Every experience we have is limited, and it points to something beyond with which it is connected, and with which it forms some sort of unity. … Every event, in the same way, points back to another, of which it is the continuation and modification, and onward to another in which it will be continued and modified; and these lines in turn lead in both directions to infinite time. This infinity of the finite, of what cannot be defined and conditioned except as finite, does not allow the intellect, which would completely embrace this definiteness and conditionedness, to come to any rest within the world of appearances.1

G) THE PRINCIPLE OF CONTINUITY

Our experience of life, William James wrote, is that of a seamless stream of spatial and temporal continuity. Based on this intuition we attribute to liquid and vaporous phenomena a similar homogeneity; but in these instances corpuscular science has disabused us of this belief by reducing the appearance of flow to the chemical activity of discrete elements. The latter thus join all the other objects whose nature it is to occupy space one at a time and place, that is, to be impenetrable by others of their kind.

Yet once we have examined these phenomena under the tenets of Leibniz’s double-aspect theory, they are apt to lose this imputed rigidity. If a drop of chloric acid can burn a hole through steel plate, this tells us something about the nature of steel which is different from what our sense of touch reveals. Then we are brought up short not so much by the ‘relativity of all things’, which is a poor descriptor of these matters, but by the continuity of, and among, all things. “It must be realised,” writes Leibniz, “that truly every body has some degree of firmness, and there is nothing so fluid that some force is not needed to bend its parts or pull them apart completely; and that every body has some degree of fluidity, so that its parts may not only be separated by a sufficiently strong force, but may also be bent inwards somewhat by however minimal a force.”2 In an ontology informed by such knowledge, the steel dissolving under the impact of a liquid becomes a symbol for the interpretation which sees nature as constructed on just one ultimate principle: that to act is indeed to be, which can also be put another way to take account of the intrinsic feature of localisation implied here: that for a force or thing to be implies that it is where it acts.

This has a peculiar relevance to the further arguments to be unfolded in the present section. Its principal purpose is to furnish supporting material to the thesis presented so far; it is designed to add depth to the arguments given there, but does not strike out on a new route. Nevertheless, and so to speak incidentally, by adding depth it also reveals unexpected perspectives which did not come out of the preceding discussions. Especially noteworthy is the fact that some of these findings bear on the fecundity of truly profound philosophical ideas beyond their own times and their own

contexts. Not the least outcome of these considerations may therefore be, that it invites us to acknowledge the enduring relevance of Leibniz’s thinking in both its metaphysical and its physics-related dimensions. These are ideas which have not, by a long shot, reached their ‘use by’ date.

1. The Labyrinth of the Continuum

Leibniz warned repeatedly of the “two labyrinths” of the human mind, into which the thinker is easily sucked, but once caught in it, may never find his way out. They are, respectively, Fate, Fortune and Freedom and the Composition of the Continuum: Time, Place, Motion, Atoms, the Indivisible and the Infinite.\(^3\) We seem to have (as it were without a deliberate agenda) traversed the first of these in Part II with Leibniz as our guide and emerged unscathed. The second, however, is still beckoning.

Before we enter it, we should however first take notice of two important criteria, namely Leibniz’s understanding of the ‘point’, mention of which occurs ubiquitously throughout all his continuum texts; and secondly what he understands by ‘the continuum’ and why, in particular, he persists in referring to it as an ‘ideal’ entity.

On the notion of the point, we have an exemplary clarification from the pen of Gueroult:

Leibniz distinguished three kinds of points: (i) the metaphysical point, which is unextended substance; it is exact and real; (ii) the mathematical point, which is the point of view from which each substance expresses the universe; it is exact but unreal – it is a modality or an aspect of real terms; and (iii) the physical point, which is the restriction of the parts of corporeal substances such that they appear as a point – the latter is not rigorously a point, but an infinitely small extension, an infinitesimal; it is real, but inexact.\(^4\)

Even though we shall be analysing its application to the continuum in detail further down, it is not superfluous to draw prior attention to some important implications, which indeed play a central role in this whole chapter. Once again we draw on Gueroult for the appropriate explication:

Points are nothing but limits and are by themselves noncontinuous. Even though they have situation, they could not constitute the extension that arises out of situation, but add continuity to a point. They do not exist by themselves, but are the extremities of bodies. To say that a point has a position is nothing other than to be able to designate where the body ends. … One exposes oneself to inextricable confusions (as Russell did) when one does not distinguish point as extremity, and point as position, place or locality that does not contain anything other than extrinsic [things], since it is reduced to the relation of exclusion from itself, which is characteristic of a point in the sense of position of term (‘place which excludes all other places’). Insofar as it is the position of a possible term, place is unextended, for there is no real extension without a real term … it is the possible principle of something extended.\(^5\)

To achieve clarification on the continuum itself, it will be helpful to bring it into juxtaposition with phenomena. These, we recall, are real in the sense of actually and physically existing. Yet on the whole, the same phenomenal entities are not real in the way we perceive them – meaning that where we see or feel or measure bodies as dis-

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\(^3\) Theod., Preface.


crete existents, we are frequently deceived by our senses (and instruments) as to the reality which pertains to these phenomena as bodies. In such instances it is ineluctably necessary for the mind to extract an order from phenomena which reflects our physical needs vis-à-vis those existents, irrespective of whether that order is real or not, since it is real enough for us as bodies. It stands to reason that a great number of such bodies, even those to which we affix identifying names, do not possess that unity which names purport to confer on them; and this includes not only such bodies as this spoon, which in Leibniz’s taxonomy is a well-founded phenomenon (i.e. physically real and present, even though not a true unity), but also such as ‘Indian Ocean’, which is a merely imaginary phenomenon. In this sense, phenomena depend to a significant extent on the mind – not for their existence, but for our understanding of what kind of existents we want or need them to be.

With respect to the continuum, these criteria can assist us by an analogous use of the term ‘ideal’. The continuum is not an existent in its own right; but rather in the meaning of the above discrimination an entity in the same category as the Indian Ocean. Yet the differences are a little more subtle, since ‘continuum’ defines a logical space: its population does not comprise physical and in some measure discrete, but wholly indeterminate parts. This has two immediate consequences for our understanding, namely that in the phenomenal world, parts make wholes, whereas in the continuum the wholes are given and are not, stringently regarded, composed of parts, but founded on the idea of parts. But to the extent that in the metaphysical perspective these parts are neither ‘real’ nor enumerable, it follows that the continuum cannot be real in the sense of ‘actual’. It has no independent existence and is merely the conceptualisation of the continuity among all things in the world. As Whitehead put it:

> It cannot be too clearly understood that some chief notions of European thought were framed under the influence of a misapprehension [consisting in] the confusion of mere potentiality with actuality. Continuity concerns what is potential; whereas actuality is incurably atomic.  

So the difficulty posed by the ‘labyrinth of the continuum’ is in the main that it must not be confused with the universe, with the world of actual existents. It pertains only to foundations and thus presents itself to us as a metaphysical terrain, or more adequately expressed, a metaphysical laboratory.

The objects we find therein comprise a very strange zoo indeed: mathematical convergences and limits; problems of instants and infinites; incorporeal substances such as minds and forces; enigmas relating to the divisibility of matter, motion, space and time; issues of transfer along infinitely fractured trajectories; conundrums relating to points and aggregates, impact, cohesion and penetration; minima and maxima; immeasurable boundaries; and finally the conservation of energy in a field whose extent remains undefined.

That all these speculative ‘entities’ owe their place on the shelves of this laboratory to the exciting developments related to infinitesimal calculus goes without saying. We

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6 A. N. Whitehead: *Process and Reality*. Corrected Ed. Macmillan, London 1979, p. 61. Whitehead’s adoption of ‘atomism’ is somewhat idiosyncratic, as many of his terms are; in this case it seems to be approximately cognate with ‘divisible’, as a later passage from the same chapter suggests: “Actual entities atomise the extensive continuum. ... In the mere extensive continuum there is no principle to determine what regional quanta shall be atomised, so as to form the real perspective standpoint for the primary data constituting the basic phase in the concrescence of an actual entity ... divisibility is what constitutes [the res verae’s] extensiveness.” Ibid, pp. 67, 69.
know that the calculus had inordinate teething problems (not finally brought under control until the 19th century); and Leibniz may have underplayed them in his enthusiasm for getting its principles adopted. But this is not an undue impediment provided one remains aware, as Leibniz was, that the calculus is an analogy for attacking metaphysical difficulties, not an open-sesame to their final resolution.

2. Natura saltus non facet

Nothing takes place suddenly, and it is one of my great and best-confirmed maxims that nature never makes a leap. I call this the law of continuity … It implies that any change from small to large, or vice versa, passes through something which is, in respect of degrees as well as of parts, in between; and that no motion ever springs immediately from a state of rest, or passes into one except through a lesser motion; just as one could never traverse a certain line without first traversing a shorter one. … All which [also] supports the judgement that noticeable perceptions arise by degrees from ones which are too minute to be noticed. To think otherwise is to be ignorant of the immeasurable fineness of things, which always and everywhere involves an actual infinity.7

If nature takes no leaps; if phenomena furnish us with a window into actual existence on a continuous slope of infinite dimensions; if magnitudes are merely the organisation of monadic entities into composite existents which are perceived as different kinds of structures in different dimensions; then the notion of a lattice-like structure of time and space becomes inadmissible. Then the concept of a real continuum moves into focus: and the notions of a vacuum, of ultimate indivisible matter particles, of time as an independent factor of existence, and of space as extension in itself, cannot be sustained:

My axiom … destroyed atoms, small lapses of motion, globules of the second element, and other similar chimeras … [and] rectifies the laws of motion. Sir, lay aside your fears about the tortoise that the Pyrrhonian sceptics have made to move as fast as Achilles. You are right in saying that all magnitudes may be infinitely subdivided. There is none so small in which we cannot conceive an inexhaustible infinity of subdivisions. But I see no harm in that or any necessity to exhaust them. A space infinitely divisible is traversed in a time also infinitely divisible. I conceive no physical indivisibles short of a miracle, and I believe nature can reduce bodies to any smallness geometry can consider.8

The axiom is one of the pillars of Leibniz’s philosophy: one of the ten basic principles on which metaphysics rests. His general formulation reads:

When the difference between two instances in a given series or that which is presupposed can be diminished until it becomes smaller than any given quantity whatever, the corresponding difference in what is sought or in their results must of necessity also be diminished or become less than any quantity whatever … This depends on a more general principle, that, as the data are ordered, so the unknowns are also ordered.9

Certain terminological as well as conceptual difficulties are involved, which need careful untangling; accordingly it is not out of place to insist on, and to stress, that the continuum, for all that it is not ‘real’, has a bearing on the real – as one scholar puts it:

7 Nouv. Ess., Preface 57.
8 Foucher (1692), W 71. – In respect of this last claim, cf. Sect. H, §4 infra.
9 Malebranche, L 351.
Although the real world is not truly continuous, it is nevertheless infinite, and so is the ideal continuum. The infinity constitutes the link between the real and ideal worlds, and it allows us to apply the continuity principle in our world as well. Infinity can be ordered or disordered, it can be continuous or discrete; therefore infinity by itself is an insufficient key to the universe. An order has to be added to it, and the order is supplied by the continuity principle, by the principle of general order. This order, the continuous order, on the other hand, would be impossible if it were not for an underlying infinity, since a finite universe cannot be ordered in the sense required by this principle.\(^{10}\)

Leibniz indeed suggests an intriguing visualisation in the passage just cited, where he speaks of mathematical theorems on conic sections. Each is a discrete, individual geometrical figure, yet note how they are derived: project a cone of light onto a screen, then insert a flat pane of glass in its path. If you ‘cut’ the cone at right angles, a circular figure will appear on your glass; but now tilt it slowly, and as you do, the circle will transform first into an ellipse, then a parabola, finally into an hyperbola.\(^{11}\) Seamless continuity.

The concept of extension already implies continuity, although in too simplistic a fashion to be of service to Leibniz’s metaphysics. The tradition holds it to be a species of continuous quantity, “the simultaneous continuous repetition of position” in Leibniz’s definition; and accordingly “a plurality, continuity and coexistence of the parts.”\(^{12}\)

But now the important rider is (as we’ve already learnt) that repetition is either discrete or continuous: but while the former poses no problems, in continuous repetition the conceptual difficulty arises that “the parts are indeterminate and can be assumed in infinite ways”.\(^{13}\) Those things to which we assign the criterion ‘real’ are discrete and therefore cannot be truly continuous inasmuch as they have determinate parts. Hence actual continuity is “an ideal thing”, not a feature of tangible reality, for bodies lack one of the characteristics which define the continuous.\(^{14}\) “In real things,” Leibniz adds, “namely bodies, the parts are not indefinite (as in space, a mental thing), but are actually assigned in a certain way.”\(^{15}\) As we saw in several places in Part II, this implies that “phenomena can therefore always be divided into smaller phenomena which could appear to more subtle animals, and one will never arrive at smallest phenomena. But substantial unities are not parts, but foundations, of phenomena.”\(^{16}\) The parts of phenomenal things are always smaller phenomena, \textit{ad infinitum} – worlds within worlds.

Note in this quotation the stress Leibniz puts on “more subtle animals”, which we might adumbrate without trouble nowadays as denoting that the world of phenomena accessible to such as bees (which can see ultraviolet light), not to mention microbes, involves something very different from our experiences.

3. Petites perceptions

But his most celebrated showcase for the continuum is undoubtedly his idea of \textit{petites perceptions} and related phenomena:

\(^{11}\) \textit{Bayle} (1687), W 66-7. For completeness’ sake, the point coincides, of course, with the origin of the light beam.
\(^{12}\) \textit{Ess. Body}, G IV 467, W 104.
\(^{13}\) \textit{Body \\& Force}, G IV 394, AG 251.
\(^{14}\) \textit{Varignon}, G IV 93, L 544.
\(^{15}\) \textit{De Volder}, G II 268, L 536.
\(^{16}\) Loc. cit.
Motion never arises immediately from rest nor is reduced to it except through a smaller motion. All this leads us to conclude rightly that noticeable perceptions also come by degrees from those which are too minute to be noticed.  

Perceptions by a mind are complex states:

[At] every moment there is in us an infinity of perceptions, unaccompanied by awareness or reflection: that is, of alterations in the soul itself, of which we are unaware because these impressions are either too minute or too numerous, or else too unvarying, so that they are not sufficiently distinctive on their own. But when they are combined with others they do nevertheless have their effect and make themselves felt, at least confusedly, within the whole. This is how we become so accustomed to the motion of a mill or a waterfall, after living beside it for a while, that we pay no heed to it.

Rescher calls the recognition of perceptions below the threshold of distinct perceivability “a bold stroke of innovative genius in the history of psychology”. It rests on the fact of memory, which retains traces of all our sensations for at least the length of time required by our perceptive faculty to decipher their meaning and import; but the subtler ones, from their weakness and their sheer quantity, enter awareness generally only if either difference comes to notice or when their combined strength makes an impact.

We are not here concerned with Leibniz’s pneumatology; or rather only to the extent that it impinges on our studies of continua. Accordingly the pains taken by Leibniz to explicate this for his contemporaries quite startling theory, cannot be pursued with the breadth it deserves. But we cannot get around noting how he brings an illuminating comparison with physical continuity into the discussion, viz.:

The state of the soul, like that of the atom, is a state of change, a tendency. The atom tends to change its place, the soul to change its thoughts; each changes by itself in the simplest and most uniform way which its state permits. Then how does it come, I will be asked, that there is such simplicity in the change of the atom and such variety in the changes of the soul? It is because the atom ... though it has parts, has nothing which causes some variety in its tendency, because we assume that its parts do not change their relations. The soul, on the other hand, though entirely indivisible, involves a composite tendency, that is to say, a multitude of present thoughts, each of which tends to a particular change ... by virtue of its essential relationship to all the other things in the world. ... And the reason for the change of thoughts in the soul is the same as that of the change of things in the universe which it represents ... [thoughts] are always images of the universe. They are after their manner worlds in abridged form, fruitful simplicities ...

17 Principia, L 636ff.
20 Cf. this account from the neurophysiological literature: “Sense organs ... admit only news of difference and are indeed normally triggered only by a change, i.e. by events or by those differences in the perceived world which can be made into events by moving into the sense organ. In other words, the end organs of sense are analogous to switches. They must be 'turned on' for a single moment by external impact. That moment is the generating of a single impulse in the afferent nerve. The threshold (i.e. the amount of event required to throw the switch) is of course another matter and may be changed by many physiological circumstances, including the state of the neighbouring end organs.” Bateson, op. cit., p. 120.
Leibniz’s reasoning here echoes the principles we have encountered in Part II on the law of the series: perceptions involve a “composite tendency” in which each state grows out of the preceding and bears the next “in its womb”; and based on the same principle, they are necessarily unique and hence confer unique identity on the monad having them. As Rescher comments: “Only on the basis of unconscious perceptions can the continuity of our mental life be assured. Otherwise our sensations and perceptions could come *ex nihilo.*” Minute perceptions – the infinity flow of experiential gradations – are clearly, for Leibniz, indispensable requisites of conscious experience.

Leibniz accordingly differentiates between perceptions and apperceptions. The former are relatively unproblematic (confusion arising generally from faulty cognitive judgement of their import); but the latter require attention to be noticed at all. In short, this consciousness is of a *global* nature, the solidification of an overall perception from many confused, criss-crossing single apperceptions. Rutherford notes that, within the domain of the sensory, then, we can establish a continuous scale of distinctness, based on the degree to which sensations faithfully convey the infinite detail of the universe. Yet even the most distinct sensory perceptions are still inherently confused, since like all representations of the universe they involve infinity.

Thus, *petites perceptions* are not only the unsensed, but also unreflective groundswell; there is no suggestion at this stage of cognitive activity; hence (Leibniz emphasises) they must be distinguished “from apperception or consciousness.” But the nature of the latter, part of the continuum of minute perceptions all the same, represents the coming and going of impressions which impinge on our sensible awareness and/or cognitive faculty. “Not only, then,” writes Anna Tymieniecka, is the cognitive aspect of our activity, which has already attained the level of objectification of the contents of our attention, grounded in the *petites perceptions,* but also the most primitive unobjectifiable ‘impressions’ or motions of our psychological level which do not crystallise into cognition but into action. Not only our cognitive functions are rooted in them, but also our volitional, aesthetic, evaluative and appreciative functions.

As Leibniz says, all our unpremeditated actions result from denser packings of minute impressions, and this applies as much to habits (which grow little by little) as to our passions and moral behaviour, which would not become dispositions without this groundswell of constant subconscious perceptions. Apperception, however, denotes immanent perception and engages both our sensory and cognitive faculties. On the whole, these features of our psychological being reflect back upon Leibniz’s principal ontological criteria. Anna Tymieniecka sums it up well in the following passage:

23 As an aside: does this not contradict the passus cited from G. Bateson? The argument is too complicated to dwell upon; but in brief, there is no contradiction because perceptions (sensations) are indeed continuous; the ‘switch’ to which Bateson refers being a kind of labour saving device by the nervous system to forestall the use of expensive resources on stimuli that are not changing.
There is only one dynamic function, the spontaneity of the petites perceptions. Leibniz calls ‘perception’ the genetic function of this spontaneity. He attributes the impulse towards change and towards the association of the petites perceptions to the operating faculty of affections. This impulse leads to their incessant variation from all points of view: changing form, volume etc. This is, however, only figuratively expressed, since there is no pattern of articulations of this variation to be set, nor forms of transformability of the petites perceptions to be established. Indeed, the petites perceptions are differentiated only with reference to potential forms; in themselves they are a mere unqualified plurality.27

Thus a conceptual relation is established by Leibniz between psychological phenomena and physical phenomena; for although it is obviously impossible to speak of ‘points’ in relation to them, yet in their genetic function we discern an analogy to the similarly specified emergence of real things, whether geometrical entities or physical bodies, from punctiform limits. But this is something the reader may wish to refer back to, when we are embarked on our journey into the labyrinth (§6 et seq., infra).

4. Relation to indiscernibility principle

Sharp-eyed readers may now wish to put the question to Leibniz, if the Principle of Continuity does not contradict the Principle of the Identity of Indiscernibles? For if, as he defines it, continuity is the passage from one thing to another by the gradual elimination and/or merging of differences, how can this be reconciled with the statement that difference is the sine qua non of discernibility (and indeed identity?).

The path to resolution of this seeming disparity is through the concept of irreducible attributes, an auxiliary of indiscernibility. To understand this, we need to cast aside the Kantian parallelism between space and time as sensory intuitions and retrieve (with Leibniz) the concept of space and time as logical relations among substances – as modalities of possibility and simultaneity. For then, as Gueroult notes,

we perceive [that] two raindrops cannot be reduced to their occupation of separate spatial locations, but rather that they possess internal differences from which we deduce their differences in position. … For disparity of position is a difference lacking differentiation, which arises from a misapprehension of the real differences, of which the former are the phenomena. Accordingly the continuity which pertains to space is thrust back into a phenomenal state.28

This might still suggest the applicability of continuity and indiscernibility selectively, i.e. one to substances and the other to their phenomena. But although Kant drew this exact conclusion in his ‘Amphiboly of Reflective Concepts’,29 it is a misunderstanding based on his failure to asportion continuity to reason rather than intuition. For reason postulates primitiae simplicie non bereft of predicates, which in their elemental indivisibility preclude the possibility of relation, including numerability; and therefore these simplices lack the very attributes – composite relationality – required for the principle of continuity to affect them. They are the minima of the world, elucidated by Leibniz in the image of many triangles of diverse orientation whose collective apices are hinged on a single point: well, where or what is this point? In truth, it is impossible to say; it does not really exist, even though it appears to have ‘function’. In a

27 Ibid, pp. 79-80.
29 Kant, Critique of Pure Reason, A260ff, B316ff.
word, it is a metaphysical point; but it is in these simplices that indiscernibility and continuity have their roots, albeit each from a different perspective.

Time is also involved to the extent that our perceptions involve successions of mental states and thus tend ineluctably to create in us an association with clock time. Yet we know from each one’s own experience that these states do not ‘tick away’ internally as measurable time – in fact, we have so little internal sense of it that we rely on objective timing devices to ascertain how much of it has actually elapsed while internally time seems constantly to fluctuate, i.e. to slow down and speed up according as the stimuli engage our attention more or less intensively:

A train of perceptions arouses the idea of duration in us, but it does not create it. Our perceptions never provide a sufficiently constant and regular train to correspond to the passage of time, which is a simple and uniform continuum like a straight line. Changes in our perceptions prompt us to think of time, and we measure it by means of uniform changes. But even if nothing in nature were uniform, time could still be determined … Knowing the rules governing non-uniform motion, we can always bring them back to uniform motions, and by this means predict what will happen through various motions in combination. In this sense time is the measure of motion, i.e. uniform motion is the measure of non-uniform motion.30

On the topic of qualities which satisfy these criteria, Leibniz enriched the technical vocabulary with an interesting new distinction, namely homogeny and homogony. Homogeny includes elements which can be made similar to each other by transformation, such as line and curve, and are thus distinguished from each other only by their degree of curvature, while otherwise similar in their constitution. Accordingly one can be a part of the other. Homogony on the other hand denotes elements which cannot become (part of) another element by continuous alteration from one state to another, e.g. instant and time, point and space, boundary and the bounded; they are incompatible and incomparable, yet related to each other in the sense that the determination of one is decisive for the determination of the other. Thus:

a common boundary of two things is an entity which is in them when they do not have a part in common. Insofar as these two things are understood to be parts of a single whole, their common boundary is called the section of the whole. It is clear from this that a boundary is not homogenous with what it bounds … [but] nevertheless homogenous, because one can disappear into the other by a process of continuous change.31

In this manner, Rombach writes, Leibniz provides “an elegant solution to an ancient issue in natural philosophy and ontology”, namely the problem of contact between two objects at each other’s boundaries.32 For it was thought that if they touch, their common elements must be identical, and if there is no common element, then no actual contact is made. As we saw, however, Leibniz postulates that at the boundary between objects, to the extent that it comprises homogenous elements, these are absorbed or “disappear in it” as if they were so contained. Accordingly:

This definition permits a qualitative differentiation even in a stringently relational context. A lower order can be in step with a higher order insofar as it fulfills the function of a boundary therein. In this way connection and continuity are established without the abandonment of different qualitative determinations … Evidently there is a relation of

30 Nov. Ess., II, xiv, §16.
orders across the dimensions, such that each layer contains possibilities for mutual determinations, e.g., lower dimensions enabling preconditions to be set for determinations in higher dimensions. ... Thus the infinity of possible two-dimensional magnitudes gives no access to the spatial dimension, yet the dimension of space cannot be articulated from within its own autonomous magnitudes without the lower category which supplies the boundary conditions.33

5. Creation ex nihilo

Leibniz’s axiom has relevance to one other criterion on which his stand might seem at first to involve a self-contradiction. For he does give as an unbreakable specification of his substances (monads) that they are all created at once, from nothing; and can only terminate their careers together, presumably into nothing again. Yet once we have looked at substances through the eyes of the double-aspect theory articulated in Part II, the contradiction dissolves itself.

For we are not, after all, looking to a material creation, as science is wont to do. When physicists endeavour to figure out how matter might have sprung from nothing, this is qualitatively a different aspect of the ‘primordial existential question’. In asking the question in the first place they are beholden to their Judaico-Christian legacy which posits such a creation at the hands of God, and this is traditionally associated with the creation of the matter content of the universe. A theory mooted in quantum physics recently might be mentioned en route of our argument, before we loop back to the creation scenario outlined in Part II. This is the surmise that the universe, rather than having a zero energy content prior to ‘creation’, is an ensemble of two kinds of force fields: namely negative attractive gravitational force and the positive mass-related force associated with Einstein’s formula $E = mc^2$. Initially homogeneous and isotropic, this model (in vague resemblance to Leibniz’s interactive inertial and active forces) proposes the ‘irreversible (entropy-governed) precipitation’ of matter in the collisions of positively and negatively charged fields. In principle, a single such disturbance would be sufficient to bring about an irrevocable escalation of ripples across the entire universe, and thus to ‘create’ the matter universe.34

Yet in spite of its consistent elaboration in mathematical physics over the years, the most important issue in this theory remains blank, namely the origin of those fields. They are simply presupposed. Nor is this its only shortcoming; for although it might adequately account for the creation of matter from (virtually) nothing, the most noteworthy absentee from this scenario is of course the agent of cognisance. The pathway from ‘blind matter’ to a ‘seeing’ and ‘understanding’ ensemble of anomalously self-constructing biochemical fibres which are additionally endowed with an extremely anomalous irritability and autonomy of responsiveness, remains closed. So the (pre-quantum science) decision of Leibniz to leave the creation of force a.k.a. agency in God’s hands cannot be said to have been superseded. Furthermore, to the extent that Leibniz’s theory also amounts to a vision of a self-constructing universe (cf. Sect. D, §3 and Sect. F, §5 of Part II), the imperative of accounting for the existence of autonomous agents of cognisance is satisfied.

33 Ibid, p. 312.
6. Into the labyrinth

The labyrinth of the continuum exercised thinkers almost from the beginnings of philosophy. Anaximander’s *apeiron* stands godfather to it, and Zeno and Democritus are two names whose immortality is in part predicated on their struggles with it, which loom behind those of early modern thinkers such as Kepler and Galileo, Descartes and Gassendi. For example, Galileo discussed the following series:35

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & \ldots \ n \\
1^2 & 2^2 & 3^2 & 4^2 & 5^2 & 6^2 & \ldots \ n^2 \\
\end{array}
\]

If both series are unending, then we are faced with a double paradox: first, that we have two sets whose contents can mapped to each other one-to-one even though one seems to overlap with the other; second, since not all numbers are squares, the first set must be the greater of the two. Galileo skates over this issue:

I do not see how it is possible to come to any decision other than to say that all numbers are infinite, the squares are infinite, and their roots are infinite … And in final conclusion, the attributes of equal, greater and less have no place in infinities, but only in bounded quantities.36

When Leibniz came to examining it, he demurred from Galileo’s opinion: “Who would deny that the number of square numbers is contained in the number of all numbers, when squares are found among all numbers? But to be contained in something is certainly to be part of it, and I believe it to be no less true in the infinite than in the finite, that the part is less than the whole.”37 For the consequence of assuming a greatest number is the same as assuming a fastest motion: it leads to self-contradiction; and thus the idea that there may be a greatest number makes a claim for something that is impossible, and asks us to comprehend a whole which is equal to its parts.38

Many paradoxes of this kind were traded in Islamic philosophy and mysticism and re-entered the stream of European philosophy via this gateway. One in particular which has relevance to this context, the *rota Algazelis*, acquired a peculiar notoriety in late scholastic speculation.39 It is essentially a ‘motion picture’ of the Galileo series, inviting us to look at two wheels of different diameter, but mounted on the same axle, each rolling on their respective surface; and then to ask how each can describe the different lengths of their paths in one revolution in the same time? Galileo came to grips with this problem in his *Two Sciences*, defining the circles as infinitangular concen-

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36 Loc. cit.
37 Pacidius, in *LoC* 179.
38 Ibid, p. 181. Compare: “If something is moved with a velocity greater than which no greater can be conceived, it will be everywhere at once.” *On Matter, Motion and the Continuum*, DSR A 58, 469.
tric polygons on the precedent furnished by Archimedes in the *Sand Reckoner*. This results in the following graphic depiction of the dilemma:  

![Diagram](image)

The geometrical demonstration involves showing that, if a circle is interpreted as a polygon with infinitely many sides, the sides and the interstices reduce to infinitely small quantities. “And just so”, Galileo explains, 

in the circles (which are polygons of infinitely many sides), the line passes over the infinitely many sides of the large circle, arranged continuously [in a straight line], is equal in length to the line passed over by the infinitely many sides of the smaller, but in the latter case with the interpositions of as many voids between them.  

This conclusion offers a way of sidestepping the problems of non-extended parts in the composition of bodies. In the tapering off of point-like particles, the common or phenomenal notions of discreteness vanish; it is no longer meaningful to think of comparing magnitudes, as in the quotation above. We shall see, however, that Leibniz will insist on retaining the attributes dismissed by Galileo; and that the idea of ratios remains valid in the infinite continuum.

When Gassendi took up this problem, he expressed his discomfort with the notion of insensible parts, preferring to treat them as hypothetical entities of use to mathematics, but not as representatives of real divisibles:

> It has already been declared before that neither this infinity of parts in the continuum nor mathematical indivisibility exist in nature, but are merely a hypothesis of the mathematicians, and that therefore in physics one should not argue on the basis of things that are not known to nature.  

This quotation comes from a section where Gassendi engages himself with Zeno’s paradoxes which, however, he is inclined to dismiss as resting on such inadmissible suppositions. He proposes that they be solved by the Epicurean theory of actually indivisible atoms. Yet, as Palmerino notes, the paradox to be solved on the hypothesis of a continuum is that bodies in uniform but unequal rectilinear motion would have to traverse the same minimal increments of time and space, leading to a logical contradic-

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40 Galilei, p. 19ff.
41 Ibid, p. 33.
tion. To resolve this problem, Gassendi takes recourse to the *rota Aristotelis*, but uses it to show that the opposite hypothesis to Galileo’s is true. Firstly, in line with Epicurus, he assumes that atoms come in different shapes and sizes, which explains the possibility of varying speeds; so that if one circle has twice the circumference of the other, both circles roll together first on the tangent of the bigger one, then on the tangent of the smaller one, with the smaller circle making contact alternately with rest points and moving points. Effectively this proceeds on the idea of mapping points between them in one-to-one correspondence, and trades on the fact that the stop-start motion on infinitely small increments would still convey continuous motion to a phenomenal observer. The idea, however, is difficult to sustain in the case of incommensurable ratios; though we have seen that this does not matter to Gassendi, since he is in any case inclined to regard the exercise as a mathematical fiction. But did he have a particular point in mind?

It seems that, on the basis of his almost unqualified admiration for both Epicurus and Galileo, Gassendi’s aim was “to turn the mathematical point into a physical indivisible; the unextended instant into a ‘timelet’, and the non-extended void into a moment of rest.” This was his response to Galileo’s elaborate disquisitions in Day 1 of the *Two Sciences* on the philosophical and mathematical problems embroiled with the conception of *de compositione continui*, in which Galileo practically defected from atomism.

There is a moral in this. The examples relate to basic problems in the study of matter, space, time and motion and do not bend themselves in the desired direction by the logical assumptions under which men like Galileo and Gassendi were forced to labour. Their *fecundity* does not become apparent until the calculus appears on the scene. Thus the mathematical physics which Galileo, and Descartes in particular, dreamed of could not be accomplished until a method has been found by which the infinite could be tamed at least conceptually. For the nature of this and similar types of problem is that they evoke the metaphor of the magnifying glass: if only we could peer at the situation in sufficient resolution, we would surely unravel its paradox. Such an instrument was the calculus – appearing too late for Galileo & Co. to be of help to them.

Such metaphysical conundrums held the most profound attraction for Leibniz and brought him to a point where he substantially and successfully managed to unravel it. On the legitimisation of the calculus in this role, he wrote:

> It has been realised that the rules of the finite work in the infinite as if there were some atoms (i.e. some assignable elements in nature), even though there are none since matter is in fact infinitely subdivided; vice versa the rules of the infinite work in the finite as if there were some infinitely little metaphysical entities, even though there is no need of them and even though the matter can never be reduced into infinitely little particles: however, since everything is ruled by reason, that is how things are.

The solution itself must be dealt with summarily, since it takes up 47 pages in English print. But it is fundamental, since it reveals how Leibniz in the end gave guid-

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44 Ibid, p. 420.
45 *Varignon*, GM IV, pp. 93-4.
46 *Pacidius*, in LoC 127-221.
ance to the future on how to deal with such problems. To preface it, however, we look first to a preparatory utterance:

Whenever it is said that a certain infinite series of numbers has a sum, I am of the opinion that all that is being said is that any finite series with the same rule has a sum, and that the error always diminishes as the series increases, so that it becomes as small as we would like.\textsuperscript{47} For numbers do not \textit{in themselves} go absolutely to infinity, since then there would be a greatest number. But they do go to infinity when applied to a certain space or unbounded line divided into parts. … Suppose to the point of division we ascribe a number always greater by unity than the preceding one, then of course the number of terms will be the last number of the series. But in fact there is no last number of the series, since it is unbounded; especially if the series is unbounded at both ends. Therefore we conclude finally that there is no infinite multiplicity, from which it will follow that there is not an infinity of things either. Or it must be said that an infinity of things is not one whole, i.e. that there is no aggregate of them.\textsuperscript{48}

If now we inspect the diagram below, we find at once that it represents the same problem that exercised Galileo and Gassendi – continuous motion interspersed with rests. It proposes a body, a being carried by continuous movement from \textit{d} to \textit{e}, which entails a concomitant movement of the radius \textit{cfd} to \textit{cge}. Accordingly the movement of the radius through the arc \textit{dhe} will be faster than at \textit{flg}.\textsuperscript{49}

The way Leibniz tackles the solution involves breaking the motion down into the behaviour of its three participating elements, the \textit{moving agent, time and distance}. This is much the same entanglement as proposed by Galileo and Gassendi; but Leibniz now queries the rationality of assuming each of these to be commensurate in their minima. Inevitably this lands us in the paradoxical situation of having to accept that the agent of motion has to leave one point and ‘jump’ to the next; or else that it touches two points at the same time; and failing either of these two possibilities, not to move at all. The second suggestion is clearly vulnerable to the objection that further minima could (indeed \textit{must}) be inserted as the agent is poised halfway between the points, if the idea of an infinity of points is to hold.

Leibniz’s demonstration – a breathtaking flight of the logical imagination – culminates in the claim that “\textit{there are no points before they are designated}”.\textsuperscript{50} Thus he identifies the flaw in all prior discussions of the subject: it is the assumption that the infinite may anteriorly be considered to be comprised of points; but this is unwarranted, for it can be nothing more than an idea. For points to be real, they must be \textit{assignable}, which requires the provision of an arbitrary compartment within the infinite. Hence points are, so to speak, created; they \textit{come into being} when e.g. a sphere touches a line or a body

\begin{itemize}
\item \textsuperscript{47} Editor Richard Arthur comments: “This is a close anticipation of the modern definition of the sum of a converging infinite series as the limit of its partial sums \(S_n\) as \(n\) \rightarrow \infty.” \textit{LoC}, 398.
\item \textsuperscript{48} \textit{Inf. Num.}, in \textit{LoC} 99-101.
\item \textsuperscript{49} \textit{Pacidius}, in \textit{LoC} p. 163.
\item \textsuperscript{50} Ibid, p. 181. \textit{Emphasis added}.
\end{itemize}
intersects another: “then the locus of intersection is a surface or a line, respectively … and in general the only extrema are those made by an act of dividing”. But the number of divisions that could conceivably be made are never made, because this would return us to the illicit assumption from which we began.\textsuperscript{51}

The upshot is that \textit{motion, distance and time are correlated punctiform continua}, none of which can be prior to the others, their coordination being indeed the mesh which explains each by the others and results in the conclusion that motion never ceases – all existents are in constant motion; and it is an illicit supposition that an existent could be in a state of ‘rest’ because then time would stand still and distance shrink to nothing. Nor is motion adequately described by change of place, since every two ‘places’ of necessity leave a wedge for an infinity of other places between themselves, making a nonsense of the term ‘minima’. Change of place of an existent being thereby equivalent to a leap ends up having to acknowledge that its own existence is impossible: it cannot move from one point to another nor stand still.\textsuperscript{52}

In sum: we have to resolve the four absurdities aforementioned: the leap, simultaneous occupation of two adjacent points, motion with rests intercalated, or no motion at all. Now the crucial argument follows:

At any moment which is actually assigned, we will say that the moving thing is at a new point. And although the moments and points that are assigned are indeed infinite, there are never more than two immediately next to each other in the same line, since indivisibles are nothing but bounds.\textsuperscript{53}

Hence, on the Cartesian crosshatch, locus and time are merely two correlated points functioning as the bounds of two infinite series meeting in that intersection; meanwhile it is clear that the assignation of another (arbitrary) ‘point’ on the scale of time would be matched by a point on the scale of loci. So these ‘points’ engender each other in the sense that the progression of time ‘creates’ the points which permit us to define the thing as being ‘in motion’, while conversely the thing in motion engenders the ‘moments’ of time, so that we can speak of a thing in a ‘state’ of motion when it passes from one ‘momentaneous’ state to another. As Christina Schneider writes:

The points of the continuum are, according to Leibniz, logically (or better put: mathematically) more complex than the continuum: they are, under the presupposition of a continuum, explained as \textit{sites of possible division}. They cannot therefore be constitutive of the continuum, because it must be understood prior to their conceptual definition and assignation what the terms ‘continuum’, ‘possible division’ and ‘site’ are intended to denote.\textsuperscript{54}

But the criterion which is decisive in preventing this correlation from engendering a concatenated structure (so that all motion would be at the same velocity) is this:

There is no part of time in which some change or motion does not occur to any part or point of a body. And so no motion stays the same through any space or time however small. Thus both space and time will be actually subdivided to infinity, just as body is. Nor is there any moment of time that is not actually assigned, or at which change does

\textsuperscript{51} Loc. cit.
\textsuperscript{52} Ibid, p. 199.
\textsuperscript{53} Ibid, p. 209.
not occur – that is, which is not the end of an old or beginning of a new state in some body. … [Hence] there is no moment of change common to each of two states, and thus no change of state either, but only an aggregate of two states.55

The conclusion in which this argument eventually culminates is that *thing, motion, time and space form an unique aggregate in their mutual interdependence.*56 And thus the perils of Galileo’s and Gassendi’s equivalence coordinations are cogently resolved.

Let us draw some consequences, which will, however, apply across the board to all the elements of the continuum. As Richard Arthur rightly points out, for Leibniz the speculations on the ‘labyrinth of the continuum’ were not primarily concerned with mathematics: the concept of substance evolving in his mind was intimately entangled with the laws of the continuum, for such problems as the basic elements of matter, space, time, things and motions figure in its web of enigmas. For, writes Arthur, “insofar as anything is continuous, its parts are indiscernible from one another and thus indefinite.”57

While we have earlier on dealt adequately with the actually infinite divisibility of matter, the discussion immediately above points to another facet of equal importance, namely that any stretch of time, no matter how small, involves some change in the world. When Arthur adds, however, that “change, on the other hand, can only be understood in bodies as an aggregate of two opposed states at two contiguous or ‘in-distant’ moments”, this is correct, yet (as we have seen) incomplete. For all three reference elements in these states: thing, space and time, form this aggregate. Change is, accordingly, *an ensemble of three interdependent elements*, and it is true to say that none can claim existence without the other existing. This is not incompatible with Leibniz’s insistence on the merely ideal reality of space and time, for it makes no difference to the principle. Whether ideal or not, space and time are integrated with motion; and since objects are ceaselessly in motion, objects exist ‘in’ time and space as our intuition assures us, even though time and space exist merely by courtesy of the objects for whose being they comprise the reference coordinates.58

At the risk of untoward repetition, the ultimate conclusion from all this can only be the principle on which all existence is grounded, on which Part II turned: that force is

55 Paceius, pp. 209-11 [italics added].
56 In the *Paceius* (incidentally one of his most splendid writings), Leibniz does not yet arrive at this conclusion, having been unable to resolve the issue of how bodies can move at the same time as they are active, and thus leaving it in God’s hands to reconcile the leap from point to point by means of transcreation. Within a few months Leibniz wrested the conception of a ‘frame of reference’ from his speculations, so that the quasi-occasionalistic intrusion of the Almighty is now replaced by the newly derived laws of motion (1678). As he writes in *Motion is Something Relative*, LoC 229, “absolute motion we imagine to ourselves”, and it is nothing but an epistemological convenience for us to consider some existents (mostly ourselves) to be immobile with respect to moving objects.
58 This may be compared with the analysis and solution of the problem at the hands of Whitehead. Consider an act of becoming in one second, he suggests: “The act is divisible into two acts, one during the earlier half of the second, the other during the later half of the second. Thus … that which becomes during the first half second presupposes that which becomes during the first quarter second, and so on indefinitely. Thus if we consider the process of becoming up to the beginning of the second in question, and ask what then becomes, no answer can be given. … The difficulty is not exalted by assuming that something becomes at each non-extensive instant of time. … The conclusion is that in every act of becoming there is the becoming of something with temporal extension; but that the act itself is not extensive in the sense that it is divisible into earlier and later acts of becoming.” Op. cit., pp. 68-9 [italics added].
that irreducible element, and indeed the only element which is entirely beyond all possibility of reduction, on penalty of reducing existence itself to nothing.

Richard Arthur draws an excellent summary:

… Instantaneous actions or tendencies to change state must exist, or … all phenomena would cease, since whatever cannot be sensed and whatever is in principle imperceptible does not exist. But it is substance … that consists in … activity that is resolvable into instantaneous changes of state or endeavours (what Leibniz later calls ‘appetitions’). … When he relocates activity in bodies, it is an inferred activity, not the observable one, that he has in mind. The endeavours are no longer those of bodies; they cannot be conceived in mass by itself. Rather they belong to the substances presupposed in body. For there must be subjects of change and activity, even if they cannot be discerned from the phenomena of motion. Thus, continued motion presupposes beings which by acting do not change, and whose actions consist in their changing relations to all other such beings, and the continuity of whose actions requires an endeavour or ap- petition at each instant.

This ties in of course with our observations in Part II on the law of the series; but its specific point is that motion, on account of its nature, cannot be associated with matter phenomena, but only with beings which possess and exhibit ‘endeavour’ (conatus, appetition). What has been defined above as a threesome aggregate – thing-in-motion, space and time – is thus nothing other than a complicated description of a moment in the law of series of a substance in which the substance abides while engendering accidents that are perceivable as phenomenal states. This bears incidentally on the footnoted observation made in Part II on the question of instantaneous states, that “each ‘state’ of a monad expresses a point of view on the universe; in this the monad takes itself to be ‘at rest’, relative to all other motion” or states of monads.

We have not quite reached the end of this story, however. Recurring once more to the Archimedean specimen, Leibniz examines the consequences of contemplating a regular polygon of $n$ sides each of length $s$ inscribed in a circle. Its length $ns$ can in principle be increased by adding further increments, so as to reduce the error of quadrature successively. Now Leibniz shows that this error ($\text{Length} - ns$) decreases as $n$ approaches infinity and becomes an infinitely reducing differential. But this brings about a conflict between infinitely small sides and an even more infinitely small differential; thus to Leibniz “it follows not only that the error is not infinitely small, but that it is nothing at all.” But if this is the case, asks Arthur, “if the differential is ‘nothing’, how can any of the results obtained by the calculus be either true or meaningful?” He finds that Leibniz’s answer is “forthright, clear and profound”:

The circle – as a polygon greater than any assignable, as if that were possible – is a fictive entity, and so are other things of that kind. So when something is said about the circle we understand it to be true of any polygon such that there is some polygon in which the error is less than any assignable amount $a$, and another polygon in which the error is less than any other definite assigned amount $b$. However there will not be a

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60 Arthur, op. cit., p. 132.

61 Part II, Note 147.

62 On the Infinitely Small, A iii 52, LoC 65.

63 Loc. cit.

64 Arthur, LoC p. LV.
polygon in which this error is less than all assignable amounts \( a \) and \( b \) at once, even if it can be said that polygons somehow approach such an entity in order.\(^{65}\)

Thus (adds Arthur) “a circle is a kind of ideal limit to a sequence of polygons, giving point to Leibniz’s conclusion that a point, having no parts, is nothing, an extremum.”\(^{66}\)

7. Minima and extrema

This example leads logically to a consideration of minima, which became the next victims of these new insights. It is not readily apparent that infinite divisibility yields no minimum unless one has already prejudged the issue by an assumption of the existence of smallest real parts (whether corpuscular, spatial or temporal). Accordingly a difference between minima and extrema must be preserved; and hence there is no final particle, no last point on a line, no final world in the Russian Doll cosmos: all these are surrogates imposed by human understanding on unassignable extrema, such as e.g. the end point of a line.

If the continuum (if there are minima in it) is other than the sum of the assumed minima, it follows that there is a part which remains when the sum of the minima has been taken away; therefore that part is greater than a minimum; for it is not less than nor equal to a minimum; therefore there are minima in it. But this is absurd, for we have already taken away all the minima.\(^{67}\)

The clear message in this is that beginnings and ends are arbitrary concepts (fictions) as well. Continuous things like space, time, motion and all matter structures cannot have either determinable magnitudes nor fixed instants at which they suddenly exist or become countable, whereas before they were not. Only comparison can reveal what we wish to know. For example it is an illicit assumption in this context to define a whole as the composition of its parts: “A whole is not what has parts, just what can have parts,” and the clue to its oneness is that “something must remain which pertains to it rather than to the other thing … [and this] can be mind itself, understanding a certain relation.”\(^{68}\) Conversely then, a continuous whole is not necessarily divided into parts, but can be. To regard them as phenomena entails therefore that “in actuals there is nothing indefinite … [and] the parts are in the actual, not in the ideal whole.” Which effectively brings us back to the observations on phenomena from which we began.\(^{69}\)

What has thus been revealed is this: minima belong to the world of real things, yet it is impossible to assign such a minimum. But in the continuum, there are no minima, only extrema, which neither can nor need be assigned. And with this crucial distinction before us, we are now in a position to see why Leibniz insists that it is permissible to

\(^{65}\) Inf. Num., A iii 498, LoC 89.
\(^{66}\) Arthur, loc. cit.
\(^{67}\) On Matter, Motion and the Continuum, DSR A 58, 470.
\(^{68}\) Inf. Num., 503, LoC 99.
\(^{69}\) The important distinction made here between ‘is’ and ‘can be’ brings to mind Dedekind’s discovery (Continuity and Irrational Numbers, 1872) that one can split a line into two portions at any point and thereby divide the set of points into two collections; from which he derived the principle that every number is an instance of what is now termed the ‘Dedekind cut’. This cut is the precise analogue of Leibniz’s assertion in the text that a metaphysical point is so to speak ‘created’ in being assigned and this point carries the notion of limit. (Significantly the point in question need not be a rational number). Cf. Morris Kline: Mathematical Thought from Ancient to Modern Times, Oxford University Press, New York 1972, pp. 985-6.
retain ratios, but impermissible to retain the notion of a descent down to an infinitesimal minimum. With a ratio there is no objection to it becoming a fictive thing. We know when it does and can draw conclusions. It is a different matter, however, to assign a limit or a minimum or a point: for the expectation of having assigned a real thing can only land us back in that inextricable labyrinth from which the above analyses have sought to preserve us.

And thus to summarise where Leibniz’s solution lies, where and how he extricated himself from the labyrinth and, equally important, in what kind of relation this achievement stands to the ontology of agency which is our underlying agenda.

The continuum, we saw, has been revealed as an imaginary construction, though this should not conduct to the false belief that therefore it does not ‘exist’ in some form or other. The whole point, rather, was to show that continuity acquires its distinctive meaning in the consideration of those indeterminate parts in which the existence of determinate existents is grounded. The latter, be they parts of matter, motion or time, retain the potential for division to infinity; but this again does not mean they are actually so divisible. This would conflict with the doctrine of extrema, which dictates that no part of matter exists as such for longer than an instant, it cannot therefore have ‘states’ or shapes, but only motion. It is as motion that they configure existents; yet one of the things we learnt earlier is that in motion there is no force. However, we already know the answer to this paradox, which is only a seeming paradox: for plainly the perduring element in matter cannot be matter, in time cannot be time, in space cannot be space and so on. For none of these are unities as such: they are unities by courtesy of the activity of substances. And so all this boils down to the very criteria on which Leibniz’s whole ontology rests. Let us recall that substances cannot influence one another; that each reflects the whole of the universe in its being and from its own perspective. Each substance (monad) is moreover part of a plenum in which all changes agree in sustaining the principle of conservation. Further that most important criterion which was highlighted in Part II that no monad leads the life of a hermit, but all are engaged in co-constructing the universe. Finally that this process is based on the release of information, and on the monodirectionalism and asymmetry of its evaluation in the continuous law of changes to monads. It is this directionality of activity which ‘creates’ time, and thus explains why time, motion and space were identified as an aggregate earlier in this section. And finally, it scarcely needs repeating, in this world, the world of monads, there are no parts; but there are extrema, and these in turn comprise the foundations of phenomena.

This entails treating rest as a special case of motion, similarity as special instances of difference, rays as special species of curves, and so on. In the perspective of infinity, these inequalities are eroded. All magnitudes tend towards a limit. Importantly the conception does not propose that ‘all things hang together’, as in our naivety we often put it, but rather that all things are founded on this connectedness. Moreover there is an edge against materialism in this of which we have already heard much, but here is a further context. The continuum is apt to reveal that atoms are not only incoherent as a concept because matter is intrinsically compounded of parts and no ultimate part is conceivable, but especially that such an assumption proposes in the first instance a fineness, an evanescence of matter such that the very term matter becomes meaning-

70 Cf. his objections to Galileo, §4, supra.
less. Matter reduced to such finality leaves matter behind: it leaves as its ultimate residue the pointillist force with which we began.

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H) SHAPES, LIMITS AND BOUNDARIES

1. Folds, fractals and figments of the imagination

Matter is, of course, formed matter. As such it abides by the definition of Descartes — until we find it necessary to resort to metaphysical principles. Formed matter, for example, displays itself in discriminable shapes and occupies specific and unique locations in space such that only one object can ever occupy the particular volume of space which at any given instant in time it factually occupies. In terms of material reality we then speak of impenetrability and resistance to displacement.

Phenomenally this is unobjectionable; but metaphysically regarded this notion is full of unstated and unchecked assumptions. Contra Descartes, for example, Leibniz maintains that “the notions of size, shape and motion are not as distinct as imagined and […] contain something imaginary and relative to our perception.” Now in one sense this is an obvious and trivial observation that anyone without any training in metaphysics could make; but it is double-barrelled as all good philosophical assertions are. For the philosopher, in adopting this commonplace truth has already thought out its consequences, which are so far-reaching that no-one before Leibniz seems to have plumbed their full depth. For from the initial *déjà-vu* statement (which would not be unexpected from the mouth of a pastry chef) that “there are few, if any, precise shapes (outlines) in nature,” to the insight that such precise shapes are in truth *impossibilities*, is a very long way conceptually.

Historically the career of this notion began in the Athens of Socrates and/or Plato, who noticed that the utmost effort to emblazon a surface with a perfect geometrical figure, e.g. a circle, was doomed to failure. Thus Socrates and/or Plato would certainly have applauded Leibniz’s claim as to the imaginary component in the locution ‘circle’.

For it is accessible to cognition and visualisable in imagination, but denied to the hand and even to nature herself. But even intellectual imagination is not as perfectly equipped as we would wish — many mental images are vague to the point of being un-serviceable, as Leibniz stressed in trying to force it to produce a mental semblance of a chiliagon. “Instead,” he writes,

> in the mind there is a thought of uniformity … we apply uniformity to this image afterwards. … Therefore it must be said, rather, that when we sense a circle or polygon, we never sense uniformity in it, but neither do we even sense a nonuniformity, that is to say, we do not remember having sensed anything nonuniform in it, since the inequality did not immediately strike us. And because of this memory we now ascribe the name of uniformity to it.72

And the longer we fix our philosophical gaze on this problem, the more intractable it becomes. From the realisation that the concept of a circle, even its mathematisation, both involve the mind for attainment of a perfect image, Leibniz advanced to a posi-

tion of no return in respect of the cognition of all and any shapes in nature, viz. that any shape whatever necessarily relies on its idealisation. He put this insight into the title of one of his papers dating from 1686: "There is no Perfect Shape at all in Bodies".73

Adams introduces the problem in the following terms:

Does shape, as a phenomenal property, belong to bodies (since after all bodies themselves are only phenomena) or only appear to them? "No determinate shape can be assigned to any body", Leibniz claims ... and then adds the qualification, "although even in an infinite series’ departure from a path [deviatio] some rules are observed by nature,"74 suggesting an infinite series of shapes in the series more and more adequately expressing reality. Every shape in the series, however, will still only be finitely complex [sic!] and for that reason among others will still only be an appearance, qualitatively different from the reality expressed, which is infinitely complex and does not literally have a shape at all.75

The argument is impeccable: The infinite regress involved in the continuous division of bodies leads to the notion of an infinite continuum. Leibniz adumbrated Descartes’ principle of the percolation of matter through the plenum, which also presupposes an infinitude of unequal spatial gaps – source of the idea of a liquid matrix which was Leibniz’s stand-by conception of the materiality of the cosmos.76 In one of his few enthusiastic encomia of Descartes’ work, Leibniz acclaimed these paragraphs as the “most beautiful and worthy of his genius”.77 The downside of the argument is, according to Leibniz, that in itself it requires an infinite regress of divisibility of both the plenum and all its parts – a mysterious cascading of infinities which leaves imagination and cognition straggling in a helpless limbo. For the continuum of matter cannot be conceived as either a smooth plenum or a collection of points. It seems inconceivable how to effect this needful accord. Leibniz’s solution to this problem is a magnificent tour de force of imaginative thinking:

There is a big difference between a perfect liquid and a body that is everywhere flexible. … If a perfectly fluid body is assumed, a finest division, i.e. a division into minima, cannot be denied; but even a body that is everywhere flexible, but not without a certain and very unequal resistance, still has cohering parts, although these are opened up and folded together in various ways. Accordingly the division of the continuum must not be considered to be like the division of sand into grains, but like that of a sheet of paper or tunic into folds. And so although there occur some folds smaller than others infinite in number, a body is never thereby dissolved into points or minima. On the contrary, every liquid has some tenacity, so that although it is torn into parts not all the parts of the parts are so torn in their turn; instead they merely take shape for some time and are transformed; and yet in this way there is no dissolution all the way down into points, even though any point is distinguished from any other by motion. It is just as if we suppose a tunic to be scored with folds multiplied to infinity in such a way that there is no fold so small that it is not subdivided by a new fold; and yet in this way no point in the tunic will be assignable without its being moved in different directions by its neighbours. … And the tunic cannot be said to be resolved all the way down into points; in-

73 A iv 310, LaC 296.
76 Cf. Descartes, Principles II 33-35.
77 Crit. Thoughts, G IV 370, L 393.
stead, although some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain mere extrema.78

Samuel Levey comments that “in Leibniz’s view, the key to this new account of the continuum is to understand that the continuum is nowhere uniform in its structure.”79

The implications are immense and cannot in this study be pursued in all their ramifications. Keeping it simple, and without pausing even to contemplate such a rich shape as a clover leaf, consider that radically simple object ‘straight line’. What we know about light beams today, those paragons of straight lines, serves amply to confirm that Leibniz was on the right track. Photons are oscillating phenomena, dependent in their propagation on the alignment of axes, perfect physical contiguity and absolute simultaneity of their rhythm of activity, none of which can be vouchsafed to escape from the interdict of infinitary discrepancies as described in the Pacidius passage. All this translates into a cognitive awareness that ‘straightness’ in its finest grain of structure is an ideal condition, a metaphysical notion, which cannot remotely be acknowledged as a reality for phenomena. Levey’s commentary on this is illuminating and deserving of quotation in extenso:

On Leibniz’s account this line segment cannot be exactly straight; rather it is divided by motion into many distinct subsegments — folded as it were into many smaller pieces, each of which is turned in a slightly different direction from its neighbours. Adjacent pieces of line are strictly discontinuous from one another and only contiguous, for the points at which they ‘touch’ are in fact points of discontinuity and preserve the continuum only in the sense that no empty space can be assigned between the touching endpoints. Those points of discontinuity … will preclude the line as a whole from forming a smooth curve. … But the smaller pieces of the line cannot be perfectly described by straight lines or smooth curves any more than could be the original, for each of them is likewise further divided into smaller folds, and so on ad infinitum.80

The suggestion of a fractal structure is close at hand, as Levey points out at once; and this elicits the conclusion that,

taking the image of the folded tunic in the most straightforward way, it will indeed be impossible to assign any traditional geometrical shape to it … The Leibnizian tunic and the simplified case of the folded line are structures that might be approximated by infinite series of geometrical figures of increasing complexity, but every shape of the sort that can be rigorously defined in traditional geometry must fail to describe the tunic or folded line exactly — and fail to do so at infinitely many different points.81

The discongruity is evidently metaphysically relevant, but by no means fatal on that account to a physics which must deal with such objects as physical existents; as Leibniz writes elsewhere:

Even if there were not, and could not be, any straight lines or circles in nature, it would be enough if it is possible for there to be figures which differ from straight lines and circles so little that the error is less than anything given. This is sufficient both for the certainty of demonstration and for its use.82

78 Pacidius, A iii 78; LoC 185–7.
80 Loc cit.
81 Ibid, p. 77.
82 Ars Magna, Ct. 429f, P 1.
Simmering in the background of this passage is an unexpected further implication: namely that the much-touted mathematisation of nature may not be so close to realisation as its chief propagandists Galileo and Descartes thought – it may indeed be ultimately un réalisable except for its applications in dynamics and its allied sciences. A remarkably prophetic insight!

2. Self-similarity, scale invariance, implicate order

One of the most fascinating features of these studies is the infinite recurrence of self-similarity and scale invariance. It is apparent that many of Leibniz’s interlocutors thought his pronouncements on this matter to be pretty bizarre; though for a generation brought up on computer simulations, none of this poses any conceptual barriers and it seems altogether fitting to bring some such illustrations into this study – the more so since (as we saw above) Leibniz scholars are already on the qui vive with respect to the virtues of such comparisons.

When Leibniz told Malebranche that “as the data are ordered, so the unknowns are also ordered” and Bayle that “objects of enquiry are ordered as the order in the data,” he was making the point that the conception of order is wider than indicated by phenomena alone. For since phenomena show order, there must necessarily be an underlying and higher (metaphysical) order from which the visible order derives; and this order in turn derives from the ‘logical objects’ which populate the continuum. Physicist David Bohm, for example, was fascinated by ‘implicate and explicate order’, which are terms with precisely the same meaning as Leibniz’s enfolding and unfolding and concern complex phenomena, whose structural influence is hidden from view. If we recall the quotation on folds from the Paedius (supra), we may find in the following a particularly impressive and illuminating specimen.

Envisage (he writes) two cylindrical bottles on a common axle, one inside the other, with the outer bottle free to rotate and the cavity between them filled with glycerine. Turning the outer cylinder, the glycerine clinging to its walls will be dragged along, but the rest offers resistance, so that all the glycerine is eventually drawn out into long threads. Now the crucial point:

If a drop of indissoluble ink is placed in the [glycerine], then it becomes possible to follow the movement … by watching how the drop is drawn out into a thread until eventually it becomes so fine as to be invisible.

Reversing the motion produces the startling result that

the fluid element will in fact retrace its steps exactly. Eventually the element will return to its original form and the droplet of ink will appear as if from nothing. ... Clearly, what was taken for a chaotic or random loss of order was in fact a hidden order of high degree that was generated out of the initial simple order [and] transformed back into the original simple order when the cylinder was reversed.

Now the Leibnizian ‘implication’ is revealed if successive drops are added while the cylinder rotates and enfolds one after another. If we vary the spots where ink drops are left to fall, then, on reversal they reappear in a line that seems as if it moves

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83 Malebranche, L 351.
84 Bayle (1687), W 66.
86 Loc. cit.
across space. “If the movement is rapid enough, this will give the impression of a particle that crosses space along a trajectory.”

The point of this exercise is to demonstrate implicate order in the glycerine, hidden in the enfolding action. “What is essential to such an order,” Bohm and his co-author write, “is the simultaneous presence of a sequence of many degrees of enfoldment with similar differences between them. . . . Such an order cannot be made explicit as a whole, but can be manifested only in the emergence of successive degrees of enfoldment.” This imagery is related conceptually to quantum physics as an explanatory route to understanding particle behaviour; for it is also possible, in this experiment, to arrange the droplets in such a way that their behaviour matches the continuous path of an electron replete with occasional orbital jumps in accordance with their quantum states.

This example may stand indeed as a crown witness to the enduring relevance of Leibniz’s continuum studies. Bohm’s own summary might have been written by Leibniz without any change whatever:

In the enfolded order, space and time are no longer the dominant factors determining the relationships of dependence or independence of different elements. Rather, an entirely different sort of basic connection of elements is possible, from which our ordinary notions of space and time, along with those of separately existing material particles, are abstracted as forms derived from the deeper order. These ordinary notions in fact appear in what is called the explicite or unfolded order, which is a special and distinguished form contained within the general totality of all the implicate orders.

To be discerned in these examples is a concern to ‘save phenomena’ at the same time as ‘ultimate reality’ is sought out – the world is as it is; but it is a structure of exceeding complexity which can and must be described, for the sake of accuracy in both its phenomenal and metaphysical dimensions, as a double-aspect ontology. Concentrating on one aspect at the expense of the other leads to aporias – a fact that needs no profound insight to be manifest to any reader of philosophical texts. The span of Leibniz’s imagination offers us one of the most remarkable solutions to the ‘eternal problems’ of philosophy, whose primary virtue is that, as one commentator proposed, it may even be true. Meanwhile we can do no better than to attend to the descriptive summation offered in Levey’s aforementioned paper, which does not mention the phrase ‘saving phenomena’, but the intention shines through nonetheless:

Sense perception presents to the subject of experience the appearance of a world of precise shapes; the manifest image (to borrow Sellar’s term) of corporeal reality is thus very much a Cartesian image of geometrically precise bodies. But the scientific image, itself only fleetingly sensed in consciousness, is of an infinitely more complex world. Corporeal reality behind the appearances is an infinitely divided fractal world, and it is only the activity of the mind, and of the imagination in particular, that smoothes over its rough edges and presents the world in experience as if it were a Cartesian geometrically uniform one. Of course, closer inspection, say perception aided by microscopes, might bring to light more of the underlying irregularities in things; but at any given scale of inspection, ‘most’ of the infinitely many details of the division of matter will fall below the threshold of sustained conscious experience. The precise shapes that appear in experience are thus imaginary, or at least involve something imaginary, insofar as the

87 Loc. cit.
uniformity they appear to contain is not actually present in nature. … Note however, that Leibniz's holding a certain notion involved in the content of experience to be imaginary does not automatically mean that there is nothing in the world behind the appearances to correspond to the experience.90

We are limited, as has been said, by the fact that we are denizens of a particular dimension, and by the further fact that our senses evolved for the primary purpose of detecting features in the environment which are important to us, but whose detailed or fine structure composition is (on the whole) irrelevant to this context. Precise shapes, straight lines and smooth curves are therefore the products of a kind of transformation mechanism with which our mind is imbued and which it performs virtually automatically. By the same token, we are also endowed with a cognitive capability which provides us with the means of self-reflective analysis of those very phenomena and thus, in imagination, to decompose (reduce) them in the effort to reveal what is normally denied to us, i.e. the aforementioned fine structure. As the scientific developments of the last 300 years have shown, such knowledge is apt to be 'useful' in the improvement of living conditions. But philosophically regarded, this same faculty enables us to represent corporeal reality in a quasi-dimensionless setting, as aspects of reality which transcend those which our immediate access denies us. In the final analysis, however (it must be said), it is surely an error to seek in those 'deeper' realities an 'only' reality. Leibniz's researches are thus of help to us in understanding that the dimensional structure of the world is a structure of aspects of the one world. That world may only be approximated by our sensory apperceptions; but as Leibniz's double-aspect ontology reveals, it is an approximation of an actually existing world, which no metaphysics purporting to report on our understanding of the truth is entitled to dismiss.

3. The infolded order

Accordingly we must now also turn to what Leibniz brought into the picture in his notes for Fardella, viz.:

… there are substances everywhere in matter, as points are in a line. And just as there is no portion of a line in which there are not an infinite number of points, there is no portion of matter which does not contain infinite substances. But just as a point is not a part of a line, but a line in which there is a point, so also a soul is not part of matter, but a body in which there is a soul. … I don't say that body is composed of souls, or that body is constituted by an aggregate of souls, but of substances.91

This has struck commentators as a typical Leibnizian 'hieroglyph'; but for us at this stage of proceedings it no longer holds any terrors. We have seen it all before, in one form or another, so that nothing more than a brief recapitulation is necessary. If points are not parts of a line but limits, then it stands to reason that we are not dealing with geometrical objects.92 Hence, logically, “even an infinity of points gathered into one will not make extension.”93 Rather, “extension arises from situation, but it adds continuity … Points have situation, but they neither have nor compose continuity.”94 Accordingly the analogy which Leibniz is drawing in the Fardella quotation suggests that, as points and lines stand in the described relationship, so this is similar to the

90 Levey, pp. 80-81.
91 Fardella, AG 104-5.
92 First Truths, I 270.
93 Des Bosses, I 597.
94 Ibid., I 598.
relationship in which souls and body matter stand – as he told Fardella, “What points are in the imaginary resolution [of the line], souls are in the true [resolution].”

We can, this passage says, resolve a line into its point-like constituents only in imagination, because points fix merely the extremities of the line. The points so to speak individuate the line or its segments, which facilitates working with them as geometrical objects, but clearly this does not imply that we are dealing with enumerable entities. Leibniz’s letter to Gabriel Wagner amplifies this point in the depiction of body as, in some ways, part of the plenary circulatory pattern even while retaining its attributes of uniqueness:

To each primitive entelechy or each vital principle there is perpetually united a certain natural mechanism, which comes to us under the name of an organic body; which mechanism, moreover, even though it preserves its form in general, remains in flux, and is, like the ship of Theseus, perpetually repaired. Nor, therefore, can we be certain that the smallest particle of matter received by us at birth remains in our body, even though the same mechanism is by degrees completely transformed, augmented, diminished, involved or evolved. Hence, not only is the soul everlasting, but also some animal always remains, although no particular animal ought to be called everlasting, since the animal species does not remain; just as the caterpillar and the butterfly are not the same animal, although the same soul is in both. Every natural mechanism, therefore, has this quality, that it is never completely destructible, since however thick a covering may be dissolved, there always remains a little mechanism not yet destroyed, like the costume of Harlequin in the comedy, to whom, after the removal of many tunics, there always remains a fresh one.

It is unfortunately still a common mistake to attribute panpsychism and other doctrines of this ilk to Leibniz, which do not conform to his explicit differentiations, of which the above is merely one instance. However, let an example illustrate the meaning of this discussion. There is a kind of ‘primeval’ image of a circle, which is a point on its periphery being set in motion and so to speak dragging a curve behind itself as it moves in accordance with the formula for its movement. Evidently neither the point nor the curve are ‘real’ in any sense, and yet the curve is being ‘generated’ in the movement. Here the curve, and thus the whole circle, are already implicit in the formula which is, of course, a kind of algorithm for the production of a circle from a point in motion. That circle is therefore the product of mathematical reason and its extension a purely imaginary thing! Ontologically speaking, the circle does not exist ‘in itself’, it is a mere phenomenon of the implied motion (conatus) of the point.

But where the ‘analogy’ makes its entry is in the following. The point is clearly not extended; yet it has a relation to extension which may be understood as a motion or may be defined in morphological terms. It is, as Leibniz describes this and similar situations, a form infolded. Appearance is the unfolded extensionality. The question that arises out of these considerations is, therefore, whether Leibniz asserts herewith an ontology of the point as a universal. This is indeed how many scholars prefer to understand it, e.g. Fritz Kaulbach, who provides a link from Leibniz to Kant in the words that the latter “refrained from the outset from such an universal ontology of the ‘point’ and proceeded instead from the ontology of appearances”. The degree of

95 Fardella, loc. cit.
96 Wagner, E 465-8, W 505-6.
relevance in this comparison will not escape the reader of Kant. Yet this scholar cut through very close to the essence of this matter:

This ‘motion’ is ontological, for it occurs in an ‘instant’, e.g., an order of that temporal dimension which lies beyond boundaries describable even in imagination. The point, understood in the sense of striving, does not fill space as if suddenly it had acquired extension, but in such a way that it comprises, in itself, the generation and production of extension. This is why Leibniz says that it cannot be depicted in or as a picture. Motion in the ‘instans’ is therefore such a punctual striving [whose determinations] relate to the character of motion in the domain of the point, which are ontological in their nature, while the corresponding determinations which rest upon the extension of phenomenal things belong to the domain of physics thinking.98

Thus Kaulbach almost, but not quite, arrived at an ontology of pure motion. He draws back from his own findings eventually by accusing Leibniz of ‘confusing’ the point from which motion and extension arise (“which makes of it a thing smaller than all possible delimitations”) with the point which resides in extended figures and appearing motion. But this is tantamount to accusing Leibniz of wanting it both ways and being unable to discern the aporia this brings about.99

The analogy to bodies is revealed, however, in Leibniz’s assertion that “there is no portion of matter which does not contain an infinity of substances”.100 But this does not lead to the conclusion that ‘therefore’ body is an aggregate of souls, as a prevalent misunderstanding has it: rather that a portion of ‘ensouled’ body is matter. In reading this together with Leibniz’s repeated claims against Descartes, that body does not consist in extension alone, we are driven to enquire what else Leibniz may be thinking of. And the answer is that there is reciprocity in the above relations between soul and body, such that a substance can lay no claim to actuality unless endowed with body. This was a conclusion we have already arrived at in Part I; but it serves well in the present environment to see a clear reinforcement.

4. The explicate order and self-organisation

As a reminder, a quote we have seen before:

In a fish pond there are many fishes and the liquid in each fish is, in turn, a certain kind of fish pond which contains, as it were, other fishes or animals of their own kinds, and so on to infinity.101

No doubt someone could write an algorithm to give us the self-repeating fish in the pond as a computer projection; and if Leibniz could see the ‘Gingerbread Man’ of the Mandelbrot Set, he would undoubtedly exclaim that this is what he meant. For the infinite regress of morphs reduplicating through an infinity of dimensional layers gives us just such a nested world-within-worlds scenario as the Fardella quotation implies. The expression ‘self-similarity’ is replaced by ‘scale invariance’ in fractal theory to denote identical iteration of morphs through all (infinite) dimensional layers. The very concept at the heart of the Leibniz quotation!

It is worthy of remarking, incidentally, that any glance at the programs required by an average computer to generate a fractal ‘world’ also serves as a demonstration of

98 Ibid., pp. 43-4.
99 Ibid, p. 46.
100 Fardella, loc. cit.
101 Ibid.
Leibniz’s great principle, *The greatest richness from the least hypotheses*, for the algorithm could not be simpler and the results not more staggeringly complex. For example, here is all one needs to generate the Mandelbrot set:

\[(x, y) \rightarrow (x^2 - y^2 + \Lambda, 2xy + B),\]

where A and B are fixed numbers. Had Leibniz known anything of this formula, he would undoubtedly have adopted it as ‘God’s Formula’. For although we cannot produce anything other than *images* with it, to his God it may well have been the secret of the creation of the cosmic structure. Indeed, as we shall see in a moment, fractals have been divulged as a generative principle with well-nigh inexhaustible applicability to living things.

Now Leibniz did not have the term ‘fractal’ in his vocabulary. But I submit that this means very little; it is the idea which counts; and at the bottom of this we need merely to ascertain whether Leibniz’s idea is the same, or at least a close relative. The answer is affirmative, as Levey’s discussions (quoted *supra*) have already strongly suggested. It is reinforced by the definitions offered in a recent textbook on fractal phenomena:

A ‘mathematical’ fractal in a certain precise sense looks the same at all scales; i.e. when examined under a microscope at no matter what magnification it will appear similar to the original object. On the other hand a ‘physical’ fractal will display this ‘self-similarity’ for only a range of magnifications or scales. The mathematical object will of course only be an accurate model within this particular range.\(^{102}\)

From this description it is evident that Leibniz’s ‘infinite fish’ will not literally (physically) reduplicate infinitely, but will presumably encounter a limit to iteration at some (as yet unknown) magnitude. Unlimited scale division is restricted to mathematics – a point made, incidentally, almost 2400 years ago by Democritus.\(^{103}\) However, it is likely that Leibniz would retort that this is an argument derived from the dimensional perspectives accessible to us: that if we were somehow able to be transported into the dimension in which atoms and electrons figure as ‘macroscopic’ bodies, we would see things in a different light. Evidently there is no way to settle this dispute.

In the previous section, we arrived at one stage on the argument that for Leibniz the creation of the cosmos entailed nothing more than the creation of monads: given certain rules of assembly, these would proceed to ‘self-organise’ appropriately. One would almost wish that he could see from some window in heaven the fulfilment of his prophecy in certain recent discoveries. An example of such a ‘law of self-organisation’ was in fact known to him, although he was not by any stretch of the imagination in a position to make that connection. For if we ask, how do elements in a complex aggregate form themselves into a coherent and self-sufficient unit, then we need go no further than this magnificently simple algorithm:\(^{104}\)

<table>
<thead>
<tr>
<th>variables:</th>
<th>A, B</th>
</tr>
</thead>
<tbody>
<tr>
<td>constants:</td>
<td>none</td>
</tr>
<tr>
<td>start:</td>
<td>A</td>
</tr>
<tr>
<td>rules:</td>
<td>A, B</td>
</tr>
</tbody>
</table>

Setting this in motion (A, B, AB, BAB, BABBAB, BABBABBAB etc.) produces the
Fibonacci series, or Golden Section, which has been known and spoken of in hushed
tones of admiration since the days of Plato. It can be depicted as a spiral (nautilus
shell, kernels of heliotropes), but of the essence for our purposes is the point that the
program displays the iterative properties which lead to self-similarity and fractal pat-
terns. All these algorithms are brilliant exemplifications of Leibniz’s principle.

But is this Leibnizian emphasis relevant? — Indubitably; for one of the major
branches of complexity science is concerned with the self-assembly by self-propagation of
forms of life: it is precisely the kind of discovery Leibniz would have applauded as an
instance of the profound wisdom of God in actualising just one law (“the least number of
hypotheses”) that can spawn innumerable instantiations of branch types (“maximal rich-
ness”) to aid organic nature to self-help in the survival struggle as well as to produce
the greatest possible variety of forms from a comparatively small handful of self-
propagative rules.

I note in support of this contention that David Green proceeds to exhibit other al-
gorithms illustrative of the growth of algae, cell differentiation in infants, the foraging
behaviour of ant colonies etc., all related to organic self-assembly from one set of
genesis. Leibniz however, did not restrict this form of creative propagation to living
things, proposing it as an universal principle and therefore applicable to pure pheno-
mena as well — recall his specimens of ‘least action’ and the brachistochrone as ex-
emplifications of the operation of divinely ordained laws to which these phenomena
adhere. Accordingly it seems apt to bring forward at this point an illustration of ‘dead’
matter organising itself in a pattern based on the Fibonacci series, which was as well
known to Leibniz as it is to us, and thus its ubiquity as an ordering principle among
phenomena would not have astonished him.

The series, as we know, can be derived very simply by writing down a sequence of
numbers, where each succeeding term is the sum of the two preceding terms, viz.:

\[
1 \ 1 \ 2 \ 3 \ 5 \ 8 \ 13 \ 21 \ 34 \ 55 \ 89 \ldots \ldots ...
\]

There are several methods of construction. The one that applies to the example we
have in mind is this: If one divides the perimeter of a circle by this ratio and draws a
diagonal from each of the resulting points, they form angles of 137.5°. With this we
are ready to attend to a neat experiment:

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105 Although the term ‘Fibonacci series’ is becoming a well-known ‘trade term’ in mathematics, the
man after whom this series is named is still woefully underrepresented in historical works, for
reasons which it is difficult to comprehend. Yet Leonardo Pisano (1170-1250), the son of Bo-
naccio, having spent many years of his life among the Muslims, brought back to Europe in his
Liber abaci (1202) and his Liber quadratorum (1226) the zero, decimal notation and algebra, surely
a momentous occasion for Europe which ought to be properly commemorated. Yet an author-
ity of the standing of Morris Kline, in his monumental 1200-page history of Mathematical
Thought (op. cit.), asserted that no mathematician worthy of the name appeared in Europe be-
 tween AD 300 and 1400 and ‘treats’ of Fibonacci in two paragraphs. This can only mean that,
since Fibonacci did not invent the zero etc., it isn’t worth dwelling on his achievement; but it
must leave us wondering about the multitude of diligent workers who survive in his pages
without having much more accomplishment to their names than one or another minor modifi-
cation to other people’s original work.

106 Green, op. cit., p. 15ff.
[The experimenters] let drops of a ferrofluid fall at the centre of a disc covered with a film of oil in which the drop floated. A magnetic field polarised the drops, so that they became little magnetic dipoles that repelled each other. ... As the drops fell at the centre of the disc, they were also exposed to a steady magnetic field that pushed them out from the centre towards the edge of the disc.107

In a word, the two-fold repellence, from the centre and from each other, forced the drops to adopt each for itself a particular vis-à-vis to all other drops, those already on the disc and those still arriving.

As the rate of adding drops [is] increased, a new drop experiences repulsive forces from more than one previous drop and the pattern changes: the initial simple symmetry of the alternative mode gets broken and a spiral pattern begins to appear. ... [if the rate is rapid] then a steady pattern emerges rapidly and successive drops quickly settle into a divergence angle of 137.5º, the spirals obeying the normal Fibonacci series.108

The illustration shows the final pattern state representative of the Fibonacci series; we recognise without difficulty the disposition of sunflower kernels. All this was, of course, completely unknown to Leibniz. But the man who divulged the secret of the brachistochrone would have recognised the identity of principle in a flash.

5. Space, time and the ‘immeasurable’

I will show here how men come to form themselves the notion of space. They consider that many things exist at once, and they observe in them a certain order of coexistence, according to which the relation of one thing to another is more or less simple. This order is their situation or distance. When it happens that one of those coexistents changes its relation to a multitude of others, which do not change their relation among themselves; and that another thing, newly come, acquires the same relation to the others as the former had, we then say it is come into the place of the former; and this change we call a motion in that body, wherein is the immediate cause of the change. ... And supposing, or feigning, that among those coexistents there is a sufficient number of them which have undergone no change, then we may say that those which have such a relation to those fixed existents, as others had to them before, have now the same place which those others had. And that which comprehends all those places, is called space. Which shows that in order to have an idea of place, and consequently of space, it is sufficient

to consider these relations, and the rules of their changes, without needing to fancy any absolute reality outside the things whose situation we consider.\footnote{Clarke, V, 47.}

That Leibniz promoted the idea of space as a relational structure is well-known, as well as the fact the he denied the existence of space-as-such. In other words, Leibnizian space is so to speak ‘created’ by objects — being extended, they convey the impression of being spatial, that is, enclosed by a spatial volume. From this we derive our sense of space and the psychological notion of it being prior and independent. This, in simplified rendering, is the conception held by Leibniz. But although it has solid textual support behind it, it is not the full story. There is more complexity to it, as shown by Richard Arthur in a paper delivered at the Fifth International Leibniz Congress. He writes that “Leibniz’s space is an explicitly instantaneous space,” pointing to a passage in which Leibniz defines space as ‘the order of existence for states which are simultaneous’.\footnote{Initia rerum, L 666.} Innovatively, and I think correctly, he reads this as designating space as an instantaneous expression of changeable collectives of phenomena, and that this cannot therefore be an enduring space. But “if space for Leibniz is simply an instantaneous order of existence, then strictly speaking it does not exist in time” and must be an idea arising in the mind. Therefore space is an ideal thing.\footnote{Richard Arthur, “On the Unappreciated Novelty of Leibniz’s Spatial Relationism”, in V. Internationaler Leibniz Kongreß: Leibniz, Tradition und Aktualität (ed. Ingrid Marchlewitz), Gottfried-Wilhelm-Leibniz-Gesellschaft, Hannover 1988, p. 28.}

Once again the labyrinth hovers over matters on which the mind desires clarification, but tends to get confusions:

The source of our difficulties with the composition of the continuum comes from the fact that we think of matter and space as substances, whereas in themselves material things are merely well-regulated phenomena,\footnote{Rémond, G III 611-13, L 656.} and space is exactly the same as the order of co-existence, as time is the order of existence which is not simultaneous. Insofar as they are not designated in extension by factual phenomena, parts consist only in possibility; there are no parts in a line except as there are fractions in unity. But if we assume that all possible points actually exist in the whole – as we should have to say if this whole were a substantial thing composed of all its parts – we should be lost in an inextricable labyrinth.\footnote{Nouv. Ess. II, xiii, §17.}

The phrase “time is the order …” is underlined by Leibniz, with the evident intention of emphasising a particular understanding he wishes to convey respecting simultaneity. How this intriguing notion is to be understood, Leibniz makes clear in another text where he writes, “space … is an order not only among existents, but also among possibles as though they existed.” This does strike us as a somewhat bizarre supposition; but Guerout has this explanation to hand:

The order of possibles in God is not a spatial order; not only does it occur only with respect to things that are without parts, unextended and real at the same time, but it is also an order to exclusion or of logical tolerance that engenders the situation, without being conditioned by it. Thus, what one calls the point of view of the monad, the point of view
from which it reflects the universe, is only the result, in the actual world, of this logical nature, and is conceivable only with respect to this existence.¹¹⁵

Important issues are explicated here in exemplary clarity. Most notably Leibniz stresses the important point that the continuum is a labyrinth because we bring insupportable assumptions forward in our endeavour to grasp its meaning. One such assumption is that phenomenal entities should be capable of being fully described, i.e. comprised of enumerable parts. But if we accept the criteria of composition, then we must discard this belief: the term ‘continuum’ bearing precisely the meaning that things do not have firm and fixed boundaries, but are at their boundaries revealed to lie in series (continuous with) with the plenum. These boundaries, accordingly, do not ‘comprise’ the points by which entities might be defined; rather the points (in principle non-enumerable owing to their continuity with the ‘surrounds’) make for foundations of these boundaries – the boundaries determined by these points are in fact part of the flux of material existence. This applies also to time, as Leibniz explained to Bourguet:

The instant is indeed the foundation of time, but since there is no one point whatsoever in nature which is fundamental with respect to all other points, … I see likewise no necessity of conceiving a primary instant. I admit, however, that there is this difference between instants and points – one point of the universe has no advantage of priority, not merely in time but in nature, over following instants.¹¹⁶

Space and time must be understood, it seems, in terms of the diffusion of spatial existents; an idea which derives of course from the underlying conception of existents being composites. Accordingly there are two kinds of continua, “the one successive, like time and motion, and the other simultaneous, that is, made up of coexisting parts, like space and body.”¹¹⁷

Now it is an empirical fact that continuity among phenomena seems frequently to be broken; but, as Drozdek observes, “The law of continuity can be fully applied to the ideal, and although it is not fully applicable to the actual, it is not thereby suspended; only its manifestation is limited.”¹¹⁸ From this criterion we may also apprehend why for Leibniz the physical extension of body must necessarily precede (be metaphysically more fundamental than) space. Drozdek explains:

Both real and ideal worlds are continuous, but continuity is understood differently. Therefore the difference is made not between discrete and continuous, but between two kinds of continuity. This should be clear from the statement that “all repetition is either discrete as where parts are discriminated … or continuous when the parts are indeterminate and can be assumed in infinite ways.”¹¹⁹ Hence, the ideal continuum is indiscriminating, the actual is discriminating. We can describe matter as discrete, meaning thereby not a discrete set but a set composed of parts, and each part is, to be sure, different from another part, and in this sense discrete or discriminate. So the gap between the

¹¹⁶ Bourguet, G III 580-3, L 664. – Loemker as editor adds the following instructive note: “The irreversibility of temporal series is thus implicit in Leibniz’s logic of existence.”
¹¹⁸ Drozdek, op. cit.
¹¹⁹ Body & Force, G IV 394, AG 251.
ideal and the actual worlds is not unbridgeable, since continuum can be found in both of them. [italics added].

In this juxtaposition two different kinds of order, two different kinds of series, are delineated; so that the extension of space undergoes in our intuition a peculiar transformation where it will seem to us that it is countable, as if made up of parts, in the same way that we think of time as countable; and both intuitions are then reinforced by our practice of ‘measuring’ both space and time, our intuition taking this succession of ‘parts’ as the ‘constituents’ of space and time:

For, since extension is a continuous and simultaneous repetition (just as duration is a successive repetition), it follows that whenever the same nature is diffused through many things at the same time, as, for example, malleability or specific gravity or yellowness is in gold, whiteness is in milk, and resistance or impenetrability is generally in body, extension is said to have place. However, it must be confessed that the continuous diffusion of colour, weight, malleability and similar things that are homogeneous only in appearance, is merely apparent and cannot be found in the smallest parts. Consequently it is only the extension of resistance, diffused through body, that retains this designation on a strict examination.

It is nonetheless not contradictory to assert that while discriminable parts populate the world of actual existents and nothing perfectly uniform can ever be found in nature, the principle of continuity holds in this world as well. Infinity pervades the world of existents as much as the ideal spectrum of metaphysics, for it is the grounding they possess in reality. Thus:

Infinity is a platform on which these two worlds meet; it enables possibilities to be realisable as existents, it enables the existents to be considered as emerging from possibilities. … Infinity is also a platform enabling an order in these two worlds. First, the realms of the possible and the actual are ordered through the continuum; and the principle of continuity expressly states the nature of this orderliness of the continuum. A harmony is only possible through a continuum, and hence, through infinity.

As Leibniz writes in confirmation: “The actual phenomena of nature are arranged, and must be, in such a way that nothing ever happens which violates the law of continuity.” In its applicability to space and time, it is logically impossible, however, for space or any part thereof to have extension per se; only what is ‘in’ space (metaphorically speaking) is divisible in so far as it is extended. Accordingly Leibniz felt induced to disinter from ancient thinking a term which had not been used since Anaximander coined it, the apeiron:

Space, by the very fact that it is divided into parts, is changeable … it is continually one thing and another thing. But the basis of space, that which is extended per se, is indivisible, and remains while changes last; … I think it best to call it the ‘immeasurable’. So it is the immeasurable which persists during continuous changes of space. … this universal space is an entity by aggregation, which is continually variable; it is composed of

120 Drozdek, op. cit.
123 Drozdek, op. cit.
124 Reply, G IV 554ff, L 583.
spaces which are empty and full, like a net, and this net continually receives new form and is so changed. What remains in that change is the immeasurable itself.\footnote{On Matter, Motion and the Continuum, DSR A 58, 469.}

It is only one step from here – from the dynamisation of extension by ‘reduction’ to its infinitist expression and his solution to the problem of the rota Algazelis – to the consideration that space is entangled in the concept of motion; namely that the motion of body must be described in terms of its \textit{expansion}:

For the more quickly a body is moved the more space it occupies at a definite time. Assuming, then, that in a plenum one body cannot be expanded without another body being contracted, and that one body cannot be contracted without another being expanded, and also that another expansion cannot be understood except by the aid of motion – assuming all this, it follows that for the same quantity of motion always to be conserved is the same as for the same quantity of matter always to be conserved.\footnote{Loc. cit.}

The foregoing with its plethora of mentions of motion shows that body and motion are not items that can be commensurately discussed in isolation from each other (as indeed we have already seen). Leibniz articulated this point in many sketches and papers; here is one such document, where his thesis of the \textit{relativity of motion} is presented as a discovery:

A remarkable fact: motion is something relative, and one cannot distinguish exactly which of the bodies is moving. Thus if motion is an affection, its subject will not be any individual body, but the whole world. Hence all its effects must also necessarily be relative. The absolute motion we imagine to ourselves, however, is nothing but an affection of our soul while we consider ourselves or other things as immobile, since we are able to understand everything more easily when these things are considered as immobile. … It follows that there is no such thing as a vacuum. For motion being relative rules out motion in a vacuum, since there is nothing by which it could be distinguished; but whatever cannot be distinguished, not even by someone omniscient, is nothing.\footnote{Motion is Something Relative, A iv 360, LoC 228.}

This might be termed the ‘equivalence hypothesis’ to distinguish it from the more familiar, but scientific rather than metaphysical, Einsteinian relativity. It purports that, if we are bereft of the impression of immobility we assign to ourselves, we can maintain in our observation of the motion of \(x\) only that it has changed place in relation to \(y\), where \(y\) may indeed be ourselves. But we can have no certainty that it is \(x\) which moves; for it might be \(y\), or both.\footnote{Cf. Leibniz on “the eye”, Part II, Note 36.} But even here Leibniz keeps his feet on the ground of physics, for in the same passage he reassures us that “when we consider motion not formally as it is in itself, but with respect to cause, it can be attributed to the body of that thing by whose contact change is brought about.”\footnote{Loc. cit. – Indeed Leibniz in this paper seems to be exercised somewhat to rebut an imaginary idealist interlocutor, for the passage quoted continues, “when someone asks me why this fire is burning, I answer, because what lit it is burning; … [but if I reply to the question] why this dog is barking, because its father barked, I should have explained nothing.”} So the causes of motion invite considerations of a different cast than are furnished by its metaphysical inspection, because it is a property intrinsic to bodies and this entails that the concepts of motion and rest permit of an unambiguous, indeed absolute, differentiation.

This exemplifies splendidly the main point to be settled here and elsewhere in this study, which is Leibniz’s espousal of a double-aspect ontology for both matter and
motion. It remains to be added that motion, being a property of objects, must be capable of being reduced to force. This is the office performed in writings cited elsewhere in this study, but it is well to keep the emphasis going and reinforce our cognizance of this as an issue on which Leibniz did not relax his stand:

I believe that if motion, or better the motive force of bodies, is something real, as it seems we must acknowledge, it is necessary for it to have a subject. For if \( a \) and \( b \) approach each other, I assert that all the phenomena involved will happen in the same way, regardless of which one the motion or rest is assigned to. … But you will not deny, I think, that each body does truly have a certain degree of motion, or if you wish, of force … from this I draw the conclusion that there is something more in nature than what geometry can determine about it [italics added]. This is not the least important of the many arguments which I use to prove that besides extension and its variations, which are purely geometrical things, we must recognise something higher, namely force.\(^{130}\)

It is force, in short, which ultimately determines whether a thing is in motion or not.\(^{131}\)

6. Apeiron: Existence, dreams and spaces

On the ‘many worlds thesis’ of Leibniz, many divergent interpretations exist; but some offend against Leibniz’s own explicit admonitions concerning their actual existence: there is not (as we’ve had ample opportunity to ascertain) a multiplicity of universes if this is taken to mean universes external to the actual universe in which we live. No similar restriction applies to the possibility of this universe being in itself a ‘multiverse’ or a manifold of universes, which would merely entail a different articulation of the one spatial domain into different dimensions; and again we have seen numerous Leibnizian utterances in affirmation of this claim. Indeed it follows as a logical consequence of his continuum theory, in regard to which we have witnessed him time and again insisting that infinite divisibility implies a kind of Russian Doll cosmos – recall his notes on Fardella that the fish in the pond has drops of water on its body that are another pond with fish in it, and so on \( \text{ad infinitum} \). An interesting aspect to this speculation of his (undoubtedly inspired by the discoveries of subvisible life under the microscope) is that nature does in fact organise animate life along such principles. Thus an intriguing piece in his \emph{Summa Rerum} invites a direct comparison with contemporary empirical knowledge, although we cannot add any empirical findings to his metaphysical speculations based on it.

To set the scene, Leibniz explains the criterion on which knowledge of existents must be based:

On due consideration, only this is certain: that we sense, and that we sense in a consistent way … [so] that a reason can be given for everything and everything can be predicted. This is what existence consists in – namely in sensation that involves some certain laws; for otherwise, everything would be like dreams.\(^{132}\)

Corollarily, our notions of space involve “that by which we separate the place and, as it were, the world of dreams from our own”. But from these two criteria we may draw the consequence that, unbeknownst to us, “infinitely many other spaces and other worlds can exist”, though \textit{internal} to the cosmos, as in the Fardella specimen; and

\(^{130}\) \emph{Huyghens}, G II 179-85, I 418.
\(^{131}\) Cf. the last paragraph of Section G, \S 5.
\(^{132}\) \emph{On Truths, the Mind, God and the Universe}, A 71, DSR p. 63.
these would exist in such a way “that between these and ours there will be no distance.”\textsuperscript{133} We recognise in this an infinitely converging series, just as $2/3 + 2/9 + 2/27$ (etc.) does not transgress beyond the finite boundary of ‘1’. But if anyone should ask if such another world or space could ‘really’ exist, says Leibniz, “[they are] simply asking if there are other minds which have no communication with ours.”\textsuperscript{134}

The interesting comparison that can be drawn here is as follows: that taken as an organic machine on Leibniz’s terms, the human body is entirely constructed and maintained by a billion-fold army of cells. In some senses, however, each of these cells is an individual, and indeed a highly complex structure and in no clearly definable way ‘inferior’ in complexity (other than sheer quantity) from the human machine. We are not in the habit of attributing a cognitive capacity to cells, since our empirical doctrines reserve such refined types of perception to creatures with nerves. But even if they have thoughts, it seems unlikely that communication of any kind whatever could be established by us with that world. The most intriguing feature of this ‘stand off’ is, however, the fact that human thoughts are (as far as we can ascertain) directly dependent on the activity of the neuron cells of our brain – posing the perplexing ‘chicken and egg’ conundrum in this environment as to which comes first? Do our thoughts, emotions and wishes coerce neuronal activity or are our thoughts the responses of neuronal ensembles to the physical changes in their environment?

We must break off, since neither the question nor any conceivable answer are intelligible as we speak. But in returning to the Leibniz/Fardella exhibit, the present comparative scenario will be seen not yet to have run its course. For cells are also constructed from smaller living things, and the ratio between human-body-and-cells to cells-and-constituents (technically referred to as ‘endosymbionts’) is roughly analogous. These smaller existents are in fact so small that they might be conceived as equipped to ‘play marbles’ with atoms. Another world nested within the world of cells, as cells are nested in our world. But this is where empirical research cuts out. We do not know if the cells of cells are made of something other than ‘matter’. Our possibilities of perception in this dimension no longer reveal the presence (if any) of ‘life’.

But Leibniz might well retort to this, as he does in a not unrelated context in the piece under consideration, that “it does not follow from this that there is not another world, or other minds which cohere among themselves in a way which is different from what holds in our case.”\textsuperscript{135} This imagery of an infinity of worlds extending upwards and downwards through an infinity of dimensions, yet all contained in the one universe which we are privileged to inhabit and (partially) to comprehend is surely one of the boldest flights of imagination bequeathed to us by any philosopher in history:

If one imagines creatures of another world, which is infinitely small, we would be infinite in comparison with them. From which it is evident that we, conversely, can be imagined to be infinitely small in comparison with the inhabitants of another world which is of infinite magnitude and yet is limited. From which it is evident that the infinite is – as indeed we commonly suppose – something other than the unlimited. This infinite would be more correctly be called the immeasurable.\textsuperscript{136}

\textsuperscript{133} Ibid, p. 65.
\textsuperscript{134} Loc. cit.
\textsuperscript{135} Ibid, p. 67.
\textsuperscript{136} On the Secrets of the Sublime, DSR A 60, 475.
7. Final considerations on matter, time, space and double-aspect theory

The biggest stumbling block to every reader of Leibniz is undoubtedly the issue of how a zero-dimensional entity (the monad) can bring about three-dimensional objects by simple aggregation. Logically this would seem to beg the point: for as we have already observed, the aggregation of any number would still, surely, occupy only a single metaphysical point? It would be tantamount to asking about creation ex nihilo, which Leibniz’s philosophy does not countenance.

But in merely asking such a question, we are on the wrong track towards its solution. Hence above every such enquiry the motto heading should appear: Refrain from confusing metaphysical with phenomenal entities.

Accordingly we will not find Leibniz stating as bluntly as this that aggregation of monads ‘produces’ extension. Rather – keeping the metaphysical and phenomenal realms cleanly apart – he tells us that extended objects are infinitely divisible. In other words, he begins with extension as a fact of phenomenal experience, and seeks from this vantage point to determine if the concept of a minimum magnitude is philosophically admissible. The answer to this is clearly “no”; but this is not the disaster which it might seem at first glance. For as we have amply seen, there cannot be an actual ‘crossover point’ from the phenomenal to the metaphysical realm; and thus divisibility implies no more than that it may proceed to a magnitude ‘as small as you please’ without attaching the notion to this a claim that it represents ‘the’ smallest possible magnitude.

So our first conclusion will be that the ‘smallest possible magnitude’ and the metaphysical point which would define the extremity of that particle do not coincide. But even the locution ‘smallest’ is already an abstraction that cannot be matched by any existing physical entity. This also holds conversely for the ‘largest’ – we recall in this connection that Leibniz wrote of the fastest possible object that it would occupy every point of the universe simultaneously.

The problem raising its head now is, whether this amounts to an explanatory deficiency. We know from Part I that Leibniz claims for every piece of matter in the universe to be vested in some way with monads; and conversely that no monad can stake a claim to existence without the vestment of a body. The shaft of this claim against idealism is self-evident. But then he further claims that space and time are imaginary phenomena –

Space is exactly the same as the order of co-existence, as time is the order of existence which is not simultaneous ...137

from which it follows that both are ideal entities, abstractions from phenomena by the mind and conveyed to us by our mind as ‘external’ uniform features of the world. This would seem to cancel out any notion of objects ‘occupying’ space or events ‘occurring in’ time. But matters are not that simple.

Space and time do exist in some fashion. Leibniz is simply being careful not to posit existences independent of those features of the world which are integrated with them. Thus space cannot exist without matter, nor time without events occurring. There are no blanks, no vacua. While it is conceivable (though metaphysically impossible) for a vacuum to exist in space, it would have to be surrounded by extended matter and thus be measurable as an empty region. However, there cannot be a vacuum in

137 Rémond, G III 612, L 656.
time, because time standing still or ‘disappearing’ would be unmeasurable and therefore unintelligible. Accordingly the conclusion to be reached is this: that space, being the order of co-existence of matter objects, exists insofar as those objects exist; and similarly with time and the events which occur with matter objects. In a word: space and time are the outcome of the creation of matter. Both exist solely in virtue of the concurrent existence of matter.

The issue is now how far we should be satisfied with these explanations. Matter and matter events being based in the phenomenal realm will not claim to have impact on metaphysical considerations. A phenomenal line segment must have a beginning and an end; but in the labyrinth of the continuum it is plain that such language is inadmissible. This is precisely what Leibniz himself says:

As concerns infinitesimal terms, it seems to me not only that we cannot penetrate to them, but that there are none in nature; that is, they are not possible. Otherwise, as I have already said, I admit that if I could concede their possibility, I should concede their being. 138

So the former is actual and imprecise; the latter exact, but fictive. But we already know that the only true entities in nature are corporeal substances; so that the question remains as stated. Moreover, if an infinitesimally small piece of matter cannot meet up with a metaphysical point, how can it be a ‘part’ in the understanding conveyed e.g. by the Monadology?

But here we actually arrive at the nexus of the argument. There is no Leibniz text which constructs phenomenal entities from simple units, i.e. from entities having no parts. The constraint on matching up infinitesimally small but still extended things with partless monads is insuperable. The answer is that the metaphysical laboratory of the continuum confers on us the insight that matter can be infinitely divided; but division is a prerogative of the phenomenal domain; and most importantly the difference here is that matter can be divided, but need not be.

We are now in a position to unravel the dilemma. The body vestment of a monad is not an acquisition. Leibniz is quite clear on this score. However, monads are not all equally vested – in fact, no two monads have exactly similar bodies. We recall from earlier Parts of this thesis that monads, being force, exhibit that force in two different kinds of expressions, and this also varies from one monad to another. But to understand this properly we need to work our back, here as before, from phenomena to basics.

In this scenario, matter is understood as the perception of resistant force, as inertia. As before, we must understand this in a plural context. A monad perceiving other monads (in the plural, as an aggregate) perceives them as active or passive agents in their varied measures. Monads comprising an organic machine (e.g. a blade of grass, an ant, fish, bird or mammal) will perceive this resistance as ‘soft’ or ‘hard’, ‘liquid’ or ‘bulky’ as the case may be and associate with that perception a distance from itself to the aggregate. This association will be reinforced by perceptions of other aggregates coming in between or perceivable behind the present object. Thus a spatial perception is engendered. But these spatial perceptions are not to be taken as ‘illusory’. Unilateral illusions are none. And exactly the same criteria pertain to time, i.e. the order of sequential and simultaneous occurrences. The point is rather that, in Leibniz’s words

138 Bernoulli, GM III 551, L 511.
it is impossible to find the principle of a true unity in matter alone or in what is merely passive ... a multitude can derive its reality only from the true unities which have some other origin and are entirely different from points, for it is certain that the continuum cannot be compounded of points. Accordingly I was forced to have recourse to an actual and so to speak animated point, in other words a substantial atom which must be formed and active in order to make a complete being.\textsuperscript{139}

In short, to speak intelligibly about phenomena is possible only within the phenomenal domain. We cannot maintain that consistency if we seek ultimate unities; for these we must enquire in a different domain, which, so long as we remain within its confines, will vouchsafe the same consistency and provide clear and unambiguous answers. Accordingly any questions pertaining to indivisible ultimate parts, if they are asked of phenomena, will lead to aporias. The realist must address questions pertaining to actual reality to the phenomena themselves and remain content with the knowledge that phenomena cannot respond to unintelligible demands. One such impossible quest is, evidently, the issue of multiplying nonspatial true unities and expecting them to produce 'space'.

A further aspect of this problem which ought to be mentioned is the genuine difficulty we have with articulating it. If phenomena and the continuum are mutually exclusive domains, the philosopher is in the unenviable position of having to expound in terms of one what the other entails. But as Anapolitanos points out, all language including philosophical terminology has semantics dictated by the phenomena themselves.\textsuperscript{140} Further, all our representations are likewise embedded in phenomena. In our consideration of Leibniz’s phenomenonaxis we noted many ambiguous and fictitious nomenclatures which are based solely on our relations to phenomena. These seem to be insuperable problems. Yet we might consider that our common experience of Euclidian space suffers from similar descriptive limitations:

What is a Euclidean three-dimensional continuum according to set theory? It is an infinite set of points $A$ together with an infinite set of particular mutual relations $R$. Such a set theoretic ordered pair $(A,R)$ can be pictured by the classical Euclidean 3D continuum $E$, so that a Cartesian one to one correspondence can be established between $(A,R)$ and $E$. To each actual point of $A$ would correspond exactly one possible point position in $E$, and to each possible point position in $E$ exactly one actual point of $A$, so that the set of actual relations of the points of $A$ would be isomorphically mapped onto the set of all possible relations of the possible point positions in $E$. $E$ can be thought of as prior to such a correspondence and therefore prior to its partition.\textsuperscript{141}

The one relation in this description which is missing is that of the text to the reader. There is no appeal to experience in this, nor to the imagination; and consequently there is no one-to-one correspondence between the words which describe the ‘space’ we live in and the immediacy of sense which that same ‘space’ conveys to us. A two-dimensional creature would hardly gain from this description the ‘feel’ we associate with space.

Our conclusion reverts to what has already been said. The moment we invoke ‘infinity’, we are identifying a state of affairs which pertains to the phenomenal domain, whereas in invoking ‘substance’ and all states of affairs which pertain to the con-

\textsuperscript{139} New System, final (unpublished) revision, HH pp. 204-5.
\textsuperscript{140} Anapolitanos, p. 113.
\textsuperscript{141} Ibid, p. 119.
tinuum we are in the metaphysical laboratory. This is the meaning, ultimately, of the need for a double-aspect theory. Only a double-aspect theory can intelligibly maintain its enquiry into both with conceptual consistency and satisfy the dictates of sufficient reason; and it can do so by the unwavering observance of the incompatibility of mapping any ‘parts’ of one to the other. Accordingly the pathway for any description of phenomenal states is from the top down, from extension down to infinitesimals. In the metaphysical domain, where the foundations have their home, phenomenal states look as strange as its denizens look to us. Nonetheless, they are, as the others, creatures of God. There are descriptive truths to be derived from the study of each: but any endeavour to commingle them is a mistake. No ‘Theory of Everything’ is possible.\(^{142}\)

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**Coda: Variations on a ground**

The evident delight of composers of Leibniz’s era in the mathematical structuring of musical materials offers an interesting opportunity for comparisons. The literature is not overly helpful in this respect: as happens to be the case with most theoretical disquisitions on music, then and now, the emotional and harmonic textures exercise both the ingenuity and belligerence of theorists. Few devote time and space to the ‘architectonic’ principles which animate composers – a rather strange state of affairs, considering that the structure of a work determines the space in which the harmonics evolve and acquire intelligibility and coherence. Put in another, more Leibnizian way: it is the law of the series of its development that puts vertical harmony into a context which gives meaning to momentaneous harmonic states. But these same architectonic principles have close relevance to the contents of the present chapter and are apt even to throw an unexpected light on them.

The kind of music we are concerned with belongs into the category of ‘art music’, and thus into a tradition which obliged the masters to publish, at least once in their lifetime (and so to speak off the beaten track of income-generating work) a ‘Kunstbuch’, a ‘masterpiece’ of their genre. These works addressed essentially the professional confraternity and/or reflected pedagogical purposes and accordingly concentrated on certain recherché forms of counterpoint: chaconne, passacaglia, fugue, ricercare, canon etc. A blanket expression often used in English, ‘variations on a ground’, may serve to explicate the nature of such exercises.\(^{143}\)

One pair of such exercises – the *passacaglia* and the *chaconne (ciacona)* – shall serve as specimens for the general type of these compositional principles. Originally a dance, the passacaglia (*passo di gallo*: ‘rooster step’) is in ¾ beat and minor tonality; it is constructed on a simple 8-bar phrase announced in the bass and then repeated unchanged for the whole duration of the piece (hence *attinato* or ‘ground’). Between the passacaglia

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142 The nearest approximation to a closure of the chain is indicated in the discussion in the *Paidius* and the ensuing paper *Motion is Something Relative* (Part III, Sect. G §5), but its upshot is of course that the closure can never be more than a momentaneous snapshot – a reality which lights up for a split second and transforms into another that same instant, never to be seen again.

143 Since several of these forms originated as dance movements, the professional restriction did not apply absolutely. Somewhat simplified specimens of such forms occur abundantly in music written for public performance, especially in ballet-opera and ceremonial functions.
and the chaconne the difference prevails that in the latter the theme is fully harmonised and therefore leads to the development of a different musical character, albeit by a similar technique to that which is now to be described.

The challenge to the composer was to write a series of embellishments on that initial phrase by embroidering it in different ways and in greater complexity on each recurrence. This is nothing other than exemplification of Leibniz’s minimax principle: the derivation of a maximal richness from the smallest conceivable cell. According to the nature of things, in the passacaglia the ‘ground’ sits firmly in the bass, while all the elaborations proceed in the upper levels; in the chaconne, the composer has liberty to spread the theme across the various instruments and/or voices of the ensemble, thus adding further complexities to the development. Thus by sheer accumulation the piece gains in aural splendour until eventually it issues in a climax where all variations return to their origin, but now with the maximum of effect achieved by the simultaneous sounding of all the voices in a pompous cadence in the major.

But how this relates to the contents of this chapter is revealed when we begin to think of music not only as mathematics in motion, but also, and perhaps primarily, as the principle of action. What is this thematic cell with which the passacaglia begins? Would Leibniz object if we called it a ‘monad’? Moreover, to the extent that the Leibnizian continuum in some of its aspects reveals a fractal world, we find that this principle is replicated in the structuring of this music. The most ready comparison is that of a little stalk in the ground which, in its growth, sends out shoots in great proliferation, each laden with leaves and further branches, each of these bearing more leaves, flowers, and so on — but ultimately they are all children of the same stem. All these clusters and their details look similar, though they are different from each other; just as the founding theme spawns innumerable offspring with close parental resemblance. At length, the bush or tree is fully mature and we may contemplate it in its fully-dressed splendour as an analogue or living metaphor of the composition.

A progress chart of just this kind (i.e. law of the series) could easily be drawn up of the typical career of a passacaglia/chaconne. It would not be unlike the algorithm mentioned above for the generation of the Fibonacci series. It may also be remarked, of the tree as much as the passacaglia, that they have no natural end point: ‘in principle’ they could go on forever proliferating their variations on the ground. It is the same principle as Leibniz’s ‘infinite fish’, where each sub-structure is apt to generate a further sub-structure and thus to proliferate into an infinite regress of morphs. The difference is that the music stops when the composer runs out of ideas or the plant has reached its maximal magnitude that can be sustained in its environment.

Now everything that has been said above applies in a certain measure to the other ‘recherché’ forms named above: thus the overall appellation variations on a ground applies in general to the use of a motto cell, which is put through a series of sparkling adventures. In the canon the theme or song accompanies itself, i.e. all variations are made from exact copies (clones) of the original, so that this is the most rigorous and spartan of these forms. In the fugue a freely invented theme acts as fellow traveller, and the skill and ingenuity of the writer is challenged in the weaving in and out of each other of these themes. It is permissible to add more voices, although there is a human limit to the complexity so attained: six-part fugues seem to be the ultimate and are correspondingly rare.144 A most important principle must be mentioned: it is not per-

144 Bach wrote such a fugue as the finale to his Musical Offering for King Frederick the Great.
mitted for any theme just to drop out. This is where the real difficulty of this ‘strict style’ is revealed: whether the theme accompanies itself (canon), proliferates itself (passacaglia), or is accompanied by another theme (fugue), the trend of the work is towards a plenum. Once all themes have been sounded, they must proceed simultaneously to the end, and of course they must all end on one euphonious cadence.

Two specific examples may illustrate the peculiar fascination inherent in all this. J. S. Bach was, historically, a kind of estuary of strict counterpoint music – the final and greatest flowering of that style. He also wrote several Kunstbücher, although by his day that fashion was almost extinct and he certainly never made any money from his publications. Beginning ten years before his death he put together a series of large-scale collections, in which he provided for every contrapuntal genre a supreme model, as if on a design to recapitulate Western musical history all by himself.145 We are here concerned with just two small pieces from the Musical Offering, the first of which I am inclined to entitle

THE LEIBNIZ CANON.

Its authentic name is ‘Canon per Tonos’. Douglas Hofstadter, who was very taken with this work, offers this description:

… it has three voices. The uppermost voice sings a variant of the Royal Theme,146 while underneath it, two voices provide a canonic harmonisation based on a second theme. The lower of this pair sings its theme in C minor (which is the key of the canon as a whole) … What makes this canon different from any other, however, is that when it concludes – or rather, seems to conclude – it is no longer in the key of C minor, but now is in D minor. Somehow Bach has contrived to modulate right under the listener’s nose. And it is so constructed that this ‘ending’ ties smoothly onto the beginning again; thus one can repeat the process and return in the key of E [and so on] … And yet magically, after six such modulations, the original key of C minor has been restored!147

However, owing to its upward progression, it is now displaced by an octave, and thus presents itself like an equation with two solutions. In the first, “the piece may be broken off in a musically agreeable way” at this juncture; but the fact that the modulations displaced the pitch by an octave means that it could also go on … forever, rising

145 These works include the Mass in B minor, 24 Preludes and Fugues in all keys; the Goldberg Variations, The Musical Offering, The Art of Fugue and several others.
146 The theme given to Bach by King Frederick II of Prussia, whom he visited in 1747.
higher by an octave at each recurrence! Obviously no voice or instrument could actually carry on after at most four or five octaves have been passed; but it is the passing along of the self-similar musical structure through an infinitude of dimensions which holds our attention – in fact it is an infinite spiral converging on a point. Ending it agreeably entails breaking off its winding ascent arbitrarily.¹⁴⁸ This raises the intriguing Leibnizian point about how long this progression may continue before it exhausts itself in a limit (since evidently the intervals between the notes get smaller on each replay in a higher ‘dimension’).¹⁴⁹

The other specimen I wish to mention is the way Bach manages to make polyphonic strands of music audible in his sonatas for one stringed instrument.¹⁵⁰ He does this by placing the notes of one melody on, the other off the beat: so although the player executes a smooth sequence of notes, the ear of the listener obeys the logic of construction and separates two strands of argument from the single melodic line. Another way of doing this is to construct two melodies with different rhythm; and since the ear tends preferentially to connect higher pitched notes, to guide it in the same way to the discrimination of discrete strands of melody. It would lead us too far into regions of aural psychology to investigate the causes; but it is for our purposes surely an obvious case of the creative use of our faculty of attentive perceptions and petites perceptions being exploited by the composer.

We seem to have arrived somewhere with respect to music where perhaps we did not expect to go. What do these recondite fantasies have to do with the pleasure of music? Where have the emotions, the affects vanished, which seem to be the raison d’être of music? But to ask such questions could be said to ask the wrong questions.

“Even the pleasures of sense,” writes Leibniz,

are reducible to intellectual pleasures, known confusedly. Music charms us, although its beauty consists only in the agreement of number and in the counting, which we do not perceive but which the soul nevertheless continues to carry out, of the beats or vibrations of sounding bodies which coincides at certain intervals.¹⁵¹

This seems disingenuousness; for he could surely not entertain the belief in the soul as ‘counting’ (what pleasure in that?). But Leibniz evidently wanted to get away from the naïve aesthetics of arousal psychology; yet since the senses do not ‘count’, this leaves only the mind as the recipient of pleasure. Accordingly “Leibniz must have at least had a glimmering of the truth. And the truth is: music is not a stimulus object but a cognitive one.”¹⁵² This idea, of music as ‘absolute music’, arose in German philosophy of the late 18th century and was characterised by Jules Combarieu as “think-

¹⁴⁸ Intriguingly Bach also included one piece called a ‘crab canon’ which can be played in the normal way as a perfectly coherent and self-contained exercise, but turns out to be a note perfect duplicate of itself when played back to front! So this is an instance where effectively we have two identical compositions nested in one. Karl Geiringer: Johann Sebastian Bach, C. H. Beck, München 1978, pp. 332-6.

¹⁴⁹ This reflects the dichotomy between practical and theoretical considerations. In theory, there is no end point, no minimum; but music is generated by vibrating air, so that there is definitely a physical limit on how close to each other the harmonic ratios may approach before they interfere with each other.

¹⁵⁰ Partitas and Sonatas for Violin solo; Six Suites for Cello solo.

¹⁵¹ Princ. Gr. §17.

ing in music, thinking with sounds, the way a writer thinks with words." This is not what the mass of average music lovers think of; although it is a suggestive line of contemplation for a philosophical mind that – when taken in its ultimate seriousness – music is a form of metaphysical enquiry which makes certain implicit features of the cosmos explicit by hijacking the aural faculties.

But this is a topic for another enquiry; for now just a single conclusion is offered, namely that the proximity of several central Leibnizian concepts to the creative practice of the great masters of music of the 16th and 17th centuries suggests a deep relevance and indeed kinship between music and philosophy. It leaves one wondering, especially in the comparison to Leibnizian philosophy, that the several resemblances of musical architectonic to central themes of his metaphysics suggests that grounding in the principle of sufficient reason is a fundamental point. This is a strain we will pick up again in our final chapter; at this point it must suffice to draw attention to those principles which echo back from these musical techniques to give an unexpectedly musical modelling to Leibniz’s philosophy of harmony, viz.

The grounding cell (monad?) as the principle of action
Its law of the series
Individuation by proliferation of morphs
Implementation of the minimax principle
The fractal nature of variation accumulation
Self-similarity of its offshoots
The infinitary progression of thematic development
The principle of convergence on a limit in modulations
Exploitation of petites perceptions and attentive apperception

Mention has been made of the demise of this art form. By the time of Kant it was ‘old hat’ and despised as much as Erasmus and his literati colleagues despised scholastic learning. But it was very much ‘in the air’ in Leibniz lifetime. If the above serves as nothing more than to show that philosophy and music did not develop in insulation from each other, but so to speak breathed the air of the same cultural and intellectual predilection, this excursus will have done its due.

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