Full-Range Moment-Rotation Behaviour of Bolted Moment End-Plate Joints

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A thesis submitted in partial fulfilment of requirements for the degree of Doctor of Philosophy

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Statement of Originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes.

I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged. Assistance received comprises the correction of grammar of Chapter 1-7 of the thesis.

Signature: [Redacted]

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Abstract

The thesis summarises research recently conducted at the University of Sydney on developing a joint model based on the component method for predicting the full-range moment-rotation behaviour of steel joints, including elastic, inelastic, post-ultimate and post-fracture response. The overall context of this work is to produce models for joints that are sufficiently simple and accurate that they can be implemented in beam-element based finite element models of steel frames, so as to be able to predict the strength of steel frames by Geometric and Material Nonlinear Analysis with Imperfections (GMNIA), or “advanced” analysis, accounting for the nonlinear behaviour and strength of joints.

The thesis first summarises the Generalised Component Model which includes elastic, inelastic and softening springs to describe the full-range behaviour of each component and the complete joint model. Having set out the equilibrium and compatibility equations, the Method is applied to bolted moment end-plate joints and compared to tests carried out on these joints as part of this thesis. Particular attention is paid to the buckling of the column web plate by presenting a component model specifically developed for this part of the bolted moment end-plate joint. Thus, the thesis contains the following parts:

1. Generalised Component Model
2. Tests on bolted moment end-plate joints
3. Component model for column web plates
4. Comparison between the Generalised Component Model and tests

Conclusions are drawn about the suitability of the Generalised Component Method for predicting the full-range moment-rotation behaviour of bolted moment end-plate joints.
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Chapter 1: Introduction

1.1 Background

In most design practices, conventionally designed structural steel joints over-simplify joint behaviour into two extreme cases (Hogan, 2009): ideally pinned (flexible) and fully rigid. An ideally pinned joint is free to rotate and therefore transmits no bending moment between the connected members, whereas a fully rigid joint provides full compatibility in rotation and transfers bending moments between connected members. However, no real joint behaves exactly like these two extreme cases (pinned or rigid) and therefore should be treated as “semi-rigid” joints (Wu, 1988), where partial bending moments are transferred and some relative rotation develops between connected members. For example, an angle cleat connection is usually assumed to be pinned, but it can resist some bending moment so the design beam dimensions may decrease due to moment redistribution. Similarly, an extended end-plate joint classified as a rigid joint may adversely affect some members if the loss of rigidity in the “rigid” construction of the joint occurs. This discrepancy may result in a joint that is either ineffective or unsafe. Therefore, its semi-rigid behaviour must be assessed to improve its robustness. Further introduction to basic issues such as joint decomposition, classification and ductile response is contained in Appendix A1.

The behaviour of a semi-rigid joint can be represented by a moment-rotation \((M - \theta)\) curve, obtained by the component method and empirical modelling. When these two methods are compared for flexibility, the component method was chosen for the research program described in this thesis. The method is currently used in Eurocode 3 (2006b), but the provisions of the Eurocode 3 based component method are limited to predictions of initial stiffness and moment capacity.
The component method specified in the early versions of Eurocode 3 could only predict a linear $M - \theta$ curve and did not include inelastic behaviour. However, the linear curve does not cover many important characteristics and is insufficient for structural analysis of many practical joints. To overcome this disadvantage, in 1998, Weynand et al. (1996) proposed an additional empirical expression to represent the nonlinear post-yield joint behaviour and this empirical segment was adopted in the current Eurocode 3 version. A theoretical method has since been developed in the early 2000s by da Silva et al. (da Silva and Girao Coelho, 2001b, da Silva et al., 2000, da Silva et al., 2002) where they added a preloaded spring to the usual elastic spring to simulate post-yield behaviour. Subsequently, Lewis (2010b) proposed a method based on the work of da Silva et al., to model the post-ultimate spring.

By following da Silva et al. and Lewis’ path, this thesis extends the component method to a general form which can model any complex joint and cover the full-range $M - \theta$ curve of any joint, including post-ultimate and post-fracture behaviours, although in this thesis the extended full-range component method formulation is applied exclusively to the bolted moment end plate connection. The full-range $M - \theta$ curve is necessary knowledge for analysing the structural collapse and earthquake/dynamic response of a building.

Another gap in our knowledge is the full-range column web buckling behaviour. Even though research into column web behaviour has already existed for more than 50 years, only a conservative method has been developed and included in Eurocode 3. This conservative method for predicting the buckling strength of a column web underestimates the potential of this component and results in unnecessary stiffening of the column. Since a relatively slender column or unstiffened column may be more cost efficient, it is worth developing an accurate method to predict the full-range column web in compression component that can lead to slender and unstiffened column design. Moreover, a column web in compression component plays a major role in some designs, such as in rigid end-plate joints, so an accurate predictive method may improve the accuracy of the joint model.
1.2 Significance

Bijlaard (2006) suggested that the cost of joints is almost 50% of the total cost of a steel structure, and this cost will further increase if column stiffeners or web doublers are installed. These additional stiffeners are usually due to an underestimation of column strength and joint flexibility. Therefore, if the joint ductility and column web behaviour can be predicted accurately, a more efficient design may substantially reduce the cost of the structure.

1.3 Objectives and Scope

The goal of this thesis is to develop a practical method to model the full-range moment-rotation ($M - \theta$) curve for structural steel joints. This method will be formulated by a closed form formulation that avoids numerical problems.

The objectives and scope specified in this research are:

- Devise a generalised component based model that extends the component method to the post-ultimate range and post-fracture range.
- Study the column web in compression component and develop an analytical model for the full-range force-displacement behaviour of this component. For other key individual components, modelling techniques are reviewed, critiqued, and modified, if necessary.
- Design and perform 13 bolted moment end-plate joint tests with four different rig setups. Observe and record the joint and key components behaviours including failure of end-plates, bolts and column webs.

Model six end-plate joints from the aforementioned experiments by Girão Coelho et al. (2004b) using the presented method and compare the model predictions with experiment data.
Chapter 2: Literature Review

2.1 Introduction

This Chapter reviews two major practical design methods, both of which focus on obtaining the joint stiffness (Section 2.2), one of the major parameters of a joint. To obtain joint stiffness means first obtaining component stiffness, which is reviewed in Section 2.3. Then, an experimental assessment (Section 2.4) and finite element (FE) study (Section 2.5) are reviewed since they can be used to acquire \( M - \theta \) data for validation and to carry out a parametric study. Although this thesis does not involve high-fidelity finite element modelling, the review of FE method is still included for the convenience of researchers in this field.

2.2 Review of joint modelling in practical design

According to Diaz et al. (2011b), in practical design the \( M - \theta \) curve of a joint can be modelled by the component method and the empirical method, each of which has the following limitations:

- The component method is a theoretical method based on the mechanics of joints and its components. It provides sufficient flexibility and reliability in joint design and is therefore adopted in design code Eurocode 3 (2006b). The history of research into this method is reviewed in Section 2.2.1.

- The empirical method is a curve fitting based method where the formula generated is relatively concise and hence is convenient to apply in practical design. However, an empirical formula is only applicable to joints having the same characteristics as those used to generate it. Therefore, this method lacks flexibility. The history of research into this method is reviewed in Section 2.2.2.

2.2.1 Component Method

The component method is based on the mechanics of a joint that is separated into its constituent components. Each component is represented by a spring with its own stiffness,
which is then assembled into a macro model, or spring model, as per its constitutive law (Figure 2.1). The spring model is simplified by two equivalent springs at the tension and compression faces, and the stiffness of each spring is calculated. Finally, by combining the stiffness of the tensile and compressive springs, the stiffness of the joint can be obtained. Based on this procedure, any joint can be modelled.

The component method was first introduced in 1983 by Wales and Rosow (1983). They provided a simple spring model for the double angle connection (Figure 2.2). This model is based on the mechanical properties of double angle components where the major source of deformation is angle bending due to tension.
In 1988, Tschemmerneegg and Humer (1988) suggested using a more complex spring model for end-plate joints; the so called Innsbruck model (Figure 2.3). It is distinctive for having a diagonal spring that was added to represent the web shear panel. This model also considered the ductility of joints where ductility is reproduced by considering the yielding and discontinuity of each spring. In this model the joint yield point is defined as when any of the springs yield, and the ultimate rotation is defined as when any spring fails. Later, the accuracy of their method was further improved (Huber and Tschemmernegg, 1998).
In 1993, the component based joint design method in Eurocode 3 (2006b) was debuted, with the result that European engineers now consider the elastic behaviour of the joints in structural design. Based on Eurocode 3 (2006b), Weynard et al. (1996) introduced a spring model for end plate joints (Figure 2.4). This model further simplifies the shear panel spring into a compressive spring and eliminates the spring for beam flange under compression. Moreover, based on a numerical parametric study and test observations, they introduced an empirical factor for the transition range between the yield point and ultimate point to reproduce the gradual yielding behaviour of a joint. However, on the level of individual components, only linear behaviours are considered, so it leads to conservative estimation of the theoretical ultimate moment and rotation for most joints. The advantage of their model is its simplicity, which is essential for a design tool.
Further development of the component method focused on the predicting ductility (post-yield and post-ultimate behaviour). In 2000, da Silva, et al. (da Silva and Girao Coelho, 2001a, da Silva and Girao Coelho, 2001b, da Silva et al., 2000, da Silva et al., 2002) added preloaded compressive springs into a simplified spring model using an energy formulation to reproduce the post-yield behaviour (not including post-ultimate behaviour). Furthermore, in 2010 this model was extended by Lewis (2010b) to gain the capability of modelling the post-ultimate range.
Del Savio et al (2009) recently proposed a new generalised model using the energy based stiffness matrix method (Figure 2.5). They transformed each component into either a compressive or tensile spring and formulated them into a stiffness matrix using the energy principle. This method did not cover the post-ultimate range.

### 2.2.2 Alternative Methods: Empirical Method

The empirical method (curve fitting) was developed in 1960 to model the behaviour of joints. Although the behaviour of joints is difficult to analyse by simple physical theories due to the complex interaction of the components, observation indicates that the $M - \theta$ curves of some types of joints are similar. Since the analysis of the complicated interactions of individual components is not required, the empirical method has become a popular and practical way to model the behaviour of joints.

According to Jones et al. (1980), the first empirical model for joints was developed by Sommer (1969), who fitted joint $M - \theta$ data with a linear curve. In 1975, Frye and Morris (1975) extended his work and introduced polynomial models for more accurate curve fitting (Eqn. (2.1)).

$$\theta_r = C_1(KM)^1 + C_2(KM)^3 + C_3(KM)^5 \quad (2.1)$$

Where $M$ is the applied bending moment, $\theta_r$ is the joint rotation, $C_1$, $C_2$ and $C_3$ are constants determined by the type of the joint considered, and $K$ is a standardisation constant determined by the type of joint and its geometrical properties. In 1986, Liu and Chen (1986) proposed an exponential model (Eqn. (2.2)) and Kishi and Chen (1986) developed a data bank for curve fitting data sources.

$$M = M_0 + \sum_{j=1}^{m} A_j \left[1 - \exp \left(-\frac{|\theta_r|}{2ja}\right)\right] + \sum_{k=1}^{n} R_k(|\theta_r| - |T_k|)H(|\theta_r| - |T_k|) \quad (2.2)$$

In Eqn. (2.2), $M$ is the bending moment, $\theta_r$ is the joint rotation, $M_0$ is initial joint moment, $a$ is the scaling factor for numerical stability, $T_k$ is the starting rotation of each step $k$, and $H$ is Heaviside’s step function. This model fits the monotonic nonlinear joint moment rotation curve very well (Chen et al., 1996).
Jones et al. (1981) proposed a model based on the cubic B-spline method by Cox (1972). This model fits the non-linear behaviour of a joint very well and circumvents the problem of negative stiffness which is common in curve fitting formulation (Lui and Chen, 1987). However, its application is limited because a large amount of sampling data is needed to obtain a suitable function (Chen et al., 1996). This problem is a common limitation in all empirical models. Thus, further development of the empirical modelling technique focused on reducing its reliance on the size of sampling data. In 1990, Kishi and Chen (1990) suggested a power model using the three or four parameter Richard-Abbott function (Richard and Abbott, 1975) (Eqns. (2.3-2.4)) to model the top and seat angle joint,

\[ M = \frac{R_1 \theta_r}{[1 + (\theta_r / \theta_0)]^{1/n}} + R_{kp} \theta_r \]  

(2.3)

\[ R_1 = R_{ki} - R_{kp} \]  

(2.4)

where \( M \) is the bending moment, \( \theta_r \) is the joint rotation, \( n \) is shape parameter, \( R_{ki} \) is the initial joint stiffness, \( R_{kp} \) is the plastic joint stiffness, and \( \theta_0 \) is the reference plastic rotation. This model is easy to operate, has sufficient accuracy and so is suitable for practical design. Furthermore, in 1994, Bahaari and Sherbourne (1997) extended this method to model end plate joints.

However, the empirical method has an inevitable drawback in that the empirical expression generated can only be applied to joints that are similar to those from the sampling data. Consequently, any new design or joint with an irregular design requires a new formulation and the generated functions must be validated. Thus the empirical methods need constant modification in order to broaden their applicability. One solution may be the hybrid method, because, by combining the component method and the empirical method, it reduces the number of variables required to be determined by curve fitting.

A hybrid method that combines a component method analysis and the empirical method was first suggested by Yee and Melchers (1985). They selected an exponential expression (Eqn.
Chapter 2: Literature Review

(2.5)) to model the shape of the end plate joints. Within this model, the stiffness parameters and bending moment capacity are obtained by analytical methods such as yield line analysis.

\[
M = M_p \left\{ 1 - \exp \left[ - \left( \frac{K_i - K_p + C\theta}{M_p} \right) \theta \right] \right\} + K_p\theta
\]

In Eqn. (2.5), \( M_p \) is the maximum bending moment, \( K_i \) is the initial stiffness, and \( K_p \) is the strain-hardening stiffness (Yee and Melchers also use \( K_p \) to represent post-buckling stiffness if web buckling occurs). \( M_p, K_i \) and \( K_p \) are obtained by analytical methods. Only the value of \( C \) is required to be empirically determined from test data.

Similarly, Kishi and Chen (1990) proposed a hybrid model to predict the bolted angle connections (angle cleat connections and top and seat angle connections). Instead of an exponential model, they used the power model (Eqn. (2.3)) to model the \( M - \theta \) curve, and plate theory to obtain the stiffness parameters, which will be discussed in Section 2.3.4. Nevertheless, the generality of the empirical method is still a disadvantage compared to the component method.

### 2.3 Analysis of Key Components

The behaviours of individual components are the basic elements of the overall model of the joint, and therefore they are essential when assessing the overall joint behaviour. Furthermore, each key component must be analysed and modelled appropriately. Although there are varieties of joint components, as shown by various spring models in Section 2.2.1, only the key components need to be considered. For example, the key components for the end-plate joints tested in this thesis are summarised in Table 2.1. These key components provide the major sources of deformation and determine the overall behaviour of joints, whereas other components are usually much stiffer, and have an insignificant effect on the joint behaviour. For other joints, the key components may differ from those listed in Table 2.1. For example, the column web in tension component deforms significantly and may cause overall failure of some joints with built-up welded column profiles. However, limited to the
experiments conducted in this thesis, only the key components listed in Table 2.1 are investigated.

Table 2.1: Key Components in joints

<table>
<thead>
<tr>
<th>Components</th>
<th>Comments</th>
<th>Types of connection</th>
</tr>
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<tbody>
<tr>
<td>Column Web in Compression</td>
<td>• Strength is estimated conservatively&lt;br&gt;• Buckling effect requires further research</td>
<td>All types</td>
</tr>
<tr>
<td>Column Web in Shear</td>
<td>• Ductile behaviour&lt;br&gt;• Key component in welded connections&lt;br&gt;• Energy absorption component for seismic loading</td>
<td>All types</td>
</tr>
<tr>
<td>Column Flange in Bending</td>
<td>• Idealised T-stub</td>
<td>All types</td>
</tr>
<tr>
<td>End-plate in bending</td>
<td>• Intensively researched since 1980s</td>
<td>End plate connection</td>
</tr>
<tr>
<td>Web Cleat Angle</td>
<td>• Major deformable part is considered as a plate under torsion</td>
<td>Angle connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Web side angle connections with or without top and seat angle)</td>
</tr>
<tr>
<td>Flange Cleat Angle</td>
<td>• Can be idealised as half T-stub</td>
<td>top and seat angle connections</td>
</tr>
<tr>
<td>Bolts</td>
<td>• Prying action may occur, amplifying the applied force</td>
<td>All types</td>
</tr>
</tbody>
</table>
In the following sections the key components listed in Table 2.1 are discussed in detail. The column web in compression discussed in Section 2.3.1 is a focus point of this thesis. Column web in shear is discussed in Section 2.3.2. Column flange in bending, end-plate in bending, and bolts are usually considered as idealised T-Stubs and are discussed altogether in Section 2.3.3. Finally, angles, including web cleat and flange cleat angles, are discussed in Section 2.3.4.

2.3.1 Column Web in Compression

Capturing the deformation of an unstiffened column web in compression is essential, especially when the column is relatively small, or slender. The deformation of a column web is caused by compression force transmitted from the beam flange, and can be treated as a patch load acting on the column flange. In Eurocode 3 (2006b), the compression force is considered as a concentrated load that is dispersed over a small area on the column web due to the column flange which serves as a bearing plate in this situation. The force transmitted onto the column web can cause it to buckle, which means it is a significant contributor to joint rotation. This phenomenon is usually considered by a strength reduction factor for a conservative estimation of the ultimate strength of a column, while the deformation of the column web remains a knowledge gap. To understand the complete range of force-displacement relationship the total rotation of a joint must be accurately estimated.

The first research into this component was by Graham et al. (1960), who tested welded beam-to-column connections to assess the criteria needed for column web stiffeners. This test series consisted of two and four way welded joints and some columns under simulated flange loads. The results from the tests illustrate the following three key facts:

1) Web buckling contributes most of the rotation in unstiffened, thin-web columns, which means the buckling effect is the major consideration of joint rotation in this instance.
2) Axial loads on columns may have almost no influence on the strength and rotation of a connection, because there was no sign of distress when twice the column’s working load
was applied axially. Bose et al. (1972b) discussed this fact and indicated that the axial load should be considered if it varies.

3) Interaction between the compression stress and tension stress are small because they are far apart, therefore a column web in compression can be considered as a rectangular plate in compression with pinned supports at loaded edges (Figure 2.6).

![Figure 2.6: EFFECTIVE AREA OF A COLUMN WEB IN COMPRESSION (SOURCE: BEG ET AL. (2004))](image)

Although these three conclusions were drawn from the tests of welded joints, it is anticipated they still hold for end-plate joints and top and seat angle joints because the forces applied onto these two types of joint are mainly transmitted from the beam flange. The difference is how the applied force spreads through the column. The ability of the force to spread through the column is represented by the angle of dispersion; it can be 1/2.5 or 1/3.5, which is obtained based on an empirical curve fitting technique (Parkers, 1952). Then, by considering this angle of dispersion, Graham et al. (1960) assumed that the load is distributed uniformly onto the web area which is idealised as a rectangular plate whose ultimate strength can be calculated by Eqn. (2.6-2.7),

\[
F_{c,wc,Rd} = t_w b_{eff} \sigma_y \tag{2.6}
\]

\[
b_{eff} = 5k + t_p \tag{2.7}
\]

where \(F_{c,wc,Rd}\) is the strength of the column web, \(t_w\) is the thickness of this web, \(b_{eff}\) is the effective width of the area, the \(\sigma_y\) is the yield strength of column steel, \(k\) is the depth of k-line, \(t_p\) is the thickness of the beam flange, and the angle of dispersion in this equation is 1/2.5, a conservative choice. It is suggested that any potential buckling area of the column web will be stiffened.
In 1970, Chen and Oppenheim (1970b) extended the study of Graham et al. (1960) to the field of column web buckling. They performed nine tests on stocky and slender columns to analyse the force-displacement \((P - \Delta)\) relationship of column web buckling. Their findings can be summarised as the following three points:

1) The \(P - \Delta\) curves for stocky and slender webs agreed with the theoretical plate buckling curves (Figure 2.8 demonstrates a typical \(P - \Delta\) curve for column web component).

\[ k = 0.8r_{root} \]
2) The predicted strength (Eqn. (2.6)) largely underestimated the true strength.

3) The yield line appears at the toe of the upper column k-line at a very early stage. Therefore, the boundary condition of the effective web plate can be assumed as simply supported along all four edges. That was also confirmed by the tests results, based on a comparison between the theoretical strength and test strength.

It is suggested that the critical buckling load of a column web should be calculated by Eqn. (2.8)

\[
P_{cr} = \frac{4\pi^2 D}{d_e} (2.8)
\]

where \( P_{cr} \) is the buckling load, \( d_e \) is the depth of the column web between the k-lines, and \( D \) is the rigidity of plate flexure. The buckling load obtained from Eqn. (2.8) was used to check for capacity because no theoretical \( P - \Delta \) curve model was obtained in their research.

In 1972, Bose et al.\cite{Bose1972a, Bose1972b} constructed a group of 2D finite element models of column webs to study the shear and compression forces on them. They suggest 1/3 as the angle of dispersion and a reduction factor (Eqn. 2.9) to consider the effect of axial load in the column,

\[
\alpha_c = 1.0 - 0.5 \left( \frac{P_c}{P_{cy}} \right) - 0.5 \left( \frac{P_c}{P_{cy}} \right)^2 (2.9)
\]

where \( \alpha_c \) is the reduction factor in the strength of the column web, \( P_c \) is the applied axial load in the column, and \( P_{cy} \) is the axial load that causes yielding in the column.

Note that the stress distributed on the column web can also be analysed by the plastic method \cite{Bose1972a, Graham1960} where the beam and column are assumed to be fully yielded. However, this method can only be used for design strength calculation and cannot determine the \( P - \Delta \) curve.
In 1982, Witteveen et al. (1982) suggested a method to determine the moment capacity of joints and made recommendations for design. Their method for designing column webs is based on the work by Graham et al. (1960) and Chen and Oppenheim (1970b). In 1993, their method was included in Eurocode 3 (2006b). In addition, Eurocode 3 also considers the interaction among column web in compression, column web in shear, plate buckling and axial load by corresponding reduction factors. The resistance of the column web of a joint in Eurocode 3 is given by Eqn. (2.10-2.11).

\[ F_{c,wc,Rd} = \omega k_{wc} b_{eff,c,wc} t_{wc} f_{y,wc} / \gamma_{M0} \]  
\[ \text{but } F_{c,wc,Rd} \leq \omega k_{wc} \rho b_{eff,c,wc} t_{wc} f_{y,wc} / \gamma_{M1} \]  

In Eqn. (2.10, 2.11), \( F_{c,wc,Rd} \) is the strength of the column web, \( \omega \) is the reduction factor for interaction with shear, \( k_{wc} \) is axial load reduction factor, \( \rho \) is the plate buckling reduction factor, \( b_{eff,c,wc} \) is the effective area width, \( t_{wc} \) is the web thickness, \( f_{y,wc} \) is the material yield stress, and the \( \gamma_{M0} \) and \( \gamma_{M1} \) terms are the partial safety factors that can be obtained in Eurocode 3 (2006b).

In 2004, Beg et al. (2004) devised an analytical method to analyse the compressive behaviour of a column web by considering the relationship between axial loading, transverse compression, and column slenderness. They calculated the ultimate deformation \( \delta_u \) from the relationship defined in Eqn. (2.12).

\[ \varepsilon_u = \frac{\delta_u}{d} \]  

In Eqn. (2.12), \( d \) is the depth of a web between the k-line and \( \varepsilon_u \) is the ultimate strain of a web in compression. \( \varepsilon_u \) was obtained by an empirical selection from a \( \varepsilon_u \) chart (Figure 2.9) where \( \varepsilon_u \) is defined as a function of axial force \( n = P/P_y \) and web slenderness \( d/(t_w \varepsilon) \) in which \( \varepsilon = \sqrt{235/f_y} \) and \( f_y \) is the yield stress of web plate. (Beg et al., 2004)
Research into the $P - \Delta$ curve of a column web component is a knowledge gap. Based on the plate buckling theory shown in Figure 2.8, a model with tri-linear curve could achieve sufficient accuracy to model the behaviour of this component, as will be explored in Chapter 4 of this thesis.

2.3.2 Column Web in Shear

Column Web in Shear has been studied by many researchers for several decades because it can be a major contribution to the ductility of a structure (da Silva and Girao Coelho, 2001b). This component provides for high energy absorption when subjected to seismic action, such as welded joints (Girao Coelho et al., 2009, Kato, 1982, Skejic et al., 2008a).

Based on the spring model suggested by Wynand et al. (1996), this component may be treated as an additional compressive spring with an infinite capacity for deformation, however, as the
experiments performed by Girao Coelho et al. (2009) have shown, Wynand’s model sometimes overestimates the shear deformation of this component, especially in joints featuring high strength steel.

Girao Coelho et al. (2009) suggested that the potential for failure by shear buckling in a high strength steel column must be considered because its sectional profile is usually slimmer than those of mild strength steel joints and slender sections are more vulnerable to shear buckling. Their study focused on the application of Eurocode 3 (2006b) to high strength shear panels; however they did not provide further investigation into the theoretical ductility model of the shear panel.

As an alternative to welded joints in seismic design, web side plate joints where both beam flanges are welded to the column draw attention. Since the beam flanges are welded to the column, the joint is rigid and should be categorised into the upper bounds of the moment rotation spectrum of the web side plate joint family. Similar to the welded connections, shear panels in the web side plate joints will undergo large deformation and therefore researchers suggested applying the web doubler for strengthening purposes when necessary.

Popov (1987) and Krawinkler and Mohassed (1987) studied the deformation of the shear panels of the web side plate joints, while Krawinkler developed a refined mathematical model for this component (Eqn. (2.13-2.17)),

\[ V_y = 0.55 F_y d_c t \] (2.13)

\[ Y_y = \frac{F_y}{(\sqrt{3} G)} \] (2.14)

\[ K_e = V / Y = 0.95 d_c t G \] (2.15)

\[ K_p = \Delta V / \Delta Y = 1.095 b_c t_c^2 G / d_b \] (2.16)

\[ V_p = 0.55 F_y d_c t (1 + 3.45 b_c t_c^2 / d_b d_c t) \] (2.17)
where \( V_y \) is the yield shear force, \( V_p \) is the ultimate shear force, \( K_e \) is the elastic stiffness, \( K_p \) is the plastic stiffness, \( \gamma \) is the angle of shear distortion, \( b_c \) and \( t_{cf} \) are the width and thickness of the column flange, \( d_c \) is the depth of column shear panel, and \( d_b \) is the depth of the beam framing into the column.

In Eurocode 3 Part 1-8 (Eurocode3, 2006b), the design plastic shear resistance \( V_{wp,Rd} \) is given by Eqn. (2.18) and the stiffness coefficient for the shear panel, \( k_1 \), can be obtained by Eqn. (2.19)

\[
V_{wp,Rd} = \frac{0.9 f_{y,wc} A_{vc}}{\sqrt{3} \gamma_{M0}} \tag{2.18}
\]

\[
k_1 = \frac{0.38 A_{vc}}{\beta z} \tag{2.19}
\]

where, \( f_{y,wc} \) is the yield stress of column web component, \( A_{vc} \) is the shear area of the column and \( \gamma_{M0} \) is the partial safety factor for the resistance of cross-sections. Both \( A_{vc} \) and \( \gamma_{M0} \) can be obtained from Eurocode 3 Part 1-1 (Eurocode3, 2005). \( \beta \) is the transformation parameter given in Clause 5.3(7) of Eurocode 3 Part 1-8 (Eurocode3, 2006b) and \( z \) is the lever arm defined in figure 6.15 of Eurocode 3 Part 1-8 (Eurocode3, 2006b).

The aforementioned models are proposed for stocky column profiles. On the other end of the slenderness spectrum, developments has been made for slender column profiles (e.g. built-up welded profiles) by Jaspart (1997) and Braham et al. (1999).

### 2.3.3 Equivalent T-stub: Column flange, End Plate and Bolt

Column flange bending, end-plate bending, and bolts in tension are usually analysed collectively as a pair of T-stubs (Figure 2.10) which determine the behaviour of the tension face of the joints. Hence, they are the most common research field in end-plate joints and have been researched extensively for several decades.

The earliest research into T-stubs may be dated back to Zoetemeijer (1974) and Agerskov (1977), but in the 1990s, this component was studied intensively. Chasten, Lu, et al (1992)
studied the prying action of bolts because this is essential to accurately determine the bolt force. They investigated the stress distribution of prying action at T-Stub flanges and provided a simplified equation to determine the magnitude and location of the prying force. Faella, Piluso, et al. (1998) analysed the influence of different types of bolts such as snug and preloaded. Subsequent research from Piluso et al. (Piluso et al., 2001a, Piluso et al., 2001b) proposed a complete theoretical model of T-stubs, where the T-stub is assumed to be fixed to a rigid surface and the web is being pulled. Their model used a multi-linear curve for the material property to take the large deformation behaviour into account, and it also considered the prying force.

At the same time, Swanson and Leon (2001) proposed another model based on the limit states of partially and fully plastic hinge forming. This model is reliable and sufficiently accurate, so it is the preferred modelling technique. **Section 6.1.1.3.1** will introduce this method in detail.

Later, Mofid et al. (2001) investigated several end-plate joints. They applied plate theory to model the end plate bending component and devised a formula using the virtual work principle. Mofid et al. (2005) formulated the ultimate strength equation to include various plate yield mechanisms and bolt fracture scenarios. Moreover, an empirical post-yield
behaviour model was suggested. Later, Shi et al. (2007) suggested an analytical formulation for the post-yield behaviour.

Group effects are considered in Cerfontaine and Jaspart (2005) and Eurocode3 (2006b). Group effects are usually present when several components are located in close proximity, such as multiple closely located bolt-rows on the tension part of the end-plate or column flange. They alter yield patterns around the influenced component leading to a reduction in stiffness.

2.3.4 Web cleat Angle and Flange Cleat Angle Joints

Angle cleat joints are flexible because the deformation sourcing from the bolt hole clearance and deformation of angle cleat can be large. The applied moment in this type of joints is firstly resisted by friction between the web cleat angles and the beam web, and subsequently the angle cleat will pick up the bending moment and start deforming (Figure 2.11 right). Angle cleat joints are flexible in design since these joints can be modified by adding flange cleat angles (top or/and seat angles) to increase stability and rigidity. The web cleat angle and flange cleat angles can be analysed by a cantilever mechanism (Figure 2.11). Note that web cleat angles are usually subjected to additional torsion which needs to be considered, in the analysis.
The earliest research conducted on angle components was in 1987 by Azizinamini et al. (1987). They divided each angle into beam segments and adopted beam deflection theory to obtain the initial stiffness (Figure 2.12).

Kishi and Chen (1990) used beam theory and plastic analysis to model top and seat angle joints. They transformed the web cleat angles to torsion resisting plates with one edge fixed, and then calculated the initial stiffness (Figure 2.13). They also suggested using a plastic hinge based model (Figure 2.11) to analyse the ultimate strength.
Jaspart (1991a) and Eurocode3 (2006b) suggested treating angle cleats in bending as equivalent T-stubs similar to the end-plate bending components. Eurocode 3 Part 1-8 Clause 6.2.6.6 states the procedures for calculating the design resistance of the angle cleat component, and Clause 6.3.2 states the method to obtain the stiffness of the angle cleat component.

Later, Mofid et al. (2002) applied this method to the end-plate attached to the top and seat angle joints. They ignored the prying action of bolts in their research on the top angle. Since it has a similar loading and boundary condition as half a T-stub, a prying action is inevitable in thin plates. Because of this similarity, the T-Stub model can be applied.

2.4 Experimental Assessment

This section is a brief review of the experiment assessments used to acquire $M - \theta$ curves of joints. The full-scale experiment is preferred since it is the most accurate method of acquiring data. However, it is expensive and hence is not suitable for parametric studies, so the results are mainly used to calibrate and validate other models such as FE models.
According to Wu (1988), the earliest experiments conducted on semi-rigid joints are by Wilson and Moore (1917) who experimented on rivet joints to assess the rigidity of joints. Between 1930 and 1970, different types of steel joints such as riveted connections, welded connections, and bolted connections, were investigated. Batho and Rowen (Batho and Rowen, 1931, Batho and Rowen, 1934, Batho and Rowen, 1936) evaluated $M - \theta$ curves for riveted and bolted joints that included double web angle cleat joints with/without top and seat angles and T-stub joints with/without web cleat angles. Young and Jackson (1934) who were contemporary with Batho and Rowen pioneered the tests of welded top plates and seat angle joints. Later on, as high tensile bolts were used more widely in industry as primary field fasteners for structural steel joints, bolted joints became popular because they are easy to erect and repair, and the bolts can be pre-stressed to increase the initial stiffness of the joint. Subsequently, joint research focused on bolted joints such as top and seat angle joints and bolted moment end plate joints. Sherbourne (1961) experimentally investigated the $M - \theta$ curves of bolted moment end-plate joints with/without column stiffeners. His experiments focused on an investigation into the efficiency of column stiffeners.

Between 1960 and 1980, the Fritz Engineering Laboratory at Lehigh University (Chen and Oppenheim, 1970b, Graham et al., 1960) performed several series of tests to study welded joints and the behaviour of the column components of the joints, and then provided recommendations for practical design. Graham et al. (1960), who performed the first series in this laboratory, tested two way and four way welded joints. In order to study column web behaviour, Chen and Oppenheim (1970b) tested nine columns subjected to concentric loads on flanges at both sides. In their experiments, they measured joint rotation by the difference in displacement between the tension and compression zones in the column.

Davison et al. (1987) tested several commonly used joints and then compared their stiffness and moment capacities. They suggested an accurate method to measure the rotation of a connection (relative rotation of the beam to the column). This method adopted a rotation bar
that is fixed onto a column with three LVDTs to calculate the change of coordinates of the rotation bar (Figure 2.14). This method was designed to measure large rotations and it avoids any influence from the curvature of the beam, but large deformations of the column components may lead to errors.

![Figure 2.14: Method of measurement of rotations (Source: Davison et al. (1987))](image)

Since these experiments were carried out on angle cleat joints where friction between the bolted components provides some resistance to joint rotation and initial stiffness, Davison et al. (1987) suggested that the torque on the bolts must be controlled. On the other hand, for the end plate joints where the bolts mainly take the axial tension force, the bolts can be fastened by hand. These experiments were included in a data bank created by Kishi and Chen (1986) and further expanded by Abadalla and Chen (1995).

In 1989, Chasten et al. (1992) tested a group of 8-bolts extended end-plate joints in a propped cantilever frame to realistically depict the behaviour of this type of joints. The loading point on a connected beam was 3.5m away from the joints to provide an appropriate shear-to-moment ratio and avoid local buckling at the loaded point. This setup was due to the high moment capacity of 8-bolts extended end-plate joints where it may lead to premature shear failure of the bolts or local buckling in the beam flange.
Later, Bose et al. (1997) tested a group of flush end plate joints that were classified as a flexible joint, by applying loading it 1.2m away from the face of the column. They suggested adopting web stiffeners at the beam load point to eliminate unexpected web buckling failure (Figure 2.15). The rig setup for their tests shown in Figure 2.15 is called a cruciform where two nominally identical beams are connecting to both sides of a column and loaded equally.

![Figure 2.15: Example of Cruciform Setup of Test Specimen (Source: Bose et al. (1997))](image)

For the connection only tests where the column is assumed to be fully rigid, the rig setup adopted by Bursi and Jaspart (1997b) can be used (Figure 2.16). Bursi and Jaspart fixed end-plate connections to a counter-beam base and loaded the beam end with screw jack.
In 1998, Cruz et al. (1998) included hundreds of tests into a database for the semi-rigid behaviour of beam-to-column connections in seismic regions. The database provided researchers ready access to experimental data for their investigations and comparisons. The included experimental data was comprehensive and hence easy to use.

Later on, Girao Coelho et al. (Girao Coelho and Bijlaard, 2007, Girao Coelho and Bijlaard, 2010, Girao Coelho et al., 2004a, Girao Coelho et al., 2004b, Girao Coelho et al., 2009) performed a series of tests to assess the ductility of end plate joints. They tested T-stubs and full scale end-plate joints. In their full scale end-plate joint experiments, they adopted an extreme rigid column to prevent any influence from column deformation. These tests assessed the performances of both mild steel and high strength steel.

They investigated the performance of various combinations of components with different strengths to find an efficient design for T-stubs where all the advantages of the components can be fully utilised. For example, high strength end-plates used in combination with low strength welds is inefficient because premature failure such as cracking may occur at the heat affected zone (HAZ) near the toe of the welds.
They also studied the effective area of bolts and different weld electrodes and found that the effective area of fully threaded bolts is less than partially threaded bolts, and suggested that the failure of welding metal or the HAZ zone dominates the strength capacity in many tests, especially when high tensile bolts and steel plates are used. They recommended using low hydrogen electrodes with a high proportion of calcium carbonate and calcium fluoride coating because it can enhance the resistance to premature failure.

Girao Coelho et al. (2004b) tested 32 T-stubs to investigate the ductility of the joints. Various T-stub configurations were tested, including simple welded T-stubs and flange stiffened T-stubs which were designed to simulate the inner end-plate bending component in end-plate joints. They also tested various types of bolts, welds, and electrodes to study T-stub failure modes. The bolts were tested by the measuring bracket (or horseshoe device in Figure 2.17 left) which can record elongations with great accuracy. The plate bending and weld performance were captured by several strain gauges fixed at the HAZ zones (Figure 2.17 right).

![Figure 2.17: Left: The rig to measure bolt elongation. Right: The location of strain gauges for plate bending (Source: Girao Coehle et al. (2004b).)
Meanwhile, Girao Coelho et al. (2004a) assessed the technique of capturing the rotation of the joint (including both column rotation and beam rotation), as shown in Figure 2.18. Column rotation can be determined by the LVDTs DT8 and DT11 in Figure 2.18, which are fixed onto the column flange, while beam rotation was calculated from Eqn. (2.20) in which the beam displacement data were captured by four LVDTs (DT1, DT2, DT3, and DT4) fixed under the beam (Figure 2.18).

\[
\theta_b = \tan^{-1}\left(\frac{\delta_{DT1}}{900}\right) - \theta_{b,el} = \tan^{-1}\left(\frac{\delta_{DT2}}{600}\right) - \theta_{b,el} = \tan^{-1}\left(\frac{\delta_{DT3}}{300}\right) - \theta_{b,el} \\
= \tan^{-1}\left(\frac{\delta_{DT4}}{100}\right) - \theta_{b,el}
\]  

(2.20)

In Eqn. (2.20), \(\theta_b\) is the beam rotation, \(\theta_{b,el}\) is the beam rotation due to elastic bending where shear deformation is ignored, and \(\delta_{DT1}, \delta_{DT2}, \delta_{DT3}\) and \(\delta_{DT4}\) are the displacements measured by the corresponding LVDTs (DT1, DT2, DT3, and DT4 in Figure 2.18). Note that beam theory may be invalid at the location of DT4 where the stress distribution was not linear. This method is easy to apply and has acceptable accuracy.
FIGURE 2.18: THE LOCATION OF LVDTS (SOURCE GIRAO COELHO ET AL. (2004A))

Finally, Girao Coelho et al. (2009) tested a stiffened column to investigate the behaviour of the column shear panel. The column was pin supported at three locations with axial loads applied at both ends of the beam (Figure 2.19).

The experiments reviewed in this section provide comprehensive background knowledge for setting up the tests, such as measuring the rotation and capturing the local deformation of a component. In Chapter 5, the test series of this thesis will be described and discussed in detail.

2.5 Finite Element (FE) Analysis

The Finite Element (FE) method is a widely used numerical method for analysing structural steel joints, and is also a convenient supplementary method for obtaining extensive data. It can be used for parametric studies and investigation of important local effects that are usually difficult to capture in experiments (Diaz et al., 2011b, Diaz et al., 2011a).

Due to limitations in computing power, early FE applications for structural steel joints were in two dimensions (Bose et al., 1972a, Chasten et al., 1992, Gebbeken et al., 1994) and 2D/3D hybrid analysis (Rothert et al., 1992, Swanson et al., 2002). Since connections usually transmit stress in three dimensions, 2D models are usually not very accurate compared to 3D models. For example, in some early 2D models, transverse stress is not considered, which is significant in many situations. In end-plate joints, the beam flange and web are always welded to the end-plate. When the tension force transmitted from the beam flange and web at the tension face, transverse stress usually exists in the end-plate. The inaccuracy of the model may be significant if a thin end-plate is used (Sherbourne and Baharri, 1994). Therefore, as computing power has improved dramatically, 3D modelling has become the nominated choice.

Early research using 3D models was carried out by Gebbeken, Rothert et al. (1994), who indicated some basic problems such as contact, pre-stresses and model simplification when simulating end-plate joints. Note that to model contact surfaces, only the penetration of
elements are considered, friction was not included in their modelling technique (Gebbeken et al., 1994, Rothert et al., 1992).

Sherbourne and Bahaari (Baharri and Sherbourne, 1996, Baharri and Sherbourne, 2000, Sherbourne and Baharri, 1994, Sherbourne and Baharri, 1997) constructed more advanced 3D models to simulate end-plate joints. They suggested some basic methods to simulate plates, bolt shanks, bolt holes, and the contact surfaces between components. Shell elements were used to model plates, such as beam webs, flanges, end-plates, column web flanges, and stiffeners. Bolt shanks were modelled by six 3D spar elements to capture the axial and bending stresses. Bolt holes were stiffened by additional thickness to simulate the stiffening effect from bolt heads and nuts. Contact was reproduced by contact elements added between the end-plate and the column flanges. However, no contact between the shanks of the bolts and the bolt holes was assumed. These models were used to construct a wide range of bolted end-plate joint models for parametric studies, including end-plate joints with/without column stiffeners and extended end-plate joints with eight bolts in the tension region. The accuracy of these simulations was acceptable, even though the shell element did not consider transverse stress. It is therefore anticipated that simulating the prying action and compression region on the contact surfaces may be improved by applying 3D brick elements.

Further progress has been made by Bursi and Jaspart (Bursi and Jaspart, 1997b, Bursi and Jaspart, 1997a, Bursi and Jaspart, 1998). They adopted 3D brick elements to replace the shell elements used in Sherbourne and Bahaari’s model (1994). Brick elements then became the main type of element in 3D FE models because they can cope with irregular shapes and are well suited for curved boundaries (Bose et al., 1997). Bursi and Jaspart (Bursi and Jaspart, 1997a) compared the accuracy of brick elements with different orders and different numbers of Gaussian integration points. Regarding the order of the brick element, they pointed out that first order elements (eight nodes brick elements) are better suited for reproducing yield lines in plasticity analysis because they are less vulnerable to numerical problems. In terms of
method of integration, full integration with incompatible modes from the ABAQUS library was the most reliable option because it can avoid shear locking problems, which arise from so called parasitic shear stresses that exist in bending-dominated problems (Bursi and Jaspart, 1998). Another integration option from the ABAQUS library is reduced integration (1 Gauss point), but this usually requires a fine mesh to provide acceptable accuracy. For example, Al-Jabri, et al (2006) constructed a 3D FE model to simulate the behaviour of a flush end plate connection in fire. They tried eight nodes bricks with reduced integration and suggested using at least three elements through the width of the plate to achieve acceptable results.

High order brick elements such as 20 nodes brick elements (also known as serendipity brick) were also investigated and shown to achieve acceptable results. It is suggested that 20 nodes brick elements will be more accurate in elastic simulation (Gantes and Lemonis, 2003). Mohamadi-shooreh and Mofid (2008) adopted 20 nodes brick elements with full integration to model a flush end-plate splice joint for its initial stiffness. It is concluded that 20 nodes brick elements with full integration suit elastic analysis very well, and 8 nodes brick elements with appropriate integration modes are more suitable for plastic analysis.

Bursi and Jaspart (Bursi and Jaspart, 1997b, Bursi and Jaspart, 1997a, Bursi and Jaspart, 1998) recommended techniques to tackle convergence problems in large displacement analysis, and they also refined the bolt modelling techniques. Their convergence study suggested that two layers of elements are essential for achieving acceptable results in inelastic bending problems. Large displacements were modelled by a natural stress-strain relationship that can be obtained by updating the area of a bolt shank by its instantaneous area in each loading step of FE simulations. The model for bolts was refined by attaching washers to the head and the nut, in order to reduce the number of contact planes (Bursi and Jaspart, 1998). They also used Agerskov’s method for bolts which embodied additional flexibilities caused by the nut and the threaded part of shank, into an effective bolt length model (Bursi and Jaspart, 1997a). The effective bolt length can be calculated from Eqn. (2.21-2.23),

where $A_b$ is the gross sectional area, $A_s$ is the tensile stress area, $B$ is the bolt force, $\Delta l_b$ is the bolt elongation, and the parameters $K_1$ and $K_4$ are defined in Eqn. (2.22, 2.23),

\begin{align*}
K_1 &= l_s + 1.43 l_t + 0.71 l_n \\
K_4 &= 0.1 l_n + 0.2 l_w
\end{align*} \quad (2.22) \quad (2.23)

In Eqn. (2.22-2.23), $l_s$ is the length of the shank, $l_n$ is the length of the nut, and $l_t$ is the length of threaded part in the bolt hole.

The effective bolt length obtained by Agerskov’s model may lead to a stiffened bolt response (Gantes and Lemonis, 2003). This effect was demonstrated by a FE simulation performed by Gantes and Lemonis (Gantes and Lemonis, 2003), who suggested lengthening the bolt by 30% to 50% to eliminate this effect. Note that Bursi and Jaspart’s model does not include the threaded stripping failure mechanism in their model, but they suggested a formula for the threaded stripping load $B_s$ (Eqn. (2.24)),

\[ B_s = \frac{5}{6} \frac{f_y}{\sqrt{3}} \frac{7}{8} \pi \eta d_t z \] \quad (2.24)

in which $f_y$ is the yield stress of the bolt, $\eta$ denotes the pitch, $d_t$ represents the diameter of the effective bolt shank, and $z$ is an assumed number of active threads between 3 and 6, for compressed nuts.

These models were constructed to simulate end plate joints. The contact problem in this type of joints is relatively simple. For example, the slipping problem, which is one of the most difficult problems in contact surface simulation, is very small and therefore ignorable. On the contrary, the slipping problem is essential in modelling T-stub joints and angle cleat joints where friction is used to transfer the load between the components in contact. Modelling these two types of joints was pioneered by Swanson et al. (2002) and Citipitioglu et al. (2002).

Swanson et al. (2002) extended FE simulation to T-stub joints where the slipping affects the initial response, but their model did not reproduce gradual slipping effects. They suggested
this error was caused by the poor consideration of bolt’s pre-tension and an inaccurate modelling of friction.

Citipitioglu et al. (2002) proposed a model to simulate the top and seat angle joint. They focused on accurately estimating slipping in their research, including friction and pre-tension bolt simulation. As development of ABAQUS, friction existing at the contact area of bolt shanks, bolt holes and clamped components can be modelled by special commands that are based on a “master-slave” algorithm. For a pre-tensioned bolt, they reproduced its behaviour by a displacement-based method, in which first a bolt that was shorter than the thickness of the plate was used, and then extended the bolt so it protruded out of the bolt hole. In these procedures, pre-tension was controlled by a non-linear force displacement relationship.

Citipitioglu et al. (2002) adopted an automatic meshing technique, suggesting that simplifying the model discretisation is unnecessary. Eight nodes brick elements with full integration and incompatible modes were used to model plates in the joints and six nodes wedge elements were used to model the core of bolts. When specifying setting the material for the bolts they encountered a convergence problem caused by severe localised strain at the corner of the bolt head. They used elastic material properties to avoid this problem and noted that the results from this linear elastic material setting have little or no effect on the overall behaviour.

Other research programs using 3D FE models for joints have been reported (Bose et al., 1997, Choi and Chung, 1996, Ju et al., 2004, Maggi et al., 2005, Pirmoz et al., 2008), but they are all based on the models already discussed. In conclusion, to create a robust 3D joint model, the following recommendations should be considered:

- Use eight nodes brick elements/six nodes wedge elements with full integration and incompatible models to construct the model.
• Contact elements must be added to faying surfaces, bolt shanks, bolt hole walls, the bottom of bolt heads and the top of nuts (if applicable). Large bolt slipping must be allowed in angle connections.

• Five layers of elements through the thickness of major bending plates and four layers through the thickness of bolt head were essential for convergence and accuracy.

• Preloading and large displacement can be modelled by special commands in ABAQUS.
Chapter 3: Generalised Component Model for Structural Steel Joints

3.1 Background

The component method considers structural steel joints as a multi-spring model containing parallel several spring series. By calculating the moment-rotation behaviour \((M - \theta)\) of the multi-spring model, the behaviour of structural steel joints can be obtained (Figure 3.1).

da Silva, Girao Coelho et al. (2000) suggested a concept to obtain closed-form equations for the \(M - \theta\) curve of dual-spring models wherein each spring is longitudinally deformable. And then, to apply this model to general cases, a procedure (da Silva and Girao Coelho, 2001b) to reduce multi-spring systems to dual-spring models was prepared. However, their method has two limitations:

1. The complexity of the equations in their method will increase dramatically with the increase in the number of springs or the complexity of individual spring behaviour (e.g. gradual yielding and post-ultimate behaviour). Due to this limitation, a bi-linear curve for each spring is recommended and the gradual-yielding behaviour is difficult to include in this method.

2. During the process of simplification, the loading condition (tension or compression) of each spring is assumed. However, it may change as the joint deforms, especially during the large deformation stage (e.g. the post-ultimate stage in which the spring has negative stiffness) or when an axial load is exerted. Lack of the consideration of the real instantaneous centre of rotation (ICR) will affect the accuracy of the \(M - \theta\) curve.

In order to incorporate the post-ultimate behaviour into da Silva’s model, Lewis (2010a) added an additional formulation for a softening spring (Figure 3.2). Lewis suggested an energy method to model the post-ultimate behaviour of the softening spring and adopted da Silva’s
procedure to derive closed-form equations for the $M - \theta$ curve. Therefore, the method has the same limitation to da Silva’s model.

This chapter outlines an approach on how to use a spring model to characterise the whole joint behaviour including post-ultimate and post-fracture effects. In this approach, the basic formulation of an individual spring will be adopted from da Silva’s and Lewis’ methods. A new procedure is prepared to overcome the limitations of existing method including:

1. Equations for the $F - \Delta$ curve of a spring series with post-ultimate behaviour are derived.
2. Equations for the $M - \theta$ curve of a multi-spring system with post-ultimate behaviour are derived. A multi-spring system consists of numbers of spring series in parallel.
3. A new concept of the instantaneous centre of rotation (ICR) is added to track the changing behaviour of each series of springs in a multi-spring system. Taking advantage of knowing the location of the ICR, the loading condition of each spring series is always known.
4. An energy based post-fracture model is presented to model the behaviour of the multi-spring system in the post-fracture range. In addition, the component fracture sequence of the multi-spring system can be predicted.

Based on these improvements, the presented model can predict the full-range $M - \theta$ curves of multi-spring models containing any number of longitudinal deformable springs.

Chapter 4 introduces an improved analytical method on predicting the full-range behaviour of column web components. Chapter 6 includes five examples to demonstrate how to use the generalised component model to characterise the whole joint behaviour.
3.2 Energy method based formulation for an individual spring (da Silva et al., 2000, Lewis, 2010a)

Based on da Silva’s work (da Silva et al., 2000), a spring with bi-linear behaviour can be disassembled into two springs with linear behaviour (Figure 3.1).

There are two different types of springs shown in Figure 3.1, an elastic spring and a plastic spring. Although both of these two springs behave linearly, the characteristics of their behaviours are different. The elastic spring is an ordinary spring whose $F - \Delta$ curve starts from the origin point and it is always active under any load. On the contrary, the plastic spring is a preloaded spring which does not deform until the applied force reaches its critical load, $P^C_p$. After activation of the plastic spring, along with the elastic spring, the plastic behaviour of the original spring can be reproduced.

The potential energy for the elastic spring in this dual-spring system is written as,

$$\Pi_e = \frac{1}{2} k_e (\Delta_e - \langle \Delta_p \rangle)^2$$

where,

$$\langle \Delta_p \rangle \equiv \begin{cases} 0, & F < P^C_p \\ \Delta_p, & F \geq P^C_p \end{cases}$$
In Eqn. (3.1), \( \Pi_e \) is the energy of the elastic spring, \( k_e \) is the stiffness of the elastic spring, \( \Delta_t \) is the total displacement of the two spring system, \( \Delta_p \) is the displacement of the plastic spring defined by Eqn. (3.2), which is zero in the elastic stage, \( F \) is the applied force and \( P_p^C \) is the critical load of activation of the plastic spring.

The potential energy for the plastic spring can be written as,

\[
\Pi_p = \frac{1}{2} k_p \left( \frac{P_p^C}{k_p} + \langle \Delta_p \rangle \right)^2
\]

(3.3)

where \( \Pi_p \) is the energy of the plastic spring and \( k_p \) is the stiffness of the plastic spring.

So the total potential energy, \( \Pi \), of the two spring system is

\[
\Pi = \Pi_e + \Pi_p
\]

(3.4)

Next, let us consider a spring with post-ultimate behaviour. This spring may be disassembled into three springs, an elastic spring, a plastic spring and a softening spring (Figure 3.2).

The elastic spring and plastic spring are the same as those shown in Figure 3.1, and the softening spring is used to reproduce the post-ultimate behaviour of the original spring. The energy equation for the elastic spring of this new spring series is modified to:

\[
\Pi_e = \frac{1}{2} k_e \left( \Delta_t - \langle \Delta_p \rangle - \langle \Delta_s \rangle \right)^2
\]

(3.5)

where,

\[
\langle \Delta_s \rangle \equiv \begin{cases} 
0, & F < P_s^C \\
\Delta_s, & F = P_s^C \text{ and subsequent } F < P_s^C 
\end{cases}
\]

(3.6)
In Eqn. (3.5), \( \Delta_p \) is the displacement of the plastic spring which is defined by Eqn. (3.2), \( \Delta_s \) is the displacement of the softening spring which is defined by Eqn. (3.6), \( F \) is the applied force and \( P_s^C \) is the critical load of activation of the softening spring.

It is suggested by Lewis(2010a) that the potential energy of a softening spring can be obtained by the following two steps:

i. Calculate the total energy by using absolute value of its stiffness.

ii. Subtract the excessive energy from the total energy. (Figure 3.3)

Therefore, the potential energy of the softening spring can be written as:

\[
\Pi_s = \frac{1}{2} |k_s| \left( \frac{P_s^C}{|k_s|} + \langle \Delta_s \rangle \right)^2 - |k_s| \langle \Delta_s \rangle^2
\]  

(3.7)

where \( \Pi_s \) is the energy of a softening spring, \( k_s \) is the stiffness of the softening spring which is negative, \( P_s^C \) is the critical load of activation of the softening spring, and \( \Delta_s \) is the displacement of the softening spring which is defined by Eqn. (3.6).

So, the total potential energy, \( \Pi \), of the tri-linear spring is

\[
\Pi = \Pi_e + \Pi_p + \Pi_s
\]  

(3.8)
3.3 Derivation of Equations for $F - \Delta$ Behaviour of Spring Series with Post-Ultimate Behaviour

Consider a spring series with $n$ springs (Figure 3.4). The total energy for this spring series can be written as:

$$
\Pi = \frac{1}{2} k_{e,eq} \left( \Delta_t - \sum_{i=1}^{n} \langle \Delta_{pi} \rangle - \sum_{i=1}^{n} \langle \Delta_{si} \rangle \right)^2 + \sum_{i=1}^{n} \frac{1}{2} k_{pi} \left( \frac{P_{pi}}{k_{pi}} + \langle \Delta_{pi} \rangle \right)^2 \\
+ \sum_{i=1}^{n} \left[ \frac{1}{2} k_{si} \left( \frac{P_{si}}{k_{si}} + \langle \Delta_{si} \rangle \right)^2 - k_{si} \langle \Delta_{si} \rangle^2 \right] - F \Delta_t,
$$

(3.9)

where $k_{ei}$ is the stiffness of an individual elastic spring, $k_{e,eq}$ is the equivalent stiffness of all the elastic springs (since elastic springs are active at all times, they can be treated as a single equivalent spring), $\Delta_t$ is the total displacement of the spring series, $\Delta_{pi}$ is the displacement of the $i^{th}$ plastic spring, $\Delta_{si}$ is the displacement of the $i^{th}$ softening spring, $k_{pi}$ is the stiffness of the $i^{th}$ plastic spring, $k_{si}$ is the stiffness of the $i^{th}$ softening spring, $P_{pi}^C$ is the critical load of activation of the $i^{th}$ plastic spring, $P_{si}^C$ is the critical load of activation of the $i^{th}$ softening spring, and $F$ is the applied force.

**FIGURE 3.4: A SERIES OF $n$ SPRINGS. EACH SPRING WITH TRI-LINEAR BEHAVIOUR CAN BE REPRODUCED BY AN ELASTIC SPRING, A PLASTIC SPRING AND A SOFTENING SPRING.**
Assume that \( P_{p1}^C < P_{p2}^C < \cdots < P_{pm-1}^C < P_{pm}^C < P_{s1}^C < \cdots \) whereby, the first plastic spring will be activated when the applied force reaches \( P_{p1}^C \). The second plastic spring will be activated once the applied force reaches \( P_{p2}^C \) and as the applied force increases, other plastic springs will be activated one after the other until the \( m^{th} \) plastic spring is activated. Then, as the applied force keeps increasing, it eventually reaches the critical load of the first softening spring which is the ultimate load of both the spring and series of springs. From this point onwards, the resistance of the first softening spring reduces as the total displacement keeps increasing. This effect leads to two assumptions:

1) If the critical load of a softening spring is larger than \( P_{s1}^C \), it will never be activated.

2) All springs except the first softening spring will unload once \( P_{s1}^C \) has been reached.

The unloading stiffness is equal to the elastic stiffness and the plastic springs have no deformation during this process.

Based on the first assumption, \( \Delta_{pm+1} \ldots \Delta_{pn} \) and \( \Delta_{s2} \ldots \Delta_{sn} \) will be zero. Thereby, the energy equation can be reduced to:

\[
\Pi = \frac{1}{2} k_{e,eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} - \Delta_{s1} \right)^2 + \sum_{i=1}^{m} \frac{1}{2} k_{pi} \left( \frac{P_{pi}^C}{k_{pi}} + \Delta_{pi} \right)^2 \\
+ \frac{1}{2} |k_{s1}| \left( \frac{P_{s1}^C}{|k_{s1}|} + \Delta_{s1} \right)^2 - |k_{s1}|(\Delta_{s1})^2 - F\Delta_t + \Pi_{aux}
\]

(3.10)

\[
\Pi_{aux} = \sum_{i=p+1}^{n} \frac{1}{2} k_{pi} \left( \frac{P_{pi}^C}{k_{pi}} \right)^2 + \sum_{i=2}^{n} \frac{1}{2} |k_{si}| \left( \frac{P_{si}^C}{|k_{si}|} \right)^2
\]

Based on the second assumption, the terms \( \Delta_{p1} \ldots \Delta_{pm} \) should be treated as constant once the first softening spring is activated and must be calculated first. To calculate the terms, an equation is required representing the \( F - \Delta \) curve before any softening spring is activated.

To obtain this pre-ultimate equation, the displacement of the first softening spring, \( \Delta_{s1} \), is temporarily set to zero. Then, the energy equation with \( m \) activated plastic springs is written as:
\[ \Pi = \frac{1}{2} k_{e, eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} \right)^2 + \sum_{i=1}^{m} \frac{1}{2} k_{pi} \left( \frac{P_{pi}^c}{k_{pi}} + \Delta_{pi} \right)^2 - F \Delta_t + \Pi_{aux} \]  

\[ \Pi_{aux} = \sum_{i=j+1}^{n} \frac{1}{2} k_{pi} \left( \frac{P_{pi}^c}{k_{pi}} \right)^2 + \sum_{i=1}^{n} \frac{1}{2} k_{si} \left( \frac{P_{si}^c}{k_{si}} \right)^2 \]  

Using the minimum total potential energy principle to find the springs’ stationary position with respect to \( \Delta_t \), the force equilibrium equation for the equivalent elastic spring:

\[ \frac{\partial \Pi}{\partial \Delta_t} = k_{e, eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} \right) - F = 0 \]  

Similarly, using the minimum total potential energy principle to find the springs’ stationary positions with respect to \( \Delta_{p1} \Delta_{p2} \ldots \Delta_{pm-1} \) and \( \Delta_m \), the force equilibrium equations for each individual plastic spring can be obtained:

\[ \frac{\partial \Pi}{\partial \Delta_{p1}} = k_{e, eq} \left( \sum_{i=1}^{m} \Delta_{pi} - \Delta_t \right) + k_{p1} \Delta_{p1} + P_{p1}^c = 0 \]  

\[ \frac{\partial \Pi}{\partial \Delta_{p2}} = k_{e, eq} \left( \sum_{i=1}^{m} \Delta_{pi} - \Delta_t \right) + k_{p2} \Delta_{p2} + P_{p2}^c = 0 \]  

\[ \vdots \]  

\[ \frac{\partial \Pi}{\partial \Delta_{pm-1}} = k_{e, eq} \left( \sum_{i=1}^{m} \Delta_{pi} - \Delta_t \right) + k_{pm-1} \Delta_{pm-1} + P_{pm-1}^c = 0 \]  

\[ \frac{\partial \Pi}{\partial \Delta_{pm}} = k_{e, eq} \left( \sum_{i=1}^{m} \Delta_{pi} - \Delta_t \right) + k_{pm} \Delta_{pm} + P_{pm}^c = 0 \]  

It is found that all these equations can be related by the common term: \( k_{e, eq} (\Delta_t - \sum_{i=1}^{m} \Delta_{pi}) \).

As a result, Eqn. (3.17) can be obtained.

\[ F = k_{e, eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} \right) = k_{p1} \Delta_{p1} + P_{p1}^c = k_{p2} \Delta_{p2} + P_{p2}^c = \ldots \]  

\[ \ldots = k_{pm-1} \Delta_{pm-1} + P_{pm-1}^c = k_{pm} \Delta_{pm} + P_{pm}^c \]  

Eqn. (3.17) is the force equilibrium equation for the spring series where all the plastic spring forces are equal:
\[ k_{p1}\Delta_{p1} + P_{p1}^C = k_{p2}\Delta_{p2} + P_{p2}^C = \cdots = k_{pm-1}\Delta_{p_{m-1}} + P_{p_{m-1}}^C = k_{pm}\Delta_{p_m} + P_{p_m}^C \quad (3.18) \]

Rearrange the terms in Eqn. (3.18) to represent the displacement of each plastic spring in terms of \( \Delta_{p1} \).

\[ \Delta_{p1} = \frac{k_{p1}}{k_{p1}}\Delta_{p1} + \left( \frac{P_{p1}^C - P_{p1}^C}{k_{p1}} \right) \quad (3.19) \]

\[ \Delta_{p2} = \frac{k_{p1}}{k_{p2}}\Delta_{p1} + \left( \frac{P_{p1}^C - P_{p2}^C}{k_{p2}} \right) \quad (3.20) \]

\[ \vdots \]

\[ \Delta_{pm-1} = \frac{k_{p1}}{k_{pm-1}}\Delta_{p1} + \left( \frac{P_{p1}^C - P_{pm-1}^C}{k_{pm-1}} \right) \quad (3.21) \]

\[ \Delta_{pm} = \frac{k_{p1}}{k_{pm}}\Delta_{p1} + \left( \frac{P_{p1}^C - P_{pm}^C}{k_{pm}} \right) \quad (3.22) \]

And add up \( \Delta_{p1} \Delta_{p2} \ldots \Delta_{pm-1} \) and \( \Delta_m \):

\[ \sum_{i=1}^{m} \Delta_{pi} = \left( \frac{1}{\langle K_{e/p} \rangle} - \frac{1}{k_{e,eq}} \right) (k_{p1}\Delta_{p1} + P_{p1}^C) - \langle C_{e/p} \rangle \quad (3.23) \]

\[ \langle K_{e/p} \rangle \equiv \begin{cases} k_{e,eq}, & F < P_{p1}^C \\ \left( \frac{1}{k_{e,eq}} + \sum_{i=1}^{m} \frac{1}{k_{pi}} \right)^{-1}, & P_{p1}^C \leq F < P_{s1}^C \end{cases} \]

\[ \langle C_{e/p} \rangle \equiv \begin{cases} 0, & F < P_{p1}^C \\ \sum_{i=1}^{n} \frac{P_{pi}^C}{k_{pi}}, & P_{p1}^C \leq F < P_{s1}^C \end{cases} \quad (3.24) \]

where, \( K_{e/p} \) is the stiffness of the spring series in elastic or plastic range and \( C_{e/p} \) is the corresponding preloading constant.

As provided by Eqn. (3.17),

\[ k_{e,eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} \right) = k_{p1}\Delta_{p1} + P_{p1}^C \quad (3.26) \]

Substituting Eqn. (3.23) into Eqn. (3.26):
\[ \left( K_{e/p} \right) \left( \Delta_t + \left\langle C_{e/p} \right\rangle \right) = k_{p1} \Delta_{p1} + P_{p1}^C \]  \hspace{1cm} (3.27)

From Eqn. (3.17):
\[ F = k_{p1} \Delta_{p1} + P_{p1}^C \]  \hspace{1cm} (3.28)

Therefore,
\[ F = \left( K_{e/p} \right) \left( \Delta_t + \left\langle C_{e/p} \right\rangle \right) \]  \hspace{1cm} (3.29)

where,
\[ \left\langle K_{e/p} \right\rangle \equiv \begin{cases} k_{e,eq}, & F < P_{p1}^C \\ \left( \frac{1}{k_{e,eq}} + \sum_{i=1}^{m} \frac{1}{k_{pi}} \right)^{-1}, & P_{p1}^C \leq F < P_{s1}^C \end{cases} \]  \hspace{1cm} (3.30)

\[ \frac{1}{k_{e,eq}} = \sum_{i=1}^{n} \frac{1}{k_{ei}} \quad (3.31) \]

\[ \left\langle C_{e/p} \right\rangle \equiv \begin{cases} 0, & F < P_{p1}^C \\ \sum_{i=1}^{n} \frac{P_{pi}^C}{k_{pi}}, & P_{p1}^C \leq F < P_{s1}^C \end{cases} \]  \hspace{1cm} (3.32)

This is the equation for the force-displacement relationship of spring series consisting of \( n \) springs and \( m \) activated plastic springs.

When the applied force just reaches \( P_{s1}^C \), based on Eqn. (3.17), the plastic deformation of each individual plastic spring can be obtained from:
\[ P_{s1}^C = k_{pi} \Delta_{pi} + P_{pi}^C \quad i = 1,2 \ldots m \]  \hspace{1cm} (3.33)

Solve Eqn. (3.33) for \( \Delta_{pi} \):
\[ \Delta_{pi} = \frac{P_{s1}^C - P_{pi}^C}{k_{pi}} \quad i = 1,2 \ldots m \]  \hspace{1cm} (3.34)

Sum up all the plastic deformation terms:
\[ \sum_{i=1}^{m} \Delta_{pi} = \sum_{i=1}^{m} \frac{P_{s1}^C - P_{pi}^C}{k_{pi}} \]  \hspace{1cm} (3.35)

Since \( P_{s1}^C, P_{pi}^C \) and \( k_{pi} \) are constants, \( \Delta_{pi} \) is a constant. Hence, \( \sum_{i=1}^{m} \Delta_{pi} \) is also a constant.
Next, substituting Eqn. (3.35) for $\sum_{i=1}^{m} \Delta_{pi}$ into Eqn. (3.10):

$$
\Pi = \frac{1}{2} k_{eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} - \Delta_{s1} \right)^2 + \frac{1}{2} |k_{s1}| \left( \frac{p_c}{|k_{s1}|} + \Delta_{s1} \right)^2
$$

$$
-|k_{s1}|(\Delta_{s1})^2 - F \Delta_t + \Pi_{aux}
$$

$$
\Pi_{aux} = \sum_{i=1}^{m} \frac{1}{2} k_{pi} \left( \frac{p_c}{k_{pi}} + \Delta_{pi} \right)^2 + \sum_{i=j+1}^{n} \frac{1}{2} k_{pi} \left( \frac{p_c}{k_{pi}} \right)^2 + \sum_{i=2}^{n} \frac{1}{2} |k_{si}| \left( \frac{p_{ci}}{|k_{si}|} \right)^2
$$

(3.36)

Applying the minimum total potential energy principle to find the springs’ stationary positions for Eqn. (3.36) with respect to $\Delta_t$:

$$
\frac{\partial \Pi}{\partial \Delta_t} = k_{eq} \left( \Delta_t - \Delta_{s1} - \sum_{i=1}^{m} \Delta_{pi} \right) - \Pi = 0
$$

(3.38)

Using the minimum total potential energy principle to find the springs’ stationary positions for Eqn. (3.36) with respect to $\Delta_{s1}$:

$$
\frac{\partial \Pi}{\partial \Delta_{s1}} = k_{eq} \left( \Delta_{s1} + \sum_{i=1}^{m} \Delta_{pi} - \Delta_t \right) - |k_{s1}| \Delta_{s1} + p_{s1}^c = 0
$$

(3.39)

Since $k_{si} < 0$, $|k_{si}| = -k_{si}$, eliminating the absolute bracket in Eqn. (3.39) and combining it with Eqn. (3.38) to obtain the force equilibrium equation:

$$
F = k_{eq} \left( \Delta_t - \Delta_{s1} - \sum_{i=1}^{m} \Delta_{pi} \right) = k_{s1} \Delta_{s1} + p_{s1}^c
$$

(3.40)

Extracting the force of the elastic springs and the force of the softening spring, an expression for $\Delta_{s1}$ in terms of $\Delta_t$ can be then obtained,

$$
\Delta_{s1} = \left[ k_{eq} \left( \Delta_t - \sum_{i=1}^{m} \Delta_{pi} \right) - p_{s1}^c \right] / (k_{eq} + k_{s1})
$$

(3.41)

Substituting Eqn. (3.41) into Eqn. (3.40):

$$
F = K_s (\Delta_t + C_s)
$$

(3.42)

where,

$$
\frac{1}{K_s} = \frac{1}{k_{eq}} + \frac{1}{k_{s1}}
$$

(3.43)
\[
\frac{1}{k_{e,eq}} = \sum_{i=1}^{n} \frac{1}{k_{ei}} \tag{3.44}
\]

\[
C_s = \sum_{i=1}^{m} \frac{p_{si}^c - \frac{p_{pi}^c}{k_{pi}}}{k_{si}} \tag{3.45}
\]

The force-displacement relationships set out above (Eqns. (3.29-3.32, 3.42-3.45)) for the elastic, plastic and softening “stages” can be succinctly written as:

\[
F = \langle K^j \rangle \left( \Delta_e + \langle C^j \rangle \right) \tag{3.46}
\]

\[
\langle K^j \rangle \equiv \begin{cases} 
K_e^j = \left( \sum_{i=1}^{n} \frac{1}{k_{ei}} \right)^{-1}, & F < p_{p1}^c \\
K_p^j = \left( \sum_{i=1}^{n} \frac{1}{k_{ei}} + \sum_{i=1}^{m} \frac{1}{k_{pi}} \right)^{-1}, & p_{p1}^c \leq F < p_{s1}^c \\
K_s^j = \left( \sum_{i=1}^{n} \frac{1}{k_{ei}} + \frac{1}{k_{s1}} \right)^{-1}, & F = p_{s1}^c \text{ and subsequent } F < p_{s1}^c \end{cases} \tag{3.47}
\]

\[
\langle C^j \rangle \equiv \begin{cases} 
C_e^j = 0, & F < p_{p1}^c \\
C_p^j = \sum_{i=1}^{m} \frac{p_{pi}^c}{k_{pi}}, & p_{p1}^c \leq F < p_{s1}^c \\
C_s^j = \sum_{i=1}^{m} \frac{p_{si}^c - \frac{p_{pi}^c}{k_{pi}}}{k_{s1}} + \frac{p_{s1}^c}{k_{s1}}, & F = p_{s1}^c \text{ and subsequent } F < p_{s1}^c \end{cases} \tag{3.48}
\]

where the superscript \( j \) on the terms \( K_e^j, K_p^j, K_s^j, C_e^j, C_p^j, \) and \( C_s^j \) represents different stiffness stages as exemplified in Figure 3.5.
3.4 Derivation of the equation for $M - \theta$ behaviour of a multi-spring system

A multi-spring system with $N$ linear series of springs (Figure 3.6) is now considered. In this system, there are $L$ spring series in the elastic or plastic stage and $M$ spring series in the post-ultimate stage such that $M + L$ is equal to the total number of the series of springs ($N$). Since all spring series have multi-linear behaviour (Figure 3.5), their combined behaviour can be also represented by a multi-linear curve, as shown in Figure 3.7 where the superscript $k = 1, 2 \ldots 5$ represents the $k$th segment ($M^k(\theta)$) of the $M - \theta$ curve. Each segment, $M^k(\theta)$, is defined by a unique stiffness value ($K^j_i$) and preloading constant ($C^j_i$) for each spring series ($i = 1, 2 \ldots N$), which are contained in the vectors,

$$K^k = \{K^j_i, i = 1, 2, \ldots N\} \quad (3.49)$$
The energy expression for the system on the $k^{th}$ linear curve of the multi-spring system can be written as:

$$H^k = \sum_{i=1}^{N} \frac{1}{2} |K_i^j|a_i - M^k\theta, \quad K_i^j \in \mathbf{K}^k \text{ and } C_i^j \in \mathbf{C}^k$$

(3.51)
where the stiffness \( (K^i) \) and preloading constant \( (C^i) \) of the \( i^{th} \) spring series are contained in \( K^k \) and \( C^k \) respectively, \( \Delta_o \) is the displacement at the centroid of the connected beam (Figure 3.8), \( h_i \) is the height of the \( i^{th} \) spring measured from the centroid of the connected beam (Figure 3.8), \( \theta \) is the rotation of the joint, and \( M^k \) is the applied moment on the \( k^{th} \) linear curve.
Using the minimum total potential energy principle to find the system’s stationary position, Eqn. (3.51) is firstly differentiated with respect to $\Delta_O$:

$$\frac{\partial \Pi^k}{\partial \Delta_O} = \sum_{i=1}^{N} K_i^j |a_i| = 0, \quad K_i^j \in K^k \text{ and } C_i^j \in C^k \quad (3.52)$$

$$a_i = \begin{cases} \Delta_O + h_i \sin \theta + c_i^j, & K_i^j = K_e^j \\ \Delta_O + h_i \sin \theta + c_i^j, & K_i^j = K_p^j \\ -\Delta_O - h_i \sin \theta + |c_s^j|, & K_i^j = K_s^j \end{cases}$$

Since $K_i^j < 0$, $|K_i^j| = -K_i^j$ and $C_i^j < 0$, $|C_i^j| = -C_i^j$, Eqn. (3.52) can be simplified to:

$$\frac{\partial \Pi^k}{\partial \Delta_O} = \sum_{i=1}^{N} K_i^j \Delta_O + \sum_{i=1}^{N} K_i^j h_i \sin \theta + \sum_{i=1}^{N} K_i^j C_i^j = 0 \quad (3.53)$$

where $K_i$ is the stiffness of the $i^{th}$ spring series of the multi-spring system and $C_i$ is the preloaded constant thereof.

Based on Eqn. (3.53), $\Delta_O(\theta)$ can be obtained:

$$\Delta_O(\theta) = -\frac{\sum_{i=1}^{N} K_i h_i}{\sum_{i=1}^{N} K_i} \sin \theta - \frac{\sum_{i=1}^{N} K_i C_i}{\sum_{i=1}^{N} K_i} \quad (3.54)$$
Using again the minimum total potential energy principle, to obtain the $M - \theta$ relationship the total potential energy (Eqn. (3.51)) is differentiated with respect to $\theta$:

$$
\frac{\partial \Pi^k}{\partial \theta} = \sum_{i=1}^{N} K_i^j a_i^j - M^k, \quad K_i^j \in K^k \text{ and } C_i^j \in C^k \quad (3.55)
$$

$$
a_i^j = \begin{cases} 
\Delta_0 h_i \cos \theta + h_i^2 \sin \theta \cos \theta + C_e^j h_i \cos \theta, & K_i^j = K_e^j \\
\Delta_0 h_i \cos \theta + h_i^2 \sin \theta \cos \theta + C_p^j h_i \cos \theta, & K_i^j = K_p^j \\
-\Delta_0 h_i \cos \theta - h_i^2 \sin \theta \cos \theta + |C_s^j| h_i \cos \theta, & K_i^j = K_s^j
\end{cases}
$$

As $K_{si} < 0$, $|K_{si}| = -K_{si}$ and $C_s^j < 0$, $|C_s^j| = -C_s^j$

$$
\frac{\partial \Pi^k}{\partial \theta} = \sum_{i=1}^{N} K_i^j (\Delta_0 h_i \cos \theta + h_i^2 \sin \theta \cos \theta + C_e^j h_i \cos \theta) - M^k = 0 \quad (3.56)
$$

Then, substituting Eqn. (3.54) for $\Delta_0(\theta)$ into Eqn. (3.56), the expression for $M(\theta)$ can be obtained:

$$
M(\theta) = -\frac{(\sum_{i=1}^{N} K_i^j h_i)^2}{\sum_{i=1}^{N} K_i^j} \sin \theta \cos \theta + \sum_{i=1}^{N} K_i^j h_i^2 \sin \theta \cos \theta
$$

$$
+ \sum_{i=1}^{N} K_i^j h_i C_i^j \cos \theta - \frac{(\sum_{i=1}^{N} K_i^j h_i)(\sum_{i=1}^{N} K_i^j C_i^j)}{\sum_{i=1}^{N} K_i^j} \cos \theta \quad (3.57)
$$

Or, concisely:

$$
M^k(\theta) = \frac{\sum_{i=1}^{N-1} \sum_{i=i+1}^{N} K_i^j K_{ii}^j (h_i - h_{ii})(h_i - h_{ii}) \sin \theta + (C_i^j - C_{ii}^j))}{\sum_{i=1}^{N} K_i^j} \cos \theta \quad (3.58)
$$

In Section 3.6, the energy for each linear curve ($V^k$, see Figure 3.9) is required for obtaining the total energy of the multi-linear curve. The energy of the $k^{th}$ linear curve can be obtained from:

$$
V^k = -\frac{a_1^k (\cos 2\theta_1 - \cos 2\theta_0)}{4} + a_1^k (\sin \theta_1 - \sin \theta_0), \quad (3.59)
$$

where
\[ a1^k = \sum_{i=1}^{N-1} \sum_{ii=i+1}^{N} K^j_i h_{ii} (h_i - h_{ii})^2 \bigg/ \sum_{i=1}^{N} K^j_i \]  
(3.60)

\[ a2^k = \sum_{i=1}^{N-1} \sum_{ii=i+1}^{n} K^j_i h_{ii} (C^j_i - C^j_{ii}) \bigg/ \sum_{i=1}^{N} K^j_i \]  
(3.61)

in which \( \theta_0 \) is the rotation at the beginning of the \( k^{th} \) linear curve and \( \theta_1 \) is the rotation at the end of the \( k^{th} \) linear curve.

By adding the energy of each linear curve, the total energy of the multi-linear curve can be obtained (Eqn. (3.62)), which is demonstrated in Figure 3.9.

\[ V(\theta_f) = \sum_{k=1}^{N_k} V^k \]  
(3.62)

where \( N_k \) is the total number of segments contained in the multi-linear curve and \( \theta_f \) is the rotation at the fracture of a component.

---

**FIGURE 3.9: THE TOTAL ENERGY (\( V \)) OF A MULTI-SPRING SYSTEM**
3.5 Instantaneous Centre of Rotation (ICR)

In Eqn. (3.58), although the $F - \Delta$ curves of all spring series are known, $K$ and $C$ are still undetermined because the loading condition (compression, tension and unloading) of each individual spring series is unknown. Therefore, to determine the $K$ and $C$, the loading condition of a spring series must be obtained first.

![Figure 3.10: The Section Moving Direction of Any Point on a Rotating Section](image)

**FIGURE 3.10: THE SECTION MOVING DIRECTION OF ANY POINT ON A ROTATING SECTION**

In order to track the loading condition of each spring series, the concept of the instantaneous centre of rotation is utilised. The definition of this point is:

- The instantaneous centre of rotation is the point whose displacement is independent of section rotation.

The moving direction of any point on the rotating section can be determined by its position relative position to this point. This is demonstrated in Figure 3.10.

The loading condition of any spring series can be analysed based on the section moving direction and its deformation history (Table 3.1).
### Table 3.1

<table>
<thead>
<tr>
<th></th>
<th>Tension</th>
<th>Compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deforming towards right</td>
<td>Tension</td>
<td>Compression unloading</td>
</tr>
<tr>
<td>Deforming towards left</td>
<td>Tension unloading</td>
<td>Compression</td>
</tr>
</tbody>
</table>

In the initial range, the ICR is at the zero displacement point ($\Delta_{ICR} = 0$, Eqn. (3.68)) and the height of the ICR can be calculated by Eqn. (3.66). Assume the section rotates in the positive clockwise direction, if a spring series is located above the ICR, it is under tension. Conversely, if it is located below the ICR, it is under compression. Then, if one of the spring series reaches the yield point, the stiffness and preloading constant of that spring series will change. The ICR will change as the stiffness and preloading constant changes (Eqn. (3.66-3.67)) in each segment ($k$) of the $M - \theta$ curve.

### 3.5.1 The height of ICR

The displacement of a point on the rotating section can be represented by:

$$\Delta_i = \Delta_0 + h_i \sin \theta$$  \hspace{1cm} (3.63)

where $\Delta_i$ is the displacement of a point on the rotating section and $h_i$ is the height of this point (Figure 3.8).

Substituting Eqn. (3.54) into Eqn. (3.63):

$$\Delta_i = \left( -\frac{\sum_i K_i^j h_i}{\sum_i K_i^j} + h_i \right) \sin \theta - \frac{\sum_i K_i^j C_i^j}{\sum_i K_i^j}$$  \hspace{1cm} (3.64)

An expression for the height of the ICR, $h_{ICR}$, can be derived, by utilising that since the ICR is independent of the section rotation, $\theta$, its displacement is a constant. Therefore,

$$\Delta_{ICR} = \left( -\frac{\sum_i K_i^j h_i}{\sum_i K_i^j} + h_{ICR} \right) \sin \theta - \frac{\sum_i K_i^j C_i^j}{\sum_i K_i^j} = \text{Constant}$$  \hspace{1cm} (3.65)

Thus, $\Delta_{ICR}$ is a constant and independent to $\theta$, if and only if
Hence, Eqn. (3.66) is the expression for height of the ICR and the expression for the displacement of the ICR, $\Delta_{ICR}$, can be simplified to:

$$
\Delta_{ICR} = -\frac{\sum_{i}^{N} K_i^j C_i^j}{\sum_{i}^{N} K_i^j}
$$

(3.67)

In the initial stage, or elastic range, $C_i^1 = 0$, and therefore,

$$
\Delta_{ICR} = \left( -\frac{\sum_{i}^{N} K_i^1 h_i}{\sum_{i}^{N} K_i^1} + h_{ICR} \right) \sin \theta = 0
$$

(3.68)

Thus, in the initial range, the ICR and the zero displacement point coincide which divides the tension and compression regions of the joint.

### 3.6 Post-fracture Behaviour

In a multi-spring system, the fracture of a spring series may not lead to failure of the whole system because the rest of the system may sustain further rotation at lower resistance level. It is therefore of interest to consider the behaviour of a multi-spring system after the fracture of one or more of its spring series.

Fracture of a spring series usually causes a sudden loss of force in the system, violates static equilibrium and induces dynamic effects. How this dynamic behaviour affects the system is of no particular interest since eventually the system will be restored to static equilibrium. Some elastic energy is converted to kinematic energy in the process, which will be assumed to be small and ignored in the analysis.

It is possible to determine the new static equilibrium point by the energy method (Figure 3.11). Each of the distinct equilibrium paths associated with one, two, three, etc. fractured components is referred to as a “fracture stage” (Figure 3.12).
FIGURE 3.11: EXAMPLE OF THE POST-FRACTURE MODEL. THE DROP IN RESISTANCE IS CAUSED BY FRACTURE OF A COMPONENT(S) OF A SPRING SERIES. THE $M - \theta$ CURVE OF THE POST-FRACTURE SYSTEM IS SHOWN IN BLUE.

FIGURE 3.12: EXAMPLE OF THE POST-FRACTURE MODEL.
Two assumptions are made in the application of the energy method in the post-ultimate range:

1) After the fracture of a component, it is assumed that the new static equilibrium point is located on the $M - \theta$ curve of a new multi-spring system with one spring series (the spring series with a fractured component) less than the non-fractured system, (or previous fractured system).

2) The total energy possessed by the post-fracture system just after returning to static equilibrium is assumed to equal the total energy possessed by the original system at incipient component fracture.

When applying the post-fracture model to end-plate connections where end-plate cracking usually dominates the fracture behaviour, three specific assumptions must be made based on experimental observations:

1) Because of the complex deformations of the joint, the bolt deformation and crack(s) propagation are observed to always be asymmetric to the centreline of the end-plate, which causes the asymmetrical fracture initiated by either bolt fracture or by end-plate tearing at the same row. In the spring model, this behaviour induces an unbalanced force distribution on the left and right parts of the same spring series, and hence, the fracture of a spring series usually occurs on one half of the spring series after the other. It is therefore assumed that fracture of a spring series only halves its stiffness and resistance, i.e. the values of $k_v, k_p, k_s, P^c_P$ and $P^c_s$ for the end-plate component are halved when fracture first occurs on one half of the end-plate and then reduced to zero when fracture occurs on the other half.

This assumption is based on experimental observation. For example, S10_0_0, which is one of the tests conducted in this thesis (Chapter 5), suffered three abrupt drops in resistance. In the first abrupt drop in resistance (Figure 3.13), the right outer part of the end-plate (the outer part of an end-plate is usually considered as a component in the top spring series) was considered as fractured (Figure 3.13 b)), since it was not stressed after
the crack propagated around the tip of the flange. On the other hand, the other side of the end-plate was considered as non-fractured (Figure 3.13 a)), since it continued transmitting force from the beam flange to the left bolt in the top bolt row. In the second abrupt drop in resistance (Figure 3.14), the left outer part of the end-plate was considered as fractured (Figure 3.14 a)), since the crack propagated to the edge of the end-plate and the left part of the end-plate could not transmit force any more. In the third abrupt drop in resistance (Figure 3.15), the left inner bolt stripped (Figure 3.15 a)) in which not force was transmitted any more. On the contrary, the right inner bolt did not strip and the right inner end-plate kept transmitting force from beam to that bolt (Figure 3.15 b)), though part of the end-plate has been torn apart. Figure 3.17 shows the component fracture sequence on the $M - \theta$ curve of S10_0_0.

![Figure 3.13: Outer End-Plate Bending Component After the First Abrupt Drop in Resistance](image)

**FIGURE 3.13: OUTER END-PLATE BENDING COMPONENT AFTER THE FIRST ABRUPT DROP IN RESISTANCE**

a) View from Left Hand Side  
b) View from Right Hand Side
a) View from Left Hand Side  
b) View from Right Hand Side

**FIGURE 3.14: OUTER END-PLATE BENDING COMPONENT AFTER THE SECOND ABRUPT DROP IN RESISTANCE**

The left outer part fractured
The right outer part was not stressed

a) View from Left Hand Side  
b) View from Right Hand Side

**FIGURE 3.15: INNER END-PLATE BENDING COMPONENT AND LEFT BOLT AFTER THE THIRD ABRUPT DROP IN RESISTANCE**

The left inner bolt stripped
The right inner part was still transmitting force

a) View from Left Hand Side  
b) View from Right Hand Side

c) View From Let Hand Side  
d) View From Right Hand Side

**FIGURE 3.16: TWO TENSION SPRING SERIES COMPLETELY FRACTURED**
2) For other components than the end-plate, after a major crack(s) occurs, the stiffness is assumed equal to zero, i.e. the resistance of the post-fracture system is assumed to be constant during each post-fracture stage.

3) After the system reaches the point where it returns to static equilibrium, it is assumed that the next component fractures (e.g. in the 1st post-fracture range in Figure 3.12, the point where the system returns to static equilibrium after fracture is $(\theta_1, M_1)$. The system suffers abrupt drop in resistance at this point and enters the 2nd post-fracture range), though the system may continue sustaining the same, or even higher, bending moment and rotating further. That is because of the instability of the system in the post-fracture range. The instability is caused by the unpredictable bolt fracture or end-plate tearing which is triggered by the unbalanced force distribution mentioned in assumption 1). Therefore, it is rational to make this conservative assumption. Moreover, as shown in Chapter 6, the predictions based on this assumption well match the experimental results.

Based on these assumptions, the post-fracture model can be obtained using the following steps:
1) The component model detects a component fracture in a spring series.

2) The total energy of the multi-spring system model at the incipient fracture stage, 
\[ V_{fi-1}(\theta_{fi-1}) \], is calculated using Eqn. (3.62) in which \( \theta_{fi-1} \) is the incipient fracture rotation. The subscript \( fi \) represents the fracture stage, as demonstrated in Figures 3.11-3.12.

3) The stiffness and strength of the fractured spring series is reduced by 50% in the post-fracture system model.

4) The total energy of the new post-fracture system \( (V_{fi}) \) at the incipient fracture rotation \( (\theta_{fi-1}) \) of the old system, \( V_{fi}(\theta_{fi-1}) \) is then calculated using Eqn. (3.62). It corresponds to the double-hatched area shown in Figure 3.11.

5) The rotation of the new post-fracture system, \( \theta_{fi} \), can be calculated using Eqn. (3.69).

6) Repeat Steps 1 to 5 until all tension spring series have failed.

The rotation of the \( i^{th} \) new post-fracture system, \( \theta_{fi} \), can be calculated as:

\[
\theta_{fi} = \frac{V_{fi-1}(\theta_{fi-1}) - V_{fi}(\theta_{fi-1})}{M_{fi}(\theta_{fi-1})} + \theta_{fi-1} \quad i = 1, 2, ... (3.69)
\]

where, \( M_{fi}(\theta_{fi-1}) \) is the moment on the new multi-spring model at the incipient fracture stage, \( (\theta_{fi-1}) \).

An example of a post-fracture analysis is shown in Figure 3.12. Three stages of component fracture occurred in the simulation and for each stage, the post-fracture system behaviour is demonstrated.

### 3.7 Application: Backbone Curve for Hysteretic Models

One of the possible applications of the full-range \( M - \theta \) model is in the earthquake field. According to Ibarra et al. (2005), in the earthquake engineering field, in order to evaluate the seismic performance of a system, hysteretic models for the structural components that the system contains, such as walls and frames, are required. The hysteretic model usually has a
non-deteriorating “backbone” curve (Figure 3.13) and hysteresis rules that reproduce the cyclic loading behaviour of the structure. The backbone curves are defined as the monotonic action-deformation curve of the structural components.

If a steel joint is considered as a structural component of a system, the backbone curve for the steel joint can be obtained by the generalised component model presented in this chapter. To exemplify the procedure, end-plate joint S10_0_0, which is described in detail in Chapter 6, is converted to the standard backbone curve introduced by Ibarra et al. (2005).

The backbone curve of end-plate joint S10_0_0 can be determined as a fit to its full-range $M - \theta$ curve by applying engineering judgement, as shown in Figure 3.14. Subsequently, the key parameters $K_e$, $a_s$, $a_c$, $M_y$, $M_c$, $M_r$, $\delta_y$, $\delta_c$, and $\delta_r$ can be determined according to the fitted curve, see Table 3.2 for the results for joint S10_0_0.
For hysteretic models, the joint with negative applied bending moment is also of great interest. By applying the generalised component model, the full-range joint behaviour with negative applied bending moment can be obtained (Figure 3.15). Then, based on engineering judgement, its backbone curve can again be determined as an appropriate fit to the full-range curve. Note that its post-fracture stiffness is the same as that of the joint with positive applied bending moment, since it has the same spring series configuration as the joint with positive applied bending moment in the 3rd fracture stage, where the spring series at the extended end-plate is completely fractured. Then, by adding the backbone curve for negative applied moment (Figure 3.15), the complete backbone curves for the end-plate joint can be obtained (Figure 3.16).
FIGURE 3.15: EXAMPLE OF CONVERTING THE FULL-RANGE CURVE TO THE BACKBONE CURVE (NEGATIVE APPLIED MOMENT)

TABLE 3.2: KEY PARAMETERS OF THE BACKBONE CURVE OF JOINT S10_0_0

<table>
<thead>
<tr>
<th>S10_0_0</th>
<th>$M_y$ (kNm)</th>
<th>$M_c$ (kNm)</th>
<th>$M_r$ (kNm)</th>
<th>$\delta_y$ (rad)</th>
<th>$\delta_c$ (rad)</th>
<th>$\delta_r$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Applied</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending Moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.21E+01</td>
<td>2.11E+02</td>
<td>6.32E+01</td>
<td>7.16E-03</td>
<td>1.44E-01</td>
<td>2.72E-01</td>
</tr>
<tr>
<td></td>
<td>$K_e$ (kNm/rad)</td>
<td>$a_s$</td>
<td>$a_c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.01E+04</td>
<td>1.00E-01</td>
<td>-1.15E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative Applied</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending Moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.99E+01</td>
<td>-1.38E+02</td>
<td>-6.93E+01</td>
<td>-6.64E-03</td>
<td>-3.30E-01</td>
<td>-3.90E-01</td>
</tr>
<tr>
<td></td>
<td>$K_e$ (kNm/rad)</td>
<td>$a_s$</td>
<td>$a_c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.50E+03</td>
<td>7.42E-02</td>
<td>-2.56E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The backbone curves of all the six end-plate joints modelled in Chapter 6 (S10_0_0_B, S20_0_0_B, FS1, FS2, and FS3) can be obtained by the same method. They are demonstrated in Appendix A3.

![Backbone Curve for S10_0_0](image)

**FIGURE 3.16: BACKBONE CURVE FOR S10_0_0**

### 3.8 Summary

The approach outlined in this chapter extends the Component Method to the post-ultimate range, and thus enables the full-range moment-rotation relationship of steel joints to be predicted. The method can be used to analyse spring models with any number of springs and does not pose numerical difficulties. It is potentially applicable to all types of joints, and has been applied to bolted moment end-plate joint in this Chapter. The proposed method accounts for fracture of components. It is shown that the full-range curves predicted by the extended component method can be closely approximate by the “backbone” curve defined by Ibarra et al. (2005).
One of the limitations of this approach is the lack of direct consideration of group effects associated with multiple spring rows. Eurocode 3 considers group effects when several components are located in close proximity. So, yield lines developed around the components are altered, leading to reductions in stiffness and resistance for involved individual components (e.g. end-plate bending and column flange bending components). However, for the joint configuration tested in Chapter 5, the column webs are strong and the weak component is the end-plate where the end-plate bending components between beam flanges are widely spaced. Therefore, group effects do not significantly influence the results. In Chapter 6, when calculating the parameters for the end-plate bending component and column flange bending component, a hybrid method is adopted to implicitly take group effects into account (Section 6.1.1.2). That is, the resistance and stiffness of these two components are calculated based on corresponding effective widths obtained from the Eurocode 3 procedure which considers group effects.
Chapter 4: Analysis of the Column Web Component Buckling

4.1 Introduction

The column web is a major component of steel joints. It bears most of the compression force transmitted from the beam flange in compression (Figure 4.1) when the steel joint is subject to bending. Consequently, this component may buckle. Therefore, the behaviour of the steel joint will be greatly affected by the buckling and post-buckling behaviour of the column web component, which has not been well understood. For example, Eurocode 3 Part 1-8 suggests stiffening this component if prone to buckling.

![Column Web Under Compression](Figure 4.1)

While thus Eurocode 3 favours a strong or stiffened column web, in practice, a weaker unstiffened column web may be suitable when it can sustain sufficient large compression forces in the post-buckling range (Figure 4.2). In this range, the resistance may increase significantly depending on the plate slenderness. Moreover, it can provide larger ductility in the post-ultimate range which is usually desirable in structural design. Therefore, it is of interest to devise a model which can predict the full-range behaviour of the column web component.
4.2 Literature Review

Since existing research has focused on the bearing load of strong or stiffened columns, few techniques for modelling the detailed behaviour of unstiffened column webs can be found in the literature. However, an analogy to steel girders subject to patch loading (Figure 4.3) can simplify the complexity of the problem. The reason for using this analogy is due to the similarity between the loading conditions and the available comprehensive body of research on patch-loaded steel girders.

FIGURE 4.2 FORCE-DISPLACEMENT OF COLUMN WEB UNDER PATCH LOADING

FIGURE 4.3: PLATES SUBJECTED TO EDGE PATCH LOADING AND SUPPORTED BY SHEAR FORCE
4.2.1 Analysis of the Elastic Buckling of Plates Subjected to Edge Loading

For the elastic buckling analysis, Khan and Walker (1972) devised a viable analytical method for the plate subject to patch loading. Their elastic buckling analysis can be extended to the elastic and inelastic post-buckling ranges by adding the second order stress function and inelastic Young’s modulus as will be described in the Chapter.

4.2.2 Eurocode 3

As a steel girder subjected to patch loading reaches its ultimate load in the post-buckling range, the most robust design method for predicting the ultimate load is given in Eurocode 3 Part 1-5 (2006a), which is derived for the design of beams subjected to patch loading. Although it is not a specific method derived for column web components in joints, its concepts can be used to predict the post-buckling behaviour and strength of the column web component in compression.

Eurocode 3 Part 1-5 (2006a) adopts a plate buckling approach and is readily understood by practicing engineers. Compared to other methods, it is the most accurate method for predicting the strengths obtained in the tests performed in University of Sydney (Chapter 5).

---

FIGURE 4.4: RESISTANCE TO PATCH LOADING OF A GIRDER (EUROCODE3, 2006A)
The procedure provided by Eurocode 3 is shown in Eqn. (4.1-4.8), where the first step is to calculate the yield resistance using Eqn. (4.1-4.3).

\[ F_y = f_{yw} l_y t_w \quad (4.1) \]

\[ l_y = S_s + 2t_f \left( 1 + \frac{f_y f_b f}{f_{yw} t_w} + m_2 \right) \leq a \quad (4.2) \]

\[ m_2 = 0.02 \left( \frac{h_w}{t_f} \right)^2 \text{ if } \overline{x}_F \geq 0.5 \text{ otherwise } m_2 = 0 \quad (4.3) \]

where \( f_{yw} \) is the yield stress of the flange, \( f_{yw} \) is the yield stress of the web. \( S_s, t_f, t_w, b_f, a, \) and \( h_w \) are defined in Figure 4.4.

The second step is to calculate the elastic buckling load using Eqn. (4.4-4.5)

\[ F_{cr} = 0.9 k_F E \frac{h_w^3}{h_w} \quad (4.4) \]

\[ k_F = 6 + 2 \left( \frac{h_w}{a} \right)^2 \quad (4.5) \]

The third and final step is to find the strength reduction factor \( \chi_F \) and ultimate resistance using Eqn. (4.6-4.8)

\[ F_{RD} = \chi_F F_y \frac{Y_{M1}}{Y_{M1}} \quad (4.6) \]

\[ \chi_F = \frac{0.5}{\overline{x}_F} \leq 1.0 \quad (4.7) \]

\[ \overline{x}_F = \sqrt{\frac{F_y}{F_{cr}}} \quad (4.8) \]

### 4.2.3 Roberts’ Mechanism Model

After passing the ultimate load, the column web component enters its post-ultimate range where the resistance decreases as the deformation continues to develop. This behaviour is ductile and therefore the component can absorb large amounts of energy. One of the methods available for obtaining this behaviour is Roberts’ mechanism model.

Roberts et al. developed a mechanism model that can predict the ultimate resistance of a girder subjected to edge loading (Roberts and Rockey, 1979, Roberts, 1981, Roberts and
Markovic, 1983, Roberts and Newark, 1997). The model assumes that the girder collapses because the web folds at the ultimate load (Figure 4.5). The solution to the model is a combination of the principle of virtual work and yield line analysis, providing conservative predictions with acceptable accuracy.

**FIGURE 4.5: COLLAPSE MODEL OF A GIRDERR UNDER EDGE PATCH LOADING**

Although Roberts’ mechanism model was originally developed to obtain the ultimate resistance of a girder subjected to edge loading, it also considered the web and flange deformation in the post-ultimate range including plastic hinges, yield line and web folding that can be used to work out the relationship between the applied patch load and the vertical displacement of the loading point.

Eqns.(4.9-4.14) are the key equations in Roberts’ mechanism model (Roberts and Newark, 1997):

\[ P_u = \frac{4M_f}{\beta} + \frac{4\beta M_w}{\cos \theta} + \frac{2c_e M_w}{\cos \theta} - \frac{2\eta M_w}{\cos \theta} \]  \hspace{1cm} (4.9)

\[ \beta^2 = \frac{M_f \alpha \cos \theta}{M_w} \]  \hspace{1cm} (4.10)

\[ \alpha = 20t_w \frac{\sigma_w}{\sigma_f} \leq \frac{d_w}{8} \]  \hspace{1cm} (4.11)

\[ \eta = \frac{(4\beta + 2c_e)M_w}{2M_w + \sigma_w t_w \alpha \cos \theta} \]  \hspace{1cm} (4.12)

\[ c_e = c + 2t_f \]  \hspace{1cm} (4.13)

\[ \frac{\cos \theta}{1 - \sin \theta} = \frac{12E_l M_w}{M_f^2} \]  \hspace{1cm} (4.14)
where $M_f$ is the resistance of the plastic hinge in the flange, $M_w$ is the resistance of a yield line in the web (in KN·m per unit length), $c$ is the length of patch loading, $\eta$ is the length of the membrane action at the junction of the web and flange and other variables are shown in Figure 4.5.

The method cannot be applied directly to the column web component because some of the important parameters such as $\alpha$ and $\beta$ cannot be confirmed with the test results of the column web component, because the column web component is much stockier and shallower than typical plates. Some modifications are therefore required.

**4.2.4 Missing Analysis Parts**

In conclusion, to obtain the full-range behaviour, the following parameters must be determined:

1. Inelastic post-buckling stiffness
2. Column web deformation at the ultimate resistance

These missing analysis parts will be investigated in this chapter and a model suitable for them will be derived.

**4.3 The Proposed Model**

The proposed model is an approximate method to predict the full-range force-displacement behaviour of a column web component. It includes elastic buckling behaviour, elastic/inelastic post-buckling behaviour and post-ultimate behaviour. The Elastic buckling analysis is based on Khan and Walker’s method (Khan and Walker, 1972), and a second order stress field (Budiansky, 1974) is added to this to accommodate nonlinear elastic post-buckling behaviour. Inelastic post-buckling behaviour is analysed using the same elastic post-buckling model with an inelastic Young’s modulus to include the reduction in stiffness caused by the gradual material yielding. Then, by using the post-buckling stiffnesses and ultimate resistance obtained by Eurocode 3, the deformation at the ultimate load can be found. The post-ultimate
behaviour is obtained by a modified model based on Robert’s mechanism method where some parameters are reformulated to suit the characteristics of the column web component.

The full-range curve for column web component in compression can be applied to the model introduced in Chapter 3 or other models based on component approach. For example, the joint model derived by including the derivation of the strain-hardening stiffness and ultimate resistance may use

### 4.3.1 Configuration of the Column Web Component

![Figure 4.6 Column Web Under Patch Loading](image)

The column web component and its loading condition are shown in Figure 4.6. Here, the origin of the coordinate system is set at the centre of the column web under the centre of the applied loading (Figure 4.6). The longitudinal axis of the column is set as the x-axis and the transvers axis of the column is set as the y-axis. Based on this coordinate system, a patch loading is applied in the direction of the y-axis. Figure 4.6 also shows the notation for the length of patch loading, which is $2C$, the length of the web plate, which is $2L$, the length of that part of the web plate is assumed to buckle during loading, $2A$, and the depth of the web plate, which is $2B$. 

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Figure 4.2 shows the response of a column web subjected to patch loading. When patch loading is transmitted from the beam, the web will be shortened by the compression exerted by the top flange. The web in-plane deformation is denoted as $v$ (Figure 4.2) and web deflection is denoted as $w$ (Figure 4.2).

4.3.2 Analysis of Elastic Post-Buckling

As discussed above, the first part of the column web deformation is the elastic buckling and the elastic post-buckling range. The key unknown parameters are the buckling load $F_{cr}$ and the elastic post-buckling stiffness $K_{p-b}$, which can be determined by a post-buckling analysis based on the energy method, as introduced in Section 4.3.2.1-4.3.2.8.

4.3.2.1 The Total Energy

The total energy of the column web can be written as:

$$\Pi = U_b + U_m + U_f - W_f$$

(4.15)

where $U_b$ is the energy due to the out-of-plane bending of the web, $U_m$ is the energy due to membrane actions in the web plate, $U_f$ is the energy due to flange bending caused by the applied patch load $P$, and $W_f$ is the work done by the applied patch load, when undergoing vertical displacement.

The web plate bending energy $U_b$ can be obtained by classic plate theory,

$$U_b = \int_{-B}^{B} \int_{-A}^{A} \frac{D}{2} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 + 2(1 - \nu) \left[ \frac{\partial^2 w}{\partial x \partial y} \right]^2 + 2 \left[ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] dxdy$$

(4.16)

where $w$ is the out-of-plane deflection of the column web, $\nu$ is the Possion’s ratio, $t_w$ is the thickness of the web plate, and $D$ is the rigidity of the web plate which can be expressed as:

$$D = \frac{E_w t_w^3}{12(1 - \nu^2)}$$

(4.17)

The web plate membrane energy $U_m$ consists of three parts.
\[ U_m = U_{m1} + U_{m2} + U'_m \]  
\[ U_{m1} \]  is the membrane energy caused by the web plate bending, and it can be obtained by classic plate theory,

\[ U_{m1} = \int_{-B}^{B} \int_{-A}^{A} t_w \left[ \frac{\partial^2 F}{\partial y^2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{\partial^2 F}{\partial x^2} \left( \frac{\partial w}{\partial y} \right)^2 + 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dx dy \]  

where \( F \) is the stress function that can be written as:

\[ F = F_0 + F_2 \]  
\[ F_0 \]  is the fundamental path of the stress function. Its second order derivatives \( \frac{\partial^2 F_0}{\partial x \partial y} \) are the statically admissible pre-buckling stress field in the web plate (Khan and Walker, 1972). \( F_2 \) is the second order term of the stress function. Its second order derivatives \( \frac{\partial^2 F_2}{\partial x \partial y} \) are the second order stress field which describes the distribution of the post-buckling stress. Higher order terms in the stress function were ignored in this analysis. The first order field \( F_1 \) of the stress function is zero (Budiansky, 1974).

\[ U_{m2} \]  is the membrane energy caused by the web plate stretching at the mid-plane,

\[ U_{m2} = \int_{-B}^{B} \int_{-A}^{A} t_w \left[ \frac{\partial^2 F}{\partial y^2} \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial^2 F}{\partial x^2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{\partial^2 F}{\partial x \partial y} \left( \frac{\partial u}{\partial x} \right) \left( \frac{\partial u}{\partial y} \right) \right] dx dy \]  

where \( u \) and \( v \) are the displacement fields in the \( x \)-axis and \( y \)-axis directions respectively.

\[ U'_m \]  is an extra energy term arising when applying a statically admissible stress field (Khan and Walker, 1972),

\[ U'_m = -\int_{-B}^{B} \int_{-A}^{A} \frac{t_w}{E} \left[ \left( \frac{\partial^2 F_0}{\partial y^2} \right)^2 + \left( \frac{\partial^2 F_0}{\partial x^2} \right)^2 + \left( \frac{\partial^2 F_2}{\partial y^2} \right)^2 \right] dx dy \]  

The strain energy associated with bending of the flange plate \( U_f \) can be obtained by:

\[ U_f = \int_{-A}^{A} \frac{E_f l_f}{2} \left( \frac{\partial^2 v(x)}{\partial x^2} \right)^2 dx \]
where $E_f$ is the modulus of elasticity of the flange, $I_f$ is the second moment of inertia of the flange, and $v(x)$ is the vertical displacement at the mid plane of the flange in terms of its x-axis coordinate.

The work done by the applied patch load can be written as:

$$W_p = \int_{-C}^{C} \frac{P}{2C} \Delta(x) dx = P \overline{v}_C$$

where $\Delta(x)$ is the vertical displacement of the applied patch load, $C$ is the half-length of the applied patch load and $P$ is the total applied patch load. $\overline{v}_C$ is the average displacement of the applied patch loading.

Therefore $F$, $u$, $v$ and $w$ are the unknown terms and will be investigated in the following sections (Sections 4.3.2.3-4.3.3.7).

### 4.3.2.2 The Half Width of the Effective Column Web Plate, $A$

In order to obtain the displacement field, the half width ($A$) of the effective column web plate (Figure 4.6) is first determined using the method proposed by Khan and Walker (1972) (Eqns. (4.25-4.32)).

\[
\begin{align*}
\frac{4\alpha^3 + 5\alpha/2}{(\alpha^4 + 5\alpha^2/4 + 17/32)} & = \frac{3ma_1\alpha^2 + 2ma_2\beta\alpha - m^3a_3 + a_4(-m\pi\alpha^2 \cos(m\pi/\alpha) + 4\alpha^3 \sin(m\pi/\alpha))}{\alpha^4(ma_1/\alpha + ma_2\beta/\alpha^2 - m^3a_3/\alpha^3 + a_4 \sin(m\pi/\alpha))} \\
\alpha_1 & = \frac{364}{27\pi^4} - \frac{2}{3\pi^2} + \frac{5}{16} \\
\alpha_2 & = \frac{4}{3\pi^2} \\
\alpha_3 & = \frac{2}{9\pi^2} \\
\alpha_4 & = \frac{364}{27\pi^5} + \frac{5}{16\pi} \\
\alpha & = A/B \\
m & = C/B \\
\beta & = L/B
\end{align*}
\]
The value of $A$ obtained from this method (174mm) was very close to the value obtained from the results (181mm) of University of Sydney tests.

### 4.3.2.3 Out-of-Plane Deflection Function $w$

The out-of-plane deflection function $w$ (Eqn. (4.33)) used in this analysis was taken from Khan and Walker’s method (1972). Its shape is confirmed by the deflection curve observed in the experiment.

$$w = q_1 t_w \left( \cos \frac{\pi x}{2A} \right) \left( \cos \frac{\pi y}{2B} + \frac{1}{4} \sin \frac{\pi y}{B} \right) \quad (4.33)$$

where $q_1$ is the amplitude which will be used later in perturbation analysis.

The first and second order derivatives of $w$ can therefore be written as:

$$\frac{\partial w}{\partial x} = q_1 t_w \left( -\frac{\pi}{2A} \right) \sin \frac{\pi x}{2A} \left( \cos \frac{\pi y}{2B} + \frac{1}{4} \sin \frac{\pi y}{B} \right) \quad (4.34)$$

$$\frac{\partial^2 w}{\partial x^2} = q_1 t_w \left( -\frac{\pi^2}{4A^2} \right) \cos \frac{\pi x}{2A} \left( \cos \frac{\pi y}{2B} + \frac{1}{4} \sin \frac{\pi y}{B} \right) \quad (4.35)$$

$$\frac{\partial w}{\partial y} = q_1 t_w \left( -\frac{\pi}{2B} \right) \cos \frac{\pi x}{2A} \left( \sin \frac{\pi y}{2B} \right) \quad (4.36)$$

$$\frac{\partial^2 w}{\partial y^2} = q_1 t_w \left( -\frac{\pi^2}{4B^2} \right) \cos \frac{\pi x}{2A} \left( \cos \frac{\pi y}{2B} + \sin \frac{\pi y}{B} \right) \quad (4.37)$$

$$\frac{\partial^2 w}{\partial x \partial y} = q_1 t_w \left( \frac{\pi^2}{4BA} \right) \sin \frac{\pi x}{2A} \left( \sin \frac{\pi y}{2B} \right) \quad (4.38)$$

### 4.3.2.4 Stress Field

The stress field on the web plate can be obtained by the perturbation analysis presented by Budiansky (1974), where the von Karman equation (Eqn. (4.39)) is used to obtain the second order stress function.

$$\nabla^4 F = \frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[ \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \quad (4.39)$$

The stress and out-of-plane deflection functions can be expanded as:

$$\{w\} = \{w_0\}_F + q_1 \{w_1\}_F + q_1^2 \{w_2\}_F + \ldots \quad (4.40)$$
Since the change in the amplitude factor \( q_1 \) is set at a small value, higher order terms will be omitted without significantly impacting on the accuracy of the solution. In the post-buckling analysis, those terms that are higher than second order can be omitted.

Therefore, by substituting Eqn. (4.40) into Eqn. (4.39) and omitting the higher order terms:

\[
\nabla^4 F_0 + \nabla^4 F_1 q_1 + \nabla^4 F_2 q_1^2 \\
= E \left[ \left( \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_1}{\partial x^2} q_1 + \frac{\partial^2 w_2}{\partial x^2} q_1^2 \right) \cdot \left( \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial^2 w_1}{\partial y^2} q_1 + \frac{\partial^2 w_2}{\partial y^2} q_1^2 \right) \right. \\
- \left. \left( \frac{\partial^2 w_0}{\partial x \partial y} q_1 + \frac{\partial^2 w_1}{\partial x \partial y} q_1^2 \right)^2 \right] \\
(4.41)
\]

Rearrange Eqn. (4.41),

\[
\nabla^4 F_0 + \nabla^4 F_1 q_1 + \nabla^4 F_2 q_1^2 \\
= E \left[ \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} q_1 + \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial x^2} q_1^2 + \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_2}{\partial y^2} q_1^2 \\
+ \frac{\partial^2 w_0}{\partial y^2} \cdot \frac{\partial^2 w_1}{\partial x^2} q_1^2 + \frac{\partial^2 w_1}{\partial x^2} q_1 + \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_2}{\partial y^2} q_1^2 + \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_2}{\partial x^2} q_1^2 \\
+ \frac{\partial^2 w_2}{\partial x^2} \cdot \frac{\partial^2 w_2}{\partial y^2} q_1^2 - \left( \frac{\partial^2 w_0}{\partial x \partial y} \right)^2 q_1 - 2 \frac{\partial^2 w_0}{\partial x \partial y} \cdot \frac{\partial^2 w_1}{\partial x \partial y} q_1^2 - 2 \frac{\partial^2 w_0}{\partial x \partial y} \cdot \frac{\partial^2 w_2}{\partial x \partial y} q_1^2 \\
- \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 q_1^2 - 2 \frac{\partial^2 w_1}{\partial x \partial y} \cdot \frac{\partial^2 w_2}{\partial x \partial y} q_1^2 - \left( \frac{\partial^2 w_2}{\partial x \partial y} \right)^2 q_1^4 \right] \\
(4.42)
\]

The equation for the fundamental term of the stress function is obtained by equating those terms of Eqn. (4.42) factored by \( q_1 \) or higher orders of \( q_1 \) to zero.

\[
\nabla^4 F_0 - E \left[ \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_0}{\partial y^2} - \left( \frac{\partial^2 w_0}{\partial x \partial y} \right)^2 \right] = 0 \\
(4.43)
\]

Similarly, the first order term of the stress function is obtained from the term of Eqn. (4.42) factored by \( q_1 \).

\[
\nabla^4 F_1 - E \left( \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} \cdot \frac{\partial^2 w_1}{\partial x^2} - 2 \frac{\partial^2 w_0}{\partial x \partial y} \cdot \frac{\partial^2 w_1}{\partial x \partial y} \right) q_1 = 0 \\
(4.44)
\]

The second order terms of the stress function is obtained from the term of Eqn. (4.42) factored by \( q_1^2 \).
\left\{ \nabla^4 F_2 - E \left[ \frac{\partial^2 w_0}{\partial x^2} \cdot \frac{\partial^2 w_2}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} \cdot \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} - 2 \frac{\partial^2 w_0}{\partial x \partial y} \cdot \frac{\partial^2 w_2}{\partial x \partial y} \right] - \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 \right\} q_1^2 = 0 \quad (4.45)

In Eqns. (4.43-4.45), the fundamental path solution of the out-of-plane deflection $w_0$, the second order solution of the out-of-plane deflection $w_2$, and the first order solution of the stress function $F_1$ are all zero (Budiansky, 1974). Therefore, Eqns. (4.43-4.45) can be reduced to:

\[ \nabla^4 F_0 = 0 \quad (4.46) \]

\[ \nabla^4 F_2 = E \left[ \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} - \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 \right] \quad (4.47) \]

The fundamental path solution for the stress function $F_0$ is given in Section 4.3.2.5 and the first order out-of-plane deflection (Khan and Walker, 1972) has been shown in Eqn. (4.33). The second order solution for the stress function $F_2$ is derived using the perturbation analysis described in Section 4.3.2.6.

### 4.3.2.5 The Fundamental Path Solution of the Stress Function $F_0$

Khan and Walker (1972) suggested the statically admissible stress field shown in Eqn. (4.48-4.54), as the fundamental path solution to the stress function to solve the energy equations. Shahabian and Roberts (1999) adopted the same stress field to solve patch loading buckling problems, which is also used in the present analysis. The statically admissible stress field can be written as:

\[ \frac{\partial^2 F_0}{\partial y^2} = \sigma_x^* = \frac{P}{t_w} \left( - \frac{3L}{8B^3} \right) y \left( 2 - \frac{C}{L} \frac{x^2}{CL} \right), \quad |x| \leq C \quad (4.48) \]

\[ \frac{\partial^2 F_0}{\partial x^2} = \sigma_x^* = \frac{P}{t_w} \left( - \frac{3L}{4B^3} \right) y \left( 1 - \frac{|x|}{L} \right), \quad |x| > C \quad (4.49) \]

\[ \frac{\partial^2 F_0}{\partial x^2} = \sigma_y^* = \frac{P}{t_w} \left( - \frac{1}{8C} \right) \left( 2 + \frac{3y}{B} - \frac{y^3}{B^3} \right), \quad |x| \leq C \quad (4.50) \]

\[ \frac{\partial^2 F_0}{\partial x \partial y} = \tau_{xy}^* = \frac{P}{t_w} \left( 3 \frac{B}{8BC} \right) \left( 1 - \frac{y^2}{B^2} \right), \quad -C \leq x \leq C \quad (4.51) \]

\[ \frac{\partial^2 F_0}{\partial y \partial y} = \tau_{xy}^* = \frac{P}{t_w} \left( 3 \frac{B}{8BC} \right) \left( 1 - \frac{y^2}{B^2} \right), \quad -C \leq x \leq C \quad (4.52) \]
\[
\frac{\partial^2 F_0}{\partial x \partial y} = \tau_{xy} = \frac{P}{t_w} \left( \frac{3}{8B} \right) x \left( \frac{y^2}{B^2} - 1 \right), \quad x < -C
\]
\[
\frac{\partial^2 F_0}{\partial x \partial y} = \tau_{xy} = \frac{P}{t_w} \left( \frac{3}{8B} \right) x \left( 1 - \frac{y^2}{B^2} \right), \quad x > C
\]

4.3.2.6 Second Order Solution for the Stress Function

The second order stress field can be determined by a perturbation analysis (Budiansky, 1974). The analysis, the first order of the out-of-plane deflection \( w_1 \) is described by Eqn. (4.33) and the first order solution of the stress functions \( F_1 \) is zero.

By substituting \( w_1 \) into the right hand side of Eqn. (4.47), the following equations (Eqns. 4.55-4.57) can be obtained:

\[
\frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} = q_1 t_w \left( -\frac{\pi^2}{4A^2} \right) \cos \frac{\pi x}{2A} \left( \cos \frac{\pi y}{2B} + \frac{1}{4} \sin \frac{\pi y}{B} \right) \]
\[
\cdot \left[ q_1 t_w \left( -\frac{\pi^2}{4B^2} \right) \cos \frac{\pi y}{2A} \left( \cos \frac{\pi y}{2B} + \sin \frac{\pi y}{B} \right) \right] = q_1 t_w \left( \frac{\pi^4}{16B^2A^2} \right) \cos^2 \frac{\pi x}{2A} \left( \cos \frac{\pi y}{2B} + \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} + \frac{1}{4} \sin \frac{\pi y}{B} \right) \left( \sin \frac{\pi y}{2B} - \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} + \frac{1}{4} \cos \frac{\pi y}{B} \right) \]

\[
\left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 = q_1 t_w \left( \frac{\pi^2}{4BA} \right) \sin \frac{\pi x}{2A} \left( \sin \frac{\pi y}{2B} - \frac{1}{2} \cos \frac{\pi y}{B} \right)^2 \]
\[
= q_1 t_w \left( \frac{\pi^4}{16B^2A^2} \right) \sin^2 \frac{\pi x}{2A} \left( \sin^2 \frac{\pi y}{2B} - \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} + \frac{1}{4} \cos^2 \frac{\pi y}{B} \right) \]

\[
\frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} - \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 = q_1 t_w \left( \frac{\pi^4}{16B^2A^2} \right) \]
\[
\cdot \left( \cos^2 \frac{\pi x}{2A} \cos^2 \frac{\pi y}{2B} + \frac{1}{4} \cos^2 \frac{\pi x}{2A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} + \frac{1}{4} \cos^2 \frac{\pi x}{2A} \sin^2 \frac{\pi y}{B} \right) \]
\[
- \sin^2 \frac{\pi x}{2A} \sin^2 \frac{\pi y}{2B} + \sin^2 \frac{\pi x}{2A} \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} - \frac{1}{4} \sin^2 \frac{\pi x}{2A} \cos^2 \frac{\pi y}{B} \right) \]

Since
\[
\cos^2 \theta = \frac{1}{2} \left( \cos 2\theta + 1 \right) \]
\[
\sin^2 \theta = \frac{1}{2} \left( 1 - \cos 2\theta \right) \]

every trigonometric term in Eqn. (4.57) can be reduced to linear terms (Eqns. (4.60-4.65)).
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\[ \frac{1}{4} \cos \frac{\pi x}{A} \cos \frac{\pi y}{B} + \frac{1}{4} \cos \frac{\pi x}{A} + \frac{1}{4} \cos \frac{\pi y}{B} + \frac{1}{4} \]  

(4.60)

\[ \frac{5}{4} \cos^2 \frac{\pi x}{2A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{2B} = \frac{5}{8} \left( \cos \frac{\pi x}{A} + 1 \right) \sin \frac{\pi y}{B} \cos \frac{\pi y}{2B} \]

\[ = \frac{5}{8} \cos \frac{\pi x}{A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{2B} + \frac{5}{8} \sin \frac{\pi y}{B} \cos \frac{\pi y}{2B} \]  

(4.61)

\[ \frac{1}{4} \cos^2 \frac{\pi x}{2A} \sin^2 \frac{\pi y}{B} = \frac{1}{16} \left( \cos \frac{\pi x}{A} + 1 \right) \left( 1 - \cos \frac{2\pi y}{B} \right) \]

\[ = \frac{1}{16} \cos \frac{\pi x}{A} \cos \frac{2\pi y}{B} + \frac{1}{16} \cos \frac{\pi x}{A} - \frac{1}{16} \cos \frac{2\pi y}{B} + \frac{1}{16} \]  

(4.62)

\[ - \sin^2 \frac{\pi x}{2A} \sin^2 \frac{\pi y}{B} = - \frac{1}{4} \left( 1 - \cos \frac{\pi x}{A} \right) \left( 1 - \cos \frac{\pi y}{B} \right) \]

\[ = \frac{1}{16} \cos \frac{\pi x}{A} \cos \frac{\pi y}{B} + \frac{1}{16} \cos \frac{\pi x}{A} + \frac{1}{4} \cos \frac{\pi y}{B} - \frac{1}{4} \]  

(4.63)

\[ \sin^2 \frac{\pi x}{2A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} = \frac{1}{2} \left( 1 - \cos \frac{\pi x}{A} \right) \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} \]

\[ = \frac{1}{2} \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} - \frac{1}{2} \cos \frac{\pi x}{A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} \]  

(4.64)

\[ - \frac{1}{4} \sin^2 \frac{\pi x}{2A} \cos^2 \frac{\pi y}{B} = - \frac{1}{16} \left( 1 - \cos \frac{\pi x}{A} \right) \left( \cos \frac{2\pi y}{B} + 1 \right) \]

\[ = \frac{1}{16} \cos \frac{\pi x}{A} \cos \frac{2\pi y}{B} + \frac{1}{16} \cos \frac{\pi x}{A} - \frac{1}{16} \cos \frac{2\pi y}{B} - \frac{1}{16} \]  

(4.65)

By substituting Eqns. (4.60-4.65) into Eqn. (4.57) and collecting like terms, the following equation is obtained

\[ \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} - \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 = q^2 \frac{1}{4} \left( \frac{\pi^4}{16B^2A^2} \right) f(x, y) \]  

(4.66)

where

\[ f(x, y) = \left[ \left( \frac{1}{2} \cos \frac{\pi x}{A} + \frac{1}{2} \cos \frac{\pi y}{B} \right) + \left( \frac{1}{8} \cos \frac{\pi x}{A} - \frac{1}{8} \cos \frac{2\pi y}{B} \right) \right. \]

\[ + \left( \frac{1}{2} \sin \frac{\pi x}{A} \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} + c_1 \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} + \frac{4}{8} \sin \frac{\pi y}{B} \cos \frac{\pi y}{B} \right) \]

\[ - \left. \frac{4}{8} \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B} \cos \frac{\pi y}{B} \right] \]

(4.67)

\[ c_1 = \frac{5}{8} \]  

(4.68)

In order to simplify the Eqn. (4.67), the factor \( c_1 \) must be changed to reduce the complexity of Eqn. (4.67). By changing the factor \( c_1 \) from \( 5/8 \) to \( 1/2 \), Eqn. (4.67) can be reduced to,

\[ f'(x, y) = \left( \frac{5}{4} \cos \frac{\pi x}{A} + \cos \frac{\pi y}{B} - \frac{1}{4} \cos \frac{2\pi y}{B} + \sin \frac{3\pi y}{2B} + \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B} \right) \]  

(4.69)
By changing the factor $c_1$ to a smaller value, i.e. from $5/8$ to $\frac{1}{2}$, $f'(x,y)$ is slightly smaller than $f(x,y)$. For example, at the maximum out-of-plane deflection point $p(0,0.239B)$, $f'(x,y)$ is 8.9% smaller than $f(x,y)$. This difference will introduce a slight error into the final result, which is deemed acceptable.

Then, by substituting Eqn. (4.66) with $f(x,y)$ replaced by $f'(x,y)$ into Eqn. (4.47), Eqn. (4.70) is obtained

$$
\nabla^4 F_2 = E \left[ \frac{\partial^2 w_1}{\partial x^2} \cdot \frac{\partial^2 w_1}{\partial y^2} - \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 \right]
$$

$$
= E q^2 w_0 \left[ \frac{\pi^4}{32B^2A^2} \right] \left( \frac{5}{4} \cos \frac{\pi x}{A} + \cos \frac{\pi y}{B} - \frac{1}{4} \cos \frac{2\pi y}{B} + \sin \frac{3\pi y}{2B} \right)
$$

$$
+ \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

Evidently, the $F_2$ function should be in the following form:

$$
F_2 = C_1 \cos \frac{\pi x}{A} + C_2 \cos \frac{\pi y}{B} + C_3 \cos \frac{2\pi y}{B} + C_4 \sin \frac{3\pi y}{2B} + C_5 \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

(4.71)

By differentiating $F_2$ four times, the fourth order derivatives of $F_2$ can be obtained:

$$
\frac{\partial^4 F_2}{\partial x^4} = C_1 \frac{\pi^4}{A^4} \cos \frac{\pi x}{A} + C_5 \frac{\pi^4}{A^4} \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

(4.72)

$$
\frac{\partial^4 F_2}{\partial y^4} = C_2 \frac{\pi^4}{B^4} \cos \frac{\pi y}{B} + C_3 \frac{16\pi^4}{B^4} \cos \frac{2\pi y}{B} + C_4 \frac{81\pi^4}{16B^4} \sin \frac{3\pi y}{2B}
$$

$$
+ C_5 \frac{\pi^4}{16B^4} \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

(4.73)

$$
\frac{\partial^4 F_2}{\partial x^2 \partial y^2} = C_5 \frac{\pi^4}{4B^2A^2} \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

(4.74)

Therefore, by adding Eqns. (4.72-4.74) appropriately, $\nabla^4 F_2$ can be also written as:

$$
\nabla^4 F_2 = \frac{\partial^4 F_2}{\partial x^4} + 2 \frac{\partial^4 F_2}{\partial x^2 \partial y^2} + \frac{\partial^4 F_2}{\partial y^4}
$$

$$
= C_1 \frac{\pi^4}{A^4} \cos \frac{\pi x}{A} + C_2 \frac{\pi^4}{B^4} \cos \frac{\pi y}{B} + C_3 \frac{16\pi^4}{B^4} \cos \frac{2\pi y}{B} + C_4 \frac{81\pi^4}{16B^4} \sin \frac{3\pi y}{2B}
$$

$$
+ C_5 \pi^4 \left( \frac{1}{A^2} + \frac{1}{4B^2} \right)^2 \cos \frac{\pi x}{A} \sin \frac{\pi y}{2B}
$$

(4.75)

Since Eqn. (4.70) and Eqn. (4.75) must be the same, the coefficients $C_1, C_2, C_3, C_4$ and $C_5$ can be determined:
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\[ C_1 = \frac{5E q_1^2 t_w^2 A^2}{128B^2} \]  
(4.76)

\[ C_2 = \frac{E q_1^2 t_w^2 B^2}{32A^2} \]  
(4.77)

\[ C_3 = \frac{-E q_1^2 t_w^2 B^2}{2048A^2} \]  
(4.78)

\[ C_4 = \frac{E q_1^2 t_w^2 B^2}{162A^2} \]  
(4.79)

\[ C_5 = \frac{E q_1^2 t_w^2}{32B^2 A^2} \left( \frac{1}{A^2} + \frac{1}{4B^2} \right)^{-2} \]  
(4.80)

Therefore, by combining the fundamental path and second order solutions, the stress field can be expressed by:

\[ \sigma_{xx} = \frac{\partial^2 F_0}{\partial y^2} + \frac{\partial^2 F_2}{\partial y^2} \]  
(4.81)

\[ \sigma_{yy} = \frac{\partial^2 F_0}{\partial x^2} + \frac{\partial^2 F_2}{\partial x^2} \]  
(4.82)

\[ \tau_{xxyy} = \frac{\partial^2 F_0}{\partial x \partial y} - \frac{\partial^2 F_2}{\partial x \partial y} \]  
(4.83)

### 4.3.2.7 Displacement Field and Average Web Deformation

The in-plane strains of an element on the web plate can be written as:

\[ \varepsilon_{xx} = \frac{1}{E} \left( \sigma_{xx} - u \sigma_{yy} \right) = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \]  
(4.84)

\[ \varepsilon_{yy} = \frac{1}{E} \left( \sigma_{yy} - u \sigma_{xx} \right) = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 \]  
(4.85)

\[ \gamma_{xxyy} = \frac{1}{G} \tau_{xxyy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \]  
(4.86)

By integrating the unit displacement in the vertical \( y \)-axis direction of the web plate, the displacement of the web plate at the top edge relative to the bottom edge can be found:

\[ v(x) = \int_{-B}^{B} \frac{\partial v}{\partial y} \, dy \]  
(4.87)

where

\[ \frac{\partial v}{\partial y} = \frac{1}{E} \left( \sigma_{yy} - u \sigma_{xx} \right) - \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 \]

By integrating Eqn. (4.87), the deflection of the top flange is obtained as

\[ v(x) = -\frac{5\pi^2 t_w^2}{64B} \left[ 1 + 2 \cos \left( \frac{\pi x}{A} \right) \right] q_1^2 \quad A \geq x > |C| \]  
(4.88)
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\[ v(x) = -\frac{B}{2Ct_w}P - \frac{5\pi^2 t_w^2}{64B} \left[ 1 + 2 \cos \left( \frac{\pi x}{A} \right) \right] q_1^2 \quad x \leq |C| \] (4.89)

The curvature of the top flange can be obtained by differentiating Eqns. (4.88-4.89) twice.

\[ \frac{\partial^2 v(x)}{\partial x^2} = \frac{5\pi^4 q_1^2 t_w^2 \cos \left( \frac{\pi x}{A} \right)}{32BA^2} \] (4.90)

The average y-axis deformation of the web plate where the patch loading is applied \( \bar{v}_C \) can be obtained by integrating \( v(x) \),

\[ \bar{v}_C = \frac{1}{2C} \int_{-C}^{C} v(x) \, dx \] (4.91)

So,

\[ \bar{v}_C = \frac{B}{2Ct_w}P + \left( \frac{10\pi t_w^2 A \sin \left( \frac{\pi C}{A} \right)}{64BC} + \frac{5\pi^2 t_w^2}{64B} \right) q_1^2 \] (4.92)

By rearranging Eqn. (4.92), \( q_1^2 \) can be expressed in terms of \( \bar{v}_C \) and \( P \):

\[ q_1^2 = C_{q12} \bar{v}_C + C_{q11}P \] (4.93)

\[ C_{q12} = \frac{64BC}{5\pi t_w^2 \left( 2A \sin \left( \frac{\pi C}{A} \right) + \pi C \right)} \] (4.94)

\[ C_{q11} = \frac{-32B^2}{5\pi E t_w^3 \left( 2A \sin \left( \frac{\pi C}{A} \right) + \pi C \right)} \] (4.95)

4.3.2.8 Total Energy and Force-Displacement Function

All the energy terms in Eqn. (4.15) can now be represented in terms of \( q_1 \) and \( P \) and will be shown from Eqns. (4.96-4.103).

By substituting Eqns. (4.35, 4.37-4.38) into Eqn. (4.16) and then integrating, the web plate bending energy term can be simplified into:

\[ U_b = q_1^2 E \pi^4 t_w^6 \left( 17B^4 + 40B^2 A^2 + 32A^4 \right) \frac{1}{6144B^3 A^3 (1 - v^2)} = C_{ub} q_1^2 \] (4.96)

By substituting Eqn. (4.34, 4.36, 4.48-4.54, 4.84-4.85 and 4.71) into Eqn. (4.18) and integrating by parts, the web plate membrane action energy can be simplified to:

\[ U_m = C_{um1} P^2 + C_{um2} P q_1^2 + C_{um3} q_1^4 \] (4.97)
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\[ C_{U_{m1}} = a_{cum} \left( 10702 \left( 1 + v - C^2 - \frac{C^2 v}{3} - v CL \right) A^3 B^6 \pi^3 ight) + 5351 \left( 1 + v - C^2 - \frac{C^2 v}{3} - 2.5 CL - v CL \right) A^5 B^4 \pi^3 \\
+ 668.9 \left( 1 + v - C^2 - \frac{C^2 v}{3} - 10 CL - v CL \right) A^7 B^2 \pi^3 + 6625 A^3 B^8 \pi^3 \\
+ 3312 A^5 B^6 \pi^3 + 414.1 A^7 B^4 \pi^3 + 111.5 A^7 C^4 \pi^3 + 278.7 A^{10} C \pi^3 \\
+ 10702(1 + v) A^4 B^6 C \pi^3 + 5351(1.833 + v) A^6 B^4 C \pi^3 \\
+ 668.9(4.332 + v) A^8 B^2 C \pi^3 - 278.7 A^7 C^3 \pi^3 + 836.1 A^8 B^2 C \pi^3 \\
+ 1784(C^4 + 2.5 C^3 L) A^5 B^4 \pi^3 + 891.8 A^6 B^2 C^4 \pi^3 - 836.1 A^9 C \pi^3 \\
+ 13377 A^5 B^4 C L \pi^3 - 2230 A^5 B^2 C^3 L \pi^3 + 6689 A^6 B^2 C L \pi^3 \]

(4.98)

\[ C_{U_{m1}} = a_{cum} \left( -26754 \left( 1 + v \right) \sin \left( \frac{C \pi}{A} \right) + v \right) A^6 B^4 E t_w^3 \\
+ 293.6(1 + v) A B^8 C E t_w^3 \pi + 146.8(1 + v) A^3 B^6 C E t_w^3 \pi \\
+ 13396 \left( 1 + v + 2(1 + v) \cos \left( \frac{C \pi}{A} \right) \right) A^5 B^4 C E t_w^3 \pi \]

(4.99)

\[ C_{U_{m3}} = a_{cum} \left( 3.402 A^8 B C^2 E t_w^5 \pi^7 + 41.85 B^8 C E^2 t_w^5 \pi^7 + 20.93 A^2 B^6 C E^2 t_w^5 \pi^7 + 74.47 A^4 B^4 C E^2 t_w^5 \pi^7 + 27.22 A^6 B^2 C E^2 t_w^5 \pi^7 \right) \]

(4.100)

\[ a_{Cum} = \frac{1}{2230 A^3 B^3 C E t_w \pi^3 (4B^2 + A^2)^2} \]

(4.101)

By substituting Eqn. (4.90) into Eqn. (4.23) and integrating, the flange bending energy can be obtained:

\[ U_f = q_1^4 \frac{25 b_f E \pi^8 t_w^7}{24576 B^3 L^3} = C_{U_f} q_1^4 \]

(4.102)

By substituting Eqn. (4.92) into Eqn. (4.24) and integrating, the work done by the applied patch load can be obtained:

\[ W_p = \frac{B}{2 C E t_w} P^2 + \left( \frac{5 \pi t_w^2 A \sin \left( \frac{\pi C}{L} \right)}{32 B C} + \frac{5 \pi t_w^2}{64 B} \right) q_1^2 P = C_{WP1} P^2 + C_{WP2} q_1^2 P \]

(4.103)

By substituting Eqns. (4.96-4.103) into Eqn. (4.15), the total energy of the system can be rewritten as:

\[ \Pi = C_{Ub} q_1^2 + C_{Um1} P^2 + C_{Um2} P q_1^2 + C_{Um3} q_1^4 + C_{Uf} q_1^4 - C_{WP1} P^2 - C_{WP2} q_1^2 P \]

(4.104)

By differentiating \( \Pi \) with respect to the amplitude \( q_1 \), the stationary point can be found:
\[ \frac{\partial \Pi}{\partial q_1} = C_{Ub} + (C_{Um2} - C_{WP2})P + 2(C_{Um3} + C_{UF})q_1^2 = 0 \]  
(4.105)

Hence:

\[ P = -\frac{2(C_{Um3} + C_{UF})q_1^2}{(C_{Um2} - C_{WP2})} - \frac{C_{Ub}}{(C_{Um2} - C_{WP2})} \]  
(4.106)

Then, by substituting Eqn. (4.93) into Eqn. (4.106) for \( q_1^2 \), the force-displacement equation can be obtained:

\[ P = K_{p-b} \bar{v} + F_{cr} \]  
(4.107)

\[ K_{p-b} = -\frac{2(C_{Um3} + C_{UF})C_{q12}}{C_{Um2} - C_{WP2} + 2(C_{Um3} + C_{UF})C_{q11}} \]  
(4.108)

\[ F_{cr} = -\frac{C_{Ub}}{C_{Um2} - C_{WP2} + 2(C_{Um3} + C_{UF})C_{q11}} \]  
(4.109)

\( K_{p-b} \) is the elastic post-buckling stiffness and \( F_{cr} \) is the elastic buckling load.

### 4.3.3 Inelastic Post-Buckling Stiffness

The early attempt to derive the inelastic behaviour of column web in compression was made by Jaspart and Maquoi (1994). They introduced a coefficient to modify the initial stiffness of the components in a joint to include strain-hardening behaviour and calculate the ultimate resistance of the joints. Similarly, in this Section, a factor is derived for the elastic post-buckling stiffness to obtain an inelastic post-buckling stiffness for the column web component.

The patch loading exerted on the top flange, which serves as a bearing plate, spreads over the effective web plate (shown as 2A in Figure 4.6) and induces membrane stress in the effective web plate. However, this membrane stress is not evenly distributed; it concentrates under the patch loading area and disperses very quickly through the depth of the web plate. Hence, it usually causes yielding in a small area of the web under the flange. The rest of the web plate usually remains elastic because the effective width of the web plate is usually wide enough to disperse the membrane stress. For example, during tests at the University of Sydney (Chapter 5), the average membrane stress for a typical specimen (e.g. 291MPa for S20_0_0_B) in the effective web plate at the maximum loading is much smaller than the material yield stress (396MPa). Therefore, it is highly unlikely that the web plate will yield.
before it begins to buckle, and so it can be reasonably assumed that yielding stems from large out-of-plane bending in the post-buckling range that leads to a reduction in post-buckling stiffness. In this section, a simple model will be introduced to take this into account.

The inelastic bending stiffness is investigated first and a simplified plate bending model is derived. This simplified model uses an inelastic Young’s modulus derived from the energy method to represent the reduction in bending stiffness at the material yielding stage. By substituting the inelastic Young’s modulus back into the elastic post-buckling analysis, the inelastic post-buckling stiffness is obtained, and by applying the inelastic post-buckling stiffness, an approximate model for the inelastic force-displacement relationship in the post-buckling range can be obtained with reasonable accuracy.

4.3.3.1 The Bending Behaviour of a Plate

The plate bending behaviour can be calculated by the basic structural analysis method.

\[ M_x = D(x_x + v x_y), \quad D = \frac{Et^3}{12(1 - v^2)} \]  \hspace{1cm} (4.110)

where \( M_x \) is the bending moment per unit length along the x-axis, \( x_x \) is the curvature along the x-axis, \( x_y \) is the curvature along y-axis, \( E \) is the Young’s modulus, \( t \) is the plate thickness and \( v \) is the Poisson’s ratio.

As a simplified model, the influence from the minor axis bending is ignored. Hence, assume \( v = 0 \): \( M_x = \frac{1}{12}Et^3 x_x \).

Based on the stress-strain distribution shown in Figure 4.7 b), in the elastic range, the relationship between bending moment per unit width and the curvature is written as,

\[ M_{elastic} = \frac{1}{12}Et^3 \chi = S_e \chi \]  \hspace{1cm} (4.111)

where \( \chi \) is the curvature of the plate due to major axis bending.
Similarly, in the plastic range (Figure 4.7 c)), the relationship between the bending moment per unit width and the curvature is written as,

\[ M_{plastic} = E \varepsilon_y \left( \frac{t^2}{4} - \frac{\varepsilon_y^2}{3\chi^2} \right) = S_e \left( \frac{3\varepsilon_y}{t} - \frac{4\varepsilon_y^3}{t^3\chi^2} \right) \]  

(4.112)

where, \( \varepsilon_y \) is the yield strain.

Also, in the strain hardening range (Figure 4.7 d)), the relationship between the bending moment per unit width and the curvature is written as,

\[ M_{sh} = E \varepsilon_y \left( \frac{t^2}{4} - \frac{\varepsilon_y^2}{3\chi^2} \right) + \frac{1}{6} E_{sh} t^2 \left( 1 - \frac{2\varepsilon_{sh}}{t\chi} \right) \left( 1 + \frac{\varepsilon_{sh}}{t\chi} \right) \left( \frac{t\chi}{2} - \varepsilon_{sh} \right) \]  

(4.113)

\[ E_{sh} = 0.02 E \]  

(4.114)

\[ \varepsilon_{sh} = 10 \varepsilon_y \]  

(4.115)

In Eqns. (4.114, 4.115), \( E_{sh} \) is the strain hardening modulus and \( \varepsilon_{sh} \) is the strain hardening strain (Shayan, 2013).
FIGURE 4.7: STRAIN AND STRESS DISTRIBUTION THROUGH THE THICKNESS OF A WEB PLATE
Then, by using Eqns. (4.111-4.115), the moment-curvature curve of a bending plate can be plotted. For instance, the moment-curvature curve of a unit width of web plate, which was tested at the University of Sydney (Chapter 5), is shown in Figure 4.9. Its material properties are listed in Table 4.1 and Figure 4.8.

**TABLE 4.1**

<table>
<thead>
<tr>
<th>$\sigma_y$</th>
<th>$\varepsilon_y$</th>
<th>$E$</th>
<th>$\sigma_u$</th>
<th>$\varepsilon_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>396 MPa</td>
<td>0.001975</td>
<td>200 GPa</td>
<td>514 MPa</td>
<td>0.0493</td>
</tr>
</tbody>
</table>

**FIGURE 4.8: THE MATERIAL PROPERTY MODEL OF THE COLUMN WEB**

### 4.3.3.2 The Simplified Inelastic Bending Model

A nonlinear inelastic plate bending model can be simplified into a trilinear model comprising an elastic, a gradually yielding and a plastic range, as shown in Figure 4.9.
The key parameters shown in Figure 4.9 such as the yield moment \( M_y \), the yield curvature \( \chi_y \), and the plastic moment \( M_p \), and can be easily determined by Eqn. (4.116, 4.117).

\[
M_y = \frac{1}{12} Et^3 \chi_y, \quad \chi_y = \frac{2\varepsilon_y}{t} \quad \text{(4.116)}
\]

\[
M_p = \frac{1}{8} Et^3 \chi_y, \quad \chi_y = \frac{2\varepsilon_y}{t} \quad \text{(4.117)}
\]

The only parameter that must be determined is the curvature at the start of the plastic moment \( \chi_p \). Assuming that the total energy of the original model at the curvature where the strain hardening commences is equal to the total energy of the simplified model at the same curvature, the plastic curvature \( \chi_p \) can be determined.

The total energy of the original model just before strain hardening can be calculated by:

\[
V_{O,p} = \int_0^{\chi_y} M_{\text{elastic}} d\chi + \int_{\chi_y}^{\chi_{\text{sh}}} M_{\text{plastic}} d\chi = \frac{271}{240} Ebt^3 \chi_y^2
\quad \text{(4.118)}
\]

The total energy of the simplified model is:

\[
V_{S,p} = \frac{1}{2} M_y \chi_y + \frac{1}{2} (M_p - M_y)(\chi_p - \chi_y) + M_p (10\chi_y - \chi_p)
\]
\[ y = \frac{1}{48} E b t^3 \chi_y (57 \chi_y - \chi_p) \quad (4.119) \]

Since \( V_{0,p} = V_{s,p} \), \( \chi_p \) can be solved and a simple formula (Eqn. (4.120)) can be obtained.

\[ \chi_p = \frac{14}{5} \chi_y \quad (4.120) \]

### 4.3.3.3 Inelastic Young’s Modulus Factor

Based on the simplified model, the stiffness at the yielding range can be derived:

\[
S_y = \left( M_p - M_y \right) / (\chi_p - \chi_y) = \frac{5}{216} b t^3 E
\]

\[
= \left( \frac{E b t^3 \chi_y}{8} - \frac{E b t^3 \chi_y}{12} \right) \left/ \left( \frac{14}{5} \chi_y - \chi_y \right) \right. = \frac{5}{216} b t^3 E
\quad (4.121)

Assuming that the change of stiffness was caused by a reduction of Young’s modulus, then the yielding stiffness can be rewritten as:

\[ S_y = \frac{S_e}{E} E_y \quad (4.122) \]

Therefore, the inelastic Young’s modulus \( (E_y) \) can be represented by the elastic Young’s modulus:

\[ E_y = \frac{S_y E}{S_e} = \frac{5}{18} E \quad (4.123) \]

By substituting the inelastic Young’s modulus into the elastic buckling analysis, inelastic post-buckling can be obtained.

### 4.3.4 Ultimate Strength and Web Deformation at Ultimate Strength

By Eurocode 3, the ultimate strength \( P_u \), can easily be predicted with reasonable accuracy. Subsequently, the web deformation at the ultimate strength can be calculated according to the elastic and inelastic post-buckling stiffnesses. In the proposed method, based on experimental observation, it is assumed that elastic post-buckling range dominates the first two thirds of the post-buckling strength, and the point at the end of the elastic post-buckling range is defined as the yield point (Figure 4.10).
The yield load $P_y$ can be hence defined by,

$$P_y = F_{cr} + \frac{2}{3}(P_u - F_{cr}) = \frac{1}{3}F_{cr} + \frac{2}{3}P_u$$  \hspace{1cm} (4.124)

where $F_{cr}$ is the critical load which is defined in Eqn. (4.109) and $P_u$ is the ultimate load which is determined by Eurocode 3 (Eqns. (4.1-4.8)).

Therefore, the web deformation at the yield point $v_y$ can be formulated as,

$$v_y = \frac{2(P_u - F_{cr})}{3K_{p-b}}$$  \hspace{1cm} (4.125)

where $K_{p-b}$ is the elastic post-buckling stiffness which is defined in Eqn. (4.108).

After the yield point, the inelastic post-buckling stiffness dominates the remaining one third of the post-buckling strength, and the point at the end of this inelastic post-buckling range is defined as the ultimate point. Therefore, the web deformation at the ultimate point $v_u$ can be formulated as Eqn. (4.126), and is associated with the ultimate strength $P_u$. 

\hspace{1cm}
\[ v_u = v_y + \frac{(P_u - F_{cr})}{3K_{p-b,y}} \] (4.126)

where \( K_{p-b,y} \) is the post-buckling stiffness obtained by substituting the inelastic Young’s modulus into the post-buckling analysis for elastic Young’s modulus.

### 4.3.5 Post-Ultimate behaviour

After the applied force reaches the ultimate strength, the column web will collapse, but this does not occur suddenly with a sudden drop in strength. Rather, the collapse will progress gradually and the load gradually reduces with the growth in deformation. This effect is caused by the column web folding and the top flange bending. In order to model this behaviour, a modified model based on Roberts’ mechanism model (Eqns. (4.9-4.14), Shahabian and Roberts (1999)) was derived.

The advantage of adopting Robert’s model as the basis is its inclusion of the applied load and web deformation, from which the force-displacement curve can be found. The disadvantage of Robert’s model is its inaccurate prediction of web deformation caused by an inaccurate prediction of the half depth of the folding web plate \( \alpha \) and the length of the bending flange \( \beta \) (\( \alpha \) and \( \beta \) is shown in Figure 4.3). Roberts’ formula is found to generally give too small predictions of \( \alpha \) and \( \beta \) for the column web application. This is the result of the empirical nature of Robert’s equation for \( \alpha \) and the configuration of the experiments from which the half folding web length \( \alpha \) was derived. Roberts’ experiments used several slender deep plate girders, whereas column web components are usually fairly stocky and deemed to be outside the range of applicability of Roberts’ equation. The length of the bending flange \( \beta \) predicted by Eqn. (4.10) does not provide accurate solutions for column web components either.

A more accurate prediction of these two parameters can be achieved based on the analytical method used in elastic post-buckling analysis that was confirmed by experimental observation. The length of the bending flange \( \beta \) can be obtained by:

\[ \beta = A - C \] (4.127)
where \( A \) is the half length of the effective web plate which can be determined by Eqns. (4.25-4.32) and \( C \) is the half length of the patch loading. \( A \) and \( C \) are also shown in Figure 4.6.

The depth of the half folding web \( \alpha \) can be defined as the distance from the top edge of the web to its maximum deflection point. The maximum deflection point can be determined from the first order derivative of the deflection function.

\[
\frac{\partial w}{\partial y} = \cos \left( \frac{\pi y}{2B} \right) + \frac{1}{4} \sin \left( \frac{\pi y}{B} \right) = 0
\]  
(4.128)

By solving Eqn. (4.128) for \( y \), \( \alpha \) can be determined:

\[
\alpha = 0.76143667B \text{ or } \alpha = \frac{3}{4}B
\]  
(4.129)

Then the initial web folding angle \( \theta_0 \) (Figure 4.11) can be determined by

\[
\theta_0 = \arcsin \left( 1 - \frac{v_u}{2\alpha} \right)
\]  
(4.130)

The incremental drop in resistance in the post-ultimate range can be found by a similar energy method to that introduced by Shahabian and Roberts (1999)

\[
\Delta P_{pu} = (4\beta M_w + 2c_e M_w - 2\eta M_w) \left( \frac{1}{\alpha \cos(\theta + \Delta \theta)} - \frac{1}{\alpha \cos \theta} \right)
\]  
(4.131)
The resistance in the post-ultimate range can be obtained:

\[ P_{pu} = P_u + \Delta P_{pu} \]  \hspace{1cm} (4.132)

and the incremental web deformation in the post-ultimate range is obtained as,

\[ \Delta \nu_{pu} = 2 \alpha [\sin \theta + \sin(\theta + \Delta \theta)] \]  \hspace{1cm} (4.133)

The web deformation in the post-ultimate range can be obtained:

\[ \nu_{pu} = \nu_u + \Delta \nu_{pu} \]  \hspace{1cm} (4.134)

Based on Eqns. (4.131-4.134), by changing the angle \( \theta \) from \( \theta_0 \) to 0°, the force-displacement curve can be obtained step by step.

### 4.4 Comparison with Test Results

Three sets of experimental data were compared to predictions obtained from the proposed model. They are from the tests described in Chapter 5, Granath and Lagerqvist (1999) and Chacón et al. (2013). The proposed model provides good predictions for the post-buckling range and post-ultimate range, but generally conservatively predicts the ultimate load which leads to discrepancies in the comparison between the predicted and experimentally observed load-displacement curves, as demonstrated in Sections 4.6.1-4.6.3.

The test specimens conducted in Granath and Lagerqvist (1999) and Chacón et al. (2013)’s tests are not prepared for evaluating joints behaviour. They are therefore different from joint components designed in accordance with Eurocode 3. However, they can be used to validate the method proposed in this chapter.

#### 4.4.1 Tests at the University of Sydney (Chapter 5)

The first set of test data are from the end-plate joint tests described in Chapter 5. Ten experimental load-displacement curves and the proposed model prediction are shown in Figure 4.12. All the experiments used the same column and beam but featured either 10mm or 20 mm thick end-plates, and were subjected to different loading conditions. Only two of
the tests (S20_0_0_B and S20_34_0_B) are directly comparable to the proposed model over the full loading range.

Six of the tests (S_10_0_0, S_10_0_0_B, S_10_34_0, S_10_19_0, S_10_0_24 and S_10_0_11) were with thin end-plates (10mm), and the column webs components in these tests were not loaded to their ultimate resistance because the tension face of the joint (End-plate Bending) failed before the compression face of the joint (Column web). After the tension face failed, the joint resistance decreased and the compression face began to unload. A comparison between the experimental data and the model prediction shows that the model fitted most of the experiments very well in the initial and post-buckling range. The only exceptional experiment with significant variation was S_10_0_24 which was one of the combined in-plane and out-of-plane bending tests where the loading plane was inclined to the major bending plane of the joint. This loading condition caused the top flange of the column and the column web to twist, so the column web was not as stiff as in the other major plane bending tests.

The remaining tests (S_20_0_0_B, S_20_34_0_B, S_20_0_11_B and S_20_0_24_B) were with 20mm thick end-plates. All the column web components in these tests passed their ultimate resistance and stayed in the post-ultimate range until their deformation reached the limit of the rig. In the major plane bending tests (S_20_0_0_B and S_20_34_0_B), the predicted ultimate displacement was 20% smaller than the experimental values, and the post-ultimate curve roughly fitted the experimental results. It is noted that the proposed model cannot predict the observed ultimate plateau (where the strength is at its ultimate but the deformation is still developing). However, the plateau is very small and insignificant compared to the extent of post-buckling and post-ultimate ranges. It appears that the overall prediction for these two tests is very good. With the combined in-plane and out-of-plane bending tests (S_20_0_11_B and S_20_0_24_B), no post-buckling range was attained and ultimate resistance was much smaller than in the major plane bending tests. Therefore, the model
does not fit the load-displacement curves of these tests. However, the post-ultimate range of the tests was still close to the model prediction.

**FIGURE 4.12: UNIVERSITY OF SYDNEY TESTS (CHAPTER 5) VS. THE PREDICTION OF THE PROPOSED MODEL**

### 4.4.2 Tests from Chacón et al. (2013)

Two tests (1VPL2500 and 2VPL2500) from Chacón et al. (2013) were compared to the model prediction (Figure 4.13 and Figure 4.14). The only difference between 1VPL2500 and 2VPL2500 was the strength of the web (325MPa for 1VPL2500 and 210MPa for VPL2500). For 1VPL2500, the model prediction (Figure 4.13) fitted the test data very well, but the model underestimated the ultimate strength (Eurocode 3 prediction). For 2VPL2500, the post-buckling stiffness was higher than the test data (Figure 4.14) but the post-ultimate range fitted the test data very well.

4.4.3 Tests from Granath and Lagerqvist (1999)

Three tests (A13p, A61p and A71p) from Granath and Lagerqvist (1999) were compared to the model prediction (Figure 4.15, Figure 4.16 and Figure 4.17). The tests were of high strength steel plate girds and revealed two major differences between the experimental data and the model prediction. First, the predictions of ultimate resistance were smaller than the test ultimate strength because the Eurocode 3 method is conservative. Second, the test data showed very soft pre-buckling behaviour. This is in contrast to the model, in which the deformation stemmed mainly from membrane action and is very small in the pre-buckling range. The reason for the observed discrepancy is difficult to explain in the absence of detailed account of the experiments.

For the remaining parts of the curves, the model generally fits the shape of the test data.


4.5 Conclusion

The proposed model in this paper provides a viable method to predict the full-range force-displacement curves for column web components. With this proposed method, most of the curve can be predicted with acceptable accuracy, especially the post-buckling and post-ultimate ranges. The method therefore enables joint designs to be optimised by considering the ductility contributed by the column web. The main weakness of the model is the relatively conservative ultimate load predictions.
Chapter 5: Experimental Assessment of End-Plate Joints

5.1 Introduction

Recent experimental assessments of steel joints have usually focused on the pre-ultimate range of moment-rotation behaviour which is from zero to the ultimate moment. However, as some experimental assessments have shown, the joints can still undergo significant bending moment and absorb large amounts of energy in the post-ultimate range, and therefore this should be assessed as part of determining the full range of behaviours.

Although the post-ultimate range was not the aim of previous experimental assessments some experiments still recorded this behaviour. These experiments can be divided into five categories according to the major contribution of post-ultimate behaviour; these categories are T-stub bending, bolt-hole elongation, shear panel deformation, column web buckling, and beam end flange buckling, as described in Sections 5.1.1-5.1.5

5.1.1 Idealised T-Stub Bending Components in Post-Ultimate Range

Idealised T-stub bending components (Figure 5.1) can be found in bolted moment end-plate joints where the T-stubs usually undergo severe deformation at flanges leading to crack(s) initiation and propagation near flange.stem welds and hence causing three stages of abrupt drops in resistance. This behaviour can be seen in the tests performed by Cabrero and Bayo (2007) and Girão Coelho et al. (2004a) and also observed and recorded in the experiments conducted in this thesis.
Chapter 5: Experimental Assessment of End-Plate Joints

5.1.2 Bolt-Hole Elongation

Bolt-hole elongation occurs in joints where the bolt-hole is subjected to large shear forces, such as the web plate joint. Može and Beg (2014) studied this component and derived theoretical models for bearing resistance, but since this component is not included in the end-plate joint, it will not be the focus of the experiments presented in this chapter.

5.1.3 Shear Panel Deformation

Shear panel deformation is usually associated with joints with stiffened column webs, such as the end-plate and welded joint with a stiffened column. Girão Coelho et al. (2009) and Skejic et al. (2008b) tested this type of joint and captured the ductile post-ultimate shear panel behaviour. Since this component is not included in the unstiffened end-plate joint, it will not be the focus of the experiments presented in this chapter.

5.1.4 Column Web Buckling

Column web buckling is a potential failure mode in end-plate joints or welded joints with unstiffened column webs. At the compression face, an unstiffened column web resists the majority of the compression transmitted from the beam flange, and the compression may cause the web plate to buckle. This behaviour is usually avoided due to lack of knowledge in this field by introducing web stiffeners, but by using the model presented in the previous chapter, the behaviour of this component can be modelled and the joint design can be
optimised further. Therefore, to acquire sufficient data to validate the proposed model, the behaviour of this component will be a focus point in the experiments presented in this chapter. As mentioned previously, little research is available in the literature on the column web component. The most recent research is from Chen and Oppenheim (1970a) who tested a column with a concentric load on its flange and found that column web buckling occurred and dominated the behaviour.

### 5.1.5 Flange Buckling at Beam End

Flange buckling at the beam end may occur in joints with very strong columns. Chen and Wang (2009) observed this phenomenon when testing high tensile end-plates. This behaviour is not the focus of the experiments described in this thesis, since the beam flange design resistance of the test specimens (798KN) is larger than the column web in compression in the same row (583kN). According to Eurocode3 (2006b), the design resistance of the beam flange \( F_{c,fb,Rd} \) can be obtained by Eqn. (5.1).

\[
F_{c,fb,Rd} = M_{c,Rd} / (h - t_{fb})
\]

where, \( M_{c,Rd} \) is the moment capacity of beam cross-section, \( h \) is the depth of the connected beam and \( t_{fb} \) is the beam flange thickness.

### 5.1.6 Scope of Tests

Based on the literature review, the experimental assessment of this chapter will focus on end-plate joints and the failure modes will be either T-stub bending or column web buckling. Deformation of the T-Stub and column web will be tracked at all times and the rig will be designed to allow the joints to rotate far into the post-ultimate range.

### 5.2 Tests Details

#### 5.2.1 Test Setup

The test series consisted of the 13 tests listed in Table 5.1. They can be divided into four groups according to their rig setups. The first group consisted of three tests: S10_0_0,
S10_0_0_B and S20_0_0_B and the specimens were subjected to major axis bending. The second group consisted of four tests: S10_19_0, S20_19_0_B, S10_34_0 and S20_34_0_B where the specimens were subjected to major axis bending and axial loading. The third group consisted of two tests: S10_0_90 and S20_0_90_B where the specimens were subjected to lateral bending actions applied about the minor axis. The last group consisted of four tests: S10_0_11, S20_0_11_B, S10_0_24 and S20_0_24_B where the specimens were subjected to diagonal biaxial bending along the major and minor axes simultaneously.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>End-plate</th>
<th>Failure Component</th>
<th>Rig (Loading Conditions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S10_0_0</td>
<td>10mm</td>
<td>End-plate</td>
<td>Bending</td>
</tr>
<tr>
<td>2</td>
<td>S10_0_0_B</td>
<td>10mm</td>
<td>End-plate</td>
<td>Bending</td>
</tr>
<tr>
<td>3</td>
<td>S20_0_0_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Bending</td>
</tr>
<tr>
<td>4</td>
<td>S10_19_0</td>
<td>10mm</td>
<td>End-plate</td>
<td>Bending with Small Tension</td>
</tr>
<tr>
<td>5</td>
<td>S20_19_0_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Bending with Small Compression</td>
</tr>
<tr>
<td>6</td>
<td>S10_34_0</td>
<td>10mm</td>
<td>End-plate</td>
<td>Bending with Large Tension</td>
</tr>
<tr>
<td>7</td>
<td>S20_34_0_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Bending with Large Compression</td>
</tr>
<tr>
<td>8</td>
<td>S10_0_90</td>
<td>10mm</td>
<td>End-plate</td>
<td>Bending against Minor Axis</td>
</tr>
<tr>
<td>9</td>
<td>S20_0_90_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Bending against Minor Axis</td>
</tr>
<tr>
<td>10</td>
<td>S10_0_11</td>
<td>10mm</td>
<td>End-plate</td>
<td>Small Diagonal Bending</td>
</tr>
<tr>
<td>11</td>
<td>S20_0_11_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Small Diagonal Bending</td>
</tr>
<tr>
<td>12</td>
<td>S10_0_24</td>
<td>10mm</td>
<td>End-plate</td>
<td>Large Diagonal Bending</td>
</tr>
<tr>
<td>13</td>
<td>S20_0_24_B</td>
<td>20mm</td>
<td>Column Web</td>
<td>Large Diagonal Bending</td>
</tr>
</tbody>
</table>

5.2.1.1 The Specimens

The specimens featured small variations in geometry for the different loading conditions so that the failure modes could be compared and analysed. The beam-column combination, end-plates, and the welds and bolts were chosen carefully, to accommodate the different types of tests.

5.2.1.1 Beam-Column Combination
All 13 specimens used the same 310UB 46.2 beam and 310UC 96.8 column (Table 5.2 and Figure 5.2) for the beam-column combination. The members performed very well in all tests as they allowed for different failure modes to be achieved simply by varying the thickness of the end-plate. The type of column chosen was slenderer than the columns chosen in comparable recent experiments where fairly stocky columns were used, so that column web buckling failure was not observed.

**TABLE 5.2**

<table>
<thead>
<tr>
<th></th>
<th>$w_f$</th>
<th>$h$</th>
<th>$t_f$</th>
<th>$t_w$</th>
<th>$r_{root}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>310UB 46.2</td>
<td>166.00</td>
<td>307.00</td>
<td>11.80</td>
<td>6.70</td>
<td>11.40</td>
</tr>
<tr>
<td>310UC 96.8</td>
<td>305.00</td>
<td>305.00</td>
<td>15.40</td>
<td>9.90</td>
<td>16.50</td>
</tr>
</tbody>
</table>

**FIGURE 5.2: I BEAM AND H COLUMN DIMENSIONS**

5.2.1.1.2 End-Plates

By varying the thickness of the end-plates, two failure modes can be achieved in the specimens, end-plate cracking and column web buckling (Table 5.1).

The specimens failing in the end-plate cracking mode were S10_0_0_B, S10_0_0, S10_19_0, S10_34_0, S10_0_90, S10_0_11, and S10_0_24, all of which had 10mm thick end-plate which were prone to bending and subsequent cracking (tearing). The bending action transmitted to
the end-plate caused cracking in the heat affected zone (HAZ) at the toe of the welds which
connect the end of the beam to the end-plate together. As the cracks grew, the strength of
the specimens dropped step by step until complete collapse of the specimen had been
attained.

The specimens with column web buckling mode were S20_0_0_B, S20_19_0_B, S20_34_0_B,
S20_0_90_B, S20_0_11_B, and S20_0_24_B. They had 20mm thick end-plates and 20mm
thick backing plates to stop the end-plate from deforming at the tension face. They failed at
the compression face by column web buckling.

5.2.1.1.3 Welds

The end-plates were welded to the beams using full strength 45° continuous fillet welds. The
welding procedure was manual metal arc welding using E48/W50X welding rods that are
widely used in industry.

5.2.1.1.4 Bolts

The end-plate was bolted to the column flanges by six M24 grade 8.8 high tensile bolts, the
strength of which was more than sufficient to transmit the applied loads from the end-plate
to the column flanges up to the ultimate joint capacity.

However, the forces in the post-ultimate range may not be applied evenly to each bolt
because, when the cracking of the end-plate produces eccentricity to the tension transmitted
from the beam flange. Thus, one bolt in a row may resist substantially more force than the
other bolt and hence fail earlier, while the other bolt will subsequently resist the (decreasing)
applied loading and fail later.

5.2.1.2 The Test Rig

The rig (Figure 5.3) was designed to contain four completely different test setups so it was
constructed with several flexible parts that can be swapped around for different test setups.
Since the assembly process involves aligning many heavy parts, it is time consuming, but this rig design was very economic and can be reused and improved for future tests.

**FIGURE 5.3: THE TEST RIG**

5.2.1.2.1 Rig Base
The rig base was used to attach the specimen to the strong floor (Figure 5.4a); it consisted of two large 30mm thick steel base plates and two steel column supports (Figure 5.4b and c). The plates were bolted to the floor with 12 M30 high tensile bolts. The two column supports consisted of 20mm thick flanges, 20mm thick stiffeners, and 30mm thick web plates. These parts were welded together in three layers with full strength 45° continuous fillet welds. During assembly, the two base plates and column supports were lined up by a theodolite to reduce any lateral disturbance. During the tests, the plates and the supports were sufficiently stiff to resist any noticeable deformation and they all remained flat after each test.
Chapter 5: Experimental Assessment of End-Plate Joints

a) Rig Base

b) Pin Support

All Welds used are the 8 mm fillet welds, category SP by E48/AW50X.
5.2.1.2.2 Reaction Frame

The reaction frame was made from 400x400x16 steel hollow sections and was available from previous test programs. It was bolted to the strong floor by four M32 high tensile shafts. The frame could resist up to 360kN of lateral loading without any noticeable deformation. Since this frame was not designed specifically for these experiments, once the load exceeded 360kN, the M32 bolts at the frame footing could fail, so all the tests performed for this thesis were designed to fail at lower loads than 360kN.

5.2.1.2.3 Loading Arrangement

The loading arrangement (Figure 5.5) consisted of a servo-controlled hydraulic jack with 1000kN loading capacity and 250mm travel distance. It was connected to the reaction frame such that its height could be adjusted for different tests. Adjusting the height was done by
theodolite is to ensure the jack only travelled along the x-axis of the rig and no lateral loading applied to the specimen.

The hydraulic jack was connected to the loading point of the specimen by a steel loading arm made from two 20mm thick steel strips strengthened by two 20mm thick steel plates. The loading arm proved strong enough for all the tests as no distortion was observed in any of the tests.

5.2.1.2.4 Lateral Braces

The specimen was restrained laterally by two braces placed at the mid to upper part of the specimen beam to control out-of-plane displacement. The contact plane between the restraints and the specimen beam was filled with Teflon plates, one of the best materials for minimising friction. This setup was sufficient for most tests, apart from the four diagonal loading tests S10_0_11, S20_0_11_B, S10_0_24 and S20_0_24_B, where the specimens were loaded at 11° or 24° to their major bending plane. In these tests, the specimens were not symmetrical along the loading axis and large out-of-plane displacements would result unless restrained. For the 11° diagonal loading tests S10_0_11, S20_0_11_B and S10_0_24, the out-of-plane displacement was effectively restrained until the late stage of the post-ultimate range. With the 24° diagonal loading test S20_0_24_B, the out-of-loading-plane displacement resulted in high later loads on the braces and associated friction in the post-ultimate range, as discussed further in Section 5.3.3.4.
5.2.1.3 Four Types of Test Rig Setups

There were four different types of test setups in this test series: a major axis bending rig setup, a major axis bending with axial loading rig setup, a lateral bending rig setup, and a diagonal (bi-axial) bending rig setup.

5.2.1.3.1 Major Axis Bending Rig Setup

The major axis bending rig setup is shown in Figure 5.6. The specimens were laid down and attached to the strong floor with the column in a horizontal position simply supported by two column supports mounted on the base plates. With this test setup, the major action on the specimen was in-plane bending.

![Figure 5.6: Major Axis Bending Tests Rig Setup](image)

5.2.1.3.2 Major Axis Bending with Axial Load Rig Setup

The major axis bending with axial loading rig was designed to simultaneously apply in-plane bending and in-plane axial loading, with the bending and axial loading changing proportionally.
The axial loading was created by introducing an angle between the loading direction and the transverse axis of the beam (Figure 5.7), so axial loading and the bending action are from the same source. This arrangement is close to a real application because the axial force is usually caused by the angle developing between the applied load and the transverse axis of the beam due to large beam rotations.

**FIGURE 5.7: THE RIG FOR BENDING WITH PROPORTIONAL AXIAL LOADING TESTS**

The angle between the loading direction and the transverse axis of the beam was created by a column extension support, which lifted one side of the column supports and tilted the specimen by an angle. With two different angles of 19° and 34°, four tests with different
loading conditions could be set up as shown in Figure 5.7. With the tilted specimens, the geometry of each rig setup was different, but could be adjusted by changing the length of the base plate strips connecting the column supports.

The axial forces in this test series were chosen to increase rather decrease the chance of failure by appropriately selecting the rotation of the specimen. Hence, S10_19_0 and S10_34_0 were rotated so that axial tension forces would be applied since these specimens were designed to fail by end-plate tearing caused by tension. However, S20_19_0_B and S20_34_0_B were rotated so that compression axial forces would develop since these specimens were designed to fail by column web buckling caused by compression.

5.2.1.3.3 Lateral Bending Rig Setup

The lateral bending rig was designed to apply lateral loading to the specimens (Figure 5.8) that were rotated at 90° from the plane of an applied load. The specimens were placed on a 400X400 Hollow Section that was bolted onto the base plate. Two purpose-designed
connecting plates were bolted at the beam end and the hydraulic jack linkage was pinned to these two plates by a 55mm diameter high tensile steel shaft.

5.2.1.3.4 Diagonal (Bi-axial) Bending Test Rig Setup

The diagonal bending test rig was designed to apply in-plane bending and out-of-plane bending to the joint simultaneously, while keeping the magnitude of this bending proportional during the test.

The diagonal bending action was achieved by rotating the specimen (Figure 5.9) to create an angle between the loading plane and the major bending plane of the beam. In order to rotate the specimens, the column support was bolted in a rotated position on the base plate, while the lengths of the base plate connecting bars were adjusted to suit this change in geometry. The angles between the loading direction and the major axis bending plane were $11^\circ$ and $24^\circ$.

![FIGURE 5.9: TOP VIEW OF THE DIAGONAL BENDING TEST RIG SETUP](image)

5.2.2 Geometrical Properties

Most geometrical properties of the specimens were the same, except for the thickness of the end-plate; a 10mm thick end-plate was used for those tests that failed due to end-plate cracking, and a 20mm thick end-plate was used for those tests which failed because the column web buckled.
FIGURE 5.10: GEOMETRY OF THE SPECIMENS

The dimensions of the specimens are shown in Figure 5.10. One end of the beam was welded to an end-plate that was bolted to the column flange. The other end of the beam was loaded at its longitudinal centreline. To apply a load at the longitudinal centreline, a sleeve (Figure 5.11a) with holes for shafts was bolted to the beam end. The holes in the sleeve held a 55mm diameter high tensile steel shaft and hence the load was applied exactly onto the longitudinal centreline of the beam. Moreover, the web near the shaft hole was stiffened by 10mm steel plates to prevent the web from buckling due to large transverse loading. Similarly, the two column ends were attached to the rig supports by the same sleeve-shaft setup (Figure 5.11b and 5.11c) to ensure the support points were also at the longitudinal centreline of the column.
The dimensions of the end-plate are shown in Figure 5.12. The 10mm and 20mm thick end-plates had the same dimensions in the top view. The beam end was at the centreline of the end-plate along the longitudinal axis in Figure 5.12, and the centreline of its bottom flange was positioned 23 mm away from the bottom edge of the end-plate.

The bolts were located 46mm away from the centre of the beam flange in the vertical direction, as shown in Figure 5.12. This distance allowed the installation of the bolts while maximising the bending stiffness of the end-plate. In the horizontal direction, as shown in Figure 5.12, the bolts were spaced 140mm apart, from centre to centre. This spacing was chosen according to the ASI Structural steel joint design guide (Hogan, 2009). For using the same guide, the distance from the bolts to the edge of the end-plate was set at 65mm.
The length of the beam was 1000mm from the centre of the pin to the surface of the end-plate, while the column was 1200mm long between the centres of the supports. The lengths of the beam and the column were consistent with those chosen for similar tests described in literature. For this length of beam, the curvature caused by bending was insignificant and the effective shear action on the joint was also insignificant compared to the bending action.

### 5.2.3 Instrumentation

In all 13 tests, the major deformations to be measured were the rotation of the loading arm, the overall joint rotation, end-plate bending, the deformation of the column tension face, and column compression web shortening.

The applied load was measured by a load cell built into the hydraulic jack.

### 5.2.4 Rotation of the loading arm

The rotation of the hydraulic jack linkage was tracked at all times when testing. The rotation determines the loading direction and hence the components of the load in the transverse and longitudinal directions. It was important to record the rotation history to calculate the true
applied load on the specimens. The rotation was measured by an inclinometer that was fixed on the loading arm at the half-way between the jack and the specimen (Figure 5.13).

![Inclinometer for the rotation of loading arm](image)

**FIGURE 5.13: THE INCLINOMETER FOR THE ROTATION OF LOADING ARM**

5.2.4.1 Joint Rotation

The joint rotation was measured by a displacement transducer at the beam end (Figure 5.14). The transducer was set perpendicular to the beam and re-positioned every 10 mins to maintain the perpendicularity as the beam rotated under increasing joint rotation. This repositioning of the transducer was considered when converting the transducer displacement to beam rotation. The calculation of the joint rotation is introduced in **Section 5.3.1**.
5.2.4.2 End-Plate Bending Deformation

Major end-plate bending occurred mainly at the extended part of the end-plate. Three transducers were positioned to measure vertical displacements at different points on this extended part of the end-plate (Figure 5.15).

FIGURE 5.15: THE DISPLACEMENT TRANSDUCERS FOR BENDING OF THE END-PLATE
5.2.4.3 Column Tension Face Deformation

Column tension face deformation consists of flange bending and web stretching. Flange bending was obtained as the difference between two transducer readings. As shown in Figure 5.16, one transducer was located at the centre of the row of bolts while the other was located at the web-supported edge of the flange. Web stretching was measured by an additional transducer at the bottom of the column (Figure 5.16). The web stretching was obtained by subtracting the readings of the bottom transducer from the readings of the transducer at the supported.

![Diagram of column tension face deformation](image)

FIGURE 5.16: THE TRANSDUCERS FOR COLUMN TENSION FACE DEFORMATION

5.2.4.4 Column Compression Face Deformation

As with the tension face, column compression face deformation was measured by two transducers. As shown in Figure 5.17, one transducer was located at the web-supported edge of the top flange and the other was positioned at the bottom of the column. The difference between these two transducer readings was the web shortening.
5.2.5 Test Procedures

All the specimens were tested by following the same procedures. After a specimen was in position and all instrumentation prepared, the loading procedure would commence. The displacement rate was set at 1 mm/sec and the data was recorded at 1 sec intervals. Then, every 10-20 minutes, the test was stopped for 2 minutes to acquire the static resistance which will be exemplified in Section 5.3.2. Since the stroke of the hydraulic jack is limited, it had to be reset after every 150mm of extension. When resetting the hydraulic jack, the specimen was unloaded and reloaded again, but this did not affect the results. Finally, after the resistance decreased to less than 25% of the ultimate resistance, the test was stopped.
5.3 Test Results

5.3.1 Definitions of Applied Moment \((M)\) and Joint Rotation \((\theta)\)

The applied moment \((M)\) and joint rotation \((\theta)\) are calculated based on measurements of the load cell in the hydraulic jack, displacement transducer DT1 (Figure 5.18) and the inclinometer attached to the loading arm to determine the direction of the applied load (Figure 5.18). Measurements of the load \((P_{LC})\), total displacement of the beam \((\delta_t)\) and rotation of the loading arms \((\theta_{LA})\) were sampling every second and this is defined as a time step. The increment of the total rotation of the beam at a time step \((\Delta \theta_t)\) can be calculated by:

\[
\Delta \theta_t = \text{atan}\left(\frac{\delta_{t,1} - \delta_{t,0}}{L}\right)
\]  

where \(\delta_{t,0}\) is the reading of the transducer DT1 at the start of the current time step, \(\delta_{t,1}\) is the reading of the transducer DT1 at the end of the current time step and \(L\) is the length of the beam. The subscripts "0" and "1" denotes the time at the start of the current time step and the time at the end of the current time step.

Therefore, the total rotation of the beam after each time step \((\theta_{t,1})\) can be obtained by adding the increment of the total rotation \((\Delta \theta_t)\) to the total rotation of the beam at the start of the current time step \((\theta_{t,0})\):

\[
\theta_{t,1} = \theta_{t,0} + \Delta \theta_t
\]  

The component of the applied load perpendicular to the beam at the end of the current time step \((P_1)\) can be then calculated by,

\[
P_1 = \cos(\theta_{t,1} - \theta_{LA,1}) \cdot P_{LC,1}
\]  

where \(P_{LC,1}\) is the reading of the load cell in the hydraulic jack at the end of the current time step and \(\theta_{LA,1}\) is the reading of the inclinometer at end of the current time step. Next, the applied moment at the end of the current time step \((M_1)\) can be then calculated.

\[
M_1 = P_1 L
\]
Figure 5.19 shows the decomposition of the increment of the total beam rotation. The increment of the joint rotation ($\Delta \theta$) is calculated by subtracting the increment of the beam rotation caused by beam deflection ($\Delta \theta_b$), as obtained from engineering bending theory (Eqn. (5.7)), from the total beam rotation ($\Delta \theta_t$),

$$
\Delta \theta = \Delta \theta_t - \Delta \theta_b
$$  \hspace{1cm} (5.6)

$$
\Delta \theta_b = \tan^{-1}\left(\frac{(P_1 - P_0)L^2}{3EI}\right)
$$  \hspace{1cm} (5.7)

where $P_0$ is the applied load perpendicular to the beam at the start of the current time step, $E$ is the Young’s modulus and $I$ is the moment of inertia of the beam. Then, the rotation of the joint ($\theta_1$) at the end of the current time step can be calculated by adding the increment of the joint rotation to the joint rotation at the start of the current time step ($\theta_0$),

$$
\theta_1 = \theta_0 + \Delta \theta
$$  \hspace{1cm} (5.8)
Chapter 5: Experimental Assessment of End-Plate Joints

The total rotation $\theta_t$ consists of the column web shear panel rotation $\theta_c$ and the connection rotation $\theta_t - \theta_c$ (Figure 5.18). Since the generalised model introduced in Chapter 3 includes both connection and web shear panel rotations, the rotation calculated from the generalised model is the total rotation of the joint. The same total rotation $\theta_t$ is adopted for the moment-rotation curves shown in later sections.

The beam shear deformation is ignored, since the shear deformation of the beam (0.002mm) is negligible by engineering theory.

5.3.2 Raw Data Processing

After the tests, the raw data were processed. Data noise was removed first and then the static resistance was acquired for later comparison. Meanwhile, the curves have also been smoothed by regression analysis and engineering judgement. A Sample moment-rotation curve is shown in Figure 5.20.
The static resistance was the same as the dynamic (original) resistance in the elastic range, but in the gradual yielding range it began to decrease until it was approximately 3% lower than the original resistance, which is the maximum difference between the static and original resistances. Finally, in the latter range, the static resistance was kept 3% lower than the original resistance. The static and dynamic curves for all tests are shown in Appendix A4.

**FIGURE 5.20: AN EXAMPLE OF THE STATIC CURVE AND THE ORIGINAL CURVE**
5.3.3 Failure Mechanism

5.3.3.1 End-Plate Bending Failure

a) Crack initiation before ultimate resistance
b) Crack opening after ultimate resistance
c) Later, the extended part was completely cut off

![Crack Propagation at End-Plate Bending Failure Stage](image)

**FIGURE 5.21: CRACK PROPAGATION AT END-PLATE BENDING FAILURE STAGE**

*Figure 5.21* shows a typical example of crack(s) initiation and propagation. *Figure 3.13-3.17* in Chapter 3 shows a typical example of three abrupt drops in resistance caused by crack(s) propagation.

Four steps can be used to describe the failure process. At the beginning, before reaching the ultimate resistance, two cracks emerged in the heat affected zone (HAZ) of the welds, opposite to the two bolts at the extended part of the end-plate (*Figure 5.22a*).
In the second step, when the joint had just passed its ultimate resistance, the two initial cracks grew wider and merged together (Figure 5.22b). This behaviour caused the first drop in resistance. In the third step, the merged crack grew around the corner of the welds, or towards the edge of the end-plate and eventually caused all of the extended part of the end-plate to be torn apart (Figure 5.22c). This behaviour caused the second drop in resistance.
In the third and final step, the crack that caused three different types of failure (Figures 5.22d, 5.22e, 5.22f) continued to propagate. This is the third stage of abrupt drop in resistance leading to the complete failure of the joint.

The three different types of final failure are a combination of four types of element failures: bolt fracture, plate tear, weld tear, and beam flange pulling out. As Figure 5.23b shows, S10_0_0_B displays mixed plate tear, flange pulling out, and bolt fracture. In this test, at one side the crack propagated around the tip of the flange and the flange on that side was pulled out. In the final step, the crack propagated to the edge of the end-plate, which lost its strength completely when the inner bolt fractured.

For S10_0_0, as shown in Figure 5.23a, final failure was caused by welds tear and the inner bolts fractured. For the bending and axial loading tests S10_19_0 and S10_34_0, as shown in Figures 5.23c and 5.23d, final failure was caused by the beam flange pulling out and the inner bolt fracturing. For the diagonal bending tests S10_0_11 and S10_0_24, as shown in Figures 5.23e and 5.23f, final failure was caused by a combination of the beam flange pulling out and welds being tear.
FIGURE 5.23: ALL THE END-PLATE BENDING TESTS AT FAILURE
5.3.3.2 Column Web Buckling Failure

Column web buckling failure was the other failure mode assessed in these tests, and S20_0_0_B, S20_19_0_B, S20_34_0_B, S20_0_11_B and S20_0_24_B failed in this mode. Web deformation usually slowly grew until the load approached the ultimate resistance, near which web buckling accelerated. Thenceforth, web buckling deformation grew quickly and the resistance reduced accordingly. Figure 5.24 shows typical column web buckling at final failure.

a) Front View  b) Side View

FIGURE 5.24: COLUMN WEB BUCKLING AT FAILURE

FIGURE 5.25: ROTATION OF THE COLUMN FLANGE DUE TO WEB BUCKLING
All the column web buckling tests ultimately failed because the high tensile bolts snapped due to the column web buckling and the associated propensity to column flange rotation (Figure 5.25). This induced rotation caused the bolts on one side to transfer much larger forces than the bolts on the other side, leading the bolts to fail one after another in the post-ultimate range.

5.3.4 Joint Behaviour

5.3.4.1 Major Axis Bending Tests (S10_0_0, S10_0_0_B & S20_0_0_B)

The first group consisted of three tests where the major axis bending test rig (Figure 5.6) was used to subject the specimens to major axis bending only. However, these three tests had end-plates that were 10 mm thick and 20mm thick, so their strengths were quite different. S10_0_0 was the weakest joint due to the 10mm thick endplate, S10_0_0_B was the medium strength joint; it had 10 mm thick end-plates strengthened by backing plates using high tensile bolts. S20_0_0_B was the strongest joint due to the end-plates being 20mm thick and backing plates being used in combination with high tensile bolts. The moment-rotation curves of all tests are shown in Figure 5.26.

![FIGURE 5.26: RESULTS FOR THE MAJOR AXIS BENDING TESTS](image-url)
5.3.4.1.1 S10_0_0 and S10_0_0_B

Figure 5.27 shows the initial test setups for these two tests, and their moment-rotation curves are plotted in Figure 5.26. S10_0_0 reached its maximum at 220.2 kNm and 0.118 rads. With help from the backing plates, S10_0_0_B gained a 2.77% increasing in strength to 226.3 kNm, but its corresponding rotation was 0.107 rad, which was 9.2% less than that of S10_0_0. The major difference between the tests was the stiffness, which was sustained to slightly higher loads when backing plates were used. The initial stiffnesses of S10_0_0 was slightly higher than that of S10_0_0_B, and at approximately 50kNm, the stiffness of S10_0_0_B decreased until it was much smaller than S10_0_0. This was due to the backing plates prevented the column flange in S10_0_0_B to yield and reduced its flexibility compared to S10_0_0.

a) S10_0_0  b) S10_0_0_B

![FIGURE 5.27: S10_0_0 AND S10_0_0_B AT INITIAL STAGE]

a) S10_0_0 at Failure  b) S10_0_0_B at Failure

![FIGURE 5.28: S10_0_0 AND S10_0_0_B AT FAILURE]
In the post-ultimate range, both specimens reduced their strength in three stages; in the first stage the strength of S10_0_0 dropped 27.3% from a maximum of 220.2 kNm to its first stage post-ultimate plateau of approximately 160kNm which is caused by end-plate cracking at HAZ on the extended part. S10_0_0_B suffered a 24.0% drop in strength from a maximum 226.3kNm to its first stage post-ultimate plateau of approximately 172kNm, again the drop was caused by the end-plate cracking at HAZ on the extended part. For the rotation range in the first post-ultimate plateau, S10_0_0 changed by 0.056rads from 0.140rads to 0.196rads, and S10_0_0_B changed by 0.062rads from 0.124rads to 0.186rads. In the second stage the strength of S10_0_0 dropped 43.8% from its first post-ultimate plateau to its second post-ultimate plateau of approximately 90kNm which was caused by tearing of the extended part and separation from the end-plate. Similarly, the strength of S10_0_0_B dropped 47.7% from its first post-ultimate plateau to its second post-ultimate plateau of approximately 75kNm. This drop in load was caused by the failure of two components, viz. one of the inner bolts stripping and the beam flange pulling out. It is worth noting that the strength drop was first initiated by beam flange pulling out leading to a drop in resistance to 130kNm. However, the joint could not sustain this moment and soon after the thread of the inner bolt stripped. Therefore, there was no strength plateau observed at 130kNm. For the rotation range in the second post-ultimate plateau, S10_0_0 changed by 0.03rads from 0.196rads to 0.226rads which was caused by crack(s) propagation along the HAZ at the root of the weld between the beam and the end-plate. Meanwhile, S10_0_0_B changed by 0.038rads from 0.189 to 0.227rads due to beam-flange pulling out (bolt stripping did not contribute to rotation). Then, in the next stage, the resistance dropped too low and both joints were considered as failures. Figure 5.28 shows the joints at failure.

5.3.4.1.2 S20_0_0_B

S20_0_0_B had 20mm thick end-plates and backing plates which greatly strengthened its performance in the tension zone. Consequently, S20_0_0_B failed in the compression zone due to column web buckling. Web buckling was insignificant until the applied action reached 280kNm in which the change in the deflection rate was small. Before web buckling occurred,
the joint first yielded as early as 220kNm and the yielding behaviour was the major source of rotation until reaching a moment of 280kNm. The moment-rotation curve for S20_0_0_B is shown in Figure 5.26.

S20_0_0_B reached its maximum moment at 292.3kNm (Figure 5.26), which was 37.2% larger than that of S10_0_0. The corresponding rotation was 0.164rads which was 39.0% larger than that of S10_0_0. In the post-ultimate range, the joint strength reduced smoothly as the column web deflection gradually increased, until finally, at 221.5kNm and 0.27rads, S20_0_0_B failed completely due to a series of fractures of the high tensile bolts. It is worth noting that the first bolt failure was at the outer bolt row as early as 232kNm. However, the failure did not cause a strength drop until the other bolt in the same row failed at 214 kNm. S20_0_0_B maintained 73.5% of its strength at the failure point and the corresponding rotation was 19.5% larger than that of S10_0_0. Therefore, for a strength and ductility viewpoint, S20_0_0_B performed significantly better than S10_0_0. Figure 5.29 shows S20_0_0_B at its initial stage (0 kNm) and at failure (214.8 kNm).

a) S20_0_0_B at initial stage

b) S20_0_0_B at Failure

FIGURE 5.29: S20_0_0_B AT INITIAL STAGE AND AT FAILURE
5.3.4.2 Major Axis Bending with Axial Loading Tests (S10_19_0, S20_19_0_B, S10_34_0 & S20_34_0_B)

The four tests in this series were designed to assess the bolted moment end-plate joint under major axis bending with axial loading. The characteristics and failure modes of the four tests are summarised in Table 5.3.

**TABLE 5.3**

<table>
<thead>
<tr>
<th>End-plate Thickness</th>
<th>Angle</th>
<th>Axial Loading Condition</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_19_0</td>
<td>10mm</td>
<td>19°</td>
<td>Tension End-plate Cracking&amp;Bolts Fracture</td>
</tr>
<tr>
<td>S10_34_0</td>
<td>10mm</td>
<td>34°</td>
<td>Tension End-plate Cracking&amp;Bolts Fracture</td>
</tr>
<tr>
<td>S20_19_0_B</td>
<td>20mm</td>
<td>19°</td>
<td>Compression Column Web Buckling</td>
</tr>
<tr>
<td>S20_34_0_B</td>
<td>20mm</td>
<td>34°</td>
<td>Compression Column Web Buckling</td>
</tr>
</tbody>
</table>

**FIGURE 5.30: RESULTS OF MAJOR AXIS BENDING WITH AXIAL LOADING TESTS**

The failure modes were largely determined by the additional axial loading. Those tests loaded in axial tension failed in the tension region, whereas those tests loaded in axial compression that failed in the compression region. **Figure 5.30** shows the moment-rotation curves for the
four tests. Figure 5.31 compares the combined axial and bending tests on 10mm end-plates with other tests featuring 10mm end-plates.

5.3.4.2.1  S10_19_0 and S10_34_0

It follows from Figure 5.31 that axial loading had very little influence on the moment-rotation curves of the 10mm thick end-plate tests. The initial stiffness and post-ultimate behaviour of S10_19_0 and S10_34_0 were similar to S10_0_0. The gradual yielding range and ultimate resistance of S10_19_0 and S10_34_0 were slightly higher than S10_0_0, so the end-plate bending failure mode was not sensitive to the axial tension force. Figure 5.32 and Figure 5.33 show the specimens S10_19_0 and S10_34_0 at initial and failure stage.

![Figure 5.31: Results of 10mm Thick End-Plate Tests](image-url)
5.3.4.2.2 *S20_19_0_B and S20_34_0_B*

In the 20mm thick end-plate tests, the axial load did not affect the initial stiffness, post-ultimate stiffness and ultimate resistance, but it did change the shapes of the curves (Figure 5.34) quite significantly. The rotations at the ultimate resistances of S20_19_0_B and S20_34_0_B were smaller than for S20_0_0_B as a result of column web buckling precipitated by the compressive axial load. Consequently, S20_19_0_B and S20_34_0_B entered their post-ultimate range much earlier than S20_0_0_B.
This behaviour was different from that obtained for S10_19_0 and S10_34_0 due to differences in the stiffness at each face. Since the column web component on the compression face is usually much stiffer than the end-plate bending component on the tension face, a stiffer column web will take more compressive axial force and will be more sensitive to axial force. Figure 5.35 and Figure 5.36 show the specimens S20_19_0_B and S20_34_0_B at initial and failure stage.
5.3.4.3 Lateral Bending Tests (S10_0_90 and S20_0_90_B)

The two tests in this series were designed to assess lateral loading of the end-plate joint, but because the specimens were very ductile they did not reach their ultimate resistance within the range of deformation afforded by the test rig. The stiffness of the 20mm thick end-plate test of S20_0_90_B was only slightly higher than for the 10mm thick end-plate test of S10_0_90 (Figure 5.37), because the major source of deformation was column flange twisting rather than end-plate bending (Figure 5.38 and 5.39).
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**FIGURE 5.37: LATERAL BENDING TESTS**

- a) S10_0_90
- b) S20_0_90_B

**FIGURE 5.38: LATERAL LOADING TESTS AT INITIAL STAGE**
5.3.4.4 Diagonal Bending Tests (S10_0_11, S10_0_24, S20_0_11_B & S20_0_24_B)

The four tests in this series were designed to assess the end-plate joint under combined in-plane and out-of-plane bending. The characteristics and failure modes of the four tests are summarised in Table 5.4 and the tests results are summarised in Figure 5.40.

**TABLE 5.4**

<table>
<thead>
<tr>
<th>End-plate Thickness</th>
<th>Vertical Angle</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_11</td>
<td>10mm</td>
<td>11°</td>
</tr>
<tr>
<td>S10_0_24</td>
<td>10mm</td>
<td>24°</td>
</tr>
<tr>
<td>S20_0_11_B</td>
<td>20mm</td>
<td>11°</td>
</tr>
<tr>
<td>S20_0_24_B</td>
<td>20mm</td>
<td>24°</td>
</tr>
</tbody>
</table>

End-plate Cracking and Bolt Fracture

Column Web Buckling
FIGURE 5.40: RESULTS FOR DIAGONAL BENDING TESTS

The rig for this test series is shown in Figure 5.9, and the front view of the rig is shown in Figure 5.41. In this test series, the longitudinal axis of the column was positioned at an angle to the loading direction to produce biaxial bending, but since the geometrical properties in the bending directions were not symmetrical, out-of-plane deformation would ordinarily occur under this loading arrangement. To eliminate out-of-plane deformation, the beam was restrained to move in the loading direction of the jack. The lateral bracing parallel to direction of loading is shown in Figure 5.41.

Higher levels of friction developed during the tests than expected, but could only be noticed in the latter stages of the post-ultimate range. Only S20_0_24_B suffered from friction exerted by the bracing in the early stages of the post-ultimate range, but the results were nevertheless useful as points in the test where static friction was overcome could be easily identified. It is recommended that in future tests, rolling contact surfaces between the lateral bracing and the specimen be used.
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FIGURE 5.41: FRONT VIEW OF THE DIAGONAL BENDING TEST RIG
5.3.4.4.1 S10_0_11 and S10_0_24

The moment-rotation curves of the two specimens are shown in Figure 5.42, while the moment-rotation curve of S10_0_0 is included in Figure 5.42 for comparison. The initial stages and failure stages of the specimens S10_0_11 and S10_0_24 are shown in Figure 5.43.

The additional out-of-plane bending affected the 10mm thick end-plate tests considerably, such that unlike the standard uniaxial bending test of S10_0_0, the initial stiffness of S10_0_11 and S10_0_24 was smaller, and the larger vertical angle greatly reduced the initial stiffness.

![Graph showing moment-rotation curves for S10_0_11, S10_0_24, and S10_0_0](image)

**FIGURE 5.42: 10MM THICK END-PLATE TESTS: S10_0_11, S10_0_24 AND S10_0_0**

The ultimate resistance of the large vertical angle test S10_0_24 was 13% smaller than S10_0_0, but the ultimate resistance of the small vertical angle test S10_0_11 was 8.7% larger than S10_0_0. This increase in strength may be caused by random variations introduced during fabrication, especially in the welds.
The shapes of the moment-rotation curves for S10_0_11 and S10_0_0 are similar, but the shape of S10_0_24 differs in the post-ultimate range due to the unexpectedly high friction between the specimen and the lateral bracing.

**FIGURE 5.43: S10_0_11 AND S10_0_24**
5.3.4.4.2  *S20_0_11_B and S20_0_24_B*

The moment-rotation curves are shown in Figure 5.44 with the moment-rotation curve of S20_0_0_B included for comparison. The initial stages and failure stages of the specimens S20_0_11_B and S20_0_24_B are shown in Figure 5.45.

![Graph showing moment-rotation curves for S20_0_11_B, S20_0_24_B, and S20_0_0_B](image)

**FIGURE 5.44: 20MM THICK END-PLATE TESTS: S20_0_11_B, S20_0_24_B AND S20_0_0_B**

The additional out-of-plane bending action also affected the ultimate strength and initial stiffness of the 20mm thick end-plate tests, such that the larger out-of-plane bending action caused a smaller ultimate strength and lower initial stiffness. It also changed the shapes of the curves indicating more gradual yielding and smooth post-ultimate response (Figure 5.344).

Compared to all the other diagonal tests, S20_0_24_B was affected more by friction between the lateral restraint and the specimen. In the post-ultimate range, friction produced a jagged moment-rotation curve due to a release of energy when static friction forces were overcome.
Since the contact surface was covered with Teflon plates, dynamic friction can be treated as essentially zero, and therefore the jagged moment rotation curve can be smoothed by a curve through the points with released static friction forces. The original moment rotation curve can be found in the Appendix A4.1.

![S20_0_11_B at Initial Stage](image1)
![S20_0_24_B at Initial Stage](image2)

![S20_0_11_B at Failure](image3)
![S20_0_24_B at Failure](image4)

**FIGURE 5.45: S20_0_11_B AND S20_0_24_B**
5.3.5 Comparison of All End-plate Bending Failure Tests

The behaviour of all end-plate bending tests was similar (Figure 5.46) in so far that all tests had three abrupt drops in resistance in the post-ultimate range. The first drop in resistance was after the ultimate resistance at around 230kNm, where two initial small cracks on the end-plate merged into a single large crack and expanded. The second drop in resistance was around 150kNm where the large crack had propagated to cause tearing of the weld leading to complete loss of strength of the extended part of the end-plate. The third drop in resistance occurred around 90kNm where the inner bolt started to fracture due to an imbalanced load. Therefore, it is possible to model the post-ultimate behaviour according to the joint behaviour of these three stages of post-ultimate response.

5.3.6 Comparison of All the Column Web Buckling Failure Tests

As Figure 5.47 Shows, the joints were greatly affected by the different loading conditions, leading to different overall shapes of the moment-rotation curves. However, the initial
stiffness, ultimate resistance, and the stiffness at the post-ultimate range were similar, and implying that this behaviour is also predictable.

\[ \Delta \delta_{DT8} - \delta_{DT14} \]  

\[ (5.9) \]

where \( \delta_{DT8} \) is the reading of DT8 and \( \delta_{DT14} \) is the reading of DT14.
The flange force \( (P) \) is the force transmitted by the flange on the tension face (Figure 5.48). The magnitude of the flange force can be approximately calculated by Eqn. (5.10).

\[
P = \frac{M}{z}
\]  

(5.10)

where \( M \) is the applied bending moment on the joint and \( z \) is the distance between the two beam flanges.

FIGURE 5.48: MEASUREMENT OF DEFORMATION AND APPLIED LOAD OF THE END-PLATE BENDING COMPONENT

End-plate bending for the 10mm thick end-plate tests was similar (Figure 5.49) in that they were all stiff in the elastic range before gradually entering a slightly varied ductile yielding range. The stiffness and strength were slightly affected by backing plates and axial loading, but the backing plates and axial loading increased stiffness in the gradually yielding range.
Compared to the 10mm thick end-plate tests, three 20mm thick end-plate tests (S20_0_0_B, S20_19_0_B and S20_34_0_B) were still in the premature stage (Figure 5.50) where the
deformation was small and could be ignored. In the other two diagonal bending tests (S20_0_11_B and S20_0_24_B), the component deformation was also affected by the column twisting, and the behaviour was quite substantially more ductile than in the other tests.

5.3.7.2 Column Web Buckling Component

Similarly, the deformation of the column web buckling component (Δ) is measured by taking the difference between two displacement transducers (DT11 and DT18) on the column (Figure 5.51).

\[
\Delta = \delta_{DT11} - \delta_{DT18}
\]  

(5.11)

where \( \delta_{DT11} \) is the reading of DT11 and \( \delta_{DT18} \) is the reading of DT18.

The flange force (\( P \)) is the force transmitted by the flange on the compression face (Figure 5.51). The magnitude of the flange force can be approximately calculated by Eqn. (5.10).

![Diagram of Column Web Buckling Component](image-url)
No experimental measurements were taken to detect the distribution of stress applied to the column. Therefore, the lever arm $z$ is assumed as the distance between the centre of gravity of the two flanges for the tested specimens. This assumption is used in Eurocode 3 Part 1-8 Figure 6.15 and Figure 6.16 (2006b), subject to satisfying Clause 6.2.7.1-(8) (Eurocode3, 2006b). By substituting the parameters of the test specimens into the strength check provided in Clause 6.2.7.1-(8) (Eurocode3, 2006b), the specimen configurations passed the check, as demonstrated in Table 5.5, where, according to Eurocode3 (2006b), $F_{Rd}$ is the design resistance which can be taken as equal to $2F_{1,Rd}$ and $F_{t,Rd}$ is given in Table 6.2 of Eurocode Part 1-8. $F_{1,Rd}$ is the bolt-row force of the idealised T-stub of the outer end-plate bending component (Figure 6.16 of Eurocode 3 Part 1-8).

**TABLE 5.5**

<table>
<thead>
<tr>
<th></th>
<th>$F_{Rd}$</th>
<th>$3.8F_{t,Rd}$</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>469kN</td>
<td>929kN</td>
<td>Pass</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>489kN</td>
<td>929kN</td>
<td>Pass</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>672kN</td>
<td>1531kN</td>
<td>Pass</td>
</tr>
</tbody>
</table>

Since column web buckling is generally not a preferred failure mode, column web stiffeners are usually added to prevent web buckling. However, column web buckling may sustain large joint rotations in the post-buckling range. Therefore, in this test series, in order to assess the column web buckling behaviour, column web shortening caused by column web buckling was recorded.

The load-shortening curves for the column web in tests featuring 20mm thick end-plate are shown in Figure 5.52. The recorded column web behaviour is typical stocky plate buckling behaviour in that it has almost infinite initial stiffness followed by positive post-buckling stiffness until it reaches the ultimate resistance, and then negative post-ultimate stiffness. However, the buckling loads were influenced by the loading conditions, and were reduced by compressive axial loading and out-of-plane bending. Axial loading hardly changed the shape,
but out-of-plane bending did. Indeed, out-of-plane bending reduces the buckling load considerably, and leads to substantially smaller post-ultimate stiffness.

The column web shortening curves for the 10mm thick end-plate tests were similar to the 20mm web shortening curves in the pre-ultimate range (Figure 5.53) because the column configurations of all tests were the same. However, in the 10mm thick end-plate tests, the column web was usually in the elastic range, so the buckled web deflection in those tests elastically unloaded when the applied load decreased in the post-ultimate range.

![Figure 5.52: Column Web Shortening of 20mm Thick End-Plate Tests](image-url)
5.4 Conclusion

All the tests were completed successfully, and all the acquired data has been presented and discussed. As shown by the data, the moment-rotation curve of the end-plate joint was affected considerably by the failure mode, especially in the post-ultimate range. End-plate bending failure tests have three distinct drops in resistance in the post-ultimate range at around similar levels of resistance, whereas the column web buckling tests had a smooth post-ultimate curve that finally failed abruptly due to the bolts fracture.

The four loading conditions had only a slight effect on the joint behaviour, so variations caused by the loading conditions in the end-plate bending failure tests was ignored. In the column web buckling failure tests, only the joint rotation at the ultimate resistance was substantially different under different loading conditions, whereas characteristics such as initial stiffness, and post-ultimate stiffness were similar.
The behaviour of individual components had a significant impact on the joint behaviour in that two distinct failure modes were achieved by changing the thickness of the end-plate. Consequently, it is important to focus on the performance of components in the design of joints.

The test rig was flexible and could be transformed to accommodate different loading conditions. The rig performed well throughout the loading range in all tests. However, in the bi-axial bending tests, the contact surfaces between the lateral restraints developed high levels of static friction which may have affected the post-ultimate response. In future tests, it is recommended that rollers are used between the contact surfaces in combination with stronger lateral restraining beams, especially if the post-ultimate range is to be accurately captured.
Chapter 6: Model Prediction and Validation

6.1 Individual Component Models

In this chapter the prediction from a generalised component model presented in chapter 3 will be compared to the experimental results for validation. Six tests of joints in uniaxial bending were used for validation where the full range moment-rotation curve and component fracture sequences will be compared.

1. Outer End-Plate Bending
2. Inner End-Plate Bending
3. Bolt in Tension
4. Column Flange Bending
5. Column Web subjected to Tension
6. Column Web Shear Panel
7. Column Web subjected to Compression

FIGURE 6.1: END-PLATE JOINT AND THE COMPONENTS IT CONTAINS

Since the input data for the generalised component model describes the behaviour of individual components, the individual component models are introduced first. The individual
components for a typical end-plate joint includes outer end-plate bending, bolt elongation, inner end-plate bending, column flange bending, column web subjected to tension, column web shear panel, and column web subjected to compression (Figure 6.1). Of these components, the deformation associated with bolt elongation, column web subjected to tension, and column web shear panel are not as significant as those of the other four components, so they were modelled by the elastic response specified in Eurocode 3. Outer end-plate bending, inner end-plate bending, column flange bending, and column web under compression are four major components with a greater impact on the overall joint behaviour, so they are modelled by a full range model covering the elastic, plastic, and post-ultimate range.

All the individual component models are then added to the generalised component model as input data that can predict the overall moment-rotation of the joint. The moment-rotation curve covers the full range of joint behaviour including elastic, plastic, and post-ultimate ranges. Moreover, as presented in Chapter 3, a post-fracture model will also be added to simulate post-fracture behaviour that is triggered by brittle failure modes such as end-plate tearing failure and tensile bolt fracture.

Although end-plate bending failure is ductile in the cracking propagation stage, it is brittle in the final stage where major cracks occur. As well as the bolt fracture failure mode, brittle failure causes dynamic effects and the discontinuity of spring rows. It also creates several abrupt drops in resistance that are too complex to consider with the pure analytical method. Therefore, for simplicity, post-fracture is modelled by the energy method which assumes that a joint will possess the same amount of energy in a stable static state before and after fracture of a spring row (Chapter 3).

Besides the two brittle failure modes, a ductile failure mode called column web buckling is also involving in the post-ultimate range. It will not trigger fracture because column web
buckling gradually softens the joint rather than create abrupt reductions in resistance, and therefore the post-fracture model will not be triggered during column web buckling until one of two brittle failure modes is detected.

In the following sections, the methods for all the individual components will be first introduced. Except for the column web buckling component, individual component models are widely available, so only minor modifications are added to produce the post-ultimate/post-fracture analysis. Finally, the generalised component model will be applied and the full-range behaviour of the six chosen specimens will be obtained and compared in Section 6.2.

6.1.1 End-plate Bending Components and Column Flange Bending Component

The inner and outer end-plate bending components and column flange component are usually considered as T-stubs, and although they are not exact T-stubs they will be firstly converted into idealised T-stubs and then the idealised T-stubs will be substituted into a suitable T-stub model to predict the force-displacement behaviour.

6.1.1.1 T-stub Bending mode

The T-stub bending mode determines the yield line pattern in the T-stub flange, which greatly affects the overall behaviour of the T-stub. Eurocode 3 (2006b) provides a sound and simple method to determine which bending mode will occur for the T-stub element. It also considers major mechanisms such as the yield line pattern at the bending flange and prying action at the bolt.

The T-stub bending mode (Figure 6.2) can be determined by finding the smallest value of plastic resistance for each bending mode (Eurocode3, 2006b). Mode 1 (Figure 6.2) plastic resistance can be calculated by:
\[ F_{T,1,Rd} = \frac{(8n - 0.5d_{Wa})M_{pl,1,Rd}}{2mn - 0.25d_{Wa}(m + n)} \]  
(without backing plate)  
\[ F_{T,bp,1,Rd} = \frac{(8n - 0.5d_{Wa})M_{pl,1,Rd} + 4nM_{bp,Rd}}{2mn - 0.25d_{Wa}(m + n)} \]  
(with backing plate)

where \( d_{Wa} \) is the diameter of the washer, \( m \) is the length from the centre of the bolt to the edge of the welds, and \( n \) is the length from the centre of the bolt to the edge of the T-stub flange, capped at \( n \leq 1.25m \).

Mode 2 (Figure 6.2) plastic resistance can be obtained by:

\[ F_{T,2,Rd} = \frac{2M_{pl,2,Rd} + n \sum F_{t,Rd}}{m + n} \]  
(6.4)

Mode 3 (Figure 6.2) plastic resistance can be obtained by:

\[ F_{T,3,Rd} = \sum F_{t,Rd} \]  
(6.5)
In Eqns. (6.1-6.4) the plastic bending resistances $M_{pl,1,Rd}$, $M_{bp,Rd}$ and $M_{pl,2,Rd}$ can be obtained by:

$$M_{pl,1,Rd} = 0.25l_{eff,1}t_f^2f_y$$  \hspace{1cm} (6.6)

$$M_{pl,2,Rd} = 0.25l_{eff,2}t_f^2f_y$$  \hspace{1cm} (6.7)

$$M_{bp,Rd} = 0.25l_{eff,1}t_f^2f_y$$  \hspace{1cm} (6.8)

where $t_f$ is the thickness of the T-stub flange, $f_y$ is the yield stress of the T-stub flange, and $l_{eff,1}$ and $l_{eff,2}$ are the total effective lengths in the T-stub which will be discussed in the next section. Note that determining the effective lengths ($l_{eff,1}$, $l_{eff,2}$) means firstly determining the bending mode; therefore, the two effective lengths must be determined iteratively. It is also recommended that the non-circular pattern effective length $l_{eff,nc}$ (calculated by either Eqn. (6.13), Eqn. (6.15) or Eqn. (6.17)) be used for the initial iteration.

The plastic resistance of bolts can be obtained by:

$$F_{t,Rd} = 0.9f_{ub}A_{be}$$  \hspace{1cm} (6.9)

where $f_{ub}$ is the ultimate tensile stress of the bolt and $A_{be}$ is the effective area of the bolt.

The plastic T-stub resistance and the bending mode can be obtained by:

$$F_{T,Rd} = \min(F_{T,1,Rd}, F_{T,2,Rd}, F_{T,3,Rd})$$  \hspace{1cm} (6.10)

Note that this Eurocode 3 T-stub resistance calculation is based on the yield line analysis of the T-stub flange. It is conservative and it does not consider the deformations at yield and ultimate load. However, the method is sufficiently accurate to determine the bending mode of the T-stub.

6.1.1.2 Converting End-plate Bending and Column Flange Bending Components to Idealised T-stubs

in Eurocode 3 (2006b), an effective length method is used to convert the inner and outer end-plate bending components and column flange bending component into idealised T-stub models. This conversion method is based on a yield line analysis.
The effective length of the idealised T-stub can be obtained by determining the minimum length of each possible yield pattern. Circular and non-circular yield patterns for bolt rows are all checked in this process. Figure 6.3-6.6.6 show examples of the yield patterns based on Srouji (1983) and Faella et al. (1999).

For the column flange bending component, bolt rows are usually considered in a group, and the circular and non-circular yield pattern of a group of bolt rows (Figure 6.3) is considered first:

\[
l_{\text{eff, cp}} = \min(mn + p, 2e_1 + p)
\]

\[
l_{\text{eff, nc}} = \min(e_1 + 0.5p, 2m + 0.625e + 0.5p)
\]

where \(e\) is the length from the centre of the bolt to the edge of the column flange, \(e_1\) is the length from the centre of the bolt to the closest end of the end-plate, \(l_{\text{eff, nc}}\) is a non-circular yield pattern and \(l_{\text{eff, cp}}\) is a circular yield pattern.

The effective lengths for bending modes 1 and 2 (\(l_{\text{eff, 1}}\) and \(l_{\text{eff, 2}}\)) can be determined as:
\[
\begin{align*}
\begin{cases}
    l_{\text{eff},1} &= \min(l_{\text{eff,nc}}, l_{\text{eff,cp}}) \\
    l_{\text{eff},2} &= l_{\text{eff,nc}}
\end{cases}
\end{align*}
\] (6.13)

For the outer end-plate bending component, bolt rows usually behave independently so are considered as individual bolt rows. The effective lengths for the circular (Figure 6.4) and non-circular (Figure 6.5) yield pattern for individual bolt rows can calculated as:

\[
l_{\text{eff,cp}} = \min(2\pi m_x, \pi m_x + w, \pi m_x + 2e)
\] (6.14)

\[
l_{\text{eff,nc}} = \min(4m_x + 1.25e_x, e + 2m_x + 0.625e_x, 0.5b_p, 0.5w + 2m_x + 0.625e_x)
\] (6.15)

Where \( m_x \) is the length from the centre of the bolt to the toe of the beam flange welds, \( w \) is the spacing between two bolts in that bolt row, \( e \) is the length from centre of the bolt to end-plate side edge, \( e_x \) is the length from the centre of the bolt to the top edge of the end plate, and \( b_p \) is the width of the end plate.

**FIGURE 6.4: EXAMPLES OF CIRCULAR YIELD PATTERN OF OUTER END-PLATE BENDING COMPONENT**
FIGURE 6.5: EXAMPLES OF NON-CIRCULAR YIELD PATTERN OF OUTER END-PLATE BENDING

For the inner end-plate bending component, bolt rows are usually considered as individual in most end-plate joints because the other inner bolt row is further away and close to the compression face. The effective lengths for the circular and non-circular yield patterns for individual bolt rows (Figure 6.6) can be calculated as:

\[ l_{\text{eff,cp}} = 2\pi m \]  \hspace{1cm} (6.16)
\[ l_{\text{eff,nc}} = \alpha m \]  \hspace{1cm} (6.17)

where \( m \) is the length from the centre of the bolt to the toe of the beam web welds, \( \alpha \) is a stiffened T-stub coefficient which can be obtained from figure 6.11 of Eurocode 3 Part 1.8 (2006b) and is a function of the position of the hole.

FIGURE 6.6: EXAMPLES OF CIRCULAR AND NON-CIRCULAR YIELD PATTERN OF INNER END-PLATE BENDING COMPONENT

For bending mode 3, an evaluation of the effective length is not covered by Eurocode 3 (2006b) because it only considers bolt failure as the T-stub failure mechanism and is not concerned with the T-stub flange bending. However, the bending of the T-stub flange is important when evaluating the displacement of the T-stub, which can be achieved by adopting the effective length for bending mode 2.
6.1.1.3 T-stub Model

By adopting the effective length method, the equivalent T-stub for inner and outer end-plate bending, and the column flange bending component, can be determined. Then, by applying the T-stub model, the force-displacement behaviour of the three equivalent T-stub components can be determined.

T-stub is a complex structural element that can be found in T-stub joints where two T-stubs are used to connect the beam flanges to the column flange. The technique for modelling this structural element has drawn increasing interest because it can be used in both end plate and top angle joints by using the equivalent T-stub conversion.

The most widely used T-stub model is from Eurocode 3 (2006b). It selectively adopts some of the most important contemporary works such as Agerskov (1977) and Jaspart (1991b) and also considers all three possible bending modes and prying forces on bolts. The method does not consider joint displacements and therefore cannot predict the full force-displacement behaviour.

More recently, Piluso et al. (2001a) suggested an analytical method to model T-stub force-displacement behaviour. They developed a theory based on the moment-curvature relationship of beam bending behaviour. It used a linear distribution of moment over the length between the plastic hinge and counter-flexural point to find the curvature distribution on the T-stub flange. By integrating the curvature over the T-stub flange, the rotation and then the displacement could be calculated.

Swanson and Leon (2001) proposed an alternative model based on the limit states of partially and fully formed plastic hinges. Compared to the model suggested by Piluso et al. (2001a), this model included the bending of bolts but did not consider strain hardening in the T-stub flange bending behaviour. However, strain hardening is usually insignificant in bending
behaviour and therefore the model prediction has sufficient accuracy. It is the preferred
modelling technique, for this thesis and is set out in full in Section 6.1.1.3.1.

The most recent model to predict the force-displacement curve is from Yu et al. (2009). They
developed a model for analysing T-stubs in fire, so paid particular attention to the joint
ductility. The model is based on the energy method and plastic hinge theory, so it considers
strain hardening for material property and necking for bolts. The method is a possible
alternative to the model from Swanson and Leon (2001).

6.1.1.3.1 Swanson and Leon (2001) Model

According to Swanson and Leon (2001), the elastic stiffness of bolts can be calculated as:

\[
K_{b,1} = \left( \frac{fd_b}{A_b E} + \frac{L_s}{A_b E} + \frac{L_{tg}}{A_{be} E} + \frac{fd_b}{A_{be} E} \right)^{-1} 
\]

\[
f = \left[ \frac{E}{K_{b,1}} - \frac{L_s}{A_b} \right] \left( \frac{d_b}{A_b} + \frac{d_b}{A_{be}} \right) 
\]

where \( f \) is a correlation factor which was discussed in Swanson’s thesis (Swanson, 1999),
\( L_s \) is the shank length of the bolt grip part, \( L_{tg} \) is the threaded length of the bolt grip part,
\( d_b \) is the nominal diameter of the bolt, \( E \) is young’s modulus, \( A_b \) is the gross bolt shank
area, \( A_{be} \) is the effective bolt shank area, and \( K_{b,1} \) is the elastic stage bolt stiffness.

So the plastic stiffness of bolts can be modelled as:

\[
K_{b,2} = \frac{1}{10} K_{eb} \quad 0.85 F_{ub} \leq F_b < 0.9 F_{ub} 
\]

\[
K_{b,3} = \frac{1}{20} K_{eb} \quad 0.9 F_{ub} \leq F_b < F_{ub} 
\]

Where \( K_{b,2} \) is the gradual yielding stiffness, \( K_{b,3} \) is the plastic stiffness, \( F_{ub} \) is the ultimate
tensile bolt strength, and \( F_b \) is the bolt force.

The elastic stiffness of the T-stub flange and bolt prying gradient can be obtained by:

\[
K_{ee,k} = \frac{12EI(3EI + K_{b,k} Y_3)}{Y_{ee,k}} 
\]
Chapter 7: Conclusion

\[ Q_{ee,k} = \frac{18EI(K_{b,k}n'm'^2\beta_b - 2EI)}{Y_{ee,k}} \]  
(6.23)

\[ Y_{ee,k} = 12EI\gamma_1 + K_{b,k}\gamma_2 \]  
(6.24)

where the subscript \( k \) on \( K_{ee,k}, Q_{ee,k}, Y_{ee,k} \) and \( K_{b,k} \) is the bolt force stage, and \( I \) is the elastic moment of inertia.

When a plastic hinge forms at the toe of the web-flange weld and the other part remains elastic, the stiffness and prying gradient can be written as:

\[ K_{pe,k} = \frac{12EI\left(3EI(K_{b,k}n'^2 + K_{h1}) + K_{b,k}K_{h1}\gamma_3\right)}{Y_{pe,k}} \]  
(6.25)

\[ Q_{pe,k} = \frac{18EI\left(K_{b,k}K_{h1}n'm'^2\beta_b + 2EI(K_{b,k}n'm' - K_{h1})\right)}{Y_{pe,k}} \]  
(6.26)

\[ Y_{pe,k} = 12EI(K_{h1}\gamma_1 + K_{b,k}(n'^2m'^2\beta_a + n'^2m'^2\beta_b) + 3EI\gamma_4) + K_{b,k}K_{h1}\gamma_2 \]  
(6.27)

When a plastic hinge forms at the bolt hole area and the other part remains elastic, the stiffness and prying gradient can be written as:

\[ K_{ep,k} = \frac{12EI(3EI(K_{h2} + K_{b,k}n'^2) + K_{b,k}K_{h2}\gamma_3)}{Y_{ep,k}} \]  
(6.28)

\[ Q_{ep,k} = \frac{18EIK_{h2}(K_{b,k}n'm'^2\beta_b - 2EI)}{Y_{ep,k}} \]  
(6.29)

\[ Y_{ep,k} = 12EI(K_{h2}\gamma_1 + K_{b,k}n'^2m'^3\beta_b + 3EId'^2)\gamma_1 + K_{b,k}K_{h2}\gamma_2 \]  
(6.30)

When plastic hinges form at the toe of the web-flange weld and the bolt hole area, the stiffness and prying gradient can be written as:

\[ K_{pp,k} = \frac{K_{h1}K_{h2} + K_{b,k}n'^2(K_{h1} + K_{h2})}{Y_{pp,k}} \]  
(6.31)

\[ Q_{pp,k} = \frac{K_{h2}(K_{b,k}n'm' - K_{h1})}{Y_{pp,k}} \]  
(6.32)

\[ Y_{pp,k} = K_{h2}\gamma_4 + K_{b,k}n'^2m'^2 + K_{h1}n' \]  
(6.33)

In Eqn. (6.20-6.33),

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\[ m' = m - 0.5d_b \]  \hspace{1cm} (6.34)
\[ n' = n + 0.5d_b \]  \hspace{1cm} (6.35)
\[ \gamma_1 = \beta_b (m'^2 + 3n'm'^2 + 3n'^2m') + n'^3 \beta_a \] \hspace{1cm} (6.36)
\[ \gamma_2 = 3n'^2m'^4 \beta_b^2 + 4n'^3m'^3 \beta_a \beta_b \] \hspace{1cm} (6.37)
\[ \gamma_3 = n'^3 \beta_a + 3n'^2m' \beta_b \] \hspace{1cm} (6.38)
\[ \gamma_4 = n'^2 + 2n'm' + m'^2 \] \hspace{1cm} (6.39)
\[ \beta_a = 1 + \frac{12EI}{G \ell_{eff} t_f n'^2} \] \hspace{1cm} (6.40)
\[ \beta_b = 1 + \frac{12EI}{G \ell_{eff} t_f m'^2} \] \hspace{1cm} (6.41)
\[ K_{h1} = \frac{E_{sh} l}{t_f} \] \hspace{1cm} (6.42)
\[ K_{h2} = \left(1 - \frac{d_{hb}}{\ell_{eff}}\right) \frac{E_{sh} l}{t_f} \] \hspace{1cm} (6.43)

In Eqn. (6.42, 6.43), \( E_{sh} \) is the strain hardening modulus, \( d_b \) is the diameter of the bolt, \( t_f \) is the T-stub bending stiffness, \( G \) is the shear modulus, \( \ell_{eff} \) is the effective length of the T-stub flange (Section 6.1.1.2), \( d_{hb} \) is the diameter of the bolt hole, \( m \) is the length from the centre of the bolt to the edge of the welds, and \( n \) is the length from the centre of the bolt to the edge of T-stub flange, capped at \( n \leq 1.25m \).

In Eqn. (6.43), the term \( \left(1 - \frac{d_{hb}}{\ell_{eff}}\right) \) is used to take the bolt hole into account for a real T-stub. However, for the idealised T-stub, when calculating the effective length \( \ell_{eff} \), the bolt hole has been already considered. Instead, for the application on idealised T-stub (outer and inner end-plate bending and column flange bending components), Eqn. (6.44) is adopted.
\[ K_{h2} = \frac{E_{sh} l}{t_f} \] \hspace{1cm} (6.44)

The plastic hinges may only be partially formed in the gradual yielding range. The flange stiffness and prying gradient with partial plastic hinges can be determined by an average weighted method. When a partial plastic hinge exists at the toe of the web-flange welds and the other part remains elastic, the stiffness and prying gradient can be obtained by:
Similarly, when a partial plastic hinge exists at bolt-hole area and the other part remains elastic, the stiffness and prying gradient can be obtained by:

\[ K_{ey,k} = \frac{K_{ee,k} + 3K_{pe,k}}{4} \]  \hspace{1 cm} (6.47)
\[ Q_{ey,k} = \frac{Q_{ee,k} + 3Q_{pe,k}}{4} \]  \hspace{1 cm} (6.48)

When a fully plastic hinge exists at the toe of the web-flange welds and a partial plastic hinge exists at the bolt-hole area, the stiffness and prying gradient can be obtained by:

\[ K_{py,k} = \frac{K_{pe,k} + 3K_{pp,k}}{4} \]  \hspace{1 cm} (6.49)
\[ Q_{py,k} = \frac{Q_{pe,k} + 3Q_{pp,k}}{4} \]  \hspace{1 cm} (6.50)

When a partial plastic hinge exists at the toe of the web-flange welds and a full plastic hinge exists at the bolt hole area, the stiffness and prying gradient can be obtained by:

\[ K_{yp,k} = \frac{K_{ep,k} + 3K_{pp,k}}{4} \]  \hspace{1 cm} (6.51)
\[ Q_{yp,k} = \frac{Q_{ep,k} + 3Q_{pp,k}}{4} \]  \hspace{1 cm} (6.52)

When partial plastic hinges exist at the toe of the web-flange welds and the bolt hole area, the stiffness and prying gradient can be obtained by:

\[ K_{yy,k} = \frac{K_{ee,k} + 3K_{pp,k}}{4} \]  \hspace{1 cm} (6.53)
\[ Q_{yy,k} = \frac{Q_{ee,k} + 3Q_{pp,k}}{4} \]  \hspace{1 cm} (6.54)

Then, the force acting on the T-stub web \( F \), and the prying force on bolt \( Q \), can be calculated incrementally:

\[ \delta F = K \cdot \delta \Delta \]  \hspace{1 cm} (6.55)
\[ \delta Q = Q \cdot \delta \Delta \]  \hspace{1 cm} (6.56)
\[ F = F_1 + \delta F \]  \hspace{1 cm} (6.57)
\[ Q = Q_1 + \delta Q \]  \hspace{1 cm} (6.58)
where $F_1$ is the force applied at the previous step, $Q_1$ is the bolt prying force at the previous step, $K$ is current flange stiffness, and $Q$ is current prying gradient. So at the initial stage, $K_{ee}$ and $Q_{ee}$ are adopted. Meanwhile, the moment at the two potential plastic hinge areas and the force applied onto the bolt can be calculated:

$$F_b = F + Q \quad (6.59)$$

$$M_A = Fm' - Qn' \quad (6.60)$$

$$M_B = Qn' \quad (6.61)$$

Then, as the displacement increased the limit states of bolt and plastic hinge will be checked. The limit state for bolts is:

$$F_{yb} = f_{yb}A_{be} \quad (6.62)$$

$$F_{ub} = f_{ub}A_{be} \quad (6.63)$$

where $F_{yb}$ is the force applied at bolt yielding, $f_{yb}$ is the yield stress of the bolt material, $F_{ub}$ is ultimate bolt strength and $f_{ub}$ is the ultimate stress of the bolt material.

The limit states for the moment at the toe of the web-flange welds are:

$$M_{yA} = \frac{f_{yleff}t_f^2}{6} \quad (6.64)$$

$$M_{pA} = \frac{f_{yleff}t_f^2}{4} \quad (6.65)$$

The limit states for moment at the bolt-hole area are:

$$M_{yB} = \left(1 - \frac{d_{bh}}{l_{eff}}\right)\frac{f_{yleff}t_f^2}{6} \quad (6.66)$$

$$M_{pB} = \left(1 - \frac{d_{bh}}{l_{eff}}\right)\frac{f_{yleff}t_f^2}{4} \quad (6.67)$$

However, for idealised T-stubs, the bolt hole was considered when calculating the effective length. So Eqn. (6.68, 6.69) will be used to substitute Eqn. (6.66-6.67).

$$M_{yB} = \frac{f_{yleff}t_f^2}{6} \quad (6.68)$$

$$M_{pB} = \frac{f_{yleff}t_f^2}{4} \quad (6.69)$$
The limit states will be checked at each incremental step. If one of the limit states is reached, the corresponding bolt stiffness, T-stub flange stiffness, and prying gradient for the next step will be used. By gradually increasing the displacement, the full force-displacement curve of the T-stub can be obtained. All the inner and outer end-plate bending components and column flange components (without backing plate) can be modelled by this method.

**6.1.1.3.2 Backing Plate**

Some specimens have backing plates installed to strengthen the column flange bending component, so this additional stiffness should be considered. The backing plate can be considered as a pinned-to-fixed end beam element (Figure 6.7), because it only strengthens the flange at the bolt hole area and takes no moment at the end near the toe of the web-flange welds. On the contrary, the T-stub flange can be considered as a fixed-to-fixed end beam element.
Since the stiffness of a pinned-to-fixed end beam element is half that of a fixed-to-fixed end beam element, the stiffness of the backing plate as a T-stub flange is calculated first by substituting the backing plate dimensions into Eqn. (6.22-6.54). The stiffness of the backing plate at different stages can be obtained by dividing the stiffness of all the backing plates at different stages by two, and then the total stiffness can be obtained by adding the column flange stiffness (Eqn. (6.70)).

\[ K_{ij} = K_{ij,cf} + 0.5K_{ij,bp} \]  
\[ \text{(6.70)} \]

where \( K_{ij} \) is the total stiffness of the column flange with a backing plate, \( K_{ij,cf} \) is the stiffness of the column flange, \( K_{ij,bp} \) is the stiffness of the backing plate calculated as T-stub flange, \( i \) and \( j \) are for the designations of the plastic hinge stage such as \( e \) for elastic behaviour, \( p \) for a full plastic hinge, and \( y \) is for a partial plastic hinge.

In addition, the limit state for the moment at the bolt hole area should be modified as:

\[ M_{yB} = M_{yB,cf} + M_{yB,bp} \]  
\[ \text{(6.71)} \]

\[ M_{pB} = M_{pB,cf} + M_{pB,bp} \]  
\[ \text{(6.72)} \]

where, \( M_{yB,cf} \) and \( M_{pB,cf} \) are the limit state moments of the column flange which are calculated by Eqns. (6.68-6.69) with the column flange properties; \( M_{yB,bp} \) and \( M_{pB,bp} \) are the limit state moments of the column flange which are also calculated by Eqns. (6.68-6.69) with the backing plate properties.

By following the procedures introduced in the previous section, the force-displacement curve for the column web flange with backing plate can be obtained.

**6.1.1.4 T-stub Resistance under Cracking at the Web-Flange Welds Toe Area**

Based on the experimental observations detainted in Chapter 5, the ultimate resistance of the T-stub is affected when the T-stub cracks, and therefore the effect of cracking should be considered when predicting the ultimate resistance of a T-stub.
Cracking is a complex mechanism that includes crack initiation, crack propagation, and dynamic effects when smaller cracks merge; these factors dramatically increase the complexity of predicting behaviour at this stage in any detail, so a simplified method is proposed.

Based on experimental observations, major cracks occurred when large deformations developed along the yield line at the HAZ zone of the end plate near the bolt-hole, where the HAZ zone is a small zone in the end-plate at the toe of the web-flange welds. Major crack(s) are caused by shear failure of the plate through its thickness. Therefore, the T-stub resistance at cracking may be determined as the shear resistance of the stress area of the end plate:

\[ F_{T,cr} = 0.6t_{ep}l_{eff}f_{uw} \]  (6.73)

where \( t_{ep} \) is the thickness of the end plate, \( l_{eff} \) is the effective length determined using Eqns. (6.14, 6.15) and \( f_{uw} \) is the ultimate weld tensile strength.

### 6.1.2 Column Web in Compression

The force-displacement behaviour for this component can be obtained by the model presented in Chapter 4.

### 6.1.3 Column Web Shear Panel

This is one of the key components in rigid joints such as welded beam-to-column joints with column stiffeners, but its behaviour is not as significant in end plate joints. Deformation of the column web shear panels in the specimens tested in this thesis were insignificant so the Eurocode 3 method for modelling this component were adopted.

The shear area of the column was calculated first, and according to Eurocode 3: Part 1.1 (2005), the shear area of the column \( A_{vc} \) can be obtained by:

\[ A_{vc} = A_c - 2b_{cf}t_{cf} + (t_{cw} + 2r)t_{cf} \geq h_{cw}t_{cw} \]  (6.74)

where \( A_c \) is the gross cross-sectional area of the column, \( b_{cf} \) is the width of the column flange, \( t_{cf} \) is the thickness of the column flange, \( r \) is the radius of the web-to-flange
transition zone of the column, \( t_{cw} \) is the thickness of the column web, and \( h_{cw} \) is the depth of the web.

The shear panel stiffness \( (K_6) \) can then be obtained:

\[
K_6 = \frac{0.38 A_{vc} E}{\beta z}
\]  

(6.75)

where \( z \) is the lever arm obtained from Figure 6.15 in Eurocode 3: Part 1.8 (2006b). \( \beta \) is the transformation parameter, and \( \beta = 1 \) for a joint with a beam connected to a single side of a continuous column.

### 6.1.4 Column Web in Tension

This component is straightforward and can be modelled as a linear elastic component by using the Eurocode 3 (2006b) method. According to Eurocode 3, the stiffness of this component \( K_5 \) can be written as:

\[
K_5 = \frac{0.7 l_{eff,cf} t_{cw} E}{d_c}
\]

(6.76)

where \( l_{eff,cf} \) is the effective length used in the column flange bending component which is defined as Eqns. (6.13), and \( d_c \) is the depth of the column web between K-zone (Figure 6.8).

![FIGURE 6.8: DEPTH OF THE COLUMN WEB BETWEEN K-ZONE](image)
6.2 Model Prediction and Validation

Six end plate joint specimens from the tests reported in Chapter 5 and Girao Coelho’s tests (Girao Coelho et al., 2004a) were modelled to obtain their full range moment-rotation \((M - \theta)\) curve and component fracture sequence. The properties of the specimens are summarised in Section 6.2.1, the behaviour of the components of these specimens are discussed in Section 6.2.2, and a comparison between the model predictions and experimental results are listed in Section 6.2.3.

Although 13 tests were carried out as part of this thesis, three particular tests were selected and their specimens were modelled (S10_0_0, S10_0_0_B and S20_0_0_B). The three tests were carried out in a standard rig where the loading direction was in-plane and perpendicular to the beam of the specimens. All the other 10 tests were carried out in non-standard rigs catering for diagonal bending, lateral bending, and combined axial load and bending, (see Chapter 5). These non-standard types of loading are not included in the proposed model at current stage.

The three specimens chosen from the tests presented in Chapter 5, henceforth referred to as “the USyd tests” had relatively small columns in order to assess how column web buckling affected the compression face, and large diameter high tensile bolts to allow for large end plate bending on the tension face. By varying the thickness of the end plates, end-plate bending and column web buckling failure modes could be achieved.

S10_0_0 had a 10mm thick end-plate that was susceptible to bending, whereas S10_0_0_B had a same 10mm thick end-plate with 20mm thick backing plates on the column flange bending component to reduce deformation of that component. Since the 10mm thick end-plates were susceptible to bending, both of these two specimens were subjected to large end plate bending deformation, while the failure mode for both specimens was cracking at the heat affected zone (HAZ) at the toe of the welds. On the other hand, S20_0_0_B had a 20mm
thick end plate with 20mm thick backing plates at the column flange bending component, so its tension face was much stronger and it failed at the compression face by column web buckling.

Girao Coelho’s tests (Girão Coelho et al., 2004b) used strong columns and medium size bolts. The strong column used in their tests was a rigid component where very little deformation occurred in each test, so their tests all failed at the tension face by either end plate bending or bolt fracture. Specimen FS1a and FS1b (or FS1a/b) were two identical tests featuring 10mm thick end plates and both failing due to end plate bending. FS2a/b were two nominally identical tests featuring 15mm end-plates and both failing due to a combination of end plate bending and bolt fracture. FS3a/b were two nominally identical tests featuring 20mm thick end plates and both failing due to end plate bending and bolt fracture.

6.2.1 Specimen Properties

The geometric properties of the six specimens are summarised in Tables 6.1-6.7.

<table>
<thead>
<tr>
<th>Tests</th>
<th>Column</th>
<th>Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profile</td>
<td>Steel Grade</td>
</tr>
<tr>
<td>S10_0_0</td>
<td>310UC 96.8</td>
<td>AS 350</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>310UC 96.8</td>
<td>AS 350</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>310UC 96.8</td>
<td>AS 350</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>HE340M</td>
<td>EN S355</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>HE340M</td>
<td>EN S355</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>HE340M</td>
<td>EN S355</td>
</tr>
</tbody>
</table>
### TABLE 6.2: END-PLATES

<table>
<thead>
<tr>
<th>Tests</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
<th>Thickness (mm)</th>
<th>Steel Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>270</td>
<td>400</td>
<td>10</td>
<td>AS 350</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>270</td>
<td>400</td>
<td>10</td>
<td>AS 350</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>270</td>
<td>400</td>
<td>20</td>
<td>AS 350</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>150</td>
<td>400</td>
<td>10</td>
<td>EN S355</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>150</td>
<td>400</td>
<td>15</td>
<td>EN S355</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>150</td>
<td>400</td>
<td>20</td>
<td>EN S355</td>
</tr>
</tbody>
</table>

### TABLE 6.3: BOLT HOLE POSITIONS (MM) (FIGURE 6.9)

<table>
<thead>
<tr>
<th>Tests</th>
<th>$e_{ep}$</th>
<th>$e_{cf}$</th>
<th>$e_{x}$</th>
<th>$m$</th>
<th>$m_2$</th>
<th>$m_x$</th>
<th>$\alpha^*$</th>
<th>$w$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>65</td>
<td>82.5</td>
<td>36</td>
<td>61.7</td>
<td>25.6</td>
<td>25.6</td>
<td>7.2</td>
<td>140</td>
<td>203</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>65</td>
<td>82.5</td>
<td>36</td>
<td>61.7</td>
<td>25.6</td>
<td>25.6</td>
<td>7.2</td>
<td>140</td>
<td>203</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>65</td>
<td>82.5</td>
<td>36</td>
<td>61.7</td>
<td>25.6</td>
<td>25.6</td>
<td>7.2</td>
<td>140</td>
<td>203</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>30</td>
<td>109.5</td>
<td>30</td>
<td>38.25</td>
<td>34.85</td>
<td>34.85</td>
<td>5.35</td>
<td>90</td>
<td>205</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>30</td>
<td>109.5</td>
<td>30</td>
<td>38.25</td>
<td>34.85</td>
<td>34.85</td>
<td>5.35</td>
<td>90</td>
<td>205</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>30</td>
<td>109.5</td>
<td>30</td>
<td>38.25</td>
<td>34.85</td>
<td>34.85</td>
<td>5.35</td>
<td>90</td>
<td>205</td>
</tr>
</tbody>
</table>

* $\alpha$ is a correlation factor for the effective length of the inner end-plate bending component, as obtained from figure 6.11 of Eurocode 3 Part 1.8 (2006b).

![Figure 6.9: Bolt-Hole Position and Height of Spring Rows](image)

a) Column Flange  

b) End-plate
TABLE 6.4: BOLTS AND WELDS (MM) \((t_w)\) IS THE LEG LENGTHS OF WEB WELDS AND \(t_f\) IS THE LEG LENGTH OF FLANGE WELDS

<table>
<thead>
<tr>
<th>Tests</th>
<th>Bolt Profile</th>
<th>Bolt Grade</th>
<th>(d_{bh})</th>
<th>(t_w)</th>
<th>(t_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>M24</td>
<td>8.8</td>
<td>26</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>M24</td>
<td>8.8</td>
<td>26</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>M24</td>
<td>8.8</td>
<td>26</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>M20</td>
<td>8.8</td>
<td>22</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>M20</td>
<td>8.8</td>
<td>22</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>M20</td>
<td>8.8</td>
<td>22</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

TABLE 6.5 HEIGHT OF EACH SPRING ROW (DISTANCE IS MEASURED FROM THE CENTROID OF BEAM. FIGURE 6.9)

<table>
<thead>
<tr>
<th>Tests</th>
<th>(h1)</th>
<th>(h2)</th>
<th>(h3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>193.5</td>
<td>103.5</td>
<td>-147.5</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>193.5</td>
<td>103.5</td>
<td>-147.5</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>193.5</td>
<td>103.5</td>
<td>-147.5</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>192.5</td>
<td>102.5</td>
<td>-147.5</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>192.5</td>
<td>102.5</td>
<td>-147.5</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>192.5</td>
<td>102.5</td>
<td>-147.5</td>
</tr>
</tbody>
</table>

TABLE 6.6: MECHANICAL PROPERTIES FOR COLUMNS AND BEAMS (MPA)

<table>
<thead>
<tr>
<th>Tests</th>
<th>Column</th>
<th>Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_y)</td>
<td>(f_u)</td>
</tr>
<tr>
<td>S10_0_0</td>
<td>396</td>
<td>512</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>396</td>
<td>512</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>396</td>
<td>512</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>335</td>
<td>530</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>335</td>
<td>530</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>335</td>
<td>530</td>
</tr>
</tbody>
</table>
TABLE 6.7: MECHANICAL PROPERTIES FOR END-PLATE, BOLT AND WELDS (MPA)

<table>
<thead>
<tr>
<th>Tests</th>
<th>End-plate</th>
<th>Bolt</th>
<th>Welds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_y$</td>
<td>$f_u$</td>
<td>$f_y$</td>
</tr>
<tr>
<td>S10_0_0</td>
<td>429</td>
<td>556</td>
<td>640</td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>429</td>
<td>556</td>
<td>640</td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>429</td>
<td>556</td>
<td>640</td>
</tr>
<tr>
<td>FS1a/b</td>
<td>340</td>
<td>480</td>
<td>856</td>
</tr>
<tr>
<td>FS2a/b</td>
<td>340</td>
<td>480</td>
<td>856</td>
</tr>
<tr>
<td>FS3a/b</td>
<td>340</td>
<td>480</td>
<td>856</td>
</tr>
</tbody>
</table>

*According to Girão Coelho et al. (2004b), the welds used in FS1a/b, FS2a/b and FS3a/b were full strength 45°-continuous fillet welds, and the electrodes used in the process were basic, soft, and low hydrogen. However, the specifications of the electrodes were not provided. Since the electrodes used in USyd tests were also basic, soft, and low hydrogen, it was assumed that the ultimate strength of the welds used in the Girão Coelho et al. (2004b) tests was the same as that in USyd tests.

### 6.2.2 Converting Multi-Linear Individual Component Force-Displacement Curves to Tri-Linear/Bi-Linear Force-Displacement Curves

By using the joint properties specified in Tables 6.1-6.7, models for the individual components can be obtained (Section 6.1 and Chapter 4). Linear models of bolts, column flanges in tension, and column web shear panels were obtained from Eurocode 3 (2006b). A full range model for the column web in compression was obtained using the model presented in Chapter 4. Full range models for end plate bending and column flange bending components were obtained using Swanson’s T-stub model (Swanson and Leon, 2001), as described in detail in Section 6.1.1.3.1.

The full-range force-displacement curves can be simplified into bi-linear and tri-linear curves. The model of the end plate bending component was represented by a bi-linear model where fracture occurred at the ultimate load (Figure 6.10), whereas the column web in compression...
component was represented by a tri-linear model because of its ductile behaviour in the post-ultimate range (Figure 6.11).

By converting full-range multi-linear curves to bi-linear or tri-linear curves, as applicable (Appendix A5), the input data (elastic, plastic, softening stiffness and corresponding critical loads) for each spring model (Chapter 3) can be obtained (Tables 6.8-6.13). Note that since Girão Coelho et al.’s tests (Girão Coelho et al., 2004b) used an oversized column (Table 6.1), the column web in tension (column web stretching) and column web in compression (column web shortening) components behaved in a rigid fashion, so the stiffness of these two column components were set to be rigid for tests FS1, FS2, and FS3.

![FIGURE 6.10: EXAMPLE OF CONVERTING A MULTI-LINEAR MODEL TO A BI-LINEAR MODEL](image-url)
### FIGURE 6.11: EXAMPLE OF CONVERTING A MULTI-LINEAR MODEL TO A TRI-LINEAR MODEL

### TABLE 6.8: S10_0_0

<table>
<thead>
<tr>
<th>Row</th>
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<th>$k_c(N/mm)$</th>
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**TABLE 6.9: S10_0_0_B**

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### TABLE 6.11: FS1

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<th>$P_p^c(N)$</th>
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<td>-</td>
<td>-</td>
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<tr>
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<td>4.30E05</td>
<td>4.48E05</td>
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<td>-</td>
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TABLE 6.13: FS3

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<td>4.48E05</td>
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6.2.3 Comparison between Model Prediction and Experimental Results

6.2.3.1 Moment Rotation Curves

By applying the spring models for each individual component (specified in Tables 6.8-6.13), the component models for the six end plate connections can be obtained. Figures 6.12-6.17 show the model predictions and experimental results. The models presented are in reasonable agreement with the experiments, throughout the loading range and including the fracture of components.

![Moment Rotation Curve S10_0_0](image)

**FIGURE 6.12: MOMENT-ROTATION CURVE FOR S10_0_0 (USYD TEST)**
Chapter 7: Conclusion

FIGURE 6.15: MOMENT-ROTATION CURVE FOR FS1 (GIRÃO COELHO ET AL., 2004B)

FIGURE 6.16: MOMENT-ROTATION CURVE FOR FS2 (GIRÃO COELHO ET AL., 2004B)
FIGURE 6.17: MOMENT-ROTATION CURVE FOR FS1 (GIRÃO COELHO ET AL., 2004B)
6.2.3.2 Component Fracture Sequence

The component fracture sequence also can be obtained (Tables 6.14-6.16). The fracture sequence predictions for specimens S10_0_0 and S20_0_0_B are accurate, and the predictions for the FS1 and FS3 test series are roughly correct as seen by comparing the failure modes of each test in each series. In the FS2 series the prediction of second stage fracture differed slightly from the experimental observation. In the second fracture stage of FS2, the 1st row end plate bending failure occurred slightly earlier in the model prediction than the 2nd row bolt failure observed in the test. In this case a small variation in the material properties may have caused a different fracture sequence. For specimen S10_0_0_B, the 2nd and 3rd stage predictions did not agree with the experimental results; actual failure at these two stages was a combination of the beam flange pulling out (Figure 6.18b), end plate cracking and bolt fracture, whereas the beam flange pulling out mechanism was not included in the model. However, the fractured rows in these two stages roughly agree with the experimental observation.

![End-Plate Failure Modes](image)

**FIGURE 6.18: END-PLATE FAILURE MODES: A) S10_0_0 AND B) S10_0_0_B**

6.3 Summary

The model presented here was in reasonable agreement with experiments for the ultimate moment and the moment-rotation curves in the elastic, inelastic, and post-ultimate ranges. Moreover, the model predictions of the component fracture sequence were also in good
agreement with the experimental observations. It was demonstrated that some improvement can be made by including the fracture of the heat affected zone in the model because this dominates end plate bending in the large deformation range.

**TABLE 6.14: FIRST STAGE FRACTURE PREDICTION**

<table>
<thead>
<tr>
<th>Tests</th>
<th>1st Stage Fracture Component</th>
<th>Model Predictions</th>
<th>Experimental Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10_0_0</td>
<td>Half End-plate in Row 1 Cracking</td>
<td>Half End-plate in Row 1 Cracking</td>
<td></td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>Half End-plate in Row 1 Cracking</td>
<td>Half End-plate in Row 1 Cracking</td>
<td></td>
</tr>
<tr>
<td>S20_0_0_B</td>
<td>Row 3, Column Web Buckling</td>
<td>Row 3, Column Web Buckling</td>
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</tr>
<tr>
<td>FS1a/b</td>
<td>Half End-plate in Row 1 Cracking</td>
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<tr>
<td>FS2a/b</td>
<td>Half End-plate in Row 1 Cracking</td>
<td>Row 2 Bolt Failure/Row 1 End-Plate Cracking</td>
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<td>FS3a/b</td>
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**TABLE 6.15: SECOND STAGE FRACTURE PREDICTION**

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<th>Model Predictions</th>
<th>Experimental Results</th>
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<td></td>
</tr>
<tr>
<td>S10_0_0_B</td>
<td>All End-plate in Row 1 Cracking</td>
<td>Row 2 Bolt Stripping/Row 1 End-plate Cracking</td>
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</tr>
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<td>S20_0_0_B</td>
<td>One Bolt in Row 1 Failure</td>
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<td>FS1a/b</td>
<td>All End-plate in Row 1 Cracking</td>
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<td>FS2a/b</td>
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**TABLE 6.16: THIRD STAGE FRACTURE PREDICTION**

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<tr>
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<td>One Bolt in Row 2 Failure</td>
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<td>S20_0_0_B</td>
<td>The Other Bolt in Row 1 Failure</td>
<td>The Other Bolt in Row 1 Failure</td>
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<td>FS1a/b</td>
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<td>FS2a/b</td>
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<tr>
<td>FS3a/b</td>
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Chapter 7: Conclusion

7.1 Summary

This thesis presents a generalised component model to predict the full-range moment-rotation curves of structural steel joints. The model used a component based method to predict the behaviour of joints in the elastic, plastic, and post-ultimate range, and an energy based method was used to extend the prediction of joint behaviour to the post-fracture range. The model is flexible and can be applied to all types of joints provided the joint components can be considered as longitudinal deformable springs.

An analysis of the full-range force-displacement behaviour of the column web in compression components is covered in this thesis. It includes a statically admissible stress field based post-buckling analysis, coupled with a semi-empirical inelastic modulus based method and the Eurocode 3 method, to predict the buckling load, yield point, and ultimate load. A modified web collapse mechanism method is used to describe post-ultimate behaviour of the web. By using the full-range force-displacement curve in the generalised component model, unnecessary column stiffeners may potentially be safely eliminated.

13 experiments on bolted moment end-plate joints were conducted in this thesis, of which three experiments with 10mm and 20mm end-plates with or without backing plates were tested by pure in-plane bending; four experiments with 10mm and 20mm thick end-plates were tested by in-plane bending combined with axial loading; two experiments with 10mm and 20mm thick plates were tested by lateral bending; four experiments with 10mm and 20mm thick end-plates were tested by diagonal (bi-axial) bending. In all these experiments, the full-range moment-rotation curves were recorded, along with the force-displacement curves of the key components such as end-plate bending and column web buckling components.
Of the experiments with the 10mm thin end-plate joints, the fracture and post-fracture behaviours were observed and recorded in detail, including three abrupt drops in resistance caused by either end-plate tear or bolt fracture. End-plate tear was due to crack(s) initiation and propagation that was discussed in Chapter 5. Bolt fracture was initiated by unbalanced bolt forces caused by uneven crack(s) propagation, as also discussed in Chapter 5.

In the experiments with 20mm thick end-plate joints, the post-ultimate behaviours caused by column web buckling were observed and recorded in detail. The buckling behaviour commenced in the pre-ultimate stage and developed slightly before reaching the ultimate load. In the post-ultimate range, the column web collapsed and started folding. Resistance in this range gradually dropped as the joint rotation increased. Finally, failure of the joints by bolt fracture on the tension face was initiated by an unbalanced bolt force distribution induced by column flange rotation.

Six bolted moment end-plate joints, including three tests conducted in this thesis and three tests performed by Girão Coelho et al. (2004b) were modelled by the presented model. The behaviours of key individual components were modelled first; the column web in compression components were modelled by the method presented in Chapter 4; the inner and outer end-plate bending, column flange bending, and column web in tension were modelled using the methods suggested in literature, but with slight modifications. Then, the individual component behaviours were applied to the generalised component models and hence the full-range moment-rotation curves for all six joints were obtained. The model predictions, including the elastic, plastic, post-ultimate, and post-fracture ranges, were in reasonable agreement with the experiments.

7.2 Future Research

The application of the generalised component model is limited to the bolted moment end-plate joint in this thesis, but it could be applied to other types of joints, e.g. angle cleat joints.
and web plate joints. Future research could investigate how this generalised component model could be applied to other types of joints, including the analyses of the key components in those joints for modelling the full-range force-displacement curves.

Crack propagation on the end-plate is not well understood. It could potentially be modelled and analysed by the finite element analysis. Based on the finite element analysis, an expression may be derived to determine the force-crack opening relationship where the deformation of the inner and outer end-plate bending components can be obtained and thus the stiffness of joints in the crack propagation range can be calculated. Meanwhile, the maximum crack opening size may be determined and then used to calculate the maximum deformation of the two end-plate components. This would be an important step towards determining the post-fracture ductility of the joint.

Full-range moment-rotation curves of joints can be incorporated into structural analysis methods, such as GMNIA analysis and earthquake analysis of steel frames. By taking full-range moment-rotation curves into account, structural analyses can achieve higher accuracy in predicting the structural response and ultimate capacity of the frame. This is an important step towards developing design-by-analysis methods for steel structures, in which the nonlinear behaviour and strength of joints are considered in the same global analysis that determines the deformation and strength of component members. Advanced analysis methods of this type are likely to become the method of design of the future. They rely on accurate full-range modelling of both members and joints. This thesis presents a major step towards developing full-range models for joints but further research is required to apply the method more generally to other types of joints and to include the derived moment-rotation curves in GMNIA frame analyses.
Appendix

A1: Introduction to Joint Decomposition, Classification and Ductile Behaviour

A joint is a structural assembly where two or more members are connected. It is usually considered to consist of two parts: column web panel and connection (Figure A1.1). Both of them may rotate when the joint is subject to bending moment.

This decomposition is based on the load path and major deformation sources in a joint. The load path starts at the connection which resists bending moment from connected beam members. The bending moment resisted by the connection is transferred into either a couple of flange forces, such as in the top and seat angle connection or end-plate connection, or a linearly distributed stress on the column flange, such as in the web side connection and web angle cleat connection. In this process, the applied bending moment also results in rotation of the connection. The couple forces transmit to the column through fasteners (bolts and welds) or faying surfaces. Ultimately, these forces will be resisted by the column web panel and cause rotation of this panel.
Although joints consist of two parts, they may be classified only by its connection part. Based on the type of components in the connection part, the joints are classified into many groups. Amid them, the typical types of joints are angle cleat joints, web side plate joints and end-plate joints.

Since the mechanical properties of the components, such as angles, plates, bolts or welds, are different, the rotational rigidities of different types of joints are different. Figure A1.2 illustrates the rotational rigidity spectrum for the three typical types of joint.

In addition, since the components may discontinue or yield during loading, the rigidity of a joint is not constant and may decrease. This rigidity reduction can be easily shown by a moment-rotation $(M - \theta)$ curve (Figure A1.3) in which the rotational rigidity may be obtained as the slope of the $M - \theta$ curve. This effect and the resulting extended $M - \theta$ curve provides the ductility of the joint.

![Figure A1.2: Rigidity Spectrum for Typical Types of Joints](image-url)

- **Pinned**
  - Angle Cleat Joint
- **Semi-Rigid**
  - Web Side Plate Joint
- **Rigid**
  - End-plate Joint
A2: Matlab Program for Generalised Component Model

A Matlab program was written to implement this method (Figure A2.1).

The input data sheet for the program contains the information of each individual component spring. It includes the number of springs in each series, number of spring series, height of each spring series, stiffness of each stage and critical load of each stage of each component.
spring. Tri-linear springs are used in this program. If necessary, the program can be extended to model multi-linear springs.

A sample input file is shown in Figure A2.2. Note that \( k_{PL} \) is the plastic stiffness of the component spring and \( k_{SO} \) is the stiffness for the post ultimate range. Therefore,

\[
\frac{1}{k_p} = \frac{1}{k_{PL}} - \frac{1}{k_e} \quad \text{and} \quad \frac{1}{k_s} = \frac{1}{k_{SO}} - \frac{1}{k_e} \quad (A2.1)
\]

![Figure A2.2: An input file example of a multi spring system in which there are three spring series and each spring series has three individual springs.](image)

The main script is called “Springs.m”. In this file, there are three parts:

1. A brief description of the main arrays in the program.
2. Read input file.
3. Call function (SIS) to calculate the \( F - \Delta \) curve of each spring series.
4. Call function (SIP) to calculate the \( M - \theta \) curve of the multi-spring system.

Function SIS (“SIS.m”) is used to obtain the combined behaviour of a spring series. There are three parts:

1. Initialisation.
2. Sort all critical loads to find the ultimate load, fracture load and the sequence of activation of springs.
3. Calculate all the parameters such as the stiffness of spring series, displacement limit at each stage, and the corresponding preloaded displacement and critical loads.
SIP ("SIP.m") is the core function to calculate the $M - \theta$ curve for a multi-spring system. There are five parts in it:

1. Initialisation, Initial stage and Main loop
2. Rotation Limit
3. Applied moment equation
4. Update

A function ("ICRini.m") used to calculate the height of ICR is called. It searches all possible height regions for a valid ICR. Figure A2.3 shows the scheme, featuring, all height regions to be checked one by one.

For example, the Matlab function firstly assumes that the ICR is located at the height region I (Figure A2.3), and therefore all four spring series in the spring system are in compression, so by substituting their compressive stiffness and preload constants into Eqn. (3.66), the height of the ICR can be calculated. If the calculated height of the ICR is also located at height region I, the ICR is valid, but if not, the Matlab function will try the next height region (height region II) by repeating this procedure, until a valid ICR is found. Only one valid ICR exists in each segment of the $M - \theta$ curve.

![Figure A2.3: Example of the height regions for locating a valid ICR in a multi-spring system](image)
The stiffness of each spring series will be determined based on its loading condition. Its loading condition can be found from Table 3.1 and the moving direction can be determined from Figure A2.4.

Rotation limits are the maximum rotations for different stages on the $M - \theta$ curve (Figure 3.7). In this part, a function called “ICR.m” is called to calculate the ICR which is based on the same scheme as that shown in Figure A2.3. Another function called “Distributor.m” is executed to allocate the stiffness, preloaded displacement and displacement limit to each spring series.

In the function “Distributor.m”, according to Table 3.1, the preloaded displacement and displacement limit for the current $F - \Delta$ curve of each spring series will be assigned to the corresponding array. If the spring series is under unloading, its $F - \Delta$ curve will be reshaped (Figure A2.4), and the displacement limit point will be changed to the corresponding zero force point (Figure A2.4). If this unloading spring series reaches its zero force point, in the next iteration, its sign will be changed from tension to compression or the other way around.

![Figure A2.4: The new F-\Delta curve of an unloading spring series](image_url)
The displacement limit is defined as the maximum displacement of the spring series in its current stage. For example, in Figure 3.5, if the spring series is in its 2nd stage \( (j = 2) \), the displacement limit of the spring series is \( \Delta_{p2} \). That means, once the displacement of the spring series reaches \( \Delta_{p2} \), the current stiffness \( (K_p^2) \) and preloading constants \( (C_p^2) \) will change to the next stage \( (j = 3) \) stiffness \( (K_p^3) \) and preloading \( (C_p^3) \) constants.

Therefore, in a multi-spring system (see Figure 3.8), since the displacement of the spring series is related to the rotation of the system, the rotation limit \( (\theta_{\text{limit},i}) \) of the \( i^{th} \) spring series can be obtained by Eqn. (A2.2)).

\[
\theta_{\text{limit},i} = \frac{\Delta_{\text{limit},i} - \Delta_0}{h_i}
\]

(A2.2)

where \( \Delta_{\text{limit}} \) is the displacement limit of a spring series in its current stage, \( \Delta_0 \) is the displacement at the centroid of the connected beam (Figure 3.8) which is calculated by Eqn. (3.64) where the height of the centroid \( (h_i, i = 0) \) is zero, and \( h_i \) is the height of the \( i^{th} \) spring series (Figure 3.8).

According to the rotation limit and attributes of each spring series, the moment at the rotation limit can be calculated based on Eqn. (3.58). For example, assume that a multi-spring system has the \( M - \theta \) curve shown in Figure 3.7. In the 1st stage (1st segment of the \( M - \theta \) curve, \( k = 1 \)), the rotation limits of all the spring series \( (\theta_{\text{limit},1}, \theta_{\text{limit},2}, ..., \theta_{\text{limit},N-1}, \theta_{\text{limit},N}) \) are first calculated by Eqn. (A2.2). The minimum of these rotation limits will be the maximum rotation of the 1st stage \( (\theta_1) \) in Figure 3.7. The corresponding moment \( M_1 \) at \( \theta_1 \) can be calculated from Eqn. (3.58).

After the multi-spring system rotates to \( \theta_1 \), the system will enter the 2nd stage (2nd segment of the \( M - \theta \) curve, \( k = 2 \)) where the stiffness and preloading constants will change from those of the 1st stage \( (K^1 \text{ and } C^1) \) to those of the 2nd stage \( (K^2 \text{ and } C^2) \). Based on \( K^2 \) and \( C^2 \), the new ICR can be determined and the rotation limit of the 2nd stage \( (\theta_2) \) can then be found using the same procedure as in the 1st stage. This process is repeated until the joint
reaches the maximum pre-fracture rotation where one of the spring series reaches its fracture displacement.

After reaching the maximum rotation, the multi-spring system enters the post-fracture range where the non-fracture system changes into a series of new systems in which the stiffness and strength of fractured spring series are reduced or eliminated (Figure A2.5). Based on the new input data, the $M - \theta$ curve of the post-fracture system is obtained, thereby producing the full-range $M - \theta$ curve.

![The input stiffnesses and critical loads of the first spring series are halved after the multi-spring system reaches the maximum rotation where the first spring series reaches the fracture displacement.](image)

**FIGURE A2.5:** CHANGE OF INPUT FILE AFTER MULTI-SPRING SYSTEM ENTERS THE POST-FRACTURE RANGE.

Note that, in Figure A2.5, the properties of the third spring in the first spring series representing the column web in tension does not change, because the force transferred by the third spring is less than its maximum value $P_s^C$. 

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For example, in Figure 3.12, after the multi-spring system reaches the maximum rotation that is the start point of the 1st stage post-fracture system $\theta_{f0}$, the input file changes into the 1st stage post-fracture system where the stiffness and critical loads of the first two components of the first spring series are halved (Figure A2.5). Based on the new input data, the $M - \theta$ curve of the 1st stage post-fracture system before $\theta_{f0}$ (see the green line in Figure 3.12) and $M_{f0}$ are obtained ($M_{f0}$ is the moment of the 1st stage post-fracture system at $\theta_{f0}$). Next, the total energy of the non-fracture system ($V_{f0}(\theta_{f0})$) and the total energy of the 1st stage post-fracture system ($V_{f1}(\theta_{f0})$) at $\theta_{f0}$ are calculated. By substituting $V_{f1}(\theta_{f0})$, $V_{f0}(\theta_{f0})$, $M_{f0}$, and $\theta_{f0}$ into Eqn. (3.69), the maximum rotation of the 1st stage post-fracture system $\theta_{f1}$ is calculated, and then the $M - \theta$ curve of the 1st stage post-fracture system is obtained (see green line in Figure A2.5). Repeating this process means the $M - \theta$ curves of

### A3: Backbone Curves of End-plate Joints

#### A3.1: Summary of the Key Parameters of the Backbone Curves

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<td>( M_y \ (kNm) )</td>
<td>( M_c \ (kNm) )</td>
<td>( M_r \ (kNm) )</td>
<td>( \delta_y \ (rad) )</td>
<td>( \delta_c \ (rad) )</td>
<td>( \delta_r \ (rad) )</td>
</tr>
<tr>
<td>------</td>
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<td>----------------</td>
</tr>
<tr>
<td>Positive Applied Bending Moment</td>
<td>1.72E+02</td>
<td>2.29E+02</td>
<td>7.84E+01</td>
<td>8.32E-03</td>
<td>2.40E-02</td>
<td>3.95E-02</td>
</tr>
<tr>
<td>( K_e \ (\frac{kNm}{rad}) )</td>
<td>( a_s )</td>
<td>( a_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.07E+04</td>
<td>1.76E-01</td>
<td>-4.71E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative Applied Bending Moment</td>
<td>-8.75E+01</td>
<td>-1.12E+02</td>
<td>-5.60E+01</td>
<td>-6.98E-03</td>
<td>-3.37E-02</td>
<td>-3.94E-02</td>
</tr>
<tr>
<td>( K_e \ (\frac{kNm}{rad}) )</td>
<td>( a_s )</td>
<td>( a_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.25E+04</td>
<td>7.31E-02</td>
<td>-7.78E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A3.2: Converting Full-Range Moment-Rotation Curves to Backbone Curves for Seismic Evaluation (Positive Applied Moment)
Note the shape of the $M - \theta$ curve for joint S20_0_0_B does not fit the general backbone curve, since S20_0_0_B has a very ductile post-ultimate range followed by a very steep post-fracture range which cannot be represented by a linear curve. Therefore, if the hysteretic model does not extend to post-fracture range, the post-ultimate range stiffness can be used as the post-capping stiffness of the backbone curve.
**A3.3: Converting Full-Range Moment-Rotation Curves to Backbone Curves for Seismic Evaluation (Negative Applied Moment)**

- **Model**
- **Backbone Curve**

![Graph showing S10_0_0 Negative Applied Moment](image1)

![Graph showing S10_0_0_B Negative Applied Moment](image2)
Appendix

**S20_0_0_B Negative Applied Moment**

\[ \theta \text{(Rads)} \]

\[ M(\text{kNm}) \]

- Model
- Backbone Curve

**FS1 Negative Applied Moment**

\[ \theta \text{(rad)} \]

\[ M(\text{kNm}) \]

- Model
- Backbone Curve
Appendix

A3.4: Backbone Curves

![Backbone Curves Graph](image)

![Backbone Curves Graph](image)
Appendix

### S20_0_0_B

![Graph of S20_0_0_B](image1)

- **M (kNm)**
- **θ (Rads)**

### FS1

![Graph of FS1](image2)

- **M (kNm)**
- **θ (rad)**
Appendix

A4: Static Curve and Original Curve

A4.1: Moment-Rotation Curves

![Graph of Static and Original Curves for S10_0_0 and S10_0_0_B](image)

- **S10_0_0**: Moment-rotation curve for S10_0_0 showing dynamic and static behavior.
- **S10_0_0_B**: Moment-rotation curve for S10_0_0_B showing dynamic and static behavior.

Static and Original curves are compared across a range of moment (M) values and rotation (θ) in radians.
Appendix

\[ M \text{ (kNm)} \]

\[ \theta \text{ (rads)} \]

**S20_0_0_B**

- **Static**
- **Original**

**S10_19_0**

- **Static**
- **Original**
Appendix

![Graph S10_34_0](image1)

![Graph S20_19_0_B](image2)
A4.2: Component Behaviour Curves

S10_0_0 End-Plate Bending

S10_0_0 Column Web Shortening
Appendix

S10_0_0_B End-Plate Bending

S10_0_0_B Column Web Shortening
Appendix

S20_0_0_B Column Web Shortening

S20_0_0_B End-plate Bending

Static

Original
Appendix

S10_19_0 End-Plate Bending

Static
Original

S10_19_0 Column Web Shortening

Static
Original

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A5: Converting Multi-Linear Curve to Bi-Linear Curve
Appendix

S10_0_0 Inner End-Plate Bending

F (kN) vs Δ (mm)

S10_0_0 Column Flange Bending

F (kN) vs Δ (mm)
Appendix

S10_0_0, S10_0_0_B and S20_0_0_B
Column Web in Compression

F (kN)

Δ (mm)
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