Asset Prices, Monetary Policy and Macroeconomic Stability

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Abstract

A focal point of macroeconomic policy analysis over the past decade has been whether central banks should respond to changes in asset prices. This thesis addresses the question from the distinct perspective of equilibrium determinacy. By obtaining the conditions for equilibrium determinacy, it is possible to ascertain whether a central bank could induce additional volatility in an economy by adopting a monetary policy rule which incorporates asset prices.

This thesis employs a New Keynesian model with a financial accelerator developed by Bernanke, Gertler and Gilchrist (1999) to analyse the effects on equilibrium determinacy. In contrast to most of the related literature, the principal finding of this thesis is that a central bank can respond to asset prices without inducing additional volatility in the economy. Moreover, responding to asset prices actually decreases the likelihood of indeterminacy. This can be attributed to the substitutability between responding to inflation and asset prices present in a New Keynesian model with a financial accelerator. The key implication is that central banks should take asset prices into account when designing monetary policy.
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1 Introduction

Large fluctuations in asset prices have been observed in most economies.\(^1\) Such episodes are normally characterised by a rapid increase followed by a sharp decline in asset prices. Fluctuations in asset prices, or, as they are sometimes called, bubbles,\(^2\) can have a substantial impact on inflation and output volatility.

It is against this backdrop that there is a long-standing debate as to whether central banks should directly respond to movements in asset prices. While there is a general consensus that the objectives of monetary policy are to ‘maintain low inflation and to limit the volatility of inflation and output’ (Gruen, Plumb and Stone 2003, p.1), the point of contention is how to best accomplish these objectives. Therefore, the question is whether by responding to asset prices a central bank can better accomplish these objectives.

It is generally agreed that even though monetary policy alone is insufficient to contain the harm caused by large fluctuations in asset prices,\(^3\) it can have a substantial impact on asset prices. However, the literature is divided over whether or not central banks should respond to asset prices in an attempt to minimise the harm to the macroeconomy.

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\(^1\)Recent notable examples include Australia, Japan, Finland, Norway, the United Kingdom and the United States. Emerging economies have also experienced substantial fluctuations in asset prices, such as Mexico and several East Asian economies in the 1990s.

\(^2\)Some of the literature discusses how an asset price bubble might be defined. One such definition is that they are episodes where fundamental factors bring about an initial increase in asset prices which is then magnified by subsequent speculative activity and the easy availability of credit. See for example Simon (2003). For the purposes of this thesis, this definition is adequate.

\(^3\)Central banks can only affect asset prices by adjusting the nominal interest rate. Fluctuations in asset prices can be affected by a number of other factors. For example, fiscal policy, the legal system and regulation of the financial sector.
Most of the debate concerns whether asset prices belong in the Taylor-type rules adopted by a large number of central banks. Proponents of a policy rule that incorporates a measure of asset prices have argued it will improve macroeconomic performance and stability by reducing distortions in investment and consumption. Conversely, critics have argued that reacting to asset prices over and above what is necessary under flexible inflation targeting will result in few, if any, additional gains. For example, Bernanke and Gertler (1999) come to the opposite conclusion to Cecchetti et al. (2000) on the desirability of responding to asset prices, despite the fact that they adopt almost identical models (this will be discussed in Section 2).

Given the considerable disagreement in the literature on whether there are additional macroeconomic benefits from responding to asset prices, analysing the determinacy conditions is another vantage point from which to assess the viability of such monetary policy rules. Indeterminacy occurs when a monetary policy rule is associated with very large sets of rational expectations equilibria (Woodford 1999). By following a monetary policy rule which responds to endogenous variables such as a Taylor-type rule, a central bank can induce indeterminacy in an otherwise determinate economy. The associated volatility in inflation and output can lead to substantial reductions in welfare.

The rationale for responding to asset prices is to improve macroeconomic stability. If incorporating asset prices in a monetary policy rule leads to additional volatility in inflation and output, this contradicts the underlying objective of responding to asset prices.
In light of this, the contribution of this thesis is to analyse the conditions for determinacy to ascertain whether central banks should take into account asset prices when designing monetary policy.

Moreover, some empirical evidence indicates that monetary volatility may be positively related to asset price volatility.\(^4\) Eichengreen and Tong (2003), for example, suggest that monetary policy volatility in a number of countries during the 1970s and 1980s may partly explain why stock markets have been more volatile in recent decades. This suggests that central banks could be inducing volatility in asset prices by following a policy rule associated with indeterminacy. Such a possibility is another reason why it is important to analyse equilibrium determinacy for a model which incorporates asset prices.

The main finding of this thesis is that central banks should take asset prices into account when designing monetary policy. By responding to asset prices, a central bank can increase the likelihood of determinacy because of the substitutability that exists between responding to inflation and asset prices. This substitutability is attributable to the financial accelerator effect\(^5\) present in the Bernanke, Gertler and Gilchrist (1999) model.

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\(^4\)Eichengreen and Tong (2003) use volatility in equity prices as a proxy for general asset price volatility in 12 countries to examine the relationship between asset market volatility and possible determinants of such volatility. The most promising determinant they find is monetary volatility, which they find to be positively related to asset price volatility in almost every country studied.

\(^5\)The financial accelerator is explained in Section 3. Essentially it works as follows. When asset prices increase, this increases the value of entrepreneurs’ net worth. An increase in net worth then reduces the premium for external finance in the next period which implies an increase investment. An increase in investment then causes asset prices to increase again in the next period and the process repeats itself.
In addition, it is found that allowing for an asset price bubble where changes in asset prices can be driven by non-fundamentals has no qualitative or quantitative impact on equilibrium determinacy. This can be explained by the way asset price bubbles have been incorporated into most models in the literature (in particular Bernanke and Gertler 1999) whereby a bubble is assumed to follow an exogenous stochastic process.

The remainder of this thesis is structured as follows. Section 2 evaluates the recent related literature on equilibrium determinacy as well as some of the literature which conducts policy simulations. Section 3 presents a variant of the Bernanke, Gertler and Gilchrist (1999) model used to calculate the determinacy of equilibrium. Section 4 is devoted to the findings of this thesis and analyses the different determinacy regions under alternative monetary policy rules and models. Section 5 concludes.
2 Recent related literature

There is extensive literature devoted to the question of whether central banks should respond to asset prices. The majority of papers on the subject have compared the effects on real activity when a central bank adopts a more traditional monetary policy rule with those when a central bank responds to asset prices by simulating a macroeconomic model which incorporates asset price fluctuations. The general consensus (although there are some prominent dissenters) is that central banks should not respond to changes in asset prices unless their movements signal a change in expected inflation.

In the limited research on determinacy when asset prices are included in the policy rule, the conclusion has also been that central banks should not respond to asset prices. However, to the author’s knowledge there has been no analysis of equilibrium determinacy for the kinds of models used in the policy simulation literature where fluctuations in asset prices propagate and amplify shocks to the macroeconomy.

This section first considers literature which addresses whether central banks should respond to asset prices from a policy simulation perspective. This is followed by a discussion of the literature which addresses the question from a determinacy perspective.

2.1 Policy simulations

The majority of research which addresses the question of whether central banks should respond to asset prices compares the effects on real activity when a central bank adopts a more traditional monetary policy rule with those when a central bank responds to asset prices.
Two of the most prominent papers on the subject are Bernanke and Gertler (1999) and Cecchetti et al. (2000).

Bernanke and Gertler (1999) argue that monetary policy should only respond to changes in asset prices when this signals a change in expected inflation. To reach this conclusion they use a modified version of the financial accelerator model developed by Bernanke, Gertler and Gilchrist (1999) which allows for exogenous bubbles in asset prices. More specifically, they develop a process for an asset price bubble which captures the speculative swings of stock markets.\footnote{The details of the model are provided in Section 4.5.} The model is simulated for a number of different scenarios including different monetary policy rules and sources of stock price bubbles.

There are two key results in the paper which suggest that central banks should not respond to changes in asset prices. First, it is found that in the event of a stock price bubble, if an inflation-accommodating\footnote{Defined as when a central bank responds to a one percentage point increase in expected inflation by raising the nominal interest rate by 1.01 percentage points.} central bank responds to stock prices, the public’s expectation that the central bank will increase interest rates in response to the bubble pushes fundamental stock prices down and output and inflation also both fall.\footnote{In the case of an aggressive inflation targeting central bank (defined as when a central bank responds to a one percentage point increase in expected inflation by raising the nominal interest rate by 2.0 percentage points), the authors find that the outcome does not differ greatly from when such a central bank responds only to inflation.} There is also a large increase in inflation variability.

The second key result is that if a central bank responds to fluctuations in asset prices arising from a mixture of fundamental and non-fundamental sources, it prevents output from rising by the amount of the increase in potential output. Essentially, such a
policy response temporarily stifles the positive impact of the increase in asset prices caused by fundamental sources. This result illuminates one of the key points of contention in the literature, that is, whether central banks can determine the source of a shock and whether or not it is fundamental.

Bernanke and Gertler (1999) propose that by focusing on the traditional goals of monetary policy a central bank can respond effectively without having to decide what is fundamental and what is not. However, whether this is a compelling reason not to directly respond to asset price bubbles is contentious. It has often been pointed out that central banks are charged with the difficult task of forecasting and estimating many variables. Moreover, some central banks respond to the output gap even though it is difficult to estimate.

Cecchetti et al. (2000) are well known proponents of a monetary policy rule which incorporates a response to asset prices. Cecchetti et al. (2000) use an almost identical model to that adopted by Bernanke and Gertler (1999) to contend their results. By assuming policy-makers seek to minimise a loss function (a weighted sum of inflation and output variability), the authors conclude that in the majority of cases central banks should respond to stock price bubbles because the loss function can be reduced by doing so. In comparing their optimal monetary policy rules with the policy rules recommended in Bernanke and Gertler (1999), they find that, regardless of the relative weights of inflation and output variability in the loss function, their rules are superior in terms of minimising loss.
The question is how Bernanke and Gertler (1999) and Cecchetti et al. (2000) obtain such different results using very similar models. The main reason is that the basic assumptions of their models are different. Bernanke and Gertler (1999) remove the output gap from the monetary policy rule when conducting their simulations, which Cecchetti et al. (2000) argue merely demonstrates that reacting to stock prices instead of the output gap results in an inferior economic performance. Interestingly, Cecchetti et al. (2000) find that after adding the output gap, the perverse impact of the bubble when a central bank is inflation-accommodating is completely eliminated. Instead, output rises modestly and inflation falls only modestly.

However, as Bernanke and Gertler (2001) point out in a subsequent paper, Cecchetti et al. (2000) do not take into account the probabilistic nature of the bubble but rather consider the single scenario of a bubble lasting five periods. In contrast, Bernanke and Gertler (2001) optimise the policy rule by considering an entire probability distribution of shocks, including shocks other than those caused by a bubble. Therefore, Bernanke and Gertler (2001) argue that the method in Cecchetti et al. (2000) will only yield an optimal policy if a central bank knows that stock prices are being driven by non-fundamentals and knows when it will burst.

Most central banks have adopted a similar position to Bernanke and Gertler (1999; 2001). However, given the considerable disagreement in the literature on whether there are additional macroeconomic benefits from responding to asset price bubbles, analysing the determinacy conditions is another vantage point from which to assess the viability of such monetary policy rules.
2.2 Determinacy of equilibrium

The literature on equilibrium determinacy in dynamic general equilibrium models dates back to the 1980s. Indeterminacy occurs when a monetary policy rule is associated with very large sets of rational expectations equilibria (Woodford 1999). A monetary policy rule which induces indeterminacy is undesirable because if a central bank adopts this rule the ‘system might be unexpectedly volatile as agents are unable to coordinate on a particular equilibrium among the many that exist’ (Bullard and Mitra 2002, p.1106). By contrast, it is assumed that agents can coordinate on a determinate equilibrium.

There are two papers which analyse the determinacy conditions for when a central bank responds to asset prices. Both Bullard and Shaling (2002) and Carlstrom and Fuerst (2007) obtain results which suggest that central banks can inadvertently induce indeterminacy in the economy by responding to asset prices.

Bullard and Schaling (2002) use the simple and small macroeconomic system adopted by Bullard and Mitra (2002) to analytically solve for the determinacy conditions when a central bank responds to ‘the percentage deviation of the general level of equity prices from the long-run equilibrium level’ in a contemporaneous monetary policy rule (Bullard and Schaling 2002, p.38). They find that for a strong enough response to deviations in asset prices, a central bank could cause real damage to the economy.

The way equity prices are incorporated into the model is essential to the conclusion reached in Bullard and Schaling (2002). Using the inverse relationship between the price of a share of aggregate...
equity and the gross nominal interest rate on a one period bond, they are able to put the equity price term in the policy rule in terms of deviations in the gross nominal interest rate. The result is that the weights given to inflation and the output gap in the policy rule are also influenced by the weight given to the coefficient on equity price deviations. The more weight placed on the equity price coefficient, the lower the weights on the inflation and output gap coefficients. In fact, as more and more weight is placed on the equity price coefficient, the inflation and output gap coefficients are driven to zero while the model specification in Bullard and Mitra (2002) remains unaffected.

The contribution made by Bullard and Schaling (2002) is an analysis of equilibrium determinacy when a central bank responds to asset prices for a simple and small macroeconomic model. However, their model incorporates asset prices in an *ad hoc* fashion and does not include credit market frictions which induce the financial accelerator effect present in the model developed by Bernanke, Gertler and Gilchrist (1999). The financial accelerator allows endogenous developments in credit markets to ‘amplify and propagate shocks to the macroeconomy’ (Bernanke, Gertler and Gilchrist 1999, p.1342).

Therefore, the result in Bullard and Schaling (2002) does not capture the effects changes in asset prices can have on the macroeconomy and the associated effect this could have on equilibrium determinacy. This thesis builds on their analysis by examining the determinacy conditions for the more complex financial accelerator model in Bernanke, Gertler and Gilchrist (1999). The foundations of the model and the mechanics of the financial accelerator are
explained in detail in Section 3.

Carlstrom and Fuerst (2007) also conclude that a central bank should not respond to asset prices on the basis that it could induce indeterminacy. Their paper focuses on the profit mechanism as a particular source of indeterminacy. The nominal interest rate in their model responds to log deviations in the share price (and no other asset prices) which is important in determining their results. Their key finding is that in a model with sticky prices a central bank that responds to share prices indirectly responds to firm profits. This is because an increase in inflation tends to lower firm profits such that when the central bank responds to share prices its overall response to inflation is weaker. This has the effect of increasing the likelihood of indeterminacy. Therefore, the larger the negative effect of inflation on profits, the more likely there will be indeterminacy of equilibrium when a central bank responds to share prices.

However, as in Bullard and Schaling (2002), there is no financial accelerator mechanism present in the model. Moreover, the monetary policy rule studied by Carlstrom and Fuerst (2007) assumes the central bank only responds to inflation and log deviations of the share price (and thus omitting the output gap). The contribution of Carlstrom and Fuerst (2007) is to highlight one particular novel source of indeterminacy. In doing so, the authors do not account for some of the other effects changes in asset prices can have on the macroeconomy. Additionally, because they omit the output gap from the monetary policy rule, there is no analysis of the interaction between the output gap, inflation and asset price terms in the policy rule in relation to equilibrium determinacy.
Both Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) have examined the impact on equilibrium determinacy when a central bank responds to asset prices in a Taylor-type monetary policy rule. However, to date there has been no analysis of the implications for determinacy when a central bank responds to asset prices in a model which allows fluctuations in asset prices to propagate and amplify shocks to the macroeconomy. This thesis incorporates asset prices in a careful fashion as well as the financial accelerator in order to analyse the effect on equilibrium determinacy when a central bank responds to asset prices.
3 Model

This thesis employs the model developed by Bernanke, Gertler and Gilchrist (1999). It is a standard New Keynesian model with one distinguishing feature: it incorporates credit market frictions, giving rise to a financial accelerator effect. While the model is not the contribution of this thesis, it is presented in this section for completeness. The model presented here is modified in a way that makes notation and timing consistent with Bernanke and Gertler (1999). This makes the determinacy conditions for the two models comparable (see Section 4).

This section introduces the agents in the model and the associated optimality conditions. The linearised equations are then provided followed by the dynamic system used to calculate equilibrium determinacy.

3.1 A New Keynesian model with a financial accelerator

The distinguishing feature of the Bernanke, Gertler and Gilchrist (1999) model is the existence of credit market frictions, giving rise to a financial accelerator effect. Apart from this addition, the model follows a standard dynamic New Keynesian framework. New Keynesian models are widely used for understanding the relationship between monetary policy, inflation and the business cycle. They have two key features: imperfect competition in the goods market and some constraint on the price adjustment mechanism. The constraint on price adjustment is normally staggered price setting introduced by Calvo (1983).
By adding a financial accelerator to the standard New Keynesian model, credit market frictions can affect the real economy. There are numerous examples of the conditions in banking systems feeding back into the real economy. Countries or regions where distressed banking systems have contributed to substantial contractions in the macroeconomy include the United States, Scandinavia, Latin America, Japan and other East Asian countries (Bernanke, Gertler and Gilchrist 1999). The Global Financial Crisis of 2007-2009 is the most recent example. Deteriorating credit conditions were not merely the product of a declining real economy but were arguably the major factor depressing economic activity.

The model assumes only fundamental factors affect asset prices and fluctuations in asset prices are caused by the financial accelerator effect. This is consistent with literature where asset price bubbles are characterised as episodes where fundamental factors bring about an initial increase in asset prices which is then magnified by subsequent speculative activity and the easy availability of credit.\(^9\) One extreme example of fluctuations in asset prices is a bubble where changes in asset prices are driven by non-fundamentals.

There are three types of agents in the model: households, entrepreneurs and retailers. Households supply labour, consume goods and services, invest their savings (for which they receive a riskless rate of return) and hold money. Entrepreneurs play a key role in the model. They produce output using labour and capital and sell the output to retailers. Capital investment is financed by the entrepreneurs’ own funds or by borrowing. Entrepreneurs survive

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\(^9\)For example, see Simon (2003).
with probability $\tau$. The entrepreneurs that do not survive make a small transfer to new entrepreneurs (which is a technical assumption so that entrepreneurs start off with some net worth) and then consume the rest. Retailers purchase wholesale goods from entrepreneurs, differentiate them costlessly and then sell them in a monopolistically competitive retail market. Not all retailers can change their prices every period in accordance with staggered price setting from Calvo (1983). The model also includes a government which conducts fiscal and monetary policy.

### 3.1.1 Household sector

Households are infinitely lived and maximise the expected present discounted value of utility defined over consumption, real money holdings and leisure:

$$
E_t \sum_{k=0}^{\infty} \beta^k \left[ \ln(C_{t+k}) + \zeta \ln \left( \frac{M_{t+k}}{P_{t+k}} \right) + \xi \frac{(1-L_{t+k})^{1-(\chi-1)}}{1-(\chi-1)} \right] \tag{3.1}
$$

where $\zeta > 0$, $\xi > 0$ are elasticities of substitution, $0 < \beta < 1$ is the discount factor and $(\chi-1)$ is the elasticity of labour supply.

Households maximise utility subject to the budget constraint:

$$
C_t + D_t + \left( \frac{M_t}{P_t} \right) = W_t L_t - T_t + \Pi_t + R_{t-1} D_{t-1} + \left( \frac{M_{t-1}}{P_t} \right) \tag{3.2}
$$

Therefore, households choose consumption ($C_t$), deposits held at financial intermediaries ($D_t$), leisure ($1 - L_t$), and money balances ($M_t/P_t$) to maximise utility subject to their budget constraint (3.2). The consumer’s budget constraint requires that consumption, savings and money holdings equal after tax ($T_t$) income. Household
income comprises real wage earnings \( (W_t L_t) \), dividends from private sector ownership \( (\Pi_t) \), the return to savings deposits held at financial intermediaries \( (R_{t-1} D_{t-1}) \) and real money balances from the previous period \( (M_{t-1}/P_t) \). The solution to the household’s problem is given by the following optimality conditions:

\[
\frac{1}{C_t} = E_t \left\{ \frac{1}{C_{t+1}} \right\} R_t
\]

\[
W_t \frac{1}{C_t} = \xi \left( \frac{1}{1 - L_t} \right)^{-1}
\]

\[
M_t \frac{P_t}{P_{t+1}} = \zeta C_t \left( \frac{R^n_t - 1}{R^n_t} \right)^{-1}
\]

Equation 3.3. is the Euler equation for optimal intertemporal allocation of consumption. According to equation 3.4 the marginal rate of substitution between consumption and leisure must equal the real wage. Equation 3.5 is the intratemporal condition requiring that the substitution between money and consumption equal the opportunity cost of holding money where \( R^n_t = R_t \frac{P_{t+1}}{P_t} \) is the gross nominal interest rate.

3.1.2 Entrepreneurial sector

Entrepreneurs produce wholesale goods using physical capital and labour. They maximise profits:

\[
E_t \sum_{k=0}^{\infty} \Lambda_{t,k} (Y_{t+k} - R^k_{t+k} K_{t+k} - W_{t+k} L_{t+k})
\]

given by the aggregate output \( (Y_{t+k}) \) minus the cost of capital \( (R^k_{t+k} K_{t+k}) \) and labour \( (W_{t+k} L_{t+k}) \).
The discount factor $\Lambda_{t,k}$ is $\beta^k C_t / C_{t+k}$.

Entrepreneurs maximise profits subject to the following technology:

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha}$$

(3.7)

where $0 < \alpha < 1$. $Y_t$ is the aggregate output of wholesale goods, $Z_t$ is an exogenous technology parameter, $K_t$ is aggregate capital purchased by entrepreneurs in the previous period $(t-1)$ and $L_t$ is aggregate labour.

The solution to the entrepreneur’s problem yields the following optimality conditions:

$$R_t^k = \frac{Y_t}{K_t} MC_t$$

(3.8)

$$W_t = (1 - \alpha) \frac{Y_t}{L_t} MC_t$$

(3.9)

where the real marginal cost of production ($MC_t$) is the Lagrangian multiplier for the period $t$. Equations 3.8 and 3.9 equate the marginal product of an input to its rental rate.

Entrepreneurs’ purchases of physical capital are financed by entrepreneurial wealth\(^{10}\) (hereinafter referred to as net worth) and borrowing. Entrepreneurial borrowing plays the key role here because of the financial accelerator effect.

The evolution of capital stock is given by:

$$K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta) K_t$$

(3.10)

where $0 < \delta < 1$ is the depreciation rate and the function $\Phi(I_t/K_t)K_t$.

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\(^{10}\)Wealth equals liquid assets plus collateral value of illiquid assets less outstanding obligations.
captures the increasing marginal adjustment costs in the production of capital such that \( \Phi'(\cdot) > 0 \), \( \Phi''(\cdot) < 0 \) and \( \Phi(0) = 0 \). This is required for capital to have a variable price. The expected price of a unit of capital \((E_t Q_{t+1})\) is then given by:

\[
E_t Q_{t+1} = \left[ \Phi' \left( \frac{E_t I_{t+1}}{K_{t+1}} \right) \right]^{-1} \tag{3.11}
\]

where capital producers have to make their investment decisions one period in advance.

Using equation 3.8, the interest rate on external funds, \( R^q_t \) is given by:

\[
R^q_t = \frac{R^k_t + Q_t(1 - \delta)}{Q_{t-1}} \tag{3.12}
\]

and depends on the rent and price of a unit of capital.

Bernanke, Gertler and Gilchrist (1999) assume that there is a costly state verification (CSV) problem. This is a source of asymmetric information between borrowers and lenders which then results in a premium for external finance.\(^{11}\) CSV makes it necessary for lenders to pay a fixed “auditing cost” which is essentially a cost paid to observe an individual borrower’s realised return (which would otherwise be unknown to the lender while the borrower can observe this at no cost).

Because the return to capital is sensitive to both aggregate and idiosyncratic risk, the ex post gross return on capital for a particular firm is also subject to an idiosyncratic shock. For a certain

\(^{11}\)The difference between the cost of funds raised externally and the opportunity cost of funds internal to the firm.
threshold value of idiosyncratic shock, an entrepreneur will be unable to repay the loan at the agreed contractual rate and thus defaults. The lending intermediary then has to pay the auditing cost and retains anything else that remains.

Entrepreneurs are risk-neutral here and care only about the mean return on wealth. Therefore, they bear all the aggregate risk which means guaranteeing the lender a return equal to the value of the riskless rate. Lenders then bear the residual risk arising from idiosyncratic shocks (which is diversifiable). When there is a rise in the expected discounted return to capital (and as a result the threshold value of idiosyncratic shock falls), entrepreneurs benefit from a lower probability of default, and as a result can take on more debt.

Therefore, this yields the following supply curve for investment finance:

\[ E_t R_{t+1} = s \left( \frac{N_t}{Q_t K_{t+1}} \right) R_t \] (3.13)

where the function \( s(\cdot) \) determines the cost of external finance \( (E_t R_{t+1}) \) relative to the cost of internal finance \( (R_t) \). \( N_t \) is the entrepreneur’s net worth and \( Q_t K_{t+1} \) is total borrowings. Therefore, the ratio \( N_t / Q_t K_{t+1} \) represents the financial condition of entrepreneurs. The cost of external finance relative to the cost of internal finance is decreasing in the financial condition of entrepreneurs, \( s'(\cdot) < 0 \) and \( s(1) = 1 \). Equation 3.13 is the key equation for the financial accelerator in the model.

Finally, the model assumes that entrepreneurs survive with probability \( \tau \). New entrepreneurs \( (1 - \tau) \) enter to replace exiting entrepreneurs. Exiting entrepreneurs make a small transfer to the
new entrepreneurs and then consume what remains.\footnote{The transfer to new entrepreneurs ensures that they start of with some net worth so they can begin operations. This assumption is inferred from the Bernanke and Gertler (1999) model; whereas Bernanke, Gertler and Gilchrist (1999) assume that entrepreneurs supplement their income by working in the general labour market. Neither of these assumptions have significant effects on the results.}

3.1.3 Retail sector

Retailers purchase wholesale goods from entrepreneurs at their nominal marginal cost and then differentiate them costlessly. These differentiated goods are then sold by retailers in a monopolistically competitive retail market. In accordance with staggered price setting from Calvo (1983), not all retailers are able to adjust their prices in the current period. The probability that a retailer changes their prices in period $t$ is given by $1 - \theta$. Therefore, retailer $z$ chooses his or her price, $p_t(z)$, to maximise expected discounted profits:

$$
E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} \left[ \frac{p_t(z)}{P_{t+k}} Y_{t+k}(z) - MC_{t+k}Y_{t+k}(z) \right] 
$$

(3.14)

where the discount factor $\Lambda_{t,k}$ is $\beta^kC_t/C_{t+k}$. $Y_{t+k}(z)$ is the demand for retail goods at this price. $\theta^k$ is the probability that the nominal price is fixed in period $k$ and $P_{t+k}$ is the aggregate price.

Retailers maximise expected discounted profits subject to:

$$
Y_t(z) = \left[ \frac{p_t(z)}{P_t} \right]^{-\epsilon} Y_t
$$

(3.15)

The first order condition for retailers is then given by:

$$
\frac{p_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \beta^k Y_{t+k} \beta^k Y_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^\epsilon}{E_t \sum_{k=0}^{\infty} \theta^k \beta^k Y_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\epsilon-1}}
$$

(3.16)
and \( p_t^* \) is the optimal price set by all retailers able to adjust prices at time \( t \).

The aggregate price in period \( t \) is given by:

\[
P_t = [\theta P_{t-1}^{1-\epsilon} + (1 - \theta) p_t^{1-\epsilon}]^{\frac{1}{1-\epsilon}}
\]

(3.17)

By approximating (3.16) and (3.17) around a zero average inflation, steady state equilibrium, we can obtain the New Keynesian Phillips Curve which provides the source of nominal price stickiness in the model:

\[
\pi_t = \kappa m c_t + \beta E_t \pi_{t+1}
\]

(3.18)

where \( \kappa = \frac{(1-\theta)(1-\theta)}{(1-\theta)} \).

### 3.1.4 Government sector

The government conducts fiscal and monetary policy. The government finances expenditure by lump sum taxes and money creation:

\[
G_t = \frac{M_t - M_{t-1}}{P_t} + T_t
\]

(3.19)

\( G_t \) follows a first order autoregressive process such that:

\[
\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g
\]

(3.20)

where \( 0 \leq \rho_g < 1 \) and \( \varepsilon_t^g \sim N(0, \sigma_g^2) \) is a serially uncorrelated random shock.
The central bank follows the monetary policy rule:

$$\log \left( \frac{R_n^t}{R_n^n} \right) = \phi_\pi \log \left( \frac{\pi_t}{\pi} \right) + \phi_y \log \left( \frac{Y_t}{Y} \right) + \phi_q \log \left( \frac{Q_t}{Q} \right)$$  \hspace{1cm} (3.21)

where the nominal interest rate responds to the log levels of inflation ($\pi_t$), output ($Y_t$) and asset prices ($Q_t$) relative to their steady state values (represented by variables with no time scripts). The coefficients on inflation, output and asset price deviations are given by $\phi_\pi \geq 0, \phi_y \geq 0$ and $\phi_q \geq 0$ respectively.

The monetary policy rule has been modified to allow the central bank to respond to asset prices. The coefficient on asset prices $\phi_q$ determines by how many percentage points a central bank responds to a one per cent increase in asset prices (relative to the steady state value).

3.2 Linearised equations

Following Bernanke, Gertler and Gilchrist (1999), the model can be linearised around the steady state. Let lower case variables denote percentage deviations from the steady state and variables without time scripts denote steady state values, the linearised equations of the model are:
Aggregate demand

\[ y_t = \frac{C}{Y} c_t + \frac{C^e}{Y} c^e_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t \]  
\[ (3.22) \]
\[ c_t = -\sigma r_t + E_t c_{t+1} \]  
\[ (3.23) \]
\[ c^e_t = q_t + k_{t+1} \]  
\[ (3.24) \]
\[ r^q_t = (1 - \vartheta)(mc_t + y_t - k_t) + \vartheta q_t - q_{t-1} \]  
\[ (3.25) \]
\[ E_t r^q_{t+1} = r_t - \psi(n_t - q_t - k_{t+1}) \]  
\[ (3.26) \]
\[ E_t q_{t+1} = \varphi(E_t i_{t+1} - k_{t+1}) \]  
\[ (3.27) \]

Aggregate supply

\[ y_t = z_t + \alpha k_t + (1 - \alpha) l_t \]  
\[ (3.28) \]
\[ y_t - l_t + mc_t - c_t = (\chi - 1) l_t \]  
\[ (3.29) \]
\[ \pi_t = \kappa mc_t + \beta E_t \pi_{t+1} \]  
\[ (3.30) \]

Evolution of state variables

\[ k_{t+1} = \delta i_t + (1 - \delta) k_t \]  
\[ (3.31) \]
\[ n_t = R^q |K_N (r^q_t - E_{t-1} r^q_t) + \frac{(1 - \tau R^k)}{\tau} y_t + n_{t-1} | \]  
\[ (3.32) \]

Shock processes

\[ g_t = \rho_g g_{t-1} + \varepsilon^g_t \]  
\[ (3.33) \]
\[ z_t = \rho_z z_{t-1} + \varepsilon^z_t \]  
\[ (3.34) \]

Monetary policy rule

\[ r^n_t = \phi_{\pi} \pi_t + \phi_y y_t + \phi_q q_t \]  
\[ (3.35) \]
The intertemporal ‘IS’ equation (3.23), New Keynesian Phillips Curve (3.30) and contemporaneous monetary policy rule (3.35) are all standard equations used in the literature on determinacy. The monetary policy rule has been modified to allow the nominal interest rate to move in response to changes in asset prices.

The additional linearised equations are from the Bernanke, Gertler and Gilchrist (1999) model and have been included for the purposes of analysing determinacy when there are credit market frictions.

In the aggregate demand block, equation 3.22 is the economy-wide resource constraint which now also includes entrepreneurial consumption, $c_e$. Entrepreneurial consumption changes in proportion to changes in asset prices and capital (3.24). Equation 3.25 is the fundamental return on assets which is given by the sum of the current return to assets $[(1 - \vartheta)(mc_t + y_t - k_t)]$ and any increase in the fundamental value of assets $[\vartheta q_t - q_{t-1}]$. This is obtained by log linearising equation 3.12 where $\vartheta = (1 - \delta)/(\alpha MC^Y_K + 1 - \delta)$.

Equation 3.26 embeds the financial accelerator from the supply curve for investment finance given by equation 3.13. It captures the influence of net worth on investment. Due to capital market frictions, the cost of external funds depends on entrepreneurs’ net worth relative to the gross value of capital $[n_t - (q_t + k_{t+1})]$. The cost of external funds decreases as this ratio increases, implying that investment will rise as a result. The parameter $\psi$ is the elasticity of the external finance premium to leverage. Equation 3.27 then captures the effect of expected investment relative to capital on the expected value of asset prices. This is obtained by log-linearising equation 3.11 where $\varphi = [\Phi(I/K)^{-1}]'/[\Phi(I/K)^{-1}]''$. 

27
In the aggregate supply block, equation 3.28 is the linearised production function from equation 3.7. Equation 3.29 gives the labour market equilibrium condition obtained by setting labour supply equal to labour demand from equations 3.4 and 3.9 respectively. The standard New Keynesian Phillips Curve derived from staggered price setting in retail sector (as discussed in section 3.1.3) is given as equation 3.30.

The evolution of capital and net worth are given by equations 3.31 and 3.32 respectively. The evolution of capital comes from equation 3.10. Entrepreneurial net worth depends mostly on the net return on assets (the first term) and the lagged value of net worth (the last term). The remaining term represents the transfer from exiting entrepreneurs where \( \tau \) is the probability of survival.

Finally, equations 3.33 and 3.34 are the exogenous disturbances to government consumption and technology which follow stationary autoregressive processes.

The way the financial accelerator operates is clear from the linearised equations. When asset prices increase, this increases the value of entrepreneurs’ net worth (equation 3.32). An increase in net worth then reduces the premium for external finance in the next period (equation 3.26) which implies an increase investment. An increase in investment then causes asset prices to increase again in the next period (equation 3.27) and the process repeats itself.
3.3 The dynamic system

Following Evans and Honkapohja (2001), Honkapohja and Mitra (2004) and Bullard and Singh (2007), the dynamic system can be written as:

\begin{align*}
B_{11}P_t &= B_{12}E_{t}P_{t+1} + B_{13}X_t \quad (3.36) \\
X_t &= RP_{t-1} + SX_{t-1} + U_t \quad (3.37)
\end{align*}

Where \( P_t = [c_t, i_t, \pi_t, r^q_t, q_t] \) is a vector of free variables, \( X_t = [k_t, n_{t-1}, g_t, z_t, q_{t-1}, E_{t-1}r^q_t] \) is a vector of predetermined variables, \( U_t = [0, 0, \varepsilon^g_t, \varepsilon^z_t, 0, 0] \) is a vector of fundamental disturbances and \( R \) and \( S \) are conformable matrices.

The matrices can then be redefined such that:

\begin{align*}
P_t &= B_1E_tP_{t+1} + CX_t \quad (3.38) \\
X_t &= RP_{t-1} + SX_{t-1} + U_t \quad (3.39)
\end{align*}

where \( B_1 = (B_{11})^{-1}B_{12} \) and \( C = (B_{11})^{-1}B_{13} \).

Letting \( \eta_{t+1} = P_{t+1} - E_tP_{t+1} \), the system can then be rewritten as:

\begin{align*}
P_t &= B_1P_{t+1} + CX_t - B_1\eta_{t+1} \quad (3.40) \\
X_{t+1} &= RP_t + SX_t + U_{t+1} \quad (3.41)
\end{align*}

The dynamic system is then written as a vector autoregressive process:

\begin{equation}
D_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = D_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} + E \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \quad (3.42)
\end{equation}
where

\[
D_1 = \begin{bmatrix}
I & -C \\
R & S
\end{bmatrix}
\quad \text{(3.43)}
\]

\[
D_2 = \begin{bmatrix}
B_1 & 0 \\
0 & I
\end{bmatrix}
\quad \text{(3.44)}
\]

\[
E = \begin{bmatrix}
0 & -B_1 \\
-I & 0
\end{bmatrix}
\quad \text{(3.45)}
\]

The Schur Decomposition is then used to find the eigenvalues for the system. The eigenvalues are obtained from the decomposition of \(D_1\) and \(D_2\). Further details are provided in Appendix A. Following Blanchard and Kahn (1980), for determinacy there must be at least as many eigenvalues inside the unit circle as there are free variables. If the number of eigenvalues inside the unit circle is less than the number of free variables in the system, the equilibrium will be indeterminate.

The determinacy regions for the monetary policy coefficients on inflation deviations (\(\phi_\pi\)), the output gap (\(\phi_y\)) and asset price deviations (\(\phi_q\)) are obtained using numerical calculations since it is difficult to obtain an analytical solution for such a large system.

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13 Because \(D_1\) is singular the Jordan Decomposition method from Blanchard and Kahn (1980) cannot be used to obtain the eigenvalues for this particular system.

14 Note that in this system it is the number of eigenvalues inside the unit circle because the system is ordered differently from Blanchard and Kahn (1980).

15 Code available on request.
4 Determinacy of equilibrium

This section first computes the determinacy region for a New Keynesian model with a financial accelerator where the central bank does not respond to asset prices. It then studies how equilibrium determinacy is impacted when the central bank responds to asset prices.

4.1 Calibrated values

Calibrated values for simulating the determinacy region have been selected from Bernanke, Gertler and Gilchrist (1999) and Bernanke and Gertler (1999). They are: \( \sigma = 1.0, \beta = 0.99, \delta = 0.025, \alpha = 0.33, \kappa = 0.086, \chi = 1.33, \tau = 0.95, \varphi = 0.25, \rho_g = 0.95, \rho_z = 1.0 \) and \( \vartheta = 0.32 \).\(^\text{16}\)

These are all fairly standard values in the literature. While the non-standard parameter values reflect the interplay between real and financial factors within the entrepreneurial sector (Bernanke, Gertler and Gilchrist 1999, p.1368). The elasticity of the external finance premium to leverage is given by \( \psi = 0.05 \) and the steady state risk spread is assumed to be given by \( R^s = 2 + R^n \), implying the risk spread is equal to approximately two percentage points. It will become apparent in the results that follow that the risk spread and the value of \( \psi \) in particular are pivotal for determinacy because they affect the external finance premium and the magnitude of the financial accelerator effect respectively. The steady state values are: \( \frac{G}{V} = 0.2, \frac{N}{K} = 0.5, \frac{C^e}{V} = 0.04. \)

\(^{16}\text{Bernanke, Gertler and Gilchrist (1999) and Bernanke and Gertler (1999) do not specify a value for } \vartheta. \text{ However, } \vartheta = \frac{(\lambda - 1)}{\alpha \epsilon [\frac{(1 - \delta)}{\epsilon} + 1 - \delta]}, \text{ where } \alpha = \frac{\epsilon}{\lambda - 1}. \text{ Using } \epsilon = 6 \text{ (to yield a fairly standard value for steady state marginal cost of approximately 0.83) we can obtain } \vartheta = 0.32.\)
4.2 A New Keynesian model with a financial accelerator

This thesis first explores how introducing a financial accelerator in a standard New Keynesian model changes the determinacy region. Here the central bank does not react to asset prices.

A model with a financial accelerator is compared with a model without a financial accelerator. The determinacy region for the contemporaneous monetary policy rule in Bullard and Mitra (2002) is used as the benchmark with no financial accelerator. The relevant parameter values from Bernanke, Gertler and Gilchrist (1999) have been substituted for those in Bullard and Mitra (2002)\textsuperscript{17} for the purpose of comparison.

![Figure 1: Determinacy region adapted from Bullard and Mitra (2002)](image1)

![Figure 2: Determinacy region with the financial accelerator and $\phi_q = 0$](image2)

Figure 1 is the determinacy region from Bullard and Mitra (2002) for a simple and small forward-looking macroeconomic model. The model comprises an intertemporal ‘IS’ equation, the New Keynesian Phillips Curve and a monetary policy rule which responds

\textsuperscript{17}In Bullard and Mitra (2002) $\sigma = 0.157$ and $\kappa = 0.024$. These have been changed to $\sigma = 1$ and $\kappa = 0.086$ to stay consistent with Bernanke and Gertler (1999).
to inflation deviations and the output gap. There is no capital, entrepreneurial consumption or financial accelerator present in the model.

The shaded regions give the policy rules associated with determinacy of equilibrium. From Figure 1 if the nominal interest rate responds to inflation deviations more than one-for-one such that \( \phi_\pi > 1 \), this will be sufficient for determinacy. This is consistent with the Taylor principle.\(^{18}\) If \( \phi_\pi < 1 \), there needs to be a stronger response to the output gap to avoid indeterminacy.

If the equilibrium is indeterminate (any point in the non-shaded region), the economy could be unexpectedly volatile if agents are unable to coordinate on one particular equilibrium given the existence of a large set of equally possible equilibria. Moreover, this large set could include equilibria in which endogenous variables respond to random events unrelated to economic fundamentals (Woodford 2003).

Consider a simple case when \( \phi_\pi < 1 \) and \( \phi_y = 0 \). Suppose there is an increase in expected inflation: that is, for some reason consumers expect that prices are going to increase. All other things being equal, this higher expected inflation will lead to a lower real interest rate because the nominal interest rate has not increased enough. This lower real interest rate will stimulate aggregate demand and fuel inflation. Therefore, such an expectation generates higher inflation and gives way to a self-fulfilling inflation spiral (Woodford 2001). However, if the central bank responds aggressively enough to inflation deviations, the resulting increase in the real interest rate will

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\(^{18}\)A broad interpretation of the Taylor principle is that a good monetary policy rule is one where the nominal interest rate responds to inflation more than one-for-one.
curb inflation and effectively prevent the occurrence of self-fulfilling expectations.

At the bounds for which the system becomes indeterminate, to achieve determinacy the central bank needs to compensate for a reduction in one of the monetary policy coefficients with a sufficient increase in the other coefficient. This can be described as a type of trade-off between responding to inflation deviations and the output gap once the monetary policy rule approaches the boundary of the determinacy region. For example, the trade-off exists such that a central bank needs to substitute a weaker response to inflation deviations with a stronger response to the output gap in order to induce determinacy. Therefore, even if $\phi_n < 1$, a central bank can still induce determinacy if it responds strongly enough to the output gap. The trade-off is not one-for-one due to the long term relationship between inflation deviations and the output gap implied by the New Keynesian Phillips Curve. The determinacy region will also be affected by the degree of price stickiness.

Figure 2 is the determinacy region when there is a financial accelerator present and no response to asset prices such that $q = 0$.

\footnote{According to the New Keynesian Phillips Curve, each percentage point of permanently higher inflation implies a permanently higher output gap of more than one per cent. Therefore, if one considers the interest rate rule purely in terms of a change in inflation, the coefficient on the output gap will need to be larger to have the same effect on the nominal interest rate as responding to inflation.}

\footnote{The value of $\kappa$ from the Phillips Curve, which is affected by the degree of price stickiness, has a significant impact on the determinacy region. A higher value of $\kappa$ increases the likelihood of indeterminacy. $\kappa$ is strictly decreasing in the index of price stickiness $\theta$ because $\kappa = \frac{(1-\theta)^2}{\theta(1-\theta)}$. Therefore, as price stickiness decreases, $\kappa$ will increase. Intuitively, if there is no price stickiness, there will be no frictions in the economy and the result will be similar to that obtained from a classical monetary model. That is, monetary policy will not affect output and therefore when $\phi_n < 1$ a central bank cannot induce determinacy by responding strongly to the output gap. However, for the purposes of examining the impact of incorporating asset price deviations in the monetary policy rule on determinacy and assuming there exists some degree of price stickiness, the qualitative implications of the results remain the same regardless of the values chosen for $\kappa$ and $\sigma$.}
The region consistent with determinacy is very similar to the one obtained using the system from Bullard and Mitra (2002). The small difference in the slopes can, in the most part, be attributed to the inclusion of entrepreneurial consumption, the financial accelerator and capital.

However, incorporating a financial accelerator effect actually increases the likelihood of indeterminacy (equivalent to an increase in the slope) which cannot be observed in Figure 2 because of the other effects working in the opposite direction. This is supported by robustness tests which show that the slope increases as the value of $\psi$ is increased from equation 3.26 (which acts to increase the financial accelerator effect because the external finance premium is more sensitive to entrepreneurs’ net worth relative to the gross value of capital). This is also consistent with the results in Hirose (2008).\footnote{Hirose (2008) uses the Bernanke, Gertler and Gilchrist (1999) model to investigate the sources of asset price fluctuations in Japan. In doing so he obtains the parameter space consistent with determinacy for $\psi$ and the monetary policy coefficient on inflation. When the external finance premium is highly elastic with respect to leverage, the economy always exhibits indeterminacy regardless of how aggressively monetary policy responds to inflation.}

Therefore, entrepreneurial consumption and capital are responsible for the decrease in the slope. If the role of capital in production is reduced\footnote{It has been observed in the literature that incorporating capital has a small effect on determinacy for contemporaneous monetary policy rules (Carlstrom and Fuerst 2005; Duffy and Xiao 2008).} and entrepreneurial consumption is eliminated,\footnote{This can be done by removing the entrepreneurial consumption from the resource constraint, equation 3.24.} the slope becomes closer to the result in Bullard and Mitra (2002). If capital’s contribution to output is reduced via the elasticity parameter $\alpha$, the slope increases. The same is true for a reduction in entrepreneurial consumption.
4.3 Asset prices in the monetary policy rule

This section examines whether a central bank can affect equilibrium determinacy by reacting to asset prices. Figure 3 is the determinacy region consistent with a monetary policy rule where the central bank reacts to deviations in inflation and asset prices from their steady state values ($\phi_y = 0$).

Figure 3: Determinacy region for a response to asset prices when $\phi_y = 0$

Figure 3 shows that a central bank can respond to deviations in asset prices without inducing additional volatility. In other words, the findings of this thesis suggest that following a monetary policy rule which incorporates asset price deviations can do no harm. Moreover, the trade-off at the boundary of the determinacy region can be exploited to the extent that values of $\phi_q < 1$ can still lead to determinacy when there is a response to asset prices.

Similar to the trade-off between responding to the output gap and inflation deviations, the trade-off between responding to asset price and inflation deviations is not one-for-one because of the relationship between inflation and asset price deviations implied by
the New Keynesian Phillips Curve. This substitutability between responding to inflation and asset price deviations is consistent with Bernanke and Gertler’s (1999) view that a strong commitment to stabilising inflation renders a response to asset prices unnecessary.

The question of importance is then what is driving this substitutability in the model. Suppose there is an increase in fundamental asset prices. This improves entrepreneurs’ balance sheets, which reduces the cost of external funds and thereby increases investment and stimulates aggregate demand, leading to further increases in asset prices. This increase in aggregate demand also drives up the price level, fuelling inflation at the same time. Therefore, in a way, responding directly to asset prices is similar to responding more aggressively to inflation.

Figure 4 shows how the determinacy region changes as the weight on the output gap coefficient $\phi_y$ is increased.

---

24 It is difficult to show this algebraically without an analytical condition for determinacy. Using the expression for marginal cost from the labour equilibrium equation, the New Keynesian Phillips Curve can be expressed as: $\pi_t = \kappa \left[ \frac{1}{1-\phi} (yt - zt - \alpha kt) + ct - yt + \beta E_t \pi_{t+1} \right]$. Where $yt = Cc_t + Cc^e_t + Ii_t + Gg_t$. Asset price deviations affect entrepreneurial consumption, investment and capital which in turn affect inflation via the above expression for the New Keynesian Phillips Curve.
The substitutability between $\phi_\pi$ and $\phi_q$ becomes increasing difficult to exploit (see bottom panels c and d in Figure 4). The intuition is that as the central bank increases the weight on the output gap, when $\phi_\pi < 1$ the additional gain from responding to asset price deviations declines because both responding to the output gap and asset price deviations are in effect different ways to respond to inflationary pressures in this model.
When the central bank is increasing the weight on the output gap coefficient, it cannot further exploit the substitutability between responding to inflation and asset price deviations by increasing the weight on $\phi_q$.

4.4 Implications for the conduct of monetary policy

Given the substitutability between responding to deviations in asset prices and inflation, the underlying question of importance is whether this can be exploited. Figure 5 shows how the determinacy region changes as the weight on the asset price coefficient, $\phi_q$ is increased.
Figure 5: The determinacy region as the weight on the asset price coefficient $\phi_q$ is increased
Figure 5 shows that indeterminacy becomes less likely when the central bank reacts to asset price deviations. As the value of $\phi_q$ increases, the determinacy region becomes more determinate, and the Taylor principle when $\phi_y = 0$ no longer holds. The cases where $\phi_q = 20$ and $\phi_y = 40$ (panels e and f) have been included purely for illustrative purposes as such weights are clearly unrealistic.

However, Figure 5 shows that the substitutability between responding to deviations in asset prices and inflation can only be exploited when the coefficient on the output gap is small. This is consistent with the reasoning that increasing the weight of the output gap coefficient diminishes the gain from responding to deviations in asset prices from a determinacy perspective, as seen in Figure 4.

The key implication of the preceding results for the conduct of monetary policy is that if a central bank elects to respond to asset prices it can do so without inducing additional volatility. This contrasts the results in Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) where the opposite is found. The difference in the results can be attributed to the financial accelerator in the model used here which drives the substitutability between responding to asset prices and inflation.

In Carlstrom and Fuerst (2007), an increase in inflation is associated with a decline in firm profits and, as a result, share prices. Therefore, inflation and share prices move in opposite directions which means the overall response to inflation is weaker when a central bank responds to share prices. Similarly, in Bullard and Schaling (2002), the central bank’s responses to deviations in inflation and equity prices move in opposite directions because any
increase in the equity price coefficient is associated with a decrease in the coefficients on inflation and the output gap and thus increases the likelihood of indeterminacy.

The conclusion in this thesis that a central bank will not necessarily induce volatility in the economy by adopting a policy rule which responds to deviations in asset prices also contrasts the view of Bernanke and Gertler (1999). The conclusion is more in line with that of Cecchetti et al. (2000). However, the results in this thesis are a different vantage point from which to answer the question of whether central banks should respond to asset prices. From the perspective of equilibrium determinacy for a New Keynesian model with a financial accelerator, not only do such monetary policy rules cause no harm, they can also make indeterminacy less likely.

4.5 A model with non-fundamental asset price fluctuations

The question of whether central banks should respond to asset prices has been, in part, motivated by the adverse macroeconomic effects of asset price bubbles. As previously noted, the Bernanke, Gertler and Gilchrist (1999) model allows only fundamental factors to affect asset prices. Bernanke and Gertler (1999) extend this model to allow non-fundamental factors to affect asset prices by allowing the market value of an asset \( s_t \) to differ from the asset’s fundamental value \( q_t \).

Consequently, the market value of an asset now determines entrepreneurial consumption and the balance sheets of firms such that:

\[
\begin{align*}
    c_t^e &= s_t + k_{t+1} \\
    E_t r_{t+1}^* &= r_t - \psi(n_t - s_t - k_{t+1})
\end{align*}
\]
Equations 4.1 and 4.2 replace equations 3.24 and 3.26 respectively. The returns to assets and capital are now determined by the following equations:

\[ s_t - q_t = \frac{(1 - \delta)}{bR^\theta} E_t(s_{t+1} - q_{t+1}) \]  \hspace{1cm} (4.3)
\[ r_t^q = (1 - \theta)(mc_t + y_t - k_t) + \vartheta q_t - q_{t-1} \]  \hspace{1cm} (4.4)
\[ r_t^s = (1 - \vartheta)(mc_t + y_t - k_t) + \vartheta s_t - s_{t-1} \]  \hspace{1cm} (4.5)
\[ E_t r_{t+1}^s = E_t r_{t+1}^q - (1 - b)(s_t - q_t) \]  \hspace{1cm} (4.6)
\[ r_t^n = \phi_\pi \pi_t + \phi_y y_t + \phi_s s_t \]  \hspace{1cm} (4.7)

where \( b = a(1 - \delta) \) and \( a \) governs the rate at which the bubble converges to zero over time. In accordance with Bernanke and Gertler (1999), \( a = 0.98 \) which means that the bubble is arbitrarily close to a ‘rational bubble’ as described in Blanchard and Watson (1982).\(^{25}\) Equation 4.3 gives the expected evolution of the bubble. The fundamental return to assets (4.4) remains the same as in equation 3.25. Equation 4.5 captures the non-fundamental return on assets (given by the sum of the current market return to assets and any increase in the market value of assets). According to equation 4.6, the expected market return and the fundamental return depend on the presence of the bubble. Finally, from equation 4.7 the nominal interest rate responds to the market value of an asset rather than the fundamental value. The details of the dynamic system are provided in Appendix B.

This thesis finds that incorporating an asset price bubble of the specifications in Bernanke and Gertler (1999), has no impact on

\(^{25}\)In broad terms, a rational bubble is a bubble that is consistent with the rational behaviour of agents.
equilibrium determinacy. This is consistent with Hirose (2008) who argues that the non-fundamental asset price fluctuations studied in the literature such as in Bernanke and Gertler (1999) incorporate the bubble only as an exogenous shock, and this does not represent agents’ beliefs in a formal and explicit way. Moreover, the financial accelerator mechanism remains the same regardless of whether there is a fundamental or non-fundamental change in asset prices. For these reasons, the conditions for determinacy of equilibrium are unaffected by adding in an asset price bubble of this type. The relevant figures have been provided in Appendix C.
5 Conclusion

The main finding of this thesis is that responding to asset prices impacts the determinacy conditions of a New Keynesian model with a financial accelerator. The results suggest that, by following a monetary policy rule which incorporates deviations in asset prices, a central bank will not induce additional volatility. In fact, responding to deviations in asset prices makes indeterminacy less likely. Central banks that are strict inflation targeters (for example, the Reserve Bank of Australia) and central banks that appear to target both inflation and the output gap (for example, the United States Federal Reserve) can potentially lower macroeconomic instability by reacting to asset prices.

It should be noted that the specifications of the model could potentially have some impact on the conditions for equilibrium determinacy. For this reason further research needs to be done on the determinacy conditions when a central bank responds to deviations in asset prices in other types of monetary policy rules. As pointed out by McCallum (1999), policy rules which react only to information observed in the current period are unrealistic given the information set available to policy-makers when they set the nominal interest rate. Consequently, further study on the determinacy conditions for policy rules which respond to current expectations, lagged values and future forecasts of the output gap and inflation and asset price deviations could be beneficial.

While the results in this thesis also indicate that it makes no difference whether asset prices change due to fundamental or non-fundamental factors from a determinacy condition perspective, this
is due to the way in which non-fundamental asset price bubbles have been incorporated into general equilibrium models. This result would not be expected to hold once the non-fundamental component of asset prices is endogenised.
References


A Method of decomposition

Following the Schur Decomposition, assume there exists square matrices $G$ and $H$ and some upper triangular matrices $\Lambda$ and $\Omega$ such that:

\[
GD_1 = \Lambda H' \quad (A.1)
\]
\[
GD_2 = \Omega H' \quad (A.2)
\]

where $GG' = G'G = I$ and $HH' = H'H = I$ are unitary matrices.

\[
D_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = D_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} + E \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \quad (A.3)
\]

To obtain the eigenvalues for the system the error vector can be omitted. Multiplying both sides by $G$ yields:

\[
GD_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = GD_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} \quad (A.4)
\]

Using A.1 and A.2:

\[
\Lambda H' \begin{bmatrix} P_t \\ X_t \end{bmatrix} = \Omega H' \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} \quad (A.5)
\]

This can then be expressed as:

\[
\Lambda \begin{bmatrix} V_t \\ W_t \end{bmatrix} = \Omega \begin{bmatrix} V_{t+1} \\ W_{t+1} \end{bmatrix} \quad (A.6)
\]
where

\[
\begin{bmatrix}
V_t \\
W_t
\end{bmatrix} = H' \begin{bmatrix}
P_t \\
X_t
\end{bmatrix}
\]  \hspace{1cm} (A.7)

The diagonal entries of $\Lambda^{-1}\Omega$ are the eigenvalues for the system such that:

\[
\Lambda^{-1}\Omega = \begin{pmatrix}
\lambda_{11} & \lambda_{12} \\
0 & \lambda_{22}
\end{pmatrix}^{-1} \begin{pmatrix}
\omega_{11} & \omega_{12} \\
0 & \omega_{22}
\end{pmatrix}
\]  \hspace{1cm} (A.8)

where $\Lambda^{-1}\Omega$ is an upper triangular matrix. Therefore, the eigenvalues are given by $\omega_{ii}/\lambda_{ii}$. 
B System incorporating non-fundamental asset price fluctuations

The system for a model where an asset price bubble can be driven by non-fundamentals is very similar to the system when there are only fundamental changes in asset prices. The key difference is that $s_t$ and $r_t^s$ are added as free variables and $s_{t-1}$ and $E_{t-1}r_t^s$ are added as predetermined variables (and $E_{t-1}r_t^q$ is no longer a predetermined variable). The dynamic system from above has been reproduced with the additional variables for convenience:

$$B_{11}P_t = B_{12}E_tP_{t+1} + B_{13}X_t \quad (B.1)$$
$$X_t = RP_{t-1} + SX_{t-1} + U_t \quad (B.2)$$

Where $P_t = [c_t, i_t, \pi_t, r_t^q, q_t, s_t, r_t^s]$ is a vector of free variables, $X_t = [k_t, n_{t-1}, g_t, z_t, q_{t-1}, s_{t-1}, E_{t-1}r_t^s]$ is a vector of predetermined variables, $U_t = [0, \varepsilon_t^g, \varepsilon_t^z, 0, 0, 0]$ is a vector of fundamental disturbances and $R$ and $S$ are conformable matrices.

The matrices can then be redefined such that:

$$P_t = B_1E_tP_{t+1} + CX_t \quad (B.3)$$
$$X_t = RP_{t-1} + SX_{t-1} + U_t \quad (B.4)$$

where $B_1 = (B_{11})^{-1}B_{12}$ and $C = (B_{11})^{-1}B_{13}$.

Letting $\eta_{t+1} = P_{t+1} - E_tP_{t+1}$, the system can then be rewritten as:

$$P_t = B_1P_{t+1} + CX_t - B_1\eta_{t+1} \quad (B.5)$$
$$X_{t+1} = RP_t + SX_t + U_{t+1} \quad (B.6)$$
The dynamic system is then written as a vector autoregressive process:

\[
D_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = D_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} + E \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \tag{B.7}
\]

where

\[
D_1 = \begin{bmatrix} I & -C \\ R & S \end{bmatrix} \tag{B.8}
\]

\[
D_2 = \begin{bmatrix} B_1 & 0 \\ 0 & I \end{bmatrix} \tag{B.9}
\]

\[
E = \begin{bmatrix} 0 & -B_1 \\ -I & 0 \end{bmatrix} \tag{B.10}
\]

The method of decomposition remains the same.
C  Figures for non-fundamental asset price fluctuations

The determinacy regions when non-fundamental factors can drive changes in asset prices are shown in Figures C.1 and C.2. The determinacy regions when only fundamental factors affect asset prices have also been included as a point of comparison.

![Graphs showing determinacy regions](image)

a: Fundamental fluctuations and $\phi_q = 0$  b: Non-fundamental fluctuations and $\phi_s = 0$

Figure C.1: The determinacy regions when a central bank does not respond to asset price deviations
The determinacy regions are identical, which indicates that there is no effect on determinacy from incorporating an asset price bubble of the type in Bernanke and Gertler (1999). If lagged asset prices are incorporated into the policy rule instead of contemporaneous asset prices as in Bernanke and Gertler (1999), as expected, this slightly increases the likelihood of indeterminacy. However, it does so for both models.