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ADVANCED CONTROL AND OPTIMIZATION FOR
FUTURE GRID WITH ENERGY STORAGE DEVICES

By

Liyan Zhang

A thesis submitted in fulfilment of the
Requirements for the degree of Master of Philosophy

Centre of Excellence in Future Energy Networks

School of Electrical and Information Engineering,

The University of Sydney

December 2015
Statement of Originality

The work and the results produced in this thesis have been carried out by the author whose name appears below. The work was carried out under the supervision of Prof. David Hill.

I declare that the material in this thesis has never been previously submitted to any institution for any other degree. The work is entirely original except where duly referenced.

Author: Liyan Zhang

Date: December 2015
Acknowledgement

The author would like to thank all who have provided help and assistance during the course of the thesis.

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The author of this thesis published the following articles during the process of the Masters Degree:

1. **Liyan Zhang, Guo Chen, Zhaoyang Dong, David Hill, K. P. Wong.** “Enhanced Optimal Distributed Consensus Control of Scattered Devices Given Security and Economic Operation Constraints”, prepare to submit to *IEEE Trans. on Smart Grid*.


3. **Liyan Zhang, Cuo Zhang, Jueyou Li, Guo Chen, David Hill.** “Modified Accelerated Consensus Optimization for Future Distribution Network”, accepted by 9th IFAC Symposium on Control of Power and Energy Systems (CPES) 2015, New Delhi, India.


Abstract

In the future grid environment, more sustainable resources, such as wind power (WP) and photovoltaic (PV) generation will be increasing steadily. Along with the soaring renewable penetration level also comes concerns over reliable, stable and economical operation of the power grid, because their inherent unpredictable and intermittent characteristics will inevitably cause adverse impacts on the system static, dynamic and economic performance simultaneously: such as jeopardising the voltage and frequency stability or deteriorating the power flow distribution, etc.

In this context, energy storage (ES) devices have been receiving growing attention because of their significant falling prices. It is anticipated in the future that energy storage facilities will be installed in most houses. Therefore, how to utilize these ES to help alleviate the problem of renewable energy (RE) sources integration has become more and more attractive. In my thesis, I will try to resolve some of the related problems from several perspectives.

First of all, a comprehensive Future Australian transmission network simulation platform is constructed in the software DIgSILENT Power Factory. Based on the analysis of different scenarios, basic conclusions can be drawn on how different levels of renewable resources would influence the static power flow distribution and system dynamic stability (including frequency, voltage and rotor angle). Then in-depth research has been done on the aspect of frequency controller design. Based on rigorous mathematical reasoning, an advanced robust $H_{\infty}$ Load Frequency Controller (LFC) is developed, which can be used to assist the power system to maintain a stable frequency when accommodating more renewables with the help of energy storage. Afterwards, I focus on the voltage and line power over-limit issues in a future
distribution network. I develop a power system sensitivity analysis based-Enhanced Optimal Distributed Consensus Algorithm (EODCA) that combines the theory of power systems and the concept of multi-agents together. The facilities can update their individual information during the iteration process based on a control law through the communication and data exchange with their adjacent neighbours so as to eventually reach a final consensus optimal value. In the following study, a Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) is proposed, with this approach it can be verified that the convergence speed is notably accelerated even for complex large dimensional systems. Furthermore, the impact of the communication network topology on algorithm dynamic performance is also studied in terms of graph theory so that guidance on how to design a relatively more optimal communication network is given as well.

Overall, in the Master thesis, I successfully provide several novel and practical solutions, algorithms and methodologies in regards to tackling both the frequency, voltage and the power flow issues in a future grid with the assistance of energy storage devices. The scientific control and optimal dispatch of these facilities could provide us with a promising approach to mitigate the potential threats that the intermittent renewables posed on the power system in the following decades.
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6. Conclusions

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1. Introduction

1.1 Motivation

As concerns about anthropogenic climate change grow in Australia, the reduced consumption of fossil energy such as coal, petroleum and oil has become one of the most important and pivotal problems waiting to be solved in modern society. In this context, the penetration level of renewable energy, such as wind power and photovoltaic generation into the future power grid has been steadily increasing in recent years in Australia [1].

In the future, a promising pattern of renewable integration in Australia appears to be unfolding. Clearly, integration of solar PV into the distribution networks due to the vast amounts of solar energy available will continue to increase. Shortages of water will render high use of hydro technologies inapplicable [2]. Actually, the geographically distributed generators (DG) like household rooftop PV in Queensland and South Australia has become quite prevalent so far. Further, in Canberra and South Australia, wind farms are being integrated into the transmission grid as well.

Although the advantages of integrating renewables seem to be quite obvious, especially from an environmental perspective, the concern over reliable, stable and economical operation of the power grid also arises. Due to the inherent unpredictable fluctuating characteristics of RE, they will inevitably cause adverse impacts [3] on system static/dynamic performance indexes and deteriorate the economic performance without the appropriate dispatching and control approaches. Therefore, it is extremely important to identify their influences on the future power network structure, and set up an effective mechanism to help to alleviate the adverse impacts as well as enhance the efficiency and economic operation of the system simultaneously.
To solve this dilemma, Energy Storage devices have been received growing attention because the price has decreased significantly. Storage can be used as an effective “buffer” to alleviate the problem brought by renewable integration if appropriate control and optimization methods are implemented. Actually, there are many different kinds of energy storages that can be utilized [4] in power system operation, ranging from high transmission level to low distribution level. Therefore, it is of high theoretical and practical value to do research on how to control and dispatch these promising devices in the future grid to assist the system to perform in a more reliable and economical way. This is the motivation for me to do research on this topic: “ADVANCED CONTROL AND OPTIMIZATION FOR FUTURE GRID WITH ENERGY STORAGE DEVICES”.

1.2 Background & Literature Review

1.2.1 The Impacts of the Renewables on Future Grid

As renewables thrive, the influences of integrating them into the grid should be further studied. There have been some research works relating to this issue from different perspectives, the following literatures are some of the significant ones:

Reference [5] is an important summary showing the impacts of the renewable resources on power delivery systems. As is known to all, conventional power distribution system is designed assuming that the substation is the only source of energy and short-circuits capacity. This may not be true in the future grid scenarios with high renewables. In this context, the author discusses some important system issues when considering the DG:

- The voltage issues: the changes of node voltages when consider DG, interaction of DG and capacitor/voltage regulator.
Protection issues: fuse coordination, feeding faults after utility protection opens, impact of DG on interrupting rating of devices, faults on adjacent feeders, fault detection, ground source impacts, single phase interruption on three phase line, recloser coordination and conductor burn down.

- Loss of power grid issues;
- Vulnerability and overvoltage issues due to islanding and coordination with reclosing.
- System restoration and network issues.

The conclusion is that, if not carefully designed, DG penetration can potentially have very significant adverse system effects, including exposing system and customer equipment to potential damage, deteriorate the power quality, decrease in reliability, extended time to restoration after outage, and potential risks to public and worker safety as well.

Apart from the systematic issues mentioned in the previous paper, there are also some interesting discussions with regards to some other aspects in engineering practice. Paper [6] studies the voltage profile changes of the load nodes when considering PV power station. In accordance with the relevant provisions, the impact factor depends primarily on a variety of different characters, including the number, location, size, and type of grid-connected PV power plants. From the paper we can draw a conclusion that we should install some reactive power support devices to help to alleviate the adverse influences that the PV exerts in voltage profile. In the paper [7], the author simulated a hypothetical network, assuming a large number of fuel cells and micro-turbines as dispersed units in low-voltage distribution network. In the study, the author simulates four categories of stability, i.e. frequency stability, oscillatory stability, transient stability and voltage stability. Based on the results and discussions, it can be concluded that DG
can improve the stability of power systems if suitable types and appropriate locations are designed. Ref. [8] proposes a multi-objective performance index for evaluating the DG impacts. Distributed generation is extensively located and sized within the IEEE-34 test feeder, wherein the multi-objective performance index is computed for each configuration. The advantage of the proposed relevance factors is its flexibility since electric utilities have different concerns about losses, voltages, protection schemes, etc. However, it is also quite obvious that some of the important indexes are neglected and not taken into account, such as the static line power transfer limitations.

Through the above mentioned literature and their references, we could obtain an overview of the impacts that the renewable generation could impose on the future electricity networks.

1.2.2 Energy Storage and its Application in Frequency Control

Use of energy storage devices has become a hot research topic recently especially considering their potential abilities in helping to maintain the stability of the system voltage and frequency. Unlike the voltage issue, which is a local power issue that has a close relationship with the reactive power injection or absorption, the frequency of a certain area implies a global performance of the active power balance between the total generation and demand. The unpredictable nature and highly intermittent outputs of the RE sources require a much higher frequency adjusting ability of the future power system to accommodate them, so it is a challenge for the traditional frequency regulation approaches. In frequency control area, to use ES is quite promising.
Some of the previous works have discussed the grid-connected energy storage and their impacts on power system frequency. Reference [9] proposes a new supplementary LFC method by using both battery energy storage and the additional cluster of mobile electric vehicles along with heat pump water heaters as controllable loads. The effectiveness has been tested by numerical simulation in the system with wind power and PV. In [10], the authors employ integrated energy storage to smooth the power output of the double fed induction generator (DFIG). The result tends to be limited because only one wind turbine and one embedded storage device are considered. In [11], the impact of the integration of battery energy storage systems on a short-term frequency control in autonomous micro grids is studied. However, as the author points out himself, it should consider “an adaptive control that optimally tunes the frequency controller parameters according to the changes occurring in the MG dynamics.” It is essentially the robustness of the system against parametric uncertainties.

In the existing literature, there are two important factors that are often disregarded. One is the parametric uncertainty due to the difficulties of precision measurement, and time-drifting character. The other is the time delay that ubiquitously exists in control channels. Both of them should be considered in practice because these inevitable phenomena could significantly degrade the system dynamic performance, especially in a relatively complex future grid environment. Although some of the literatures have discussed the time delay problem (e.g. [12]) in LFC application, the parametric uncertainties of the energy storage has not yet been addressed and analysed simultaneously in the designing process.

1.2.3 Distributed Consensus Optimization for Distributed Energy Resources

Having realized the potential adverse impacts that these renewables will pose on electricity grid especially distribution network, some researchers have proposed some possible
solutions to better manage and control these devices. Compared with the traditional centralized control in transmission networks, it is more challenging to utilize large numbers of these small distributed energy resources (DER) safely, effectively and economically in low voltage distribution networks compared with high-voltage transmission grid. In distribution networks, the grid tends to be more vulnerable without the support of synchronous generators and all the stabilizers and facilities in transmission network.

In the existing control patterns, DERs would need to be centrally controlled by the system operator to maximize the total profit. However, in practice, for geographically dispersed DER, the centralized dispatch mode will inevitably lead to the difficulty of transferring all individual DER information (including operating status, cost, and revenue) to the dispatch centre, huge investment cost for high-bandwidth communication channels between the centre and each DER. Besides, the ubiquitous time delay in the information transmission channel also makes it easy to get false/bad data, introducing the potential failure in control process. Thus, although the centralized approach is an easier way to obtain globally optimal solution, it is obviously impractical. On the other hand, the decentralized dispatch without communications can only achieve optimal decision results for each DER, but not a global optimum for the entire system. In order to implement a practical, flexible, and scalable multi-agent dispatch that require lower communication investment, a distributed dispatch scheme should be suitable for this practical problem. By coordinating individual decision-making of agents who control several DER, the optimal dispatch strategies can be achieved by only exchanging information between neighbouring DER, rather than sharing global information.

The mathematical theory of multi-agent and distributed consensus algorithm could be adopted to tackle this problem. Actually, the utilization of the distributed consensus algorithm
has been studied intensively by control theorists and the progress can be identified as follows. In [13], the author proposes a general framework for parallel and distributed computation over a set of processors. A wide range of references includes [14] tackling continuous-time consensus, [15]- [16] investigating discrete-time versions, and [17] where asynchronous implementation of consensus algorithms is discussed. The papers [18]-[20] treat randomized consensus via gossip communication, achieving consensus through quantized information, consensus over random graphs and average consensus through memory- less erasure broadcast channels, respectively. In [21], the authors solve a multi-agent unconstrained convex optimization problem through a novel combination of average consensus algorithms with sub-gradient methods. More recently, the paper [22] further takes local constraint sets into account.

Although distributed algorithms are relatively developed in theory, the application into industrial problem is still preliminary so far. Here are some applications of these distributed algorithms into power system analysis and their advantages and limitations: Reference [23] uses optimal power flow to compare the two methods and shows that intelligent distributed voltage and reactive power control of the DG gives similar results to those obtained by centralized management in terms of the potential for connecting increased capacities within existing networks. Therefore, if suitable incentive mechanism is designed, the distributed approach would be promising instead of using centralized one. Papers [24-25] propose a robust approach to coordinate customers’ resources and manage voltage rise in residential LV networks. The suggested coordination approach in this paper includes both localized control strategy, based on local measurement, and distributed control strategy based on consensus algorithm. The approach can completely avoid maximum permissible voltage limit violation. However, the drawback is that the author neglects the other constraints such as the power flow limit and the economic
objectives. The focus of paper [26] is to develop a distributed control algorithm of PVs in a
distribution network. To this end, the cooperative control methodology from network control
theory is used to make a group of PV generators converge and operate at certain (or the same)
ratio of available power, which is determined by the status of the distribution network and the
PV generators. The proposed control only requires asynchronous information intermittently from
neighbouring PV generators. It is shown that the analysis and design methodology has the
advantages that the corresponding communication networks are local, their topology can be time
varying, and their bandwidth may be limited. The utilization of energy storage is neglected in
this paper, which could be improved in future study. The paper [27] illustrates the feasibility of
implementing the multi-agent system scheme in the control/optimization problem. The prototype
problem considered is the dispatching of distributed generators on a distribution feeder to
provide voltage support. The popular Control Net Protocol (CNP) for Multi-Agent System (MAS)
has been adopted to facilitate distributed control. This paper illustrates that characterization of
the optimal solution is necessary in order to develop a protocol for the MAS to implement. It also
shows that MAS facilitates a model-free control procedure, as it can monitor the local
sensitivities. Test results show that the MAS-based control scheme is effective in obtaining the
solution for the prototype problem. Nonetheless, the MAS-based method needs fast
communication among the DGs in order to assure fast response during emergency conditions.
Ref. [28] analyses an extended multi-agent convex optimization problem in which the agents are
to collectively minimize a global objective function subject to a global inequality constraint, a
global equality constraint, and a global constraint set. The objective function is defined by a sum
of local objective functions, while the global constraint set is produced by the intersection of
local constraint sets. Specifically, the author studies two scenarios: one is with the absence of the
equality constraint, and the other one is that the local constraint sets are identical. Then he devises two distributed primal-dual sub-gradient algorithms. These algorithms can allow the agents to asymptotically agree on optimal solutions and optimal values of the optimization problem under the Slater’s condition. This approach is quite new and it seems that can be implemented into the power system scenarios as well. However, the strict requirement of the linear constraint in the definition limits its scope of application.

Drawing on these papers, it can be seen that the research on the application of distributed optimization algorithms on future grid with DER and ES is still preliminary so far. Hence continued research may further explore the development of the existing approaches and the combination with future power systems.

1.3 Contributions of the Thesis

The main contributions of this thesis are:

1. A comprehensive simulation platform for Australian future grid with a high penetration level RE scenario is built in DIgSILENT, which includes both steady state power flow scanning and dynamic stability analysis considering different RES penetration as well as DSM uptakes. Simulation experiments are performed to demonstrate the impacts of renewables on future power transmission system in several typical scenarios.

2. A novel control block diagram of the closed-loop LFC system with ES and wind power module has been proposed. Two important factors that exist in future power network, i.e. parametric uncertainty as well as ubiquitous time delay in control channel, are considered simultaneously.
3. The unpredictable nature and highly intermittent outputs of the RE sources require a much higher frequency adjusting ability of the future power system to accommodate them, so it is a challenge for the traditional frequency regulation approaches. The problem of how to maintain the frequency stability under a much more complicated environment should be studied. Based on mathematical LMI theory, a Robust H∞ controller is designed to deal with the problem. The effectiveness of the controller is tested in Matlab Simulink software package.

4. A sensitivity analysis based-Enhanced Optimal Distributed Consensus Algorithm (EODCA) that combines the theory of power systems and the concept of multi-agents is proposed. The effectiveness of the proposed algorithm is verified: it can help the power distribution company to maximize its net profits with very limited information channel built among devices instead of setting up a control centre. Furthermore, the optimal solutions can also guarantee the safety operation of the distribution network at the same time.

5. Based on the traditional ADMM theory in mathematics, I propose a Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) to solve the practical control problem in future power distribution networks. Compared with a latest version of the distributed optimization algorithm called DLPDS, we can see that MC-ADMM performs much better in terms of the speed, the accuracy and the ability to handle larger systems with more devices.

6. The influence of the communication network topology on the calculation performance is studied. Laplacian eigenvalue plays a crucial role and a conclusion is made that an optimum topology for communication network connections exists.

1.4 Organization of the Thesis
The thesis is structured in three parts that all relate to the future scenarios of renewable integration and energy storage utilization. Each part focuses on an aspect of this problem, namely: **Impacts on Power System with RE and ES (Chapter 2); Frequency Control with RE and ES (Chapter 3); Optimal Dispatch with RE and ES (Chapter 4-5)**. Following are the details of each chapter:

**Part 1- Impacts on Power System with RE and ES (Chapter 2)**

**Chapter 2**-constructs a comprehensive platform in the software DIgSILENT, based on which various scenarios of Australian future grid can be tested. Some typical situations including both static power flow scanning calculation and short circuit dynamic simulations (frequency, voltage and rotor angle behaviour) with/without ES are performed to identify the influence on the Australian transmission network under different levels of RE/ES sources penetration and different levels of DSM uptakes with multiple levels of granularity.

**Part 2-Frequency Control with RE and ES (Chapter 3)**

**Chapter 3**- develops the dynamic model of the grid-connected ES and the closed-loop frequency-domain block diagram. Then a novel controller is proposed based on rigorous linear matrix inequalities (LMI) theory to ensure the robustness and stability of the system, when taking into account the parametric uncertainty and random time delay in control channels in the complex environment. Comprehensive case studies are performed to demonstrate the effectiveness of the proposed Controller.

**Part 3-Optimal Dispatch with RE and ES (Chapter 4-5)**
Chapter 4- proposes a novel Enhanced Optimal Distributed Control Algorithm (EODCA) based on power system sensitivity analysis to regulate the scattered energy storage devices while the overall economic performance and network security constraints are considered. The comprehensive case study also demonstrates the effectiveness of the proposed method.

Chapter 5- proposes a Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) to demonstrate the effectiveness of accelerating the convergence speed. Afterwards, some guidance is given on how to design a better communication network by innovatively analysing the performance of network topology.

Chapter 6- Concludes the whole thesis and discusses the future work.
2. Future Grid Simulations in South Eastern Australia and the Impacts Analysis of High Renewable Penetration

Large thermal power plants have traditionally underpinned the whole operation of conventional power systems. The traditional planning and control models will be challenged by all new features of the future grid: renewable energy (RE) sources, distributed generation (DG), energy storage devices (ES) and new type of loads, etc. Meanwhile, other changes, such as new control mechanisms, will be applied to the demand side as well. In all the changes above, the role of modelling and analysis related to security for large numbers of scenarios remains of central importance. In this context, it inspired us to analyse the various scenarios under the future grid infrastructure based on the 14-Generator model for National Electricity Market (NEM) in Australia. In this Chapter, I firstly construct a platform in the software DIgSILENT Power factory, based on which various scenarios of Australian future grid network security can be simulated, then I choose some typical situations including both static power flow calculation without ES and short circuit dynamic simulations (frequency, voltage and rotor angle behaviour) to identify the influence on the Australian transmission network under different levels of RE sources penetration and different levels of DSM uptakes with multiple levels of granularity.

2.1 The Future Grid Modelling of South Eastern Australia in DIgSILENT

The starting point of my studies is the observation that to the best of my knowledge, no international research so far has developed/provided a standardized comprehensive modelling framework and platform for the Australian future grid close to what we have been accustomed to for the existing systems. We need to develop a comprehensive tool and platform that can reflect the real situation in the future that can also deal with the potential problems in future power system: e.g. power flow calculation, stability analysis, economic dispatch, security and reliability
simulation, etc. We use the analysis software DIgSILENT as providing us a satisfactory approach in modelling and tackling renewable energy generations.

2.1.1 The Simplified 14-Generator Model of the South-Eastern Australia Power System

The simplified 14-Generator, 50 Hz system model for the South-Eastern (SE) Australian grid is used as the base model here. For convenience, it has been divided into 5 areas which stand for different states and regions in Australia [29]:

- Area 1: Snowy Mountains
- Area 2: New South Wales
- Area 3: Victoria
- Area 4: Queensland
- Area 5: South Australia

Areas 1 and 2 are more closely coupled. Thus, there are in essence 4 main areas and hence 3 inter-area modes, as well as 10 local-area modes. Australian Electricity Market Operator (AEMO) has proposed this network to 16 zones for the NEM as shown in the Fig. 1.
Fig. 1 The Simplified 14-Generator Model of the SE Australia Power System

The more detailed parameter settings of the platform are given in the following tables.

The transmission line parameters are listed in the following table 1:

Table 1 Transmission Line Parameters: Values per Circuit

<table>
<thead>
<tr>
<th>From bus/to bus</th>
<th>Line No.</th>
<th>Line $r+jx; b$ $(p.u. \text{ on } 100\text{MVA})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>102 217</td>
<td>1,2</td>
<td>0.0084 0.0667 0.817</td>
</tr>
<tr>
<td>102 217</td>
<td>3,4</td>
<td>0.0078 0.0620 0.760</td>
</tr>
<tr>
<td>102 309</td>
<td>1,2</td>
<td>0.0045 0.0356 0.437</td>
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<td>102 309</td>
<td>3</td>
<td>0.0109 0.0868 0.760</td>
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The transformer ratings and reactance are demonstrated in the following table 2:

Table 2 Transformer Ratings and Reactance

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<th>Buses</th>
<th>Number</th>
<th>Rating, each Unit (MVA)</th>
<th>Reactance per transformer % on per unit</th>
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<td>555.6</td>
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The parameters of the 14 generators are listed in Table 3.

Table 3 Generator Parameters

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<th>MPS_2</th>
<th>VPS_2</th>
<th>LPS_3</th>
<th>YPS_3</th>
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<td>0.21</td>
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<td>1.70</td>
<td>1.70</td>
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</tbody>
</table>

With the given parameter settings of the electrical components, we can perform simulations with the platform to test different scenarios and cases in future grid environment.

2.1.2 The Enhanced Analysis Model with Renewables in Future Grid Scenarios
To make a model which can serve the purpose of the future study [30], the base 14-Generator Model was rebuilt in DIgSILENT environment with the following enhancements:

- The capability to add a solar thermal, PV or wind farm at all buses of the grid, including possible grid extensions;
- The traditional generator models have also been enhanced to allow a wider range of dynamic studies than just small-disturbance stability;
- The capability to include the demand-side representation of load, storage and demand-response at city level.

The modelling framework aims to be very flexible for considering scenarios of RE placement, randomly or from specific proposals, and optimizing which ones will provide the best overall performance. In this chapter, we illustrate this flexibility with additional RE at nodes in the four main areas. The traditional generators are replaced by RE sources with different percentages of their capacity to see their influence on system performance.

According to the practical data in Australia, we can select the following nodes to accommodate the renewable energy sources.

SA:  Node 501; NPS_5; Wind Farm Lake Bonney
VIC:  Node 301; LPS_3; Wind Farm Waubra
NSW: Node 204; MPS_2; Wind Farm Capital
QLD:  Node 401; TPS_4; Photovoltaic Power Station

The geographic positions of the four power stations are denoted in the Fig.2:
The scenarios are divided into two categories, i.e. 1) Power flow calculation under steady operation conditions and the 2) Transient analysis under short circuit conditions. Both of the two categories are performed within different penetration levels of sustainable energy sources.

Firstly, we divide daily load curve into three different levels, i.e. light load level (0 am-8 am, 22 pm-0 pm), medium load level (9 am-17 pm) and heavy load level (18 pm-21 pm) respectively. In each scenario, the penetration of the wind farm and photovoltaic varies from 0% to 40%. In different time points in a day, the power outputs of the wind farms and PV stations are drawn from the real-time data of AEMO online reports. From the practical power output data, we can convert the corresponding capacity factor of the different types of generators throughout the day. The following Figs. 3-4 is the practical real-time PV power output data and the corresponding capacity factor in a day. We can use the PV power output capacity factor (the
hourly PV power output data divided by the corresponding total capacity of the PV station) to show the PV power generation ability within a day at different time points, as illustrated in Fig. 3

Fig. 3 The Real-time PV Power Output within a Day
In each penetration level, we calculate the power flow in a day (24 time points) of the system with real-time varying load level and wind/PV power outputs to test whether the transmitted power are out of limit in transmission lines.

To simply represent the change of actual load within a day’s period, The daily load curve are divided into three different levels, i.e. light load level(0 am-8 am, 22 pm-0 pm), medium load level (9 am-17 pm) and heavy load level (18 pm-21 pm) respectively. In each scenario, the wind and photovoltaic farms replace traditional generators in 10%, 20%, 30% and 40% of their capacity at each specified nodes, namely Node 501, Node 301, Node 204 and Node 401. By calculation, the penetration levels are 3.23%, 6.46%, 9.68% and 12.92% accordingly in terms of the energy generated.

In different time points of a day, the power outputs of wind/PV farms are drawn from the real-time data of online reports [31] (Wind Farm Performance and University of Queensland Solar Photovoltaic Data) and scaled to different scenarios. From the practical power output data, the corresponding capacity factors of different generators have been converted throughout the day.

2.1.3 Demand Side Management Model with Energy Storage Integrated

The demand side management model is also analysed in this study under future grid scenario. In order to do that, firstly, the market simulation is done by PLEXOS, which is a well-known market simulation software. The scenario is based on the dispatch process from AEMO. The model is designed based the residential level then granulated into city levels of the NEM future grid model according to the requirement for different study purposes. Specifically
speaking, the model that we designed in a decision making optimization programming where the
decision can be computed at each household in order to achieve the objective of minimizing
electricity bills. Mathematically, the model can be described as an optimization problem where
the objective function and constraints can be described as follows,

\[ \min \quad C_i = \sum_{k=1}^{K} \lambda(k)(A_i^k + E_i^k - P_{pv,i}^k) \]  

\[ s.t. \quad A_i^k \in [m_i^k, M_i^k] \]  

\[ E_i^k = SOC(k+1) - SOC(k) \]  

\[ Q_i^- \leq E_i^k \leq Q_i^+ \]  

\[ P_{pv,i}^k \geq 0 \]  

Where \( C_i \) denotes the electricity bill of the user \( i \) for the studies time period, normally a
day and \( K=24 \). Here is the list of denotation:

\( \lambda(k) \): the rate of purchasing electricity from/selling electricity to the grid at time slot \( t \);

\( A_i^k \): the electricity consumption of all appliances of user \( i \) within time slot \( k \);

\( P_{pv,i}^k \): the photovoltaic system generation of user \( i \) within time slot \( k \);

The constraint \( A_i^k \in [m_i^k, M_i^k] \) shows that the total consumption of household \( i \) in time
slot \( k \) should between the minimum and maximum consumption value;

\( E_i^k = SOC(k+1) - SOC(k) \) and \( Q_i^- \leq E_i^k \leq Q_i^+ \) represent that constraints on the state of charge
$SOC(k)$ and maximum charge/discharge rate $Q_i^-$ and $Q_i^+$ respectively. The term $A_i^k + E_i^k - P_{pv,i}^k$ actually equals to the amount of electricity that is exchanged with the grid from user $i$ in time slot $k$, denoted by $P_{g,i}^k$. Therefore it indicates that the user $i$ is purchasing the electricity from the grid at time slot $k$ when $P_{g,i}^k > 0$; and the user $i$ is sending back the electricity to the grid when $P_{g,i}^k < 0$. The price is assumed to be Time of Use (ToU) and the consumers are only considered as the price taker at this stage.

Later this demand side model is granulated into the city level in order to analyse the scenarios when the demand side management is considered in the future grid stability study.

### 2.2 Steady-State Calculation and Analysis

The primary analysis tool for steady-state operation is power flow analysis, where the voltages (magnitude and phase), line power flows and losses in the system are determined. This analysis is widely used for both operation and planning studies throughout the system i.e. both transmission and distribution systems.

#### 2.2.1 Renewable Thriving

It is well known that Australia has an abundance of renewable energy resources, such as wind, solar, geothermal etc. The CSIRO Australia has formulated several scenarios for the Australian grid in 2050 [32]. Among these one called “Renewables Thrive” represents the increasing to high usage of renewable energy sources. The purpose of this study that is to reveal with the integration of the DG and ES (and residential sized), how could the impact of ‘renewable thriving’ under different penetration level of RE.
Therefore, the scenarios are formulated at the same location under different level of renewable penetration and then compared to find out whether the power flow of the system is converged or not.

Fig. 5 The Power Flow Distribution under 6.46% Renewables without Energy Storage
The Fig. 5-6 shows the power flow on inter-connected transmission line under the RE penetration level of 6.46% and 12.92% without ES respectively and Fig. 7 demonstrates the none-convergence period becomes longer with the penetration level increases. It can be seen that under higher penetration level of renewables without energy storage integrated, the power flow cannot converge in certain periods, i.e. 11:00-21:00. It is because the intermittent characteristic of these renewables may incur imbalance between active power generation and consumption.

2.2.2 Rise of Prosumers and Energy Storage Devices
A second scenario in [32] is called “Rise of Prosumers”. Prosumers are the residential customers who also supply their own electricity. The sustained high retail prices of electricity, falling costs of solar roof-top panels and increasingly innovative financing and product packaging from energy services companies leads to large increases in the scale of on-site generations. In this study the demand-side model has been aggregated into city level. Parameters in this model can capture the percentages of uptake for the technologies involved.

Fig. 8 The Power Flow Distribution under 12.92% Renewables with 40% Energy Storage
The Fig. 8-9 shows when there is accumulated energy storage integrated into some critical nodes (such as Sydney city), the none-convergence problem of power flow could be dramatically alleviated, which can be seen from Fig. 6 and Fig. 8 that the power flow non-converge period time is reduced significantly. Also with the penetration level of energy storage increases (From Fig. 8 to Fig. 9), the supporting function also strengthens, making the non-converge time shorter (from 2 hours to 1 hour). Therefore, the optimal dispatch of energy storage can indeed help the system survive from the harsh situations in the future with high penetration level of intermittent renewable resources.

2.2.3 Impact of Location of the Distribution RE and ES
The simulation also concludes that with the same energy storage modelling, same renewable energy generation penetration level and the same energy storage level, the impact of the energy storage to the grid could vary according to the location of the load.

Fig. 10 The Power Flow Distribution under 12.92% Renewables with 80% ES at Node 212
It can be seen that when energy storage devices are integrated in node 509 (remote load), power flows in 8 pm converge. On the other hand, as shown in Fig. 10, when the energy storages are placed in node 212 (load centre), the power flows are not converged at 8 pm. This could indicate with same level of renewable energy penetration and ES uptake, the stability diverse for different locations.

2.3 Dynamic Simulation and Analysis

A fault such as a common short circuit event in a power system leads to a dynamic condition. Such an incident can start a variety of different dynamic phenomena in the system such as instability and/or oscillations. Hence studies with various dynamic models in future grid are essential as well. Specifically, power system dynamic analysis deals with the power system response to small and large disturbances. This usually relates to dynamics of the system around some equilibrium point (possibly quasi-static). The steady-state analysis of these dynamic equilibrium using dynamic power flow models and associated indices can demonstrate the dynamic behaviour of the grid a lot. Thus, the security scanning tools and framework are based on steady-state and dynamic analysis. For each scenario considered, the framework can be used to make comprehensive security assessment through diverse scanning tools and finally, the assessment report can be generated.

Power system dynamic analysis aims to analyse the ability of an electric power system to regain a state of operating equilibrium after a physical disturbance for a given initial operating condition [34]. The basic security requirement after balancing (energy, power and ramping) is
maintaining adequate stability margins (angle, voltage, frequency) for specified contingencies. Power system stability for classical grids can be divided into three types [34], i.e. voltage stability, rotor angle stability and frequency stability.

In this section, the preliminary results of the RE integration on system stability are presented. The dynamic analysis is done under different RE penetration levels without considering ES in this chapter whilst the control and dispatch methodologies of ES will be discussed in the following chapters from the frequency, voltage and power flow perspectives. As mentioned above, different stabilities have been simulated and assessed in DlgsILENT.

2.3.1 Voltage Stability with RE integration

Voltage stability [34] analysis can be classified as large disturbance voltage stability and small disturbance voltage stability. Large disturbance voltage stability refers to the system’s ability to maintain steady voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. Determination of large-disturbance voltage stability requires the examination of the nonlinear response of the power system over a period of time sufficient to capture the performance. Small disturbance voltage stability refers to the system’s ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. This form of stability is influenced by the characteristics of loads, continuous controls, and discrete controls at a given instant of time.

In my research, voltage stability analysis is studied with both large disturbance and small disturbance, where a short circuit is selected for the large disturbance and a reduction on the system load is used for the small disturbance analysis.
It can be seen from Fig. 12 that generally speaking, as the penetration level of RE increases, the voltage dynamic performance deteriorates significantly. The reason is that most of the REs are inverter-based and running in the condition of constant power factor. When the penetration level increases, i.e. the active power injected soars, the compensation reactive power derived from the power electronic circuits are insufficient to support the voltage stability. Therefore, the voltage tends to be more vulnerable and susceptible to disturbances when more REs are connected to the power system.

![Fig. 12 Voltage Stability with Different RE Penetration under Small Disturbance](image)

(a) 6.46% RE Penetration (b) 12.92% RE Penetration

**2.3.2 Rotor Angle Stability with RE integration**

Rotor angle stability [34] refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. It
depends on the ability to maintain/restore equilibrium between electromagnetic torque and mechanical torque of each synchronous machine in the system.

Compared with voltage stability, the RE does not in this case show much impact on rotor angle stability. Even though there are some wings after the disturbance, they are still acceptable value and the system can achieve to a new stable situation. We can see the dynamic behaviour is insensitive with regard to the increasing penetration level of RE, but further studies would be needed.

Fig. 13 Rotor Angle Stability with Different RE Penetration under Small Disturbance

(a) 6.46% RES Penetration (b) 12.92% RES Penetration

2.3.3 Frequency Stability with RE integration

According to the standard definition [34], frequency stability refers to the capability of an inter-connected power system to maintain steady frequency around the rated value after a significant active power imbalance between generation and load. It depends on the ability to
maintain/restore equilibrium between system generation and load, with minimum unintentional loss of load. Instability occurs in the form of sustained frequency swings leading to the tripping of generating units and/or loads.

Therefore, in the dynamic study, we test the system frequency behaviour after a disturbance caused by load step change. The Fig. 14 shows the result.

![Fig. 14 Frequency Stability with Different RE Penetration Level under Small Disturbance](image)

(a) 6.46% RE Penetration (b) 12.92% RE Penetration

It can be seen that the frequency stability of the system decreases with the RE penetration level increases under disturbance and the final value of frequency is also worse.

However, another interesting and noteworthy phenomenon is that under higher RE penetration level, the decreasing speed (dynamic process) becomes faster compared with lower level situation. This is caused by the inverter-based RE reducing the rotation inertia of the system compared with conventional rotating generators so that the active power balancing process is accelerated. Since the energy storages are also connected to the system with power
electronic based equipment, it inspires us to design an appropriate controller to utilize this characteristic to assist the traditional AGC units to maintain the frequency stable in future grid scenarios with high penetration level of intermittent RE to balance the active power. This is also the central discussion topic in Chapter 3.
3. Robust $H_\infty$ Load Frequency Control (LFC) of Future Power Grid with Energy Storage Devices

As illustrated in chapter 2, the integration of RE will inevitably cause adverse impacts on both system static power flow distribution and dynamic performance such as jeopardizing the frequency and voltage stability [35]. In this chapter, I will focus on the frequency perspective. Unlike the voltage quality which is a local power issue that has a close relationship with the reactive power injection or absorption, the frequency of a certain area implies a global performance of the active power equilibrium between the total generation and consumption. The unpredictable nature and highly intermittent outputs of the RE sources require a much higher frequency adjusting ability of the future power system to accommodate them, so it is a challenge for the traditional frequency regulation approaches. Meanwhile, the utilization of energy storage (ES) can play a positive and noteworthy role in dealing with the above problems. Firstly, the dynamic model of the grid-connected ES and the closed-loop frequency-domain block diagram is developed. Then a novel controller is proposed based on linear matrix inequalities (LMI) theory to ensure the robustness and stability of the system, when taking into account parametric uncertainty and random time delay in control channels in the complex environment. The system can utilize ES to participate in Load Frequency Control (LFC) process so as to assist conventional thermal units. Finally, comprehensive case studies are performed to demonstrate the effectiveness of the proposed methods.

3.1 Dynamic Modelling of the Grid-Connected Battery Energy Storage

There are many different kinds of ES that can be utilized in power system operation. In the literature [3], the merits and limitations of available choices of storage devices are studied.
We can conclude that for high power grid-connected storage level (sec/min), the Lithium-ion battery energy storage (BES) remains the best compromise between performance and cost.

In practice, the ESA (Energy Storage Association) of USA has given the guidance of using energy storage in frequency regulation. It points out that in the group of “ancillary services” provide in the open market management of the grid, frequency regulation has the highest value, which typically takes minutes and battery electricity storage has the capacity for doing this job in milliseconds, making it an ideal and competitive choice. The practical examples are various, such as the AES Energy Storage Angamous Battery Energy Storage System (BESS) in North Chile; the AES Energy Storage Laurel Mountain Battery Energy Storage in Belington; the AES Los Andes Battery Energy Storage System (BESS) in Atacama; the Kaheawa Windfarm Dynamic Power Resource (DPR) Energy Storage on the island of Maui, etc. These are all successful examples of using grid-connected energy storage devices to help the power system to operate more reliable and economical.

The following battery photo Fig.15 is an example: it shows the 12MW A123 lithium battery at AES Gener’s Los Andes substation in the Atacama Desert in Chile used for frequency regulation [36].
Fig. 15 The Lithium Battery Storage in Chile

The equivalent circuit of the battery ES can be represented by a DC/AC converter connecting an equivalent battery, which is composed of a set of batteries in parallel/series connection as shown in Fig. 16[37].

Fig. 16 Equivalent Circuit of Battery Energy Storage
The battery can provide both active power and reactive power as the power system requires. In order to enable the battery energy storage being easily used in the stability analysis and controller design, the model should be transformed into the frequency domain, which has been derived in the previous work [37]. The active power increment $\Delta P_{ES}$ transferred through the converter is controlled relying on the system measured frequency deviation, which can be expressed as:

$$\Delta P_{ES} = \frac{K_p}{1+sT_{ES}} \cdot \Delta f$$  \hfill (6)

### 3.2 Traditional PI Controller and its Limitations

The system cannot work at all without appropriate controllers. The conventional PI control scheme shown in Fig. 17 is one of the most widely used approaches to regulate the frequency in practical interconnected power systems.

![Fig. 17 Conventional PI LFC Controller](image)

However, the traditional PI controller has quite a few limitations: most importantly, it cannot tackle these two important factors in future grid scenarios:
➤ One is the parametric uncertainty due to the difficulties of precision measurement, time-varying characters under different working conditions and the lack of sufficient knowledge to describe some certain system parameters in an accurate value;

➤ The other factor is the ubiquitous time delay that exists not only in the control channels but also in the data propagation, switching, accessing and queuing process.

Both of them should be considered in practice because these inevitable phenomena could significantly degrade the system dynamic performance. Besides, with one set of PI coefficients, the system would lose the stability when time delay accumulates to a certain level or system parameter variation occurs. Therefore, the robustness of the power grid cannot be guaranteed in the complex environment in the future when we consider the cyber-physical power system. Although some of the references have discussed the time delay problem (e.g. [12]) in LFC application, the parametric uncertainties of the energy storage has not yet been addressed and analysed simultaneously in the design process.

In the following sections, a novel $H_\infty$ Controller is designed based on Linear Matrix Inequalities (LMI) Theory to help to solve this problem.

The reason to choose $H_\infty$ control is that it is readily applicable to problems involving multivariate systems with cross-coupling between channels, and $H_\infty$ techniques can also be used to minimize the closed loop impact of a perturbation, which is quite suitable for tackling the problem discussed in the thesis. Besides, the $H_\infty$ controller has the rigorous mathematical foundation to guarantee its effectiveness theoretically, and the commercial software is also available to support $H_\infty$ controller synthesis, making it an ideal and promising approach in the future.
3.3 Closed Loop Diagram of the LFC System with ES

3.3.1 Block Diagram of the System with Energy Storage

A two-area system is considered here, as shown in Fig. 18 where the topology of the New England 39-bus power system has been augmented with energy storage integrated.

Fig. 18 The Topology Structure of the IEEE 10-Unit 39-Bus Two-Area Test System (with BES and WP)
For the purpose of committing LFC, a comprehensive model has to be built. The grid-connected battery ES station together with conventional AGC units are controlled together to change their injection power so as to follow the load disturbances. Without loss of generality, the block diagram for above mentioned two-area system is abstracted from Fig. 18 to Fig. 19:

Fig. 19 The Block Diagram of the LFC System with Battery Energy Storage, Wind power Module and Time Delay

Here, the AGC units are lumped as a single equivalent generating component separately for the purpose of analysis. We assume that there is one battery ES in area 1 and two in area 2. The method proposed below is also applicable if more energy storage devices are integrated. The
active power unbalance could either come from load change, or from the fluctuation of REs such as wind turbine output. Note that the network induced time delay $\tau(t)$ exists in some parts of the communication channels and the parametric uncertainty is also considered here, which will be discussed in mathematical modelling later.

### 3.3.2 Analysis of Parametric Uncertainty and Time Delay

In order to make an effective design, the differences between the simplified model and the real objects are called the model uncertainties. In addition to the uncertainties that emerge when simplifying the model, because of the lack of sufficient knowledge, difficulties on precision measurement would also introduce model uncertainties [38]. For example, the uncertainty of some parameters, such as the time constants of generators and ES, are very difficult to be described in a precise value. Furthermore, most likely these parameters are time-varying according to different conditions in the process of operation. Then, the actual system model should be described with a linear model plus a time-varying uncertainty part. The linear part is called the nominal model of the system, while the time-varying part reflects the reality of the system in practical situation. Generally, the parameters vary in a specific bounded area.

Another important factor that affects the system dynamic performance is the ubiquitous time delay that is scattered in the control network. In particularly in future smart grid environment, the widely usage of general-purpose communication network (e.g. the Internet or Ethernet) would make the network-induced delay non-negligible. Thus it can significantly degrade the system performance.

The main reasons for the network-induced delay are as follows:
1) Data queue waiting time. When the network is busy or the collision of the packet happens, the data has to wait for sending until the network is idle again.

2) Information generation. At transmitting end, the information has to wait for packaging to enter the network.

3) Transmission. The transmission time for the data in the medium depends on the packet size, the bandwidth and transmission delay.

Table 4 lists some of the typical access control modes and their corresponding delay characteristics and parameters.

<table>
<thead>
<tr>
<th>Network</th>
<th>Ethernet</th>
<th>CAN Bus</th>
<th>TP Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access Mode</td>
<td>Random Access</td>
<td>Random Access</td>
<td>Token Passing</td>
</tr>
<tr>
<td>Network</td>
<td>IEEE 802.3</td>
<td>CSMA/AMP</td>
<td>IEEE 802.4</td>
</tr>
<tr>
<td>Protocol</td>
<td>CSMA/CD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Packet</td>
<td>Maximum 1500</td>
<td>Maximum 8</td>
<td>Maximum 504</td>
</tr>
<tr>
<td>Delay</td>
<td>Random; Unbounded</td>
<td>Random; Bounded</td>
<td>Random; Bounded; Periodic</td>
</tr>
</tbody>
</table>

As is shown in Table 4, the delay characteristics are different in different forms of control networks. The following research is for Ethernet, but the methodology and framework also apply to other control networks. Although Ethernet has its advantages of high data transfer rates, low power consumption, easy installation and compatibility, the disadvantages, however, are also
quite obvious. When the serious heavy load condition happens, the time delay is random and unbounded, which puts a relatively higher requirements for maintaining the stability and robustness of the control method and system.

Without loss of generality and for the purpose of analysis, we simplify the distributed network-induced delays as a single delay module that exists in parts of the control channels, which can be used for simulating the impacts on the system frequency dynamic performance.

3.3.3 The State-Space Formulation of the Proposed System with Parametric Uncertainty and Time Delay

Based on the closed-loop diagram and the above analysis, the state-space equations with parametric uncertainty and time delay can be mathematically derived as follows through linearization near the steady operating point of the system for small disturbance stability analysis:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau(t)) + (B_2 + \Delta B_2(t))u(t) + B_1w(t) \\
z(t) &= Cx(t) + D_{11}w(t) + D_{12}u(t)
\end{align*}
\]

(7)
\[
A = \begin{bmatrix}
-\frac{D_1}{M_1} & 0 & -\frac{1}{M_1} & 0 & 0 & -\frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{R_i T_{G1}} & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{T_{CH1}} & -\frac{1}{T_{CH1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{K_{b1}}{T_{ES1}} & 0 & 0 & -\frac{1}{T_{ES1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
T & 0 & 0 & 0 & 0 & 0 & -T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{M_2} & -D_2 & 0 & \frac{1}{M_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{R_i T_{G2}} & -\frac{1}{T_{G2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{CH2}} & -\frac{1}{T_{CH2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & b_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{K_{b2}}{T_{ES2}} & 0 & 0 & 0 & -\frac{1}{T_{ES2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{K_{b3}}{T_{ES3}} & 0 & 0 & 0 & 0 & -\frac{1}{T_{ES3}} \\
\end{bmatrix}
\]

\[
A_d = \begin{bmatrix}
0 & 0 & 0 & \frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & \frac{1}{M_2} & \frac{1}{M_2} \\
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{T_{G1}} \\
0 & 0 \\
\vdots & \vdots \\
0 & 0 \\
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{T_{G2}} \\
0 & 0 \\
\vdots & \vdots \\
0 & 0 \\
\end{bmatrix}
\]
Here,

the system state variables $X=[x_1, \ldots, x_{12}]^T=[\Delta f_1, \Delta Y_1, \Delta P_{G1}, \Delta P_{ES1}, \int ACE_1 dt, \Delta P_{tie}, \Delta f_2, \Delta Y_2, \Delta P_{G2}, \int ACE_2 dt, \Delta P_{ES2}, \Delta P_{ES3}]^T$.

Where $\Delta f_i (i=1,2)$ is the frequency;

$\Delta Y_i (i=1,2)$ is the valve opening variation of AGC units;

$\Delta P_{Gi} (i=1,2)$ is the active power variation of traditional AGC units;

$\Delta P_{ESi} (i=1,2,3)$ is the power variation of ES;

$\int ACE_i dt (i=1,2)$ is the integral of $ACE$ signals as this ensures zero steady-state error to step-change load disturbances;

$\Delta P_{tie}$ is the power variation of the tie line that connects two areas.

The disturbance variables $w(t)=[w_1, w_2]^T=[\Delta P_{L1}, \Delta P_{L2}]^T \in L_2[0, \infty)$, i.e. square-integrable;

Control variables $u(t)=[u_1 u_2]^T$;

Control output variables $z(t)=[z_1, z_2]^T=[ACE_1, ACE_2]^T$.

The system matrix $A, A_d, B_2, B_1, C, D_{11}, D_{12}$ are real-valued constant matrices of appropriate dimensions that describe the nominal system, $\Delta A(t), \Delta A_d(t), \Delta B_2(t)$ are real time-varying matrix functions representing parametric uncertainty. $D_i (i=1,2)$ is the load frequency characteristic coefficient; $M_i, R_i, T_{Gi}$ and $T_{Chi} (i=1,2)$ are the inertia time constant of the AGC
units, the speed regulation due to governor action, the governor time constant, and the turbine
time constant respectively; $K_{bi}$ and $T_{ESi}$ ($i=1,2,3$) are frequency response coefficients of ES and
equivalent time constant of the batteries respectively; $T$ is the tie line synchronization coefficient;
$\tau(t)$ is the equivalent time delay that satisfying:

$$0 \leq \tau(t) < \infty, \dot{\tau}(t) \leq \rho < 1$$

(8)

With regard to the parametric uncertainty, it is assumed that they can be modelled as:

$$[\Delta A(t) \Delta A_d(t) \Delta B_2(t)] = HF(t)[E_1 E_2 E_3]$$

(9)

Where $H$, $E_1$, $E_2$, $E_3$ are known as real constant matrices, and $F(t)$ is an unknown time-
varying uncertainty matrix function that satisfying

$$\|F(t)\| \leq 1, \forall t \in [0, +\infty)$$

(10)

The matrices $H$, $E_1$, $E_2$, $E_3$ specify how the uncertain parameters in $F(t)$ affects the
nominal matrix of system (7), which implies that they essentially reflect the structure of the
system. In this study, we assume that the variation ranges of the following parameters are:

$$\begin{align*}
\frac{1}{T_{Gi}} & \in \left[\frac{1}{T_{Gi}}, \frac{1}{T_{Gi}} + \Delta_{Gi}\right], i = 1, 2 \\
\frac{1}{T_{CHj}} & \in \left[\frac{1}{T_{CHj}}, \frac{1}{T_{CHj}} + \Delta_{CHj}\right], j = 1, 2 \\
\frac{1}{T_{ESk}} & \in \left[\frac{1}{T_{ESk}}, \frac{1}{T_{ESk}} + \Delta_{ESk}\right], k = 1, 2, 3
\end{align*}$$

(11)

Through the matrix decomposition, we can acquire:
\[ H = \begin{bmatrix}
0 & \Delta_{G1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta_{CH1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Delta_{ES1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta_{G2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Delta_{CH2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \Delta_{ES2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_{ES3} & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ E_J = \begin{bmatrix}
-1/R_i & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_{R_i} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1/R_i & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{E_2} & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{E_3} & 0 & 0 & 0 & -1 \\
\end{bmatrix} \]

\[ E_2(i,j)=0; (i\in\{1,2,\ldots,5\}, j\in\{1,2,\ldots,12\}) \]

\[ E_J = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

3.4 Controller Design based on LMI Theory

3.4.1 Summary of the LMI Methods

The system cannot work at all without a suitable controller. Since Zames first proposed \( H_\infty \) robust control method in 1981, this problem has aroused tremendous attention and developed rapidly. Not only does the \( H_\infty \) robust control have a rigorous mathematical foundation, but also the controller has been proved to be a great success in industry application.

A control system can be abstracted as the generalized object \( G(s) \) that will be controlled by \( K(s) \) as shown in Fig. 20.
The goal of the $H_\infty$ robust control is to design a real regular function controller $K(s)$ to ensure internal stability of the generalized object. In other words, $K(s)$ has to internal stabilize the whole system first, then inhibit the input disturbance $w(t)$ optimally. Here, "internal stability" is a term in robust control theory, which guarantees all the signals are bounded in the system given the fact that the input signals are bounded wherever they connected into the system. Under this constraint, the controller has to minimize the input disturbance, i.e. guarantees the transfer function (from the disturbance input $w(t)$ to control output $z(t)$) $G_{wz}(s)$’s $H_\infty$ norm would not exceed a pre-given small positive constant $\gamma$, which means $\|z(t)\|_2 < \gamma \|w(t)\|_2$, $\forall w \in L_2[0, +\infty)$ under the zero initial condition $x(t) = 0$ ($t \in [-d, 0]$). Here, $x(t)$ is the system state; $d$ is the delay time; $\|z(t)\|_2$ is the signal’s $L_2$ norm, which essentially represents the energy of the signal and can be mathematically defined as $\|z(t)\|_2 = \|Z(j\omega)\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \text{tr}(\dot{Z}^*(j\omega)\dot{Z}(j\omega))d\omega}$. $L_2[0, +\infty)$ denotes a set of the time-domain signals that are $L_2$ norm bounded. The controller obtained at this time is called a suboptimal robust $H_\infty$ controller, with $\gamma$ expresses the capability of the whole system to suppress the external input interference $w(t)$ and is called “interference suppression degree”. The smaller $\gamma$ is, the better robust performance and stability of the system will have. If we solve the
optimization problem to minimize $\gamma$ so as to get the smallest value of the interference suppression degree, then the controller obtained at this time is called optimal robust $H_\infty$ controller.

In this LFC problem, the $H_\infty$ control method has to be used for time delayed system after concerning the parametric uncertainties. The traditional robust control often deals with this problem by solving algebraic Ricatti equation or solving the simplified Lyapunov equation. However, this approach is difficult to calculate, especially the second Ricatti equation is difficult to converge when solving two simultaneous Ricatti equations, which limits the application scope of this method greatly.

In recent years, with the development of the linear matrix inequality (LMI) theory, especially after the propose of interior point method and MATLAB-LMI Tool Box, LMI-based controller design has drawn more and more attention from control engineers, and it has been successfully applied in the practical systems.

The delay-independent and delay-dependent problems are two different issues and have to adopt different methods when analysing the stability character of the system. As mentioned above, this study is based on Ethernet so the controller design in the delay-independent situation is focused and deeply studied. In general, the stability conditions are more conservative under delay-independent situation, because the system can be stable with any large time delay as long as it satisfies delay-independent stability conditions, which nevertheless could be too strict for general small-delay system. With regards to the delay-dependent problem, it refers to the stability character of the system depends on the length or other characteristics of the delay, namely it depends on the length of time delay itself, thus is called delay-dependent design. For
some small-delay systems that the upper bound of the delay has been identified, the controller design could be simplified. However, there are quite a few merits for delay-independent controller; for example, it allows the delay to be uncertain, random and unknown, which means that the system can be stable without knowing the accurate information of the time delay, as well as guaranteeing the robustness of the system with any uncertain situations. As the future smart grid is wide-area distributed and the structure of the grid varies a lot geographically, it would be better to consider the more severe and extreme situation. Therefore, the following discussion mainly focuses on the controller design under delay-independent conditions, i.e. assumes that the time delay is random and unbounded.

3.4.2 LMI Based Design of the Robust Controller

It is better to design a full-state feedback controller, because the more states we can get, the better the dynamic performance of the system will be. The $H_\infty$ part of the controller is depicted as Fig. 21:

![Diagram of H_\infty Part of Full-State Feedback Control](image)

Fig. 21 The $H_\infty$ Part of Full-State Feedback Control

In order to design the controller, a lemma should be introduced first:

Lemma 1[39]: For a system with time delay:
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + A_d x(t - \tau(t)) + Bw(t) \\
z(t) &= Cx(t) + Dw(t)
\end{align*}
\] (12)

And a pre-given constant $\gamma$, assuming the network delay $\tau(t)$ satisfies the condition (8), then if there exist two positive definite matrixes $P$ and $S$ such that the following LMI holds ($<0$ indicates the matrix is negative definite):

\[
\begin{bmatrix}
A^T P + PA + S & PB & C^T & PA_d \\
B^T P & -\gamma I & D^T & 0 \\
C & D & -\gamma I & 0 \\
A_d^T P & 0 & 0 & -(1 - \rho)S
\end{bmatrix} < 0
\] (13)

Then the system is robust stable, and the interference suppression degree character is $\gamma$. This lemma is used to determine whether the designed system has the $H_\infty$ performance.

**Lemma 2**[40] Given the matrix $Y$, $D$, $E$ with appropriate dimensions where $Y$ is symmetrical, then

\[
Y + DFE + E^T F^T D^T < 0
\] (14)

For all the matrix $F$ that satisfies (10), if and only if there exists a constant $\epsilon > 0$, such that

\[
Y + \epsilon DD^T + \epsilon^{-1} E^T E < 0
\] (15)

**Lemma 3 (Schur Theorem):** Given the symmetric matrix $S \in R^{n \times n}$, $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, in which the $S_{11} \in R^{r \times r}$, $S_{12} \in R^{r \times (n-r)}$, $S_{21} \in R^{(n-r) \times r}$, $S_{22} \in R^{(n-r) \times (n-r)}$, then the following three conditions are equivalent:

\[a) \ S < 0;\]
b) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

c) $S_{22} < 0, S_{11} - S_{12}^T S_{22}^{-1} S_{12}^T < 0$.

Here we propose a novel theorem that can be used to design the full-state feedback $H_\infty$ controller based on the above mentioned three lemmas. With this approach, the $\gamma$ robust performance can be guaranteed both with the parametric uncertainties bounded by (9)-(10) and time delay subjected to (8).

**Theorem 1:** consider a system (7) that the time delay satisfies (8), parametric uncertainties are bounded by (9)-(10) and $(A, B_2, C_2)$ is stabilized and can be detected. Then for a pre-given positive constant $\gamma$, we can design a memoryless full-state feedback controller $u(t) = Kx(t)$ if there exist a positive scalar $\varepsilon$, two positive definite matrixes $X, Q$ and a matrix $Y$ such that the LMI (16) holds.

In particular, if $(X, Y, Q)$ is a feasible solution of (16), $u(t) = YX^{-1}x(t)$ is one of the $\gamma$-suboptimal full-state feedback controllers of the system. This problem can be solved using LMI Toolbox [41].

\[
\begin{bmatrix}
\Psi & B_1 & (CX + D_{12}Y)^T & A_d & (E_1X + E_2Y)^T \\
B_1^T & -\gamma I & D_{11} & 0 & 0 \\
CX + D_{12}Y & D_{11}^T & -\gamma I & 0 & 0 \\
A_d^T & 0 & 0 & -(1 - \rho)Q & E_2^T \\
E_1X + E_2Y & 0 & 0 & E_2 & -\varepsilon I
\end{bmatrix} < 0
\] (16)

Where

\[
\Psi = AX + B_2Y + (AX + B_2Y)^T + Q + \varepsilon HH^T
\] (17)
Proof: From (7) and $u(t)=Kx(t)$, we can get:

\[
\begin{align*}
\dot{x}(t) &= \bar{A}x(t) + \bar{A}_d x(t-\tau(t)) + \bar{B}w(t) \\
z(t) &= \bar{C}x(t)+\bar{D}w(t)
\end{align*}
\]  

(18)

Where, $\bar{A} = (A + \Delta A + B_2K + \Delta B_2K)$; $\bar{A}_d = (A_d + \Delta A_d)$; $\bar{B} = B_1$; $\bar{C} = C + D_{12}K$; $\bar{D} = D_{11}$.

Apply the system (18) into the lemma 1, we can get the robust stable conditions of the system is that the following LMI holds:

\[
\begin{bmatrix}
\bar{A}^T P + P\bar{A} + S & PB & \bar{C}^T & P\bar{A}_d \\
B^T P & -\gamma I & D^T & 0 \\
\bar{C} & D & -\gamma I & 0 \\
\bar{A}_d^T P & 0 & 0 & -(1-\rho)S
\end{bmatrix} < 0
\]  

(19)

Replace the relevant parameters, we can get:

\[
Y + DFE + E^TF^TD^T < 0
\]  

(20)

Where

\[
Y = \begin{bmatrix}
(A + B_2K)^T P + P(A + B_2K) + S & PB_1 & (C + D_{12}K)^T & PA_d \\
B_1^T P & -\gamma I & D_{11}^T & 0 \\
C + D_{12}K & D_{11} & -\gamma I & 0 \\
A_{\bar{d}}^T P & 0 & 0 & -(1-\rho)S
\end{bmatrix} ;
\]

(21)

\[
D = \begin{bmatrix}
PH \\
0 \\
0 \\
0
\end{bmatrix} ; E = \begin{bmatrix}
(E_1 + E_2K)^T \\
0 \\
0 \\
E_2^T
\end{bmatrix}
\]

According to lemma 2, there exists a constant $\varepsilon>0$, such that $Y + \varepsilon DD^T + \varepsilon^{-1}E^TE < 0$, i.e.
Where \( \Xi = (A + B_2K)^T \, P + P(A + B_2K) + S + \varepsilon PHH^TP + \varepsilon^{-1}(E_1 + E_3K)^T (E_1 + E_3K) \)

With lemma 3, we can prove the (17) equals to:

\[
\begin{bmatrix}
\Xi & P B_i & (C + D_{12}K)^T & PA_d & \varepsilon^{-1}(E_1 + E_3K)^T E_2 \\
B_i^T P & -\gamma I & D_{11}^T & 0 & 0 \\
C + D_{12} K & D_{11}^T & -\gamma I & 0 & 0 \\
A_d^T P & 0 & 0 & -(1-\rho)S E_2^T & E_2 \\
E_1 + E_3 K & 0 & 0 & E_2 & -\varepsilon I
\end{bmatrix} < 0 \quad (23)
\]

Left multiply and right multiply the matrix

\[
\begin{bmatrix}
P^{-1} & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
0 & 0 & 0 & P^{-1} \\
0 & 0 & 0 & I
\end{bmatrix}
\]

Then define \( X=P^{-1} \), \( Y=KP^{-1} \), \( Q=P^{-1}SP^{-1} \), then Formula (23) can be changed as equation (16). We can complete the proof.

3.5 Case Studies

3.5.1 System Description

In order to verify the effectiveness of the proposed control method, the two-area LFC system derived from the New England 39-bus system in section 3.3.1 is studied. The parameters are shown in Table 5.
Table 5 Parameters of the Two-Area LFC System Case Studies

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Area1</th>
<th>Area2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional frequency deviation coefficient $b_i$ (p.u.MW/Hz)</td>
<td>4.89</td>
<td>5.12</td>
</tr>
<tr>
<td>Speed regulation due to governor action $R_i$ (Hz/p.u.MW)</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Inertia time constant of the AGC units $M_i(s)$</td>
<td>6.23</td>
<td>5.77</td>
</tr>
<tr>
<td>Load frequency characteristic coefficient $D_i$ (p.u.MW/Hz)</td>
<td>0.87</td>
<td>1.23</td>
</tr>
<tr>
<td>Frequency response coefficient of ES $K_{bi}$ (p.u.MV/Hz)</td>
<td>3.00</td>
<td>2.00;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.50</td>
</tr>
<tr>
<td>Governor time constant $T_{Gi}(s)$</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>Turbine time constant $T_{CHi}(s)$</td>
<td>0.56</td>
<td>0.45</td>
</tr>
<tr>
<td>Equivalent time constant of batteries $T_{ESi}(s)$</td>
<td>0.12</td>
<td>0.20;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>Parametric uncertainty of $\Delta_{Gi}$ (s^{-1})</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>Parametric uncertainty of $\Delta_{CHi}$ (s^{-1})</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>Parametric uncertainty of $\Delta_{ESi}$ (s^{-1})</td>
<td>1.90</td>
<td>0.50;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td>Tie line synchronization coefficient $T_i$ (p.u.MV/Hz)</td>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td>Time delay bound $\rho$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Interference suppression degree $\gamma$</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>
The New England 39-bus test power system has 46 connection lines. Generator No.6 is a wind generator and there is one grid-connected battery energy storage station in area 1 and two ES in area 2. Bus 15 and 16 are low-voltage nodes, while the two lines that connects bus 14 and 15, 16 and 17 divide the whole system into two separate areas. The total capacity of the system is 627.5 MW. The required frequency regulation capacity is set to 8% of the total loads, namely 50.2 MW in which the conventional AGC plants and ES account for 80% (40.16 MW) and 20% (10.02 MW) respectively. Use the proposed approach, $H_\infty$ controller is:

$$K = \begin{bmatrix} -90.54 & -9.70 & -36.28 & 0.03 & -0.56 & -142.58 & 32.82 & -0.24 & -0.32 & 0.01 & 0.06 & 0.07 \\ 8.93 & 0.58 & 1.25 & -0.48 & 0.05 & -68.22 & -236.97 & -3.66 & -11.55 & -0.20 & -0.24 & -0.39 \end{bmatrix}$$

Set the step-test load fluctuation vector

$$w_1(t) \& w_2(t) = \begin{cases} 0 & ;0 \leq t \leq 1 \\ 0.3 & \& 0.1 \text{ p.u.} & ;1 \leq t \leq 15 \\ 0.4 & \& -0.2 \text{ p.u.} & ;15 \leq t \leq 30 \end{cases}$$

(24)

The power output variation of the traditional AGC units and the energy storages of the two-area system are shown in the Fig. 22.
From Fig. 22, we can conclude that the step load change is adjusted by both AGC units and ES at the first transient process while the final increment/decrement of the load is carried by generators. This is reasonable because the energy storage cannot supply or absorb power perpetually.

3.5.2 Comparison of Control Methods

First, the comparison of different control methods, namely the traditional PI controller and the proposed $H_{\infty}$ controller is simulated. Fig. 23 shows the $ACE_1$ performance curve using different controllers under two different scenarios: one with no time delay while the other with 0.8s time delay.
It can be concluded from Fig. 23 that as the time delay increases to 0.8s, PI control loses its stability if we use the same coefficients all the time. This demonstrates that the robust performance of the system is quite poor. Due to the unpredictable and random character of the time delay, we cannot keep on adjusting the PI coefficients all the time in practical operation. By contrast, the proposed $H_\infty$ controller can always help the system to be robustly stable without changing any parameters. This is a noteworthy advantage.

### 3.5.3 Effects of Duration and Forms of Time Delay on System Dynamic Characteristics

To test the controller further, different delay types are adopted. Fig. 24 shows the influence of different forms of time delay on $ACE_1$ performance using $H_\infty$ controller.
It can be seen that for all the cases, the overall conclusion is that the proposed control method can ensure the stability of the system no matter what type of delay is experienced. To be specific, from the simulation result we can conclude that for the constant type of time delay, the system dynamic performance deteriorates when the delay increases (From 0.5s to 1s): we can see that overshoot and oscillation period both increases. For the sine-type time delay and the random time delay, the system dynamic performance behaves more unpredictably because the delay changes continuously. However, as long as the delay satisfy the restrictions and is bounded, the system will eventually come back to its normal status.

3.5.4 Effects of Parametric Uncertainty on System Dynamic Characteristics
In order to test the robustness of the designed controller against parametric uncertainty, two most hash scenarios are selected, i.e. according to (11), all of these parameters reach their upper limit simultaneously or, reach their lower limit simultaneously. The Fig. 25 shows the influence of parametric uncertainty on $ACE_1$ performance using $H_\infty$ controller.

![Fig. 25 The Influence of Parametric Uncertainty on $ACE_1$ Performance using $H_\infty$ Controller](image)

It can be seen that the system can maintain robustly stable no matter how the certain parameters of the model varies, as long as they do not exceed an pre-known boundary as (11) specifies.

**3.5.5 Effects of Energy Storage on System Accommodation of Intermittent RE**

The following study assumes that the active power unbalance comes from the fluctuation of wind power outputs. The variation of wind power output module is simulated with band-
limited white noise and a low-pass filter [42], as shown in Fig. 19. The Fig. 26 shows the $ACE_1$ performance under two scenarios: with or without the participation of energy storage.

Fig. 26 The $ACE_1$ Performance under two Scenarios: with or without the Participation of ES

From Fig. 26, the ES has reduced the overall fluctuation interval by 55.8%. This demonstrates that with an appropriate controller, the ES could play a positive role in helping the system respond more accurately and rapidly towards the integration of unpredictable intermittent renewables.

In the next Chapter 4, we will focus on the voltage issues and power flow problem in future distribution network and a novel distributed consensus algorithm will be designed to resolve the problem.
4. The Enhanced Optimal Distributed Consensus Control of Scattered Devices Considering Security and Economic Operation Issues

As we can see from the above chapter 3, the intermittent characteristics of wind or PV power will have an adverse impact on system frequency stability. Actually, apart from the frequency aspect, the increasing penetration level of intermittent renewable generations will also inevitably cause adverse impact on other indexes such as power flow distribution, causing reverse power flow and node voltage/line power over-limit problems. Therefore, novel dispatching strategies are essential to be developed and studied. In this chapter, I propose a novel Enhanced Optimal Distributed Control Algorithm (EODCA) based on power system sensitivity analysis to regulate the scattered energy storage devices in order to alleviate these problems while the overall economic performance and network security constraints are considered. These scattered devices can be regarded as small “agents” who can update their individual information during the iteration process based on control law through the communication and data exchange with their adjacent neighbours so as to eventually reach a final consensus optimal value. The comprehensive case study also demonstrates the effectiveness of the proposed method: not only can it help to find the global economic optimal solutions in a distributed manner, but also it can guarantee the secure operation of the distribution network and enhance the robustness of the system as well.

4.1 Mechanism Analysis of Energy Storage Installation

In smart grid environment, increasing small-scale Distributed Generations (DG), such as Photovoltaic (PV) Panels or Combined Heat and Power (CHP) Generators are installed and connected into the distribution network. For example, the household rooftop PV in Queensland and South Australia has become quite prevalent. However, the conventional distribution system
is designed based on the assumption that the substation is the only source of energy. In the new situation, this system has to face new challenges. For example, reverse power flow, voltage and transmission power over-limit problems may frequently occur [5] [8]. Furthermore, the common radial structure of the distribution grid is more vulnerable to disturbances compared with the high-voltage transmission network that has more stabilizers, protections and controllers in it. Therefore, the security operation of the distribution system is an important issue given increasing DG integration.

On the other hand, the scattered Energy Storage (ES) devices have been received growing attention because of their falling price. They can be utilized as an effective “buffer” to help the system alleviate these problems if an appropriate control approach is adopted.

Generally speaking, there are two mechanisms of installing and manipulating of these DGs. These devices can be purchased and installed by the customers themselves, or, alternatively, set up by local companies, such as the “ aggregator” in the future. The main merit of the former scheme is its flexibility and no extra costs/control processes for utilities. Users’ electricity bills can be reduced through Demand Response (DR). Thus, plenty of recent literature (e.g. [43]) have discussed how to design a suitable incentive mechanism for motivating the households to perform better DR [44]. However, most of them use the simplified power balance constraints instead of precise power flow calculation. For a practical system, both the electrical topology and power flow distribution issues are crucial and should be considered properly. What is more, there are some drawbacks of this mechanism noteworthy as well:

- The cost of the ES device is so high that it is somehow unbearable for individuals to install in the foreseeable future;
Most importantly, without a well-designed policy and highly developed mature market mechanism, the enormous individual profit-driven behaviours, even vicious competitions would put serious threat to power system security operation process (such as forming another peak load, etc.);

By contrast, the adoption of the concept “aggregator” provides a promising perspective. These aggregators are essentially some local distribution companies. They purchase energy from the grid and sell it to the local residents so as to gain profits; furthermore, it is their responsibility to maintain the security operation of the whole system. In this scheme, the devices are purchased, installed and manipulated by the aggregators, which can overcome the demerits of the former scheme. For example:

- The aggregators have the financial ability for installing expensive storage devices;
- They can optimize PV and ES size and location where needed, so that the waste of resources and the blind investment behaviours of individuals can be avoided;
- Comprehensive control strategy for these devices can guarantee the security operation of the whole system, avoiding serious safety hazards posed by the individuals;
- Last but not the least, for the customers, they do not have any extra burdens; on the contrary, through appropriate control strategy, the net profit of the aggregators can increase in spite of new investment. This trend indicates the decrease of the electricity price in turn in a long run. Undoubtedly, this win-win mechanism is promising.

The limitations are the suitable control method should be adopted to ensure the secure operation of the system; otherwise, it is neither practical nor reliable to control numerous devices. Besides, the net profit of the power distribution company should be guaranteed to motivate the facility construction and renewable integration.
Lately, the theory of multi-agent and distributed consensus algorithm has received much attention. However, the application of distributed algorithms in power system is still quite preliminary so far. The over simplification of the electrical system and power flow calculation in existing research may incur critical information loss and cannot reflect the real situations accurately[28][45]. In this context, this chapter includes the power system theory to enhance the distributed consensus algorithm. With a more accurate model of the system calculation, two key indexes in future grid scenarios, i.e. both the power flow and the voltage issues are considered simultaneously to make the whole optimization process more objective, detailed and practical.

The rest of the Chapter is organized as follows: three different forms of control schemes are analysed and then mathematical modelling of the practical problem is derived in Section 4.2. In Section 4.3, the prototype of a sub-gradient based distributed consensus algorithm is briefly introduced. In Section 4.4, we propose the linearization enhancement based on power system sensitivity analysis, which makes the algorithm applicable for dealing with the non-linear constraints and issues existing in this problem. Comprehensive case studies are presented in section 4.5 to test the performance of the proposed Enhanced Optimal Distributed Consensus Algorithm (EODCA), and verify its effectiveness compared with different scenarios (e.g. the random behaviour or centralized dispatch).

4.2 Problem Description and Mathematical Modelling

4.2.1 Three Control Schemes in Distribution Network

There are three different control structures shown in Fig. 27. We can also analyse the drawbacks and merits of each configuration:
In traditional power system operation, the optimal power flow (OPF) strategy is the most widely used approach to dispatch the generators in a network. It is essentially a centralized balancing control. The control centre gathers the information from all the devices and then performs an OPF optimization. The control signals are sent back to these distributed devices again to set their power outputs. The advantage is that the highly developed algorithm can guarantee the global optimum. However, for a future distribution system that have numerous and
geographically dispersed DGs, such centralized controls have a number of weaknesses: First, it is too expensive to build all these information channels and the control centre. Secondly, the centralized control centre is also the weakest point in the system especially when huge amount of information and data congests in it, once attacked or overloaded, the system may collapse immediately. Third, in future distribution grid with a very high renewable penetration level, the amount of small scale devices is so large that the “curse of dimensionality” would overwhelm traditional centralized algorithms, resulting in the calculation speed dropping significantly, or even make them unsolvable.

The second scheme is decentralized control, which is the totally different to the centralized scheme. For example, the maximum photovoltaic power tracking (known as MPPT), constant voltage and frequency (VF) with droop mode, or the feeder power flow control mode are such controls. The controls are performed within each individual device as shown in (b) of Fig. 27. Although it is simple and cheap to realize, i.e. no need to invest in communication channels at all, yet the main flaw is that it only uses the information of the target device itself so only an individual optimum can be achieved.

In contrast with them, the last scheme is called distributed control. It can be seen that a local communication network is adopted to link the distributed devices. Specifically, a device should incorporate whatever information available from its neighbouring device or critical nodes into its control law to make an optimal decision. Despite the relatively more complicated controller design process, the distributed control scheme is quite suitable for tackling numerous scattered devices. Not only can it use the information exchange in local communication network to achieve a global optimum, but also it enhance the robustness of the system with respect to intermittency and latency of its feedbacks, tolerate connection and disconnection of network
components with a suitable design. Furthermore, it helps to avoid the “curse of dimensionality” and saves fixed cost for building redundant infrastructures. Therefore, it combines the positive features of both centralized and decentralized controls while limiting their disadvantages, making it the most promising approach for future distribution networks.

4.2.2 Problem Description

In this study, we consider the specific scenario in the future that a power distribution company or an aggregator installs the infrastructure for the customers, which means that the profit of this distributor should be maximized through comprehensive optimal dispatching approach.

The concept of “multi-agents” and the consensus algorithm can be adopted because these distributed facilities can be regarded as small “agents”, i.e. they can exchange information with their neighbours through the communication channel built between them. Apart from the security of operation issues, the economic performance should be taken into account simultaneously. Through the proposed Enhanced Optimal Distributed Consensus Algorithm (EODCA) in the following sections, these “agents” can update their individual information separately during the iteration process only use limited information instead of gathering all the data from all the devices in traditional ways. Regardless of what initial value they take, they can reach the consensus optimal value in the end, i.e. collaboratively achieve a same goal: to maximize the certain profits while ensure the secure operation (avoid the voltage/transmission power over limit problem) of the distribution networks simultaneously.

4.2.3 Mathematical Modelling

The mathematical model of the problem is formulated as follows:
Define $x=(P_{PV,1}, P_{PV,2}, \ldots, P_{PV,N}, P_{ES,1}, P_{ES,2}, \ldots, P_{ES,M})$;

**Objective function $f$ (Max net profit of the aggregator):**

$$f(x) = \sum_{k=2}^{K} \rho_D P_{D,k} - \rho_b P_b + \sum_{n=1}^{N} [\rho_s P_{PV,n} - C_{PV}(P_{PV,n})] - \sum_{m=1}^{M} C_{ES}(P_{ES,m})$$  \hspace{1cm} (25)

Where $\rho_D$ is the retail price selling to the customers; $\rho_b$ is the wholesale price selling to the aggregators; $P_{D,k}$ is the load of the node $k$; $P_b$ is the total power bought from external grid; $\rho_s$ is the financial subsidies set by the external grid/government to absorb and stimulate sustainable energy generation; $P_{PV,n}$ is the power output of the $n$th PV; ($n \in N$); $P_{ES,m}$ is the charging/discharging power of the $m$th ES ($m \in M$); $C_{PV}(P_{PV,n})$ is the cost function of the $n$th PV, which can be written as the quadratic form[45]:

$$C_{PV}(P_{PV,n}) = a_{PV} P_{PV,n}^2 + b_{PV} P_{PV,n}$$  \hspace{1cm} (26)

$C_{ES}(P_{ES,m})$ is the cost function of the $m$th ES, which can be written as the quadratic form[45]:

$$C_{ES}(P_{ES,m}) = a_{ES} P_{ES,m}^2 + b_{ES} P_{ES,m}$$  \hspace{1cm} (27)

$K$ denotes the set of nodes that have loads connected; $N$ denotes the number of PV; $M$ denotes the number of ES.

**Constraints:**

1) Total power balance:

$$P_k + \sum_{n=1}^{N} P_{PV,n} + \sum_{m=1}^{M} P_{ES,m} = \sum_{k=2}^{K} P_{D,k}$$  \hspace{1cm} (28)
2) The power limitation of the PV output:
\[
P_{PV,n} \leq P_{PV,n} \leq \overline{P}_{PV,n}; \forall n \in N
\]  
(29)

Where \( P_{PV,n} \) and \( \overline{P}_{PV,n} \) are the lower and upper limits of the \( n \)th PV power output separately.

3) The power limitation of the storage charging/discharging:
\[
P_{dCH,m} \leq P_{ES,m} \leq P_{CH,m}, \forall m \in M
\]  
(30)

Where \( P_{CH,m} \) is the maximum charging power of \( m \)th storage while \( P_{dCH,m} \) is the maximum discharging power of the \( m \)th storage. \( P_{ES,m} > 0 \) indicates charging whilst \( P_{ES,m} < 0 \) denotes discharging.

4) The power flow equation of the distribution network:
\[
\begin{align*}
    P_i &= P_{D,i} + P_{PV,i} + P_{ES,i}; \\
    Q_i &= Q_{D,i} + \alpha P_{PV,i} + \beta P_{ES,i}; \\
    P_i - U_i \sum_{j=1}^{I} U_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) &= 0; \quad i = 2,3,...I \\
    Q_i - U_i \sum_{j=1}^{I} U_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) &= 0
\end{align*}
\]  
(31)

Where \( P_i \) is the algebraic sum of (generation/load) active power injected to node \( i \); \( Q_i \) is the algebraic sum of (generation/load) reactive power injected to node \( i \); \( \alpha \) is the required (predetermined) power factor of PV; \( \beta \) is the required (predetermined) power factor of ES; \( I \) denotes the set of the nodes in this system; \( G_{ij} \) is the conductance between node \( i \) and \( j \); \( B_{ij} \) is the susceptibility between node \( i \) and \( j \); \( \delta_{ij} \) is the phase angle difference between node \( i \) and \( j \).

5) The voltage limitation of the system:
The line power limitation of the distribution system:

\[
[P_1^l \ P_2^l \ ... \ P_l^l \ ... \ P_L^l]^T = A[P_1^l \ P_2^l \ ... \ P_l^l \ ... \ P_L^l]^T;
- P_{\text{max},l} \leq P_l^l \leq P_{l,\text{max}}, \forall l \in L
\]  

Where \( L \) denotes the number of lines in the system; \( A \in \mathbb{R}^{L \times L} \) is the matrix that denotes the topology relation; \( P_l^l \) is the line flow of the system; \( P_{\text{max},l} \) is the maximum transmission limit of the \( l \)th line.

The traditional OPF problem has been successfully resolved by centralized approaches, such as interior point method [46]. However, as is analysed before, in future grid situations, it is better to solve the problem in a distributed way to overcome the drawbacks of centralized methods. It means that during the iteration process of each “agent”, the distributed PV/battery updates their decision variables only based on information from itself and neighbours, its local objective function, local upper/lower constraints and a global constraint (power flow). Neither do they know the global objective function nor the upper/lower range information of other agents except their neighbours. In order to solve this problem, we should reformulate the above mentioned model to the standard format.

By substituting (28) into the objective function, we can get:

**Max net profit \( f \)**

\[
f(x) = \sum_{k=2}^{K}(\rho_{p} - \rho_{k}P_{D,k})
+ \sum_{n=1}^{N}[\rho_{p}P_{PV,n} + \rho_{p}P_{PV,n} - C_{PV}(P_{PV,n})] + \sum_{m=1}^{M}[\rho_{b}P_{ES,m} - C_{ES}(P_{ES,m})]
\]

\(0.95 \leq U_i \leq 1.05; \forall i \in I\)  

(32)
Note that we can eliminate the first term, because it is a constant value without considering the demand response. It is irrelevant and indifferent to the objective function. Then the model can be expressed as:

\[
\text{Min } f(x) = \sum_{n=1}^{N} \left[ C_{pV}(P_{pV,n}) - \rho_b P_{pV,n} - \rho_s P_{pV,n} \right] + \sum_{m=1}^{M} \left[ C_{ES}(P_{ES,m}) - \rho_b P_{ES,m} \right]
\]

\[= \sum_{i=1}^{N'} f_i(x_i); \quad N' = N + M \tag{35}\]

Constraints (29) and (30) are local constraints that are only known by agent \( i \) itself. It can be mathematically expressed as:

\[x \in \bigcap_{i=1}^{N'} X_i \tag{36}\]

Where \( X_i \) represents the separate local constraint set of each agent \( i \).

Constraint (33) is essentially a global linear inequality constraint and can be simplified easily to the following format:

\[g(x) = A^T x \leq 0 \tag{37}\]

Constraint (31) and (32) will be discussed later in Section 4.4.

4.3 Prototype of Sub-Gradient based Distributed Consensus Algorithm

In the latest research work [28], the authors proposed a sub-gradient based algorithm to realize distributed optimization considering inequality and equality constraints. Here in this section, we briefly introduce the prototype of the distributed method and then analyse its shortcomings when combining the algorithm with practical power system problems.
Firstly, let us analyse a network of separate agents marked by $V := \{ 1, 2, \ldots, N \}$. The objective of this multi-agent group is to optimize the common goal cooperatively, which in the standard form of the problem is described as:

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{N} f_i(x), \quad s.t. \; g(x) \leq 0; \; h(x) = 0; \; x \in \bigcap_{i=1}^{N} X_i \tag{38}$$

Here $f_i$ is the separate objective of each agent while the final objective function is the sum of them. It is assumed that these multi-agents operate synchronously. The constraint $g(x) \leq 0$ is satisfied for all $l \in \{1, \ldots, m\}$, i.e. it is a global inequality constraint. The constraint $h(x) = Ax - b$ is known to all the agents and denotes a global equality constraint. $X_i$ represents the separate local constraint set of each agent $i$.

The communication connection relations of the network is depicted by an adjacency matrix $A(k)$ and its elements represent the weight assigned between the edge $i$ and $j$. The communication graph has to satisfy three assumptions: “Non-Degeneracy”, “Balanced Communication” and “Periodical Strong Connectivity” which can be referred to [28].

The adjacency matrix $A(k)$ denotes the communication network connection between all the different agents. If the agent $i$ and agent $j$ have the communication connection and the information exchange, then the element $a_{ij}>0$; otherwise $a_{ij}=0$. In regards to the value of $a_{ij}$, it is assigned according to the average total amount of information exchanged between the agent $i$ and agent $j$. Usually the diagonal element $a_{ii}$ is larger than the other elements and the matrix is symmetrical because $a_{ij}=a_{ji}$. In addition, the matrix has to satisfy both the “Non-Degeneracy” and the “Balanced Communication” conditions, which means the sum of the all the elements in each row and each column equals $1$. 
In the algorithm, firstly each agent $i$ chooses any initial state $x^i(0) \in X_i$, $\mu^i(0) \in \mathbb{R}^{n_i}$. Then at each step $k \geq 0$, each agent $i$ generates the next value $x^i(k+1), \mu^i(k+1)$ according to (39):

$$
\begin{align*}
\text{Consensus:} & \quad v^i_x(k) = \sum_{j=1}^{N} a^i_j(k) x^j(k); \\
\text{Projection:} & \quad v^i_\mu(k) = \sum_{j=1}^{N} a^i_j(k) \mu^j(k);
\end{align*}
$$

$$
\begin{align*}
x^i(k+1) &= P_{x_i} [v^i_x(k) - \alpha(k) D^i_x (k)]; \\
\mu^i(k+1) &= P_{M_i} [v^i_\mu(k) + \alpha(k) D^i_\mu (k)];
\end{align*}
$$

(39)

where $a^i_j(k)$ is the corresponding element in adjacency matrix $A(k)$ which denotes the communication connection between agents; $P_{x_i}$ is the projection operation onto the set $X_i$ (respectively, $P_{M_i}$ is the projection operation onto the set $M_i$); the scalars $\alpha(k) > 0$ are step-sizes and should satisfy: $\lim_{k \to \infty} \alpha(k) = 0$; $\sum_{k=0}^{\infty} \alpha(k) = +\infty$; $\sum_{k=0}^{\infty} \alpha(k)^2 < +\infty$. The notations are:

$$
D^i_x (k) \equiv DL_{v^i_x(k)} (v^i_x(k)), \quad D^i_\mu (k) \equiv DL_{v^i_\mu(k)} (v^i_\mu(k)).
$$

Although the prototype algorithm can solve the optimization problem in a distributed manner, it cannot be applied directly in the proposed model due to the highly non-linear voltage magnitude constraints (31) and (32). In the following section, a novel power system sensitivity analysis based-linearized approach is developed to enhance the algorithm, making it compatible with the revised piecewise linearized voltage constraints during the iteration process.

4.4 Linearized Enhancement based on Power System Sensitivity Analysis

4.4.1 The Limitations of the Existing Methodology

In the algorithm proposed in [28] described in the previous section, $h(x) = Ax - b$ is required, which means that the equality constraint should be the linear combination of the state variables.
Whilst in our model, the power flow constraint (31) combined with limits (32) is highly non-linear. Besides, (32) is essentially an inequality constraint that can be written as $x=h(u)$. Due to the non-convex characteristic of the $h$, the uniqueness of the mapping $x\rightarrow u$ cannot be guaranteed. Which means that the implicit function $u=h^{-1}(x)$ doesn’t exist at all. Therefore, the combination of (31) with (32) cannot be written as $u=h^{-1}(x)\leq 0$, which indicates it impractical to use the previous algorithm directly.

Therefore, the linearization of the power flow equation (31) should be adopted. Although the DC power flow method is the most straightforward way, the inherent and insurmountable drawback is only the active power flow can be calculated whilst the voltage profile and the reactive power calculations are neglected, which is a pivotal security operation problem in this scenario.

In order to make the problem solvable, more theory of the power systems should be considered and adopted. The brand new approach we propose here is based on the power system sensitivity analysis method. We call it **Enhanced Optimal Distributed Consensus Algorithm** (EODCA, for short) to find a saddle point of the Lagrangian function $L$ over $X\times M$ and the optimal value.

### 4.4.2 The Power System Sensitivity Analysis based Piecewise Linearization Modification and Enhancement

Since during two steps of the iteration processes, the updated variation of renewable power outputs injected into the nodes are quite small, so we can regard the renewable power output injected into the nodes as a certain form of disturbance, and through the Taylor Series Expansion, the sensitivity matrix can be deduced.
For a system that is working under stable operation status, the equation (31) can be written as:

\[ W_0 = f(X_0, Y_0) \]  

(40)

Where, the \( W_0 \) is the active and reactive power injected into the nodes, \( X_0 \) is the vector that consists of the node voltage and angle; \( Y_0 \) denotes the parameters of the network itself under normal operation status.

If the injected power changes \( \Delta W \), or the network changes \( \Delta Y \), the state variables should also changes \( \Delta X \). Therefore, the (40) can be written as:

\[ W_0 + \Delta W = f(X_0 + \Delta X, Y_0 + \Delta Y) \]  

(41)

Perform the Taylor series expansion of (41), we can get:

\[ W_0 + \Delta W = f(X_0 + \Delta X, Y_0 + \Delta Y) = f(X_0, Y_0) + f_x'(X_0, Y_0)\Delta X + f_y'(X_0, Y_0)\Delta Y + \frac{1}{2}[f_{xx}''(X_0, Y_0)(\Delta X)^2 + 2f_{xy}''(X_0, Y_0)\Delta X \Delta Y + f_{yy}''(X_0, Y_0)(\Delta Y)^2] + ... \]  

(42)

When the disturbance variables and the status variables are small, we can neglect the \((\Delta X)^2\) and the high-order terms. Besides, due to the fact that \( f(X, Y) \) is the linear function of \( Y \), so \( f_{yy}''(X, Y) = 0 \). Therefore, the (42) can be simplified as:

\[ W_0 + \Delta W = f(X_0, Y_0) + f_x'(X_0, Y_0)\Delta X + f_y'(X_0, Y_0)\Delta Y + f_{xy}''(X_0, Y_0)\Delta Y \Delta X \]  

(43)

By substituting (40) into the above equation, we can get:

\[ \Delta W = f_x'(X_0, Y_0)\Delta X + f_y'(X_0, Y_0)\Delta Y + f_{xy}''(X_0, Y_0)\Delta Y \Delta X \]  

(44)
Therefore, the linear relation between the status variables with the disturbance variables and the network changing variables is:

\[
\Delta X = [f_x(X_0, Y_0) + f_y(X_0, Y_0)\Delta Y]^{-1}[\Delta W - f_x(X_0, Y_0)\Delta Y]
\]  

(45)

If we consider a time-invariant network, i.e. \( \Delta Y = 0 \), (45) becomes:

\[
\Delta X = [f_x(X_0, Y_0)]^{-1}\Delta W = S_0\Delta W
\]  

(46)

where:

\[ f_x(X_0, Y_0) = \left. \frac{\partial f(X, Y)}{\partial X} \right|_{X=X_0, Y=Y_0} = J_0 \]

(47)

\( J_0 \) is the Jacobian matrix when the load flow calculation finishes and the \( S_0 \) is called the sensitivity matrix.

In this problem, we only consider the magnitude limitation of the voltage. Through partitioning of the matrix, (46) can be written as:

\[
\begin{bmatrix}
\Delta \theta \\
\Delta U
\end{bmatrix} = J_0^{-1}
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = S_0
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(48)

Further, in the problem proposed, the power factor of PV and ES is predefined as a fixed value, i.e. \( Q = \alpha P \), hence we can get:

\[
\Delta U = (C + \alpha D)\Delta P = S\Delta P = S\Delta x
\]  

(49)

This is very important because the relation is linear now (essentially piece-wise linear). During the iteration process, (49) can guide the algorithm to find a suitable direction of changing
To be specifically, during the iteration step $i$, after an agent updates its information based on consensus process of (48), i.e.

\[
x'(k+1) = \sum_{j=1}^{N} a_j'(k)x'(k) - \alpha(k)D_i'(k)
\]

\[
\mu'(k+1) = \sum_{j=1}^{N} a_j'(k)\mu'(k) + \alpha(k)D_{\mu}'(k)
\]

Then the (49) should be calculated. Since the $\Delta U$ is bounded by (32), i.e.:

\[
\Delta U \leq \min\{1.05 - U_{\text{step}i}^{\text{step}i}, U_{\text{step}i}^{\text{step}i} - 0.95\}
\]

So the $\Delta x$ is bounded because of (49) by simply projecting the vector onto the feasible region, which is a convex set given by linear inequalities (52).

\[
S'\Delta x = \Delta U \leq \min\{1.05 - U_{\text{step}i}^{\text{step}i}, U_{\text{step}i}^{\text{step}i} - 0.95\}
\]

Inequality (52) should replace the simple Max-Min restrictions in the projection operator of the (39) in the prototype of the algorithm, which can be used to restrain the power outputs of the renewables in some critical period when the voltage constraints are violated even with the help of ES so as to guarantee the voltage security operation issues.

\[
\begin{cases}
P_{x_i} : & S'\Delta x_{\text{step}i} \leq \min\{1.05 - U_{\text{step}i}^{\text{step}i}, U_{\text{step}i}^{\text{step}i} - 0.95\}; \\
x'(k+1) = P_{x_i}x'(k+1); \\
P_{M_i} : & \mu_{\text{step}i} \geq 0; \\
\mu'(k+1) = P_{M_i}\mu'(k+1);
\end{cases}
\]

In conclusion, the flow chart of the above proposed power system sensitivity analysis based-EODCA can be described as:
4.5 Case Studies

4.5.1 System Description
In order to demonstrate the effectiveness and the characteristics of the proposed method, the algorithm is coded in a Matlab environment and simulations are performed with MatPower toolbox. The topology shown in Fig. 29 is a future distribution network, where at bus 9, 12 and 13, the scattered rooftop PVs and energy storages (ES) are integrated. The local distribution company has the ownership of installing and dispatching these devices and only limited information channels are built between adjacent devices for the sake of saving costs. Meanwhile, apart from the inherent incentive of obtaining more profits, another important responsibility of
the power distribution company is to guarantee the safety operation of the whole distribution network at any time point of the day, i.e.: both the transmission power in all lines and the voltage at all nodes should not exceed the maximum and the minimum boundaries.

Therefore, the enhanced optimal distributed consensus algorithm can be implemented here to help to achieve these goals from a brand new perspective.

The system parameters are summarized as follows in the Table 6:

Table 6 System Parameters

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Values</th>
<th>System Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>3</td>
<td>$P_{PV_{max_base}}$ (MW)</td>
<td>0.9</td>
</tr>
<tr>
<td>$M$</td>
<td>3</td>
<td>$P_{ES_{max_charge}}$ (MW)</td>
<td>1.2</td>
</tr>
<tr>
<td>$I$</td>
<td>15</td>
<td>$P_{ES_{max_discharge}}$ (MW)</td>
<td>-1.2</td>
</tr>
<tr>
<td>$L$</td>
<td>14</td>
<td>$P_{max,1}$ (MW)</td>
<td>19</td>
</tr>
<tr>
<td>$K$</td>
<td>15</td>
<td>$P_{max,2}$ (MW)</td>
<td>15</td>
</tr>
<tr>
<td>$a_{PV}$ ($/MWh^2$)</td>
<td>70</td>
<td>$P_{max,3}$ ~ $P_{max,14}$ (MW)</td>
<td>6</td>
</tr>
<tr>
<td>$b_{PV}$ ($/MWh$)</td>
<td>65</td>
<td>$\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td>$a_{ES}$ ($/MWh^2$)</td>
<td>43</td>
<td>$\beta$</td>
<td>0.2</td>
</tr>
<tr>
<td>$b_{ES}$ ($/MWh$)</td>
<td>82</td>
<td>$\rho_s$ ($/MWh$)</td>
<td>250</td>
</tr>
<tr>
<td>$\rho_b$ ($/MWh$)</td>
<td>180</td>
<td>$\rho_D$ ($/MWh$)</td>
<td>270</td>
</tr>
</tbody>
</table>

As is seen from the graph, the information channels only exist between the neighbouring devices. Therefore, the “adjacency matrix” can be formulated as the following format, with the
assigned weights as the elements that satisfy both the “Non-Degeneracy” and the “Balanced Communication” conditions:

\[
a = \begin{bmatrix}
\frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\
\frac{1}{6} & \frac{4}{6} & \frac{1}{6} & 0 & 0 & 0 \\
0 & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & 0 & 0 \\
0 & 0 & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & 0 \\
0 & 0 & 0 & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \\
0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6}
\end{bmatrix}
\]

In our study, we assume that the maximum value of each PV power output equals 0.9 MW from 8:00 am to 6:00 pm and it drops to zero during other periods in a day. With regards to the loads, the base load of each node is given in the following Table 7. And the load profile ratio curve in a day is varying and has peaks and valleys. So in our study, we assume the loads vary according to this trend shown in the Fig. 30.
Table 7 The Base Load of Each Node

<table>
<thead>
<tr>
<th>Node Number</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Active</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power/MW</td>
<td>1.59</td>
<td>1.29</td>
<td>1.02</td>
<td>1.90</td>
<td>0.35</td>
<td>1.52</td>
<td>1.54</td>
</tr>
<tr>
<td>Base Reactive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power/MVar</td>
<td>0.28</td>
<td>0.47</td>
<td>0.30</td>
<td>0.34</td>
<td>0.18</td>
<td>0.22</td>
<td>0.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node Number</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Active</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power/MW</td>
<td>1.47</td>
<td>0.69</td>
<td>0.81</td>
<td>0.44</td>
<td>1.90</td>
<td>1.60</td>
<td>1.11</td>
</tr>
<tr>
<td>Base Reactive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power/MVar</td>
<td>0.35</td>
<td>0.32</td>
<td>0.39</td>
<td>0.36</td>
<td>0.33</td>
<td>0.32</td>
<td>0.79</td>
</tr>
</tbody>
</table>

4.5.2 The Consensus Behaviour of the Agents during Iteration Process

First of all, we can observe the optimal consensus behaviour of the six agents during the iteration process. In each time point in a day, the algorithm is performed with randomly chosen initial values of the agents. Here we observe a specific time point: time=11.

From Fig. 31-32, we can see that during the iteration process, each agent updates its information separately regardless of the initial values they choose, until final consensus is reached. Besides, the black solid line indicates the global optimal value obtained by centralized control approach (interior-point method) while the other colourful lines show the final optimal value obtained by proposed distributed consensus control method. From the result we can see
that through limited information and data exchange between the neighbouring devices, the same
global optimized value can be acquired both from centralized and distributed approaches.

Fig. 31 The Consensus Behaviour of the Total Objective Function (time=11)
4.5.3 The Objective Function (Net Profit of the Company) in three Different Scenarios

In the following sections we compare the objective function (Net Profit of the Company) in a day in three different scenarios:

1) Without renewables at all;

2) With renewables behave randomly without control;

3) With renewables and they perform EODCA.

Firstly, if the devices behave randomly without control at all, the power outputs of renewables are shown in Fig. 33.

---

**Fig. 32 The Consensus Behaviour of Different Agents (time=11) in EODCA**

---

**Fig. 33 The Chaotic Power Outputs of the Devices in a Day without Control**
In contrast, with the EODCA, we can get the optimal power outputs of the devices, as depicted in Fig. 34.

Fig. 34 The Optimal Power Outputs of the Devices in a Day with EODCA

And the Fig. 35 indicates the Net profit of the company in a day in these 3 scenarios.
We can conclude that without renewables, the power distribution company can always get positive profits in current electricity markets. However, the situation changes when renewables are integrated. If we let these devices behave randomly without any regulation, sometimes the profit can be more than that without renewables but sometimes, the profit is less than zero, which means a deficit happens. In contrast, we can see the red line represents for the net profit obtained using EODCA, which is strictly larger than zero and the profit obtained without renewables. It is very useful in decision making process for companies in engineering practice.

4.5.4 The Comparison of Line Flows in three Scenarios

Here we compare the power flow in all the lines of the distribution network in above mentioned three scenarios.
From the simulation results in Fig. 36-38 we can see clearly that, if there is no renewables at all, the power flows in distribution lines are all within their upper and lower limits. However, if the unregulated chaotic behaviours of the renewables happen at some time points, the reverse power flow of some distribution lines may violate the security boundaries and thus may cause system overload problems. Through the EODCA, the power flow in distribution lines remains inside the safety area and the reverse power flow problem is also alleviated.

Fig. 37 The Active Power Flow Profile in a Day with Chaotic Renewable Behaviours without Control
4.5.5 The Comparison of the Voltage Profiles in three Scenarios

Here we compare the voltage magnitude profile of all the nodes of the distribution network.

From the simulation results of Fig. 39-41, we can see that, when the renewable penetration level is 0%, the voltage in each node remains between 0.95 \( p.u. \) and 1.05 \( p.u. \). However, the chaotic behaviour of the sustainable devices would undermine the voltage profile, making it over upper limit of the safety voltage range at some time points. After using EODCA, all the voltage remains inside the safety area of the system.
Fig. 39 The Voltage Profile of Each Node in a Day without Renewables

Fig. 40 The Voltage Profile of Each Node in a Day with Chaotic Renewable Behaviours without Control
In the next Chapter 5, we will discuss on how to improve the calculation speed and efficiency of the proposed algorithm. Besides, the influence of the communication network topology will also be examined.
5. Modified Consensus Direction Method of Multipliers (MC-ADMM) and Communication Network Topology Analysis on Performance

As mentioned in the above chapter 4, the number of distributed photovoltaic and energy storage devices is increasing in future distribution networks. The research in chapter 4 focuses on developing a distributed consensus optimization algorithm that can optimize these devices to ensure the secure operation of the system. Units in the system exchange information through a communication network to maximize the net profit cooperatively.

In this Chapter, firstly I will focus on improving the efficiency and calculation speed of the optimization algorithms. I propose a Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) to demonstrate the effectiveness of accelerating the convergence speed. Afterwards, I also try to give some guidance on how to design a better communication network by innovatively analysing the performance in terms of network topology using graph theory. Through the simulation of several typical topologies of communication network respectively, a conjecture of the optimal topology for a communication network is given.

5.1 Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM)

In this section, we develop an algorithm called Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) to solve the problem. It is based on the distributed optimization algorithm proposed in [47].

The mathematical model is quite similar with that in the previous chapter. Specifically, the same objective function is given in (25). However, here in this section, the simplified model is used, which means we only consider the power flow balance and limitation constraints (28),(29),(30) and (33) due to the requirement of the MC-ADMM.
The objective of this multi-agent group is to optimize the common goal below cooperatively, namely objective function (25) and constraints (28),(29),(30),(33) can be written as:

\[
\min_{x \in \mathbb{R}^n} \sum_{i=1}^{2N} f_i(x_i), \quad \text{s.t.} \quad \sum_{i=1}^{2N} a_i x_i \leq b; \quad x \in \bigcap_{i=1}^{N} X_i
\]  \hspace{1cm} (54)

In this practical model, \( f_i(x_i) = \alpha_i x_i^2 + \beta_i, \) \( \alpha_i > 0, \) \( N=M. \) The iteration steps of MC-ADMM are as follow:

1). Given initial variables \( x_i^{(0)} \in \mathbb{R}, y_i^{(0)} \in \mathbb{R}^{2M}, p_i^{(0)} = 0, \) for each agent \( i, i \in V = \{1, 2, \ldots, 2N\}, \) set step index \( k=1; \)

2). for all \( i \in V, \) perform (55)-(57) in parallel:

\[
p_i^{(k)} = p_i^{(k-1)} + c \cdot \sum_{j \in N_i} (y_i^{(k-1)} - y_j^{(k-1)})
\]  \hspace{1cm} (55)

\[
x_i^{(k)} = \arg \min_{x_i \in X_i} \left\{ f_i(x_i) + \frac{c}{4} \frac{1}{|N_i|} \| 1 \frac{1}{c} (a_i x_i - \frac{b}{2N}) - \frac{1}{c} p_i^{(k)} + \frac{1}{c} (y_i^{(k-1)} + y_j^{(k-1)}) \|_2^2 \right\}
\]  \hspace{1cm} (56)

\[
y_i^{(k)} = \frac{1}{2} \frac{1}{|N_i|} \left\{ \sum_{j \in N_i} (y_i^{(k-1)} + y_j^{(k-1)}) - \frac{1}{c} p_i^{(k)} + \frac{1}{c} (a_i x_i^{(k)} - \frac{b}{2N}) \right\}
\]  \hspace{1cm} (57)

3). Set \( k=k+1; \)

4). Repeat steps 2) and 3) until \( \| x_i^{(k)} - x_i^{(k-1)} \| \leq \varepsilon. \)

In this problem, (56) can be expressed as:
\[
\begin{align*}
    x^{(k)}_i &= \frac{1}{2\alpha_i + a^T_i a_i} \left[ \frac{a^T_i b}{4N_i | N_i | c} + \frac{a^T_i p^{(k)}_i}{2| N_i | c} - \beta_i - \frac{\sum_{j \in N_i} a^T_i y^{(k-1)}_j}{2| N_i | c} \right]_X_i \\
\end{align*}
\]

\([*]_{X_i}\) is the projection operation onto the set \(X_i\); \(N_i = \{ j \in V | i \neq j \}\), i.e. all agents except itself \(i\); parameter \(\varepsilon\) is a pre-defined constant value which stands for a computational accuracy. Adaptive parameter \(c\) will be discussed later.

5.2. Case Studies & Algorithm Analysis

5.2.1 System Description

In order to demonstrate the effectiveness and the characteristics of the proposed method, the algorithm is coded in Matlab environment. The comparison algorithm is the latest proposed one called Distributed Lagrangian Primal-Dual Sub-gradient algorithm (DLPDS) in [28]. The dimension of the independent variable \(x\) represents the number of distributed devices, i.e. if we choose dimension=10, the topology would be demonstrated as Fig. 42. If the dimension increases, the number of devices would be increased as well, but the basic radial structure remains the same, which means the system and algorithm is easy to extend to a larger distribution network.

![Fig. 42 Future Distribution Network with Energy Storages and Distributed Generators (Dimension=10)]
5.2.2 Comparison between Algorithms

First of all, we choose dimension of independent variables: 10; \( \varepsilon = 0.1 \); Maximum iteration steps: 5000. Then we implement the calculation with two algorithms. Fig. 43-44 shows the dynamic convergence performance with MC-ADMM and DLPDS, from two perspectives, namely convergence performance of the different agents and the convergence performance of the relative error.

Fig. 43 The Dynamic Convergence Performance of Different Agents in MC-ADMM and DLPDS (Dimension=10)
Fig. 44 The Dynamic Convergence Performance of the Relative Error (Independent Variables and Objective Function) in MC-ADMM and DLPDS (Dimension=10)

From Fig. 43-44, we can conclude that for lower dimension cases, although both of the two algorithms (MC-ADMM and DLPDS) can guarantee the convergence of the optimal value and adequate accuracy, the convergence speed of MC-ADMM is much faster than DLPDS (426 versus 4460 steps). Note that the reason for oscillation is that the DLPDS runs into the high-accuracy area (namely relative error is less than 0.02) and it is not so stable compared with MC-ADMM.

In order to compare the two algorithms especially their ability to calculate higher dimension of independent variables, we gradually increase the dimension of $x$. The results are shown in Fig. 45-46.

- Dimension of independent variables: 20; $\varepsilon=0.1$; Maximum iteration steps: 10000.
Fig. 45 The Dynamic Convergence Performance of the Relative Error (Independent Variables and Objective Function) in MC-ADMM and DLPDS (dimension=20)

- Dimension of independent variables: 40; \( \varepsilon = 0.1 \); Maximum iteration steps: 20000

Fig. 46 The dynamic convergence performance of the relative error (independent variables and objective function) in MC-ADMM and DLPDS (dimension=40)
From the above figures, we can see that when the dimension of independent variables increases, both the speed of the two Algorithms (MC-ADMM and DLPDS) slows down. However, only MC-ADMM are much faster than the DLPDS and it can guarantee adequate accuracy (pre-defined $\varepsilon=0.1$) whilst DLPDS cannot. It can be noticed that the reason for smooth DLPDS curve is that the iteration stops at the maximum iteration number and it is far from the high-accuracy area. Table 8 lists the detailed calculation indexes of these two algorithms.

<table>
<thead>
<tr>
<th>Dimension of $x$</th>
<th>Max Iteration steps</th>
<th>DLPDS</th>
<th>MC-ADMM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Steps to reach $\varepsilon$</td>
<td>Computation time/$s$</td>
</tr>
<tr>
<td>10</td>
<td>5000</td>
<td>4460</td>
<td>16.0</td>
</tr>
<tr>
<td>20</td>
<td>10000</td>
<td>Max</td>
<td>261.2</td>
</tr>
<tr>
<td>40</td>
<td>20000</td>
<td>Max</td>
<td>3844.3</td>
</tr>
<tr>
<td>80</td>
<td>40000</td>
<td>Max</td>
<td>19249.5</td>
</tr>
</tbody>
</table>

From the Table 8, we can draw the following conclusions:
Calculation time: when the dimension of independent variables multiples, both the
calculation time of the two Algorithms (MC-ADMM and DLPDS) increases. However, the MC-
ADMM maintains to be much faster than the DLPDS.

Iteration numbers used to reach the same accuracy (pre-defined $\varepsilon=0.1$): when the
dimension of independent variables multiples, both the iteration numbers used of the two
Algorithms (MC-ADMM and DLPDS) increases. However, the MC-ADMM can guarantee the
high accuracy within limited iteration numbers whilst DLPDS doesn’t guarantee high enough
accuracy within acceptable iteration steps.

The reason why MC-ADMM performs much faster than DLPDS, especially in a large
system, can be explained from a calculation perspective.

First of all, the most time-consuming process is the information updating process in each
iteration step. In MC-ADMM, we can see from the (56) that in each iteration step $k$, MC-ADMM
only requires the $x_i^{(k)}$ of agent $i$ ($i=1\ldots N$) to update. This means for a system with $N$ agents, each
iteration step we only need $N$ agents to update their status: the calculation is a simple scalar
operation and the dimension is only $N$.

By contrast, in DLPDS, during each iteration step $k$, we can see from the (39) that
actually it requires each agent $i$ ($i=1\ldots N$) to generate a vector that has $N$ elements, i.e. $N$-
dimensional. Then, each agent updates their information according to the control law, which
means that the complex matrix operation requires $N$ vectors (each one is $N$-dimensional)
calculating simultaneously. Therefore, the memory usage of the computer is high-demanding and
the calculation speed is slowed down significantly. The situation gets worse at an exponential
rate when $N$ increases so that the superiority of MC-ADMM is even more obvious when dealing with larger systems.

5.2.3 Comparison between Different Communication Network Topologies

In this section, we will analyse the different types of communication network topologies on calculation performance, under intact and fault conditions. We consider three typical connecting strategies for communication network, i.e. Radial, Circular and Random, as shown in Fig. 47:

Fig. 47 Three Different Types of Communication Network Topology (dimension=10)
Under intact condition:

When the network is intact and everything runs normally, we can see the dynamic convergence performance of the relative error (objective function) in three different communication network topologies (Dimension of independent variables: 10; \( \varepsilon = 0.1 \); Maximum iteration steps: 1000) as shown in Fig. 48.

**Fig. 48** The Dynamic Convergence Performance of the Relative Error (Objective Function) in Three Different Communication Network Topologies (dimension=10)

From the Fig. 48, we can conclude that the topology of the communication network has a profound influence on convergence speed of the algorithm.

Compared with the radial structure, the circular topology accelerates the convergence because some of the paths will be shortened due to the end-to-end characteristic.
However, the speed of random structure does not necessarily accelerate the algorithm, which will be discussed later in section 5.3-5.4 this Chapter.

➢ Under fault condition:

In Fig. 49, we can observe what will happen if the communication network link breaks down due to hacker attack or natural disasters.

![Graph showing dynamic convergence performance under fault condition](image)

**Fig. 49 The Dynamic Convergence Performance of the Relative Error (Objective Function) under Fault Condition with two Communication Network Topologies and two Algorithms**

From the Fig. 49, we can conclude that the connectivity is quite crucial, if the fault happens to make the communication network into two separate parts, then both of the DLPDS and the MC-ADMM cannot guarantee the convergence and the optimal value to be reached, because some of the crucial information cannot reach some agents during iteration process. However, when the network is connected, both the DLPDS and the MC-ADMM can help the
convergence to optimal value. Hence, the random topology is more robust than the circular network.

5.2.4 The Parametric Sensitivity Analysis of MC-ADMM

The last question is to determine how to choose parameter $c$ in MC-ADMM. Fig. 50 shows the calculation performance with regard to the value of $c$. (Dimension of independent variables: 10; $\varepsilon=0.1$; Maximum iteration steps: 1000; Radial Topology)

![Graph showing dynamic convergence performance](image)

**Fig. 50** The dynamic convergence performance of the relative error (objective function) with different parameter $c$

From the Fig. 50, we can see that the parameter $c$ has a profound influence on convergence rate of the algorithm. If $c$ is constant, it is neither appropriate to choose $c$ too large nor too small. It’s very important to choose the proper parameter $c$. Besides, we can see that if
we choose $c$ increases gradually with the iteration process, the result is better than maintaining it the constant value all the time.

5.3 Topology Types of the Communication Network

In section 5.2.3, we conclude that the type of communication network plays a pivotal role in influencing the calculation performance. In this section, we are going to discuss this topic thoroughly based on graph theory and give some useful conclusions on designing a better network.
In the distribution power grid, the users are usually connected to the feeder line in sequence. However, the communication network can be designed to have different physical topology compared with power network. The result of MC-ADMM algorithm shows the convergence speed of optimization highly relies on the structure of the communication network.

Fig. 51 Examples of Typical Topologies of Communication Networks
To analyse the impact of communication network topology on the calculation speed of MC-ADMM, a series of numerical simulation have been designed. Six scenarios with different typical network connections demonstrated in Fig. 51 are proposed in this study [48, 49].

➢ **Linear network**

The simplest topology, all “agents” are connected one by one. Most of the feeder systems are formed as this kind of topology. A linear network has high expandability. To add a new “agent” into the network, the operator only needs to connect this new “agent” to the edge of the network and it is not necessary to change the network topology. On the contrary, an “agent” is only able to communicate with another “agent” via the neighbours between them.

➢ **Ring network**

A ring network is a network with circular topology. Each “agent” incorporates a receiver and a transmitter to send the information to the next “agent” in the ring. Compared with the linear network, the ring network increases the transmission speed because some of the paths will be shortened due to the end-to-end characteristic. However, in ring network it’s difficult to add a new “agent” into the original system. Also as a disadvantage, it’s hard to identify the fault location when accident happens.

➢ **Star network**

In a network with star topology, each “agent” is connected to a central “agent” called hub with a point-to-point connection. The star topology is considered the easiest topology to design and implement. The central “agent” is playing a very important role during the information exchanges. Thus, there is high risk for the whole network to collapse when a failure destroy this
hub “agent”. An advantage of the star topology is the simplicity of adding additional nodes to the existing network.

- **Small World network**

  Small world network is a special network topology that arose in social science and was modelled analytically by Duncan Watts and Steven Strogatz in 1998[50]. In a network with the characteristic of small world topology, most of the “agents” are not neighbours of one another, but can be reached from most of other “agents” by a small number of steps. That is to say, in a small world network the average distance between two “agents” is far smaller than the number of “agents” that exists in the network.

- **Scale Free network**

  The most notable characteristic in a scale-free network is the relative commonness of “agents” with a degree that greatly exceeds the average[51]. Differently with the Poisson distribution degree in most random networks, the degree distribution of scale free network obey the power law. Namely, most of the “agents” in a scale free network have only small number of connections but several “agents” have extremely high degree. The ascendancy of these critical “agents” makes this kind of network robust to random failures, but can be very vulnerable to targeted attacks.

- **Fully connected network**

  A fully connected network is also known as a complete graph. It’s a network in which each of the nodes is connected to each other. This kind of network provides a high degree of reliability due to the multiple paths for data transportation between nodes. On the contrary, it's
very expensive to set up a network with this topology, since it requires extremely large numbers of direct links.

5.4 Influence of the Different Communication Network Topology on Calculation Performance

To illustrate the difference of calculation speed among typical communication networks mentioned above, a numerical simulation is performed. The topology of the network is the same with Fig. 42 and MC-ADMM is utilized to solve this problem. The simulation results are elucidated in Fig. 52.

![Iteration Performance of MC-ADMM under Different Communication Network Topologies](image)

Fig. 52 Iteration Performance of MC-ADMM under Different Communication Network Topologies

From the Fig. 52 it can be seen that there are two phases during the convergence process, descent phase and fluctuation phase. In the descent phase, all curves experience the decreasing to
the optimal solution with different velocities. Then after several fluctuations, they converge to the optimal value. However, it can be clearly seen that the iteration steps and converge speed are totally different. In some of the topologies, the curve decrease very fast and draw a severe tails, and others draw very long tails. On the other hand, several curves fluctuate dramatically and take further steps to converge.

To identify the reasons of disparity in convergence performance, several basic characteristics of network topology in graph theory are introduced.

- **Degree**

  In graph theory, the degree of a node is the number of edges incident to the node\([52]\). The degree of a node \(v\) denoted \(\text{deg}(v)\). The degree expresses the adjacency relationship between \(v\) and other nodes, which shows the significance of it in the network. The degrees of each node in the network topologies used in simulation were calculated.

- **Laplacian eigenvalue**

  A Laplacian eigenvalue is the eigenvalue of a Laplacian matrix\([53]\). For a graph \(G\) with \(n\) nodes, its Laplacian matrix \(L\) is defined as:

  \[
  L = D - A 
  \]  

  (59)

  where \(D\) is the degree matrix and \(A\) is the adjacency matrix of the graph. In terms of node degrees, it can be written as:

  \[
  L_{i,j} := \begin{cases} 
  \text{deg}(v_i) & i = j \\
  -1 & i \neq j \text{ and } v_i \text{ is adjacent to } v_j \\
  0 & \text{otherwise}
  \end{cases}
  \]  

  (60)
The Laplacian eigenvalues can be used to find many properties of the graph. One of the most important theorem and applications in spectral graph theory is to calculate the approximate sparsest cut of a graph through the second-largest Laplacian eigenvalue[52].

It can be approved that the second-smallest Laplacian eigenvalues, namely $\lambda_{n-1}$, can reflect connectivity of a network[52]. Connectivity is one of the basic concepts of graph. It represents the minimum number of nodes or edges that need to be removed to disconnect the remaining nodes from each other. If a graph has a large $\lambda_{n-1}$ this means nodes in this graph are highly connected.

The distributions of $\lambda_{n-1}$ in the typical network topologies are demonstrated in Fig. 53.

![Fig. 53 Distribution of $\lambda_{n-1}$ in Typical Topologies](image)

From the introductions above, it can be known that, MC-ADMM algorithm requires “agents” share the information with their neighbours during the calculation. Thus, in a graph with higher connectivity, “agents” can finish information exchange in fewer steps. This can explain the
difference of descent velocity in simulation result. From Fig. 53 it can be seen that fully connected graph has the biggest $\lambda_{n-1}$, that's why it takes the least iteration numbers in descent phase. On the contrary, the scale free network topology used in this simulation has the lowest connectivity. So the iteration curve decreases very slow and takes almost five times iteration steps more than that in fully connected topology.

The distribution of degree in a topology is also found to affect the calculation speed. In [54] Li and Pan have proved a lemma:

$$\lambda_2 \geq D_2$$  \hspace{1cm} (61)

Where $D_2$ is the second-largest degree in the graph. They also pointed out that it is difficult to find a graph with $\lambda_2 = D_2$, most of the graph has a difference between $\lambda_2$ and $D_2$. This can be classified as an extremal graph[55] problem in graph theory, which means to search for a graph with largest number of edges with certain property. A graph with $\lambda_2 = D_2$ can be called an extremal graph here. Then the smaller difference between $\lambda_2$ and $D_2$, the closer a graph is to an extremal graph.

The reason of long term fluctuation is analysed as follows. It may be caused by the redundancy information between “agents” communication during the distributed optimization. But from the simulation result, it can be deduced that the difference between $\lambda_2$ and $D_2$ affecting to the fluctuation level during iteration. The distribution of difference between $\lambda_2$ and $D_2$ in each topology is shown in Fig. 54. It is not hard to find, the larger difference there is, the more intense fluctuation it has. In other words, if a graph is closer to an extremal graph, it can reduce the level of fluctuation during iteration.
From the discussions above, it is known that connectivity of a communication network will influence the calculation speed. But it doesn’t mean a network with a higher connectivity will gain the faster converge speed, it also has the risk to get into a long term fluctuation. Therefore, there might exist an optimum topology for communication network in distributed PV-ES combined system. Such topology can help to improve the calculation speed of distributed algorithms. In this case, the ideal topology should have maximum $\lambda_{n-1}$ and minimum difference between $\lambda_2$ and $D_2$. With such a communication network, an optimum iteration curve with the highest descent velocity and the smallest fluctuation level can be gained. Also this means the fastest calculation speed of distributed algorithm can be achieved.
6. Conclusions

In this Master thesis, I investigated several problems in the future smart grid environment related to higher renewable generation. In particular I have emphasised the impact of the renewable power on balancing and stability, use of storage and some control problems arising. Specifically, I proposed several practical and promising methods using energy storage devices to better control and dispatch these intermittent resources to mitigate the adverse impact. The summary and conclusions are listed as follows on a chapter by chapter basis:

In Chapter 2, A comprehensive simulation test platform for the Australian future grid with high penetration level of RE and ES is built in DIgSILENT. The platform includes steady state power flow calculation scanning, dynamic stability analysis considering different penetration levels as well as DSM uptakes.

The simulation results show that in steady state the penetration level of RE and the locations of ES play a very important role. These two factors will impact the result of power flow distribution greatly. In the aspect of voltage stability, the performance deteriorates significantly due the lack of reactive power. Result shows rotor angle stability is insensitive with regard to the RE penetration level. In terms of frequency stability, during disturbance the frequency dynamic speed becomes faster because of the increasing inverter based RE.

In Chapter 3, A novel block diagram of the closed-loop LFC system with ES and wind power module has been proposed. Two important factors that exist in future power network, i.e. parametric uncertainty as well as ubiquitous time delay in control channel, are considered simultaneously. Based on rigorous mathematical LMI theory, a Robust $H_\infty$ controller is designed
to deal with the above mentioned problems and meanwhile it maintains the frequency stability of the system.

Through a comprehensive case study of the two-area practical system, we can verify that the proposed controller outperforms the conventional PI one: it can guarantee the robustness of the system as long as the parametric uncertainty and the time delay are bounded, without changing the coefficients of the controller itself all the time. The case studies have also demonstrated that with the appropriate controller, ES can be quite effective to the overall power system frequency regulation when the system contains intermittent renewable power generation sources.

In Chapter 4, we develop a sensitivity analysis based-enhanced optimal distributed consensus algorithm (EODCA) that combines the theory of power system itself and the concept of multi-agents together.

The effectiveness of the proposed algorithm is verified: Firstly, it can help the power distribution company to maximize its net profits with very limited information channel built among devices instead of setting up a control centre, and the optimal value is the same with that of the centralized approach. This scheme can help the power distribution company save the costs and achieve a better economic goal. Furthermore, the optimal solutions can also guarantee the safety operation of the distribution network simultaneously, i.e. to alleviate the reverse power flow problem and ensure the transmission power in all lines and the voltages of all nodes are restrained within the engineering required envelopes. Therefore, the EODCA can be utilized to improve the penetration level of the sustainable energy integration in the future. The more numbers of renewables connected into the distribution network, the more cost can be reduced.
In Chapter 5, we propose and utilize a Modified Consensus Alternating Direction Method of Multipliers (MC-ADMM) to solve a practical problem in future power distribution networks. Besides, we also analysed the influence of the communication network topology on the calculation performance.

Compared with a latest version of the distributed optimization algorithm called DLPDS, we can see that MC-ADMM performs better in terms of the speed, the accuracy and the ability to handle larger systems with more devices. Simulation result shows the distributed optimization algorithm speed in this model is highly sensitive to the topology of communication networks. By analysing the degrees and Laplacian eigenvalues of these networks, a conjecture is made. The descent velocity during the iteration is determined by the second-largest Laplacian eigenvalue, and the level of fluctuation is related to the difference between the second-largest Laplacian eigenvalue and second-largest degree. Therefore, we can give the hypothesis that there might be an optimum topology for communication network connections.

**General Conclusion and Future Work:**

As we can see from the above discussions, the integration of highly intermittent RE will pose potential adverse impacts on future grid operation and stability. Therefore, with the smart grid technologies such as the advanced control approach and distributed optimization methods proposed in this thesis, we can have a promising way to tackle this problem with energy storage devices.

I present these findings, conclusions and algorithms as fundamental contributions in helping us to face the upcoming challenges for future power grids.
There are quite a few interesting topics for further work with regard to the topic of integrating renewables and applying storages. For example, how to mathematically design and verify an optimum communication network topology that can minimize the distributed calculation time; how to design an output feedback frequency controller instead of the full-state feedback one, that can not only tackle time delay and parametric uncertainty but can also deal with the data disorder and loss problems in the future situation? These topics are all challenging and require a further research.
References


