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Graduate School of Business
The University of Sydney

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CREW SIZE DETERMINATION FOR TERMINAL QUEUING OPERATION OF A BUS ROUTE

William Zhu

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ABSTRACT: High demand for public transport and hectic traffic conditions subjects the bus operations on a bus route in cities of developing countries to a terminal queuing process: buses and crews are placed into queues at terminals waiting for an assignment of a trip after they complete a trip. For this type of bus operation, complementary guidelines for real-time bus and crew scheduling are discussed, followed by the development of formulae for the relationship between hired and daily crew sizes based on the work requirements by bus schedules and holiday variations, which can be used to determine minimum crew sizes to cover work requirements. A case study on a depot in the Bangkok bus transit system was used to illustrate the formulae. The results show the insufficiency of crew sizes employed by the Bangkok Mass Transit Authority (BMTA), verifying the inefficiency of the current operation.

AUTHOR: William Zhu, BSc MSc PhD
Research Fellow
Institute of Transport Studies

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CONTACT: Institute of Transport Studies
Graduate School of Business
The University of Sydney NSW 2006
Australia

Telephone: +61 2 550 8631
Facsimile: +61 2 550 4013

1. **Introduction**

Bus and crew schedules are generally addressed in the literature as separate issues with the former as one of the input into the latter. The practice of bus and crew scheduling by allocating trips to buses and crews prior to real operations is popular in cities where planned bus and crew schedules can be tightly controlled in real operations because of stable traffic conditions and a predictable demand for public transport. Conventional bus scheduling allocates daily trips to buses to meet both timetable and maintenance requirements, and is normally solved by mathematical programming or network approaches. With bus schedules as an input, crew scheduling is carried out subsequently to cover the timetable, and allow for lunch break requirements. Most existing approaches for crew scheduling make use of set covering and set partitioning algorithms (Mitra and Welsh 1981, Parker and Smith 1981, Ryan and Roster 1981, Ward et al 1981, Heurgon 1972, 1975). An historical review of earlier work on crew scheduling is given in Wren (1981b). Recent work on bus and crew schedules can be found in Wren (1981a), Rousseau (1983), Daduna and Wren (1985), and Desrochers and Rousseau (1990).

In many developing countries, however, the demand for urban public transport is very high, public transport being the only affordable travel mode for most urban poor. The bus transit system is one of the efficient means of moving large numbers of people with considerable flexibility. Due to high demand and short headways between successive trips, the pre-assignment of trips to buses before the start of the daily operation (i.e. conventional bus scheduling) is impractical for cities like Bangkok where there is a hectic traffic environment. Instead, the bus operation is subject to a terminal queuing process: buses and crews are placed into queues at terminals waiting for an assignment to a trip after they complete a trip. The advantage of the terminal queuing process is that it can easily be adapted to the unpredictable traffic conditions, and headways can be adjusted according to the on-going circumstances, e.g. delayed arrival of buses, absence of drivers, breakdowns and accidents.

The terminal queuing process subjects the bus and crew to one component in terminal queuing. It is not economical to change drivers among buses during an operation day because of short headways. This bundles bus and crew scheduling in a specific way which is different from the literature. Because of the shift existence for drivers, preparing a bus schedule does not imply automatically generating a crew schedule. Further, trip frequency determination for multiple routes is not practical in cities like Bangkok since buses are already fully used on their original route, and are unable to operate on other routes during an operating day. For this type of terminal queuing operation in transit systems, our task is therefore to prepare different bus and crew schedules for one route.

Bus and crew schedules are normally generated based on a trip frequency schedule. A trip frequency schedule, in the sense of a timetable, is the set of number of trips in each hour. Crew sizes, in terms of number of drivers, are normally based on crew schedules. Since bus schedules are served as inputs to the crew schedules, the determination of crew sizes will take into account the results of bus scheduling. The determination of crew sizes is also called operator workforce planning (MacDorman et al., 1984) or manpower planning (Wijngaard 1983).

Workforce planning has received increased attention in the transit industry as the labour cost can be saved through proper crew size management. Analytical approaches to workforce planning are available through mathematical programming (e.g. Koutsopoulos and Wilson, 1987). In the paper of Koutsopoulos and Wilson (1987), they provide an historical view of workforce planning in the transit industry with comments on the work of MacDorman et al. (1982, 1984, 1985). Further, they present the workforce planning as three hierarchically interrelated models: a strategic model, a tactical model, and an operational model, implemented with non-linear programming formulations. The three models are defined as follows:

A Strategic Model:: emphasises the decision on overall workforce size, hiring level, and vacation allocation for a planning period (usually a year or longer). To arrive at a decision across the planning period, the planning period is divided into several seasonal subperiods

A Tactical Model:: emphasises the determination of extraboard operator size to cover the uncertainty beyond the schedule requirements for specific days of a week.

An Operational Model:: emphasises the assignment of specific times of a day to available extraboard operators. This is the daily assignment of extraboard operators.

While the models developed in Koutsopoulos and Wilson (1987) reveal significant research interest and application potential by non-linear programming models, they suffer several shortcomings. First, they reveal no relationship between hired crew size and daily crew sizes for weekdays and weekends in the strategic model. Second, the minimisation of expected overtime in the objective functions of the models may lead to infeasible solutions in the sense that daily required work may not be covered by the scheduled workforce. This is because the daily work requirement is only expressed in the expected overtime functions as the difference between the available cover and work requirement. Third, the expected overtime in the objective functions of the models are difficult to derive in practice. In the paper of Hickman et al. (1988), normal distributions used for the random variables of required work and extra work are untested. In the paper of Kaysi and Wilson (1990), the binomial distribution for the number of open runs in a period reveals no conditional probability on open runs from the preceding period. Finally, the results of the strategic model in Hickman et al. (1988, Table 3) vary significantly among scenarios. This raises the question on the stability of the models. It is questionable to establish the confidence on implementing the optimal solutions if the models are not stable. Probably sensitivity analysis could be further carried out.

In practice, transit agencies rely heavily on traditional ad hoc methods to determine appropriate crew sizes. In Bangkok, the number of drivers employed by the Bangkok Mass Transit Authority (BMTA) is 1.8 times the existing bus fleet size for each route. The daily crew sizes are judged by route managers using experienced rules of thumb. This has led to the situation that on many routes buses were released with fluctuating headways and had to terminate operation in the afternoon peak due to the lack of drivers.

In this paper, instead of developing mathematical programming formulation, we will discuss optimal crew size determination based on the relationship between crew sizes, bus schedules and day-off policies. The crew sizes are classified as hired crew size, and daily crews for weekdays and weekends. The determination of hired and daily crew sizes is actually the subject in the strategic and tactical models of Koutsopoulos and Wilson (1987) respectively. We will focus on workforce planning for a specific scheduling period only (e.g. for a season). Workforce planning for a year is not considered because of the changing pattern of demand throughout the whole year. To understand the relationship between crew sizes and bus schedules, we will first briefly discuss complementary guidelines for the real-time bus and crew scheduling for the terminal queuing operations, followed by the development of formulae for determining optimal crew sizes with the objective of minimising the number of operators. A case study is carried out for determining the optimal crew sizes for a depot in Bangkok. Although the formulae are developed for the Bangkok case, the idea can be extended to a general bus transit system with terminal queuing operation.

2. **Bus and Crew Schedules**

Bus and crew schedules are prepared based on a trip frequency schedule (Zhu, 1991). In general, a trip frequency schedule can be generated by dividing an operation day [0.T] into m intervals $[T_{i-1}, T_i]$, $T_0 = 0$, $T_m = T$. For each interval $[T_{i-1}, T_i]$, which can be simply called time period i, a number of trips, K_{ki}, for terminal k (k=1,2), will be determined. The set $\{ K_{ki} \}$ for all time periods and both terminals constitutes a trip frequency schedule. In trip frequency scheduling, various constraints should be considered: limited bus fleet and crew sizes, the availability of buses from time periods to time periods, maintenance, limited parking space, depot location, etc. Since timetables are not practical in cities with unstable traffic condition, trip frequency schedules can serve as a guideline to release buses according to on-going circumstances for the terminal queuing operations.

Trip frequency scheduling has been extensively dealt with in Zhu (1991) using an integer programming approach and heuristic algorithms. By the Greedy Assignment Algorithm developed by Zhu (1991), the number of buses required from each terminal in each time period can be determined. These required buses have to start from the depot at an appropriate time in order to begin operation during the specified time period and thus indicate the starting time of buses from the depot. That is, if L_{ki} buses are required from the depot at terminal k in time period i, then by estimating the travel time t_{ki} of buses from the depot to terminal k, $L_{ki}/2$ buses can be sent from the depot at time T_{i-1}-t_{ki} and $L_{ki}/2$ buses can be sent from the depot at time $(T_{i-1}+T_i)/2-t_{ki}$. The number of required buses divided by two is based on the releasing strategy that buses need to be released from depots every half an hour (assuming the durations of each time period are one hour). For example, L_{ki}=10, time period i = [5:00,6:00]. The travel time of buses from the depot to terminal k requires one hour. Then $L_{ki}/2$=5 buses should be released from the depot at time 5:00-1:00=4:00 am and $L_{ki}/2$=5 buses should be released from the depot at time (5:00+6:00)/2-1:00=4:30.

Because of the terminal queuing operation, bus and crew scheduling only determines the number of buses and crews required to enter and exit the operation in each time period. The starting time of a bus will automatically imply the starting time of a driver. Because the working time of crews cannot exceed certain limits, the crews need to be scheduled with shifts. Generally, the crew of a route having normal operation is divided into two shifts: the first shift performs duties in the morning shift; the second shift performs duties in the afternoon shift. The crew for a route of 24-hour operation is divided into three shifts: morning shift, afternoon shift and night shift. This paper mainly studies the crew size problem for routes having normal operation. It is not difficult to extend the proposed idea to a route having 24-hour operations.

By the Greedy Assignment Algorithm (Zhu, 1991), the number of buses required from the depot can be determined to be N_1 for the first shift and N_2 for the second shift. N_2 is the additional bus requirement for the second shift such that $N_1 + N_2 \leq N$. Let M be the number of daily drivers and O be the number of daily overtime drivers (the determination of M and O will be discussed in the next section). Since for the first shift, N_1 drivers are required, at most M-N_1 drivers can be scheduled for the second shift. The M-N_1 drivers together with the O overtime drivers should be able to cover the N_1+N_2 buses in the second shift, i.e., $M-N_1+O \geq N_1+N_2$. Therefore the daily crew size M and daily overtime crew size O should cover the total shift duties $2N_1+N_2$, i.e.,

$$M+O \geq 2N_1+N_2 \tag{1}$$

and at least $O = 2N_1+N_2-M$ drivers are required to perform overtime duties. $2N_1 + N_2$ indicates the number of duties to be performed during a day.

Now suppose the crew size M and the number of overtime drivers O are adjusted such that $M+O = 2N_1+N_2$. Starting time can be assigned to the drivers. Not only should the starting time of the drivers be arranged according to the bus schedule but other aspects should be taken into account, such as the drivers' preference, distance of drivers' house to the depot and the number of overtime drivers, as well. When all these aspects are considered, the driver list can be sorted and rearranged by starting time. From the sorted list, the starting time for the first-shift drivers can be assigned according to the starting time of buses, and the starting time of the second-shift drivers can be assigned according to the scheduled trips K_{1p}, $K_{1,p+1}$, ... at terminal 1 (suppose terminal 1 a shift relief point) in time period p, p+1, ... and so on until all the second-shift drivers are assigned. The way of assigning the drivers would be assigning $K_{1p}/2$ at time T_{p-1}, $K_{1p}/2$ drivers at time $T_{p-1}+(T_p- T_{p-1})/2$, $K_{1,p+1}/2$ drivers at time T_p, ... , and so on (this is particular to the case where drivers will be assigned buses every half an hour for hourly time periods).

In reality, the way of assigning the second-shift drivers and the determination of the relief point should be more flexible taking into account the drivers' preference and convenience. At the relief point, the second-shift drivers are allowed only to replace the first-shift drivers who do not work overtime.

At the end of the operation, both buses and crews should be sent back to the depot according to the trip frequency schedule. The end time of drivers should also be arranged such that priority is given to the overtime drivers. That is, if starting from time period i, there is an indication in the trip frequency schedule that S_{1i} buses should be sent to the depot from terminal 1 (suppose terminal 1 is close to the depot), overtime drivers who arrive at terminal 1 in time period i should stop work and send their corresponding buses to the depot. If more than S_{1i} overtime drivers arrive at terminal 1 in time period i, only S_{1i} overtime drivers are allowed to stop work. The same process is applied to the next time periods until all the overtime drivers have finished work and then the second-shift drivers can be stopped according to the trip frequency schedule, based on a first-arrive-first-assigned criterion.

The number of buses that should be sent to the depot from terminal 1 can be calculated from the number of remaining buses. That is, if starting from time period i there are remaining buses R_{1j} afterwards (j=i,i+1, ... , m) at terminal 1, then $S_{1i} = \max \{ R_{1j} \mid i \le j \le m \}$ buses should be sent to the depot in time period i. The number of buses required to be sent to the depot in the next time period depends on the buses actually sent in time period i. If the number of buses actually sent from the arrival of the overtime drivers in time period i is S'_{1i}, then the number of buses to be sent to the depot in time period i+1 would be $S_{1,i+1} = \min \{R_{1j} - S'_{1i} \mid i+1 \le j \le m\}$. This process is repeated until the end of the operation.

To avoid the overtime exceeding the working intensity of the drivers, the overtime drivers should be assigned in turns. This can be on a daily or weekly basis. The daily (weekly) turn would be to ask the overtime drivers to perform normal duties the next day (week) and replace them with another group of drivers who are willing to work overtime. This should be reflected by dividing the overtime drivers into several groups (at least two groups) and assigning these groups to work overtime by turns. The problem of the detailed assignment of actual drivers to perform the duties day by day and week by week while balancing working time and income is the crew rostering problem.

3. Daily and Hired Crew Sizes

Daily crew size is dependent on the bus requirements in each time period, which is determined by bus schedules in the terminal queuing operations. Because of holiday and vacation variation for drivers, the hired crew size of a route for a certain period should be larger than the daily crew sizes. For the simplicity, we adopt the day-off policy of having at least one day off per week for drivers, which is implemented in most Asian countries. However, the basic idea can similarly be extended to other day-off policies.

In order to distinguish the daily crew sizes between the weekdays and weekends, indices w and e are used to represent weekdays and weekends respectively:

M_w - daily crew size of weekdays
O_w - daily overtime crew size of weekdays
M_e - daily crew size of weekends
O_e - daily overtime crew size of weekends

N_{w1} - number of buses required for the first shift of weekdays
N_{w2} - number of additional buses required for the second shift of weekdays
N_{e1} - number of buses required for the first shift of weekends
N_{e2} - number of additional buses required for the second shift of weekends

As indicated by Inequality (1) in the previous section, $N_w \equiv 2N_{w1}+N_{w2}$ and $N_e \equiv 2N_{e1}+N_{e2}$ are the numbers of duties to be performed by drivers, on weekdays and weekends respectively.

Denote Φ as hired crew size and φ the total number of hired drivers who are willing to work overtime, which is called the hired overtime crew size. Since each driver will work at most 6 days per week according to the at-least-one-day-off policy, the total number of available working days per week by a crew of size of Φ will be at most 6Φ. On the other hand, having daily crew sizes M_w for weekdays and M_e for weekends, the total number of required working days for a week will be $5M_w + 2M_e$. Since the total number of available work days should cover the required working days,

$$6\Phi \geq 5M_w + 2M_e \qquad (2)$$

Similarly, each overtime driver will work at most 6 days per week and will be assigned by turns. Therefore, the total number of overtime days per week by a overtime crew of size φ will be at most 3φ. The total number of required overtime days for a week will be $5O_w + 2O_e$, given the daily overtime crew sizes O_w for weekdays and O_e for weekends. Since the total number of available overtime days should cover the required overtime days,

$$3\varphi \geq 5O_w + 2O_e \qquad (3)$$

From the Inequalities (2) and (3), the minimum hired and overtime crew sizes can be determined based on the daily crew sizes (M_w, M_e) and the daily overtime crew sizes (O_w, O_e) by

$$\Phi = \lceil (5M_w + 2M_e) / 6 \rceil \qquad (4)$$
$$\varphi = \lceil (5O_w + 2O_e) / 3 \rceil \qquad (5)$$

where, ∪x′ is the smallest integer greater than or equal to the number x.

Furthermore, from Inequality (1), the relationships between the daily overtime crew size and the daily crew size can be stated as

$$M_w + O_w \geq 2N_{w1} + N_{w2} = N_w \qquad (6)$$
$$M_e + O_e \geq 2N_{e1} + N_{e2} = N_e \qquad (7)$$

5 x Inequality (6) + 2 x Inequality (7) gives $5(M_w+O_w) + 2(M_e+O_e) \geq 5N_w + 2N_e + \tilde{N}$. On the other hand, Inequality (2) + Inequality (3) leads to $6\Phi+3\varphi \geq 5(M_w+O_w) + 2(M_e+O_e) = \tilde{N}$.

Therefore, the relationship between the hired crew sizes and the total number of duties on weekdays and weekends can be determined by

$$6\Phi + 3\varphi \geq \tilde{N} \qquad (8)$$

where, $\tilde{N} + 5N_w+2N_e$ represents the total number of duties for both weekdays and weekends.

From Inequality (8), the hired crew sizes can be determined based on the number of duties for weekdays and weekends. For instance, if φ is specified to be at least α percent of Φ, then $6\Phi+3\alpha\Phi \geq \tilde{N}$. Therefore, the minimum hired crew size is determined by $\Phi=\lceil \tilde{N}/(6+3\alpha)\rceil$ and the minimum hired overtime crew size is determined by $\varphi=\lceil \alpha\tilde{N}/(6+3\alpha)\rceil$.

If the daily overtime drivers are specified to be at most α_w percentage of the daily weekday drivers and α_e percentage of the daily weekend drivers, then from (6) and (7) we have $M_w + \alpha_w M_w \geq N_w$ and $M_e + \alpha_e M_e \geq N_e$. Therefore, the minimum daily crew sizes based on the bus requirements can be determined by

$$M_w = \lceil N_w / (1+\alpha_w)\rceil \qquad (9)$$
$$M_e = \lceil N_e / (1+\alpha_e)\rceil \qquad (10)$$

respectively for weekdays and weekends. The minimum daily overtime crew sizes is thus determined by

$$O_w = \lceil \alpha_w N_w / (1+\alpha_w)\rceil \qquad (11)$$
$$O_e = \lceil \alpha_e N_e / (1+\alpha_e)\rceil \qquad (12)$$

respectively for weekdays and weekends.

The specification of the percentages α_w and α_e should be such that the determined daily crew sizes will not contradict the hired crew sizes. That is, the specified daily crew sizes should satisfy Inequalities (2) and (3):

$$6\Phi \geq 5M_w + 2M_e \geq \frac{5N_w}{(1+\alpha_w)} + \frac{2N_e}{(1+\alpha_e)} \quad (13)$$

$$3\varphi \geq 5O_w + 2O_e \geq \frac{5\alpha_w N_w}{(1+\alpha_w)} + \frac{2\alpha_e N_e}{(1+\alpha_e)} \quad (14)$$

From the above inequalities, the ranges for α_w and α_e can be determined. For example, if a predetermined value α_w is given, the range of α_e is given by

$$\frac{(1+\alpha_w)(2N_e-6\Phi)+5N_w}{6(1+\alpha_w)\Phi-5N_w} \leq \alpha_e \leq \frac{3(1+\alpha_w)\varphi-5\alpha_w N_w}{(1+\alpha_w)(2N_e-3\varphi)+5\alpha_w N_w} \quad (15)$$

Or, if $\alpha_w=\alpha_e=\alpha$ is specified, the range of α is given by

$$\frac{(\tilde{N} - 6\Phi)}{6\Phi} \leq \alpha \leq \frac{3\varphi}{(\tilde{N} - 3\varphi)} \quad (16)$$

$\alpha_w = \alpha_e$ implies that $O_w/M_w = O_e/M_e$, which implies that the ratios of daily overtime crews and daily crews are equal on weekdays and weekends. From Inequalities (6) and (7), to minimise the labour cost, the daily crew sizes should be such that $M_w + O_w = N_w$ and $M_e + O_e = N_e$. Thus

$$\frac{M_w}{M_e} = \frac{O_w}{O_e} = \frac{N_w - M_w}{N_e - M_e} \quad (17)$$

or

$$M_w/M_e = N_w/N_e \quad (18)$$

Substituting this into Inequality (2) gives

$$6\Phi \geq 5\frac{N_w}{N_e} M_e + 2M_e \quad (19)$$

or

$$M_e \leq \frac{6\Phi N_e}{5N_w + 2N_e} = \frac{6\Phi N_e}{\tilde{N}} \tag{20}$$

and

$$M_w = M_e \frac{N_w}{N_e} \leq \frac{6\Phi N_w}{\tilde{N}} \tag{21}$$

Further substituting $O_w/O_e = M_w/M_e = N_w/N_e$ into (3) results in

$$3\varphi \geq 5N_w \frac{O_e}{N_e} + 2O_e \tag{22}$$

or

$$O_e \leq \frac{3\varphi N_e}{5N_w + 2N_e} = \frac{3\varphi N_e}{\tilde{N}} \tag{23}$$

and

$$O_w = O_e \frac{N_w}{N_e} \leq \frac{3\varphi N_w}{\tilde{N}} \tag{24}$$

It can be seen from Inequalities (20), (21), (23), and (24) that the daily crew and daily overtime crew sizes are based on the hired crew sizes Φ and φ and the number of duties N_w and N_e. Since the daily crew sizes, M_w and I_w for weekdays, should be determined such that $M_w + O_w = N_w$ are satisfied, Inequality (21) can be used to determine the maximum daily crew size by $M_w = \lfloor 6\Phi N_w/\tilde{N} \rfloor$ if the policy is to fully utilise the daily crews (where $\lfloor a \rfloor$ is the largest integer smaller or equal to the number a). Then the daily overtime drivers should be determined by $O_w = N_w - M_w$. The value O_w should not be determined by $O_w = \lfloor 3\varphi N_w/\tilde{N} \rfloor$ (Inequality (24)) because extra daily overtime drivers may be introduced if the hired overtime crew size φ is sufficiently large such that $\lfloor 3\varphi N_w/\tilde{N} \rfloor \geq N_w\text{-}M_w$. In addition, the hired overtime crew size should be large enough such that $\lfloor 3\varphi N_w/\tilde{N} \rfloor \geq N_w\text{-}M_w$ once M_w is determined. This indicates that the minimum hired overtime crew size should be based on $\lfloor 3\varphi N_w/\tilde{N} \rfloor \geq N_w - M_w$, once M_w is determined.

Inequality (8) gives us the impression that the hired overtime crew size can be determined by $\varphi = \cup(\tilde{N}-6\Phi)/3'$ once the hired crew size is determined. This expression may sometimes be too tight or even infeasible for determining the daily crew sizes because of the elimination of the decimal portion during rounding off. This fact can be illustrated by considering $(\tilde{N}-6\Phi)/3$ as an integer and $6\Phi N_w/\tilde{N}$ a real number. Thus $M_w + O_w \leq \lfloor 6\Phi N_w/\tilde{N} \rfloor + \lfloor 3\varphi N_w/\tilde{N} \rfloor < 6\Phi N_w/\tilde{N} + 3\varphi N_w/\tilde{N} = 6\Phi N_w/\tilde{N} + 3[(\tilde{N}-6\Phi)/3]N_w/\tilde{N} = N_w$. This is contradictory since M_w and I_w should be such that $M_w + O_w \geq N_w$. This indicates that Inequality (8) can not be used to evaluate the hired overtime crew size φ. The actual determination of the minimum hired overtime crew size should also be based on the daily crew sizes M_w and M_e, i.e., φ should be such that $\lfloor 3\varphi N_w/\tilde{N} \rfloor \geq N_w - M_w$ and $\lfloor 3\varphi N_e/\tilde{N} \rfloor \geq N_e - M_e$.

From the above discussion, the hired and daily crew sizes are interrelated and the daily crew sizes depend not only on the daily duties but on the hired crew sizes as well. On the other hand, the hired crew sizes can be determined based on the daily crew sizes or daily duties. Overtime crews should be provided when the normal crew sizes are not sufficient to perform all the daily duties. The overtime crews are defined as part of the normal drivers who are willing to work overtime. The relationships between the daily and hired crew sizes based on the duty requirements give precise ideas on how to determine the optimal crew sizes based on the daily duties. The selection of appropriate formulae should be based on the strategies for the crew management. Furthermore, mathematical programming models can be formulated based on the formulae developed above, taking into account the minimisation of total labour cost.

4. Extraboard Crew

The optimal crew sizes, in this paper, are the minimum crew sizes covering the duties to be performed. In reality, extraboard drivers will be required to deal with uncertainty: absence, breakdown, accidents and etc. The determination of hired extraboard drivers falls in the category of the tactical model of Koutsopoulos and Wilson (1987). In their model, the objective is to minimise the expected overtime, which can be expressed as the function of the absenteeism rate. Since different expressions of the overtime functions and different distribution used in the overtime functions may lead to different solutions, the confidence level on covering the absence by the solutions cannot be gained through the mathematical modelling approach. In this paper, we introduce simple formulae for extraboard crew size determination based on probability theory.

To determine the extraboard crew size, the absence probability p should be known, which can be estimated by the average percentage of absent drivers. The estimation of the absence rate can be collected according to the day of week. Here, we use the average weekly absence rate. If we can assume that the absence probabilities of each driver are independent, the probability that at most x drivers do not come to work is then given by the binomial distribution (Ross, 1985):

$$p(x) = P(X \leq x) = \sum_{j=0}^{x} \binom{\Phi}{j} p^{j}(1-p)^{\Phi-j} \qquad (25)$$

where, Φ = crew size

1-p(x) is the probability that at least x drivers will not come to work. If x extraboard drivers are used, the confidence that all absent drivers are covered by the extraboard drivers is p(x). In other words, the risk that not all absent drivers will be covered by the x extraboard drivers is 1-p(x).

Given a confidence level α, what is the minimum number of reserve drivers such that the probability that all absent drivers will be covered by the extraboard drivers will be not less than α? The answer is to increase x until $p(x) \geq \alpha$, with an initial value zero of x. The value of p(x) can be either calculated or found in binomial distribution tables.

In many bus transit systems, several bus routes will use a bus depot. For these routes, reserving extraboard drivers for individual routes is not economical. If the number of extraboard drivers is determined for all routes of a depot, the extraboard crew size can be reduced. In general, suppose there are R routes sharing the depot and on route r, there are m_r drivers and the probability of having absent drivers is p_r (r=1,2, ..., R). Then the overall absence probability of each individual for the depot is

$$p = (p_1 m_1 + p_2 m_2 + ... + p_R m_R) / m \qquad (26)$$

where $m = m_1 + m_2 + ... m_R$.

Therefore, the probability that at most x drivers do not come to work at the depot is given by the binomial distribution (25) with the overall absence probability in Inequality (26).

Using the overall absence probability p, a reduction can be compared between the total extraboard crew size for all routes and the extraboard crew size for the depot: given a confidence level α, the minimum number of extraboard drivers can be determined for the depot, say x, and for each route, say x_r (r=1,2, ... , R). Thus $(x_1+x_2+...+x_R)$ - x extraboard drivers can reduced using the policy of reserving extraboard drivers for the depot.

Due to the fact that the crew size Φ is large and p is small, the Poisson distribution can be used to approximate the probability p(x):

$$p(x) = \sum_{j=0}^{x} e^{-\lambda} \lambda^{j}/j! \qquad (27)$$

where, $\lambda = np$.

Example

Suppose R=4 bus routes use a depot and the number of drivers on each route is 50, i.e. $m_1=m_2=m_3=m_4=50$. The absence probabilities are 5% for all routes, i.e. $p_1=p_2=p_3=p_4=5\%$. Then m=4x50=200, p=5%. From Formula (27), p(14)=0.9165 and p(15)=0.9513. Therefore, a minimum of 15 extraboard drivers are required for the depot at the confidence level of α =95%.

If extraboard drivers are reserved for individual routes, 5 extraboard drivers will be required for each route at the confidence level of α =95% as calculated by Formula (27): p(4)=0.8912 and p(5)=0.9580. Therefore, a total of 4x5=20 extraboard drivers are required when they are reserved for each route. Compared with the minimum extraboard drivers for the depot, the number of extraboard drivers can be reduced by 5, i.e. a reduction of 25%.

The hired extraboard crew size is determined based on the hired crew size. The daily extraboard crew size determination can be carried out in a similar manner which is discussed in Section 3. If the absence probabilities of individual drivers are collected by day of week, the daily extraboard crew sizes can be determined by day of week using the Poisson probabilities. The hired extraboard crew size can be determined by the similar discussion to the normal hired crew size determination in Section 3.

5. A Case Study

Early in 1991, a case study was carried out for a depot of the Bangkok Mass Transit Authority (BMTA), the major supplier of the Bangkok public transport. The depot under study was used by 7 bus routes as listed in Table 1.

Table 1 Bus requirements and crew sizes

ROUTE	W-BUS	D-BUS	%	MCREW	BMTACREW	EXTRABOARD
4	70	63	8.19	120	126	14
47	35	31	12.54	60	63	11
72	40	34	10.45	68	72	11
89	27	24	8.09	47	48	6
22(S)	30	25	9.47	51	54	8
102	20	17	7.79	34	36	5
205	50	44	9.08	86	90	11
TOTAL	272	238		466	489	66
AVERAGE	38.86	34.00	9.37	66.57	69.86	9.43

Legend: W-BUS - number of buses operated on weekdays
D-BUS - number of buses operated on weekends
% - percentage of absent drivers
MCREW - hired crew size by the formulae in this paper
BMTACREW - number of drivers by the BMTA formula
EXTRABOARD - hired extraboard crew size with the actual absence percentage

In Table 1, the number of buses used on weekdays and weekends, and the percentages of absent drivers are also listed, which were collected for the month of January 1991. From these data, the number of minimum hired crew sizes for each route are calculated by Formula (8), based on the BMTA policy that buses used for weekdays and weekends are fully operated in the morning shift. The numbers of hired extraboard drivers are calculated by formula (27) for individual routes at the confidence level of 90%. These data are listed in columns MCREW and EXTRABOARD of Table 1 respectively. The column MBTACREW shows the crew sizes for each route calculated by the BMTA empirical formula: number of drivers = 1.8 x bus fleet size of each route.

From Table 1, the total number of reserve drivers is 66 for the depot when extraboard drivers are reserved for individual routes. If extraboard drivers are reserved for the depot, the total number of extraboard drivers is calculated to be 52 by Formulas (26) and (27), at the same confidence level 90%. Therefore, 66-52=14 extraboard drivers can be reduced, a reduction of (66-52)/66 = 21.21% for reserving extraboard drivers for the depot.

Using the BMTA formula, a total of 489 drivers are required (Table 1). Compared with the total number of drivers and extraboard drivers determined by the formulae in this paper, there is a shortage of 466+52-489 = 29 drivers. This explains why in many cases buses had to terminate the operation in the afternoon peak; there were not sufficient drivers [1].

A single bus route of the depot, Route 47, was further considered for determining the optimal crew sizes. The number of hired drivers of Route 47 is $\Phi = 62$ by the BMTA formula. Crews of Route 47 were divided into two shifts. With a bus fleet size of 35, a trip frequency schedule was produced according to the trip frequency scheduling method of Zhu (1991). According to the complementary bus scheduling method of Section 2, 35 buses have to be used in the first shift on weekdays. On weekends, 24 buses are required in the first shift and 5 buses are required in the second shift. That is,

$$N_{w1} = 35,\ N_{w2} = 0,\ N_{e1} = 24, \text{ and } N_{e2} = 5$$

Thus, $N_w = 70$, $N_e = 53$, and $\tilde{N} = 456$.

In 1991, 65% of the total drivers were willing to work overtime in BMTA. Thus the hired overtime crew size $\varphi = 62 \text{x} 65\% = 40$. If the daily crews are fully used, the maximum daily crew sizes have to be determined by $M_w = 57$ and $M_e = 43$. The daily overtime crew sizes become $O_w = 13$ and $O_e = 10$.

Based on Inequality (8), the minimum hired crew sizes can be determined as $\Phi = 58$ and $\varphi = 38$ (α=65%), with corresponding maximum daily crew sizes $M_w = 53$, $I_w = 17$, $M_e = 40$, and $I_e = 13$ (by Inequalities ((20), (21), (23), and (24)).

According to the absence probability of Route 47 in Table 1, the number of hired extraboard drivers is determined to be 12 with confidence level α=90%. Therefore, the total number of drivers required is 58+12=70. Compared with the crew size 63 determined by the BMTA formula, 7 more drivers are required. This verifies the fact that there were not sufficient drivers on Route 47 and very often some buses had to terminate operation in the afternoon peak due to the lack of drivers.

The discussion above shows that the empirical formula used by BMTA is mathematically infeasible in the sense of covering all required work, though in reality the uncovered work can

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[1] *BMTA sets up a policy that all buses should be fully operated in the morning on weekdays. With this policy, the second-shift drivers have to make up the absence in the morning shift. Therefore, a lack of drivers occurs in the afternoon.*

be abandoned without considering the passenger demand on streets. In order to prevent this, BMTA should either revise their bus and crew scheduling methodology or take some disciplines to prevent the high percentage of absenteeism. The policy of fully operating all buses in the morning may lead to having not enough drivers in the afternoon. It is thus recommended that the second shift drivers do not make up for the absent drivers of the first shift, unless it is absolutely necessary. In this way, the crew management can be much easier, and the heavy load in both peak periods can be better balanced. Furthermore, schedules of the second shift can be more manageable when there is less shortage of drivers.

6. Conclusions

While sophisticated mathematical programming techniques are widely used in finding optimal solutions for the operational problems of bus and crew scheduling, and workforce planning, this paper discusses complementary guidelines for real-time bus and crew scheduling for the terminal queuing operations in the transit systems of developing countries. Formulae for the relationship between daily and hired crew sizes, the daily bus requirements, and holiday and vacation variation are developed. This approach overcomes the shortcoming of the mathematical programming techniques where coefficients are difficult to estimate and solution insight to the real problem cannot be sufficiently gained. The developed formulae highlights the decision insight on optimal crew sizes and thus offers an alternative and efficient way to deal with optimal crew size determination with confidence level indication.

The advantage of this paper's approach for the crew size determination is that it is simple but can offer insight into the real-time crew size management. In addition, it can be easily implemented on any low-level computing equipment, which is suitable for developing countries. Further study can consider mathematical programming formulations to minimise the total labour cost (salary + overtime pay + fringe benefit), based on the relationships developed in this paper.

A case study shows that the developed formulae produce a better solution than the empirical formulae used by the Bangkok Mass Transit Authority (BMTA). Therefore, the approach in this paper can be used as a guideline for BMTA to revise their crew management methodology in order to alleviate the awkward situation that buses have to terminate operation during afternoon peak. Although the study was carried out based on the terminal queuing operation in the Bangkok bus transit system, the basic ideas can be easily extended to other bus transit systems.

References

Daduna J.R. and Wren A. (Editors, 1987) Computer-Aided Transit Scheduling, *Proceedings of the Fourth International Workshop on Computer-Aided Scheduling of Public Transport*, Institut für Informatik of the Universität der Bundeswehr, Germany.

Desrochers M. and Rousseau, J.M. (Editors, 1990) Computer-Aided Transit Scheduling, *Proceedings. of the Fifth International Workshop on Computer-Aided Scheduling of Public Transport*, Montréal, Canada.

Heurgon, E. (1972) Un Probleme de Recouvrement. L'habillage des Horraines d'une ligne D'Autobus, *Revue Francaise d'Automatique Informatique and Recherche Operationelle*, No. V-1.

Heurgon, E. (1975) Preparing duty roster for bus routes by computer, *Proc. of the First International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Chicago.

Hickman M.D., Koutsopoulos H.N., and Wilson H.M. (1988), Strategic model for operator work-force planning in the transit industry, *Transportation Research Record*, No. 1165.

Kaysi I. and Wilson H.M. (1990) Scheduling transit extraboard personnel, *Transportation Research Record*, No. 1266.

Koutsopoulos H.N., and Wilson H.M. (1987) Operator workforce planning in the transit industry, *Transportation Research*, Vol 21A, No. 2.

MacDorman L.C. and MacDorman J.C. (1982) The transit extraboard: Some opportunities for cost saving. *Paper presented at the APTA Annual meeting*.

MacDorman J.C., Schwager D.S. and MacDorman L.C. (1984) Reducing the cost of absenteeism through workforce optimization, *Paper presented at the APTA Western Conference*.

MacDorman L.C. (1985) Extraboard management: procedures and tools, *National Cooperative Transit Research and Development Program, Synthesis of Transit Practice Report*, No. 5.

Mitra, G., and Welsh, A.P.G. (1981) A Computer Based Crew Scheduling System using a Mathematical Programming Approach, *Proc. of the Second International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Leeds, England.

Parker, M., and Smith, B. (1981) Two Approaches to Crew Scheduling, *Proc. of the Second International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Leeds, England.

Ross, M.S. (1985) *Introduction to probability models*, International Edition, Academic Press, Inc., Orlando, Florida.

Rousseau, J.M. (editor) (1983) Computer Scheduling of Public Transport 2, *Proc. of the Third International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Montréal, Canada.

Ryan, D.M., and Foster, B.A. (1981) An Integer Programming Approach to Urban Scheduling, *Proc. of the Second International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Leeds, England.

Wijngaard J. (1983) Aggregation in manpower planning, *Management Science*, No. 29.

Wren, A. (editor) (1981a) Computer Scheduling of Public Transport: Urban Passenger Vehicle and Crew Scheduling, *Proc. of the Second International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Leeds, England.

Wren, A. (1981b) General Overview of the use of Computers in Scheduling Buses and their Crews, *Proc. of the Second International Workshop on Automated Techniques for Scheduling of Vehicle Operations for Urban Public Transportation Services*, Leeds, England.

Zhu, W. (1991) *Bus scheduling management in Bangkok*, Unpublished Doctoral Dissertation, Diss No. IE-91-1, Asian Institute of Technology, Bangkok, Thailand.