

**The Term Structure of Interest Rates in a simple Stochastic Growth
model: Evidence from Australian data**

by

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ABSTRACT

This paper presents a theoretical relationship between the yield curve and future economic growth in a simple stochastic growth model. The derived relationship implies that, in a simple competitive production economy, the slope of the yield curve predicts future output growth. This predictive content of the yield curve is tested using Australian data by employing a vector autoregression (VAR) method. As a way to examine the conventional view that the predictive content of the yield curve is mainly due to the liquidity effect of monetary policy operating on short term interest rates, Granger causality test is performed. The short rate is found to fail to Granger cause either the spread or the long rate. This finding does not support the conventional view that the predictive content of the yield curve is primarily due to the conduct of monetary policy.

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I. Introduction

It has been long recognised that financial market variables, such as stock prices and interest rates, contain a significant amount of information about the future state of the economy. Recently, there has been a growing consensus, especially in the US, that the term structure of interest rates, or more loosely interest rate spreads, contain considerable information about future economic activity.

A number of researchers [e.g. Stock and Watson (1989), Harvey (1989)] have pointed out that whenever the yield curve was inverted (i.e. whenever short rates exceeded long rates) the economy subsequently went into recession. On the other hand, an upward sloping yield curve was found to foreshadow strong economic activity. These results raise the question as to why the interest rate spread is such a good predictor of future real economic activity.

This paper aims to examine the yield spread variables between the long term and short term interest rates as a predictor of real economic growth in Australia. In particular, it provides a theoretical rationale underlying the predictive power of yield spreads using a basic dynamic equilibrium model. The organisation of this paper is as follows.

Following a brief survey on the literature on the predictive ability of the the term structure for the future output, a theoretical relationship linking the slope of the yield curve to the expected growth in real output is derived in a simple model of dynamically competitive production economy. This theoretical relationship provides a framework for the empirical analysis to be followed in the subsequent section. After discussing the data, we examine the predictive power of the yield spread for real economic growth using vector autoregression (VAR) methodology. Finally, we attempt to examine what the data suggests is the reason for the predictive power of the yield spread. The proposition that we test is whether the predictive power of the yield spread stems from the effect of monetary policy operation on the short term interest rate or not. To investigate this, the Granger causality between the short and long end of the yield spread

and also between the short end and the spread is tested. The concluding remarks then follow.

The formal link between asset markets and real activity was first noted by Fisher (1907). Since the development of the intertemporal capital asset pricing model (ICAPM) by Merton (1973), asset pricing models and, in particular, models of the term structure, have been treated in a general equilibrium framework assuming rational expectations. [See Lucas (1978), Brock (1982), Cox, Ingersoll and Ross (1985a, 1985b) and Breeden (1979, 1986)]. The equilibrium asset pricing theories, in particular, the consumption based asset pricing model, have suggested a relationship between the asset returns and expected future consumption paths.

While many researchers looked at the term structure and consumption in the continuous time framework, the discrete time version of these theories has also been well developed. [See Harvey (1988), Fisher and Richardson (1990), for example]. However, the relationship between term structure of interest rates and output growth has not received as much theoretical consideration. Very recently, Hu (1993) derived a theoretical link between the yield spread and real output growth by extending the continuous time stochastic version of the equilibrium asset pricing model.

The growth of the equilibrium approach, however, has not been confined to asset pricing theories. In macroeconomics, the dynamic equilibrium approach has been applied to business cycles and growth most notably by Lucas (1975, 1977, 1980), Kydland and Prescott (1982), Long and Plosser (1983) and, King, Plosser and Rebelo (1988) and Campbell (1994).

This emergence of the equilibrium approach has opened the theoretical possibility for relating the real term structure to consumption growth. For recent works, see Harvey (1988) and Salyer (1994).

Salyer (1994) derives parametric relations between the term structure of interest rates and production in the framework of a simple equilibrium stochastic growth model. This paper also takes a similar approach, utilising a discrete

time stochastic growth model, so as to show the link between the real term structure of interest rates and expected growth in real output.

On the empirical side, a number of studies have examined the relationship between movements in the yield spread and real output. Stock and Watson (1989) examined the predictive power of two interest rate spreads: the difference between the six month commercial paper rate and the Treasury bill rate with the same maturity (also referred to as the paper-bill spread), and the difference between the ten year and one year Treasury bond rates. They found that the paper-bill spread is an exceptionally good predictor of future real activity. This finding has been confirmed by Bernanke (1990) and further by Friedman and Kuttner (1992). Other spread variables have also been investigated. Bernanke and Blinder (1992) claim that the spread between the federal funds rate and a long term rate is an “extremely” good predictor of US economic growth.

There are numerous studies that have empirically examined the predictive power of the spread between long term and short term riskless interest rates. They include Harvey (1988, 1989), Chen (1991), Estrella and Hardouvelis (1991), and recently Hu (1993) and Plosser and Rouwenhorst (1994). They all have shown that the term structure of the US and/or other industrialised nations can be used to predict real economic growth.

In Australia, Lowe (1992) and Alles (1995) have examined the spread between short term and long term interest rates as a predictor of future real activity. Lowe finds that the spread between the nominal interest rate on 180 day bank bills and 10 year government bonds predicts the rate of change in real activity for forecast horizons of one to two years. Alles also presents evidence that the Australian term structure is a good predictor of cumulative growth in real GDP. Both Alles and Lowe find that the yield spread contains no predictive information for future growth in output prior to the third quarter of 1982 and that the yield spread is less effective in predicting quarterly, as opposed to annual, growth rates of output.

Many arguments have been given as to why movements in the yield spread are related to future growth of real output or domestic demand. There are two main but different views concerning the economic rationale for the predictive power of the spread between short term and long-term rates.

The first attributes the predictive power to the effectiveness of monetary policy through short-run price stickiness. Typically, the monetary authority's actions to influence interest rates occur at the short end of the term structure. According to this view, a tight monetary policy causes short term rates to increase relative to long term rates which will lead to a flat or negative slope of the yield curve. Higher short term rates will deter consumption and investment, which will eventually dampen economic activity. Consequently, a flat or inverted yield curve is associated with a future downturn in economic activity.

In fact, a number of economists share the view that a good way to judge the stance of monetary policy is to look at the slope of the yield curve. Studies that interpret the predictive power of the yield curve as a result of the transmission mechanism of monetary policy include Bernanke (1990), Bernanke and Blinder (1992) and Lowe (1992), among others. For example, Lowe asserts that the predictive power of the spread is mainly due to the liquidity effect, which is the possible negative response of the interest rate to a contemporaneous rise in the money supply.

The argument is that movements in short term interest rates as a consequence of monetary policy actions underlie the predictive power of the yield spread for future real activity because real activity responds to movements in short term interest rates with a lag.

The other view is that the predictive power of the yield spread is a consequence of economic agents' expectations about the future state of the economy and intertemporal utility maximising behaviour. Harvey (1988, 1989), Hu (1993) and Plosser and Rouwenhorst (1994) are representative examples of this point of view.

This paper extends these previous Australian studies by providing a simple but clear theoretical case as well as further econometric investigation. While the former Australian studies use exclusively single equation OLS econometric methods this study employs the vector autoregression (VAR) method which has been well adopted in examining dynamic relationships among macroeconomic variables.

II. Model

In this section, the link between the term structure of interest rates and real output is analytically shown within the framework of a stochastic growth model. It is well recognised that interest rates are a crucial variable in dynamic equilibrium models, and as a consequence a natural extension is to consider the term structure within such a framework. Recently, the relationship between the term structure of interest rates and real economic growth has been analysed further both theoretically and empirically by Salyer (1994) and Plosser and Rouwenhorst (1994) among others.

This section attempts to analytically consider the link between the term structure of interest rates and real output growth in the context of stochastic growth model, a variant of the Brock-Mirman type economy.

We consider a simple intertemporal general equilibrium model as follows.

Preferences

Consider a representative agent maximising the life time utility

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln C_t \right] \quad (1)$$

Note that for this isoelastic log utility the coefficient of relative risk aversion is unity.

Production/Technology

Consider the production function

$$Y_t = A_t K_t^\alpha \quad (2)$$

where A and K denotes technology and capital respectively.

This specification¹ exhibits decreasing returns to scale with fixed labour, $N_t = 1$, so the capital is the only input.

We assume that A_t is lognormally² distributed with mean 0 and constant variance.

Capital accumulation

The evolution of the capital stock is described as follows.

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (3)$$

The gross investment I_t is a decision variable at time t . This may be interpreted as a gestation lag of one period before investment becomes available as an input to production in the next period.

We assume that capital is assumed to depreciate fully each period, i.e. $\delta = 1$. Incomplete depreciation of capital was considered by King, Plosser and Rebelo (1988) and Campbell (1994) but it makes the solution only approximate due to nonlinearity, which makes the model difficult to solve. Therefore, we specify

$$K_{t+1} = I_t \quad (3')$$

Efficiency condition for capital use

The gross rate of return on a one period investment in capital is equal to the marginal product of capital under complete depreciation (which, in equilibrium, is equal to $1+r_{1t}$). That is,

$$1 + r_{1t} = [\alpha A_{t+1} K_{t+1}^{\alpha-1}] \quad (4)$$

Note that β discounts utility units while $1/R_{t+1}$ discounts consumption (real net cash flows).

Resource constraint

Since labour is assumed to be fixed, all we require is that, assuming no output is wasted,

$$C_t + I_t = Y_t$$

¹ This is a concave production function. For the use of this type of specification; see Balvers et al. (1990) and Salyer (1994).

Optimal Decisions for the Planner

For the log utility and the production function (2), the Lagrangian function is formed as

$$L = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t [A_t K_t^\alpha - C_t - K_{t+1}]$$

where λ_t is the Lagrange multiplier attached to the period t resource constraint $Y_t - C_t - I_t = 0$.

The first order conditions are

$$\beta^t C_t^{-1} = \lambda_t \quad (5)$$

$$\lambda_{t+1} [\alpha A_{t+1} K_{t+1}^{\alpha-1}] = \lambda_t \quad (6)$$

$$A_t K_t^\alpha - C_t - K_{t+1} = 0 \quad (7)$$

$$\text{TVC: } \lim_{t \rightarrow \infty} \beta^t \lambda_t K_{t+1} = 0 \quad (8)$$

The Solution to the system

The solution to the dynamic system is of the form³

$$C_t = C_t(K_t, A_t)$$

$$K_{t+1} = K_t(K_t, A_t)$$

In particular, we conjecture that

$$C_t = \Theta_1 A_t K_t^\alpha \quad (9)$$

$$K_{t+1} = \Theta_2 A_t K_t^\alpha \quad (10)$$

Using (5) and (6) to obtain

$$\beta \alpha A_{t+1} K_{t+1}^{\alpha-1} = \left(\frac{C_{t+1}}{C_t} \right)$$

Using (9), we can re-write this as

$$\beta \alpha A_{t+1} K_{t+1}^{\alpha-1} = \frac{\Theta_1 A_{t+1} K_{t+1}^\alpha}{\Theta_1 A_t K_t^\alpha}$$

² For a lognormal random variable X_{t+1} : $\log(E_t X_{t+1}) = E_t \log X_{t+1} + 1/2 \text{var}_t(\log X_{t+1})$

³ McCallum (1989, pp21-22) and Campbell (1994, pp470-71) also use this conjecture.

$$\beta\alpha / K_{t+1} = \frac{1}{A_t K_t^\alpha}$$

Substitute (6) in and re-arrange to obtain

$$\Theta_2 = \alpha\beta$$

To find Θ_1 substitute (9) and (10) into (7), so that

$$\Theta_1 A_t K_t^\alpha + \Theta_2 A_t K_t^\alpha = A_t K_t^\alpha$$

$$A_t K_t^\alpha (\Theta_1 + \alpha\beta) = A_t K_t^\alpha$$

$$\Theta_1 = 1 - \alpha\beta$$

So the implicit solution (9) and (10) becomes

$$C_t = (1 - \alpha\beta) Y_t \quad (11)$$

$$K_{t+1} = \alpha\beta Y_t \quad (12)$$

which are the decision rules expressed as the functions of output.

The Relationship between the real interest rate and output growth

It can be shown that within the stochastic growth model the interest rate is linearly linked to the real output growth. To see this, take the logs of (2) and using lower case letters to denote logarithms

$$y_t = a_t + \alpha k_t \quad (2')$$

Recalling $1 + r_{1t} = [\alpha A_{t+1} K_{t+1}^{\alpha-1}] \quad (4)$

Take the logs of (4) and using lower case letters

$$r_{1t} = \ln \alpha + a_{t+1} + (\alpha - 1)k_{t+1} \quad (4')$$

where we have used the approximation, $\ln(1+r_{1t}) \approx r_{1t}$.

Update (2') one period and substitute into (4') to obtain

$$r_{1t+1} = \ln \alpha + y_{t+1} - k_{t+1}$$

Take logs of (12)

$$k_{t+1} = \ln \alpha\beta + y_t$$

and substitute in the previous equation to obtain

$$r_{1t+1} = -\ln \beta + \Delta y_{t+1}$$

or

$$r_{1t+1} = -\ln \beta + \ln (Y_{t+1}/Y_t) \quad (13)$$

Equation⁴ (13) relates the risk-free real interest rate linearly to real output growth. Since β is typically less than 1, the negative of $\log \beta$ will be positive, hence implying a positive relationship between the one period interest rate and one period output growth.

First Order Condition for Optimal Consumption

To derive the condition for optimal consumption, update (5) one period and substitute the result into (6) to obtain

$$\beta C_{t+1}^{-1} [\alpha A_{t+1} K_{t+1}^{\alpha-1}] = C_t^{-1} \quad (14)$$

Recall again (4)

$$1 + r_{1t} = [\alpha A_{t+1} K_{t+1}^{\alpha-1}]$$

Substitute the equation (4) into (8) and take the expectations conditional on information at time t to obtain

$$C_t^{-1} = \beta (1+r_{1t}) E_t \{ C_{t+1}^{-1} \} \quad (15)$$

This is the stochastic Euler equation for intertemporal choice of consumption for the above problem.

The Relation between the term structure and consumption growth

To see the relation between the term structure and output more formally, we now assume that the purchase price of a bond is 1 unit of consumption, which can be written as, assuming log utility

$$(1+r_{1t})^{-1} = \beta C_t E_t \{ C_{t+1}^{-1} \} \quad (16)$$

Similarly, for a bond with maturity n , where the yield is known at t , it follows that

$$(1+r_{nt})^{-n} = \beta^n C_t E_t \{ C_{t+n}^{-1} \} \quad (17)$$

Now assume that consumption is lognormal and homoskedastic

$$\ln E_t(C_{t+1}^{-1}) = - E_t (\ln C_{t+1}) + (1/2) \text{var}_t(\ln C_{t+1}) \quad (18)$$

⁴ It can be verified that the equation (13) also results from the use of the Cobb-Douglas production function with constant returns to scale.

Take the logs⁵ of the equations (16), and substitute in (12) to obtain

$$r_{1t} = -\ln\beta + E_t(\ln C_{t+1} - \ln C_t) - (1/2)\text{var}_t(\ln C_{t+1}) \quad (19)$$

Similarly, take the log of equation (17), update (18) $n - 1$ periods and substitute it to obtain

$$r_{nt} = -\ln\beta + (1/n) E_t(\ln C_{t+n} - \ln C_t) - (1/2n)\text{var}_t(\ln C_{t+n}) \quad (20)$$

This equation has also been derived by Breedon (1986) and Fisher and Richardson (1992).

The yield differential between the long term and the short term rates in terms of consumption can be obtained by subtracting (19) from (20)

$$\begin{aligned} r_{nt} - r_{1t} &= (1/n) E_t(\ln C_{t+n} - \ln C_t) - E_t(\ln C_{t+1} - \ln C_t) \\ &\quad - (1/2n)\text{var}_t(\ln C_{t+n}) + (1/2)\text{var}_t(\ln C_{t+1}) \end{aligned} \quad (21)$$

The Relationship between the term structure and expected future output growth

Now recall that the decision path for consumption in the competitive equilibrium can be expressed as a function of output.

$$C_t = (1 - \alpha\beta) Y_t \quad (11)$$

This relationship implies that the economic agent consumes a fixed proportion of output, with the proportionality factor being the product of his or her rate of time preference and the technology parameter α . Intuitively, this is analogous to the formula derived by Hu (1992) in the continuous time asset pricing framework.

Now forward (5) by one and n periods respectively to obtain

$$C_{t+1} = (1 - \alpha\beta) Y_{t+1} \quad (22)$$

$$C_{t+n} = (1 - \alpha\beta) Y_{t+n} \quad (22')$$

Take logs of these equations and subtract (5) from (16) and (16') respectively to obtain

$$\ln(C_{t+1}/C_t) = \ln(Y_{t+1}/Y_t) = \ln Y_{t+1} - \ln Y_t \quad (23)$$

$$\ln(C_{t+n}/C_t) = \ln(Y_{t+n}/Y_t) = \ln Y_{t+n} - \ln Y_t \quad (23')$$

⁵ Here, we use that for small x , $\ln(1+x) \approx x$

This implies that the growth rate of consumption and output is the same over the steady state in a competitive economy.

We can note that the variance of log consumption is equal to the variance of log output.

$$\begin{aligned}\text{var}_t(\ln C_{t+n}) &= \text{var}_t(\ln(1-\alpha\beta)Y_{t+n}) \\ &= \text{var}_t(\ln(1-\alpha\beta) + \ln Y_{t+n}) \\ &= \text{var}_t(\ln Y_{t+n})\end{aligned}$$

Substitute (23) and (23') into (21) to obtain

$$\begin{aligned}r_{nt} - r_{1t} &= (1/n) E_t(\ln Y_{t+n} - \ln Y_t) - E_t(\ln Y_{t+1} - \ln Y_t) \\ &\quad - (1/2n)\text{var}_t(\ln Y_{t+n}) + (1/2)\text{var}_t(\ln Y_{t+1})\end{aligned}\quad (24)$$

Since both log consumption and log output are homoscedastic we can write

$$\begin{aligned}r_{nt} - r_{1t} &= (1/n) E_t(\ln Y_{t+n} - \ln Y_t) - E_t(\ln Y_{t+1} - \ln Y_t) \\ &\quad - (1/2n)\sigma^2 + (1/2)\sigma^2\end{aligned}\quad (25)$$

The equation (25) can be re-written, suppressing the constant terms, as

$$\begin{aligned}r_{nt} - r_{1t} &= (1/n) E_t[\ln Y_{t+n} - \ln Y_{t+n-1} + \ln Y_{t+n-1} - \ln Y_{t+n-2} + \ln Y_{t+n-2} - \ln Y_{t+n-3} \\ &\quad + \dots + \ln Y_{t+1} - \ln Y_t] - E_t[\ln Y_{t+1} - \ln Y_t]\end{aligned}$$

or

$$r_{nt} - r_{1t} = (1/n) E_t[\Delta \ln Y_{t+n} + \dots + \Delta \ln Y_{t+1}] - E_t[\Delta \ln Y_{t+1}]\quad (26)$$

Equation (26) relates the yield differential between an n period and a 1 period bond to the expected average of future output growth for n periods less the 1 period ahead output growth. For a growing economy, the real term structure will be positively related to expected future output growth.

The above relationship links the real term structure to the expected future growth rate of output. Although in empirical studies, we use the nominal term

spread this can well approximate the real term spread in an economy with relatively stable inflation.⁶ This relationship will be served as a theoretical basis for the empirical section that follows.

III. Empirical Analysis

1). Preliminary Data Analysis

The slope of the yield curve can be represented as the spread between the long-term and short-term yields on government securities or default-free bonds. The short and long term yields data used are the annualised quarterly percentage returns on 13 weeks treasury notes (TN13) and 10 year treasury bond (TB10Y).

We can compute the slope of yield curve as follows.

$$\text{SPREAD1} = r_{10y}^g - r_{13w}^g$$

Alternatively, the spread between the short term bank-accepted bill (BB180) and long term government bond can be considered. Although the two yields variables are somewhat different in terms of the risk structure, it is nevertheless worth considering if the predictability about the future growth of real output matters. Lowe (1992) claims that this measure of spread is best at predicting future changes in inflation.

$$\text{SPREAD2} = r_{10y}^g - r_{180d}^b$$

Throughout the section both measures of spreads will be used concurrently and their relative performances as a predictor of GDP.

The main data series used in this study are taken from the DX Database and include the short-term (BB180, TN13) and long-term interest rate (TB10Y), real GDP(A) at 89/90 prices. All the series are seasonally adjusted and are quarterly observations although monthly data series are also used in the section for the Granger causality test. The sample period to be used in this

⁶ Professor Kingston pointed out that a further research area in this regard would be to examine the predictive power of the index bonds.

study is 1982:3 to 1995:4. (for monthly data, from January 1980). This effectively covers the post deregulatory period.

Table 1: Summary Statistics for real GDP growth and Yield spreads

Variable	Mean	S.D.	Autocorrelations coefficients					
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_8	ρ_{12}
TN13	11.50	3.80	.91	.82	.71	.59	.12	.03
BB180	12.32	4.03	.94	.85	.77	.65	.22	.06
TB10Y	12.19	2.31	.93	.86	.79	.72	.38	.14
GDP growth	2.92	2.55	.79	.58	.31	.03	.28	-.16
Spread 1	0.69	2.17	.86	.68	.52	.36	-.13	-.03
Spread 2	-0.19	2.17	.90	.73	.60	.45	-.02	-.06

The summary statistics in Table 1 reveal the followings. First of all, the mean of the 10 year treasury bill is greater than the short-term 13 weeks T-notes implying the existence of a term premium. On the other hand, the mean of the 180 day bank-accepted bill is slightly above that of the long-term government bond with the differential possibly implying a risk premium. So spread 2 has a negative mean while spread 1 has a positive mean. This indicates that, on average, the yield curve for the riskless bonds generally slopes upward

Figure 1 : GDP growth and Lagged Yield *Spread 1* in Australia

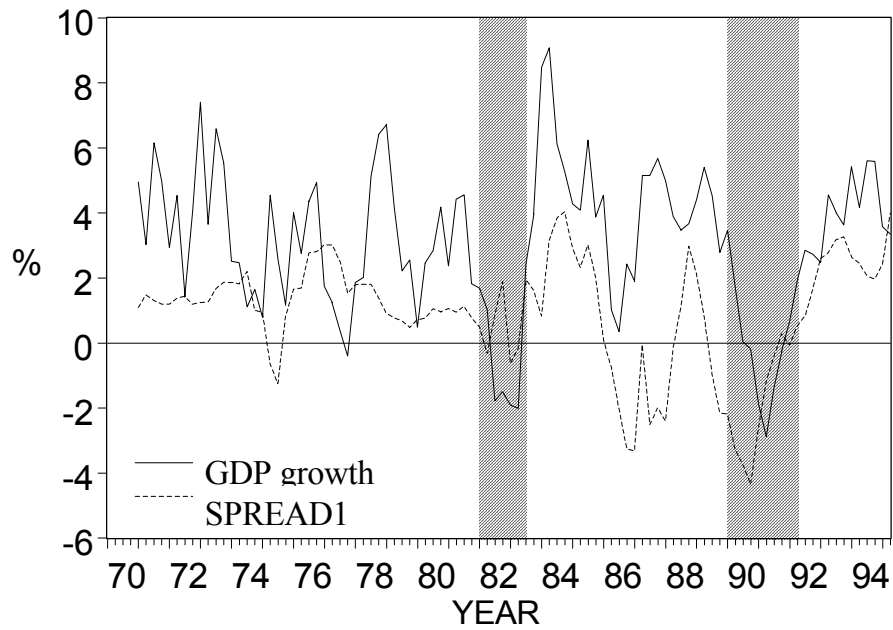
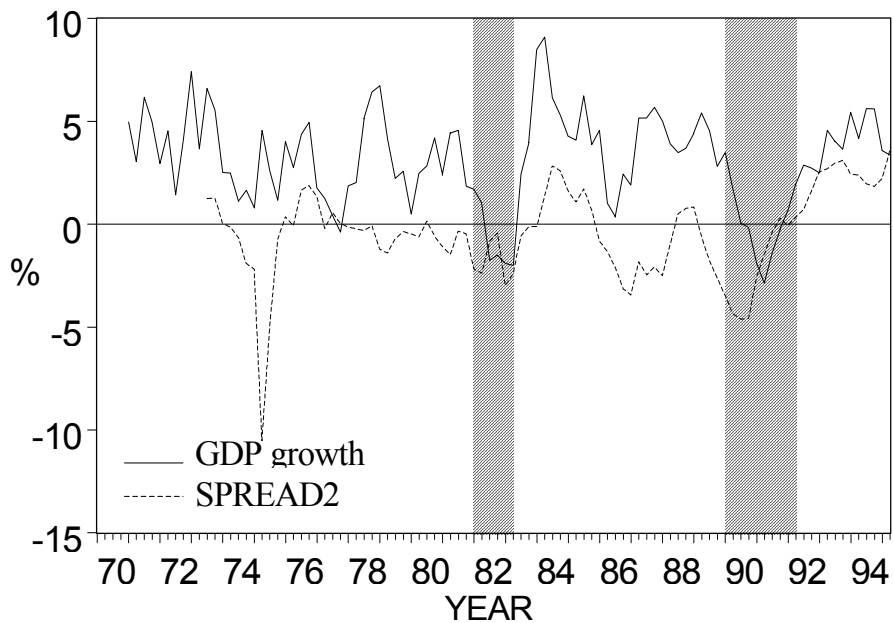


Figure 2: GDP growth and Lagged Yield *Spread 2* in Australia



Note: Shaded areas represent major recessionary periods since 1980.

for the sample period. The 180 day bill is also more volatile⁷ than either of the other two bond yields.

But it is to be noted that despite the difference in the relative volatilities of the selected short term rates the two measures of spread share the same degree of volatility.

Although the autocorrelations functions provide an indication of whether the series are stationary or not, a formal test for stationarity of the series is now employed. To test for the stationarity of the spreads and GDP growth the augmented Dickey-Fuller test is performed, which is based on the following regression model

$$\Delta x_t = \tilde{\mu} + \tilde{\beta} \cdot t + \tilde{\alpha} \cdot x_{t-1} + \sum_{i=1}^p \tilde{\phi}_k \Delta x_{t-p} + \tilde{\varepsilon}_t$$

where p is the number of augmentation terms included to ensure approximately white noise Gaussian residuals, $\tilde{\varepsilon}_t$.

Table 2: ADF Unit-root Test results.

Series	Sample Period	p	t-statistics
TN13	70:1 - 95:2	0	-1.23
BB180	70:1 - 95:2	6	-1.23
TB10Y	70:1 - 95:2	0	-1.34
Spread1	70:1 - 95:2	4	-3.31**
Spread2	72:3 - 95:2	1	-3.47**
GDP	70:1 - 95:2	0	-1.89
GDP growth	70:1 - 95:2	4	-2.97*

Note: - ** and * denote the rejection of the null hypothesis of unit root at 5% and 10%, respectively.

- We follow the sequential procedure suggested by Dolado et al (1990).

⁷ Using 90 day bank-accepted bill was considered but ruled out because it is even more volatile than the 180 day bill. Harvey (1989) argues that volatility of a financial variable is of a nuisance in reflecting the predictive information about output growth. For using the 90 day bill rate, see Alles (1995).

The test results indicate that the measures of spread and output growth are both stationary processes as expected a priori.

2). *VAR analysis*

The econometric method to be used in the subsequent section is the vector autoregression (VAR). The primary objective of using the VAR approach in this section is to examine the response of output growth to a shock to the yield spread.

We present a bivariate VAR model, including GDP and each of the spread variables.

A VAR model with deterministic terms can be represented as follows.

$$z_t = A_0 + A_1 z_{t-1} + \dots + A_p z_{t-p} + e_t \quad (1)$$

where z_t is a 2×1 vector of variables containing spread and output growth.

$$\text{Let } z_t = \begin{bmatrix} \text{sp}_t \\ \dot{y}_t \end{bmatrix}$$

Following Sims (1980) it can be shown that the above VAR system is the reduced form of some underlying structural system of equations.

The VAR model can be written, using lag polynomials, as

$$A(L)z_t = A_0 + e_t \quad (2)$$

where $A(L) = I_n - A_1 L - \dots - A_p L^p$;

and $L^i z_t = z_{t-i}$.

The implied moving average (MA) representation of z_t can be written as

$$\begin{aligned} z_t &= \mu + A(L)^{-1} e_t \\ &= \mu + \sum_{s=0}^{\infty} \psi_s e_{t-s} \end{aligned} \quad (3)$$

where $\mu = [A(L)]^{-1} A_0$

This can be transformed into the recursive form

$$z_t = \sum_{s=0}^{\infty} \psi_s^* \varepsilon_{t-s} \quad (4)$$

by using a lower triangular matrix S where $\psi_i^* = \psi_i S^{-1}$ and $\varepsilon_t = S\varepsilon_t$, so that the impulse response functions to a shock to the orthogonalised innovations, ε_i , can be generated.

However, a major drawback⁸ of this technique is that the choice of the S matrix is not unique, so that a different ordering of the variables in z will typically lead to a different S , thus altering the impulse response functions and innovation accounting decompositions. Sims (1981) suggests trying various plausible orderings of the variables and to check for robustness and consistency. Although the yield spread is not likely to be most endogenous, the practice of ordering it last has been popularly adopted. [See Bernanke and Blinder (1992) and Friedman and Kuttner (1993) for adopting this practice]. This paper also follows this practice. But the results were found to be robust to changes in ordering indicating that the spread variables are almost orthogonal to output growth.

In order to determine an appropriate order of VAR models for the quarterly VAR system, a sequential Sims' Likelihood Ratio test and Akaike's Information Criterion (AIC)⁹ are used to choose appropriate order (p) of the VAR. However, we are guided by the Sims' LR test as it suggests more parsimonious lag lengths in all cases. In fact, it is well known that the AIC has a tendency to overestimate the correct order of p . We adopt lag lengths of 4 for the bivariate VAR model.

We now examine the predictive ability of spread using a VAR model. The results from the VAR can be compared with the results from the single equation econometric studies undertaken by Lowe (1992) and Alles (1995).

From the variance decomposition table, both the spreads explain more than 12 and up to 14 % of the forecast error variance of quarterly GDP growth of forecast horizons between 12 to 20 quarters.

⁸This was critically reviewed by Cooley and ReLoy (1985).

⁹ That is, to minimise $-\ln$ likelihood + number of parameters.

Table 3 : Variance Decomposition of GDP growths : bi-variate VAR

Decomposition of Forecast Error Variance of GDP growth		
A One S.D. Shock due to	Spread1	Spread2
4	1	2
8	10	10
12	13	13
16	13	13
20	14	13

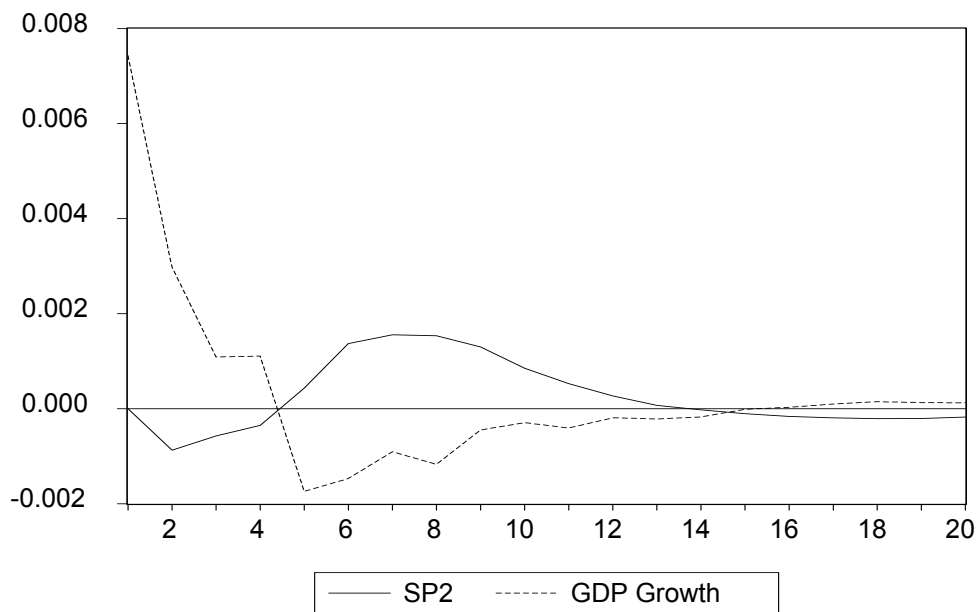
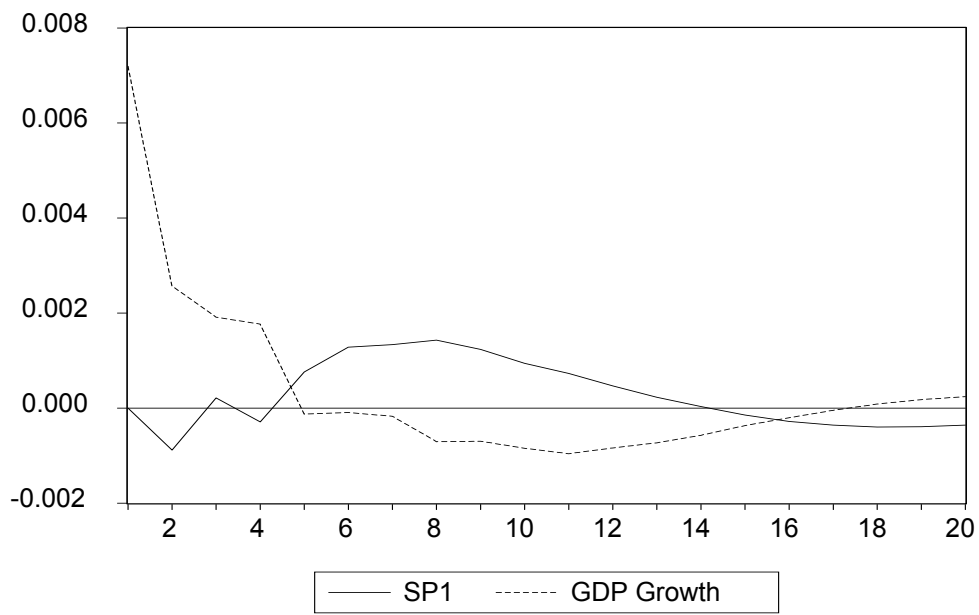
Figures 3.1 and 3.2 exhibit impulse responses of quarterly GDP growth to an exogenous shock to Spread 1 and Spread 2, respectively.

The impulse responses of GDP growth confirm the results of variance decomposition. In particular, it can be noted that the response of GDP growth to a shock to the spread variables persists for more than 12 quarters, with the most impact occurring between 6 and 8 quarters.

While this result is broadly consistent with the findings of the previous Australian studies that the predictive power of the yield spread is relatively persistent up to 9 quarters, our results indicate more persistent impacts of the yield spread on the output growth.

Although the predictive power of the yield spread found above appears consistent with the prediction of the model presented in this paper, it is not clear whether it stems from real forces driving the economy as implied in the model or by monetary policy as has been argued by researchers such as Bernanke and Blinder (1992) and Lowe (1992).

Figure 3 : Impulse Responses of quarterly Output growth



To empirically examine this issue, Granger causality test is performed. The underlying idea is as follows. If the predictive power is due to monetary policy

which affects real output due to short-run price stickiness, it is plausible to assume that the much of the predictive content must be due to the variations in the short end of the spread as the long rate is less directly influenced by monetary policy. If this is the case, the short rate must be exogenous and hence Granger causally the prior.

3). *Granger Causality Tests*

In examining the relative predictive content between the short and long interest rates we first need to consider the possibility for cointegration between the long and short rates. Although the Johansen's system approach¹⁰ is also feasible, for bivariate case the single equation test, such as the Engle-Granger type test can be readily used, which is to test for the stationarity of the cointegrating regression.

For Granger causality test between the long term and short term rates, we consider two cases. Since it is well known that for nonstationary data standard F-statistics do not have standard limiting distribution, we consider the Granger causality test for the first differences of the data. However, it has also been shown that, although the level variables contain unit root, Granger causality test can be consistently performed in the cointegrated VAR processes. [See Lütkepohl and Reimers (1992) and Toda and Phillips (1993)].

To develop the test of the Granger causality in the cointegrated VAR model, let x_t and y_t denote the levels of the short term and the long term rates, which are respectively $I(1)$, so that Δx_t and Δy_t are stationary.

Then a bivariate VAR of order p can be written as

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \sum_{i=1}^p \begin{bmatrix} a_{11,i} & a_{12,i} \\ a_{21,i} & a_{22,i} \end{bmatrix} \begin{bmatrix} x_{t-i} \\ y_{t-i} \end{bmatrix} + \mu + \varepsilon_t$$

¹⁰ The Johansen cointegration test indicates there is a cointegrating relationship only at the 20 % level.

where $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is assumed to be an independently and identically distributed Gaussian process with the mean and variance $(0, \Lambda)$.

This representation can be re-arranged as a vector error correction (VEC) representation

$$\begin{bmatrix} \Delta x_t \\ \Delta y_t \end{bmatrix} = \sum_{i=1}^{p-1} \Gamma_i \begin{bmatrix} \Delta x_{t-i} \\ \Delta y_{t-i} \end{bmatrix} - \Pi \cdot \begin{bmatrix} x_{t-p} \\ y_{t-p} \end{bmatrix} + \mu + \varepsilon_t$$

where $\Gamma_i = -(I_k - A_1 - A_2 - \dots - A_i)$ for $i = 1, \dots, p-1$.

and $\Pi = I_k - A_1 - A_2 - \dots - A_p = \alpha\beta'$

and $A_i = \begin{bmatrix} a_{11,i} & a_{12,i} \\ a_{21,i} & a_{22,i} \end{bmatrix}$, $i = 1, \dots, p$.

The test of the null hypothesis that x_t does not Granger cause y_t is a test of:

$$a_{12,i} = 0, \quad \forall i = 1, \dots, p$$

and conversely the test of Granger noncausality of y_t is a test of:

$$a_{21,i} = 0, \quad \forall i = 1, \dots, p$$

Lütkepohl and Reimers (1992) maintain that the Wald statistic has an asymptotic $\chi^2(p)$ distribution if the cointegration rank $r = 1$ or 2 . If $r = 0$, the VAR coefficients may be estimated in first differences and the resulting Wald statistic for the above coefficient restrictions has an asymptotic $\chi^2(p-1)$ distribution.

That is, the test of noncausality is consistent regardless of the cointegration rank r in bivariate case. The following tables report the causality test results for the two aforementioned cases. (i.e., first differenced VAR and cointegrated VAR). For sufficient sample size, we use monthly observations from January 1980 to June 1995.

The test results do not support the hypothesis that the short term rate Granger causes the long term rate. Rather, long term rate appears to Granger cause short term rate with a possible feedback, as the short term rate is significant only at the 10 % level.

Table 4.1: Granger Causality in the stationary VAR model. ($r = 0$)

Sample 1980:01 - 1995:06		
Lags = 5	F - Statistic	P-value
H_0		
Δr^L does not Granger Cause Δr^S	2.38**	0.04
Δr^S does not Granger Cause Δr^L	1.19	0.32

Note: The lag length test selects $p = 5$. The critical values have χ^2 distribution.
** significant at 5 % level.

Table 4.2 : Granger Causality tests in the cointegrated VAR. ($r = 1$)

Sample 1980:01 - 1995:06		
Lags = 6	F - Statistic	P-value
H_0		
Δr^L does not Granger Cause Δr^S	2.56**	0.021
Δr^S does not Granger Cause Δr^L	1.82*	0.099

Note: The lag selection test applied, assuming a maximum lag of 10, in unrestricted VAR models in levels suggests $p = 6$. *, ** significant at 10% and 5% level.

Therefore the causality tests give the same results whether we specify the model as a stationary VAR or cointegrated VEC model. This is consistent with the evidence presented by Lütkepohl and Reimers (1992) using the US term structure data.

We now examine the causality between the spread and short term rate. Since the spread is already found to be stationary while the short term rate contains a unit root, the appropriate procedure is to difference the short term rate and then to perform the causality test. The test result from the stationary VAR model indicates that the spread Granger causes the short term rate, as presented below.

The causality test easily cannot reject the hypothesis that the short term rate does not Granger cause the yield spread. Whereas the hypothesis that the short term rate does not Granger cause the yield spread cannot be rejected.

Table 5: Granger Causality between Spread and Short end.

Sample 1980:01 - 1995:06		
Lags = 7	F - Statistic	P-value
H₀		
SP does not Granger Cause Δr^S	2.71**	0.01
Δr^S does not Granger Cause SP	1.15	0.33

Note: Lag length test indicates VAR order of 7. ** indicates significance at 5 % level.

Therefore, the proposition that the movement of the spread is due to the movement of the short end is not well supported by data. This suggests that the predictive power of spread is not mainly due to movements in the short term interest rate. .

From the above set of the causality tests we find that the long term rate and the spread, respectively, Granger cause the short term rate.

While there is evidence that short term rate also Granger causes long term rate, short term rate does not Granger cause the spread. These results do not support the argument that the movement of the short term rate is the main source of the predictive power of the spread due to monetary policy.

IV. Conclusion

This paper has examined the dynamic relationship between the yield curve and future output growth in Australia. A theoretical rationale for relating the yield curve to the expected growth of future output is provided. In this model, bonds are priced in a basic stochastic growth model with production but without nominal variables. On the empirical side, this paper presented an evidence that the yield spread predicts future real economic growth, which is consistent with the theoretical model. This is supported by the variance decomposition and impulse responses from a bivariate VAR model. To examine whether the variations in the short end of the yield curve is largely attributed to the predictive power for output growth, Granger causality test was performed. The Granger causality test indicates that the short term rate fails to Granger cause the spread while there is a two way causality between the short and long term rates.

Whether the predictive power is due to real forces or monetary policy is an area for further and on-going research. Although this issue remains to be resolved, it seems clear that the yield curve provides useful information about future economic activity, that can be incorporated into the formulation of current monetary policy.

REFERENCES

- Alles, L (1995), "The Australian Term Structure as a Predictor of Real Economic Activity", *The Australian Economic Review*, 4th Quarter, 71-85.
- Balvers, R. J., Cosimano, T. F. and B. McDonald (1990), "Predicting Stock Returns in an Efficient Market", *Journal of Finance*, 4, 1109-1128.
- Barro, R. J. (1976), "Rational Expectations and the Role of Monetary Policy", *Journal of Monetary Economics*, 2, 1-32.
- Barro, R. J. (1981), "The Equilibrium Approach to Business Cycles", In *Money, Expectations and Business Cycles*, New York, Academic Press.
- Bernanke, B. S. (1990), "On the Predictive Power of Interest Rates and Interest Rate Spreads", *New England Economic Review*, Nov/Dec, 51-68.
- Bernanke, B. S. and A. Blinder (1992), "The Federal Funds Rate and the Channels of Monetary Transmission", *American Economic Review*, 82, 901-921.
- Breedon, D. T. (1979), "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", *Journal of Financial Economics*, 7, 265-296.
- Breedon, D. T. (1986), "Consumption, Production, Inflation and Interest Rates: A Synthesis", *Journal of Financial Economics*, 16, 3-39.
- Brock, W. and L. Mirman (1972), "Optimal Economic Growth and Uncertainty: the Discounted Case", *Journal of Economic Theory*, 4, 479-513.
- Brock, W. (1982), "Asset Prices in a Production Economy", in J. McCall (eds): *The Economics of Information and Uncertainty*, Chicago, University of Chicago Press.
- Campbell, J. Y. (1994), "Inspecting the Mechanism: An Analytical approach to the Stochastic Growth Model", *Journal of Monetary Economics*, 33, 463-506.
- Chen, Nai-fu (1991), "Financial Investment Opportunities and the Macroeconomy", *Journal of Finance*, 46, 529-44.
- Cooley, T. F. and S. F. LeRoy (1985), "Atheoretical Macroeconometrics: A Critique", *Journal of Monetary Economics*, 16, 283-308.
- Cox, J. and Ingersoll, J. and S. Ross (1985a), "An Intertemporal General Equilibrium Model of Asset Prices", *Econometrica*, 53, 363-384.

- Cox, J. and Ingersoll, J. and S. Ross (1985b), "A Theory of the Term Structure of Interest Rates", *Econometrica*, 53, 385-408.
- Dolado, J., Jenkinson, T. and S. Sosvilla-Rivero (1990), "Cointegration and Unit Roots", *Journal of Economic Surveys*, 3, 249-272.
- Estrella, A. and G. A. Hardouvelis (1991), "The Term Structure as a Predictor of Real Economic Activity", *Journal of Finance*, June, 555-576.
- Fisher, I. (1907), *The Rate of Interest*, New York, Macmillan.
- Fisher, L. A. and P. A. Richardson (1992), "Stochastic Trends in Consumption and the Term Structure of Interest Rates", *Journal of Macroeconomics*, 14, 289-304.
- Friedman, B. M. and K. N. Kuttner (1993), "Why Does the Paper-Bill Spread Predict Real Economic Activity?", in Stock, J. H. and M. W. Watson (eds) : *Business Cycles, Indicators, and Forecasting*: NBER Conference on Research in Business Cycles, Chicago, University of Chicago Press.
- Hardouvelis, G. A. (1988), "The Predictive Power of the Term Structure During Recent Monetary Regimes", *Journal of Finance*, 43, 339-356.
- Harvey, C. R. (1988), "The Real Term Structure and Consumption Growth", *Journal of Financial Economics*, 22, 305-334.
- Harvey, C. R. (1989), "Forecasts of Economic Growth from the Bond and Stock Markets", *Financial Analysts Journal*, 45, 38-45.
- Hu, Z. (1993), "The Yield Curve and Real Activity", *IMF Staff Papers*, 40(4), December, 781-806.
- King, R., Plosser, C. and S. Rebelo (1988), "Production, growth and business cycles I: The basic neoclassical model", *Journal of Monetary Economics*, 21, 195-232.
- Kydland, F. and E. Prescott (1982), "Time to build and aggregate fluctuations", *Econometrica*, 50, 1345-70.
- Long, J. B. and C. I. Plosser (1983), "Real Business Cycles", *Journal of Political Economy*, 91, 39-69.
- Lowe, P. (1992), "The Term Structure of Interest Rates, Real Activity and Inflation", *Reserve Bank of Australia Research Discussion Paper*, No.9204.

- Lucas, R. E. (1972), "Expectations and the Neutrality of Money", *Journal of Economic Theory*, 4, 103-124.
- Lucas, R. E. (1975), "An Equilibrium Model of the Business Cycle", *Journal of Political Economy*, 83, 1113-44.
- Lucas, R. E. (1977), "Understanding business cycles", *Carnegie Rochester Conference Series on Public Policy*, Vol 5. North Holland, Amsterdam.
- Lucas, R. E. (1978), "Asset Prices in an Exchange Economy", *Econometrica*, 46, 1426-1445.
- Lucas, R. E. (1980), "Methods and Problems in Business Cycle theory", *Journal of Money, Credit and Banking*, 4, 696-715.
- Lütkepohl, H. and H. Reimers (1992), "Granger-causality in cointegrated VAR Processes - the Case of the Term Structure", *Economics Letters*, 40, 263-68.
- Merton, R. C. (1973), "An Intertemporal Capital Asset Pricing Model", *Econometrica*, 41, 867-887.
- McCallum, B.T. (1989), "Real Business Cycles Models", in R. Barro (eds): *Readings in Business Cycles theory*, Cambridge, Harvard University Press.
- Plosser, C. I. and K. G. Rouwenhorst (1994), "International Term Structures and Real Economic Growth", *Journal of Monetary Economics*, 33, 133-155.
- Salyer, K. D. (1994), "The Term Structure of Interest Rates within a Production Economy: A Parametric Example", *Journal of Macroeconomics*, Fall, 16, 729-34.
- Sims, C.A. (1980), "Macroeconomics and Reality", *Econometrica*, 48, 1-46.
- Sims, C. A. (1981), "An Autoregressive Index Model for the US 1948-1975", in J. Kmenta and J. B. Ramsey (Eds): *Large-Scale Macroeconometric Models*, 283-327, Amsterdam, North Holland.
- Stock, J. H. and M. W. Watson (1989), "New Indexes of coincident and leading indicators", in Olivier J. Blanchard and Stanley Fischer (eds): *NBER Macroeconomics Annual 1989*, MIT Press, Cambridge, 4, 351-393.
- Toda, H. and P. Phillips (1993), "The Spurious effect of Unit roots in VectorAutoregressions: An analytical study", *Journal of Econometrics*, 59, 229-256.