The Macroeconomic Effects of Quantitative Easing

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Statement of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any other degree or diploma at the University of Sydney or at any other educational institutions, except where due acknowledgment is made in the thesis.

Any contribution made to the research by others, with whom I have worked at the University of Sydney or elsewhere, is explicitly acknowledged in this thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project’s design and conception or in style, presentation and linguistic expression is acknowledged.

Richard Swain
Abstract

The recent crisis has raised two key macroeconomic issues. First, has the quantitative easing policy pursued by the Federal Reserve had an effect on output, employment and prices? Second, whether ‘quantitative easing,’ is a mechanism through which monetary policy may continue to be able to stimulate economic activity despite the presence of the zero lower bound and various financial market frictions. This paper surveys the recent empirical evidence of the policy having a substantial impact on various interest rates in the United States because of certain financial frictions. It then uses this evidence to analyse the macroeconomic effects of the quantitative easing policy by simulating a New Keynesian macroeconomic model shown to closely fit the U.S. economy. It is concluded that the quantitative easing policy has had an impact on output, prices and employment, irrespective of any plausible financial frictions arising from the GFC. The result also demonstrates that the policy could be used as a monetary policy instrument. The paper ends with an examination of the numerous avenues of research that must be pursued before a firm conclusion can be made regarding the use of quantitative easing as a viable instrument of monetary policy.
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I. INTRODUCTION

The Great Financial Crisis (GFC) is considered the largest macroeconomic crisis since the Great Depression (Stiglitz, 2010; Reinhart and Rogoff, 2008). Following excessive risk taking by global financial markets and the subsequent meltdown in the American sub-prime mortgage market, world financial markets became paralysed by a wave of uncertainty (Astley et al., 2009; Covitz et al., 2009). The collapses of Lehman Brothers and AIG further shook confidence in the integrity of the world’s financial system, leading to substantial increases in interest rate spreads during the period (Taylor, 2009; Mishkin, 2011). The flow-on effects of these financial difficulties were soon felt in the wider economy, resulting in severe decreases in international trade and economic growth not seen since the Great Depression (Astley et al., 2009). Governments felt obliged to act to ensure financial stability in the short term, at the very least, but then to reduce unemployment and increase growth in the longer term.

Central banks also came under pressure to respond to the financial paralysis and economic woes. At first they responded with the very conventional policy response of cutting their target interest rates. This was an attempt at reducing the level of interest rates across the maturity spectrum, thereby encouraging borrowing, investment and spending (Taylor, 2009). As the crisis worsened, a key question that arose was whether monetary policy could remain effective even in the worst possible scenario for policymakers of unsustainable levels of government debt, persistent and abnormally high yield spreads, a zero nominal policy rate and deflation.¹

A variety of answers to this question had already been proposed after Japan’s deflationary period, including zero interest rate commitments, quantitative easing and asset buyback programs (Bernanke and Reinhart, 2004). Only quantitative easing was the most vigorously pursued and so forms the focus of this thesis.²

¹ Unsustainable government debt is defined as a debt to GDP ratio that cannot be maintained at trend growth rates without politically unfeasible tax increases or spending cuts (Domar, 1944).
² A zero interest rate commitment occurs when the central bank states that it will not raise the target nominal interest rate until certain conditions are satisfied. Examples include the zero interest rate policy commitment of the Bank of Japan (ZIRP) between 1999-2005 (Eggertsson and Woodford, 2003; Okina and Shiratsuka, 2004) and the recent commitment by the Federal Reserve (Federal Reserve, 2011d).
Quantitative easing (hereafter ‘QE’) is a monetary policy instrument defined as a ‘package of unconventional policy measures designed to absorb the shocks hitting the economy by making use of both the asset and liability sides of the central bank balance sheet’ (Shiratsuka, 2009). It absorbs shocks by reducing long-term interest rates and stresses within the financial system brought on by financial crises. It is argued that QE reduces interest rates through three core channels: altering private sector expectations about the future course of monetary policy, reducing the total supply of long-term securities, which increases prices and lowers yields, and reducing illiquidity concerns by providing a consistent buyer of long-term securities (Chung et al., 2011). It also reduces stress by providing sufficient liquidity to embattled financial institutions and the central bank acting as a market maker in certain paralysed asset markets. By doing so, the central bank aims to facilitate the expansion of credit and investment, thereby assisting the economic recovery.

Before the current crisis, the most significant implementation of QE was by the Bank of Japan between 2001 and 2006. In an attempt to combat persistent deflation, falling output and financial instability, the Bank of Japan augmented its traditional operating procedure by injecting significant amounts of central bank reserves, through the purchase of government securities, to achieve certain targets for current account balances. The Bank of Japan also allowed a wider range of entities to borrow at the official target rate and committed to maintaining sufficient liquidity if required (Kimura et al., 2003).

The Federal Reserve and the Bank of England have also recently aggressively pursued variants of this policy. After reducing its target interest rate to 0.5% in March 2009, the Bank of England began purchasing two hundred billion pounds worth of (almost entirely government) securities of varying maturities through the issuing of reserves (Joyce et al., 2010). The purchases amount to approximately 14% of nominal GDP (Figure 1). In conjunction with other short-term liquidity provision programs, the Federal Reserve has also implemented QE by purchasing of $1.25 trillion dollars worth of mortgage-backed securities (hereafter ‘MBS’) and $600 billion of treasury securities as well as the

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3 Examples include the Term Auction Facility and the Term Securities Lending Facility.
use of short-term liquidity provision services to certain stressed securities markets (Figure 2) (Mishkin, 2011).

*Figure 1*

![Chart 1: Bank of England assets to 2 June 2010](chart1.png)

![Chart 2: Bank of England liabilities to 2 June 2010](chart2.png)

![Chart 3: Cumulative QE asset purchases by type: amounts outstanding](chart3.png)

![Chart 4: Cumulative gilt purchases by maturity](chart4.png)

Source: Joyce et al. (2010)
Recent empirical evidence indicates that these two policies reduced a variety of interest rates by 50-
100 basis points (Gagnon et al., 2011; Joyce et al., 2011). However, little is known about their 
macroeconomic effects especially given the difficulties in financial markets. This thesis examines a 
version of the postulated transmission mechanism for QE to uncover the full range of its potential 
macroeconomic effects, both theoretically and in relation to the Federal Reserve’s policy. It is found
that QE has improved economic welfare by stimulating aggregate demand. A further conclusion is that QE appears to be a valid instrument for monetary policy to remain effective even in very difficult economic conditions.

The rest of this thesis is structured as follows. Section 2 provides an evaluation of the empirical evidence and the theoretical models used to ascertain QE’s effect on key macroeconomic variables. Section 3 presents a transmission mechanism for how QE could stimulate output and prices. Section 4 sets out the model that will be used to investigate QE’s effects. Section 5 presents the results from simulations of this model. Section 6 contains a discussion of the implications of these results for the future conduct of monetary policy and section 7 concludes.
II. LITERATURE REVIEW

Three key strands of the overwhelmingly large literature examining alternative instruments for monetary policy are relevant to the questions raised in this thesis: the theoretical justifications for QE, the empirical evidence of QE’s effectiveness when implemented and the different models that have been used to analyse unconventional monetary policy.

2.1. Theoretical Justifications for QE

It is well-established that under normal circumstances the monetary authority’s primary instrument for achieving their objective(s) is a short-term interest rate (Mishkin, 2007). However, as demonstrated during the Great Depression, the Japanese experience between 1990-2006 and the current crisis, this methodology can become problematic due to the zero lower bound on nominal interest rates. Blinder (2000) identifies seven alternatives ranging from purchasing government securities with a longer maturity to purchasing foreign securities to depreciate the exchange rate. Bernanke and Reinhart (2004) and Klyuev et al. (2009) narrow these to the two most plausible ‘unconventional’ techniques: actions which shape expectations of future interest rates and expanding the size or altering the composition of the central bank’s balance sheet (QE). These techniques are not mutually exclusive. As seen in the Federal Reserve’s policy, the expansion of its balance sheet may be an action taken to credibly signal future policy actions.

Auerbach and Obstfeld (2005) demonstrate that if QE involves a credible commitment to raise the money supply permanently, then inflation expectations rise, lowering the real interest rate and hence stimulating aggregate demand. After incorporating a financial sector, they also identify that banks’ beliefs regarding the permanency of the monetary base expansion determines QE’s effectiveness. Their crucial insight is that QE’s effectiveness is primarily determined by how temporary private sector participants expect the policy to be.\(^4\) This result demonstrates that, theoretically, QE can enable the monetary authorities to stimulate economic activity even after the zero lower bound is reached.

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\(^4\) A temporary policy refers to the situation where the monetary authority injects large amounts of high-powered money through the purchase of securities before completely reversing the injection. This is in contrast to a permanent increase which entails an injection leading to a
Nevertheless, each time QE has been used, it has been perceived or explicitly stated to be temporary, raising a key theoretical problem. There appears little consensus in the literature regarding whether this problem precludes QE from being effective. Bernanke, Reinhart and Sack (2004) extensively discuss the ways in which a temporary version of QE could work in a ‘liquidity trap’ and make two significant contributions. First, they emphasise that in order for the monetary authorities to shape interest rate expectations by policy commitments, their statements must be credible. One method of sending a credible signal is to undertake actions such as temporarily increasing the size of the central bank’s balance (Krugman, 2000). In contrast to a pure policy commitment, which suffers from a time inconsistency and so a credibility problem (Kydland and Prescott, 1977; Barro and Gordon, 1983), such an action is credible because of the risk involved in these asset purchases and the delay before the central bank’s position could be reversed. Second, they demonstrate that QE may have a direct impact on interest rates through imperfections in financial markets. This motivates part of the analysis in section 4.

In contrast to Bernanke, Reinhart and Sack (2004), Eggertsson and Woodford (2003, 2004) conclude that upon reaching the lower bound on the nominal interest rate, any increase in the monetary base which does not signal a future change in interest rate policy will ‘neither stimulate real activity or halt deflation.’ The reason is twofold. Forward looking agents will anticipate the reversal of the policy, as occurred for Japan between 2001-2006, and will therefore adjust their economic decisions accordingly. Moreover, at the zero lower bound, bonds and the monetary base become substitutes so agents will respond to asset purchases by increasing their holdings of money. This implies that the model’s general equilibrium is independent of the supply of bonds and money at the zero lower bound. Another crucial observation is that the financial imperfections required to generate any portfolio rebalancing effect may in fact generate countervailing effects. Theoretically, the stimulatory influence of the portfolio rebalancing effect is therefore ambiguous. Unfortunately, their illustrative model only contains a representative consumer and no financial frictions and none of these plausible

temporary increase in the growth of the money supply but allowing ‘base drift’ after that. The money supply is not returned back to the level it would have been without the injection of funds.

5 This is defined as a period of deflation when the nominal interest rate is at zero such that money and short-term bonds become perfect substitutes (Krugman et al., 1998)
countervailing effects. This limits the validity of their conclusions regarding the portfolio rebalancing channel and is the impetus for this thesis to examine this issue in section 4.

Even after incorporating financial market frictions, it still remains theoretically unclear whether QE would have an unambiguously positive influence on economic activity. Curdia and Woodford (2011) incorporate borrowing frictions and heterogeneous households and demonstrate that a temporary use of QE will only stimulate aggregate demand if it enables the central bank to extend the optimal level of credit to the private sector. This conclusion is also reached by Gertler and Karadi (2011) using a slightly modified framework which incorporates financial intermediaries. Importantly, Curdia and Woodford (2011) find that the provision of reserves beyond this optimal level of credit does not alter the path of the endogenous variables in equilibrium and so QE cannot have any effect. Such an irrelevance proposition was also found in the context of bank lending (Martin et al., 2011). However, they abstract from non-financial firms and capital accumulation which is not desirable given that one of the main objectives of QE is to stimulate investment by lowering long-term interest rates. Another limitation is that no firm micro-foundations are provided for the credit spread which should be derived from profit maximisation by financial intermediaries’ under imperfect information (Bernanke et al., 1999).

It is apparent that the theoretical effects of QE depend on both the degree of financial imperfections and whether it signals a change in the future course of the policy rate. Heterogeneity in households, lending constraints on banks, preferred habitats for financial market participants and borrowing constraints for households and firms may all enable a temporary use of QE to stimulate demand, primarily by altering inflation or interest rate expectations (Clouse et al., 2003). It is therefore important to survey empirical evidence on whether such frictions exist to a sufficient extent that QE has been effective when implemented.
2.2. Empirical Analyses of QE’s Effectiveness

The empirical literature analysing the policy’s effectiveness, including the relative importance of the identified financial market imperfections through which it can operate, can be separated into those examining the Japanese experience between 2001-2006 or the current implementation in the US, UK and Japan.

2.2.1. Japan’s Experience between 2001-2006

Ugai (2006) provides an excellent summary of numerous articles examining the relative strengths of the various transmission mechanism channels for the Japanese episode. Considerable variation in results amongst the papers is found. Nevertheless, the overall conclusion is that QE had moderate effects on interest rates and other asset yields through the portfolio rebalancing effect and by providing credibility to the monetary authority’s low interest rate commitment.

Upon re-examining the period, Shiratsuka (2009) also concludes that the policy had an effect on short-term interest rates (e.g. the Tokyo interbank offered rates) and credit spreads. Another finding is that QE helped stabilise the financial system by providing financial institutions with sufficient liquidity while they repaired their balance sheets. This is supported by Oda and Ueda (2005) using a small macroeconomic model with no-arbitrage asset pricing components. These authors find that the policy had small effects on interest rates and mainly acted to provide credibility to the Bank of Japan’s zero interest rate policy (ZIRP). However, Oda and Ueda (2005) also conclude that the policy had uncertain effects on the risk premium in long-term bond yields. The paper’s methodology does have the drawback that they abstract from many potentially confounding effects in their model, casting doubt on their conclusions. An example is failing to control for binding liquidity constraints on firms.

A key issue that remains, despite this strong evidence regarding interest rates, is that after considering evidence from a variety of statistical methods, Ugai (2006) finds contradictory estimates of QE’s effect on economic activity. The reason for this appears to be the choice of econometric methodology. Those studies which utilise sign-restricted vector auto-regressions find QE had small but significant
effects (Kamada and Sugo, 2006; Schenkelberg and Watzka, 2011) while those using alternative methodologies do not (Okina and Shiratsuka, 2004; Baba, Nishioka, Oda, Shirakawa, Ueda and Ugai, 2005). This reinforces the need to use a macroeconomic model to ascertain the effects of QE.

Despite these contradictory results, these papers provide the necessary empirical evidence to justify incorporating further financial frictions within a fully specified macroeconomic model. They also identify the important factors behind why QE has a definite effect on interest rates but an uncertain effect on output and inflation. Kamada and Sugo (2006) conclude that this is due to (1) a significant deterioration in private sector balance sheets due to asset price declines and (2) large disruptions to traditional economic relationships (financial and non-financial), once the zero lower bound is reached, which severely impedes economic activity. Support for these conclusions is found in the estimated debt overhang of firms and households by Shirakawa (2001) and the finding by Fujiwara (2006) that there was a structural break in both the Japanese economy and monetary policy’s effectiveness in the 1990s. The insights of these papers will be used to motivate some aspects of the model presented in section 4.

2.2.2. Current Experience of Quantitative Easing

In an analysis of QE by the Bank of England, Meier (2009) and Joyce et al. (2010) find that the large scale purchase of gilts (government bonds) led to a decline in yields immediately following the policy announcement of between 40-100 and 55-120 basis points respectively over the 5-25 year segment of the yield curve. Joyce et al. (2010) also identify significant movements in corporate bond yields but uncertain effects of the policy on equity prices and the sterling exchange rate. While providing further evidence of QE’s effects on interest rates, the use of a one to three day timeframe is clearly inadequate for assessing the policy’s macroeconomic effects.

Gagnon et al. (2010) use a similar empirical methodology to Joyce et al. (2010) to identify the effect of the Federal Reserve’s policy on yields across the maturity spectrum. They conclude that the yields on the assets targeted by the policy fell cumulatively by 120 basis points on average, after all the relevant policy announcements occurred. The authors also find that the component of QE that
involved bond purchases lowered the term premium on government bonds by 52 basis points. These findings are supported by Krishnamurthy and Vissing-Jorgensen (2011) who use an event study approach to identify sharp decreases in nominal rates for long-term safe assets and a rise in inflation expectations, implying an even larger drop in real interest rates. Lenza et al. (2010) corroborates these findings using a Bayesian VAR to estimate the policy’s effects on yields and spreads over many different asset classes. Furthermore, when specifically focussing on the MBS purchases, Hancock and Passmore (2010) conclude that the asset purchases removed risk premiums due to the financial crisis from mortgage rates primarily through providing greater market liquidity.

Using a wide array of methodologies, the empirical literature demonstrates that QE does indeed have an effect on financial market conditions and interest rates. This establishes that sufficient financial market frictions exist for QE to stimulate economic activity, though there is little consensus regarding the size and direction of this effect. As a result, a macroeconomic model which replicates the United States economy very well and is capable of being augmented to capture the essence of QE is required for this thesis to achieve its aims.

2.3. Modelling Unconventional Monetary Policy

The seminal paper on modelling unconventional monetary policy actions is Eggertsson and Woodford (2003). The authors use a somewhat standard New Keynesian framework in their analysis with the main elements including a representative household, monopolistic competition in the goods market, price adjustment as in Calvo (1983) and non-separable real money balances in the utility function.⁶ In terms of conditions it is assumed that:⁷

\[ U'(C_t, \frac{M_t}{P_t}, \xi_t) > 0 \text{ for all } \frac{M_t}{P_t} \leq m(Y_t; \xi_t) \]  

(2.1)

---

⁶ Non-separable money balances implies that the marginal utility of consumption does not depend on money balances (Gali, 2008). To illustrate the point, a non-separable utility function would be of the form \( U(C_t, \frac{M_t}{P_t}, N_t) = \frac{(1-\phi)C_t^{1-\phi} + \frac{M_t^{1-\phi}}{P_t}}{1-\phi} - \frac{N_t^{1+\phi}}{1+\phi} \) whereas a separable utility function would be of the form \( U(C_t, \frac{M_t}{P_t}, N_t) = C_t^{1-\phi} + \frac{M_t^{1-\phi}}{1-\phi} - \frac{N_t^{1+\phi}}{1+\phi} \) (Gali, 2008).

⁷ \( C_t \) denotes the level of consumption, \( \frac{M_t}{P_t} \) is the level of real money balances held by households and \( \xi_t \) is a vector of exogenous disturbances (e.g. external finance premium or net worth shocks) (Eggertsson and Woodford, 2003). The satiation level of real money balances at different income levels is given by \( m(Y_t; \xi_t) \).
Household optimisation yields the following two inequalities with the ‘complementary slackness’ condition that at least one must hold with equality at any time (Eggertsson and Woodford, 2003).\(^8\)

\[
U_m' \left( C_t, \frac{M_t}{P_t}, \xi_t \right) = 0 \text{ for all } \frac{M_t}{P_t} > m(Y_t; \xi_t)
\]

(2.2)

\[
\frac{M_t}{P_t} \geq L(Y_t, i_t; \xi_t)
\]

(2.3)

\[
i_t \geq 0
\]

(2.4)

This implies that once the satiation level \(m(Y_t; \xi_t)\) is reached,\(^9\) the nominal interest rate must be zero and so the equilibrium becomes independent of the actual monetary base level so long as it exceeds the amount required for the satiation level to bind. In their model, QE is represented by a choice of monetary base function where the central bank supplies excess liquidity over that which is required to maintain the nominal interest rate at zero. The authors therefore conclude that unless the policy alters expectations of future policy rates (and hence interest rates using the expectations approach) then it will have no effect on consumption, investment and output. However, this conclusion relies on the underlying assumption that households have a satiation level for real money balances.

Despite Eggertsson and Woodford (2003) clearly identifying the main conditions for the Japanese QE policy to theoretically succeed, their model cannot be used for three reasons. First, the specification of the monetary base rule does not accord with the current policy reality. The current policy is not targeting the size of outside money but instead involves purchasing set quantities of assets to influence yields across the yield curve. Second, Doh (2010) suggests that the lack of financial imperfections, such as investors having preferred habitats and credit rationing, is the reason behind the communication channel being the only channel available in the model. Given the empirical evidence on these imperfections, a failure to include them would severely limit the analysis. Third, even if the policy altered financial market perceptions of future interest rates this would not lead to an

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\(^8\) \(L(Y_t, i_t; \xi_t)\) is the money demand function arising from the households’ optimisation problem with real money balances in the utility function. The nominal interest rate is denoted by \(i_t\).

\(^9\) The satiation level is defined as the minimum level of money balances at which the marginal utility from holding real money balances is zero such that the nominal interest rate also equals zero.
unambiguous negative and positive change in interest rates and output respectively. Relationships within financial markets and between financial markets and the real economy may have been severely disrupted (Kamada and Sugo, 2006) and such factors must also be accounted for.

Much attention has been given to financial market and other frictions within the context of a New Keynesian model. Smets and Wouters (2007) construct a more sophisticated model than Eggertsson and Woodford (2003) including adjustment lags and costs, investment specific technology shocks, wage shocks and price shocks. An excellent feature of this paper is that the authors establish the validity of these additions by showing how their model achieves a tighter fit with the actual data compared to unrestricted VAR models.

Nevertheless, the empirical evidence suggests that it would be better to employ a model with many more frictions to ascertain QE’s effects. For example, Smets and Wouters (2007) do not explicitly model one key financial market imperfection which has been identified as another channel through which monetary policy can function (the credit channel) (Mishkin, 2007). Bernanke et al. (1999) label this imperfection the ‘financial accelerator’ which describes the increase in the external finance premium as the ratio of borrowings to borrowers’ net worth rises due to higher moral hazard and adverse selection problems. While this mechanism is included in Smets and Wouters (2007) as a shock disturbance, Bernanke et al. (1999) provide insights into the microeconomic foundations and macroeconomic implications of this mechanism. In order to achieve the most complete analysis possible, it is therefore necessary to incorporate these insights into the Smets and Wouters (2007) model.

Gilchrist et al. (2009) extend the Smets and Wouters (2007) model by explicitly incorporating the external finance premium equation of Bernanke et al. (1999). An alternative framework is Iacoviello (2005) who extends Bernanke et al. (1999) by including real estate which is used by consumers and firms as collateral for loans and for production respectively. Given the importance of housing in both the debt-fuelled consumption increases between 2004-2007 and the GFC, this model would appear to be tailor-made to the questions under consideration. However, the model only contains a technology
shock, a housing preference shock and an inflation shock, none of which provide a plausible description of the crisis which QE was instituted to counteract. For example, no analysis of the crisis is known which suggests that the sudden decrease in housing prices and the resulting economic turmoil was due to a shock to consumers’ preferences regarding housing. Instead, a more reasonable proposition is the sub-prime mortgage collapse induced falls in all asset prices, including house prices. In turn, this caused a sharp decrease in entrepreneurs’ net worth and an increase in uncertainty and risk aversion and hence yield spreads (Blanchard, 2010). The Gilchrist et al. (2009) model contains one of the broadest set of dynamics in the literature including shocks to net worth and yield spreads and matches the actual data well (Pagan and Robinson, 2011). As such, this model is more capable of replicating the path of the U.S. economy with and without QE. It therefore represents the best starting point for any theoretical analysis of the policy.

It appears that while the transmission mechanism for QE has been extensively discussed there has not been sufficient research which convincingly identifies empirically the long-term effects of the policy on key macroeconomic variables or embeds components of the suggested transmission mechanism into an augmented New Keynesian DSGE model to investigate its effects. While addressing both would be desirable, this thesis will only focus on the second issue by augmenting an existing model to include many other factors which have been identified as crucial to the policy’s success.
Before proceeding to an analysis of how QE could affect output and prices it is first important to set out the underlying theoretical framework upon which the transmission mechanism for QE is based. Monetary policy has come to be viewed as the pre-eminent instrument for (1) counter-cyclical demand management to achieve stable inflation and (2) ensuring financial system stability in countries without self-regulation or prudential supervision. This in large part was due to fiscal policy being discredited following the high inflation levels of the 1970s and the realisation that the monetary policy stance could be varied much more quickly and easily (Mishkin, 1995; Pringle, 1995). While monetary base targeting was experimented with, it was quickly abandoned because of concerns about its impact on the stability of financial markets and interest rates as well as the stability of the money demand relation. The current consensus is that central banks should use one instrument, a short-term interest rate, to achieve their objective(s), typically low inflation (Goodhart, 1987; Lewis and Mizen, 2000: 339).

3.1. Traditional transmission mechanism for monetary policy

A central bank is able to achieve its goals due to the existence of various channels through which an alteration to the policy rate, by open market operations, affects the ultimate output and inflation variables. These channels have been grouped into three categories: the interest rate channel, the exchange rate channel and the credit channel (Mishkin, 1995).

The interest rate channel is the process whereby a lower policy rate induces a fall in yields across the yield curve through arbitrage in financial markets and competition in the banking sector. As a result, borrowing costs fall10 and any liquidity/solvency constraints become less stringent, leading to a rise in consumption and investment. Investment also rises due to higher expected future demand and lower expected future borrowing costs (Lucas and Prescott, 1971; Bernanke, 1983). Furthermore, arbitrage between debt and equity markets implies that the lower yields on debt induce a rise in equity prices.

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10 If the policy was to lower yields in debt markets through the described mechanism then the funding costs for firms would be reduced directly if they had access to those markets. If not, then the lower yields would lower the opportunity cost of borrowing funds for financial intermediaries, set at the risk-free rate $R^r$ due to portfolio diversification (Bernanke et al., 1999), resulting in lower borrowing costs for borrowers in a competitive financial sector. At the individual firm level, the policy should therefore stimulate investment, ceteris paribus.
This raises output further by inducing higher consumption through wealth effects (Dvornak and Kohler, 2003) and higher investment by placing upward pressure on Tobin’s Q (Tobin, 1969).

The exchange rate channel relates to return-maximising investors moving capital to other countries in response to the lower returns offered in the domestic country following a fall in the policy rate. This causes an increase in the supply of domestic currency in the foreign exchange market and so a currency depreciation. A lower nominal currency implies a fall in the real exchange rate under sticky prices and the purchasing power parity doctrine (Balassa, 1964). The economy’s international competitiveness will rise as a result, leading to higher output.

The credit channel comprises two parts due to the different effects of an interest rate change on the suppliers and purchasers of credit. On the supply side, lower rates imply that banks face lower funding costs, increasing the amount of capital available for them to lend (bank lending effect). Borrowers also have a higher net worth which reduces the adverse selection/moral hazard issues inherent in a debt relationship (balance sheet effect) (Mishkin, 1995). Both effects increase the willingness of lenders to provide credit which stimulates investment and consumption (Bordon and Weber, 2010). From the borrowers’ perspective, higher net worth lowers consumers and firms’ perceptions about the possibility of financial distress which causes them to decrease their level of precautionary saving and increase consumer durable, housing and business investment (Mishkin, 2007: 61).

3.2. Transmission mechanism for Quantitative Easing

There exists a significant literature on how variations in the size and composition of the central bank’s balance sheet may affect investment, consumption or net exports and therefore activity levels. This transmission mechanism is located within the traditional mechanism for monetary policy because QE is viewed as an additional instrument for changing certain interest rates when the policy instrument faces the zero lower bound constraint. This section consists of Figure 3 which schematically represents the transmission mechanism followed by a brief statement of each identified channel.
Figure 3

Purchase of Financial Assets using the creation of central bank reserves

- Lower duration premia
- Lower liquidity premia
- Lower default risk premia
- Information regarding future macroeconomic/financial market variables
- Credible commitment regarding policy rate
- Portfolio Rebalancing by private investors
- Repair of Private Sector Balance Sheets (Consumer, Firms & Banks)

- Risk Premia
- Future Course of the Official Target Rate

- Lower nominal interest rates
- Inflation Expectations
- Expectations of an Economic Recovery

- Lower real interest rates
- Reduced Credit Rationing
- Opportunity for additional fiscal stimulus
- External Finance Premium

- Net Exports
- Lending & Investment
- Consumption

Higher Prices, Output and Employment

Source: Ugai (2006); Krishnamurthy and Vissing-Jorgensen (2011)
3.2.1. Long-Term Interest Rates

A widely accepted approach to the determination of long-term nominal interest rates is that they are determined by the expected value of the risk-free rate (typically the policy rate) over the period before maturity and a variety of risk premia (Clouse et al., 2003; Blinder, 2010). To fix ideas, one tractable way of representing this is:\(^{11}\)

\[
i_t^L = \frac{1}{N} \sum_{z=0}^{N-1} E_t(i_{t+z}) + \theta_t^L
\]

(3.1)

This component of QE’s transmission mechanism can be clearly understood using these two elements.

3.2.1.1. Future Course of the Policy Rate

It has been established that under perfect financial markets, unless the policy altered perceptions regarding the future course of interest rates, then it will have no effect (Eggertsson and Woodford, 2003). The policy could achieve this by using the large-scale purchase of financial assets to provide the necessary credibility to a commitment to maintain interest rates at very low levels for an extended period of time (Clouse et al., 2003). Credibility would be obtained because the Federal Reserve would incur substantial losses if it raised interest rates before removing the excess liquidity by selling the acquired assets.\(^{12}\) The sheer scale of the asset purchases also signals the Federal Reserve’s resolve in their policy actions (Krishnamurthy and Vissing-Jorgensen, 2011). At the very least, the institutional framework surrounding the policy implies a significant time delay between the decision to begin removing the injected reserves and an increase in the policy rate. This delay was indeed seen in the Japanese experience of QE (Bernanke and Reinhart, 2004).

While the above mechanism would operate to reduce nominal interest rates, only if real interest rates are reduced will consumption and investment respond (Lucas and Prescott, 1971; Boyle and Guthrie, 2003). With constant inflation expectations, the nominal and real interest rates would move one-for-

\(i_t^L\) is the long-term bond rate with a maturity of T periods at time t; \(i_t\) is the short-term rate at time t (the policy rate) and \(\theta_t^L\) is the risk-premium on the long-term bond with the given maturity (Clouse et al., 2003).

\(\theta_t^L\) is the risk-premium on the long-term bond with the given maturity (Clouse et al., 2003).

\(\theta_t^L\) is the risk-premium on the long-term bond with the given maturity (Clouse et al., 2003). Recently, the Federal Reserve was given the power to pay interest on both required reserves and excess reserves by the Emergency Economic Stabilization Act of 2008 (Federal Reserve, 2008a). While the new interest rate could be raised above the policy rate, this would be theoretically and practically equivalent to a rise in the policy rate (Bernanke, 2009). Large capital losses would therefore also occur if this alternative action was taken.
one as per the Fisher equation\(^\text{13}\) so that the manipulation of expectations using QE would reduce real, long-term interest rates. Moreover, considerable concern has been expressed regarding the policy’s negative effect on the monetary authority’s ability to control inflation (Feldstein, 2010) and there is evidence that this raised inflation expectations each time it has been used (Krisnamurthy and Vissing-Jorgensen, 2011; Ugai, 2006). Another channel is therefore through raising inflation expectations which would reduce real interest rates further.

3.2.1.2. Risk Premia

Long-term interest rates are also determined by the risk premia term \((\theta^I_t)\). In general, this term depends on illiquidity, default and duration risk. Duration risk refers to the asset price fluctuations investors are exposed to due to changes in inflation and interest rates over the maturity of the bond. Despite a risk averse investor demanding a premium for this risk, the large-scale purchase of longer term treasury securities reduces their relative supply and therefore lowers the total amount of duration risk in the market. With market segmentation according to the preferred habitat theory,\(^\text{14}\) this should result in a lower overall premium. This effect would be strengthened by the policy providing a credible signal regarding the future course of interest rates, alleviating some of the uncertainty regarding future interest rate changes.

While the default and liquidity premiums are not relevant for government securities, a fall in the liquidity premium certainly represents another channel for reducing interest rates on MBS and corporate debt. Following the realisation of the losses on sub-prime mortgages, there was a substantial fall in US house prices due to a decline in household demand for housing and a large increase in foreclosures (Mishkin, 2011; Kamin and DeMarco, 2010). This was accompanied by a significant decline in liquidity in the MBS market due to heightened risk aversion (Hancock and Passmore, 2011). The purchase of $1.25 trillion worth of MBS and the reinvestment of interest repayments involved the Federal Reserve acting like a ‘buyer of last resort’ leading to an increase in liquidity in the market

\(^{13}\)\(i_t = E_t(\pi_{t+1}) + r_t\), where \(i_t\) is the nominal interest rate, \(E_t(\pi_{t+1})\) is the expected inflation rate next period and \(r_t\) is the real interest rate.

\(^{14}\)The preferred habitat theory is based upon investors only trading in securities within certain maturity bands or classes. This results in very similar assets ceasing to be perfect substitutes for each other. Their rates of return will therefore not be equalised as there is limited arbitrage between the long-term to the short-term markets (Juttner and Hawtrey, 1997)
through private investors becoming confident that the market would receive strong, ongoing central bank support (Joyce et al., 2010). Liquidity premiums should therefore fall, reducing these assets’ interest rates. Hancock and Passmore (2011) present convincing evidence that this channel has been operative over 2008-2010.

3.2.2. Portfolio Rebalancing Effect

In addition to the signalling and risk premia channels, QE can also lower long-term nominal interest rates by altering the supply of various assets in the financial system. If investors have portfolio preferences or preferred habitats, then the reduction in the supply of certain assets will induce them to purchase substitutes to return their portfolios to their desired composition. The resulting increase in demand for these substitutes will reduce their yield (Gagnon et al., 2010). This process would be assisted by arbitrage between the various asset markets by speculators. Consequently, the targeted purchases by the Federal Reserve\textsuperscript{15} will not only lower the yields on these assets but will lead to broader falls in yields.

3.2.3. Private Sector Balance Sheets, the Supply of Credit and the External Finance Premium

The GFC led to a significant deterioration in the balance sheets of financial institutions which had a significant impact on the amount of borrowing and lending occurring in the economy. Financial institutions suffered large capital losses which certainly reduced their willingness and/or ability to lend, even if the losses did not endanger their survival. Much research on the effect of such a ‘credit crunch’ has been undertaken, especially in relation to the Great Depression and Japan’s experiences between 1990-2006. Friedman and Schwartz (1963) argue that the lack of liquidity in the banking sector during the Great Depression, brought on by a crisis of confidence, led to a large decrease in lending and the money supply. Bernanke (1983) found that the issue of lender solvency constrained credit flows in the Great Depression even after 1933, with small to medium businesses and homeowners particularly affected. The experience in the home loan market was particularly severe with some lenders nearly exiting the market (Bernanke, 1983).

\textsuperscript{15}In the current context, the Federal Reserve has concentrated its government bond purchases in the longer maturity end of the yield curve in an attempt to increase the demand for these bonds, pushing up prices and hence lowering yields (Doh, 2010).
Firms also experienced large deteriorations in their balance sheets which increased the level of deliberate credit rationing and borrowing costs. Due to informational asymmetries in credit markets, it is argued that lenders will use credit rationing based on observable borrower characteristics to allocate credit because of moral hazard and adverse selection considerations (Walsh, 2003; Stiglitz and Weiss, 1981). The information asymmetries also lead to an external finance premium due to the existence of a ‘state verification cost’ (Walsh, 2003; Townsend, 1979). The premium compensates the lender for bearing this cost (Bernanke & Gertler, 1989, 1995) and is closely linked to borrowers’ net worth (Bernanke et al., 1999). In times of crisis, this premium rises, which in turn raises the effective borrowing costs of firms and hence discourages investment (Bernanke et al., 1999).

QE may reduce credit rationing and the external finance premium in a number of ways. First, through the direct purchase of corporate debt and treasury securities from firms, the central bank bypasses these effects and provides the necessary cash for these institutions to invest. Furthermore, QE generates asset price increases which would improve the liquidity and quality of firms’ balance sheets. While there appears little evidence of financial institutions rationing credit to large firms (Fisher, 2010), these balance sheet improvements would lower borrowing costs and improve credit availability to small/medium enterprises. In turn, this would assist the recovery of investment and employment.

Second, the provision of liquidity through the purchase of MBS, corporate debt and government securities would remove any balance sheet constraints on financial institutions’ ability to lend. The purchases also provided the financial system with enormous amounts of reserves, alleviating illiquidity concerns and so improving firms’ access to capital for investment.

Third, the policy largely involves the purchase of government bonds and MBS with small purchases of corporate debt. The increase in demand should raise the prices of these assets, increasing the net worth of those holding them. At the same time, the sale of these assets by firms alters the liquidity of their balance sheet. This would reduce the state verification cost of lenders as fire-sales of illiquid assets would not need to be conducted as much (Bernanke et al., 1999). Consequently, the net worth of firms...
and the quality of their net worth should rise, lowering the external finance premium and hence borrowing costs.

The higher asset prices induced by the policy would also improve consumer balance sheets causing an increase in consumption through wealth effects and the removal of liquidity constraints. Through the purchase of MBS and government securities, the Federal Reserve attempted to improve equity and house prices, raising household wealth (Hancock and Passmore, 2011). Estimates of these wealth effects indicate that a $1 increase in housing and equity wealth increases consumption by between $0.02-0.08 (US and UK) and $0.04-0.08 (US, UK and Canada) respectively (Dvornak and Kohler, 2003) which suggests this channel would be significant even after the GFC.

In conjunction with these wealth effects, the purchases would improve an individual’s access to credit, especially in relation to housing which was severely damaged by the sub-prime mortgage collapse. QE does this by providing substantial amounts of reserves. This would improve the liquidity and quality of the financial intermediaries’ balance sheets, lowering the amount of credit rationing to consumers. Indeed, the Federal Reserve has explicitly stated that the purchase of $1.25 tn worth of MBS is aimed at improving the state of the housing market by allowing individuals to borrow again (Federal Reserve, 2008). Furthermore, given that households' net worth significantly affects their access to credit (Iacoviello, 2005), the higher asset prices would enable indebted households to borrow more.

3.2.4. Government Spending/Taxation

Both the US and UK versions of the policy involve a substantial purchase of government securities (5% and 14% of nominal GDP for the US and UK respectively), funded by the electronic creation of reserves. Practically speaking, these reserves pay a significantly lower interest rate than long-term government securities (Federal Reserve, 2008a; 2011c). By purchasing and holding government debt, even for a certain period of time, the central bank is essentially monetising the debt, thereby reducing the interest payments on the debt. Lower debt repayments would allow the government to lower current taxes or not raise future taxes as much while still satisfying its intertemporal budget constraint. It has been suggested that such a policy could create expectations of lower government taxes in the
future (Bernanke and Reinhart, 2004). Consequently, consumption would rise due to wealth effects arising from the negative relation between consumer wealth and taxation. Alternatively, current government expenditure could rise and/or future government expenditure would not need to be reduced as much. This would improve output and employment which would then lead to higher consumption as households gained more income.

3.2.5. Expectations regarding future macroeconomic variables

In conjunction with the other channels, private agents’ expectations regarding the future may be altered by the policy. For example, forward looking households could come to expect that the policy will lead to an improvement in employment and wage outcomes. In turn, this would lead to higher spending in the near-term. The plausibility of this channel can be seen in QE’s effect on inflation expectations (Krisnamurthy and Jorgensen, 2011; Ugai, 2006).

3.2.6. Exchange Rates

In an era of open capital markets, the effect of monetary policy on economic activity through the exchange rate is well documented and represents another major channel whereby QE could affect output and prices (Mishkin, 2007; Mishkin, 1995; Bernanke et al., 2004; Coenen and Wieland, 2003; Bordon and Weber, 2010). In the short-run, a nominal depreciation equates to a real exchange rate depreciation and so an improvement in net exports (Krugman, 2000). Foreign investment is thereby made more expensive as the currency depreciates which should encourage firms to invest domestically. Firms are also more likely to purchase their capital products from domestic firms if there are domestic substitutes, owing to the real exchange rate depreciation. A similar analysis applies to consumers with a currency depreciation making it more likely for consumers to purchase domestic products and services. The nominal depreciation potentially induced by the policy would therefore stimulate economic activity.
Unfortunately, there appears no unambiguous relationship between QE and the exchange rate because of the numerous contradictory effects the policy can have. In the one direction is the role of the carry-flow trade.\footnote{The carry flow trade is the process whereby investors borrow in a currency with low domestic interest rates and then use these borrowed funds to purchase assets in countries with higher interest rates in order to maximise their returns (Debelle, 2006).} The purchase of government securities and other financial assets provides investors with a zero-yielding asset in return for an asset with a low but positive yield. As a result, return-maximising investors would seek to use the newly acquired cash to invest in assets which are yielding the highest return. Presently, these high-return assets are located outside of the countries employing QE due to stagnant output growth and high unemployment. QE therefore necessitates an increase in the supply of domestic currency on the foreign exchange market, which should lead to a nominal depreciation.

Portfolio rebalancing effects and expectations of higher future inflation will strengthen this effect. If investors have preferred portfolios to maximise returns, the sale of government bonds for cash disrupts this balance. Investors would respond to this by purchasing more of the desired assets (both domestic and international) to restore the desired ratios. This in turn requires that domestic currency is supplied for foreign currency, resulting in a currency depreciation, \textit{ceteris paribus}. Concerns regarding future inflation would also induce investors to sell because of the anticipated erosion of the currency’s purchasing power, implying a nominal depreciation.

However, QE may lower investors’ risk appetites and confidence, placing a constraint on the carry flow trade effect and leading to an influx of capital into ‘safe’ currencies. Higher inflation expectations may also induce purchases of currencies in anticipation of future interest rate rises. Further, QE may result in retaliatory policies by foreign governments due to a perception that it is a ‘beggar they neighbour’ policy.

It appears that there were declines in the $US/£UK nominal exchange rates immediately after each phase of QE was announced but since 2010, these exchange rates have returned to their pre-policy levels. Given the apparently inconclusive evidence and the lack of empirical research, this particular channel will not be incorporated in the model. A careful empirical examination of QE’s effect on exchange rates lies beyond the scope of this thesis but represents an avenue for future research.
IV. STRUCTURAL MODEL

The base model which this thesis draws upon is the model developed by Gilchrist et al. (2009) who augment the Smets and Wouters (2007) model (hereafter ‘SW model’) with a ‘financial accelerator.’ The SW model is a medium-scale macroeconomic model similar to the canonical New Keynesian model (Gali, 2008). It incorporates imperfect competition, limits on the ability of firms to adjust prices as in Calvo (1983) and a variety of additional non-financial frictions which are relevant to the questions analysed in this thesis. These include: variable capital utilisation, wage and price indexation, labour unions generating a wage mark-up and time-dependent wage determination.

While there were a number of competing models, the SW model was chosen as the base model for analysing the potential macroeconomic effects of the policy for two reasons. First, it fits the US data very well and embodies a micro-founded framework, thereby avoiding the Lucas (1976) critique as much as possible. Second, it is widely accepted as a benchmark model for monetary policy analysis (Adjemian et al., 2008).

Nevertheless, the SW model has the crucial limitation that it does not contain sufficient financial market imperfections to ascertain whether QE can have an effect on output and prices. This somewhat limits its application to QE. Financial frictions are vital because of the empirical evidence and the result by Curdia and Woodford (2010) that under perfect competition, separable money balances with a satiation level and complete financial markets, an irrelevance proposition may be proven for the excess supply of bank reserves.

Gilchrist et al. (2009) introduces an external finance premium to the SW model by assuming there is an information asymmetry between borrowers and lenders leading to a wedge between the return to capital and the risk-free rate. This extension was selected from a variety of candidates because it contains two of the main shocks believed to have caused the GFC: a shock to the external finance premium (i.e. yield spreads) and a shock to the net worth of firms and consumers (Blanchard, 2010). The model also closely fits the data (Pagan and Robinson, 2011) and so presents a plausible framework for the experiments outlined in section 5.
Nevertheless, to comprehensively examine QE and the various channels through which it could work it was necessary to augment the SW model further. In this section, the basic underlying structure will be set out first, followed by the log-linearised equations and then the various augmentations to the Gilchrist et al. (2009) version of the SW model.

4.1 Outline of Smets and Wouters (2007)

There are five agents in the model: households, labour unions, intermediate goods firms, final goods firms and the government. Households seek to maximise their objective utility function by choosing consumption, the amount of labour to supply to labour unions, bonds, investment and capital utilisation.

Within each industry type, labour unions aggregate the homogeneous household labour, differentiate it and then sell it to labour companies, subject to time-dependent wage adjustment based on Calvo (1983). They distribute any profit back to the households that make up the union. The labour companies combine the differentiated labour into a homogeneous aggregate labour bundle which they then sell to intermediate goods firms.

Intermediate goods firms seek to maximise profits by renting capital and differentiated labour and selling their differentiated goods to final goods producers. In turn, final goods producers seek to maximise profit in a perfectly competitive market by repackaging intermediate goods and selling them to consumers, investors and the government. The following outline draws heavily on the supporting appendix to the SW model.

4.1.1 Households

There exists a continuum of infinitely lived households which maximise the expected present discounted value of utility given by:\(^{17}\)

\[
E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{1}{1-\sigma_c} (C_{t+s}(j) - \lambda C_{t+s-1})^{1-\sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_I} b_{t+s}(j)^{1+\sigma_I} \right)
\]

(4.1)

\(^{17}\) The omission of money balances from the utility function is not crucial. It is assumed that the central bank supplies reserves to satisfy the level of money demand at the desired level for the policy rate.
where $\sigma_c$ is the co-efficient of relative risk aversion, $\sigma_l$ is the elasticity of substitution of leisure and $\lambda$ is the degree of external consumption habits.

Each household seeks to maximise their utility by choosing consumption ($C_t(j)$), hours worked ($L_t(j)$), investment ($I_t(j)$), capital utilisation ($u_t(j)$) and bonds ($B_t(j)$) subject to an intertemporal budget constraint, a capital accumulation constraint and a capital utilisation constraint.

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\epsilon_t^B K_{t+s}^B P_{t+s}} - T_{t+s}$$

$$\leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{t+s}^h(j)L_{t+s}(j)}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}} + \frac{R_{t+s}^h u_{t+s}(j)K_{t+s-1}(j)}{P_{t+s}} - a(u_{t+s}(j))R_{t+s-1}(j)$$

$$K_t(j) = (1 - \delta)K_{t-1}(j) + \epsilon_t^I \left[ 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j)$$

$$K_t^B(j) = u_t(j)K_{t-1}(j)$$

The intertemporal budget constraint requires the net present value of consumption, investment, the costs of adjusting the capital utilisation rate $\left( a(u_{t+s}(j))R_{t+s-1}(j) \right)$ and bond purchases to equal the net present value of after tax bond income $\left( \frac{R_{t+s-1}(j)}{P_{t+s}} \right)$, labour income $\left( \frac{W_{t+s}^h(j)L_{t+s}(j)}{P_{t+s}} \right)$, labour union dividends $\left( \frac{Div_{t+s}}{P_{t+s}} \right)$ and capital income $\left( \frac{R_{t+s}^h u_{t+s}(j)K_{t+s-1}(j)}{P_{t+s}} \right)$. $\epsilon_t^B$ represents an exogenous shock to bond returns.

The capital accumulation constraint incorporates an increasing marginal adjustment cost for investment $\left( \left[ 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \right] \right)$ with $S'(\gamma) = S''(\gamma) = 0, S''' > 0$ and an investment specific technology shock $\left( \epsilon_t^I \right)$. The capital utilisation constraint indicates that the level of capital services available to be used by intermediate goods firms $\left( K_t^B(j) \right)$ is the product of the previous period’s capital stock $\left( K_{t-1}(j) \right)$ and the utilisation rate $\left( u_t(j) \right)$.

---

18 $W_{t+s}^h(j)$ is the wage household $j$ receives for supplying their labour to union $j$.

19 $P_{t+s}$ is the return on a unit of capital services.
The households’ problem yields the following first order conditions with the j index dropped because all households are homogeneous and so will all make the same choices.

\[
\frac{w^h_t}{P_t} = -\left[\left(\frac{1}{1-\sigma_c}(C_t - \lambda C_{t-1})^{1-\sigma_c}\right)\exp\left(\frac{\sigma_c-1}{1+\sigma_i}L_{t}^{1+\sigma_i}(\sigma_c - 1)L_t^{\sigma_i}\right)\right]\frac{\xi_t}{\xi_t}
\]

(4.5)

\[
\xi_t = \exp\left(\frac{\sigma_c - 1}{1 + \sigma_i}(C_t - \lambda C_{t-1})^{-\sigma_c}\right)
\]

(4.6)

\[
\xi_t = \beta e^h R_t E_t \left[\frac{\xi_{t+1}}{\xi_{t+1}}\right]
\]

(4.7)

\[
\xi_t = \xi^{k^i} e_t^i \left(1 - S\left(\frac{L_t}{L_{t-1}}\right) - S'\left(\frac{1}{L_{t-1}}\right)\right) + \beta E_t \left[\xi^{k^i} e_{t+1} S'\left(\frac{L_{t+1}}{L_t}\right)^2\right]
\]

(4.8)

\[
\xi^{k^i} = \beta E_t \left[\xi_{t+1} \left(\frac{R_t^k}{P_t^k} u_{t+1} - a(u_{t+1})\right) + \xi^{k^i} e_{t+1}(1 - \delta)\right]
\]

(4.9)

\[
\frac{R_t^k}{P_t} = a'(u_t)
\]

(4.10)

Equation 4.5 is a standard labour supply equation which links the real wage received by the households \(\left(\frac{w^h_t}{P_t}\right)\) with the marginal rate of substitution between consumption and leisure. Equation 4.6 and 4.7 are standard Euler equations relating the expected marginal utility of current consumption with the expected marginal utility of future consumption, where \(\xi_t\) is the Lagrange multiplier associated with the budget constraint in the optimisation problem. Equations 4.8 and 4.9 determine the optimal choice of investment by relating the marginal cost with the marginal benefit. \(\xi^{k^i}\) refers to the Lagrange multiplier on the capital accumulation constraint. Equation 4.10 relates the marginal cost of changing the capital utilisation rate \(a'(Z_t))\) to the higher capital income that is derived from supplying more capital services to intermediate goods producers.
4.1.2 Labour Market

Labour unions take the homogeneous labour from households and differentiate it before on-selling it to labour companies. Labour companies then aggregate the differentiated labour using a constant elasticity of substitution (CES) function. Finally, they sell the labour good to intermediate goods producers in a perfectly competitive market. The demand for each unit of differentiated labour is derived from the profit maximisation problem of the labour companies:

$$\max_{L_t} \ W_t L_t - W_t(j)L_t(j) \quad \text{s. t.} \quad L_t = \left[ \int_0^1 \left( L_t(j) \right)^{\frac{1}{1+\lambda_{w,t}}} \, dj \right]^{1+\lambda_{w,t}}$$

(4.11)

where \( W_t \) and \( L_t \) represent the aggregate wage and labour respectively. \( W_t(j) \) and \( L_t(j) \) are the wages paid to and labour demanded from labour union \( j \). \( \lambda_{w,t} \) is the elasticity of substitution between the differentiated labour bundles from the different unions.

The first order condition of this problem yields the labour demand function for each union’s differentiated labour:

$$L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1}{\lambda_{w,t}}} L_t$$

(4.12)

The labour company’s problem under perfect competition yields the equation for aggregate wages as a function of industry wages.

$$W_t = \left[ \int_0^1 \left( W_t(j) \right)^{\frac{1}{\lambda_{w,t}}} \, dj \right]^{\lambda_{w,t}}$$

(4.13)

Labour unions take this demand for their labour as given and seek to maximise their profits which they then return to households. The labour unions are subject to time dependent wage adjustment which means that they can only adjust wages with probability \( 1 - \zeta_t^w \) each period as in Erceg et al. (2000). The union seeks to maximise the expected discounted present value of the differential between the wage it pays to households and the wage it offers to the labour company over the period when its wage will be fixed by selecting the optimal wage \( \bar{W}_t(j) \).
subject to the labour demand constraint:

\[ L_t(j) = \left( \frac{W_t(j)}{W_t(j)} \right)^{\frac{(z+1)\lambda_{ws}}{1-\alpha}} L_t \]  

(4.15)

It is assumed that unions are owned by households and so they discount the future at the households’ stochastic discount factor \( \frac{\beta^z \Xi_{t+s} \Pi_t}{\Xi_t \Pi_{t+s}} \). Wage indexation is present in the model so that when a firm cannot re-optimize, the nominal wage they receive grows at the deterministic growth rate of technology \( (\gamma) \) and a weighted average of last period’s inflation and the steady state inflation rate \( \left( \pi_{t+1-1} \pi_{s}^{1-s} \right) \).

\[ W_{t+s}(j) = \bar{W}_t(j) \left( \prod_{i=1}^{s} \gamma_{t+s-1}^{iw} \pi_{s}^{1-s} \right) \]  

(4.16)

The first order condition is given below after substituting in the household labour supply decision derived above and multiplying by the optimal wage:

\[ E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^z \Xi_{t+s} \Pi_t}{\Xi_t \Pi_{t+s}} L_{t+s}(j) \frac{1}{\lambda_{w,t+s}} \left( 1 + \lambda_{w,t+s} \right) W_{t+s}^h - X_{t,s} \bar{W}_t(j) = 0 \]  

(4.17)

where

\[ X_{t,s} = \left\{ \begin{array}{ll} 1 & \text{for } s = 0 \\ \prod_{i=1}^{s} \gamma_{t+s-1}^{iw} \pi_{s}^{1-s} & \text{for } s = 1, \ldots, \infty \end{array} \right. \]  

(4.18)

Essentially, equation 4.18 captures the fact that the fixed wage is only indexed to inflation and the deterministic growth rate in future periods. Equation 4.17 relates the optimal price to a weighted average of wages and is used to determine the path of the real wage around the steady state.
4.1.3 Final Goods Firms

Final goods producers seek to maximise profits by purchasing intermediate goods, packaging them and selling them to consumers, investors and the government. They operate under perfect competition, taking the price of output as given. Consequently, their maximisation problem becomes:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$  \hspace{1cm} (4.19)

subject to$^{20}$

$$1 = \int_0^1 G \left( \frac{Y_t(i)}{Y_t} ; \lambda_{p,t} \right) di$$  \hspace{1cm} (4.20)

The first order conditions are:

$$P_t = \frac{\mu_t}{Y_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di$$  \hspace{1cm} (4.21)

$$P_t(i) = \mu_t G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{1}{Y_t}$$  \hspace{1cm} (4.22)

Solving out for $\mu_t$ (which is the Lagrange multiplier) yields a demand schedule for each differentiated, intermediate good:

$$Y_t(i) = Y_t G^{-1} \left[ \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di \right]$$  \hspace{1cm} (4.23)

4.1.4 Intermediate Goods Firms

Intermediate goods firms hire capital and labour in the current period, taking the utilisation rate as given, and produce differentiated goods which they then sell to final goods producers. Each firm has some product market power, which introduces a price mark-up, but is only able to alter their prices each period with a given, exogenous probability as in Calvo (1983). Firms seek to maximise profit

20 $G \left( \frac{h(i)}{h_t} ; \lambda_{p,t} \right)$ is a constant-returns-to-scale variety aggregator which is able to replicate any demand curve including a demand curve with a non-constant elasticity (Kimball, 1995). The constant-elasticity aggregator of Dixit and Stiglitz (1977) is a special case of this general aggregator.
\[
\max_{K_t^L, L_t} p_t(i) Y_t(i) - W_t L_t - R_t^k K_t^L
\]  
(4.24)

subject to

\[
Y_t(i) = \epsilon_t^a (K_t^L(i))^a (y^t L_t(i))^{1-a} - y^t \Phi
\]  
(4.25)

where \(0 < \alpha < 1\). In 4.24 and 4.25, \(Y_t(i)\) is the intermediate goods output of firm \(i\), \(\gamma\) is the labour augmenting technology growth rate, \(\epsilon_t^a\) is a technology shock and \(\Phi\) is the fixed production costs. The firm’s optimisation problem becomes:

\[
L = \max_{K_t^L, L_t} \left[ p_t(i) Y_t(i) - W_t L_t - R_t^k K_t^L \right] - \Theta_t(i) \left[ Y_t(i) - \epsilon_t^a (K_t^L(i))^a (y^t L_t(i))^{1-a}\right]
\]  
(4.26)

The first order conditions yield demand schedules for both capital and labour as follows:

\[
\Theta_t(i) y^{t(1-a)}(1-a) \epsilon_t^a (K_t^L(i))^a (L_t(i))^{-a} = W_t
\]  
(4.27)

\[
\Theta_t(i) y^{t(1-a)}(a) \epsilon_t^a (K_t^L(i))^{a-1} (L_t(i))^{1-a} = R_t^k
\]  
(4.28)

Combining these yields the relationship between capital services and labour:

\[
K_t^L = \frac{\alpha \cdot W_t}{1 - \alpha R_t^k L_t}
\]  
(4.29)

Using this and the result that the marginal cost for each firm is equal to the Lagrange multiplier (Sydsaeter & Hammond, 2008) yields and expression for the firms’ marginal cost

\[
\Theta_t = MC_t = \alpha^{-a}(1-a)^{-(1-a)} W_t^{1-a} (R_t^k)^a y^{(1-a)}(\epsilon_t^a)^{-1}
\]  
(4.30)

Note that the marginal cost \(MC_t\) is the same across firms because they are all homogeneous and so they all make the same price, capital and labour choices in equilibrium.

The firm faces time-dependent pricing and so they seek to maximise the discounted flow of profits over the expected interval when the price cannot be changed. As with labour unions, firms are owned by households and so they discount the future using the households’ stochastic discount factor. Further, when fixed their price is indexed to the deterministic growth rate and a weighted average of past and steady state inflation \(\prod_{t=1}^{\delta} \gamma_{t+1-t}^{-1} n_{t+1}^{-1}\).
This yields the optimisation problem:

\[
\max \left( \frac{\max \prod_{t=0}^{\infty} \left( \frac{\beta^t \sum_{s=0}^{\infty} P_t}{n P_{t+s}} \right) \left( \prod_{i=1}^{2} W s_{i-1} \right)^{s_{i-1} - 1} M C_{t+s} \right)}{Y_{t+s}(t)} \right)
\]  

subject to the demand function for the firm’s goods

\[
Y_{t+s}(t) = Y_{t+s} G'^{-1} \left( \frac{P_t(i) X_{t+s}}{P_{t+s}} \right) \int_0^1 \left( \frac{Y_{t+s}(i)}{Y_{t+s}} \right) Y_{t+s}(i)
\]

This problem is equivalent to maximising the following objective function subject to equation 4.35\(^21\)

\[
\max \left( \frac{\max \prod_{t=0}^{\infty} \left( \frac{\beta^t \sum_{s=0}^{\infty} P_t}{n P_{t+s}} \right) \left( \prod_{i=1}^{2} W s_{i-1} \right)^{s_{i-1} - 1} M C_{t+s} \right)}{Y_{t+s}(t)} \right)
\]

Defining

\[
\tau_{t+s} = \int_0^1 \left( \frac{Y_{t+s}(i)}{Y_{t+s}} \right) Y_{t+s}(i)
\]

allows one to redefine equation 4.32 as

\[
Y_{t+s}(i) = Y_{t+s} G'^{-1} \left( \frac{P_t(i) X_{t+s}}{P_{t+s}} \right) \tau_{t+s}
\]

The first order condition of from this optimisation problem is:

\[
E_t \sum_{s=0}^{\infty} \left( \frac{\beta^t \sum_{s=0}^{\infty} P_t}{n P_{t+s}} \right) \left( \prod_{i=1}^{2} W s_{i-1} \right)^{s_{i-1} - 1} M C_{t+s} \frac{1}{G'^{-1}(\xi_{t+s})} G'(x_{t+s}) Y_{t+s}(i) = 0
\]

where

\[
x_{t+s} = G'^{-1}(\xi_{t+s}) = G'^{-1} \left( \frac{P_{t+s}^l}{P_{t+s}^s} \right)
\]

Combining this condition with the maximisation conditions for final goods producers yields an expression for the aggregate price index.

\(^{21}\)\(x_{t,s}\) is defined by equation 4.18
\[ P_t = (1 - \xi_p)G^{-1} \left( \frac{P_t}{P_t} \tau_t \right) + \xi_p P_{t-1} \left( \frac{\pi^{1-p}_{t-1} \pi_s^{1-\rho_p}}{P_t} \right) G^{-1} \left( \frac{P_{t-1} \pi^{1-p}_{t-1} \pi_s^{1-\rho_p}}{P_t} \right) \tau_t \] (4.38)

Log-linearising equations 4.36 and 4.38 around a zero inflation steady state and then substituting out for the optimal price level yields a version of the New Keynesian Phillips Curve (equation 4.42).

4.1.5 Government Sector

The government sector conducts fiscal policy and monetary policy. Fiscal policy is financed through lump-sum taxation and issuing one period bonds. The government must satisfy the following budget constraint every period:

\[ P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \] (4.39)

Monetary policy is conducted by supplying households with whatever money stock is required to produce the desired nominal interest rate. This is found using the money demand schedule of households. The central bank reacts to economy wide variables using the following Taylor rule:

\[ \frac{R_t}{R_s} = \left( \frac{R_t}{R_s} \right)^{\frac{\psi_2}{\psi_1}} \left( \frac{\psi_1}{\psi_2} \right)^{\psi_2} \left( \frac{Y_t}{Y_t^{f}} \right)^{1-\psi_2} \left( \frac{Y_t^{f}}{Y_t^{f}} \right) e^{\varepsilon^m} \] (4.40)

where \( \varepsilon^m \) is the monetary policy shock and \( Y_t^{f} \) is the level of output obtained under fully flexible prices and wages.
4.2 Outline of Gilchrist et al. (2009) Augmentations

Gilchrist et al. (2009) introduce an external finance premium of the kind developed by Bernanke et al. (1999) into the SW model in the following way. Instead of intermediate goods producers, entrepreneurs hire capital and labour from households and financial institutions to produce differentiated intermediate goods which they then sell to final goods producers. Entrepreneurs are assumed to survive to the next period at an exogenous rate (θ) with those who do not survive simply consuming their net worth. In the next period, new entrepreneurs enter so that the stock of entrepreneurs remains constant over time.22

Entrepreneurs finance their capital and labour purchases through their borrowings and net worth.23 In the Gilchrist et al. (2009) model (hereafter ‘GOZ model’) it is assumed that there is a costly state verification problem for financial institutions. The entrepreneurs may observe the return on their capital but financial institutions cannot, unless they pay an ‘auditing cost’ which is assumed to be a proportion of the gross return on the entrepreneur’s capital (Bernanke et al., 1999: 1350). This information asymmetry generates an external finance premium (S_c) which is negatively related to the ratio of entrepreneurs’ net worth to debt. As a result of this augmentation, the price for capital24 and resource constraint equations must be altered and the investment specific technology shock is removed. Extra equations describing the relationship between the external finance premium and

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22 It is assumed that a very small transfer of wealth from households to entrepreneurs occurs so that these new entrepreneurs can start business.

23 In the SW model, entrepreneurs borrow from financial institutions who can perfectly verify the return on capital and so in equilibrium firms pay out the full return on capital (R^c) to households.

24 Equation 4.50 may be derived by realising that the return on capital in the presence of the external finance premium depends on the net cost after depreciation of a unit of capital and the marginal revenue that is gained by that unit of capital. The optimality condition in Smets and Wouters (2007) is \( Q_t = E_t \left[ \beta \left( \frac{E_{t+1}}{E_t} \right) \left( Q_{t+1}(1 - \delta) + u_{t+1}r^{t+1}_{t+1} - a(u_{t+1}) \right) \right] \). With an external finance premium, \( S_c \) enters this condition to reflect the extra cost the firm faces when borrowing to purchase a unit of capital. The cost drives a wedge between the return on the risk-free asset (long-term interest rate) and private capital (Drautzburg and Uhlig, 2011). The condition implies that in the steady state \( R^c = \frac{S_c}{\beta} \).

(1 - \delta) = \left( \frac{\bar{E}_t}{E_t} \right) \left( 1 - \delta \right) \) as entrepreneurs’ return on capital is altered by their degree of leverage. Log-linearising the adjusted optimality condition yields the equation above (Appendix, Section K). Essentially, in the log-linearised equation the real interest rate (r_t - \( E_t \pi_{t+1} \)) is replaced with the new, effective real interest rate (r_t - E_t \pi_{t+1} + S_c). Due to the profit maximisation of firms \( R^c = m p_k \) for all t so the rest of equation 4 in Smets and Wouters (2007) is consistent with the formulation in equation 15 of Gilchrist et al. (2009).
entrepreneurs’ net worth (equation 4.53)\(^{25}\) and the evolution of this net worth over time (equation 4.54)\(^{26}\) are also incorporated.

### 4.3 Dynamic Equations of Gilchrist et al. (2009)

The first order conditions set out in section 4.1 can be log-linearised around the model’s steady state using Uhlig (1995) to give the following dynamic equations\(^{27}\) where \(\tilde{x}_t\) denotes the log deviation of a variable from its steady state.

\[
\tilde{x}_t = (1 - \alpha) \tilde{x}_t + \alpha x^*_t - e^*_t \tag{4.41}
\]

\[
\tilde{r}_t = \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \tilde{r}_{t-1} + \left[ \frac{\beta y (1 - \sigma_c)}{1 + \beta y (1 - \sigma c)} \right] E_t \tilde{r}_{t+1} - \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \left( \frac{(1 - \xi_p) (1 - \beta y (1 - \sigma_c) \xi_p)}{\xi_p ((\phi_p - 1) \xi_p + 1)} \right) \tilde{m}_t + e^*_t \tag{4.42}
\]

\[
\tilde{w}_t = \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \tilde{w}_{t-1} + \left[ \frac{\beta y (1 - \sigma_c)}{1 + \beta y (1 - \sigma c)} \right] (E_t \tilde{w}_{t+1} + E_t \tilde{r}_{t+1}) - \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \tilde{r}_t - \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \left( \frac{1}{1 - \xi_p} \right) (\tilde{e}_t - \tilde{e}_{t-1}) + e^*_t \tag{4.43}
\]

\[
\tilde{e}_t = \left[ \frac{\sigma_c - 1}{\sigma_c (1 + \frac{2}{\gamma})} \right] \left( \frac{w^* t}{\xi_p} \right) (\tilde{e}_t - \tilde{E}_t) - \left[ \frac{1}{\sigma_c (1 + \frac{2}{\gamma})} \right] (\tilde{e}_t - \tilde{E}_t) + e^*_t \tag{4.44}
\]

\[
\tilde{m} p k^*_t = \tilde{r}_t = \tilde{w}_t + \tilde{t}_t - \tilde{k}_t \tag{4.45}
\]

\[
\tilde{k}_t = \tilde{r}_t + \tilde{u}_t \tag{4.46}
\]

\[
\tilde{u}_t = \left[ \frac{1 - \psi}{\psi} \right] \tilde{r}_t = \left[ \frac{1 - \psi}{\psi} \right] \tilde{m} p k_t \tag{4.47}
\]

\[
\tilde{t}_t = \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \tilde{t}_{t-1} + \left[ \frac{\beta y (1 - \sigma_c)}{1 + \beta y (1 - \sigma_c)} \right] E_t \tilde{t}_{t+1} + \left[ \frac{1}{1 + \beta y (1 - \sigma_c)} \right] \left( \frac{1}{1 - \xi_p} \right) \tilde{q}^*_t + e^*_t \tag{4.48}
\]

\(^{25}\) The relationship between the external finance premium and entrepreneurs’ net worth is defined as \(\tilde{S}_t = \left( \frac{\rho d}{\rho y} \right) e^*_t \). After transforming both sides, the following expression was obtained: \(e^*_t = e^{\psi (k_t + \xi + \gamma + \phi)} e^*_t\). A first order approximation of both sides of this equation around the model’s steady state yields equation 4.53.

\(^{26}\) \(\tilde{t}_t = \frac{\kappa}{\mu} (\tilde{t}_t - E_{t-1} \tilde{t}_t) + E_{t-1} \tilde{t}_t^b + \theta \tilde{t}_{t-1} + \tilde{e}^*_t\) is an alternative way of expressing the evolution of net worth over time. The other representation was used for simplicity.

\(^{27}\) When deriving equation 4.50 from equation 4.9 one needs to utilise the fact that based on equation 4.10, \(\tilde{r}_t = a (u_t)\) in the steady state. A complete derivation is given in the Appendix (Section K).
The steady state of the model and the shock processes are described in the appendix (Section D and F respectively). For a more detailed description of all these equations, the reader is referred to those in both Smets and Wouters (2007) and Gilchrist et al. (2009).

4.4 Augmentations to Gilchrist et al. (2009)

As it stands, the GOZ model is unable to capture all of the channels through which QE could affect economic activity levels. For example, the model does not contain a portfolio rebalancing effect. The model must therefore be augmented.

The augmentations are in three levels so that a number of different effects can be isolated and examined. First, a signal effect for the provision of excess reserves was incorporated into the GOZ
model. Second, a signal effect and portfolio rebalancing effect were added to the GOZ model. Finally, a variable elasticity of the external finance premium to entrepreneurs’ net worth was introduced to determine whether QE’s effects were robust to changes in the yield spread arising from financial system shocks and independent of entrepreneurs’ net worth.

4.4.1 Signal Effect in GOZ model

It was felt that the most effective way to incorporate the signalling channel\textsuperscript{28} was to alter the Taylor rule. The central bank is assumed to set the nominal interest rate in the following way:

\[
\frac{R_t^p}{R_s^p} = \left( \frac{R_t^{p_{t-1}}}{R_s^p} \right) \rho_t^p \left[ \left( \frac{\pi_t}{\pi_s} \right) \psi_1 \left( \frac{Y_t}{Y_s} \right) \right]^{1-\rho_t^p} \left( \frac{\frac{X_t}{v_{t-1}}}{\frac{X_t}{v_{T}}} \right) \left( \frac{1}{E_{R_t-1}} \right)^\zeta r_t^p
\] (4.58)

The nominal interest rate set by the monetary authority reacts to the lagged value of the central bank balance sheet ($E_{R_t}$). After taking expectations of both sides, equation 4.58 implies that a large central bank balance sheet today will lead private sector participants to expect a lower policy rate tomorrow, ceteris paribus. This signal is credible because the large-scale purchase of assets takes considerable time to implement and unwind (Shiratsuka, 2009). The results were not sensitive to this specification as similar dynamic impulse response functions were obtained when using different lags on the excess reserves variable.

It is assumed that the central bank chooses the size of its balance sheet according to a specific rule. Despite the apparent variation between the different versions of the policy as implemented by the Federal Reserve, the Bank of England and the Bank of Japan, they have all fundamentally involved the central bank independently choosing the level of government securities and other financial assets through the creation of central bank reserves. Furthermore, there is little difference between a central bank rule for the targeting of a certain monetary aggregate (involving periodic injections or removal of liquidity) and the targeting of a level of asset purchases and so a specific rule is a valid representation of the QE policy (Curdia and Woodford, 2011).

\textsuperscript{28} This is defined as the large-scale purchase of assets sending a credible signal that there has been a change in the way the central bank responds to economic fluctuations in the short to medium term.
It is assumed that the rule takes the following form following Gertler and Karadi (2011):

\[ ER_t - ER_* = \kappa (S_t - S_*) \]  \hspace{1cm} (4.59)

In terms of deviations from the steady state:\(^{29}\)

\[ \bar{ER}_t = \kappa \left( \frac{S_\star}{ER_*} \right) \bar{s}_t = \frac{\kappa \left( \frac{\kappa}{N} \right)^X \bar{s}_t}{ER_*} \]  \hspace{1cm} (4.60)

One of the primary reasons for QE was a belief that the substantial increases in yield spreads between 2008-2009 were doing significant damage to economic activity. Indeed, based on Federal Reserve minutes it is clear that the extent of monetary policy accommodation was tied to the state of the economy and in particular, credit spreads and availability (Federal Reserve, 2011d; Bernanke, 2009). This is captured by assuming that the central bank reacts to the deviation of the external finance premium from its steady state value when determining the size of its balance sheet (equation 4.59).\(^{30}\)

While there was some diversification in the types of assets purchased, the overwhelming majority of purchases were in two very narrow asset classes for both the US and the UK (Figures 1 and 2) and so it was deemed sufficient to just consider the overall size of the central bank balance sheet instead of purchases in specific asset classes. A more detailed division must be left for future research.

### 4.4.2 Portfolio rebalancing effect in GOZ model

The first step to introducing both effects into this model is to include a one year nominal interest rate into the model in addition to the policy rate. It is assumed that the financial firms which borrow from the central bank and lend to households and entrepreneurs, face transaction costs which generate a wedge between the policy rate and the new interest rate. Practically speaking, this appears reasonable

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\(^{29}\) Given the definition of the external finance premium, \(S_\star = \left( \frac{\kappa \sigma}{N} \right)^X \). As shown in Smets and Wouters (2007), \( Q_* = 1 \) so this simplifies to \( S_\star = \left( \frac{\kappa}{N} \right)^X \cdot ER_* \), was set to 24.30362 which represents a monotonic transformation of the average value over the 2007-2008 period (Federal Reserve, 2011a). Alternative transformations were investigated but little changes to the results were obtained.

\(^{30}\) It appears that the Federal Reserve could have also been targeting the output gap as well as yield spreads using QE. Consequently, an investigation into whether a decision rule incorporating the output gap generated different dynamics was conducted \( \left( \frac{\kappa \sigma}{N} \right)^X \). In the GOZ model, the external finance premium impulse response is negatively correlated with the output gap impulse response. Hence, this alternative formulation did not generate quantitatively important differences.
because the Federal Reserve only supplies liquidity to certified financial institutions. Households and firms therefore cannot lend and borrow at the risk-free policy rate. While there are a number of interpretations for this new nominal rate, for current purposes, the best interpretation is the one year government securities interest rate. This approach is similar to that of Drautzburg and Uhlig (2009) except that the bond spread is assumed to be time invariant.31

Using the expectations approach to interest rate determination, this new interest rate is determined by a weighted average of the current and expected future policy rates in the next three quarters. Persistence is also introduced based on empirical evidence of OECD interest rates (Romero-Avila, 2007). The results were not sensitive to the assumed degree of persistence or the particular maturity chosen for the long-term interest rate.32

\[ R_t = (R_{t-1})^{p_R} \left[ \left( \left( R_t^p \right) \left( R_{t+1}^p \right) \left( R_{t+2}^p \right) \left( R_{t+3}^p \right) \right)^{\frac{1}{3}} \right]^{1-p_R} \] (4.61)

In the steady state, the only difference between the rates is the assumed 13 basis points spread based on the 1990-2011 average yield spread between the one year treasury security and the federal funds rate (Federal Reserve, 2011) so that:

\[ R_* = R_p^p + 0.000325 \] (4.62)

It should be noted that this means that the issue of the zero lower bound on the interest rate is less problematic because agents make decisions based on the new nominal interest rate which remains positive even when the policy rate reaches zero. This circumstance is supported by recent evidence from a number of countries where the policy rate is now effectively zero (Federal Reserve, 2011b). The results were not sensitive to the size of this yield spread.34

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31 Introducing the long-term interest rate implies that the Euler equation for households is altered to \[ \Xi_t = \beta e_t^\pi R_t E_t \left[ \frac{\pi_t}{\pi_{t+1}} \right] \] instead of \[ \Xi_t = \beta e_t^\pi R_t E_t \left[ \frac{\pi_t}{\pi_{t+1}} \right] \].
32 Simulations using a six-month and five-year interest rate did not yield quantitatively different results to those reported.
33 The 0.000325 is calculated by annualising the steady state real policy rate in the GOZ model which yields a value of 4.8%. The 13 basis point yield spread is then added before converting this back into a quarterly interest rate, providing the number used for the simulations.
34 Simulations were performed using an average yield spread of 200 basis points (reflecting the difference between deposit rates and the federal funds rate) and 300 basis points (representing the 1990-2011 average spread between the AAA yield and the federal funds rate) without significantly changing any of the results.
The new nominal rate allows the portfolio rebalancing and risk premium channels to be included relatively easily (equation 4.63). It is assumed based on the empirical evidence surveyed in section 2 that there is a negative relationship between the size of the central bank balance sheet and the long-term nominal interest rate. The size of the central bank balance sheet is determined as before (equation 4.59). In this formulation, only one channel is incorporated because under an alternative formulation the various effects could be aggregated together anyway (Appendix section J).

\[ R_t = (R_{t-1})^{\rho_r} \left[ \left( \frac{1}{E} \right)^{\rho_r} \left( \frac{1}{E} \right)^{\rho_r} \right]^{\rho_f} \]

The precise mechanism through which these effects arise will not be explicitly modelled because this thesis desires to take the empirical evidence on these effects as given and then establish the resulting impact on the economy. Unfortunately, this approach is susceptible to the Lucas (1976) critique because the way that rational investors respond to the provision of central bank reserves may change with QE. This thesis attempts to account for such changes but future research should empirically estimate these effects, if possible.

4.4.3 Augmented GOZ model with QE and a variable external finance premium

Much literature has been generated on the way financial institutions respond to variations in output. The conclusion reached is that risk premia, a class into which the external finance premium falls, varies counter-cyclically over the business cycle (Juttner and Hawtrey, 1997; 590). There has been some recent discussion in the literature regarding the counter-cyclical nature of the external finance premium and how this increases in periods of financial stress (Justiniano et al., 2011). Key factors which contribute to this include: (1) the capital hoarding decisions of financial institutions as they seek to repair their balance sheets, (2) internally or externally imposed improvements in monitoring and lending standards by financial institutions which create a stronger relationship between entrepreneurs’ net worth and the external finance premium and (3) the increase in bad loan write-offs - a phenomenon seen during the GFC and for Japan in the 1990s - which undermines financial institutions’ balance sheets and makes them more sensitive to variations in the capital-debt ratio of their clients (Blanchard,
2010). In the GOZ model, this is equivalent to suggesting that \( \chi \) (the parameter governing the way financial institutions respond to entrepreneurs’ net worth) is variable and dependent on output growth. Incorporating this feature into the model involves specifying the relationship between the external finance premium and entrepreneurs’ net worth in the following manner:

\[
S_t = \left( \frac{Q_t K_t}{N_t} \right)^{\chi_t} e^{\varepsilon_t^{fd}}
\]  

(4.64)

As shown in the Appendix (Section G) this implies that:

\[
\hat{s}_t = -\chi_t \left( \hat{\eta}_t - \hat{\eta}_t^{ks} - \hat{K}_t \right) + \hat{\chi}_t (\log(S_t)) + \varepsilon_t^{fd}
\]  

(4.65)

To be consistent with the evidence, \( \chi_t \) should be pro-cyclical. It should also contain a habit component because the institutional structure of financial institutions implies that they should only respond partially to current economic developments. One tractable and concise way of specifying this is:\(^{35}\)

\[
\hat{\chi}_t = \rho_{\chi} \hat{\chi}_{t-1} - \omega(\hat{\chi}_t - \hat{\chi}_{t-1}); \quad \rho_{\chi} = 0.8, \omega = 100
\]

(4.66)

---

\(^{35}\) The particular value of \( \omega \) was chosen to roughly match the data from the U.S. economy during 2008-2009. Over both the 1990-2007 period and the 2001-2007 period the average quarterly growth rate was 0.7%. Between Mar 2008 and June 2009 growth fell to an average of -0.9% per quarter (RBA, 2011). When compared with the increase in the BAA spread of 72% (taking into account endogenous movements because of changes to entrepreneurs’ net worth) this implies the given value. The results were not sensitive to large changes in this variable (Section 5, Tables 4 & 5).
V. SIMULATION RESULTS

5.1 Calibration of Model Parameters

A number of parameter values needed to be chosen before simulating the GOZ model using Dynare. The parameter values used by the authors in the provided mod file were used (Appendix, Section A). Extensive testing was conducted using two alternative sets of parameters (Appendix, Section A) and it was established that the results were not sensitive to this choice.

The new parameters in the Taylor rule (4.58) \( c \) and the long-term interest equation (4.63) \( \phi \) were set at 0.15 and 0.1 respectively and were obtained by approximate estimates of the signal provided by the central bank and the elasticity of long-term interest rates to the size of the Federal Reserve’s balance sheet.\(^{36}\) The response of excess reserves to the external finance premium was set at 2, which is lower than the value of 10 set in Gertler and Karadi (2011). It was felt this value was a better approximation based on the data.\(^{37}\)

In further extensions of the GOZ model it was necessary to calibrate the responsiveness of the effect of the central bank’s balance sheet on the long-term interest rate to output growth as well as the degree of counter-cyclical movement in \( \gamma \). No literature providing guidance on reasonable values for these parameters could be found and so a range of values was utilised.

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\(^{36}\) As detailed in Section 2, Krishnamurthy and Vissing-Jorgensen (2011), Joyce et al. (2010) and Gagnon et al. (2010) all find that the policy decreased long-term nominal interest rates by around 60 basis points and lowered expectations regarding the future course of the federal funds rate by around 75 basis points. Given the steady state value of the nominal policy rate in the model of 4.8%, this implies an elasticity of approximately 0.15 between the policy rate and the central bank balance sheet based on the increase in the Federal Reserve’s balance sheet (Federal Reserve, 2011a). The 1962-2011 average for a 1 year Treasury security is 5.829% (Federal Reserve, 2011b) which suggests an elasticity of around 0.1 between the long-term interest rate and the central bank balance sheet based upon the same increase in the Federal Reserve’s balance sheet. Further simulations indicated that the results were not sensitive to alternative calibrations of these two parameters.

\(^{37}\) The average BAA-10 year treasury security spread between July 2008-2011 was 3.43%, which is a 72% increase compared to its 2004-June 2008 average of 1.995% (Federal Reserve, 2011b). The average size of the Federal Reserve’s balance sheet was $2290953 bn over Sep 2008-2011 compared to an average of $891105 bn between 2007- Sep 2008, representing a 157% increase (Federal Reserve, 2011a). The value of \( \kappa \) is therefore approximately equal to 2. Using the 2002-Sep 2008 average for the Federal Reserve’s balance sheet yields an increase of 179%, which implies \( \kappa = 2.48 \). Simulations using this value did not yield substantially different results.
5.2 Simulation Results

Before proceeding to a discussion of the simulation results, it is important to discuss how these simulations relate to the GFC. The GFC was characterised by substantial losses in net worth for individuals, financial institutions and non-financial institutions due to the collapse of the housing market in the US, as well as a substantial increase in borrowing costs, which was reflected in the rise in spreads for a large variety of assets (Taylor, 2009, Mishkin, 2011). Within the context of the GOZ model, the financial crisis can be plausibly replicated by considering a shock to both the external finance premium and entrepreneurs’ net worth. Due to the log-linearised nature of the equations, the magnitude of the shocks generating the GFC can be easily replicated by altering the variance of these shocks. The model also contains exogenous variations in the returns on the bonds held by households. However, the authors state that this shock has similar effects as other external finance premium shocks considered by Bernanke et al. (1999) (Smets and Wouters, 2007: 589) which implies that using this shock will not add much to the analysis. Moreover, this shock has been criticised for being impossible to interpret (Chari et al., 2009).

One difference between the crisis and these simulations is that the zero lower bound on the nominal interest rate appears to remain binding (Federal Reserve, 2011b) while this feature is entirely absent from the GOZ model. The presence of the zero lower bound presents a problem to the validity of the log-linearisation technique used in this thesis because, once this constraint binds, expectations regarding future interest rates and inflation must be formed in a non-linear way.

The method employed in this thesis overcomes this difference in three ways. First, it is assumed that the central bank balance sheet acts as a signal for the future policy rate implying that, if the zero lower bound binds, then under the policy it will bind for a longer period of time than that dictated by the original policy rule. Second, despite the policy rate reaching its zero lower bound, longer-term interest rates have remained positive which leaves open the portfolio rebalancing channel for the policy, even if the signalling channel is unavailable. Third, it is well established that log-linearisation may be utilised even in the presence of the zero lower bound except the final simulation output must be
adjusted (Adolfson et al., 2011; Woodford, 2003; Bodenstein et al., 2010). However, after adjustment, the qualitative properties of the various impulse response functions are unaltered and the quantitative properties are largely unaltered as well (Gertler and Karadi, 2011; Bodenstein et al., 2010). This suggests that none of the following conclusions would be affected by this omission. Subsequent research should utilise the algorithm suggested by Bodenstein et al. (2010) to assess the quantitative impacts of the zero lower real bound on QE’s effectiveness.

5.2.1 Response to a shock to net worth and the external finance premium

The impulse response functions (IRFs) to a one standard deviation innovation to both the net worth \( \varepsilon_{t}^{nw} \) and external finance premium shocks \( \varepsilon_{t}^{fd} \) are as expected (Appendix Section B, Figure B.1). As the system has been log-linearised, the response of each variable is just the sum of the two impulse response functions. Output, investment and employment all fall substantially and the central bank responds by decreasing the policy rate which in turn lowers the long-term interest rate. While the central bank conducts the traditional function of monetary policy by reducing the interest rate in response to these developments, it is insufficient to return output to its steady state value before twenty periods.

5.2.2 Signal Effect in GOZ model

After the signal channel for the size of the central bank’s balance sheet is introduced, output, employment, investment and consumption deviate less from the economy’s steady state. This conclusion is based upon nearly all the relevant variables registering smaller negative deviations. In particular, inflation is stabilised around its steady state value. This suggests that one of the crucial benefits of the policy is that it can prevent an economy being caught in a deflationary spiral by stabilising inflation expectations. This seems to match the actual inflation response with inflation recording one quarter of significant declines (1% per month) before returning to the 2000-2010 decade average of 0.2% per month (BLS, 2011).

\[\text{The improvement in the quantitative properties of the impulse response functions was not affected by the choice of lag in the Taylor rule. Simulations using a two or four quarter lag returned differences of second order importance. A further simulation using the arithmetic average of a two and four quarter signal also did not alter the results substantially.}\]
5.2.3 GOZ model with signal and portfolio rebalancing effects

Incorporating the portfolio rebalancing effect, in addition to the signalling effect, does result in an improvement in the policy’s effectiveness (Appendix Section B, Figure B.1). Consumption, investment, output and employment all have smaller deviations with the implementation of QE. The reason for the improvement is that QE directly induces a lower long-term nominal interest rate through the portfolio rebalancing effect and makes the central bank respond more aggressively, *ceteris paribus*, to declines in output and inflation. In turn, this raises inflation expectations and so reduces the real interest rate. The intuition is that the response of the policy rate is expected to be larger and for a longer period of time due to the signalling effect. In turn, agents anticipate higher marginal costs in the future and so higher inflation. The lower real interest rate stimulates consumption and investment relative to when QE is not implemented, raising output. Romer (1992, 1993) presents convincing evidence that this real interest rate transmission mechanism generated output increases post-1933, during a period of similar financial distress, adding credibility to these simulation results.

The higher investment generates a higher capital stock. Due to a stable depreciation rate this creates a persistent increase in capital, raising the marginal productivity of labour and so wages and employment according to the assumptions on the production function ($F_{rK}^L(K,L) > 0$). These results appear in line with preliminary empirical work by Baumeister and Benati (2010), adding plausibility to these simulations.

The simulations also demonstrate two further results in relation to QE. First, they show that the central bank is not required to lower the policy rate by as much because of the increase in inflation expectations. The result suggests that the use of QE can not only help an economy in a liquidity trap but may also help an economy avoid a liquidity trap if implemented aggressively at the beginning of a GFC-like crisis.

The second result is that despite these improvements, the policy leads to substantially higher inflation and inflationary expectations over a very extended horizon (between 40-60 quarters). The implication is that while the policy may be able to generate reasonable improvements in output and employment,
these come at the cost of higher inflation in the medium to long-term. While beneficial during a deflationary period, like the Japanese economy between 2001-2006 and the U.S. economy between Aug-Dec 2008, this would prove detrimental if inflation remained robustly positive after a GFC-like financial crisis. However, this conclusion depends on the specific ‘exit strategy’ the central bank is assumed to follow based on its policy rule (equation 4.59). The adoption of a more rapid removal of excess reserves tied to improvements in output could avoid this result. Such dynamic regime switches are unfortunately beyond the scope of this thesis because they cannot be easily incorporated into a log-linearised framework. The reason is that such regime switches violate the assumption of a log-linearised framework that the underlying identities hold throughout all time.

5.2.4 GOZ model with variable elasticity of external finance premium to entrepreneurs’ net worth

The addition of a variable external finance premium does have an effect on the welfare improvements from QE. The simulation results indicate that the presence of counter-cyclical financial distortions only marginally diminishes QE’s effectiveness. Over a policy horizon of 20 quarters, the addition of a signalling function leads to smaller improvements in output, investment, employment and inflation, although the change is not quantitatively large. QE also continues to stimulate investment, though the effect is now quite small. Furthermore, the observed external finance premium is significantly lower, which is one indicator that the policy also reduces the degree of financial stress (Appendix Section B, Figure B.2). The mechanism through which these improvements come about is unchanged.

The same conclusions are reached when the signal and portfolio rebalancing channels are included (Appendix Section B, Figure B.2). Ultimately, it seems that QE is not affected too much by changes in financial institutions’ lending standards as they seek to repair their balance sheets over time. These results also highlight that financial frictions enable monetary policy to remain effective at the zero lower bound without resorting to permanent increases in the money base contrary to Eggertsson and Woodford (2003).
5.2.5 Introduction of a variable influence of the central bank balance sheet

There has been some discussion that significant financial crises like the GFC result in significant alterations in financial market participants’ behaviour especially in regard to their risk-appetites and hoarding of capital (Gale and Yorulmazer, 2011; Kamada and Sugo, 2006). If this were the case then it could be that the purchase of financial assets only resulted in a large increase in financial institutions’ holdings of cash without the subsequent rebalancing of portfolios. Based on the money base statistics for the US over the period 2007-2011 there appears some justification for saying this (Federal Reserve, 2011). Investigating the policy’s effect under these conditions represents a valuable extension to the research already undertaken.

It is postulated that the efficacy of the central bank’s purchases depends on the growth rate (equation 5.1). Equations 5.1-5.3 are designed to capture two correlated factors. First, very low/negative levels of growth induce greater financial market segmentation and/or financial distortions which reduce the effect of targeted asset purchases on the broad spectrum of interest rates (Blanchard, 2010; Baba et al., 2004). Second, they capture the liquidity hoarding behaviour of financial institutions so that in a severe downturn they do not use the reserves provided to purchase substitute assets. Instead, they hold the cash for precautionary purposes (Gale and Yorulmazer, 2011). The use of one proxy is reasonable because these underlying behavioural and institutional variables should be strongly correlated. The derivation of the dynamic, log-linearised version of this equation is contained in the Appendix (Section H).

\[
R_t = (R_{t-1})^{\rho_r} \left[ \left( (R_t^p)(R_{t+1}^p)(R_{t+2}^p)(R_{t+3}^p) \right)^{\frac{1}{4}} \left( \frac{1}{\bar{\Omega}_tER_t} \right) \right]^{1-\rho_r} 
\]  
(5.1)

After log-linearisation this equation becomes:

\[
\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \left[ 0.25\left( \hat{r}_t^p + \hat{r}_{t+1}^p + \hat{r}_{t+2}^p + \hat{r}_{t+3}^p \right) - \theta (\bar{E}R_t + \hat{\Omega}_t) \right] 
\]  
(5.2)

with,

\[
\hat{\Omega}_t = \rho_\Omega \hat{\Omega}_t + \theta (\hat{\nu}_t - \hat{\nu}_{t-1}) \; ; \; \rho_\Omega = 0.9, \theta = 0.05 
\]  
(5.3)
The results of the various simulations that were conducted suggest that these factors do have an influence. They do not however generate large quantitative departures from the case that featured a constant effect of the central bank’s balance sheet on the target interest rate (Appendix Section B, Figure B.3).  

Nevertheless, the simulations do highlight the role of a variety of factors which could lead to the policy either succeeding or failing. QE may induce changes in financial market participants’ behaviour, fundamentally altering the way specific asset markets operate, such as the bond market. An example would be that the central bank’s purchase of bonds and other financial assets induces a lengthy period of unjustified optimism about an economy’s future prospects or unjustified fear regarding future inflation. These would lead to asset price falls and increases in the targeted long-term interest rates. This thesis attempts to capture such factors with this variable factor but a more fully-fledged derivation is beyond its scope and should be the subject of future research.

5.2.6 Asset purchases of different maturities

Another important question to ask regarding QE is if the central bank was only able to target purchases at a single maturity length, could it still use QE to stimulate output? The experiment that is envisaged is where consumption and investment decisions are influenced by an interest rate not directly targeted by policymakers through some error of judgment or institutional restriction. For simplicity, only three different maturity lengths were incorporated and it was assumed that real spending decisions were affected by the one year interest rate. The results were not sensitive to either of these assumptions.

39 Further simulations using different values for $\Theta$ demonstrate that the size of these financial market effects is negatively related to the policy’s effectiveness. In none of the simulated cases ($\Theta \in \{0.1,0.25,0.3\}$ for both a constant and variable $\gamma$ did the policy generate welfare losses. It was also investigated if adverse changes in financial participants’ behaviour caused directly by the enlarging of the central bank balance sheet could result in the policy having a negative impact on welfare. This is based upon the injection of central bank liquidity increasing expectations regarding future inflation. Investors would therefore sell assets, anticipating falling asset prices as a consequence of rises in the future policy rate to combat the higher inflation. This would place upward pressure on yields and this could well overcome the initial policy effect, resulting in stable or higher yields. The effect was incorporated by altering equation 5.3 to $\tilde{q}_t = \rho q_{t-1} + \Theta(\tilde{y}_t - \tilde{y}_{t-1}) + \Gamma E\tilde{R}_t$; $\Gamma \in \{0.05,0.1,0.15,0.2\}$. In all of the cases considered, the policy’s implementation still led to welfare improvements.
Essentially, these follow the expectations approach except that interest rate persistence is imposed to match the empirical evidence (Romero-Avila, 2007). An advantage of this approach is that it captures the stylised fact that long-term interest rates demonstrate a relatively higher level of persistence compared to short-term rates.

The results are that the best maturity to target is the one which affects the consumption and investment decisions of agents in the economy, and this is the medium-term interest rate under the current assumptions. The reason is that asset purchases of the correct maturity leads to a proportionally greater decline in the interest rate which directly affects consumer choices. Importantly, even with a large degree of persistence in long-term interest rates, purchases of other maturities still spill over enough that the correct real interest rate falls, stimulating economic activity (Appendix Section C).

The result that little change occurs whether the central bank targets either a six month, a twelve month or a five year rate supports two further conclusions. First, even if the Federal Reserve purchased the wrong assets, sufficient spill-over effects were present that the policy still yielded welfare improvements. Second, supposing that institutional or political factors prevent the monetary authorities from targeting certain assets, it can still use QE to improve welfare in a GFC-like downturn.

\[
R_{t}^{short} = (R_{t-1}^{short})^{\rho_r} \left[ \left( R_{t}^{short} \right) \left( R_{t+1}^{short} \right) \right]^{1-\rho_r}
\] (5.4)

\[
R_{t}^{med} = (R_{t-1}^{med})^{\rho_r} \left[ \left( R_{t}^{med} \right) \left( R_{t+2}^{med} \right) \right]^{1-\rho_r}
\] (5.5)

\[
R_{t}^{long} = (R_{t-1}^{long})^{\rho_r} \left[ \left( R_{t}^{long} \right) \left( R_{t+4}^{long} \right) \left( R_{t+8}^{long} \right) \left( R_{t+12}^{long} \right) \right]^{1-\rho_r}
\] (5.6)

40 An alternative simulation was conducted where the long-term interest rate assumed to effect households’ behaviour was the 5 year interest rate. Purchases of the 5 year interest rate yielded the largest welfare improvements.
5.2.7 Welfare Effects of Different Policy Rules

In light of the public releases associated with QE not revealing the precise variables the Federal Reserve was reacting to, the impact of QE if the central bank uses different decision rules was another issue to be addressed. It was therefore necessary to analyse a variety of rules for setting the size of the central bank balance sheet. Seven alternative rules were selected according to an assessment of what central banks, including the Federal Reserve, appear to be reacting to when implementing QE: inflation, yield spreads and the output gap. These rules are listed below (see the Appendix section I for the dynamic versions). The relative weights in each rule were chosen so that a combined percentage deviation in the target variables induced a $\kappa\%$ deviation in the size of the central bank balance sheet.

These rules should be seen as special cases of the general rule:

$$ER_t - ER_s = \phi(S_t, \Delta S_t, Y_t, \Delta Y_t, \pi_t, ER_t)$$

A more detailed examination of the optimal form of this rule is an issue for further research.

\[
ER_t - ER_s = S_\kappa \left( \frac{S_t}{S_t} \right)^{\frac{\kappa}{2}} \left( \frac{Y_t}{Y_{t-1}} \right)^{-\frac{\kappa}{2}}
\]  
(5.7)

\[
ER_t - ER_s = S_\kappa \left( \frac{S_t}{S_t} \right)^{\frac{\kappa}{2}} \left( \frac{Y_t}{Y_{t}} \right)^{-\frac{\kappa}{2}}
\]  
(5.8)

\[
ER_t - ER_s = S_\kappa \left( \frac{Y_t}{Y_{t}} \right)^{-\kappa}
\]  
(5.9)

\[
ER_t - ER_s = S_\kappa \left( \frac{S_t}{S_{t-1}} \right)^{\kappa}
\]  
(5.10)

\[
ER_t - ER_s = S_\kappa \left( \frac{S_t}{S_{t-1}} \right)^{\frac{\kappa}{2}} \left( \frac{Y_t}{Y_{t-1}} \right)^{-\frac{\kappa}{2}}
\]  
(5.11)

\[
ER_t - ER_s = S_\kappa \left( \frac{S_t}{S_{t}} \right)^{\frac{\kappa}{2}} \left( \frac{Y_t}{Y_{t-1}} \right)^{-\frac{\kappa}{2}} \left( \frac{\pi_t}{\pi_{t-1}} \right)^{-\frac{\kappa}{2}}
\]  
(5.12)
Simulations using these seven different rules demonstrate that the effectiveness of QE is affected by the type of rule adopted. Unfortunately, no derivation of the welfare loss function using a second order Taylor approximation of the Smets and Wouters (2007) utility function around the model’s steady state has been undertaken (Debortoli et al., 2011). Given the presence of wage and price rigidities and the form of the utility function in the SW model, a welfare loss measure like that derived by Erceg et al. (2000) was used instead. This assumption is consistent with the literature (Bean et al., 2010; Chadha et al., 2009; Curdia et al., 2011; Debortoli et al., 2010). The welfare loss over twenty quarters was used to ascertain which decision rule yielded the greatest welfare increase over the relevant policy horizon. This particular technique was used because of the temporary nature of QE and the use of only two of the shocks in the GOZ model.

Table 1

<table>
<thead>
<tr>
<th>Welfare Improvements of alternative policy rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy</td>
</tr>
<tr>
<td>External Finance Premium (EFP)(4.59)</td>
</tr>
<tr>
<td>Output Gap and EFP(5.7)</td>
</tr>
<tr>
<td>EFP and $\Delta Y$ (5.8)</td>
</tr>
<tr>
<td>Output Gap (5.9)</td>
</tr>
<tr>
<td>$\Delta$EFP(5.10)</td>
</tr>
<tr>
<td>$\Delta$EFP and $\Delta Y$ (5.11)</td>
</tr>
<tr>
<td>EFP, Output Gap and Inflation (5.12)</td>
</tr>
<tr>
<td>EFP with lag (5.13)</td>
</tr>
</tbody>
</table>

$E_{t} - E_{*} = S_{t}(E_{t-1}^{\frac{\ell}{E_{*}}})^\frac{s}{S_{t}} (1-\rho^s)$ (5.13)

41 The functional form of the welfare loss function used was $-\frac{1}{2} E_t \sum_{i=0}^{T} \beta^t \left( (y_{t+i} - y_{t+i}^f)^2 + (\pi_{t+i}^p)^2 + (\pi_{t+i}^w)^2 \right)$ (Erceg et al., 2000; Gali, 2008). The results were not sensitive to alternative specifications of the welfare function which altered the weight on the output gap or added additional objectives such as the external finance premium based on what appears to have been the Federal Reserve’s objectives. In total, three other measures were employed: Measure 1 = $-\frac{1}{2} E_t \sum_{i=0}^{T} \beta^t \left( (y_{t+i} - y_{t+i}^f)^2 + \pi_{t+i}^w \right)$ (Clarida et al., 1999; Ravenna and Walsh, 2006), Measure 2 = $-\frac{1}{2} E_t \sum_{i=0}^{T} \beta^t \left( (y_{t+i}^f)^2 + (\pi_{t+i}^p)^2 + (\pi_{t+i}^w)^2 \right)$ and Measure 3 = $-\frac{1}{2} E_t \sum_{i=0}^{T} \beta^t \left( (y_{t+i} - y_{t+i}^f)^2 + (\pi_{t+i}^w)^2 + (\pi_{t+i}^p)^2 \right)$ (Onatski and Williams, 2004; Adolfson et al., 2008).
Table 2  
Variable Effect of Central Bank Balance Sheet

<table>
<thead>
<tr>
<th>Policy</th>
<th>Constant $\chi$</th>
<th>Variable $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Finance Premium (EFP)(4.59)</td>
<td>0.218</td>
<td>0.220</td>
</tr>
<tr>
<td>Output Gap and EFP(5.7)</td>
<td>0.268</td>
<td>0.219</td>
</tr>
<tr>
<td>$EFP$ and $\Delta Y$ (5.8)</td>
<td>0.098</td>
<td>0.120</td>
</tr>
<tr>
<td>Output Gap (5.9)</td>
<td>0.292</td>
<td>0.217</td>
</tr>
<tr>
<td>$\Delta EFP$ (5.10)</td>
<td>-0.058</td>
<td>-0.048</td>
</tr>
<tr>
<td>$\Delta EFP$ and $\Delta Y$ (5.11)</td>
<td>-0.052</td>
<td>-0.040</td>
</tr>
<tr>
<td>EFP, Output Gap and Inflation (5.12)</td>
<td>0.197</td>
<td>0.166</td>
</tr>
<tr>
<td>EFP with lag (5.13)</td>
<td>0.203</td>
<td>0.217</td>
</tr>
</tbody>
</table>

As Table 1 and 2 demonstrate, the rules which contain the external finance premium and/or the output gap are the most effective. Essentially, these rules ensure that the central bank adopts a more aggressive stance to a combination of an external finance premium shock and a net worth shock, which implies that the deviation of output from its steady state value is lower.

In terms of the current use of QE by the Federal Reserve, there appears little evidence that it was reacting to either the change in yield spreads or output growth. Consequently, these simulations suggest that QE has had an effect regardless of the exact policy rule used by the Federal Reserve between 2008-2010.42

5.2.8 Welfare Effects of Fiscal versus Monetary Policy

Simulations with fiscal instead of monetary policy as the active instrument were also conducted to provide an initial conclusion regarding which policy would be more effective in dealing with a GFC-like financial crisis. The GOZ model contains a fiscal policy shock and this shock was used as one representation of a fiscal policy. It was assumed that this fiscal spending shock was financed in such a way that it did not affect labour supply decisions, the sustainability of the fiscal deficit or the real interest rate in the medium-run. While simplistic, it was thought this was a simple way of capturing any direct effect of fiscal stimulus. As table 3 demonstrates, fiscal policy generated similar welfare

---

42 The results of simulations using a range of different weights in the welfare objective function, including those used by Erceg et al. (2000), were practically identical to those reported. None of the conclusions drawn by this thesis are affected by this assumption.
improvements and under certain calibrations, QE was the preferred alternative. This suggests that QE represents a viable alternative to fiscal stimulus if the latter is not feasible because of political machinations and an unsustainable budget deficit.

Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant $\chi$</th>
<th>Variable $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fiscal</td>
<td>Monetary</td>
</tr>
<tr>
<td>$\sigma = 1.0188$</td>
<td>0.236</td>
<td>0.246</td>
</tr>
<tr>
<td>$\sigma = 1.3$</td>
<td>0.635</td>
<td>0.425</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>2.047</td>
<td>0.913</td>
</tr>
</tbody>
</table>

5.2.9 Sensitivity Testing

To establish the credibility of the results obtained above, the sensitivity of QE’s effect was examined along a number of dimensions. These included: the degree of relative risk aversion of consumers which affects the marginal rate of inter-temporal substitution, the elasticity of the long-term interest rate to the size of the central bank balance sheet, the size of the signalling effect, the variability of the external finance premium to output innovations and the size of the portfolio rebalancing effect.

On the whole, the results obtained in this thesis were not qualitatively sensitive to the specification of these and other parameters (Tables 4 and 5). Minor changes were obtained but they were not sufficient to negate the conclusion that QE always generates welfare improvements.
### Table 4
*Constant Effect of Central Bank Balance Sheet*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant χ</th>
<th>Variable χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.246</td>
<td>0.231</td>
</tr>
<tr>
<td>σ_c = 1.3</td>
<td>0.425</td>
<td>0.401</td>
</tr>
<tr>
<td>σ_c = 2</td>
<td>0.913</td>
<td>1.065</td>
</tr>
<tr>
<td>χ = 0.04</td>
<td>0.364</td>
<td>-</td>
</tr>
<tr>
<td>ρ_r = 0.98</td>
<td>0.319</td>
<td>0.259</td>
</tr>
<tr>
<td>ω = 150</td>
<td>-</td>
<td>0.215</td>
</tr>
<tr>
<td>ω = 200</td>
<td>-</td>
<td>0.220</td>
</tr>
<tr>
<td>ρ_χ = 0</td>
<td>-</td>
<td>0.220</td>
</tr>
<tr>
<td>ρ_χ = 0.95</td>
<td>-</td>
<td>0.781</td>
</tr>
<tr>
<td>R*_s = Rp + 0.0025 (100 basis point spread)</td>
<td>0.245</td>
<td>0.230</td>
</tr>
<tr>
<td>R*_s = Rp + 0.005 (200 basis point spread)</td>
<td>0.245</td>
<td>0.230</td>
</tr>
<tr>
<td>5 year interest rate</td>
<td>0.212</td>
<td>0.166</td>
</tr>
<tr>
<td>Original parameters from GOZ^41</td>
<td>24.429</td>
<td>106.522</td>
</tr>
<tr>
<td>Re-estimated parameters</td>
<td>0.373</td>
<td>0.261</td>
</tr>
<tr>
<td>ε_t^nw = ε_t^nw ; ε_t^nw ~ N(0, σ_{nw})</td>
<td>0.076</td>
<td>0.061</td>
</tr>
</tbody>
</table>

---

^41 These large numbers are explained by the large persistence in the net worth shock relative to the base case.

### Table 5
*Variable Effect of Central Bank Balance Sheet*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant χ</th>
<th>Variable χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.218</td>
<td>0.220</td>
</tr>
<tr>
<td>σ_c = 1.3</td>
<td>0.361</td>
<td>0.379</td>
</tr>
<tr>
<td>σ_c = 2</td>
<td>0.710</td>
<td>1.001</td>
</tr>
<tr>
<td>χ = 0.04</td>
<td>0.347</td>
<td>-</td>
</tr>
<tr>
<td>ρ_r = 0.98</td>
<td>0.316</td>
<td>0.257</td>
</tr>
<tr>
<td>ω = 150</td>
<td>-</td>
<td>0.216</td>
</tr>
<tr>
<td>ω = 200</td>
<td>-</td>
<td>0.214</td>
</tr>
<tr>
<td>ρ_χ = 0</td>
<td>-</td>
<td>0.201</td>
</tr>
<tr>
<td>ρ_χ = 0.95</td>
<td>-</td>
<td>0.767</td>
</tr>
<tr>
<td>R*_s = Rp + 0.0025 (100 basis point spread)</td>
<td>0.218</td>
<td>0.220</td>
</tr>
<tr>
<td>R*_s = Rp + 0.005 (200 basis point spread)</td>
<td>0.217</td>
<td>0.219</td>
</tr>
<tr>
<td>5 year interest rate</td>
<td>0.215</td>
<td>0.168</td>
</tr>
<tr>
<td>Original parameters from GOZ</td>
<td>23.249</td>
<td>106.227</td>
</tr>
<tr>
<td>Re-estimated parameters</td>
<td>0.307</td>
<td>0.245</td>
</tr>
<tr>
<td>ε_t^nw = ε_t^nw ; ε_t^nw ~ N(0, σ_{nw})</td>
<td>0.070</td>
<td>0.059</td>
</tr>
</tbody>
</table>
Tables 6 and 7 also demonstrate that the qualitative effect of QE was not sensitive to the size of the shocks which demonstrates that the policy could be welfare-enhancing whether the financial crisis was mild or severe.

**Table 6**

*Constant Effect of Central Bank Balance Sheet*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant $\chi$</th>
<th>Variable $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.246</td>
<td>0.231</td>
</tr>
<tr>
<td>Shocks with $2\sigma_{\text{shocks}}$</td>
<td>0.982</td>
<td>0.922</td>
</tr>
<tr>
<td>Shocks with $4\sigma_{\text{shocks}}$</td>
<td>3.929</td>
<td>3.690</td>
</tr>
<tr>
<td>Shocks with $5\sigma_{\text{shocks}}$</td>
<td>6.139</td>
<td>5.765</td>
</tr>
<tr>
<td>Shocks with $10\sigma_{\text{shocks}}$</td>
<td>24.557</td>
<td>23.061</td>
</tr>
<tr>
<td>Shocks with $20\sigma_{\text{shocks}}$</td>
<td>98.229</td>
<td>92.243</td>
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</tbody>
</table>

**Table 7**

*Variable Effect of Central Bank Balance Sheet*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant $\chi$</th>
<th>Variable $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.218</td>
<td>0.220</td>
</tr>
<tr>
<td>Shocks with $2\sigma_{\text{shocks}}$</td>
<td>0.873</td>
<td>0.880</td>
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<tr>
<td>Shocks with $4\sigma_{\text{shocks}}$</td>
<td>3.493</td>
<td>2.588</td>
</tr>
<tr>
<td>Shocks with $5\sigma_{\text{shocks}}$</td>
<td>5.457</td>
<td>5.499</td>
</tr>
<tr>
<td>Shocks with $10\sigma_{\text{shocks}}$</td>
<td>21.830</td>
<td>21.996</td>
</tr>
<tr>
<td>Shocks with $20\sigma_{\text{shocks}}$</td>
<td>87.318</td>
<td>87.985</td>
</tr>
</tbody>
</table>
VI. IMPLICATIONS FOR THE FUTURE CONDUCT OF MONETARY POLICY

The results of the simulations conducted support two conclusions. First, that QE can generate welfare improvements if incorporated into the policy response to a large financial crisis. Second, that the QE policy implemented by the Federal Reserve since 2009 has prevented output, employment and investment from falling as far as it could have done. These findings support the assertions of Bernanke (2009) that monetary policy can remain effective even in the presence of severe stress in the financial system and the presence of the zero lower bound. They are also consistent with Baumeister and Benati (2010) and Chung et al. (2011) who find that QE has had a positive effect on output since 2009. Even if the signal effect of the size of the central bank balance sheet is set to zero and significant frictions are incorporated, QE is still able to affect economic activity. This contradicts the findings of Curdia and Woodford (2011) and Eggertsson and Woodford (2003), highlighting the importance of financial frictions.

Another implication of the results is that there appears little downside risk to QE. Even if QE had no effect on nominal interest rates but resulted in higher inflation expectations, as seen in the simulations, then this would unambiguously reduce real interest rates and so stimulate economic activity.

It has been argued that firms and households in a severe downturn alter their behaviour to debt minimisation instead of profit and utility maximisation respectively (Koo, 2009) because of a greater aversion to debt. This would result in the private sector reducing their debt levels despite QE ensuring an ample supply of funds at a very low long-term interest rate. The excessive consumption growth relative to income growth between 1995-2006, leaving many with high or unsustainable leverage ratios, certainly makes this factor plausible (Lee, 2011; Barnes and Young, 2003; Barba and Pivetti, 2009). There is also empirical evidence of this occurring in Japan in the 1990s (Lee, 2011), during the 1973-75 US recession (Mishkin et al., 1977), during the Great Depression (Mishkin, 1978) and in the current context (Fisher, 2010; Lee, 2011). However, this does not create a downside risk for QE. Rather, the simulation results suggest that QE may still remain effective. Alternatively, QE would be
effective only insofar as it enabled firms and consumers to repair their balance sheets faster by providing them with greater refinancing options at a lower interest rate.

Another argument is that heightened uncertainty causes delays in consumption and investment decisions by risk averse agents. Bernanke (1983) and Romer (1990) have suggested that such a mechanism was responsible for the immediate drops in consumption and investment independent of interest rates seen during the Great Depression. The simulations lead to the conclusion that QE would still generate welfare improvements even if this was currently in play. The reason is that QE enables the central bank to signal its future policy movements and increases liquidity in asset markets, reducing uncertainty. Indeed, it would be valuable if future research examined this issue.

A third implication of these results is that it would be beneficial for the central bank to react to yields spreads, exemplified by the external finance premium in the GOZ model. The postulated rule for the size of the central bank’s balance sheet implies the addition of the lag of the yield spread into the Taylor rule. Simulations with just this effect demonstrated that the central bank could bring about welfare improvements by doing so. The simulations show that QE can generate welfare improvements even in the presence of fluctuations in spreads beyond the influence of QE. However, these simulations are only preliminary evidence and a more detailed investigation into the source of these fluctuations is required before a final conclusion can be made.

Nevertheless, a number of issues need to be considered before any general conclusion regarding QE as an instrument of monetary policy can be made. The first of these issues is the need to empirically estimate the behavioural changes discussed by Koo (2009) and Romer (1990). If they existed, one prediction would be that the extra liquidity provided to financial institutions by QE would simply be held within the banking system. There is some evidence of this based on the reserve statistics for the U.S. banking sector (Federal Reserve, 2011). If this was empirically established then a subsequent issue would be to incorporate these into the GOZ model in a more micro-founded fashion. One potential method could be to introduce a disutility cost for households to hold large quantities of debt which was dependent on consumption or output.
A second issue is the need to more rigorously estimate the parameters in the augmentations to the GOZ model. If the range of values used for the parameters was in fact too low then an increase in the central bank balance sheet could induce an overall rise in interest rates. This raises the corollary that the efficacy of QE could also depend on different economic institutions unique to a particular country. In these circumstances, a recommendation for QE could not be given in quite the same way as interest rate setting with an explicit inflation target has been.

A third issue is the role of external financial markets which could alter the way that the transmission mechanism of the policy works. In this thesis, a closed economy was assumed because of the ambiguous theoretical effects of QE on the exchange rate. However, there is some empirical evidence of a depreciation of the $U.S. immediately after QE was implemented, highlighting the importance of this issue for future research. This is especially so given the effect of QE is not clear.

The impact of the zero lower bound is a further issue that must be addressed. This thesis abstracted from this issue because (1) the solution methods for instituting a lower bound constraint involved the same method as this thesis except with a manual adjustment of the resulting impulse response functions and (2) the impulse response functions were unchanged except that they were more exaggerated when the constraint bound. The addition of the factors described in the preceding paragraphs may alter these properties but again this is for future research.

Another issue is whether QE alters the indeterminacy region of the GOZ model. The inclusion of a central bank balance sheet size equation is identical to making the central bank respond to the yield spread within the context of the GOZ model. However, this may induce greater instability/stability in inflation expectations. This would be a large drawback or another advantage of QE if present.

A final issue is that QE does not do anything to address the ‘flawed institutions and practices of the current financial regime, often referred to as the New Financial Architecture (NFA)’ which is argued to have caused the crisis (Crotty, 2009). While outside the scope of this thesis, reforming of financial market regulations remains a crucial issue for the policy response to the GFC. Future research should certainly consider QE’s effect on the breadth and speed of such reforms.
VII. CONCLUSION

This thesis contributes to the existing literature by examining the macroeconomic effects of QE in the U.S. context and assessing the effects of the policy adopted by the Federal Reserve. While there has been some research on the effect of the policy on the yields of particular assets, little conclusive empirical or theoretical work has been conducted on the possible effects of the policy in the presence of financial market imperfections. The results obtained indicate that the policy can and has raised welfare by significantly reducing the effects of a net worth and yield spread shock to the U.S. economy.

Instituting the policy upon reaching the zero lower bound on the policy rate would not eliminate these welfare gains as the portfolio rebalancing channel could still operate. This is in contrast to Eggertsson and Woodford (2003) who assert that upon reaching the zero lower bound, the size of the central bank’s balance sheet becomes irrelevant if it does not credibly signal a lower policy rate in the future.

The simulations involving QE undertaken in this thesis also demonstrate that the policy is likely to be a valuable instrument for a central bank in addition to a short-term interest rate. This conclusion is only provisional because of the need to conduct further research on especially two issues: (1) the variable elasticity between interest rates and the central bank balance sheet and (2) the effect of relaxing the closed-economy assumption.
### VIII. APPENDIX

#### A. Parameter Values

<table>
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<th>Parameter</th>
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<th>Alternative 2</th>
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<td>0.98</td>
<td>0.5067</td>
</tr>
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<td>$\rho_{uw}$</td>
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<td>0.4984</td>
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---

44 Reported parameters in Gilchrist et al. (2009).
45 Parameters estimated using an alternative external finance premium series.
Figures for impulse response functions of variables to net worth and external finance premium shocks.

**Figure B.1**
GOZ Model with long-term interest rate, signal effect and signal/portfolio rebalancing effect (constant $\chi$)
Figure B.2
GOZ Model with long-term interest rate, signal effect and signal/portfolio rebalancing effect (variable $\chi$)
Figure B.3
GOZ Model with long-term interest rate, signal and portfolio rebalancing effects and a variable influence of the central bank balance sheet

### Constant $\chi$

#### Consumption

![Graph showing consumption with different scenarios]

#### External Finance Premium

![Graph showing external finance premium with different scenarios]

#### Inflation

![Graph showing inflation with different scenarios]

#### Output

![Graph showing output with different scenarios]

### Variable $\chi$

#### Consumption

![Graph showing consumption with different scenarios]

#### External Finance Premium

![Graph showing external finance premium with different scenarios]

#### Inflation

![Graph showing inflation with different scenarios]

#### Output

![Graph showing output with different scenarios]
C. Targeting Different Maturities

Figure C.1
GOZ model with long-term interest rate and signal/portfolio rebalancing effects

<table>
<thead>
<tr>
<th>Constant $\chi$</th>
<th>Variable $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
</tr>
<tr>
<td><strong>External Finance Premium</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Constant
- Base
- 6m
- 12m
- 5y
D. Smets and Wouters (2007) Steady State

The steady state of the Smets and Wouters (2007) model is set out below.

\[
R^k_s = \frac{1}{\beta} - (1 - \delta) = \frac{1}{\beta} y^{\sigma c} - (1 - \delta)
\]

\[
w^s = \left( \frac{\alpha^a(1 - \alpha)^{(1-a)}}{\phi_p(R^k_s)^a} \right) = \left( \frac{\alpha^a(1 - \alpha)^{(1-a)}}{(1 + \lambda_p)(R^k_s)^a} \right) = (1 + \lambda_w)w^s
\]

\[
\frac{i}{k} = \gamma - (1 - \delta)
\]

\[
\frac{l}{k} = \left[ 1 - \frac{1 - \alpha}{\alpha} \right] \frac{R^k_s}{w^s}
\]

\[
k = \phi_p \left( \frac{1}{k} \right)^{a-1}
\]

\[
\frac{i}{\bar{y}} = \frac{i}{k} \frac{k}{\bar{y}}
\]

\[
c = \bar{y} = 1 - g - \frac{i}{\bar{y}}
\]

\[
z = \frac{R^k_s k}{\bar{y}}
\]

\[
\frac{w^h}{c^s} = \frac{1 - \alpha}{\phi_w} \left( \frac{z y}{\bar{y} c} \right) = \frac{1}{(1 + \lambda_w)} \left( \frac{z y}{\alpha} \right)
\]

The steady state of the Gilchrist et al. (2009) model is identical except that:

\[
R^k = \left( \frac{K}{N} \right)^x \frac{1}{\beta} - (1 - \delta) = \left( \frac{K}{N} \right)^x \frac{1}{\beta} y^{\sigma c} - (1 - \delta)
\]

All the other steady state equations are adjusted accordingly to reflect this new steady state return to capital.

There are 7 shocks in the SW model.

Technology Shock

$$\epsilon_t^q = (1 - \rho_a)\epsilon_t^a + \rho_a\epsilon_{t-1}^a + \epsilon_t^a; \epsilon_t^a \sim N(0, \sigma_a)$$

Investment-specific technology Shock

$$\mu_t = (1 - \rho_\mu)\mu_s + \rho_\mu\mu_{t-1} + \epsilon_t^\mu; \epsilon_t^\mu \sim N(0, \sigma_\mu)$$

Bond Return Shock

$$\epsilon_t^b = (1 - \rho_b)\epsilon_t^b + \rho_b\epsilon_{t-1}^b + \epsilon_t^b; \epsilon_t^b \sim N(0, \sigma_b)$$

Government Spending Shock

$$\tilde{g}_t = (1 - \rho_g)g_s + \rho_g\tilde{g}_{t-1} + \rho_g\epsilon_t^a + \epsilon_t^g; \epsilon_t^g \sim N(0, \sigma_g)$$

Monetary Policy Shock

$$\epsilon_t^m = \rho_m\epsilon_{t-1}^m + \epsilon_t^m; \epsilon_t^m \sim N(0, \sigma_m)$$

Price Mark-up Shock

$$\epsilon_t^p = (1 - \rho_p)\epsilon_t^p + \rho_p\epsilon_{t-1}^p + \theta_p\epsilon_{t-1}^p + \epsilon_t^p; \epsilon_t^p \sim N(0, \sigma_p)$$

Wage Mark-up Shock

$$\epsilon_t^w = (1 - \rho_w)\epsilon_t^w + \rho_w\epsilon_{t-1}^w + \theta_w\epsilon_{t-1}^w + \epsilon_t^w; \epsilon_t^w \sim N(0, \sigma_w)$$
F. Specification of shocks in Gilchrist et al. (2009)

There are 8 shocks in the Gilchrist et al. (2009) model.

Technology Shock
\[ \varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \varepsilon_t^a; \varepsilon_t^a \sim N(0, \sigma_a) \]

Bond Return Shock
\[ \varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \varepsilon_t^b; \varepsilon_t^b \sim N(0, \sigma_b) \]

Government Spending Shock
\[ \hat{g}_t = \rho_y \hat{g}_{t-1} + \rho_{gy} \varepsilon_t^g + \varepsilon_t^g; \varepsilon_t^g \sim N(0, \sigma_y) \]

Monetary Policy Shock
\[ \varepsilon_t^m = \rho_r \varepsilon_{t-1}^m + \varepsilon_t^m; \varepsilon_t^m \sim N(0, \sigma_m) \]

Price Mark-up Shock
\[ \varepsilon_t^p = \rho_p \varepsilon_{t-1}^p - \theta_p \varepsilon_{t-1}^p + \varepsilon_t^p; \varepsilon_t^p \sim N(0, \sigma_p) \]

Wage Mark-up Shock
\[ \varepsilon_t^w = \rho_w \varepsilon_{t-1}^w - \theta_w \varepsilon_{t-1}^w + \varepsilon_t^w; \varepsilon_t^w \sim N(0, \sigma_w) \]

External Finance Premium Shock
\[ \varepsilon_t^{fd} = \rho_{fd} \varepsilon_{t-1}^{fd} + \varepsilon_t^{fd}; \varepsilon_t^{fd} \sim N(0, \sigma_{fd}) \]

Net worth Shock
\[ \varepsilon_t^{nw} = \rho_{nw} \varepsilon_{t-1}^{nw} + \varepsilon_t^{nw}; \varepsilon_t^{nw} \sim N(0, \sigma_{nw}) \]
G. Derivation of the dynamic equation for the external finance premium with a variable elasticity

The external finance premium with a variable elasticity may be expressed as follows based on Bernanke et al. (1999).

\[ S_t = \left( \frac{Q_t K_t}{N_t} \right)^{\chi_t} \epsilon_t^{f_d} \]

In the steady state,

\[ S_s = \left( \frac{Q_s K_s}{N_s} \right)^{\chi_s} \]

This implies that

\[ \log(S_s) = -\chi_s(\log(N_s) - \log(Q_s) - \log(K_s)) \]

Using the method proposed in Uhlig (1995) the variables may be expressed in terms of percentage deviations from their steady state values.

\[ S_s e^{\beta t} = \left( \frac{Q_s e^{t_0} K_s e^{ar{\theta} t}}{N_s e^{ar{n}_t}} \right)^{\chi_s e^{\beta t}} \epsilon_t^{f_d} \]

\[ S_s e^{\beta t} = (e^{ar{t}_0 + \bar{\theta} t - \bar{n}_t})^{\chi_s (1 + \beta t)} \epsilon_t^{f_d} \]

Using the approximation that \( e^{\tilde{\beta} t} \approx 1 + \tilde{\beta} t \),

\[ S_s e^{\beta t} = e^{\chi_s (1 + \beta t)} \left( e^{\bar{t}_0 + \bar{\theta} t} \right)^{\chi_s (1 + \beta t)} \epsilon_t^{f_d} \]

Since \( \tilde{\beta}_t Y_t \approx 0 \), this may be simplified to:

\[ S_s e^{\beta t} = e^{\chi_s (\bar{t}_0 + \bar{\theta} t - \bar{n}_t)} \left( \frac{Q_s K_s}{N_s} \right)^{\chi_s e^{\beta t}} \epsilon_t^{f_d} \]

Therefore, using the steady state relationship:

\[ e^{\beta t} = e^{\chi_s (\bar{t}_0 + \bar{\theta} t - \bar{n}_t)} \left( \frac{Q_s K_s}{N_s} \right)^{\chi_s e^{\beta t}} \epsilon_t^{f_d} \]

Taking the natural logarithm of both sides yields the following dynamic equation:

\[ \delta_t = -\chi_s (\tilde{n}_t - \bar{\theta}_t - \bar{\theta}_t) - \chi_s \tilde{\beta}_t (\log(N_s) - \log(Q_s) - \log(K_s)) + \epsilon_t^{f_d} \]

Using the steady state relationship, another form for this equation is:

\[ \delta_t = -\chi_s (\tilde{n}_t - \bar{\theta}_t - \bar{\theta}_t) + \tilde{\beta}_t (\log(S_s)) + \epsilon_t^{f_d} \]

Where \( \log(S_s) = \log \left( \frac{K_s}{N_s} \right)^{\chi_s} \) based on the definition of the external finance premium.
H. Derivation of the dynamic equation for interest rate with a variable effect of the central bank balance sheet

\[ R_t = (R_{t-1})^\rho_r \left[ \left( \frac{1}{\Omega_t} \right) \right]^{1-\rho_r} \]

In the steady state,

\[ R_s = (R_s)^\rho_r \left[ \left( \frac{1}{\Omega_s} \right) \right]^{1-\rho_r} \]

\[ R_s = (R_s^p) \left( \frac{1}{\Omega_s} \right)^\rho \]

Using the method in Uhlig (1995), we can express the original equation in terms of deviations from this steady state:

\[ e^{\hat{R}_t} = (e^{\hat{R}_t})^{\rho_r} \left[ \left( \frac{1}{\Omega_s} \right) \right]^{(1-\rho_r)} \left[ e^{0.25(t^p + \hat{t}^p) + \hat{t}^p} (e^{-\Omega_R_t})(e^{-\Omega_t}) \right]^{1-\rho_r} \]

Using the steady state relationship,

\[ e^{\hat{R}_t} = (e^{\hat{R}_t}) \left[ e^{0.25(t^p + \hat{t}^p) + \hat{t}^p} (e^{-\Omega_R_t})(e^{-\Omega_t}) \right]^{1-\rho_r} \]

Using the fact that \( e^{\hat{x}_t} \approx 1 + \hat{x}_t \) and \( \hat{x}_t \hat{y}_t \approx 0 \) this can be simplified to:

\[ \hat{\tau}_t = \rho_r \hat{\tau}_{t-1} + (1 - \rho_r)[0.25(\hat{\tau}^p_{t+1} + \hat{\tau}^p_{t+2} + \hat{\tau}^p_{t+3}) - \Omega_t] \]
I. Log-linearised, dynamic versions of different monetary policy decision rules

\[ \bar{E}_R_t = \left( \frac{\kappa}{2} \right) \left( \frac{S_r}{E_R} \right) (\hat{\delta}_t - (\hat{y}_t - \hat{y}_{t-1})) \]

\[ \bar{E}_R_t = \left( \frac{\kappa}{2} \right) \left( \frac{S_r}{E_R} \right) (\hat{\delta}_t - (\hat{y}_t - \hat{y}_{t}')) \]

\[ \bar{E}_R_t = - \left( \frac{S_r}{E_R} \right) \kappa (\hat{y}_t - \hat{y}_{t}') \]

\[ \bar{E}_R_t = \left( \frac{S_r}{E_R} \right) \kappa (\hat{\delta}_t - \hat{\delta}_{t-1}) \]

\[ \bar{E}_R_t = \left( \frac{\kappa}{2} \right) \left( \frac{S_r}{E_R} \right) ((\hat{\delta}_t - \hat{\delta}_{t-1}) - (\hat{y}_t - \hat{y}_{t}')) \]

\[ \bar{E}_R_t = \left( \frac{\kappa}{3} \right) \left( \frac{S_r}{E_R} \right) ((\hat{\delta}_t - (\hat{y}_t - \hat{y}_{t}')) - \hat{\pi}_t) \]

\[ \bar{E}_R_t = \rho_p \bar{E}_R_{t-1} + \kappa (1 - \rho_p) \left( \frac{S_r}{E_R} \right) \hat{\delta}_t \]
J. Alternative formulation with separated portfolio-rebalancing and risk premia

The original formulation was:

\[ R_t = (R_{t-1})^{\rho_{pr}} \left[ \left( (R_P^t)(R_{t+1}^P)(R_{t+2}^P)(R_{t+3}^P) \right)^{\frac{1}{4}} \left( \frac{1}{ER_t} \right)^{1-\rho_{pr}} \right] \]

After log-linearisation following the method proposed by Uhlig (1995):

\[ \hat{r}_t = \rho_r \hat{r}_{t-1} + \left( 1 - \rho_r \right) \left( \frac{1}{4} (\hat{\rho}_t^P + \hat{\rho}_{t+1}^P + \hat{\rho}_{t+2}^P + \hat{\rho}_{t+3}^P) \right) - \theta ER_t \]

The long-term nominal interest rate could alternative be formulated as follows with \( \theta_{t}^{RP} \) and \( \theta_{t}^{PRB} \) reflecting the influence of the risk premium and investor portfolio preferences on the interest rate respectively:

\[ R_t = (R_{t-1})^{\rho_{pr}} \left[ \left( (R_P^t)(R_{t+1}^P)(R_{t+2}^P)(R_{t+3}^P) \right)^{\frac{1}{4}} (\theta_t^{RP})^{\varphi_1} (\theta_t^{PRB})^{\varphi_2} \right]^{1-\rho_{pr}} \]

After log-linearisation this yields:

\[ \hat{r}_t = \rho_r \hat{r}_{t-1} + \left( 1 - \rho_r \right) \left( \frac{1}{4} (\hat{\theta}_t^P + \hat{\theta}_{t+1}^P + \hat{\theta}_{t+2}^P + \hat{\theta}_{t+3}^P) \right) + \varphi_1 \hat{\theta}_t^{RP} + \varphi_2 \hat{\theta}_t^{PRB} \]

Now supposing that the central bank balance sheet affects the risk premium and the portfolio rebalancing variable in the following way one can see that the original formulation can be obtained by simply redefining variables.

\[ \hat{\theta}_t^{RP} = \mu_1 + \omega ER_t \]
\[ \hat{\theta}_t^{RP} = \omega ER_t \]
\[ \hat{\theta}_t^{PRB} = \mu_2 + \psi ER_t \]
\[ \hat{\theta}_t^{PRB} = \psi ER_t \]

Using these equations one derives the relationship that:

\[ \theta = \varphi_1 \omega + \varphi_2 \psi \]

The separation of the two effects does little for ascertaining the influence of the policy on US macroeconomic variables or the effectiveness of QE in general and hence the simplified version is utilised.
K. Derivation of Q equation for Smets and Wouters (2007) and Gilchrist et al. (2009)

The optimality condition for capital accumulation in Smets and Wouters (2007) is given by:

\[ \Xi^k = \beta E_t \left[ \Xi_{t+1} \left( \frac{R^k_{t+1}}{P^k_{t+1}} u^k_{t+1} - a(u^k_{t+1}) \right) + \Xi^k_{t+1}(1 - \delta) \right] \]

Dividing both sides by \( \Xi^k_t \), re-arranging, dividing through by the stochastic growth rate \( \gamma \sigma c \) and defining \( \bar{Q}_t = \frac{\Xi^k_t}{\Xi^k_t}, \bar{\beta} = \frac{\beta}{\gamma \sigma c} \) and \( \xi_t = \Xi_t \gamma \sigma c \) yields:

\[ Q_t = E_t \left[ \bar{\beta} \left( \frac{\xi_{t+1}}{\xi_t} \right) \left( Q_{t+1}(1 - \delta) + u^k_{t+1} r^k_{t+1} - a(u^k_{t+1}) \right) \right] \]

Dividing both sides by \( \left( \frac{\xi_{t+1}}{\xi_t} \right) \) yields:

\[ E_t \left( \frac{Q_t \xi_t}{\xi_{t+1}} \right) = E_t \left[ \bar{\beta} \left( Q_{t+1}(1 - \delta) + u^k_{t+1} r^k_{t+1} - a(u^k_{t+1}) \right) \right] \]

This can be re-expressed as:

\[ E_t \left( e^{\ln(Q_t \xi_t/\xi_{t+1})} \right) = \bar{\beta} E_t \left[ e^{\ln(Q_{t+1})(1 - \delta)} + e^{\ln(r^k_{t+1})} u^k_{t+1} - a(u^k_{t+1}) \right] \]

The second last equation implies that as \( Q_* = u_* = 1 \) and \( a(u_*) = 0 \), in the steady state:

\[ 1 = \left( \bar{\beta}(1 - \delta) + r^k \right) \]

To derive out the final equation, it is necessary to use a first order taylor expansion on both sides around the zero inflation steady state.

Let \( f(\ln(q_t), \ln(\xi_{t+1}), \ln(\xi_t)) = \left( e^{\ln(Q_t \xi_t/\xi_{t+1})} \right) \)

\[ f(\ln(q_t), \ln(\xi_{t+1}), \ln(\xi_t)) \approx f(\ln(q_t), \ln(\xi_t), \ln(\xi_t)) + f'(\ln(q_t), \ln(\xi_t), \ln(\xi_t))(\ln(q_t) - \ln(q_t)) + (\ln(\xi_{t+1}) - \ln(\xi_t)) + (\ln(\xi_t) - \ln(\xi_t)) \]

Now, \( f(q_t, \ln(\xi_t), \ln(\xi_t)) = f(q_t, \ln(\xi_t), \ln(\xi_t)) = 1 \) by definition so:

LHS = \( E_t (1 + \hat{q}_t - \hat{\xi}_{t+1} + \hat{\xi}_t) \) where \( \hat{x}_t \) is the log-deviation of variable x from its steady state value.

Let \( g(\ln(q_{t+1}), \ln(Q_{t+1}), \ln(r^k_{t+1}), u_{t+1}) = \left( e^{\ln(Q_{t+1})(1 - \delta)} + e^{\ln(r^k_{t+1})} u_{t+1} - a(u_{t+1}) \right) \)

\[ g(\ln(q_{t+1}), \ln(Q_{t+1}), \ln(r^k_{t+1}), u_{t+1}) \approx g(\ln(q_t), \ln(r^k_t), u_t) + g'_1(\ln(q_t), \ln(r^k_t), u_t)(\ln(q_{t+1}) - \ln(q_t)) + g'_2(\ln(q_t), \ln(r^k_t), u_t)(\ln(r^k_{t+1}) - \ln(r^k_t)) + g'_3(\ln(q_t), \ln(r^k_t), u_t)(u_{t+1} - u_t) \]

Now, using the definition of g:

\[ g'_1(\ln(q_t), \ln(Q_t), \ln(r^k_t), u_t) = (1 - \delta) \]

\[ g'_2(\ln(q_t), \ln(r^k_t), u_t) = e^{\ln(r^k_t)} = r^k \]

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\[ g'(\ln(q^*_t), \ln(r^k_t), u_t) = r^k_t - a'(u_t) = 0 \]

where the last equality is obtained by equation 4.10.

Also, in the steady state,
\[ g(\ln(q^*_t), \ln(r^k_t), u_t) = \bar{\beta} \left( Q_t (1 - \delta) + u_t r^k_{t+1} - a(u_t) \right) = 1 \]

So,
\[ \text{RHS} = g(\ln(q^*_{t+1}), \ln(r^k_{t+1}), u_{t+1}) \approx 1 + r^k_{t+1} (1 - \delta) \hat{q}_{t+1} \]

Hence,
\[ E_t (\hat{q}_t - \xi_{t+1} + \xi_t) = \bar{\beta} E_t \left( r^k_{t+1} + (1 - \delta) \hat{q}_{t+1} \right) \]

which implies, using the definition of \( \bar{\beta} \) from the steady state relationship:
\[ \hat{q}_t = E_t (\xi_{t+1}) - \xi_t + \frac{r^k_t}{1 - \delta} + r^k_t E_t \left( \frac{\xi_{t+1}}{r^k_{t+1}} \right) + \frac{(1 - \delta)}{1 - \delta} + r^k_t \hat{q}_{t+1} \]

We must now use the Euler equation for households to get an expression for \( E_t (\xi_{t+1}) - \xi_t \).

Dividing both sides of equation 4.7 through by the stochastic growth trend yields:
\[ \xi_t = \bar{\beta} \xi_{t+1} R_t \left[ \frac{\xi_{t+1}}{r_{t+1}} \right] \]

Log-linearising this using the same first order taylor expansion technique results in:
\[ E_t (\xi_{t+1}) - \xi_t = -(\tilde{R}_t - E_t (\tilde{R}_{t+1})) \]

Substituting this back into the \( Q \) equation yields its final form.

The presence of an external finance premium generates a wedge between the long-term interest rate (which is effectively the policy rate) and private capital (Drautzburg and Uhlig, 2011).

With an external finance premium as in Gilchrist et al. (2009), the optimality condition becomes:
\[ Q_t = E_t \left[ \frac{\bar{\beta}}{S_t} \left( \frac{\xi_{t+1}}{\xi_t} \right) \left( Q_{t+1} (1 - \delta) + u_{t+1} r^k_{t+1} - a(u_{t+1}) \right) \right] \]

This implies in the steady state that
\[ 1 = \left[ \frac{\bar{\beta}}{S_t} \left( (1 - \delta) + r^k_t \right) \right] \]

Applying the same first order Taylor expansion technique as before yields
\[ E_t (\hat{q}_t + \hat{s}_t - \xi_{t+1} + \xi_t) = \frac{\bar{\beta}}{S_t} E_t \left( r^k_t \hat{r}^k_{t+1} + (1 - \delta) \hat{q}_{t+1} \right) \]

Substituting in the log-linearised Euler equation yields equation 4.50, remembering that profit maximisation by intermediate producers implies that \( \hat{r}^k_{t+1} = \hat{m} \hat{r}^k_{t+1} \) and the new steady state relationship.
Bibliography


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