Behavioural implications of preferences, risk attitudes and beliefs in modelling risky travel choice with travel time variability

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Abstract

The appropriate interpretation of a behavioural outcome requires allowing for risk attitude and belief of an individual, in addition to identification of preferences. This paper develops an Attribute-Specific Extended Rank-Dependent Utility Theory model to better understand choice behaviour in the presence of travel time variability, in which these three important components of choice are empirically addressed. This framework is more behaviourally appealing for travel time and travel time variability research than the traditional approach in which risk attitude and belief are overlooked. This model also reveals significant unobserved between-individual heterogeneity in preferences, risk attitudes and beliefs.

Keywords: Traveller behaviour, stated choice, travel time variability, preference, risk attitude, belief

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1 Introduction

Travel time variability (i.e., random variations in travel time due to uncertainty in travel demand and supply), a feature of transport systems, is gaining in interest as congestion and system unreliability become daily occurrences and a major concern for service providers and politicians. Understanding traveller choice behaviour with travel time variability has gained increasing attention, as has empirically estimating the value of willingness to pay (WTP) for reduced travel time variability (see e.g., Small *et al.*, 1999; Bates *et al.* 2001). One of the significant milestones is the incorporation of Expected Utility Theory (EUT) into the representation of travel time variability, known as Maximum Expected Utility (MEU)[1] (Noland and Small 1995), which involves a choice process in which the alternative with the highest value of expected utility is preferred. Since Noland and Small's seminal paper in 1995, this has become the standard approach in travel time variability studies (see Li *et al*. 2010 for a review).

Travel time variability leads to multiple possible travel scenarios for repeated experiences of a trip (e.g., from home to work), where each scenario is experienced up to a probability of occurrence. This 'probabilistic' influence of travel time variability is reflected in many stated choice (SC) experiments for investigating the effects of travel time variability. There are two typical representations for an alternative associated with travel time variability per respondent' choice set: (1) as the extent and frequency of delay relative to normal travel time (e.g., one out of five chance of a 5-minute delay), and (2) a travel time distribution (e.g., a probability of 0.6 for arriving on time, 0.3 for arriving later by 10 minutes, and 0.1 for arriving earlier by 5 minutes, using three points as an example[2]). The latter form is preferred, given that it takes into account the entire distribution.

The traditional frameworks (e.g., MEU) for analysing travel time and travel time variability are typically established as a linear utility specification, which cannot reveal respondents' real risk attitudes (e.g., risk averse or seeking), but implicitly assumes risk neutrality. The attitude towards risk will have an impact on choice. For example, a risk-averse agent would prefer a sure alterative (which has a 100 percent chance of occurrence) to a risky alternative (which has multiple possible outcomes) with the equal expected value; however these two alternatives would be indifferent under the assumption of risk neutrality. Another behavioural limitation of the traditional approaches is that the original probabilities given in the experiments are directly used to weight corresponding outcomes. Evidence from psychology and behavioural economics (e.g., Quiggin 1982; Diecidue and Wakker 2001) has shown that in many cases, the raw probabilities provided in the experiments were transformed by subjects, and this transformation is influenced by individuals' beliefs (optimistic or pessimistic). As two important components of the psychological perspective of decision making, risk attitude and belief should also be accommodated, along with preference.

This paper develops an Attribute-Specific Extended Rank-Dependent Utility Theory (AS_ERDUT) model to reveal some important psychological factors (e.g., risk attitude and belief) of decision making in the presence of travel time variability, and consequently provide a richer representation of choice behaviour. The AS_ERDUT model allows for a systematic

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[1] MEU adopts linear probability weighting of EUT within the linear utility specification of Random Utility Maximisation (RUM).

[2] Some studies used five time levels per respondent alternative (e.g., Small *et al*. 1999), or 10 levels (e.g., Bates *et al.* 2001)

treatment of three key components of decision making - preferences, risk attitudes and beliefs. Under such a framework, we also reveal significant unobserved heterogeneity in preferences, risk attitudes and beliefs across sampled individuals, which is the first in the literature to our knowledge.

2 Existing Travel Time Variability Valuation Research: An Overview and Some Comments

Given random occurrences both in terms of demand and supply side effects, travel time variability is embedded in most transport systems.[3] Bates *et al.* (1987) classified travel time variability into three categories: i) inter-day variability caused by seasonal and day-to-day variations (such as demand fluctuations, accidents, road construction and weather changes), ii) inter-period variability which reflects the impact of differences in departure times and the caused changes in congestion, and iii) inter-vehicle variability mainly due to individual driving styles and traffic signals. Noland and Polak (2002) use similar categories to represent travel time variability; in particular differences in travel time from day-to-day, over the course of the day and even from vehicle to vehicle. Bates *et al.* (2001) further added that on the demand side, after considering seasonal effects, day-of-week effects and other systematic variations, the residual day-to-day variations are essentially random, whilst the randomness on the supply side is mainly due to incidents (e.g., vehicle breakdowns and signal failures).

The majority of travel time variability studies have investigated day-to-day variations in travel times, and have explicitly defined travel time variability as the random variation in travel time (see e.g., Bates *et al.* 2001; Hollander 2006), so as to emphasise the stochastic feature of travel time variability. In addition to free flow time and congested time, Noland and Small (1995), Bates *et al.* (2001) and others added an additional component of time, namely travel time variability, which represents the randomness in travel times over repeated trips. The concept of variability suggests that individuals have to make their travel decisions under uncertain circumstances with respect to the travel time; hence they are not able to predict the exact travel time or arrival time before starting their trips, given a departure time. Noland and Polak (2002) emphasised that the distinction between travel time variability and congestion is linked in that travellers have difficulty in predicting the former (e.g., caused by unforeseen road accidents or service cancellations) from day to day, while they can to some extent predict the variation in travel time due to congestion (e.g., peak hours vs. off-peak hours).

With the presence of travel time variability, the anticipated or expected trip time when departing might be different from the actual time after the trip is completed. As such, decision makers are faced with a decision context where there is a chance of arriving earlier, later or at the same time as the preferred arrival time, given a departure time. There thus exist certain probabilities of occurrence that for a given trip, a traveller or shipment will arrive either earlier, on-time or later than expected or planned, and assuming decision markers understand this, it is likely that these probabilities will somehow be utilised in the decision-making process. Within such a conceptual decision-making framework, the 'risk' (where multiple possible outcomes are associated with probabilities of occurrence) is therefore

[3] Although this paper focuses on passenger transport, it is important to note that such effects are not just within the domain of passenger movements, but will also impact upon road freight, which has wider implications for freight forwarder decisions, whose decisions may in turn impact on the road network and hence influence the decisions of other travellers.

embedded within the process. Psychological studies have found that a subject or respondent may show one of three typical attitudes towards risk (averse, neutral or taking) when decision making is subject to risk (e.g., Fox and See 2003). If a decision maker is risk averse, a sure alternative would be preferred to a risky alternative (with multiple possible outcomes) of equal expected value; if risk taking, a risky alternative is preferred to a sure alternative of equal expected value; and if risk neutral, the decision maker is indifferent between two alternatives of equal expected value.

The majority of recent travel time variability valuation studies (see Li *et al.* 2010 for a review) are established on Maximum Expected Utility (MEU), a theory proposed by Noland and Small (1995) where the attribute levels of travel time are weighted by the corresponding probabilities of occurrence, to address the fact that travel time variability leads to multiple possible travel times for a trip. The scheduling model and the mean-variance model, typically developed empirically within the stated choice theoretic framework, are two dominant approaches to empirical measurement of the value of time variability (Small *et al.* 1999; Bates *et al.* 2001). Under MEU, a scheduling model and a mean variance (MV) model are given in equations (1) and (2) respectively.

$$E(U)\_Scheduling = \beta_{E(T)}E(T) + \beta_{ESDE}E(SDE) + \beta_{ESDL}E(SDL) + \beta_{Cost}Cost + ... \quad (1)$$

$$E(U)\_MV = \beta_{E(T)}E(T) + \beta_{SD}SD(T) + \beta_{Cost}Cost + ... \quad (2)$$

In a scheduling model, $E(U)$ is a linear function of the expected travel time ($E(T)$), the expected schedule delay early ($E(SDE)$) which is the amount of time arriving earlier than the preferred arrival time (PAT) weighted by its corresponding probability of occurrence, the expected schedule delay late ($E(SDL)$) which is the amount of time arriving later than the preferred arrival time weighted by its corresponding probability of occurrence, and other attributes such as cost. In a mean-variance model, *SD(T)*, the standard deviation of travel time, is used instead of schedule delay.

MEU is the dominant behavioural theory used in the investigation of travel time variability. However MEU, based on linear utility maximisation[4] which implicitly assumes risk neutrality, is incapable of providing insights into individuals' attitudes towards risk, which is a key component of the psychological perspective of decision making. In addition to the linear utility specification, MEU also adopts the linear probability weighting function of Expected Utility Theory in which the original probabilities are directly used to weight potential outcomes. Allais (1953) in his paradox suggests that designed probabilities given in choice experiments are in reality transformed by respondents. To account for the perceptual translation of agents (i.e., the transformation of original probabilities), non-linear probability weighting was introduced by a number of authors to transform the analyst-provided probabilities into chooser perceptions. Therefore, the transformed probability, rather than the original probability, should be used as the weight, if such perceptual conditioning exists.

[4] A number of studies (e.g., Small *et al.* 1999) included the squared time attribute (e.g., *squared E(SDE)*) in their scheduling model. The reason why the squared attribute is included in these travel time variability valuation studies is that it is a convenient way to test that marginal disutility is a function of the level of an attribute; however they have not linked this assumed non-linearity to risk attitude explicitly. By contrast, the non-linearity (risk attitude) parameter is empirically estimated in this paper.

Ignoring the transformation of probabilities is another major limitation of the traditional approach to travel time variability, in addition to ignoring risk attitude.

The two important components of decision making (i.e., risk attitude and transformation of probabilities) can be accommodated in a general model, and we have chosen the form: $U=\sum_m \frac{p_m^\gamma}{[p_m^\gamma+(1-p_m)^\gamma]^{\frac{1}{\gamma}}}\times\frac{x_m^{1-\alpha}}{1-\alpha}$, where ($\frac{p_m^\gamma}{[p_m^\gamma+(1-p_m)^\gamma]^{\frac{1}{\gamma}}}$) addresses the transformation of probability through a non-linear probability weighting function, and ($\frac{x_m^{1-\alpha}}{1-\alpha}$) addresses the attitude towards risk through a non-linear utility specification.). When $(1-\alpha)=1$ and $\gamma=1$, where $(1-\alpha)$ is the risk attitude parameter and γ is the probability weighting parameter, this model reduces to a MEU model. Therefore, MEU is a special case of this modelling framework. The original model (i.e., Rank-Dependent Utility Theory) and its extension developed in this paper are introduced in Section 3 and Section 4 respectively.

3 Rank-Dependent Utility Theory

Rank Dependent Utility Theory (RDUT) is an appealing description of risky choices, which recognises that "a psychological weight attached to an event, …, usually differs from the probability of that event", when a choice is made under risk (Wu and Gonzalez 1999, p.74), and hence defines non-linear (non-additive) probabilities as the decision weight. A RDUT is capable of addressing risk attitudes through its non-linear utility function and the transformation of probabilities according to personal beliefs (optimistic or pessimistic).

With regard to the non-linear utility specification, constant *absolute* risk aversion (CARA, an exponential specification) and constant *relative* risk aversion (CRRA, a power specification) are two options. The CRRA form rather than CARA is used in this study, given that CARA is usually a less plausible description of the attitude towards risk than CRRA (see Blanchard and Fischer 1989). Blanchard and Fischer (1989, p.44) further explained that "the CARA specification is, however, sometimes analytically more convenient than the CRRA specification, and thus also belongs to the standard tool kit". Moreover CRRA often delivers "a better fit than alternative families" (Wakker 2008, p.1329).[5] A popular CRRA form is $U(x_m)=\frac{x_m^{1-\alpha}}{1-\alpha}$), where x_m is the m^{th} outcome of an alternative with multiple possible outcomes, associated with p_m chance of occurrence; *Alpha* (α) needs to be estimated; the calculated value of (1-α) indicates the attitude towards risk (averse, neutral or taking).

Prior to the contribution of Quiggin, (1982) who developed RDUT, the non-linear probability weighting function ($w(p)$) was applied independent of outcomes, = referred to as a separable probability weighting function, where the utility function of an outcome is directly weighted by its transformed probability based on its own probability and the probability weighting function (i.e., $U(x_m)*w(p_m)$). One of the most famous probability weighting functions was

[5] Although there are some differences, in both CARA and CRRA, the *Alpha* parameter has a similar interpretation of its sign in terms of risk aversion/seeking.

proposed by Tversky and Kahneman (T&K) (1992), that is, $w(p_m) = \frac{p_m^\gamma}{[p_m^\gamma + (1-p_m)^\gamma]^{\frac{1}{\gamma}}}$ where γ is a parameter to be estimated, which determines the curvature of the probability weighting function. For values of 0<γ<1, the weighting function has an inverse S-shape with overweighting of low probabilities, and underweighting of high probabilities; for values of 1<γ<2, the weighting function shows a S-shape with underweighting of low probabilities, and overweighting of high probabilities; for values of $\gamma \geq 2$, a convex probability weighting curve will be shown. If γ=1, the probability weighting function is linear (i.e., $w(p) = p$). Appendix A illustrates how the shape changes theoretically over the possible γ values.

Quiggin (1982) suggested that the transformed probabilities should also be determined by the rank of the outcomes in terms of preferences, (an idea developed further by Yaari 1987), and proposed an innovative model in which the cumulative probability distribution is transformed based on the rank of outcomes (i.e., cumulative probability weighting of Rank-Dependent Utility Theory, or decision weights). A RDUT model is shown in equation (3).

$$RDUT(x) = \sum_m \pi(p_m)U(x_m) \quad (3)$$

The utility function follows a non-linear specification (e.g., $U(x) = \frac{x^{1-\alpha}}{1-\alpha}$). $\pi(p)$, defined in equation (4), is the cumulative probability weighting function or decision weight which transforms the cumulative probability distribution based on the rank of outcomes, rather than transforming each probability (p) separately, where all potential outcomes for an alternative are ranked in increasing order of preference (from worst to best i.e., $U(x_1) < U(x_2) < ... < U(x_n)$).

$$\pi(p_m) = w(p_m + p_{m+1} + ... + p_n) - w(p_{m+1} + ... + p_n) \text{ for m=1, 2, …, n-1; and } \pi(p_n) = w(p_n) \quad (4)$$

w(p) is a non-linear probability weighting function; and the decision weight of the best outcome is equal to the transformed probability by the probability weighting function ($\pi(p_n) = w(p_n)$).

Unlike the probability in Expected Utility models, which is additive, RDUT allows $w(p_1 + p_2) \neq w(p_1) + w(p_2)$, where p_1 and p_2 are the probabilities of two independent outcomes. Decision weights under RDUT are not solely based on original probabilities, but also on the rank ordering of all possible outcomes. That is, the decision weight ($\pi(p_m)$) for the m^{th} outcome depends on its original probability (p_m) and its ranking position (see equation 4). Given an increasing order of rank, the decision weight ($\pi(p)$) of the best outcome is the same as $w(p)$. This characteristic of RDUT is also capable of revealing beliefs. If the estimated probability weighting function is convex (i.e., $w(p) < p$), the probability of the best outcome is always underweighted, suggesting conservatism or pessimism. On the contrary, a concave curve (i.e., $w(p) > p$) reveals optimism.

In the transport literature, Koster and Verhoef (2012) tested the cumulative probability weighting function of RDUT ($\pi(p_m)$) within a linear utility specification. Michea and Polak (2006) in a major contribution investigated train travellers' decision making within different behavioural frameworks including RDUT[6], using stated preference data collected by Bates *et al.* (2001), where the respondents were asked to choose between two train services with different travel time variability. Michea and Polak (2006) is an excellent pioneering work which demonstrated how alternative behavioural theories can be applied for parameter estimation in a descriptive model of choice between probabilistic alternatives with multi-attribute outcomes.

Both Michea and Polak (2006) and Koster and Verhoef assumed preference homogeneity in which fixed rather than random parameters were used. In Hensher and Li (2012), both the non-linear utility specification and cumulative decision weights of RDUT are empirically addressed within a mixed multinomial logit (MMNL) framework where significant unobserved heterogeneity in the travel time parameter was revealed across their sampled respondents. Like preferences, individuals' attitudes towards risk and beliefs may also be heterogeneous. Using a stated choice data set for a tollroad project conducted in Australia, this study develops the Attribute-Specific Extended RDUT within which to investigate unobserved between-individual heterogeneity in preferences, risk attitudes and beliefs.

4 Modelling Framework: Attribute-Specific Extended RDUT

As introduced in the previous section, a RDUT model has two key components: (1) a non-linear utility specification showing risk attitudes and (2) cumulative probability weighting (or decision weight) accounting for beliefs. The CRRA form that we used is $U = \frac{x^{1-\alpha}}{(1-\alpha)}$, where x is the travel time in this study which is associated with risk, given that each alternative (trip) has three possible outcomes (arriving early, on time and late).

The other component of RDUT (i.e., cumulative probability weighting) requires ranking all potential outcomes for an alternative. In the choice experiment used in this study, there are three possible travel times for each alternative: 'arriving *x* minutes earlier than expected', 'arriving *y* minutes later than expected', and 'arriving at the time expected'. The on-time travel time is the sum of free flow, slowed down and stop/start/crawling times directly shown in the experiment[7]; the actual travel time of early arrival is the on-time travel time minus *x* minutes while the actual travel time of late arrival is the on-time travel time plus *y* minutes. Each of these times has a corresponding probability of occurrence shown in the experiment, and we refer to these times as on-time (On_T), early (E_T) and late (L_T)[8] and their probabilities

[6] Michea and Polak had problems with Tversky and Kahneman's probability weighting function, and an alternative function was used in the RDUT model.

[7] Free flow as described as 'can change lanes without restriction and drive freely at the speed limit'. Slowed down time was described as 'changing lanes is noticeably restricted and your freedom to travel at the speed limit is periodically inhibited. Queues will form behind any lane blockage such as a broken down car'. Stop/start/crawling time is described as 'can only change lanes if others let you in. Consistently braking and accelerating in stop-start traffic.'

[8] In the choice experiment used in this study, there are three possible travel times for each alternative: 'arriving *x* minutes earlier than expected', 'arriving *y* minutes later than expected', and 'arriving at the time expected'. The on-time travel time is the sum of free flow, slowed down and stop/start/crawling times directly shown in the experiment; the actual travel time of early arrival is the on-time travel time minus *x* minutes while the actual travel time of late arrival is the on-time travel time plus *y* minutes.

as P_{On}, P_E and P_L.[9] Based on the RDU model in equation (3), three outcomes are ranked from the worst (1) to best (3): 1 = late arrival (L), 2 = early arrival (E) and 3 = on-time arrival (On) with the preference of $U(L_T) < U(E_T) < U(On_T)$. The RDUT decision weights are given in equation (5).

$$\pi(P_L) = 1 - w(P_E + P_{On}), \pi(P_E) = w(P_E + P_{On}) - w(P_{On}), \pi(P_{On}) = w(P_{On}) \quad (5)$$

We used T&K's non-linear probability weighting function (*w(p)*): $w(p) = \frac{p^{\gamma}}{[p_m^{\gamma} + (1-p)^{\gamma}]^{\frac{1}{\gamma}}}$, where γ needs to be estimated, which illustrates how the original probabilities shown in the experiment are transformed into decision weights (see equation 5). The acceptable range for γ is that γ has to be greater than 0; while α can be either negative or positive.

By combining the non-linear utility specification and decision weights, the model following the original RDUT framework is given in equation (6).

$$RDUT(Time) = \frac{[\pi(P_L)L_T^{1-\alpha} + \pi(P_E)E_T^{1-\alpha} + \pi(P_{On})On_T^{1-\alpha}]}{(1-\alpha)} \quad (6)$$

In a standard RDUT model, where normally there is only one attribute (e.g., the price of lottery which is often used in controlled laboratory experiments), the attribute-specific parameter (or marginal utility weight) is set equal to "1", where the attitude towards risk and choice behaviour are explained by the risk attitude parameter (1-α) only. When the travel time is associated with the risk attitude parameter, the implied risk attitudes are: risk taking if $(1-\alpha)<1$, risk averse if $(1-\alpha)>1$, and risk neutral if $(1-\alpha)=1$. However, in addition to the travel time attribute, the experiment used in this study also has the travel cost attribute (=running (fuel) cost + toll cost), which also needs to be included in the utility function. In order to calculate empirical willingness to pay, the attribute-specific parameters for both *Time* and *Cost* have to be estimated. Therefore, we need to improve the original RDUT model; the improved model is referred to as the *Attribute-Specific Extended* RDUT (AS_ERDUT) model. The overall utility expression for the proposed model is given in equation (7b) with the AS_ERDUT component defined in equation (7a).

$$AS\_ERDUT = \beta_{E(T)} \frac{[\pi(P_L)L_T^{1-\alpha} + \pi(P_E)E_T^{1-\alpha} + \pi(P_{On})On_T^{1-\alpha}]}{(1-\alpha)} \quad (7a)$$

$$U = AS\_ERDUT + \beta_{Cost}Cost + \beta_{Tollasc}Tollasc \quad (7b)$$

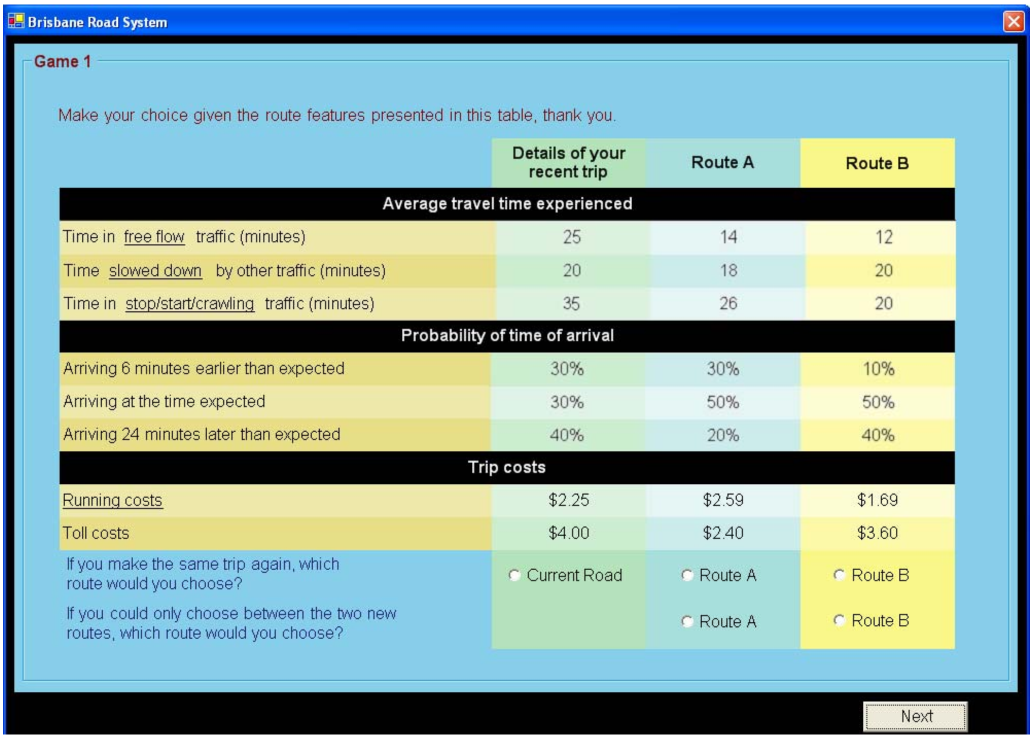
[9] Although there are three time components (free flow, slowed down, stop/start/crawling) in the choice experiment (see Figure 1), they are related only to the average travel time (i.e., on-time arrival). Hence, it is not possible to have preserved the three components in the absence of such information for the early and late travel times (noting that this is the only data we have). We have as result, added the components so that we can use the probabilities associated with early, late and on-time trip times to obtain, the probability weighting function. The add-up of three components into a total travel time applies to all models in this paper. The supplementary questions in the data set show that 79% of respondents actually added up the travel time components; and 81% added up the travel cost components (running cost and toll cost). Therefore it is reasonable to use a total time and total cost attribute rather their components.

Tollasc is the dummy variable to indicate whether a specific alternative is a tolled road; $\beta_{E(T)}$, β_{Cost} , and $\beta_{Tollasc}$ are the *Expected Time*, *Cost* and other parameters to be estimated; *Alpha* (α) needs to be estimated, and the calculated value of (1-α) indicates the attitude towards risk; and RDUT decision weights ($\pi(P_L)$, $\pi(P_E)$ & $\pi(P_{On})$) are given in equation (5).

Our proposed model is essentially a modified and improved framework over RDUT and Random Utility Maximisation (RUM) in which, for the attribute(s) associated with risk or probabilities of occurrence (e.g., the travel time in this case), we embed a functional form that accounts for attribute risk (through a risk attitude parameter) and probability of attribute level occurrence (through decisions weights), as well as preference (through the attribute-associated parameter); while the remaining non-risky attributes (with 100 percent chance of occurrence for an alternative within a choice set) maintain a linear-additive form under RUM (e.g., the cost attribute).

5 A Toll Road Project Data

The stated choice (SC) survey was conducted in Australia for a toll road project. In each choice task (see Figure 1 for an illustrative example), three alternatives were defined where the first alternative is the revealed preference (RP) alternative - a trip described by its current attribute levels; and two SC alternatives (i.e., route A and route B) that are pivoted around the knowledge base of travellers (i.e., the RP alternative), each of which may have a toll or no toll for part or all of the trip. The two new alternatives offer levels of the same set of attributes associated with the RP alternative (including a zero toll if the current route is a non-tolled route).


Brisbane Road System

Game 1

Make your choice given the route features presented in this table, thank you.

	Details of your recent trip	Route A	Route B
Average travel time experienced			
Time in free flow traffic (minutes)	25	14	12
Time slowed down by other traffic (minutes)	20	18	20
Time in stop/start/crawling traffic (minutes)	35	26	20
Probability of time of arrival			
Arriving 6 minutes earlier than expected	30%	30%	10%
Arriving at the time expected	30%	50%	50%
Arriving 24 minutes later than expected	40%	20%	40%
Trip costs			
Running costs	$2.25	$2.59	$1.69
Toll costs	$4.00	$2.40	$3.60
If you make the same trip again, which route would you choose?	Current Road	Route A	Route B
If you could only choose between the two new routes, which route would you choose?		Route A	Route B

Next

Figure 1: Illustrative stated choice screen of this design (three travel scenarios per alternative)

Each alternative has three travel scenarios - 'arriving *x* minutes earlier than expected', 'arriving *y* minutes later than expected', and 'arriving at the time expected'. Respondents were advised that *departure time remains unchanged* and that each of the reported trip times is associated with a corresponding probability[10] of occurrence, to indicate that travel time is not fixed but varies from time to time. The survey firm that collected the data went to great lengths, with the interviewer present, to explain what this meant for each respondent. For example, the 30% associated with a 6-minute earlier arrival relative to the expected arrival time (i.e., taking the average travel time of 58 minutes consisting of 14-minute free flow time, 18-minute slowed down time and 26-minute stop/start/crawling time with 50% chance of occurrence) for Route A in Figure 1was explained as 'for every 10 trips you might take, 3 out of the 10 trips the travel time will be 6 minutes less than the 58 minutes stated above as the average time experienced, or a trip time of 52 minutes'. The descriptive statistics for the time and probability variables are given in Table 1.

Table 1: Travel Times and Probabilities of Occurrence (13,440 cases)

Variable	Mean	Std.Dev.	Minimum	Maximum
P_E	0.25	0.11	0.1	0.4
P_L	0.25	0.11	0.1	0.4
P_{on}	0.50	0.15	0.2	0.8
$E_{\Delta T}$	4.80	3.14	0	18
$L_{\Delta T}$	9.60	6.28	1	36
On_T	39.29	16.58	10	119
E_T	34.48	14.98	7	115
L_T	48.89	21.09	11	150

Notes: P_E, P_L and P_{on} are probabilities for arriving early, late and on time, On_T is the average or expected travel time (the sum of three components: free flow, slowed down and stop/start times), $E_{\Delta T}$ and $L_{\Delta T}$ are the amounts of time associated with arriving earlier and later than expected which are designed and presented in the experiment. E_T is the actual travel time for early arrival (=On_T - $E_{\Delta T}$); L_T is the actual travel time for late arrival (=On_T + $L_{\Delta T}$).

For all attributes except the toll cost, minutes arriving early and late, and the probabilities of arriving on-time, early or late, the values for the stated choice alternatives are variations around the values for the current trip. The vehicle running cost for car travel and any toll cost for the specific trip in question were also included in the SC attributes. In the definition provided to the respondent, running costs include only fuel costs for cars (at Au$1.50 a litre or 15c/kilometre), the most commonly perceived cost for the marginal trip, plus toll costs - the amount of money spent for a specific trip assuming the trip occurred using a toll route. Given the lack of exposure to tolls for many travellers in the study catchment area, the toll levels were fixed over a range, varying from no toll to Au$4.20, with the upper limit determined by the trip length of the sampled trip.

The survey was designed to capture a large number of travel circumstances, to determine how each individual trades-off different levels of travel times and trip time reliability with various levels of proposed tolls and vehicle running costs in the context of tolled and non-tolled roads. The survey has five major sections: the introduction to the survey task and background on the study; questions describing a current or recent trip in terms of travel times and cost (including tolls if paid); the SP experiment (16 screens); a series of attitudinal questions seeking views on the broader set of quality benefits of toll and freeway roads; and socio-economic questions. Sampling rules were imposed on three trip length segments: 10 to 30 minutes, 31 to 45 minutes, and more than 45 minutes (capped at 120 minutes). Sampling by

[10] The probabilities are designed and hence exogenously induced to respondents, similar to other travel time variability studies.

the time of day that a trip commences was also included, defining the peak[11] as trips beginning during the period 7-9 am or 4.30-6.30pm. All non-peak trips were treated as off peak in the internal quota counts.

There are three versions of the experimental design depending on the trip length, with each version having 32 choice situations (games) blocked into two subsets of 16 choice situations each. In generating the designs, the free flow, slowed and stop/start/ crawling times were set to five minutes if the respondent entered zero for their current trip. It is important to understand that the distinction between free flow, slowed down and stop/start/crawling time is solely to promote the differences in the quality of travel time between various routes – especially a tolled route and a non-tolled route, and is separate to the influence of total time.

Fieldwork took place in November 2008. In total, 280 Australian commuters were sampled for this study. The experimental design method of D-efficiency used herein is specifically structured to increase the statistical performance of the models with smaller samples than are required for other less-efficient (statistically) designs such as orthogonal designs.

6 Unobserved Heterogeneity in Preferences, Risk Attitudes and Beliefs and the Implied Behavioural Outcome

A mixed multinomial logit (MMNL) model is estimated within an AS_ERDUT framework (equation 7), where distributions are assumed for *Alpha* (the value of (1-*Alpha*) indicates ***risk attitude***), *Gamma* (cumulative probability weighting which reveals ***belief***) which allows for unobserved heterogeneity at the individual level, and the *expected time* parameter (i.e., ***preference***). With regard to the *cost* parameter, it can also be random. However there is a large literature (such as Revelt and Train 1998) that argues for keeping one of the parameters fixed in the ratio to derive willingness to pay. Daly *et al.* (2012) have discussed this recently and have expressed concerns when both numerator and denominator are random: "some popular distributions used for the cost coefficient in random coefficient models, including normal, truncated normal, uniform and triangular, imply infinite moments for the distribution of WTP, even if truncated or bounded at zero" (p.19). Given the distributions that provided statistically and behaviourally acceptable parameter estimates (see below), the *cost* parameter is assumed to be non-random in the MMNL model in order to avoid the potential problems associated with taking the ratio of two random variables, following the advice of Sillano and Ortuzar (2005).

We have allowed for correlations between the random parameters. We tested different distributions to represent these random parameters (such as normal, lognormal, triangular and skewed normal distribution), and within the AS_ERDUT framework, the model with unconstrained normal distributions for *Alpha*[12] , an unconstrained triangular distribution for

[11] The way we handle trips that are partly in the peak: a trip is peak if 60 percent or more of the trip falls within the peak period.

[12] A reviewer commented that it would problematic if (1-*Alpha*)=0 or *Alpha*=1, given that (1-*Alpha*) defines the denominator of the marginal utility specification for AS_RDUT, and given that a normal distribution is used to represent the random parameter of Alpha. While it is true that a value of 1 is theoretically possible, we are unaware of a convenient distribution in which '1' is excluded. Therefore, in reality, we cannot apply a distribution in which the value of (1-*Alpha*) has a zero probability of being '0'.

Gamma and a constrained triangular distribution[13] for the *expected time* parameter has the best performance in terms of the model fit. Given that three random parameters used in the model are assumed to be correlated, the standard deviations are no longer independent. To assess this we have decomposed the standard deviation parameters into their specific and interaction standard deviations. The preferred model which allows for the correlation between random parameters is given in Table 2.

To illustrate the behavioural advantage of the non-linear AS_ERDUT model, a linear model is also estimated with $(1-\alpha)=1$ (i.e., linear utility specification assuming risk neutrality) and $\gamma=1$ (i.e., linear probability weighting ignoring the transformation of probabilities based on personal beliefs). The linear MMNL model is reported in Appendix B. A likelihood ratio test is used to statistically compare the two models. The calculated test statistic is 1416.84. Given the critical value (chi-square), with six degrees of freedom, of 16.82 at the 99 percent confidence level, the non-linear model (Table 1) delivers a statistically better fit than the linear model (Appendix B), which in turn suggests that the non-linear model which addresses risk attitudes and the transformation of probabilities based on personal beliefs better explains the choice data, by providing a richer understanding of travellers' choices and behavioural responses.

[13] Although a constrained triangular distribution (in which its mean equals its spread, and the standard deviation is the spread/√6) can ensure that all *expected time* parameters across the sample of respondents are negative, this is only true if the random parameters are uncorrelated. In our model, we allow for correlated parameters which can induce movement in the distribution across the positive and negative range even under the mean=spread constraint for one of the parameters. Although a normal distribution can also be constrained and a lognormal distribution can produce all positive or negative individual parameters, they may have some serious problems when estimating models (see Cherchi 2009 for a review).

Table 2: Mixed multinomial logit (MMNL) within an AS_ERDUT framework
(Estimated using Nlogit5)

Variable	Coefficient	*t*-Ratio
Nonrandom parameters:		
Status quo constant	2.0509	20.07
Cost ($)	-0.0899	-1.69
Tollasc	-1.0605	-5.37
Means for random parameters:		
Alpha (α)	1.4105	67.54
Gamma (γ)	1.1524	3.79
Expected Time (minutes)	-2.9531	-16.81
Diagonal elements of Cholesky matrix:		
Alpha (α)	4.1650	25.61
Gamma (γ)	1.2276	2.92
Expected Time (minutes)	1.2566	2.28
Below diagonal elements of Cholesky matrix:		
Gamma (γ) x Alpha (α)	-0.6688	-0.43
Expected Time (minutes) x Alpha (α)	0.3864	0.98
Expected Time (minutes) x Gamma (γ)	-2.9531	-16.81
Standard deviations of parameter distributions		
Alpha (α)	4.1650	25.61
Gamma (γ)	1.3979	2.64
Expected Time (minutes)	3.2325	8.85
No. of observations	4,480	
Information Criterion : AIC	3,893.20	
Log-likelihood	-1,935.62	
Pseudo R-squared	0.46	

Note: α has an unconstrained normal distribution, γ has an unconstrained triangular distribution, and *expected time* has a constrained triangular distribution (although for the latter, see footnote 13 for implications of correlated random parameters on constrained distributions).

The underlying components of the variances for the random parameters are the standard deviations (σ1, σ2, σ3) and the correlations ρ12, ρ13, and ρ23. Hensher *et al.* (2005, Section 16.7) provides details of how these are used to calculate the Cholesky matrix elements reported in Table 2. Since the "Cholesky parameters" are not directly interpretable, given that the results are arbitrarily based on the order chosen[14], we report in Table 3 the implied variances, covariances and the correlations which are independent of the order chosen, and which can be unambiguously interpreted.

Table 3: Correlation and variance-covariance elements of the random parameter matrix
Note: Let c(i) be the i^{th} row of C. The variance is the square root of c(i)'c(i). The correlation is c(i)'c(j) / sqr(c(i)'c(i) * c(j)'c(j)).

	Correlations			Variances and Covariances		
	alpha	gamma	Expected time	alpha	gamma	Expected time
alpha	1.0			0.00032		

[14] Although the ordering of the variables does produce small differences, it would be a mistake to attribute this to something other than finite sample variation, affected by the size of the data set and the number of replications (points) in the simulations.

gamma	-0.478	1.0		-0.0042	4.9046	
expected time	0.119	-0.659	1.0	0.0015	-0.1661	0.0490

The estimated parameter for the status quo constant (i.e., the constant for the current trip) is positive, which suggests, after accounting for the observed influences, that sampled respondents, on average, prefer their current trip attribute package relative to the two stated choice alternatives. *Tollasc* is negative, which indicates that, on average after accounting for the time and cost of travel, other factors bundled into a 'toll road quality bonus' are less desirable for a tolled route than a non-tolled route, mainly due to the lack of exposure to tolls for our sampled respondents[15]. With regard to the preference (or taste) parameters, both the unconditional[16] mean *expected time* and *cost* parameters are negative[17], which is as expected.

The unconditional mean *Alpha* estimate is 1.4105, which is statistically significant with a *t*-ratio of 67.54. As the *expected time* parameter is negative, a risk attitude parameter ($(1-\alpha)$) less than one suggests decreasing marginal disutility, where the utility of a risky alternative would be higher (less negative) than a sure alternative with the same expected value. This indicates, on average a risk-taking attitude at the sample-population level. The standard deviations (or spread) of three random parameters (*expected time*, *Alpha*, and *Gamma*) are statistically significant when the full diagonal and off-diagonal (correlated) influences are taken into account. The findings suggest that there is statistically significant unobserved heterogeneity in *expected time* (preferences), *Alpha* (risk attitudes), and *Gamma* (transformation of probabilities according to personal beliefs) across the sampled Australian car commuters.

The unconditional distribution of expected time is calculated as -2.9531 +0.3864*t-2.9531*t +3.2325*t where t is the triangular distribution. The allowance for correlation between random parameters (represented by the additional elements +0.3864*t-2.9531*t – see Hensher et al. 2005 for details on how correlated influences are included) results in a distribution that is not single-sign satisfied despite the initial imposition of the mean equal the spread, and hence the sign condition no longer holds. The standard deviation is 0.274 around a mean of -2.953.

However, for Alpha, its unconditional unbounded distribution crosses the positive and negative domains, suggesting the mix of risk-taking ($1-\alpha<1$) and risk-averse attitudes ($1-\alpha>1$). Senna (1994) assumed that his sampled commuters with flexible arrival times are risk averse when making risky time-related decisions, where the assumed risk attitude parameter is 1.4 (>1), and his sampled commuters with fixed arrival times are risk taking with the assumed parameter of 0.5 (<1). The mix of risk-taking and risk-averse attitudes revealed by

[15] This empirical study was undertaken in a city with little exposure to toll roads. In contrast, in Sydney where there has been high exposure to tolls over the last 15 years, we found a significant positive parameter associated with Tollasc.

[16] Within a MMNL modelling framework where random parameters are estimated over the sample population from a number of draws (e.g., random or intelligent draws), Hensher *et al.* (2005) defines 'unconditional' estimates as follows: "Parameter estimates estimated at the sample-population level are called *unconditional* parameter estimates, as the parameters are not conditioned on any particular individual's choice pattern but rather on the sample population as a whole. The process of estimating *unconditional* random parameters is similar to the estimation process of non-random parameters in the MNL and M[MN]L models; that is, maximization of the simulated LL [log-likelihood] function over the data for the sample population (p. 631)."

[17] The estimated mean *expected time* parameter is negative, given the constrained distribution.

the MMNL model may be attributed to commuters with a fixed arrival time and commuters with flexible arrival times, both sampled in our study. This finding is in line with the finding of Hensher *et al.* (2011) based on the same choice data set.

The unconditional distribution of *Gamma* has an empirically estimated range of 0.4317-1.8805 (based on 1.1524+1.3979*t-0.6688*t). Such a range produces two types of probability weighting curvatures: an inverse S-shaped curvature with over-weighting of low probabilities and under-weighting of medium to high probabilities for values of $0.4317 \leq \gamma < 1$ (see Figure 2a for an illustrative example when $\gamma = 0.4317$), and an S-shaped curvature with under-weighting of low to medium probabilities and over-weighting of high probabilities for $1 < \gamma \leq 1.8805$ (see Figure 2b for an illustrative example when $\gamma = 1.8805$). The estimated range of the unconditional *Gamma* distribution suggests that an original probability may be under-weighted ($w(p) < p$) or over-weighted ($w(p) > p$), according to the specific value of the probability itself and the specific value of *Gamma*. Given the way that the decision weights are defined (see equation 5), a decision weight of the best outcome (i.e., on-time scenario in this study) is the same as its transformed probability by non-linear probability weighting (that is, $\pi(P_{On}) = w(P_{On})$). Given the range of the unconditional *Gamma* distribution, the raw probabilities of on-time travel may be under-weighted or over-weighted, or directly used as the weights ($\gamma = 1$, linear probability weighting). This suggests the mix of pessimistic and optimistic beliefs of the sampled respondents. However, when there is the transformation of probabilities (either $0.4317 \leq \gamma < 1$ or $1 < \gamma \leq 1.8805$), medium probabilities always tend to be underweighted. In Figure 3, we plotted the distribution of on-time arrival probabilities shown to the respondents, where over 66 percent of on-time arrival probability values are between 0.4 and 0.6 (medium probabilities). This suggests that there is a higher chance that the on-time arrival probabilities given in the experiment would be under-weighted, which in turn implies stronger conservative (or pessimistic) beliefs.

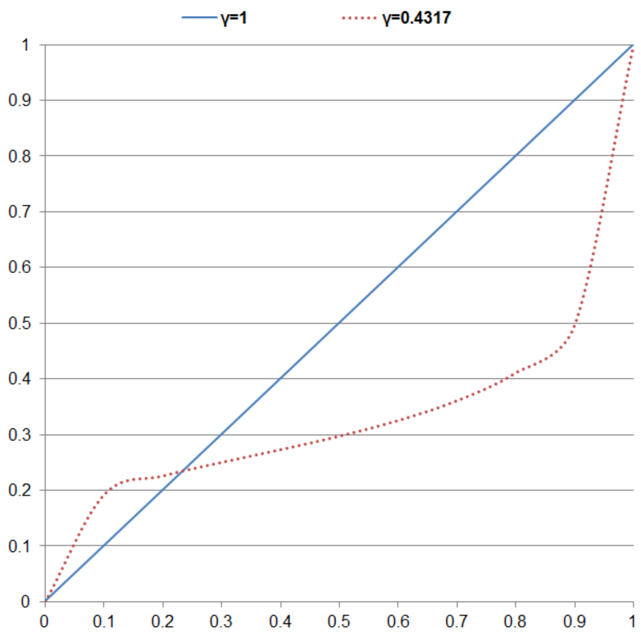


Figure 2a: Inverse S-shaped probability weighting function for $0.4317 \leq \gamma < 1$

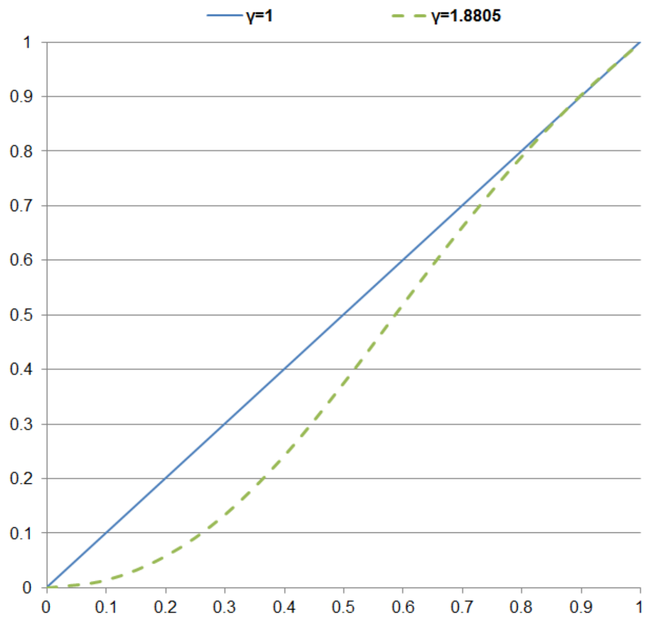


Figure 2b: S-shaped probability weighting function for $1 < \gamma \leq 1.8805$

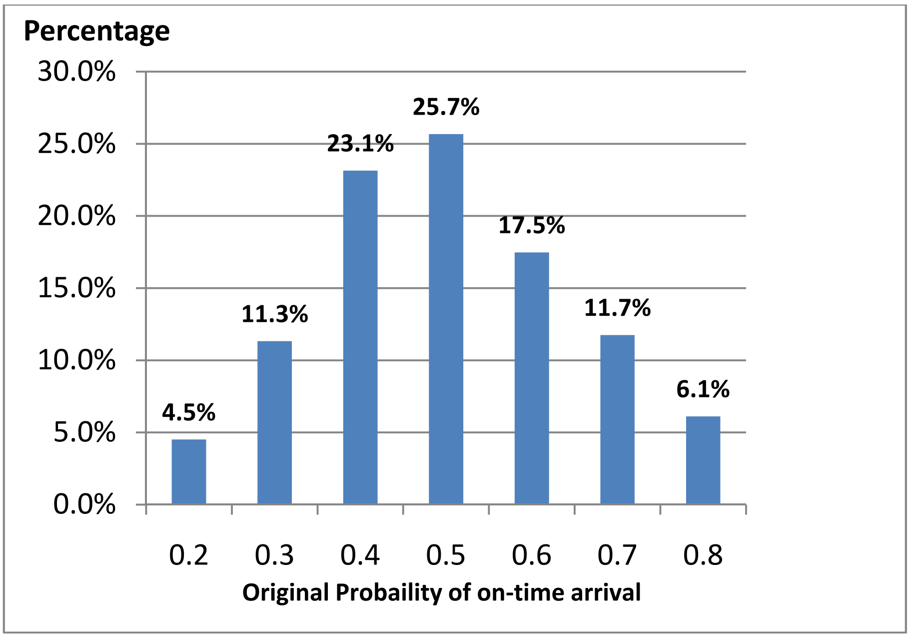


Figure 3: The distribution of on-time arrival probabilities in the choice experiment (13,440 cases)

The appropriate interpretation of a behavioural outcome also requires the attitude towards risk, and belief of, an agent (see e.g., Wakker 2004; Dickinson 2009). The AS_ERDUT model reveals that the sampled car commuters in this study exhibit heterogeneity in risk attitudes when making risky travel choices, as well as beliefs. In an earlier paper, Hensher *et al*. (2011), applied the non-linear probability weighting function separably, i.e., the transformation of probabilities only determined by original probabilities, independent of outcomes. The cumulative way of probability weighting in this paper is capable of revealing individuals' beliefs, which extends Hensher *et al*. (2011).

7 Conclusions

The paper introduced a modelling framework associated with what we refer to an Attribute-Specific Extended Rank-Dependent Utility Theory, in which three important components (i.e., preference, risk attitude and belief) of decision making are empirically addressed, when making risky travel choices in the presence of travel time variability. More importantly, unobserved between-individual heterogeneity in preferences, risk attitudes and beliefs is revealed in a single model. The AS_ERDUT model with a non-linear utility function and cumulative probability weighting delivers a substantial improvement in model fit compared to a traditional model established on the linear utility specification and linear probability weighting. The evidence suggests that the AS_ERDUT framework is more behaviourally appealing for travel time and travel time variability research than the traditional approach in which risk attitude and belief are overlooked.

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Appendix A: Curvatures of Tversky and Kahneman's Probability Weighting over Different γ Values

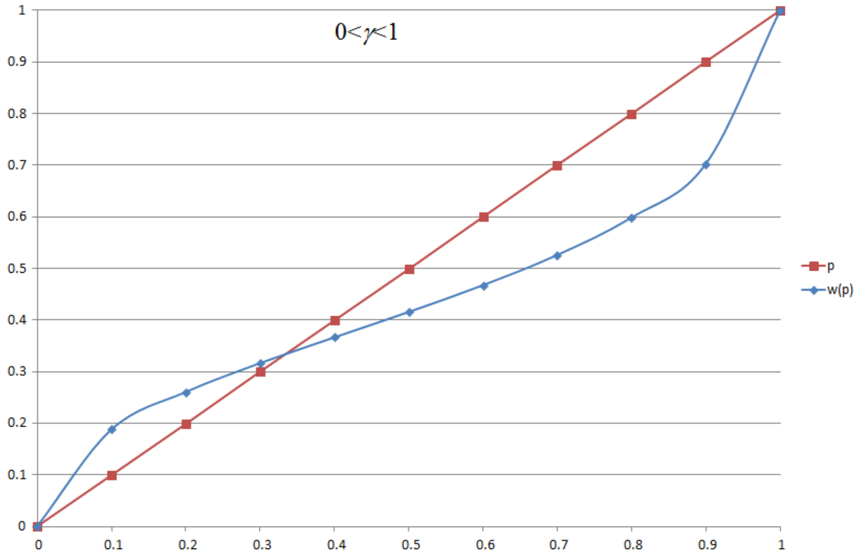

Figure A1: Probability weighting Curvature for 0<γ<1

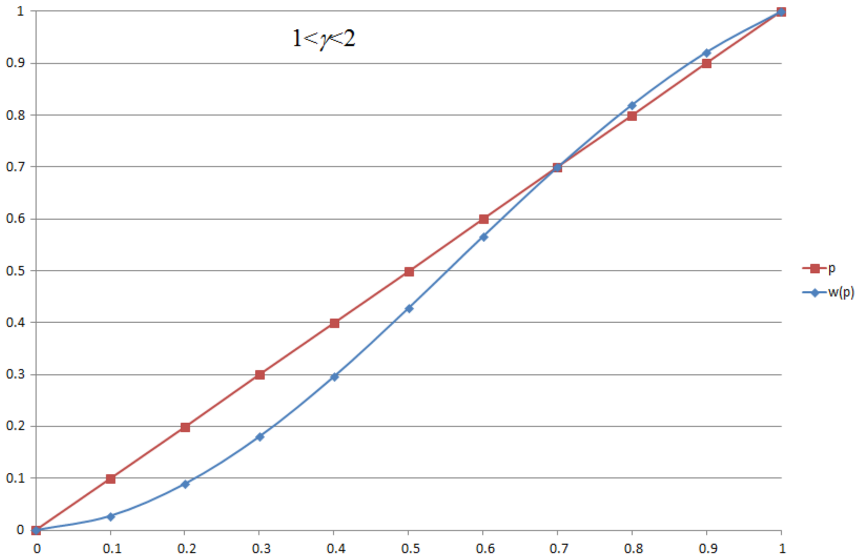

Figure A2: Probability weighting curvature for 1<γ<2

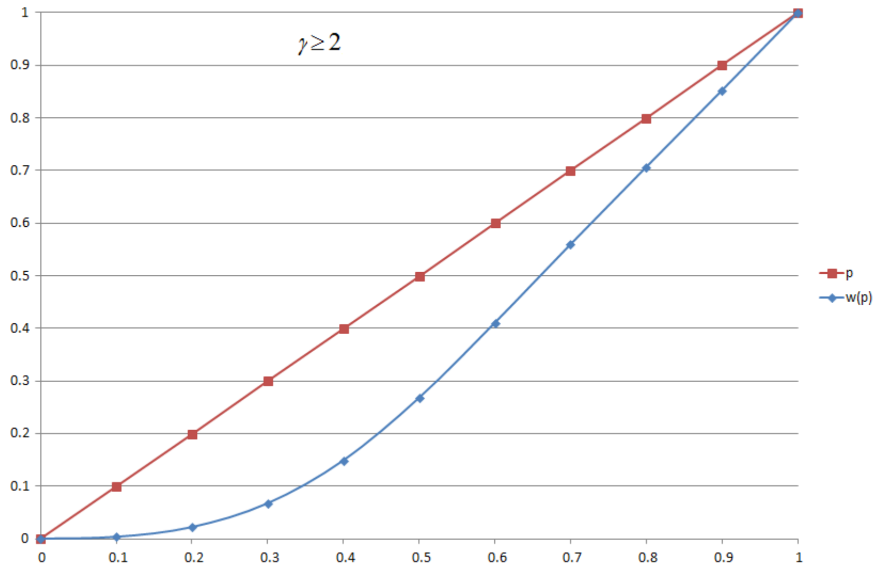

Figure A3: Probability weighting curvature for $\gamma \geq 2$

Appendix B: Linear MMNL Model

Variable	Coefficient	*t*-Ratio
Nonrandom parameters:		
Status quo constant	0.4978	7.09
Cost ($)	-0.2930	-16.87
Tollasc	-0.2861	-2.93
Means for random parameters:		
Alpha (α)	Assumed to be 0	-
Gamma (γ)	Assumed to be 1	-
Expected Time (minutes)	-0.1158	-31.12
Standard deviations (or spread) for random parameters:		
Alpha (α)	-	-
Gamma (γ)	-	-
Expected Time (minutes)	0.1158	31.12
No. of observations	4,480	
Information Criterion : AIC	6,714.90	
Log-likelihood	-3,352.46	
Pseudo R-squared	0.32	

Bios

Zheng Li is Senior Research Fellow in Transportation at the Institute of Transport and Logistics Studies (ITLS) in The University of Sydney Business School. Zheng's main research interests include willingness to pay valuation, advanced non-linear travel choice models, and transport policy. Zheng has published over 20 journal articles, the majority of which are published in the top Transportation and Logistics journals (e.g., *Transportation Research Parts A, B, D and E, Transportation Science*, *Transportation*), and has presented papers at a number of international conferences. In 2010, Zheng was awarded the Institute of Transport and Logistics Studies Prize for research excellence in transport or logistics.

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