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Contrasts of Relative Advantage Maximisation with Random Utility Maximisation and Regret Minimisation

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Abstract

This paper discusses the theoretical properties and the empirical application of an improved version of the 'relative advantage maximising' (RAM) model. This model shares several desirable features of a set of models based on random regret minimisation (RRM), such as parsimony and choice set dependence. Although model fit differences are small, a preliminary comparison shows that the RAM model empirically outperforms the standard random utility maximisation (RUM) model, the RRM model, and a hybrid RUM–RRM model in all eight data sets analysed. The paper concludes with a discussion of the marginal willingness to pay (WTP) measures derived from the RAM model.

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1.0 Introduction

Ever since discrete choice modelling became popular in the 1970s, the majority of applications have used a context-independent specification to represent the utility of an alternative. By context independence, it is usually assumed that the overall utility of an alternative is determined by the level of its attributes multiplied by a weighting parameter which accounts for the marginal (dis)utility input of that attribute. The modelled component of utility is then described by the sum of these individual part-utilities. For reasons of convenience and analytical tractability, the context-independent, linear additive form remains popular even today. In this standard random utility maximisation (RUM) specification, the utility of an alternative is assumed to be invariant to the presence or absence of other alternatives, whether in the choice set or in the real world.

This is not to say that other representations of utility have not been considered in the literature (see Leong and Hensher (2012a) for a review). More recently, a random regret minimisation (RRM) approach, which is essentially a context-dependent utility specification, has been advocated as a behavioural alternative to the context-independent RUM model (Chorus *et al.*, 2008; Chorus, 2010, 2012). In the RRM environment, rather than assuming that people choose the alternative that provides the highest pay-off in terms of the part-utilities described earlier, the main hypothesis is that people seek to minimise the negative emotions associated with decision making. Quite plausibly, opting for any chosen alternative creates a negative emotion or feeling of loss when the attributes of that chosen alternative fare worse than those of the non-chosen alternatives. Hence, the preference for an alternative in random regret minimisation explicitly makes references to the attribute values of the competing alternatives in the choice set.

Another fairly robust empirical finding to emerge from the psychology and marketing literature is that of a heuristic known as extremeness aversion (Simonson and Tversky, 1992; Tversky and Simonson, 1993). If an extreme alternative is defined as one with both the best value on some subset of attributes, and the worst value on other attributes, then a specific form of extremeness aversion known as the compromise effect is said to occur when the inclusion of an extreme alternative in the choice set causes the pair-wise choice share of the compromise or the in-between alternative to go up relative to the other extreme alternative.[1] It is normally supposed that a 'betweenness inequality' holds in decision making, in which the middle alternative loses relatively more than an existing extreme alternative when another extreme alternative is introduced (Tversky and Simonson, 1993). Hence, the compromise effect can be seen as a violation of the betweenness inequality and its existence is generally attributed to a consequence of loss-aversive behaviour (Kivetz *et al.*, 2004).

Models that account for the compromise effect have been suggested in the literature (Tverksy and Simonson, 1993; Kivetz *et al.*, 2004; Chorus, 2010; Rooderkerk *et al.*, 2011). In this paper, we examine the performance of another model which, like the RRM model, also explains the compromise effect; one that Tversky and Simonson (1993) first called a 'componential context model'. Their model representation was later

[1]This notion of 'extremeness' might be distinguished from 'dominating/dominated' alternatives, in which all attributes of an alternative are better/worse than a competing alternative.

rechristened as a ‘relative advantage model’ by Kivetz *et al.* (2004). In the context of discrete choice modelling, we prefer to term this model as a ‘relative advantage maximising’ or RAM model, as it will become readily apparent in the description of the model that the relative advantage of an alternative enters into its utility function as a context-dependent component to be maximised.

The paper is organised as follows. Section 2 discusses the RRM model and the RAM model. Section 3 discusses the empirical results and some policy implications of this work. Section 4 provides the conclusions.

2.0 Models of Context Dependence

2.1 The Random Regret Minimisation (RRM) model

Regret-based theories and models are built on the premise that people aim to minimise anticipated regret when making a choice. The RRM model was primarily developed to analyse riskless choice involving multi-attribute alternatives and, in this context, regret is said to occur when the attributes of a non-chosen alternative perform better than the attributes of the chosen alternative (Chorus, 2012).

To describe the Chorus (2010) version of the RRM model, first define, in equation (1), the binary or pair-wise regret associated with considering alternative j as opposed to alternative j':

$$\mathrm{reg}(j, j') = \sum_k \ln[1 + \exp(\beta_{j'k} X_{j'k} - \beta_{jk} X_{jk})]. \tag{1}$$

As a matter of notation, X_{jk} refers to the value of attribute k in alternative j and β_{jk} is its corresponding taste parameter. The total regret associated with alternative j is the sum of binary regrets over all alternatives j' in choice set S; that is, equation (2):

$$\mathrm{reg}(j) = \sum_{\substack{j' \in S \\ j' \neq j}} \mathrm{reg}(j, j'). \tag{2}$$

The RRM model can be estimated by observing that minimising the regret function is equivalent to maximising the negative of regret.

In the RRM model, preferences for each alternative depend not only on the attribute values for that alternative, but also on the relative performance of these attributes against their counterpart attribute levels in all the other alternatives in the choice set. In other words, preferences for an alternative are context dependent or choice-set specific. The RRM allows preferences to change even if an alternative’s attribute levels remain constant from choice set to choice set, as long as there is a change in the attribute levels of any of the other alternatives to which an alternative is being compared.

Arising from the convexity property of the regret function, it has been observed that the RRM model is able to capture semi-compensatory behaviour[2] as well (Chorus,

[2]Semi-compensatory behaviour occurs when a disproportionately large improvement in an attribute is required to offset a given decline in the performance of another attribute. If the decline in the performance of the latter attribute is large enough, no amount of improvement in the first attribute will compensate sufficiently.

2010). The extent to which an improvement in an attribute can compensate for the deterioration in the value of another attribute depends very much on its value relative to the other alternatives. For example, an improvement in an attribute that is already far superior to its counterparts in the other alternatives leads to a minimal reduction in regret, while a worsening of another attribute that was comparing poorly to begin with can lead to a substantial increase in regret. This property also allows the RRM model to explain the compromise effect. As defined earlier, since extreme alternatives contain both best- and worst-performing attributes in the choice set, the higher amount of regret engendered by their worst-performing attributes, described in equation (1), is not fully offset by their best-performing attributes, and this can lead to the extreme alternative being relatively less preferred and the compromise alternative being relatively more preferred.

Parsimony is another frequently cited advantage of the RRM model (Chorus, 2010). The RRM model captures context dependence without the need to estimate additional parameters beyond what is needed for the linear additive multinomial logit (MNL) RUM model. Despite these desirable properties, empirical work has shown that the RRM model delivers a somewhat mixed performance in terms of improving the goodness of fit relative to the standard RUM models, especially for stated preference data (Chorus, 2012).

One attempt to refine the RRM model is to introduce heterogeneity in decision making along the dimension of how attributes are processed. For example, in a hybrid RRM/linear additive RUM model, respondents are assumed to process a subset of attributes according to RRM, and the remaining attributes of the alternatives according to a linear additive processing rule (Chorus *et al*., 2013). If it is assumed that attributes $1, \ldots, m$ of alternative j are processed according to linear additive RUM and attributes $m+1, \ldots, K$ are processed according to RRM, then the observed component of utility can be described by equation (3):

$$V_j = \beta_{0,j} + \sum_{k=1,\ldots,m} \beta_{jk} X_{jk} - \sum_{\substack{j' \in S \\ j' \neq j}} \sum_{k=m+1,\ldots,K} \ln[1 + \exp(\beta_{j'k} X_{j'k} - \beta_{jk} X_{jk})]. \tag{3}$$

As there is no theory to determine *a priori* which attributes are RUM-processed and which are RRM-processed, one approach that can be adopted is to search over all possible RUM/RRM combinations of attributes to find the one combination that results in the best model fit.

2.2 The Relative Advantage Maximisation (RAM) model

2.2.1 Model specification

Like the RRM model, the RAM model assumes that each alternative is assessed against all other alternatives in the choice set. However, one key difference between RAM and RRM is that the RAM model explicitly considers the disadvantages and advantages of an alternative, with the advantages of an alternative expressed as a ratio to the sum of advantage and disadvantage.

The RAM model described and estimated in this paper is conceptually similar to the model discussed by Kivetz *et al*. (2004). The main differences are the functional form used to represent the advantage and disadvantage function, as well as the imposition of an additional assumption of symmetry between advantage and disadvantage.

Before describing the RAM model itself, it will be useful first to define the disadvantage of j over j' for an attribute k, denoted by $D_k(j, j')$. We depart from the piecewise functions used by Kivetz *et al.* (2004), and suggest that an improvement over the original RAM model might be to note that regret and disadvantage are practically synonymous, and so the smoothed regret function proposed in Chorus (2010) may be used to represent $D_k(j, j')$, such as in equation (4):

$$D_k(j, j') = \ln[1 + \exp(\beta_{j'k} X_{j'k} - \beta_{jk} X_{jk})]. \tag{4}$$

How the advantage variable $A_k(j, j')$ is to be specified is derived from various assumptions about the model. In the simplest case, if symmetry between advantage and disadvantage is assumed — that is, if the advantage of alternative j over alternative j' with respect to an attribute k is the corresponding disadvantage of j' over j with respect to the same attribute — then equation (5) follows:

$$A_k(j, j') = D_k(j', j) = \ln[1 + \exp(\beta_{jk} X_{jk} - \beta_{j'k} X_{j'k})]. \tag{5}$$

This assumption of symmetry in the advantage and disadvantage functions allows a parsimonious representation for the RAM model and will be used for the remainder of this paper.

Having now defined $A_k(j, j')$ and $D_k(j, j')$, we proceed to define $A(j, j')$, the binary advantage of alternative j over alternative j', and $D(j, j')$, the binary disadvantage of j over j', according to equation (6):

$$A(j, j') = \sum_k A_k(j, j') \quad \text{and} \quad D(j, j') = \sum_k D_k(j, j'). \tag{6}$$

The relative advantage of alternative j over alternative j', denoted by $RA(j, j')$, may now be defined in equation (7) as:

$$RA(j, j') = \frac{A(j, j')}{A(j, j') + D(j, j')}. \tag{7}$$

By this definition, $RA(j, j')$ only takes values in the open interval between zero and one.

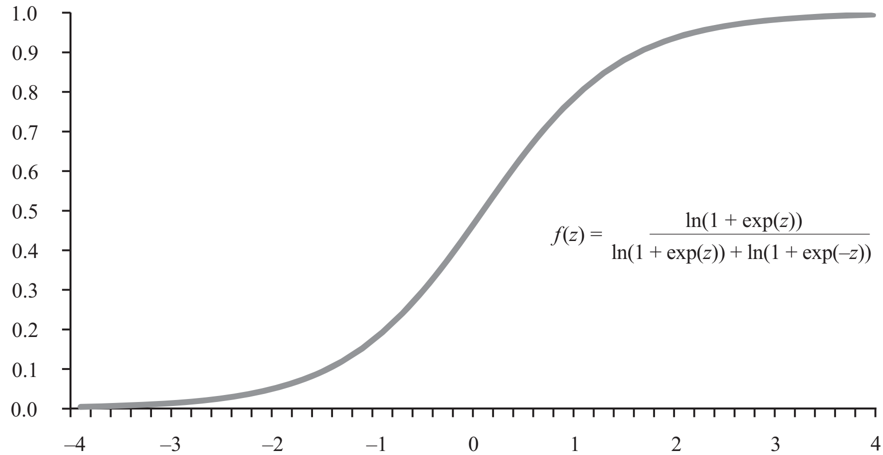
A graph of the function

$$f(z) = \frac{\ln(1 + \exp(z))}{\ln(1 + \exp(z)) + \ln(1 + \exp(-z))}$$

is plotted in Figure 1 as a means of visualising $RA(j, j')$ in the case of a single attribute. Where the alternatives contain multiple attributes, changing the value of one attribute holding all else constant, is akin to adding a constant to both the numerator and the denominator, and this does not fundamentally alter the shape of the function.

It is immediately obvious from Figure 1 that $RA(j, j')$ follows an S-shaped curve reminiscent of the value function from prospect theory (Kahneman and Tversky, 1979), and that it captures the notion of concavity in gains and convexity in the losses. This is a rather nice result which suggests that the use of the symmetry assumption $A_k(j, j') = D_k(j', j)$ is not entirely inappropriate. The difficulties in model estimation experienced by Kivetz *et al.* (2004) may have been related to their use of a kinked relative advantage function, and this issue is avoided as the improved $RA(j, j')$ is now a smooth function.

Figure 1
Graph of the $R(j, j')$ Function



The observed component of utility for alternative j is then written as a linear combination of the linear additive RUM component and the relative advantage component, as shown in equation (8):

$$V_j^{RAM} = \beta_{0,j} + \sum_k \beta_{jk} X_{jk} + \sum_{\substack{j' \in S \\ j' \neq j}} RA(j, j'). \tag{8}$$

Unlike the standard RRM model (for example, Chorus, 2010), the RAM model allows for a combination of context-independent ('inherent') preferences and context-dependent preferences, which is consistent with Kivetz *et al.*'s (2008) hypothesis that preferences may not be entirely context dependent. The RAM model acknowledges that preferences are to a certain extent shaped by the choice context, but also allows each person a set of context-free, innate preferences which are brought to bear on each choice situation.

For this paper, it will be assumed that all attributes which appear in the context-independent RUM component will also be included in the relative advantage component of V_j^{RAM}. It might also be observed that both the context-independent component and the relative advantage component are for the moment given equal weights in V_j^{RAM}. In future extensions of the RAM model, the weight of the relative advantage component of utility can be conditioned by means of certain socio-economic characteristics or even choice-set characteristics, using a multiple heuristics approach in the utility function (Leong and Hensher, 2012b).

Following standard practice, the total utility for an alternative is the sum of the observed component and an unobserved error component ε_j, or $U_j = V_j + \varepsilon_j$. We assume that ε_j is i.i.d. EV type I distributed, so that all models estimated are of the MNL form.[3] Extensions of the model to account for random parameters will be left for future work.

[3]Note, however, that the RAM model does not display the IIA property because of the embedded context-dependent effects.

3.0 Empirical Application

3.1 Description of the data sets

Eight stated choice (SC) data sets are used in the empirical application. The first seven data sets (DS 1–DS 7) are from five Australian and two New Zealand toll road studies that were conducted between 1999 and 2008. The choice experiments involved each sampled respondent answering sixteen choice scenario questions. In each choice question, the respondent was required to make a choice among three alternatives, one of which was described by a recent trip, and the other two were unlabelled alternatives defined by attribute levels pivoted off of the recent (or reference) trip profile. DS 8 involved mode choice in a context of travelling within the Central Business District in which a proposed Metro system is offered as one of the alternatives. Each of the six choice questions put to the respondents in this data set were described by labelled alternatives, *viz*. bus, rail, proposed metro, walk, and taxi. All eight surveys were conducted as computer-aided personal interviews (CAPI).

We briefly describe each data set below. Further details of the toll-road data sets are provided in Hensher *et al*. (2012). The metro data set is extensively described in Hensher *et al*. (2011).

3.1.1 Data set 1

Undertaken in 2008, this study uses a D-efficient experimental design structured to increase the statistical performance of models with relatively smaller samples than are required for other less-efficient (statistically) designs such as orthogonal designs (see, for example, Rose *et al*., 2008). In total, 280 car commuters (with less than 120 minutes' trip length) were sampled.

The three alternatives shown in each choice set were described in terms of free flow time, slowed down time, stop/start/crawling time, running cost, toll cost, and travel time variability (see Figure 2). For all attributes except the toll cost, the values for the SC alternatives are variations around the values for the current trip.

3.1.2 Data sets 2–7

DS 2 (Australia, 2000), DS 3 (New Zealand, 1999), DS 4 (Australia, 2005), DS 5 (Australia, 2004), DS 6 (Australia, 2004), and DS 7 (New Zealand, 2007) used a survey similar to that shown in Figure 3. An orthogonal design was used in DS 2 and DS 3, and a D-efficient design was used for DS 4–DS 7. All studies allowed the disaggregation of trip cost into the running cost and the toll cost. In terms of travel time, respondents in DS 3 and DS 4 were given three time components, which were free flow time, slowed down time, and stop/start/crawling time; while respondents in DS 2, DS 5, DS 6, and DS 7 were given two time components, which were free flow time and congestion time.

In all the seven toll-road data sets, we exclude trip time variability in all model estimation, given previous evidence using these data sets that, with the exception of DS 1, the variability attribute was poorly specified and often not statistically significant. Hence in DS 1, DS 3, and DS 4, the attributes of the alternatives that are modelled in the utility function are: free flow time (FF), slowed down time (SDT), stop/start/crawling time (SST), running cost (RC), and toll cost (TC). In DS 2, DS 5, DS 6, and DS 7, the attributes modelled in the utility function are: free flow time (FF), congestion time (CT), running cost (RC), and toll cost (TC).

Figure 2
Illustrative Screenshot of Choice Experiment Used in Study 1

Game 5

Illustrative Choice Experiment Screen

Make your choice given the route features presented in this table, thank you.

	Details of your recent trip	Route A	Route B
Average travel time experienced			
Time in free flow traffic (minutes)	20	14	12
Time slowed down by other traffic (minutes)	20	18	20
Time in stop/start/crawling traffic (minutes)	20	26	20
Probability of time of arrival			
Arriving 9 minutes earlier than expected	30%	30%	10%
Arriving at the time expected	30%	50%	50%
Arriving 6 minutes later than expected	40%	20%	40%
Trip costs			
Running costs	$2.25	$3.26	$1.91
Toll costs	$2.00	$2.40	$4.20
If you make the same trip again, which route would you choose?	Current Road	Route A	Route B
If you could only choose between the two new routes, which route would you choose?		Route A	Route B

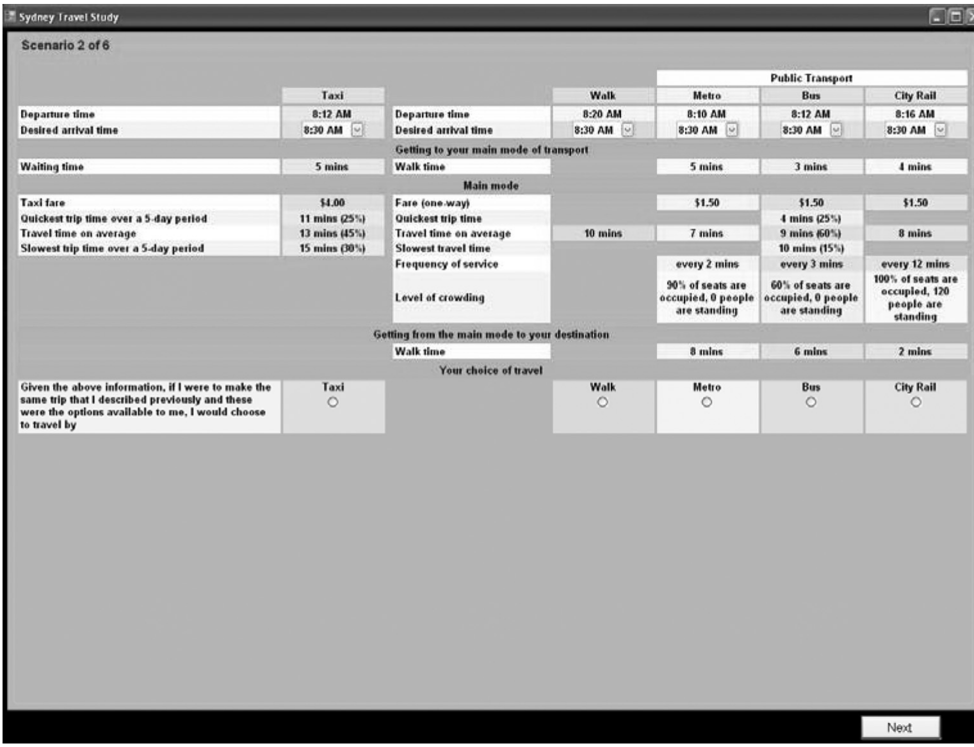
Figure 3
Illustrative Screenshot of Choice Experiment Used in Studies 2–7

Practice Game

Make your choice given the route features presented in this table, thank you.

	Details of your recent trip	Route A	Route B
Time in free flow traffic (minutes)	15	21	12
Time slowed down by other traffic (minutes)	10	10	8
Time in stop/start/crawling traffic (minutes)	2	2	3
Trip time variability (minutes)	+/- 8	+/- 9	+/- 8
Taxi fare	$30.70	$27.63	$18.42
Toll costs	$4.00	$0.00	$0.70
If you make the same trip again, which route would you choose?	Current Road	Route A	Route B
If you could only choose between the two new routes, which route would you choose?		Route A	Route B

Figure 4
Illustrative Screenshot of Choice Experiment Used in Study 8



The sample size for car commuters ranges from 57 in DS 5 to 280 in DS 1. Given that the sampled New Zealand's car commuters had no tolling experience before they were interviewed, self-reported toll costs are only available for Australian studies.

3.1.3 Data set 8

DS 8 provides some variation from the choice contexts of the previous data sets. This mode choice study used a Bayesian D-efficient design (see, for example, Bliemer *et al*., 2008), with screenshots illustrated in Figure 4. Among other attributes, the five labelled alternatives were described in terms of access and egress modes (where applicable), one-way cost, and attributes that attempt to capture travel time variability. As with the toll-road studies, respondents were asked to provide information, either real or perceived, related to the levels of the relevant alternatives for a recent trip that they undertook. The SC experiment then 'pivots' the attribute levels of the various alternatives around this recent trip profile. The sample size for this data set is 269. For this data set, the attributes modelled in the utility function are fare and average travel time.

3.2 Empirical results

As a result of the unlabelled nature of the choice experiments in each of the toll-road data sets, we modelled taste parameters as being generic across all alternatives. For convenience,

Table 1
Summary of Overall Goodness of Fit of all Models

	DS 1 Australia 2008	*DS 2 Australia 2000*	*DS 3 NZ 1999*	*DS 4 Australia 2005*	*DS 5 Australia 2004*	*DS 6 Australia 2004*	*DS 7 NZ 2007*	*DS 8 Australia 2009*
				Log-likelihood				
Linear additive RUM	−3434.58	−1862.23	−1694.93	−2670.14	−847.75	−3031.58	−1631.79	−2278.73
RRM	−3439.32	−1951.54	−1691.55	−2683.70	−847.55	−3044.06	−1639.22	−2285.18
Hybrid RRM–RUM	−3434.79	−1872.09	−1690.49	−2670.20	−846.53	−3029.70	−1631.14	−2277.98
Symmetric RAM	−3433.77	−1854.09	−1688.83	−2664.50	−847.29	−3027.75	−1630.32	−2276.73

the cost and time parameters in DS 8 were also modelled generically. All models were estimated with Nlogit 5.0.

As reported in previous studies (see, for example, Chorus, 2012), we find that the full RRM model, compared to the standard linear additive RUM, offers a very small improvement in model fit in some, but not all, of the data sets considered (see Table 1 for a summary of model fit across all model types). Out of the eight data sets studied, the RRM model only fits DS 3 and DS 5 slightly better. We also report the results of the best performing hybrid RRM–RUM model, where some attributes are processed according to regret minimisation and the rest according to linear additive RUM. The empirical evidence for the hybrid model is also somewhat mixed. In DS 3, DS 5, DS 6, and DS 7, some marginal improvement over the RUM model in the log likelihood ratios can be observed, while in the other data sets, the hybrid model performs slightly worse than the RUM model. In all data sets, however, the hybrid model is a better performer compared to the full RRM model.

Turning to the symmetric RAM model, though, we find some improvement in model fit (albeit small) compared to the standard RUM, the RRM, and the hybrid RRM–RUM models, in almost all of the data sets studied, with the single exception that the hybrid model outperforms the RAM model in DS 5.[4] This improvement can be somewhat larger, as seen in DS 2, DS 3, and DS 4, even if it is modest in some other cases. Overall, this initial finding from a small sample of data sets is noteworthy considering that all models contain the same number of estimated parameters. Besides the seven unlabelled toll-road studies with three alternatives per choice task, it is also worth mentioning that even when the choice context is varied (DS 8), the RAM model remains the best performer among all the models compared.

Our results stand in contrast to the conclusion obtained by Kivetz *et al*. (2004). They found that the RAM model was a consistently poorer performer on a series of consumer choice data sets. As alluded to in Section 2.2, Kivetz *et al*. (2004) had to circumvent the difficulties of estimating a highly non-linear disadvantage function by imposing *a priori* restrictions on certain parameters in the disadvantage function and then employing a grid search to find an optimal estimate for other key parameters. With the suggested improvement in the specification of the functional form of the RAM model, and with advances in software, such arbitrary restrictions and grid searches are no longer required.

[4]Results for the symmetric version of the RAM model, using piecewise functions as advocated by Kivetz *et al.* (2004), are available from the corresponding author upon request.

We also find that the estimation of the model was not particularly sensitive to the choice of starting values, despite the highly non-linear nature of the utility function. While the actual run times are highly dependent on the choice of starting values, in our experience, none of the run times exceeded three minutes.

An examination of the parameter estimates (see Tables A.1 to A.8 in Appendix 1) shows that the symmetric RAM model is able to replicate the results of the RUM model, where the latter model produces parameter estimates that are of the expected sign and are statistically significant at the 5 per cent level. Moreover, the *z*-ratios of the estimates in the RAM model are comparable to the RUM, the RRM, and the hybrid models. The only instance where the RRM model might be said to be preferable to the RAM model occurs in DS 2, where the RC parameter weight is statistically significant only in the RRM model, and not in any of the other three models. However, in this case, the RRM model displays a large deterioration in the LL statistic, which raises concerns about the predictive validity of the RRM model.

3.3 Marginal willingness to pay measures (value of travel-time savings)

3.3.1 Linear additive RUM model

The value of travel-time savings (VTTS) in \$/person-hour or the marginal willingness to pay for a one unit reduction in travel time is given in equation (9) by:[5]

$$\mathrm{VTT}S_j = 60 \times \frac{\partial V_j/\partial(\mathrm{time})}{\partial V_j/\partial(\mathrm{cost})}. \tag{9}$$

Equation (9) is the ratio of the marginal utility with respect to time to the marginal utility with respect to cost. In the toll-road data sets, since there are generally two cost components modelled in the utility function, $\partial V_j/\partial(\mathrm{cost})$ can be expressed as a weighted average of $\partial V_j/\partial RC_j$ and $\partial V_j/\partial TC_j$, as shown in equation (10):

$$\frac{\partial V_j}{\partial(\mathrm{cost})} = \frac{RC_j}{RC_j + TC_j} \times \frac{\partial V_j}{\partial RC_j} + \frac{TC_j}{RC_j + TC_j} \times \frac{\partial V_j}{\partial TC_j}. \tag{10}$$

For the RUM model, $(\partial V_j/\partial RC_j) = \beta_{RC}$ and $(\partial V_j/\partial TC_j) = \beta_{TC}$.

In DS 2, DS 5, DS 6, and DS 7, where the only time components modelled are *FF* and *CT*, the VTTS measure is obtained as a weighted average of $\partial V_j/\partial(FF_j)/\partial V_j/\partial(\mathrm{cost})$ and $\partial V_j/\partial(CT_j)/\partial V_j/\partial(\mathrm{cost})$, as in equation (11):

$$\mathrm{VTTS}_j = 60 \times \left[\frac{FF_j}{FF_j + CT_j} \times \frac{\partial V_j/\partial(FF_j)}{\partial V_j/\partial(\mathrm{cost})} + \frac{CT_j}{FF_j + CT_j} \times \frac{\partial V_j/\partial(CT_j)}{\partial V_j/\partial(\mathrm{cost})}\right]. \tag{11}$$

In DS 1, DS 3, and DS 4, where the *FF*, *SDT*, and *SST* time attributes are modelled in the utility function, the VTTS expression is simply an extension of equation (11) — that is, equation (12):

$$\mathrm{VTTS}_j = 60 \times \left[\frac{FF_j}{TT_j} \times \frac{\partial V_j/\partial(FF_j)}{\partial V_j/\partial(\mathrm{cost})} + \frac{SDT_j}{TT_j} \times \frac{\partial V_j/\partial(SDT_j)}{\partial V_j/\partial(\mathrm{cost})} + \frac{SST_j}{TT_j} \times \frac{\partial V_j/\partial(SST_j)}{\partial V_j/\partial(\mathrm{cost})}\right], \tag{12}$$
$$TT_j = FF_j + SDT_j + SST_j.$$

[5]A multiplication by sixty is appropriate, since the time attributes were presented in minutes.

3.3.2 The RRM model and the hybrid RRM–RUM model

As regret minimisation is equivalent to maximising the negative of regret, the choice problem in the full RRM model may be stated as choosing an alternative j to maximise $U_j = V_j + \varepsilon_j$, where $V_j = -\text{reg}(j)$ and ε_j is i.i.d. EV type I.

Where an attribute is processed according to random regret minimisation, the marginal utility $\partial V_j / \partial X_{jk}$ with respect to X_{jk} will be specific to the alternative and also specific to the choice set, as the attribute value of the alternative and its counterpart values in all competitor alternatives enter into this expression. Therefore, equation (13) follows:

$$\frac{\partial V_j}{\partial X_{jk}} = \sum_{\substack{j' \in S \\ j' \neq j}} \frac{\beta_k}{1 + \exp[\beta_k (X_{jk} - X_{j'k})]}. \tag{13}$$

By appropriate substitution of $\partial V / \partial X_{jk}$ in equation (13) into equations (10) to (12), the VTTS expressions in the case of the full RRM follow analogously.

In the hybrid RRM–RUM model, the expressions for the partial derivatives of V_j with respect to X_{jk} will either follow equation (13) if the attribute is RRM-processed or will simply be β_k if the attribute is RUM-processed. Again, by the appropriate substitution, the VTTS expressions can be derived from equations (10) to (12).

3.3.3 The RAM model

Recall the observed component of utility in the RAM model, replicated in equation (14):

$$V_j^{RAM} = \beta_{0,j} + \sum_k \beta_k X_{jk} + \sum_{\substack{j' \in S \\ j' \neq j}} R(j, j'). \tag{14}$$

As a result of the advantage function $A(j, j') = \sum_k A_k(j, j')$ and the disadvantage function $D(j, j') = \sum_k D_k(j, j')$ appearing in $R(j, j')$, the partial derivatives of $R(j, j')$ and V_j^{RAM} with respect to X_{jk} will be a function of all attributes of all alternatives, and not simply a function of attribute k alone. First, equation (15) follows directly from the definitions of $A(j, j')$ and $D(j, j')$:

$$\frac{\partial A(j, j')}{\partial X_{jk}} = \frac{\beta_k}{1 + \exp[-\beta_k (X_{jk} - X_{j'k})]} \quad \text{and} \quad \frac{\partial D(j, j')}{\partial X_{jk}} = \frac{-\beta_k}{1 + \exp[\beta_k (X_{jk} - X_{j'k})]}. \tag{15}$$

Equation (16) follows:

$$\begin{aligned} \frac{\partial V_j^{RAM}}{\partial X_{jk}} &= \beta_k + \sum_{\substack{j' \in S \\ j' \neq j}} \frac{\partial R(j, j')}{\partial X_{jk}} \\ &= \beta_k + \sum_{\substack{j' \in S \\ j' \neq j}} \frac{D(j, j') \dfrac{\partial A(j, j')}{\partial X_{jk}} - A(j, j') \dfrac{\partial D(j, j')}{\partial X_{jk}}}{[A(j, j') + D(j, j')]^2}. \end{aligned} \tag{16}$$

Again, by appropriate substitutions into equations (10) to (12), the VTTS for the RAM model may be obtained.

3.3.4 Estimates and discussion

As a result of context dependency, each alternative in each choice set will have a unique VTTS, but following Chorus *et al.* (2013) and restricting the VTTS calculations only to those alternatives actually chosen by the respondents, the VTTS distribution for each model type for the toll-road data sets was simulated using the Krinsky and Robb (1986) procedure with 5,000 replications. A summary of the VTTS estimates is presented in Table 2. These estimates are intended as a comparison across the four model types, rather than a representation of respondents' actual VTTS values, since hypothetical attribute values generated in the experimental design are inserted into the VTTS equations for the RRM, hybrid RRM–RUM, and RAM models.

Between the symmetric RAM model and the linear additive RUM model, there is nothing much to suggest that the mean VTTS estimates are systematically higher or lower in one model compared with the other. However, a more appropriate comparison might be made between the RAM and RRM VTTS estimates, since these models are most similar to each other in terms of requiring the attribute values of all competitor alternatives to enter into the VTTS expression of the alternative being considered. In this regard, there is again nothing much (at least in the toll-road data sets) to suggest that the RAM model is producing consistently higher or lower mean VTTS estimates than the RRM model, although the much lower estimate from the RRM in DS 2 might be treated with more caution in light of the significantly poorer model fit of the RRM and the wider 95 per cent confidence interval of the estimates. The *z*-ratios (mean/standard

Table 2
Summary of VTTS Measures Across Model Types

	DS 1 Australia 2008	*DS 2 Australia 2000*	*DS 3 NZ 1999*	*DS 4 Australia 2005*	*DS 5 Australia 2004*	*DS 6 Australia 2004*	*DS 7 NZ 2007*
Linear additive RUM							
Mean ($/person-hour)	12.26	13.25	9.15	12.85	16.23	14.51	12.92
z-ratio	8.64	36.85	4.27	10.59	7.61	20.91	11.51
95% confidence interval	9.48–15.04	12.55–13.96	4.95–13.35	10.48–15.23	12.05–20.40	13.15–15.87	10.72–15.11
RRM							
Mean ($/person-hour)	13.04	8.67	10.02	11.41	16.34	14.74	13.50
z-ratio	8.33	2.36	3.16	10.87	7.54	20.21	10.39
95% confidence interval	9.97–16.11	1.46–15.88	3.81–16.24	9.35–13.47	12.09–20.58	13.31–16.17	10.95–16.05
Hybrid RRM–RUM							
Mean ($/person-hour)	12.94	11.53	8.86	11.76	15.57	15.80	14.59
z-ratio	8.47	32.83	3.55	10.77	7.28	22.94	11.02
95% confidence interval	9.95–15.94	10.85–12.22	3.96–13.75	9.62–13.90	11.38–19.76	14.45–17.14	12.00–17.18
Symmetric RAM							
Mean ($/person-hour)	12.17	13.51	8.79	12.87	16.18	14.52	12.86
z-ratio	10.73	35.22	6.67	11.90	8.38	22.42	12.70
95% confidence interval	9.95–14.39	12.76–14.27	6.21–11.37	10.75–14.99	12.39–19.97	13.25–15.79	10.88–14.85

Note: VTTS estimates presented in the currency of the country where the experiment was conducted.

deviation) of the RAM VTTS estimates are comparable to the RUM and hybrid models, and in all cases are higher than their counterpart RRM z-ratios, suggesting that the RAM model offers a good degree of precision in estimating VTTS.

In terms of comparisons across different heuristics, tests of differences between pairs of models RUM–RRM, RUM–Hybrid, RUM–RAM, RRM–Hybrid, RRM–RAM and Hybrid–RAM show that the differences in mean VTTS between models is small and statistically insignificant, with the exception of the Hybrid–RUM and Hybrid–RAM model pair in DS 2. However, in practice, only mean VTTS values are generally used, and the differences in means do reveal substantial time-benefit differences when applied to specific projects. For example, in DS 4, the hybrid model predicts a mean VTTS of \$11.76, while the RAM predicts a mean VTTS of \$12.87.

Since the attribute values of all available alternatives enter the RAM-based VTTS equation, the RAM VTTS measures will generally change when the attributes of alternatives that compete with a considered alternative change. This is fully in line with the notion that the RAM model, together with the RRM and hybrid models, implies choice set-specific preferences. We could not agree more with the argument advocated by Chorus *et al.* (2013) that this allows for a richer interpretation of the implied trade-offs that are made as choice set composition is varied.

Allowing preferences to be choice set-specific may seemingly imply that models like the RAM, RRM, and hybrid RRM–RUM are less suitable for the derivation of VTTS measures. After all, the VTTS measures are now a function of hypothetical attribute values, which are a function of the experiment design. For the policy analyst, the task of deciding an appropriate VTTS estimate appears to be even more challenging than before. However, a careful assessment of the VTTS equation will reveal that the range of policy options may actually be expanded under the assumption of choice set-specific preferences. The policy maker can influence VTTS quite substantially simply by framing and appropriately defining alternatives and choice sets in the public eye. For example, to increase VTTS and perhaps the chance of a transport project being approved, the policy maker could paint an alternative scenario with very poor attribute values (if nothing is done), so that the transport project, in comparison, will appear to have a very large relative advantage compared with the status quo or no-improvement alternative.

4.0 Conclusion

This paper has introduced and discussed an improved version of the RAM model. In light of relatively unsuccessful attempts at estimating a RAM model in the past (Kivetz *et al.*, 2004), the major innovation of this paper has been to suggest an easily estimable form of the RAM model, based on the smoothed regret function of the RRM model. The proposed symmetric RAM model is just as parsimonious as the RUM, RRM, and hybrid RRM–RUM models. A preliminary comparison of the RAM model with these other models reveals that even though model fit differences are small, there is a lot of potential in the RAM model in terms of providing a better fit for the data and in obtaining more precise model outputs, such as willingness to pay measures. More fundamentally, the results indicate a need seriously to consider and incorporate context-dependent effects into a literature that has hitherto mainly relied on context-independent models.

It is very early days yet for the RAM model and a number of very fruitful areas of research can be pursued. The analysis in this paper is based on a small sample of data sets with some, but not much, variation in choice context. While the results are highly suggestive, further testing of the symmetric RAM model in other data sets — for example, in revealed preference data where attribute values across alternatives could be quite similar to one another — would be necessary. Allowing for heterogeneous weights on the context-independent RUM component and the context-dependent relative advantage component is another potential avenue of inquiry.

We have also assumed that all attributes attended to in the context-independent RUM component of the model are also attended to in the relative advantage component. This assumption can also be tested in future work by allowing some subset of attributes to appear in either one component only. Where data on attribute processing are available, such as whether a respondent reported ignoring an attribute (see Hensher (2010)) for details of such models), the link between attribute processing and the relative advantage can also be explored.

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Appendix 1

In the results for the hybrid RRM–RUM model, (U) refers to attribute processing by the linear additive RUM rule and (R) refers to attribute processing by the RRM rule.

In all tables in this Appendix, ** denotes significance at the 5 per cent level and *** denotes significance at the 1 per cent level.

Table A.1
Estimation Results from DS 1

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)
FF (min)	−0.0516***	−0.0332***	−0.0515***	−0.0427***
	(−7.36)	(−7.05)	(−7.35) (U)	(−7.84)
SDT (min)	−0.0723***	−0.0472***	−0.0725***	−0.0599***
	(−9.92)	(−9.36)	(−9.94) (U)	(−10.12)
SST (min)	−0.0805***	−0.0549***	−0.0804***	−0.0669***
	(−14.28)	(−13.50)	(−14.26) (U)	(−14.98)
RC ($)	−0.3425***	−0.2171***	−0.3417***	−0.2866***
	(−9.19)	(−8.54)	(−9.17) (U)	(−10.88)
TC ($)	−0.2770***	−0.1813***	−0.1823***	−0.2302***
	(−12.46)	(−12.46)	(−12.50) (R)	(−13.19)
ASCs				
$\beta_{0,curr}$	0.9116***	0.9156***	0.9294***	0.9076***
	(17.62)	(18.32)	(18.64)	(18.62)
Number of observations	4,480	4,480	4,480	4,480
Initial LL (constants only)	−3694.75	−3694.75	−3694.75	−3694.75
Model LL	−3434.58	−3439.32	−3434.79	−3433.77

Table A.2
Estimation Results from DS 2

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)
FF (min)	-0.1256^{***} (−14.97)	-0.0308^{***} (−5.21)	-0.1214^{***} (−14.56) (U)	-0.1030^{***} (−14.39)
CT (min)	-0.1192^{***} (−14.04)	-0.0358^{***} (−7.23)	-0.0686^{***} (−13.22) (R)	-0.0983^{***} (−13.32)
RC ($)	Not statistically significant	-0.2727^{***} (−3.05)	Not statistically significant	Not statistically significant
TC ($)	-0.5568^{***} (−21.42)	-0.2328^{***} (−17.96)	-0.5281^{***} (−21.14) (U)	-0.4578^{***} (−21.62)
ASCs				
$\beta_{0,\mathrm{curr}}$	0.5984^{***} (8.55)	0.6599^{***} (11.46)	0.6105^{***} (8.74)	0.5991^{***} (8.13)
$\beta_{0,\mathrm{alt\ A}}$	0.1434^{**} (2.02)	0.1348 (1.93)	0.1432^{**} (2.02)	0.1447^{**} (2.04)
Number of observations	2,352	2,352	2,352	2,352
Initial LL (constants only)	−2192.64	−2192.64	−2192.64	−2192.64
Model LL	−1862.23	−1949.69	−1872.09	−1854.09

Table A.3
Estimation Results from DS 3

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)
FF (min)	Not statistically significant	Not statistically significant	Not statistically significant	Not statistically significant
SDT (min)	-0.0788^{***} (−6.45)	-0.05448^{***} (−6.02)	-0.0543^{***} (−6.01) (R)	-0.0650^{***} (−6.62)
SST (min)	-0.1701^{***} (−9.76)	-0.1160^{***} (−9.29)	-0.1701^{***} (−9.68) (U)	-0.1389^{***} (−9.42)
RC ($)	-0.2597^{***} (−4.05)	-0.1653^{***} (−3.45)	-0.2605^{***} (−4.05) (U)	-0.2214^{***} (−6.35)
TC ($)	-0.8152^{***} (−13.39)	-0.5757^{***} (−13.08)	-0.5796^{***} (−13.11) (R)	-0.7037^{***} (−20.83)
ASCs				
$\beta_{0,\mathrm{curr}}$	1.0937^{***} (14.72)	1.0784^{***} (14.62)	1.0851^{***} (14.69)	1.0561^{***} (14.77)
$\beta_{0,\mathrm{alt\ A}}$	0.2295^{***} (2.84)	0.2319^{***} (2.86)	0.2324^{***} (2.86)	0.2321^{***} (2.74)
Number of observations	2,432	2,432	2,432	2,432
Initial LL (constants only)	−1897.63	−1897.63	−1897.63	−1897.63
Model LL	−1694.93	−1691.55	−1690.49	−1688.83

Table A.4
Estimation Results from DS 4

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)
FF (min)	-0.0761^{***}	-0.0504^{***}	-0.0507^{***}	-0.0649^{***}
	(−7.94)	(−7.86)	(−7.89) (R)	(−8.87)
SDT (min)	-0.1167^{***}	-0.0754^{***}	-0.1163^{***}	-0.0985^{***}
	(−13.60)	(−12.92)	(−13.58) (U)	(−15.26)
SST (min)	-0.1684^{***}	-0.1114^{***}	-0.1682^{***}	-0.1418^{***}
	(−20.29)	(−19.03)	(−20.28) (U)	(−21.37)
RC ($)	-0.5629^{***}	-0.4142^{***}	-0.5649^{***}	-0.4732^{***}
	(−10.16)	(−10.77)	(−10.19) (U)	(−11.04)
TC ($)	-0.5041^{***}	-0.3390^{***}	-0.5055^{***}	-0.4307^{***}
	(−22.85)	(−21.84)	(−22.84) (U)	(−25.69)
ASCs				
$\beta_{0,curr}$	0.3621^{***}	0.3578^{***}	0.3571^{***}	0.3405^{***}
	(6.49)	(6.34)	(6.33)	(6.23)
Number of observations	4,864	4,864	4,864	4,864
Initial LL (constants only)	−3537.96	−3537.96	−3537.96	−3537.96
Model LL	−2670.14	−2683.70	−2670.20	−2664.50

Table A.5
Estimation Results from DS 5

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)	$\hat{\beta}$ (z-ratio)
FF (min)	-0.0351^{***}	-0.0238^{***}	-0.0351^{***}	-0.0280^{***}
	(−8.37)	(−7.96)	(−8.36) (U)	(−8.46)
CT (min)	-0.0351^{***}	-0.0242^{***}	-0.0243^{***}	-0.0282^{***}
	(−7.40)	(−7.17)	(−7.19) (R)	(−8.24)
RC ($)	-0.1166^{***}	-0.0770^{***}	-0.1172^{***}	-0.0937^{***}
	(−6.54)	(−6.36)	(−6.55) (U)	(−7.29)
TC ($)	-0.1575^{***}	-0.1083^{***}	-0.1083^{***}	-0.1261^{***}
	(−9.34)	(−9.49)	(−9.48) (R)	(−9.72)
ASCs				
$\beta_{0,curr}$	-0.5491^{***}	-0.5837^{***}	-0.5597^{***}	-0.5521^{***}
	(−5.17)	(−5.59)	(−5.32)	(−5.30)
$\beta_{0,alt\ A}$	0.2396^{***}	0.2394^{***}	0.2409^{***}	0.2397^{***}
	(2.79)	(2.79)	(2.80)	(2.77)
Number of observations	912	912	912	912
Initial LL (constants only)	−976.78	−976.78	−976.78	−976.78
Model LL	−847.75	−847.55	−846.53	−847.29

Table A.6
Estimation Results from DS 6

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)
FF (min)	−0.0683***	−0.0465***	−0.0472***	−0.0553***
	(−17.71)	(−16.92)	(−17.04) (R)	(−19.05)
CT (min)	−0.0904***	−0.0631***	−0.0898***	−0.0733***
	(−28.53)	(−26.50)	(−28.70) (U)	(−29.68)
RC ($)	−0.3159***	−0.2091***	−0.2131***	−0.2554***
	(−14.40)	(−14.15)	(−14.34) (R)	(−14.81)
TC ($)	−0.3633***	−0.2578***	−0.3693***	−0.2948***
	(−28.74)	(−27.77)	(−29.69) (U)	(−29.69)
ASCs				
$\beta_{0,curr}$	0.0920**	0.0122	0.0614	0.0897**
	(2.17)	(0.30)	(1.47)	(2.08)
Number of observations	3,888	3,888	3,888	3,888
Initial LL (constants only)	−4271.07	−4271.07	−4271.07	−4271.07
Model LL	−3031.58	−3044.06	−3029.70	−3027.75

Table A.7
Estimation Results from DS 7

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)
FF (min)	−0.0994***	−0.0622***	−0.0998***	−0.0804***
	(−14.67)	(−14.31)	(−14.74) (U)	(−16.07)
CT (min)	−0.1271***	−0.0837***	−0.1280***	−0.1028***
	(−10.06)	(−9.51)	(−10.12) (U)	(−11.00)
RC ($)	−0.4671***	−0.2872***	−0.4475***	−0.3806***
	(−10.27)	(−9.51)	(−9.95) (U)	(−11.21)
TC ($)	−0.6488***	−0.4175***	−0.4385***	−0.5240***
	(−21.95)	(−21.28)	(−21.27) (R)	(−22.17)
Number of observations	1,840	1,840	1,840	1,840
Initial LL (constants only)	−1918.79	−1918.79	−1918.79	−1918.79
Model LL	−1631.79	−1639.22	−1631.14	−1630.32

Table A.8
Estimation Results from DS 8

	Linear additive RUM	*RRM*	*Hybrid RRM–RUM*	*Symmetric RAM*
Attribute	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)	$\hat{\beta}$ (*z*-ratio)
Fare ($)	−0.4648***	−0.1686***	−0.4630***	−0.2612***
	(−18.83)	(−18.61)	(−18.86) (U)	(−18.63)
Travel time (min)	−0.1341***	−0.0455***	−0.0509***	−0.0740***
	(−16.66)	(−16.89)	(−17.38) (R)	(−16.36)
ASCs				
ASC_bus	−0.3894***	−0.4449***	−0.3987***	−0.3891***
	(−5.11)	(−5.85)	(−5.24)	(−5.19)
ASC_metro	0.4132***	0.3383***	0.3890***	0.4171***
	(6.89)	(5.61)	(6.43)	(6.87)
Number of observations	1,614	1,614	1,614	1,614
Initial (constants only)	−2440.98	−2440.98	−2440.98	−2440.98
Model LL	−2278.73	−2285.18	−2277.98	−2276.73