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Spinning galaxies within the large scale structure of the Universe

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A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy

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Abstract

Finding links between the large scale structure of the Universe and galaxy formation presents an important challenge for cosmology. The properties of dark matter halos in N-body simulations, in particular the spin angular momentum, can provide these links. This thesis is an in depth study of the alignments of halo spin direction within filaments in the large scale structure. Filaments in the halo and galaxy distribution of the Millennium simulation were identified using two simple methods and a difference between the spin orientation of low and high mass halos with the axis of filaments was uncovered. The evolution of these alignments and other aspects of halo spin suggested an ongoing process of angular momentum acquisition. This process was found to be largely reliant on the anisotropic infall of satellite halos. The spin of dark matter halos tends to become increasingly parallel to the axis of filaments and this change is driven by major mergers between halos traveling orthogonal to the axis of filaments. This new scenario of the build-up of dark matter halo spin could see significant consequences in theories of galaxy formation.
Statement of Originality

This thesis describes work carried out in the Sydney Institute for Astronomy, within the School of Physics, University of Sydney, between March 2009 and July 2013. The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree or diploma at this university or any other institution.

Holly E. Trowland

Date

Included Publications

Chapter 4 of this thesis contains a reproduction of a paper that has been published in a peer-reviewed journal. The paper consists of research conducted by myself, in consultation with my supervisors Geraint Lewis and Joss Bland-Hawthorn. The original text of the paper is presented in the Appendix.

The Cosmic History of the Spin of Dark Matter Halos within the Large-scale Structure

Additional Publications

The following papers are are listed as forming a small part of my thesis. However, my contributions to them were minor.

The SAMI IFU Galaxy Survey

The Sydney-AAO Multi-object Integral field spectrograph

First Science with SAMI: A Serendipitously Discovered Galactic Wind in ESO 185-G031
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Chapter 1

Introduction

This is your one opportunity to do something that no one has ever done before, and that no one will copy throughout human existence. And if nothing else, you will be remembered as the one guy who ever did this. This one thing.

1.1 Cosmology

Cosmology has in the past been the realm of the philosopher. Every culture has its creation myth, from the Maori myth of the sky father and the earth mother which were separated to create the world, to the still popular seven day creation of the heaven and earth in the Book of Genesis. Without the aid of telescopes, the true nature and origins of the Universe was impossible to discern. Modern cosmology is a precision science based on astronomical observation and scientific theory.

The Universe is spatially flat, homogeneous and isotropic on large scales and not only expanding but expanding at an accelerating rate. The luminous matter that makes up the stars and the galaxies accounts for only a small fraction of the whole matter-energy budget of the Universe, leaving the rest as the dark component; dark energy and dark matter. Although these quantities remain mysterious, several key properties are known. Dark matter is the stuff that makes up most of the mass of the Milky Way and the large scale structure. Dark energy is causing the expansion of the Universe to accelerate by its negative pressure. The currently accepted cosmology of dark matter and dark energy is called ΛCDM. This model is the simplest that accounts for the large scale structure formation and the expansion history of the Universe.

In the following Section I will outline the key cosmological principles that underlie this set of studies.

1.1.1 Gravity

Although gravity is the weakest of the four fundamental forces, it is the only one which effectively acts over long distances and so is the most relevant to the formation of the large scale structure. Gravity is best represented with General Relativity (GR) but can also be successfully approximated with Newton’s law of universal gravitation.

In 1687, English mathematician Sir Isaac Newton published Philosophiæ Naturalis Principia Mathematica, which hypothesizes the inverse-square law of universal gravita-
tion. In his own words, “I deduced that the forces which keep the planets in their orbs
must [be] reciprocally as the squares of their distances from the centers about which they
revolve” (Newton, 1687). Expressed mathematically, the force $F$ between two masses $m$
and $M$ separated by distance $r$ is;

$$F = \frac{GmM}{r^2}.$$  \hspace{1cm} (1.1)

The proportionality constant, $G$ is called the gravitational constant and has now been
measured to 1 part in 8,300 (Mohr et al., 2012). Newton’s theory enjoyed great suc-
ess in explaining Keplarian orbits but it could not account for the precise astronomical
observations of the precession of the perihelion of Mercury.

The solution to this inaccuracy came about in 1916 when Einstein published the
general theory of relativity (GR; Einstein 1916. In GR, rather than acting as a force,
the action of gravity is instead described in terms of geometry. The presence of matter
and energy induces curvature in the four dimensional space-time of the Universe and this
curvature determines the path that particles will follow. This theory is encapsulated in
the Einstein field equations, a set of 10 equations which are as follows

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$  \hspace{1cm} (1.2)

This is a simple mathematical statement that the geometry of space-time is equivalent
to matter/energy. The geometry is given by the Einstein tensor $G_{\mu\nu}$ (defined in terms of
the Ricci curvature tensor and the scalar curvature $G_{\mu\nu} \equiv R_{\mu\nu} - g_{\mu\nu}R/2$, where $g_{\mu\nu}$ is the
metric) and the matter/energy is given by the the stress energy tensor $T_{\mu\nu}$.

Einstein demonstrated that GR is consistent with the measurements of the precession
of the perihelion of Mercury as well as other tests such as the deflection of light as it bends
around the sun and gravitational redshift. Newtonian gravity is a good approximation
for GR for weak fields and slow speeds so it is only in these special cases when GR can
be tested.

1.1.2 The Cosmological Model

The Cosmological Principle states that our observational location in the Universe is in
no way unusual or special; on a large enough scale, the Universe looks the same in all
directions (isotropy) and from every location (homogeneity) (Milne, 1933). It is derived
from the Copernican Principle, which is the assumption that we are not privileged ob-
servers. The Cosmological Principle is a working assumption that has been confirmed
in $\geq 100$Mpc scales with large galaxy redshift surveys, such as the 2-degree field Galaxy
Redshift Survey (2dFGRS Peacock et al. 2001; Cole et al. 2005, shown in Figure 1.1) and
the Sloan Digital Sky Survey (SDSS, Abazajian et al. 2009). On small scales, however,
the cosmological principle does not hold. There are superclusters, clusters, groups and
galaxies as well as the complicated network of filaments, sheets and voids which form the
cosmic web.

The isotropy and homogeneity of the Universe make it possible to describe space
time with the Friedmann Lemaître Robertson Walker (FLRW) metric. This is an exact
solution of Einstein’s field equations of general relativity and can represent a space that
is expanding or contracting (although distance measurements of galaxies show that space
is in fact expanding). This metric can be understood in terms of a ‘fundamental observer’
Figure 1.1: The observed distribution of galaxies in the universe as measured by the 2dF Galaxy Redshift Survey. The cosmic web has a spongy structure, with galaxies clustered together in filaments and clusters, leaving great areas of space as voids. Picture credit: http://www2.aao.gov.au/2dFGRS.

who is at rest with respect to the matter in their vicinity. The fixed coordinates of such an observer are called comoving coordinates. The line element in FLRW-space is

$$\text{ds}^2 = -dt^2 + a^2(t) \left[ d\chi^2 + S^2_k(\chi)(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1.3)$$

where $\chi$, $\theta$ and $\phi$ are comoving coordinates. $a(t)$ is the time-dependent scale factor which characterizes the large scale expansion or contraction of the Universe and Hubble’s constant is defined by $H(t) = \dot{a}/a$. The scale factor defines the conversion between the coordinate separation and spatial separation of two fundamental observers on a hypersurface of constant time. The term $S_k(\chi)$ is topology-specific and is given by

$$S_k(\chi) = \begin{cases} 
\sin \chi & \text{if } k = 1 \\
\chi & \text{if } k = 0 \\
\sinh \chi & \text{if } k = -1
\end{cases}, \quad (1.4)$$

where $k$ describes the overall spatial curvature of hyper-surfaces of constant time. $k = 1/0/-1$ defines a closed/flat/open Universe. The Friedman equations are derived from the Einstein field equations and are as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3};$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3};$$

$$\dot{\rho} = -\frac{3\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right).$$

These equations can be used for any cosmological fluid although some notable fluids are the perfect fluid, matter (non-relativistic particles), radiation (relativistic particles) and
dark energy. In the equations, $\rho$ is the fluid density, $p$ is the pressure and $\Lambda$, $G$ and $c$ are the cosmological constant, the gravitational constant and the speed of light respectively. The dot denotes a derivative with respect to times. $\rho$ and $p$ for a particular fluid are related by the equation of state $w$,

$$p = w\rho.$$  \hspace{1cm} (1.5)

A positive equation of state implies that the material becomes hotter under compression (work is done to compress the fluid). Conversely, for a negative equation of state, work is done to stretch the fluid, like stretching a rubber band. Some examples of values of the equation of state include,

$$w = \begin{cases} 
1/3 & \text{radiation} \\
0 & \text{matter} \\
-1 & \text{cosmological constant} 
\end{cases}. \hspace{1cm} (1.6)$$

The energy densities of these quantities are commonly normalized by the critical density such that $\Omega_i \equiv \rho_i/\rho_{\text{crit}}$ and the critical density for closure is

$$\rho_{\text{crit}}(t) = \frac{3H^2(t)}{8\pi G}. \hspace{1cm} (1.7)$$

A simple solution to the Friedman equations is when the Universe is assumed to be spatially flat and matter dominated,

$$a(t) = \left(\frac{3}{2}H_0 t\right)^{2/3}. \hspace{1cm} (1.8)$$

In this Universe, there is expansion but the rate of expansion is decelerating ($\ddot{a} < 0$). In a Universe with a component that has the equation of state $w < -1/3$ in the Friedman equations, there would be accelerating expansion ($\ddot{a} > 0$).

1.1.3 \textit{$\Lambda$CDM}

Currently, our best and simplest model of cosmology is the $\Lambda$ Cold Dark Matter ($\Lambda$CDM) model. In this model, cosmic acceleration is caused by the vacuum energy or dark energy inherent in empty space, otherwise known as the cosmological constant with an equation of state $w = -1$.

The cosmological constant was introduced by Einstein (1917) in order to reconcile preconceived notions of a static Universe. In a matter dominated Universe, the Universe is expanding and the rate of expansion is decelerating (Equation 1.8). An expanding universe seemed physically implausible to Einstein so he added the term $\Lambda$ to balance this expansion. The Einstein field equations permit this addition of an arbitrary constant proportional to the metric, $\Lambda g_{\mu\nu}$. Subsequent to the addition of $\Lambda$ into the cosmological model, Hubble discovered that there exists a linear relationship between distance and redshift of nearby galaxies. This is interpreted as an expansion of the Universe at a rate given by the Hubble constant, $H_0$. The discovery destroyed previous assumptions of a static Universe and removed the need for a cosmological constant in GR. Einstein is said to have told George Gamow that introducing the cosmological constant was his “greatest blunder”. However, the cosmological constant was not such a terrible blunder; it has
been reintroduced since observations of type 1a supernovae (Riess et al., 1998; Perlmutter et al., 1999) showed an acceleration of the expansion of the Universe. The cosmological constant now represents the dark energy that is driving this accelerated expansion.

Supernova surveys are just one of the observational probes used to pin down the values of the parameters in ΛCDM. Unfortunately, there have to be many different observational probes because the parameters of ΛCDM contain many degeneracies and datasets have to be combined in order to break these. For example, Big Bang Nucleosynthesis (BBN) finds the constraints on the baryon fraction, while galaxy clusters are used to estimate the baryonic to dark matter ratio. There is the matter power spectrum measured in the Cosmic Microwave Background (CMB), Baryon acoustic oscillations (BAOs) and gravitational lensing. An example of how the degeneracies are broken and concordance values are found is Figure 1.2. The ΛCDM model can be completely described by only six parameters: physical baryon density ($\Omega_b h^2$); the physical cold dark matter density ($\Omega_c h^2$); the dark energy density ($\Omega_\Lambda$); the amplitude of primordial scalar curvature perturbations, ($\Delta^2_\zeta$ at $k = 0.002$Mpc$^{-1}$); the power-law spectral index of primordial density (scalar) perturbations ($n_s$); and the reionization optical depth ($\tau$). Spatial flatness is assumed ($\Omega_b + \Omega_c + \Omega_\Lambda = 1$) which implies the value of the Hubble constant $H_0 = 100$hkms$^{-1}$Mpc$^{-1}$. The latest values of these parameters are in the WMAP 9-year results where the CMB data is combined with BAO and Hubble constant measurements (Hinshaw et al., 2012).

There are other parameters that are important in describing the dynamics of the
Universe and structure formation within it. The rate of expansion of the Universe today is given by the Hubble constant \( H_0 \), the value of the local slope of Hubble's distance-redshift relationship. The Hubble constant is often rescaled as the quantity \( h \equiv H_0/100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \). The Hubble constant today is now very accurately measured, for example Freedman et al. (2012) measured \( H_0 = 74.3 \pm 1.5\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \) using Hubble Space Telescope (HST)/Wide-Field Planetary Camera 2 (WFPC2) observations. Another useful parameter is \( \sigma_8 \), the normalization of the matter density power spectrum (the amplitude of the (linear) power spectrum on the scale of \( 8\,h^{-1}\,\text{Mpc} \)).

Although ΛCDM is a simple and powerful model, there are many other cosmological models about. Cold dark matter is a well accepted and supported theory, but the inclusion of Λ dark energy is a bit more contentious. The cosmological constant suffers from fine tuning problems and what's called the 'coincidence problem'. It is often assumed that the quantum vacuum is equivalent to the cosmological constant. If the Universe is described by an effective local quantum field theory down to the Planck scale, then we would expect a cosmological constant of the order of \( M_{\text{pl}}^4 \). However, the measured cosmological constant is smaller than this by a factor of \( 10^{120} \). This is known as a fine tuning problem because if this discrepancy is to be canceled out, it must be canceled out by a term that leaves exactly the right amount of dark energy as it is observed. The coincidence problem is that we appear to be living in a very special time where \( \Omega_m \simeq \Omega_\Lambda \). Extensions of the ΛCDM model attempt to solve these problems by adding extra degrees of freedom, for example quintessence where the equation of state of dark energy is allowed to vary through time.

### 1.1.4 Dark Matter

As well as dark energy, cold dark matter is an important component in ΛCDM. Of the energy density of the Universe, about 71.9% is dark energy, 23.5% is dark matter and only 4.6% is ordinary baryonic matter. Dark matter is a key player in large scale structure formation. It forms both the larger framework (the cosmic web) and also binds together individual galaxies and groups of galaxies into dark matter halos. Galaxies and clusters are the visible portion of the mass but only sit in the middle of the vast gravitational wells that dark matter halos create, like snow on mountain tops.

The first evidence for dark matter came from observations of the Coma cluster (Zwicky, 1937). This cluster was shown to have an exceptionally large mass to light ratio and that ionized gas was insufficient to bind the cluster, hinting that there was an unseen mass that held the cluster together. Dark matter was not an accepted candidate for this discrepancy until the discovery of the flattening of galactic rotation curves of nearby spiral galaxies (Bosma, 1978; Rubin et al., 1978). These rotation curves showed that despite decreasing observable mass from the center of the galaxy, the speed of rotation did not slow so there must be extra mass that resides in the outskirts to keep the rotation Keplerian. Dramatic evidence of dark matter can be seen in the Bullet Cluster (1E0657-558) (Figure 1.3) where two clusters have collided, separating the dark matter from the visible matter. The dark matter has been found through analysis of gravitational lensing which is sensitive to the underlying mass distribution and the gas is visible in X-ray (Clowe et al., 2006; Bradač et al., 2008). During the collision the dark matter passed through relatively undisturbed since its only interaction is by gravity, but the gas has formed a bullet-shaped shock front and does not lie at the center of mass.

The identity of dark matter remains a mystery. The primary properties of dark matter
Figure 1.3: The matter in galaxy cluster 1E 0657-56, the “Bullet Cluster”, is shown in this composite image. Clouds of hot x-ray emitting gas shown in red and the mass distribution as mapped out with gravitational lensing is shown in blue. The image shows that there has been a collision between two clusters that has produced a shock front in the gas. The collision has also caused the visible matter to be shifted from the center of mass since the dark matter has not interacted with the cluster gas except by gravity. The clear separation of the visible matter and the bulk of the mass is considered direct evidence that dark matter exists. Picture credit: Clowe et al. (2006); Markevitch (2006)

...are that it has mass but is not visible. It has been suggested that dark matter could be baryonic Jupiter sized objects such as planets or brown dwarfs, collectively known as Massive Compact Halo Objects (MACHOs). These have been ruled out since they would be detectable from their microlensing effect on extragalactic sources (Alcock et al., 2000; Calchi Novati et al., 2009; Tisserand et al., 2007). More likely candidates are some exotic particle outside of the standard model of physics, known as Weakly Interacting Massive Particles (WIMPS). WIMP-like particles are predicted by R-parity-conserving supersymmetry, a popular type of extension to the standard model, although none of the large number of new particles in supersymmetry have been observed. Lastly, it has been proposed (Milgrom, 1983) that dark matter might not exist at all but its effects are from deviations from Newtonian gravity. However, Modified Newtonian Dynamics (MOND) requires the inclusion of free parameters to correctly calculate gravitational forces on galactic scales and these terms vary from galaxy to galaxy. Although the true identity of
dark matter is unknown, simply including an unseen mass in the standard model is much more plausible than changing the theory of gravity for each galaxy.

Dark matter can be characterized by its ‘temperature’ - cold, warm or hot. These different types of dark matter damp density perturbations through free streaming in different ways. Free streaming damping means that the gravitation of the density perturbations of those particles cannot bind them below their free streaming length, and therefore density perturbations smaller than this length cannot form. The free streaming length is the length that those particles can travel until they become non-relativistic. CDM particles are assumed to become non-relativistic very early, having thus a small free streaming length. This, in turn, means that density perturbations on small scales can be formed leading to a bottom-up scenario of structure formation. Hot dark matter smooths out fluctuations in the total matter density even on galaxy cluster scales, which leads to strong bounds on their mass and density (Bond et al., 1984; Primack and Blumenthal, 1984). The first objects which can form are huge supercluster-size pancakes, which are theorized to somehow fragment into galaxies, in a top-down scenario of structure formation. The best candidate for the identity of hot dark matter is the neutrino (Olive and Turner, 1982). Warm dark matter behaves in a way somewhere in between these two extremes. Observations are consistent with a bottom-up hierarchy of structure formation where small structures form first then merge into bigger structure (Davis et al., 1985), so cold dark matter is more likely.

1.2 The Large Scale Structure

The large scale structure refers to the way that dark matter and galaxies clusters in the Universe. Below the level of isotropy (∼ 100Mpc), matter forms a complex web of superclusters, clusters and groups of galaxies as well as the network of filaments, sheets and voids. The origins and evolution of these structures are discussed in this Section.

1.2.1 Seeds of Large Scale Structure

The large scale structure of the Universe as we see it today has grown from the gravitational collapse of tiny departures from regularity in an otherwise completely smooth and infinitely large cosmic background. These seeds of large scale structure were put in place during the era of inflation when the Universe increased in size at an exponential rate, increasing in volume by a factor of $10^{78}$. This inflationary period happened very early in the history of the Universe, from $10^{-36}$ seconds after the Big Bang to sometime between $10^{-33}$ and $10^{-32}$ seconds. Following the inflationary period, the universe continued to expand, but at a slower rate.

Inflationary theory was first proposed by Guth (1981) as a solution to the horizon and flatness problems. The horizon problem is that the temperature of the Cosmic Microwave Background (CMB) is the same across the whole sky. The CMB is the first light to escape the opaque fog of the early Universe. However, the different parts of the Universe at this time could not have been causally connected and could not have reached thermal equilibrium to produce the remarkably isotropic temperature signal that is detected. The flatness problem is that the density of matter and energy in the early Universe must have been very finely tuned to produce a Universe that today is extremely flat ($k = 0$).
These problems, posed by Dicke (1961), led Guth (1981) to propose a rapid expansion of space soon after the big bang, termed inflation. Inflation not only solved the horizon and flatness problems but explains the absence of observations of magnetic monopoles and (most importantly here) provides a mechanism for planting the seeds of large scale structure.

As a direct consequence of inflation, all of the observable Universe originated in a small causally connected region. The time previous to inflation is not properly understood, but there is no reason to think that space was isotropic and homogeneous on all scales before inflation. The rapid expansion of space had the effect of smoothing out any inhomogeneities as well as the curvature of space. Although inflation smoothed out the Universe it also introduced small perturbations in the cosmic density field ($\rho$) via quantum fluctuations. The perturbations are best discussed in terms of the overdensity $\delta$:

$$
\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}.
$$

(1.9)

Inflation itself was driven by a scalar field, called the *inflaton*. Just as quantum fluctuations allow the creation of particle-antiparticle pairs of virtual particles, fluctuations appear the inflaton field. Briefly, a quantum perturbation to the inflaton field on a particular scale will exit outside the Hubble radius during inflation. When this happens it causes a fixed perturbation in the curvature of space-time corresponding to that scale, which will become the perturbations in the density field of the universe. Once inflation is over, the Hubble radius begins to grow again and the density perturbations re-enter the horizon and begin to grow according to linear theory.

Figure 1.4: Fluctuations in the CMB temperature, as determined from nine years of WMAP data, about the average temperature of 2.725K. The fluctuations are at the level of only a few parts in $10^5$. Picture credit: NASA/WMAP Science Team (http://map.gsfc.nasa.gov)
1.2.2 The Growth of Structure

The small scale density fluctuations frozen in place during inflationary times have since been amplified into the dark matter halos and large scale structure of the Universe today. This process is simple to understand; a region whose initial density is slightly higher than the mean will attract the surrounding matter slightly more strongly than average. The region will continue to grow as it pulls even more matter towards it, amplifying the density fluctuation. At early times when $\delta \ll 1$, the growth is governed by linear physics and it is the physical size of the region that increases with time due to the overall expansion of the Universe. Once the perturbation reaches $\delta \approx 1$, it breaks away from the overall expansion and starts to collapse. This is the time when non-linear physics kicks in.

The emergence of structure from the initial perturbations was pioneered by Jeans (1902). In this paper, Jeans was concerned with the stability of spherical nebula where gravity would cause the nebula to contract while thermal energy causes the nebula to expand. This theory can be applied to collisionless dark matter to model its collapse and the early history of the formation of the large scale structure. The Jeans length $\lambda_J$ is defined as the critical wavelength that separates oscillatory and exponentially-growing density perturbations. On scales smaller than $\lambda_J$, the sound crossing time is shorter than the gravitational free-fall time, allowing the build-up of a pressure force that counteracts gravity. On these scales, the patch of dark matter cannot collapse and instead the density excess oscillates. On larger scales, the pressure gradient force is too slow to react to a build-up of the attractive gravitational force and the patch of dark matter can collapse, monotonically increasing in density excess. Perturbations in different fluid components such as matter and radiation, or collisionless and collisional matter may grow at different rates. For example, dark matter, which interacts negligibly with radiation and baryonic matter, can grow even if baryonic perturbations are supported against collapse by their pressure. This leads to the formation of the formation of the large scale structure in dark matter well before structure in the baryonic matter, setting up an invisible framework for the Universe.

The basis of Jeans result of an oscillatory and a growing solution of the density excess is the set of three fundamental fluid equations involving the vectorial flow $\mathbf{v}$ and pressure $p$ of the density field and induced gravitational potential $\Phi$.

\[
\begin{align*}
\frac{D\rho}{Dt} &= -\rho \nabla \cdot \mathbf{v} & \text{Continuity equation} \\
\frac{D\mathbf{v}}{Dt} &= -\frac{\nabla p}{\rho} - \nabla \Phi & \text{Euler’s equation} \\
\nabla \Phi &= 4\pi G \rho & \text{Poisson’s equation}
\end{align*}
\]  

(1.10)

denoting the convective derivative by $D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$. Perturbations of the field are found by perturbing these equations by deconstructing each parameter into its mean value and a residual term (e.g. $\rho = \rho_0 + \delta \rho$), where the residual is called a perturbation if it is small. Perturbing equations 1.10 and taking the result for just the small perturbation term gives;
\[
\frac{d\delta \rho}{dt} = -\nabla \cdot \delta \mathbf{v} \\
\frac{d\mathbf{v}}{dt} = -H \delta \mathbf{v} - \frac{\nabla \delta p}{\rho_0} - \nabla \delta \Phi \\
\nabla \delta \Phi = 4\pi G \delta \rho.
\]

(1.11)

Hubble’s constant, \( H \) appears here because Hubble’s law for the expansion of space is used, \( \mathbf{v}_0 = H \mathbf{x} \). These are the linearized equations for the growth of perturbations. The equations are written in Eulerian coordinates (specific locations in the space through which the fluid flows as time passes), however it makes more sense to transform to comoving coordinates that take into account the expansion of space. The quantities are transformed with the scale factor as follows:

\[
r = \frac{x}{a}; \quad u = \frac{\delta \mathbf{v}}{a}; \quad \nabla_r = a \nabla.
\]

(1.12)

Combining the fluid Equations (1.11) and defining the speed of sound as \( c_s^2 \equiv \frac{\delta p}{\delta \rho} \), brings us to the growth equation, the key relationship in the linear theory of structure formation;

\[
\ddot{\delta \rho} + 2H \dot{\delta \rho} = \delta \rho \left( 4\pi G \rho_0 - \frac{c_s^2 \nabla^2 \delta \rho}{a^2} \right).
\]

(1.13)

This equation governs the gravitational amplification of density perturbations. The growth equation has two types of solution; exponential growth (where the growth factor is \( D(a) = \delta(a)/\delta(a_i) \), defined in terms of the overdensity at some time, \( a_i \)) or standing sound waves. At the border between these solutions is the Jeans length:

\[
\lambda_J = c_s \sqrt{\frac{\pi}{G \rho}}
\]

(1.14)

It is the wavelength where we switch from exponential growth for long-wavelength modes to standing sound waves for short wavelengths (when the overdensity stops growing with the background and collapses on itself).

Linear perturbation theory is only relevant when the perturbations are small \( (\delta \ll 1) \). Once the perturbations grow to a size comparable to the mean density, Equations (1.11) are no longer relevant. This is the phase of non-linear growth. During the linear regime, perturbations on each scale grow independently of each other. When the perturbations grow bigger, they start to self-interact which causes the phases to couple and the growth is no longer simple. In the non-linear regime, the power spectrum is still a very useful statistic. Since only the amplitude of the wave modes is used, not the phase information, it doesn’t matter that the phases are no longer coherent.

Since the equations of motion (1.10) are non-linear, and we have only solved them in the limit of linear perturbations, the exact evolution of the density field is usually modeled using an N-body simulation; the details of these simulations are discussed in Chapter 2. Although the full development of the gravitational instability cannot be solved exactly without N-body simulations, there are some specific cases that can be approximated, as discussed in the next Section.
1.2.3 Hierarchical Structure Formation

When linear theory breaks down, the physics of the Universe is still fairly straightforward: Newtonian gravity is still the dominant force and there is not yet tricky gas dynamics. There are simple models for the collapse of matter that have strong predictive power.

Spherical Collapse

An overdense sphere is a very useful nonlinear model, as it behaves as an isolated region of space-time and can be treated as a closed sub-Universe. According to Newton’s shell theorem, any spherically symmetric perturbation will clearly evolve at a given radius in the same way as a uniform sphere containing the same amount of mass. Since the internal configuration of the mass need not be uniform, density henceforth refers to mean density. Integrating the Newtonian equation \( \frac{d^2 r}{dt^2} = -\frac{GM}{r^2} \) gives the first integral of motion:

\[
\frac{1}{2} \left( \frac{dr}{dt} \right)^2 - \frac{GM}{r} = E
\]

where \( E \) is the total energy of the perturbation. This equation can be solved for the values of the overdensity at different times. The spherical collapse model was first proposed by Gunn and Gott (1972) and solutions of the model for \( \Lambda = 0 \) as well as for a Universe with a dark energy component are examined in Mo et al. (2010). There are three interesting epochs in the development of a spherical overdensity:

**Turnaround:** The sphere breaks away from the general expansion and reaches a maximum radius. Theory predicts that the linear density at turnaround is \( \delta_{\text{lin}} = 1.06 \). Turnaround also represents the breakdown of linear theory, in that it represents the time when our volume containing the perturbation breaks away from the background expansion (but has not yet collapsed to form a gravitationally bound structure).

**Collapse:** If only gravity operates, then the sphere will collapse to a singularity, although the contraction is always disrupted by the dissipative processes in virialization. A spherical top-hat perturbation collapses when its linear overdensity exceeds the threshold of \( \delta_c \approx 1.686 \). This threshold has only a weak dependence on cosmological parameters (e.g. Lacey and Cole 1994; Eke et al. 1996). Objects which collapse at a given redshift have the *characteristic mass scale for collapse*, \( M_*(z) \). The variance of linear density fluctuations at a given mass scale \( M \) is related to the linear power spectrum \( P(k, z) \) at redshift \( z \) by

\[
\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k, z) \tilde{W}_{\text{TH}}^2(k, M),
\]

where \( \tilde{W}_{\text{TH}}(k, M) \) is the Fourier transform of a spherical top-hat window function of comoving size \( R = (3M / 4\pi\bar{\rho})^{1/3} \), and \( \bar{\rho} \) is the comoving mean mass density of the Universe. At a given redshift, the typical mass scale \( M_*(z) \) to collapse from a 1\( \sigma \) fluctuation is hence given by the implicit solution of

\[
\sigma(M_*, z) = \delta_c.
\]

For example, the characteristic mass for collapse at \( z=0 \) is \( 5.89 \times 10^{12} h^{-1}M_\odot \), which is the scale of a small group of galaxies.

**Virialization:** This is the process of converting the kinetic energy of collapse into random motions. The virial theorem states that equilibrium is achieved when the kinetic
energy ($K$) is related to the potential ($V$) by $V = -2K$. This happens when the sphere has collapsed by a factor of 2 from maximum expansion. The linear overdensity at this time is $\delta_{\text{lin}} = 1.58$. Simulations show that the end state of virialization is a halo with a centrally-concentrated mass distribution, characterized by the Navarro Frenk and White profile (NFW, Navarro et al. 1997):

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

(1.18)

where $\rho_0$ and the scale radius $r_s$ are parameters which vary from halo to halo. The NFW profile is characterized by two parameters, an overall normalization (set by the halo mass) and the concentration $c$, defined as the ratio of the virial radius to the scale radius. There is much debate in the literature regarding the NFW profile, particularly its inner slope. But CDM simulations generically predict a cuspy inner profile, with slope between $-1.5$ and $-1$, with the exact slope a matter of much dispute (Navarro et al., 2004b; Graham et al., 2006). Observations, at face value, indicate a slope between $-1$ and 0, though there is much debate about that as well.

**Anisotropic Collapse**

Although the spherical collapse model is a simple and useful model, the real distribution of matter in the Universe is far from isotropic. In fact, gravity not only sets any overdense perturbation into a runaway collapse but it has an amplifying effect on any asphericity in the initial matter configuration. This was found in early studies (Lynden-Bell, 1964; Lin et al., 1965; Icke, 1973) who investigated the evolution of homogeneous ellipsoidal configurations in an expanding FRW Universe and concluded that the predominant morphologies are flattened and elongated.

This idea was generalized by Zel’Dovich (1970b) as the kinematic formalism that deals with the anisotropic collapse for generic cosmological circumstances. The Zel’dovich approximation is a straightforward approximation scheme to particle dynamics. It is even shown to be exact in the case of one-dimensional motion, as well as in the linear regime. The approximation was shown in Zel’Dovich (1970a) to apply even at early non-linear times. To do the approximation, first write the particle positions in terms of comoving coordinates

$$x(t) = q + D(t) \nabla \Phi(q)$$

(1.19)

where $q$ is the Lagrangian (fixed) coordinate and $x$ is the Eulerian (comoving with the fluid) coordinate. The time dependent function $D(t)$ is the growth rate of linear density perturbations and the time independent spatial function $\Phi(q)$ is related to the linearly extrapolated gravitational potential.

For a patch of matter, its collapse can be characterized by the eigenvalues of the deformation tensor,

$$D_{jl} = \frac{\partial^2 \phi}{\partial q_j \partial q_l}$$

(1.20)

Which is a real symmetric matrix with eigenvalues $\lambda_1 > \lambda_2 > \lambda_3$, its eigenvectors define a set of three principal (orthogonal) axes. Applying the mass conservation relation,

$$\bar{\rho} d^3 q = \rho(x) d^3 x$$

(1.21)
to equation (1.19) gives the contraction or expansion along the three principal axes:

$$\rho(x) = \frac{\bar{\rho}}{[1 - D_+(t)\lambda_1(q)][1 - D_+(t)\lambda_2(q)][1 - D_+(t)\lambda_3(q)]} \quad (1.22)$$

Collapse therefore takes place first along the axis defined by the shortest axis, and forms a flattened *pancake*. The subsequent collapse is determined by the second largest eigenvalue which produces a filament and eventually a spherical clump. This mechanism of anisotropic collapse helps give rise to the filamentary nature of the cosmic web. At the point when $D_+(t)\lambda_1(q) = 1$ the density goes infinite (although gas heating stops this from happening physically) and the Zel'dovich approximation breaks down. This pancaking picture is a top-down scenario of structure formation, where large structures form first and then break down into smaller ones.

### 1.2.4 Statistical Measures of Large Scale Structure

There are many different measures of the nature of the large scale structure of the Universe. Perhaps the most useful one is the *power spectrum* which captures the spatial distribution of structure in the Universe into a single function. It is a two-point statistic that quantifies the amount of clustering present as a function of scale in Fourier space. The power spectrum can be derived from the two-point correlation function,

$$\xi(r) = \langle \delta(x)\delta(x + r) \rangle. \quad (1.23)$$

For a given radius $r$, the correlation function is a function of one variable (distance, $x$) which describes the probability that two overdensities are separated by this particular distance. It can be thought of as a lumpiness factor - the higher the value for some distance scale, the more lumpy the Universe is at that distance scale. The power spectrum is then the Fourier transform of the correlation function,

$$P(k) \equiv \langle |\delta_k|^2 \rangle, \quad (1.24)$$

which expresses the amplitude of a wave mode of wavenumber $k$. The power spectrum of the CMB measured by the WMAP satellite (Hinshaw et al., 2012) is shown in Figure 1.5. The most prominent features here are the peaks. The first peak in the power is at around $1^\circ$: density fluctuations on the scale of $1^\circ$ are the most dominant. ‘Wiggles’ at higher multipole moments were imprinted by acoustic oscillations of baryons in the early Universe.

Acoustic peaks are not limited to the CMB. The distribution imprinted in the CMB has been propagated into the clustering patterns of galaxies in the evolved Universe via the mechanisms discussed previously in this Section. Because of the characteristic size of perturbations in the CMB (as determined by the first peak in the power spectrum), the clustering of galaxies too has a particular length scale associated. This makes BAOs useful as a standard ruler which is used to test the evolution of the angular diameter distance across redshifts (Seo and Eisenstein, 2003; Blake and Glazebrook, 2003).

The light from the CMB does not travel an easy path towards our telescopes. It is affected by both the Sunyaev-Zel'dovich effect (SZ, Sunyaev and Zeldovich (1972)) and the Integrated Sachs-Wolfe effect (ISW, Sunyaev and Zeldovich (1972)) due to the intervening large scale structure. In the SZ effect, changes in apparent temperature of the CMB are
caused by inverse Compton scattering of CMB photons by hot electrons inside massive dark matter halos. This method is used for the detection of clusters. In the ISW effect, CMB photons are red/blueshifted due to traveling in and out of potential wells produced by perturbations in the matter density field. In a matter dominated universe, these shifts in wavelength would be balanced out, but with the presence of dark energy, there would be some residual effect.

Gravitational lensing, in particular weak lensing where the images are well removed from the caustics, is highly dependent on the distribution of mass in the universe. Light emitted by galaxies in the background is distorted as it passes through the gravitational potential of massive objects along the line of sight. This causes the images to change in magnification and also additional ellipticity which is related to shear in the geodesic. These changes can be used to derive the individual masses of clusters and as a complementary probe to redshift space distortions in determining the growth of structure.

### 1.3 The Build Up of the Spin of Galaxies and Dark Matter Halos

The formation and evolution of dark matter halos is intimately connected to the surrounding large scale structure. The individual properties of each halo; mass, density profile, shape and spin are dependent on the halo’s history and environment. In this Section I discuss the potential mechanisms that are responsible for the build up of angular momentum in dark matter halos and in turn, galaxies.
### 1.3.1 Tidal Torque Theory

The generally accepted theory of the origin of the initial angular momentum of dark matter halos, known as Tidal Torque Theory (TTT), is that it is the result of gravitational interactions between protohalos at the time of turn-around, just before they collapsed into virialized objects. A protohalo is loosely defined as the small patch of matter in some volume that is destined to end up as a virialized halo. The gravitational force between a protohalo and its nearest neighbor would stretch the protohalo radially, much like the ocean tides from the Moon’s pull on the earth. These tidal forces result in asymmetries in the shape of the protohalo, which could end up being very irregular. The tidal forces acting on an irregular protohalo would pull it more in certain directions, resulting in a torque that is dependent on the local dark matter landscape. Below is an overview of the theory, as initially worked out by Hoyle (1949); Peebles (1969); Zel’Dovich (1970b); Doroshkevich (1970).

The aim in TTT is to calculate the angular momentum of a protohalo in the linear regime, post-recombination. We consider an expanding Universe filled with (pressureless) dust with small density fluctuations that grow by gravitational instability. The local overdensities are defined by

\[ \rho(r, t) = \rho_0(t)[1 + \delta(r, t)]. \]  

\( \delta \) is the fractional deviation away from the mean density \( \rho_0 \) and the distance \( r \) is defined in terms of the scale factor \( a(t) \) and a co-moving coordinate, \( r = a(t)x \). The angular momentum is defined in the usual way, \( J = p \times v \). For a protohalo at time \( t \) with center of mass position \( \bar{r} \) and velocity \( \bar{v} \) the angular momentum is,

\[ J(t) = \int_{V_L} [r(t) - \bar{r}(t)] \times [v(t) - \bar{v}(t)] \rho_0 a^3 d^3r \]  

(1.25)

Here \( V_L \) is the Lagrangian volume of the matter of the protohalo. The velocity of the center of mass does not contribute to the angular momentum and will hence be ignored. In co-moving coordinates, the angular momentum is,

\[ J(t) = \rho_0 a^5(t) \int_{V_L} [1 + \delta(x, t)] [x - \bar{x}] \times \dot{x} d^3x \]  

(1.26)

This is an exact expression of the angular momentum and it is clear that it depends on the shape of the volume, \( V_L \). The co-moving position can be written in terms of the cosmological expansion of space \( a(t)q \) and some displacement due to the perturbations \( S(q, t) \),

\[ r(q, t) = a(t)x(q, t) = a(t)[q + S(q, t)] \]  

(1.27)

where \( q \) is a Lagrangian coordinate, defined as the initial value of \( x \). We want to express the angular momentum in terms of this Lagrangian coordinate and must use the Jacobian, \( J = ||\partial x/\partial q|| \). Conservation of mass demands \( \rho d^3x = \rho_0 d^3q \) so using Equation (1.25) the Jacobian is \( J^{-1}(q, t) = 1 + \delta[x(q, t)] \). This Jacobian introduces the limit of linear growth, \( \langle \delta^2 \rangle \ll 1 \), into the theory, when \( J = 0 \). The limit of this theory is reached when the mapping \( q \rightarrow x \) becomes ambiguous as the growth departs from linearity. Under this change of coordinates the angular momentum is,

\[ J(t) = a^2(t)\rho_0 a^3_0 \int_{V_L} [q - \bar{q} + S(q, t) - \bar{S}] \times \dot{S} d^3q \]  

(1.28)
Now we need an expression for the displacement due to the density perturbations, \( S(q, t) \). The fractional density deviation is a separable function, \( \delta(x, t) = D(t)\delta_0(x) \) and so is the gravitational potential, \( \Phi(x, t) = (D(t)/a(t))\Phi_0(x) \). The acceleration is proportional to the gradient of the potential (since \( F = m\ddot{x} = -m\nabla\phi \) and so are the velocity and the displacement vectors since they are just integrations of the acceleration. Thus we can write the displacement caused by the density fluctuations as

\[
S(q, t) = -D(t)\nabla\Phi(q). \tag{1.30}
\]

This is known as the Zel'dovich approximation (see also Section 1.2.3). This Equation (1.30) is strictly only valid for the linear case when \( \langle\delta^2\rangle \ll 1 \). This is only satisfied at very early times and it ceases to describe the evolution of matter distribution long before galaxies are formed and the growth of the fluctuations becomes non-linear. This is where the strength of the Zel'dovich approximation lies; it can be extrapolated from the linear regime to apply to early non-linear times. Further, to minimize the nonlinearity of structure growth, White (1984) suggests we apply Equation (1.30) not to the actual density field but to a smoothed out field. This is obtained by assuming that small nonlinear structures have negligible influence on the evolution of large scale structure. Practically, this smoothed field is obtained by applying a window function on the protogalactic scale. This filters out the small scale non-linear effects so that only quasi-linear effects remain.

The first order approximation to the angular momentum is obtained by substituting Equation (1.30) into Equation (1.29),

\[
J^{(1)}(t) = -a^2(t)\dot{D}(t)\rho_0 a_0^3 \int_V [q - \bar{q}] \times \nabla\phi \, d^3q \tag{1.31}
\]

This shows that the growth rate of angular momentum is \( L \propto a^2(t)D(t) \). This can be simplified by noting that in the matter dominated era, \( a \propto (t/t_0)^{2/3} \) and adopting the convention that matter fluctuations grow as the expansion rate before turn around time, \( D = a \) so that \( L \propto \frac{2}{3} t_0^{-2} t \) in an Einstein-de-Sitter Universe. Thus the angular momentum grows linearly with time in this approximation.

It is worth mentioning the result of the second order approximation to Equation 1.29 using the Zel’dovich approximation, even though it is not the lowest order non-zero approximation.

\[
J^{(2)}(t) = a^2(t)\rho_0 a_0^3 \int_V [q - \bar{q}] \times \dot{x} \, d^3q \tag{1.32}
\]

Peebles (1969) used this result to estimate a galaxy’s angular momentum, which was found to grow in proportion to \( t^{5/3} \).

By the divergence theorem, the integral in Equation (1.31) can be expressed as an integral over the surface of the volume, \( \Sigma \),

\[
J^{(1)}(t) = -a^2(t)\dot{D}(t)\rho_0 a_0^3 \int_{\Sigma} \phi(q) [q - \bar{q}] \times dS \tag{1.33}
\]

If the Lagrangian volume is chosen to be spherical, as was done in Peebles (1969), this integral turns out to be zero since \( q' \equiv q - \bar{q} \) is perpendicular to the surface. This was first pointed out in Zel'Dovich (1970b). In general the Lagrangian volume is not spherical but is deformed by the neighboring fluctuations so the angular momentum is non-zero to
first order. If we expand the potential by a Taylor series about $q' = 0$ to second order we have,

$$
\phi(q) = \phi(0) + q_i \left( \frac{\partial \phi}{\partial q_i} \right)_{q' = 0} + \frac{1}{2} q_i q_j \left( \frac{\partial^2 \phi}{\partial q_i \partial q_j} \right)_{q' = 0}
$$

However, as pointed out in Porciani et al. (2002a), there is no a priori reason for truncating the expansion like this but it was also shown that including more terms does not increase the accuracy of the theory. Substituting this expansion into Equation (1.31) and using the definition of the cross product in terms of the completely antisymmetric tensor $a \times b = \epsilon_{ijk} a_j b_k$, gives

$$
J(1)(t) = -a^2 \dot{D}(t) \epsilon_{ijk} \int_{V_L} \rho_0 a^3 d^3q
$$

We can write the definitions of the inertia tensor $I_{lk}$, which measures of an object’s resistance to changes in its rotation rate and the deformation tensor $D_{jl}$, which is a measure of the change of shape of the protohalo due to neighboring fluctuations,

$$
I_{lk} = \rho_0 a^3 \int_{V_L} q'_l q'_k d^3q
$$

$$
D_{jl} = \left( \frac{\partial^2 \phi}{\partial q_j \partial q_l} \right)_{q' = 0}
$$

Using these definitions, the first order approximation of the angular momentum is,

$$
J^{(1)}(t) = -a^2 \dot{D}(t) \epsilon_{ijk} I_{lk} D_{jl}
$$

This equation only deals with the traceless parts of $I_{lk}$ and $D_{jl}$, that is $I_{ii}$ and $D_{ii}$ do not appear in the equation. The traceless part of the deformation tensor is $T_{ij} = D_{ij} - \frac{1}{3} D_{ii} \delta_{ij}$ which is called the tidal or shear tensor that describes the tidal stresses of the protohalo in a gravitational field. Similarly, the traceless part of the inertia tensor is, $I_{ij} - \frac{1}{3} I_{ii} \delta_{ij}$. Practically, the inertial tensor is calculated by summing over the $N$ particles in the protohalo of mass $m$ each,

$$
I_{ij} = m \sum_{n=0}^{N} q_{i}^{(n)} q_{j}^{(n)}
$$

with $q^{(n)}$ the position of the $n$th particle with respect to the center of mass.

Equation (1.38) means that to first order the angular momentum depends on the coupling of the tidal field exerted on the protohalo by its neighbors and the shape of its mass distribution ie,

$$
J_i \propto \epsilon_{ijk} T_{jl} I_{lk}
$$

The principal axes of rotation $\omega$, is defined by the eigenvalue equation, $I \omega_i = \lambda_i \omega_i$. If we re-orientate the coordinate system to coincide with the principle axes, the inertial tensor in this frame will be diagonal. In the special case when the principal axes of the inertial tensor aligns with the principal axes of the deformation tensor, both of these tensors will be diagonal in the new coordinate system. According to Equation (1.38), in this special
case the angular momentum will be zero. In particular, if the protohalo is a sphere the inertia tensor will be diagonal with identical entries so the angular momentum is zero. Also, if the volume containing the protohalo has an equi-potential surface of $\phi$ then the deformation tensor will be diagonal with identical entries so the angular momentum is again zero. However, in general the principal axes of $I$ and $D$ will not coincide and the angular momentum to first order will be non-zero.

In the principle axis frame of the tidal tensor, where $\lambda_i$ are the eigenvalues of the tidal field,

\[
J_1 \propto (\lambda_2 - \lambda_3) I_{23} \\
J_2 \propto (\lambda_3 - \lambda_1) I_{31} \\
J_3 \propto (\lambda_1 - \lambda_2) I_{12}
\]

By definition, $\lambda_3 \leq \lambda_2 \leq \lambda_1$ so $\lambda_3 - \lambda_1$ is the largest coefficient, making $J_2$ the largest component of $J$ so that halo spin is preferentially aligned with the second eigenvector of the tidal field.

The key result from TTT in Equation (1.40) assumes that the tidal and inertial tensors are completely uncorrelated, which has been shown to not always be true (Lee and Pen, 2000; Porciani et al., 2002a). If there is some correlation, the preferred direction of halo spins discussed above may be a small effect.

In summary, TTT finds that the spin of protohalos in the early stages of structure formation is expected to be correlated with the local tidal field and the inertia tensor. If this correlation survives to low redshift, spin is expected to be aligned with the intermediate axis of the tidal field. The cosmic web is the manifestation of the tidal field, filaments in particular are regular, symmetric morphologies which on large scales exhibit a uniform tidal field. Thus it is expected that the orientation of halo spin today should retain some correlation with the direction of filaments and halos should be aligned with each other over short distances.

### 1.3.2 Acquiring Spin Through Accretion

Tidal torque theory provides a comprehensive framework for explaining the origin of galactic angular momentum. A limitation of the theory is that it assumes that all the matter in a given halo at $z = 0$ was in that same halo in the past. This is not the case in general because over time galaxies merge and accrete satellites. It was shown in Bett and Frenk (2012) that it is not uncommon for the direction of the spin of a halo to completely flip over in its lifetime and this phenomenon is caused by minor and major mergers and even close halo flybys. Merger events also play a significant part in the transfer of angular momentum, as discussed in (Gardner, 2001; Vitvitska et al., 2002; Maller et al., 2002) where it was shown that the acquisition of spin modeled by mergers and by TTT both produce the same characteristic spin distribution. Further to this, Peirani et al. (2004) showed that the spin properties of halos depend on the halo’s merging history and Sharma et al. (2012) found that the universal shape of angular momentum distributions seen in simulations is found to be generically produced as a result of mergers.

Although tidal torques determine the spin at very early times, the late time spin is significantly influenced by the galaxy’s particular merger history. In particular, Vitvitska et al. (2002) claims that the spin parameters of halos that had major mergers after $z = 3$
should be considerably larger than halos that have not, but the gradual accretion of low mass satellites does not have much affect. This implies that elliptical galaxies, which are the result of spiral galaxy mergers, should rotate faster than their spiral counterparts. However, this is not backed up by the observational or simulation data which indicate the spin parameter of ellipticals is on average a factor of 10 times smaller than that of a spiral. This is a major discrepancy between this model and observation. However, this discrepancy may be rectified by considering only relaxed halos. D’Onghia and Navarro (2007) proved that equilibrium dark matter halos show no significant correlation between spin and merging history and that it is the virialization process that causes the halos to spin down due to the redistribution of mass and angular momentum.

1.3.3 The Build Up of Spin in Galaxies

Dark matter is relatively simple to model and simulate because its only interaction is through gravity. The baryonic matter that makes up the visible Universe has to obey far more complex laws in physics, making it very difficult to model and simulate fully. In the standard picture of galaxy formation, the gas inside virialized dark matter halos cools radiatively and collapses while conserving its angular momentum, resulting in the formation of centrifugally supported disks (White and Rees, 1978; Fall and Efstathiou, 1980; White, 1984). If the specific angular momentum of the baryons is conserved during galaxy formation, there is enough angular momentum to make galaxy discs with the observed size. But numerical simulations discovered a flaw in this standard picture; gas was found to lose a significant fraction of its angular momentum, resulting in disks which were too small in size, a problem known as the angular momentum catastrophe (for example see Navarro and Benz 1991). The cause of the problem is that due to efficient cooling, the gas is accreted as dense clumps, which during mergers loses its angular momentum via dynamical friction.

A second problem is that even if the angular momentum is assumed to be conserved, one cannot explain the exponential nature of disk galaxies. This is the angular momentum distribution problem. It is assumed (and shown using simulations with non-radiative gas by Sharma and Steinmetz 2005) that the angular momentum distribution of disks is similar to that of the dark matter. In CDM simulations (Bullock et al., 2001), this results in excess mass near the center compared to an exponential disk and an excess of low angular momentum material which forms a bulge. In contrast, many observed disc galaxies do not have bulge components (e.g. Allen et al. 2006; Cameron et al. 2009; Kautsch 2009) and have angular momentum distributions that lack the significant amount of low angular momentum that is found in dark matter haloes (van den Bosch et al., 2001). Assuming the verity of the cold dark matter paradigm, it would seem that the resolution of the angular momentum problem requires either the ejection or redistribution of low angular momentum material.

Another area where gas shows a difference from dark matter is the misalignment between their spin vectors. In non-radiative hydrodynamical simulations, the angular momentum of gas in galactic halos is found to be misaligned with respect to dark matter with a mean angle of 20° (van den Bosch et al., 2002; Sharma and Steinmetz, 2005). In simulations with star formation and feedback the galactic disks are also found to be misaligned, with a median angle of 30° (Bett et al., 2010).

Although the build up of spin in galaxies and the formation of disk galaxies is not well
understood, progress could be made with greater understanding of the build up of spin in dark matter halos.

1.4 Measurements of Spin

Using the formalism of Tidal Torque Theory in Section (1.3.1), predictions of the statistical properties of halo spin can be tested using real data sets and N-body simulations. TTT is used to predict the average value of angular momentum or the spin parameter and to compare predictions of the magnitude and direction of angular momentum with results from N-body simulations. The theory is also tested by measurements of correlations of the spin magnitude and direction with factors associated with the galaxy like mass and star formation and overdensity of the local environment. Also the theory predicts interactions between galaxies like spatial correlations in orientation and magnitude of spin vector, especially correlations around cosmic voids. Where the predictions from TTT fail, there is room for the merger scenario of angular momentum build up. In this Section I review how the spin of dark matter halos is measured in N-body simulations and observations, and what the spin depends on.

1.4.1 The Spin Parameter

A useful parameter to measure the amount of spin a dark matter halo contains is the spin parameter $\lambda$, introduced in Peebles (1969) which is a measure of how supported a halo is by rotation. That is the ratio between the observed angular velocity of a galaxy $\omega$ and the angular velocity needed for rotational support $\omega_0$:

$$\lambda = \frac{\omega}{\omega_0} = \frac{|J|/(MR^2)}{\sqrt{GM/R^3}}$$

(1.41)

for a halo with angular momentum $J$ and mass $M$. $G$ denotes Newton’s gravitational constant. The dimensionless spin parameter in terms of the gravitational binding energy $E$ is,

$$\lambda = \frac{|J|\sqrt{E}}{GM^{2/5}}.$$  

(1.42)

In principle, given the spin parameter $\lambda$, the value of the global specific angular momentum, $|J|/M$, can be determined by using an assumed energy content for the halo in Equation 1.42. In practice, however, this is not a straightforward procedure. For example, the energy of a halo in a crowded region is somewhat ambiguous because it depends on the environment. This led Bullock et al. (2001) to redefine the spin parameter $\lambda'$ as

$$\lambda' = \frac{|J|}{\sqrt{2MV R}},$$

(1.43)

given the angular momentum $J$ inside a sphere of radius $R$ containing mass $M$, and where $V$ is the halo circular velocity at radius $R$, $V^2 = GM/R$. This spin parameter reduces to the earlier definition of $\lambda$ (Equation 1.42) when measured at the virial radius of a truncated singular isothermal halo.
Numerical simulations (Barnes and Efstathiou, 1987; Frenk et al., 1988; Quinn and Zurek, 1988; Gardner, 2001; Bullock et al., 2001) find both $\lambda$ and $\lambda'$ to be approximately log-normally distributed,

$$P(\lambda') = \frac{1}{\lambda'\sqrt{2\pi}\sigma} \exp\left(-\frac{\ln^2(\lambda'/\lambda'_0)}{2\sigma^2}\right).$$

The spin parameter has been found (Lemson and Kauffmann, 1999; Cervantes-Sodi et al., 2008) not to depend on cosmology or environment. Both Knebe and Power (2008) and Muñoz-Cuartas et al. (2011) find a mass dependence of the spin parameter at high redshift but not at low redshift.

Analytically, the median value of the classical spin parameter was found using Gaussian statistics to be $\lambda = 0.07^{+0.04}_{-0.05}$ in Steinmetz and Bartelmann (1995). Although many approximations have been made, this is a fairly good match to measurements of the spin parameter in numerical simulations, which can be as small as 0.05 for ellipticals and as high as 0.5 for spirals (Efstathiou and Jones 1979; Barnes and Efstathiou 1987). However, calculations of this type based on Equation (1.38) depend on the misalignment of the principal axes of the inertial ($I$) and tidal ($T$) tensors. Assuming no correlation at all would lead to relatively large spins whereas a strong correlation between $I$ and $T$ would lead to spins that are too small. The inertial and shear tensors were found to be strongly aligned in Porciani et al. (2002b), with the galaxy spin due to only 10% deviation from perfect $I$ and $T$ alignment.

Also, predictions from N-body simulations are dependent on the effective time that spin-up from tidal torques should be stopped. It is shown in Porciani et al. (2002a) that non-linear effects can account for some of the discrepancies between analytical and simulated values for the angular momentum, and this is reflected in an uncertainty of the choice of time that linear interactions are stopped. This is some time before turn-around and can be chosen to be around $z \sim 3$.

### 1.4.2 Spin Alignments Between Dark Matter Halos

Since the spins of halos are aligned with the large scale structure, there should be some degree of coherence between the direction of spin of two neighboring halos. It is not clear if this alignment is strong enough to be detected even in N-body simulations. Heavens et al. (2000a), Porciani et al. (2002a), Faltenbacher et al. (2002) and Bailin and Steinmetz (2005) see no strong alignment whereas Hatton and Ninin (2001) do see a significant alignment. In contrast, several claims have been made of spiral galaxy spin alignments in observations (Pen et al., 2000; Slosar et al., 2009; Lee, 2011). If these alignments can be seen in observations but not in dark matter simulations then it is a possible indication that the spins of the luminous galaxies are not aligned with their dark matter halos.

A correlation between $I$ and $T$ has ramifications for the possibility of finding a correlation between the spin and the tidal tensor. Assuming a correlation between $I$ and $T$ tends to weaken the correlation between $J$ and $T$, as shown in Porciani et al. (2002b). In Lee and Pen (2000) the $J - T$ correlations are parametrized through the expression,

$$\langle \hat{J}_i \hat{J}_j | T \rangle = \frac{1 + c}{3} \delta_{ij} - c \hat{T}_{ik} \hat{T}_{kj}$$

where $c$ is the correlation parameter ($0 < c < 3/5$, for uncorrelated to completely correlated $I$ and $T$ and $c = 3/5$ for perfect $I$ and $T$ correlation) and $\hat{T}_{ij}$ is the trace-free unit
shear tensor. A nonzero value of $c$ implies the existence of a spin-spin correlation due to the spatial correlation of the local shears. Using N-body simulations, Lee and Pen (2000) found that $c = 0.24 \pm 0.02$ and that the spin axes is correlated with the intermediate axes of the shear tensor. On the other hand Porciani et al. (2002b) found a much lower value for a different simulation, $c = 0.07 \pm 0.04$ which implies the spin and shear are virtually uncorrelated. It was concluded that there is perhaps only a very weak tendency for the protohalo spin to be perpendicular to the major axes of $\mathbf{T}$ and for the spin to be aligned with the intermediate axes. Most literature still makes the assumption that spin and shear are uncorrelated.

This is significant when considering the possible spatial correlation of spins. In Pen et al. (2000) these correlations are analytically approximated using the second order spin correlation function, $\eta(r)$ which is defined as

$$\eta(r) \equiv \langle |\hat{L}(\mathbf{x})\hat{L}(\mathbf{x} + \mathbf{r})|^2 \rangle - \eta_0$$

This is approximated by

$$\eta(r) \simeq \frac{c^2}{6} \xi_R^2(r)$$

where $c$ is the correlation parameter as before and $\xi_R(r) \simeq r^{-1}$ is the correlation of the top-hat convolved density field. Thus for a small value of $c$ the spin-spin correlation is tiny and it decreases rapidly over larger scales. For a few $h^{-1}\text{Mpc}$ and the value of $c$ from Lee and Pen (2000), the spatial correlation is of the order 1% but for the value of $c$ from Porciani et al. (2002b) the spatial correlation is much less, 0.1%. The spatial correlations are also observed by Pen et al. (2000) using the Tully catalog of 12,122 spiral galaxies. The spin vector for each galaxy is determined using the position angle and the axial ratio. It is found that the spin-spin correlation is of the order of 1% at $1 \ h^{-1}\text{Mpc}$, but this is far from
being accurate because of the high statistical noise. This correlation has consequences for weak gravitational lensing, where the key assumption is that galaxies are randomly orientated. If the correlation is significant, galaxy alignments could have an effect on the distortion of galaxy shapes and the interpretation of the lensing effect, as discussed in Crittenden et al. (2001). This effect could be detected by looking at correlations between galaxy ellipticities in the nearby Universe where a lensing interpretation for any observed signal is highly improbable.

A different test of spin correlations is done using the Galaxy Zoo project (Land et al., 2008) where the public classifies the morphology and in particular, the spin chirality of galaxies in the Sloan Digital Sky Survey (SDSS, York et al. 2000). The spin chirality is the observed sense of rotation from lagging spiral arms, either ‘S’ or ‘Z’ shape corresponding to spin vectors pointing along the line of sight either towards us or away from us. An example of galaxies used in their sample are shown in Figure (1.6). It has been shown Pasha and Smirnov (1982) that the galaxy is actually rotating contrary to intuition, with leading spiral arms 4% of the time, but otherwise this classification is fairly accurate. One bit of information for each galaxy is obtained of unit spin vector projected along our line of sight and this is used to look for correlations in the spin chirality. It was found that there is a correlation at separations less than 0.5 Mpc. However since this test uses only one bit information the results are not completely reliable. More information of the magnitude and direction of the spin is needed to find any conclusive results.

1.4.3 Halo Spin Alignments with the Large Scale Structure

Spatial correlations are of particular interest within the largest scale cosmological structures like on sheets or filaments and around voids. Measuring spin correlation is difficult because it is an inherently local phenomena, effective over only a few megaparsecs, due to the nearby tidal fields. On a sheet or around a void however one would expect a large scale ($\sim 10 \text{Mpc}$) coherent orientation from the large scale tidal field. In a sheet, assuming $I$ and $T$ are uncorrelated, it is expected that the spin vectors should be perpendicular to the minor axes of the sheet the galaxies are embedded in. This is due to the correlation between the spin and the shear. It is shown analytically in Lee (2004) that the stronger the intrinsic shear-spin correlation the more inclined the galaxy spins relative to the sheet. It has been shown that spins lie preferentially in the plane of sheets in simulations (Navarro et al., 2004a; Hahn et al., 2007b; Zhang et al., 2009; Libeskind et al., 2012).

Similarly, around voids we expect that the rotation axes of galaxies should lie preferentially on the void surface. This has been observed by Trujillo et al. (2006) in the 2dFGRS and SDSS catalogs but there was a null result from the SDSS catalog alone (Slosar and White, 2009). A correlation has been seen in simulations by Brunino et al. (2007); Cuesta et al. (2008), where halos located on the shells of the largest cosmic voids have angular momenta that tend to be preferentially perpendicular to the direction that joins the center of the halo and the center of the void.

Filaments are a particularly interesting place to look at the mergers and accretion of halos because of the large scale cosmic flows where matter escapes from the voids to the walls then towards filaments and along their axis to the nodes. This picture makes filaments an interesting region of bulk flow and accretion. In simulations it has been detected that the spin direction of dark matter halos points parallel with the axis of filaments for low mass halos and perpendicular to the axis for high mass halos (Faltenbacher et al.,
In observations there has been a tentative detection of some weak correlation with filaments (Jones et al., 2010) but no significant detection has been found to date. The evolution of halo spin with respect to filaments and sheets was explored by Hahn et al. (2007a) who found no change in the orientation of spin over cosmic time. In observations (Tempel et al., 2013), the spin axes of bright spiral galaxies have a weak tendency to be aligned parallel to filaments while the spin axes of elliptical/S0 galaxies are preferentially aligned perpendicular to their host filament. The spin alignment of 100 galaxies in filaments has been simulated (Hahn et al., 2010) to find that it is the massive galaxies that have their spins aligned with the filament axis.

More generally, there are direct correlations between the spin axis and the local tidal field. There has been confirmations of the predicted alignment of halo spin with the intermediate axis of the tidal field from TTT (Hahn et al., 2010) and with the nearby distribution of matter (Paz et al., 2008).

### 1.4.4 Measuring the Spin of Galaxies

In galaxy formation, the dark matter undergoes collisionless virialization through violent relaxation whereas the gas collapses through shocks. Although the dark and baryonic matter collapse in very different ways, it is often implicitly assumed that galaxy disks are perpendicular to the spin vector of their host halos. This assumption has been the basis of many studies, such as the studies relating to galaxy alignments and weak lensing, Croft and Metzler (2000); Heavens et al. (2000a); Catelan et al. (2001). This assumption is quite hard to test since observationally we only see the galaxy orientation and in simulations it is problematic to model baryonic matter at the same time as dark matter, so we usually only have a measurement of the halo spin.

Hydrodynamical simulations on individual galaxy scales (van den Bosch et al., 2003; Sharma and Steinmetz, 2005; Bett et al., 2010) have shown that the specific angular momentum of baryons remains close to that of dark matter and that the galaxy angular momentum is generally about 20° misaligned with the dark matter halo and the dimensionless spin parameter distributions are the same between both types of matter. This means that dark matter halo spin is a fairly good proxy for galaxy spin, so some understanding of the spins of galaxies may be gleamed from dark matter-only simulations. The spin of a dark matter halo depends mainly on two things; the initial torques driven by the surrounding landscape at early times, and the accretion and merger history of the halo.

Observations of galaxy spin alignments in the large scale structure to date have only been through inferred galaxy spin orientations from observed disk galaxy shape. For example, Lee and Erdogdu (2007) used the Tully catalog of nearby spirals (Nilson, 1974; Lauberts, 1982) to infer spin from the axial ratio (to find an alignment with the tidal field) and Slosar et al. (2009) used the apparent sense of spiral rotation in the Galaxy Zoo catalog. Direct measurements of galaxy rotation have been done with integrated field units (IFU) although only one galaxy is targeted at a time and it is not feasible to conduct a survey of large scale structure with direct spin measurements. However, a new multi-object IFU instrument has been developed which will enable a survey of $10^{4-5}$ galaxies in a volume limited sample (Bland-Hawthorn et al., 2011; Croom et al., 2012). There will soon be a huge influx of galaxy spin data, which has never been sampled before.
in such high volumes. In order to get the most out of these observations and to direct future surveys, the dark matter halo spin must be better understood.

Problematically, the spin parameter cannot be measured easily from observations, as none of $L$, $E$ or $M$ can be measured directly. To observe the spin parameter of a galaxy indirect methods must be used, such as the method outlined in Jimenez et al. (2003). Here the particular profile for the dark halo and baryonic disk is assumed and fitted to rotation curve data. Another simplistic approximation for $\lambda$ from observational data is described in Hernandez and Cervantes-Sodi (2006) where the density profile is assumed to be isothermal so the rotation curve is flat. The spin parameter is approximated by,

$$\lambda \simeq \sqrt{2V_d^2 R_d \over GMh}$$

Clearly, this is a very approximate method of finding the spin parameter, but it can be used for any sample of galaxies and no detailed modeling of the rotation curve and matter distributions is needed, such as done in Tonini et al. (2006). The spin parameter is reduced to the simple expression depending only on the radius of the disk, $R_d$ and the rotation curve $V_d$ using the Tully-Fisher relation between the disk mass $M_d$ and the rotation velocity $V_d$ is also assumed, $M_d = A_{TF} V_d^{3.5}$.

$$\lambda = 21.8 {R_d/kpc \over (V_d/km \ s^{-1})^{3/2}}$$

Using this approximation, the spin parameter for the Milky Way is $\lambda = 0.0234$ and for the data set of 304 late-type spiral galaxies, $\langle \lambda \rangle = 0.0645$ Hernandez and Cervantes-Sodi (2006).

Spin is a key factor in galaxy formation and is fundamental in shaping the properties of galaxies. Star formation is dependent of the density, so the more a galaxy is supported by spin, the less dense it is and the less efficient star formation. A correlation of spin with star formation rate is investigated in Berta et al. (2008) using the SDSS catalog and the approximation of the spin parameter in Equation (1.48). This is combined with determinations of detailed star formation histories using the optimal parameter extraction method MOPED (Heavens et al., 2000b). The results show that a higher spin parameter indicates a higher rate of star formation, contrary to initial predictions. This could also be due to the galaxy merger effects; that recently merged galaxies are likely to have a higher spin and spur a period of star formation.

As well as the effects on star formation, spin is shown to be anticorrelated with mass. A study by Cervantes-Sodi et al. (2008) of the SDSS sample finds that lower disk masses have a broader and generally higher distribution of spins than high mass galaxies. A similar study by Berta et al. (2008) which uses the Tully-Fisher relation only once in the approximation of the spin parameter and not to determine the galaxy mass, finds the same kind of anti-correlation. This is expected because in Hernandez and Cervantes-Sodi (2006) it is shown that the spin parameter plays a big part in determining the Hubble type of a galaxy which quantitatively describes the morphological type. This analysis uses the same approximation for the spin parameter, Equation (1.49) to directly compare the value of $\lambda$ and the Hubble type for galaxies in the ASSSG sample. It was found that spin is closely related to the Hubble type, which is also highly dependent on the galaxy mass. Thus we expect that the spin of a galaxy will be dependent on the mass of the galaxy.
1.5 Thesis Outline

Currently there are two schools of thought on the origins of angular momentum in dark matter halos. One school believes that it is the product of a primordial interaction between over-densities in the early Universe. The other school proposes that it is later interactions between formed dark matter halos and slow accretion in the recent Universe which is responsible for the angular momentum properties of dark matter halos today. This thesis aims to cut through the debate and get to the source of dark matter halo angular momentum.

Early work on the origins of angular momentum in dark matter halos focused on gravitational interactions in the very early Universe, called Tidal Torque Theory. This theory stood up well in tests using N-body simulations in the linear regime (Sugerman et al., 2000) but did not hold so well during non-linear times. Porciani et al. (2002a) and Porciani et al. (2002b) found that while TTT holds up well in the linear regime, non-linear influences cause a significant deviation of the angular momentum direction and the amplitude of the angular momentum grows by more than a factor of two between $z = 3$ and $z = 0$.

More recently, investigations have been made on the role of late interactions between dark matter halos and their surrounding structure. It was shown in Gardner (2001); Vitvitska et al. (2002); Maller et al. (2002) that the acquisition of spin modeled by mergers and by TTT both produce the same characteristic spin distribution. Further to this, Peirani et al. (2004) showed that the spin properties of halos depend on the halo’s merging history and Sharma et al. (2012) found that the universal shape of angular momentum distributions seen in simulations is found to be generically produced as a result of mergers. It is clear that mergers and accretion are fundamental in the build up of angular momentum in dark matter halos but it is not clear exactly how this affects the halo spin direction.

In this thesis, I explore the build up and evolution of the spin of dark matter halos within N-body simulations. The goal is to understand the angular momentum content of dark matter halos and its relationship to the large-scale structure of matter that the halos reside in. By understanding how the spin of dark matter halos is related to the surrounding large scale structure, the build up of spin in dark matter halos and in galaxies can be better understood.

This thesis has three main parts, Chapters 2 and 3 describe the tools used in analyzing the make up of the large scale structure of the Universe, Chapter 4 looks at dark matter halo angular momentum properties and their evolution and Chapter 5 looks further at the build up of angular momentum through mergers.

For this analysis I have used publicly available data from the Millennium simulation. This simulation and N-body simulations in general are described in Chapter 2. Central to the analysis in this thesis is the identification of dark matter halos. The fundamentals of halo identification are given in this chapter.

To see how dark matter halos fit into their environment, the filaments in the Millennium simulation are picked out using two different methods. These methods are described in Chapter 3. Both methods are relatively simple but fresh adaptations of filament finding methods used in the field.

In Chapter 4, I use the methods described in Chapters 2 and 3 to investigate the evolution of the orientation and magnitude of dark matter halo angular momentum within
the large scale structure since $z=3$. In particular, I look at the evolution of the alignment of halo spins with filaments and with each other, as well as the spin parameter, which is a measure of the magnitude of angular momentum. This exploratory work uncovers the state of angular momentum in dark matter halos today and in their recent past. Several surprising results point to holes in the current understanding of the build up of angular momentum in dark matter halos. This work has been published in a peer-reviewed journal.

The research in Chapter 4 led to a more close look into the build up of angular momentum in dark matter halos; in Chapter 5, I study the merging histories of dark matter halos in the context of spin. I look first at the infall of halos onto filaments and how these relative motions could affect merging histories. Then I look directly at how mergers change the spin orientation of dark matter halos. The work in this Chapter has led to a new model of the build up of angular momentum in dark matter halos, primarily through mergers.

A discussion of my findings and proposed future studies are presented in Chapter 6.
Chapter 2

N-body Simulations

It shone, pale as bone
as I stood there alone.
And I thought to myself how the moon
that night, cast its light
on my heart’s true delight
and the reef where her body was strewn.
- Grim Fandango (Video Game 1998)

2.1 Introduction

N-body simulations provide a means of probing the nonlinear stage of structure formation. The basic mechanism by which a N-body simulation operates is simple to understand but requires a lot of finesse to work efficiently. The first step is to discretize the matter density in the Universe by breaking it up into particles and assigning each a position, velocity and mass. Then the simulation box is evolved forward under gravity until a later time.

Luckily, Newtonian gravity and electromagnetic radiation share the same inverse square relationship with distance. This enabled the first N-body simulations to be run using an experimental set up of light bulbs and photocells (Holmberg, 1941). The first modern simulations of large scale structure, computationally integrating the gravitational force, were done in the ’70s (Press and Schechter, 1974; Miyoshi and Kihara, 1975; Aarseth et al., 1979). Since then, there has been a proliferation of computing technology that has taken simulations to new highs with extremely fine resolution and an excess of $10^{10}$ particles.

The simulation used in Chapters 4 and 5 is the publicly available Millennium Simulation (Springel et al., 2005). This is a dark matter simulation in a $500 h^{-1}$Mpc box, suitable for studying the large scale structure in the ΛCDM Universe. This simulation has far higher resolution than any simulation I could have run locally (although I did create initial conditions and run a simulation for pedagogical purposes, see Figure 2.1). A high particle resolution was needed because high resolution halos are required to get an accurate measurement of halo spin and a large box size was needed to study the large scale structure of the Universe, making Millennium perfect for my needs.

An overview of the concepts and procedures involved in the running of N-body simulations is presented in the following Chapter. The basic methodology (generating the initial conditions and running the simulation through integrating the equations of motion)
is outlined and the effects of discretization are touched upon. The first step of analysis - halo finding - is described, and lastly the details of the Millennium simulation and merger trees.

2.2 Initial Conditions

To start a simulation, particles need to be assigned initial positions. These positions are taken from a Gaussian random field, imprinted with the initial linear power spectrum (or correlation function). Several codes, such as cmbfast (Seljak and Zaldarriaga, 1996), camb (Lewis et al., 2000) and cmbeasy (Doran, 2005) can compute the linear matter power spectrum and transfer function for a given set of cosmological parameters.

The basic method of generating initial conditions is as follows:

1. Setup a Cartesian grid with dimensions corresponding to the volume of the simulated universe. Periodic boundaries will be apply.

2. Compute the perturbation field using the inverse Fourier transform of the power spectrum.

3. Compute the perturbation of the velocities, following the Zeldovitch approximation.

4. Apply these perturbations to the Cartesian grid.

The initial distribution of particles can be set in two possible configurations, both of which incorporate periodic boundary conditions. In the grid configuration, the particles are randomly distributed onto a lattice. In the glass configuration, the particles are randomly distributed then evolved by gravity, but with the gravitational force acting repulsively instead of attractively, until the forces on each particle are balanced. To generate the initial conditions for the simulation, the matter power spectrum is applied to this initial configuration. This can be done using the Zel'dovich approximation, which gives the equations of motion derived from first order Lagrangian perturbation theory:

\[ x = q + D(t)\phi(q)v = a \frac{dD}{dt}\phi(q) \]  

The starting redshift where the initial conditions are sampled must be set sufficiently high such that the final distribution of particles is unaffected by the choice of method and all modes sampled by the simulation box are linear up until that epoch. If the starting redshift is too low the formation of the smallest scale structures is delayed, but if the starting redshift is too high, force errors are introduced. Starting too late (or allowing for not enough expansion before extracting physical information from the simulation) will delay the collapse of the first halos acting as seeds for further structure formation. Starting too early will introduce force errors, errors due to the discreteness in the mass distribution and time integration errors because they all accumulate as the ratio of the final expansion factor to the initial expansion factor increases.
2.3 Discretization

The particles put into position by the initial conditions are essentially a Monte Carlo sampling of the underlying smooth mass distribution. Because of this discretization of the density field, limits are imposed on scales larger than the box size as well as the inter-particle separation.

Due to the finite box size and periodic boundary conditions, the $k$-space at large scales will be poorly sampled, leading to random fluctuations around the desired power spectrum at large scales. The smallest sensible box size is that for which the length of a side corresponds to the scale that has just begun to go non-linear at the final epoch of the simulation.

The most important effect caused by discreteness in simulations occurs when two particles get very close to each other. As their separation goes to zero, the gravitational force diverges which yields unphysical results such as the numerical depression of structure. Softening is introduced to describe the forces more accurately, particularly for nearby particles. The usual way to smooth the forces is to replace the gravitational potential for each particle with that of a theoretical model. The Plummer model is usually used, which models the gravitational potential distribution in a spherical halo of matter as,

$$\Phi(r) = \frac{-GM}{\sqrt{r^2 + \epsilon^2}}$$

This removes the singularity at $r = 0$. The equations of motion actually integrated are:

$$F_{ij}(r) = Gm_i m_j \frac{r}{(r^2 + \epsilon^2)^{3/2}}.$$  (2.3)

The softening length $\epsilon$ must be small so that the potential is not substantially biased. Softening essentially smoothes out small scale physics that cannot be resolved in the simulation but for large distances, the small number has little effect. In addition to solving problems with close particle interactions, smoothing the potential reduces the ‘graininess’ of the particle distribution, thereby making the potential of the model system more similar to that of a system with a smooth density distribution.

It is obvious that $\epsilon$ cannot be too large: this would result in substantial bias of the potential, and would also impose strong constraints on the spatial resolution of structural features of the system. Merritt (1996) proposed a criterion for choosing the softening length based on the minimization of the mean irregular force acting on a particle. In a study of dark matter halo spins, it is necessary that the softening length must be fairly small in order to get high resolution data from each particle to calculate the total angular momentum accurately.

2.4 Integrating the Equations of Motion

A N-body simulation works by calculating the acceleration of each particle due to gravity and then transforming this into particle motion over some small increment in time (the timestep), then recalculating the accelerations and so on. Taking the brute-force approach, the number of calculations grows as the number of particles squared, which is computationally unfeasible for the number of particles involved in cosmological simulations.
Figure 2.1: A sample n-body simulation output, undertaken by the author for pedagogical soundness. These are the particle positions at z=0. This simulation was run in a $50h^{-1}\text{Mpc}$ box with $128^3$ particles. Initial conditions were generated on a grid using a power spectrum produced by CMBEASY (Doran, 2005). The particles were evolved from $z=20$ to $z=0$ using GADGET2 (Springel et al., 2001b).

One of the simplest ways to break this problem into manageable pieces is the Particle-Mesh (PM) approach. Calculating the potential at a given point in space involves summing the potential produced by every piece of mass in the simulation. This is equivalent to convolving the density field with a $1/r$ potential kernel. Convolutions are easy in Fourier space where they become simple multiplications. So as long as we are able to compute the Fourier transform, the computation time can be drastically reduced. The PM approach does lose out in accuracy though, since the continuous density field must be sampled onto a discreet 3D grid before performing the discrete Fast Fourier Transform (FFT), so it is not well suited for work with high spatial resolution. Although decreasing the size of the mesh would improve the resolution, if the number of grid points becomes larger than the number of particles, even the increased speed from using FFT would not outweigh the increased time from the greater number of calculations. PM methods typically scale like $O(M \log M)$ where $M$ is the number of grid points. Although some studies have argued that N-body integrators that employ softenings that are smaller than the mean interparticle separation are unreliable (e.g. Melott et al. (1997)), if the number of grid points
Another way to cut down the number of calculations is the Tree method. This method exploits the principle that an error in the position of a particle will not significantly change the gravitational acceleration induced on another particle as long as the distance between them is much greater than the error. The simulation volume is divided into different regions and the gravitational acceleration due to one region is calculated assuming that all particles lie at the region’s center of mass. This method has no intrinsic resolution limit but can be substantially slower than Fourier based methods. Tree-code methods typically scale like $O(N \log N)$ where $N$ is the number of particles.

Gravitational N-body simulations are Hamiltonian systems. A symplectic integrator (like the commonly used ‘leapfrog’) is an exact solution to a discrete Hamiltonian system that is close to the Hamiltonian system of interest. This works well for simulations with a fixed time-step but when conserving computational resources with individual time-steps, a formally symplectic integration scheme is not possible. This is due to the pairwise coupling of particles. However, the potential between two particles can be partitioned into a long-range and short-range part. Because the Hamiltonian has been separated, but in practicality maintains a comparable accuracy to fully symplectic schemes, this is called quasi-symplectic. Unfortunately, symplecticity can not be maintained if individual and adaptive time-steps are assumed (Quinn et al., 1997).

A popular, publicly available N-body code used in the Millennium run is GAlaxies with Dark matter Gas intEracT (gadget2, Springel et al. (2001b)). This is a massively parallel TreeSPH code, designed to follow a collisionless fluid with the N-body method and an ideal gas by Smooth Particle Hydrodynamics (SPH). To compute the gravitational force this code combines the Tree and Particle Mesh methods into a hybrid, ‘TreePM’ (Xu, 1995). The integration is based on a quasi-symplectic scheme where long and short range forces can be computed with different time steps.

### 2.5 Identifying Halos

The output of an N-body simulation is several snapshots of the particle data taken at particular times. Identifying dark matter halos in the particles is one of the first challenges in the analysis of this data. There are many ways of defining and finding halos; some of them are simple and some are very sophisticated but all of them are useful in their own way. In the early days of halo finding, the spherical-overdensity (SO) method (Press and Schechter, 1974) and friends of friends (FoF) algorithm (Davis et al., 1985) were the standard techniques. These halo finders worked well at identifying isolated field halos but are unable to pick out substructure within larger host halos. There have since been many more sophisticated halo finders developed that attempt to cope with this problem often using parallel computation; (Springel et al., 2001a; Gill et al., 2004; Neyrinck et al., 2005; Shaw et al., 2007; Knollmann and Knebe, 2009), although the foundation of nearly all these codes is SO or FoF. Algorithms based on SO aim at identifying peaks in the matter density field. About these centers, spherical shells are grown out to the point where the density drops below some threshold. Algorithms based on FoF connect and link particles together that are close to each other. They afterwards determine the center of the grouping. A comparison of several modern halo finders is presented in Knebe et al. (2011).
**Friends of Friends** The friends of friends algorithm is one of the simplest and quickest ways of picking out groups from the diffuse haze of particles. A FoF group is the largest group of particles that are all within some separation, the *linking length* from another particle in that group. The linking length ($b\overline{\ell}$, where $\overline{\ell}$ is the mean inter-particle separation) is the only free parameter in this method and is usually taken to be $b = 0.2$.

For a large number of particles and a given value of $b$, the FoF algorithm defines the boundary of a halo as corresponding to an isodensity surface. The overdensity of this surface is $\delta_{\text{fof}} \simeq 2b^{-3}$ (Frenk et al., 1988). For the most commonly used value of $b = 0.2$, this corresponds to an enclosed overdensity of $\delta \simeq 180$. This value is close to the virial overdensity predicted by the spherical collapse model in the Einstein-De Sitter cosmology and is usually regarded as a justification for using $b = 0.2$ in analyses of simulations.

The definition of a FoF halo does not incorporate the velocity information of the particles, and it cannot determine if a group of particles is gravitationally bound. Another disadvantage is that it may identify two separate groups as one, if they are linked by a thin bridge of particles. The FoF algorithm does not assume any particular shape and can therefore better match the generally triaxial mass distribution. FoF groups are a useful tool in dark matter halo finding and in very approximate studies, could be taken to be the halos themselves. Usually, the FoF groups are taken as the first step in halo finding and other, more complex algorithms are used to find halos and substructure within the FoF groups.

**SUBFIND** A powerful tool to find not only virialized structure but also subhalos within those structures is SUBFIND (Springel, 1999), which was used to identify halos in the Millennium simulation. This method builds upon the initial list of FoF groups by finding saddle points and local minima in the density field and identifying locally overdense regions. It does this in a hierarchical manner; the generated sets can have nested subsets, sub-subsets and so on. This top-down approach to structure finding reflects the hierarchical nature of large scale structure formation so it is particularly appropriate to use in this context. The idea of hierarchical clustering predates SUBFIND and was first introduced by the code ISODEN (Pfitzner et al., 1997).

Once the group boundaries have been found from the density contours, each substructure candidate is subjected to a gravitational unbinding procedure. If the remaining bound part has more than 20 particles, the subhalo is kept for further analysis and some basic physical properties (angular momentum, maximum of its rotation curve, velocity dispersion, etc.) are determined. An identified subhalo is extracted from the FOF halo, so that the remainder formed a featureless background halo which was also subjected to an unbinding procedure.

### 2.6 The Millennium Run

The simulations used in the analysis of Chapters 4 and 5 are part of the Millennium Run using data from the Millennium simulation (Springel et al., 2005), a very large dark matter simulation of the concordance $\Lambda$CDM model. The Millennium simulation was run in parallel at the Computing Center of the Max-Planck Society in Garching, Germany, over 512 processors for 28 days. The output of the simulation can be visualized with the projected dark matter density field to show the web like nature of the large scale structure.
The Millennium simulation followed $2160^3 \simeq 1.0078 \times 10^{10}$ particles from redshift $z = 127$ to the present in a periodic box of 500 $h^{-1}\text{Mpc}$ on a side (which gives a particle mass of $8.6 \times 10^8 h^{-1}\text{Mpc}$). The initial conditions were created by displacing particles from a homogeneous, ‘glass-like’ distribution. The simulation was run using a customized version of \texttt{gadget2} (Springel et al., 2001b), using the ‘TreePM’ method (Xu, 1995) for evaluating gravitational forces. The gravitational force law was softened isotropically on a co-moving scale of $5 h^{-1}\text{Kpc}$ (Plummer-equivalent) using a spline kernel. This is 46.3 time smaller than the mean particle separation. The cosmological parameters of the $\Lambda$CDM simulation were chosen to be consistent with a combined analysis of the 2dFGRS (Colless et al., 2001) and first-year WMAP data (Spergel et al., 2003): $\Omega_m = \Omega_d m + \Omega_b = 0.25$, $\Omega_b = 0.045$, $h = 0.73$, $\Omega_\Lambda = 0.75$, $n = 1$, and $\sigma_8 = 0.9$. $\Omega_m$ is the total matter density, $\Omega_b$ is the baryon density and $\Omega_\Lambda$ is the dark energy density at the present day. The Hubble constant is parametrized as $H_0 = 100 h \text{km} s^{-1} \text{Mpc}^{-1}$, while $\sigma_8$ is the root-mean-square linear mass fluctuation within a sphere of radius $8 h^{-1}\text{Mpc}$ extrapolated to $z = 0$. Further details on these parameters are given in Section 1.1.2.

At each output time, FoF groups were identified which contain one or several subhalos,
found by SUBFIND. The FoF groups played no direct role in determining the merger trees or in the semi-analytic models. The first subhalo in the FoF group typically contains 90% of the mass of the group (De Lucia and Blaizot, 2007). The halo data is available at http://gavo.mpa-garching.mpg.de/MyMillennium Lemson and Virgo Consortium (2006).

The milli-Millennium simulation is a smaller version of the Millennium simulation, with the same cosmology and particle mass ($8.6 \times 10^8 h^{-1}\text{Mpc}$) but in a smaller box ($62.5 h^{-1}\text{Mpc}$). The milli-Millennium simulation was used in this thesis for demonstrating the capabilities of the filament finding methods and for all other results, the full Millennium or a subsection of this simulations was used. Data is also available on-line from the more recently run Millennium II simulation (Boylan-Kolchin et al., 2009). This is a higher resolution simulation of the same number of particles as Millennium, in a smaller box ($100 h^{-1}\text{Mpc}$).

### 2.6.1 Merger Trees

![Merger Tree Diagram](image)

Figure 2.3: Schematic organization of the merger tree in the Millennium Run. The merger tree connects subhalos that are contained within FoF groups. Each halo knows its descendant, and its most massive progenitor. Possible further progenitors can be retrieved by following the chain of next progenitors. In a similar fashion, all halos in a given FoF group are linked together. Picture credit: Springel et al. (2005).
Our own galaxy, the Milky Way (MW) and its neighbor Andromeda (M31) are part of the Local Group of galaxies and each have their own dark matter halos and a system of satellite galaxies. In particular, the large and small Magellanic clouds, visible from the southern hemisphere, orbit the MW. These irregular galaxies appear to be being ripped apart by the MW’s tidal forces, and will eventually merge completely with the MW. These merging events will contribute to the build up of mass and angular momentum of the MW, as well as its structure composition. Eventually, even the MW and M31 will merge together to create a giant elliptical galaxy (Cox and Loeb, 2008).

To understand the physics behind galaxy evolution, we have to accurately know the evolution of the dark matter halos in terms of hierarchical merging at each time step. The hierarchy of halo mergers can be represented by merger trees. A Merger tree thus describes the sequence in which halos merge and grow and sometimes split and die. Most of the growth of a halo can be viewed as a sequence of mergers of halos, termed progenitors, with the rest of the assembled mass considered smooth accretion of dark matter flowing onto the halo. Every node in the merger tree represents a halo and an edge connected to the node tells about the halo’s descendants and ancestors. An example of a visualization of a merger tree in the Millennium simulation is given in Figure 2.3.

In the Millennium simulation, merger trees are constructed from the subhalos found by SUBFIND, FoF halos play no direct roll. It is required that subhaloes have at the most one descendant, ruling out the possibility of finding halos that split in two, which is inconsistent with the bottom-up theory of structure formation. If more than one descendant is found, the fragmentation of the subhalo is likely to be a transient event, with the descendants merging back into one halo again. When events like this are allowed, it leads to a false measure of the merger rate. Each descendant is allowed to have more than one progenitor. There are two scenarios for finding the descendant of a subhalo:

1. For many subhaloes, the descendant can be found trivially: all particles in a subhalo at snapshot \( S_n \) may belong to a single subhalo at the subsequent snapshot \( S_{n+1} \), in which case this subhalo is clearly the descendant of the subhalo at the previous snapshot.

2. There is also the possibility that particles belonging to one subhalo at \( S_n \) may be distributed over more than one subhalo at \( S_{n+1} \). We still require each subhalo to have at the most one descendant, which is taken to be the halo that contains the core of the progenitor. This is found by calculating the binding energy of the subhalo at \( S_n \). The possible descendant with the most bound particles (the core of the subhalo) is said to be the descendant.

Sometimes a descendant can not be found in the snapshot \( S_{n+1} \) but can be found in \( S_{n+2} \). This happens when SUBFIND fails to locate a subhalo because it is passing through the dense center of a larger system. This kind of “flickering is a problem for a lot of halo finders that trace substructure but can be addressed with phase-space merger-tree codes (eg. Behroozi et al. (2013)), where individual particles are tracked across time steps.

An example of a simplified Millennium merger tree is shown in Figure 2.3. This tree follows the progenitors of one of the subhalos in a FoF group and illustrates the pointers used in the Millennium catalog. The first progenitor (the most massive progenitor) contains most of the mass of the subhalo in the previous snapshot. The other progenitors are often satellites to the first progenitor when they merge (minor mergers), but if two
or more of the progenitors are comparable in mass there is said to have been a major merger. This is usually taken to be when the mass ratio of the first progenitor to the next progenitor is less than three.

2.7 Further Analysis

The structure in the N-body dark matter particles (Figure 2.1), the N-body density field (Figure 2.2) and even the galaxy distribution (Figure 1.1) shows a web like nature. There are narrow threads of matter, densely knotted in places, stitching together the massive superclusters. The filamentary nature of the Universe today is a manifestation of its history and continuing evolution. The dark matter halos that reside in this web are intimately tied to its evolution and this will be reflected in the properties of the halos, such as angular momentum.

This thesis focuses on the analysis rather than the running of N-body simulations. I build upon the halo catalogs and merger trees already made by the Millennium team by identifying filaments in the large scale structure and searching for clues on the build up of angular momentum of dark matter halos.
Chapter 3

Finding Features in the Large Scale Structure

*Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry.
- Richard P. Feynman*

3.1 Introduction

On scales of a few Megaparsecs to a few hundred Megaparsecs, the matter distribution of the Universe is remarkably anisotropic, forming a hierarchical web-like structure. This structure emerged as a result of the growth through gravity of small amplitude primordial density fluctuations. Evolution under gravity has shaped the initial perturbations into complex patterns and structures in the density field. The way in which the web-like structure was formed is hierarchical in nature and this is illustrated in the growth of filamentary structure in Figure 3.2. Small structures formed first and then subsequently merged into larger structures. As a result, structure exists on various scales. The web-like pattern can be understood through the tendency of matter to contract and collapse in an anisotropic manner. As a result, massive voids are formed, surrounded by walls and filaments which converge at massive clusters. Untangling the cosmic web into these separate components is a challenge because the voids, walls, filaments and clusters are all part of the continuous density field and are formed entwined with each other.

In this Chapter I present two completely different methods for identifying features in the large scale structure. The first looks at the shape of the smooth density field by calculating the Hessian and the second links massive clusters by taking the point positions of dark matter halos. Both of these methods have been created by myself and are based on existing methods.

The Density Field method transforms the halo distribution to a smooth density field then calculates the Hessian matrix at a particular scale length. Eigenvalues of the Hessian classify each point in space to be blob, filament, sheet or void. This method is based on the work of Aragón-Calvo et al. (2007a); Hahn et al. (2007b); Forero-Romero et al. (2009); Zhang et al. (2009).

The Cylinder Extraction method uses the halo distribution directly without the need for finding the density field. In an iterative process, it searches around high mass halos for candidate cylinders that match specific shape and density requirements. A cylinder
could be a filament if it links together two massive halos. This method is based closely on the method described in Zhang et al. (2009).

### 3.2 Feature Finding Methods

![Figure 3.1: Dark matter halos through a $5h^{-1}\text{Mpc}$ slice of the milli-Millennium simulation. The size of the dots is proportional to the size of the virial radius of the halos. The milli-Millennium simulation was used for illustration purposes and the same slice is shown in all figures throughout this Chapter, details of the simulation are in Section 2.6.](image)

Patterns and features in the large scale structure are often very difficult to pick out. Although dark matter halos have been well studied and defined, other features in the large scale structure are not so clearly determined. Feature finders that pick out the clusters, voids, sheets and filaments often yield very different results, depending on what their definition of the feature is. For example, finding voids in the matter distribution initially seems a simple problem; voids are just the absence of matter. However, remarkably different results arise from using different void finding algorithms and different definitions, as was illustrated in “The Aspen-Amsterdam void finder comparison project” (Colberg et al., 2008). This project compared the ability of 13 different void finders to locate a particular void within the Millennium simulation. The basic results of all the void finders
Figure 3.2: The hierarchical growth of filamentary structure. The red dots are halos in filaments and the black dots are all other halos. The panels show filaments at $z=0,1,2,3$. The filaments are found using the density field method, and only halos with more than 500 particles are shown.
agreed; they all identified a void and located the center in approximately the same place. Although they agreed on the basics, the details of the voids they found varied greatly depending on the methods used. They found very different results for the size, shape and density profile of the void. The disagreement was a result of the differences in definitions of what a void is. The only two points that were generally agreed upon are; (i) voids are very underdense in their centers (approaching around 5 per cent of the mean density) and (ii) voids often have very steep edges. The void finders can be broadly grouped into two categories, those that rely on the dark matter distribution and those which rely on the sparser galaxy or halo distributions. This general categorization of void finders can be extended to all sheet and filament finders as well, they either use the smooth dark matter distribution or rely on point sets.

The simplest definition of a sheet is an object that has collapsed along one of its dimensions, in accordance with the pancake model (Zel’Dovich, 1970b). Although sheets are simple morphologies, there is no well defined identification method. Sheets have been identified in the same manner as filaments, using the shape of the density field (Aragón-Calvo et al., 2007a; Hahn et al., 2007b) and from the local galaxy distribution (Noh and Lee, 2006).

A filament is formed when a sheet collapses along another of its dimensions. There have been more extensive efforts in creating filament finding algorithms than searching for sheets in the large scale structure. As with the void finders, filament finders can be generally put into one of two categories: those that rely on the smooth dark matter distribution and those which rely on point sets. Filaments in the matter distribution are usually found using the shape of the density field, which is obtained from the Hessian matrix (Hahn et al., 2007b; Zhang et al., 2009) or in a similar manner from the potential field (Forero-Romero et al., 2009). Some variations to this basic method have been implemented, for example the Smoothed Hessian Major Axis Filament Finder (Bond et al., 2010) and the Skeleton (Sousbie et al., 2008) The skeleton is formed by lines parallel to the gradient of the field, which connect the saddle points to local maxima of the field.

Filament finders that use the smooth density field work best for N-body simulations where the density field can be reconstructed from the particle positions. For observations of galaxies or the dark matter halo distribution, a different sort of filament finder is needed. An early method of filament identification that uses a point set is the minimal spanning tree (Barrow et al., 1985). The minimal spanning tree is unique for a given point set and it connects all the points. However, because it connects all of the points, when the number of galaxies is large the MST is fuzzy and it describes mainly the local nearest neighbor distribution. A more recent method to identify filaments from a point distribution is the Candy model (Stoica et al., 2005). The distribution of high mass clusters is often the starting point of filament finders. Filaments are found that form bridges between these high mass nodes (Colberg et al., 2005; González and Padilla, 2010).

### 3.3 Features in the Density Field

In N-body simulations, the particles form a discretized version of the density field. The shape of the density field at each point can be used as a guide to classify a point as belonging to a blob, filament, sheet or void. The shape is defined by the principal components of the local curvature, found by convolution of the Hessian matrix with the density
field. This method will be called the *Density Field method* in this thesis. The method requires that the density be defined at all points on a grid, which can be found using the Delaunay Tessellation Field Estimator (DTFE) algorithm for finding the continuous density field from a discrete point set. The method presented in this Section is the one used in Trowland et al. (2013), included in Chapter 4. For completeness, it is described in detail here.

### 3.3.1 The Delaunay Tessellation Field Estimator

The continuous density field is obtained using the DTFE method using the dark matter halo distribution (see van de Weygaert and Schaap (2007); Schaap and van de Weygaert (2000); Schaap (2007)). The DTFE method can be summarized in three steps:

1. From the distribution of points the Delaunay tessellation is constructed, which is a volume covering division of space into mutually distinct Delaunay tetrahedra. A Delaunay tetrahedron is defined by the set of four points whose circumscribing sphere does not contain any of the other points in the generating set.

2. The local density at each point is calculated from the volume of the Voronoi cells (the dual of the Delaunay tessellation) and the mass of the contained halo.

3. The density within each Voronoi cell is interpolated, assuming the density field varies linearly.

The DTFE method is useful when looking for geometrical features in the density field because it automatically adapts to variations in density and geometry. This method is sensitive to the local geometry of the point distribution. This allows them to trace anisotropic features such as encountered in the cosmic web. Other methods such as cloud-in-cell or triangular-shaped-cloud are not adaptive and may not fully capture the anisotropic nature of the cosmic web.

For the analysis in Chapter 4, a section of the full Millennium simulation was used. The DTFE was carried out on the dark matter halo distribution weighted by halo mass (An example is shown in Figure 3.1) with vacuum boundary conditions and a buffer region around the box. Vacuum boundary conditions were necessary because not the whole Millennium box was used so the boundaries were not periodic. This meant that a buffer region around the box was necessary to allow for Voronoi cells that overlap with the boundaries of the box.

This buffer region was made to be at least as big as the maximum distance between nearest neighbor halos so that no Voronoi cells constructed leaked outside the filled region. For larger smoothing scales, the buffer was at least as big as 2\(\sigma\). For the 2 and 3.5 \(h^{-1}\text{Mpc}\) scales the buffer was 7 \(h^{-1}\text{Mpc}\) and for the 5 \(h^{-1}\text{Mpc}\) scale the buffer was 10.5 \(h^{-1}\text{Mpc}\). The buffer region was also used in the smoothing of the density field then discarded. The density smoothed at different scales is shown in Figure(3.3).

### 3.3.2 Structure Identification

Different morphologies in the continuous density field are found from the principal components of the local curvature. The curvature is dependent on the scale at which the
density field is examined due to the hierarchical nature of structure formation. The structures that are uncovered are representative of the scale at which they are found and very different structures dwell at different scales.

To find structure at a particular scale $S$, smoothing the density field is done by convolving with a spherically symmetric Gaussian filter,

$$
\rho_s(x) = \int dy \rho(y) G_s(x,y).
$$

(3.1)

Here $\rho(y)$ is the Fourier transform of the DTFE density and the Gaussian filter at scale $s$ is defined by,

$$
G_s = \frac{1}{(2\pi\sigma_s^2)^{3/2}} \exp \left( -\frac{(y-x)^2}{2\sigma_s^2} \right)
$$

(3.2)

The curvature of the density field is given by the Hessian matrix of second derivatives at each point,

$$
H_{\alpha\beta} = \frac{\partial^2 \rho_s(x)}{\partial x_\alpha \partial x_\beta}
$$

(3.3)

The second derivatives can be found while simultaneously smoothing the field by making
use of an identity of the convolution;
\[
\frac{d}{dx} (f \ast g) = \frac{df}{dx} \ast g = f \ast \frac{dg}{dx}.
\] (3.4)

Applying this to Equation 3.1 gives,
\[
\frac{\partial^2}{\partial x_\alpha \partial x_\beta} \rho_s(x) = \int dy \rho(y) \frac{\partial^2}{\partial x_\alpha \partial x_\beta} G_s(x,y).
\] (3.5)

Thus, the Hessian of the smoothed density field is simply given by the convolution of the DTFE density and the second derivative of the Gaussian (the so-called ‘Mexican Hat wavelet’).
\[
H_{\alpha\beta} = \frac{1}{\sigma_s^4} \int dy \rho(y) [(x_\alpha - y_\alpha)(x_\beta - y_\beta) - \delta_{\alpha\beta} \sigma_s^2] G_s
\] (3.6)

The eigenvalues of the Hessian quantify the curvature of density at a particular point, in the direction of the corresponding eigenvector and are arranged so that \(\lambda_1 \geq \lambda_2 \geq \lambda_3\). A positive eigenvalue indicates that the shape of the density field is concave up and a negative is concave down. The density field may now be classified uniquely into blob, filament, sheet or void regions according to the eigenvalues of this Hessian. The eigenvalue sign criteria for each region is as follows:

- **Blob**: All negative \(\lambda_3 < 0; \lambda_2 < 0; \lambda_1 < 0\)
- **Filament**: Two negative, one positive \(\lambda_3 < 0; \lambda_2 < 0\)
- **Sheet**: Two positive, one negative \(\lambda_3 < 0\)
- **Void**: All positive

It can be useful to classify every point into one of these features as was done in Zhang et al. (2009), and an alternative approach is to pick out only the best features like in Aragón-Calvo et al. (2007a). The decomposition of volume into features is shown in Figure 3.4 on the scale of \(2 h^{-1}\text{Mpc}\). The filament and sheet morphologies dominate the volume, with blob regions taking up the least volume. The relative volume fractions do not change much over scale.

Morphological features are defined using only the eigenvalues of the Hessian. The direction of the eigenvectors are also used to assign a directionality to filaments and sheets. The direction of the axis of a filament is the direction of the eigenvector corresponding to the positive eigenvalue, and the normal direction of a sheet is the direction of the eigenvector corresponding to the negative eigenvalue. The features discussed in this paper have been found choosing the smoothing scales of 2.0, 3.5 and 5.0 \(h^{-1}\text{Mpc}\). These scales have been chosen to match with the visual classification of structure at \(2 h^{-1}\text{Mpc}\) (Hahn et al., 2007b) and to explore the scales above that. The comoving smoothing scales are kept constant for different redshifts in order not to bias the results with preconceived assumptions about filament formation.

This feature finding algorithm uniquely identifies regions into blob, filament, sheet or void depending only on the scale. The simulation volume used in Chapter 4 is broken down into 3.5% blob region, 40.9% filament, 46.1% sheet and 9.5% void by volume.
Figure 3.4: The volume of the simulation is uniquely classified into features of the large scale structure using the dark matter density field. Here the classification of the volume is shown through the shading: blob regions are blue, filaments are red, sheets are orange and voids are cyan. The features have been found on the scale of $2\ h^{-1}\text{Mpc}$.

### 3.4 The Cylinder Extraction Method

Although it is straightforward to extract the density field from N-body simulations, in observational or incomplete data it can be difficult to obtain. When the density field is not well reconstructed, the Density Field method described in the previous section can not be used to find filaments. Observational data gives only the positions of galaxies and to a lesser extent, halos and groups. In order to find filaments using this data, a method utilizing the point distribution must be used. The method described in this Section follows closely the method described in Zhang et al. (2009), which is based on the Candy model, proposed by Stoica et al. (2005) and the filament extraction technique in Colberg et al. (2005). The Candy model reconstructs filaments by connecting individual segments that are found in a basic point distribution. It was originally proposed for the detection of road networks in remote sensing (Stoica, 2001; Stoica et al., 2002). The method uses a marked point process, where segments serve as marks.

The advantages of using this method are that each halo belongs to a particular filament, whereas in the Density Field method each cell is given a structure classification and a halo is said to belong to a filament if it is in a filament cell. The filaments in the
Cylinder Extraction method each have a defined length and diameter.

The requirements that the point set have to fulfill can be adjusted to fit the context where they are used. For the purposes of this thesis, segments are built using the dark matter halo distribution from N-body simulations. The candidate segments are chosen to be cylinders containing an overdensity, ranging in length from \([L_{\text{min}}, L_{\text{max}}]\) and a radius in the range \([R_{\text{min}}, R_{\text{max}}]\). The mean density within the segment should be at least \(N\rho\) times that of the average mass density of all halos. Finally, a segment should have at least \(N_{\text{min}}\) member halos. Choosing values for these parameters is somewhat arbitrary. The same values are used here as in Zhang et al. (2009), where it was found that the results are robust to substantial changes in these parameters. The values of the free parameters are set as follows:

\[
\begin{align*}
L_{\text{min}} &= 3 \, h^{-1}\text{Mpc} \\
L_{\text{max}} &= 10 \, h^{-1}\text{Mpc} \\
R_{\text{min}} &= 1 \, h^{-1}\text{Mpc} \\
R_{\text{max}} &= 3 \, h^{-1}\text{Mpc} \\
N\rho &= 5 \\
N_{\text{min}} &= 5
\end{align*}
\] (3.7)

To obtain an accurate measurement of spin, only halos with more than 500 particles were used in the analysis of the next Chapters. For consistency, filaments were found using this same set of halos with greater than 500 particles. In the following I describe the successive steps of the Cylinder Extraction method used to find filaments in these halos.

1. Candidate filament segments around the most massive halo in the box are formed by searching for other halos with distance between \(L_{\text{min}}\) and \(L_{\text{max}}\). The most massive halo in the primary node of all filaments formed around it. Candidate filament segments are formed by these halo pairs.

2. For each cylindrical segment, the average mass density of the filament is found, \(\rho\), which is defined as

\[
\rho = \frac{\sum_{k=1}^{N} M_k}{\pi R^2 L}.
\] (3.8)

Where \(L\) is the length of the segment (the distance between the two halos) and \(N\) is the number of halos within the cylinder that connects the two halos that define the segment. \(M_k\) is the mass of halo \(k\) that resides inside the cylinder. The radius of the cylinder \(R\) is determined such that the average mass density \(\rho\) is maximum, within the minimum \(R_{\text{min}}\) and the maximum \(R_{\text{max}}\).

3. We first consider the segment with the highest mass density that has at least \(N_{\text{min}}\) member halos and mass density greater than \(N\rho\) times the average mass density of the simulation box.
4. For that segment, the secondary node is found by searching for the most massive halo (apart from the primary node) that has at least $N_{\text{min}}$ halos between it and the primary node, within the segment cylinder. Those halos that lie between the primary and secondary halos are considered to be inside a filament and are taken out of consideration for belonging to other filaments or being node halos. The axis of the filament is the line connecting the primary and the secondary nodes.

5. Once the first filament has been found, the other candidate segments around the primary are tested for filaments, repeating steps 3 and 4.

6. When all the filaments have been found around the most massive halo in the box, the next most massive halo is taken to be the primary node and so on, until all the filaments in the box have been found.

### 3.5 Comparison

The two methods described above for the detection of filaments in N-body simulations are based on two completely different ideas. The Density Field method is a more elegant method that classifies structure using only the shape of the density field. It depends only of the choice of scale. The Cylinder Extraction method is more specific to the problem of filament finding. It uses the dark matter halo distribution directly and constructs cylindrical segments that terminate in high mass node halos. Since the foundations of these methods are so different, their results are not expected to be identical.

Because each filament finding method uses a very different concept, the results of each method will be different. The Density Field method identifies each point in space as being a filament or not. A halo can be said to be in a filament if it lies in a filament cell and the axis of that filament is in the direction of the eigenvector corresponding to the positive eigenvalue of the Hessian. This method is simple but it does not find whole filaments. The Cylinder Extraction method actually seeks out full filaments and the axis of the filament is the line connecting the two nodes of the filament. It is possible to compare the filament direction of individual halos that are found to be in filaments with the two methods.

A simple comparison of the methods is the fraction of halos that each determines to be in filaments and in nodes. In the Density Field method, 58% of halos are in filaments and 20% are in nodes (with 21% in sheets and 1% in voids). In the Cylinder Extraction method, 35% of halos are in filaments and 11% are in nodes.

To illustrate the differences and similarities between the filaments found using the Density Field and the Cylinder Extraction methods, the filament and node halos and the axis of filaments through a $5h^{-1}\text{Mpc}$ slice are plotted in Figure 3.5. In this figure the red dots are the halos in filaments and their size is indicative of their mass. The blue dots are the halos in blob regions (in the density field method, top left panel) and the nodes of filaments (Cylinder Extraction method, top right panel). It is apparent that the halos identified as being in a filament or node in one method are not necessarily the same halos identified in the other method. The Density Field method tends to identify more halos as belonging to filaments than the Cylinder Extraction method. This may be because of the restriction on the minimum number of halos that may belong to a filament in the Cylinder Extraction method, which causes filaments to be selected mainly in high density regions. The Density Field method allows filaments to be identified in low density regions.
Figure 3.5: Dark matter halos and filament axis in a $5\, h^{-1} \text{Mpc}$ slice through the milli-Millennium simulation. The top left panel shows halos in the slice, colored by their feature classification determined by the density field method and the top right panel shows filament halos found using the Cylinder Extraction method. In both of these panels the blue dots are halos in blobs or nodes and the red dots are the halos in filaments. The orange dots in the top left panel are halos in sheets. The bottom left panel shows the axis of filaments determined by the density field method and the bottom right panel shows the filament axis found using the Cylinder Extraction method. The axes are placed at the position of each halo that is inside a filament.
because it is only concerned with the shape of the density field, not the value. There is
some agreement between halos selected in the two methods though, most notably that
the two highest mass halos (at the top of the figure) are identified as node halos in both
methods.

The projected filament axis in the lower two panels of Figure 3.5 further illustrates
how different results are for the two methods. In the Density Field method (the lower
left panel), a halo is in a filament if it sits in a filament cell in the volume. This leads to
many neighboring halos having completely different filament axis directions. Conversely,
neighboring halos in the Cylinder Extraction method may sit inside the same filament,
so they share the same filament axis.

This comparison shows that the filament classification of dark matter halos is often
different depending on which method of filament identification has been used. However,
it is not clear which method is better at extracting filaments from the cosmic web. Figure
3.6 represents an important result from Chapter 4: that the spin low and high mass halos
are aligned differently with the axes of filaments and this has evolved throughout cosmic
history. This result was obtained using the Density Field method (left panel of Figure
3.6) but could have been obtained using the Cylinder Extraction method (right panel of
Figure 3.6). Both methods find that for $z = 0$, low mass halos spin preferentially parallel
to filaments with high mass halos spin orthogonal. The main difference between the results
obtained using the two methods is that in the Density Field method, as we go to high
redshift snapshots, the alignment for low mass halos gets systematically more orthogonal.
This effect is not so pronounced for filaments found using the Cylinder Extraction method.
This is perhaps because of the number of fixed parameters that have to be set in this
method. Another interesting difference is that the Cylinder Extraction method identifies
more high mass halos to be in filaments, even at high redshifts. Overall, the two methods
give similar results, especially for low redshift filaments.
Figure 3.6: The alignment of the spin vectors of dark matter halos with the axis of filaments. The alignment is characterized by the median of the directional cosine and the shaded error regions are the $1\sigma$ error of the median. \textit{Left} – Filaments are found using the density field method, smoothed at the $2\, h^{-1}\text{Mpc}$ scale. \textit{Right} – Filaments are found using the Cylinder Extraction method.
Chapter 4

The Alignment of Dark Matter Halo Spin with Large Scale Features

If my calculations are correct, when this baby hits eighty-eight miles per hour, you're going to see some serious shit.
- Doc Brown, Back to the Future (1985)

4.1 Introduction

The formation and evolution of galaxies within dark matter halos involves many interconnected processes. Gravitational instability, gas cooling, star formation, feedback and halo mergers all play a part in shaping the properties of galaxies. The dark matter halo that the galaxy sits inside is also fundamental to molding the characteristics of galaxies. For example, the efficiency of star formation varies drastically for galaxies inside halos of different mass (Navarro and Steinmetz, 2000). Also, galaxy properties vary with the environment of the halo; galaxies in dense environments are more massive, more gas-poor and more bulge dominated and have fewer young stars than those in low density regions (Kauffmann et al., 2004). The existence of links between the properties of halos and galaxies with environment suggest that galaxy formation is entwined with the formation of the cosmic web.

The spin of dark matter halos is particularly important in determining the final properties of their resident galaxies. Galactic spin plays a large part in determining morphology since it is responsible for supporting galactic disks. The spin of baryons falling into the center of dark matter halos is well conserved so that the spin of galaxies is well aligned with the spin of their host halo (Sharma and Steinmetz, 2005). If there are alignments in the spins of dark matter halos in N-body simulations then it is expected that there should be corresponding alignments in large samples of galactic spin. Furthermore, since galaxy properties are linked to properties of dark matter halos and their assembly histories, the way in which the spin of dark matter halos is build up may reveal secrets of galaxy formation.

The era in which spin alignments of observed galaxies, not just dark matter halos in N-body simulations, is nearly upon us. There have been tentative detections of galaxy spin alignment with filaments (Jones et al., 2010; Tempel et al., 2013), however these studies were done without direct measurements of the spin. Photometric measurements of galaxy shape were made in order to infer the orientation of spin and although the spin
and shape are very well correlated (Bett et al., 2007), this is not a direct measurement of
spin and the spin orientation for elliptical galaxies is difficult to reconstruct.

Direct measurements of galaxy rotation can be done with integrated field units (IFU)
although only one galaxy is targeted at a time and it is not feasible to conduct a survey of
large scale structure with direct spin measurements. The SAURON and ATLAS projects
were only able to measure the spins of on the order of 48-260 galaxies. However, a new
multi-object IFU instrument has been developed which can target several galaxies at once
using a special type of optical fibre. SAMI (Bland-Hawthorn et al., 2011; Fogarty et al.,
2012; Croom et al., 2012) is the first of this type to be able to produce kinematic maps
of huge numbers of galaxies. A survey is planned using this instrument to target 3000
galaxies and the next generation instrument HECTOR (Lawrence et al., 2012) is planned to
target $10^{4-5}$ galaxies. The kinematic measurements from this vast future survey will not
only be able to give precise directions of halo spin to test for spin alignments of galaxies
but will be able to quantify the angular momentum content of stellar dominated systems
like elliptical and lenticular galaxies.

Figure 4.1: SAMI velocity fields of early type galaxies. Most of these galaxies are fast
rotators with clear signatures of rotation. An example of a slow rotator is the fifth galaxy
in on the top row; there is not a signature of rotation in this galaxy. Picture credit:
Fogarty et al. (2013, in prep).

Understanding the formation mechanisms of elliptical and lenticular galaxies (generally
grouped as Early Type Galaxies, ETGs) is a crucial subject in astrophysics. The stellar
kinematics of several ETGs have been mapped by the SAURON Emsellem et al. (2007)
and ATLAS Emsellem et al. (2011) surveys, and more recently with the SAMI instrument
(Croom et al., 2012). Some examples of kinematic maps of ETGs made with SAMI
(Fogarty et al., 2013, in prep) are shown in Figure 4.1 The magnitude of the spin is
quantified by the $\lambda_R$ parameter, which can be derived from the first two stellar velocity
moments and can be used as a robust estimator of the apparent specific angular moment
(in stars) of galaxies. The galaxies can be divided into two groups, the fast rotators \((\lambda_R > 0.1)\) which are disk dominated systems and the slow rotators \((\lambda_R < 0.1)\) which are supported by internal dispersion. Slow and fast rotators tend to be classified as ellipticals and lenticulars, respectively, but although this is often the case, using the morphological classification scheme does not always identify the slow or fast rotators. The slow rotators exhibit complex stellar velocity fields and often include stellar kinematically distinct cores, and fast rotators which have regular velocity fields. Slow rotators tend to be brighter and more massive but only make up about 20\% of all ETGs, with fast rotators making up the majority, 80\% (Emsellem et al., 2011). The differences in ETG rotation structure could be due to differences in their formation history so studies on their relative fraction in different environments could bring to light clues on galaxy formation. Direct measurements of galaxy spin are required to investigate the properties of slow/fast rotating galaxies and to investigate this on a large scale, much bigger and better surveys of galaxy spin will need to be undertaken.

4.2 Markov Chain Monte Carlo

A statistical method used in this Chapter is the Markov Chain Monte Carlo (MCMC). This is a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution. The state of the chain after a large number of steps is then used as a sample of the desired distribution. The Metropolis-Hastings algorithm has been implemented in this work to fit a function of only one variable to a probability distribution. Although the MCMC was designed for multi-dimensional models, this is still an instructive demonstration and it gives a robust result.

The idea of MCMC is to calculate the expectation value of some function by integrating the function weighted posterior distribution \(p(X|D, I)\) (which is the probability of \(X\) given data \(D\) and prior information \(I\)). The integration is performed by Monte Carlo where the procedure is to pick \(n\) points uniformly randomly distributed in a volume of parameter space \(X\). The integral is then estimated by evaluating the function weighted posterior distribution at these points (see Gregory (2005) for the full theorem). These points need not be independent, which is where the Markov Chain comes in.

The Markov Chain in the Metropolis-Hastings method constructs a kind of random walk through the parameter space such that the probability for being in a region of this space is proportional to the posterior density for that region. The new sample \(X_{t+1}\) depends on the previous sample \(X_t\) according to the transition probability which has the remarkable property of generating samples of the parameter space that have the same probability density as the desired posterior.

The basic Metropolis-Hastings algorithm is simple:

1. Initialize \(X_0\) at \(t=0\)

2. Obtain a new sample \(Y\) from the proposal distribution. The simplest proposal distribution is a Gaussian with mean equal to the current sample \(X_t\). This will make the probability density decrease with distance away from the current sample.

3. Calculate the Metropolis ratio, \(r = \frac{p(Y|D, I)}{p(X_t|D, I)}\). If \(r \geq 1\) then set \(X_{t+1} = Y\), otherwise
set \( X_{t+1} = X_t \). This is done by sampling a random variable \( U \) from a uniform distribution between 0 and 1. If \( U \leq r \) then set \( X_{t+1} = Y \), otherwise set \( X_{t+1} = X_t \).

4. Repeat steps 2 and 3

This method has been implemented in this Chapter in order to estimate the most likely parameters of the function in Equation 4.1 when it is fitted to different data. Here the parameter space is \( X = [c] \) and a Gaussian proposal distribution was used.

\[
P(\cos \theta) = (1 - c) \sqrt{1 + \frac{c}{2}} \left[ 1 - c \left( 1 - \frac{3}{2} \cos^2 \theta \right) \right]^{-3/2}.
\] (4.1)

An example of samples from the MCMC and the distribution of those samples are shown in Figure 4.2. They show that after about 200 iterations of burn in, the most likely value for \( c \) is \(-0.304^{+0.005}_{-0.006}\).

Figure 4.2: The results from my simple one dimensional Markov Chain Monte Carlo simulation for the parameter \( c \) in Equation 4.1. The data used is the cosine of the angle between halo velocity and filament axis for low mass halos \( \log M/M_\odot = 11.6 - 12.2 \) at \( z=0 \). Left – A sequence of 1000 samples from the MCMC. Right – The comparison of the MCMC samples, excluding the first 200 which are treated as the burn in period. The red shaded area is the 1\( \sigma \) region.
4.3 The Cosmic History of the Spin of Dark Matter Halos Within the Large Scale Structure


It is a study on the history of dark matter halo spin, particularly the alignments within filaments in the large scale structure. All analysis was preformed by myself in consultation with my supervisors, Geraint Lewis and Joss Bland-Hawthorn; the text was my own. The full text of the paper in its published format is provided in the Appendix.

4.3.1 Introduction

The large-scale structure of the universe observed today has formed by a long history of gravitational collapse, gradual accretion, and mergers. Through these processes a filamentary, sponge-like structure has emerged. The distribution of galaxies and their motions provides clues on how they formed, and together with galactic angular momentum data, the emergence of the intricate large-scale structure can begin to be explained.

Before we can determine what spin tells us about the formation of large-scale structure, the mechanisms of angular momentum buildup need to be well understood. The initial spin of early dark matter proto-halos can be predicted analytically (White, 1984); however, these predictions are largely limited to the regime of linear structure formation. To track the angular momentum buildup through more recent cosmic history, N-body simulations of cold dark matter must be used. These simulations give full information on the dark matter halos which can be used to form a hypothesis on the buildup of galaxy angular momentum on cosmological scales. However, on cosmological scales it is not yet feasible to simulate the gas component to track the angular momentum buildup of galaxies directly (although Hahn et al. (2010) simulated 100 disk galaxies in a filament to find an alignment of galaxy spin with filaments).

Hydrodynamical simulations on individual galaxy scales (van den Bosch et al., 2003; Sharma and Steinmetz, 2005; Bett et al., 2010) have shown that the specific angular momentum of baryons remains close to that of dark matter and that the galaxy angular momentum is generally about 20° misaligned with the dark matter halo. This means that dark matter halo spin is a fairly good proxy for galaxy spin, so some understanding of the spins of galaxies may be gleamed from dark matter-only simulations. The spin of a dark matter halo depends mainly on two things: the initial torques driven by the surrounding landscape at early times, and the accretion and merger history of the halo.

The initial spin of dark matter halos is given through a mechanism known as “tidal torque theory”, pioneered by Hoyle (1949), Peebles (1969) and Zel’Dovich (1970b). This theory proposes that the initial spin of a proto-halo early in its formation in the linear regime of structure formation depends on its shape and the tidal forces exerted from the surrounding structure, so the spin is dependent on the local dark matter landscape. The greatest effects of tidal torquing happen at the time of turn-around, just before the proto-halos have collapsed to virialized objects. A halo that was torqued in this manner should retain some memory of the tidal field where it formed, and this has been confirmed through N-body simulations and galaxy catalogs (eg. Lee and Pen, 2001; Porciani et al., 2002a; Lee and Erdogdu, 2007). The cosmic web is the manifestation of the tidal field,
filaments in particular are regular, symmetric morphologies which on large scales exhibit a uniform tidal field. Thus, it is expected that the orientation of halo spin today should retain some correlation with the direction of filaments and halos should be aligned with each other over short distances.

Since the epoch of tidal torquing, halo spins have been substantially influenced by mergers and accretion. It was shown in Bett and Frenk (2012) that it is not uncommon for the direction of the spin of a halo to completely flip over in its lifetime and this phenomenon is caused by minor and major mergers and even close halo flybys. Satellite accretion has been proposed to be the main contributor of angular momentum and it has been shown that by neglecting tidal torques and considering mergers alone the distribution of the magnitude of spin can be reproduced (see Gardner, 2001; Vitvitska et al., 2002; Maller et al., 2002).

To figure out how accretion has influenced dark matter halo spin and what spin can reveal about the formation of large-scale structure, several authors have investigated an alignment of spin with the cosmic web using N-body simulations and galaxy catalogs. In simulations, it has been found that spins are aligned on shells around voids, lying preferentially on the void surface (Brunino et al., 2007; Cuesta et al., 2008). It has been shown that spins lie preferentially in the plane of sheets in simulations (Navarro et al., 2004a) and along the axis of filaments (Faltenbacher et al., 2002; Aragón-Calvo et al., 2007c; Hahn et al., 2007b; Zhang et al., 2009). In observations, there has been a tentative detection of some weak correlation with filaments (Jones et al., 2010) but no significant detection has been found to date. The evolution of halo spin with respect to filaments and sheets was explored by Hahn et al. (2007a) who found no change in the orientation of spin over cosmic time.

Since the spins of halos are aligned with the large-scale structure, there should be some degree of coherence between the direction of spin of two neighboring halos. It is not clear if this alignment is strong enough to be detected even in N-body simulations. Heavens et al. (2000a), Porciani et al. (2002a), Faltenbacher et al. (2002) and Bailin and Steinmetz (2005) see no strong alignment, whereas Hatton and Ninin (2001) do see a weak alignment for halos with separation $1 h^{-1}$Mpc. The number of halos in these studies is too low to see a strong correlation. In contrast, several claims have been made of spiral galaxy spin alignments in observations (Pen et al., 2000; Slosar et al., 2009; Lee, 2011). If these alignments can be seen in observations but not in dark matter simulations, then it is a possible indication that the spins of the luminous galaxies are not aligned with their dark matter halos.

In addition to the orientation, the magnitude of the spin may reveal secrets of the large-scale structure. The spin parameter is a dimensionless measure of the amount of rotation of a dark matter halo and it has been found (Lemson and Kauffmann, 1999; Cervantes-Sodi et al., 2008) not to depend on cosmology or environment. Both Knebe and Power (2008) and Muñoz-Cuartas et al. (2011) find a mass dependence of the spin parameter at high redshift but not at low redshift.

Observations of galaxy spin alignments in the large-scale structure to date have only been through inferred galaxy spin orientations from observed disk galaxy shape. For example, Lee and Erdogdu (2007) used the Tully catalog of nearby spirals (Nilson, 1974; Lauberts, 1982) to infer spin from the axial ratio (to find an alignment with the tidal field) and Slosar et al. (2009) used the apparent sense of spiral rotation in the Galaxy Zoo catalog. Direct measurements of galaxy rotation have been done with integrated
field units (IFU) although only one galaxy is targeted at a time and it is not feasible
to conduct a survey of large-scale structure with direct spin measurements. However, a
new multi-object IFU instrument has been developed which will enable a survey of $10^{4-5}$
galaxies in a volume limited sample (Bland-Hawthorn et al., 2011; Croom et al., 2012).
There will soon be a huge influx of galaxy spin data, which has never been sampled before
in such high volumes. In order to get the most out of these data and to direct future
surveys, the dark matter halo spin must be better understood.

Our paper is organized as follows. First, the method is described in Section 4.3.2. Here,
we describe the set of simulations used, we discuss the characteristic mass scale for halo
collapse, and describe the method used for finding features in the large-scale structure.
Theoretical predictions from Tidal Torque theory are discussed in Section 4.3.3 and the
results of alignment of halo spin with filaments and the alignment of neighboring halos
spins are presented in Section 4.3.4. The halo-halo spin alignment is explored in Section
4.3.5 and results of the evolution of the spin parameter in are presented in Section 4.3.6.
Lastly, we summarize and discuss our results in Section 4.3.7.

4.3.2 Method

N-Body Simulation

Since any relic alignments of spin with the large scale structure are expected to be weak,
a large simulation volume and high resolution are needed. To this end, the publicly
available Millennium simulation of Springel et al. (2005) was used. This simulation is of a
cubic volume $500 h^{-1}$Mpc on a side containing $2160^3$ particles using the GADGET-2 code
(Springel, 2005). This gives a particle mass of $8.6 \times 10^8 h^{-1} M\odot$. A $\Lambda$CDM cosmology is
chosen and the parameters are $\Omega_m = 0.25$, $\Omega_b = 0.045$, $\Omega_\Lambda = 0.75$, $h = 0.73$, $n = 1$ and
$\sigma_8 = 0.9$.

The halo catalog was built by Springel et al. (2005) by first using the simple friends-
of-friends group (FOF) finder (Davis et al., 1985) to attempt to select structure in the
particle distribution and then finding the virialized subhalos within the FOF groups using
SUBFIND (Springel et al., 2001a). The SUBFIND algorithm first identifies subhalo
candidates within each FOF halo using dark matter density and then removed particles
that are not gravitationally bound to the subhalo candidate. The most massive sub-
halo typically contains most of the mass of the corresponding FOF object, and so can
be regarded as the self-bound background halo itself, with the remaining subhalos as its
substructure. The halo catalog used in this paper includes all virialized halos, including
subhalos. There are 184,891 FOF halos and 213,799 halos in total. Bett et al. (2007)
found that angular momentum and shape parameters of a halo were subject to numerical
biases if it contained fewer than approximately 300 particles. To be safe from random
effects from outer halo particles, spin measurements are only made on halos with more
than 500 particles in this paper.

The density field must be calculated from the halo distribution in order to find fila-
ments. The maximum size of the density field was chosen to be $1024^3$ voxels to be within
computational limits. To get convergence in the results at this resolution, a smaller box
than the full Millennium simulation must be used.

The resolution of the density field was tested using several $100 h^{-1}$Mpc sample cubes.
As the resolution of the density field was raised from $128^3$ to $1024^3$ voxels, the alignment
Figure 4.3: Left: The distribution of dark matter halos in a volume of the simulation where the large scale structure has been dissected into its component features. Haloes in blob regions are colored black, filament halos are dark gray, sheet halos are light gray and halos in voids are outlined in black. The size of the dots are proportional to the virial radius of the halo and the volume shown is $100 \times 100 \times 5 \, h^{-1} \text{Mpc}$. Right: The volume of the simulation is uniquely classified into features of the large scale structure using the dark matter density field. Here the classification of the volume is shown through the shading: blob regions are black, filaments are dark gray, sheets are light gray and voids are white. The features have been found on the scale of $2 \, h^{-1} \text{Mpc}$.

between halo spin and the resulting filaments became stable above a certain threshold. For smoothing lengths 2.0, 3.5 and $5.0 \, h^{-1} \text{Mpc}$ (Gaussian smoothing is used for finding filaments on different scales), the minimum resolution for stable features is $0.4 \, h^{-1} \text{Mpc}/\text{cell}$. For a grid of $1024^3$ voxels, the maximum box size is $400 \, h^{-1} \text{Mpc}$. To ensure the resolution was more than sufficient, a box of size $300 \, h^{-1} \text{Mpc}$ was chosen.

For smoothing on $1.0 \, h^{-1} \text{Mpc}$ scales, a finer grid must be used and the maximum cell size is $0.2 \, h^{-1} \text{Mpc}$ so a $200 \, h^{-1} \text{Mpc}$ box was used for this scale. At smaller scales than $1 \, h^{-1} \text{Mpc}$ the box size required is too small so there are not enough halos for useful results. The following results display no cosmic variance when a different sample of the same size is chosen. There are 4,027,242 halos in our $300 \, h^{-1} \text{Mpc}$ box and 932,961 halos with more than 500 particles from which a reliable spin measurement could be made. The halos in a $5 \, h^{-1} \text{Mpc}$ slice through the simulation volume are shown in Figure 4.3.

Snapshots are taken at several points throughout the simulation. Here we have used the snapshots at redshift 0, 0.99, 2.07 and 3.06 (rounded to 0, 1, 2, 3).

**Characteristic Mass**

In structure formation, there is a characteristic mass scale for collapse, $M_*(z)$. A spherical top-hat perturbation collapses when its linear overdensity exceeds a value of $\delta_c = 1.686$. The variance of linear density fluctuations at a given mass scale $M$ is related to the linear
power spectrum $P(k, z)$ at redshift $z$ by

$$\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 \, P(k, z) \, \tilde{W}^2_{\text{TH}}(k, M),$$

(4.2)

where $\tilde{W}_{\text{TH}}(k, M)$ is the Fourier transform of a spherical top-hat window function of comoving size $R = (3M / 4\pi\bar{\rho})^{1/3}$, and $\bar{\rho}$ is the comoving mean mass density of the universe. At a given redshift, the typical mass scale $M_\ast(z)$ to collapse from a $1\sigma$ fluctuation is hence given by the implicit solution of

$$\sigma(M_\ast, z) = \delta_c.$$  

(4.3)

The calculated values of characteristic mass at redshift 0, 1, 2 and 3 are 5.89, 0.273, 0.0132, $4 \times 10^{-5}$, respectively in units of $10^{12}M_\odot$.

**Quantifying the large scale structure**

Morphological features in large scale structure may be classified into four general categories: blobs, filaments, sheets and voids. This analysis uses the curvature of the density field to identify each of these features in N-body simulations.

Firstly, the density field is obtained using the Delaunay Tessellation Field Estimator (DTFE) method using the dark matter halo distribution (see van de Weygaert and Schaap (2007); Schaap and van de Weygaert (2000); Schaap (2007)). The DTFE method can be summarized in three steps: (1) from the distribution of points the Delaunay tessellation is constructed, which is a volume covering division of space into mutually distinct Delaunay tetrahedra. A Delaunay tetrahedron is defined by the set of four points whose circumscribing sphere does not contain any of the other points in the generating set. (2) The local density at each point is calculated from the volume of the Voronoi cells (the dual of the Delaunay tessellation) and the mass of the contained halo. (3) The density within each Voronoi cell is interpolated, assuming the density field varies linearly. The DTFE method is useful when looking for geometrical features in the density field because it automatically adapts to variations in density and geometry.

The DTFE was carried out with vacuum boundary conditions and a buffer region around the box. This buffer region was made to be at least as big as the maximum distance between nearest neighbor halos so that no Voronoi cells constructed leaked outside the filled region. For larger smoothing scales, the buffer was at least as big as $2\sigma$. For the 2 and 3.5 $h^{-1}\text{Mpc}$ scales the buffer was $7 h^{-1}\text{Mpc}$ and for the 5 $h^{-1}\text{Mpc}$ scale the buffer was $10.5 h^{-1}\text{Mpc}$. The buffer region was also used in the smoothing of the density field then discarded.

Smoothing the density field to some scale $s$ is done by convolving with a spherically symmetric Gaussian filter,

$$\rho_s(x) = \int d\mathbf{y} \rho(\mathbf{y}) G_s(x, \mathbf{y}).$$

(4.4)

Here $\rho(\mathbf{y})$ is the Fourier transform of the DTFE density and the Gaussian filter at scale $s$ is defined by,

$$G_s = \frac{1}{(2\pi\sigma_s^2)^{3/2}} \exp\left(-\frac{(y-x)^2}{2\sigma_s^2}\right).$$

(4.5)
The curvature of the density field is given by the Hessian matrix of second derivatives at each point,

\[ H_{\alpha\beta} = \frac{\partial^2 \rho_s(x)}{\partial x_\alpha \partial x_\beta} \]  (4.6)

The second derivatives can be found while simultaneously smoothing the field by making use of an identity of the convolution; \( \frac{df}{dx}(f * g) = \frac{df}{dx} * g = f * \frac{dg}{dx} \). Applying this to Equation 4.4 gives

\[ \frac{\partial^2 \rho_s(x)}{\partial x_\alpha \partial x_\beta} = \int dy \rho(y) \frac{\partial^2}{\partial x_\alpha \partial x_\beta} G_s(x,y). \]  (4.7)

Thus, the Hessian of the smoothed density field is simply given by the convolution of the DTFE density and the second derivative of the Gaussian (the so-called ‘Mexican Hat wavelet’). \[
H_{\alpha\beta} = \frac{1}{\sigma_s^4} \int dy \rho(y) [(x_\alpha - y_\alpha)(x_\beta - y_\beta) - \delta_{\alpha\beta} \sigma_s^2] G_s \]  (4.8)

The eigenvalues of the Hessian quantify the curvature of density at a particular point, in the direction of the corresponding eigenvector. A positive eigenvalue indicates that the shape of the density field is concave up and a negative is concave down. The density field may now be classified uniquely into blob, filament, sheet or void regions according to the eigenvalues of this Hessian. The eigenvalue sign criteria for each region is as follows,

<table>
<thead>
<tr>
<th>Region</th>
<th>Eigenvalue Sign Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blob</td>
<td>All negative</td>
</tr>
<tr>
<td>Filament</td>
<td>Two negative, one positive</td>
</tr>
<tr>
<td>Sheet</td>
<td>Two positive, one negative</td>
</tr>
<tr>
<td>Void</td>
<td>All positive</td>
</tr>
</tbody>
</table>

It can be useful to classify every point into one of these features as was done in Zhang et al. (2009), and an alternative approach is to pick out only the best features like in Aragón-Calvo et al. (2007a). The decomposition of volume into features is shown in Figure 4.3 on the scale of \( 2 h^{-1}\text{Mpc} \). The filament and sheet morphologies dominate the volume, with blob regions taking up the least volume. The relative volume fractions do not change much over scale.

Morphological features are defined using only the eigenvalues of the Hessian. The direction of the eigenvectors are also used to assign a directionality to filaments and sheets. The direction of the axis of a filament is the direction of the positive eigenvalue, and the normal direction of a sheet is the direction of the negative eigenvalue. The features discussed in this paper have been found choosing the smoothing scales of 2.0, 3.5 and 5.0 \( h^{-1}\text{Mpc} \). These scales have been chosen to match with the visual classification of structure at \( 2 h^{-1}\text{Mpc} \) (Hahn et al., 2007b) and to explore the scales above that. The comoving smoothing scales are kept constant for different redshifts in order not to bias the results with preconceived assumptions about filament formation.

This feature finding algorithm uniquely identifies regions into blob, filament, sheet or void depending only on the scale and quality of features required.

### 4.3.3 Alignment of halo spin with the cosmic web

Halo particles can be loosely bound, following stochastic paths, but adding up each particles angular momentum gives the net effect of a halo spin. Spin is calculated by adding
Figure 4.4: The direction of dark matter halo spin vectors (top left), velocity vectors (top right) and filament axis (bottom). The velocities show a coherent flow along filament axis whereas spin vectors are much more random and not obviously aligned. Shown is a slice of the simulation $100 \times 100 \times 5 \, h^{-1}\text{Mpc}$ and all vectors have been normalized to have the same length.

up the angular momentum of each particle ($i$) in the halo, simply defined as the cross product of the distance of the particle from the halo’s center of mass ($\mathbf{r}$) and the particles velocity ($\mathbf{v}$) with respect to the center of mass;

$$ \mathbf{J} = \sum_{i=0}^{N} \mathbf{r}_i \times m_i \mathbf{v}_i $$

(4.9)

In order to get a reliable measurement of halo spin, only the halos with more than 500 particles have been included. The unit spin vectors are shown in the top panel of Figure 4.4 but there is no obvious alignments with each other or with the large scale structure (as defined by the axis of filaments, shown on the bottom panel).
From tidal torque theory (TTT), the spin of dark matter halos is expected to be correlated with the local tidal field \( T_{ij} \equiv \partial_i \partial_j \phi \) and the inertia tensor \( I_{ij} \). During the linear regime (assuming that \( T \) and \( I \) are uncorrelated), the first order result from TTT (White, 1984) is,

\[
J_i \propto \epsilon_{ijk} T_{jl} I_{lk},
\]

where \( \epsilon_{ijk} \) is the Levi-Civita symbol. In the principle axis frame of the tidal tensor, where \( \lambda_i \) are the eigenvalues of the tidal field,

\[
\begin{align*}
J_1 &\propto (\lambda_2 - \lambda_3) I_{23} \\
J_2 &\propto (\lambda_3 - \lambda_1) I_{31} \\
J_3 &\propto (\lambda_1 - \lambda_2) I_{12}
\end{align*}
\]

\( \lambda_3 \leq \lambda_2 \leq \lambda_1 \) so \( \lambda_3 - \lambda_1 \) is the largest coefficient, making \( J_2 \) the largest component of \( \mathbf{J} \) so that spin is preferentially aligned with the second eigenvector of the tidal field. The cosmic web is a manifestation of the potential \( \phi \), related by the Poisson equation, \( \nabla^2 \phi = 4\pi G \rho (x) \). Our definition of a filament (having two negative eigenvectors of the Hessian of density) translates into a region where there are two positive eigenvectors of the tidal tensor. The second eigenvector of the tidal field points in a direction orthogonal to the filament (the minor axis of the tidal field is the axis of the filament) and so we expect that halo spin should point in a direction orthogonal to the axis of the filament.

The result from TTT in Equation 4.10 assumes that \( T \) and \( I \) are completely uncorrelated, which has been shown to be not always true (Lee and Pen, 2000; Porciani et al., 2002a). If there is some correlation, the preferred direction of halo spins discussed above may be a small effect. The alignment would also be greatly affected by merger and accretion events that have happened during nonlinear structure growth.

An expression for the relation between the unit spin vector \( \hat{\mathbf{J}} \) and the unit traceless tidal field \( \hat{T} \) was proposed in Lee and Pen (2000, 2001):

\[
\langle \hat{J}_i \hat{J}_j | T \rangle \equiv \frac{1}{3} \frac{c}{c - \frac{3}{5}} \delta_{ij} - c \hat{T}_{ik} \hat{T}_{kj},
\]

where \( c \in [0, 3/5] \) is the correlation parameter to measure the strength of the intrinsic spin-shear alignment with the nonlinear modifications taken into account. When \( c=0 \) it corresponds to the case when nonlinear effects have completely broken down initial spin-shear correlations and when \( c=3/5 \) it is the ideal case when \( I \) is independent of \( T \).

Lee et al. (2005) derived an expression using Equation 4.11 for the Probability Density Function (PDF) of the orientations of the galaxy spin vectors relative to the tidal spin tensors:

\[
P(\cos \alpha, \cos \beta, \cos \theta) = \frac{1}{2\pi} \prod_{i=1}^{3} (1 + c - 3c\hat{\lambda}_i^2)^{-1/2} \times \left[ \frac{\cos^2 \alpha}{1 + c - 3c\hat{\lambda}_1^2} + \frac{\cos^2 \beta}{1 + c - 3c\hat{\lambda}_2^2} + \frac{\cos^2 \theta}{1 + c - 3c\hat{\lambda}_3^2} \right]^{-3/2}.
\]

(4.12)

Where \( \hat{\lambda}_i \) are the eigenvalues of \( \hat{T} \) and \( \alpha, \beta \) and \( \theta \) are the angles between the unit spin vector and the major, intermediate and minor axis of the tidal field, respectively.
To quantify the preferred alignment of halo spins orthogonal to filament axis, we calculate $P(\cos \theta)$ which is the PDF of the cosine of the angle between spin axis and the minor axis of the tidal field which defines the axis of filaments. Filament regions are defined as having two positive and one negative eigenvector. They also must satisfy the traceless condition of $\sum_i \hat{\lambda}_i = 0$ as well as the unit magnitude condition of $\sum_i \hat{\lambda}_i^2 = 1$. Therefore the eigenvalues in filament regions can be approximated by $\hat{\lambda}_1 = \hat{\lambda}_2 = 1/\sqrt{6}$ and $\hat{\lambda}_3 = -2/\sqrt{6}$. Using these values in Equation 4.12 gives

$$P(\cos \theta) = (1 - c) \sqrt{1 + \frac{c}{2} \left[ 1 - c \left( 1 - \frac{3}{2} \cos^2 \theta \right) \right]^{-3/2}}.$$  \hspace{1cm} (4.13)

If halo spins are oriented completely randomly then $c = 0$ and the PDF is flat. If halo spins are preferentially orthogonal to filaments then $c > 0$ and the function increases with $\cos \theta$. Although tidal torque theory restricts $c$ to positive values, other effects could be in play that cause halo spins to be aligned parallel with filaments, which would cause a negative value of $c$.

### 4.3.4 Alignment of halo spin and velocity with filaments

The alignment between a filament and the spin of the halos that make it up is simply given by the cosine of the angle $\theta$ between the two vectors and the absolute magnitude is taken because the filament is only defined by an axis, not a particular direction. The distribution of $|\cos \theta|$ for all halos in filaments at redshift 0 and 3 is shown in Figure 4.5.
Figure 4.6: A comparison of two ways to quantify the degree of alignment of halo spin with the large scale structure. The data points are for halos in mass bins at $z=0$, scale $= 2.0 \, h^{-1}\text{Mpc}$ where the same mass bins can be seen in the red line in Figure 4.7 ($\log(M) = 11.63 - 12.21, 12.21 - 12.79, 12.79 - 13.37, 13.37 - 13.95, 13.95 - 14.53$). The error bars of $c$ are the $1\sigma$ errors of the MCMC fit and the error bars of $\langle |\cos \theta| \rangle$ are the standard error of the median. The dashed lines are for random spin orientations.

where the number of halos in each bin of $|\cos \theta|$ is normalized to make the area under the graph unity. The shape of this distribution can be quantified in two ways; the median value or by fitting a function to the curve.

Since the distributions shown in Figure 4.5 are clearly non-Gaussian, the median rather than the mean would be the more useful statistic (although the mean was used by eg. Zhang et al. (2009); Aragón-Calvo et al. (2007c)). The standard error of the median was found by bootstrap re-sampling and finding the standard deviation of the re-sampled medians. The distributions can also be fitted to the probability density function of Equation 4.13 to find the correlation parameter $c$ of the intrinsic spin-shear alignment which characterizes the shape of the distribution. The fit was done using a Markov chain Monte Carlo and two examples of such a fit is shown as the red lines in Figure 4.5.

These above two methods are compared in Figure 4.6 for some example points (halos in mass bins at $z=0$, scale $= 2.0 \, h^{-1}\text{Mpc}$ which are the same mass bins as the red line in Figure 4.7). There is a one-to-one correlation of the two parameters so either could be used. We have chosen to use the correlation parameter $c$ in this paper since it is theoretically motivated by TTT.

The value of $c$ indicates the strength of the alignment of halo spins with the orientation of filaments, and also the intrinsic alignment of spin with the tidal field. If the halos generally have spins parallel to filament axis $c$ is negative, conversely, if the halo spin are generally orthogonal to filament axis then $c$ will be positive. The error of $c$ is the standard deviation of the value which maximizes the likelihood of the fit of the PDF to
Figure 4.7: The alignment of dark matter halo spin with filaments over cosmic time. Alignment is characterized by the parameter $c$ of the fit of Equation 4.13 to the distribution of $|\cos \theta|$, where positive $c$ indicates orthogonal alignment and negative $c$ indicates parallel alignment. The panels show filaments found in different smoothing scales: 1.0 (top left) 2.0 (top right), 3.5 (bottom left) and 5.0 $h^{-1}$Mpc (bottom right). At high redshift all spins are orthogonal to filaments but recent times, low mass halos have a parallel alignment with filaments. The dashed line is the expected distribution for random halo spins and the shaded regions are the 1$\sigma$ errors. The red line is for $z=0$, yellow line is $z=1$, blue is $z=2$ and green line is $z=3$. 
Figure 4.8: The distribution of alignments of halo spin with the axis of filaments for low mass \((M < M_\ast, \text{ solid lines})\) and high mass \((M > M_\ast, \text{ dashed lines})\) halos. These halos are at \(z=0\) and filaments are found using smoothing scale \(3.5 \, h^{-1}\text{Mpc}\). This mass division corresponds to the red line in the bottom left panel of Figure 4.7 where the first two points of that figure are the low mass halos and the other points are the high mass halos.

The alignment of halo spin vectors with filaments is shown in Figure 4.7. The alignment distribution has been fitted to find \(c\) for halos in bins of mass and for halos at different redshifts. For all smoothing scales, it can be seen that at \(z=0\) the alignment is weakly parallel (negative \(c\)) for low mass halos in filaments (mass less than about \(M_\ast = 5.89 \times 10^{12} M_\odot\)) and orthogonal (positive \(c\)) for high mass halos. This is illustrated in Figure 4.8. At higher redshifts the alignment becomes more orthogonal for all halo masses. There are less halos in the high mass bins at high redshift because the high mass halos have not had time to form yet. The result of Faltenbacher et al. (2002); Aragón-Calvo et al. (2007c); Hahn et al. (2007b) and Zhang et al. (2009) that halo spins generally lie along the axis of filaments is driven by the low mass halos at \(z=0\). This is demonstrated in Figure 4.5 where the alignment distribution for all halos at \(z=0\) is shown. The alignment is preferentially parallel because of the high number of low mass halos that exhibit parallel alignment.

The affects of smoothing scale on the halo spin alignment with filaments show something about the formation of filaments. For redshift 0 (the red line in Figure 4.7), halos seem to be best aligned at a large smoothing scale while high redshift halos are best aligned at small scales. If an orthogonal alignment is an indicator that a halo formed inside a filament topology, then this shows that filaments grow in size over time.

Figure 4.9 shows the effect of taking into account the characteristic mass. Here we can compare halos between redshifts at equivalent stages of collapse. When the this is accounted for, almost all the points overlap within their errors. This means that halos at a similar stage in their collapse have the same degree of preferential alignment with filaments over cosmic time. A halo that is just starting to collapse \((M = M_\ast)\) at redshift
Figure 4.9: The alignment of dark matter halo spin with filaments over cosmic time for halos in bins of halo mass normalized by the characteristic mass. The alignment for a $2h^{-1}$Mpc scale is shown here.

$2$ has a similar probability of orthogonal alignment with its filament as a halo that is just starting to collapse at redshift $1$ or $0$. However, no assumptions were made about the evolving scale of filaments and the smoothing scale was kept constant at $2.0h^{-1}$Mpc. Even with a constant scale, this similarity between alignments at different times shows that the build up of spin is closely linked with a halo’s formation.

When substructure is discounted by taking the most massive subhalo in each FOF group, there is practically no change in the alignments.

Although the $c$ parameter was introduced in the context of spin alignments with the tidal field (manifested by filaments in the large scale structure), it can also be used as a more general measure of alignment. The distributions of $|\cos \theta|$ where $\theta$ is the angle between halo center of mass velocity and filament axis is also well-fit by the PDF in Equation 4.13. Again, a negative value of $c$ means a parallel alignment and a positive value is orthogonal alignment.

All panels of Figure 4.10 show a parallel alignment which is stronger for high mass halos. This shows streaming of halos of all masses down filaments into massive clusters.

This streaming can be seen in the velocity vectors of halos in some filaments in the middle panel of Figure 4.4, where vectors are pointed along filaments towards clusters. However, some filaments display bulk motions where the entire filament is moving towards some attractor. To see the extent of these bulk motions, they have been subtracted in Figure 4.11 by subtracting the mass-weighted average velocity of halos by halo mass found within the smoothing scale on which the filaments were found. When bulk motions are discarded, an orthogonal motion remains. The apparent streaming of halos down filaments was wholly caused by bulk motions of entire filaments, and this bulk flow is generally along the axis of filaments. The relative motions can be seen in Figure 4.12.
Figure 4.10: The alignment of dark matter halo velocity with filaments. For all redshifts, halos are parallel aligned with filaments which demonstrates a streaming motion of halos down bulk flows. Alignment is characterized by the $c$ parameter of Equation 4.13 where $\theta$ is the angle between halo velocity and filament axis. Lines are colored as in Figure 4.7.
Figure 4.11: The alignment of dark matter halo velocity with filaments on the scale of $2.0\,h^{-1}\text{Mpc}$ where bulk motions have been subtracted. Colored lines are for different redshifts as in Figure 4.7.

Figure 4.12: The alignment of dark matter halo velocity with the local bulk motion on the scale of $2.0\,h^{-1}\text{Mpc}$. Colored lines are for different redshifts as in Figure 4.7.

the alignment of halo velocity with the flow of the local bulk motion. (Bulk motions have been subtracted from halo velocities here.) Low mass halos are moving slightly orthogonal to the flow and high mass halos have no preferred direction of motion. This reflects how bulk motions have been removed: high mass halos were given more weight than low mass halos and so the residual motions of high mass halos once bulk flow is removed is minimal.

The enlargement of filaments over time that was seen in the spin alignments is also visible in the way the bulk flows are aligned. The low mass halos at $z=0$ (red line in Figure 4.10) are more strongly aligned at large smoothing scales and the low mass halos at high redshifts are most aligned at small smoothing scales. If filaments are chutes where halos are channeled into clusters then these low mass halos are evidence for the growth of the size of filaments over time. The high mass halos on the other hand are generally less aligned at large smoothing scales for all redshifts which is seen as a flattening of the curves. This may be due to the inclusion of some cluster halos when the smoothing scale is broadened which would introduce random velocities into the sample.

Although both halo spin and velocity are somewhat aligned with filaments, these alignments are not strong enough so that there is a significant alignment between a halo’s spin and velocity.

### 4.3.5 Halo-halo spin alignment

Tidal torque theory predicts that as well as being aligned with the large scale structure, halo spins should be aligned with each other. This is usually tested by simply taking the average of the dot product of pairs of halo spins separated by distance $r$:

$$
\eta(r) = \langle |\mathbf{J}(x) \cdot \mathbf{J}(x + r)| \rangle.
$$

(4.14)
A second quantity used by Pen et al. (2000) and Bailin and Steinmetz (2005) is

\[
\eta_2(r) = \langle |\hat{\mathbf{J}}(x) \cdot \hat{\mathbf{J}}(x + r)|^2 \rangle - \frac{1}{3},
\]

(4.15)

These quantities are plotted in the top panels of Figure 4.14, where at very small halo separations \(r < 0.3 \, h^{-1}\text{Mpc}\) there seems to be a parallel alignment of halo spins.

Figure 4.13: An example of a distribution of halo-halo spin alignments. \(P(|\hat{\mathbf{J}}(x) \cdot \hat{\mathbf{J}}(x + r)|)\) is the number of halo pairs in each bin of \(\hat{\mathbf{J}} \cdot \hat{\mathbf{J}}\). This example is for halos that are separated from 0.06 to 0.1 \(h^{-1}\text{Mpc}\), which is the second data point from the left in Figure 4.14. The thin line is the actual distribution and the thick line is a straight line fit. There is a significant deviation from random spin orientations here, shown by the positive slope of the straight line.

However, both of these quantities rely on taking an average over all the halo pairs in each bin of separation. The mean is a useful value when dealing with a peaked distribution, but none of the actual distributions of \(|\hat{\mathbf{J}}(x) \cdot \hat{\mathbf{J}}(x + r)|\) has an apparent peak (an example of one of these distributions is Figure 4.13, where \(P(\hat{\mathbf{J}} \cdot \hat{\mathbf{J}})\) is the number of halos in each bin normalized so that the area under the curve is unity). A fairer way of dealing with these noisy distributions is to fit a straight line and see if there is any deviation from randomness. The slope of the best fit line indicates if more halos are aligned parallel or orthogonal to each other.

\[
P(\hat{\mathbf{J}}(x) \cdot \hat{\mathbf{J}}(x + r)) = m|\hat{\mathbf{J}}(x) \cdot \hat{\mathbf{J}}(x + r)| + c.
\]

(4.16)

A positive slope \(m\) of the best fit line means there are more parallel aligned halo pairs, a negative \(m\) means they are more orthogonal and \(m = 0\) means the halos have random alignment. The values of \(m\) that maximized the likelihood of fitting a straight line to the distributions are shown in the bottom panel of Figure 4.14.

The shape of the plot of the slope (bottom panel of Figure 4.14) is similar to the shape of the plots of the conventional statistics. This is expected since they are effectively measuring the same thing but in a slightly different way. Halo spins are aligned parallel for halo separations under 0.3 \(h^{-1}\text{Mpc}\). This alignment has not been seen before in simulations because it exists only on very small scales which have not before been examined. It has
Figure 4.14: The alignment of neighboring halo’s spins, for halos separated by distance r. Three different statistics are used: $\eta$ from Equation 4.14 (top), $\eta_2$ from Equation 4.15 (middle), and $m$, the slope of the distribution of the halo-halo spin alignment (bottom, an example is shown in Figure 4.13). The dashed lines are for random halo alignments and the shaded regions are the $1\sigma$ errors.

Figure 4.15: The alignment of neighboring friends-of-friends halo’s spins. There is no alignment at any scale.
however been seen in galaxy surveys, for example Galaxy Zoo (Slosar et al., 2009) found alignment for galaxies closer than $0.5h^{-1}\text{Mpc}$. The alignment exists on the scale of substructure within clusters. If only the most massive subhalo in each FOF group is taken (the substructure is thrown out), then there is no significant alignment at any scale (Figure 4.15). Here there are no halos at small separations and there is no significant alignment at any scale. Only the subhalos within large clusters exhibit any halo-halo spin alignment, although it is weak.

### 4.3.6 Evolution of spin parameter

The spin parameter is a measure of the amount of angular momentum contained in a halo. It was defined in Bullock et al. (2001) as,

$$\lambda' \equiv \frac{|\mathbf{J}|}{\sqrt{2MV}}$$  \hspace{1cm} (4.17)

given the angular momentum $\mathbf{J}$ inside a sphere of radius $R$ containing mass $M$, and where $V$ is the halo circular velocity at radius $R$, $V^2 = GM/R$.

The distribution of $\lambda'$ over the halos in our sample is shown in Figure 4.16. It is well fit by a log-normal distribution,

$$P(\lambda') = \frac{1}{\lambda'\sqrt{2\pi\sigma}} \exp \left( -\frac{\ln^2(\lambda'/\lambda'_0)}{2\sigma^2} \right). \hspace{1cm} (4.18)$$

The fit was done using a Markov chain Monte Carlo maximum likelihood analysis. For all halos with more than 500 particles at $z=0$ the best fit values are $\lambda'_0 = 0.02900_{-0.00005}^{+0.00006}$, $\sigma = 0.604_{-0.002}^{+0.001}$ and at $z=3$ $\lambda'_0 = 0.02940_{-0.0001}^{+0.00008}$, $\sigma = 0.576 \pm 0.002$. The distributions at both these redshifts over all halos in the snapshots are nearly identical.

![Figure 4.16: The distribution of the spin parameter at z=0. The histogram is the data and the red (smooth) line is a log-normal fit (Equation 4.18) where the best fit values are $\lambda'_0 = 0.02900_{-0.00005}^{+0.00006}$, $\sigma = 0.604_{-0.002}^{+0.001}$.](image)

When halos are binned by mass, the spin parameter at high redshift shows a mass dependence while there is no mass dependence at $z=0$, as shown in the left hand side of
Figure 4.17. Here the spin parameter is characterized by the mid point of the log-normal distribution, $\lambda'_0$. The spin parameter over all redshifts is only the same for low mass ($M < 10^{12} h^{-1} M_\odot$) halos but there are far more low mass than high mass halos. Since low mass halos dominate, the average distributions over all halos at the different redshifts look the same. At high redshift, there is a tendency for the spin parameter to be smaller for high mass halos.

This redshift dependency can be characterized by a power relationship between $\lambda'_0$ and mass at each redshift:

$$\lambda'_0 \propto M^{a(z)}.$$  \hspace{1cm} (4.19)

The more negative the value of $a$, the stronger the correlation and $a = 0$ is no correlation at all. The redshift dependence of $a$ is shown in Figure 4.18. The lines for halos with $> 500$ particles and $> 1000$ particles overlap in Figure 4.18 whereas the line for halos with $> 100$ particles does not. This shows that halos with more than 100 particles are susceptible to errors from particles in the outer regions and the cut off of only using halos with more than 500 particles is justified.

Knebe and Power (2008) found that mass binning and selection criteria for relaxed halos has almost no effect on this correlation. We did find a small effect when a different halo catalog was used. Instead of using all the subhalos, only the most massive subhalo (with more than 500 particles) in each friends-of-friends halo was used. Most of the mass of the FOF halo is in the most massive subhalo so it can be regarded as the background halo itself. When substructure is disregarded, we find that there is a stronger mass dependency of the spin parameter at almost all redshifts (the green line in Figure 4.18 is below the corresponding orange line which includes all substructure). The spins of subhalos are greatly affected by interactions and merger events so may be out of equilibrium.

Mass dependence of the spin parameter at high redshift was first found by Knebe and Power (2008), who looked at $z=1$ and $z=10$. When extrapolating the linear trend of $a(z)$...
with redshift, we predict a much stronger correlation, \( a(z = 10) \simeq -3 \) whereas they find \( a(z = 10) = -0.059 \pm 0.171 \). Our results agree more closely with Muñoz-Cuartas et al. (2011) who found \( a(z = 2) \simeq -0.03 \). For halos in different environments (blobs, filaments, sheets and voids), the trends are the same.

When halo mass is scaled by characteristic mass in the right hand side of Figure 4.17, we find that halos at similar stages of collapse at \( z=0 \) and \( 1 \) have the same spin parameter (the orange and red lines overlap). At high redshift, halos at similar stages of collapse have a higher spin parameter (At \( \log M/M_* = 3 \) for example, the green \( (z=3) \) point lies above the points for \( z=2 \) and \( z=1 \)). This may be the result of accretion and merger events decreasing the spin of halos. At \( z=3 \), halos have retained much of their initial spin but by \( z=1 \), similar halos have experienced accretion that has lowered their spin parameter.

![Figure 4.18: The mass dependence of the spin parameter over redshift. The mass dependence, \( a \) is the slope of the straight dashed lines in Figure 4.17. The red (lowest) line includes all halos with more than 100 particles, orange line includes 500 particles and blue line includes 1000 particles. The green line is for the halo catalog which doesn’t include substructure. There is a linear trend of stronger mass dependence at higher redshift.](image)

**4.3.7 Summary and Conclusions**

Using the Millennium N-body simulation, we have tracked the evolution of dark matter halo angular momentum alignments with the large scale structure, with each other and the evolution of the spin parameter. We have used the shape of the density field to find filaments of \( 2h^{-1}\text{Mpc} \) in scale in the large scale structure. The alignment between dark matter halo spin and the axis of filaments was characterized by the shape of the distribution of \(|\cos(\theta)|\) where \( \theta \) is the angle between the two vectors. The distribution was fitted to the PDF of Equation 4.1 to find the free parameter \( c \) which characterized the strength of parallel or orthogonal alignment.

We found that angular momentum vectors of dark matter halos since \( z=3 \) are generally orthogonal to filaments but high mass halos have a stronger orthogonal alignment than low mass halos. At \( z=0 \) the spins of low mass halos have become parallel to filaments, whereas high mass halos keep their orthogonal alignment.
An interpretation of this is that at early times all halo spins were aligned orthogonal to filaments, as TTT predicts. High mass halos especially are well aligned because they have had their maximal expansion more recently and so will have been tidally torqued for longer. They usually exist close to clusters where the infall of dark matter is almost isotropic and so the nett effect from mergers and accretion is minimal. Low mass halos, however, are vulnerable to being disturbed by mergers and accretion which is usually assumed to have the effect of randomizing the spin orientation. This leaves unexplained why low mass halos at low redshift exhibit a parallel alignment with filaments.

We found that filaments are regions of bulk flow. When bulk flows are included there is a clear trend for halos to travel parallel to filaments, and high mass halos travel with the best alignment. When bulk flows on the scale of the filaments are subtracted, an orthogonal alignment to filaments remains, particularly for low mass halos. This shows that entire filaments themselves are moving towards attractors and on small scales there is only orthogonal motion. There was also an orthogonal motion of low mass halos with the bulk flow but no alignment of high mass halos out of the bulk flow.

The motions of halos relative to the bulk flow could affect how matter is accreted onto them and the spin orientation this would cause. Orthogonal motion to the bulk flow and filaments by low mass halos could cause low mass halos to accrete matter preferentially in one direction. High mass halos traveling with the bulk flow would experience accretion differently, and this could cause the difference in spin orientation.

Filaments at large smoothing lengths at low redshift contain halos with the best aligned spins and bulk motion, while at high redshift it is filaments at small smoothing lengths that contain the best aligned halos. This shows that filaments are growing in size over time. Because of the nature of the way that the filaments were found (using Gaussian smoothing), this enlargement tells more about the width of the filaments rather than the length. This is complimentary to Sousbie et al. (2009) where filament length is discussed and it was found that there is a general dilation of filaments that began larger and a shrinking, fusion and disappearance of the smaller filaments.

We found an alignment only between the spin orientation of very close neighboring halos. Only at separations of less than $0.3 \, h^{-1}\text{Mpc}$ do halos exhibit any mutual parallel alignment of their spin axis. The halo finding method used in the Millennium simulation has enabled us to see this small scale alignment. In the Millennium simulation, the subfind algorithm was used to identify substructure in friends-of-friends groups, and the subhalos are counted as halos. This means that alignments between very close halos can be probed, not just alignments between the friends-of-friends groups.

Lastly, we tracked the evolution of the spin parameter from $z=3$ to now and its dependence on halo mass. This was done by finding the center of the log-normal distribution of the spin parameter. There is a mass dependence of the spin parameter at $z=3$ but not at low redshift and the spin parameter is lower overall at high redshift. The spin parameter follows a power law with halo mass at high redshift but is independent of mass at $z=0$.

Future work will bridge the gap between idealistic CDM simulations and real galaxy observations. To do this we will generate mock galaxy catalogs and use only the data that would be available in a real survey, see if any alignments of galaxy spin orientations could be seen in the universe. This could be used to plan a survey using new multi-object IFU instruments (Bland-Hawthorn et al., 2011; Croom et al., 2012).
4.4 Discussion

The paper reproduced in the previous Section explored how dark matter halos spin in relation to the large scale structure and how this has evolved though time. It dealt with three main aspects; the alignment of spin with filaments, the alignment of the spins of dark matter halos with each other and the evolution of the spin parameter. The central result was that low mass halos spin parallel to filaments and high mass halos spin perpendicular but this changes with time. It had been found previously at z=0, the spin of low mass halos is parallel to filaments and high mass halos spin orthogonal to filaments (Aragón-Calvo et al., 2007c). During publication of the paper in Section 4.3, it was found that the mass at which parallel alignment turns to orthogonal, changes with redshift Codis et al. (2012). This finding confirmed my result of a continually evolving spin alignment with filaments.

Since publication, the work in this paper has been cited several times. It has been mentioned in new observational studies Tempel et al. (2013); Lee (2013). The scale of filaments was examined in the paper in Section 4.3 where it was found that halos are better aligned with small scale filaments at high redshift. This work was mentioned in Aragon-Calvo (2013) where the scale of filaments was also studied. Onions et al. (2013) and Libeskind et al. (2013) also mentioned my work in their analysis of spin alignments with the large scale structure.

In the paper of Section 4.3 it was suggested that work needs to be done to bridge the gap between idealistic cold dark matter simulations and real galaxy observations. In order to design and interpret future galaxy surveys, predictions need to be made of what might be found about galaxy spin.

4.5 Alignments of Galaxies

It is not clear what alignments, if any, exist in the spins of observed galaxies. It has been shown in van den Bosch et al. (2002); Sharma and Steinmetz (2005) that the spins of hydrodynamically simulated galaxies were fairly well aligned with their host dark matter
Figure 4.20: Disk galaxy kinematic map made by SAMI. Gas velocity map for ESO 185-G031 (top left) shown with the best-fit disk model (top right). The residual map, found by subtracting the model from the observed velocity map (bottom left), shows a very strong deviation from the rotating disk model in the region of the galactic wind. This region is co-spatial with the broadened emission lines seen in the FWHM map (bottom right). Picture credit: Fogarty et al. (2012)

halos. It is therefore expected that observed galaxies should be aligned with large scale structure in the same way that simulated dark matter halos are. If these alignments are detected then it is a success for the predictive power of N-body simulations, and if not then there is something missing to the theory of either how gas falls into dark matter halos to form galaxies or the way that luminous matter traces out the large scale structure.

It has been shown in observed galaxy catalogs that the spin axes tend to be correlated with the large scale features they are in Kashikawa and Okamura (1992); Navarro et al. (2004a); Trujillo et al. (2006). On the other hand, Slosar and White (2009) found that there were no correlations between galaxies and the voids they are contained in. In filaments there have been tentative detections of alignments. Jones et al. (2010) found that the spin axes of a small sample of edge on SDSS galaxies were oriented perpendicular to filaments. To the contrary, Tempel et al. (2013) found that the spins of spiral galaxies are oriented parallel to filaments and the spins of elliptical/SO galaxies are orthogonal to filaments.

The source of these disagreements could be the difficulty in measuring the inclination angles of galaxies or in fully defining features in the large scale structure. Current and future surveys utilizing new multi-object Integral Field Unit (IFU) technology could change the first of these problems by making direct measurements of galactic spin in a large survey.

These multi-object IFUs are made possible by a new fiber optic technology called
hexabundles. Hexabundles are multi-fiber imaging bundles and have resulted in an imaging device that can be replicated many times in a given field and positioned in a similar way to that used for single-fiber survey instruments in the past. Their use is demonstrated in Figure 4.19 where the 61-fiber configuration of a hexabundle is compared to the single fiber aperture of traditional galaxy survey technology. Measuring the spectra from each of the fibres of the hexabundle results in spatially resolved data that can be used to pursue the science goals of understanding star formation, gas flows and AGN activity. Galactic rotation can be measured directly using hexabundle technology from the kinematic data (an example of a galaxy gas kinematic map is shown in Figure 4.20), combined with the inclination angle of spirals. This new technology means that we can now consider angular momentum in gas-rich systems (e.g. spirals, dwarf irregulars) and stellar dominated systems (e.g. SOs, ellipticals, dwarf spheroidals). Presently, a galaxy survey is taking place using the multi-object IFU, SAMI (Sydney-AAO Multi-object IFU) (Bland-Hawthorn et al., 2011; Fogarty et al., 2012; Croom et al., 2012), which comprises of 13 hexabundle IFUs deployable over a 1 degree field-of-view. This survey will target 3,500 galaxies within the next two years.

Future surveys using a larger scale instrument, hector (Lawrence et al., 2012), will target 50,000-100,000 by end of the decade. This instrument is still in the developmental phase but it has been proposed to have 117 IFUs positionable over a 3 degree field of view. The implications of this technology are that soon there will be a massive influx of directly measured galactic spin data which can be used to test the predictions from N-body simulations.

In order to predict what such a future survey should see, the semi-analytic galaxies in the Millennium simulation (see Section 2.6) were used to create a mock catalog, emulating the results of a real galaxy survey. The technique of semi-analytic modeling takes the approach of treating the various physical processes associated with galaxy formation using approximate, analytic techniques. The primary advantage of the semi-analytic approach is that it is computationally inexpensive compared to full gas and dark matter N-body simulations. The disadvantage of this technique is that it involves a large degree of approximation in the analytic models.

The semi-analytic analysis was of the Millennium simulation was performed by De Lucia and Blaizot (2007); Croton et al. (2006). The analysis was based on the merger tree of dark matter halos and included analytical models for physical process such as: gas infall and cooling; reionization; black hole growth; AGN outflows; star formation and supernova feedback. Using the best parameters for these processes, the dark matter halos were populated with galaxies. The results of this analysis are available publicly at http://gavo.mpa-garching.mpg.de/MyMillennium.

To create a mock catalog of semi-analytic galaxies, a lightcone of galaxies was taken from the Millennium simulation. This lightcone was taken to simulate a galaxy survey spanning a contiguous one radian solid angle of the sky. The redshift restricted volume was taken to be all redshifts up to $z = 0.09$. The dimensions of this cone were thus taken to be $r = 200/\text{h}^{-1}\text{Mpc}$ and depth $370\text{h}^{-1}\text{Mpc}$, from the snapshot at $z=0$. Redshift distortions were added by hand and the sample consists only of spirals (bulge to total luminosity $B_{\text{bulge}} - B > 1.56$) with maximum apparent B magnitude of 20. The light cone of galaxies is shown in Figure 4.21.

The methods for finding filaments in the mock data set had to be adjusted in order to emulate handling real data. It was assumed that the galaxy positions and redshifts could
be measured accurately and that galaxy spin was oriented the same way as their dark matter halos. However, it was not assumed that the mass of the galaxies’ dark matter halos was known, and this piece of information has been central to both filament finding methods.

In place of halo mass, circular velocity $v_{\text{circ}}$ was used for the mock catalog. This is the velocity that a star in a galaxy must have to maintain a circular orbit at a specified distance from the center ($R$), assuming a symmetric gravitational potential. The rotation curve in a function $v_{\text{circ}}$ for a galaxy and can be measured over a range of distances from the center of the galaxy. This can be measured from the gas and young stars in spiral galaxies using multi-object IFUs. Due to spherical symmetry, gravitational acceleration due to the mass internal to the orbit ($M(R)$) must match centripetal acceleration;

$$\frac{v_{\text{circ}}^2}{R} = \frac{GM(R)}{R^2}, \quad (4.20)$$

and therefore the circular velocity is related to the mass by

$$v_{\text{circ}} = \sqrt{\frac{GM(R)}{R}}, \quad (4.21)$$

making circular velocity a strong proxy for halo mass. Instead of using the mass of the dark matter halos in the mock galaxy catalog, the values of maximum circular velocity from the Millennium database were used. $V_{\text{max}}$ occurs at roughly twice the scale radius.
and for halos with 500 particles and this isn’t too much larger than the softening in the simulation. This may introduce error in the measurement of $V_{\text{max}}$ for the smallest halos.

Figure 4.22: Alignment of halo spin with filaments for the mock catalog of galaxies. The top row shows the results using filaments found using halo mass and the bottom row shows filaments found using circular velocity as a proxy for halo mass. On the left is the alignment of halos with filaments using the Cylinder extraction method. On the right filaments were found using the Density field method.

In the Density Field method, this was used in the weighting of the galaxies when constructing the density field by DTFE. Instead of the mass of the halo that the galaxy is the center, the maximum value of the circular velocity was used. The filaments were found from the Hessian the same way as described in Chapter 3.

In the Cylinder Extraction method as described previously, dark matter halos were ranked by their mass, cylinders constructed around them and secondary nodes were found to be high mass halos. Instead of halo mass, the maximum circular velocity was used to rank the galaxies. Filaments were found using exactly the method described in Chapter 3, with the maximum value of $v_{\text{circ}}$ substituted for mass.

There were 606,189 galaxies in the catalog in total, 379,808 were found to be in filaments in the Cylinder Extraction method and 340,625 were found to be in filaments using the Density field method when circular velocity is substituted for halo mass. Both methods find a significant if weak parallel alignment of halo spin with filaments (as shown in Figure 4.22). The Density Field method gives slightly stronger results compared to the Cylinder Extraction method when using either halo mass ($<\cos(\theta)> = 0.5070 \pm 0.0008$ compared to $<\cos(\theta)> = 0.5048 \pm 0.0008$ ) or circular velocity ($<\cos(\theta)> = 0.5079 \pm 0.0008$ compared to $<\cos(\theta)> = 0.5053\pm0.0008$). Circular velocity serves well as a proxy to halo mass and actually gives a marginally stronger result for both methods.

In a future galaxy survey of a large number of galaxies, especially if galaxies in filaments are targeted specifically, there is likely to be some parallel alignment of galactic spin with
Figure 4.23: The number of galaxies needed to see an alignment of galaxy spin with filament axis. The Cylinder Extraction method was used on the mock catalog, sampling fewer galaxies by restricting the distance to the furthest galaxies. The error bars here are the 1σ errors of the median. More than about 60,000 galaxies in filaments (the second point from the left) are needed before a significant signal is achieved.

filaments. The alignment predicted is weak ($\langle \cos(\theta) \rangle = 0.504 - 0.508$) but clearly present. This prediction assumes that galactic spin is well aligned with dark matter halo spin (an assumption supported by Sharma and Steinmetz (2005)) and can be tested using accurate measurements of the positions, redshifts and rotation curves of galaxies. More than 60,000 galaxies in filaments will need to be targeted, according to Figure 4.23. This estimate was achieved by further limiting the volume of the mock galaxy catalog to reduce the sample size. The point furthest to the right in this figure is for the whole mock catalog and the volume was limited for the other points by limiting the depth of the lightcone. The median is inconsistent with random alignments ($\cos(\theta) = 0.5$) when there are more than about 60,000 galaxies (the point second from the left).
Chapter 5

The Origins of Dark Matter Halo Spin Inside Filaments

In order to be irreplaceable one must always be different.
- Coco Chanel

5.1 Introduction

The initial spin of an overdense patch which will one day become a dark matter halo is predicted by Tidal Torque Theory (TTT). In this theory, halo spin is caused by a misalignment of its inertia tensor and the local gravitational tidal tensor at the time of the protohalo’s maximum expansion. The spin direction should be initially correlated with the principal axes of the local tidal tensor. However, mergers can significantly disrupt halos, especially low mass halos, which could have the effect of randomizing halos spin orientation. Mergers could potentially have the effect of creating a new preferred spin direction if the infall of satellite halos is not isotopic. A mass-dependence of halo spin alignment as found in Sugerman et al. (2000); Bailin and Steinmetz (2005); Codis et al. (2012); Trowland et al. (2013) is an indication that TTT is not the only force at work in aligning halo spin, and mergers could play an integral part.

Dark matter halos are constantly under a barrage of infalling material, some of it is by smooth accretion and some of it by minor and major mergers. The merger rate of a halo is dependent on the descendant halo mass, progenitor mass ratio and redshift, and can be empirically predicted from the Millennium simulations (Fakhouri et al., 2010). Mergers can significantly alter a halo’s properties. A merger impacts the dynamical state of the halo, creating an un-relaxed configuration for a time after the merger (Power et al., 2012). In the long term, mergers can change a halo’s mass, concentration, shape and spin. Wong and Taylor (2012) worked backwards from these properties to recreate the mass assembly histories of dark matter halos. Mass assembly is best characterized by halo age, and secondly by the acceleration or deceleration of growth.

It is generally assumed that the flow of matter in filaments is along the axis, and a flow such as this has angular momentum oriented orthogonal to the axis. If this matter is accreted onto a halo it would add orthogonal angular momentum to the halo’s spin. A secondary effect is the infall of material from voids or walls onto filaments which would add angular momentum parallel to the axis of filaments. In order to figure out how merging can affect halo spin, these flows need to be understood in more detail.
This Chapter builds on the work of the previous Chapter. I search for the reasons why halos exhibit certain properties in filaments and what is the source of the difference in spin orientation of high and low mass halos. The first Section looks at how halos flow around filaments and the second Section focuses on mergers and how they could be the source of some halo spin alignments.

5.2 The Infall of Halos onto Filaments

In order to investigate how angular momentum is built up by halos in filaments, their behavior and movements within those filaments must first be studied. The primary theory governing the formation and evolution of filaments is the pancaking effect (see Section 1.2.3 for an overview). In this theory, collapse of an overdense region depends on its shape and takes place first along the shortest axis, creating a flattened out pancake shape. the pancake then collapses along the next shortest axis creating a filament and then along the last axis, ending up in a clump.

The anisotropic theory of collapse involves flows first from voids towards sheets, then along the plane of sheets to filaments, then along the axis of filaments to clusters. The flows of halos or galaxies along filaments has not attracted a lot of attention so far in theory or observation due to the difficulty in measuring accurately the galaxy velocities from observations. The first stage of the pancaking effect, the flow from voids onto sheets, has been confirmed in the velocities of the Millennium galaxies (Noh and Lee, 2006, 2007). However, it is not clear exactly how the galaxies progress in the next stage of their journey; from filaments to clusters. In Trowland et al. (2013, Chapter 4 of this thesis), it was found that the velocity vectors of dark matter halos are aligned strongly parallel to the axis of filaments, suggesting a flow of halos along filaments. It was also found that when that bulk flow is subtracted, an orthogonal alignment of halo velocity with filament axis remains, which is a signature of the infall of halos onto filaments. In this Section, those results are built upon to create a clearer picture of how galaxies and halos flow onto and along filaments.

5.2.1 Bulk Flows of Halos Along Filaments

The pancaking effect of filaments can be studied by looking at how halos behave around filaments. Filaments were found using the Cylinder Extraction method which defines the filament axis as a line between two high mass halos; the primary and the secondary nodes (see Chapter 3 for details on the Cylinder Extraction method). The position of halos inside filaments with respect to the axis and the nodes gives an idea of what kind of structure the filaments have. The mean position of halos in filaments is plotted in Figure 5.1 for several different redshifts. It shows that the average position of halos in filaments gets progressively closer to both the axis and the nodes as time goes on. The effect of this is to create more concentration about the axis and clumps close to the nodes at low redshift. Halos flow closer to the axis and then along the axis towards the nodes, as predicted by pancaking theory.

Another way of seeing the flow towards the axis and then the nodes of filaments is in Figure 5.2. The left hand panel shows the median velocity alignment of halos with filaments as a function of how far they are from the axis. The halos far from the axis
Figure 5.1: Left: The mean distance of halos from the axis of filaments. Right: The mean distance of halos from the nodes of filaments, where distance is normalized by the total length of the filament. Error bars are standard error of the mean.

Figure 5.2: Median alignment of halo velocity (Left) and spin (right) with filament axis for halos of varying distances away from the axis. The red line is for $z = 0$, yellow line is $z = 1$, blue is $z = 2$, and green line is $z = 3$. The shaded areas are the standard error of the median and were found by bootstrap re-sampling and finding the standard deviation of the re-sampled medians.
at $z=0$ (the red line) are not moving with any particular orientation with respect to the axis but the halos that are close in are moving along the axis. At higher redshifts (yellow, blue and green lines), halos that are far off from the axis are actually moving orthogonal to it; they are still in the process of falling into the filaments.

Also shown in Figure 5.2 is the alignment of halo spin with the axis. Consistent with the results in Chapter 4, halos at low $z$ are generally spinning parallel to the filament (because the low mass halos dominate the sample), while at high $z$ the alignment is orthogonal. Halos far away from the axis have no particular alignment at all. This shows that the most interesting behaviors of halos in filaments happens close to the axis, either because this is where the geometry is the best or because this is where the most accretion events happen. There is no difference in the alignment of halo velocity or spin for halos near or far away from the nodes of filaments.

### 5.2.2 Formation Time

The formation time of a dark matter halo is taken to be the redshift when half of its final mass (at $z=0$) was first assembled into a single object. The merger tree is used to find the earliest snapshot when a halo’s most massive progenitor had more than half of its final mass, and this is taken as the formation redshift. This is the most commonly used definition of formation time (e.g. Lacey and Cole, 1993; van den Bosch, 2002; Gao et al., 2004; Wechsler et al., 2006). This definition of halo formation time is useful because it indicates when the main body of a halo was assembled but it is not the only definition (Li et al., 2008)

It was found that halos that form at high redshift are streaming along filaments more coherently than halos that formed more recently (see the top panel of Figure 5.4). This could indicate that early forming halos have had enough time to fall into filaments and are now flowing along the axis. This is supported by Figure 5.3, which shows that early forming halos do sit closer to the axis and nodes of filament than late forming halos.

### 5.2.3 Halo Spin and Formation Time

An important result in Trowland et al. (2013) was that high mass halos were spinning with a different orientation to filaments than low mass halos. According to Tidal Torque
Figure 5.4: Median alignment of halo velocity (Left) and spin (Right) with filaments at $z=0$ for halos of varying formation time. The red line is low mass halos and the yellow line is for high mass halos. The shaded areas represent the standard error of the median.

Theory (TTT, Section 1.3.1), halo alignments should be imprinted early on in their life so halos that form late should have a better alignment because they haven’t had a chance to be disturbed between their formation and now. From the right hand panel of Figure 5.4 this seems to be true; halos that formed before $z=1$ have no significant spin alignment with filaments. On the other hand, TTT also predicted that halo spins should be aligned orthogonal to the axis of filament, and this is only true for high mass halos that formed after $z=1$. The low mass halos have spins that are parallel to the axis.

As shown in the left hand panel of Figure 5.3, high mass halos have formed more recently than low mass halos. This might lead one to think that the difference in spin alignment between low and high mass halos is due to their different formation time. However, there is an intrinsic difference between low and high mass halos that is not caused by their generally different formation times. Low mass halos have spin that is more parallel to filaments than high mass halos, regardless of formation time.

5.2.4 Infalling Halos

It has been shown in Section 5.2.1 that halos travel closer to filament axes and towards the nodes. In this Section, the way that halos make this journey is examined in more detail. The way that halos fall onto filaments could affect their final properties and also the way that filaments are built up over time.

The alignment of halo velocity over time is tracked to understand the path of the infalling halos. The progenitors of all the halos that end up in filaments at $z=0$ are tracked back in time. The filament axis is taken to be in the same direction as it is at $z=0$ and $\theta$ is taken to be the angle between the progenitor’s velocity and this axis. Fixing the filament axis allows the same set of halos that are in filaments at $z=0$ to be tracked back in time, instead of finding a new set of halos in filaments for each snapshot. The downside is that
it does not allow for any change in filament orientation over time. A more sophisticated filament finding algorithm would be needed to track the change in filament direction over time. The Cylinder Extraction method finds filaments in each snapshot independently but a further developed method might track those filaments between snapshots. Such a filament finder does not yet exist but could be the subject of future work.

The results can be seen in Figure 5.5. The velocity of both high and low mass halos is orthogonal to filaments from \( z = 3 \) until \( z \approx 0.5 \) when it becomes parallel. This can be interpreted as a transition between an infall of halos onto filaments at \( z > 0.5 \) and the streaming of halos along the filament axis at \( z < 0.5 \). The black line tracks the halos that are low in mass \( M < 10^{12.77} h^{-1} M_{\odot} \) at \( z=0 \) and the blue line is for the high mass halos \( M > 10^{12.77} h^{-1} M_{\odot} \). High mass halos transition from infalling to steaming slightly earlier than low mass halos and end up streaming with better alignment down the filaments. This is consistent with the results of Chapter 4, in particular Figure 8 of Trowland et al. (2013), which showed that high mass halos have better velocity alignment with filaments. That result was from taking all halos in filaments at each snapshot. By tracking the progenitors instead of simply taking all the halos in the snapshot, the infall phase of filament growth and the transition into streaming down the axis becomes apparent.

Another way to build the picture of halo infall onto filaments is to test for rotation of halos about the filament axis. Again the progenitors to halos in filaments at \( z=0 \) are tracked back in time and the filament axes are fixed to the orientation of the \( z=0 \) filaments; the rotation angle is illustrated in Figure 5.6. It is the angle about the axis of the filament that the halo has rotated through, compared to its position at \( z=0 \). Examples of the rotation angle history for a few halos are shown in Figure 5.11 and Figure 5.12. All of these halos but one have rotated by more than 70° since \( z=3 \). Rotating by more than

Figure 5.5: Median velocity alignment of halos over time. Taking all halos in filaments at \( z=0 \), their main progenitors are tracked. Black line is low mass halos at \( z=0 \) and blue line is high mass halos.
Figure 5.6: Rotation angle. The angle about the filament axis that a halo has rotated through since $z = 0$.

$90^\circ$ is quite common, it is achieved by $10.8\%$ of all halos. Rotating by more than $180^\circ$ is more rare, done by $1.3\%$ of the population and rotating more than a full circle is very rare, done by only $0.07\%$ of the population. It is not common for a halo to wind around a filament axis as it travels along.

Figure 5.7: The infall of halos onto filaments over cosmic history. The halos in filaments are found at $z=0$ and their progenitors are tracked back in time. $\theta$ is the angle between the orbital angular momentum (with respect to the filament axis) of halos in filaments and the axis of the filament. Since all the values of the median are below $\cos(\theta) = 0.5$, orbital angular momentum is aligned orthogonal to the axis. Error bars are the standard error of the median.

A characteristic of the way that halos infall onto filaments is the alignment of the halo’s orbital angular momentum with the filament axis. If the halos are orbiting the axis of the filament then the orbital angular momentum of halos in filaments is parallel.
to the filament axis \( (\cos(\theta) > 0.5) \). If the halos are instead proceeding directly along the axis of filaments or are falling onto the filament, the orbital angular momentum will be orthogonal to the axis \( (\cos(\theta) < 0.5) \). The halos in filaments are found at \( z=0 \) and their progenitors are tracked back in time to see how the alignment evolves as halos fall into filaments. The orbital angular momentum for halos in filaments was measured from the nearest point on the filament axis to the halo. The results are shown in Figure 5.7. For all redshifts, \( \cos(\theta) < 0.5 \) so the alignment is generally orthogonal which means that halos are not orbiting about the axis of the filament, confirming the result of the previous paragraph that halos generally do not rotate about the axis of filaments. At low redshift, the orthogonal alignment improves, which means that halos are either falling onto filaments or traveling onto their axis with less orbital motion.

A picture of how a typical halo moves with respect to a filament axis has emerged. At high redshifts the halo moves in closer to the axis and then after \( z=0.5 \) its motion is more along the axis than towards it. The halo doesn’t rotate much about the filament axis as it moves, it takes a direct path towards the axis and then towards the node. This simple picture of halo motion with respect to filaments is attractive, but it is anchored in the Millennium Simulation. Testing the genericness of this process in different simulations and in observational data is the next step in future work.
5.3 The Origins of Halo Spin

In ΛCDM the most massive structures grow through the continuous infall of subhalos. In this scenario the halo grows fast at high redshift, where major merger are more common due to the fact that the masses of the halos are lower at higher redshifts and the environment is generally denser, and the accretion slows down at lower redshifts, dominated by the infall of less massive structures. In addition, cosmological simulations show that about 40% of the material of a halo is accreted by smooth accretion (Genel et al., 2010).

A major merger can significantly change the angular momentum of a dark matter halo. Orbital angular momentum is transferred to the remnant’s internal angular momentum during a merger. Statistically, the net angular momentum of a large number of mergers would be zero (if infall occurs from random directions). However, due to the low number of major mergers that occur during a halo lifetime, randomization is ineffective and there remains an imprint on the halos spin by the final (and often only) major merger.

Studies on the effects of mergers and smooth accretion on halo spin have largely focused on the magnitude of angular momentum. For example, Peirani et al. (2004) found that halos that have experienced mergers end up with slightly more angular momentum than those that have only undergone smooth accretion, and D’Onghia and Burkert (2004) argued that halos with a quiet merging history might not acquire enough angular momentum to host late-type spiral galaxies. It is not clear what effect a halo’s merging history has on the direction of its spin. It was found that a major merger can cause a halo to completely flip its direction of spin (Bett and Frenk, 2012), although it is not necessary to undergo a merger to produce a spin flip. In this Section I look at how mergers and accretion events can affect change in the direction of the spin of dark matter halos.

5.3.1 The Infall of Subhalos Onto Halos

The way that halos merge can have a large impact on the halo’s angular momentum. It was found in Aubert et al. (2004) that the infall of subhalos takes place preferentially in the plane perpendicular to the direction defined by the spin of the halo. This study was restricted to halos of mass > 5 × 10^{12} h^{-1}M_\odot, where substructure was found using their own method, called ADAPTAHOP. I have done a similar study using the Millennium halos and subhalos.

For all the Millennium halos with more than 500 particles, their progenitors are tracked back in time in order to log the merger events. For each merger, the ‘mother’ is taken to be the higher mass halo and the ‘satellite’ is taken to be the lower mass halo. There is no mass limit on the satellite halos. For each merger, the dot product between the orbital angular momentum of the satellite halo and the spin of the mother halo is calculated in the snapshot immediately preceding the merger. All mergers, major and minor are included. If the dot product < 0.5 then the satellite halo is approaching from an orbit about the poles and if the dot product > 0.5 then the orbit is about the equator.

The average value of the dot product is <\cos \theta >= 0.5616 ± 0.0002 which favors the scenario of satellites orbiting the equator of the mother halo before merging. When the mother halos are split by mass, the low mass halos experience more equatorial mergers (<\cos \theta >= 0.5714 ± 0.0002) than high mass halos (<\cos \theta >= 0.5344 ± 0.0005). Low mass halos are those with mass less than the characteristic mass, \(M = 10^{12.77} h^{-1}M_\odot\) and high mass halos have mass greater than the characteristic mass.
A polar merger can be defined to be a merger that has \( \cos(\theta) < 0.5 \) and an equatorial merger is one with \( \cos(\theta) > 0.5 \). Halos that very recently experienced a polar merger (as measured in the snapshot after the merger) have spins that are more parallel aligned with filaments \( < \cos(\theta) > = 0.512 \pm 0.002 \) than halos that had equatorial mergers \( < \cos(\theta) > = 0.504 \pm 0.002 \). They have the same velocity alignment. This suggests that it is polar mergers, not the more common equatorial mergers, that are driving the perpendicular alignment of low mass halos, however since these measurements were taken soon after a merger, the halos are un-relaxed and the effects may be transitory.

Another way of approaching the same problem is to see how the satellites of Friends of Friends (FoF) halos behave. The orbital angular momentum of each subhalo within a FoF group was measured with respect to the central FoF halo. It was found that the alignment between the satellite’s orbital angular momentum and the central halo’s spin was \( < \cos \theta > = 0.6193 \pm 0.0004 \), clearly favoring orbits in the equatorial plane.

The orbit of a merging satellite about the equator of the mother can be understood in the same way as the orbit of moons about the planets of the solar system. Tidal forces between the mother and satellite create a bulge on the satellite. A satellite on an inclined orbit, passing above and below the equator of the mother, will have its tidal bulge constantly moving up and down. The movement of the bulge leads to friction. That friction acts to try to stop the satellite from going up and down, that is, to decrease its inclination, or to make it orbit around the equator of the mother.

The Infall of Subhalos Onto Halos in Filaments

In order to see how mergers are altering the spins of halos in filaments, I look at the trajectory of the satellite halo that is about to merge with the mother halo. As before, filament orientation is kept fixed at its \( z=0 \) direction and the properties are measured in the snapshot immediately preceding the merger event. The alignment of the satellite velocity with the filament axis is orthogonal, \( < \cos \theta > = 0.4925 \pm 0.0004 \). This goes against the general trend of halos streaming along filaments. The orbital angular momentum of the satellite, with respect to the mother, when it is just about to merge is parallel to the filament axis \( < \cos \theta > = 0.5037 \pm 0.0008 \). Mergers that involve satellites with parallel orbital angular momentum would cause the mother’s spin to become more parallel with the filament axis. From these results, it is expected that mergers should cause halos spin to become more parallel to filaments.

5.3.2 Changes in the Direction of Halo Spin

The effect of mergers on halo spin can be studied by tracking the orientation of the spin over a halo’s lifetime. Here, I focus on halos in filaments and track the alignment of dark matter halos with their host filament axis since \( z=3 \). To see the effect of mergers, the change in alignment is measured for a particular time interval, or ‘event’ in the halo’s life. If the event involved a large change in the halo’s mass then it is said to be a major merger, if not then it taken to be an accretion event. The fractional mass change is:

\[
\Delta \mu(t, \tau) = \frac{M(t) - M(t - \tau)}{M(t)}
\]  

(5.1)
Figure 5.8: The distribution of events over a time scale of $\tau = 0.6 Gyr$. The horizontal dotted line separates events that result in a parallel alignment (points above the line) and events that result in a more orthogonal alignment (below the line). The vertical dotted line separates events that end up with a higher halo mass (to the right) and a lower halo mass (to the left). The central region becomes too dense and is represented by a colour contour map instead of points.
and a major merger occurs when $\Delta \mu > 0.3$. The change in alignment over an event is

$$\Delta |\cos(\theta)| (t, \tau) = |\cos(\theta)|(t) - |\cos(\theta)|(t - \tau).$$

(5.2)

Where $\theta$ is the angle between the filament axis at $z=0$ and the halo’s main progenitor spin at time $t$. The filament axis is always kept to be at its orientation at $z=0$ so that progenitors always have an axis to measure their alignment from, even if they may not be in a filament in one particular snapshot.

As discussed in Bett and Frenk (2012), defining a time scale over which to measure an event is a difficult task. In that paper, the time $\tau = 0.5$ was used, which relates to the half mass radius for Milky Way sized halos. In my work, a time scale of 0.6Gyr is chosen. The lookback time between the Millennium snapshots varies from 0.19 to 0.38Gyr so the events can span over more than three snapshots. To find the halo property in question, it is linearly interpolated between snapshots before and after the time $t - \tau$. A typical time for a low redshift merger is about 3Gyr (McCavana et al., 2012), which means that events will not usually span the entire time of a merger.

Figure 5.9: The mean change in spin alignment per event of duration $\tau$. The red line is the average change for all events, the yellow line is for low mass halos that undergo a major merger during the event time and the blue line is for high mass halos that undergo a major merger. The thin shaded regions represent the standard error of the mean.

The events lasting 0.6Gyr are shown in Figure 5.8. There are a huge number of events for the progenitors of all halos in filaments at $z=0$ that have more than 500 particles. In the figure there are more events where $\Delta \mu > 0$ than $\Delta \mu < 0$ which reflects the tendency for halos to grow in mass. It is all but indistinguishable from the figure but there are slightly more events that leave the halo more parallel to its filament than events that leave the halo more orthogonal. $<\Delta |\cos(\theta)|> = 0.0116 \pm 0.0004$. In particular, events that are classified as major mergers have halo spins ending up significantly more parallel to filaments; the
difference in alignment for major merger events is $\langle \Delta | \cos(\theta) | \rangle = 0.052 \pm 0.001$. This effect is driven by the low mass halos (mass less than the characteristic mass at $z=0$, $10^{12.77} h^{-1} M_\odot$). For low mass halos and major merger events, $\langle \Delta | \cos(\theta) | \rangle = 0.065 \pm 0.002$ and for high mass halos $\langle \Delta | \cos(\theta) | \rangle = 0.031 \pm 0.001$. Low mass halos are more easily disturbed by mergers, so the effect is more pronounced.

These results show that halo spin generally becomes more parallel to filaments over time and this effect is driven by the low mass halos undergoing major mergers. The trend for halo spin to become more parallel to filaments can also be seen in Figure 5.10. The progenitors of halos in filaments at $z = 0$ are tracked back in time. The filament directions are kept at their $z = 0$ orientations and low and high mass halos are defined as having mass lower or higher than $10^{12.77} h^{-1} M_\odot$ at $z=0$. The spin of the progenitors of both low and high mass halos becomes more parallel with filaments over time. Low mass halos transition from orthogonal to parallel alignment earlier than high mass halos, at $z \approx 1$. The high mass halos transition from orthogonal to slightly parallel alignment at late times ($z \approx 0.1$). The trend for halo spin to become increasingly parallel is driven by accretion events, particularly for low mass halos.

![Figure 5.10](image.png)

Figure 5.10: The median alignment of the spins of halos in filaments over time. The halos in filaments are found at $z=0$ and their main progenitors are tracked to see how the average alignment changes. The black line is the low mass halos and the blue line is the high mass halos.

These results depend on the time scale chosen for events. When the time scale is varied (Figure 5.9) the average change in alignment varies significantly. For longer timesteps, the average change in alignment for all events increases. This is expected since it is a cumulative effect. For all values of timestep, the low mass halo merger events produce a more parallel alignment than a typical event.
5.3.3 A Typical Halo’s Life

An idea was suggested in Codis et al. (2012) that it is typical for low mass halos to form with a parallel spin alignment to filaments, which is preserved until $z=0$. The typical spin behavior for high mass halos is said to be the product of a merger between progenitors travelling along the filament. The orbital angular momentum of the progenitors is in the plane perpendicular to the filament since their motion is along it, resulting in a halo that has spin perpendicular to the filament. This idea was supported by the observed winding of dark matter particles about the axis of filaments in dark matter only N-body simulations.

This idea is not supported by the results in this Chapter. In Section 5.3.2 it was found that the spin of dark matter halos becomes more parallel to filaments after major mergers. It is not typical for a halo to end up with spin more orthogonal to the filament axis after a major merger.

This parallel alignment is a result of halos falling into the filaments instead of halos streaming along it. As shown in Section 5.3.1, infalling halos bring orbital angular momentum that is parallel to the filament axis, causing the merger remnant to spin more parallel to the filament than before the merger.

To further evaluate this model, I extracted halos from the milli-Millennium simulation and tracked their progenitors back in time to see what really constitutes a ‘typical’ halo. A few examples of halos in filaments that have experienced major mergers are shown in Figure 5.11. The main progenitors of the halo and the progenitors of the most recent major merger (defined by a mass ratio less than 10) are both displayed. The halos shown are a very high mass halo, a Milky Way mass halo and a low mass halo. All of these halos undergo a major merger between $z=1$ and $z=2$. The spin orientation with respect to the filament axis exhibits typical behavior; it varies a lot in the early stages of the halo’s life when it is low in mass and stabilizes somewhat later on (see Panel 2). For the highest and lowest mass halos, a merger occurs with a satellite that is traveling more orthogonally to the filament than the mother halo (see Panel 3). These mergers result in a halo that has its spin pointing more parallel to the filament axis ($|\cos(\theta)|$ increases). The Milky Way mass halo experiences a merger with a satellite that is traveling at about the same orientation to the axis. That merger caused the spin orientation to change from parallel alignment to orthogonal alignment, demonstrating that not all mergers cause a more parallel alignment of halo spin. The filament axis was kept at its $z=0$ orientation for these measurements.

Many halos have not experience major mergers since $z=3$. Figure 5.12 shows three examples of such halos. Although these halos have not had major mergers, their spin orientations are still quite variable over time. These variations could be caused by minor mergers, slow accretion and even close fly-bys of high mass halos.

These ‘typical’ halos were chosen to demonstrate some characteristic behaviors that are implied by the results of the previous sections. Any alignments in halo spin are very weak and thus are not expected to be obvious in one individual halo’s behavior.
Figure 5.11: The history of individual halos from the milli-Millennium simulation that have experienced major mergers since $z=3$. The halos masses at the top are at $z=0$. (1) The position of the halo at $z=0$ (red) and its progenitors at previous redshifts, with dot size representing mass and color representing redshift. The black line indicates the filament axis. (2) The alignment of the halo spin with the filament. Color is representative of halo mass. (3) The alignment of the halo velocity with the filament. (4) The rotation angle of the halo from its position at $z=0$ with respect to the filament axis.
Figure 5.12: The history of individual halos that have not experienced major mergers. The panels are the same as Figure 5.11.
5.4 Conclusions

In this Chapter, I have delved further into the mysteries of the origin of dark matter halo spin. The results of Chapter 4 showed that halo spin is an evolving property, intimately linked to the halo's environment. In order to find out what is causing this evolution, I looked at how halos are flowing onto and along filaments and how mergers affect spin orientation.

A simple model of halo flow in filaments was found. Over time, halos get increasingly close to the axis and nodes of filaments. At early times, the flow of halos is dominated by halos falling onto the filament and at late times the flow is along the filament axis towards the nodes. The halos generally proceed directly along their path towards the axis and the node, with no time spent rotating around the filament axis. Halos that form early on have had time to fall onto filaments and are streaming along the axis with better alignment than late forming halos.

The spin of dark matter halos is not dependent on their formation time. High mass halos form more recently than low mass halos but it is not this difference between the two groups that is responsible for their different spin orientation. Low mass halos always spin with more parallel alignment to filaments than high mass halos, regardless of their formation time. The halos that are closest to the axes of filament have the strongest spin alignment. It is not clear if this is because of the regular geometry there or the occurrence of anisotropic accretion.

The role of mergers and accretion in the evolution of spin orientation of dark matter halos in filaments was examined. It was found that satellite halos generally merge with the mother halo along her spin equator. This was found in the alignment of the satellite orbital angular momentum with the mother's spin \( \langle \cos \theta \rangle = 0.5616 \pm 0.0002 \) and also in the alignment of FoF satellite halos with the spin of the central halo in the FoF group's spin \( \langle \cos \theta \rangle = 0.6193 \pm 0.0004 \).

For halos in filaments, the orbital angular momentum of a satellite halo that is just about to merge is actually aligned parallel to the filament axis. This alignment is atypical for the entire halo population; halos in filaments generally have orbital angular momentum, as measured from the closest point on the filament, pointing orthogonal to the filament. This parallel orbital angular momentum alignment for halos that are just about to merge says that there is something special about these halos and could be the source of the evolution of spin alignment of halos in filaments.

The general trend for halos in filaments is for the spin to become increasingly parallel with the axis over time. This is true for both high and low mass halos although low mass halos transition from orthogonal to parallel spin alignment at an earlier time \( (z \approx 1) \). This trend is driven by major merger events. The average change in spin alignment with the filament axis for a major merger event (on a timescale of 0.6Gyr) is \( \langle \Delta | \cos(\theta) | \rangle = 0.052 \pm 0.001 \), and for all events the average change in alignment is \( \langle \Delta | \cos(\theta) | \rangle = 0.0116 \pm 0.0004 \). The spin of low mass halos in particular become more parallel to filaments after a major merger. The average change in alignment for low mass halos during a major merger event is \( \langle \Delta | \cos(\theta) | \rangle = 0.065 \pm 0.002 \).

A new model of the acquisition of halo spin in filaments has emerged. The initial spin of dark matter halos in filaments is set by Tidal Torque Theory, preferring an orthogonal orientation of halo spin with the axis. Over time, mergers with halos falling onto the filament cause the spin to become more and more parallel with the filament axis. Low
mass halos are more susceptible to having their spin orientation altered by a merger and become parallel to the filament sooner than high mass halos. The spin of high mass halos does eventually become parallel to filaments but it takes a longer time and more merger events. This simple model for the acquisition of spin for halos in filaments explains the differences found in Chapter 4 between the spin orientation of low and high mass halos.

This new model can be confirmed by further inquiry in the role of mergers and accretion in the build up of angular momentum. The role of slow accretion is not yet clear, as well as how minor mergers compare to major mergers. A drawback of the filament finding method used to get these results was that filaments must be found in each snapshot independently. This means that halos could be in a filament in one snapshot but not the next. In order to work around this, filaments were only found at $z=0$ and their orientation was held in place for earlier times. Although filament orientation is not expected to change a lot over time, this is a disadvantage. To better study the evolution of halo angular momentum in filaments, a new filament finding algorithm must be created that finds filament that are continuous through time.
Chapter 6

Conclusions and Future Work

*I cannot count my day complete
’Til needle, thread and fabric meet.
- Author Unknown.

This thesis has contributed to the understanding of the nature of dark matter halo spin and how angular momentum is acquired. This is important work on its own, but also because it adds to the understanding of the large scale structure of the Universe in general and the formation of galaxies. The angular momentum of dark matter halos in N-body simulations is intimately linked with the filamentary environment, and understanding exactly how it is linked can provide insights into the build up of filaments in the large scale structure. The formation of rotationally supported disk galaxies is a huge topic of research in modern astronomy and the way that angular momentum in a galaxy’s dark matter halo is acquired could have a huge impact on the way that disks are formed. Even early type galaxies exhibit properties that are dependent on angular momentum, as demonstrated by the differences between the fast and slow rotators. In the following Chapter, I summarize the achievements of the present work and suggest future directions to extend these results.

Dark matter halo angular momentum has been studied in this thesis within the context of the filamentary large scale structure. Filaments in the large scale structure can be intangible and ill-defined things. There is not solid definition of a filament, unlike a dark matter halo, which makes them particularly hard to pick out in N-body simulations. Many simple (Barrow et al., 1985; Stoica et al., 2005; Colberg et al., 2005; Hahn et al., 2007b; Zhang et al., 2009; González and Padilla, 2010) and more complex (Aragón-Calvo et al., 2007a,b; Sousbie et al., 2008; Forero-Romero et al., 2009; Aragón-Calvo et al., 2010) filament finders have been developed. I have further developed and implemented two simple filament finding algorithms. The Density Field method uses the Hessian of the density field to find filament regions. A filament is defined as being a region where the density field is saddle shaped. My implementation of this method was used in Chapter 4 of this thesis. The Cylinder Extraction method was based on the Candy method, described in Zhang et al. (2009) and the filaments found using this method were used in Chapter 5 of this thesis. These methods are useful and simple tools for studying the filamentary structure of the Universe but for more detailed future studies, one of the more complex filament finding algorithms could be used.

The published paper in Chapter 4 was a first look at the alignments and evolution of dark matter halo spin. This paper used the Millennium N-body simulation to track
the evolution of dark matter halo angular momentum alignments with the large scale structure, mutual halo spin alignment and the evolution of the spin parameter. The Density Field method was used to find filaments of $2 \, h^{-1}$Mpc in scale in the large scale structure. The main results from this paper are listed:

- There is a difference in the alignment of halo spin with the axis of filaments between high and low mass halos. The spin vectors for low mass halos were parallel to filaments at $z=0$ while the spins of high mass halos were orthogonal.

- This alignment evolved over time; the spins of all halos tended to become increasingly parallel to filaments as time went on. An explanation for this difference between low and high mass halos was found in Chapter 5, as well as a reason why halo spins would tend to become more and more parallel to filaments.

- Filaments are regions of bulk flow. Dark matter halos flow along the axis of filaments, towards massive clusters. When this bulk flow is subtracted, an infall of halos orthogonal to the axis remains. The bulk flow suggests an enlargement of filaments over time. This was implied by looking at the bulk flow for filaments found at different scales.

- Filaments at large smoothing lengths at low redshift contain halos with the best aligned spins and bulk motion, while at high redshift it is filaments at small smoothing lengths that contain the best aligned halos. This shows that filaments are growing in size over time. Because of the nature of the way that the filaments were found (using Gaussian smoothing), this enlargement tells more about the width of the filaments rather than the length.

- An alignment was found between the spin vectors of very close ($< 0.3 \, h^{-1}$Mpc) neighboring halos. These nearby neighbors showed a slight parallel alignment, but only when subhalos were included in the sample. There are no alignments between neighboring Friends-of-Friends groups.

- The spin parameter is not currently mass dependent but it has been in the past. The spin parameter is a measure of how well a halo is supported by angular momentum and follows a log-normal distribution. It was found that the center of the spin parameter distribution is mass dependent at high redshift but not mass dependent at $z=0$. Further research needs to take place in order to find the reasons for this mass and redshift dependence.

A preliminary study of what results a future galaxy survey using an instrument like HECTOR (Lawrence et al., 2012) could expect was conducted at the end of Chapter 4. I found that more than 60,000 galaxies in filaments would need to be targeted in order to see an alignment of spin orientation with the large scale structure. This study compared the two filaments finding methods on a mock light cone of spiral galaxies where it was assumed that the circular velocity of the galaxies could be measured. Both methods found that the spins of more than 60,000 galaxies would need to be measured in order to find a statistically significant parallel alignment with filament axes. Further studies using mock galaxy catalogs should be conducted to determine the best possible target selection for future galaxy surveys. More complex filament finding methods could be used to refine
the filament selection. It was found in Chapter 5 that halos close to the axis of filaments exhibit a stronger parallel alignment, and this should be taken into account for target selection. Other factors should also be considered for target selection, such as galaxy color, luminosity and morphology. Further studies with mock galaxy catalogs could bring down the number of galaxies needed to be observed and increase the significance of any alignments measured.

In Chapter 5 of this thesis, explanations for the results of the previous Chapters were sought after. Primarily, an explanation for the difference in spin orientation between low and high mass halos (found in Chapter 4) was needed. The results from this chapter were found using the Millennium simulation and the Cylinder Extraction method of finding filaments. The main results from this Chapter are listed:

- Over time, halos get increasingly close to the axis and nodes of filaments. At early times, the flow of halos is dominated by halos falling onto the filament and at late times the flow is along the filament axis towards the nodes. The halos generally proceed directly along their path towards the axis and the node, with no time spent rotating around the filament axis. Halos that form early on have had time to fall onto filaments and are streaming along the axis with better alignment than late forming halos.

- The difference between the spin orientation of high and low mass halos with filaments is not caused by the gap in formation times between low and high mass halos. Low mass halos form earlier than high mass halos, giving them more time between their formation and $z=0$ to be disturbed by accretion than high mass halos. This is not the effect responsible for their different spin alignment with filaments though; low mass halos of all formation times are aligned parallel to filaments and even early forming high mass halos are aligned orthogonal to filaments.

- It was found that satellite halos generally merge with the mother halo along her spin equator. This was found in the alignment of the satellite orbital angular momentum with the mother’s spin and also in the alignment of FoF satellite halos with the spin of the central halo in the FoF group’s spin.

- For halos in filaments, the orbital angular momentum of a satellite halo that is just about to merge is actually aligned parallel to the filament axis. This alignment is atypical for the entire halo population; halos in filaments generally have orbital angular momentum, as measured from the closest point on the filament, pointing orthogonal to the filament.

- The general trend for all halos in filaments is for the spin to become increasingly parallel with the axis over time. This trend is driven by major merger events when halo spin increases in parallel alignment significantly, especially for low mass halos. The average change in spin alignment with the filament axis for a major merger event (on a timescale of 0.6Gyr) is $< \Delta |\cos(\theta)| = 0.052 \pm 0.001$, and for low mass halos during a major merger event it is $< \Delta |\cos(\theta)| = 0.065 \pm 0.002$. Low mass halos are more vulnerable to the disturbances of mergers and their spin orientation becomes parallel sooner than high mass halos.
A new model of the acquisition of halo spin in filaments has emerged. The initial spin of dark matter halos in filaments is set by Tidal Torque Theory, preferring an orthogonal orientation of halo spin with the axis. Over time, mergers with halos falling onto the filament cause the spin to become more and more parallel with the filament axis. Low mass halos are more susceptible to having their spin orientation altered by a merger and become parallel to the filament sooner than high mass halos. The spin of high mass halos does eventually become parallel to filaments but it takes a longer time and more merger events. This simple model for the acquisition of spin for halos in filaments explains the differences found in Chapter 4 between the spin orientation of low and high mass halos.

This new model can be confirmed by further inquiry in the role of mergers and accretion in the build up of angular momentum. The role of slow accretion is not yet clear, as well as how minor mergers compare to major mergers. A drawback of the filament finding method used to get these results was that filaments must be found in each snapshot independently. This means that halos could be in a filament in one snapshot but not the next. In order to work around this, filaments were only found at z=0 and their orientation was held in place for earlier times. Although filament orientation is not expected to change a lot over time, this is a disadvantage. To better study the evolution of halo angular momentum in filaments, a new filament finding algorithm must be created that finds filament that are continuous through time.

This thesis presents a new model of the acquisition of halo spin in filaments. The initial spin of dark matter halos in filaments is set by Tidal Torque Theory, preferring an orthogonal orientation of halos spin with the axis. Over time, mergers with halos falling onto the filament cause the spin to become more and more parallel with the filament axis. Low mass halos are more susceptible to having their spin orientation altered by a merger and become parallel to the filament sooner than high mass halos. This simple model for the acquisition of spin for halos in filaments explains the differences between the spin orientation of low and high mass halos.

This merger scenario for the acquisition of spin could be confirmed in further analysis of N-body simulations. The different roles of smooth accretion, minor and major mergers is not yet clear. More research needs to be done to uncover what the consequences of this model are for galaxy formation and how it could cause the differences between fast and slow rotating ellipticals.
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Chapter 7

Appendix

The following paper,
The Cosmic History of the Spin of Dark Matter Halos within the Large-scale Structure
has been reproduced in Chapter 4 and is printed in its original format here.
THE COSMIC HISTORY OF THE SPIN OF DARK MATTER HALOS WITHIN THE LARGE-SCALE STRUCTURE

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ABSTRACT

We use N-body simulations to investigate the evolution of the orientation and magnitude of dark matter halo angular momentum within the large-scale structure since $z = 3$. We look at the evolution of the alignment of halo spins with filaments and with each other, as well as the spin parameter, which is a measure of the magnitude of angular momentum. It was found that the angular momentum vectors of dark matter halos at high redshift have a weak tendency to be orthogonal to filaments and high-mass halos have a stronger orthogonal alignment than low-mass halos. Since $z = 1$, the spins of low-mass halos have become weakly aligned parallel to filaments, whereas high-mass halos kept their orthogonal alignment. This recent parallel alignment of low-mass halos casts doubt on tidal torque theory as the sole mechanism for the buildup of angular momentum. We see evidence for bulk flows and the broadening of filaments over time in the alignments of halo spin and velocities. We find a significant alignment of the spin of neighboring dark matter halos only at very small separations, $r < 0.3 \, \text{Mpc} \, h^{-1}$, which is driven by substructure. A correlation of the spin parameter with halo mass is confirmed at high redshift.

Key words: cosmology: theory – large-scale structure of universe

Online-only material: color figures

1. INTRODUCTION

The large-scale structure of the universe observed today has formed by a long history of gravitational collapse, gradual accretion, and mergers. Through these processes a filamentary, sponge-like structure has emerged. The distribution of galaxies and their motions provides clues on how they formed, and together with galactic angular momentum data, the emergence of the intricate large-scale structure can begin to be explained.

Before we can determine what spin tells us about the formation of large-scale structure, the mechanisms of angular momentum buildup need to be well understood. The initial spin of early dark matter proto-halos can be predicted analytically (White 1984); however, these predictions are largely limited to the regime of linear structure formation. To track the angular momentum buildup through more recent cosmic history, N-body simulations of cold dark matter must be used. These simulations give full information on the dark matter halos which can be used to form a hypothesis on the buildup of galaxy angular momentum on cosmological scales. However, on cosmological scales it is not yet feasible to simulate the gas component to track the angular momentum buildup of galaxies directly (although Hahn et al. 2010 simulated 100 disk galaxies in a filament to find an alignment of galaxy spin with filaments).

Hydrodynamical simulations on individual galaxy scales (van den Bosch et al. 2003; Sharma & Steinmetz 2005; Bett et al. 2010) have shown that the specific angular momentum of baryons remains close to that of dark matter and that the galaxy angular momentum is generally about $2^\circ$ misaligned with the dark matter halo. This means that dark matter halo spin is a fairly good proxy for galaxy spin, so some understanding of the spins of galaxies may be gleaned from dark matter-only simulations. The spin of a dark matter halo depends mainly on two things: the initial torques driven by the surrounding landscape at early times, and the accretion and merger history of the halo.

The initial spin of dark matter halos is given through a mechanism known as “tidal torque theory,” pioneered by Hoyle (1949), Peebles (1969), and Zel’Dovich (1970). This theory proposes that the initial spin of a proto-halo early in its formation in the linear regime of structure formation depends on its shape and the tidal forces exerted from the surrounding structure, so the spin is dependent on the local dark matter landscape. The greatest effects of tidal torquing happen at the time of turnaround, just before the proto-halos have collapsed to virialized objects. A halo that was torqued in this manner should retain some memory of the tidal field where it formed, and this has been confirmed through N-body simulations and galaxy catalogs (e.g., Lee & Pen 2001; Porciani et al. 2002; Lee & Erdogdu 2007). The cosmic web is the manifestation of the tidal field, filaments in particular are regular, symmetric morphologies which on large scales exhibit a uniform tidal field. Thus, it is expected that the orientation of halo spin today should retain some correlation with the direction of filaments and halos should be aligned with each other over short distances.

Since the epoch of tidal torquing, halo spins have been substantially influenced by mergers and accretion. It was shown in Bett & Frenk (2012) that it is not uncommon for the direction of the spin of a halo to completely flip over in its lifetime and this phenomenon is caused by minor and major mergers and even close halo flybys. Satellite accretion has been proposed to be the main contributor of angular momentum and it has been shown that by neglecting tidal torques and considering mergers alone the distribution of the magnitude of spin can be reproduced (see Gardner 2001; Vitvitska et al. 2002; Maller et al. 2002).

To figure out how accretion has influenced dark matter halo spin and what spin can reveal about the formation of large-scale structure, several authors have investigated an alignment of spin with the cosmic web using N-body simulations and galaxy catalogs. In simulations, it has been found that spins are aligned on shells around voids, lying preferentially on the void surface (Brunino et al. 2007; Cuesta et al. 2008). It has been shown that spins lie preferentially in the plane of sheets in simulations (Navarro et al. 2004) and along the axis of filaments.
Since any relic alignments of spin with the large-scale structure are expected to be weak, a large simulation volume and high resolution are needed. To this end, the publicly available Millennium simulation of Springel et al. (2005) was used. This simulation is of a cubic volume 500 Mpc $h^{-1}$ on a side containing $2160^3$ particles using the GADGET-2 code (Springel 2005). This gives a particle mass of $8.6 \times 10^8 M_{\odot}$ $h^{-1}$. A ΛCDM cosmology is chosen and the parameters are $\Omega_m = 0.25$, $\Omega_b = 0.045$, $\Omega_{\Lambda} = 0.75$, $h = 0.73$, $n = 1$, and $\sigma_8 = 0.9$.

The halo catalog was built by Springel et al. (2005) by first using the simple friends-of-friends group (FOF) finder (Davis et al. 1985) to attempt to select structure in the particle distribution and then finding the virialized subhalos within the FOF groups using SUBFIND (Springel et al. 2001). The SUBFIND algorithm first identifies subhalo candidates within each FOF halo using dark matter density and then removed particles that are not gravitationally bound to the subhalo candidate. The most massive subhalo typically contains most of the mass of the corresponding FOF object, and so can be regarded as the self-bound background halo itself, with the remaining subhalos as its substructure. The halo catalog used in this paper includes all virialized halos, including subhalos, although spin measurements are only made on halos with more than 500 particles in order to minimize random effects from outer halo particles. There are 184,891 FOF halos and 213,799 halos in total.

For this analysis, a 300 Mpc $h^{-1}$ section of the full Millennium simulation was used. This smaller section was chosen so that the resolution of the density field was high enough to be able to find features in the large-scale structure. This was tested using several 100 Mpc $h^{-1}$ sample cubes. As the resolution of the density field was raised from $128^3$ to $1024^3$ voxels, the alignment between halo spin and the resulting filaments became stable above a certain threshold. For smoothing lengths 2.0, 3.5, and 5.0 Mpc $h^{-1}$ (Gaussian smoothing is used for finding filaments on different scales, see Section 2.3), the minimum resolution for stable features is 0.4 Mpc cell$^{-1}$. For a grid of 1024$^3$ voxels, the maximum box size is 400 Mpc $h^{-1}$. To ensure that the resolution was more than sufficient, a box of size 300 Mpc $h^{-1}$ was chosen. For smoothing on 1.0 Mpc $h^{-1}$ scales, a finer grid must be used and the maximum cell size is 0.2 Mpc, so a 200 Mpc $h^{-1}$ box was used for this scale. At smaller scales than 1 Mpc $h^{-1}$ the box size required is too small, so there are not enough halos for useful results. The following results display no cosmic variance when a different sample of the same size is chosen. There are 4,027,242 halos in our 300 Mpc $h^{-1}$ box and 932,961 halos with more than 500 particles from which a reliable spin measurement could be made. The halos in a 5 Mpc slice through the simulation volume are shown in Figure 1.

Snapshots are taken at several points throughout the simulation. Here we have used the snapshots at redshifts 0, 0.99, 2.07, and 3.06 (rounded to 0, 1, 2, and 3).

2.2. Characteristic Mass

In structure formation, there is a characteristic mass scale for collapse, $M_c(z)$. A spherical top-hat perturbation collapses when its linear overdensity exceeds a value of $\delta_c = 1.686$. The variance of linear density fluctuations at a given mass scale $M$ is related to the linear power spectrum $P(k, z)$ at redshift $z$ by

$$\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k, z) \overline{W}_\text{TH}^2(k, M),$$

where $\overline{W}_\text{TH}(k, M)$ is the Fourier transform of a spherical top-hat window function of comoving size $R = (3M / 4\pi \bar{\rho})^{1/3}$, and $\bar{\rho}$ is the mean density.
and \( \bar{\rho} \) is the comoving mean mass density of the universe. At a given redshift, the typical mass scale \( M_\star(z) \) to collapse from a 1\( \sigma \) fluctuation is hence given by the implicit solution of

\[
\sigma(M_\star, z) = \delta_c. \tag{2}
\]

The calculated values of characteristic mass at redshifts 0, 1, 2, and 3 are 5.89, 0.273, 0.0132, and \( 4 \times 10^{-3} \), respectively, in units of \( 10^{12} M_\odot \).

### 2.3. Quantifying the Large-scale Structure

Morphological features in large-scale structure may be classified into four general categories: blobs, filaments, sheets, and voids. This analysis uses the curvature of the density field to identify each of these features in \( N \)-body simulations.

First, the density field is obtained using the Delaunay Tessellation Field Estimator (DTFE) method using the dark matter halo distribution (see van de Weygaert & Schaap 2000; Schaap 2007). The DTFE method can be summarized in three steps: (1) from the distribution of points the Delaunay tessellation is constructed, which is a volume covering division of space into mutually distinct Delaunay tetrahedra. A Delaunay tetrahedron is defined by the set of four points whose circumscribing sphere does not contain any of the other points in the generating set. (2) The local density at each point is calculated from the volume of the Voronoi cells (the dual of the Delaunay tessellation) and the mass of the contained halo. (3) The density within each Voronoi cell is interpolated, assuming the density field varies linearly. The DTFE method is useful when looking for geometrical features in the density field because it automatically adapts to variations in density and geometry.

The DTFE was carried out with vacuum boundary conditions and a buffer region around the box. This buffer region was made to be at least as big as the maximum distance between nearest neighbor halos so that no Voronoi cells constructed leaked outside the filled region. For larger smoothing scales, the buffer was at least as big as 2\( \sigma \). For the 2 and 3.5 Mpc scales the buffer was 7 Mpc and for the 5 Mpc scale the buffer was 10.5 Mpc. The buffer region was also used in the smoothing of the density field then discarded.

Smoothing the density field to some scale \( s \) is done by convolving with a spherically symmetric Gaussian filter:

\[
\rho_s(x) = \int dy \rho(y) G_s(x, y). \tag{3}
\]

Here, \( \rho(y) \) is the Fourier transform of the DTFE density and the Gaussian filter at scale \( s \) is defined by

\[
G_s = \frac{1}{(2\pi \sigma_s^2)^{3/2}} \exp\left(-\frac{(y-x)^2}{2\sigma_s^2}\right). \tag{4}
\]

The curvature of the density field is given by the Hessian matrix of second derivatives at each point:

\[
H_{s\beta} = \frac{\partial^2 \rho_s(x)}{\partial x_\alpha \partial x_\beta}. \tag{5}
\]

The second derivatives can be found while simultaneously smoothing the field by making use of an identity of the convolution:

\[
\frac{\partial}{\partial x_\alpha}(f * g) = \frac{df}{dx_\alpha} * g = f * \frac{dg}{dx_\alpha}. \tag{6}
\]

Applying this to Equation (3) gives

\[
\frac{\partial^2 \rho_s(x)}{\partial x_\alpha \partial x_\beta} = \int dy \rho(y) \frac{\partial^2}{\partial x_\alpha \partial x_\beta} G_s(x, y). \tag{6}
\]

Thus, the Hessian of the smoothed density field is simply given by the convolution of the DTFE density and the second derivative of the Gaussian (the so-called Mexican Hat wavelet):

\[
H_{s\beta} = \frac{1}{\sigma^2} \int dy \rho(y) \left[ (x_\alpha - y_\alpha)(x_\beta - y_\beta) - \delta_{\alpha\beta}\sigma^2 \right] G_s. \tag{7}
\]
The eigenvalues of the Hessian quantify the curvature of density at a particular point, in the direction of the corresponding eigenvector. A positive eigenvalue indicates that the shape of the density field is concave up and a negative is concave down. The density field may now be classified uniquely into blob, filament, sheet, or void regions according to the eigenvalues of this Hessian. The eigenvalue sign criteria for each region is as follows:

- **Blob**: All negative
- **Filament**: Two negative, one positive
- **Sheet**: Two positive, one negative
- **Void**: All positive

It can be useful to classify every point into one of these features as was done in Zhang et al. (2009), and an alternative approach is to pick out only the best features like in Aragón-Calvo et al. (2007a). The decomposition of volume into features is shown in Figure 1 on the scale of 2 Mpc $h^{-1}$. The filament and sheet morphologies dominate the volume, with blob regions taking up the least volume. The relative volume fractions do not change much over scale.

Morphological features are defined using only the eigenvalues of the Hessian. The direction of the eigenvectors are also used to assign a directionality to filaments and sheets. The direction of the axis of a filament is the direction of the positive eigenvalue, and the normal direction of a sheet is the direction of the negative eigenvalue. The features discussed in this paper have been found choosing the smoothing scales of 2.0, 3.5, and 5.0 Mpc $h^{-1}$. These scales have been chosen to match with the visual classification of structure at 2 Mpc $h^{-1}$ (Hahn et al. 2007b) and to explore the scales above that. The comoving smoothing scales are kept constant for different redshifts in order not to bias the results with preconceived assumptions about filament formation.

This feature-finding algorithm uniquely identifies regions into blob, filament, sheet, or void depending only on the scale and quality of features required.

3. ALIGNMENT OF HALO SPIN WITH THE COSMIC WEB

Halo particles can be loosely bound, following stochastic paths, but adding up each particles angular momentum gives the net effect of a halo spin. Spin is calculated by adding up the angular momentum of each particle ($i$) in the halo, simply defined as the cross product of the distance of the particle from the halo’s center of mass ($r$) and the particles velocity ($v$) with respect to the center of mass:

$$ J = \sum_{i=0}^{N} \mathbf{r}_i \times m_i \mathbf{v}_i. $$

(8)

In order to get a reliable measurement of halo spin, only the halos with more than 500 particles have been included. The unit spin vectors are shown in the top panel of Figure 2 but there is no obvious alignments with each other or with the large-scale structure (as defined by the axis of filaments, shown on the bottom panel).

From the tidal torque theory (TTT), the spin of dark matter halos is expected to be correlated with the local tidal field ($\mathbf{T} = T_{ij} \equiv \partial_i \partial_j \phi$) and the inertia tensor ($\mathbf{I} = I_{ij}$). During the linear regime (assuming that $\mathbf{T}$ and $\mathbf{I}$ are uncorrelated), the first order result from TTT (White 1984) is

$$ J_i \propto \epsilon_{ijk} T_{jk} I_{ik}. $$

(9)

Figure 2. Direction of dark matter halo spin vectors (top), velocity vectors (middle), and filament axis (bottom). The velocities show a coherent flow along filament axis whereas spin vectors are much more random and not obviously aligned. Shown is a slice of the simulation $100 \times 100 \times 5$ Mpc $h^{-1}$ and all vectors have been normalized to have the same length.
where $\epsilon_{ijk}$ is the Levi-Civita symbol. In the principal axis frame of the tidal tensor, where $\lambda_i$ are the eigenvalues of the tidal field:

$$
J_1 \propto (\lambda_2 - \lambda_3) I_{23}
$$

$$
J_2 \propto (\lambda_3 - \lambda_1) I_{31}
$$

$$
J_3 \propto (\lambda_1 - \lambda_2) I_{12}
$$

$\lambda_3 \leq \lambda_2 \leq \lambda_1$ so $\lambda_3 - \lambda_1$ is the largest coefficient, making $J_2$ the largest component of $J$ so that spin is preferentially aligned with the second eigenvector of the tidal field. The cosmic web is a manifestation of the potential $\phi$, related by the Poisson equation, $\nabla^2 \phi = 4\pi G \rho(x)$. Our definition of a filament (having two negative eigenvectors of the Hessian of density) translates into a region where there are two positive eigenvectors of the tidal tensor. The second eigenvector of the tidal field points in a direction orthogonal to the filament (the minor axis of the tidal field is the axis of the filament) and so we expect that halo spin should point in a direction orthogonal to the axis of the filament.

The result from TTT in Equation (9) assumes that $T$ and $I$ are completely uncorrelated, which has been shown not to always be true (Lee & Pen 2000; Porciani et al. 2002). If there is some correlation, the preferred direction of halo spins discussed above may be a small effect. The alignment would also be greatly affected by merger and accretion events that have happened during nonlinear structure growth.

An expression for the relation between the unit spin vector $(\hat{J})$ and the unit traceless tidal field $(\hat{T})$ was proposed in Lee & Pen (2000, 2001):

$$
(J_i \hat{J}_j | T) \equiv \frac{1 + c}{3} \delta_{ij} - \hat{c}_i \hat{c}_j,
$$

where $c \in [0, 3/5]$ is the correlation parameter to measure the strength of the intrinsic spin-shear alignment with the nonlinear modifications taken into account. When $c = 0$ it corresponds to the case when nonlinear effects have completely broken down initial spin-shear correlations and when $c = 3/5$ it is the ideal case when $I$ is independent of $T$.

Lee et al. (2005) derived an expression using Equation (10) for the Probability Density Function (PDF) of the orientations of the galaxy spin vectors relative to the tidal spin tensors:

$$
P(\cos \alpha, \cos \beta, \cos \theta) = \frac{1}{2\pi} \prod_{i=1}^{3} \left(1 + c - 3c\lambda_i^2\right)^{-1/2} \times \left[\frac{\cos^2 \alpha + \cos^2 \beta + \cos^2 \theta}{1 + c - 3c\lambda_i^2 + 1 + c - 3c\lambda_i^2 + 1 + c - 3c\lambda_i^2}\right]^{-3/2},
$$

where $\lambda_i$ are the eigenvalues of $\hat{T}$, and $\alpha$, $\beta$, and $\theta$ are the angles between the unit spin vector and the major, intermediate, and minor axes of the tidal field, respectively.

To quantify the preferred alignment of halo spins orthogonal to filament axis, we calculate $P(\cos \theta)$ which is the PDF of the cosine of the angle between spin axis and the minor axis of the tidal field that defines the axis of filaments. Filament regions are defined as having two positive and one negative eigenvalue. They also must satisfy the traceless condition of $\sum \lambda_i = 0$ as well as the unit magnitude condition of $\sum \lambda_i^2 = 1$. Therefore the eigenvalues in filament regions can be approximated by $\lambda_1 = \lambda_2 = 1/\sqrt{6}$ and $\lambda_3 = -2/\sqrt{6}$. Using these values in Equation (11) gives

$$
P(\cos \theta) = (1-c) \sqrt{1 + c} \left[1 - c \left(1 - \frac{3}{2} c \cos^2 \theta\right)\right]^{-3/2}.
$$

If halo spins are oriented randomly then $c = 0$ and the PDF is flat. If halo spins are preferentially orthogonal to filaments then $c > 0$ and the function increases with $\cos \theta$. Although the tidal torque theory restricts $c$ to positive values, other effects could be in play that cause halo spins to be aligned parallel with filaments, which would cause a negative value of $c$.

3.1 Alignment of Halo Spin and Velocity with Filaments

The alignment between a filament and the spin of the halos that make it up is simply given by the cosine of the angle $\theta$ between the two vectors and the absolute magnitude is taken because the filament is only defined by an axis, not a particular direction. The distribution of $|\cos \theta|$ for all halos in filaments at redshifts 0 and 3 is shown in Figure 3 where the number of halos in each bin of $|\cos \theta|$ is normalized to make the area under the graph unity. The shape of this distribution can be quantified in two ways; the median value or by fitting a function to the curve.

Since the distributions shown in Figure 3 are clearly non-Gaussian, the median rather than the mean would be the more useful statistic (although the mean was used by, e.g., Zhang et al. 2009 and Aragón-Calvo et al. 2007b). The standard error of the median was found by bootstrap resampling and finding the standard deviation of the resampled medians. The distributions can also be fitted to the PDF of Equation (12) to find the correlation parameter $c$ of the intrinsic spin-shear alignment which characterizes the shape of the distribution. The fit was done using a Markov chain Monte Carlo (MCMC), and two examples of such a fit are shown as the red lines in Figure 3. The two methods above are compared in Figure 4 for some example points (halos in mass bins at $z = 0$, scale = 2.0 Mpc which are the same mass bins as the red line in Figure 5). There is a one-to-one correlation of the two parameters, so either could
be used. We have chosen to use the correlation parameter $c$ in this paper since it is theoretically motivated by TTT.

The value of $c$ indicates the strength of the alignment of halo spins with the orientation of filaments, and also the intrinsic alignment of spin with the tidal field. If the halos generally have spins parallel to filament axis, $c$ is negative, and conversely, if the halo spin are generally orthogonal to filament axis, then $c$ will be positive. The error of $c$ is the standard deviation of the value that maximizes the likelihood of the fit of the PDF to the distribution. From the value of $c$ found for all the halos at $z = 0$ ($c = -0.035 \pm 0.004$) and for $z = 3$ ($c = 0.129 \pm 0.009$), the general trend is that halos are aligned orthogonal to filaments at high redshift and aligned parallel at low redshift.

The alignment of halo spin vectors with filaments is shown in Figure 5. The alignment distribution has been fitted to find $c$ for halos in bins of mass and for halos at different redshifts. For all smoothing scales, it can be seen that at $z = 0$ the alignment is weakly parallel (negative $c$) for low-mass halos in filaments (mass less than about $M_\star = 5.89 \times 10^{12} M_\odot$) and orthogonal (positive $c$) for high-mass halos. This is illustrated in Figure 6. At higher redshifts, the alignment becomes more orthogonal for all halo masses. There are fewer halos in the high-mass bins at high redshift because the high-mass halos have not yet had time to form. The result of Faltenbacher et al. (2002), Aragón-Calvo et al. (2007b), Hahn et al. (2007b), and Zhang et al. (2009) that halo spins generally lie along the axis of filaments is driven by the low-mass halos at $z = 0$. This is demonstrated in Figure 3 where the alignment distribution for all halos at $z = 0$ is shown. The alignment is preferentially parallel because of the high number of low-mass halos that exhibit parallel alignment.

The affects of smoothing scale on the halo spin alignment with filaments show something about the formation of filaments. For redshift 0 (the red line in Figure 5), halos seem to be best aligned at a large smoothing scale while high-redshift halos are best aligned at small scales. If an orthogonal alignment is an indicator that a halo formed inside a filament topology, then this shows that filaments grow in size over time.

Figure 7 shows the effect of taking into account the characteristic mass. Here we can compare halos between redshifts at equivalent stages of collapse. When the this is accounted for, almost all the points overlap within their errors. This means that halos at a similar stage in their collapse have the same degree of preferential alignment with filaments over cosmic time. A halo that is just starting to collapse ($M = M_\star$) at redshift 2 has a similar probability of orthogonal alignment with its filament as a halo that is just starting to collapse at redshift 1 or 0. However, no assumptions were made about the evolving scale of filaments and the smoothing scale was kept constant at 2.0 Mpc. Even with a constant scale, this similarity between alignments at different times shows that the buildup of spin is closely linked with a halo’s formation.

When substructure is discounted by taking the most massive subhalo in each FOF group, there is practically no change in the alignments.

Although the $c$ parameter was introduced in the context of spin alignments with the tidal field (manifested by filaments in the large-scale structure), it can also be used as a more general measure of alignment. The distributions of $|\cos \theta|$ where $\theta$ is the angle between halo center of mass velocity and filament axis is also well-fit by the PDF in Equation (12). Again, a negative value of $c$ means a parallel alignment and a positive value is orthogonal alignment.

All panels of Figure 8 show a parallel alignment that is stronger for high-mass halos. This shows streaming of halos of all masses down filaments into massive clusters.

This streaming can be seen in the velocity vectors of halos in some filaments in the middle panel of Figure 2, where vectors are pointed along filaments toward clusters. However, some filaments display bulk motions where the entire filament is moving toward some attractor. To see the extent of these bulk motions, they have been subtracted in Figure 9 by subtracting the mass-weighted average velocity of halos by halo mass found within the smoothing scale on which the filaments were found. When bulk motions are discarded, an orthogonal motion remains. The apparent streaming of halos down filaments was wholly caused by bulk motions of entire filaments, and this bulk flow is generally along the axis of filaments. The relative motions can be seen in Figure 10 in the alignment of halo velocity with the flow of the local bulk motion. (Bulk motions have been subtracted from halo velocities here.) Low-mass halos are moving slightly orthogonal to the flow and high-mass halos have no preferred direction of motion. This reflects how bulk motions have been removed: high-mass halos were given more weight than low-mass halos and so the residual motions of high-mass halos once bulk flow is removed is minimal.

The enlargement of filaments over time that was seen in the spin alignments is also visible in the way the bulk flows are aligned. The low-mass halos at $z = 0$ (red line in Figure 8) are more strongly aligned at large smoothing scales and the low-mass halos at high redshifts are most aligned at small smoothing scales. If filaments are chutes where halos are channeled into clusters, then these low-mass halos are evidence for the growth of the size of filaments over time. The high-mass halos on the other hand are generally less aligned at large smoothing scales for all redshifts which is seen as a flattening of the curves. This may be due to the inclusion of some cluster halos.
Figure 5. Alignment of dark matter halo spin with filaments over cosmic time. Alignment is characterized by the parameter $c$ of the fit of Equation (12) to the distribution of $|\cos \theta|$, where positive $c$ indicates orthogonal alignment and negative $c$ indicates parallel alignment. The panels show filaments found in different smoothing scales: 1.0 (top left), 2.0 (top right), 3.5 (bottom left), and 5.0 Mpc $h^{-1}$ (bottom right). At high redshifts all spins are orthogonal to filaments, but recent times low-mass halos have a parallel alignment with filaments. The dashed line is the expected distribution for random halo spins, and the shaded regions are the 1$\sigma$ errors. The red line is for $z = 0$, yellow line is $z = 1$, blue is $z = 2$, and green line is $z = 3$.

(A color version of this figure is available in the online journal.)

when the smoothing scale is broadened which would introduce random velocities into the sample.

Although both halo spin and velocity are somewhat aligned with filaments, these alignments are not strong enough so that there is a significant alignment between a halo’s spin and velocity.

### 3.2. Halo–Halo Spin Alignment

Tidal torque theory predicts that as well as being aligned with the large-scale structure, halo spins should be aligned with each other. This is usually tested by simply taking the average of the dot product of pairs of halo spins separated by distance $r$:

$$ \eta(r) = \langle |\hat{J}(x) \cdot \hat{J}(x + r)| \rangle. $$

A second quantity used by Pen et al. (2000) and Bailin & Steinmetz (2005) is

$$ \eta_2(r) = \left( \langle |\hat{J}(x) \cdot \hat{J}(x + r)|^2 \rangle - \frac{1}{3} \right). $$

These quantities are plotted in the top panels of Figure 12, where at very small halo separations ($r < 0.3$ Mpc $h^{-1}$) there seems to be a parallel alignment of halo spins.

However, both quantities rely on taking an average over all the halo pairs in each bin of separation. The mean is a useful value when dealing with a peaked distribution, but none of the actual distributions of $|\hat{J}(x) \cdot \hat{J}(x + r)|$ has an apparent peak (an example of one of these distributions is Figure 11, where $P(\hat{J} \cdot \hat{J})$ is the number of halos in each bin normalized so that the area under the curve is unity). A fairer way of dealing with
these noisy distributions is to fit a straight line and see if there is any deviation from randomness. The slope of the best-fit line indicates whether more halos are aligned parallel or orthogonal to each other:

$$P((\mathbf{J}(x) \cdot \mathbf{J}(x + r))) = m \mathbf{J}(x) \cdot \mathbf{J}(x + r) + c.$$  \hspace{1cm} (15)

A positive slope \(m\) of the best-fit line means there are more parallel aligned halo pairs, a negative \(m\) means they are more orthogonal, and \(m = 0\) means the halos have random alignment. The values of \(m\) that maximized the likelihood of fitting a straight line to the distributions are shown in the bottom panel of Figure 12.

The shape of the plot of the slope (bottom panel of Figure 12) is similar to the shape of the plots of the conventional statistics. This is expected since they are effectively measuring the same thing but in a slightly different way. Halo spins are aligned parallel for halo separations under 0.3 Mpc \(h^{-1}\). This alignment has not been seen before in simulations because it exists only on very small scales which have not previously been examined. It has, however, been seen in galaxy surveys; for example, Galaxy Zoo (Slosar et al. 2009) found alignment for galaxies closer than 0.5 Mpc. The alignment exists on the scale of substructure within clusters. If only the most massive subhalo in each FOF group is taken (the substructure is thrown out), then there is no significant alignment at any scale (Figure 13). Here there are no halos at small separations and there is no significant alignment at any scale. Only the subhalos within large clusters exhibit any halo–halo spin alignment, although it is weak.

4. EVOLUTION OF SPIN PARAMETER

The spin parameter is a measure of the amount of angular momentum contained in a halo. It was defined in Bullock et al. (2001) as

$$\lambda' = \left| \frac{\mathbf{J}}{\sqrt{2MV^2}} \right|$$  \hspace{1cm} (16)

given the angular momentum \(\mathbf{J}\) inside a sphere of radius \(R\) containing mass \(M\), and where \(V\) is the halo circular velocity at radius \(R, V^2 = GM/R\).

The distribution of \(\lambda'\) over the halos in our sample is shown in Figure 14. It is well fit by a log-normal distribution:

$$P(\lambda') = \frac{1}{\lambda' \sqrt{2\pi} \sigma} \exp \left( -\frac{\ln^2(\lambda'/\lambda_0)}{2\sigma^2} \right).$$  \hspace{1cm} (17)

The fit was done using an MCMC maximum likelihood analysis. For all halos with more than 500 particles at \(z = 0\) the best-fit values are \(\lambda'_0 = 0.02900_{-0.00005}^{+0.00005}, \sigma = 0.664_{-0.002}^{+0.001}\) and at \(z = 3, \lambda'_0 = 0.02940_{-0.0001}^{+0.0001}, \sigma = 0.576 \pm 0.002\). The distributions at both redshifts over all halos in the snapshots are nearly identical.

When halos are binned by mass, the spin parameter at high redshift shows a mass dependence while there is no mass dependence at \(z = 0\), as shown in the left panel of Figure 15. Here the spin parameter is characterized by the mid point of the log-normal distribution, \(\lambda_0'\). The spin parameter over all redshifts is only the same for low-mass \((M < 10^{12})\) halos, but there are far more low-mass than high-mass halos. Since low-mass halos dominate, the average distributions over all halos at different redshifts look the same. At high redshift, there is a tendency for the spin parameter to be smaller for high-mass halos.

This redshift dependency can be characterized by a power relationship between \(\lambda'_0\) and mass at each redshift:

$$\lambda'_0 \propto M^{a(z)}.$$  \hspace{1cm} (18)

The more negative the value of \(a\), the stronger the correlation and \(a = 0\) is no correlation at all. The redshift dependence of \(a\) is shown in Figure 16. The lines for halos with > 500 particles and > 1000 particles overlap in Figure 16 whereas the line for halos with > 100 particles does not. This shows that halos with more than 100 particles are susceptible to errors from particles.
in the outer regions and the cut off of only using halos with more than 500 particles is justified.

Knebe & Power (2008) found that mass binning and selection criteria for relaxed halos has almost no effect on this correlation. We did find a small effect when a different halo catalog was used. Instead of using all the subhalos, only the most massive subhalo (with more than 500 particles) in each FOF halo was used. Most of the mass of the FOF halo is in the most massive subhalo, so it can be regarded as the background halo itself. When substructure is disregarded, we find that there is a stronger mass dependency of the spin parameter at almost all redshifts (the green line in Figure 16 is below the corresponding orange line which includes all substructure). The spins of subhalos are greatly affected by interactions and merger events and so may be out of equilibrium.

Mass dependence of the spin parameter at high redshift was first found by Knebe & Power (2008), who looked at $z = 1$ and $z = 10$. When extrapolating the linear trend of $a(z)$ with a redshift, we predict a much stronger correlation, $a(z = 10) \approx -3$ whereas they find $a(z = 10) = -0.059 \pm 0.171$. Our results agree more closely with Muñoz-Cuartas et al. (2011), who found $a(z = 2) \approx -0.03$. For halos in different environments (blobs, filaments, sheets, and voids), the trends are the same.

When halo mass is scaled by characteristic mass in the right panel of Figure 15, we find that halos at similar stages of collapse at $z = 0$ and 1 have the same spin parameter (the orange and red lines overlap). At high redshift, halos at similar stages of collapse have a higher spin parameter (At log $M/M_\ast = 3$, for example, the green ($z = 3$) point lies above the points for $z = 2$ and $z = 1$). This may be the result of accretion and merger events decreasing the spin of halos. At $z = 3$, halos have retained much of their initial spin but by $z = 1$; similar halos have experienced accretion that has lowered their spin parameter.
Figure 9. Alignment of dark matter halo velocity with filaments on the scale of 2.0 Mpc where bulk motions have been subtracted. Colored lines are for different redshifts as in Figure 5. (A color version of this figure is available in the online journal.)

Figure 10. Alignment of dark matter halo velocity with the local bulk motion on the scale of 2.0 Mpc. Colored lines are for different redshifts as in Figure 5. (A color version of this figure is available in the online journal.)

5. SUMMARY AND DISCUSSION

Using the Millennium N-body simulation, we have tracked the evolution of dark matter halo angular momentum alignments with the large-scale structure, with each other and the evolution of the spin parameter. We have used the shape of the density field to find filaments of 2 Mpc in scale in the large-scale structure. The alignment between dark matter halo spin and the axis of filaments was characterized by the shape of the distribution of \( |\cos(\theta)| \) where \( \theta \) is the angle between the two vectors. The distribution was fitted to the PDF of Equation (12) to find the free parameter \( c \) which characterized the strength of parallel or orthogonal alignment.
We found that angular momentum vectors of dark matter halos since $z = 3$ are generally orthogonal to filaments, but high-mass halos have a stronger orthogonal alignment than low-mass halos. At $z = 0$, the spins of low-mass halos have become parallel to filaments, whereas high-mass halos keep their orthogonal alignment. An interpretation of this is that at early times all halo spins were aligned orthogonal to filaments, as TTT predicts. High-mass halos especially are well aligned because they have had their maximal expansion more recently and so will have been tidally torqued for longer. They usually exist close to clusters where the infall of dark matter is almost isotropic, and so the net effect from mergers and accretion is minimal. Low-mass halos, however, are vulnerable to being disturbed by mergers and accretion which is usually assumed to have the effect of

Figure 13. Alignment of neighboring friends-of-friends halo’s spins. There is no alignment at any scale.

Figure 14. Distribution of the spin parameter at $z = 0$. The histogram is the data and the red (smooth) line is a log-normal fit (Equation (17)) where the best-fit values are $\lambda_0 = 0.02900^{+0.00006}_{-0.00005}$, $\sigma = 0.604^{+0.012}_{-0.002}$.

(A color version of this figure is available in the online journal.)

Figure 15. Redshift evolution of the spin parameter ($\lambda_0'$). The red line is for $z = 0$, yellow is $z = 1$, blue is $z = 2$ and green is $z = 3$. The shaded regions are the 1σ confidence intervals. Left: at high redshift, the spin parameter is less and there is a mass dependency. Right: mass bins are normalized by the characteristic mass.

(A color version of this figure is available in the online journal.)

Figure 16. Mass dependence of the spin parameter over a redshift. The mass dependence, $a$ is the slope of the straight dashed lines in Figure 15. The red (lowest) line includes all halos with more than 100 particles, the orange line includes 500 particles, and the blue line includes 1000 particles. The green line is for the halo catalog that does not include substructure. There is a linear trend of stronger mass dependence at higher redshifts.

(A color version of this figure is available in the online journal.)
randomizing the spin orientation. Why low-mass halos at low redshift exhibit a parallel alignment with filaments remains unexplained.

We found that filaments are regions of bulk flow. When bulk flows are included there is a clear trend for halos to travel parallel to filaments, and high-mass halos travel with the best alignment. When bulk flows on the scale of the filaments are subtracted, an orthogonal alignment to filaments remains particularly for low-mass halos. This shows that entire filaments themselves are moving toward attractors and on small scales there is only orthogonal motion. There was also an orthogonal motion of low-mass halos with the bulk flow but no alignment of high-mass halos out of the bulk flow.

The motions of halos relative to the bulk flow could affect how matter is accreted onto them and the spin orientation this would cause. Orthogonal motion to the bulk flow and filaments by low-mass halos could cause low-mass halos to accrete matter preferentially in one direction. High-mass halos traveling with the bulk flow would experience accretion differently, and this could cause the difference in spin orientation.

Filaments at large smoothing lengths at low redshift contain halos with the best aligned spins and bulk motion, while at high redshift it is filaments at small smoothing lengths that contain the best aligned halos. This shows that filaments are growing in size over time. Because of the nature of the way that the filaments were found (using Gaussian smoothing), this enlargement tells more about the width of the filaments rather than the length. This is complimentary to Sousbie et al. (2009) where filament length is discussed and it was found that there is a general dilation of filaments that began larger and a shrinking, fusion and disappearance of the smaller filaments.

We found an alignment only between the spin orientation of very close neighboring halos. Only at separations of less than 0.3 Mpc h^{-1} do halos exhibit any mutual parallel alignment of their spin axis. The halo finding method used in the Millennium simulation has enabled us to see this small-scale alignment. In the Millennium simulation, the SUBFIND algorithm was used to identify substructure in FOF groups, and the subhalos are counted as halos. This means that alignments between very close halos can be probed, not just alignments between the FOF groups.

Lastly, we tracked the evolution of the spin parameter from $z = 3$ to now and its dependence on halo mass. This was done by finding the center of the log-normal distribution of the spin parameter. There is a mass dependence of the spin parameter at $z = 3$ but not at low redshift and the spin parameter is lower overall at high redshift. The spin parameter follows a power law with halo mass at high redshift but is independent of mass at $z = 0$.

Future work will bridge the gap between idealistic CDM simulations and real galaxy observations. To do this we will generate mock galaxy catalogs and use only the data that would be available in a real survey to see if any alignments of galaxy spin orientations could be seen in the universe. This could be used to plan a survey using new multi-object IFU instruments (Bland-Hawthorn et al. 2011; Croom et al. 2011).

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