Fabulous fun or fierce frustration?

Initial experiences with learning a computer algebra system

MARY COULAND, UNIVERSITY OF SYDNEY

Within university mathematics departments worldwide there is much debate about the introduction into teaching and learning of computer algebra software. Research to date has largely been carried out within the constructivist theory of learning paradigm. The impact of experiences with computer algebra software on student approaches to learning, and on the pedagogic decision making of teachers, is not well documented. To investigate these aspects of the situation requires an alternative viewpoint: learning can be regarded as a continuing interaction, mediated by cultural tools, between people with individual histories who are working towards their own goals in certain social, institutional and physical contexts. A survey and interview study of the learning experiences of first-year university students in mathematics is underway, building on previous work which established patterns of relationships between approaches to studying mathematics, conceptions of mathematics, and achievement. The first stage of the analysis of results indicates that a positive experience with the computer algebra software Mathematica is related to a strong computing background, a cohesive conception of mathematics and a deep approach to studying mathematics, and is negatively associated with a surface approach to study.

Mathematics education at universities is challenged by major changes in technology. Computer algebra systems (CASs) that are now commercially available are able to perform the routine calculations and symbolic manipulations that traditionally have been the bread and butter of first-year mathematics subjects. Publications concerned with the impact of CASs on the university scene have largely consisted of descriptions and evaluations of innovative teaching programs, for example several case studies (Houston 1994), and research reports of teaching experiments. In the latter, comparisons are usually made between the achievements of groups taught using CASs and groups taught with traditional pen and paper methods. One of the earliest and most widely quoted studies is that of Kathleen Heid (1988), and one of the most recent is by David Meel (1998). These and other papers indicate that students taught in a CAS environment achieve the same
or higher levels of mathematical skills (as measured by pen and paper tests), and in many cases an improved level of conceptual understanding. Very little has been published concerning students' own evaluations of their first experiences with CASs, and in particular how those experiences are shaped by individual learning styles. Indeed, whether or not those experiences might influence a student's approach to studying mathematics remains unresearched. The current study is part of a larger enterprise designed to investigate the experiences of first-year students and their lecturers at a city university, during the incorporation into the teaching and learning of a particular CAS, Mathematica. The methodology of the larger study will be ethnographic, with interviews and observations forming the chief source of data. However, in order to link the study to previous work, and to assist in the selection of students for interview, a survey was carried out and the results of that survey are the focus of this report.

THEORETICAL BACKGROUND

Competing paradigms, viewed from a distance

As noted by Keeves and Stacey, in recent years in Australia there has been 'an emerging awareness that mathematics education is influenced by the social and cultural context in which it takes place and that the social and cultural characteristics of teachers and students affect outcomes' (Keeves and Stacey 1999, p.209). Allied with this is a challenge by the alternative paradigms of the interpretive view and of critical theory to the position held by positivism as the dominant research paradigm (Kanes and Atweh 1993). One way of making sense of these conflicting views is to set them in opposition, and seek a position outside of both camps from which to make comparisons. Anna Sfard (1996) did this very convincingly in her comparison of 'acquisitive' versus 'participatory' models of learning. The acquisitive models include those based on information processing, which treat knowledge as an entity that can be acquired through learning; the learner acquires concepts and skills (assumed to be transferable across contexts), while passing through stages of development. Participatory models regard knowledge as a social construct, with the context being of paramount importance:

Thinking is intricately interwoven with the context of the problem to be solved. The context includes the problem's physical and conceptual structure as well as the purpose of the activity and the social milieu in which it is embedded. (Rogoff 1984, p.2)

Sfard was able to achieve the comparison of acquisitive versus participatory models by utilising a tool of cognitive linguistics (Lakoff 1996, Lakoff and Johnson 1980) which identifies the metaphors used in public and professional discourses and the way they combine to give a conceptual system. By an analysis of the keywords and phrases that appear in professional journals she identified and contrasted the metaphors of learning as acquisition, involving terms such as concept, notion, fact, accumulation, grasp ... with the metaphor of learning as participation, which uses words like reflective discourse, collective reflection, legitimate peripheral participation ... The latter position, with its avoidance of the
objectification of items of knowledge and its questioning of the separation of subject and object is an example of the influence of post-modernist perspectives (Lechte 1994).

In such a climate, the researcher interested in the learning of mathematics might choose to locate their work in one or other of these metaphorical world views, or to seek a position that attempts to bridge the two. This is what Sfard advocates: 'It is my deep belief that [the] most powerful theories are those that stand on more than one metaphorical leg' (1996, p.408). But one may ask whether this is epistemologically possible. It is an aim of the larger study to investigate this issue.

A possible bridge: from phenomenography to traditional psychology via John Biggs

An alternative to the positivist view is to abandon attempts to obtain direct descriptions of the world and aim instead to collect and classify people's experiences of various aspects of it. This is the focus of phenomenography (Marton 1981, Marton and Booth 1997), which aims not to categorise people in terms of their possessing or not possessing certain concepts or skills, but instead to identify categories of description for the conceptions people have about perceived phenomena, derived from their individual experiences. In an educational context, these phenomena include the content that is studied, and the learning situation itself. In this approach, learning is viewed as a change in internal relationship between person and world, and, as such, learning cannot be studied in a way that abstracts it from the context of what is being learned (Marton and Booth 1997, p.115). These experiences are nondualistic, and are considered neither mental nor physical objects: 'The subject's experience of the object is a relation between the two' (Marton 1994, p.91). It is usually found that the categories of descriptions form a hierarchy of increasing complexity.

Researchers using a phenomenographic approach have described various aspects of learning at university level. The earlier studies from the seventies, reviewed by Dahlgren (1984), were largely based on text reading experiments. In these cases, the categories of description that arose from an analysis of the students' answers were considered significant because 'in each case, the outcome does not amount to the retention or non-retention of a disembodied fact that has no meaning beyond itself. Instead, the phenomenon is invested with a specific meaning that both reflects and colours how the phenomenon is thought about' (p.31). Investigations of students' intentions as they undertook various learning tasks have found motives to be related to the outcomes of learning. Marton and Saljo (1984) identified deep (searching for meaning) and surface (focusing on the text itself) approaches to learning in studies of students asked to read texts as part of learning experiments. This description is developed more fully in terms of awareness in Marton and Booth (1997). Learners differ from each other in their simultaneous awareness of more and more aspects of the learning situation: the text itself, the meaning of the text, the meaning of the phenomenon. This would appear to be placing the deep and surface approaches on the same continuum. Further, the student's perception of the learning task and their interaction with the subject material constitute the relationship which is then seen to be of a deep or surface kind. The knower is not seen as separated from the
known, so that the same person may choose to adopt a deep approach in some situations and a surface approach in another, depending on their interpretation of the context.

In contrast to this continuum, Biggs (1987) suggests that while it would not be possible to exercise deep and surface strategies simultaneously, deep and surface motives may be present at the same time. For example, someone may rote learn certain items (diagrams, definitions ...) as a first step but fully intend to invest them with meaning at a later stage. He regards these motives and strategies as personality characteristics located in the student, not in the interaction between the learner, the content, and the context of the learning situation. John Biggs developed his theory of student learning from an entirely different direction from that of the phenomenographers described above. As he explained himself, his background is in the traditional psychological field that uses information processing as a base model for student behaviour (Biggs 1991). In Sfard’s classification scheme, his work belongs to the acquisitive metaphor. In this tradition, basic personality factors can be measured and are assumed to be stable across situations, leading to an interaction between person and situation that is responsible for the way that learners react to particular demands. Over decades of research with students of various ages and in several countries, Biggs refined an original questionnaire containing ten scales to two instruments, the LPQ, or Learning Process Questionnaire, for use with secondary students, and the SPQ, or Study Process Questionnaire, for use with tertiary students. Each has six subscales derived from three motive and three strategy subscales. The subscales are: deep motive, deep strategy, surface motive, surface strategy, achieving motive, and achieving strategy. The LPQ and SPQ have been widely tested and the reliabilities are reported as ‘very satisfactory’ (Biggs 1987, p.32). Research with these questionnaires has led to a model of student learning involving presage, process and product variables, which could be seen to include aspects of Marton’s scheme without wholly incorporating it. As Biggs summarised the differences, his model incorporates both person variables and situational variables, while he believed that the view of Marton and others overemphasises the role of situational factors. Moreover, since the two approaches address different questions (the one about student approaches to a task and the resulting outcome, and the other more concerned with predicting academic success), Biggs claimed there was no conflict between the two research approaches and models (Biggs 1987, p.92).

The role of goals: another bridge?

The theory developed by Biggs does not stop with the creation of the LPQ and SPQ. Acknowledging that students, whose personality characteristics predetermine them towards a certain combination of motives and strategies, still choose to use different strategies in certain circumstances, and may change their motives for doing so, Biggs proposed that a person’s metalearning is responsible for the interaction between person-related factors and situational factors. Metalearning, which he likened to metacognition, involves ‘awareness of the learning processes one may use, and executive control in deploying them effectively’ (Biggs 1987, p.97). Metalearning appears to develop with
maturity and experience, especially experience that forces one to monitor one’s own thought. Although goal setting is not explicitly a part of Biggs’ presage-process-product model, it may be assumed that choosing goals is part of establishing motives, and matching strategies to motives is within the role of metalearning.

Simone Volet and others (Volet and Lawrence 1990; Volet and Chalmers 1992) have undertaken more explicit investigations of the importance of goals in student learning in higher education. Both of these sets of studies build on concepts established in cognitive psychology as well as the work of the Gothenberg school of phenomenography. In terms of Sfard’s analysis, they would appear to belong to learning as acquisition, with an emphasis on the acquisition of facts and meanings. As appropriate for a relatively new discipline (the study of learning in higher education) that is developing its own methodologies and conceptual structures, the paper by Volet and Lawrence reports that its findings provide support for theories from various areas: cognitive psychologists’ emphasis on appropriate goals for effective problem-solving, activity theorists’ focus on goal-directed actions, and the relations between motives and strategies reported by Biggs and by Marton. There is a sense here that a bridge may be being built across Sfard’s divide by focusing on the dynamic nature of how students change their goals and actions over time and in response to their perceptions of both their own achievements and of the demands of the learning environment. Volet and Lawrence (1990) described situations in which students adjusted towards lower goals when faced with failure to achieve a previously intended goal, and also situations which involved changes in the opposite direction in response to a realisation of the personal relevance of the content of a course.

Is something missing?

While all the approaches described above have offered much to the investigation of student learning, it can be argued that they show only part of the picture. What appears to be missing from the analyses is consideration of the roles played by other actors in the social system that constitutes a learning institution: the teachers and lecturers. Without this aspect, measures of achievement are not questioned. Neither are the choices of learning and assessment tasks – choices that teachers and lecturers make every day. While the methods of both phenomenography and of cognitive psychology permit investigations of the behaviour and instructional choices of teachers and lecturers, to investigate the dynamic nature of the interactions between learners, teachers and institutions may require a methodology more appropriate to the investigation of social systems. This is particularly the case when changes in the basic components of the learning environment, such as those occasioned by the introduction of a new technology, are the focus of interest. This is the aim of the larger study, of which the current report is the first phase, the starting point.

A starting point incorporating aspects of the models of learning described above will anchor the new work in existing research and provide directions for further investigations.
PREVIOUS INVESTIGATIONS OF APPROACHES TO LEARNING MATHEMATICS AT UNIVERSITY

Recent work at the University of Sydney (Crawford, Gordon, Nicholas and Prosser (1994 1998a and b) has resulted in two questionnaires, the Conceptions of Mathematics Questionnaire and the Approaches to Study Questionnaire. The latter is based on the SPQ developed by John Biggs (1987), with two kinds of changes: the wording of several questions has been changed to bring out study approaches relevant to mathematics at university level, and questions from the Achieving Approach subscale have been dropped. The Conceptions of Mathematics Questionnaire is based on the earlier phenomenographic study (Crawford et al 1994), and has two scales indicating a fragmented conception of mathematics (mathematics is seen as a collection of rules and formulas), and a cohesive conception of mathematics (mathematics is seen as a logical system for generating new knowledge). Crawford et al reported statistically significant correlations between these two sets of scales, namely a positive correlation between a surface study approach and a fragmented conception, and a positive correlation between a deep study approach and a cohesive conception (1998b). When other variables such as course experiences and prior academic ranking were included, two clusters of students were identified. These are described as follows (1998a, p.465):

This analysis suggests that students holding cohesive conceptions of mathematics adopt deep approaches to learning mathematics, and have very different interpretations of learning mathematics at university. They perceive the learning environment as more satisfactory and fulfilling than do students reporting fragmented conceptions. Moreover, these students achieve at a higher level in their university study of mathematics than those students holding fragmented conceptions of mathematics and adopting surface approaches to learning.

The authors in that study emphasised that their model of student learning is not causal and deterministic, but instead indicates ongoing interrelationships between prior and post experiences and understandings, perceptions and study approaches, and the teaching and learning context.

AIMS OF THE CURRENT STUDY

The aims of the current study included:

(a) to find out if patterns of relationships between approaches to studying mathematics and conceptions of the subject itself, as identified by Crawford and others, could be established for similar students in another institution,

(b) to identify groups of students to ensure that interviews in the next stage of data collection were conducted with a representative sample (assuming the patterns in (a) were indeed found),

(c) to identify qualitatively different sets of experiences with Mathematica, and

(d) to investigate whether or not students from different groups reported qualitatively different experiences with Mathematica.

In this paper I report on the progress made towards (a), (c) and (d).
COUPLAND

THE SURVEY

First two questionnaires

The survey included two of the questionnaires used by Crawford et al (1998a): the Approaches to Learning Mathematics Questionnaire and the Conceptions of Mathematics Questionnaire. Also included was a third questionnaire, Experiences of Mathematica, which was developed from several sources as described below. Several open-ended questions were included to gain a broader picture of study approaches and experiences. Biographical data was also requested: age, sex, language spoken most of the time at home, course of study, prior mathematics studied, and a self-assessment of computing background. Students were also asked for their student number to allow access to assessment information.

Experiences of Mathematica questionnaire

Students in another first-year mathematics subject at the same institution completed an anonymous evaluation towards the end of the previous semester. One of the questions asked was ‘Overall, how would you describe your initial experiences with Mathematica?’ The responses formed the basis of many of the items in the questionnaire. Items were also constructed to reflect some of the metaphors for technology identified by Kissane, Kemp and Bradley (1995) in a study of first-year university student attitudes to graphing calculators. Several items were written to find out if students using a surface/deep approach to study might follow through to similar approaches to learning Mathematica. In addition, some items were written to find out about practical aspects of the students’ experiences, for example whether or not they experienced problems with access to the computers or with using the hardware.

COLLECTING THE DATA

The students

The target population for the sample consisted of three first-year semester 1 mathematics classes: a mainstream calculus course Mathematics 1, Mathematical Modelling for Science, and Mathematical Modelling 1 (for engineering students). Classes were surveyed at the beginning of semester 2 of 1999. The questionnaires were given out in classes in which students in the target population would normally be found: either the next mathematics subject in the sequence or the same subject for those who had not passed in first semester. The best response rate (nearly 70%) was from students who had studied the subject Mathematics 1 in the previous semester, and this is the group whose responses are reported below. The responses for Mathematics 1 were checked by subject grade and by degree course of enrolment to see whether or not an identifiable group of students was underrepresented. One particular degree course (Bachelor of Science in Mathematics) was underrepresented, and students who had achieved passes or fails in the subject rather than credits, distinctions or high distinctions did appear to be underrepresented. It is understandable that students who had failed the course, or only just passed it, may
well have been absent on the day of the data collection if they had dropped out or transferred to another course. They may also have been unwilling to participate if they had had a negative experience in mathematics the previous semester.

**The planned learning experiences with Mathematica for this class (Mathematics I)**

The students in this class had ten one-hour *Mathematica* laboratory sessions over the semester. In these labs they had the choice of working individually or in small groups on prepared *Mathematica* notebooks, which had been written to demonstrate the application of CASs to solving real-world problems. The broad range of contexts included applications such as finance (compound interest, extending to present value and future value), the design of a theme park ride (mechanics) and the design of software to assist in an embroidery factory (Bezier curves and free-form computer-aided design). All the examples were based on actual case studies. The students were also required to work individually on a *Mathematica* assignment which contributed towards their assessment in the subject. The assignment concerned differential equations and population growth.

**RESULTS FROM THE FIRST TWO QUESTIONNAIRES**

The first two questionnaires were summarised using the subscales from previous studies as mentioned above. The results are shown in Table 1.

![Table 1 - Correlations between study process variables and conceptions of mathematics variables](image-url)

<table>
<thead>
<tr>
<th>Study process variables</th>
<th>Conceptions of mathematics variables</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Fragmented</td>
</tr>
<tr>
<td>Surface approach</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td>Deep approach</td>
<td>-0.12</td>
</tr>
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<td></td>
<td>-0.12</td>
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**Note:** Figures in *italics* are from Crawford et al (1998b), N=300. Figures in *bold* are from the current study, N=54.

The correlations are all in the expected directions. There does seem to be a difference, however, in the strengths of the relationships between a surface approach to study and the conceptions of mathematics. Possible reasons for this include (a) genuine differences in the attitudes and behaviours of the students at the two institutions, and (b) lack of reliability in some of the measures. There is also the possibility that the construct of 'a surface approach' may be problematic: it may be less stable than previously thought. Indeed it is quite likely that the interactions between context, including purposes and goals, and the study approach chosen may make it inappropriate to attempt to measure a surface...
approach with this questionnaire. Further investigation is warranted, both with larger groups of students (for breadth) and with interviews (for depth).

RESULTS FROM THE THIRD QUESTIONNAIRE

The third questionnaire, the Experiences of *Mathematica* questionnaire, contained 56 items designed to collect information about various aspects of the students’ experiences. To begin to make sense of the responses, descriptive statistics and graphs for each item were inspected. It was decided to look first at an overall experience scale, by deleting certain items. These were items such as number 9 ‘I had difficulty using the printer’, with which most people disagreed; and also items such as number 24, ‘*Mathematica* should be integrated more fully into tests and exams to avoid the tedious calculations and algebra’, which asked for an opinion or a judgement rather than reflection on one’s own immediate experience. Several of the remaining items were then reverse coded, so that a high score reflected a positive experience with *Mathematica*. As it was felt to be important to relate students’ previous computer experience with the *Mathematica* experience result, a score of computing background was created. To construct this score, one ‘point’ was given for each ‘yes’ to the answers ‘Do you have access to a computer at home?’, ‘Do you regard yourself as a competent user of any of the following software? (spreadsheets, word processing, mathematical software one point each). This was then added to a score for the answer to the question ‘Overall, how would you rate your own computing background and experience?’, with one for ‘very limited’, two for ‘adequate for my study needs’, three for ‘more than adequate for my study needs’, and four for ‘extensive’. The computing background score is then a number from zero to eight.

The correlations of all these measures are shown below in Table 2.

**Table 2 – Correlations between study process variables, conceptions of mathematics variables, Mathematica experience and computing background**

<table>
<thead>
<tr>
<th></th>
<th>Fragmented conception of maths</th>
<th>Cohesive conception of maths</th>
<th>Surface approach</th>
<th>Deep approach</th>
<th>Computing background</th>
<th>Mathematica experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmented conception</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohesive conception</td>
<td>-0.11</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface approach</td>
<td>0.10</td>
<td>-0.28</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep approach</td>
<td>-0.12</td>
<td>0.61</td>
<td>-0.17</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computing background</td>
<td>-0.10</td>
<td>0.09</td>
<td>-0.15</td>
<td>0.12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mathematica experience</td>
<td>0.05</td>
<td>0.31</td>
<td>-0.36</td>
<td>0.29</td>
<td>0.40</td>
<td>1</td>
</tr>
</tbody>
</table>
These results indicate that a positive first experience with *Mathematica* is associated with a strong computing background, a cohesive conception of mathematics, and a deep approach to studying mathematics; and is negatively associated with a surface approach to study.

**Correlations with measures of achievement**

Two measures of achievement in the subject Mathematics 1 were used to look for relationships with the variables that have already been described. These were the raw exam mark out of 120, and the mark for the *Mathematica* assignment out of 10. The exam mark correlated positively with a cohesive conception of mathematics (at 0.23) and negatively with a fragmented conception of mathematics (at -0.28). All other correlations were less than 0.25 in magnitude. The *Mathematica* assignment mark correlated negatively with a fragmented conception of mathematics (-0.31) but other correlations were less than 0.25 in magnitude. Interestingly, the correlations between *Mathematica* assignment marks and both *Mathematica* experience and computing experience were quite low (0.12 and 0.03 respectively).

**Variables by gender and by home language**

In Mathematics 1 there were 28 males and 26 females. Sorting another way, 31 students stated that the language spoken most of the time in their home was English, whereas 23 spoke another language. Descriptive statistics for these groups on the variables of interest were obtained, scaled to a common scale, and are shown below in Figures 1 and 2 on the following pages. Differences by gender do not appear to be large, although there appears to be a tendency for females in the group to score more highly on fragmented conceptions and surface approach, with males scoring more highly on cohesive conceptions and deep approach. There is a noticeable difference between English and bilingual students on the cohesive conceptions of mathematics and on the deep approach to study. This difference is in the same direction for the achievement variables and for the computing background and *Mathematica* experience variables. It will be important to explore these differences in the follow-up interviews.
Figure 1: Variables by Gender
Figure 2: Variables by Home Language
FUTURE DIRECTIONS

The results of this preliminary study provide direction for further investigations. Groups of students that are likely to have different experiences with Mathematica can now be identified, with approaches to studying mathematics and computing background being important variables. In targeting students for interview, these considerations will be kept in mind. The study also has identified variables that are likely to be important in the next stage of the analysis, when results from several classes are combined. Further dimensions of the experience of students learning Mathematica for the first time may arise in the analysis of the qualitative data that was obtained in the study, and it is fitting to end with some of the students' own words. At the end of the survey, students were asked to write a reply to 'Overall, how would you describe your experiences with Mathematica in your first semester?' The responses included many of the types of comments that had been collected in the previous semester and which had been incorporated into the questions in the third questionnaire. As an example of the full range, here are two, a positive and a negative summing-up which illustrate the title of this paper:

‘Useful to learn a mathematical computer package. Good to see how to use it with real life problems. Useful in checking answers from maths exercises. Assignment also demonstrated Mathematica’s use with real world problems. Good to be involved with other students and swap and compare Mathematica techniques.’

‘Awful. The material on how to use Mathematica is too complicated. It was in essay form and a lot of professional words, very hard to understand. I wonder if they can simplify the language and give more examples. Instead of talking about what’s going on with the computer, explain why we are doing this.’

REFERENCES


