APPENDIX D: A NON-LINEAR ELASTIC-PLASTIC MODEL FOR SAND

The model is isotropic, so it is convenient to formulate it in terms of mean and deviator stresses, which are also invariants.

D1. Elastic Behaviour

The stress-strain law may be written in terms of the tangent bulk modulus, K_t, and the tangent shear modulus, G_t, as follows:

$$\begin{pmatrix} dp' \\ dq \end{pmatrix} = \begin{bmatrix} K_t & 0 \\ 0 & 3G_t \end{bmatrix} \begin{pmatrix} d\varepsilon_p \\ d\varepsilon_q \end{pmatrix} \tag{D-1}$$

where

$$dp' = 1/3\,[d\sigma'_1 + d\sigma'_2 + d\sigma'_3]$$

$$q = \tfrac{1}{\sqrt{2}} \sqrt{(d\sigma'_1 - d\sigma'_2)^2 + (d\sigma'_2 - d\sigma'_3)^2 + (d\sigma'_3 - d\sigma'_1)^2}$$

$$d\varepsilon_p = (d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3)$$

$$d\varepsilon_q = \tfrac{\sqrt{2}}{3} \sqrt{(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2}$$

and $d\sigma_1'$, $d\sigma'_2$ and $d\sigma'_3$ are increments of principal stresses, and $d\varepsilon_1$, $d\varepsilon_2$ and $d\varepsilon_3$ are increments of principal strain.

D2. Bulk Modulus, K_t

Following the suggestions of Naylor et al. (1981) and Lee and Salgado (2000), it is assumed that the tangent bulk modulus, K_t, is given by:

$$K_t = D_s (p')^{n_k} (p_a)^{(1-n_k)} \tag{D-2}$$

where p_a = atmospheric pressure

D_s and n_k are material constants, with n_k usually having a value of about 0.5

Therefore K_t increases as a power law function of mean effective stress.

D3. Shear Modulus, G_t

D3.1 Small strain shear modulus, G_o

Following Hardin and Black (1966), it is assumed that:

$$G_o = C_g\left(\frac{(e_g - e_o)^2}{1 + e_o}\right)(p_a)^{(1-n_g)}(p')^{n_g} \qquad \text{(D-3)}$$

where C_g, n_g and e_g, are material constants,

and e_o = initial void ratio

D3.2 Secant Shear Modulus, G_s

Lee and Salgado (2000) extended the work of Fahey and Carter (1993), by proposing an expression for the secant shear modulus, as:

$$\frac{G_s}{G_o} = \left(1 - f\left(\frac{\sqrt{J_2} - \sqrt{J_{2o}}}{\sqrt{J_{2max}} - \sqrt{J_{2o}}}\right)^g\right)\left(\frac{I_1}{I_{1o}}\right)^{n_g} \qquad \text{(D-4)}$$

where, the exponents, g and n_g, and the factor, f, are all material constants,

$I_1 = 3p'$

$J_2 = 1/6\,[(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_1 - \sigma'_3)^2]$

For triaxial conditions, $J_2 = 1/3\,(q)^2$ (D-5)

In equation D-4, I_{1o} and J_{2o} are the values of the stress invariants at the commencement of monotonic loading.

If $J_{2\,max}$ is the maximum value of the second stress invariant which is attainable at the current mean stress, $J_{2\,max}$ can be found from the appropriate yield function,
e.g. if $q = Mp'$, then

$$J_{2max} = \frac{1}{3}q_{max}^2 = \frac{1}{3}(Mp')^2 \qquad \text{(D-6)}$$

where M = current gradient of the failure envelope in q-p′ space

p′ = current mean stress

However it should be noted that other failure criteria (e.g. Mohr-Coulomb) could be used to determine $J_{2\ max}$ for general stress conditions.

D3.3 Tangent Shear Modulus, G_t

For conditions of simple shear (τ-γ), Fahey and Carter (1993) demonstrated that

$$\frac{G_t}{G_o} = \frac{\left(G_s / G_o \right)^2}{\left[1 - f(1-g)\left(\frac{\tau}{\tau_{max}} \right)^g \right]} \qquad \text{(D-7)}$$

By analogy it is proposed that the generalised form of this equation should be

$$\frac{G_t}{G_o} = \frac{\left(G_s / G_o \right)^2}{\left[1 - f(1-g)\left(\frac{\sqrt{J_2} - \sqrt{J_{2o}}}{\sqrt{J_{2max}} - \sqrt{J_{2o}}} \right)^g \right]} \qquad \text{(D-8)}$$

Equations D-2 and D-8 define the generalised forms of K_t and G_t to be substituted into the elastic stress-strain law, equation D-1.

Limit on G_t

It may also be necessary to place a limit on G_t as it reduces with increasing deviator stress. If a limit is placed, the following condition must be satisfied:

$$\left(\frac{G_t}{G_o} \right) \geq r_g$$

where r_g is a material parameter.

$r_g = 0$ corresponds to no limit on the reduction of G_t with deviator stress

$r_g = 1$ corresponds to no reduction in G_t with deviator stress

D4. PLASTIC BEHAVIOUR

Refer to the basic model described in Appendix C.