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EQUILIBRIUM AND ADVERSE
SELECTION
by
Colin Rose
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ABSTRACT

The nature of equilibrium in markets with adverse selection evoked considerable interest following George Akerlof's famous paper on the market for lemons. Whereas Akerlof argued that markets with adverse selection may yield no equilibrium, Charles Wilson has argued that multiple equilibria may result. In this paper, it is shown that if the distribution of quality follows some standard distribution, then a unique equilibrium will result. In the (less plausible) context of multiple-equilibria, conditions are derived under which both buyers and sellers will prefer higher price-equilibria.
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Equilibrium and Adverse Selection

In his seminal paper on the market for lemons, Akerlof illustrates how a market with adverse selection may lead to market breakdown. In follow-up papers, Wilson (1979, 1980) provides a framework for the analysis of markets with adverse selection, and these represent the state of the art in lemon-style models. In particular, Wilson argues that:

- markets with adverse selection may be characterised by multiple stable equilibria. As such, the assumption of a unique equilibrium in models of adverse selection may be inappropriate.
- these equilibria can always be ranked according to the Pareto criterion in order of ascending price: that is, both buyers and sellers always prefer higher price-equilibria to lower ones.

Such results are surely impressive. Moreover, they have crossed the Rubicon from journal to post-graduate\(^1\) text. Using a combination of theoretical and numerical analysis, we argue over two sections that:

Section 1 the possibility of multiple equilibria is most unlikely.

Section 2 as a theoretical redress, high-price equilibria will only Pareto-dominate low-price equilibria if average quality is a convex function of price. Numerical analysis suggests that it is generally concave.

The Akerlof-Wilson Model

Akerlof considers a market with asymmetric quality information: buyers are unable to ascertain the quality of goods (used cars) before they purchase, whereas sellers are aware of the quality, but have no way of making buyers believe them. As is standard in this literature, the absence of signalling and search is assumed. Each agent has the following utility function:

\[
U = U(c, n, t, q) = c + t q n
\]

where \(c\) is consumption of other goods, \(n\) is a discrete binary variable representing consumption of used cars (\(n = 0\) or \(n = 1\)), \(q \in \{q_l, q_h\}\) is the quality of the car consumed with density \(f(q)\), and \(t \in \{t_l, t_h\}\) is a parameter that measures the relative valuation of a car of quality \(q\) for consumption of other goods, with density \(h(t)\). Finally, let \(p\) denote the price of used cars, and let the price of other goods be unity.

Wilson suggests that \(U = U(c, q, t)\). But crucially, this is a market with asymmetric information, such that buyers cannot discriminate between lemons (low quality cars), and melons (high quality cars). Thus, since buyers cannot discriminate between good and bad, they can NOT use \(q\) as a choice variable over which they maximise utility. Hence, we must return again to maximising w.r.t. quantity \(n\), rather than w.r.t. quality \(q\). This does not cause this analysis to differ from that of Wilson.

The Supply Side

As per Wilson, all sellers/owners are assumed to have the same valuation parameter \(t \in \{t_l, t_h\}\), which we denote by \(t\). Ownership implies that sellers know the quality of the cars they own, so sellers simply \(\max U = c + t q n\) subject to an income constraint. Equating the MRS with the relative price ratio yields \(\eta q = p\) where \(q \in \{q_l, q_h\}\). Hence, for an owner with car of quality \(q\), the condition for positive supply is \(\eta q \leq p\). Stated differently, an owner will sell her car if and only if \(q \leq p/\eta\). Then, at any price, the supply of cars may be thought of as the proportion of cars for which this is true:

\[
\mathcal{S}(p) = \text{Prob}\{q \leq \eta p / \eta q\} = \int_{q_l}^{\eta p / \eta q_1} f(q) dq \quad \text{for } p > \eta q_1
\]

As per Wilson (1980), the average quality of cars at price \(p\) is:

\[
q^*(p) = E\{q \mid q \leq \eta p / \eta q\} = \frac{\int_{q_l}^{\eta p / \eta q} q f(q) dq}{\mathcal{S}(p)} \quad \text{for } p > \eta q_1 \quad \text{(where } E \text{ is the expectation operator)}
\]

The Demand Side

The very essence of a lemons-model with asymmetric quality information is that buyers are unable to ascertain the quality of goods before they purchase. Since buyers cannot choose \(q\), they maximise expected-utility. Thus, the problem for buyers is to \(\max U^* = c + q^* n\) subject to an income constraint (where \(q^*\) is given by \(\mathcal{S}\)). Equating the marginal rate of substitution with the relative price ratio yields \(q^* = p\). Hence, the condition for positive demand is \(q^* \geq p\). Then, at any price, the demand for cars may be thought of as the proportion of buyers for which this is true:

\[
\mathcal{D}(p) = \text{Prob}\{q^* \geq p / q^*(p)\} = \int_{p / q^*(p)}^{\mathcal{S}(p)} h(t) dt \quad \text{for } p < q^*(p)
\]

\[
0 \quad \text{otherwise}
\]

\(^1\)I take great pleasure in thanking Jeff Sheen for his pervasive mentorial presence. Warren Henne, Jeff Sheen and Alan Woodland all provided helpful insights.\(^2\)

\(^1\)For instance, Louis Philips devotes over 10 pages of \textit{The Economics of Imperfect Information} to discuss Wilson's paper, whereas Jean Tirole (pg. 110) makes mention of multiple equilibria and Pareto dominance of high price equilibria in \textit{The Theory of Industrial Organization} in an exercise modelled on Wilson's paper.
Section 1: Multiple Equilibria are Most Unlikely

From equation (1), we see that the supply curve is monotonically increasing in price. Hence, the possibility of multiple equilibria requires that the demand curve cut the supply curve in at least 2 places (practically 3). This in turn implies that the demand curve must contain an upward-sloping segment, in addition to the standard downward-sloping segment. From equation (2), we can determine the condition for an upward-sloping demand curve:

$$\frac{dD(p)}{dp} > 0 \text{ iff } \frac{dpq(p)}{dp} = 1 \left[ \frac{dq(p)}{dp} - \frac{p}{q^2(p)} \right] < 0$$

If we denote by \( \varepsilon \) the price elasticity of average quality, then the requirement for an upward-sloping demand curve is simply:

$$\varepsilon = \frac{dq(p)}{dp} - \frac{p}{q^2(p)} > 1$$

Summarising the above conditions, and deriving \( \varepsilon \) using equation (2), one obtains after a few lines of less than pretty algebra:

$$\frac{dD(p)}{dp} \geq 0 \text{ iff } \varepsilon \leq 1$$

where \( \varepsilon = \frac{\frac{d}{dp} \left( \frac{p}{q^2(p)} \right)}{\frac{d}{dp} \left( \frac{q}{p} \right)} \)

The result for \( \varepsilon \) is not quite as meaningless as it might at first seem. As can be seen from diagram 1, the first part of \( \varepsilon \) (the fraction) is simply the area bound by the rectangle, divided by the (shaded) area bound by the curve. All we can say about the second part (within the brackets) is that it must necessarily be non-negative\(^2\). Unfortunately, these insights do not tell us whether \( \varepsilon > 1 \). By applying numerical methods\(^3\) to equation (6), computer-generated plots of \( \varepsilon \) vs. \( p \) were derived for all the standard frequency distributions, each of which underwent a systematic and comprehensive test of different parameter values. The results are somewhat surprising, if only for the consistency of each distribution, irrespective of the chosen parameters.

\(^2\)From equation (1), \( q^2 \) is non-decreasing in price. Hence, \( \frac{dq^2}{dp} \geq 0 \). Since \( p \) and \( q^2 \) are necessarily non-negative, it follows that \( \varepsilon \) is also non-negative.

\(^3\)The distribution function of many distributions cannot be expressed without an integral sign, which is of course the raison d'être for the tables found at the rear of statistic texts. The same applies to the calculation of \( q^2 \). In such cases, numerical integration techniques may be used. Calculations were performed using Mathematica. Mathematica was used to check the results to a desired level of accuracy (up to 50 digits of precision). Graphs were produced using Theorist.

The following table lists the frequency distributions which were tested, and provides a summary of our results:

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Conditions</th>
<th>( \varepsilon \leq 1 )</th>
<th>( \varepsilon = 1 )</th>
<th>( \varepsilon \geq 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Chi-squared</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Exponential</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Lognormal</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>( \checkmark ) if ( \beta &gt; 1 )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
<tr>
<td>Uniform</td>
<td>( \checkmark ) if ( \beta &lt; 1 )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td></td>
</tr>
</tbody>
</table>

Typical plots\(^4\) of \( \varepsilon \) vs. \( p \) for the gamma, chi-squared, exponential, lognormal and normal distributions are illustrated in the middle column of diagram 2, with their pdfs. We use the word typical, for our analysis suggests that it is not possible to generate plots of \( \varepsilon \) vs. \( p \) that are qualitatively different to those illustrated, irrespective of the parameters chosen. This can be easily seen by means of a 3D plot, by plotting price on the x-axis, the distribution's parameter on the y-axis, and the elasticity on the z-axis. Of course, this only works if the distribution has but one parameter. For distributions with two parameters, it seems sensible to hold the location or scale parameter constant, and vary the shape parameter, and this technique is used where necessary in diagram 2. The beta distribution \( B(\alpha, \beta) \) is capable of producing somewhat more diverse results, and diagram 3 illustrates these (\( \beta > 1 \), \( \beta = 1 \), \( \beta < 1 \) each with varying \( \alpha \)). The uniform distribution is captured via a beta distribution \( B(\alpha, \beta) \) with \( \alpha = 1 \), and \( \beta = 1 \) (see centre graph in diagram 3).

The central issue here is whether or not a distribution can generate multiple equilibria. From equation (1), we note that the supply curve is monotonically increasing. By keeping this in mind, and then referring to Table 1, and equation (2), it follows that:

* \( \varepsilon < 1 \) \( \text{p} \)

If \( f(q) \) follows a gamma-distribution, then the demand curve is always downward sloping (and never upward sloping), and hence cannot generate multiple equilibria. The same applies if \( f(q) \) follows a chi-squared, exponential\(^5\) or lognormal distribution, or even the beta distribution \( B(\alpha, \beta) \) for parameter \( \beta > 1 \).

* \( \varepsilon = 1 \) \( \text{p} \)

If \( f(q) \) follows a uniform distribution, or a beta distribution \( B(\alpha, \beta) \) with parameter \( \beta = 1 \), then the demand-curve is perfectly elastic, and hence cannot generate multiple equilibria.

* \( \varepsilon > 1 \) \( \text{p} \)

If \( f(q) \) follows a beta distribution \( B(\alpha, \beta) \) with \( \beta < 1 \), then for \( p < q^* \) the demand-curve is always upward sloping. But for \( p \geq q^* \), demand is zero. Thus, both the demand curve and the supply curve are upward sloping, and hence may generate multiple equilibria. However, this case is perhaps trivial, for when we look at the underlying pdf of quality when \( \beta < 1 \), we see that it is most unrealistic (see last column of diagram 3). We shall consequently ignore this possibility.

\(^4\)In all plots, we set \( q_0 = 0 \).

\(^5\)If the gamma distribution always yields \( \varepsilon < 1 \) \( \text{p} \), then so must the chi-squared and exponential distributions, for they are just special cases of the gamma distribution.
<table>
<thead>
<tr>
<th>Distribution</th>
<th>Typical 2D Plot</th>
<th>3D Plot</th>
<th>( \beta &gt; 1 \Rightarrow \varepsilon &lt; 1 )</th>
<th>( \beta = 1 \Rightarrow \varepsilon = 1 )</th>
<th>( \beta &lt; 1 \Rightarrow \varepsilon &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gamma</strong></td>
<td><img src="#" alt="Gamma Plot" /></td>
<td><img src="#" alt="Gamma 3D Plot" /></td>
<td>( f(q) = \frac{\gamma q^{\gamma - 1}}{\Gamma(\gamma)} e^{-\gamma q} )</td>
<td>( \gamma &gt; 0 )</td>
<td>( \gamma = 1 )</td>
</tr>
<tr>
<td>Shape parameter ( \gamma &gt; 0 )</td>
<td></td>
<td></td>
<td>Scale parameter ( \gamma &gt; 0 )</td>
<td>Scale parameter ( \gamma &gt; 0 )</td>
<td>Scale parameter ( \gamma &gt; 0 )</td>
</tr>
<tr>
<td>( 0 &lt; q &lt; \infty )</td>
<td></td>
<td></td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
</tr>
<tr>
<td><strong>Chi-squared</strong></td>
<td><img src="#" alt="Chi-squared Plot" /></td>
<td><img src="#" alt="Chi-squared 3D Plot" /></td>
<td>( f(q) = 2^{q/2} e^{-q} \frac{1}{\sqrt{2\pi q}} )</td>
<td>( \gamma &gt; 0 )</td>
<td>( \gamma = 1 )</td>
</tr>
<tr>
<td>Shape parameter ( \nu &gt; 0 )</td>
<td></td>
<td></td>
<td>Scale parameter ( \nu &gt; 0 )</td>
<td>Scale parameter ( \nu &gt; 0 )</td>
<td>Scale parameter ( \nu &gt; 0 )</td>
</tr>
<tr>
<td>( 0 &lt; q &lt; \infty )</td>
<td></td>
<td></td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
</tr>
<tr>
<td><strong>Exponential</strong></td>
<td><img src="#" alt="Exponential Plot" /></td>
<td><img src="#" alt="Exponential 3D Plot" /></td>
<td>( f(q) = e^{-\frac{q}{\beta}} \beta )</td>
<td>( \beta &gt; 0 )</td>
<td>( \beta = 1 )</td>
</tr>
<tr>
<td>Scale parameter ( \beta &gt; 0 )</td>
<td></td>
<td></td>
<td>Scale parameter ( \beta &gt; 0 )</td>
<td>Scale parameter ( \beta &gt; 0 )</td>
<td>Scale parameter ( \beta &gt; 0 )</td>
</tr>
<tr>
<td>( 0 &lt; q &lt; \infty )</td>
<td></td>
<td></td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
</tr>
<tr>
<td><strong>Lognormal</strong></td>
<td><img src="#" alt="Lognormal Plot" /></td>
<td><img src="#" alt="Lognormal 3D Plot" /></td>
<td>( f(q) = \frac{1}{q \sigma \sqrt{2\pi}} e^{-\frac{\ln(q) - \mu}{2\sigma^2}} )</td>
<td>( \sigma &gt; 0 )</td>
<td>( \sigma = 1 )</td>
</tr>
<tr>
<td>Shape parameter ( \sigma &gt; 0 )</td>
<td></td>
<td></td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
</tr>
<tr>
<td>( 0 &lt; q &lt; \infty )</td>
<td></td>
<td></td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
</tr>
<tr>
<td><strong>Normal</strong></td>
<td><img src="#" alt="Normal Plot" /></td>
<td><img src="#" alt="Normal 3D Plot" /></td>
<td>( f(q) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} )</td>
<td>( \sigma &gt; 0 )</td>
<td>( \sigma = 1 )</td>
</tr>
<tr>
<td>Location parameter ( \mu &gt; 0 )</td>
<td></td>
<td></td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
<td>Scale parameter ( \sigma &gt; 0 )</td>
</tr>
<tr>
<td>( -\infty &lt; q &lt; \infty )</td>
<td></td>
<td></td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
<td>3D plot: plane denotes x-y plane with scale parameter constant at ( \gamma = 0.8 ), and ( i = 2 ).</td>
</tr>
</tbody>
</table>

*The variable "quality" has been truncated at zero.*

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**Beta Distribution**

\( B(\alpha, \beta) \)

\[ f(q) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} q^{\alpha-1} (1-q)^{\beta-1} \]

\[ \text{Diagram 3} \]

**Uniform Distribution**

\[ f(q) = \frac{1}{b-a} \]

---

**Column 1**

- \( \beta > 1 \) with:
  - \( \alpha < 1 \)
  - \( \alpha = 1 \)
  - \( \alpha > 1 \)

**Column 2**

- \( \beta = 1 \) with:
  - \( \alpha < \beta \)
  - \( \alpha = \beta \)
  - \( \alpha > \beta \)

**Column 3**

- \( \beta < 1 \) with:
  - \( \alpha < 1 \)
  - \( \alpha = 1 \)
  - \( \alpha > 1 \)

In all of the above:

- \( q_0 = 0 \)
- \( \Gamma = 1 \)
That leaves the normal distribution. As can be seen from Diagram 2 or 4, the normal distribution yields \( \varepsilon > 1 \) for a low range of prices \( p \in (0, p^*) \), and thereafter \( \varepsilon < 1 \) for a higher range of prices \( p \in (p^*, \infty) \). In this example, \( p^* = 3 \). By equation \( \theta \), this implies that the demand curve is upward sloping for \( p \in (0, p^*) \), and downward sloping (or zero demand) for \( p \in (p^*, \infty) \). Stated differently, the demand curve will be a hill-shaped function of price: upward-sloping at low prices, and downward-sloping at high prices, with a single hump, and a peak at \( p^* \). Moreover, this result holds irrespective of the distribution assumed for preferences \( k(i) \), since preferences do not enter equation \( \theta \).

May we stress 2 points: firstly, of the distributions considered, a normal distribution for quality is the only distribution that could possibly generate multiple equilibria, since it is the only distribution that can yield a demand curve with both downward- and upward- sloping segments. Secondly, if we then use this distribution, the resulting demand curve will always contain a single hump. This is important, for it implies that the possibility of multiple equilibria is quite remote. See Diagram 5 below: on the left-hand side is a 're-print' of the demand and supply curves as drawn by Wilson (1979, 1980). On the right-hand side, a computer-generated demand- and supply-curve is shown for the case where the distribution of quality \( f(q) \) is normal\(^6\), and the distribution of preferences \( k(i) \) is uniform\(^7\), although the latter is irrelevant. Since supply is a monotonically increasing function of price (see \( \theta \)), whereas demand is hump-shaped, and since demand must exceed supply at \( p = 0 \) (as per Diagram 5), it appears that multiple equilibria cannot be generated.

### Section 2

**Higher-Price Equilibria are not necessarily Pareto-optimal**

If we argue, as has been done, that multiple equilibria are essentially not feasible, then a discussion of the Pareto-ranking of such equilibria is perhaps a little pointless. Nevertheless, the concept is an intriguing one; we shall play the Devil’s advocate, and suppose in this section that such equilibria can exist.

Following Wilson, the optimal condition for buyers \( p = u^p \) may be represented by a diagram in \( p, q \) space for any agent, with preference \( r \approx [c, \ell] \), and this we do in Diagram 6. The resulting ray from the origin, with slope \( i \) may be interpreted as the locus of points along which the ratio between cars and other goods is equated with the relative price of their quality.

Section 2: Higher-Price Equilibria are not necessarily Pareto-optimal

If we argue, as has been done, that multiple equilibria are essentially not feasible, then a discussion of the Pareto-ranking of such equilibria is perhaps a little pointless. Nevertheless, the concept is an intriguing one; we shall play the Devil’s advocate, and suppose in this section that such equilibria can exist.

Following Wilson, the optimal condition for buyers \( p = u^p \) may be represented by a diagram in \( p, q \) space for any agent, with preference \( r \approx [c, \ell] \), and this we do in Diagram 6. The resulting ray from the origin, with slope \( i \) may be interpreted as the locus of points along which the ratio between cars and other goods is equated with the relative price of their quality.

The curve \( q^* = q(p, i) \) (given by equation \( \theta \)) represents the possible market combinations of price and average quality. If we limit ourselves to extremes, we could draw \( q^* = q(p, i) \) in the following 3 ways:

- as strictly concave in price
- as a function linear in price
- as strictly convex in price

Suppose \( q^* = q(p, i) \) is strictly concave in price (as per Diagram 6, note that \( q^* \) is on the horizontal axis). If at any price \( p \), the Carlinian market combination of price and average-quality \( q^* \) lies left of the line \( p = u^p \) for \( r \approx [c, \ell] \), agent \( r \) will have zero/impartial/positive demand respectively at that price.

For example, we see from Diagram 6 that at price \( p = p^* \), the market combination is \( (q^*, p^*) \). Hence, for agent \( r \), a purchase at price \( p^* \) yields a net expected marginal utility\(^8\) of \( u^p - p^* = (q^2 - q^1) \) which may be interpreted as a measure of consumer surplus. Stated differently, any agent’s consumer surplus is maximised when the gap between his locus line \( p = u^p \) and the market combination line \( q^* \) is greatest. Given that we have drawn the curve \( q^* = q(p, i) \) as strictly concave, the necessary and sufficient requirement for this agent’s consumer surplus measure to be maximised is that the slopes of the two curves be equal; that is, by turning the graph on its side, when \( dq^* / dp = db \). Since \( b \) is distributed over \( (c, \ell) \) with density \( k(b) \), this is clearly impossible for all \( b \). Thus, a price change, whether it be up or

---

\(^6\)The parameters used here are the same as those in Diagram 4 (the elasticity diagram above). In that diagram, we argued that there exists some \( p^* \) at which \( \varepsilon \) changes from inelastic to elastic, and that in this example \( p^* = 3 \). As expected then, we see in Diagram 4 that at \( p = 3 \), the demand curve changes from upward sloping to downward sloping.

\(^7\)For this example, we assumed \( \varepsilon = 1, \ell = 3 \), and hence that \( k(0) = 0.5 \).

\(^8\)Recall that expected utility is given by: \( U' = \varepsilon + u^{p'}. \)
down, cannot possibly be Pareto-optimal since some buyers will certainly be made worse off. This is easy to see from Diagram 6: if the equilibrium price is \( p_{e,w} \), then the consumer surplus of agent \( i \) is at a maximum. If the equilibrium price is \( p_{e,w} \), then the consumer surplus of agent \( i \) is at a maximum. Clearly then, as the equilibrium price rises from \( p_{e,w} \), the utility of agent \( i \) falls, whilst that of agent \( i \) increases, which is of course not Pareto-optimal. We cannot even argue that buyer's collectively prefer higher-price equilibria for this will depend on the frequency distribution assumed for \( h(i) \). In general, it will require that we find the price at which the collective consumer surplus of all buyers is maximised:

\[
\max \int_{\Omega} \left[ q^* - \min\{q^*, \bar{q}(i)\} \right] \, d\lambda
\]

This concludes the discussion if \( q^* = q^*(p) \) is strictly concave. As is easily seen, if \( q^* = q^*(p) \) is convex or strictly convex, then all buyers will in fact prefer a higher price. If \( q^* = q^*(p) \) contains both convex and strictly concave sections, then we need to make qualified comments such as, "Within the convex segment of \( q^* = q^*(p) \), all buyers will prefer higher prices".

If standard distributions for quality cannot generate multiple equilibria (as was argued in Section 1), then a numerical analysis of the concavity or convexity of average quality may seem somewhat pointless. Nevertheless, the issue is important: in the absence of multiple-equilibria, a higher price implies that markets no longer clear, and that excess supply results. Under this scenario, if average quality is linear or strictly convex, all buyers will prefer this higher disequilibrium price. Sellers, however, face a trade-off between the higher price and a reduction in the probability of making a sale. If the probability loss is sufficiently small, sellers will also prefer the higher disequilibrium price, and if so, this high price may not converge down to the equilibrium price. So ends the theoretical discussion of this section. Typical plots of average quality are shown in Diagram 7 and Diagram 8.

In economic analysis, the assumption of a unique equilibrium is still very much the status quo. We have argued that if quality follows a standard distribution, the possibility of multiple equilibria is extremely remote. Consequently, this paper finds no evidence to cause us to change the status quo of the unique equilibrium, in markets with adverse selection.

\(^9\)Wilson obtains his 'always Pareto dominating' argument by drawing \( q^* = q^*(p) \) with a strictly concave section and a convex section, and then by drawing price comparisons only within the convex section.

\(^{10}\)Once again, the beta distribution yields slightly more diverse results. The parameter's in Diagram 6 are the same as those in Diagram 3, where the underlying pdf is illustrated for each case. Thus, to view the pdf, simply refer to Diagram 3.
<table>
<thead>
<tr>
<th>$\beta &gt; 1$</th>
<th>$\beta = 1$</th>
<th>$\beta &lt; 1$</th>
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**Beta Distribution**

$f(q) = \frac{\alpha^{\beta-1}(1-q)^{\alpha-1}}{\Gamma(\alpha)\Gamma(\beta)}$

**Diagram 8**

**Uniform Distribution**

**Standard Normal Distribution**

Let $Z$ be $N(0,1)$ with

- probability density function: $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
- distribution function: $\Phi(z) = \int_{-\infty}^{z} \phi(t) \, dt$

Let $c$ and $d$ denote lower and upper truncations of this distribution respectively. By a well-known lemma:

If $Z$ is $N(0,1)$, then $\int_{c}^{d} z\phi(z) \, dz = \Phi(d) - \Phi(c)$. Similarly, $\int_{c}^{d} z\phi(z) \, dz = \Phi(d) - \Phi(c)$.

Hence, $\int_{c}^{d} z\phi(z) \, dz = \Phi(c) - \Phi(d)$. 

Thus $E[Z | c \leq Z \leq d] = \int_{c}^{d} z - \frac{1}{\Phi(d) - \Phi(c)} \, dz = \frac{\Phi(c) - \Phi(d)}{\Phi(d) - \Phi(c)}$
Generalising to the Normal Distribution
Let $Q$ denote the random variable of quality, where $Q$ is $N(\mu, \sigma^2)$.

Then $Z = \frac{Q - \mu}{\sigma} \Rightarrow Q = \mu + \sigma Z$

From equation $\Phi$:

$Q(\mu, \sigma) = E[Q | 10 \leq Q \leq 15]$

(truncated at zero)

where

$c = \frac{0 - \mu}{\sigma} = -\frac{\mu}{\sigma}, \hspace{1em} d = \frac{15 - \mu}{\sigma}$

$\Rightarrow$

$Q = \mu + \sigma E[Z | 10 \leq Z \leq 15]$

From equation $\Phi$:

$\varepsilon = \frac{\int_{c}^{d} f(q) dq}{\int_{e}^{f} f(q) dq}$

If $f(.)$ denotes a normal distribution $N(\mu, \sigma^2)$, and if truncation implies that $q_e = 0$, then $\varepsilon$ may be expressed as follows:

$\Rightarrow$

$\varepsilon = \frac{\int_{c}^{d} \frac{f(q)}{f(\mu)} dq}{\int_{c}^{d} \frac{f(q)}{f(\mu)} dq} - 1$

In other words, when the distribution of quality is normal, the price elasticity of average quality $\varepsilon$ may be expressed in terms of both the standard normal density function, and the standard normal distribution function. The latter may be expressed in terms of the error function, which is found in many computer packages.

---

1. This result (see $\Phi$) may of course be derived directly from the definition $\varepsilon = \frac{K(p) \cdot P}{K(p)}$ where $K(p)$ is given by equation $\Phi$. The derivation is less than elegant.

2. It turns out that numerical integration yields the same result as the above method (where the above method takes advantage of built-in functions in Mathematica). The use of these built-in functions serves as a useful check on the results.
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