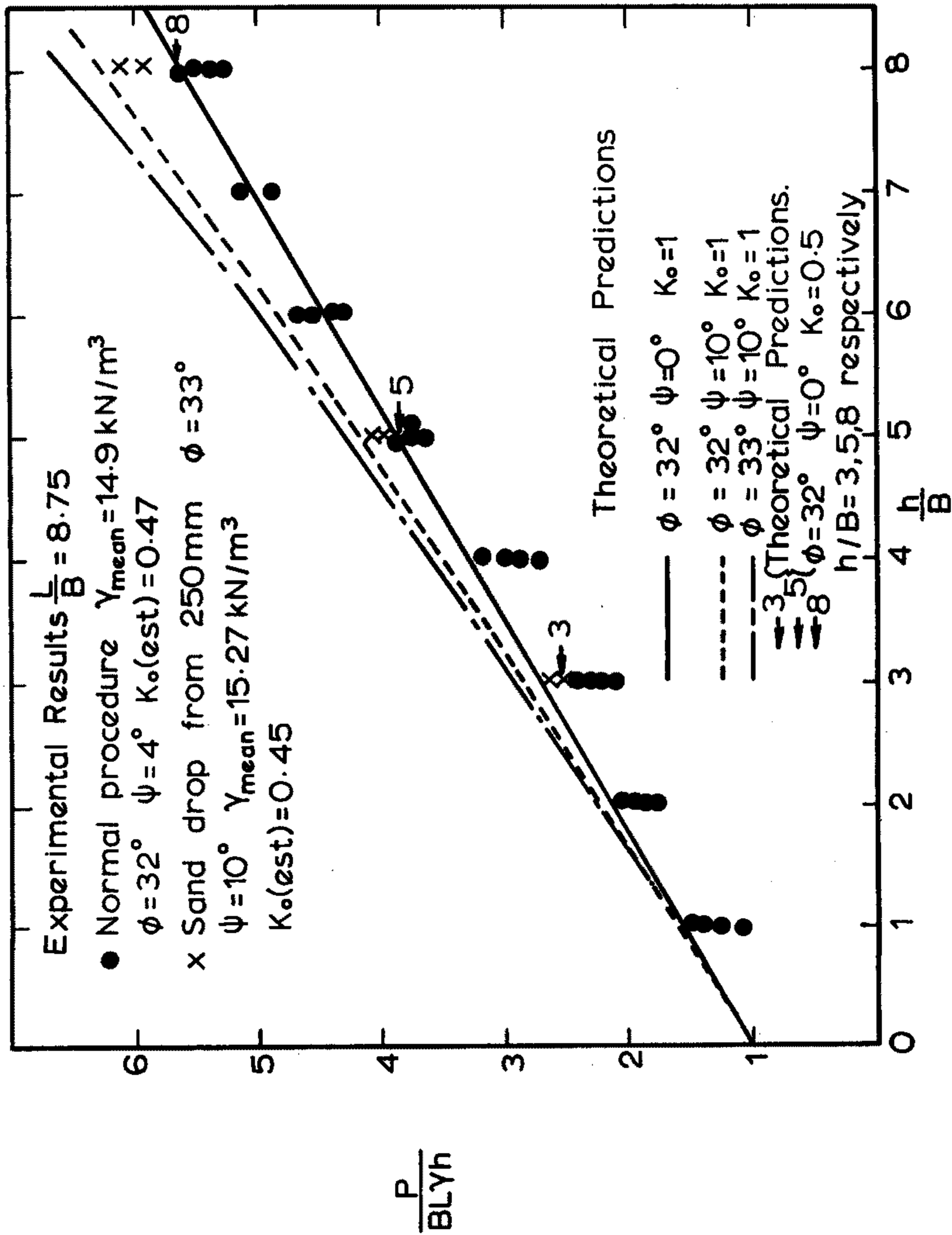


FIG. 8.25 COMPARISON OF EXPERIMENTAL AND THEORETICAL COLLAPSE LOADS.

thermore, the magnitude of this effect appears to increase with embedment ratio, as predicted in Chapter 7.

In a number of cases (over a range of embedment ratios), an anchor was unloaded after collapse and then reloaded again until collapse occurred. This procedure was repeated three or four times for each such case and it was found that:

- (1) the apparent elastic stiffness decreased slightly for each subsequent loading;
- (2) the extent of the inelastic deformation increased; and
- (3) the ultimate bearing pressure appeared to be unaffected by this cyclic loading, as shown for the case where $h/B = 2$ in Fig. 8.26.

Tests numbers 36 to 44 were performed for an anchor with embedment ratio $h/B = 3$ for a range of aspect ratios L/B in an attempt to ascertain the likely effect of shape upon the ultimate bearing pressure. The dimensionless bearing capacities obtained from these tests are plotted against aspect ratio in Fig. 8.27, and it can be seen that the anchor shape has a significant effect upon the ultimate bearing pressure. In particular, it was found that the ultimate bearing pressure for $L/B = 1$ is 120% above that for $L/B = 8.75$, while even at $L/B = 5$ the ultimate bearing capacity is 10% above that for $L/B = 8.75$, indicating that at this aspect ratio, shape is still of some importance. This result may be compared with the findings for purely cohesive soils, where the ultimate bearing pressure was independent of aspect ratio for $L/B \geq 5$.

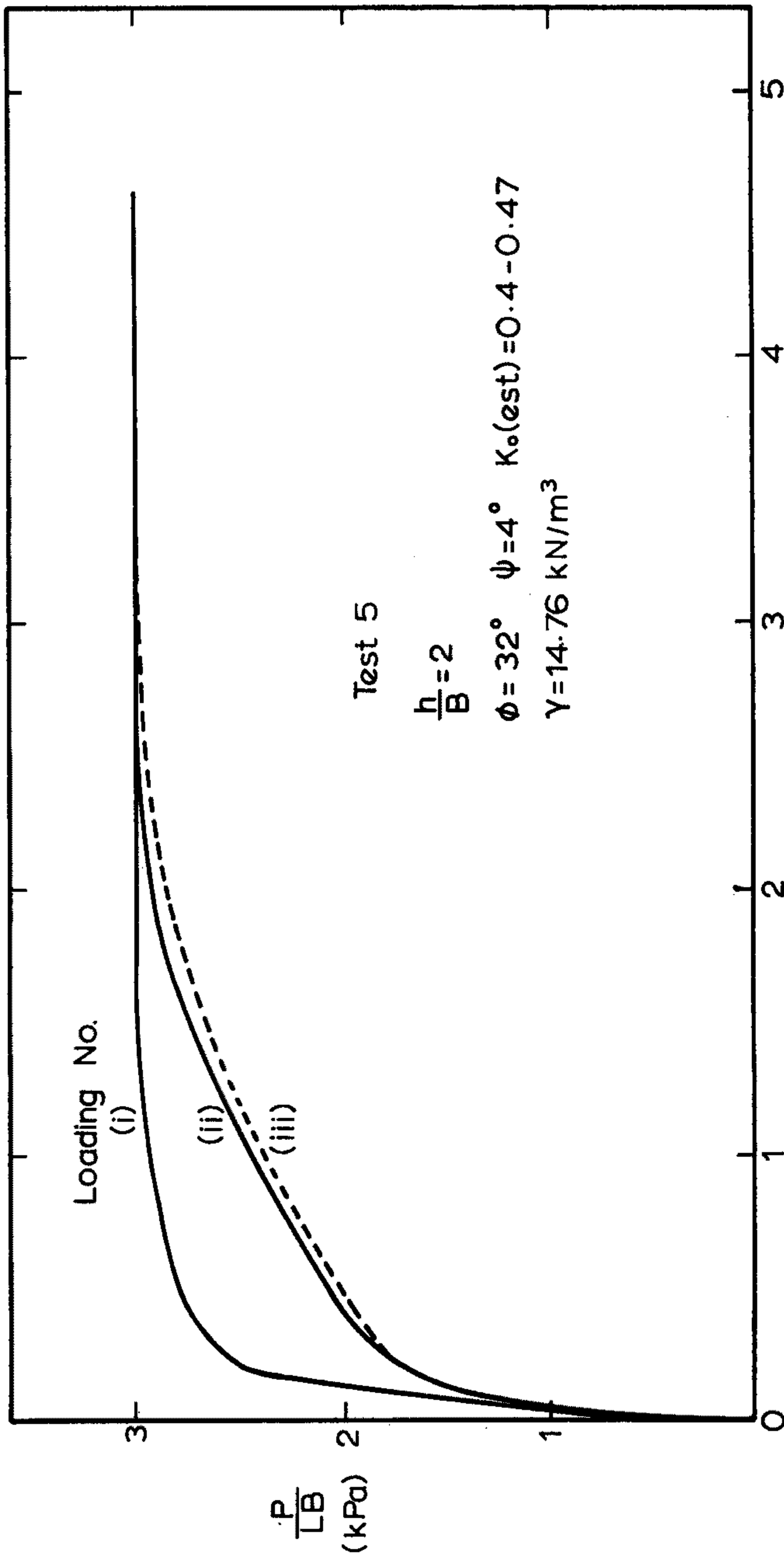


FIG. 8.26 EFFECT OF RELOADING ON LOAD-DEFLECTION BEHAVIOUR

$$\frac{h}{B} = 2 \quad \frac{L}{B} = 8.75$$

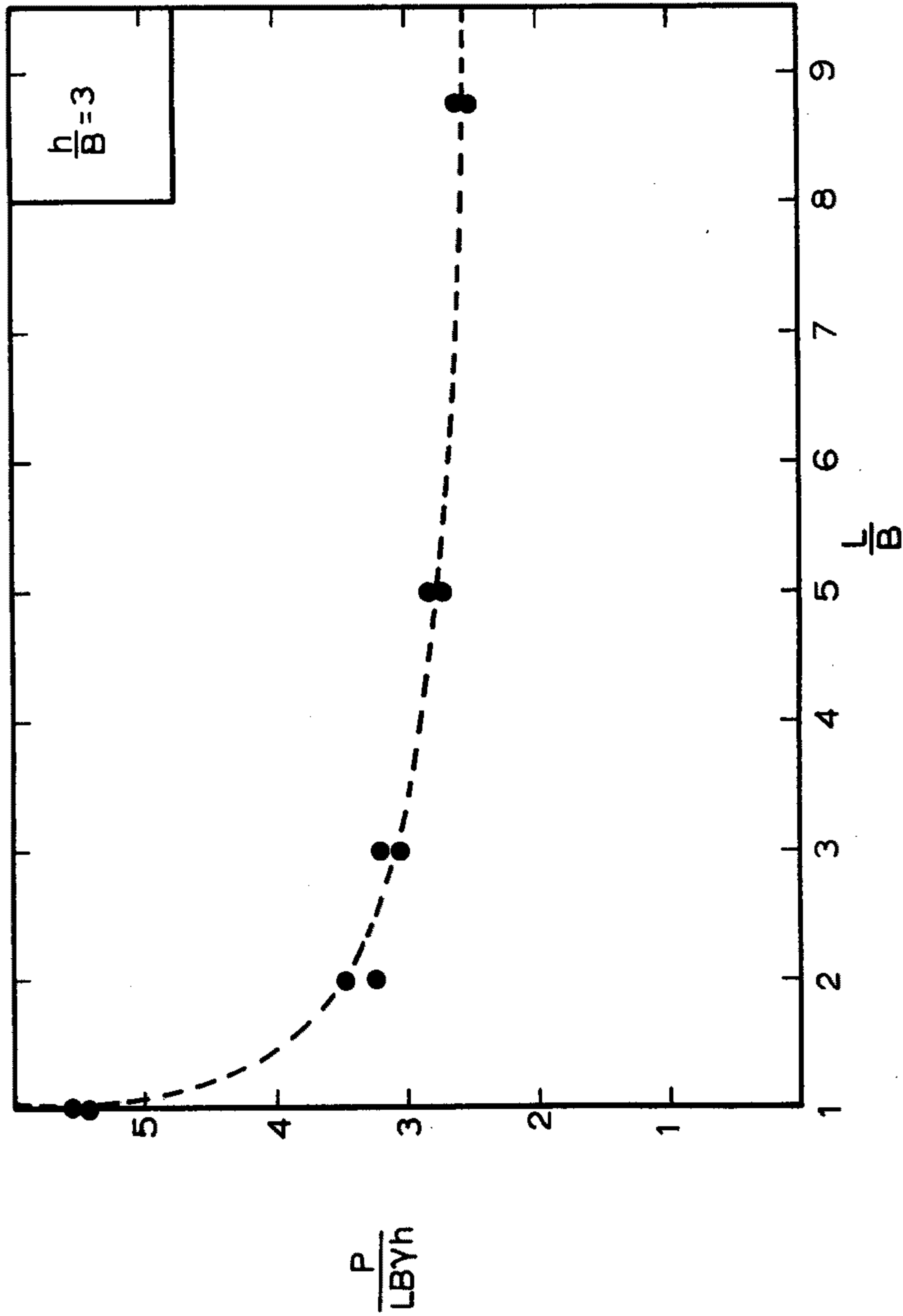


FIG. 8.27 EFFECT OF ASPECT RATIO L/B UPON EXPERIMENTAL COLLAPSE LOAD.

8.3.5 Discussion and Comparison with Theoretical Predictions

In Chapter 7, it was predicted that for horizontal anchors in sand, the initial collapse of the anchor (for $\psi \approx 0$) would not involve significant surface deformation while for $K_0 \leq 1$ it was predicted that collapse would be preceded by extensive contained plastic deformation; both of these general observations have been confirmed by this series of tests.

Direct comparison of the experimental and theoretical load deflection curves is limited because of the uncertainty regarding the value of K_0 , and the theoretical assumption that:

- (1) the elastic modulus was independent of stress state and consolidation conditions; and
- (2) the angle of dilatancy ψ was independent of the strain.

These restrictions are such that one could not reasonably expect to reproduce the experimental load deflection behaviour, since in practice the elastic modulus will vary with the stress state while the dilatancy of each point will decrease from the maximum value at the peak strength to zero at the residual strength, thereby affecting the general soil response to the loaded anchor. However, despite these restrictions, a comparison can be made between the experimental load deflection curve and the theoretical load deflection curves obtained in Chapter 7 for $\phi = 30^\circ$, $\psi = 0^\circ$, $K_0 = 0.5$ as shown in Fig. 8.24. Also shown in this figure is the theoretical load deflection for $\phi = 30^\circ$, $\psi = 0^\circ$, $K_0 = 1$ and it can be seen that the predicted effect of K_0 upon the load deflection curve is in general agreement with the experimental observa-

tions. Furthermore, it will be recognised that while there are noticeable differences between the experimental and theoretical curves, the theoretical prediction for $K_0 = 0.5$ is in error by less than 15% at any displacement level and hence would be quite adequate for many engineering applications.

The theoretical idealisation involving the assumption of constant elastic modulus and angle of dilatancy would not be expected to seriously effect the ultimate collapse load (particularly for relatively loose sands), since at collapse the elastic parameters are largely irrelevant while a significant portion of the soil mass will generally have reached its residual strength and hence the dilatancy, ψ , will be zero. Comparison of the experimental ultimate bearing pressures with the theoretical prediction for $\phi = 32^\circ$, $\psi = 0^\circ$, $K_0 = 1$ (see Fig. 8.25) indicates encouraging agreement. As discussed in Chapter 7, the value of $K_0 = 0.5$ does not greatly alter the collapse load, however, comparison of the predicted collapse load where $\phi = 32^\circ$, $\psi = 0^\circ$, $K_0 = 0.5$ and $h/B = 3, 5$ and 8 indicated even better agreement with the experimental results.

The dimensionless anchor bearing capacities shown in Fig. 8.25 for anchors resting in the denser sand deposit ($\phi = 33^\circ$, $\psi = 10^\circ$, $K_0 = 0.4 - 0.45$, $\gamma = 15.27 \text{ kN/m}^3$) may be compared with the theoretical prediction for $\phi = 33^\circ$, $\psi = 10^\circ$, $K_0 = 1$. As might be expected, the experimental results ($K_0 = 0.4 - 0.45$) fall below the prediction for $K_0 = 1$; however, the general trend of increasing significance of dilatancy with increasing embedment ratio is observed, while the discrepancy between the experimental results and the theoretical predictions is less than 10%.

8.4 COMPARISON OF THEORETICAL PREDICTIONS WITH PUBLISHED EXPERIMENTAL DATA

8.4.1 Circular Anchors in Clay

As stated in Chapter 2, a large number of model and field pullout tests have been performed with circular anchors in clay. In some cases, special care was taken to prevent the development of suction or adhesion between anchor and soil (e.g. Adams and Hayes, 1967; Davie and Sutherland, 1977). In other cases, the test results were corrected to make allowance for the estimated suction forces between the anchor and the soil (e.g. Spence, 1965; Langley, 1967; and Ali, 1968) while in the remaining cases (e.g. Turner, 1962; Paterson and Urie, 1964), no consideration appears to have been given to the effect of suction. The dimensionless bearing capacities obtained from these various tests are plotted against embedment ratio in Fig. 8.28. Unfortunately, insufficient data exists for a direct comparison of the predicted and experimental loads at the various 'k2', 'k3' etc. displacements, however, the predicted 'k4' and 'kl0' failure loads given in Chapter 7 are shown in Fig. 8.28 to allow some assessment of the suitability of failure loads predicted on this basis.

The wide scatter of experimental results indicates the general uncertainty involved in estimating the uplift capacity of anchors with regard to the definition of failure itself and the determination of the relevant soil cohesion c , and for model tests, the determination of the relevant embedment ratio h/B . In cases where details of the anchor behaviour prior to failure have been given, it would appear that failure was preceded by extensive contained plastic deformation, as predicted

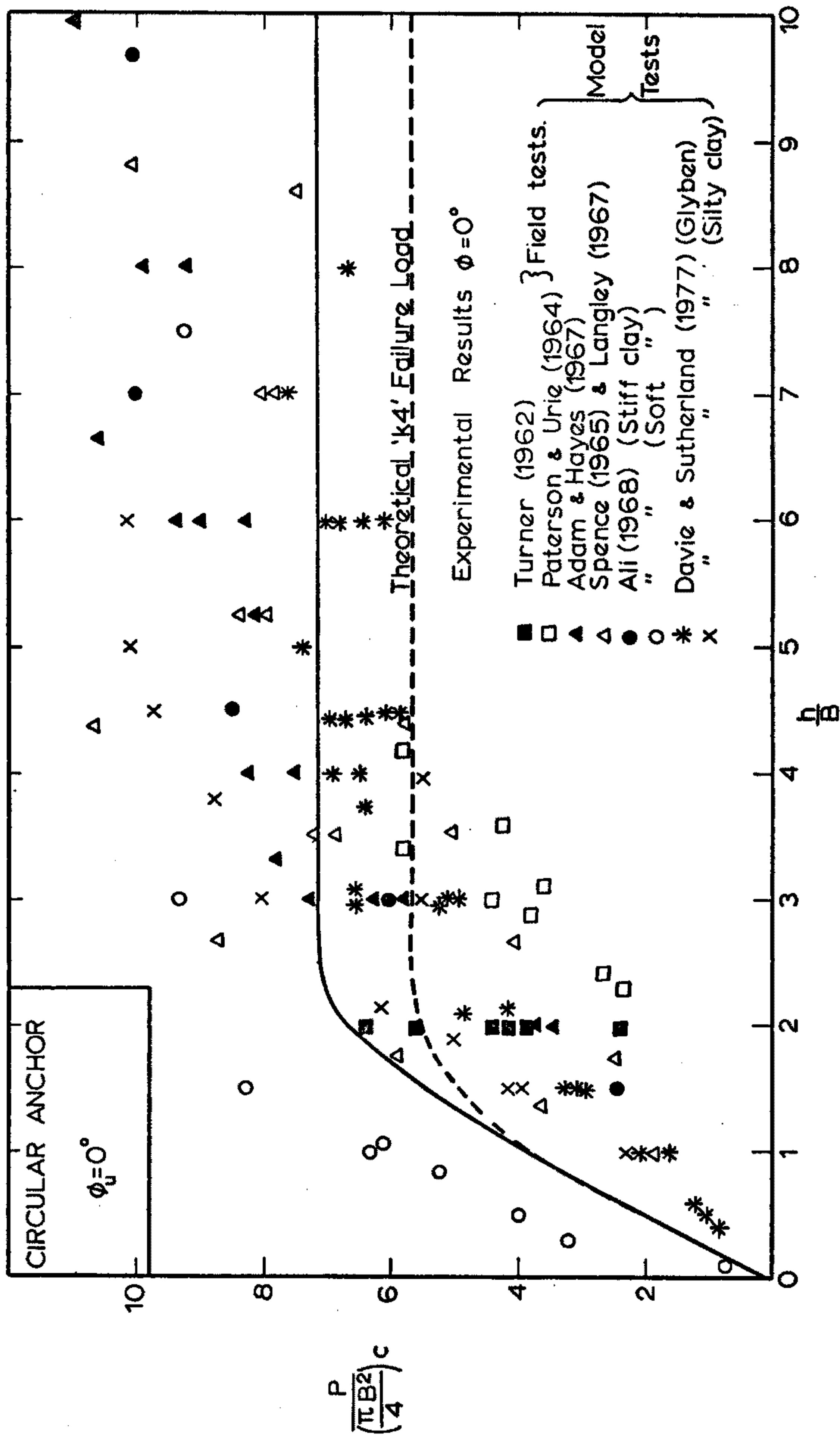


FIG. 8.28 COMPARISON OF EXPERIMENTAL 'COLLAPSE' LOADS WITH PREDICTED
'k4' FAILURE LOAD.

in Chapter 7.

It is difficult to draw any firm conclusions from the comparison of experimental and theoretical failure loads given in Fig. 8.28: however, two observations can be made. Firstly, for embedment ratios less than 2, the majority of experimental results lie below the predicted failure load. In many of these cases, it would appear that the anchor behaviour was dominated by tensile failure arising from flexure of the soil above the anchor. This accords with the experience gained in section 8.2 with rectangular and strip anchors in purely cohesive soil, indicating that before an accurate prediction of the uplift capacity of very shallow anchors in purely cohesive material can be made, it will be necessary to make provision in the analysis for the formation of tensile cracks. Secondly, it will be observed that for anchors with embedment ratios greater than 3, the 'k4' definition of failure generally provides a conservative estimate of the failure load, while the 'k10' definition of failure lies within the scatter of reported failure loads.

8.4.2 Strip Anchors in Clay

Swain (1976) performed two plane strain pullout tests on deep, isolated triangular under-reamed anchors. In the first of these tests (Test 4) the clay was normally consolidated under a pressure of 216 kPa and the pullout tests were then performed with this surcharge pressure acting. In the second test (Test 5) the clay was consolidated under a pressure of 1318 kPa; the pressure was then reduced to 188 kPa and the pullout tests performed on the overconsolidated clay ($OCR = 7$) with this surcharge acting. The relevant soil parameters

are shown in Table 8.5.

TABLE 8.5

SOIL PARAMETERS FROM SWAIN'S TESTS

Test No. (Swain)	Consolidation Initial kPa	Pressure Final kPa	OCR	c_u kPa	E_u kPa	E_u^* kPa	K_o
4	216	216	1	38.6	3850	770	0.654
5	1318	188	7	98.9	9500	4750	0.669

The dimensionless 'k2', 'k3', 'k4', 'k5' load levels obtained by Swain are shown in Fig. 8.29. The basic prediction that can be obtained for the case of immediate breakaway (from Chapter 7, Fig. 7.7) is also shown in Fig. 8.29. However, in this case allowance must also be made for the effect of the surcharge upon the load carrying capacity. In Chapter 7, the appropriate equation for an isotropic material was $q = cF_c + \sigma_{vo}$ provided that $q \leq cF_c^*$ where F_c , and F_c^* were the immediate, and no breakaway, bearing capacity factors respectively; c is the cohesion and σ_{vo} the surcharge. This approximate equation is based upon the assumption that the soil, being isotropic, will have the same modulus under expansion and compression and hence, at a given displacement, the decrease in compressive stress below the anchor will be the same (for a very deep anchor) as the increase in compressive stress above the anchor. However, Swain's elastic modulus beneath the anchor, E_u^* , was observed to be one fifth and one half the modulus above the anchor, E_u , in the normally consolidated and over consolidated states respectively (Swain, 1976, p. 5.4) and hence some modification must be made to the isotropic equation in making the prediction. It would seem reasonable that if the modulus below the anchor is significantly reduced, then

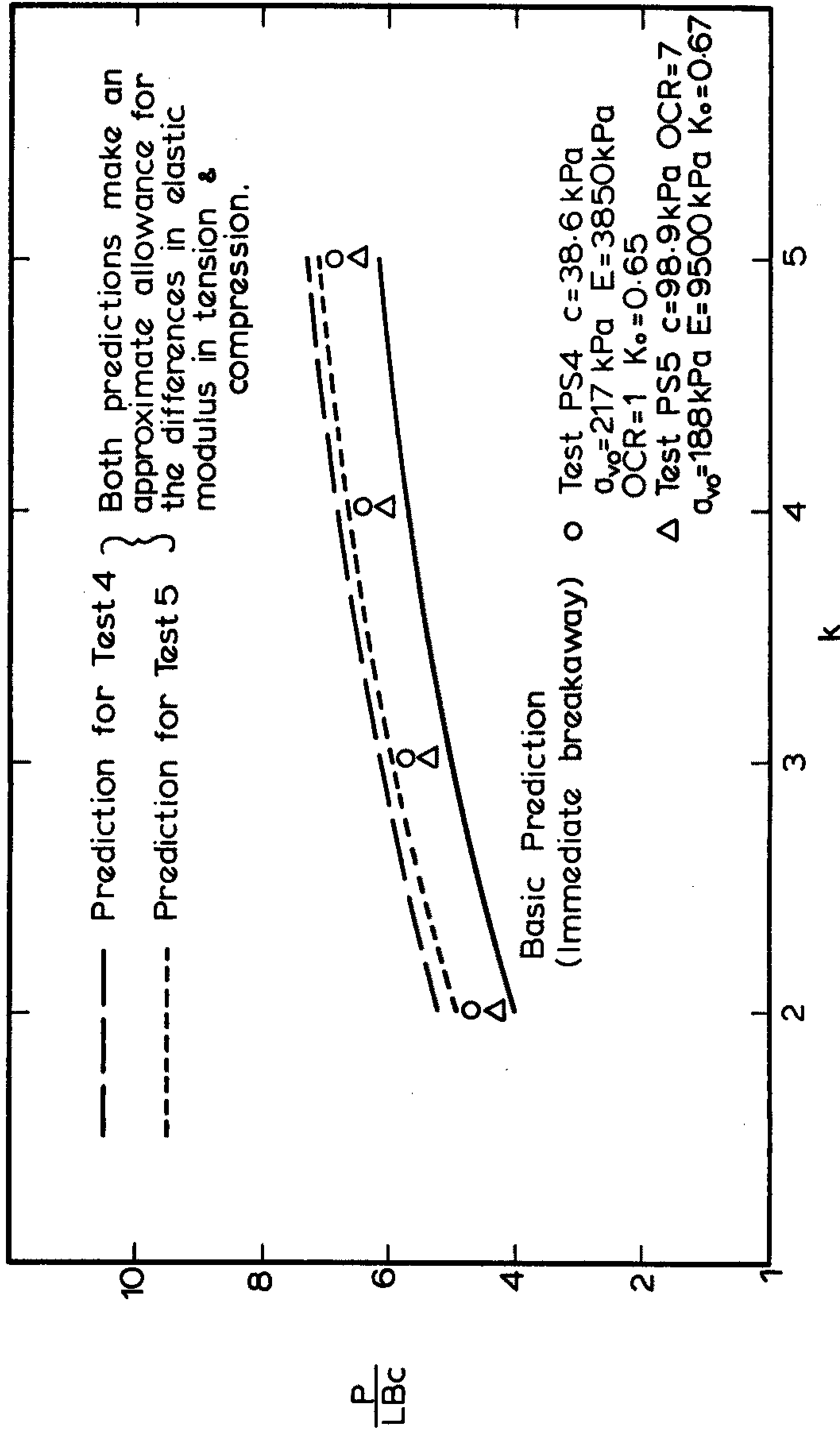


FIG. 8.29 COMPARISON OF SWAINS RESULTS WITH THEORY

the decrease in compressive stress (and consequently in load carrying capacity at any particular displacement) will also be reduced, and in the limiting case as E_u^*/E_u tends to zero the load carrying capacity will approach the immediate breakaway solution. Consequently, the isotropic equation may be replaced by the approximate equation:

$$q = cF_c + (E_u^*/E_u) \cdot \sigma_{vo}$$

provided that $q \leq cF_c^*$ (as defined by Eq. 7.2b)

This leads to the approximate predictions shown in Fig. 8.29. It will be observed that these approximate predictions at the various displacement ('k2', 'k3', 'k4' etc.) levels are in reasonable agreement with the experimental results. It should be noted that if no allowance is made for the effect of different moduli above and below the anchor, then the prediction for the normally consolidated clay would have overestimated the load at each displacement by approximately 68%, however, for the over consolidated clay the error would have been 38%. The major difference between the two cases lies, firstly, in the difference in the ratio E_u^*/E_u from 1/5 to 1/2 and secondly, in the fact that the ratio σ_{vo}/c varies from 5.6 for the normally consolidated clay to 1.9 for the over consolidated clay. Clearly, the implication of this is that caution must be adopted in applying the bearing capacity factors of Chapter 7 to anisotropic clay subjected to high surcharge or overburden pressures. However, these results also indicate that predictions for the immediate breakaway case (ie. $\sigma_{vo}/c \rightarrow 0$) are likely to be reasonably accurate for materials which have significantly different moduli in expansion and compression.

It is of interest to note that the theoretical collapse pressure for an infinitely deep anchor $q_u = 11.42c$ would overestimate the actual collapse load by 20% and 30% respectively for Tests 4 and 5.

8.4.3 Rectangular Anchors in Sand

Das and Seeley (1975a) have performed pullout tests for rectangular anchors ($h/B \leq 5$) in dry sand ($\phi = 31^\circ$, $\psi = ?$, $\gamma = 14.8$ kN/m³, $K_o = ?$) and their results for $L/B = 5$ are shown in dimensionless form in Fig. 8.30. For comparative purposes, the average values (for each embedment ratio) of the results in the present series of tests (see Table 8.4, Fig. 8.25 and section 8.3) are also shown in Fig. 8.30, along with the theoretical predictions for the three cases $\phi = 31^\circ$, $\psi = 0^\circ$, $K_o = 1$; $\phi = 31^\circ$, $\psi = 10^\circ$, $K_o = 1$; and $\phi = 31^\circ$, $\psi = 31^\circ$, $K_o = 1$. It will be observed that the bearing capacities obtained by Das and Seeley are in some cases considerably higher than both the theoretical predictions for the basic case, and the results obtained for $L/B = 8.75$ in the present series of tests. There would appear to be two partial explanations for this.

Firstly, the dilatancy of the sand used by Das and Seeley is unknown. However, it will be seen from both the theoretical predictions and the author's test that the dilatancy can considerably increase the ultimate load carrying capacity. For $\phi = 31^\circ$, the dilatancy would be unlikely to be greater than 10° and hence the experimental results can, at best, be reasonably compared with the theoretical prediction for $\phi = 31^\circ$, $\psi = 10^\circ$. The second factor likely to affect the experimental result is the shape of the anchor. From the author's tests, it can be seen that there is a considerable difference between the results obtained for $L/B = 8.75$ and $L/B = 5$, and this difference may be as high as 15%. Allowing for the possible combination of the effects of dilatancy (say $\psi = 10^\circ$) and shape, the discrepancy between the experimental results of Das and Seeley and the theoretical

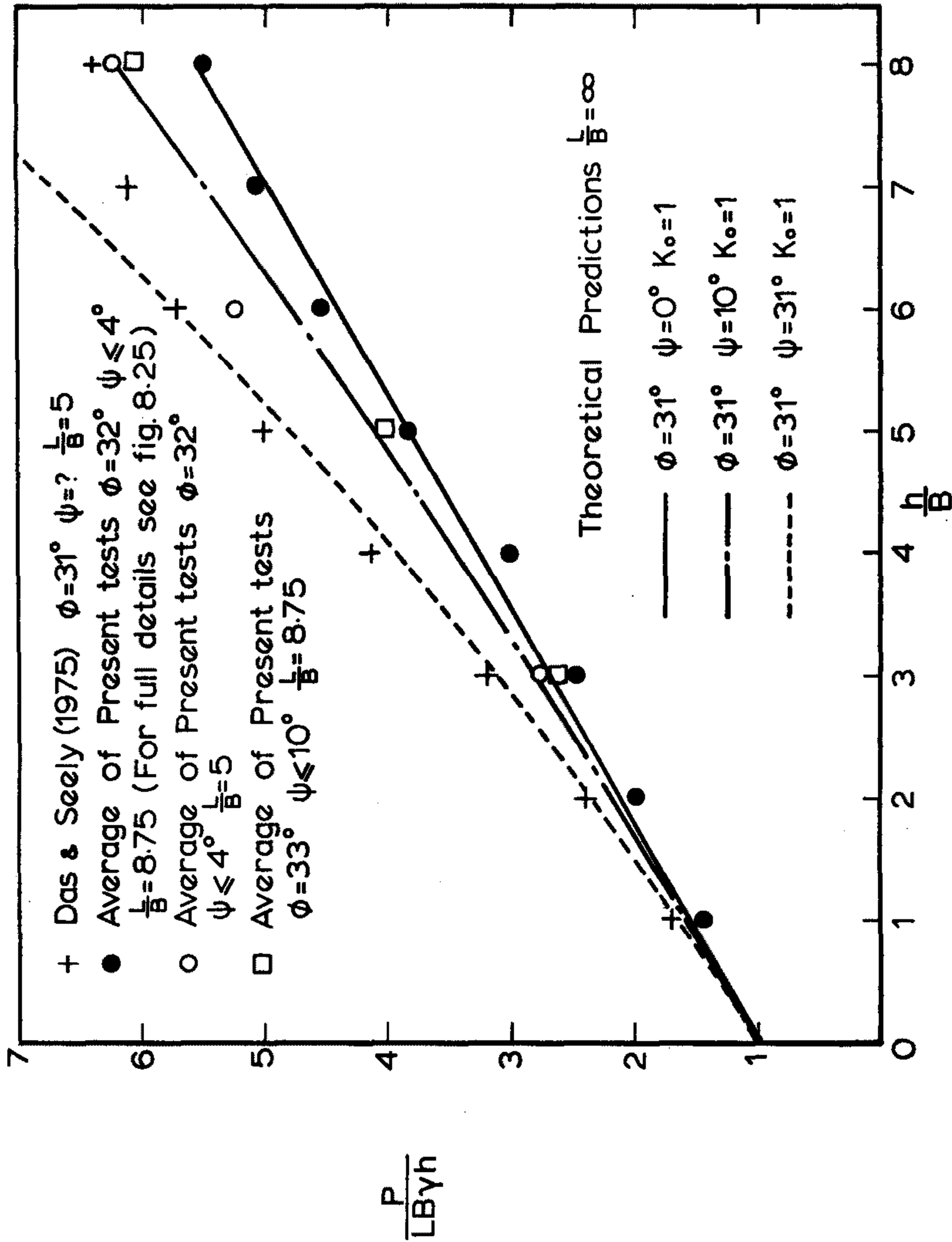


FIG. 8.30 COMPARISON OF EXPERIMENTAL AND THEORETICAL COLLAPSE LOADS.

predictions for a strip anchor does not seem unreasonable and in any case, these results clearly indicate that the theoretical results presented in Chapter 7 lead to a conservative estimate of the collapse load for model anchors in the normally consolidated dry sand used by Das and Seeley.

8.5 CONCLUSIONS

In this chapter, the experimental results obtained by the author from model anchor pullout tests in overconsolidated Kaolin and normally consolidated dry sand have been presented and compared with the theoretical predictions of Chapter 7. These comparisons were then supplemented by further comparisons between published model and field data for circular and strip anchors in clay as well as rectangular anchors in sand.

In general, the experimentally observed behaviour was found to be in quite good agreement with the theoretical predictions, although a number of limitations of the theoretical approach were detected. In particular, it was found that for shallow anchors ($h/B \leq 2.5$) in clay, the theoretical bearing capacities exceeded those observed experimentally and this was attributed to the dominant effect of flexural tensions upon the behaviour of shallow anchors. This finding indicates the need to include provision for the formation of tension cracks in the analysis of shallow anchors. With regard to the load carrying behaviour of rectangular anchors in clay, the experimental results indicated that anchors with an aspect ratio equal to or greater than 5 behave as if they were strip anchors and direct application of the plane strain results for the various 'k2', 'k3' failure

loads would appear to be appropriate. It should be emphasised that the majority of these tests were performed for the limiting case where the soil-structure interface was not permitted to sustain tensile tractions and it should not be inferred that the reasonable agreement between the theoretical and experimental results for this case necessarily justifies the use of the modified bearing capacity factors for the case where breakaway does not occur immediately. Indeed, consideration of the experimental results obtained by Swain (1976) for normally and overconsolidated clay subjected to a surcharge indicates that if different apparent elastic moduli are observed in expansion and compression, then the approximate approach suggested in Chapter 7 for including the effect of surcharge must be modified to allow for this difference in moduli. It will be noted that the importance of the difference in compression and expansion moduli increases with dimensionless overburden pressure σ_{vo}/c and would not be expected to have a serious effect upon the failure load where there is immediate breakaway ($\sigma_{vo} = 0$). Accordingly, the predicted failure load for the immediate breakaway case should give a conservative estimate of the actual failure load for $\sigma_{vo}/c > 0$, and it is recommended that in practice, the bearing capacity of anchor plates in clay should be determined on the assumption of immediate breakaway.

The behaviour of anchor plates in sand was found to be significantly affected by the anchor shape, and for aspect ratios $L/B \leq 5$, the experimental results were considerably greater than those predicted from the plane strain analysis. However, for anchors with an aspect ratio $L/B = 8.75$ resting in normally consolidated loose sand, encou-

raging agreement between the theoretical and experimental collapse loads was observed. The overall agreement between the theoretical and experimental load deflection curves was not quite as good, and while this may be partially due to an experimental value of K_0 which was less than that assumed in the theoretical prediction, it is likely that the assumption of an elastic modulus which was independent of stress state also contributed to the discrepancy.

Despite the quite good agreement reported between the predicted and observed failure loads for a strip anchor in over consolidated clay and normally consolidated loose sand, it should not be considered that this necessarily proves the adequacy, for all cases, of the method of analysis used. Clearly, compensating errors might be involved whose effects have not been fully appreciated. Furthermore, in some cases (e.g. in clay subjected to higher surcharge pressures, or in dense sand) the implications of the simplifying assumptions regarding an elastic modulus which is independent of stress state and an angle of dilatancy which is independent of strain may be of far greater significance than for the cases considered in this chapter. Nevertheless, the encouraging agreement between experimental and theoretical failure loads must give some degree of justification to the proposed method for the analysis of soil-structure interaction, and to the results obtained from this analysis.

CHAPTER NINE
THE BEHAVIOUR OF
VERTICALLY EMBEDDED
STRIP ANCHORS

9.1 INTRODUCTION

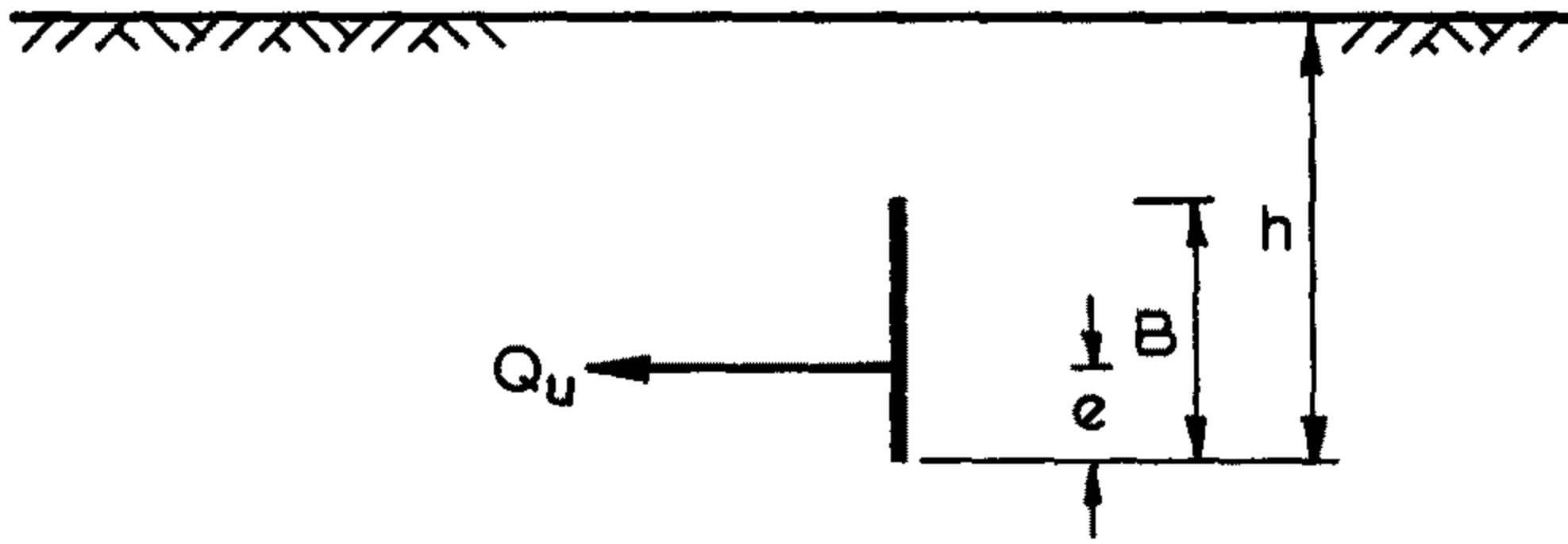
Following from the investigation of horizontal anchor behaviour presented in Chapter 7, it is of some interest to determine the effect of anchor orientation upon the behaviour of the anchor. Accordingly, in this chapter, the behaviour of a vertical strip anchor will be investigated and from consideration of the differences and similarities in behaviour between horizontal and vertical anchors some assessment of the significance of anchor orientation will be made.

The basic problem to be analysed involves a vertical anchor of height (ie. width) B buried at a depth h below the soil surface, as shown in Fig. 9.1a.

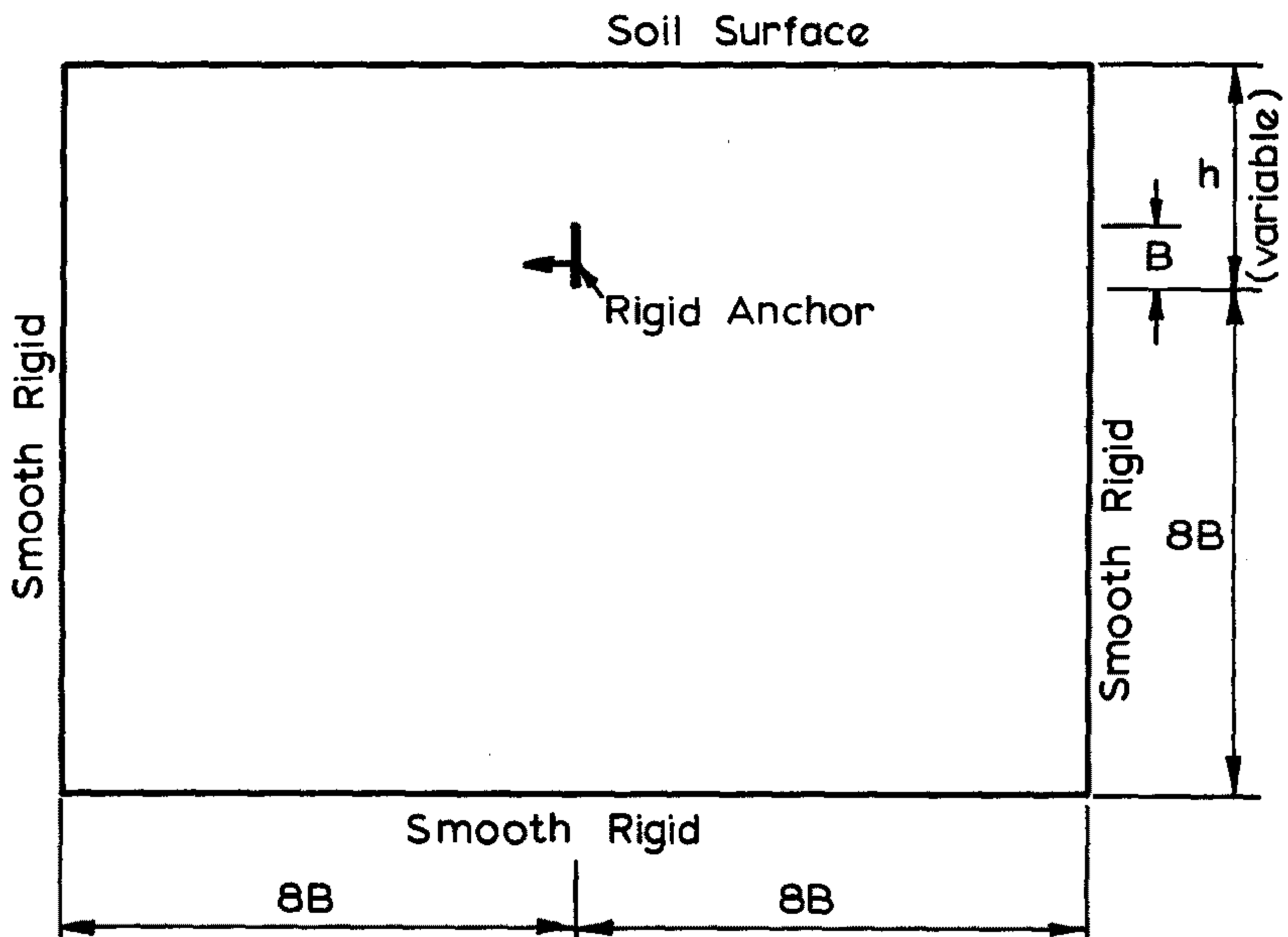
In the finite element idealisation of this problem shown in Fig. 9.1b, smooth rigid lateral boundaries were adopted at a distance $8 B$ from the plane containing the anchor plate, while the soil layer was assumed to extend a distance $8 B$ below the bottom of the anchor plate and was underlain by a smooth rigid base.

The analyses of anchor behaviour were performed for rigid displacement, without rotation, of the anchor plate. This corresponds to the case where the applied load P is applied at the eccentricity e necessary to prevent rotation of the anchor, where typically $0.33 B \leq e \leq 0.5 B$.

The ultimate collapse load Q_u is given by the equation



(a) Basic Problem



(b) Finite Element Idealisation

FIG. 9.1 VERTICAL ANCHOR UNDER PULLOUT LOAD

$$Q_u = A \cdot q_u + F_f \quad (9.1)$$

where A is the area of the anchor plate;

q_u is the average bearing pressure per unit area acting on the anchor plate; and

F_f is the force due to friction on the anchor cable as well as the frictional force acting on the top and bottom of the anchor due to the anchor's finite breadth and the weight of the anchor block. The friction force F_f will not be considered in this thesis.

The ultimate anchor bearing pressure q_u depends primarily upon the soil cohesion c , angle of internal friction ϕ , unit weight γ and the overburden pressure, q_h , at the level of the anchor (ie. $q_h = \gamma h$). As in the case of horizontal anchors, it is generally convenient to separate the effects of cohesion and self-weight by adopting an approximate expression for q_u based upon the assumption that the two effects are independent and may be superimposed. Thus, for cohesive frictional materials the ultimate bearing pressure may be written in the form

$$q_u = F_s (c F_c + \gamma h F_\gamma) \quad (9.2)$$

where F_s is a factor introduced to make allowance for errors due to the assumption of superposition implicit in Eq. 9.2 and where

$$F_c = \left(\frac{q_u}{c} \right)_{\gamma=0} \quad \text{and} \quad F_\gamma = \left(\frac{q_u}{\gamma h} \right)_{c=0}$$

are dimensionless bearing capacity factors which are functions of the

angle of internal friction ϕ , angle of dilatancy ψ , anchor roughness, embedment ratio and, in the case of F_γ , the coefficient of lateral earth pressure at rest, K_0 .

By definition, the bearing capacity factor F_γ assumes a trivial (zero) value where $\phi = 0^\circ$. Consequently, if it is assumed that the soil structure interface cannot sustain tensile tractions, then the assumption of superposition does not hold for purely cohesive materials. In this special case the effect of self weight upon the ultimate bearing pressure will be included in the determination of the bearing capacity factor F_c .

The complex nature of vertical anchor behaviour due to the lack of symmetry, the wide range of parameters and the complicated interaction of these parameters makes a full parametric study of vertical anchor behaviour impossible. Rather, an assessment of the significance of the various parameters will be made by consideration of various parametric cross-sections.

In the following sections, consideration will first be given to the effect of numerical simplifications without which it would not have been feasible to perform the following parametric study.

Secondly, the behaviour of smooth anchors in a purely cohesive soil will be discussed for the two limiting cases of immediate breakaway ($\gamma = 0$) and no breakaway ($\gamma h/c \rightarrow \infty$), as well as for the transition between the limiting cases ($0 \leq \gamma h/c \leq \infty$). The discussion will then be extended to the behaviour of anchors resting in a weightless cohesive - frictional material. The effect of anchor roughness will be

considered briefly.

Thirdly, the behaviour of a rigid anchor in a cohesionless material will be considered for the basic case of a smooth anchor where ψ and K_o are zero and unity respectively. The effects of the dilatancy, anchor roughness and initial stress state will then be considered separately to indicate the likely effect of each parameter upon the basic bearing capacity factor F_γ .

Approximate predictions of collapse loads based upon the assumptions that:

- (1) the bearing capacity factor F_c and F_γ can be superimposed for cohesive frictional materials; and
- (2) the effect of various factors (e.g. roughness, K_o , ψ) upon the bearing capacity factor F_γ can be superimposed;

will be compared with results obtained from direct finite element calculations performed for 'typical' combinations of these parameters, and the applicability of the approximate bearing capacity factors will be assessed.

Finally, the bearing capacity factors obtained in the previous sections will be compared with published experimental data.

Unless otherwise stated, it may be assumed that all analyses presented in this chapter are for the basic case of a smooth rigid vertical anchor resting on a material with angle of dilatancy $\psi = 0^\circ$ and coefficient of lateral earth pressure at rest $K_o = 1.0$.

As in Chapter 7, the finite element results were obtained for $E/c = 500$ and $E/\gamma h = 333$ in the case of cohesive and purely frictional materials respectively. However, the finite element results have been presented in a non-dimensional form throughout this chapter and are independent of the actual ratios E/c or $E/\gamma h$. It may be anticipated that the load deflection curves would be partially dependent upon Poisson's ratio, ν , and the depth of the soil layer below the anchor, D .

9.2 NUMERICAL DETAILS

As far as possible, the same numerical procedures adopted in the analysis of horizontal anchors (Chapter 7) were adopted for the following analysis of vertical anchors. However, due to the lack of symmetry in the general case of a vertical anchor, and due to limitations on the total number of finite elements for which it would be economically feasible to perform non-linear analyses, it was necessary to adopt a finite element mesh (as shown in Fig. 9.2) which, although involving a larger total number of elements, is less refined than the mesh used in the horizontal anchor analysis. To offset the disadvantages of a less refined mesh in regions of high stress gradient, additional potential rupture lines were introduced as shown in Fig. 9.2b. This basic mesh was adopted for all embedment ratios and the dimensions of the variable bays i , j , k etc. are given in Table 9.1.

To ascertain the accuracy which could be expected from this mesh a number of test cases were performed. For example, Fig. 9.3 shows the load displacement curve obtained for a smooth anchor with embedment ratio $h/B = 1$ resting in a weightless cohesive material.

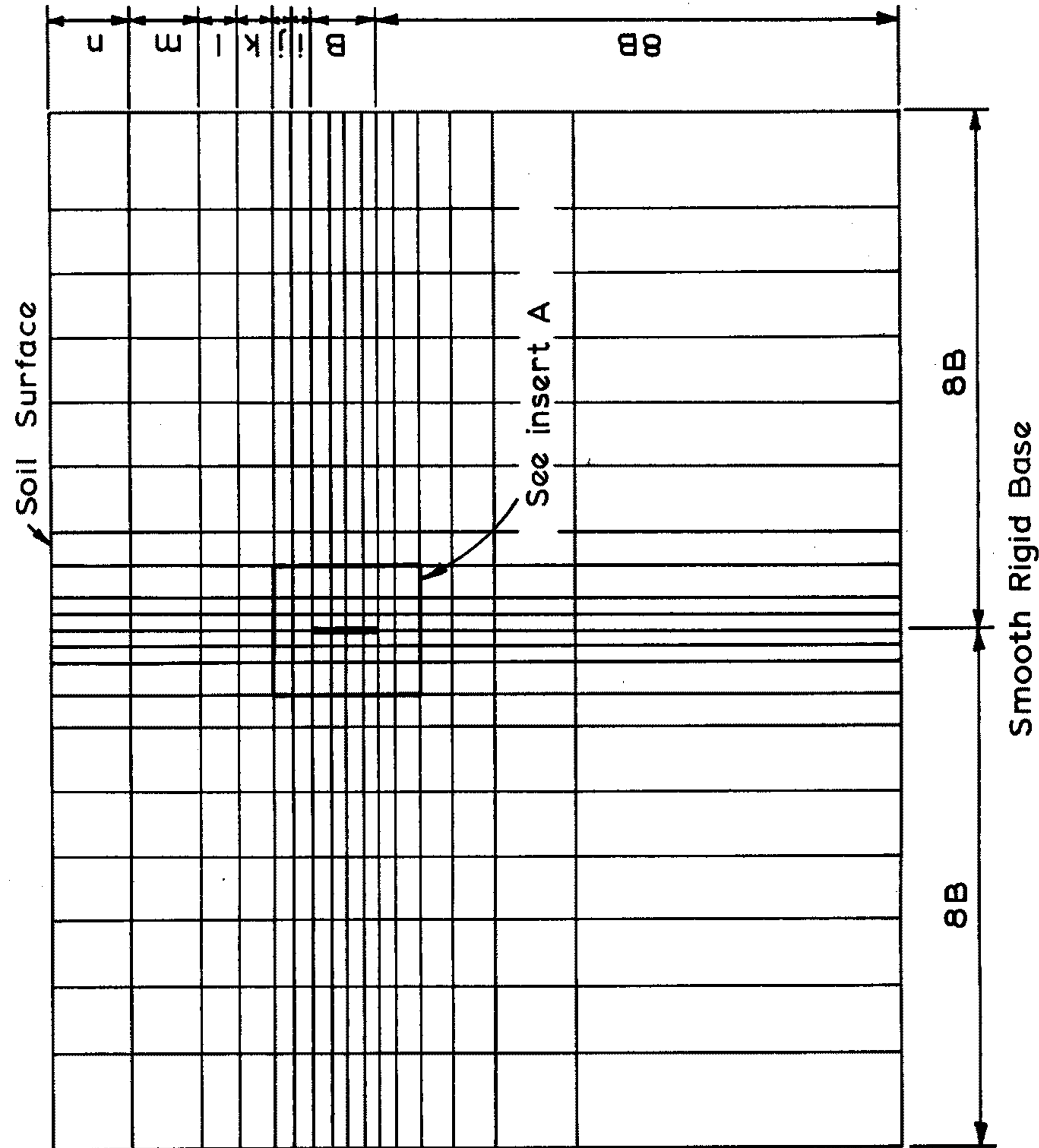


FIG.9 .2a BASIC FINITE ELEMENT MESH

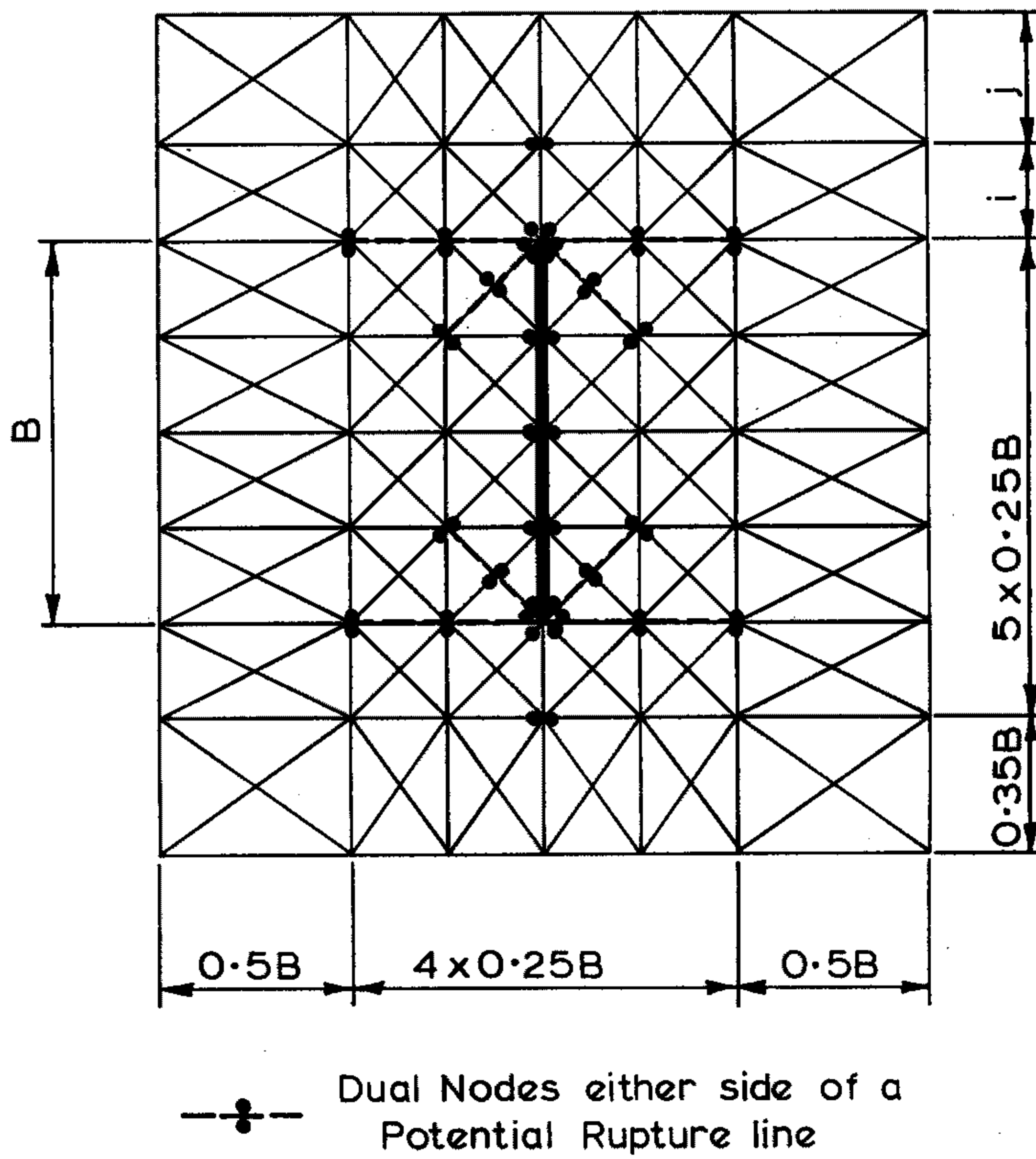


FIG.9. 2b INSERT A TO FIGURE 9.2a

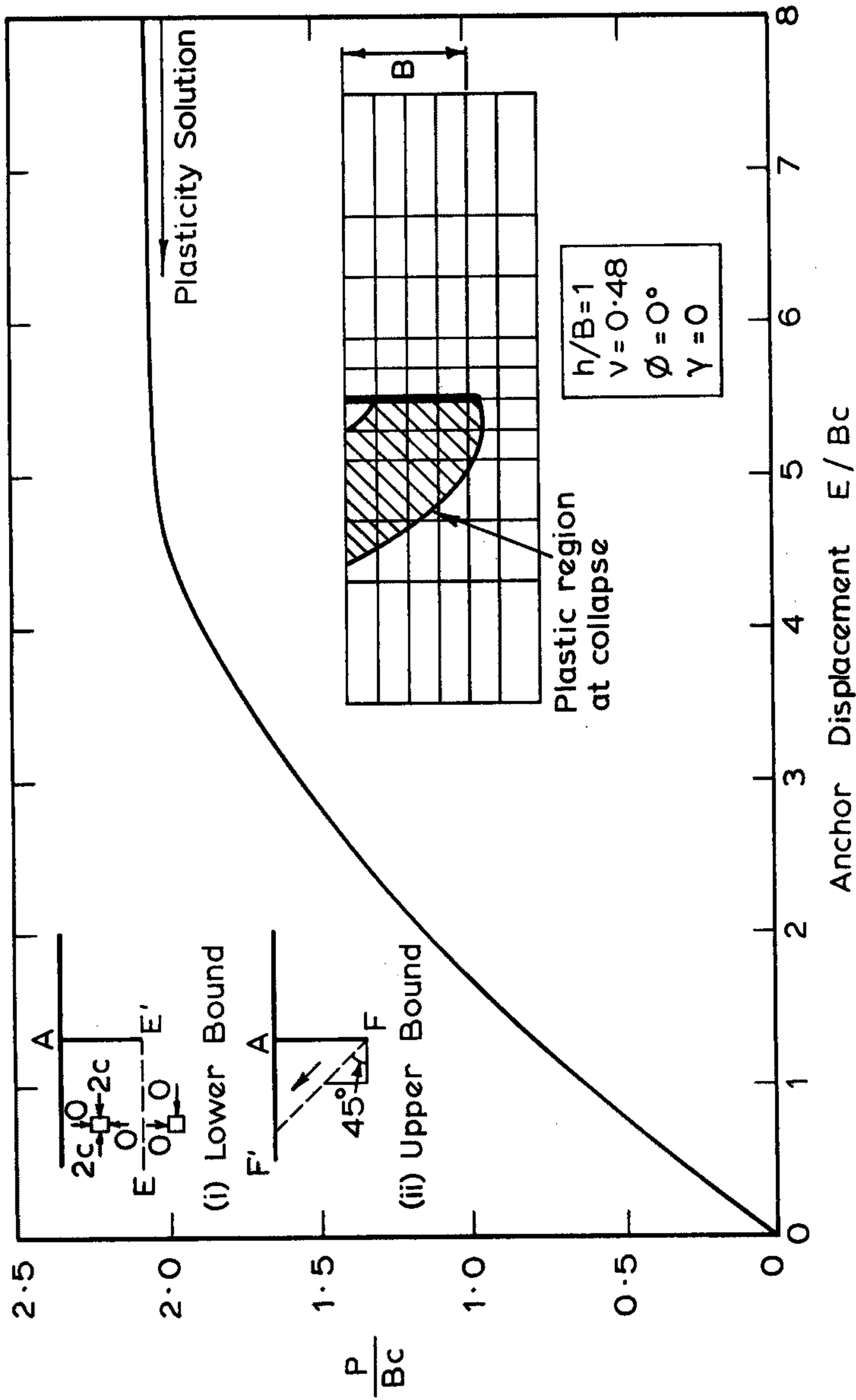


FIG. 9. 3 LIMITING CASE OF AN ANCHOR AT THE SURFACE-IMMEDIATE BREAKAWAY

TABLE 9.1

VARIABLE MESH DIMENSIONS FOR VERTICAL ANCHOR ANALYSIS

See Figure 9.2

h/B	i	j	k	ℓ	m	n
1	-	-	-	-	-	-
1.5	0.25B	0.25B	-	-	-	-
2	0.25B	0.35B	0.4B	-	-	-
3	0.25B	0.35B	0.4B	0.5B	0.5B	-
5	0.25B	0.35B	0.5B	0.65B	1.05B	1.2B
8	0.25B	0.35B	0.5B	0.65B	1.25B	4.0B
∞	0.25B	0.35B	0.5B	0.65B	1.25B	5.0B

The exact collapse load for this case may be obtained from plasticity theory by considering the lower and upper bound solution indicated in Figs. 9.3(i) and 9.3(ii). In the lower bound solution the horizontal stress in front of the anchor is $2c$ while the vertical stress is zero. The line EE' is a stress discontinuity and the material below EE' and behind the anchor is unstressed. The upperbound solution is obtained by considering the work done by the rigid movement of the wedge AFF' parallel to the velocity discontinuity FF' , where the material outside the region AFF' is stationary. The collapse load obtained from the finite element analysis exceeded this plasticity solution by 2.5% while the velocity field at collapse indicated that failure was associated with an upward rigid movement of a wedge of material at the kinematically correct angle of 45° .

To determine the likely effect of the reduction in mesh re-

finement and the increase in the number of potential rupture lines, analyses were performed for an infinitely deep anchor and a surface footing using the appropriate portions of the mesh shown in Fig. 9.2. The load deflection curves obtained from these analyses as well as those obtained from identical analyses performed using the basic mesh adopted for horizontal anchors, are shown in Fig. 9.4.

From these curves it will be appreciated that the error in collapse load obtained using the less refined mesh is only slightly greater than that obtained for the basic mesh used in Chapter 7 and the collapse loads from the mesh are still only approximately 6% above the Prandtl solution of $5.14 Bc$ and the upperbound of $11.42 Bc$ obtained from classical plasticity for the surface footing and deep anchor respectively. The greatest effect of the combination of a less refined mesh and more potential rupture lines was to alter the early elastoplastic load deflection behaviour. Consequently, the effect of the less refined mesh upon the bearing capacity factor will be greatest where there is extensive contained plastic deformation prior to collapse necessitating the adoption of a 'practical' (k_4) definition of failure. Thus the most severe effect may be expected to arise for the case of an infinitely deep anchor in a weightless cohesive material; however, comparison of the load deflection curves using the meshes shown in Figs. 7.2 and 9.2 respectively for otherwise identical analyses of this particular case indicated that at worst the difference in applied load for a given displacement was less than 10% while the ' k_2 ' (and similarly, the ' k_3 ' and ' k_4 ') loads differed by less than 3%.

As a final check on the suitability of the finite element mesh the limiting case of a smooth vertical anchor with an embedment

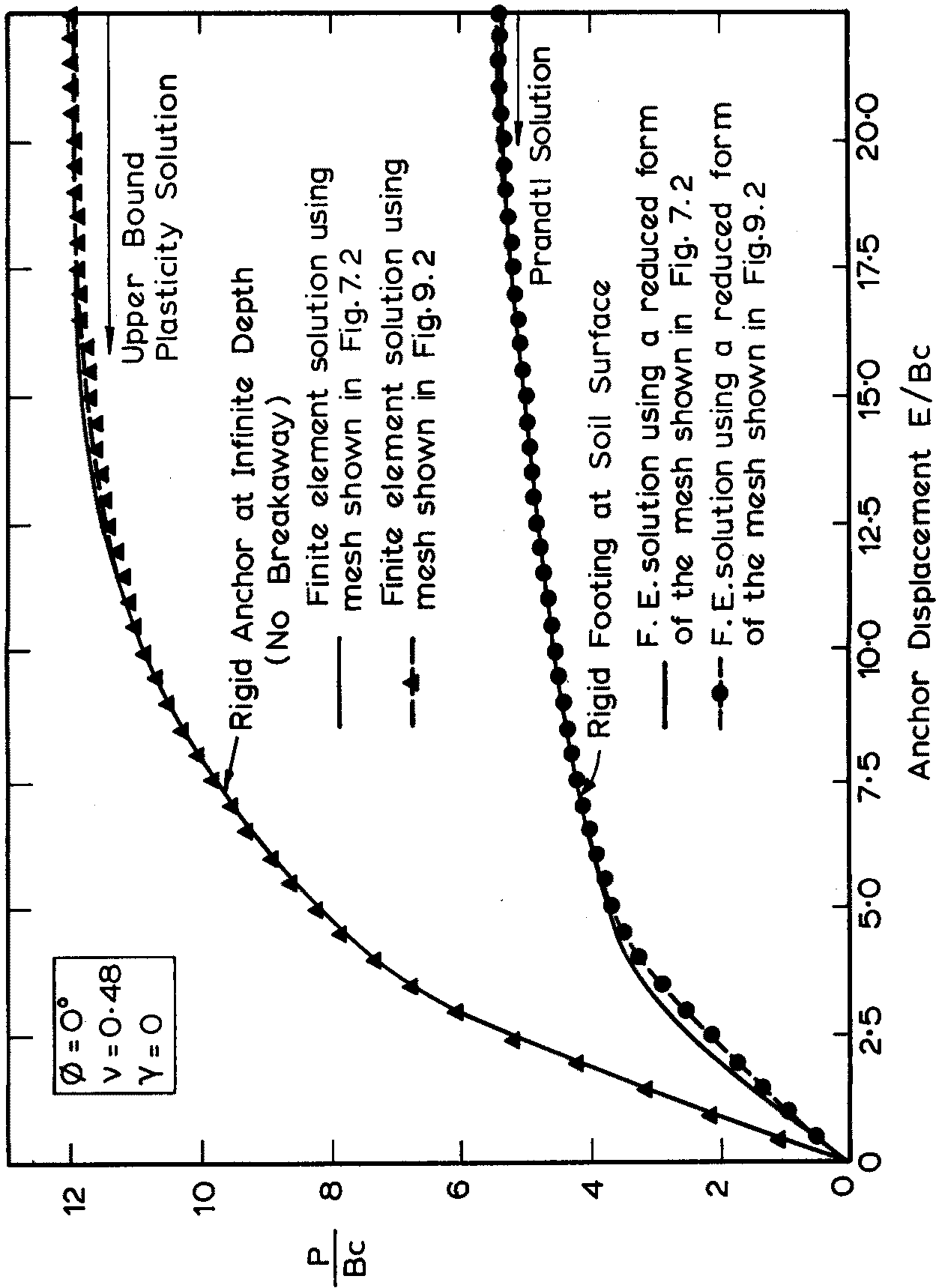


FIG. 9.4 EFFECT OF FINITE ELEMENT MESH ON LOAD DISPLACEMENT CURVES

ratio of one, resting in a purely frictional material (ie. $c = .0008 \gamma h$) with $\phi = 30^\circ$, $\psi = 0^\circ$, $K_0 = 1$, was analysed and the load displacement curve is shown in Fig. 9.5. From this figure it will be observed that collapse was clearly defined at a load 6% above that determined by taking the difference between the conventional passive and active pressures acting on a smooth retaining wall. It was found that approximately 2.4% of this error in collapse load can be attributed to an underestimation of the active force on the anchor while 3.6% of the error may be attributed to an overestimate of the passive force acting on the anchor. The relatively large contribution of error in the active force to the total anchor load was attributed to the fact that significant active failure had occurred at low load (and displacement) levels while significant passive failure only occurred in the period just prior to collapse. Consequently the plastic elements within the active region were in a plastic state at relatively low load levels for a large number of load (displacement) increments allowing significant accumulation of round-off and other numerical error arising from the nature of the 'initial stress' approach. It was in fact found that the contribution of the active force on the wall was in error by just over 0.5% when the active region became plastic as compared to the error of more than 2.4% at collapse of the anchor. The plastic region and velocity field at collapse for this case are shown in Fig. 9.6 where the dual velocities at the anchor interface correspond to the velocity of a particle just in front and just behind the anchor.

Displacement defined analyses were performed, and it was generally found that suitable increments in displacement were $0.5 Bc/E$ and $0.16 \gamma h B/E$ for cohesive and purely frictional materials respectively. However, for purely frictional materials with a low value of K_0 it was

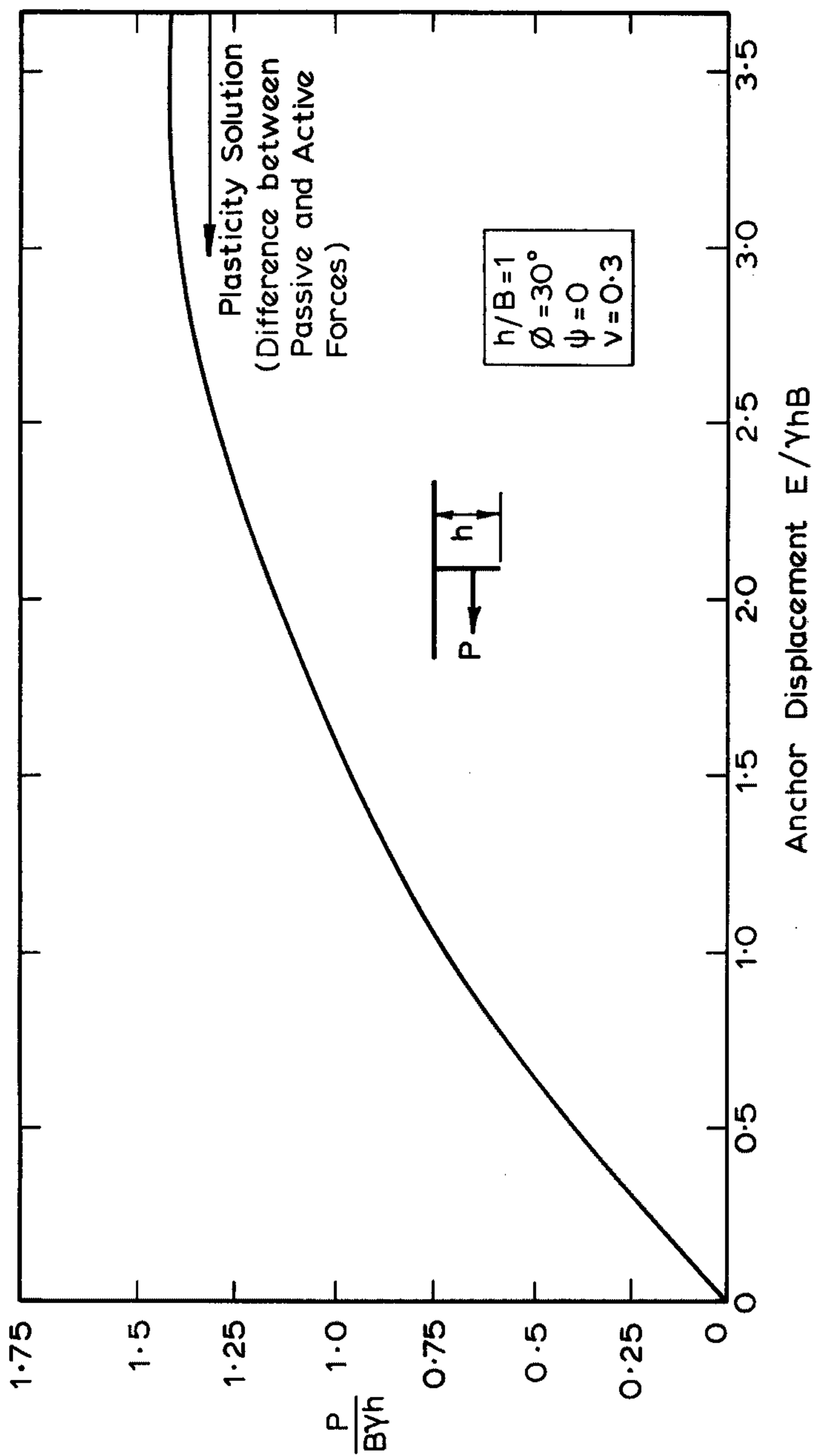
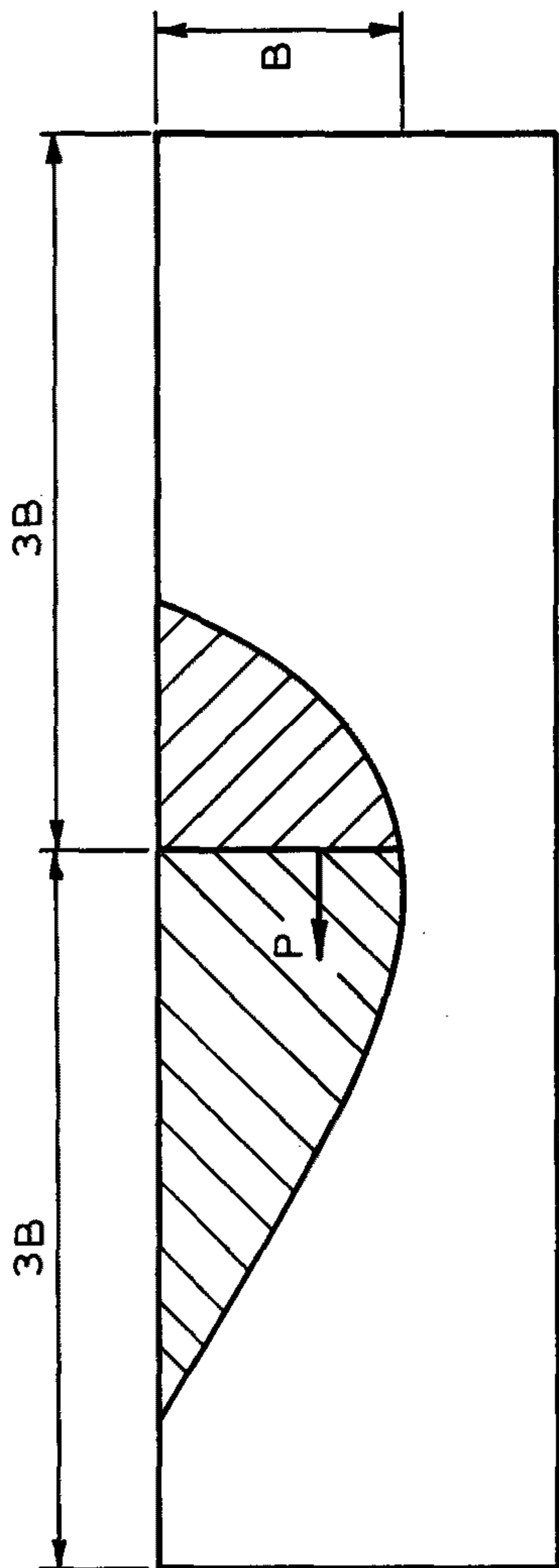
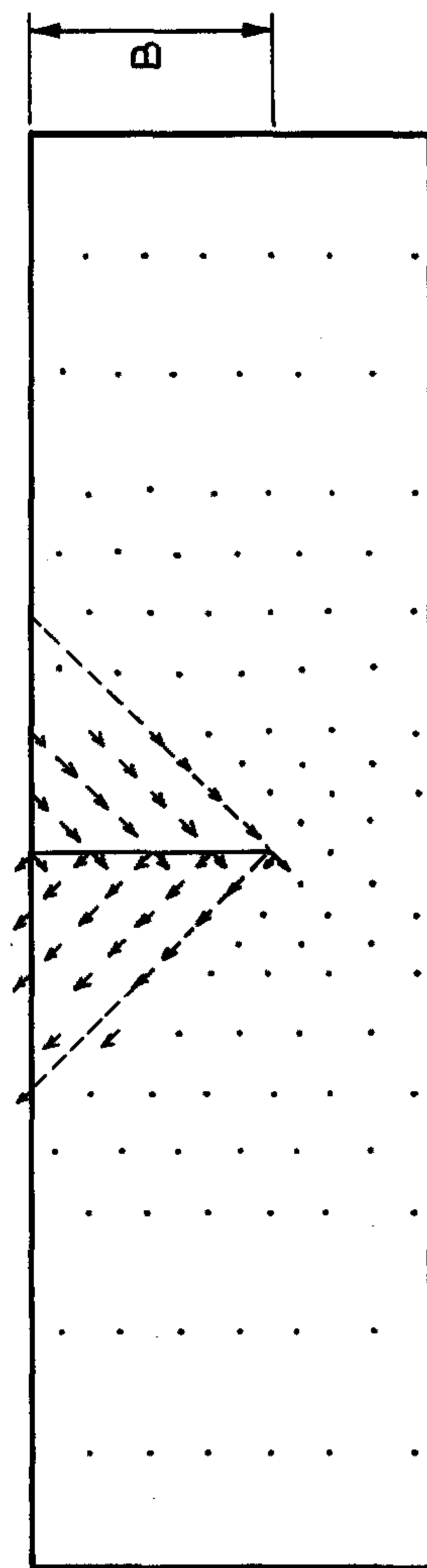


FIG.9.5 LOAD DISPLACEMENT CURVE FOR A SMOOTH VERTICAL ANCHOR



(a) Plastic Region at Collapse



(b) Velocity Field at Collapse

FIG. 9.6 BEHAVIOUR OF SHALLOW ANCHOR $h/B=1, \phi=30^\circ, c=0$

necessary to adopt a smaller increment size of $0.033 \gamma hB/E$ to ensure numerical suitability of elements in zones of active failure. Typically, the selected increment sizes were such that more than 50 increments were required to either cause collapse or to reach a practical failure load in cohesive materials, while more than 100 (and for moderately deep anchors several hundred) load steps were required to reach failure for anchors in purely frictional materials. The adoption of larger displacement increments was attempted for purely frictional material, however in general this was found to lead to an unacceptable numerical instability of elements within the active regions behind the anchor.

The convergence criteria applicable to displacement defined analyses (as described in Chapter 6) were satisfied to better than 2% in these analyses. As a check on the adequacy of the selected step size for purely frictional materials (ie. $0.16 \gamma hB/E$), the case of a smooth vertical anchor shown in Fig. 9.5 (ie. where $\phi = 30^\circ$, $\psi = 0^\circ$, $K_0 = 1$) was re-analysed using a step size of $0.033 \gamma hB/E$ and it was found that this five fold reduction in step size reduced the collapse load by less than 1%. The step size adopted for cohesive material was the same as that adopted for horizontal anchors and the comments upon its suitability given in Chapter 7 may also be considered to apply in this chapter.

In the analysis of rough anchors slip can only occur across a velocity discontinuity and hence the apparent adhesion c_k and angle of friction ϕ_k between the anchor and soil were taken to be the appropriate values acting along a velocity discontinuity between the soil and structure as given in Eq. 5.40.

Finally, for reasons of numerical convenience, at very low load levels the analysis of 'cohesionless' soils with weight were performed using a cohesion $c = 8.3 \times 10^{-5} \gamma h$. As discussed in regard to horizontal anchors, provided $K_0 = 1$ this cohesion may be considered to be equivalent to a very small surcharge which does not directly contribute any shear resistance to anchor pullout and as such it was shown that the contribution of the cohesion in the case of horizontal anchors could have been ignored when determining the bearing capacities factors F_γ . In the analysis of vertical anchors, it was only necessary to adopt a cohesion 1/100th of that adopted in the horizontal anchor analyses, and hence it may be neglected in the determination of bearing capacity factors.

9.3 ANCHORS IN A COHESIVE SOIL

9.3.1 Purely Cohesive Soil (Undrained Loading)

The behaviour of vertical anchors resting in a purely cohesive soil mass is in many respects similar (and for $h/B \rightarrow \infty$ is identical) to that already described for horizontal anchors (see Chapter 7). In particular, two types of anchor behaviour were observed, namely that of a shallow and deep anchor. Considering firstly the situation where there is no surcharge or body force, and where the soil-structure interface cannot sustain tensile tractions (ie. where immediate breakaway occurs), then shallow anchor behaviour was characterised by clearly defined collapse loads associated with flow of the plastic material towards the soil surface. Alternatively, deep anchor behaviour was characterised by extensive elasto-plastic deformation prior to collapse associated with contained plastic flow within a plastic region which gradually extends to

encompass the entire anchor, allowing collapse by means of plastic flow from the front to the back of the anchor. The two types of behaviour can be appreciated from Fig. 9.7 which shows the load displacement curves obtained for a range of embedment ratios. It will be seen that clearly defined collapse loads were obtained for embedment ratios $h/B = 1, 1.5$ and 3 . For $h/B = 5$ and ∞ the load displacement curves were still steadily rising when loading ceased after extensive deformation, indicating the need for the adoption of a practical definition of failure. The velocity field and plastic regions at collapse obtained for shallow anchors are shown in Figs. 9.8 and 9.9. Not surprisingly, the behaviour of very shallow anchors (e.g. $h/B = 1, h/B = 1.5$) was found to be dominated by the proximity of the soil surface in that the growth of the plastic region was primarily directed towards the soil surface and the collapse of the anchor was associated with a fairly rigid movement of a wedge of a material in front of the anchor towards the surface. Consequently, little plastic deformation occurs prior to collapse and collapse itself is very clearly defined. However, the influence of the soil surface upon anchor behaviour decreases rapidly with increasing embedment ratio, and an embedment ratio $h/B = 3$ may be considered to mark the transition from shallow to deep anchor behaviour. In this case, the early growth of the plastic region (and the associated plastic velocities) was essentially symmetric with respect to the centre line of the anchor and at the practical ' k_4 ' failure load ($P/Bc = 5$, which is equivalent to 90% of the final collapse load) the plastic region although extensive can be seen to be only slightly affected by the proximity of the soil surface. Consequently, considerable contained plastic deformation is observed and if a 'practical' definition of failure is adopted then the ' k_2 ', ' k_3 ', ' k_4 ', ' k_5 ', loads are the same as those obtained for

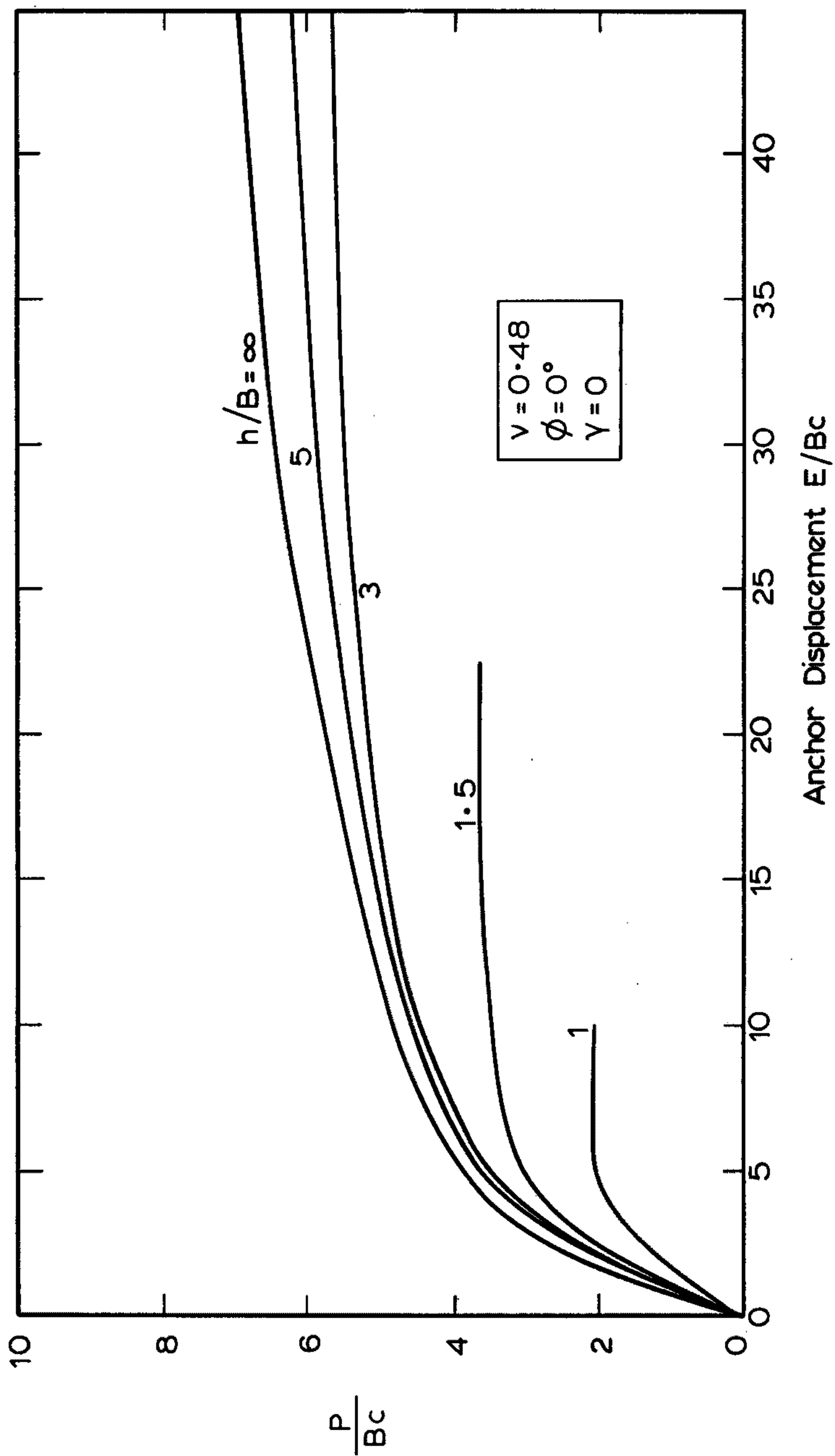


FIG.9. 7 LOAD DISPLACEMENT CURVES FOR VERTICAL ANCHORS - IMMEDIATE BREAKAWAY

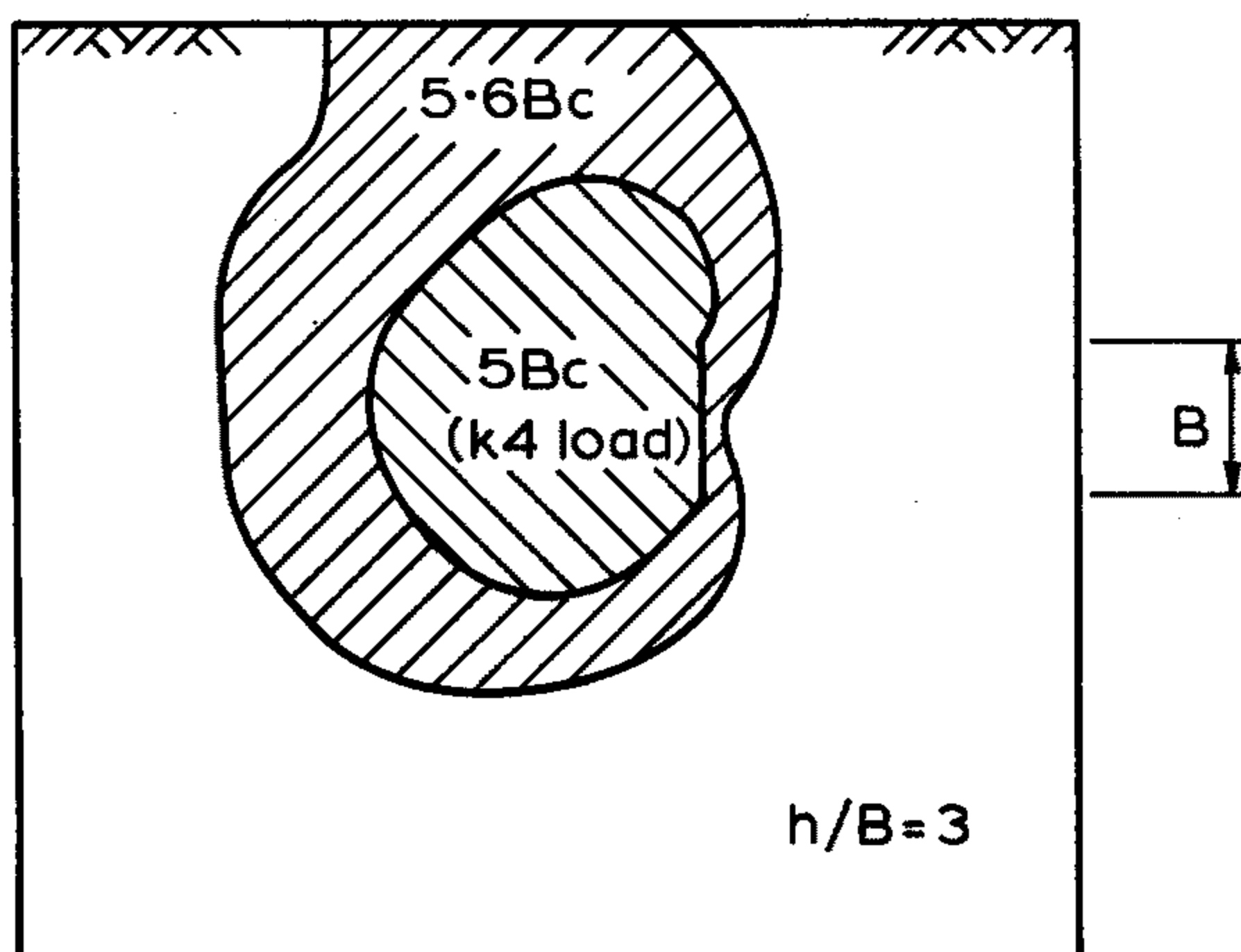
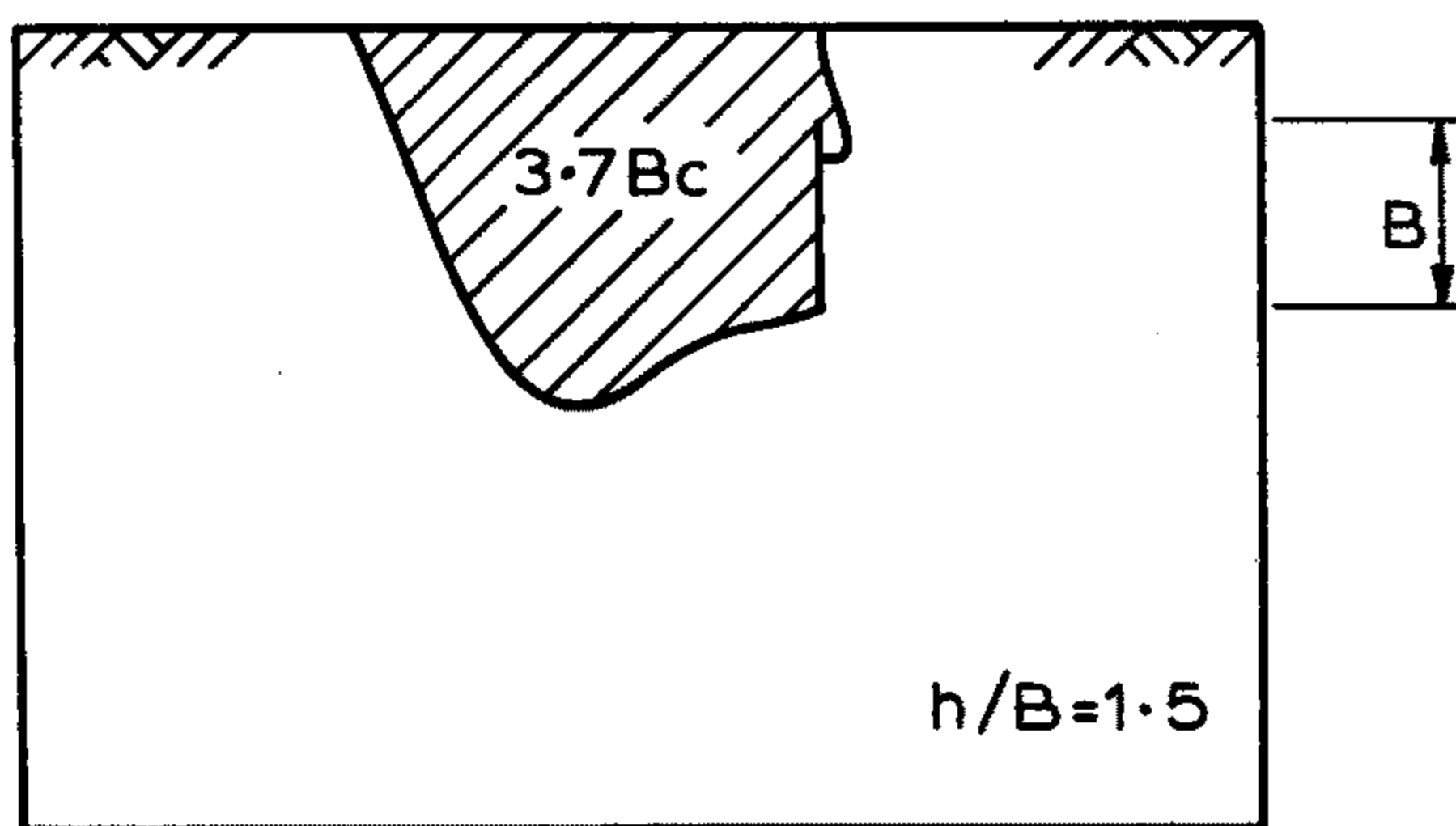
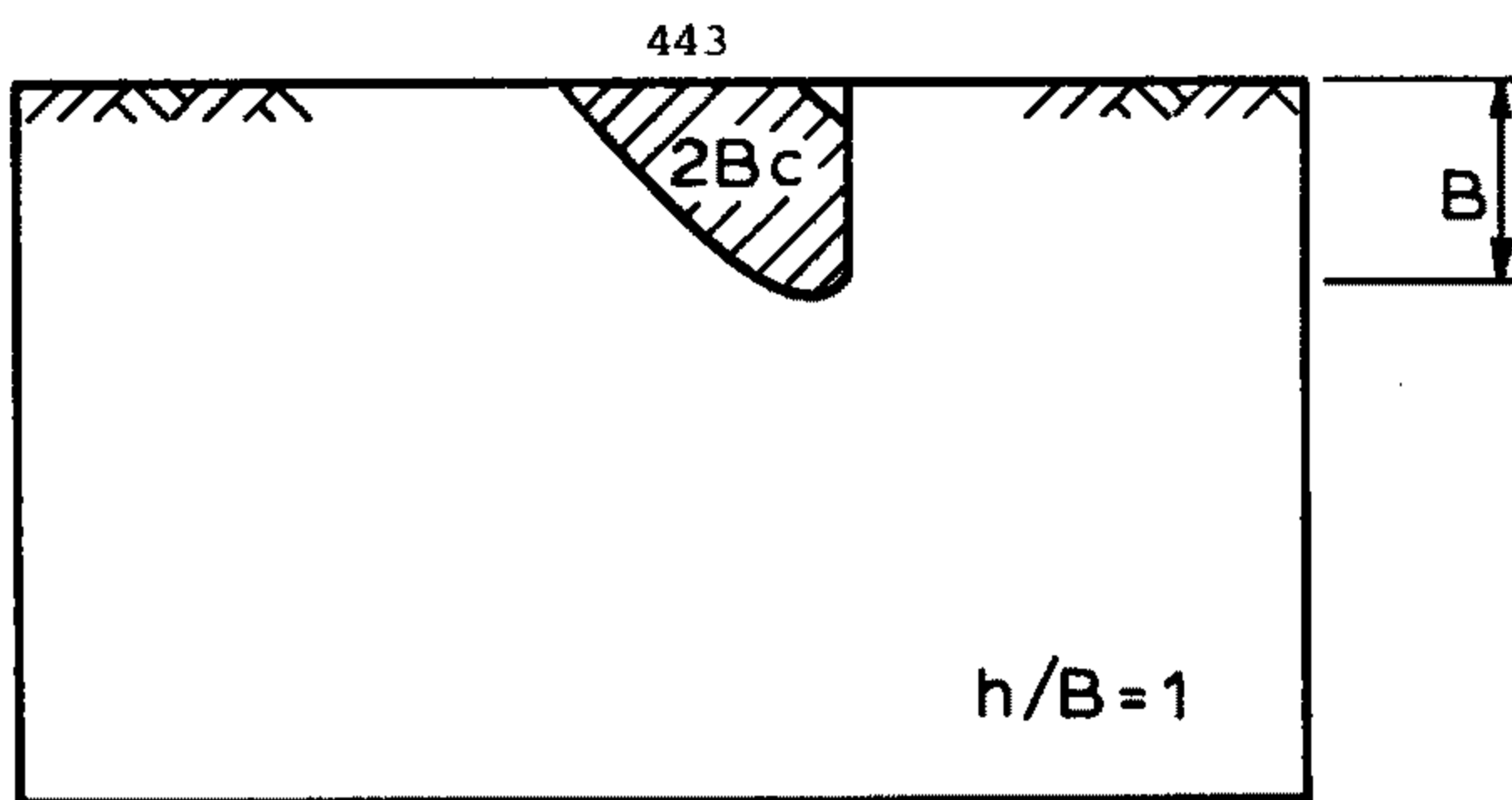


FIG.9.8 PLASTIC REGIONS AT COLLAPSE $\phi=0^\circ, \gamma=0$

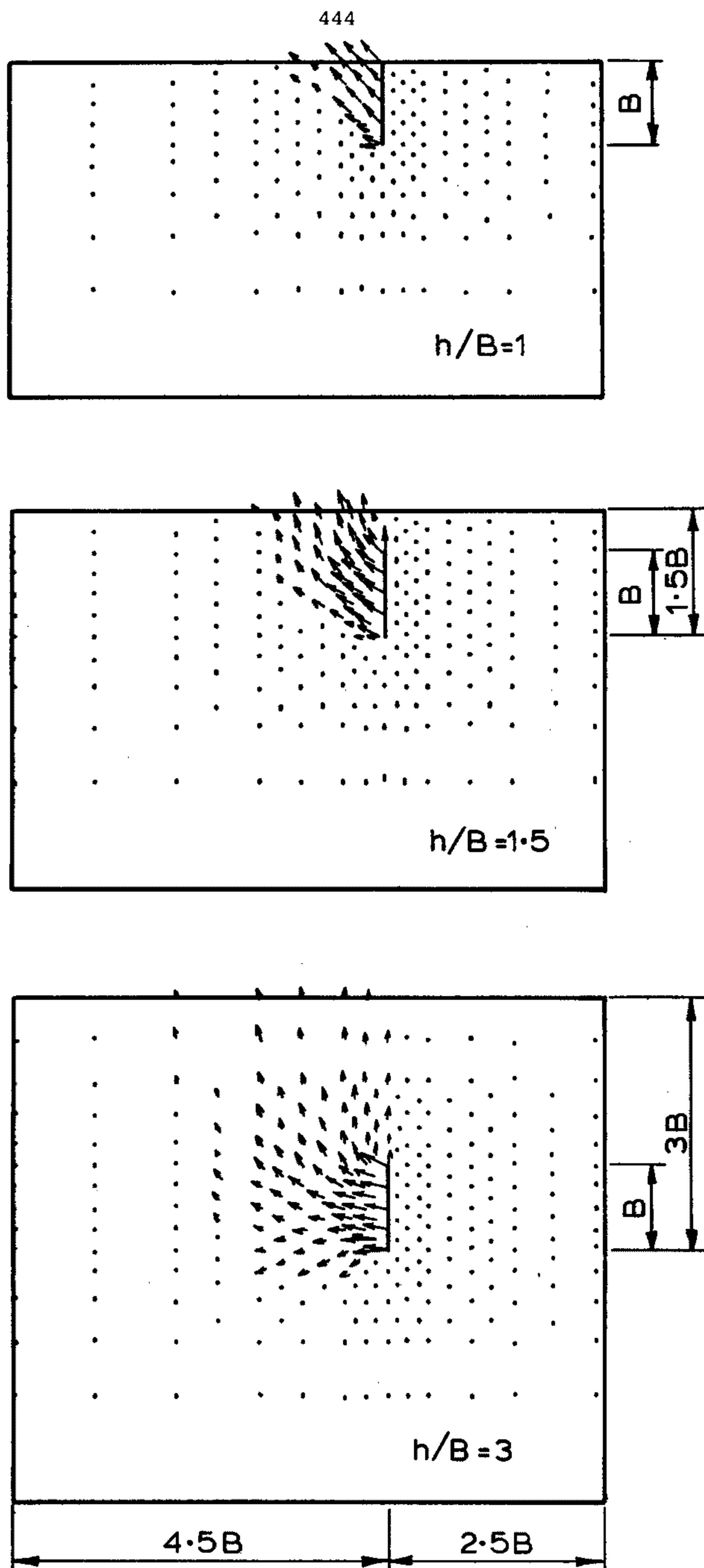


FIG.9 . 9 VELOCITY FIELDS AT COLLAPSE $\phi=0^\circ, \gamma=0, \psi=0^\circ$

much deeper anchors (see Fig. 9.10). However, the behaviour of this anchor is still influenced by the soil surface and final collapse is associated with plastic flow of material to the soil surface, although there are indications of impending flow around the anchor.

For embedment ratios greater than five the plastic region and velocity fields are clearly those of a deep anchor and the general behaviour is essentially the same as already shown in Chapter 7 for deep horizontal anchors.

In the limiting case of a fully bonded anchor (ie. where there is no breakaway of the structure from the soil) the velocity field and stress distribution are antisymmetric about the anchor place. Furthermore, for very shallow anchors ($h/B \leq 1.5$) the velocities and stresses on the passive side of the anchor are almost identical to those observed in passive regions for the immediate breakaway case.

The collapse loads for both shallow and deep bonded anchors were quite clearly defined, although at the limiting embedment ratios (ie. $h/B = 1$, $h/B = \infty$) the finite element collapse loads were approximately 6% above the values obtained from classical plasticity theory. To allow for this error, the bearing capacity factors (F_c) shown in Fig. 9.10 for the fully bonded anchor were obtained by reducing the finite element collapse loads by 6%.

For a fully bonded anchor the transition from shallow to deep anchor behaviour was, as in the case of immediate breakaway, observed at an embedment ratio of 3. At this depth the deep anchor collapse load was obtained while the plastic region and velocity fields

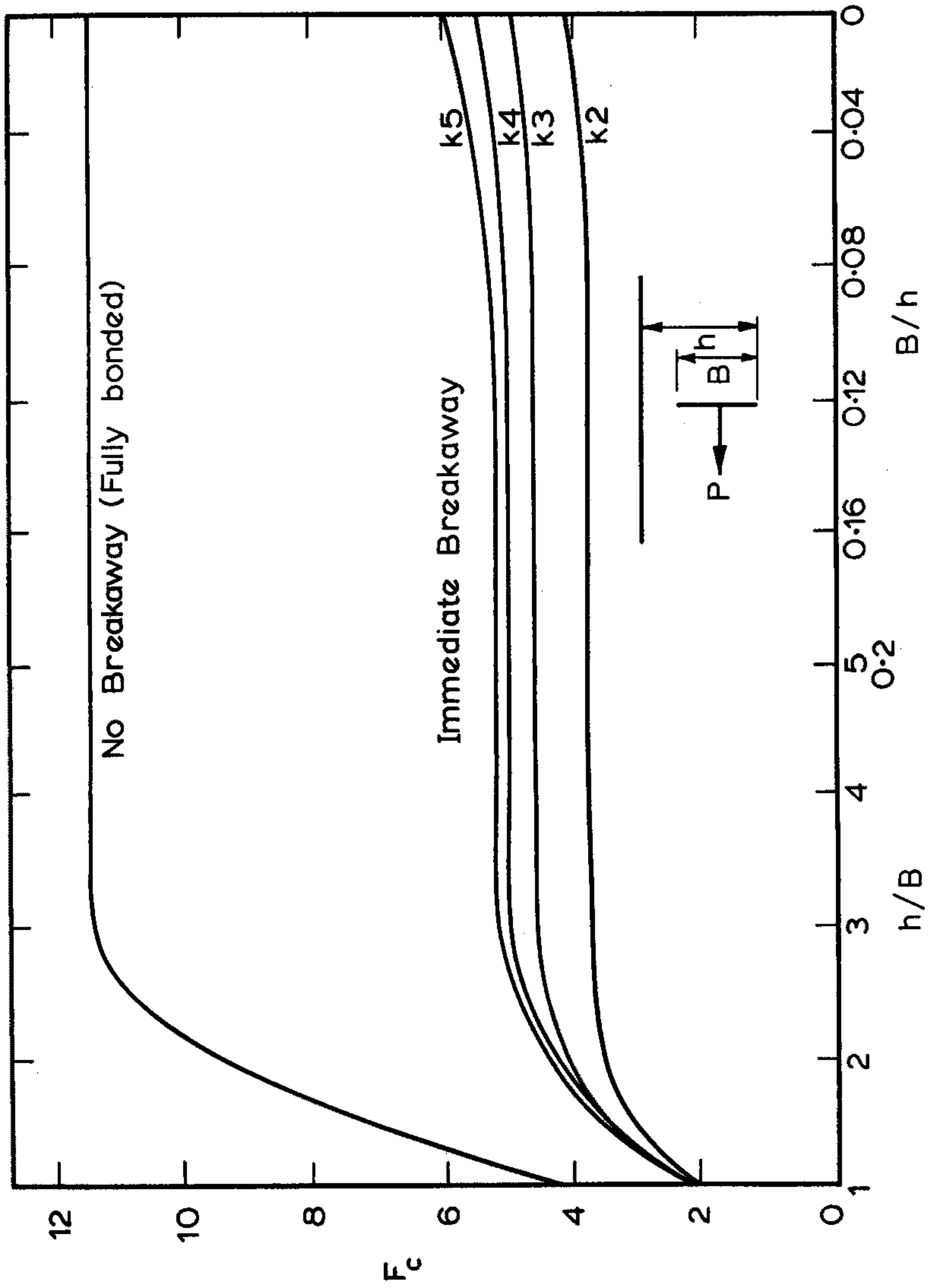
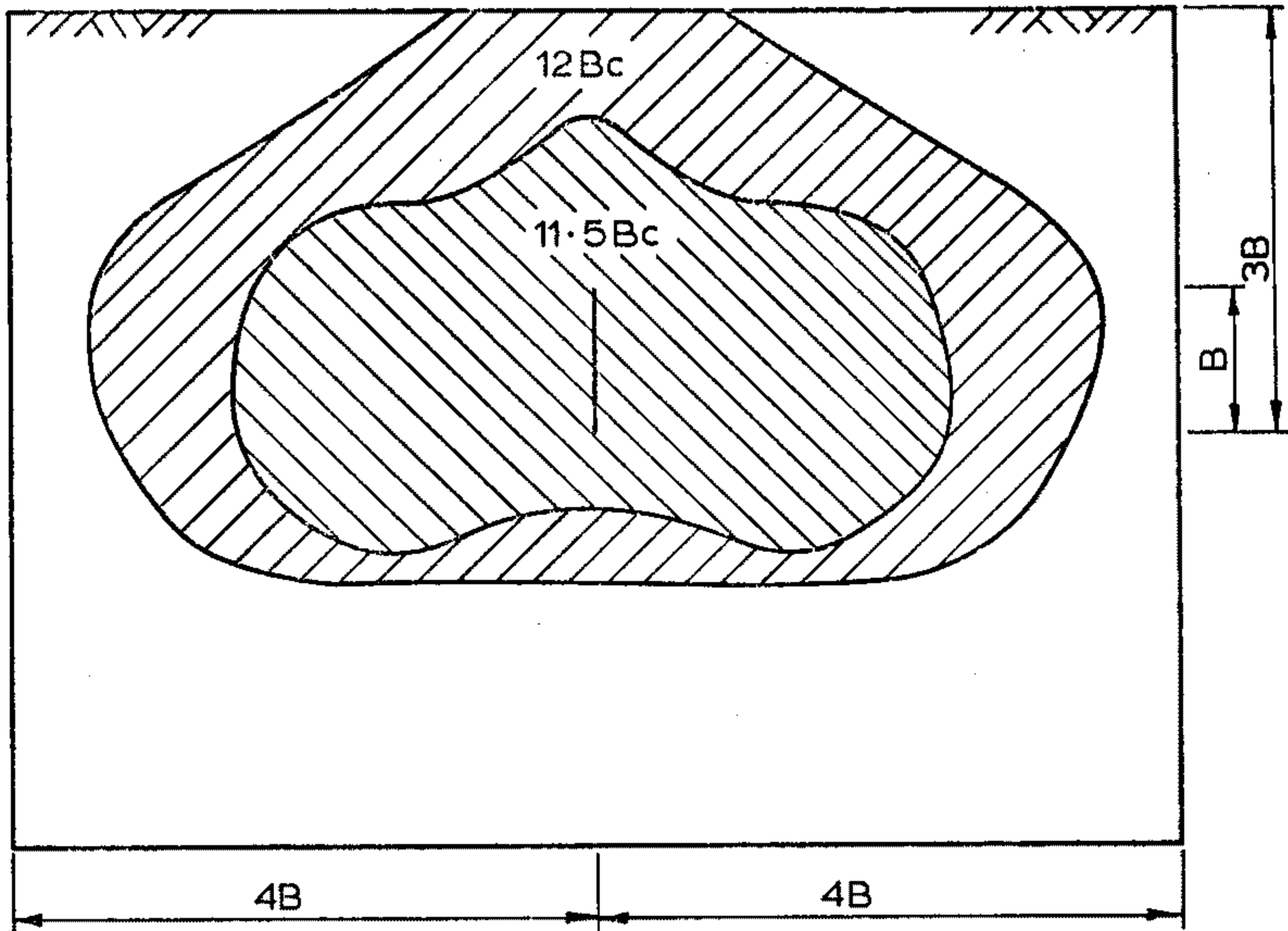


FIG.9.10 BEARING CAPACITY FACTOR F_c FOR VERTICAL ANCHOR $\phi=0^\circ$

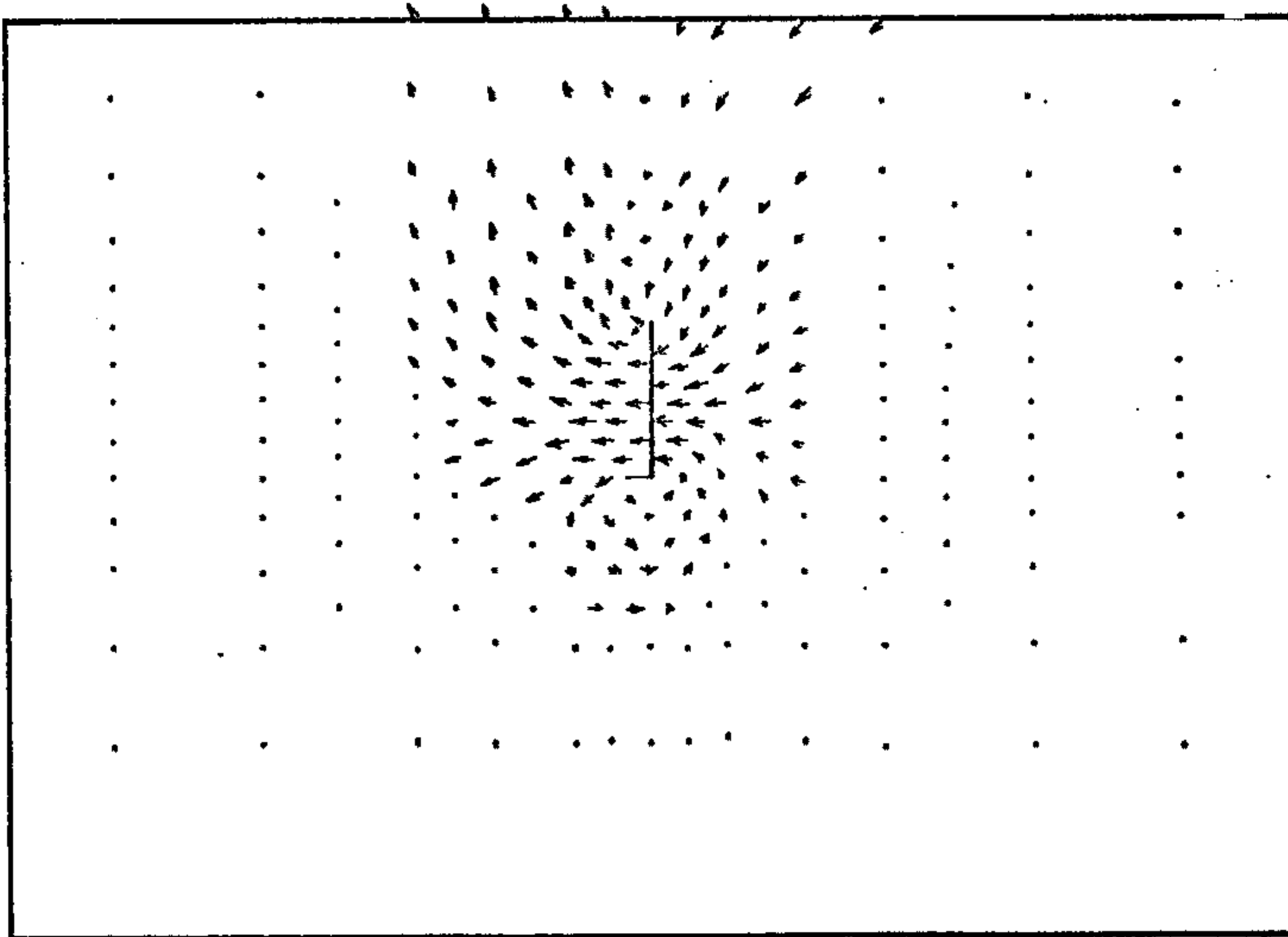
indicated that failure was largely associated with flow around the anchor although the behaviour was slightly affected by the proximity of the surface as illustrated in Fig. 9.11.

Anchors with an embedment ratio greater than 3 behaved entirely as deep anchors and the free surface appeared to have no significant affect upon the behaviour of the anchor at collapse.

For a general $c - \phi$ material, or a purely frictional material, the influence of soil self-weight upon the bearing capacity of an anchor can be considered by superposition of the bearing capacity factors F_c , and F_γ as indicated in Eq. 9.2. However, the limiting case of a purely cohesive material with weight must be considered as a special case since the definition of $F_\gamma = (P/B\gamma h)_{c=0}$ breaks down when $\phi = 0^\circ$. Two situations may arise for a purely cohesive soil. Firstly, if the overburden pressure is sufficient to prevent the development of tensile tractions between the soil and structure, then the bearing capacity is as given for the fully bonded (no breakaway) case in Fig. 9.10, and is independent of the soil self-weight. Alternatively, if the soil structure interface cannot sustain tensile traction, and breakaway does occur during the loading procedure then the affect of self-weight must be included in the dimensionless bearing capacity factor F_c which is then a function of both embedment ratio and dimensionless density $\gamma h/c$. The limiting cases of $\gamma = 0$ (immediate breakaway) and $\gamma h/c = \infty$ (no breakaway) have already been considered and it remains to investigate the transition between the two types of behaviour for intermediate values of $\gamma h/c$. To illustrate this effect the load deflection curves for an anchor with embedment ratio $h/B = 3$ are shown in Fig. 9.12, for a range of values of $\gamma h/c$. It



(a) Plastic Region



(b) Velocity Field

FIG.9. 11 BEHAVIOUR OF AN ANCHOR AT COLLAPSE $h/B=3$
NO BREAKAWAY, $\phi = 0^\circ$

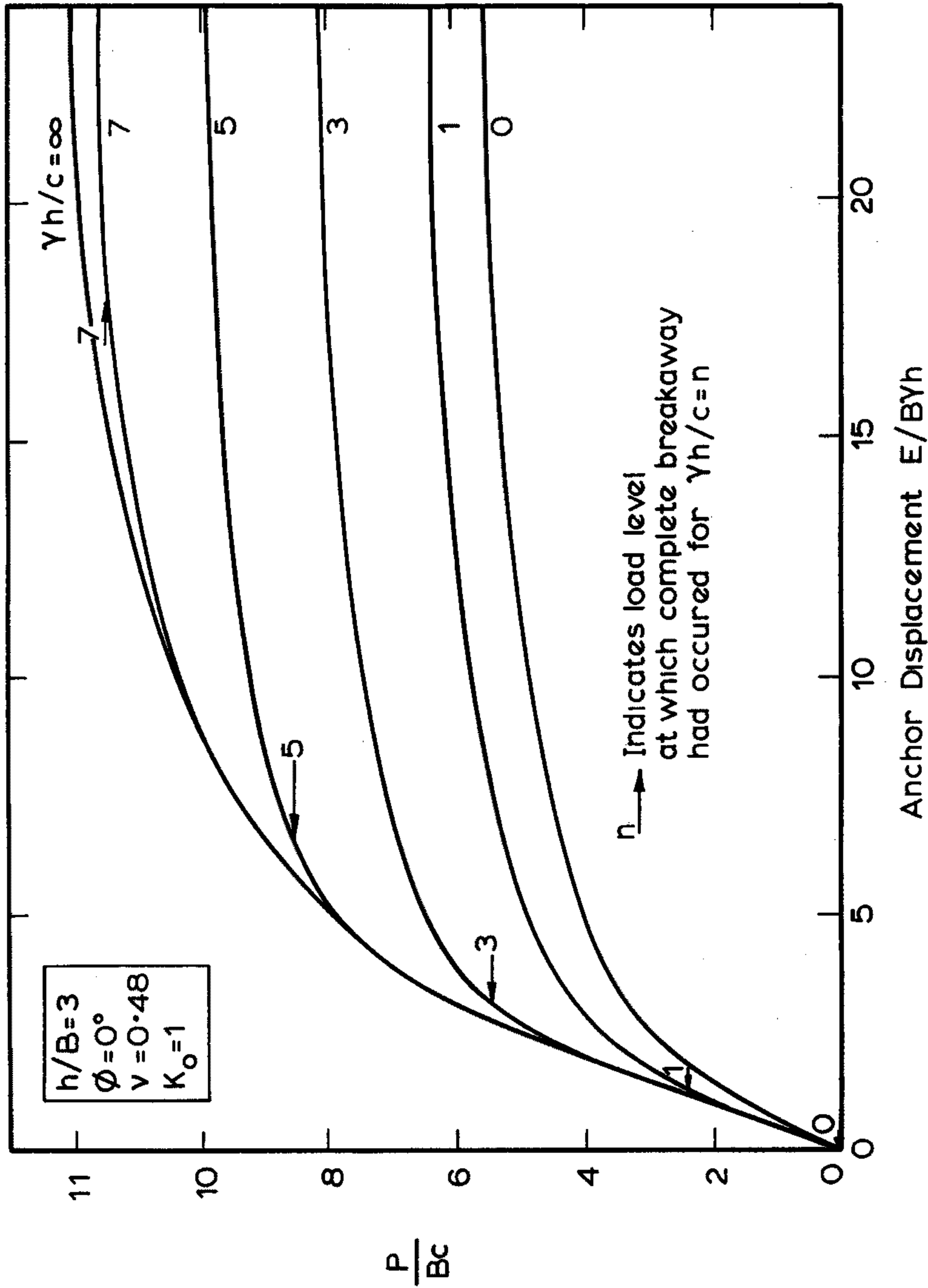


FIG.9.12 LOAD DISPLACEMENT CURVES FOR ANCHOR IN PURELY COHESIVE MATERIAL
WITH WEIGHT -EFFECT OF BREAKAWAY

will be observed that in each case collapse was reasonably clearly defined, although for lower values of $\gamma h/c$ the practical ('k4') failure load was up to 10% below the actual collapse load. Furthermore, the increase in ultimate collapse load with $\gamma h/c$ is approximately constant indicating that at this embedment ratio, the behaviour of the anchor is primarily that of a shallow anchor since, as already discussed in Chapter 7, the ultimate collapse load of a very deep anchor is independent of $\gamma h/c$ (although the practical failure load will be dependent upon $\gamma h/c$).

In fact the variation of practical failure load with $\gamma h/c$ is approximately linear until the limiting value (corresponding to the no-breakaway condition) is reached, as shown in Fig. 9.13. The rate of change in the practical failure load with $\gamma h/c$ is approximately constant for a given embedment ratio, however the rate of increase, s , is itself a function of the embedment ratio varying, from $s = 0.5$ for $h/B = 1$, to $s = 0.96$ for $h/B = 3$ and $s = 1.0$ for $h/B = \infty$. Thus to sufficient accuracy the bearing capacity factor F_c is given by

$$F_c = F_{c_0} + s \cdot \frac{\gamma h}{c} \quad (9.3)$$

provided that $F_c \leq F_c^*$

where
$$F_c = F_c \left(\frac{h}{B}, \frac{\gamma h}{c}, \phi = 0 \right)$$

$$F_{c_0} = F_c \left(\frac{h}{B}, \frac{\gamma h}{c} = 0, \phi = 0 \right)$$

$$F_c^* = F_c \left(\frac{h}{B}, \frac{\gamma h}{c} = \infty, \phi = 0 \right)$$

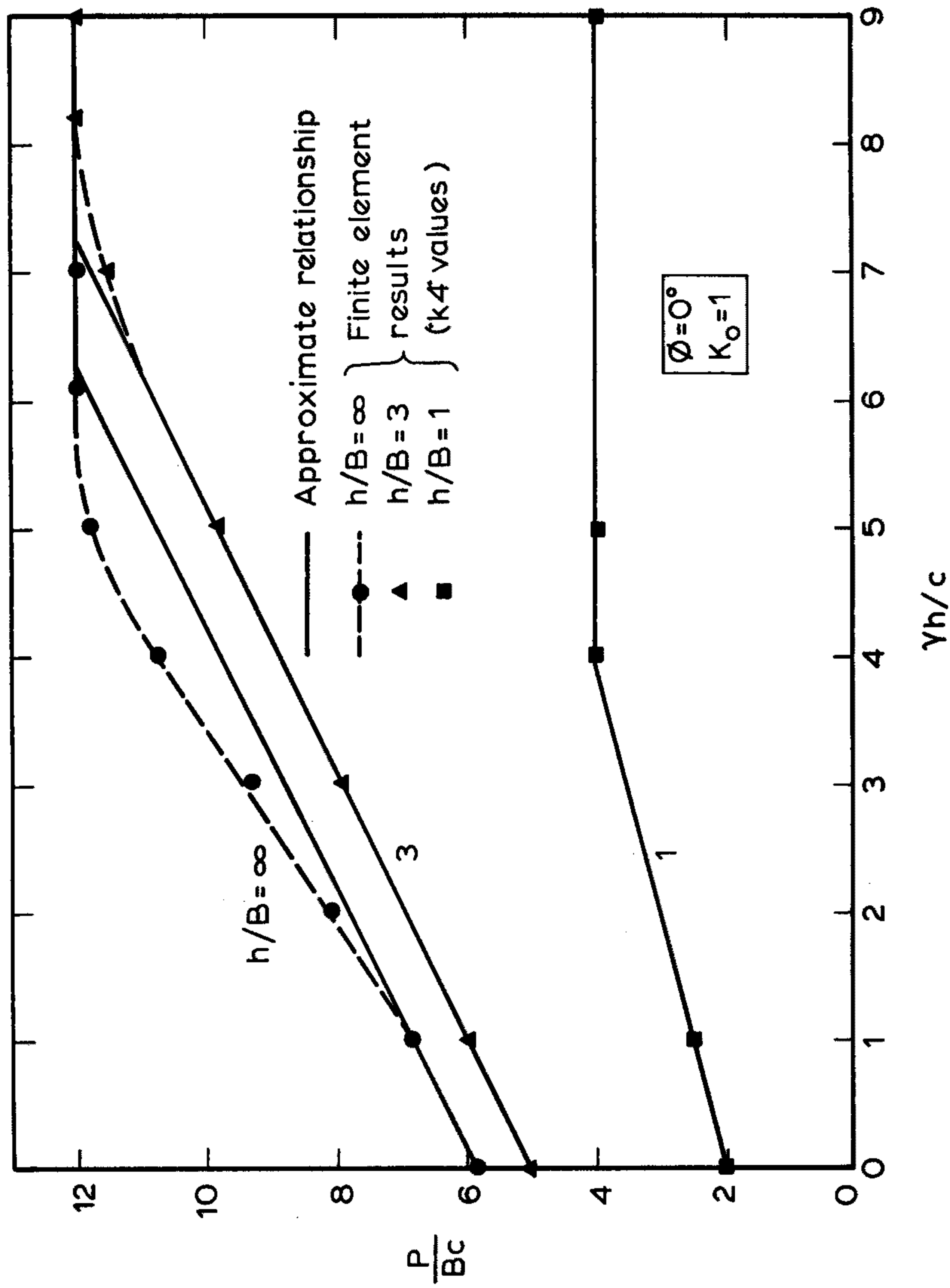


FIG.9.13 EFFECT OF $\gamma h/c$ UPON BEARING CAPACITY OF A VERTICAL ANCHOR

Note that F_c^* is the bearing capacity for the fully bonded case and that the parameter s ($0.5 \leq s \leq 1$) is itself a function of the embedment ratio h/B .

9.3.2 Cohesive-Frictional Soil (Drained Loading)

In broad outline, the behaviour of a cohesive frictional material is similar to that of a purely cohesive material. In particular, for material which deforms plastically with zero volume change (ie. $\psi = 0^\circ$) the velocity fields for cohesive frictional materials, both at collapse and at corresponding 'k3', 'k4' etc. load levels, are essentially the same as those shown for purely cohesive materials. In fact, this is to be expected since the form of the velocity field is mainly dependent on the geometry and the dilatancy angle.

The size of the plastic region at either a given load level or even at load levels corresponding to comparable 'k3' or 'k4' conditions were found to increase with friction angle ϕ as can be appreciated by comparison of the plastic region at the 'k4' load level and at collapse for $\phi = 0^\circ$ (shown in Fig. 9.8) with the plastic regions at the 'k3' and 'k4' load levels for a material with $\phi = 30^\circ$ (shown in Fig. 9.14). Three additional observations can be made. Firstly, the plastic region of failure extends much further below and behind the anchor for higher friction angles. Secondly, the behaviour of an anchor with embedment ratio $h/B = 3$ at the 'k4' practical failure load is that of a shallow anchor, and involves breakout of the plastic region to the soil surface. Finally, the plastic region at 'k3' and 'k4' loads for an anchor with embedment ratio $h/B = 5$ is only slightly influenced by the proximity of the soil surface and considerable additional load (and hence

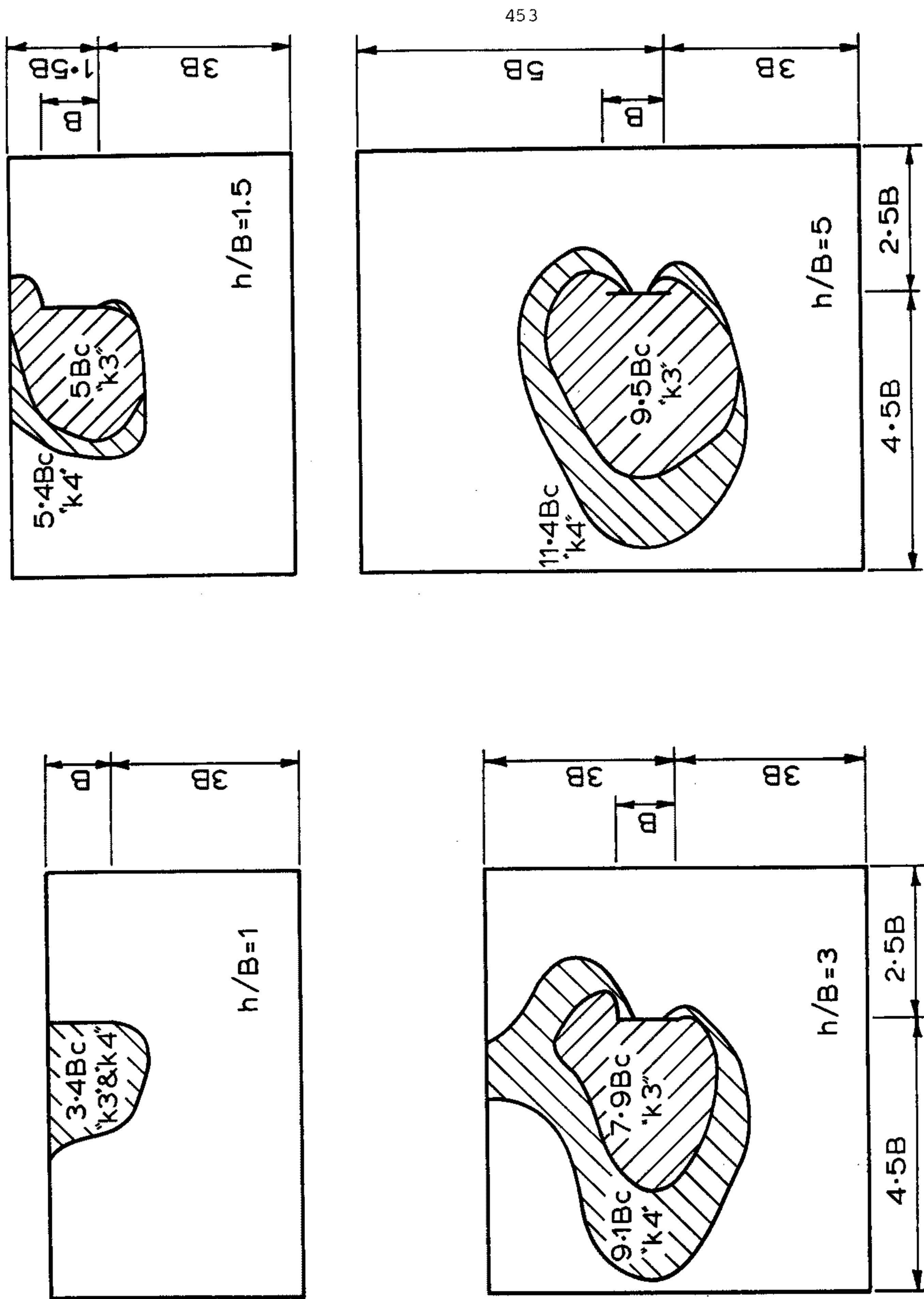


FIG.9.14 PLASTIC REGIONS AT 'k3' AND 'k4' LOAD LEVELS $\phi=30^\circ$, $\psi=0^\circ$, $\gamma=0$

contained plastic deformation) would be required to cause collapse of the anchor, since the rate of growth of the plastic region behind the anchor is significantly less than that in front of the anchor.

Fig. 9.15 shows the bearing capacity factor F_c obtained on the basis of a practical 'k4' failure load and it can be seen that the bearing capacity factor F_c and the critical depth at which 'deep anchor' behaviour is observed both increase with friction angle ϕ . (This observation has previously been inferred by consideration of plastic regions). In particular, it will be noticed that the clear distinction between shallow and deep anchor behaviour for a purely cohesive material is no longer evident for material with a high angle of internal friction.

The bearing capacity factors for an anchor in a cohesive frictional material (ie. $\phi > 0^\circ$) with weight can be obtained by superposition of the bearing capacity factor F_c and F_γ as indicated by Eq. 9.2 and as illustrated by example in section 9.5.

9.3.3 Effect of Anchor Roughness ($\gamma = 0$, $\psi = 0^\circ$)

From prior consideration of the velocity fields corresponding to failure of a smooth vertical anchor (see Fig. 9.9) it will be recognised that the embedment ratio has a marked effect on the velocity of soil adjacent to the anchor. In particular, for shallow anchors ($h/B \leq 2$) there is a significant component of velocity tangential to the anchor plate and consequently, the roughness of the anchor may be expected to lead to an appreciable increase in load carrying capacity. For example, the ultimate load carrying capacity of an anchor with em-

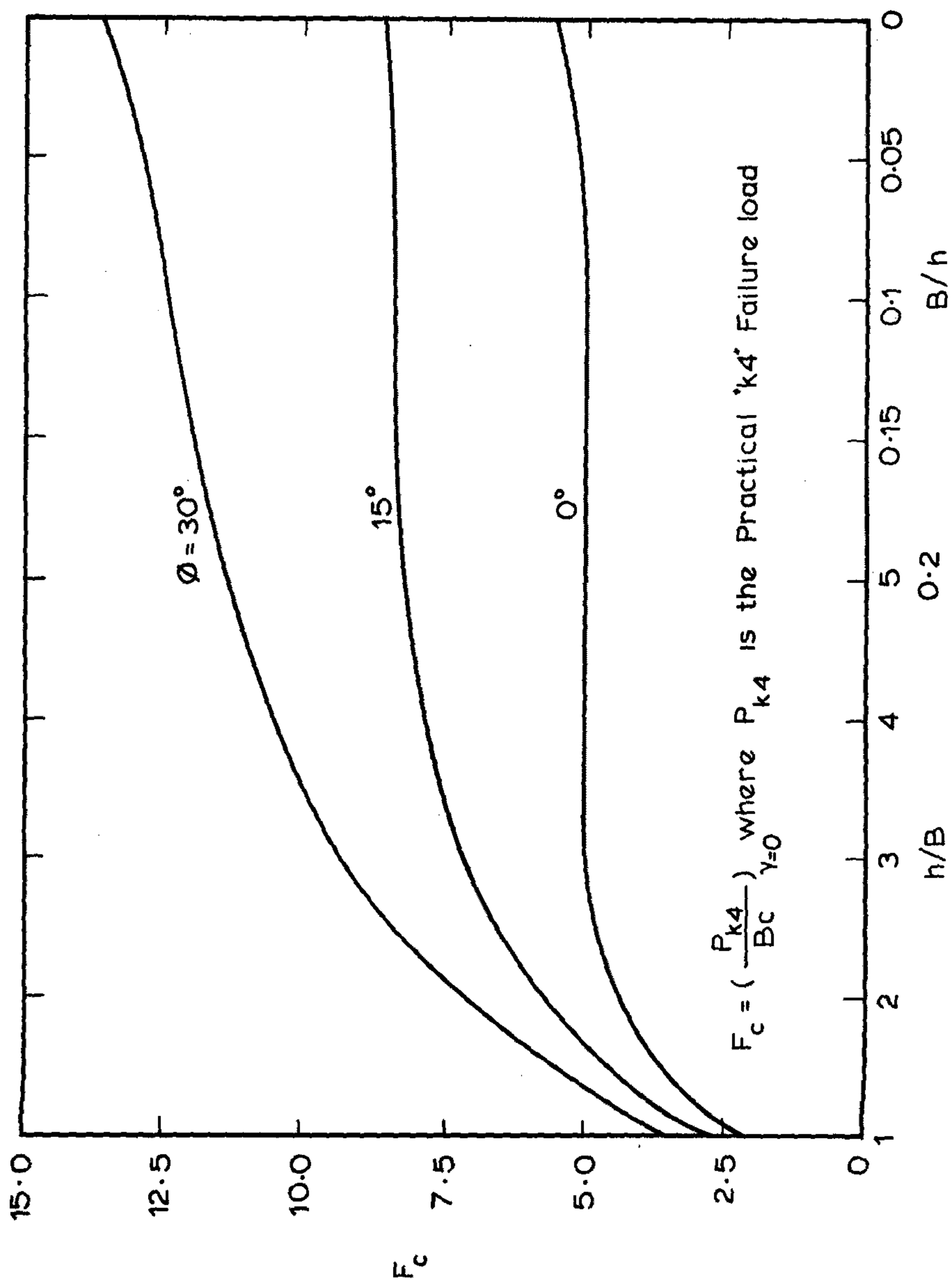


FIG.9.15 BEARING CAPACITY FACTOR F_c FOR VERTICAL ANCHOR - $\phi = 15^\circ, 30^\circ$ IMMEDIATE BREAKAWAY

bedment ratio $h/B = 1.5$ resting in a purely cohesive soil and having an adhesive interface was found to be approximately 30% above the value obtained for a smooth interface. However, the velocity fields also suggest that the significance of roughness will reduce dramatically with embedment ratio. At embedment ratios greater than two, the velocity field for a smooth anchor is predominantly perpendicular to the anchor plate, and it was found that for an anchor with an embedment ratio $h/B = 3$ the effect of an adhesive, and an adhesive-frictional interface (for an anchor resting in a soil with $\phi = 0^\circ$ and $\phi = 30^\circ$ respectively) increased the ultimate load carrying capacity by only 2% in the worst case.

9.4 COHESIONLESS SOIL WITH WEIGHT

The ultimate behaviour of vertical anchors in cohesionless soil is dependent upon the interaction of a wide range of soil parameters, which include angle of internal friction ϕ , angle of dilatancy ψ , coefficient of lateral earth pressure at rest, K_0 , and the angle of friction between soil and anchor (ie. anchor roughness effects). With such a range of parameters it is not feasible to perform a full parametric study, and attention will be restricted primarily to the basic case of a smooth rigid anchor resting in a material which deforms with constant volume change (ie. $\psi = 0^\circ$), and has a coefficient of lateral earth pressure at rest $K_0 = 1$. The effect of anchor roughness, soil dilatancy and initial stress state upon the behaviour of the anchor will then be considered briefly.

9.4.1 Basic Case - The Effect of h/B and ϕ (Drained Loading)

In the foregoing analysis of anchors in cohesive frictional soil it was found that the clear distinction between shallow and deep anchor behaviour which was evident for a purely cohesive material became increasingly less clearly defined as the angle of internal friction increased. This trend is still more apparent for cohesionless materials and even at considerable depth (ie. $h/B = 8$) the anchor behaviour is influenced by the proximity of the soil surface. To illustrate this, the load deflection curves obtained for a material with $\phi = 30^\circ$ are shown in Fig. 9.16 for a range of embedment ratios. The corresponding plastic regions and velocity fields at failure are shown in Figs. 9.17 and 9.18 respectively.

For embedment ratios up to and including five, failure was clearly defined and involved breakout of the plastic region to the soil surface, signifying shallow anchor behaviour. The load deflection curve for the anchor with $h/B = 8$ was still rising when the practical 'k4' failure load was reached, however the evidence suggests that eventual collapse of this anchor would also be associated with breakout of the plastic region to the soil surface. This contention is supported by the presence of elastic wedges of material above the anchor for embedment ratios of 3 and 5. In particular, it can be seen that for $h/B = 5$ these wedges very nearly joined and in such an event would involve a 'tunnel' of plastic material extending from the soil surface down to the back of the anchor and then up again to the soil surface. This mode of failure is clearly suggested by the velocity field which shows collapse of the soil into the region behind the anchor, and movement of the soil in front of the anchor towards the sur-

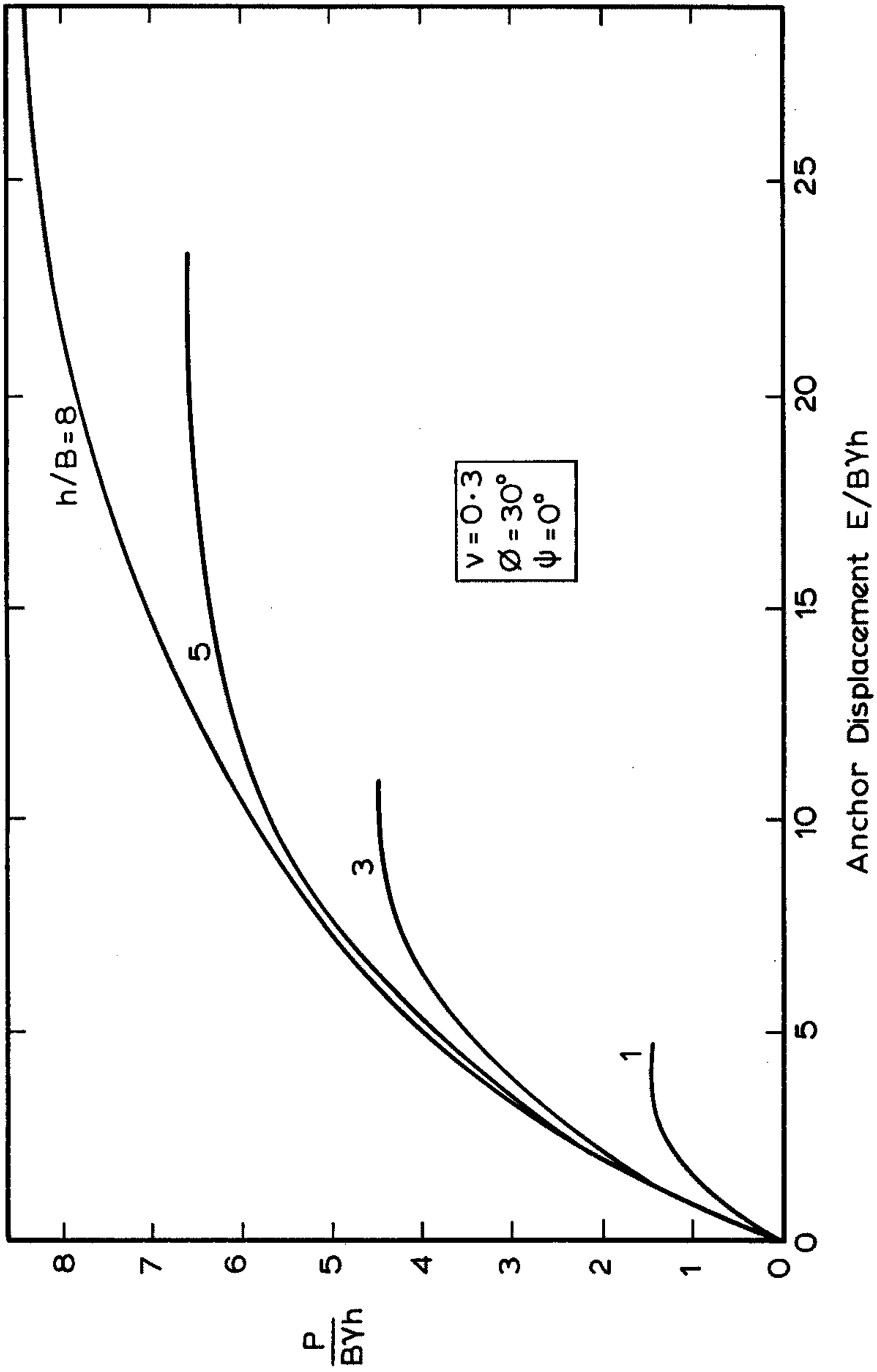
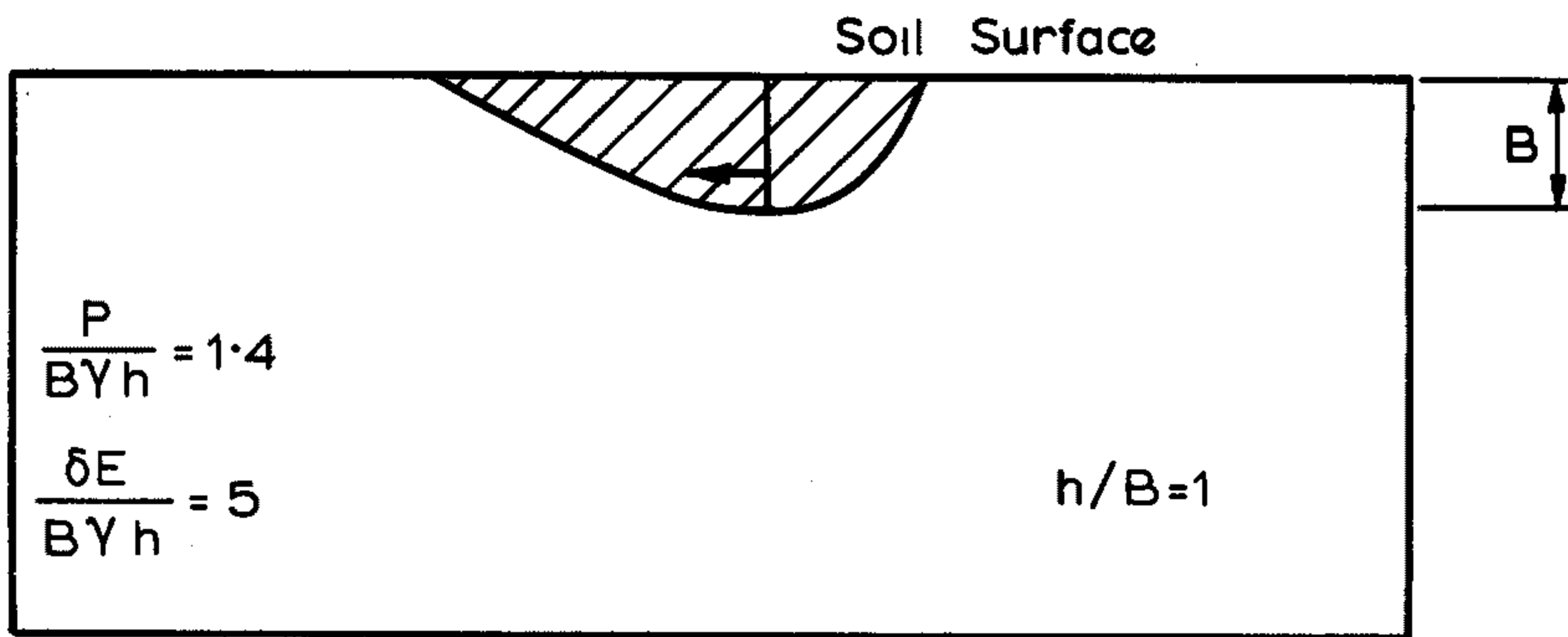
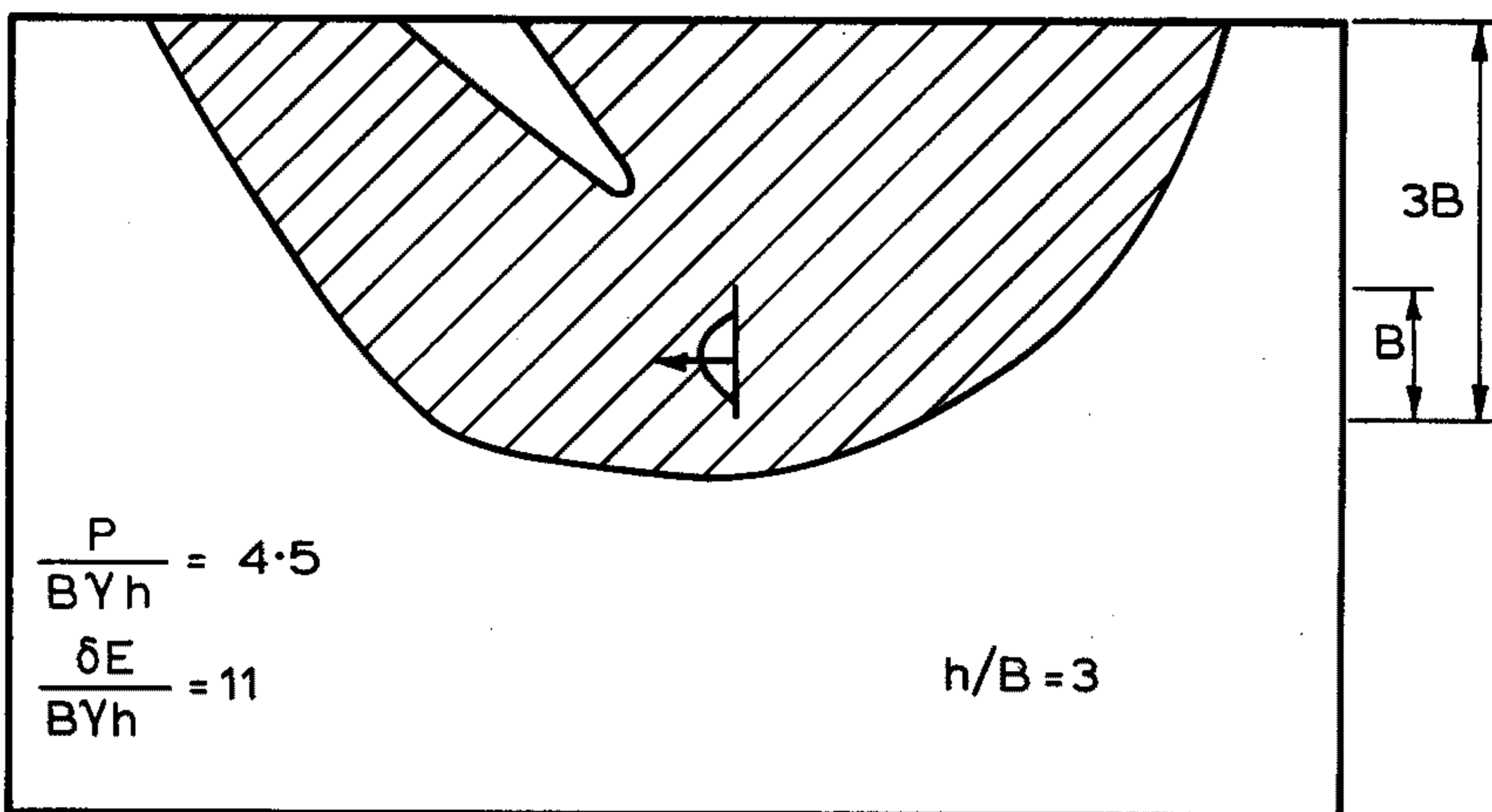


FIG.9.16 LOAD DEFLECTION CURVES FOR SMOOTH VERTICAL ANCHORS - $\phi = 30^\circ$



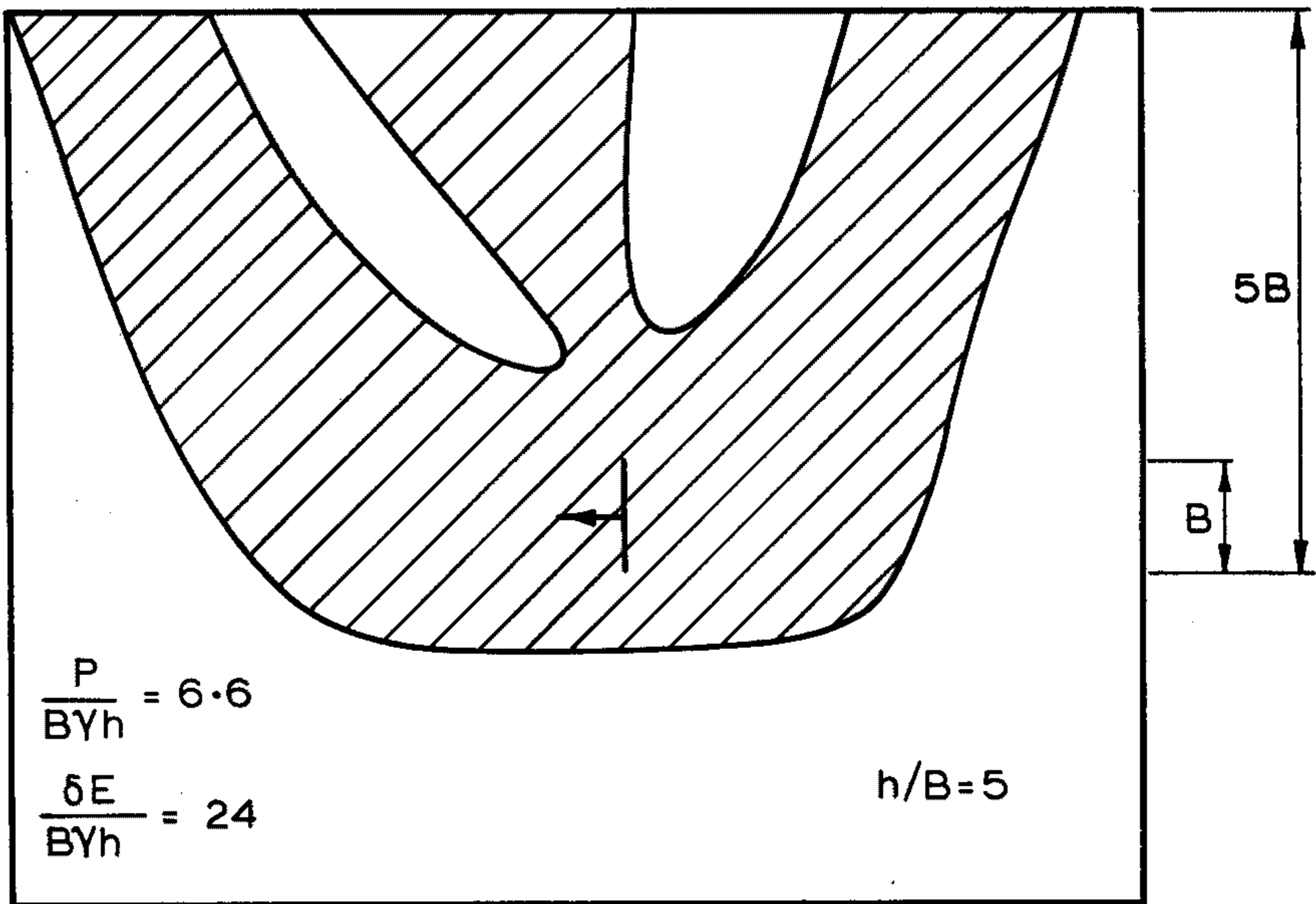
(a)



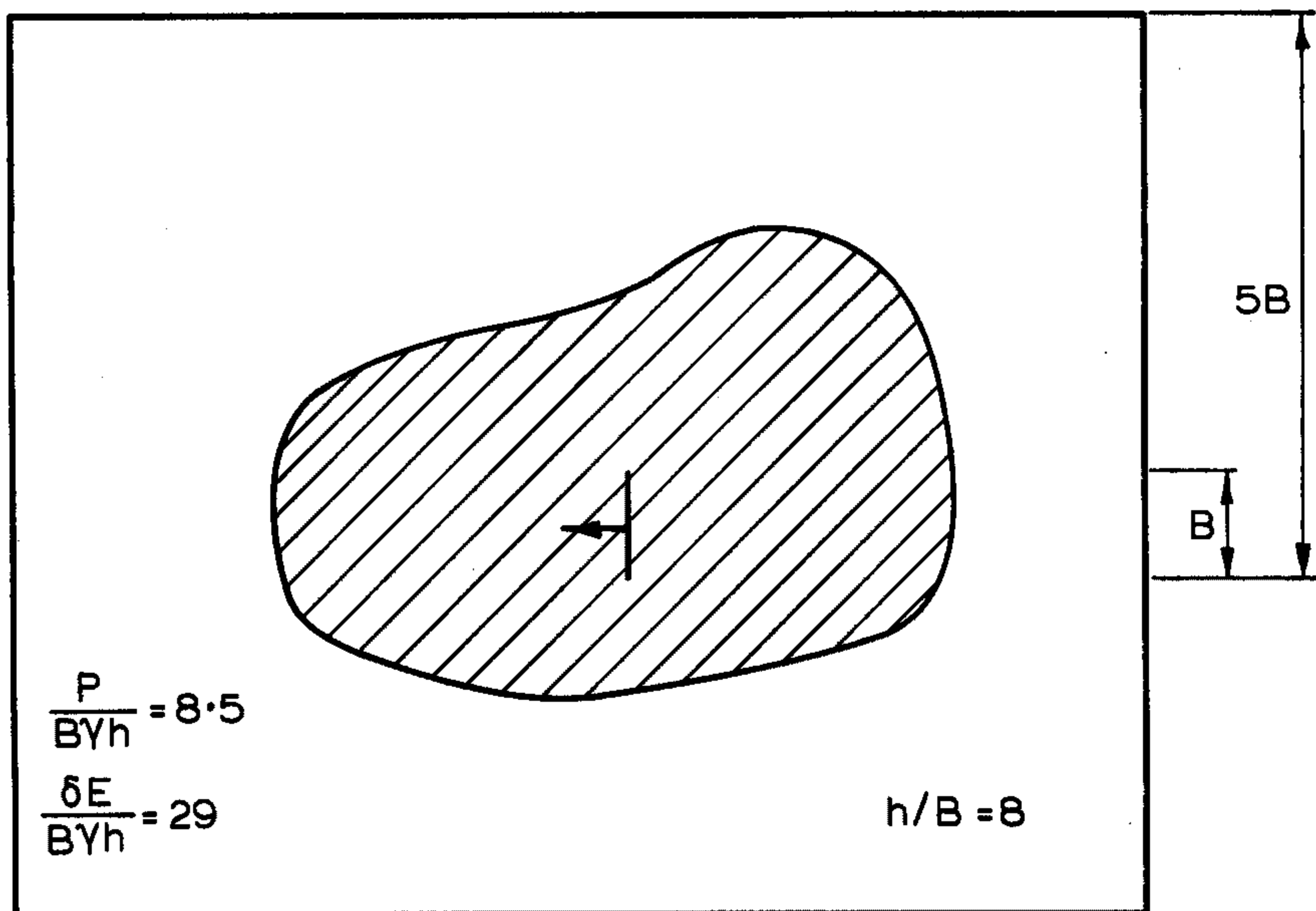
(b)

FIG.9. 17a&b PLASTIC REGIONS AT FAILURE $c=0, \phi=30^\circ, \psi=0^\circ, K_0=1$

Soil Surface

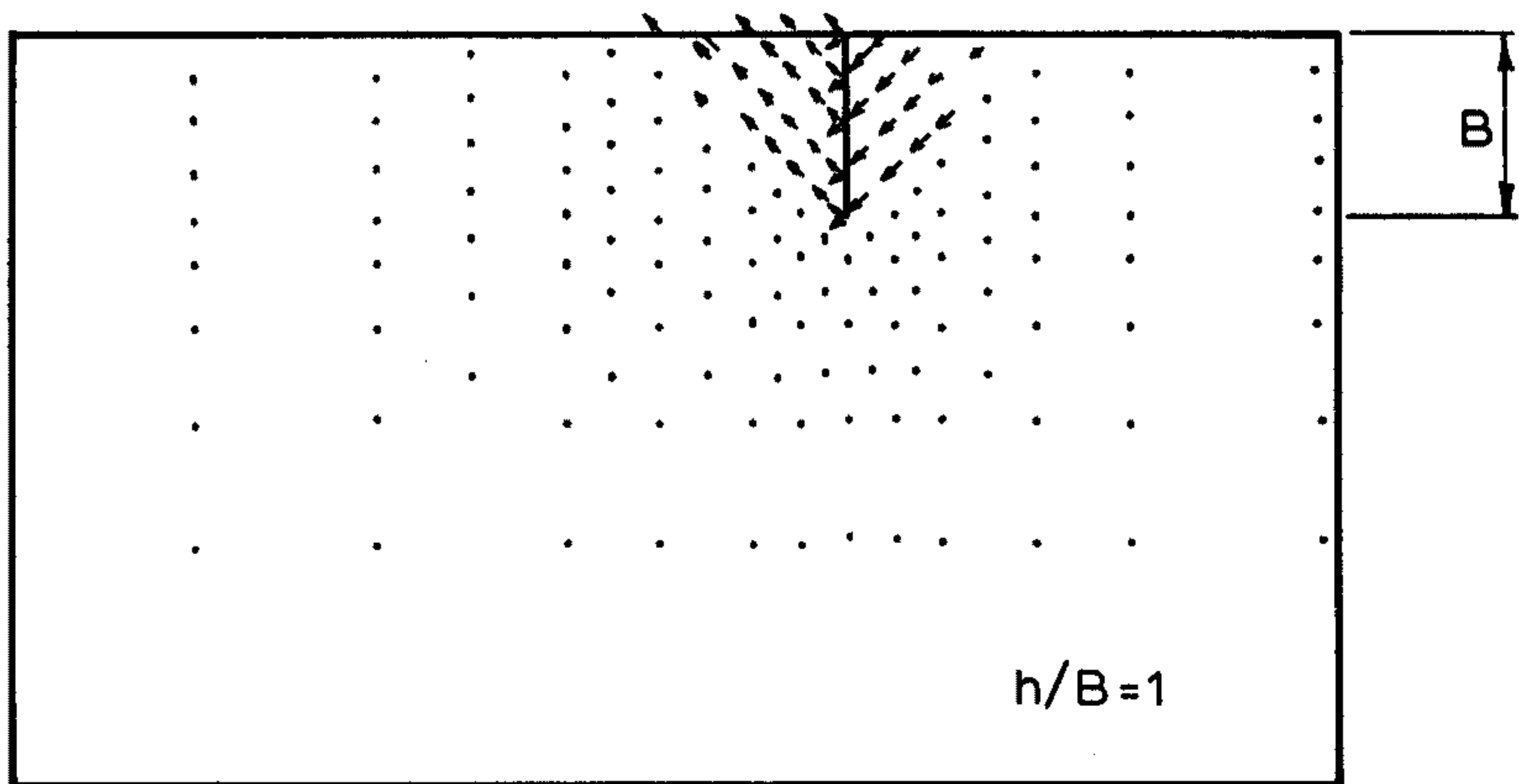


(c)

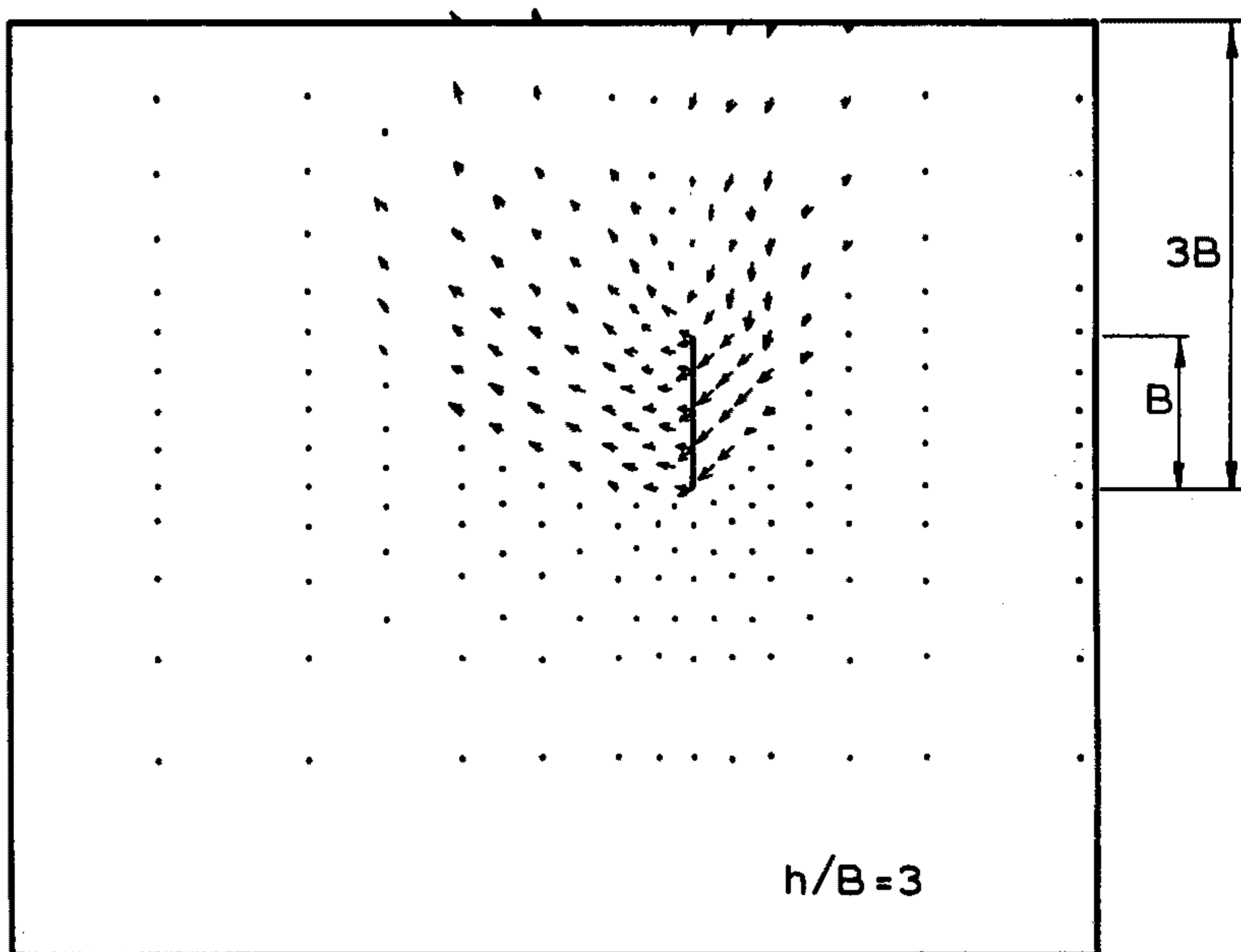


(d)

FIG.9. 17 c&d PLASTIC REGIONS AT FAILURE $c=0, \phi=30^\circ, \psi=0^\circ, K_o=1$



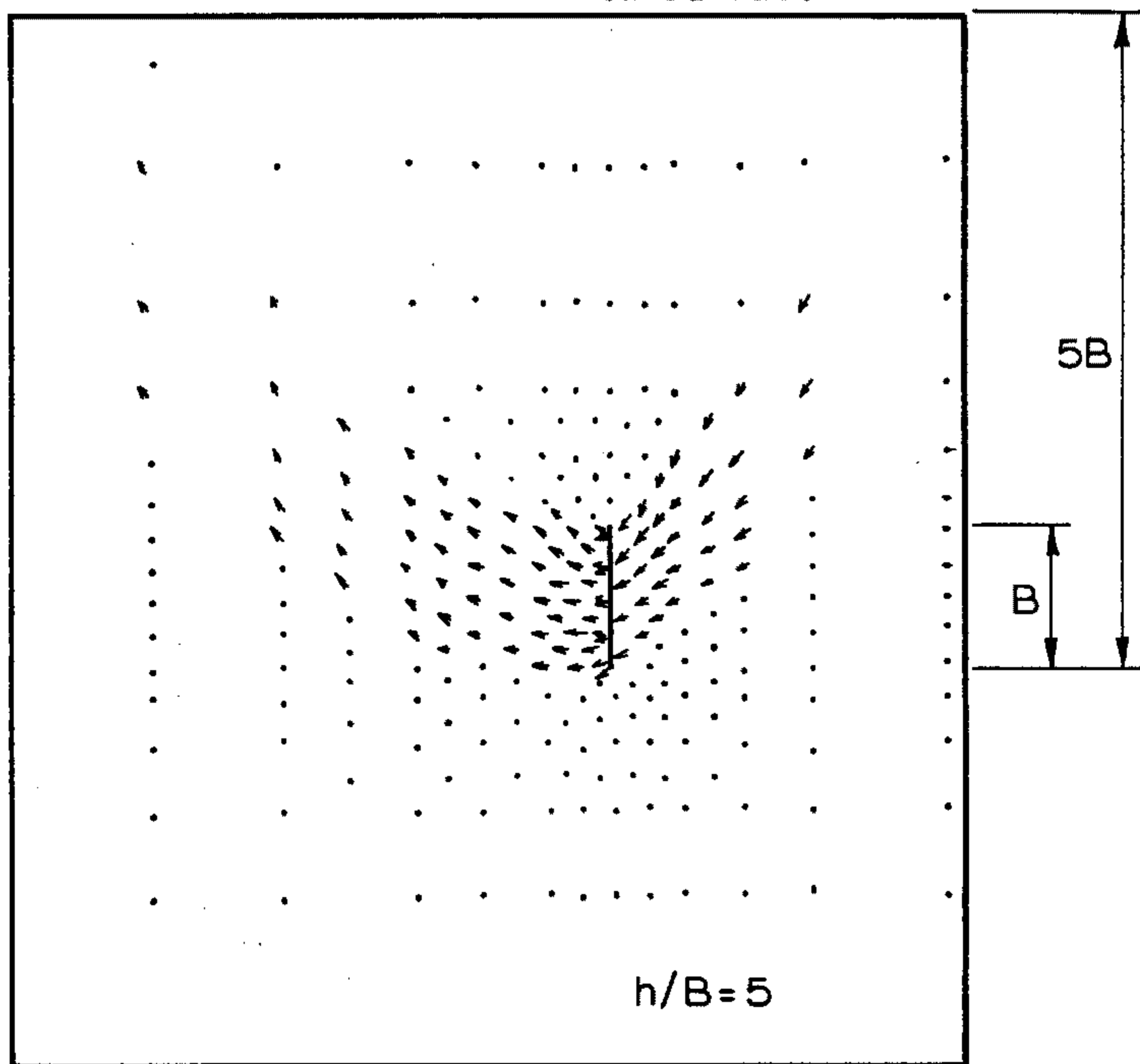
(a)



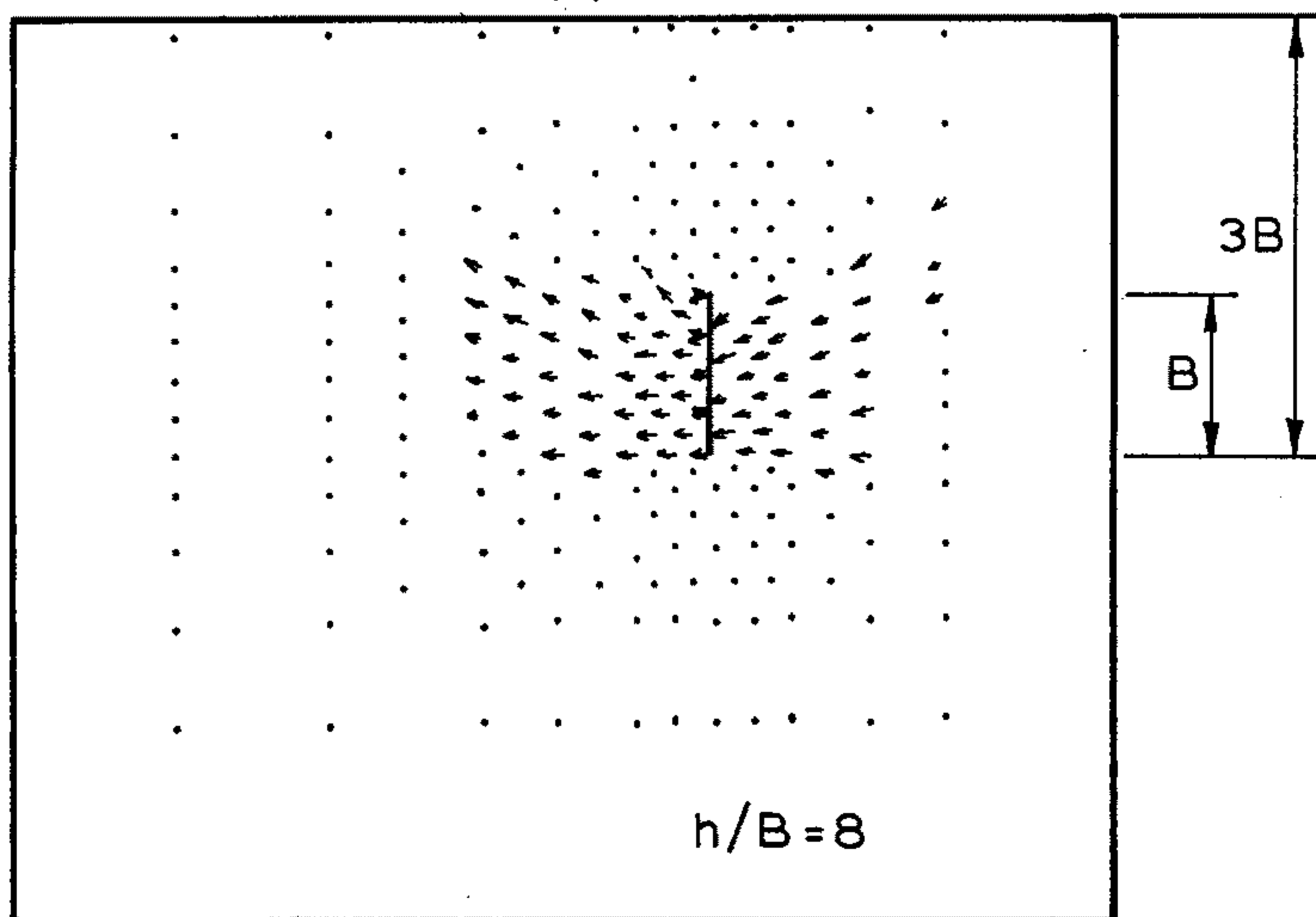
(b)

FIG. 9. 18. a & b VELOCITY FIELDS AT FAILURE
 $c = 0$, $\phi = 30^\circ$, $\psi = 0^\circ$, $K_0 = 1$

Soil Surface



(c)



(d)

FIG.9.18 c&d VELOCITY FIELDS AT FAILURE $c = 0$
 $\phi = 30^\circ, \psi = 0^\circ, K_o = 1$

face along the 'tunnel' of plastic material. Two observations can be made from the plastic region and velocity field at the practical ('k4') failure load for the case where $h/B = 8$. Firstly, there is a large region in which contained plastic flow is occurring. This region extends upward towards the soil surface behind the anchor, and directly outwards in front of the anchor. Secondly, although the anchor is entirely surrounded by plastic material the velocity fields indicate movement along the suggested 'tunnel' of plastic material and there is no evidence of material flowing from front to back of the anchor (as would be expected for 'deep anchor' behaviour). Clearly the implication of these observations is that even though the occurrence of extensive contained plastic deformation makes it necessary to adopt a practical definition of failure (as was the case of deep anchors in cohesive materials), the ultimate collapse of the anchor would be associated with breakout of plastic material to the soil surface, and as such must be considered to indicate shallow anchor behaviour. This conclusion is of some practical importance since it suggests that anchors with an embedment ratio greater than or equal to eight are capable of sustaining very large applied loads prior to complete collapse. Consequently, the actual performance of the anchor will be dominated by the contained plastic deformation prior to collapse rather than collapse itself.

Bearing capacity factors deduced from the actual collapse loads for $h/B \leq 5$ and from the practical 'k4' failure load for $h/B > 5$ are shown in Figs. 9.19 and 9.20 for $\phi = 15^\circ$, 30° and 45° . With regard to these figures, it should be noted that no solution was obtained for $h/B = 8$ where $\phi = 45^\circ$ because of the very high cost that would have been involved in performing this analysis. Thus the dotted portion of the curve for $\phi = 45^\circ$ and $h/B = 8$ in Figs. 9.19 and 9.20 is an extrapolation of the

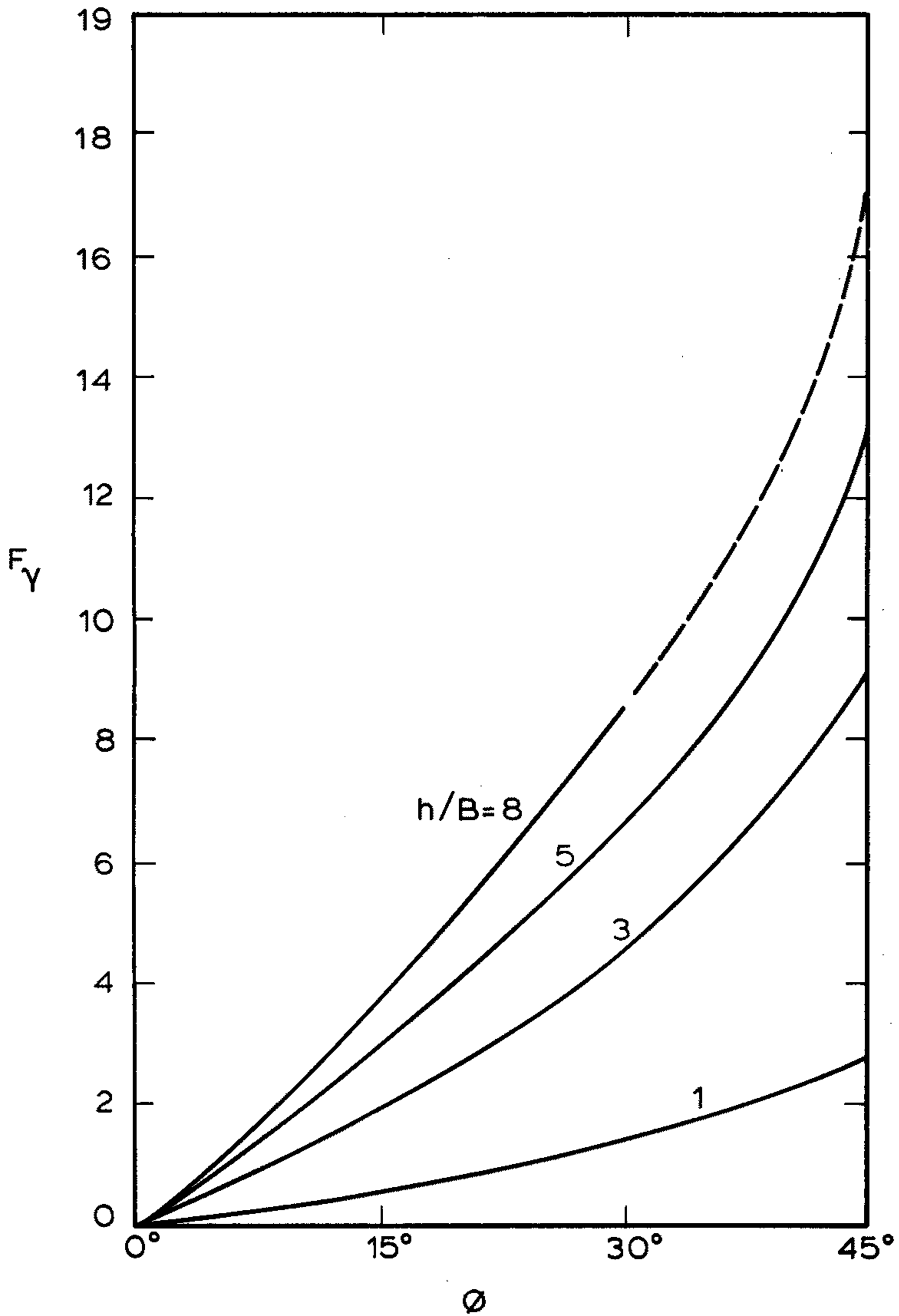


FIG.9.19 VARIATION IN BEARING CAPACITY FACTOR
WITH $\emptyset$ -VERTICAL ANCHORS $K_0=1, \psi=0$, SMOOTH

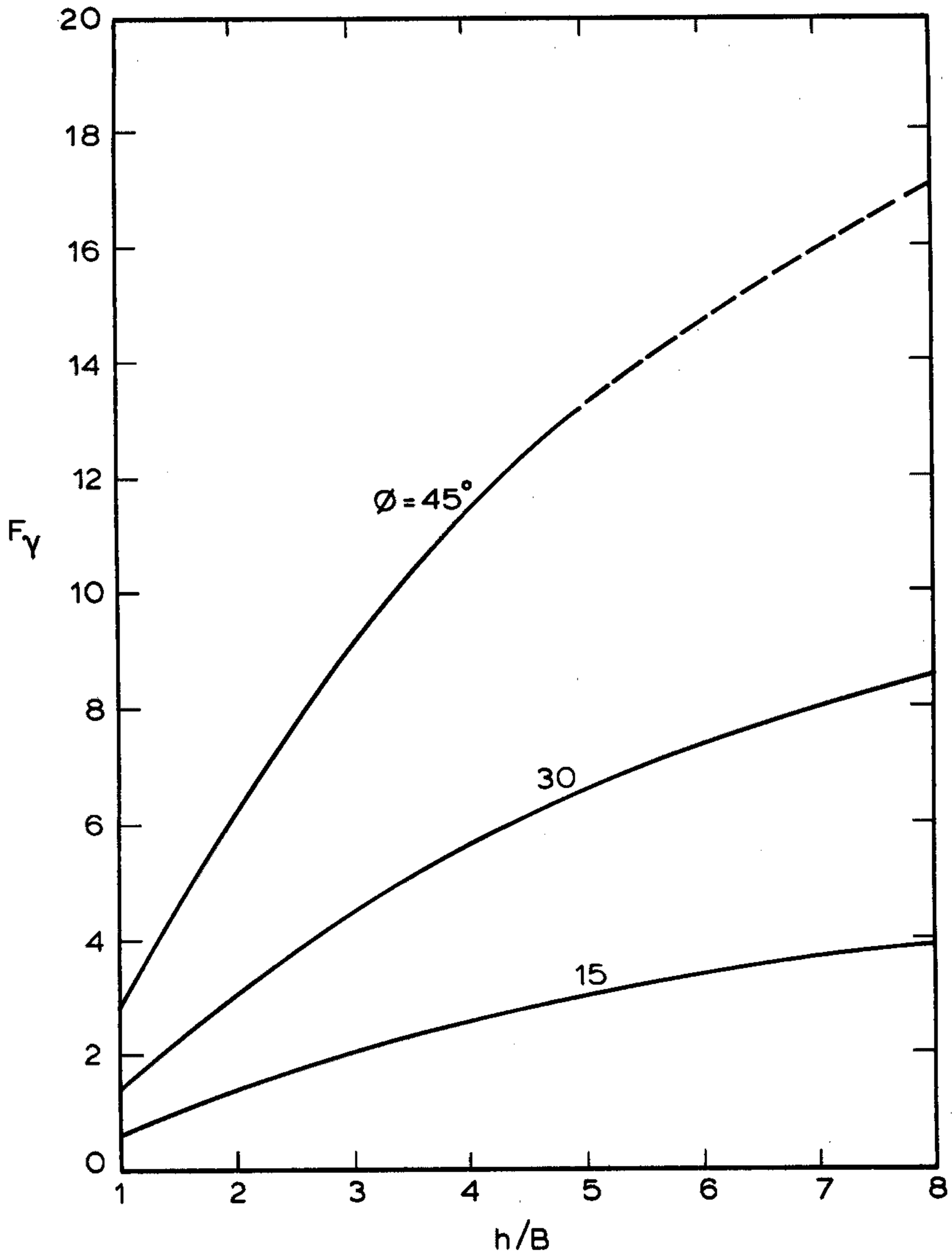


FIG. 9. 20 BEARING CAPACITY FACTOR F_γ FOR SMOOTH VERTICAL ANCHORS $K_o=1, \psi = 0^\circ$

earlier portion of the curve, and should be viewed with some caution.

Two features emerge from Figs. 9.19 and 9.20. Firstly, the bearing capacity factor F_γ increases dramatically with friction angle and it may be anticipated that the actual failure load will increase even more dramatically with ϕ once the effect of the dilatancy angle (which is usually greater than zero for high values of ϕ) is included in the analysis. Secondly, the rate of increase in bearing capacity factor F_γ with embedment ratio is such that for a given depth h there is an optimum anchor width B for which the actual ultimate load carrying capacity of the anchor, ie. $P = B\gamma h F_\gamma$ is at a maximum. For a wide range of friction angles ($15^\circ \leq \phi \leq 45^\circ$) the optimum width is approximately $0.5h$ and thus the provision of an anchor width greater than $0.5h$ will not increase the ultimate load carrying capacity of a smooth anchor.

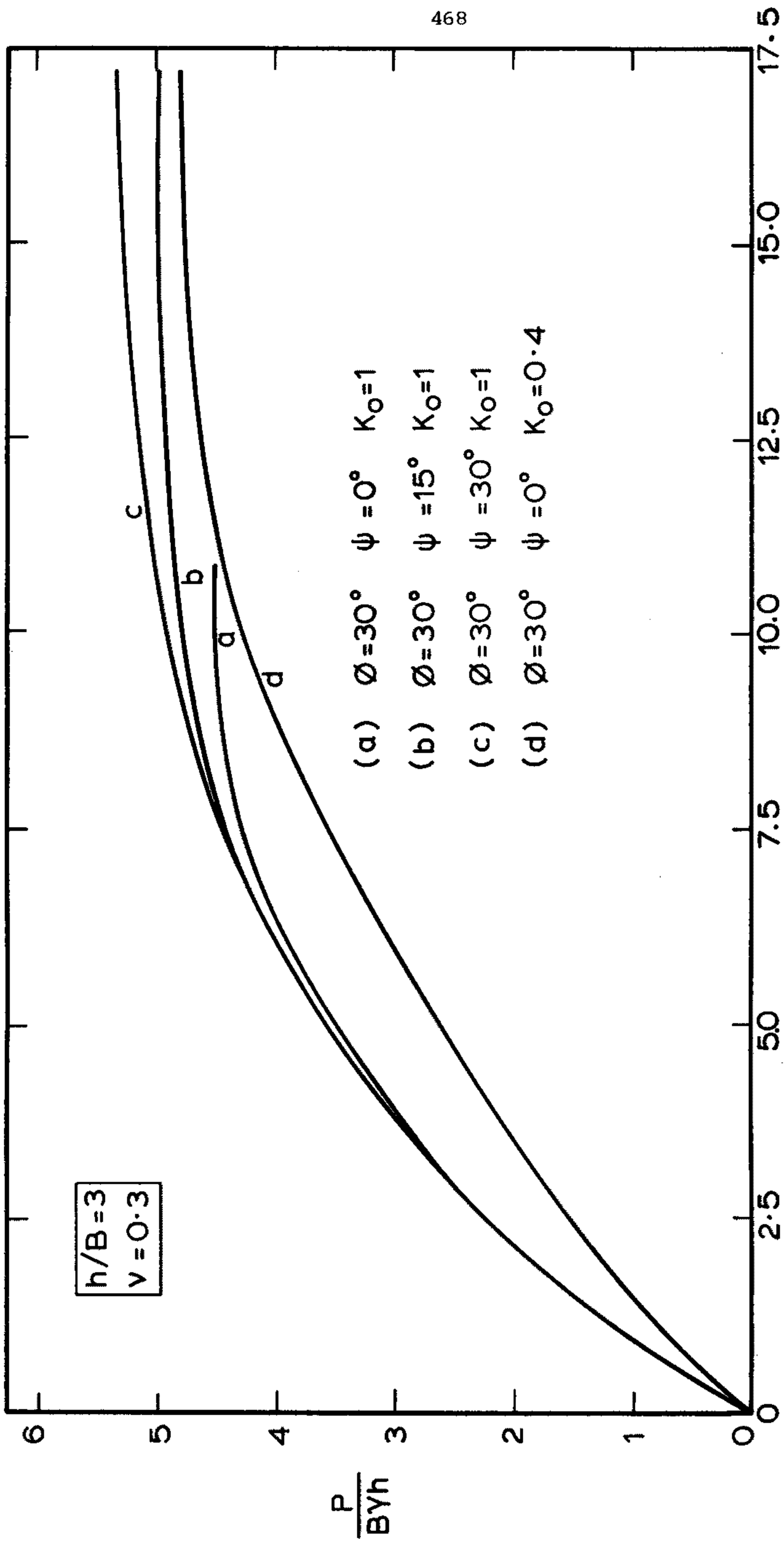
These results also confirm that the approximate, empirical rule suggested by Terzaghi (1943) (namely, that for anchors with embedment ratio less than 2, the anchor can be analysed as a retaining wall extending to the soil surface) gives conservative collapse loads to an accuracy of better than 10% for embedment ratios less than 4. However, this approach is quite unconservative for embedment ratios much greater than 4. It must be emphasised here that the above conclusions regarding the applicability of Terzaghi's approach are for a smooth anchor resting in a material where $\psi = 0^\circ$; it will become apparent from section 9.4.3 that much greater errors may be encountered in the application of Terzaghi's approach to rough anchors.

9.4.2 Effect of Dilatancy and Initial Stress State

As previously discussed in relation to horizontal anchor behaviour, it is known from classical plasticity theory (Davis, 1968) that the collapse load for a material with an associated flow rule ($\phi = \psi$) is an upper bound to the collapse load for $\psi < \phi$. Consequently the load carrying capacity of an anchor may be expected to increase with increasing angle of dilatancy, as is clearly indicated by the load displacement curves shown in Fig. 9.21 for an anchor with $h/B = 3$, $\phi = 30^\circ$ and where $\psi = 0^\circ$, 15° and 30° respectively.

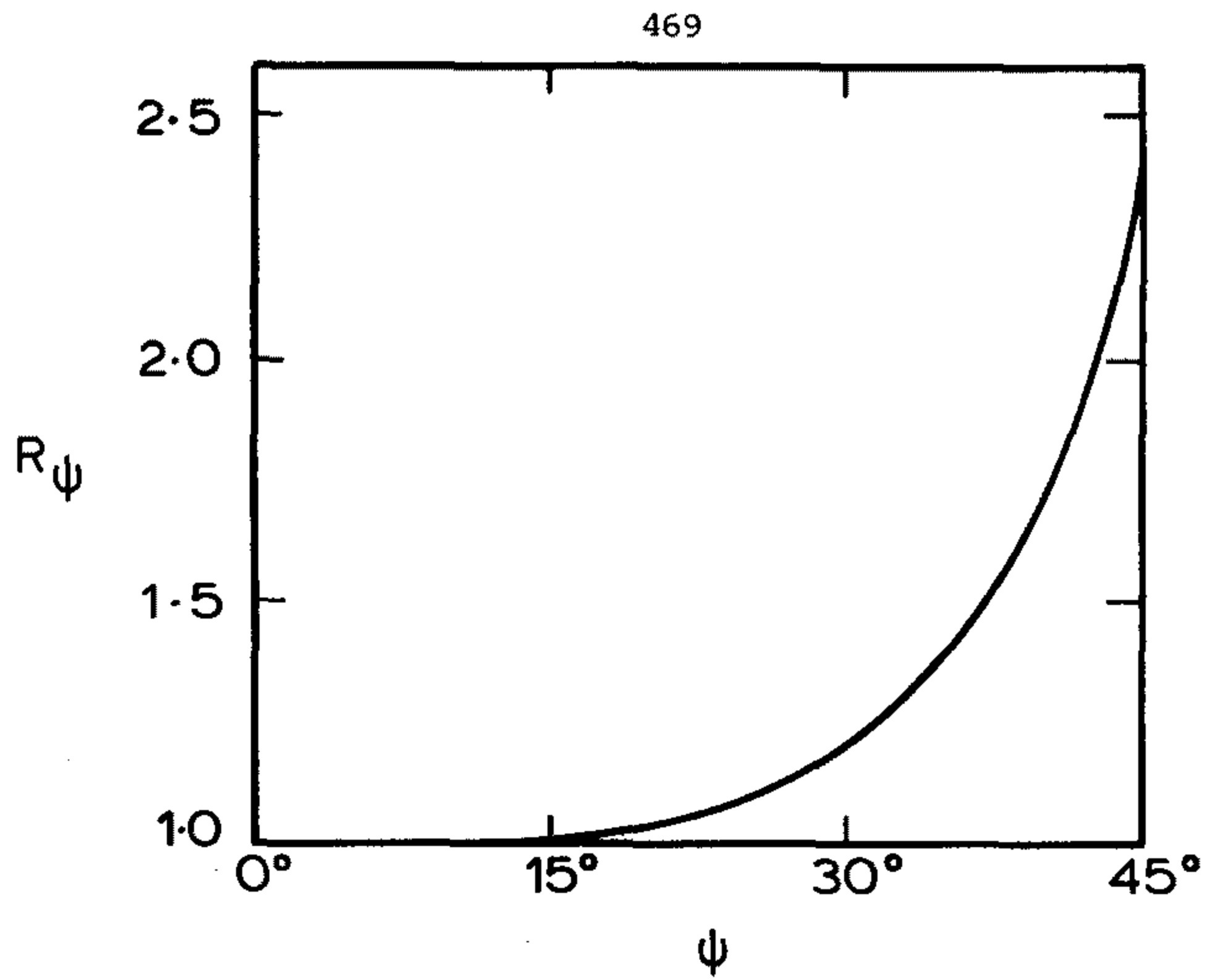
The effect of dilatancy may be conveniently incorporated in bearing capacity calculations by expressing the dimensionless bearing capacity factor F'_γ in the form $F'_\gamma = F_\gamma \cdot R_\psi$ where F_γ is the bearing capacity factor obtained for the basic case where $\psi = 0^\circ$; and R_ψ is a correction factor which will depend upon the parameters ϕ , ψ and h/B . The correction factor R_ψ is given in Figs. 9.22(a) and 9.22(b) for the cases where $h/B = 3$ and $0 \leq \psi = \phi \leq 45^\circ$ and $h/B = 3$, $\phi = 30^\circ$ $0 \leq \psi \leq \phi$. From this figure it will be observed that the influence of dilatancy upon the bearing capacity of the anchor increases dramatically with the angle of internal friction, leading to approximately a 20% increase in bearing capacity for $\psi = \phi = 30^\circ$ and to a 135% increase for $\psi = \phi = 45^\circ$. For a constant value of friction angle, the bearing capacity can be seen to increase approximately linearly with dilatancy angle ψ .

For shallow anchors, the extent of the plastic region at collapse is not greatly affected by the dilatancy of the material; however a comparison of the velocity fields shown in Figs. 9.18(b) and 9.23 for the case where ($h/B = 3$, $\phi = 30^\circ$), $\psi = 0^\circ$ and $\psi = 30^\circ$ clearly indicates

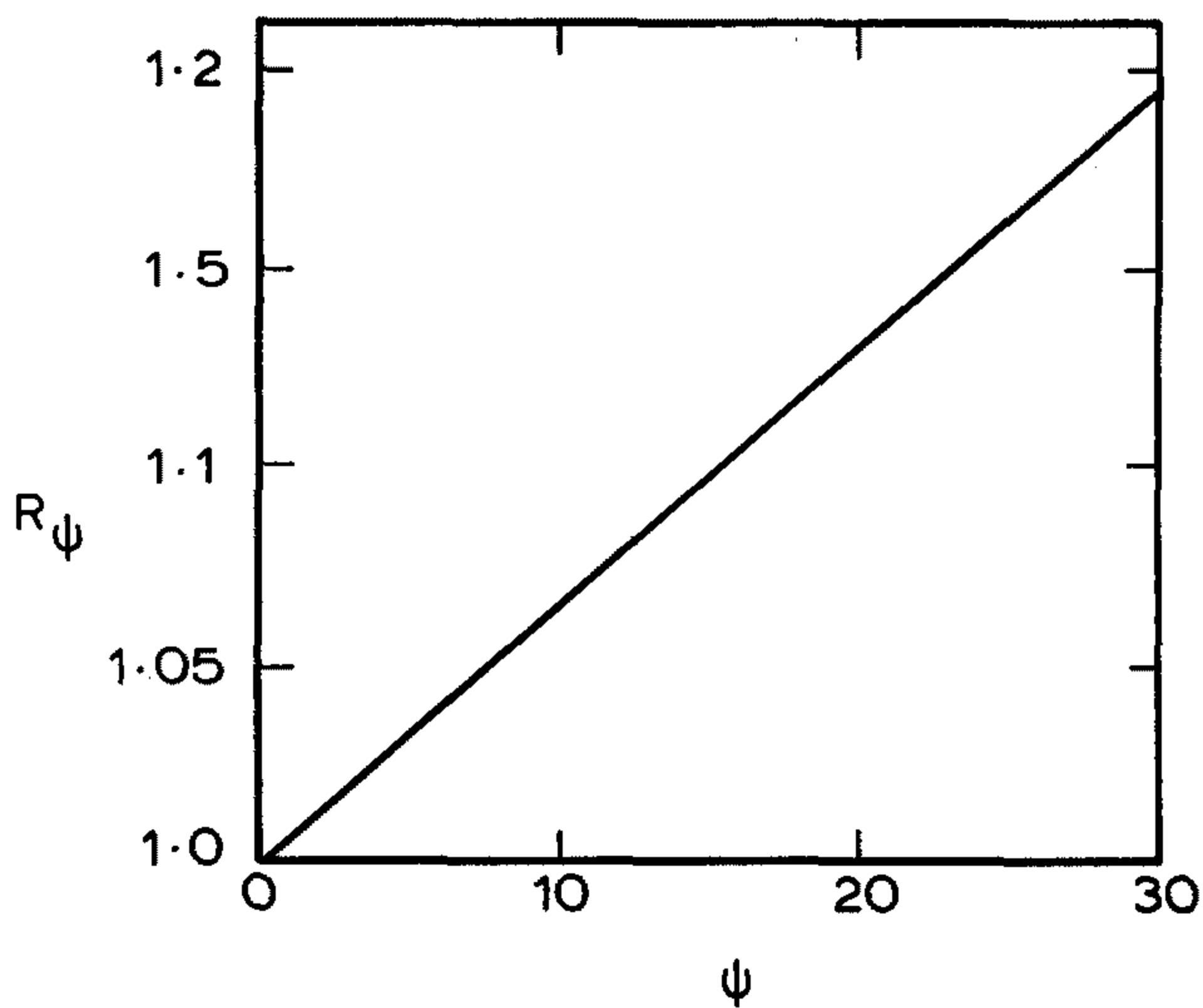


Anchor Displacement $E/(BYh)$

FIG.9.21 EFFECT OF DILATANCY AND K_o UPON LOAD DISPLACEMENT BEHAVIOUR OF A VERTICAL ANCHOR



(a) Associated Flow Rule $\phi = \psi$



(b) Constant $\phi = 30^\circ$

FIG.9.22 CORRECTION FACTOR FOR THE EFFECT OF ψ UPON A VERTICAL ANCHOR $h/B=3$

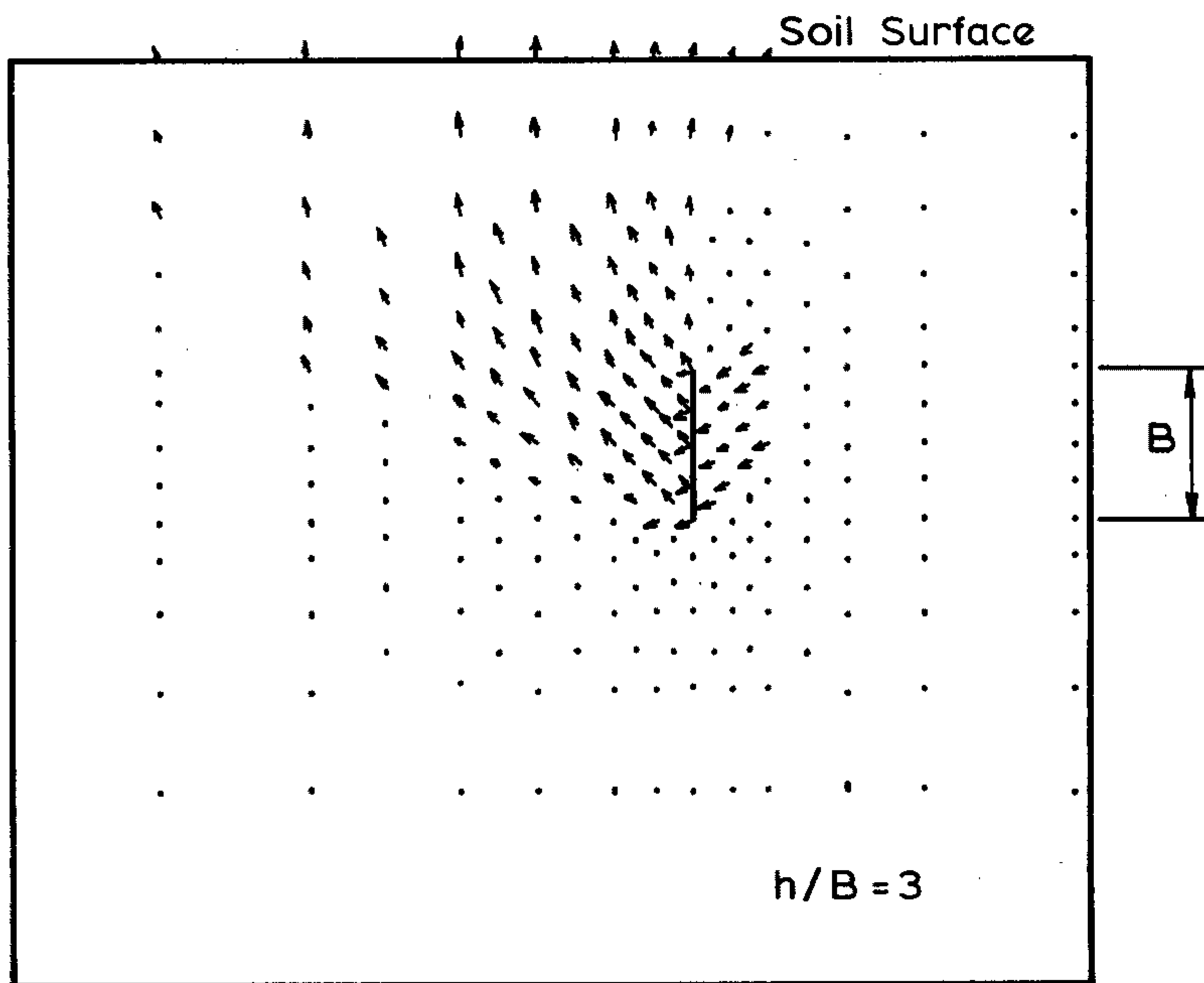


FIG. 9.23 VELOCITY FIELD AT FAILURE $c = 0, \phi = 30^\circ$
 $\psi = 30^\circ, K_o = 1$

that the velocity fields are significantly altered by the dilatancy of the material. In particular, the collapse of a dilatant material is associated with extensive movement of the deforming region in front of the anchor towards the soil surface. Thus the dilatancy of the material leads to far more apparent movement of the soil surface in front of the anchor. In addition, the dilation of the soil induces arching stresses above and towards the rear of the anchor, and this inhibits the flow of material from the soil surface to the back of the anchor.

From the foregoing discussion it is apparent that the predicted collapse load calculated from the basic bearing capacity factor $F_{\gamma} = (P/B\gamma h)_{\psi=0}$ may considerably underestimate the actual collapse load for dense granular materials with high dilatancy and friction angles. For moderately shallow anchors ($h/B \leq 5$, say) an approximate estimate of the actual collapse load may be obtained using the correction factor R_{ψ} given for $h/B = 3$. Experience gained with horizontal anchors would indicate that the influence of dilatancy upon the collapse load increases with increasing embedment ratio and hence approximate predictions based on the values of R_{ψ} for $h/B = 3$ would be expected to give a conservative estimate of the collapse load for deeper anchors ($h/B > 3$).

In the chapter on horizontal anchors, it was found that for a particular anchor geometry a variation in the initial stress state (in terms of coefficient of lateral earth pressure at rest K_0) effectively changed the load path to failure thereby altering the load displacement curve and, in some cases, the collapse load. This general observation is equally applicable to the behaviour of vertical

anchors, however, the details of the effect of K_0 upon the collapse load differ significantly between the two cases. In particular, for a material deforming with constant volume ($\psi = 0^\circ$) a reduction in the coefficient of lateral earth pressure, K_0 , tended to reduce the collapse load for horizontal anchors, whereas for vertical anchors a reduction in K_0 was found to increase the collapse load relative to that obtained with $K_0 = 1$.

To illustrate this the load displacement curves obtained for a vertical anchor with $h/B = 3$, $\phi = 30^\circ$ where $K_0 = 1$ and 0.4 are shown in Fig. 9.21. A comparison of the two curves indicates that the reduction in K_0 leads to an increase in the plastic deformation observed prior to collapse as well as to an increase of 7% in the collapse load itself. The velocity fields obtained in these two cases were not significantly different however, the extent of the plastic region at collapse was considerably altered (as may be appreciated by an examination of Figs. 9.17(b) and 9.24). For comparative purposes, an analysis was performed for $h/B = 1$, $\phi = 30^\circ$, $\psi = 0$ where $K_0 = 1$ and $K_0 = 0.4$. At this embedment ratio it was found that the load displacement curve was significantly altered by the initial stress state, however the actual collapse load remained unaltered. Thus it may be anticipated that while the value of the coefficient of lateral earth pressure significantly affects the load displacement behaviour of the anchor, it does not greatly alter the collapse load for shallow vertical anchors in normally consolidated granular material.

It should be emphasised that although the collapse load for an associated material is an upper bound to the collapse load for all

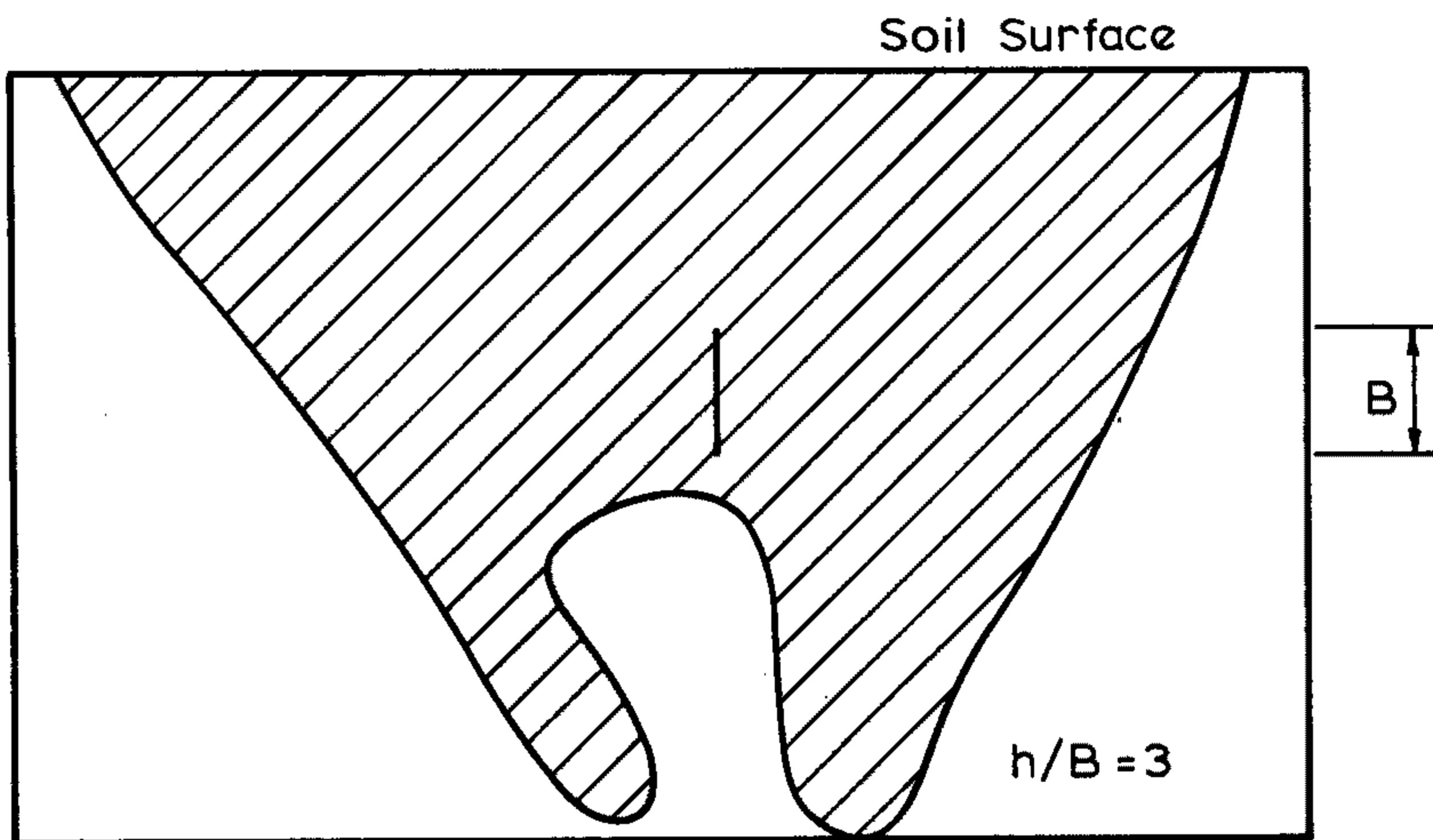


FIG. 9.24 PLASTIC REGION AT FAILURE $c = 0, \phi = 30^\circ, \psi = 0^\circ, K_o = 0.4$

values of ψ and K_o , the collapse load obtained for the basic case ($K_o = 1$, $\psi = 0^\circ$) does not necessarily represent the lowest possible collapse load that may be obtained for a given geometry and angle of internal friction ϕ . It is conceivable that there exists some initial stress state for which the actual collapse load may be less than that obtained with $K_o = 1$. Consequently, the bearing capacity factors for the basic case should only be applied after consideration of the likely effects of K_o and ψ ; and while it is considered that the basic case will generally give a conservative estimate of the collapse load, caution should be adopted for values of K_o significantly larger or smaller than one.

9.4.3 Effect of Anchor Roughness

In Chapters 3 and 7 it was found that both the elastic and ultimate behaviour of horizontal anchor plates were largely insensitive to anchor roughness and accordingly it may be anticipated that the behaviour of deep vertical anchors will also be insensitive to anchor roughness since, in the limiting case of an infinitely deep anchor, the direction of anchor pull is irrelevant. It is, however, well known that the behaviour of a retaining wall is significantly affected by wall roughness (Lee and Herrington, 1972; Chen, 1975) indicating that there must be a transition in behaviour corresponding to the changing effect of roughness with increasing embedment ratio. To quantify this variation in behaviour, analyses were performed for a rough anchor in a granular material with $\phi = 30^\circ$, $\psi = 0^\circ$ and $K_o = 1$. The effect of roughness may be expressed in terms of a correction factor R_R , which corresponds to the ratio of the bearing capacity for a rough anchor to that for a smooth anchor. This correction factor R_R is shown in Fig.9.25

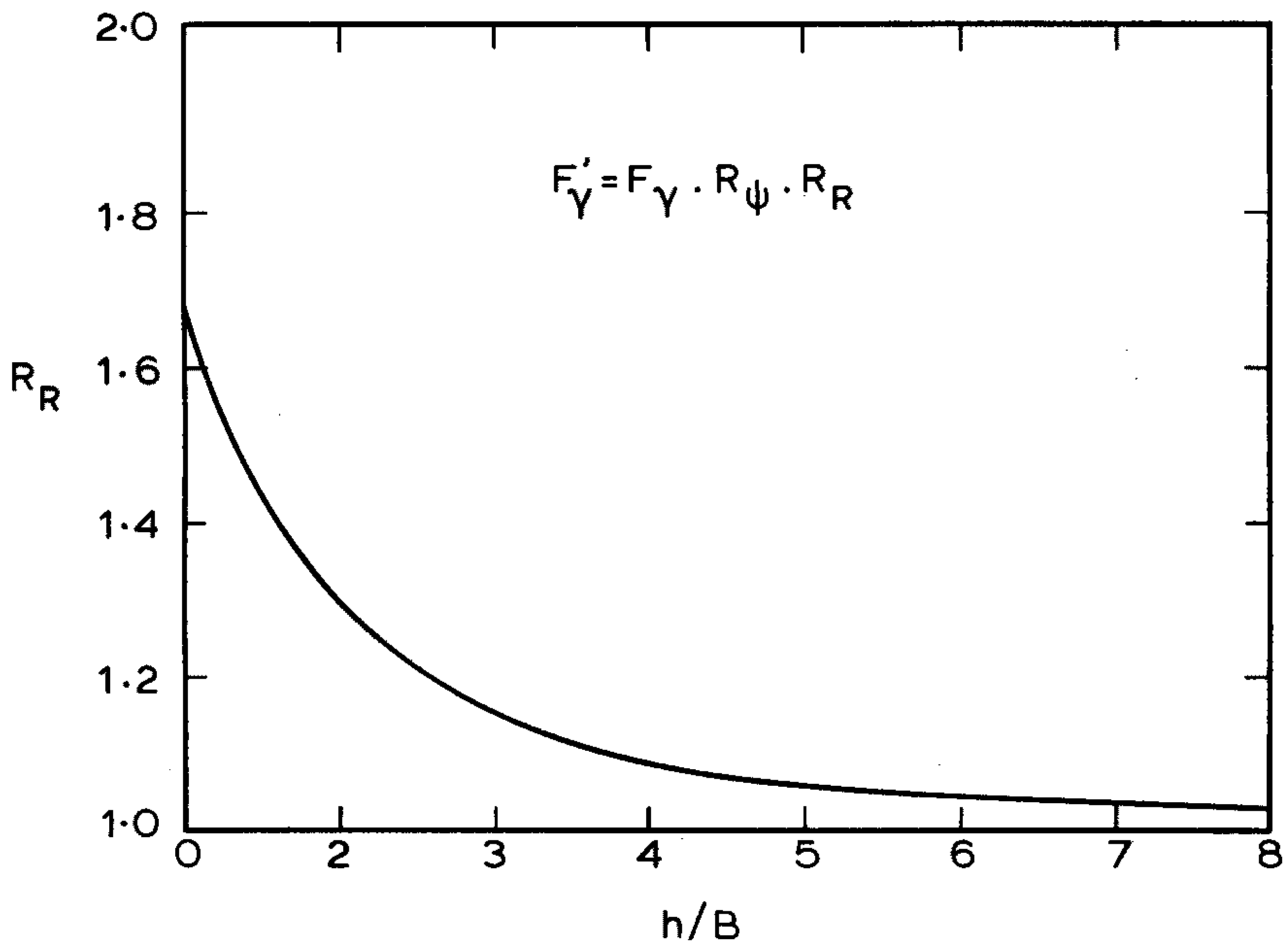


FIG. 9.25 CORRECTION FACTOR FOR THE EFFECT OF
ROUGHNESS $\phi=30^\circ, \psi=0^\circ$

as a function of the embedment ratio. For a material with $K_o = 1$ and $\psi = 0^\circ$ the increase in bearing capacity with anchor roughness decreases rapidly with depth from a 68% increase for $h/B = 1$ to a 15% increase for $h/B = 3$ and less than a 6% increase for $h/B = 5$. This rapid decrease in the effect of anchor roughness could have been anticipated from consideration of the velocity field shown in Fig. 9.18 where it can be seen that the natural movement of soil particles adjacent to the front (and to a lesser extent the back) of a smooth anchor is at 45° to the direction of anchor movement for $h/B = 1$, but tends to be almost perpendicular to the direction of anchor movement for embedment ratios greater than or equal to three. Accordingly, the additional restraint of anchor roughness only leads to a significant alteration in anchor behaviour for embedment ratios less than three.

In the foregoing discussion the effects of dilatancy and roughness have been considered in isolation. However, a comparison of the velocity fields obtained for the two cases where $\psi = 0^\circ$ (Fig. 9.18) and $\psi = \phi = 30^\circ$ (Fig. 9.23) indicates that there is a much larger component of velocity tangential to the smooth anchor in the latter case; consequently it may be expected that the effect of roughness will be much greater for the dilatant material. Indeed, an analysis performed for a rough anchor (where $h/B = 3$, $\phi = \psi = 30^\circ$, $K_o = 1$) gave a correction factor $R_R = 1.67$ which indicates a 67% increase in the bearing capacity of the rough anchor as compared to a smooth anchor in the same material. Thus, the effect of roughness, while decreasing with embedment ratio, increases with angle of dilatancy due to both an increase in the apparent angle of roughness, ϕ_k (from approximately 26° for $\phi = 30^\circ$, $\psi = 0^\circ$, to 30° for $\phi = \psi = 30^\circ$) as well as to the different

mode of failure.

The ultimate collapse of a rough anchor in an associated material is preceded by significant contained plastic flow, necessitating the adoption of a practical ('k4') failure load. For $h/B = 3$, $\phi = \psi = 30^\circ$ this failure load corresponds to a correction factor $R_R = 1.13$ which is in fact quite close to the value obtained for a material which deforms plastically with zero volume change (ie. $\psi = 0^\circ$). Thus, although in the rigorous applied mechanics sense the interaction between dilatancy and anchor roughness is significant, in terms of practical failure loads it would appear that, to a first approximation, the bearing capacity factor F_γ is given by

$$F'_\gamma = F_\gamma \cdot R_\psi \cdot R_R \quad (9.4)$$

where $F_\gamma = \left(\frac{P}{B\gamma h}\right)_{\psi=0^\circ, \text{ smooth}};$

R_ψ is the correction factor for the effect of dilatancy alone (ie. smooth anchor); and

R_R is the correction factor for roughness alone (ie. $\psi = 0^\circ$).

For example, in the particular case of a rough anchor with $h/B = 3$, $\phi = \psi = 30^\circ$, $K_o = 1$ the predicted failure load obtained using Eq. 9.4 in conjunction with Figs. 9.19, 9.22 and 9.25 overestimates the practical failure load by 2%, although the actual collapse load is underestimated by 47%.

It is of some interest to note that the application of Terzaghi's rule (Terzaghi, 1943) for the prediction of the ultimate load carrying capacity of shallow, rough anchors ($1 \leq h/B \leq 2$) would lead to estimates of collapse load which are unconservative by up to

15% within this range of embedment ratios. The situation is the opposite of that already observed for smooth anchors, and arises because the significance of anchor roughness decreases very rapidly with embedment ratio when $\psi = 0^\circ$. Consequently, predictions of the ultimate load that can be carried by a buried anchor which are based on the behaviour of a retaining wall buried to the same depth will over-estimate the contribution of anchor roughness to the ultimate load carrying capacity. Provided that the application of Terzaghi's approach is restricted to the suggested range of embedment ratios (ie. $1 \leq h/B \leq 2$), the unconservative prediction would not be expected to lead to serious practical difficulties since the maximum error of 15% would generally be accommodated by the factor of safety adopted in anchor design. In practice the dilatancy angle ψ may be expected to be greater than zero, thereby enhancing the actual load carrying capacity of the anchor. In fact, the effect of a non-zero value of ψ will be twofold; firstly, the dilatancy will directly increase the bearing capacity and secondly, the effect of anchor roughness will be enhanced by soil dilatancy.

In the limiting case of a rough anchor in an associated material ($\psi = \phi$) Terzaghi's approach would again lead to a conservative estimate of the anchor collapse load because of the increasing significance of dilatancy with embedment ratio, and the enhancement of the roughness effect due to the soil dilatancy.

9.5 APPLICATION OF THE BEARING CAPACITY FACTORS FOR VERTICAL ANCHORS

In this chapter, the study of the complex behaviour of vertical anchor plates has been facilitated by isolating the effects upon

anchor behaviour of each of the more significant parameters. Some attempt has been made to quantify the effects of these parameters upon the failure (or collapse) load of the anchor by expressing the bearing capacity in the form

$$q = cF_c + \gamma h F_\gamma$$

where $F_c = \left(\frac{P}{Bc}\right)_{\gamma=0}$ is a dimensionless failure load obtained for the limiting case where the soil density γ is equal to zero; and where $F_\gamma = \left(\frac{P}{B\gamma h}\right)_{c=0}$ is the dimensionless failure load for a cohesionless material ($\phi > 0$).

However, application of this expression involves an implicit assumption that the effect of cohesion and self-weight are independent of each other and hence may be superimposed for cohesive frictional material with weight. Furthermore, the bearing capacity factors F_c and F_γ were determined for a particular selection of the secondary parameters (ie. dilatancy, initial stress state and anchor roughness). The influence of some of these secondary parameters was then incorporated by taking the product of the basic bearing factor F_γ with a series of correction factors for the effect of each of the individual secondary parameters; that is

$$F'_\gamma = F_\gamma \cdot R_\psi \cdot R_R \cdot R_K \quad (9.5)$$

where $F_\gamma = \left(\frac{P}{B\gamma h}\right)_{c=0, \psi=0^\circ, K_o=1, \text{ smooth}}$

$$R_\psi = \frac{\left(\frac{P}{B\gamma h}\right)_{c=0, K_o=1, \text{ smooth}}}{F_\gamma}$$

$$R_R = \frac{\left(\frac{P}{Byh}\right)}{F_Y} c=0, \psi=0^\circ, K_o=1$$

and $R_K = \frac{\left(\frac{P}{Byh}\right)}{F_Y} c=0, \psi=0^\circ, \text{smooth}$

This simplification introduces a second level at which superposition is implicitly assumed, and it is clearly of some importance to restate the restriction associated with these bearing capacity factors, and to consider their applicability to problems involving realistic combinations of the various parameters.

In the determination of the bearing capacity factor F_c , it has been assumed that the anchor is smooth, the initial stresses were hydrostatic ($K_o = 1$) and that the soil deformed plastically with zero volume change. The effect of roughness was considered and, except for very shallow anchors, was found to have little effect on anchor behaviour. The assumptions regarding initial stress state and volumetric behaviour were considered appropriate for the prediction of the bearing capacity in many cohesive materials, although caution is clearly required when there is significant deviation from the assumed conditions. For example, a non-hydrostatic initial stress state may be expected to lead to much larger plastic deformation prior to collapse thereby altering the practical failure load, and for a non-associated cohesive-frictional material it may cause either an increase or decrease in the actual collapse load. In the limiting case of a purely cohesive material the bearing capacity factor F_Y is undefined and it is necessary to define the factor F_c as a function of the parameter $\gamma h/c$. A series of approximate curves have been presented for the particular case where the initial stress state is hydrostatic.

The bearing capacity factor F_γ was obtained for a smooth anchor in a soil which deformed plastically with zero volume change ($\psi = 0^\circ$) and has a hydrostatic initial stress state ($K_o = 1$). Consequently, the bearing capacity factors F_c and F_γ may be directly combined to predict the failure load of a smooth anchor in a cohesive frictional material where $K_o = 1$ and $\psi = 0^\circ$. To illustrate this, consider an anchor with embedment ratio $h/B = 3$, resting in a cohesive frictional soil where $\phi = 30^\circ$, $\psi = 0^\circ$, $K_o = 1$, $\gamma h/c = 1$. From Fig. 9.15 the bearing capacity factor corresponding to the practical 'k4' failure load for a weightless cohesive material is $F_c = 7.2$. From Fig. 9.19 the bearing capacity factor corresponding to collapse of a purely frictional material is $F_\gamma = 2$, and hence

$$q = P/B = cF_c + \gamma hF_\gamma$$

gives $P/Bc = 7.2 + 2.0 \gamma h/c = 9.2$

This prediction may be compared with the actual 'k4' failure load $P/Bc = 10$ obtained from a direct finite element analysis for this problem, as shown in Fig. 9.26. It will be observed that the predicted failure load underestimates the actual 'k4' failure by 8% and this discrepancy may be largely attributed to error in the assumption of superposition. As suggested in regard to horizontal anchors, it may be anticipated that the superposition errors will be least for shallow anchors where both the weightless cohesive and purely frictional material exhibit clearly defined collapse, and involve relatively little contained plastic deformation.

In considering the effect of dilatancy, roughness, and the initial stress state upon the behaviour of purely frictional material, correction factors R_ψ , R_R , and R_K were obtained for a restricted number of cases. In particular, R_ψ is given in Fig. 9.22 for a smooth anchor with $h/B = 3$;

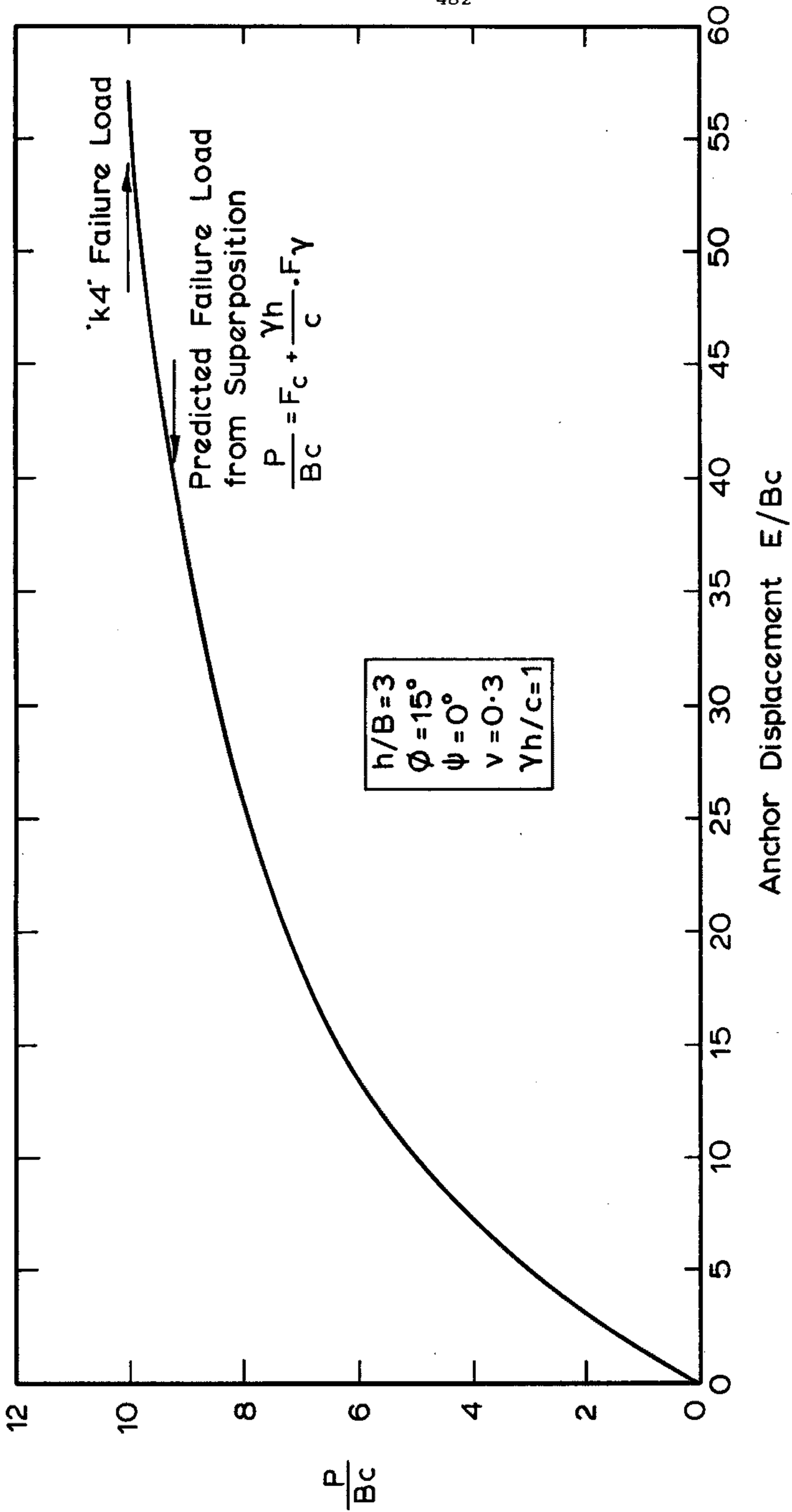


FIG. 9.26 LOAD DISPLACEMENT CURVE FOR COHESIVE FRICTIONAL SOIL WITH WEIGHT

R_R is given in Fig. 9.25 for $\phi = 30^\circ$, $\psi = 0^\circ$; and R_K is given in section 9.4.2 as 1.0 for $h/B = 1$ and 1.07 for $h/B = 3$ where $K_O = 0.4$, $\phi = 30^\circ$, $\psi = 0^\circ$ (by linear extrapolation of these results R_K may, to a first approximation, be taken to be 1.14 for $h/B = 5$). Thus an approximate prediction of the bearing capacity factor F_Y for a given combination of parameters may be obtained from Eq. 9.5 viz. $F'_Y = F_Y \cdot R_\psi \cdot R_R \cdot R_K$ where the basic bearing capacity factor F_Y is given in Fig. 9.19 and where it is assumed that the correction factors R_ψ , R_R and R_K obtained for the special case outlined above are generally applicable.

To illustrate the applicability of bearing capacity factor F'_Y determined from Eq. 9.5, consider a 'typical' case of a rough rigid anchor resting in a granular material with properties $\phi = 35^\circ$, $\psi = 12^\circ$ and $K_O = 0.4$. Solutions were obtained from direct finite elements analyses for three embedment ratios (viz. $h/B = 1, 3, 5$) and the dimensionless 'k4' failure load as well as the dimensionless collapse load for these cases are given in Table 9.2.

TABLE 9.2

EFFECT OF SUPERPOSITION - ESTIMATED AND DIRECTLY
CALCULATED BEARING CAPACITY FACTORS

h/B	Predicted					From Full Analysis	
	F_Y	R_ψ	R_R	R_K	F'_Y	$(\frac{P}{Byh})_{k4}$	$(\frac{P}{Byh})_{collapse}$
1	1.8	1.14	1.68	1.00	3.4	3.2	3.4
3	5.7	1.14	1.15	1.07	8.0	7.8	10.3
5	8.2	1.14	1.06	1.14	11.3	11.7	14.5

Comparison of the dimensionless collapse load with the collapse load predicted from Eq. 9.5 indicates that except for the very shallow anchor (ie. $h/B = 1$) the predicted value of F_γ underestimates the actual value by approximately 22%. This finding is of special significance when $h/B = 3$ since the correction factors adopted were directly applicable to this case, indicating that the discrepancy is largely due to interaction effects between the various parameters.

In this regard, it has been seen in earlier sections that anchor roughness, dilatancy and initial stress state all have quite different effects upon the general behaviour of the anchor. The interaction between these various effects will now be considered.

For a rough anchor in a soil with parameters $\phi = 35^\circ$, $\psi = 12^\circ$, and $K_o = 0.4$, anchor collapse was preceded by extensive contained plastic deformation and the practical 'k4' failure loads given in Table 9.2 are significantly below the actual collapse load (and, coincidentally, are quite close to the predicted collapse load). The occurrence of extensive contained plastic deformation may be attributed to the effect of initial stress and dilatancy, and in fact the plastic region of collapse shown in Fig. 9.27(a) is quite similar to that obtained in Fig. 9.24 for $\phi = 30^\circ$, $\psi = 0^\circ$, $K_o = 0.4$. The combined effects of roughness and dilatancy may be assessed by comparison of the velocity field shown in Fig. 9.27(b) for $h/B = 3$, $\phi = 35^\circ$, $\psi = 12^\circ$ with those shown in Figs. 9.18(b) and 9.23 for $\phi = 30^\circ$, $\psi = 0^\circ$ and $\phi = \psi = 30^\circ$ respectively. It will be observed that although the value of ψ is only approximately 1/3rd the value of ϕ , the effect of dilatancy upon the velocity field is quite comparable with that obtained for the associated material (Fig. 9.23),

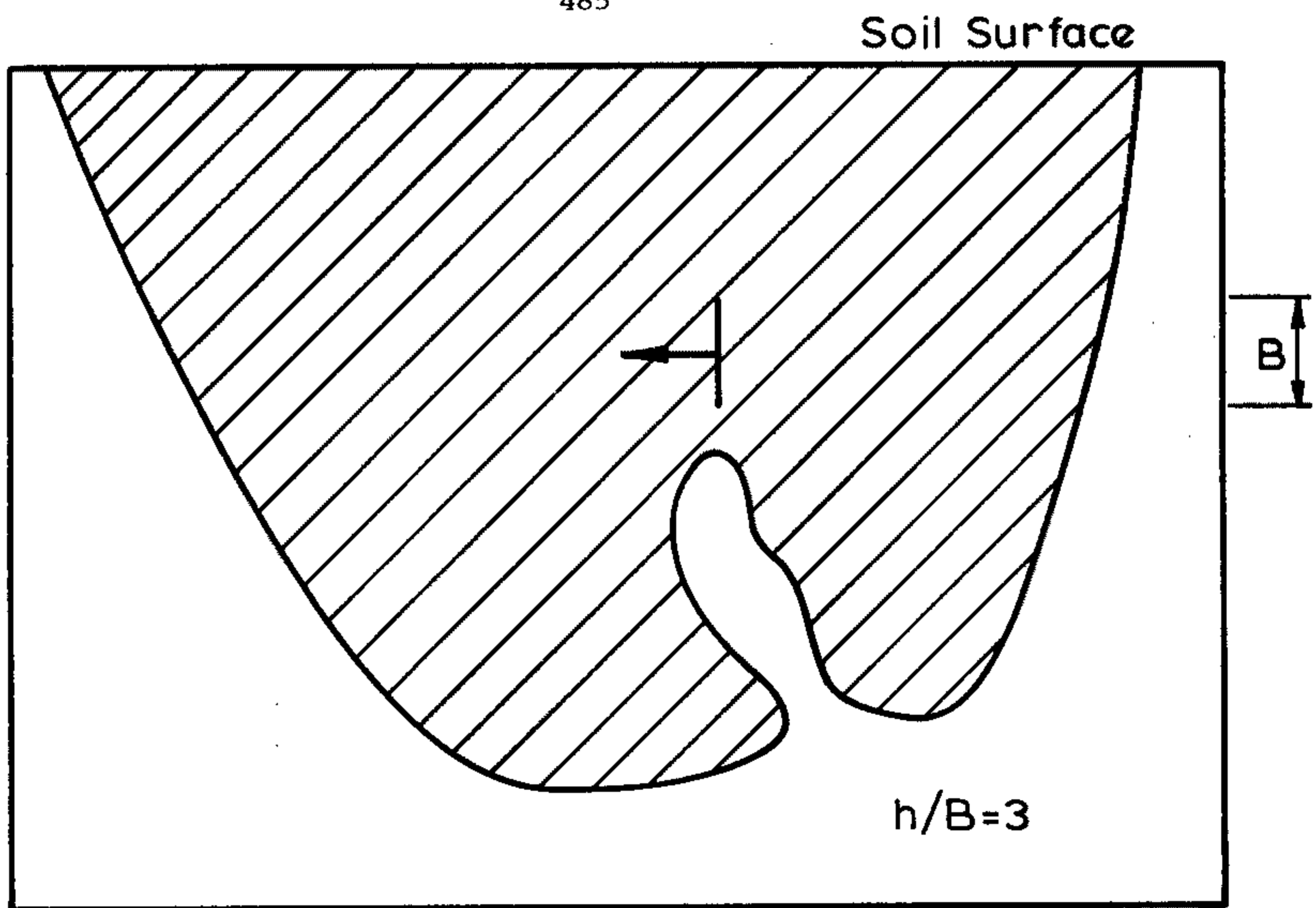


FIG.9.27a PLASTIC REGION AT COLLAPSE $\phi=35^\circ, \psi=12^\circ$
 $K_0=0.4$ (ROUGH)

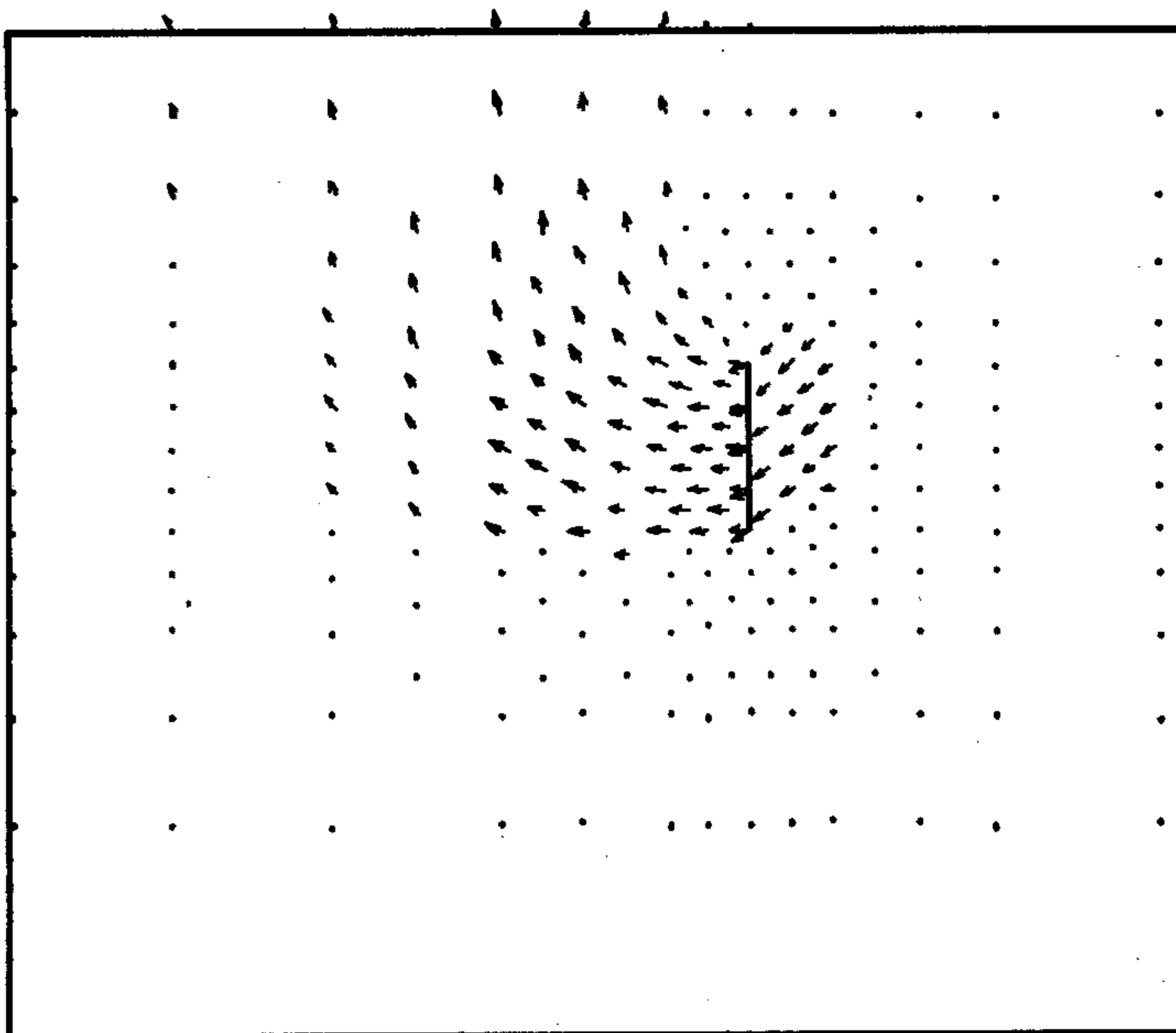


FIG.9.27b VELOCITY FIELD AT COLLAPSE $\phi=35^\circ, \psi=12^\circ$
 $K_0=0.4$ (ROUGH)

while the combination of roughness and dilatancy has placed further restrictions upon the deformation of surrounding soil resulting in a quite rigid lateral movement of a wedge of material directly in front of the anchor.

Thus, it may generally be anticipated that the effects of dilatancy, roughness and (to a lesser extent) initial stress state will enhance each other and consequently the prediction of collapse load based on simple superposition of these basic parameters will be conservative.

A more accurate prediction of the collapse load could be obtained if the correction factors were evaluated for various combinations of parameters. For example, in the case where $h/B = 3$ an upper estimate of the collapse load can also be obtained by replacing the correction factor $R_R = 1.15$ obtained for $\phi = 30^\circ$, $\psi = 0^\circ$ by the value $R_R' = 1.67$ obtained for $\phi = \psi = 30^\circ$, giving a predicted collapse load of $P = 11.6 B\gamma h$, which is 13% above the actual collapse load. Clearly this procedure could be repeated for more appropriate combinations of roughness, friction angle and dilatancy angle had it been feasible to obtain such a comprehensive range of solutions for R_R ; however, for problems where this additional accuracy is warranted it would, in any case, be desirable to perform a full analysis for the particular parameters involved.

9.6 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

While a number of model and field pullout tests on vertical anchors have been reported in the literature, most of these investiga-

tions have been directed primarily at the effect of embedment ratio upon the ultimate pullout capacity for a selected cohesion c , angle of internal friction ϕ and unit weight γ . Consequently, the information regarding the roughness parameters c_k , ϕ_k , initial geostatic stress state (ie. K_0), soil dilatancy ψ , and elastic modulus E is not sufficient to allow detailed comparison of experimental and theoretical load deflection behaviour of particular cases. Rather, in this section, an attempt will be made to compare the general trends observed experimentally with the theoretical predictions which can be made within the limitations of the experimental data.

9.6.1 Vertical Anchor in Cohesive Material

The paucity of information on vertical anchor behaviour is most notable in regard to strip anchors resting in cohesive soils and indeed, the only relevant experimental data of which the author is aware is that obtained by MacKenzie (1955).

MacKenzie performed small scale model tests using a wooden block with 1.6mm thick steel plates on the front and back, and overall dimensions 25mm x 25mm x 250mm. Two series of tests were performed using a soft Houston clay at different moisture contents. In each case the soil was compacted around the anchor by hand. The anchor was loaded incrementally at time intervals of 16 minutes with the entire test being carried out over a time period ranging from two to five hours. It might be expected that the parameters appropriate to tests performed over such a prolonged period would lie somewhere between the drained and undrained values, however, the only information given by MacKenzie concerned the undrained shear strength of the material. In particular, the cohesion

was stated to be 3.9 kPa and 21.4 kPa for the first and second series respectively, where these values represent an average of results obtained from direct shear tests and conventional triaxial tests which typically differed as much as 80% of the mean value (e.g. for Series A: $2.3 \text{ kPa} < c < 5.5 \text{ kPa}$). These experimental results form the basis of the empirical design approach proposed by Tschebotarioff (1973).

The dimensionless practical 'k4' failure loads obtained from MacKenzie's tests are shown in Fig. 9.28. Also shown in Fig. 9.28 are the theoretical practical 'k4' failure loads for the cases where $\phi = 0^\circ$ and $\phi = 15^\circ$, since it is anticipated that the actual value of ϕ would lie between these limits. Unfortunately, the uncertainties in the experimental results are such that it is not possible to form any clear conclusion regarding the applicability of the theoretical predictions; however, two observations can be made from MacKenzie's results. Firstly, in the case of shallow anchors, failure was associated with the development of tension cracks as well as an upward vertical movement of the anchor. The corresponding 'k4' failure loads were consistently lower than predicted from the theoretical analysis, although the effects of tension cracking and the upward movement of the anchor would not be expected to be as significant in field tests where the difference in density between the anchor and the displaced soil would tend to reduce upward movement, and where the greater overburden pressure would inhibit the development of tension cracks. Secondly, failure was generally associated with rupture of the material as the anchor punched into the clay, while there was relatively little plastic flow around the anchor into the void which formed once breakaway had occurred.

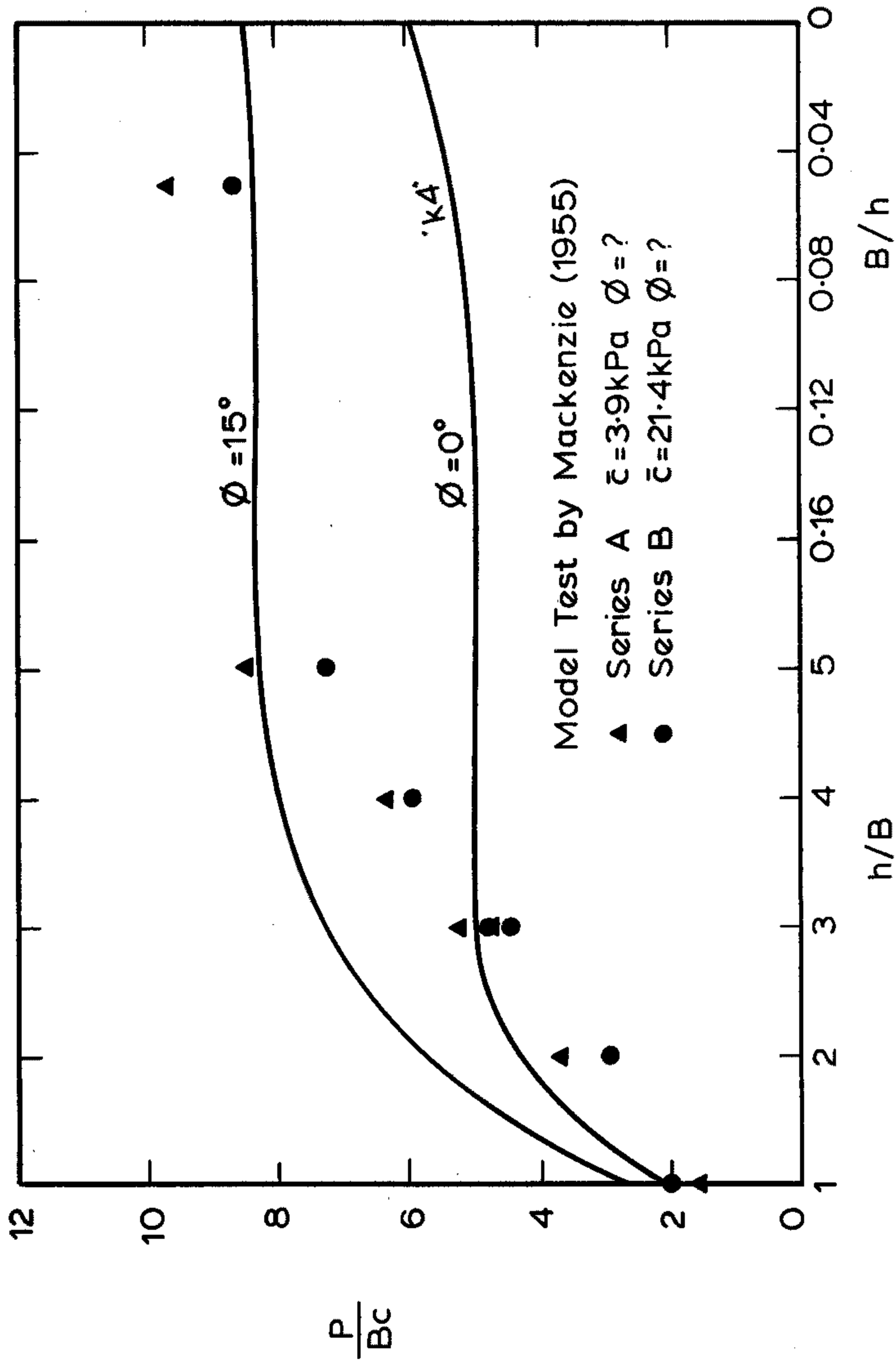


FIG.9.28 COMPARISON OF THEORETICAL AND EXPERIMENTAL "k4" FAILURE LOADS FOR VERTICAL ANCHORS IN CLAY

From a qualitative standpoint, the general behaviour of vertical anchors observed by MacKenzie is in reasonable agreement with observations made from the finite element analysis as typified by the extent of the plastic regions and velocity fields at collapse shown in Figs. 9.8 and 9.9.

9.6.2 Vertical Anchor in Granular Material

A more meaningful comparison between experimental and theoretical results can be obtained for granular material although comparisons are still complicated by uncertainty regarding many material parameters. In particular, it would appear that all the published results (Krebs Ovesen and Strømman, 1972; Neely et al., 1973; Das and Seely, 1975; Smith, 1962, 1966) were obtained for normally consolidated granular material and hence K_o would be expected to be given, at least approximately, by Jaky's relation (1944) $K_o = (1 - \sin \phi)$ and hence $30^\circ \leq \phi \leq 38.5^\circ$ corresponds to $0.5 \geq K_o \geq 0.38$. Each published test was performed for a 'rough' anchor, however except for Neely et al., (1973) no details are given regarding the angle of contact friction ϕ_k between the anchor and soil, although, on the basis of previous experience (e.g. Meyerhof and Adams, 1967) ϕ_k may be expected to lie in the range $\phi/2 \leq \phi_k \leq 2\phi/3$ as compared with the theoretical value $\tan \phi_k = \cos \psi \cdot \sin \phi / (1 - \sin \psi \sin \phi)$. Finally, no mention is made in any paper of the angle of dilatancy ψ (ie. the volumetric behaviour of the material is not considered).

For comparative purposes, in the theoretical analyses, K_o was taken to be 0.4 for all values of ϕ ; a perfectly rough anchor was assumed to have the theoretically derived constant friction angle ϕ_k

given by $\tan \phi_k = \cos \psi \sin \phi / (1 - \sin \phi \sin \psi)$; and the angle of dilatancy was taken to be zero and 12° for loose and dense materials respectively.

Considering firstly the behaviour of anchors in loose sand ($\phi = 30^\circ$), the ultimate pullout capacity obtained by Krebs Ovesen and Strøman (1972), for a 'rough' anchor are shown in dimensionless form in Fig. 9.29 and may be compared with the theoretical predictions obtained for the basic case ($K_o = 1$, smooth), the predictions allowing for initial stress state ($K_o = 0.4$, smooth) and the predictions for a perfectly rough anchor ($K_o = 0.4$, rough). It will be observed that the experimental results are bracketed by the two limiting cases of $K_o = 1$, smooth and $K_o = 0.4$, rough ($\phi_k = 26.5^\circ$). The actual parameters relevant to the experimental results might be expected to lie between these two limiting cases (for example, $K_o = 0.5$ and $15^\circ \leq \phi_k \leq 20^\circ$ would seem reasonable) and consequently the theoretical limits obtained for this case may be considered to be quite acceptable.

Consideration of the experimental results obtained for dense sands is complicated by the range of the frictional angle for which results have been obtained (ie. $32^\circ \leq \phi \leq 38.5^\circ$) and the additional uncertainty as to the dilatancy behaviour of the material. To provide a reasonable comparison an analysis was performed for the 'typical' case of a rough anchor ($\phi_k = 30^\circ$) resting in a granular material with $\phi = 35^\circ$, $\psi = 12^\circ$ and $K_o = 0.4$ and the results are shown in Fig. 9.30. This selection of parameters may be considered to be slightly unconservative since the actual contact friction angle ϕ_k might be expected to lie in the range $18^\circ \leq \phi_k \leq 23^\circ$, while the dilation angle $\psi = \phi/3$ was assumed to be constant at all levels of strain, whereas in reality ψ would be

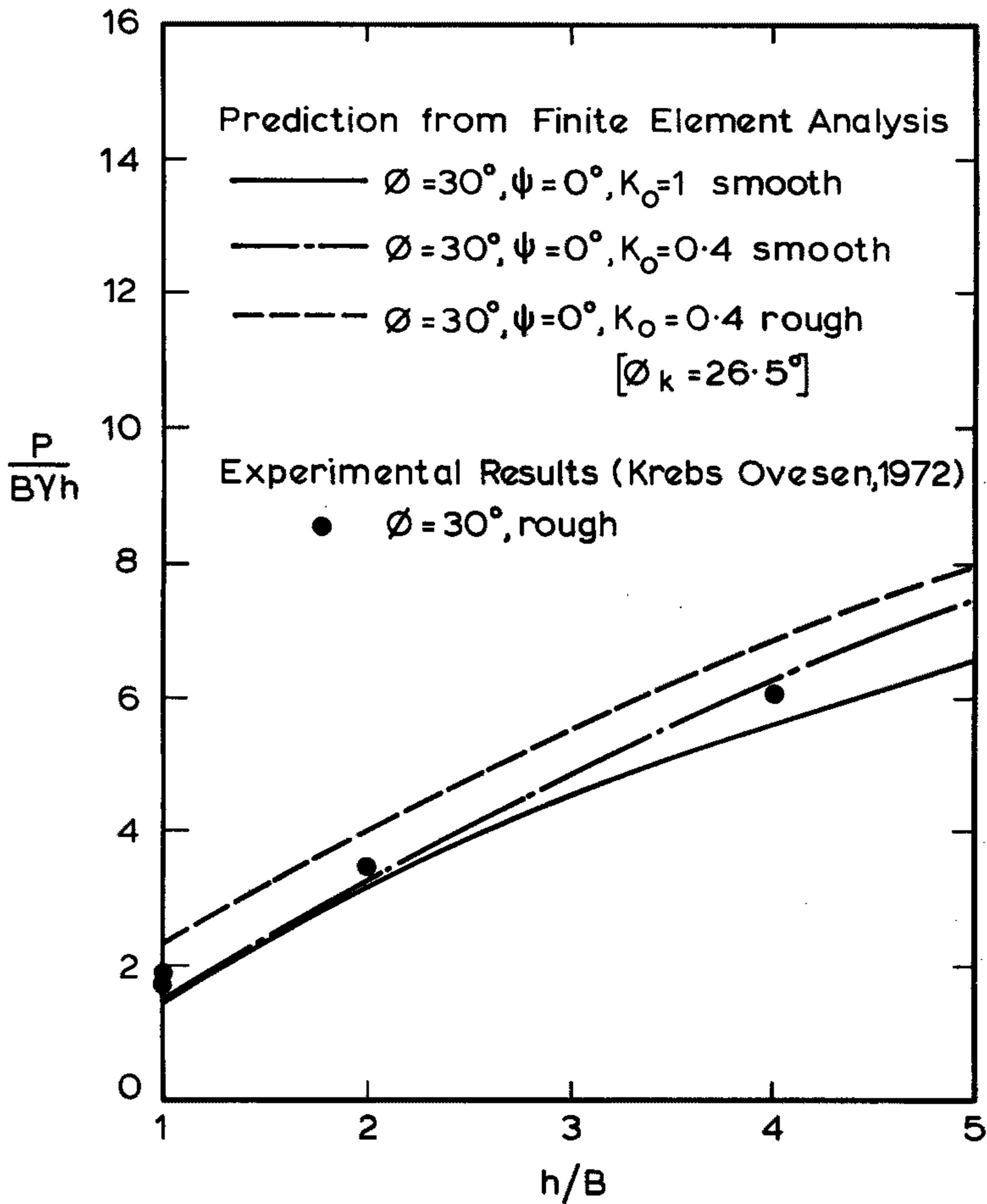


FIG.9. 29 COMPARISON OF EXPERIMENT WITH THEORY

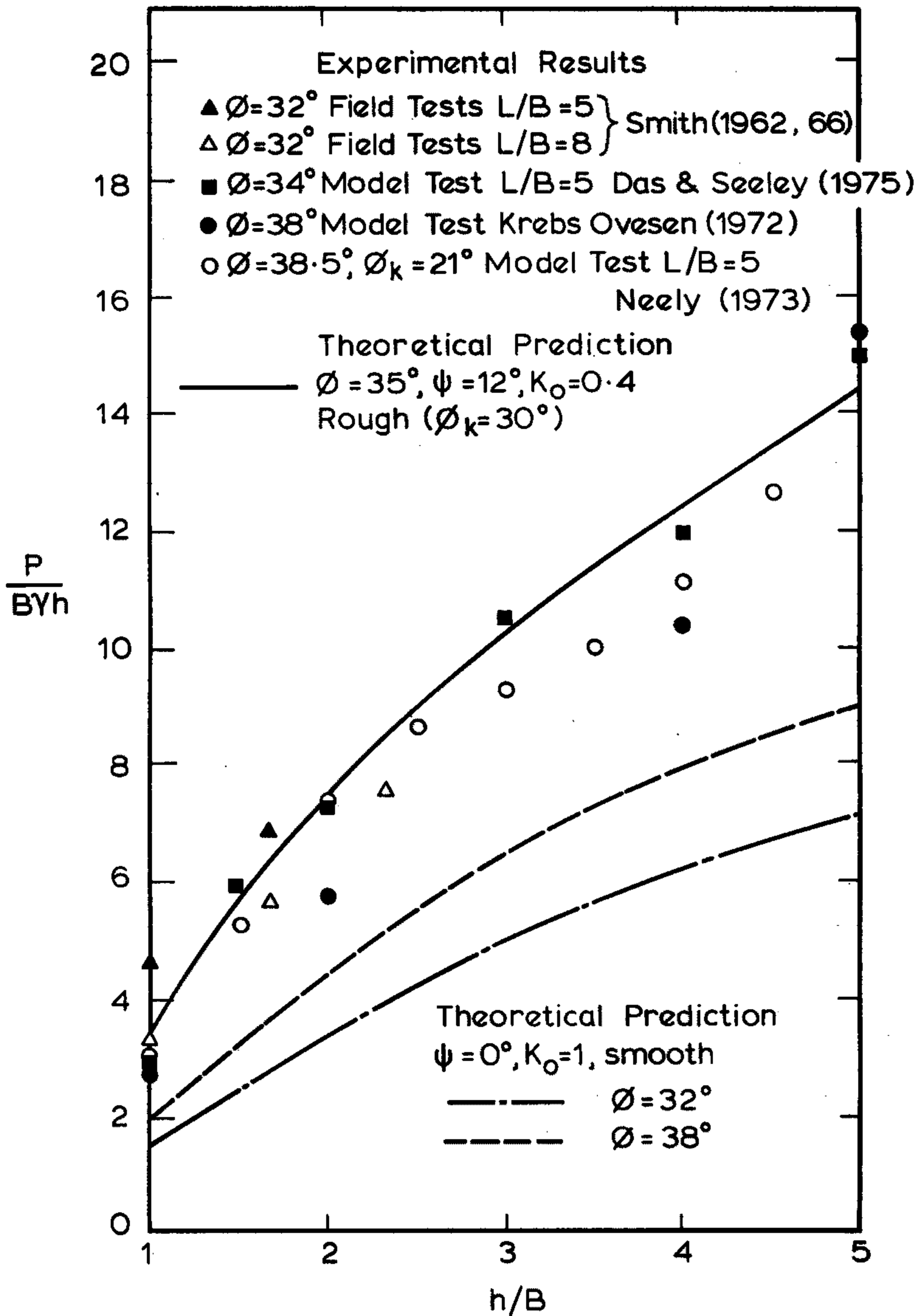


FIG. 9.30 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

expected to decrease to zero as failure approached. Despite these limitations, comparison of the typical theoretical case with the experimental results obtained for dense sand indicates encouraging agreement. It is of particular interest to note the significant effect that the consideration of roughness, initial stress and dilatancy have upon the predicted collapse loads as well as the significant effect of density, and hence dilatancy, upon the experimental collapse loads.

9.7 DISCUSSION AND CONCLUSIONS

In this chapter, consideration has been given to the effects of a wide range of parameters upon the behaviour of vertical anchors.

It has been found that the behaviour of anchors in cohesive materials is dominated by extensive contained plastic deformation and in an engineering sense practical failure will occur at a load level significantly below the ultimate collapse load. This finding accords with a similar observation which has been made for horizontal anchors (see Chapter 7) and it may be concluded that with the obvious exception of very shallow anchors, the behaviour of anchors in a weightless cohesive material is largely independent of the orientation of the anchor.

As in the case of horizontal anchors, the behaviour of vertical anchors in cohesionless material was found to be dominated by the proximity of the soil surface for the range of embedment ratios considered ($1 \leq h/B \leq 8$). In particular, anchors with embedment ratios

$h/B \leq 5$ exhibited clearly defined collapse involving plastic flow towards the soil surface although, for a non-dilatant material, relatively little deformation was actually observed at the surface (in fact, for $h/B \geq 3$ initial failure may not even be apparent from the surface profile). For embedment ratios greater than five (ie. $h/B = 8$) anchor behaviour was characterised by significant contained plastic deformation, necessitating the adoption of a practical 'k4' definition of failure, and although the analysis was not carried to complete collapse for this anchor, the form of both the velocity field and plastic region suggested that eventual failure would be associated with plastic flow to, and from, the soil surface. The extent to which the soil strength must be mobilised prior to collapse, and the much greater significance of plastic distortion (as opposed to rigid body motion of the plastic material) results in much higher dimensionless collapse loads for the general case of a vertical anchor as compared with a similar horizontal anchor. Thus it may be concluded that, except for very shallow anchors in soils with low angles of internal friction ϕ , a vertical anchor tends to be more efficient than a comparable horizontal anchor, and this trend becomes even more apparent when the effect of the secondary parameters (ie. anchor roughness, soil dilatancy and initial stress state) are considered.

As in the case of horizontal anchors, it was found that the dilatancy of a granular material has a very marked effect upon the general behaviour of the anchor. In terms of the accuracy of predicted collapse loads the important role played by soil dilatancy indicates that solutions obtained for a material with an associated flow rule may lead to very unconservative estimates of the collapse load of a realistic material (ie. where $0 \leq \psi \leq \phi$, and typically $0 \leq \psi \leq \phi/3$)

while approaches which neglect dilatancy altogether (ie. assume $\psi = 0^\circ$) may significantly underestimate the collapse load. An additional effect of dilatancy was to increase the deformation of the soil mass at collapse, leading to a far more extensive disturbance of the soil surface than was obtained for the non-dilatant material.

The effect of roughness upon anchor behaviour has been found to depend upon the orientation of the anchor. In fact from Chapters 3 and 7 it is quite clear that anchor roughness has no significant effect on the behaviour of horizontal anchors, however depending upon the embedment ratio and material properties (c , ϕ , ψ) roughness can considerably enhance the load carrying capacity of vertical anchors. The effect of roughness decreases with embedment ratios, and for non-dilatant materials roughness has very little effect for embedment ratios greater than 3 and 5 for weightless and cohesionless material respectively. However, the effects of roughness and dilatancy interact giving much higher collapse loads than would be expected from a simple combination of the two effects, while at the same time increasing the range of embedment ratios over which roughness is of importance.

Consideration of the initial stress state was restricted to two values of K_0 (ie. $K_0 = 1$ and $K_0 = 0.4$) and it was found that a reduction in K_0 resulted in significant contained deformation prior to collapse while increasing the extent of the plastic regions at collapse. The collapse load itself was enhanced slightly by the change in initial stress for buried anchors ($h/B > 1$). For $h/B = 1$ the collapse load was unaltered by the change in initial stress state.

The applicability of the bearing capacity factors presented in this chapter to the prediction of collapse loads for 'typical' combinations of the parameters was investigated by the comparison of these predictions with the collapse loads obtained from direct analysis. From these comparisons, it was seen that the behaviour of vertical anchors is far from straightforward and involves complicated interaction between a wide range of parameters. Consequently, it may be anticipated that an accurate prediction of anchor behaviour for complex combinations of parameters could only be obtained by a direct analysis for the particular case. The bearing capacity factors presented in this chapter can, however, be of some use in hand calculations of the bearing capacity, and on the basis of the results presented in section 9.5 may be expected to generally (although not necessarily) underestimate the actual bearing capacity by less than 25%. Thus, providing the charts are used with due regard to the restrictions under which they were obtained, they may be expected to give an acceptable estimate of the bearing capacities that would be obtained from a full analysis. Finally, comparisons of theoretically predicted collapse loads with available experimental data showed encouraging agreement while indicating the importance of determining the dilatancy and initial stress state of the material as well as the contact roughness of the anchor in making reasonable predictions of anchor behaviour.

CHAPTER TEN

SOIL - STRUCTURE INTERACTION

ANALYSIS - SOME EXAMPLES

10.1 INTRODUCTION

The techniques developed in previous chapters for the analysis of anchor plate behaviour may also be used to give greater insight into the behaviour of a wide range of soil-structure interaction problems. In an attempt to illustrate some of these applications, three particular idealised cases involving interaction between various structural components and the soil will be examined in this chapter. The three problems to be considered are:

- (i) The interaction between a tied-back retaining wall and its anchor;
- (ii) The interaction between anchor plates within a group; and
- (iii) The usefulness of pile groups in modifying the behaviour of soil slopes.

Each of these problems is sufficiently complex to warrant its own intensive investigation, and it will be necessary here to restrict attention to a few of the many significant factors influencing the performance of the respective soil-structure systems.

10.2 ANCHOR-WALL INTERACTION

10.2.1 Background

Studies of tied-back wall systems have been reported by a number of investigators (e.g. Hanna and Matallana, 1970; Hanna and Abu Taleb, 1971; Clough et al., 1972; Egger, 1972; Tsui, 1974; Clough and Tsui, 1974; Murphy et al., 1975; Broms and Stille, 1975; Clough and

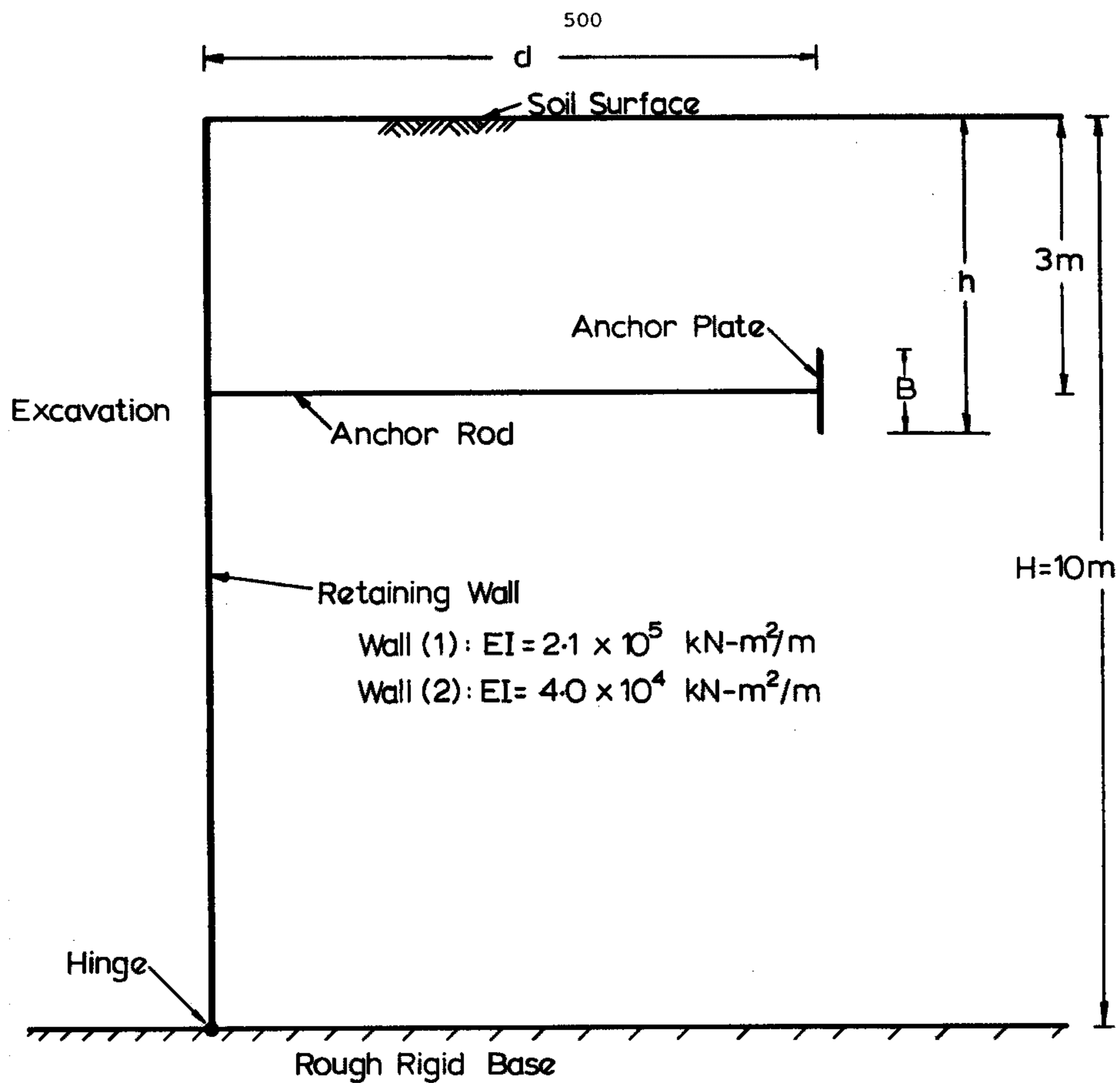
Mana, 1976; Sills et al., 1977). In some of these cases, the finite element method has been used to obtain a clearer understanding of tied-back wall performance and good agreement has been reported in cases where it was possible to compare predicted and observed behaviour (e.g. Clough et al., 1972; Murphy et al., 1975). In previous analyses the influence of wall stiffness and anchor prestress have been considered and the general behaviour of a tied-back wall has been compared with the behaviour of a braced wall. A non-linear elastic soil model has usually been adopted.

In this section, the interaction between a retaining wall and a single strip anchor will be investigated. In particular, consideration will be given to the effect of the distance between the anchor and wall upon the performance of the anchor-wall system. The effect of wall flexibility and anchor size will also be considered briefly.

10.2.2 Problem Description

A hypothetical design involving a 10m excavation down to a firm base stratum was considered (see Fig. 10.1). It was assumed that the retaining wall would be socketed into the base stratum thereby preventing lateral or vertical movement, although the wall was free to rotate about this point. Thus the restraint against overturning of the wall was entirely provided by the anchor. The anchor was not prestressed in this analysis. Two cases were considered involving different wall stiffness $(EI)_B$; namely,

$$\text{Wall (1): } (EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m} \quad \text{and}$$



Soil Properties

$$E = 9000 \text{ kPa}$$

$$\nu_u = 0.48$$

$$\phi_u = 0$$

$$\gamma = 18 \text{ kN/m}^3$$

$$K_o = 1$$

FIG.10-1 BASIC GEOMETRY AND PROPERTIES.

$$\text{Wall (2): } (EI)_B = 4.0 \times 10^4 \text{ kN-m}^2/\text{m}$$

These stiffnesses differ by a factor of 5. Actual walls with similar stiffness values would be a 0.5m thick concrete slurry wall and a 0.3m thick concrete slurry wall respectively.

The anchor plate was located at a distance 'd' (metres) from the wall and was connected to the wall 3m below the soil surface by a flexible but inextensible anchor rod.

The anchor plate itself had a stiffness $(EI)_a = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$, a width (ie. height) B and was buried to a depth h below the surface as shown in Fig. 10.1.

The soil was assumed to be homogeneous isotropic clay acting under undrained conditions. An elasto-plastic soil model was adopted and the relevant soil parameters were $E = 9000 \text{ kPa}$, $\nu = 0.48$, $\phi = 0^\circ$, $\gamma = 18 \text{ kN/m}^3$ and $K_o = 1$. For the purposes of comparing the performance of different anchor arrangements at 'working load', it was assumed that the soil had an undrained cohesion $c = 30 \text{ kPa}$. However, to determine the factor of safety, the undrained cohesion was reduced until collapse of the wall occurred for each anchor arrangement.

10.2.3 Finite Element Analysis

The finite element idealisation used in this analysis is shown in Fig. 10.2 for the case where the distance from the wall to the anchor 'd' was 13.5m. A similar arrangement was adopted for other values of d,

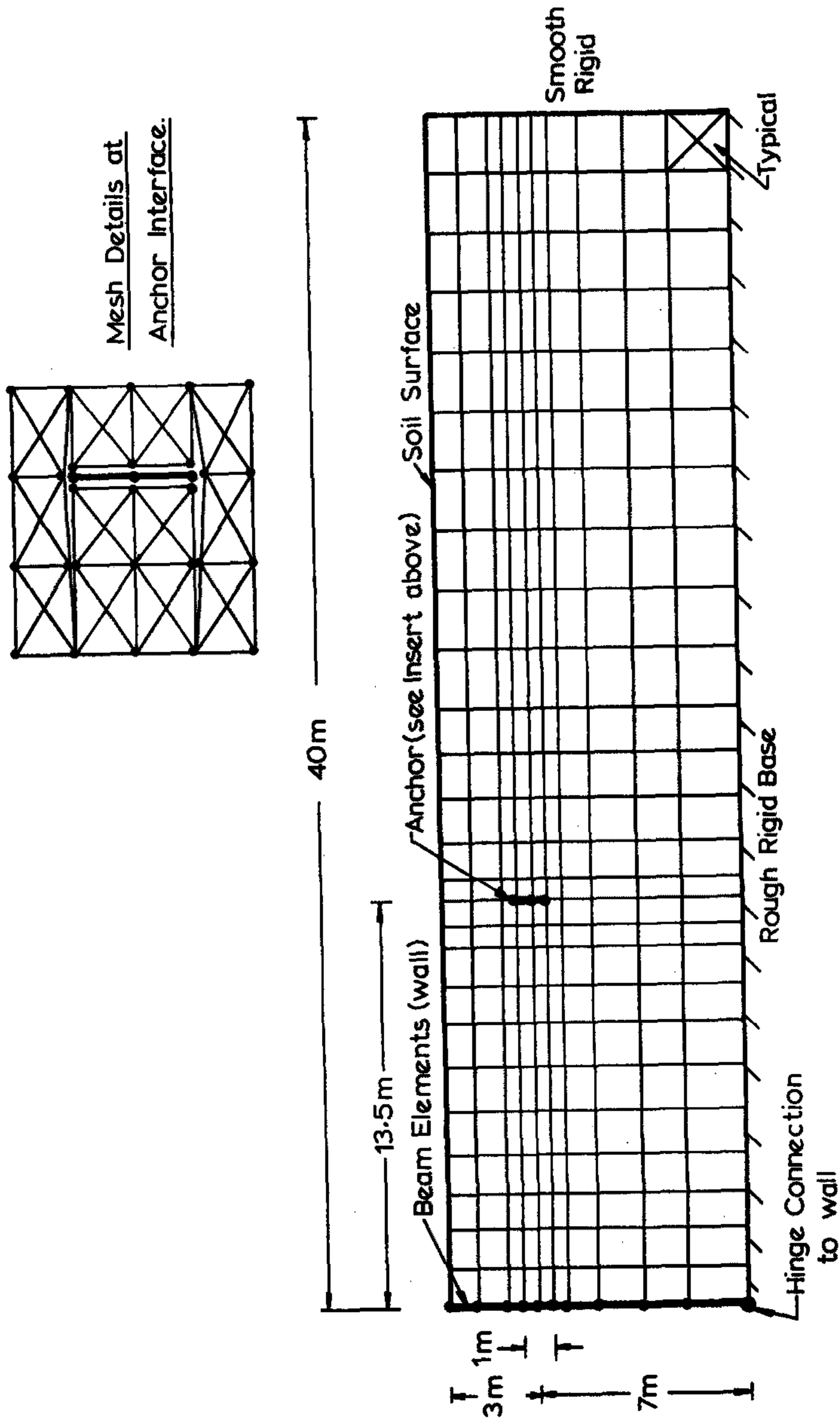


FIG.10-2 TYPICAL FINITE ELEMENT MESH ADOPTED FOR THE ANALYSIS OF
RETAINING WALL - ANCHOR INTERACTION.

although the bay dimensions differed slightly. The wall and anchor were modelled using beam elements while the soil was modelled using constant strain triangles. To reduce the number of parameters which needed consideration, both the wall and the anchor were assumed to have a smooth interface. Provision was made for:

- (1) independent movement of the soil at the front and back of the anchor (see insert to Fig. 10.2);
- (2) slip between the top and bottom of the anchor and soil (ie. potential rupture surfaces were provided near the anchor tips); and
- (3) breakaway of the soil from behind either the anchor or the retaining wall once the horizontal pressure has been reduced to zero.

The physical construction sequence was assumed to involve construction of the retaining wall and anchor; connection of the wall to the anchor; and then excavation of the clay in front of the wall. This construction procedure was simulated by developing the initial K_0 stress state within the soil and then incrementally removing the lateral pressure acting on the front of the wall. This approach gives a more realistic representation of the construction sequence than an incremental density approach; however, the approach still suffers from the limitation that the effect of the stiffness of the soil in front of the anchor prior to its excavation is not directly included in the analysis. Rather, the 'soil pressure' acting on the front of the wall is assumed to be a linear function of the depth of excavation. This restriction is not considered to be serious within the context of this investigation (since we are primarily concerned with the relative per-

formance of different anchor arrangements); although it is recognised that in actual design analyses it may be desirable to adopt a more sophisticated construction sequence (e.g. see Clough, 1972; Desai, 1975).

10.2.4 Behaviour of Anchor - Wall Systems at 'Working Loads'

The performance of a tied-back wall under 'working load' conditions was considered for various anchor arrangements. The 'working load' condition was assumed to correspond to the case of full (10m) excavation where the soil has an undrained cohesion $c = 30 \text{ kPa}$ and unit weight $\gamma = 18 \text{ kN/m}^3$.

Firstly, consider the behaviour of the more rigid of the two walls (ie. where the flexural stiffness $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$). The bending moment diagram and deformation of the wall are shown in Fig. 10.3(a) for the limiting case where the anchor is rigidly fixed at some location remote from the wall. The corresponding horizontal pressure distributions are shown in Fig. 10.3(b). The bending diagram obtained from the finite element analysis (which involved no plastic failure) may be compared with the bending moment diagram obtained from statics for the limiting pressure distribution given by:

- (i) Full lateral earth pressure at rest acting on the wall; and
- (ii) Rankine active pressures acting on the wall (with allowance for a tension crack extending to the depth $2c/\gamma$).

It will be observed that by comparison with the maximum bending moment obtained from the finite element analysis, case (i) overestimates the

Case	Tie Rod Force kN per m
(i). K_0 Pressure Distribution	429
(ii). Rankine Active Pressure Distribution	126
(iii). Finite Element	436

Wall: $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$

Soil: $c = 30 \text{ kPa}$
 $E = 9000 \text{ kPa}$
 $\gamma = 18 \text{ kN/m}^3$

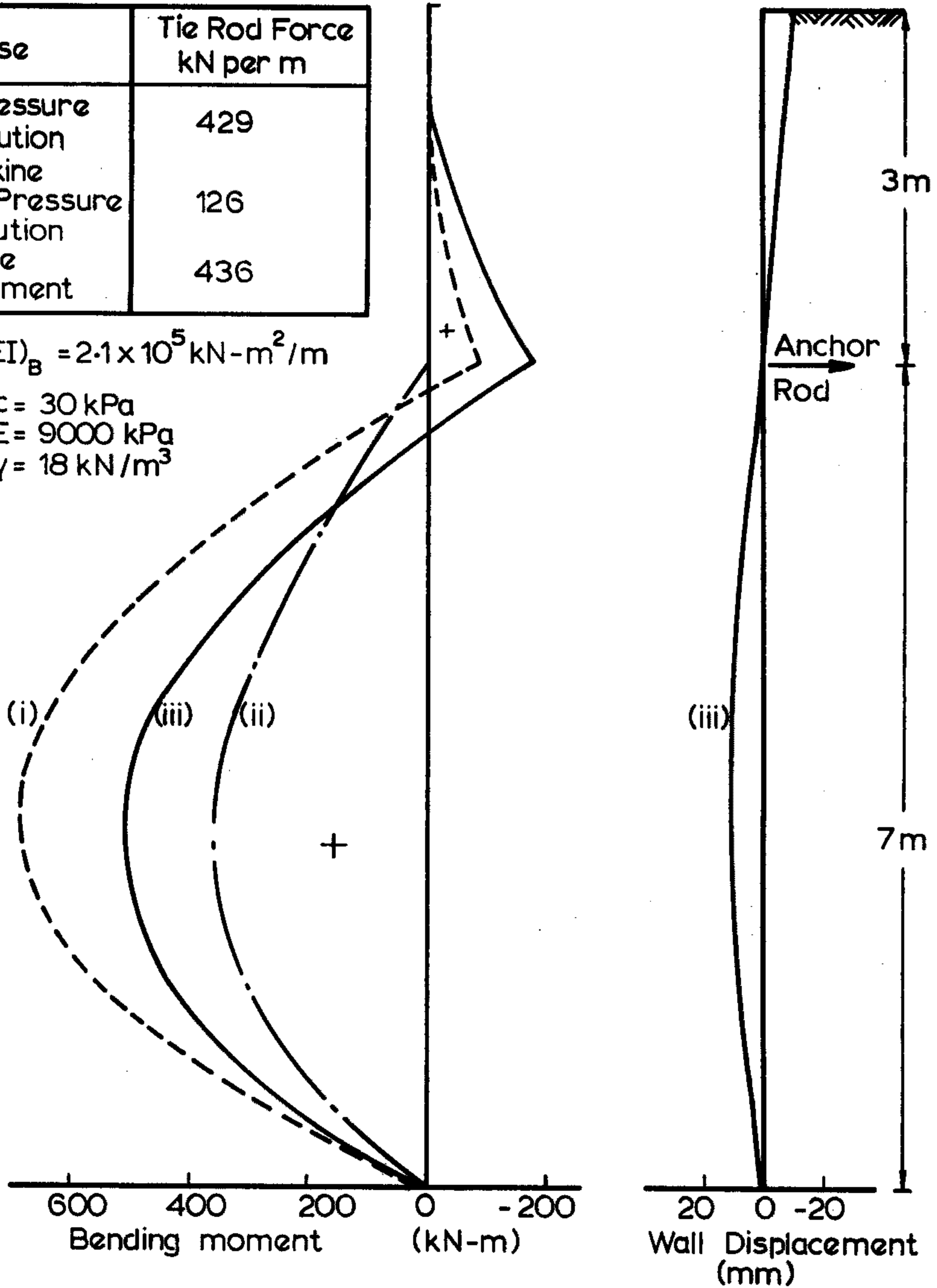


FIG.10.3a BEHAVIOUR OF A WALL WITH A RIGIDLY
FIXED ANCHOR.

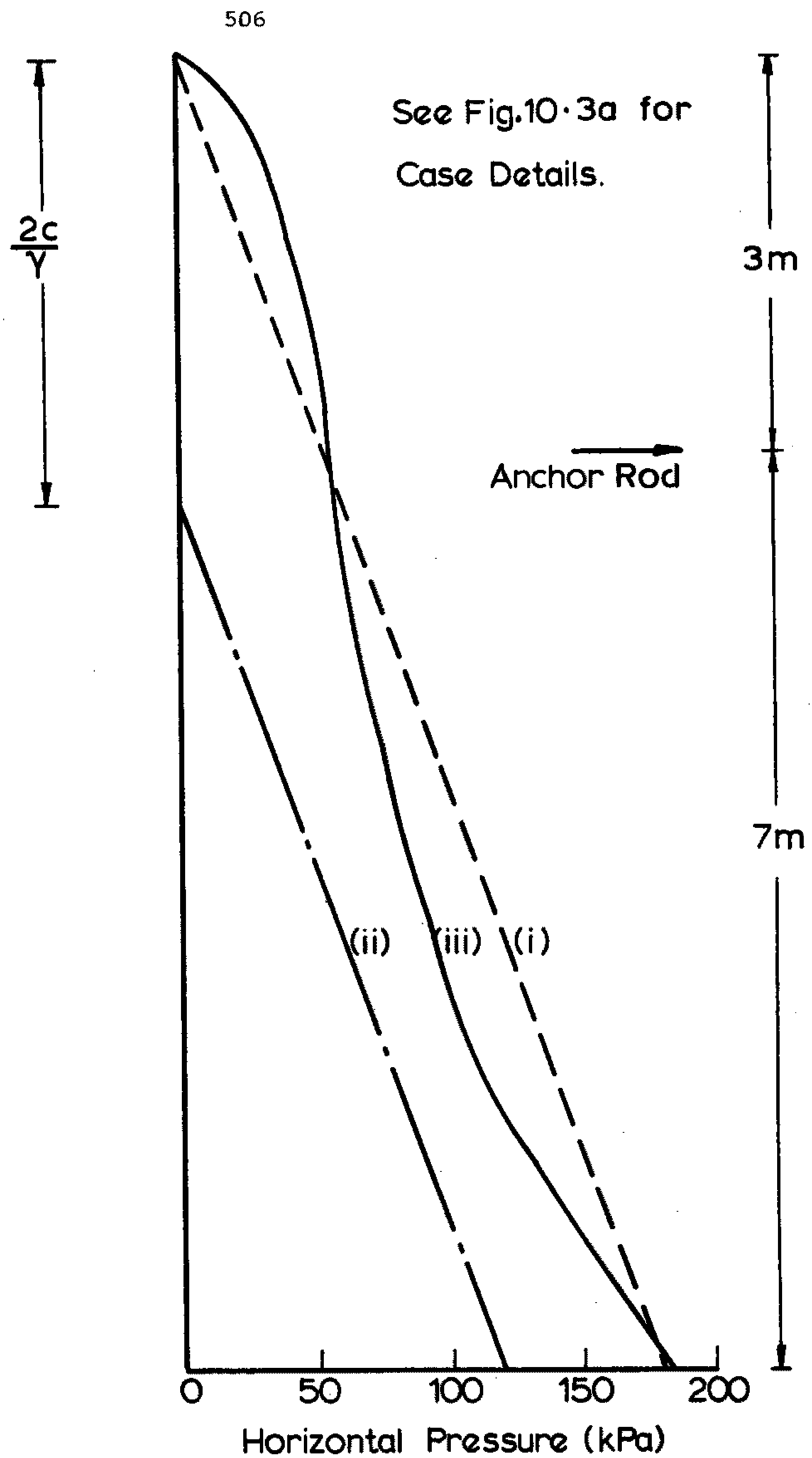


FIG.10.3b PRESSURE DISTRIBUTION BEHIND A RIGID WALL
WITH A RIGIDLY FIXED ANCHOR

bending moment by 38% while case (ii) underestimates the maximum moment by 28%. Furthermore, the deformation of the wall is such that above the anchor, it moves back into the soil (by up to 10mm) thereby increasing the pressure acting on the wall [see Fig. 10.3(b)] in this region as compared to either case (i) or (ii). Correspondingly, the negative bending moment obtained from a finite element analysis is considerably greater than that predicted for either case (i) or case (ii).

For the rigidly fixed anchor, the pressure acting on the wall was everywhere compressive. However, in general tensions may develop between the wall (or anchor) and the soil. If it is assumed that the soil-structure interface cannot sustain tensile tractions then it is necessary to make provision for breakaway of the structure from the soil once the lateral pressure ceases to be compressive (ie. when it reaches zero). The effect of breakaway upon the deformed shape and bending moment diagram is illustrated in Figs. 10.4(a) and 10.4(b) for the case where the wall (1) [with $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$] is separated from its supporting anchor (with $h = 3.5\text{m}$, $B = 1\text{m}$) by a distance $d = 13.5\text{m}$. (The significance of this choice of distance will be discussed shortly). A comparison of the deformation and bending moments for the two cases where breakaway was and was not permitted indicates that if the soil-structure interface can sustain tensile tractions then this will reduce the deformation of the wall (compared to the no tension case) by up to 20% and will increase the maximum bending moment by 5%. From Fig. 10.4(a), it will be seen that when breakaway is permitted, a tension crack develops behind the wall to a depth of 3.5m which is in reasonable agreement with the depth predicted from Rankine's theory (viz. $2c/\gamma = 3.3\text{m}$).

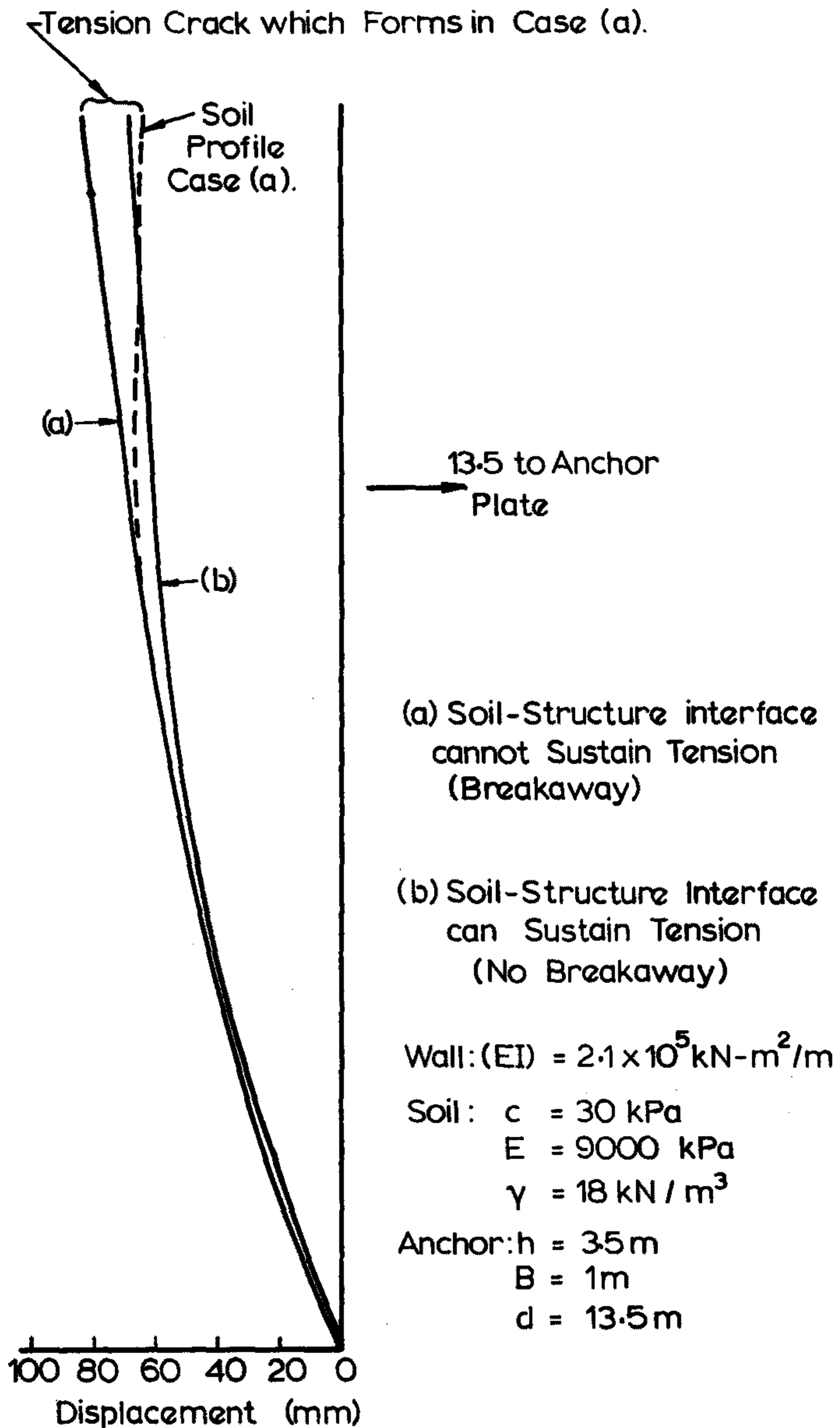


FIG.10.4a EFFECT OF BREAKAWAY BEHIND THE WALL
UPON WALL DISPLACEMENT PROFILE.

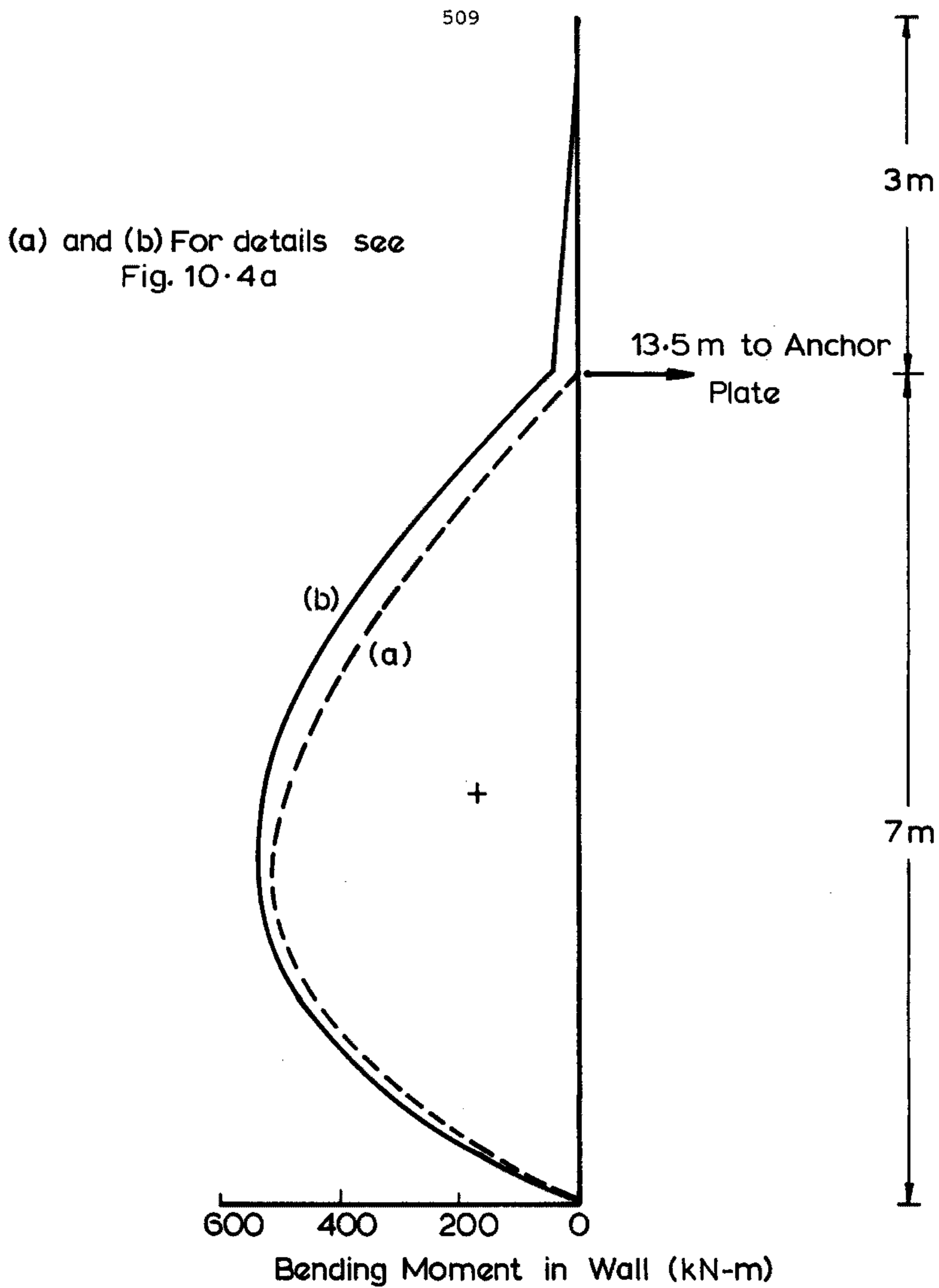


FIG.10·4b EFFECT OF TENSION BETWEEN SOIL AND WALL UPON BENDING MOMENT.

The situation in which the soil-structure interface cannot sustain tension is considered to be the most practical case and will be assumed in all subsequent analyses.

Fig. 10.5 shows the displacement profile at 'working loads' (ie. $c = 30 \text{ kPa}$, $\gamma = 18 \text{ kN/m}^3$, full excavation) of a wall with $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$ supported by an anchor with $h/B = 3.5$ and $B = 1\text{m}$ for the three cases:

- (1) Where the anchor is just contained within an assumed Rankine 'active' zone behind the wall (ie. $d = 6.5\text{m}$);
- (2) Where the anchor is outside the Rankine 'active' zone but where the Rankine 'active' zone behind the wall intersects an assumed Rankine 'passive' zone in front of the anchor (ie. $d = 10\text{m}$); and
- (3) Where the anchor is located at the minimum recommended distance (British Civil Eng. Code of Practice No. 2, 1951; Terzaghi; 1943) from the wall corresponding to the closest position such that the Rankine 'active' zone behind the wall and the Rankine 'passive' zone in front of the anchor (and extending to the soil surface) do not intersect. Hence $d_{\min} = H + h$ for $\phi = 0^\circ$, and in this case $d = 13.5\text{m}$.

The plastic regions at working load corresponding to these three cases are shown in Fig. 10.6, while the bending moment diagram for $d = 13.5\text{m}$ and $d = 6.5\text{m}$ are given in Fig. 10.7 and the results are summarised in Table 10.1. It was found that the bending moments and anchor rod forces were largely

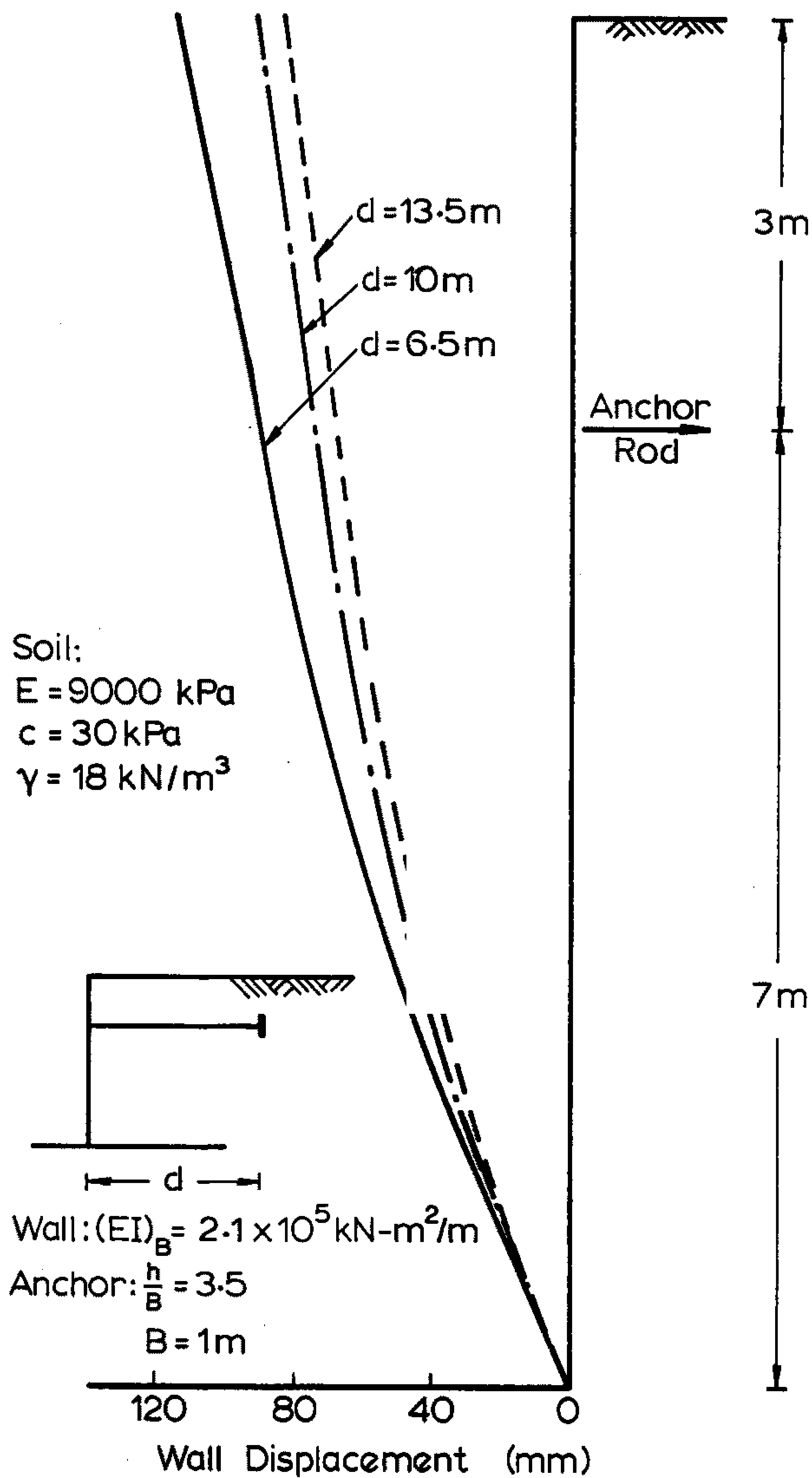


FIG.10.5 EFFECT OF DISTANCE FROM WALL TO ANCHOR UPON WALL DISPLACEMENT AT WORKING LOAD.

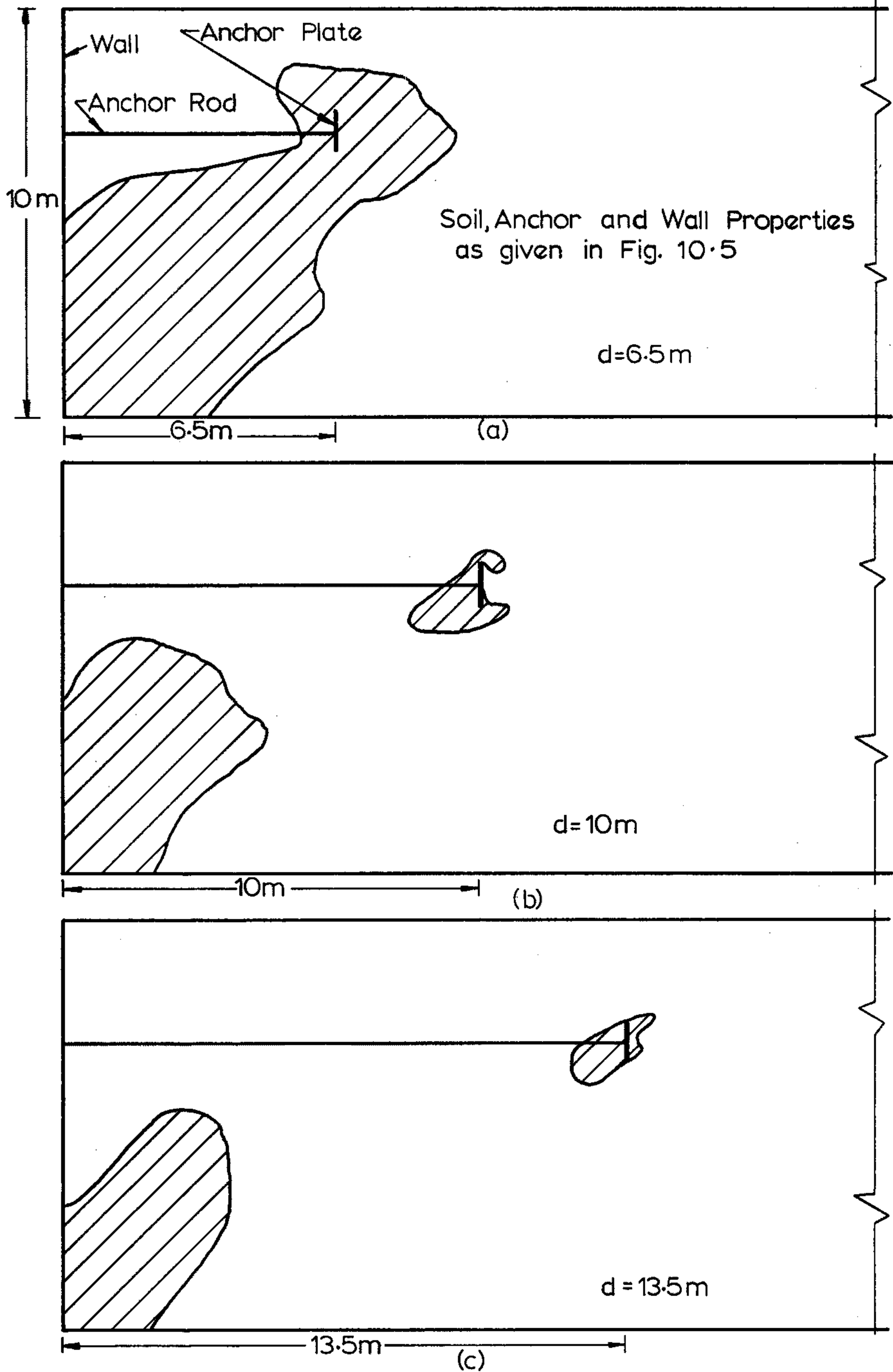


FIG.10.6 PLASTIC REGIONS AT WORKING LOAD

Wall Properties

(1) $EI = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$

(2) $EI = 4.0 \times 10^4 \text{ kN-m}^2/\text{m}$

Anchor Properties

$h = 3.5\text{m}$

$B = 1\text{m}$

Soil Properties

$E = 9000 \text{ kPa}$

$c = 30 \text{ kPa}$

$\gamma = 18 \text{ kN/m}^3$

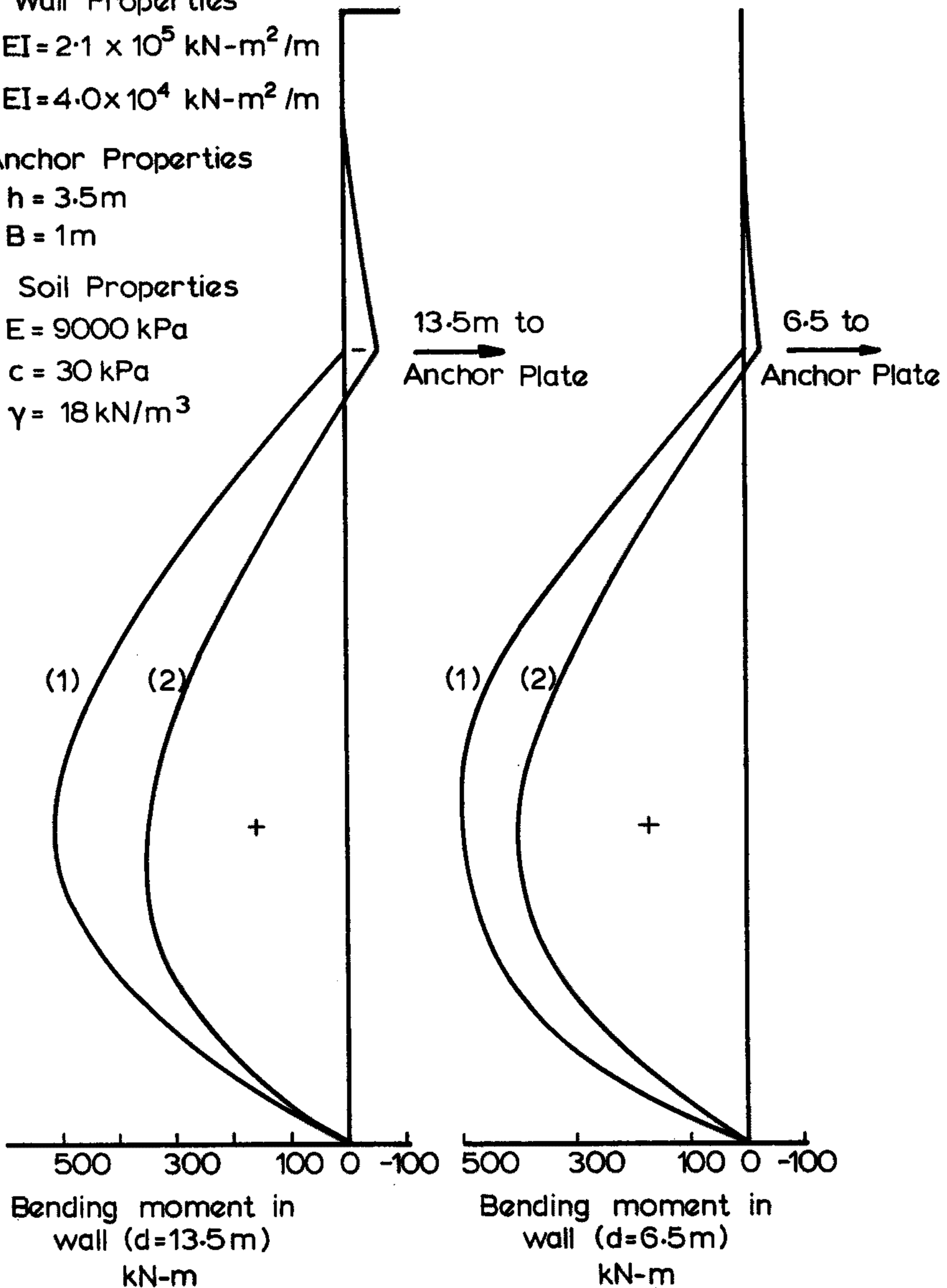


FIG.10.7 EFFECT OF WALL FLEXIBILITY AND ANCHOR POSITION UPON BENDING MOMENT.

TABLE 10.1

COMPARISON OF THE PERFORMANCE OF DIFFERENT ANCHOR-WALL SYSTEMS

h (m)	B (m)	d (m)	(EI) _B kN-m ² /m	Max Moment kN-m/m		Axial Force kN/m	Lateral Displacement of the Wall (1) (mm)			Factor of Safety
				-ve	+ve		4m	7m	10m (2)	
-	-	∞	2.1 x 10 ⁵	175	505	436	10	0	-10	(3)
3.5	1	13.5	2.1 x 10 ⁵	0	510	171	49	68	84	1.36
3.5	1	10.0	2.1 x 10 ⁵	0	505	168	54	75	92	1.25
3.5	1	6.5	2.1 x 10 ⁵	0	500	168	63	90	115	1.11
3.5	1	13.5	4.0 x 10 ⁴	55	350	166	76	73	67	1.36
3.5	1	6.5	4.0 x 10 ⁴	25	405	166	100	103	95	1.03
4.0	2.0	14.0	2.1 x 10 ⁵	15	530	245	42	55	62	1.45

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Notes: (1) positive outward displacement

(2) level above bottom of wall at which displacement was determined

(3) rigidly fixed anchor.

unaffected by the distance to the anchor. In fact, the maximum bending moment was (in each case) almost identical to that obtained for a completely rigid anchor (see Fig. 10.3) although the actual distribution of bending moment was quite different in this case. In each case the anchor rod force (ie. 168 kN/m for $d = 6.5\text{m}$; 168 kN/m for $d = 10\text{m}$; 171 kN/m for $d = 13.5\text{m}$) was considerably lower than that obtained from the finite element analysis of a rigidly fixed anchor. It should be noted that the maximum bending moment and the anchor rod force were 39% and 36% respectively above the values predicted from a conventional hand analysis assuming Rankine active pressure and a rigidly fixed anchor.

Although the distance between anchor and wall did not greatly influence the bending moment and anchor rod force, this distance did noticeably affect the extent of plastic failure as well as the deformation of the wall. In particular, the maximum displacement for $d = 6.5\text{m}$ and $d = 10\text{m}$ were respectively 37% and 9% greater than that for $d = 13.5\text{m}$. Thus the deformation of the wall increases rapidly with decreasing distance 'd' and it will be observed from Fig. 10.6 that the plastic region behind the wall and around the anchor increases significantly with decreasing distance. As might be expected, this suggests that the wall-anchor system becomes increasingly less stable as the distance between wall and anchor decreases. This effect of the distance 'd' upon anchor-wall stability will be discussed in section 10.2.5.

The influence of wall flexibility upon the wall deformations and the bending moment diagram is illustrated in Figs. 10.7 and 10.8 for two anchor positions (ie. $d = 6.5\text{m}$ and $d = 13.5\text{m}$) and for wall stiffnesses given by

Distance to Anchor

—— $d = 6.5\text{m}$

----- $d = 13.5\text{m}$

Wall Stiffness

(1): $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$

(2): $(EI)_B = 4 \times 10^4 \text{ kN-m}^2/\text{m}$

Soil Properties

$E = 9000 \text{ kPa}$

$c = 30 \text{ kPa}$

$\gamma = 18 \text{ kN/m}^3$

Anchor Details

$h = 3.5\text{m}$

$B = 1\text{m}$

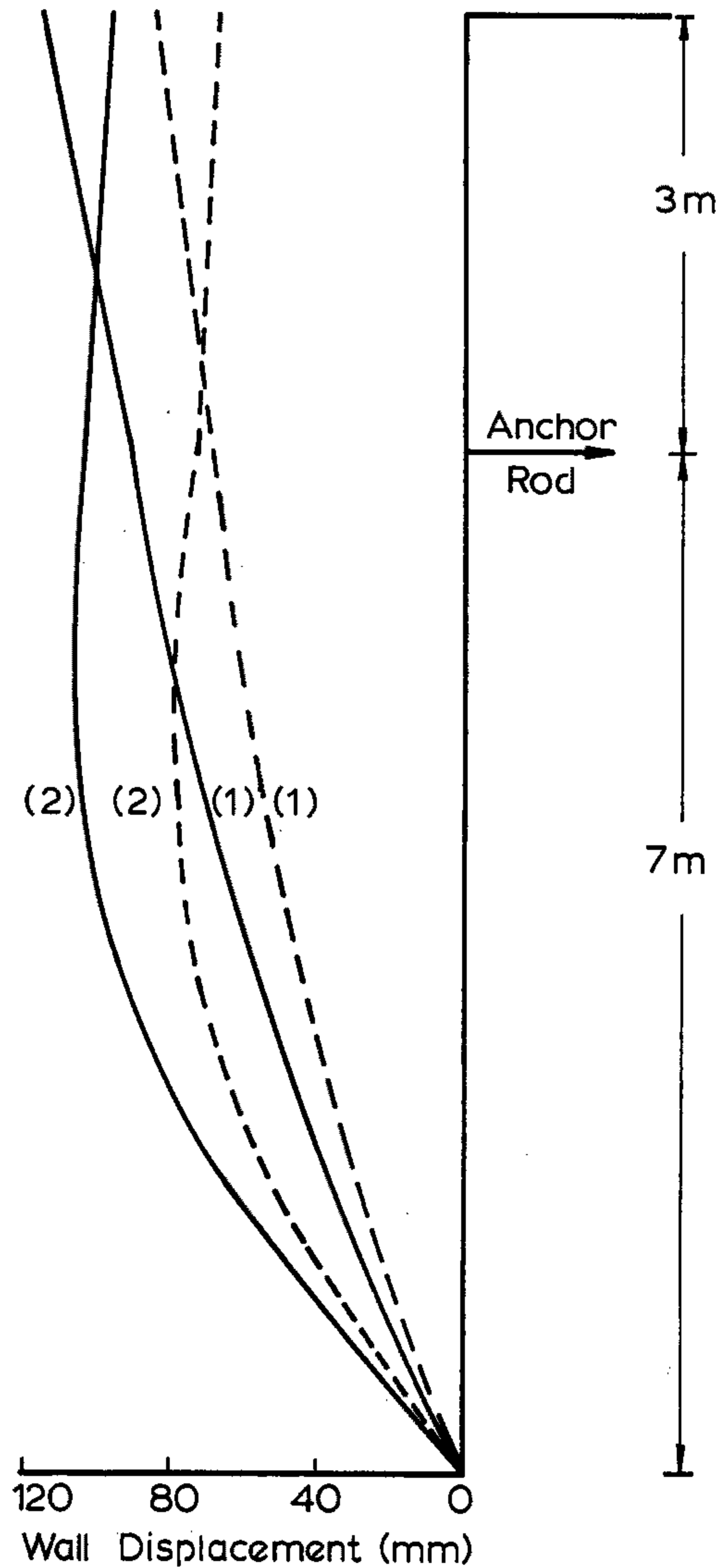


FIG.10·8 EFFECT OF WALL FLEXIBILITY AND ANCHOR POSITION UPON WALL DEFORMATION.

$$\text{Wall (1): } (EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m} \quad \text{and}$$

$$\text{Wall (2): } (EI)_B = 4.0 \times 10^4 \text{ kN-m}^2/\text{m}.$$

It will be observed that the wall flexibility has a significant effect upon both the deformed shape and bending moment diagram for the wall. In particular, it was found that for a given anchor arrangement, an increase in wall stiffness $(EI)_B$ led to:

- (1) An increase in the likelihood of a tension crack developing (and hence, to a decrease in negative bending moment);
- (2) A decrease in wall deformation below anchor rod level;
- (3) An increase in wall displacement above the anchor rod; and
- (4) An increase in positive bending moment.

The relative influence of flexibility upon the performance of the wall was fairly similar for the two cases where $d = 6.5\text{m}$ and $d = 13.5\text{m}$; however, the actual magnitude of the deformations were up to 32% greater for $d = 6.5\text{m}$ than for $d = 13.5\text{m}$. Furthermore, the influence of wall flexibility appeared to decrease with decreasing distance to the anchor plate. For example, a five-fold decrease in stiffness reduced the maximum bending moment by 30% for $d = 13.5\text{m}$ and by 20% for $d = 6.5\text{m}$ (see Fig. 10.7).

It may be anticipated that the performance of the retaining wall-anchor system depends upon an interaction between the relative stiffness of the wall and the response of the anchor to loading (which itself depends primarily upon the parameters h , B and d). To illus-

trate the effect of anchor size upon the performance of the wall, an analysis was also performed for wall (1)

$$(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$$

supported by an anchor with $h = 4\text{m}$ and $B = 2\text{m}$ located at the minimum recommended distance of $d = 14\text{m}$ from the wall. The details regarding the performance of the wall are given in Table 10.1 and are shown in Fig. 10.9.

By comparing the results shown in Fig. 10.9 with those obtained for a rigidly fixed anchor (see Fig. 10.3) it will be appreciated that increasing anchor fixity leads to:

- (1) A decrease in wall deformation. For example, doubling the size of the anchor plate from $B = 1\text{m}$ to $B = 2\text{m}$ (which, for a constant anchor rod level at 3m , reduced the anchor embedment ratio from 3.5 to 2) reduced the maximum displacement by 26%.
- (2) Reduces the likelihood of breakaway while increasing the negative bending moment.
- (3) Increases the force applied to the anchor. For example doubling the anchor width (e.g. $B = 1\text{m}$ to $B = 2\text{m}$) increased the anchor force by 46% while the anchor force for the rigidly fixed anchor was 255% above that for $B = 1\text{m}$; and
- (4) Alters the shape of the bending moment diagram but does not significantly alter the maximum bending moment.

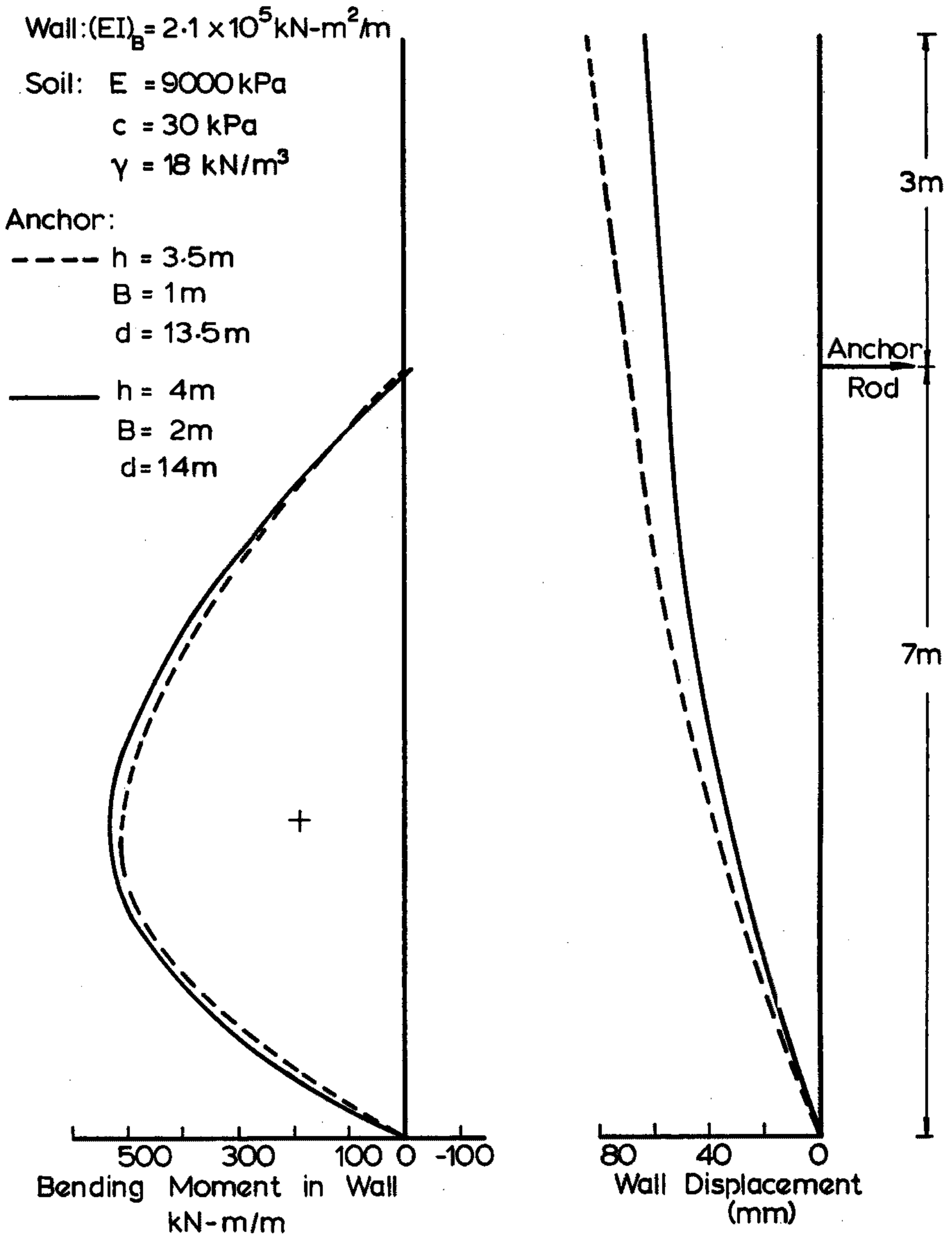
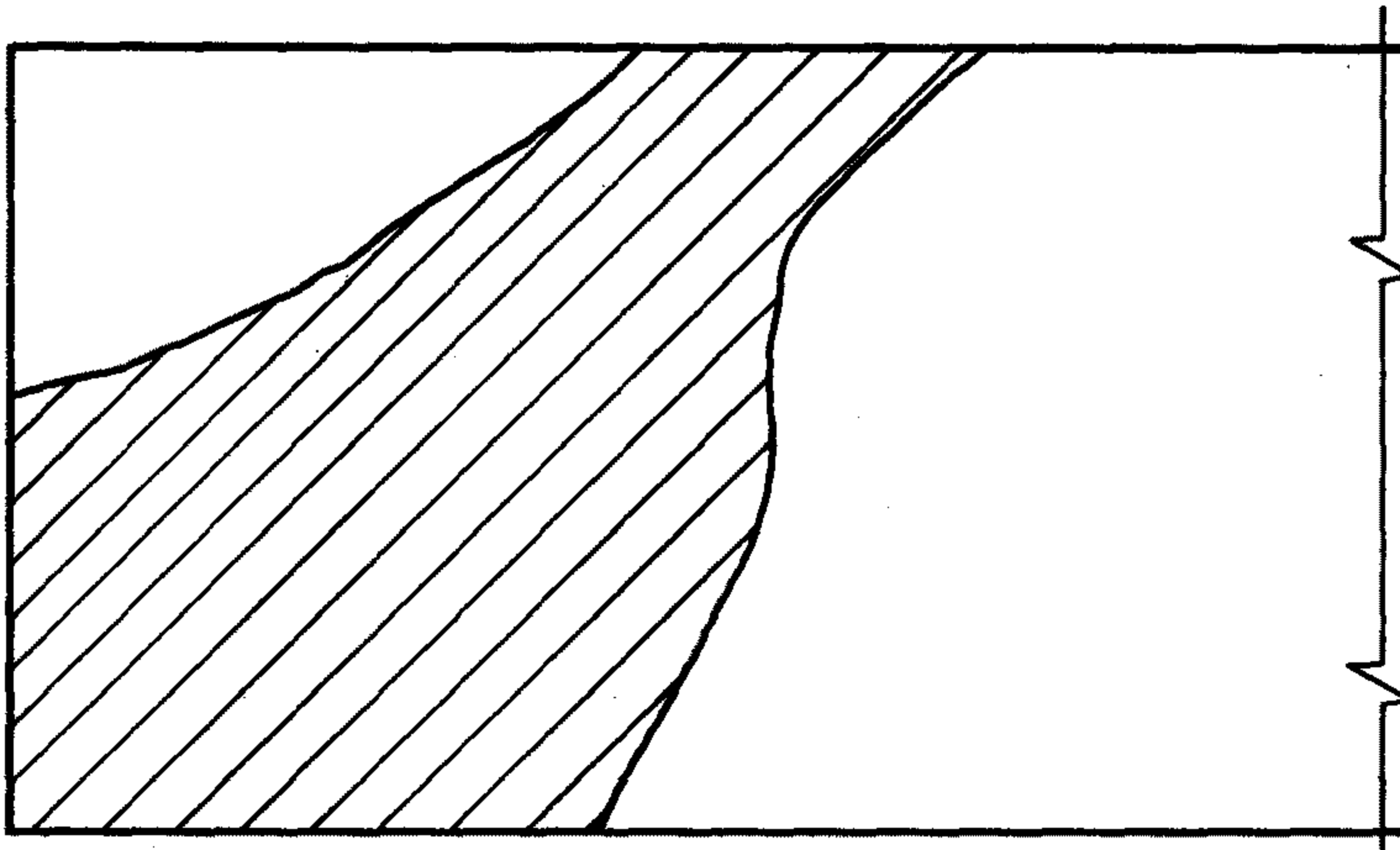


FIG.10·9 EFFECT OF ANCHOR SIZE UPON WALL PERFORMANCE.

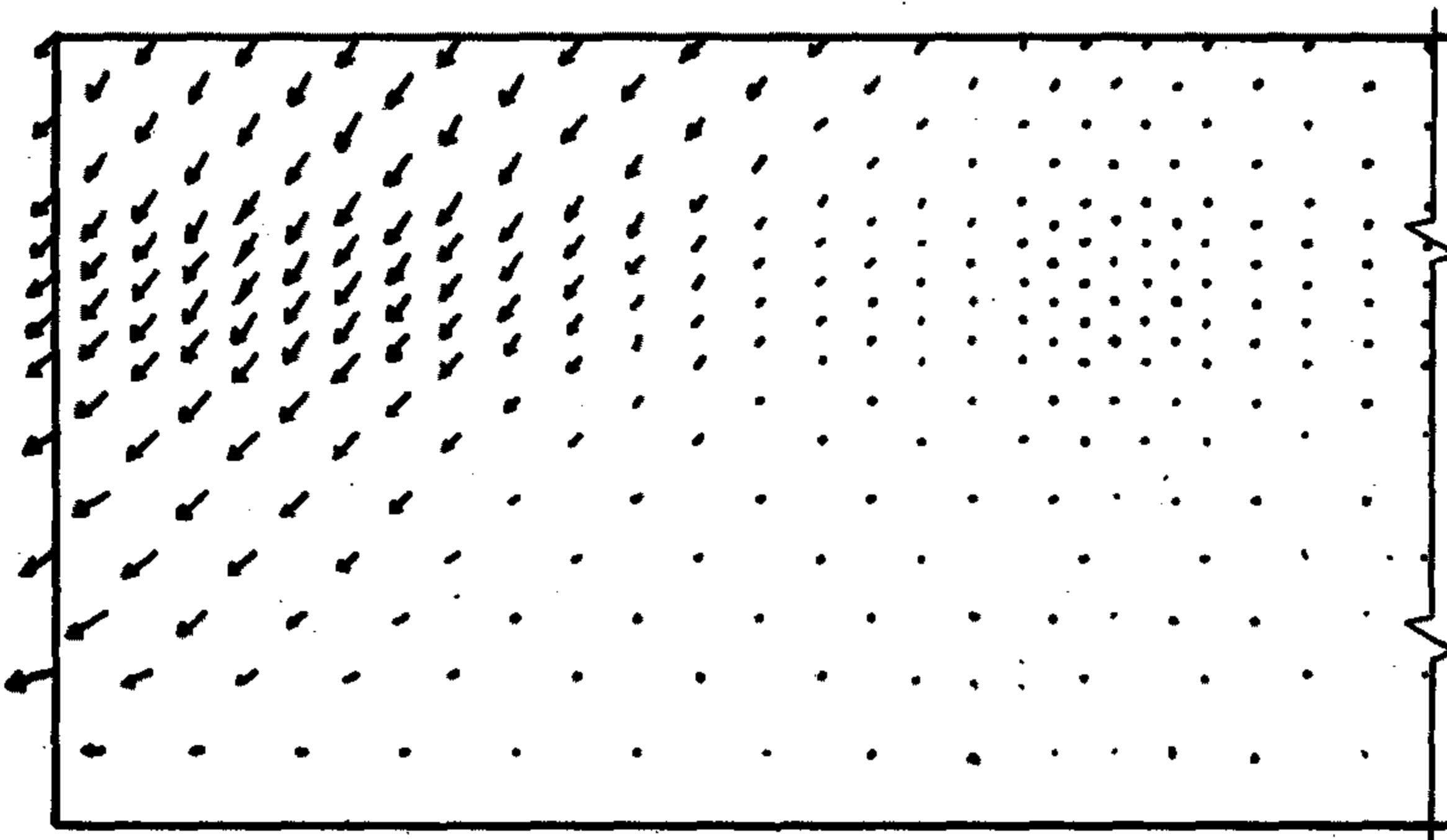
10.2.5 Effect of Anchor Location Upon Anchor-Wall Stability

For an anchor-wall system in a purely cohesive material, the factor of safety may be defined to be the ratio of the mobilised shear strength under working conditions (in this case $H = 10\text{m}$, $c = 30\text{ kPa}$, $\gamma = 18\text{ kN/m}^3$, $\phi = 0^\circ$) to the mobilised shear strength at which failure would occur under the same loading conditions (ie. $H = 10\text{m}$, $\gamma = 18\text{ kN/m}^3$, $\phi = 0^\circ$). This definition of the safety factor is appropriate to retaining wall design since the major uncertainty lies in the determination of soil strength rather than in the determination of the loading condition (H and γ here). Accordingly analyses were performed for a number of soil cohesions to determine the likely effects of a soil strength upon the performance of the four anchor-wall arrangements where $d = 0, 6.5, 10$ and 13.5m . The case where $d = 0$ corresponds to an unsupported cutting (ie. no anchor) and was analysed as a check upon the accuracy which could be expected from the numerical procedure. In this case collapse of the cutting occurred (at full excavation, ie. $H = 10\text{m}$, $\gamma = 18\text{ kN/m}^3$) for a cohesion $c = 44\text{ kPa}$ (ie. $\gamma h/c = 4.09$) which is 7% below the cohesion $c = 47\text{ kPa}$ (ie. $\gamma h/c = 3.83$) predicted from a conventional slip circle analysis (e.g. Taylor, 1948). This result implies that, relative to the assumed working load conditions (ie. $c = 30\text{ kPa}$), the factor of safety of an unsupported slope would be 0.68. For comparative purposes, the plastic region and velocity field at collapse for the unsupported wall are given in Fig. 10.10.

The lateral displacement at the top of the wall during excavation is shown in Fig. 10.11 for a range of soil cohesions and anchor positions. The extent of the plastic region after full excavation is



(a) Plastic Region at Collapse.



(b) Velocity Field at Collapse.

FIG.10-10 FAILURE OF AN UNSUPPORTED CUTTING.

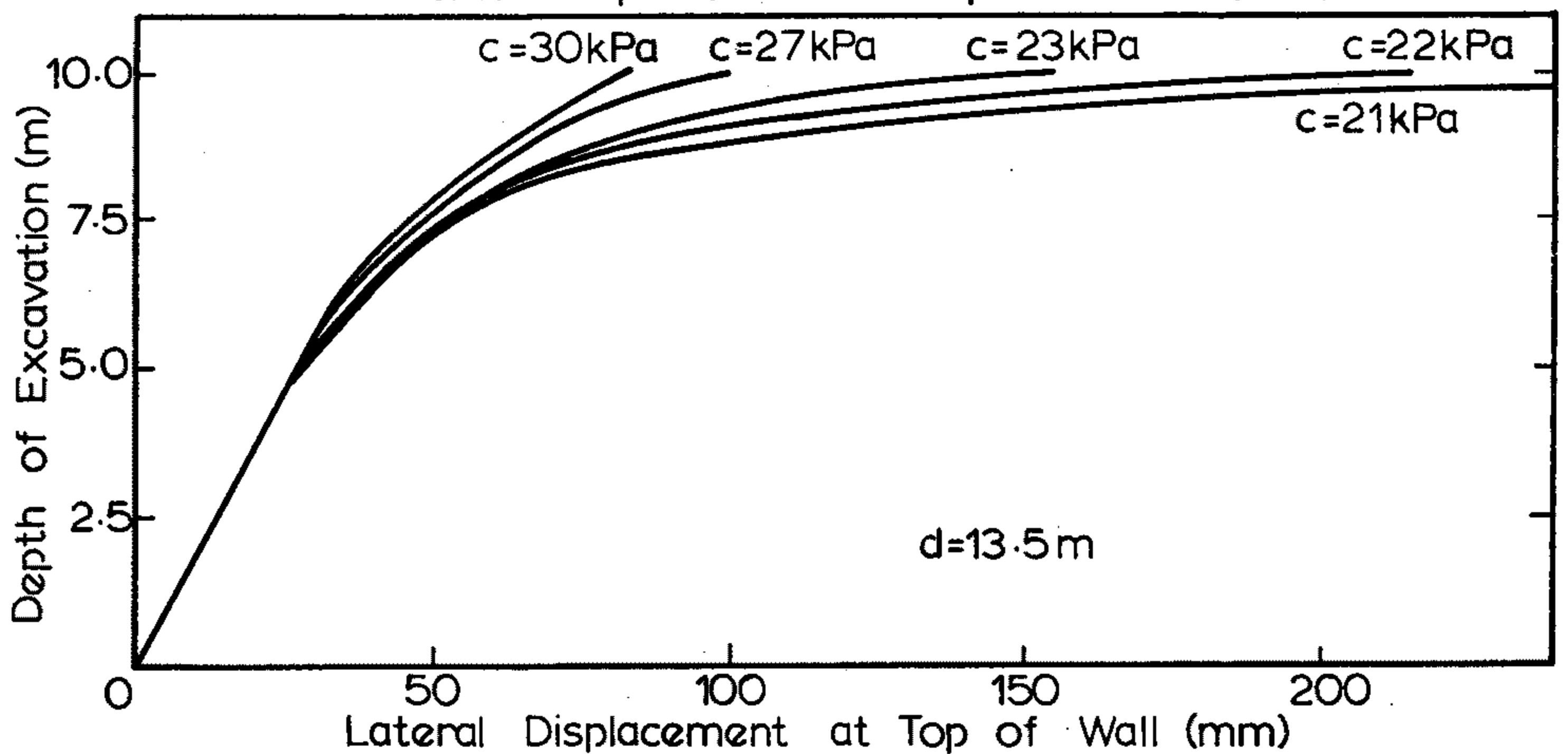
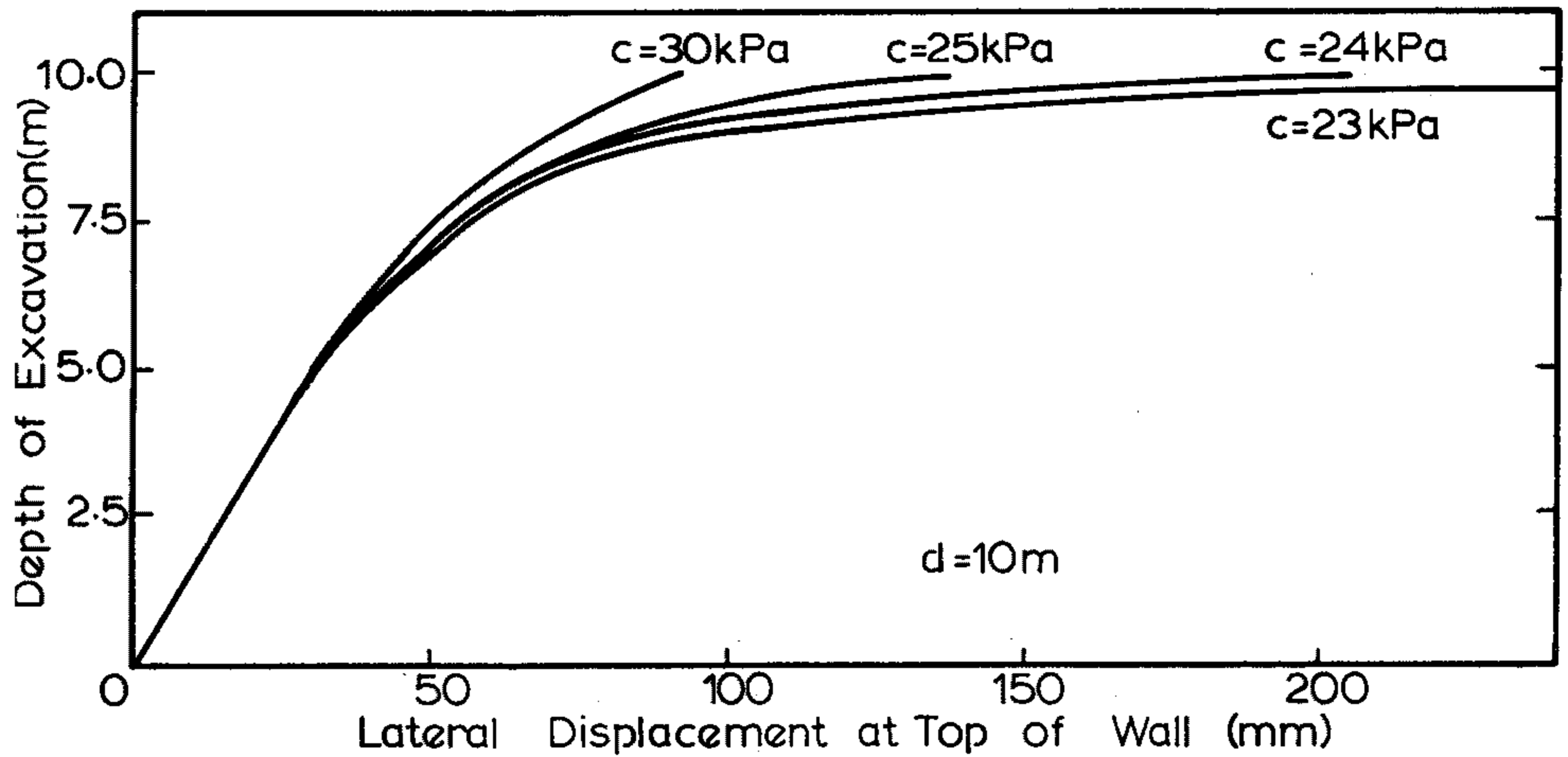
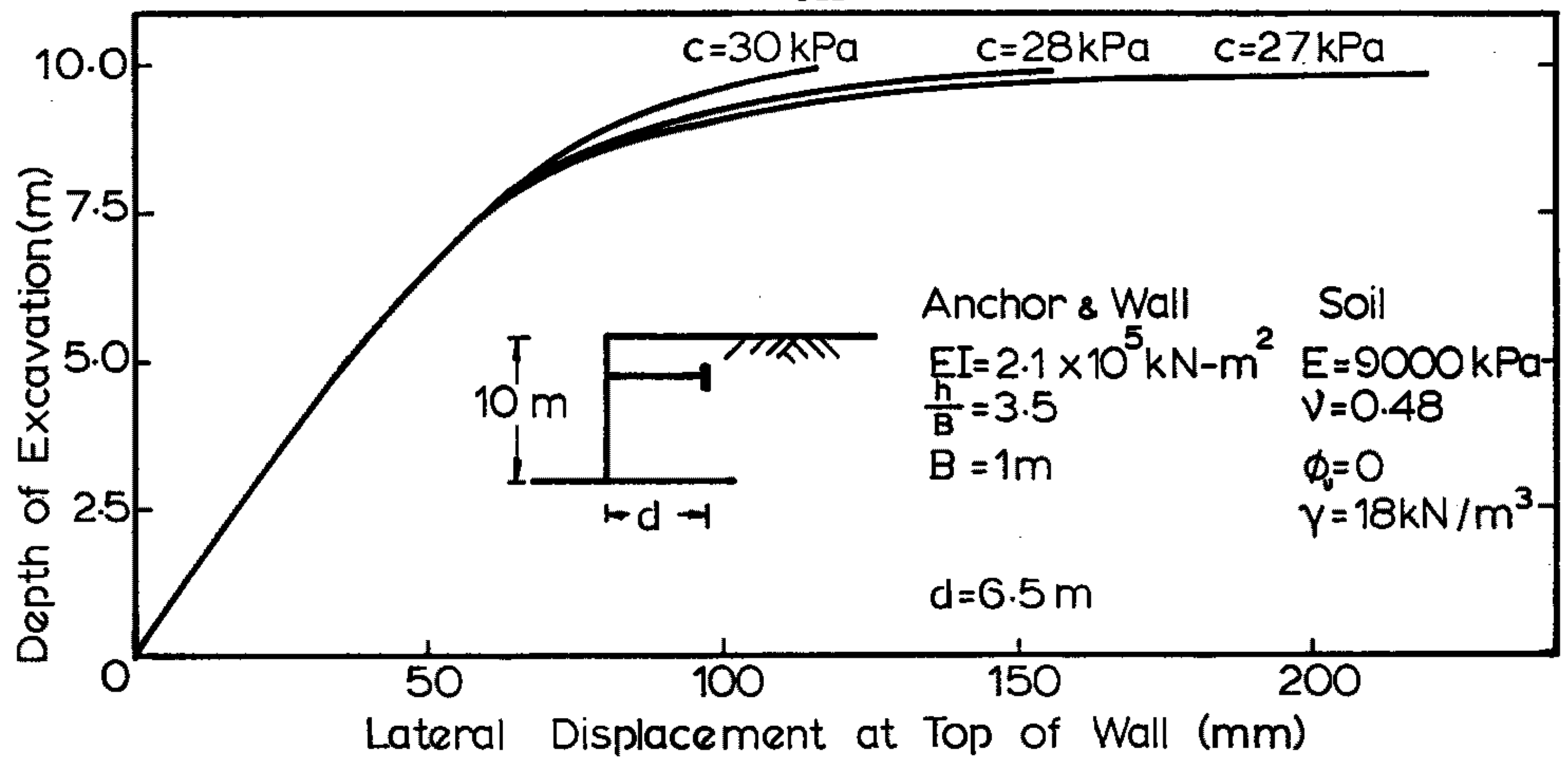
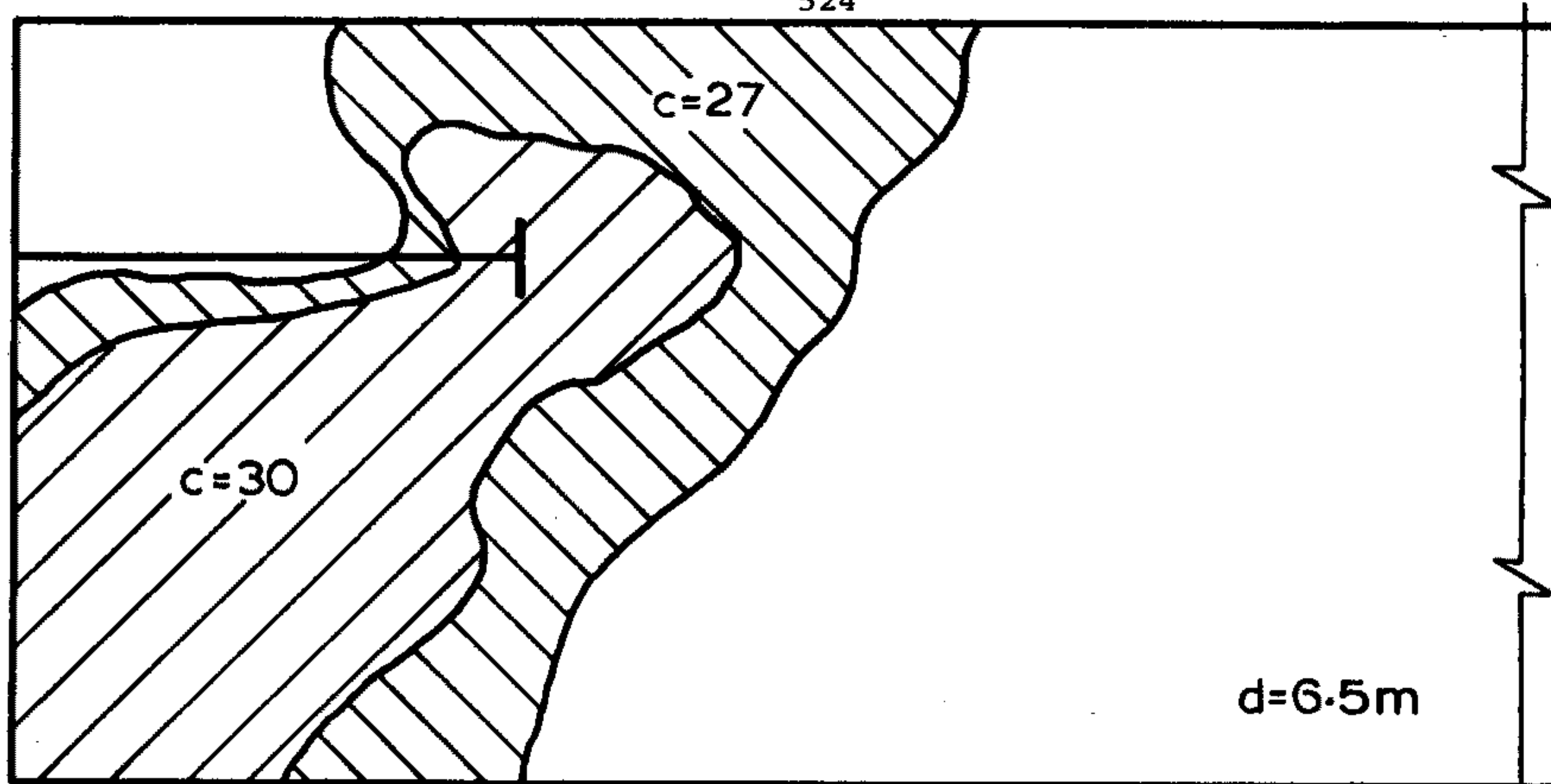


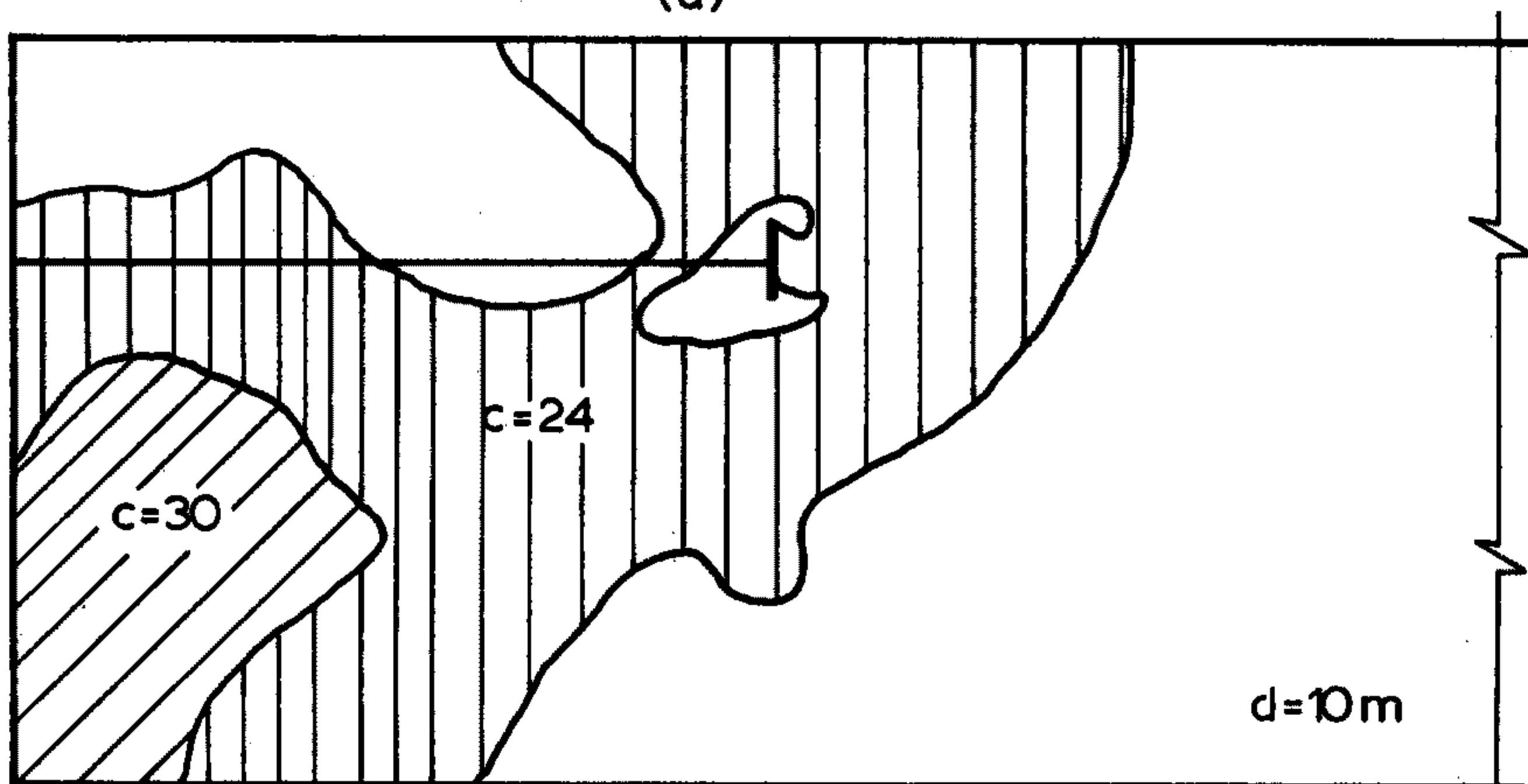
FIG.10.11 BEHAVIOUR OF WALL - ANCHOR SYSTEM DURING EXCAVATION.

shown in Fig. 10.12. From these two figures it may be inferred that collapse of the wall (after excavation) would occur at cohesions $c = 27, 24$ and 22 for anchors located at a distance $d = 6.5, 10$ and 13.5m respectively. This result would indicate that under working conditions (ie. $c = 30 \text{ kPa}$) the factor of safety for the three systems (ie. for $d = 6.5, 10$ and 13.5m) would be $1.11, 1.25$ and 1.36 respectively. The velocity fields at collapse are shown in Fig. 10.13.

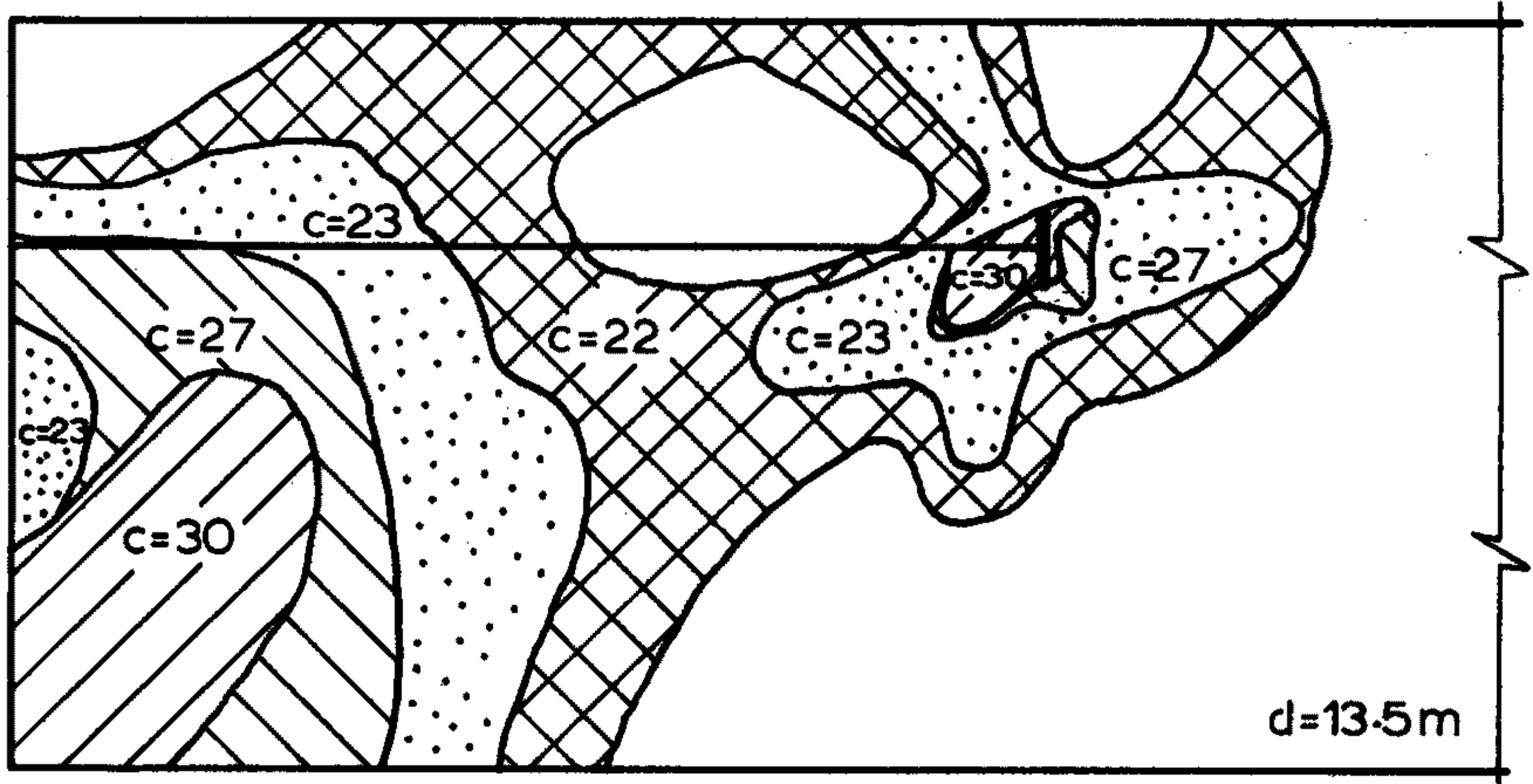
Conventional theory would suggest that the performance of a wall-anchor system where the anchor lies within the Rankine active wedge behind the wall (ie. $d = 6.5$) would be similar to that observed for an unsupported cutting. However, it was found that the factor of safety of the system was 1.11 for $d = 6.5\text{m}$ compared to 0.68 for an unsupported cutting. This increase in stability may be attributed to an interaction between the anchor and wall arising from the action of the anchor plate during excavation. Thus during excavation, a 'passive' region develops in front of the anchor while an 'active' region develops behind the wall. As excavation continues, these two incompatible stress fields must interact. (Simplistically, one may consider that as the 'active' region extends towards the anchor, the 'passive' similarly extends towards the wall and the region where these fields intersect cannot be either 'active' or 'passive'). The effect of this interaction between the behaviour of the anchor and wall is to force the failure surface to develop behind the region influenced by the compression in front of the anchor, as can be seen from Figs. 10.12 and 10.13, thereby increasing the stability of the anchor-wall system. However, it should be appreciated that in this case, failure of the anchor-wall system is dominated by active failure behind the retaining wall and the general behaviour is only modified by the existence of the anchor.



(a)

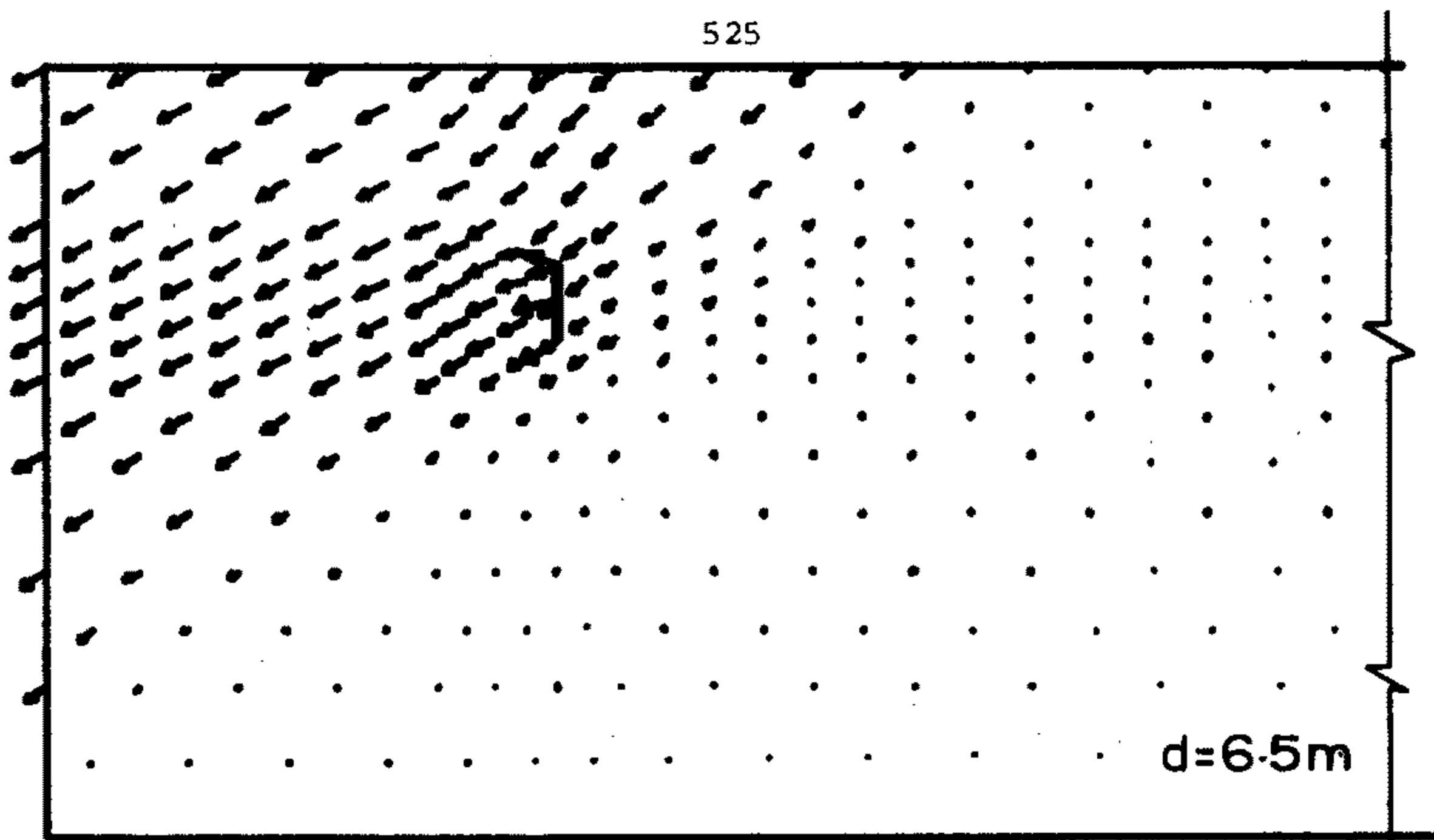


(b)

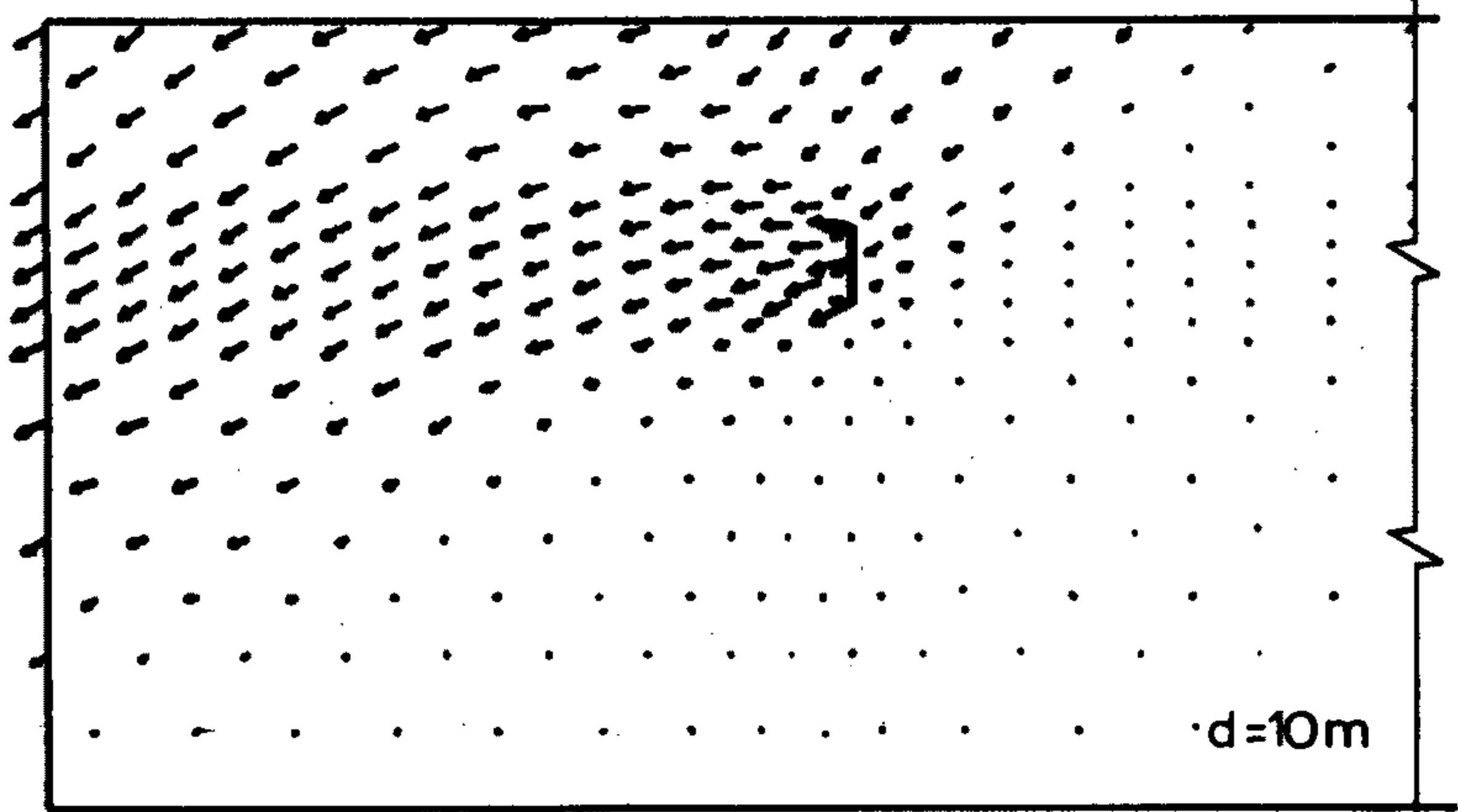


(c)

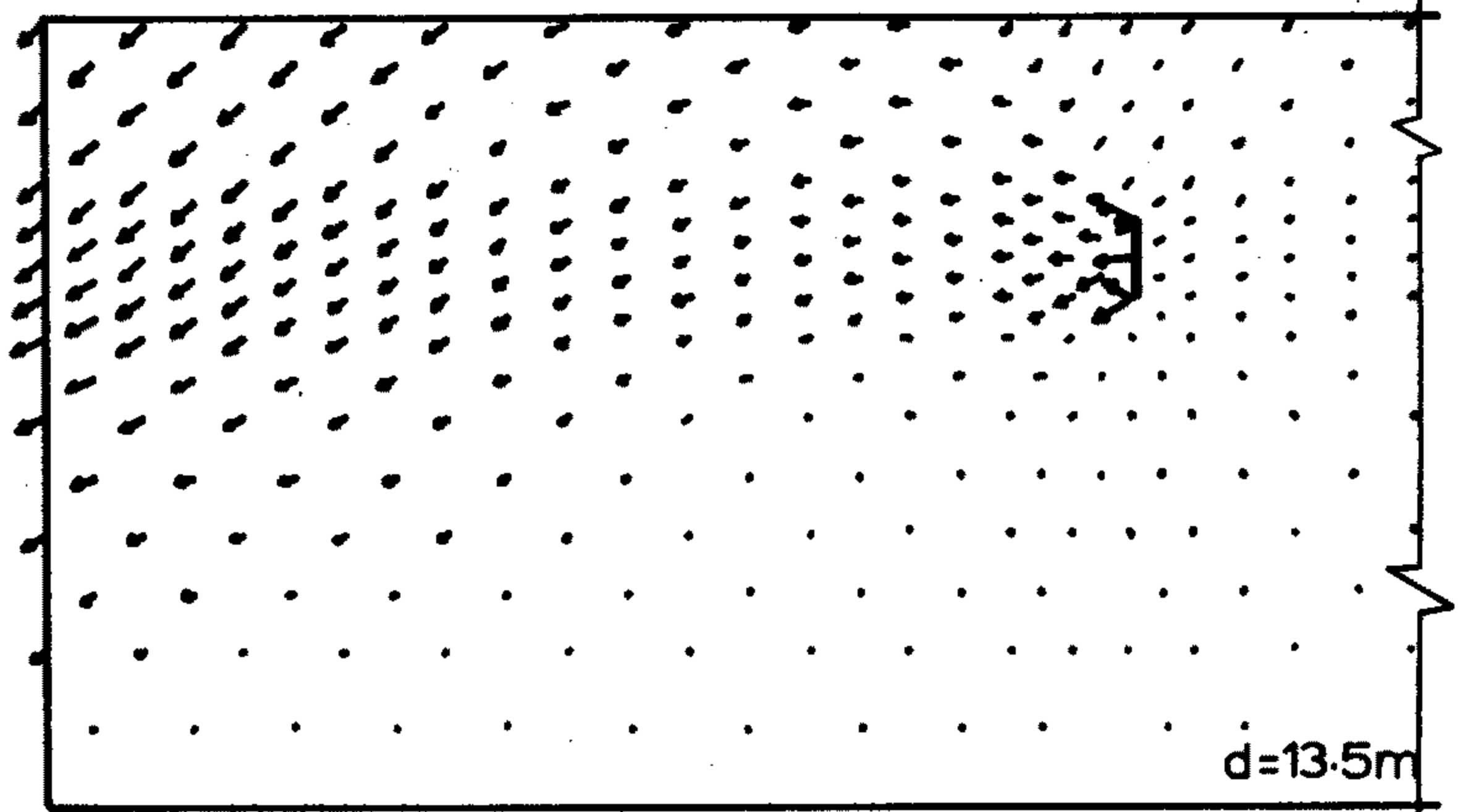
FIG.10.12 PLASTIC REGIONS AT COLLAPSE



(a)



(b)



(c)

FIG.10.13 VELOCITY FIELDS AT COLLAPSE.

In the second case, where $d = 10\text{m}$, the interaction between the stress field associated with failure of the wall and anchor prevents the normal active failure wedge developing behind the wall and forces the failure surface to extend such that it includes both wall and anchor. This leads to a further increase in the stability of the anchor-wall system and it may be considered that failure of the entire system corresponds to simultaneous failure of the two components (ie. the behaviour of both the wall and anchor are significantly influenced by each other).

In the third case, where $d = 13.5\text{m}$ [ie. at the minimum distance suggested by Terzaghi (1943) and the British Civil Eng. Code of Practice No. 2 (1951)] the behaviour of the combined system is dominated by the anchor behaviour which is only slightly modified by the presence of the wall. Thus it may be considered that collapse of the anchor is subsequently followed by collapse of the wall and hence the stability of the anchor-wall system is largely (although not entirely) governed by the stability of the anchor alone.

For example, the failure load of an isolated anchor is given in Chapter 9 by $Q_u = B(cF_c + syh)$ where

$$F_c = 5.2 \quad \text{for } h/B = 3.5$$

from section 9.3.1

$$s = 0.96$$

Hence for an isolated anchor subjected to an applied load Q where $\gamma = 18 \text{ kN/m}^3$, $h = 3.5\text{m}$, $B = 1\text{m}$, the cohesion at which failure would be expected to occur is given by

$$c = \frac{Q/B - s_y h}{F_c}$$

For the anchor force at working load $Q = 171 \text{ kN/m}$ this would imply collapse of the anchor for $c = 21.2 \text{ kPa}$ and a factor of safety of 1.41 which is only 4% above that observed for the combined system (ie. 1.36). In fact, the force in the anchor rod reduces as failure approaches and at collapse the anchor force of 161 kN/m^3 is 8% below the value predicted for failure of an isolated anchor.

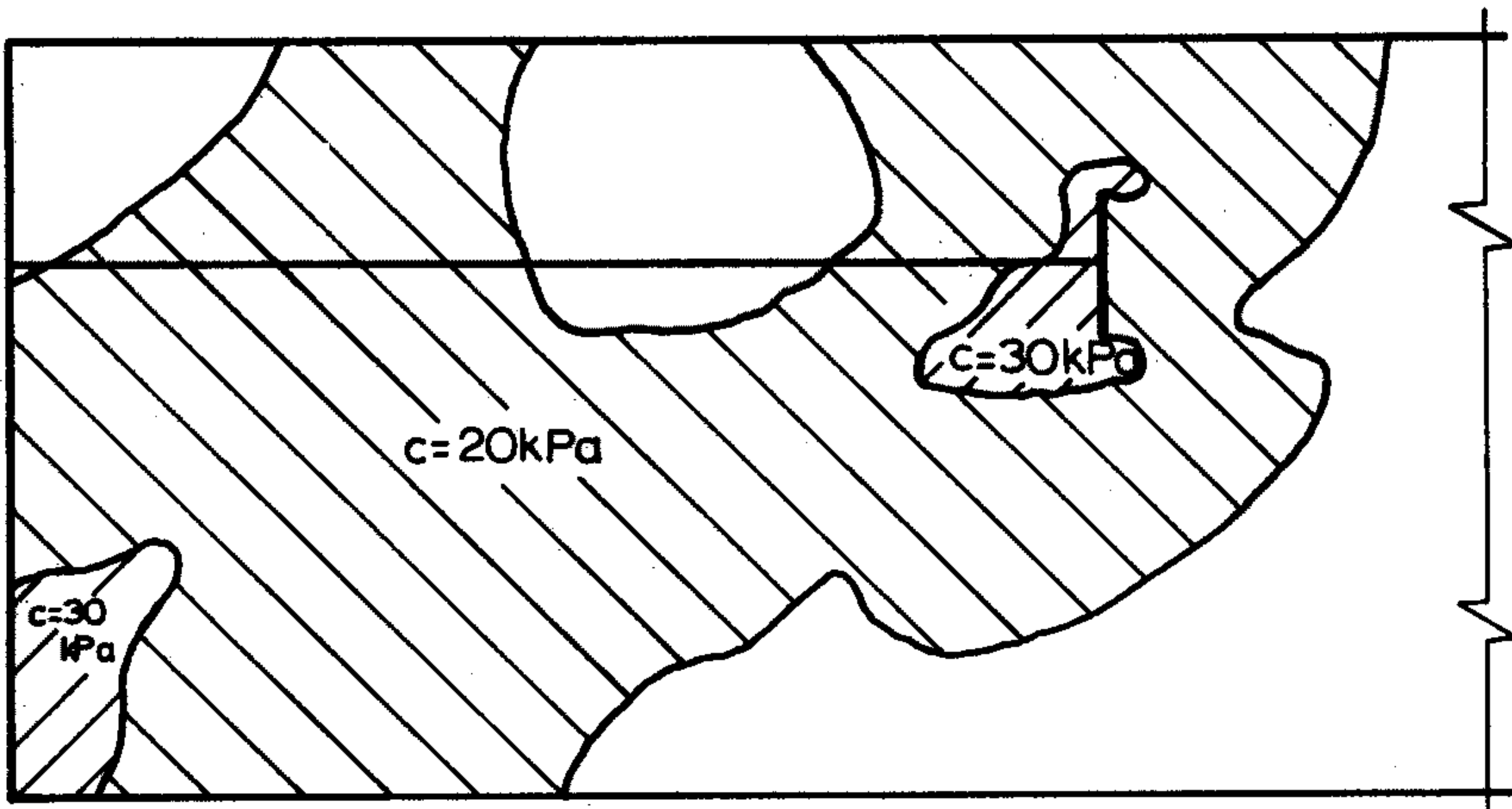
Similar stability analyses were performed for the more flexible wall [ie. Wall (2) with $(EI)_B = 4 \times 10^4 \text{ kN-m}^2/\text{m}$] and it was found that compared to the performance of Wall (1) [$(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$] the five fold reduction in wall stiffness did not alter the stability of the anchor-wall system for $d = 13.5\text{m}$ but did reduce the stability of the system for $d = 6.5\text{m}$ as indicated in Table 10.1. This situation is not surprising and serves to underline the fact that for $d = 13.5\text{m}$ the anchor and wall behaved largely as isolated units while for $d = 6.5\text{m}$ the wall behaviour dominated the performance of the entire system.

The stability analyses discussed above have all been concerned with an anchor-wall system in which the anchor size is small with respect to the cutting (ie. $B/H = 0.1$) and it is of some interest to determine the effect of increasing the anchor size. To investigate this effect, an analysis was performed for an anchor with $h = 4\text{m}$ and $B = 2\text{m}$ (ie. $B/H = 0.2$), located at the minimum recommended distance (Terzaghi, 1943) $d = h + H = 14\text{m}$ from the wall. The plastic region and velocity fields at collapse obtained from this analysis are shown

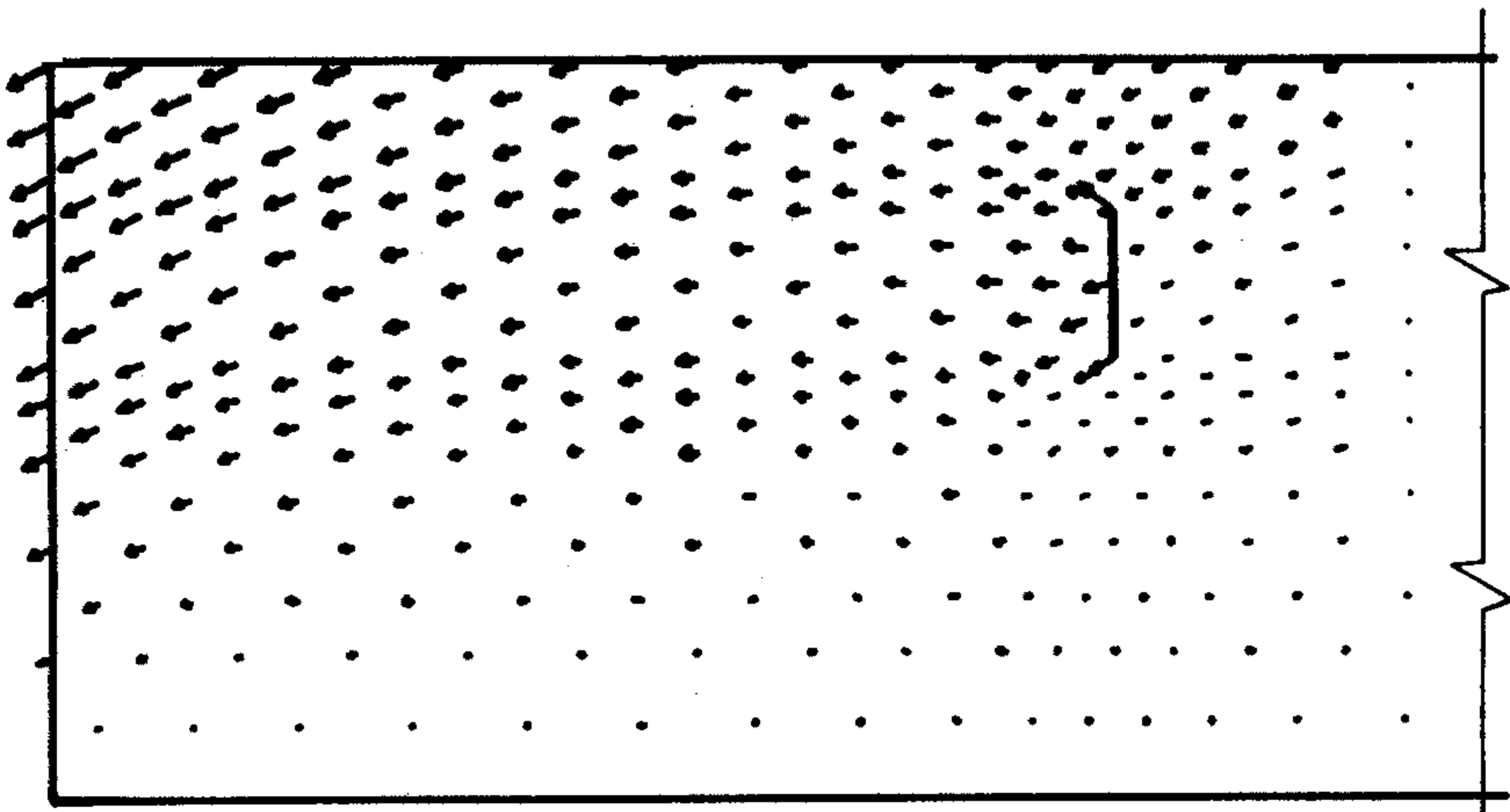
in Fig. 10.14. A comparison of the velocity field in Fig. 10.14(b) with that obtained for $h = 3.5$, $B = 1\text{m}$, $b = 13.5$ [ie. Fig. 10.13(c)] and that obtained for an unsupported wall [Fig. 10.10(b)] indicated that for the larger anchor ($B = 2\text{m}$) there is a much greater interaction between wall and anchor at collapse than was observed for the smaller anchor ($B = 1\text{m}$) even though both anchors were located at the appropriate recommended distance $d = h + H$. In particular, it was found that the anchor with $B = 1\text{m}$, $d = 13.5\text{m}$ behaved almost as if it were an isolated anchor and the subsequent collapse of the wall appears to be largely unaffected by the actual deformation near the anchor. However, for $B = 2\text{m}$ the factor of safety (of 1.45) is well below that predicted for an isolated anchor (using the results from Chapter 9) of 1.87 (where $Q = 245\text{ kN/m}$, $F_c = 4.4$, $s = 0.73$, $h = 4\text{m}$, $B = 2\text{m}$). Thus it can be seen that while increasing the anchor size did increase the stability of the anchor-wall system, this increase in stability is much lower than would be predicted assuming that there is no interaction between the wall and anchor. Consequently, it would appear that the recommended minimum distance between anchor and wall does not necessarily ensure that there is no interaction between the two components.

10.2.6 Conclusions

In the foregoing section, the influence of a number of parameters upon the performance of a retaining wall tied back to a single anchor (or line of anchors) has been briefly examined. Specifically, the effects of wall flexibility, anchor size and the separation between wall and anchor have been considered. In these studies, the soil was assumed to be homogeneous, and isotropic, exhibiting an undrained



(a) Plastic Region at Collapse.



(b) Velocity Field at Collapse.

FIG.10.14 BEHAVIOUR OF WALL - ANCHOR SYSTEM AT COLLAPSE.

WALL: $(EI)_B = 2.1 \times 10^5 \text{ kN-m}^2/\text{m}$ ANCHOR: $h=4\text{m}$ $B=2\text{m}$ $d=14\text{m}$

(purely cohesive) behaviour and obeying a Mohr-Coulomb failure criterion.

From the results of this study it would appear that:

- (1) The performance of the anchor-wall system depends upon an interaction between the relative stiffness of the wall and the fixity of the anchor (ie. the response of the anchor to loading). Thus the bending moments within the wall tended to increase with increasing wall stiffness, anchor size and distance to the anchor, although of these factors, the wall stiffness is the most significant.
- (2) The deformation of the wall was far more sensitive to the location and fixity of the anchor than were the bending moments.
- (3) The force applied to the anchor is largely governed by the elastic response of the anchor which depends primarily on the embedment ratio and height of the anchor. Neither the distance between wall and anchor nor the wall flexibility had any significant effect on the anchor rod force.
- (4) The stability of the anchor-wall system decreases with the distance between the anchor and the wall; however, even for very close anchors (e.g. $d/H = 0.65$) the presence of the anchor increased the stability of the cutting significantly. It is anticipated that the stabilizing effect of the anchor would depend both upon the dimensionless distance to the anchor, d/H , and the relative size of the anchor, B/H , although more investigation would be required to quantify these effects.

- (5) The minimum recommended distance between anchor and wall (Terzaghi, British Civil Eng. Code of Practice No. 2, loc cit.) does not ensure that there is no interaction between wall and anchor. In particular, the results of this investigation would suggest that the interaction between the wall and anchor depends both on the distance to the anchor [$d/H = 1 + h/H$ (for $\phi = 0^\circ$)] and the relative size of the anchor (B/H).

10.3 INTERACTION BETWEEN ANCHORS WITHIN A GROUP

10.3.1 Background

To date, research into anchor group behaviour (e.g. Yilmaz and Hanna, 1971; Neely, 1972a; Hanna et al., 1972; Larnach, 1973; Werner, 1975) has been confined to an experimental investigation into the behaviour of small groups of model anchors in sand. However, a similar problem, involving the prediction of shielding effects for long piles within a group of piles, has been analysed for a number of cases by Yegian and Wright (1973). By judicious selection of boundary conditions, Yegian and Wright predicted the load-displacement behaviour of piles within an infinite row or column at a spacing of three times the pile diameter, and for an isolated group of two piles at a spacing of twice the pile diameter. In these analyses, a non-linear hyperbolic expression was used to describe the relationship between stress and strain in the clay soil.

In this section, similar boundary conditions to those adopted by Yegian and Wright will be used in the analysis of anchor groups

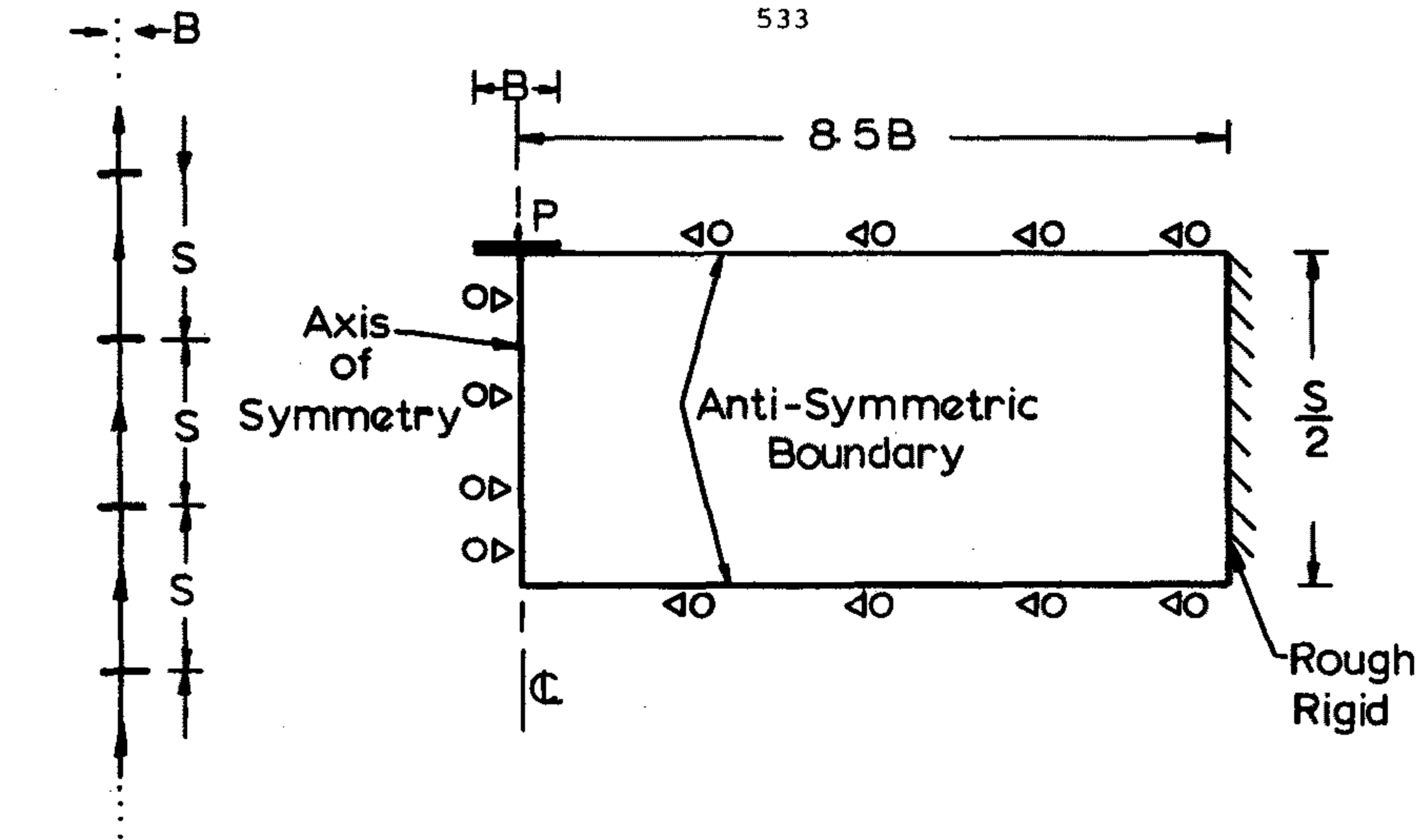
in clay, assuming an elasto-plastic soil model. In particular, the efficiency (with respect to ultimate capacity) of an infinite row and column of anchors will be considered for a range of spacings S/d . Subsequently, a specific configuration of three (infinite) rows of anchors will be examined to determine the shielding effect of the leading anchor. In this context, an infinite row of anchors consists of a group of anchors in parallel (ie. lying in a single plane and loaded perpendicular to that plane) while an infinite column consists of a group of anchors in series (ie. the load is applied along the axis of the group) as indicated in Fig. 10.15. Strictly speaking, the results in this section only apply to plane strain plate anchors; however, it is anticipated that the efficiency factors R_s obtained for this case would be reasonably applicable to anchors of finite length and to slender piles.

10.3.2 Finite Element Analysis

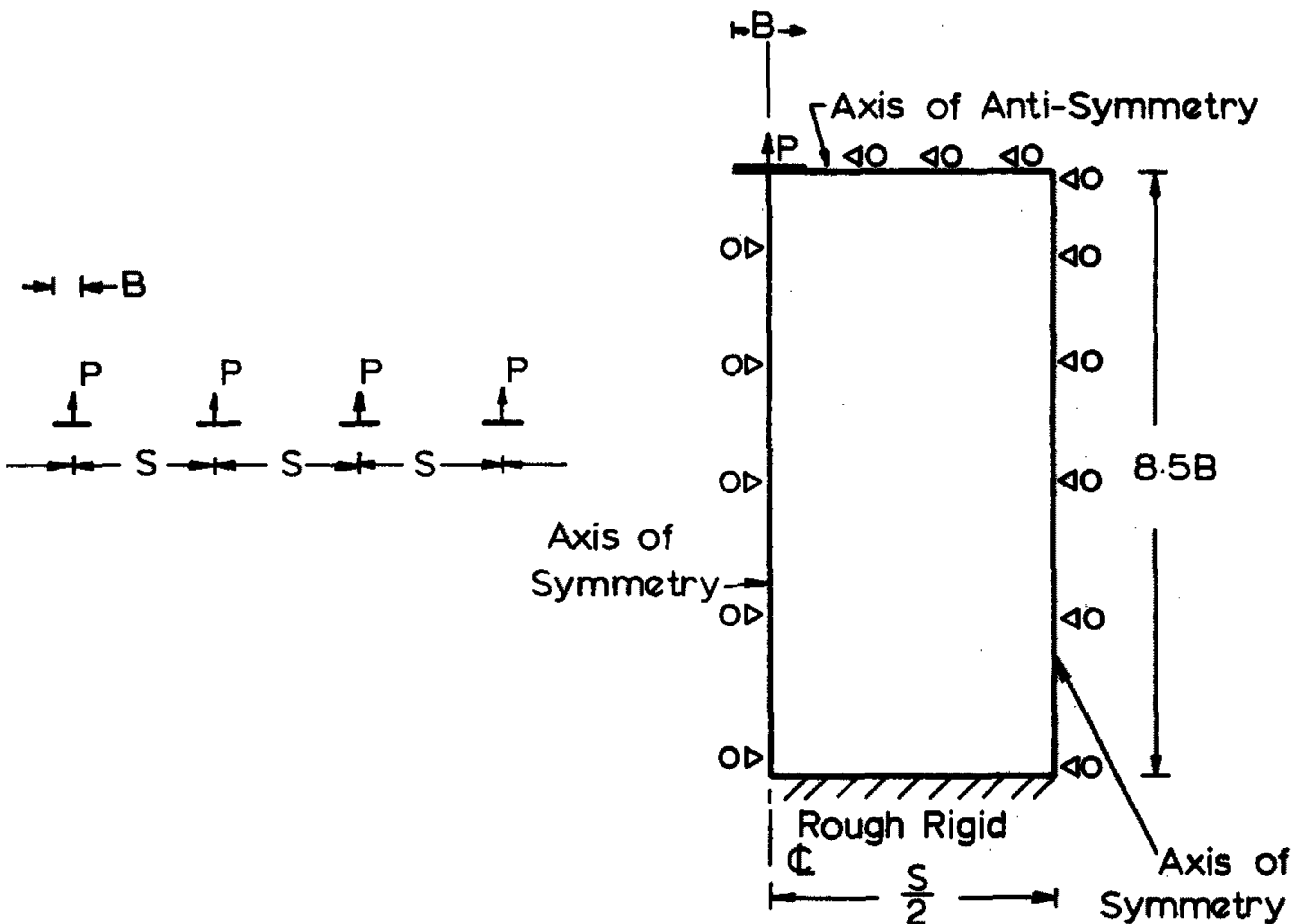
The two basic problems to be considered are:

- (1) An infinite column of anchors subjected to an applied load along the axis of the group; and
- (2) An infinite row of anchors subjected to an applied load perpendicular to the group.

The finite element idealisations of these problems are shown in Fig. 10.15 where 'S' is the centre to centre distance between anchors, 'B' is the width of each anchor and 'P' is the load applied to each anchor. Since the anchor groups are infinite, the problems can be reduced to the consideration of a single anchor by an appropriate choice of boun-

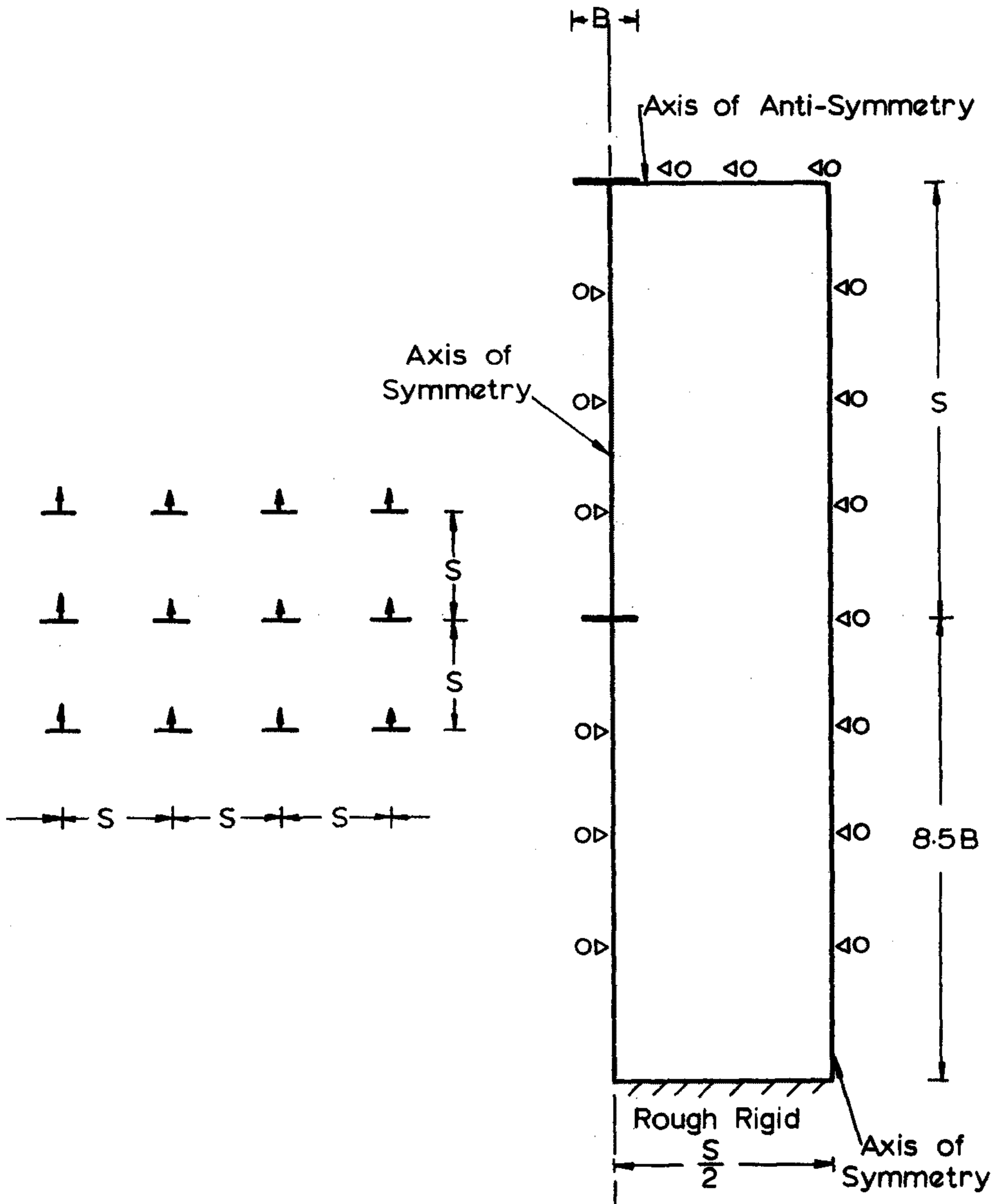


(a) Infinite Column of Anchors.



(b) Infinite Row of Anchors.

FIG.10-15 a&b BOUNDARY CONDITION FOR ANCHOR GROUPS.



(c) 3x Infinite Row of Anchors

dary conditions. The finite element mesh adopted was basically the same as that shown in Fig. 7.3(b) for the case of a strip footing. Bays of elements were removed where necessary to allow analysis of groups with different anchor spacings.

The third problem analysed consisted of three infinite rows of anchors perpendicular to the direction of anchor pull [as shown in Fig. 10.15(c)]. In this case it was necessary to examine the behaviour of both the first (or last) and second anchor within a column; the remaining anchors could be replaced by appropriate boundary conditions. The finite element mesh in this case was constructed by removing unnecessary bays and then repeating units such as those shown in Fig. 7.3(b).

The techniques adopted in this analysis were basically the same as described in Chapter 7 for an isolated anchor.

An elasto-plastic soil model was adopted and the relevant soil parameters were $E/c = 500$, $\nu = 0.48$, $\phi = 0^\circ$, $\gamma = 0$.

10.3.3 Effect of Spacing Upon the Efficiency of Anchors Within a Group

In the following discussion, the behaviour of an anchor within a group will be expressed in terms of the mean pressure q acting on the anchor and the dimensionless displacement $\delta E/Bc$ of the anchor relative to its original position. The efficiency of an anchor, R_s , is defined as the ratio of the mean pressure q_u acting on the anchor at collapse to the mean pressure acting on an isolated anchor (at infinite depth) as

determined from Chapter 7.

$$\text{ie. } R_s = \frac{q_u (S/B)}{q_u (S \rightarrow \infty)}$$

The relative performance of anchors at a spacing 'S' in an infinite column of anchors may be assessed from Fig. 10.16. As the anchor spacing decreases, the apparent stiffness of the anchor $P/\delta E$ decreases rapidly. For example, a 50% reduction in spacing from $S/B = 16$ to $S/B = 8$ reduced the elastic stiffness by 37%. These results suggest that elastically, the interaction between anchors within an infinite (column) group is quite significant even for relatively large anchor spacings. However, the same is not true of the ultimate behaviour. In fact, the collapse load for $S/B = 16$ is virtually identical to that obtained for an isolated anchor, while a 50% reduction in spacing to $S/B = 8$ only reduced the ultimate collapse load by 5%. For spacing S/B less than 8 the efficiency of the individual anchor decreases rapidly as shown in Fig. 10.18. For spacings greater than $8B$ failure of the anchor was predominantly associated with flow from the front to the back of the anchor. For spacings less than $8B$ failure was associated with the shearing of a column of material containing the anchors past the adjacent intact material. This mechanism gives rise to a simple upper bound such that $q/c = 2S/B$.

The actual load carrying capacity of the anchor, expressed in the dimensionless form $\frac{q/c}{S/B}$, is given in Table 10.2 for a range of spacings, and it will be appreciated that for $S/B \leq 6$ the finite element results indicate that bearing capacity predictions based on this upper bound would be appropriate.

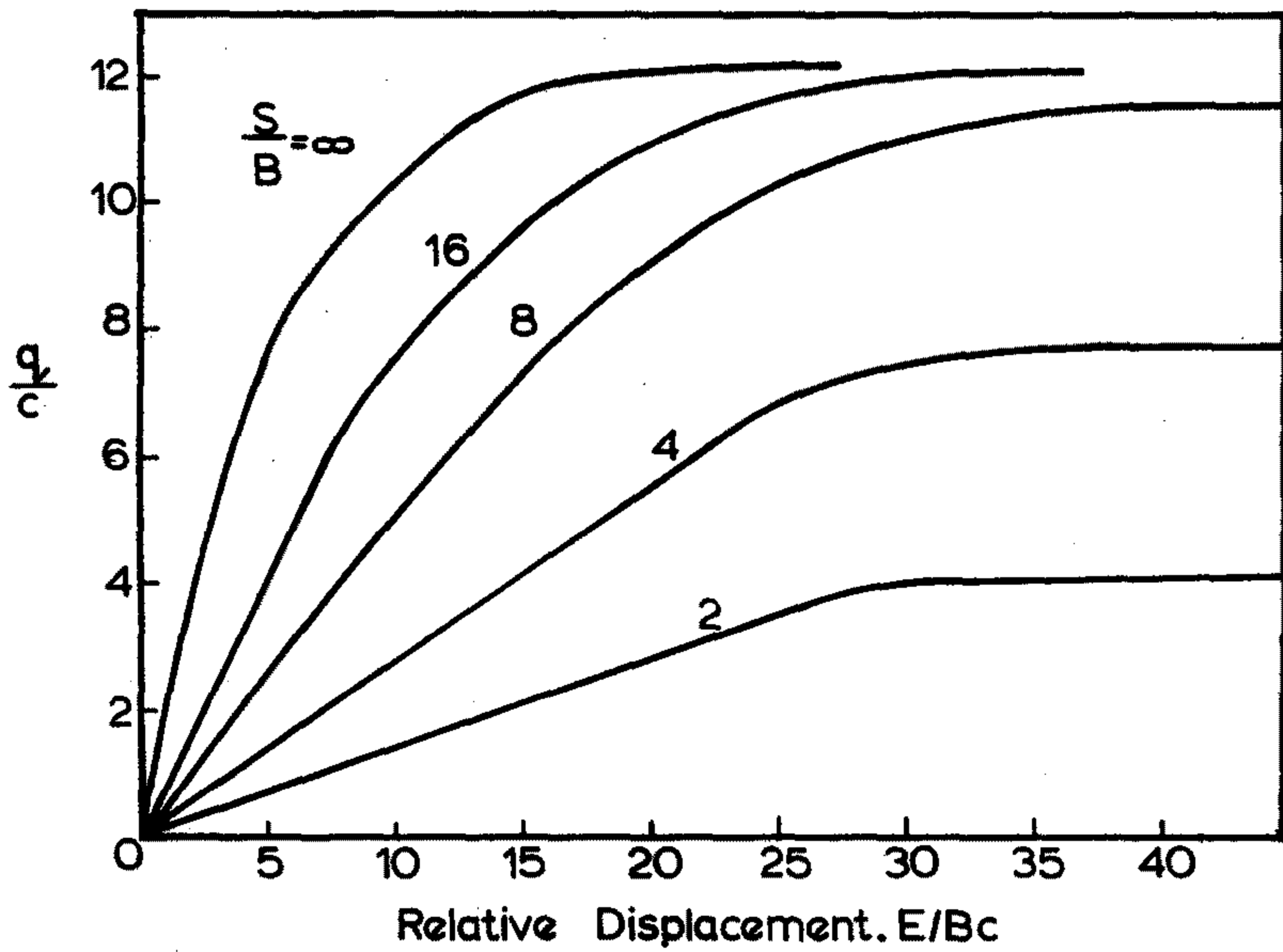


FIG.10.16 BEHAVIOUR OF AN INFINITE COLUMN OF ANCHORS.

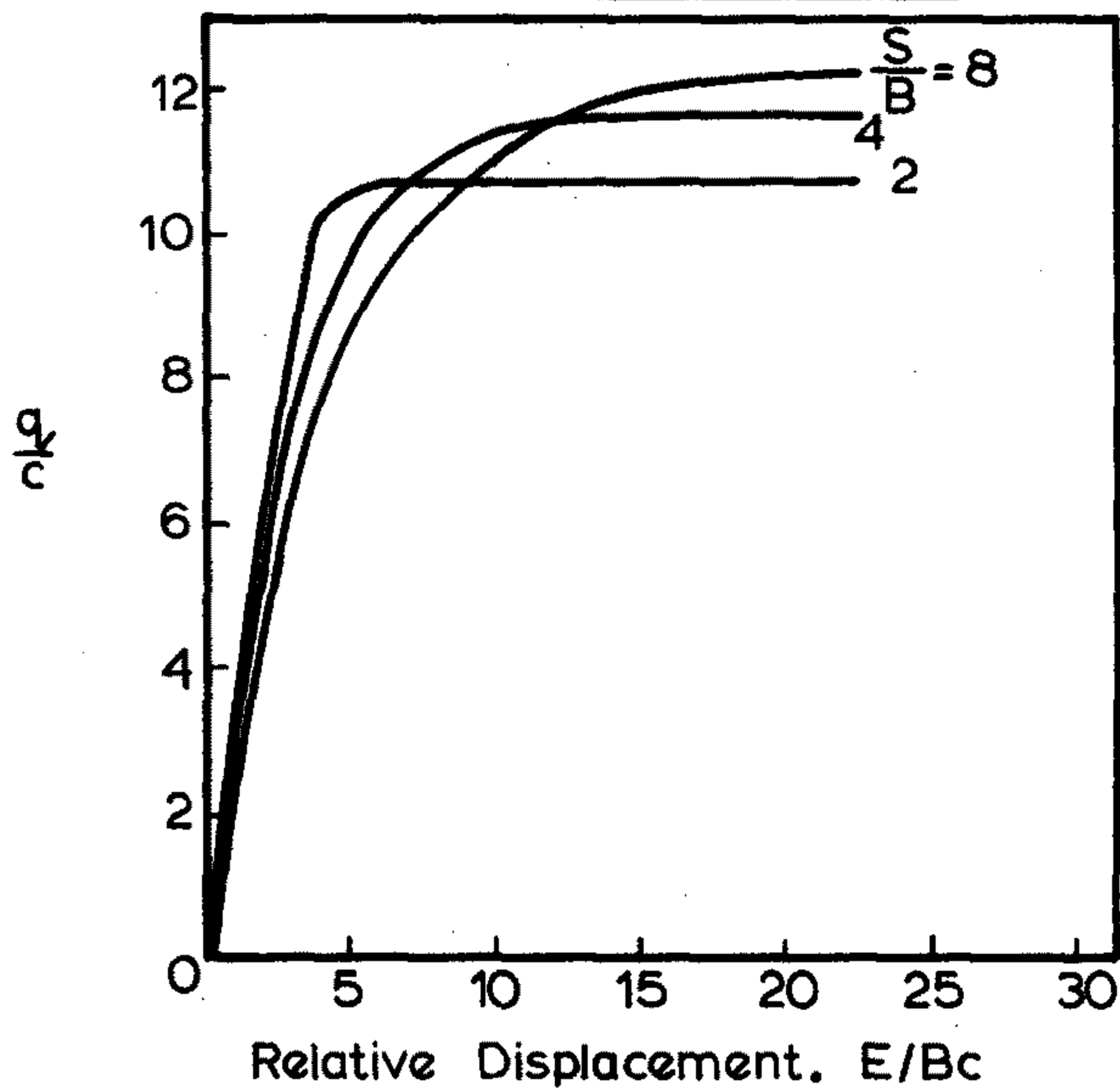


FIG.10.17 BEHAVIOUR OF AN INFINITE ROW OF ANCHORS.

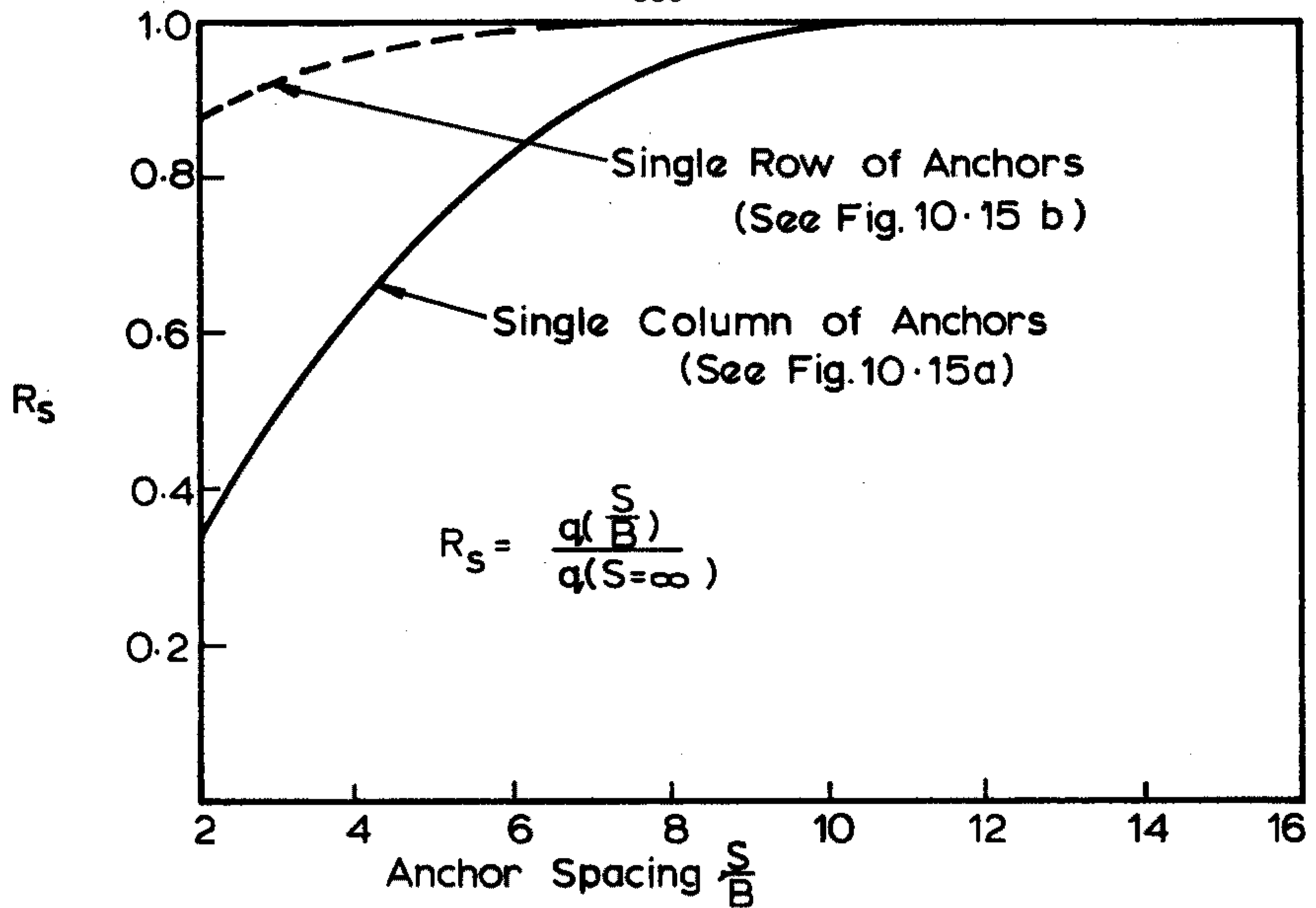


FIG.10.18 EFFICIENCY OF AN INFINITE COLUMN AND ROW OF ANCHORS

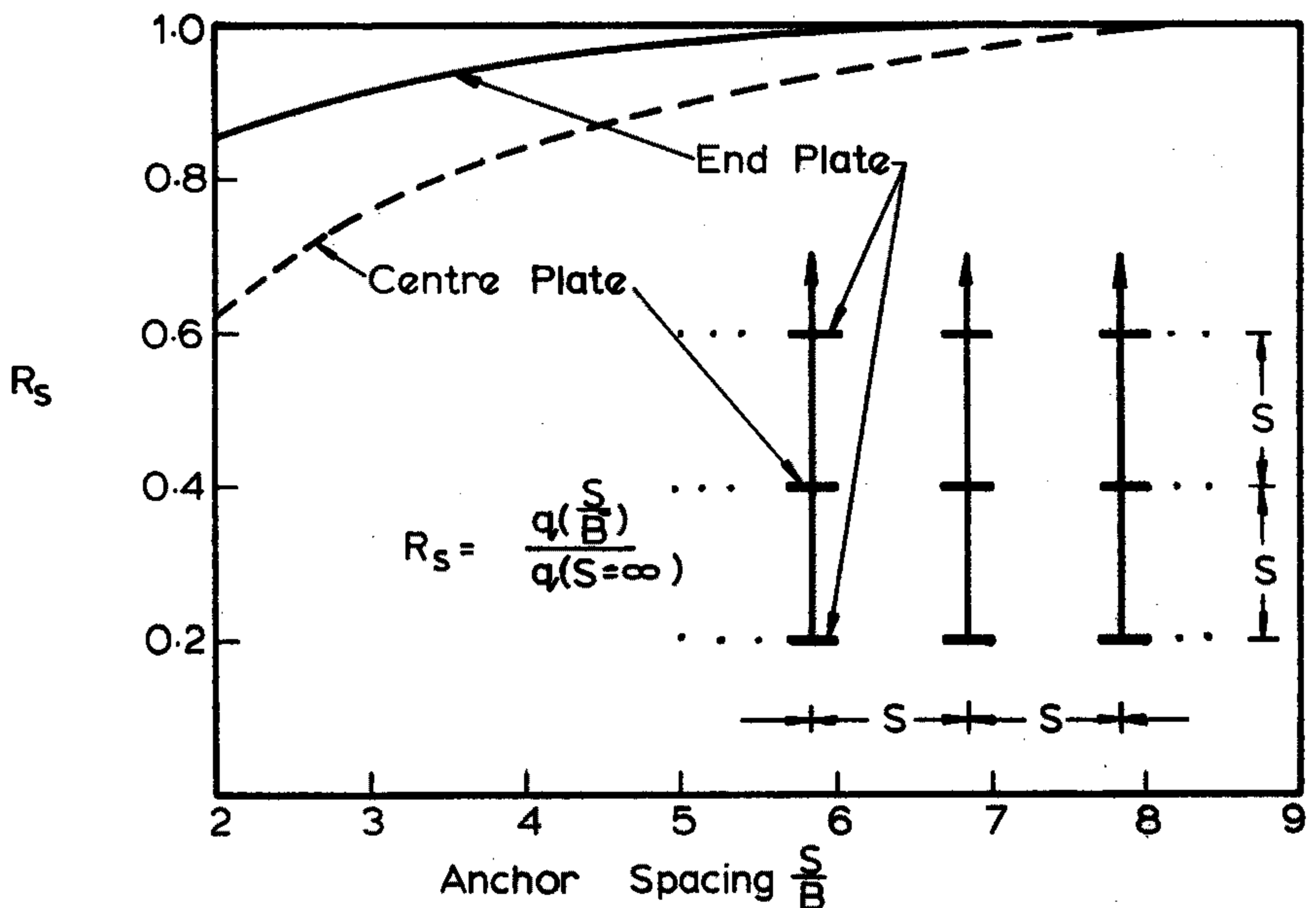


FIG.10.19 EFFICIENCY OF A 3x INFINITE ANCHOR GROUP

TABLE 10.2

EFFECT OF SPACING UPON DIMENSIONLESS BEARING CAPACITY

S/B	$\frac{q/c}{S/B}$
2	1.97
4	1.90
8	1.43
16	0.75

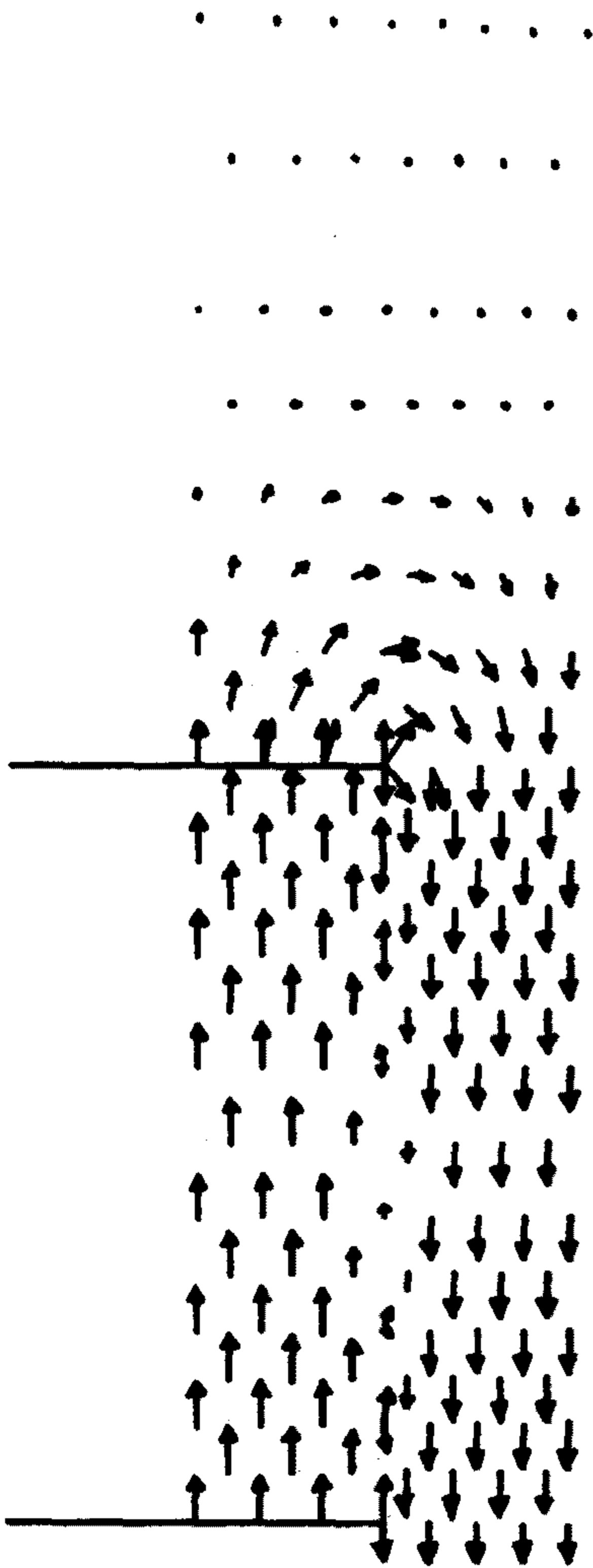
The relative performance of anchors at a spacing 'S' in an infinite row may be assessed from Fig. 10.17. In particular, it will be observed that the apparent stiffness of the anchor $P/\delta E$ increases with decreasing anchor spacing while the ultimate collapse load decreases very slowly with spacing. The elastic behaviour of a row of anchors is in stark contrast to that observed for a column of anchors. It will be recalled from Chapter 3 that the behaviour observed for a row of anchors is a direct result of the incompressible nature of the material.

The ultimate bearing capacity of anchors within either infinite row or column groups, expressed in terms of the anchor efficiency R_s are shown in Fig. 10.18. From these results it may be concluded that

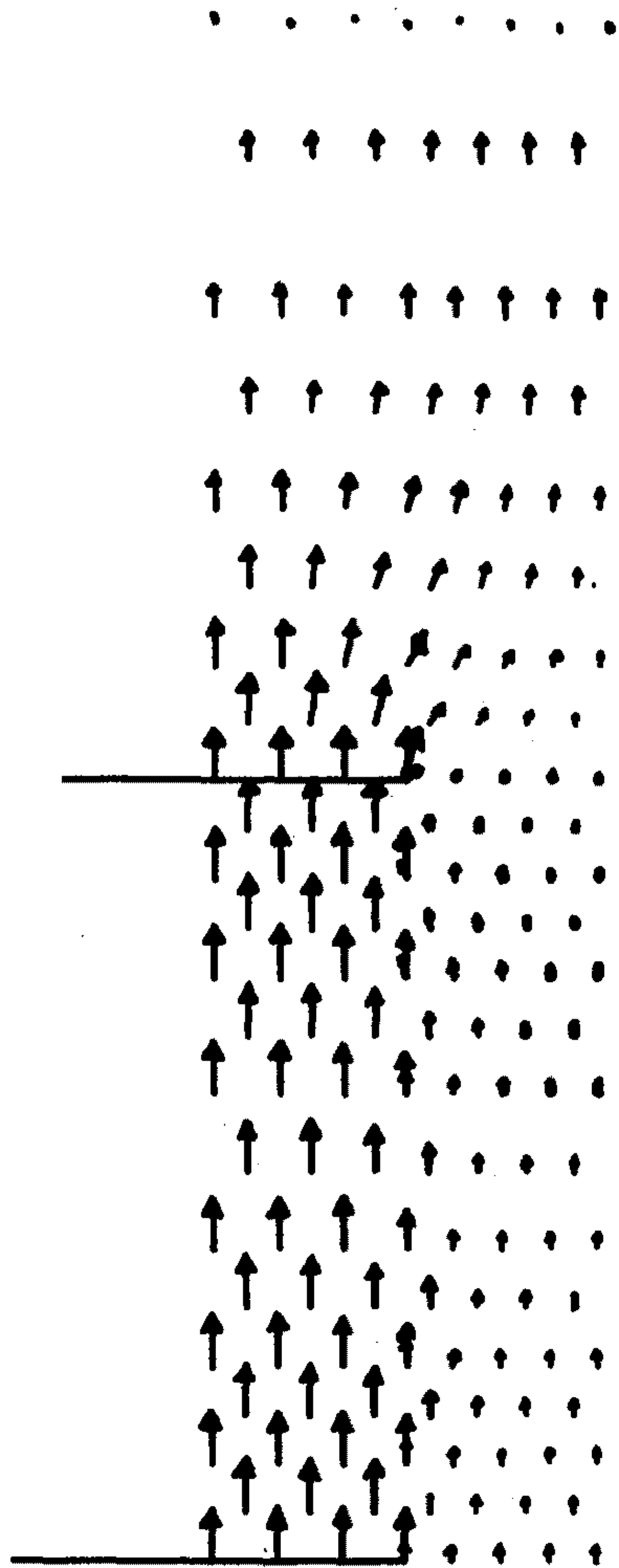
- (1) In either case, anchors with a spacing $S/B \geq 8$ have collapse loads approximately equal to that of an isolated anchor; and
- (2) Placing anchors in series has a far greater effect upon anchor efficiency than placing them in parallel.

For the particular case when $S = 3B$ the efficiencies predicted (see Fig. 10.17) for an infinite row and column of anchors (ie. $R_s = 0.92$ and 0.5 respectively) are in close agreement with the efficiencies predicted by Yegian and Wright for piles with the same spacing (ie. $R_s = 0.96$ and 0.55 respectively). The fact that Yegian and Wright's results were obtained using a hyperbolic stress-strain behaviour suggests that the precise nature of the soil model is not critical for this type of problem.

The findings of this analysis raised some questions as to the efficiency of multiple anchors. To investigate this aspect of behaviour, an anchor group consisting of an infinite row of multiple anchors (involving 3 anchors in series) was analysed and the resulting efficiency was plotted against anchor spacing S/B in Fig. 10.19. It was found that the behaviour of the front (or rear) anchor within the group was largely unaffected by the adjacent anchors, since the efficiency of these anchors was only slightly less than that obtained for a single row of anchors. However, the central anchor was significantly shielded by the leading and end anchors resulting in a much lower efficiency. It will be appreciated from the foregoing discussion that this reduction in efficiency is primarily due to the shielding of the other anchors in series rather than to the influence of adjacent multiple anchors. The shielding effect of the leading anchors may be qualitatively assessed from the velocity field obtained for $S/B = 2$ and $S/B = 4$ as shown in Figs. 10.20 and 10.21 respectively. In particular, it will be observed that for $S/B = 2$ the soil flows around from the front of the anchor and then straight past the column of material contained by the anchors. Clearly, the mode of failure at the end anchors



(a) Velocity Field



(b) Displacement Field

FIG.10.20 VELOCITY AND DISPLACEMENT FIELD FOR
3 x INFINITE GROUP - $S=2B$.

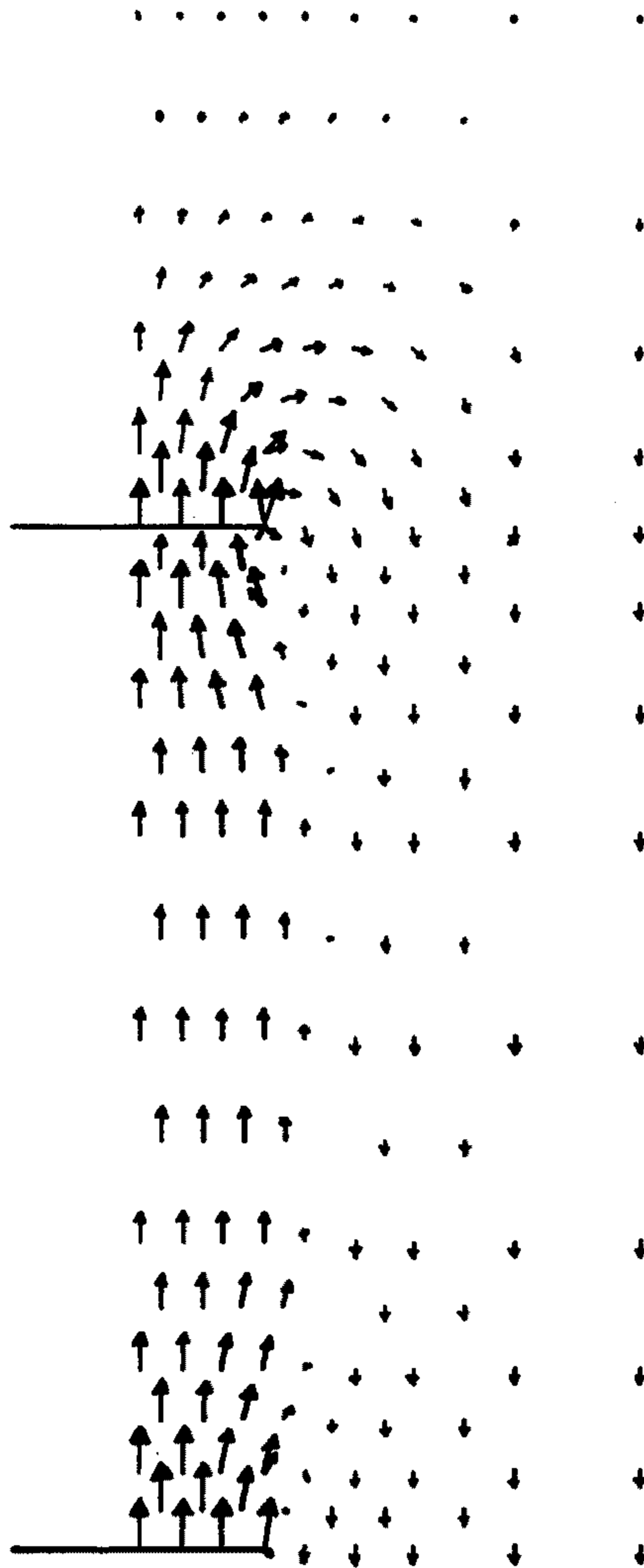


FIG.10-21 VELOCITY FIELD FOR 3x INFINITE GROUP
S=4B.

is significantly different from that exhibited at the central anchor. As the spacing between the leading and central anchor increases, the tendency for material to flow past the central anchor is reduced and is replaced by a tendency for the material to flow from the front to the back of each anchor individually. Thus the velocity field shown in Fig. 10.21 can be seen to mark the transition between these two types of behaviour. When $S/B = 8$, the velocity field for each anchor is almost identical to that shown in Fig. 7.16(a) for an isolated anchor plate.

In general, the velocity field (ie. the rate of change in displacement over a load increment) and displacement field (ie. the total displacement at a particular load level) are quite similar in appearance; however this is not always the case. For example, in the analysis of a 3x infinite group of multiple anchors, the displacement field just prior to collapse is markedly different from the velocity field at collapse, as shown in Fig. 10.20. This difference underlines the importance of a genuine plasticity approach rather than a bi-linear model since the nature of the elastic and plastic deformations are markedly different. (A more detailed discussion of possible limitations of bi-linear and multi-linear approaches is given by Davis and Booker, 1977).

10.3.4 Conclusions

Although the foregoing analysis of group anchor behaviour was somewhat limited in scope, a number of important trends were observed. In particular, it was found that, for anchor groups in clay:

- (1) The efficiency of an anchor is not seriously affected by the proximity of other anchors within the same plane. Indeed, in the short term (ie. for an incompressible materials) the apparent stiffness of the anchor may be enhanced by the proximity of similar anchors.
- (2) The elastic response and ultimate bearing capacity of anchors in series (e.g. multiple anchors) are both significantly affected by the proximity of adjacent anchors. The ultimate bearing capacity was unaffected for spacings greater than $8B$ however the elastic response was still noticeably reduced for spacings up to and beyond $16B$. Even for a group of anchors where only three anchors were in series, the apparent stiffness and the ultimate bearing capacity of the central anchors were noticeably reduced by the proximity of adjacent anchors.

10.4 THE EFFECT OF PILE GROUPS UPON SLOPE STABILITY

10.4.1 Background

In the past, piles have been used to increase slope stability in very soft clay (e.g. Broms, 1972) and to stabilise landslides by dowell action (e.g. Fukuoka, 1977). However, despite the growing acceptance of piles as a method of stabilising a soil mass, there has been very little research into the usefulness and applicability of these procedures. Indeed, present design methods appear to be based upon limited empirical experience and relatively crude analytical techniques (e.g. see Poulos and Davis, 1978; Fukuoka, 1977).

In this section, a finite element technique for the analysis of the undrained behaviour of soil slopes reinforced by pile groups will be proposed. This technique arises directly from the soil-structure interaction theory of Chapter 5. The proposed technique has the advantage that it is possible to include the effects of failure within soil or pile as well as soil flow past the piles within a single analysis. Thus the deformation and stability of a slope may be assessed for various pile arrangements and stiffnesses, while the influence of interaction between adjacent piles is implicitly included in the analysis.

Application of the proposed technique will be illustrated by considering the effects of pile arrangement and stiffness upon the performance of an idealised slope.

10.4.2 Method of Analysis

The application of the finite element method to the prediction of slope stability for an elasto-plastic material is quite straightforward (e.g. Snitbhan and Chen, 1976). However, extension of the method to allow the analysis of a slope reinforced by piles is complicated by the three dimensional nature of the pile group. In principle, this type of problem could be analysed using three dimensional finite elements, however the cost of performing such an analysis is prohibitive. Alternatively, if the pile group extends parallel to the slope for a reasonable distance then the general problem may be analysed, in an approximate sense, by using a conventional two dimensional (plane strain) soil-structure interaction analysis, provided some allowance is made for the three dimensional nature of the real situation.

In particular, it is necessary to consider:

- (1) The correlation between the elastic response of a pile group and the idealised plane strain pile;
- (2) The determination of the limiting pressure which may act on the piles within a group and the provision for soil flow 'through' the plane strain pile, corresponding to slip or flow of the soil between and past the piles once this limiting pressure is attained; and
- (3) Provision for the shielding effect of piles within a group upon the limiting pressure the pile will experience once the soil begins to flow past the pile.

In the proposed analysis, the soil and the piles are considered separately, being related by nodal forces which ensure compatibility and equilibrium requirements are satisfied. Thus, if the limiting pressure acting on the pile at a point has not been reached, then the displacement of both soil and pile must be identical. Once the limiting pressure is reached at a point then flow of the soil around the pile will occur, and this may be simulated by allowing slip of the soil past (ie. through) the pile at that point, as illustrated in Fig. 10.22. The numerical formulation for this analysis is basically the same as that presented in Chapter 5 for the general analysis of soil-structure interaction problems.

An approximate correlation between the elastic response of a real pile group with the idealised (plane strain) pile may be obtained by:

- (1) Determining the lateral response of the real pile group

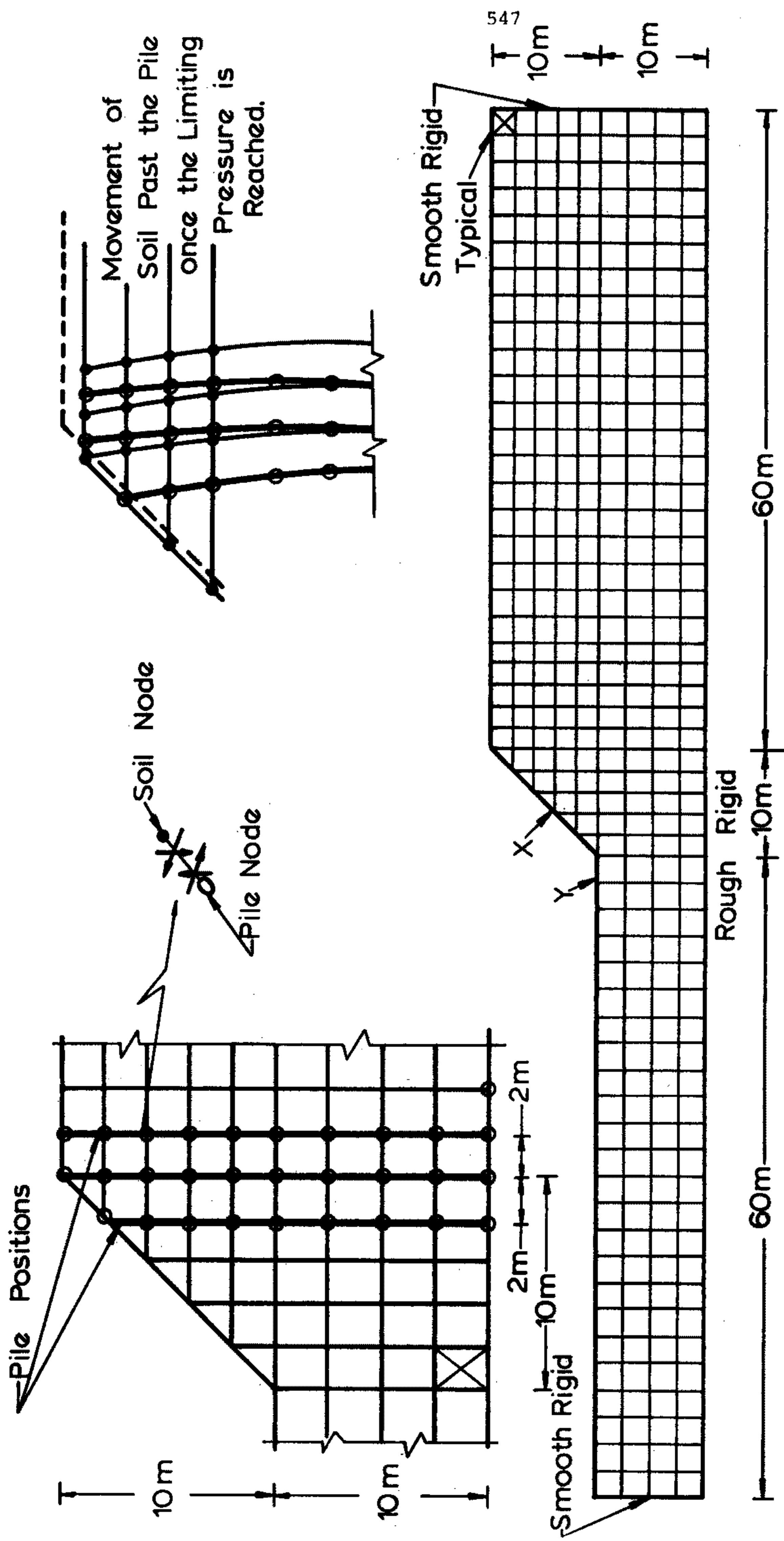


FIG.10.22 FINITE ELEMENT IDEALISATION OF SOIL SLOPE.

from an elastic pile group analysis (e.g. Poulos, 1971)

- (2) Determining the lateral response of the idealised pile arrangement for a range of stiffnesses (e.g. from an elastic finite element analysis); and
- (3) Equating the lateral response of the real pile group with the lateral response of the idealised pile group to give an equivalent stiffness of the plane strain piles.

The value of the limiting soil pressure which can act at any particular point along an isolated pile depends both upon the shape of the pile and the depth below the surface to the point in question. A summary of the various solutions for the ultimate soil pressure acting on a pile at considerable depth below the surface has been given by Broms (1964a). It will be recognised from the foregoing chapters that the limiting pressure for a thin, wide pile (ie. rectangular in section with width $\gg$ thickness) would correspond to the ultimate bearing pressure which may act on an anchor plate (viz. $q_u/c = 11.4$). However, this value is too high for a circular pile and a limiting pressure of $q/c = 9$ appears to be more generally accepted (although the value may vary between 8 and 12 depending upon pile shape and roughness). The variation in ultimate bearing pressure with depth ranges from $q/c = 2$ at the surface to the 'deep' pile value at a depth of approximately 3 pile diameters (Poulos and Davis, 1978). In the proposed analysis, the pressure of the soil acting on the pile is manifest as nodal forces required to ensure compatibility of pile and soil deflections. Thus a limiting nodal force, corresponding to the limiting pressure over a region of the pile, can be specified for each node and once this value is attained then compatibility requirements are no longer enforced at that point, thereby allowing the soil to flow (ie. slip) past the pile.

The value of the limiting nodal force may vary from point to point, allowing freedom to select the distribution of ultimate bearing pressure considered appropriate.

The shielding effect of piles within a group upon the ultimate bearing pressure may be determined by performing analyses such as described in section 10.3. In particular, the efficiencies ' R_S ' obtained for anchor groups in section 10.3 may be used to give an approximate estimate of shielding effects of piles upon the ultimate bearing pressure. As previously mentioned, a comparison between the shielding effect predicted for circular piles by Yegian and Wright (1973) and that predicted by the author for strip anchors (ie. slender rectangular piles) indicated agreement to better than 10%. Thus the ultimate bearing pressure at a point of an isolated pile may be approximately modified for the effect of group interaction by using the efficiency factor R_S . Clearly, this modification to the limiting pressure which may act on a pile leads to an identical modification to the limiting nodal force which may be attained at each point on the idealised (plane strain) pile, and hence the effects of group interaction upon soil flow past the pile groups may also be incorporated into the analysis.

10.4.3 Problem Description and Finite Element Idealisation

A hypothetical 10m high slope in soft clay [layer (a)] resting on a 10m thick clay stratum [layer (b)] and underlain by a rough rigid base, was considered [see Fig. 10.23(a)]. The slope was assumed to have a 1:1 grade. Two soil profiles were examined, namely, a homogeneous soil mass and two layered soil mass. The soil parameters for these two pro-

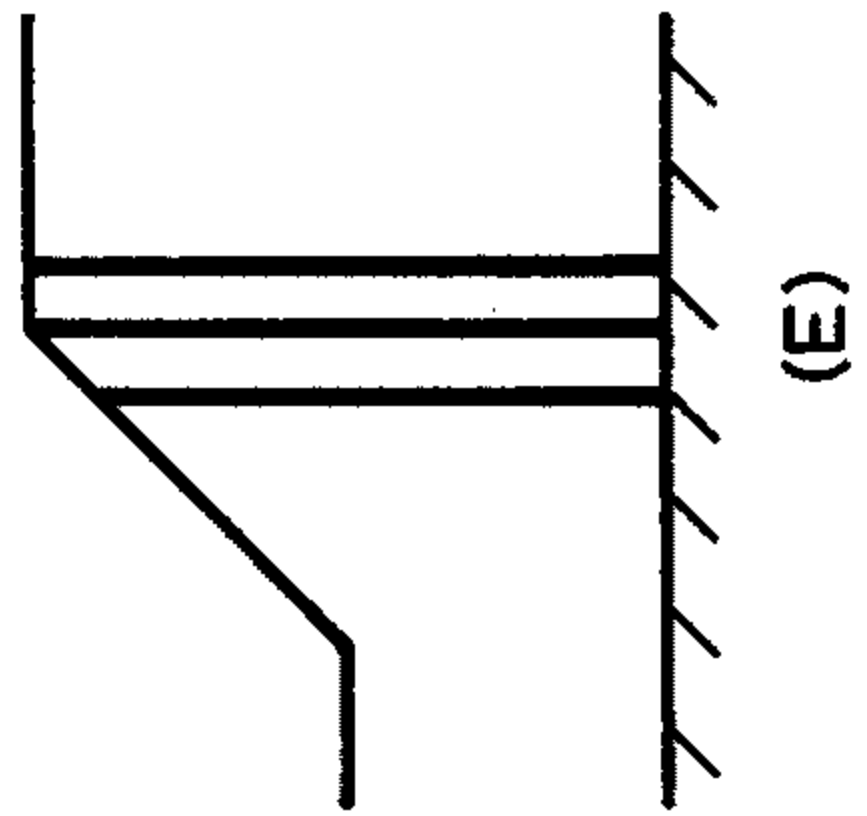
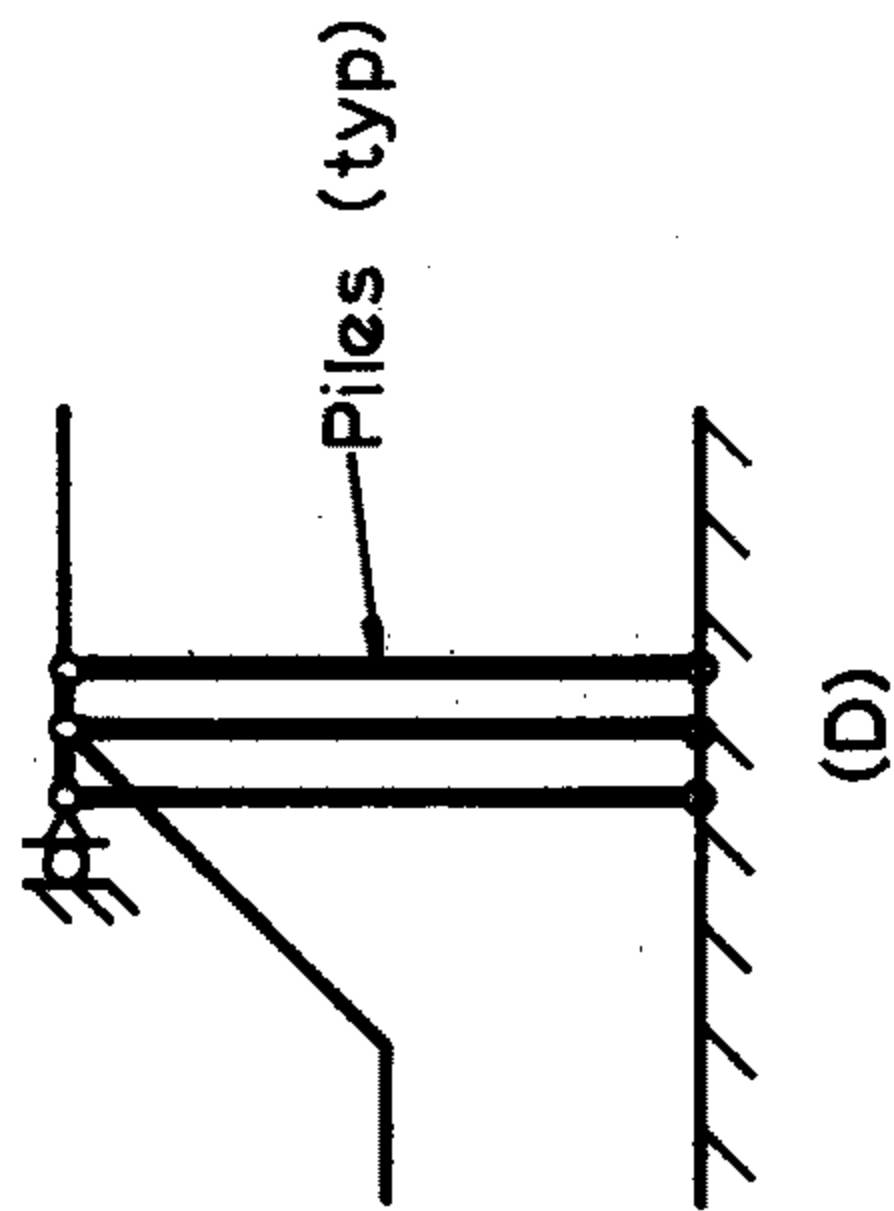
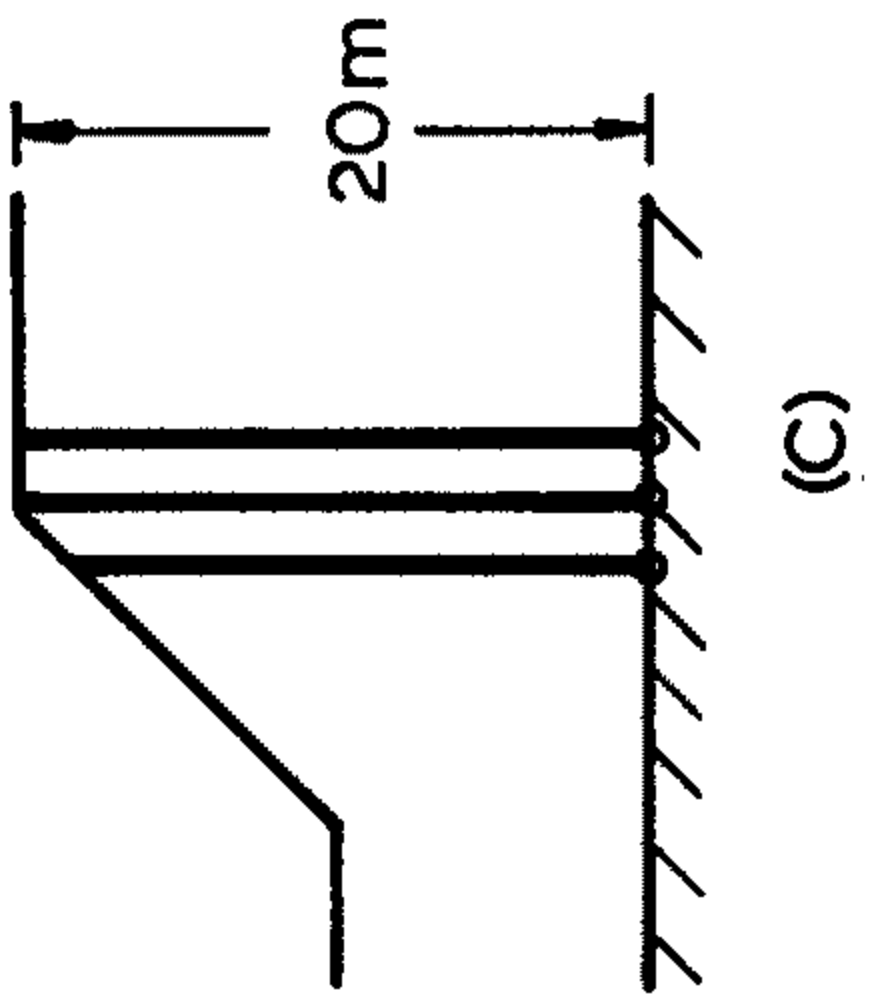
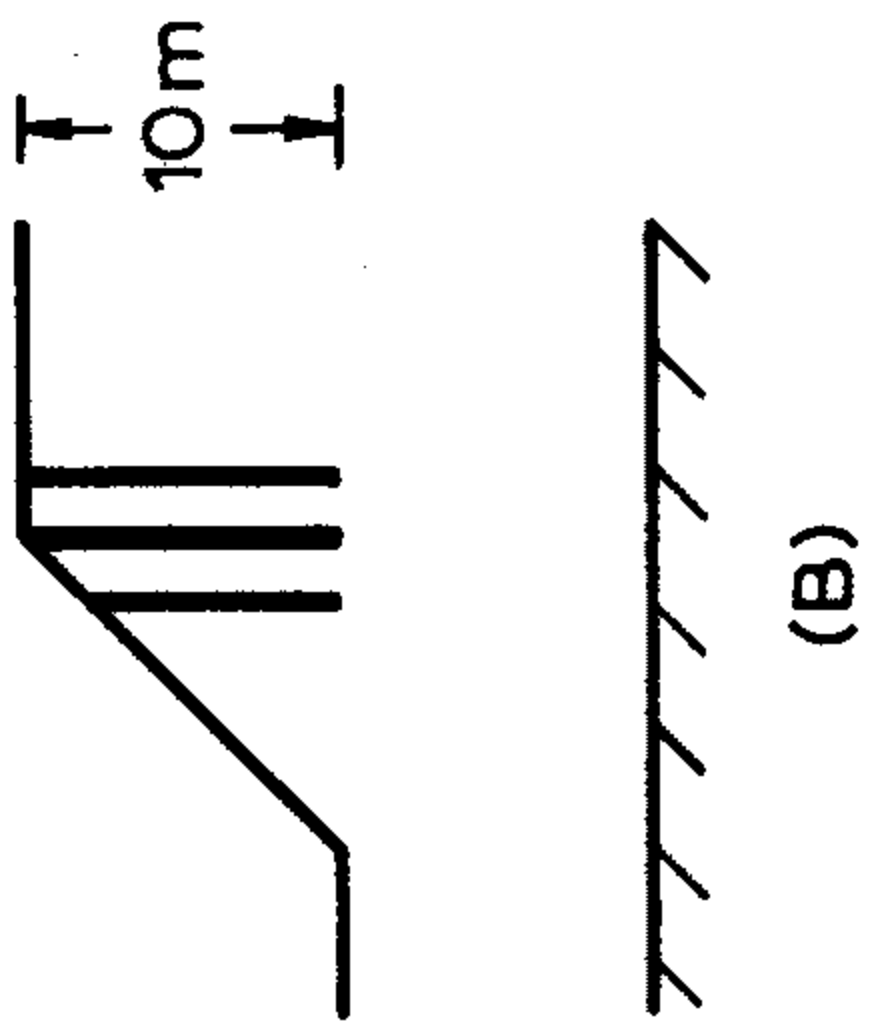
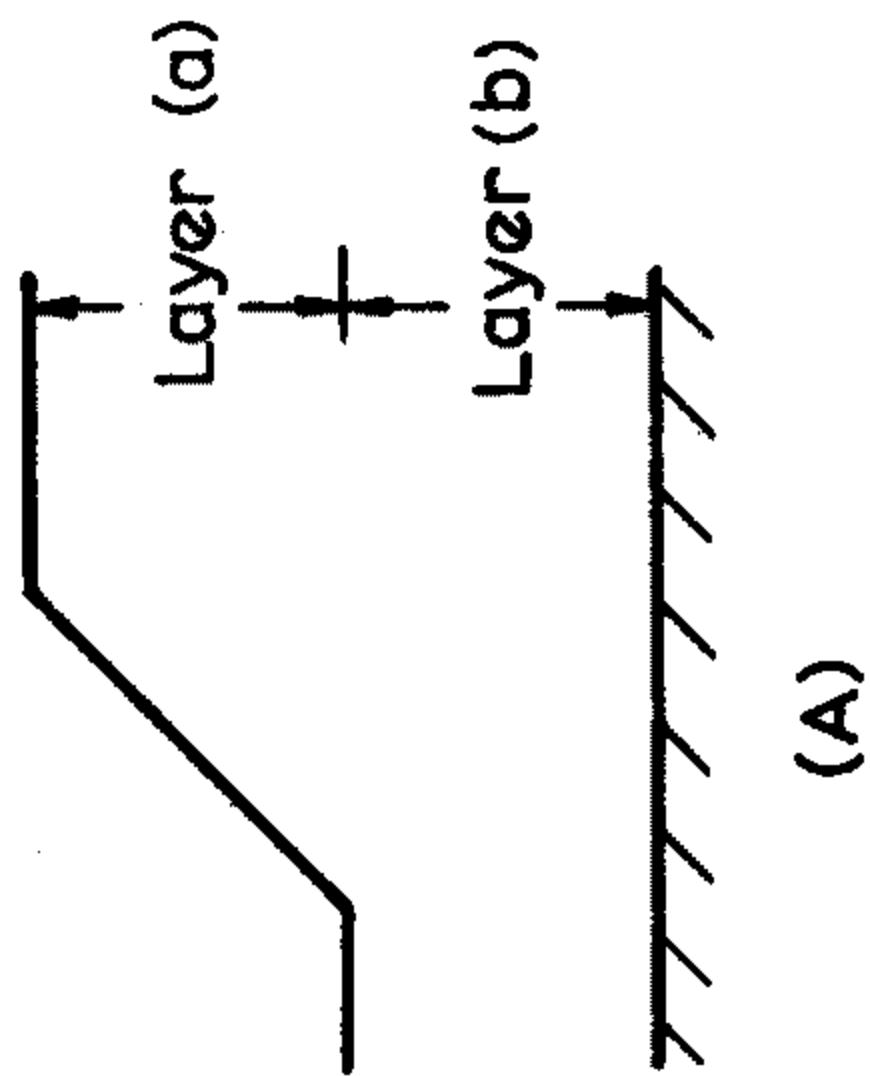


FIG.10.23 PILE ARRANGEMENT DETAILS

files are given in Table 10.3 and the finite element idealisation of the soil mass is shown in Fig. 10.22.

TABLE 10.3

SOIL PARAMETERS FOR THE SLOPE

Soil Type	Layer (see Fig. 10.23a)	E (kPa)	ν	c (kPa)	ϕ
Homogeneous	a & b	9000	0.48	30	0°
Layered	a	9000	0.48	30	0°
	b	18000	0.48	60	0°

To illustrate the effect of the piles upon the performance of the slopes, a particular pile group consisting of three rows of piles with a centre to centre spacing of 2m was examined. For this spacing, two pile diameters, d , were considered; namely, $d = 0.5\text{m}$ and $d = 1.0\text{m}$, corresponding to a dimensionless spacings S/d of four and two respectively. The effect of shielding upon the ultimate bearing pressure ($q/c = 9$ for an isolated pile) was determined from the analysis given in section 10.3 (ie. see Fig. 10.19).

The piles within the group were subject to a variety of restraint conditions as shown in Fig. 10.23 and summarised in Table 10.4.

For purposes of comparison, three stiffnesses (EI) were considered for the idealised pile: $EI = 2.1 \times 10^4 \text{ kN-m}^2/\text{m}$

$$EI = 4.2 \times 10^5 \text{ kN-m}^2/\text{m}$$

$$EI = 8.4 \times 10^6 \text{ kN-m}^2/\text{m}$$

This corresponds to a 400 fold range in stiffness.

TABLE 10.4

PILE CASES CONSIDERED

Case	Pile Length (m) (From top of slope)	Pile Boundary Conditions	
		Top	Bottom
A	0	No Pile	
B	10	free	free
C	20	free	pinned
D	20	No lateral displacement	pinned
E	20	free	fixed

To facilitate the comparison of the relative performance of different pile systems, an incremental density construction sequence was adopted. This approach has the advantage of simplicity, however, caution is required in interpreting the physical meaning of displacements obtained from the analysis. This does not create serious difficulties when comparing the relative performances of different systems although careful consideration must be given to the physical implications of any construction sequence when it is desired to predict actual deformations for comparison with experimental observation.

In addition, consideration must be given to the method by which the piles are inserted and the condition of the embankment or slope at the time of insertion. For example, significant negative friction forces may develop between pile and soil if end bearing piles are inserted before incrementing the soil density. For the purposes of this

analysis, it was assumed that no negative friction developed between pile and soil, since this was considered to be the most realistic situation. However, it should be recognised that the development of negative skin friction may significantly alter the performance of the pile-soil system. It is anticipated that negative friction would have a beneficial effect upon the performance of the slope provided it does not induce failure within the piles.

Finally, in the following analyses, it has been assumed that no failure would take place within the pile. This simplifying assumption was made to reduce the number of parameters to be studied, and it is recognised that in practice the moment resistance of the pile may be a major factor influencing the performance of the pile reinforced slope. As previously stated, the provision for the formation of plastic hinges within the pile(s) is quite straightforward.

10.4.4 Effect of Piles Upon the Deformation and Stability of the Slope

The results obtained from the analyses of slope behaviour may be subdivided into four series as indicated in Table 10.5. In particular Table 10.5 summarises the reduction in the lateral displacement (R_L) at locations 'X' and 'Y' (shown in Fig. 10.22) as well as the reduction in vertical heave (R_H) at location 'Y' obtained for the various cases where:

$$R_H \text{ or } R_L = \frac{\text{displacement obtained for a particular case with piles}}{\text{displacement of the same point when there are no piles}}$$

To allow a reasonable comparison to be made between the effect of various pile arrangements upon the performance of the slope, the displacement ratios (R_H , R_L) and maximum bending moment M_{Max} were compared for a ty-

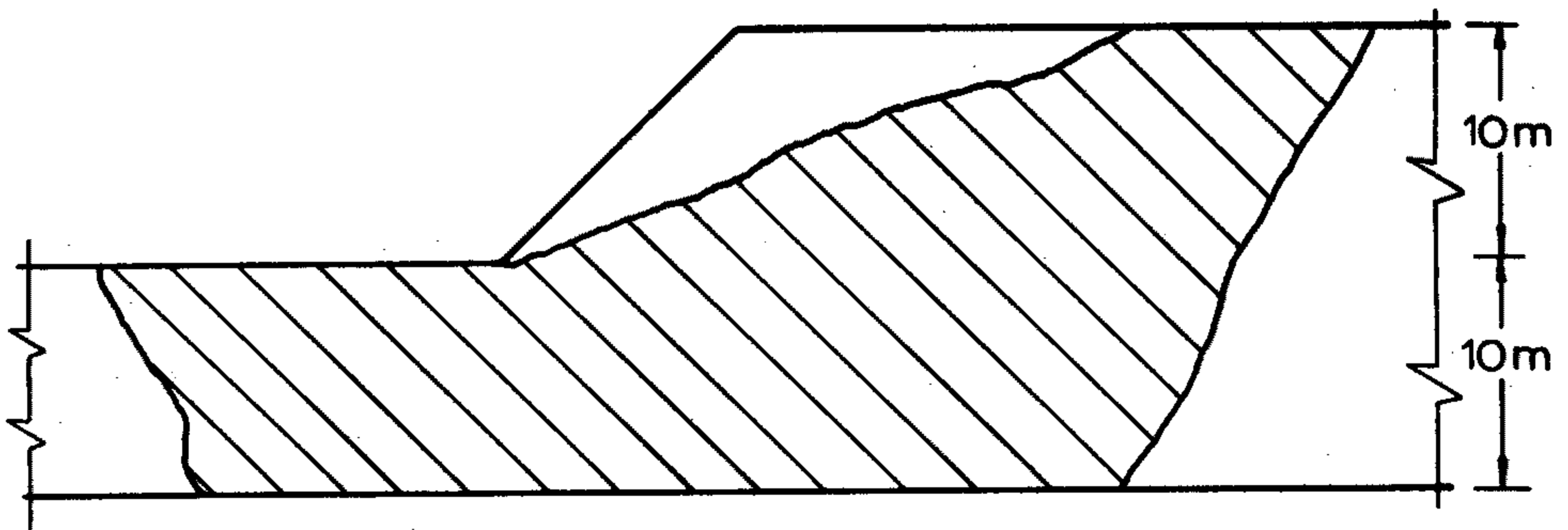
TABLE 10.5

COMPARISON OF VARIOUS PILE ARRANGEMENTS

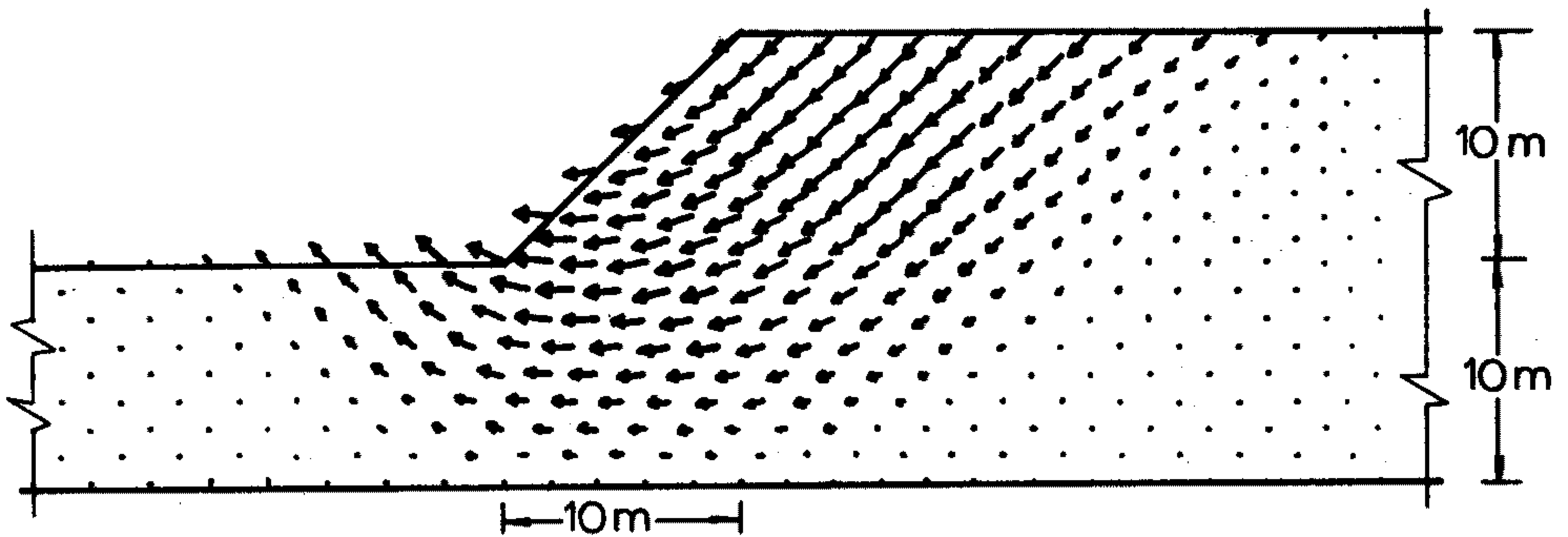
Series	Soil	Slope	Arrangement (see Fig. 10.23)	Spacing S/d	$\frac{(EI)_p}{E_s L^4}$	Reduction in Displacement Elastic @ $\gamma = 16 \text{ kN/m}^3$			Maximum moment @ $\gamma = 16 \text{ kN/m}$ (kN-m/m)	Maximum moment when $\delta_L (@X) = 250 \text{ mm}$ (kN-m/m)	% increase in Factor of Safety
						$R_L @ X$	$R_L @ X$	$R_L @ Y$	$R_H @ Y$		
(i)	Homogeneous	1:1	A	-	0	1.0	1.0	1.0	1.0	-	0
				4	1.46×10^{-5}	0.993	.85	.80	.72	132	4.0
				4	2.91×10^{-4}	0.972	.67	.53	.29	700	12.5
				4	5.83×10^{-3}	0.905	.58	.38	.13	1280	22.6
(ii)	Homogeneous	1:1	A	-	0	1.0	1.0	1.0	1.0	-	0
				4	2.33×10^{-4}	0.986	.97	.99	.99	18.4	0.6
				4	1.46×10^{-5}	0.993	.85	.80	.72	132	4.0
				4	1.46×10^{-5}	0.733	.54	.58	.36	176	12.5
				4	1.46×10^{-5}	0.968	.74	.64	.47	481	10.1
(iii)	Homogeneous	1:1	A	-	0	1.0	1.0	1.0	1.0	-	0
				2	2.91×10^{-4}	0.972	.67	.53	.29	735	13.1
				2	2.91×10^{-4}	0.505	.29	.33	.10	2420	94.5
				2	2.91×10^{-4}	0.770	.47	.34	.09	3090	37.6
(iv)	2 Layers	1:1	A	-	0	1.0	1.0	1.0	1.0	-	0
				4	1.46×10^{-5}	0.975	.92	1.0	.99	187	23.0

pical value of soil unit weight $\gamma = 16 \text{ kN/m}^3$. Similarly, the maximum moments within the piles were compared for a loading corresponding to a lateral displacement at the point 'X' of 250mm, which generally corresponded to the situation just prior to collapse of the slope. Finally, the increase in the factor of safety (relative to that for an unreinforced slope) is also shown for the various pile arrangements. In fact the collapse loads obtained for the homogeneous slope, $\gamma = 16.5 \text{ kN/m}^3$ may be directly compared with value of $\gamma = 16.8 \text{ kN/m}^3$ obtained from Taylor's charts, indicating agreement to within better than 2%. The plastic region and velocity field at collapse for the 1:1 unreinforced homogeneous slope are shown in Fig. 10.24.

Fig. 10.25 shows the deformed profile of the homogeneous slope with $\gamma = 16 \text{ kN/m}^3$ for the four cases considered in series (i). From this, it can be seen that pile stiffness significantly alters the lateral displacement of the slope, and the heave at the toe of the slope; however, the rate of improvement with increasing pile stiffness is relatively slow. It will be appreciated from Fig. 10.24 that the failure of the slope is associated with a deep seated movement of the soil. Accordingly, the introduction of piles has the greatest effect on the deformation at the toe of the slope and the least effect upon the lateral movement of the slope itself. This situation arises because the piles (even for large stiffnesses) tend to move with the soil, and although they offer some restraint to the deep seated movement, the rotation of the pile about the base still allows quite significant deformation of the slope. In fact, for pile arrangements such as shown in Fig. 10.23(b) and Fig. 10.23(c), the piles moved with the soil mass to such an extent that failure of the slope could be deemed to have occurred before any soil flow, relative to the pile, was evident.



(a) Plastic Region



(b) Velocity Field.

FIG.10.24 PLASTIC REGION AND VELOCITY FIELD AT FAILURE OF A 1:1 SLOPE.

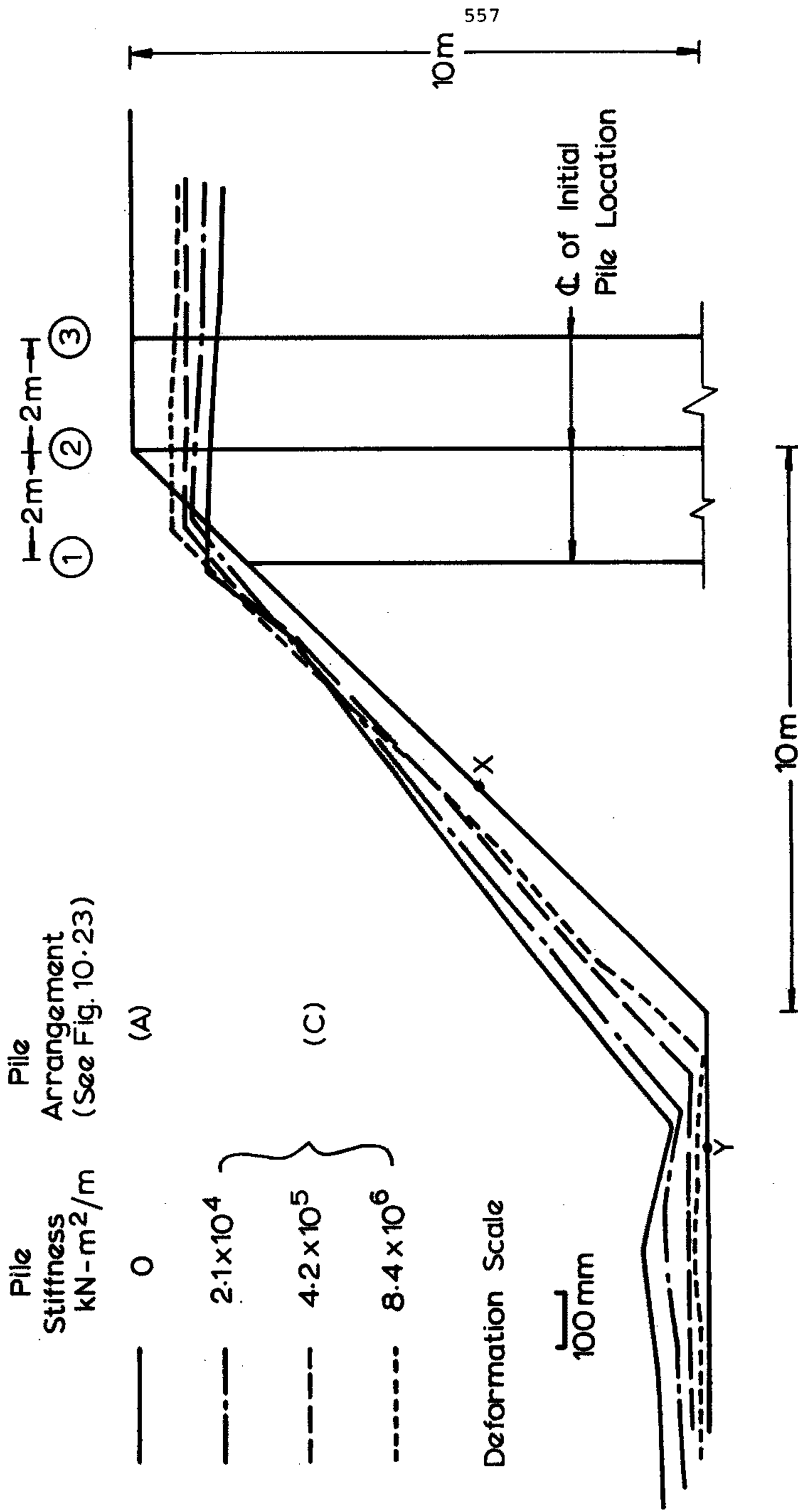
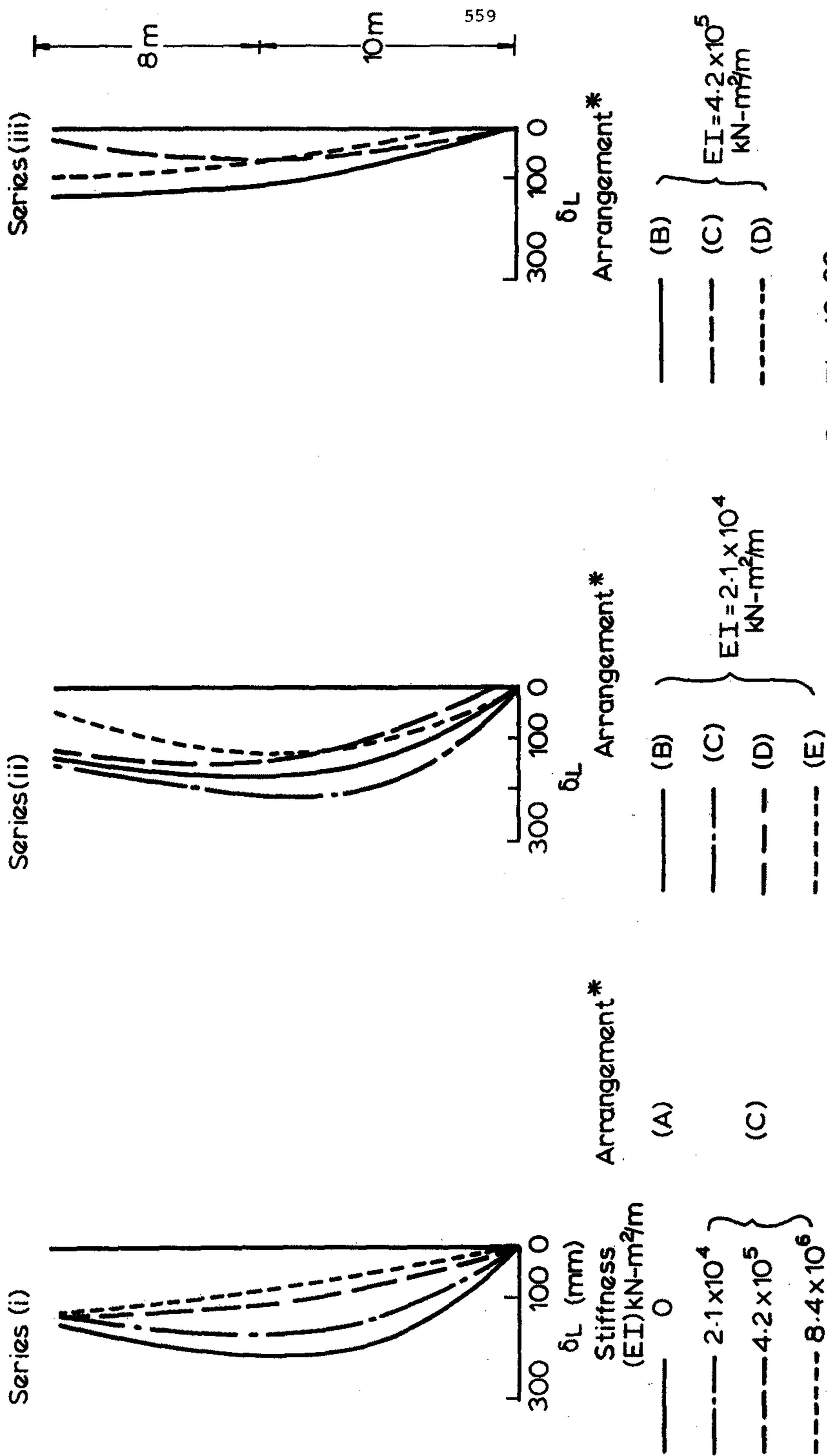


FIG.10.25 EFFECT OF PILE STIFFNESS UPON SLOPE PROFILE - SERIES (i).

The effect of pile stiffness upon the lateral deformation of the soil adjacent to the leading row of piles may be assessed from Fig. 10.26 and it can be seen that while increasing the pile stiffness significantly decreases the lateral deformation near the centre of the layer, it has very little effect on the deformation at the top of the slope. On the other hand, the bending moments within the pile (see Fig. 10.27) increase dramatically with increasing stiffness. Finally, it is of some interest to note that a twentyfold increase in pile stiffness only increased the slope stability by about 9%.

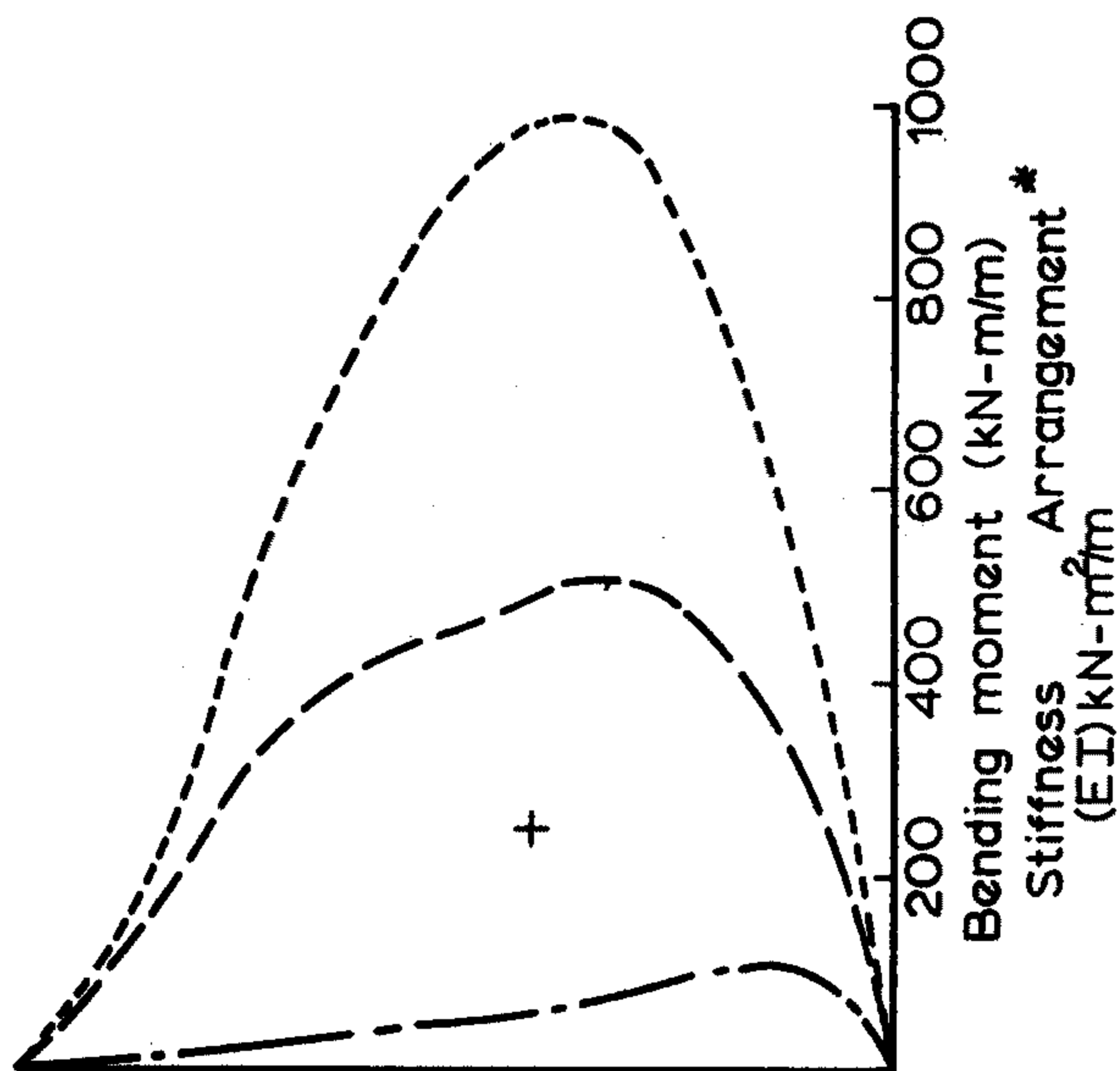
The effectiveness of piles in reducing the deformation and increasing the stability of slopes depends both on the pile stiffness and the fixity of the pile. To illustrate this point, Figs. 10.28 and 10.29 show the slope profile obtained for various pile arrangements where the idealised (plane strain) pile has an equivalent stiffness $EI = 2.1 \times 10^4 \text{ kN-m}^2/\text{m}$ [series (ii)] and $4.2 \times 10^5 \text{ kN-m}^2/\text{m}$ [series (iii)] respectively. The corresponding lateral deformation (for $\gamma = 16 \text{ kN/m}^3$) at the location of the leading pile is shown in Fig. 10.26. Considering firstly the case where the piles have stiffness $EI = 2.1 \times 10^4 \text{ kN-m}^2/\text{m}$, it will be seen that the partially penetrating pile (arrangement B) only reduces the slope deformations by less than 3% while increasing slope stability by less than 1%. This result confirms the intuitively obvious proposition that piles which do not intersect the failure surface will not have any meaningful effect on slope performance. However, even fully penetrating piles pinned to the base stratum only increase the slope stability by about 4% compared to a 10% increase when the piles were fully restrained and a 12.5% increase for a pile pinned at the base and restrained against lateral displacement at the pile head. The importance of the provision for flow of the soil past (ie. through) the idealised



*See Fig. 10-23

FIG.10-26 EFFECT OF PILE ARRANGEMENT AND STIFFNESS UPON LATERAL DISPLACEMENT δ_L - AT LOCATION OF PILE 1.

Series (i)



(A)

0

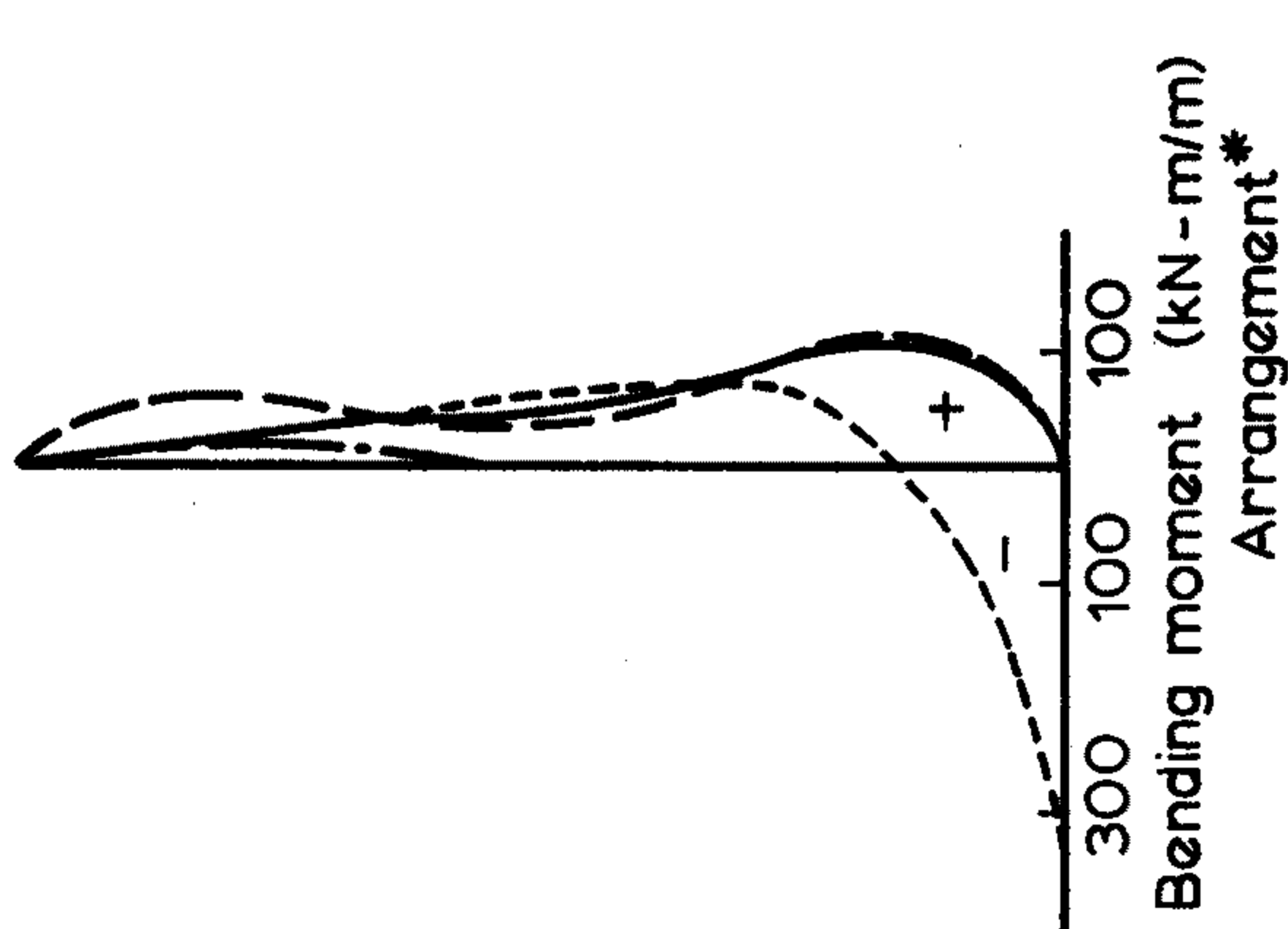
2.1×10^4

4.2×10^5

8.4×10^6

(C)

Series (ii)



(B)

(C)

(E)

(D)

$EI = 2.1 \times 10^4$
 $\text{kN-m}^2/\text{m}$

* See FIG.10-23

FIG10-27 EFFECT OF PILE ARRANGEMENT & STIFFNESS UPON BENDING MOMENT IN PILE 1.

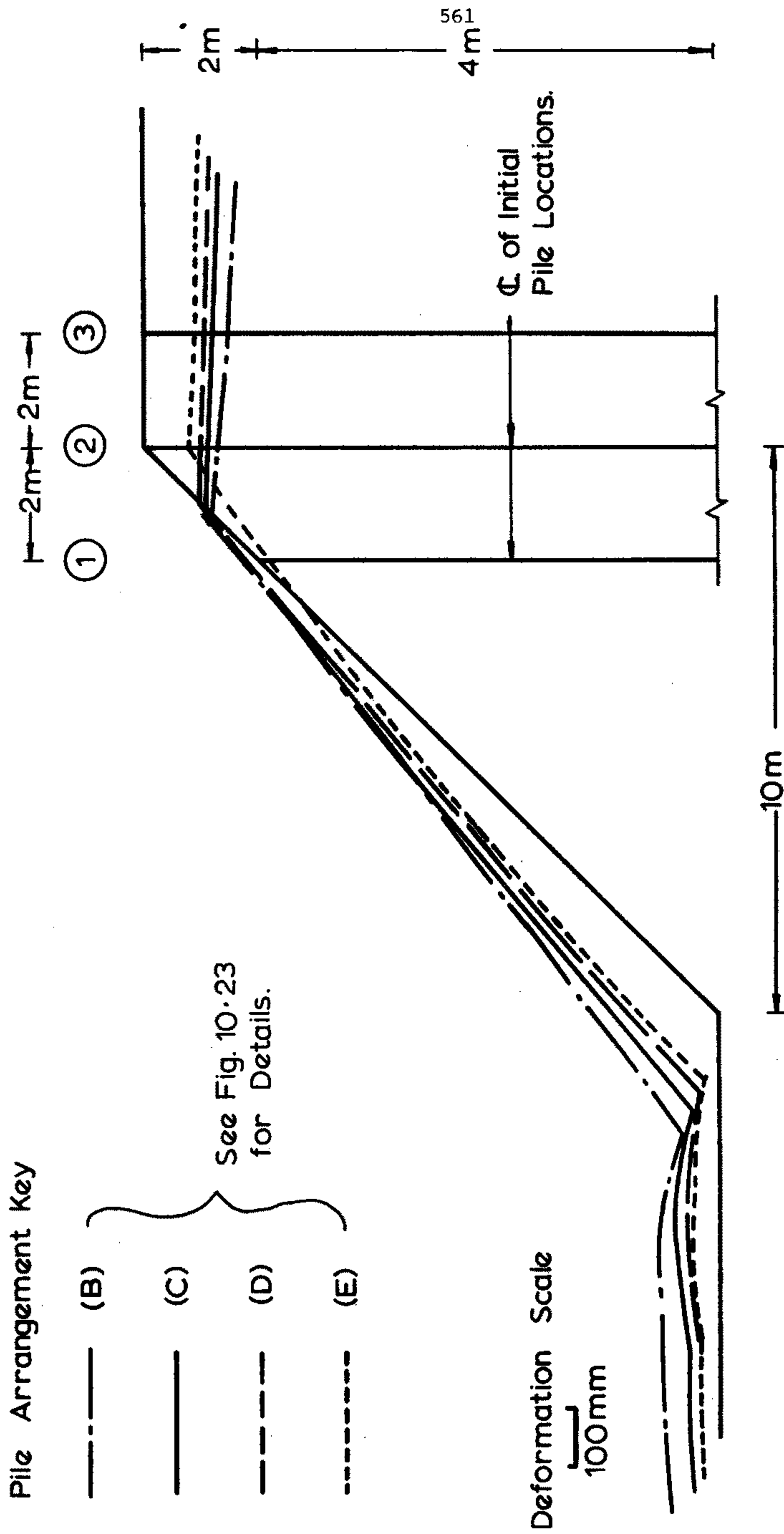


FIG.10.28 EFFECT OF PILE FIXITY UPON SLOPE PROFILE - $\gamma = 16 \text{ kN/m}^3$
PILE STIFFNESS $EI = 2.1 \times 10^4 \text{ kN-m}^2/\text{m}$ - SERIES(ii).

Pile Arrangement Key

- (B)
- - - (C)
- - - (D)

See Fig.10.23
for Details.

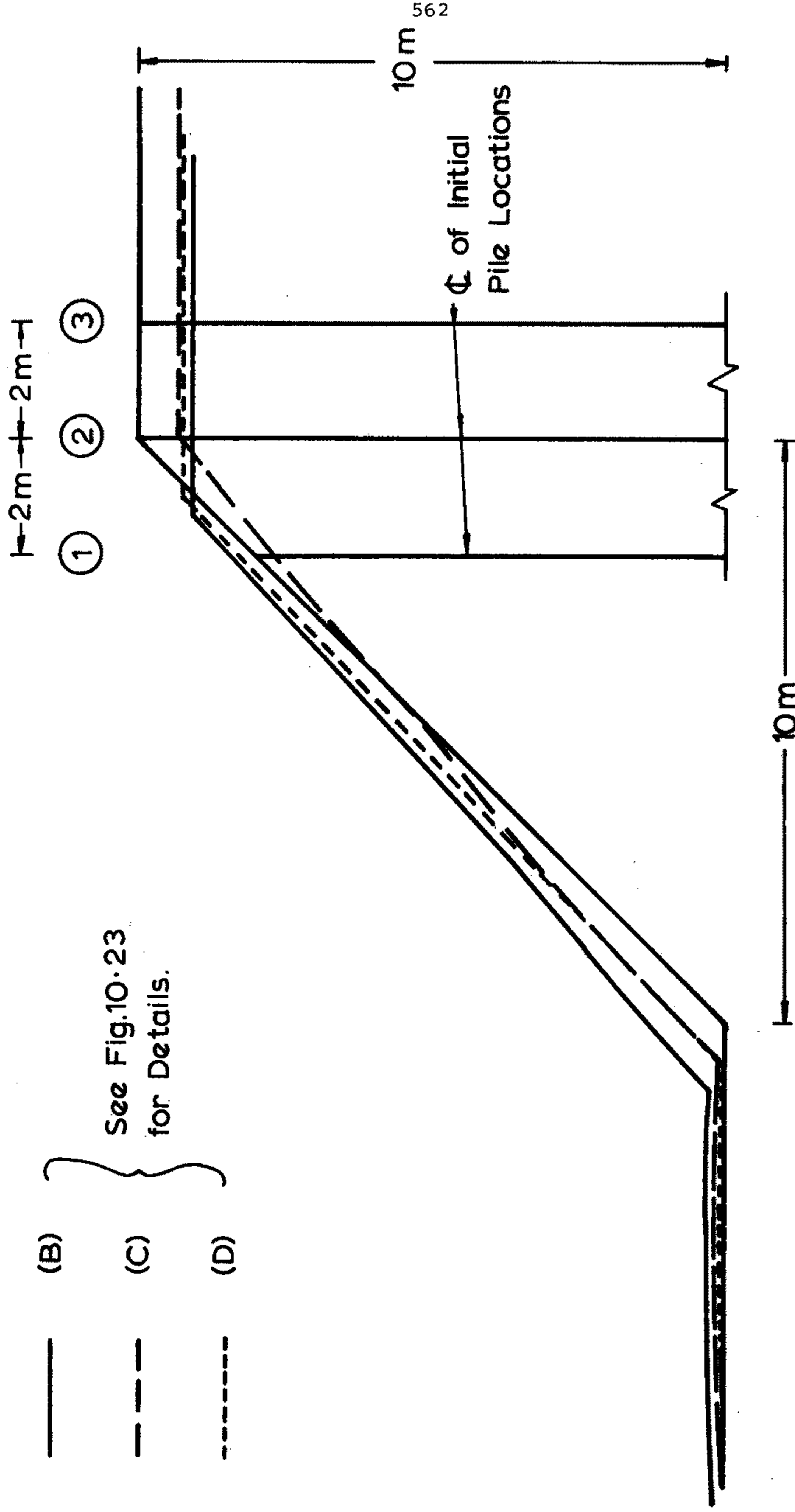


FIG.10.29 EFFECT OF PILE FIXITY UPON SLOPE PROFILE - $\gamma = 16 \text{ kN/m}^3$
PILE STIFFNESS $EI = 4.2 \times 10^5 \text{ kN-m}^2/\text{m}$ - SERIES (iii).

pile increases with the degree of pile restraint, and pile stiffness. In fact, at this equivalent pile stiffness (which approximately corresponds to 0.5m diameter concrete piles at 2m centre to centre spacings), quite significant deformation of the piles was observed, and consequently there was very little flow of the soil past the piles.

As might be expected, the bending moments (see Fig. 10.27) in the partially penetrating pile were negligible. Significant bending moments were obtained for the fully penetrating pile with a pinned base (arrangement C) although these moments would generally not be large enough to cause failure of the pile (assuming a reasonably reinforced concrete pile). The provision for lateral restraint at the pile head (arrangement D) altered the bending moment distribution (compared with arrangement C) but did not greatly increase the maximum bending moment. On the other hand, the maximum bending moment for the fully built-in pile (arrangement E) was approximately three times that obtained for either arrangements C or D.

An examination of slope performance for pile arrangements C, D and E using a stiffness $EI = 4.2 \times 10^5 \text{ kN-m}^2/\text{m}$ [ie. a twenty-fold increase in stiffness compared to series (ii)] indicated the same general trends as already described for the more flexible piles, although the differences between the various arrangements were magnified. In particular, pile arrangement D (in which lateral displacement at the head of the pile is prevented) had the most beneficial effect on both the deformation and the stability of the slope.

The restraint provided by the prevention of rotation at the base of the pile (arrangement E) was found to decrease the slope defor-

mations and increase the slope stability quite significantly. However, the bending moments induced at the built-in base are very large and it may be anticipated that the level of restraint assumed in this analysis would not be achieved in practice.

The foregoing analyses have all been performed for a homogeneous slope, but it may be anticipated that the effectiveness of piles in reducing slope deformations and increasing slope stability will depend greatly upon the soil profile. To illustrate the possible effect of non-homogeneity, a two-layered system was analysed as indicated in Fig. 10.30. The deformations of the slope in the layered case were less than those for the homogeneous case. Because of the different nature of the failure surface, the reduction in displacement along the slope (at $\gamma = 16 \text{ kN/m}^3$) due to the introduction of piles was less than 8% compared to a reduction of up to 15% for the homogeneous case. By contrast, for the layered case the introduction of piles (arrangement C, $EI = 2.1 \times 10^4 \text{ kN-m}^2/\text{m}$) increased the factor of safety of the slope by 23% (relative to the same slope and layering without piles), as compared to only a 4% increase in factor of safety using similar piles in the homogeneous soil. The bending moment diagram (see Fig. 10.30) is also significantly different in the two cases, and it may be inferred that the distribution of bending moments within a pile depends upon both the location of the failure surface and the soil profile.

In fact, the shape of the bending moment distribution observed in practice varies significantly from case to case. The observed deformation of a 20m long by 0.32m diameter pipe pile at the site of the Higashitomo landslide (Fukuoka, 1977) indicated that the pile was bent in single curvature and the general shape of the bending moment diagram

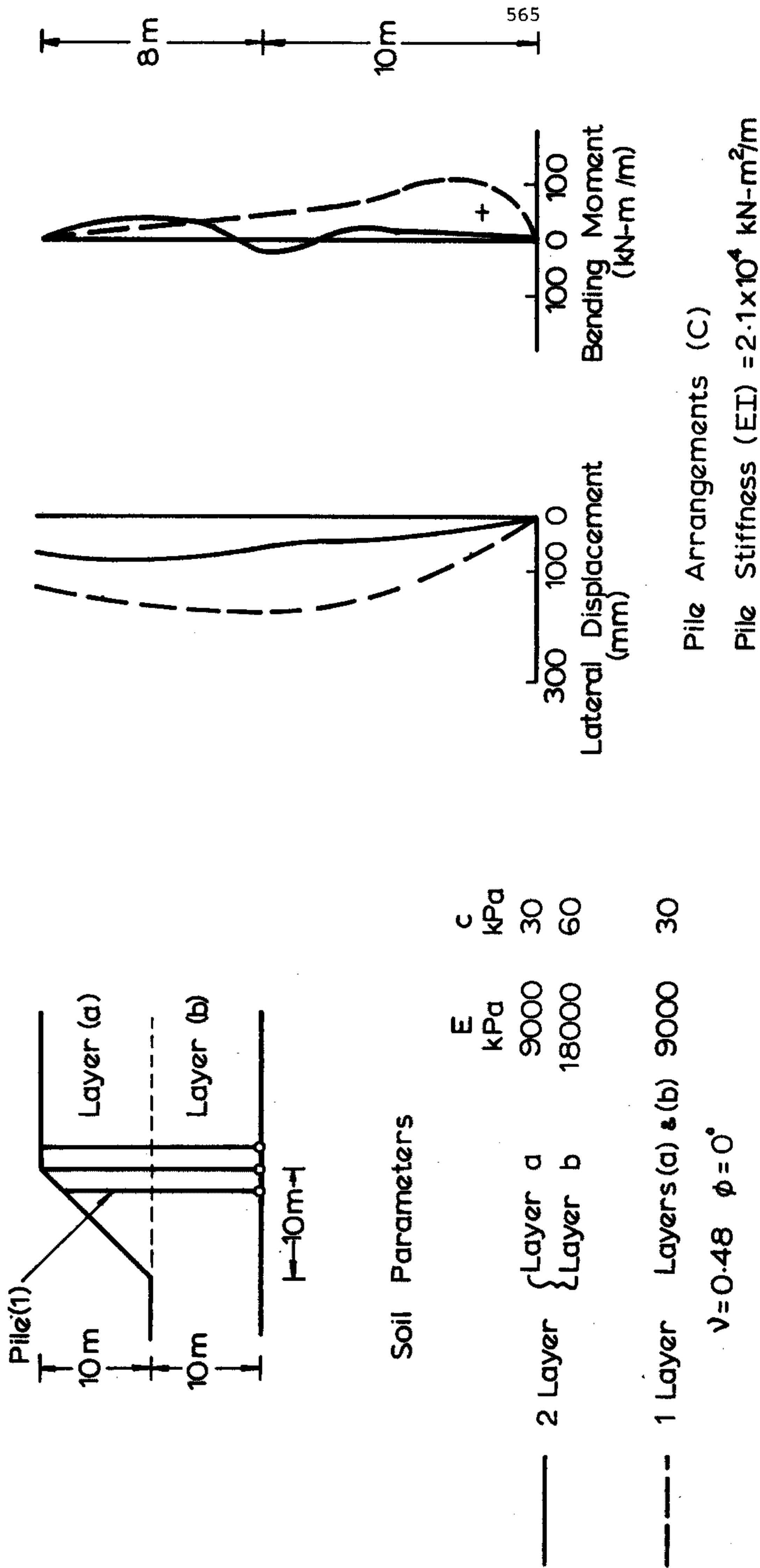


FIG.10.30 EFFECT OF HOMOGENEITY UPON DEFORMATION AND BENDING MOMENTS AT PILE 1
 $\gamma = 16$ kN/m³ SERIES(iv)

was similar to that shown for a homogeneous slope in Fig. 10.30. On the other hand, field measurements relating to a 20m long by 0.4m diameter pipe pile at the site of the Higashitomo landslide (Fukuoka, 1977) indicated that the pile was bent in double curvature and the bending moment distribution involved significant positive and negative bending moments. This type of behaviour is consistent with a soil profile in which the soil modulus increases significantly with depth. Unfortunately, very little soil data is given in either case and hence it was not possible to obtain a direct comparison between theoretical predictions and field observations.

10.4.5 Conclusions

A technique which may be used for predicting the performance of pile-reinforced clay slopes has been presented. This approach has advantages in that both the deformation and stability aspects of the behaviour may be considered simultaneously and without the need for arbitrary assumptions regarding the deformed shape of the pile or the location of the failure surface. In fact, provision can be made for plastic failure in the soil and pile, and in an approximate sense, provision can also be made for the flow of soil past the piles.

A brief parametric investigation into the effect of piles upon the performance of a particular slope geometry was undertaken. With regard to the problem considered, the following conclusions would appear to be warranted:

- (1) The effect of piles upon slope deformation and stability increases very slowly with increasing pile stiffness;

consequently, it may be necessary to use very stiff piles to obtain any significant improvement in slope performance.

- (2) The effectiveness of piling is enhanced by provision of restraint at the top or bottom of the pile; however the bending moment also increases rapidly with both pile stiffness and pile restraint indicating that failure of the pile itself needs careful consideration in these cases. It was found that the most effective pile restraint corresponded to arrangement D (Fig. 10. 23) where lateral displacement was prevented at both the top and bottom of the pile.
- (3) The effectiveness of piling is enhanced in cases where the soil stiffness and strength increases with depth, while the bending moment distribution within the piles appears to depend both upon the geometry of the slope and the soil profile.

The results of this investigation raise some questions as to the effectiveness of piles in improving the performance of slopes. In particular, the results indicate that the performance of the slope varies significantly with pile arrangement, stiffness and restraint condition as well as with the soil profile. However, because of the complex interaction between the various parameters influencing the pile-slope behaviour, it is very difficult to form general conclusions regarding the effectiveness of using piles to enhance slope performance. For the same reasons, it would appear that empirical generalisations and extrapolations should also be viewed with considerable caution.

CHAPTER ELEVEN
SUMMARY AND CONCLUSIONS

11.1 INTRODUCTION

This thesis has been concerned with the development and application of techniques for the analysis of soil-structure interaction problems. In particular, analytical methods involving the classical theories of elasticity and plasticity as well as the finite element load path method have been used to investigate the behaviour of anchor plates.

The motivations for the detailed study of anchor plate behaviour presented in this thesis were two fold: firstly, to obtain a better understanding of the behaviour of anchor plates; and secondly to develop techniques and experience based on the anchor problem which could be extended to more complex problems. The factors affecting both the analysis and the behaviour of anchor plates are common to more complex soil-structure interaction problems. Consequently, attention has been directed primarily at the analysis of an isolated anchor plate because the results of numerical load path analyses could be compared with analytical results obtained using the classical theories of elasticity and plasticity for certain cases, and with experimental data obtained from model and field tests. Clearly, if reliable solutions cannot be obtained for this relatively simple problem then one cannot place confidence in the analysis of more complicated forms.

Subsequently, the techniques and experiences obtained in the analysis of anchor-soil behaviour have been utilised to highlight the effect of a number of the more significant soil or structural parameters upon the behaviour of three complex soil-structure interaction problems.

In the following section, the major findings of this thesis will be summarised. General conclusions will then be presented.

11.2 SUMMARY

A method of analysis for horizontal anchors in an elastic soil has been proposed (Chapter 3). This method is different to previous approaches used for the prediction of anchor behaviour and allows for the consideration of many factors which could not be conveniently incorporated into earlier methods. These factors include the effect of anchor rigidity, layer depth, anchor-soil interface condition, breakaway of the anchor from the underlying soil and interaction between anchors within a large group.

This approach has been applied to the cases of strip, circular and rectangular anchors in a homogeneous, isotropic elastic soil, and solutions have been presented and discussed for a wide range of material and geometric parameters.

In Chapter 4, the use of classical plasticity theory for the prediction of collapse loads was discussed and a number of limit solutions for horizontal strip anchors in a purely cohesive material were presented. The usefulness of classical plasticity was restricted by the fact that solutions could only be obtained for certain well defined problems in highly idealised materials. In addition, classical plasticity theory does not provide any information regarding the effect of load path or initial stress state upon the collapse load, nor does it provide any information regarding the deformation and behaviour of the

soil prior to collapse. However, classical plasticity solutions are particularly useful as a benchmark against which the accuracy of numerical collapse loads may be assessed.

In subsequent chapters, the use of a finite element load path approach for the analysis of soil-structure interaction problems was investigated. In Chapter 5, the underlying assumptions and the development of the incremental equations for an elasto-plastic material were outlined and consideration was given to the use of economical techniques for performing non-linear analyses. A general method was then proposed for the analysis of soil-structure interaction problems. This approach retains the well known advantages of sub-structuring and the initial-stress technique while making provision for plastic failure of both soil and structure as well as for breakaway or shear failure at a frictional-dilational soil-structure interface. The significance of these factors was then illustrated by considering a number of examples.

In Chapter 6, the general features of an elasto-plastic load path analysis were examined, and particular consideration was given to the factors affecting the numerical accuracy of collapse loads. Based on this experience, two techniques were presented for the prediction of collapse loads. The first is a simple extrapolation technique; and the second attempts to overcome the inhibition of free plastic flow inherent in the usual stiffness formulation of the finite element method, by permitting the formation of velocity discontinuities in regions of high stress and velocity gradient. Both techniques give a better estimate of the collapse load for a particular problem than would otherwise have been obtained with the same degree of numerical sophistication.

The general behaviour of horizontal anchor plates was investigated (Chapter 7) using the techniques for soil-structure interaction analysis which were developed in the preceding chapters. The effects of soil cohesion, angle of internal friction, body force and embedment ratio were examined and a set of basic bearing capacity factors was determined. The effects of secondary parameters such as soil dilatancy, the coefficient of lateral earth pressure at rest, anchor roughness and soil layer depth were also considered.

In many cases it was found that, although the collapse load was easily identified, significant deformation occurred before the ultimate collapse load was reached. In these cases, the 'safe foundation load', obtained by dividing the collapse load by the factor of safety, would produce deformations in excess of those predicted by an elastic analysis. Consequently, to allow the adoption of a conventional engineering approach to foundation design (ie. separate deformation and stability analyses) it was found necessary to define the 'failure' load (which may be less than or equal to the actual collapse load) such that reasonable settlement predictions could be made at the maximum working load determined from bearing capacity considerations. In this context, it was found convenient to define the predicted failure load as the load which would give rise to a displacement four times that predicted by an elastic analysis. Failure loads defined on this basis were found to be largely insensitive to elastic parameters of the soil (e.g. Young's modulus and Poisson's ratio) and to the depth of the soil layer beneath the anchor.

The critical embedment ratio (ie. anchor depth/anchor width), at which a transition from shallow to deep anchor behaviour was observed

varied with material properties. For cohesive materials this critical embedment ratio ranges from four for $\phi = 0^\circ$ to more than eight for ϕ greater than 15° . For purely frictional material, the critical embedment ratio was greater than eight for all values of ϕ considered (ie. $15^\circ \leq \phi \leq 45^\circ$).

Soil dilatancy was found to have a significant effect on the general behaviour of the soil-anchor system, and in particular upon the ultimate collapse load. The effect of dilatancy increases rapidly with embedment ratio and it may be concluded that analyses of anchor behaviour in moderately dense granular material may be considerably in error if the effect of dilatancy is neglected.

The initial stress state within the soil was also found to have a notable effect upon anchor behaviour and, under certain conditions, failure to consider this effect will lead to an unconservative estimate of the collapse load. On the other hand, anchor roughness and the depth of the layer below the anchor both had negligible effect upon the ultimate behaviour of horizontal anchors.

It was found that overburden pressure, as well as any adhesion or suction between anchor and soil, may increase the bearing capacity significantly. In these cases, failure of the anchor was usually clearly defined and occurred at, or shortly after, complete breakaway of the anchor from the underlying soil. In purely cohesive material, it was found that there was a theoretical limit to the contribution of overburden pressure (or of adhesion and suction) to the bearing capacity. Furthermore, from a practical viewpoint, it would appear that the contribution of these factors to the bearing capacity

is affected by the different response of soil in expansion and compression, the possibility of cavitation of the pore water and the uncertainty as to the magnitude of any adhesion or suction between anchor and soil.

From a comparison of bearing capacity factors obtained for circular and strip anchors in a purely cohesive material, it was inferred that the strip corresponds to the most critical shape for an anchor. Consequently, the use of bearing capacity factors presented for a strip would lead to a conservative estimate of the collapse load for rectangular (or circular) anchors. Indeed, experimental investigations indicate that below a limiting aspect ratio, the bearing capacity of rectangular anchors increases with decreasing aspect ratio and reaches a maximum for a square anchor. This trend was confirmed by the author's model tests (Chapter 8).

In Chapter 8, the experimental results from model anchor pullout tests in overconsolidated clay and normally consolidated sand were presented and compared with the theoretical predictions of Chapter 7. These comparisons were then supplemented by further comparisons between published model and field data for circular and strip anchors in clay as well as for rectangular anchors in sand.

In general, the experimentally observed behaviour was found to be in quite good agreement with the theoretical predictions, although a number of limitations of the theoretical approach were detected. Specifically, it was found that for very shallow anchors (with an embedment ratio less than 2.5) in clay, the theoretical bearing capacity exceeded those observed experimentally, and this was

attributed to the dominant effect of flexural tensions upon the behaviour of shallow anchors. With regard to the bearing capacity of rectangular anchors in clay, the experimental results indicate that anchors with an aspect ratio equal to or greater than five behave as if they were strip anchors. A decrease in aspect ratio below five corresponds to an increase in bearing capacity but this is largely offset by the increase in importance of tension cracking upon anchor performance. In fact, the compensating factors are such that it appears (somewhat fortuitously) that in practice the bearing capacity factors for strip anchors would give a reasonable prediction of bearing capacity for both rectangular and circular anchors in clay.

The behaviour of anchor plates in sand was found to be significantly influenced by the anchor shape, and for aspect ratios less than or equal to five, the experimental bearing capacities were notably greater than those predicted from a plane strain analysis. However, encouraging agreement between theoretical and experimental bearing capacities was observed for anchors with aspect ratios greater than eight.

The behaviour of vertical anchors was investigated (Chapter 9) using the soil-structure interaction theory of preceding chapters, and a series of bearing capacity factors were presented. In general, the effect of the various parameters upon the predicted behaviour of vertical anchors was similar to that observed for horizontal anchors. The actual magnitude of the bearing capacity factor (for a given embedment ratio) was usually higher for vertical anchors, although the bearing capacity of deep anchors in a cohesive weightless material was independent of the orientation of the anchor.

Two important differences between the behaviour of vertical and horizontal anchors were observed. Firstly, a reduction in the value of the coefficient of lateral earth pressure at rest tended to increase the bearing capacity of vertical anchors while reducing the bearing capacity of horizontal anchors. Secondly, the effect of anchor roughness upon anchor behaviour has been found to depend upon the orientation of the anchor. From Chapters 3 and 7 it is quite clear that anchor roughness has no significant effect on the behaviour of horizontal anchors; however depending upon the embedment ratio and material properties, roughness can considerably enhance the load carrying capacity of vertical anchors. The effect of roughness decreases with embedment ratio and increases with the dilatancy of the material. In particular, the effects of roughness and dilatancy interact giving much higher collapse loads than would be expected from a simple combination of the two effects, while at the same time increasing the range of embedment ratios over which the roughness is of importance.

Comparisons of the predicted collapse loads for vertical anchors with available experimental data showed encouraging agreement, while indicating the importance of determining the dilatancy and initial stress state of the material, as well as the contact roughness of the anchor, in making reasonable predictions of anchor behaviour.

In Chapter 10, the techniques and experience of the foregoing chapters were utilised in the analysis of three significant soil-structure interaction problems.

Firstly, the influence of anchor position, wall flexibility

and anchor dimensions upon the performance of a tied-back retaining wall was considered. From this analysis, it was found that the performance of the anchor-wall system depends upon both the relative stiffness of the wall and the location and size of the supporting anchor. For instance, the bending moments were most affected by the wall flexibility whereas the maximum deformation of the wall was more sensitive to the location and size of the anchor plates. It was also found that the magnitude of the anchor rod force was governed by the anchor dimension and embedment ratio. The anchor rod force did not vary appreciably with the distance between anchor and wall.

As might be expected, the stability of the anchor-wall system decreased with decreasing distance between the anchor and wall; however, the presence of the anchor increased the stability of the cutting significantly even when the anchor was located within the Rankine 'active' zone behind the wall. Thus, the stabilising effect of anchors which were closer than the recommended minimum distance between anchor and wall (British Civil Eng. Code of Practice No. 2, 1951) was considerably greater than would be anticipated from conventional engineering considerations (e.g. wedge analysis or slip circle). On the other hand it would appear that the adoption of the recommended spacing between anchor and wall does not ensure that there is no interaction between wall and anchor. Indeed, the results of this investigation would suggest that the interaction between wall and anchor depends both on the distance to the anchor, and on the size of the anchor plate relative to the height of the wall.

Secondly, the interaction between strip anchors within large groups was investigated for the particular case of deep anchors resting

in a purely cohesive soil mass. It was found that the efficiency of an anchor was not seriously affected by the proximity of other anchors within the same plane. However, both the elastic and ultimate efficiency of anchors in series (ie. the components of a multiple anchor) were significantly reduced by the proximity of adjacent anchors.

Finally, an approximate method for predicting the performance of pile reinforced clay slopes was presented. This approach has the advantage that both the deformation and stability aspects of behaviour may be predicted from a single analysis. Furthermore, this approach avoids the necessity for arbitrary assumptions regarding the deformed shape of the pile or the location of the failure surface. Provision is made in the analysis for plastic failure within both the soil and pile, as well as for flow of the soil past the piles.

A parametric investigation into the effect of piles upon the performance of a particular slope was described. For the problem considered, it was necessary to use very stiff piles to obtain any significant improvement in slope performance. The effectiveness of a particular pile arrangement was improved by either restraining the pile against lateral deflection at the top, or against rotation at the base; however, this leads to relatively high bending moments within the pile, and the possibility of failure in the pile itself needs careful consideration in these cases. The effectiveness of piling was enhanced in cases where the soil modulus and strength increased significantly with depth, although even in this case the results of this investigation raised some questions as to the effec-

tiveness of piles in stabilising soil slopes.

11.3 CONCLUSIONS

In this thesis, a method has been developed for the analysis of soil-structure interaction problems. It has been demonstrated that this method of analysis may conveniently be adopted in the analysis of quite complex problems. The method involves an incremental load path analysis of the soil-structure system and hence the entire load history of the system may be followed from installation to working load conditions, and may then be pursued until complete collapse of the system takes place. By adopting numerical modelling techniques, unsatisfactory deformations of a soil-structure system may be predicted at load levels which would appear to be quite safe from conventional stability considerations.

Particular attention has been given to the difficulties involved in predicting the stability of soil-structure systems and a number of practical techniques for economically obtaining accurate collapse loads, using a finite element load path analysis, have been proposed.

The behaviour of anchor plates has been studied in considerable detail. Specifically, a method for the analysis of horizontal anchors in an elastic soil was developed and utilised in the analysis of strip, circular and rectangular anchor plates. The results of a parametric study were presented in the form of influence charts which may be used to predict the elastic displacement of anchor plates for a wide range of geometric and material parameters. Extension of

the proposed method of analysis to allow consideration of multiple anchors, multi-layer soil strata or cross-anisotropic soils is quite straightforward.

The general behaviour of both horizontal and vertical anchor plates was studied for a wide range of material and geometric parameters. In many cases, it was found that unsatisfactory performance of the anchor occurred at load levels well below the actual collapse load. In these cases, a practical definition of failure was adopted. Bearing capacity factors were presented and it is anticipated that these may be used to obtain a reasonable estimate of the failure load for many problems. Comparison between the theoretical predictions and the behaviour observed from model and field tests in both clay and sand indicated encouraging agreement. Furthermore, the theoretical analysis confirmed previous suggestions, based on experimental data, that the interaction between factors such as soil dilatancy, initial stress state and anchor roughness may have a significant effect upon anchor behaviour. These factors have been ignored in previous methods of analysis.

With regard to the undrained behaviour of anchors in clay, the contribution of overburden pressure (and, similarly, of adhesion or suction between the anchor and soil) to the bearing capacity of the anchor was considered; however it is recommended that in practice, the bearing capacity of anchor plates in clay should be determined on the assumption of immediate breakaway between the back of the anchor and the soil.

From the results of the investigation into anchor behaviour,

it does appear that the techniques developed in this thesis may be applied successfully to practical foundation engineering problems.

These techniques have also been used to give insight into the behaviour of three complex soil-structure systems. It has not been possible to undertake detailed investigation into the performance of these systems; however, the effects of some of the more significant parameters have been highlighted. It is anticipated that work performed in this thesis will form the basis of a more detailed investigation into problems of this type. In particular, it is hoped that the profession can be encouraged to make greater efforts to monitor and record the performance of soil-structure systems so that direct comparison can be made with theoretical predictions.

The objective of this thesis has been to develop techniques and experience which could be conveniently and economically utilised in the analysis of a wide range of soil-structure interaction problems. However, numerous extensions could be made to the present analysis. For example, consideration of the soil as a visco-elastic-plastic material is possible, while several, more refined, theories of plastic failure (e.g. Roscoe, Schofield and Wroth, 1958; Lade and Duncan, 1975) could be adopted. The incorporation of consolidation and large strain effects into this analysis is also possible, although it should be recognised that the computational costs of such a comprehensive analysis would be extremely high, and it is considered that these additional refinements of the analysis would probably only be justified for specific cases where extensive investigation, analysis and subsequent monitoring of the soil and soil-structure system is to be undertaken.

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ABBREVIATIONS

ASCE	American Society of Civil Engineers
ECSMFE	European Conference on Soil Mechanics and Foundation Engineering
ICSMFE	International Conference on Soil Mechanics and Foundation Engineering
JGED	Journal of the Geotechnical Division, ASCE
JSMFD	Journal of the Soil Mechanics and Foundations Division, ASCE
JSCE	Japanese Society of Civil Engineers
SM and FE	Soil Mechanics and Foundation Engineering.

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