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This thesis has been accepted for the award of the degree in the Faculty of Engineering
CYCLIC RESPONSE OF
OFFSHORE PILE GROUPS

by

Caroline M. Hewitt

Thesis presented for the Degree of
Doctor of Philosophy

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SYNOPSIS

A numerical analysis of pile groups under both static and cyclic axial loading is presented. The solution procedure does not use interaction factors to approximate the effect of a loaded pile on the response of neighbouring piles but analyses the group directly, effecting a complete simultaneous solution for pile-soil-pile interaction between all piles in the group.

This analysis can handle groups of dissimilar piles, which may be of non-uniform diameter, arranged in any configuration. The settlement of the pile cap and the axial deflection and the distribution of pile-soil shear stress and load along the pile can be determined at each load increment up to failure, including the effects of slip between the pile and the soil.

Two series of tests of model piles and pile groups in overconsolidated clay subjected to static and cyclic axial loading are described. In the first series, performed on single piles and on 2-pile, 4-pile and 8-pile groups, axial loads were applied, and group settlements, group loads and pile strains were measured. The second series, performed on single piles, was carried out to determine the cyclic degradation characteristics of the clay.

Values of Young's modulus for the clay and the degradation parameters for use in the theoretical analyses were backfigured from the results of tests on single piles. Comparisons are presented between the theoretically predicted and experimentally observed responses to cyclic loading of single piles and pile groups.
PREFACE

This thesis is submitted to the University of Sydney, Australia, for the degree of Doctor of Philosophy. The work described in this thesis was carried out by the candidate in the School of Civil and Mining Engineering at the University of Sydney under the supervision of Professor H.G. Poulos.

In accordance with the By-Laws of the University of Sydney governing the requirements for the degree of Doctor of Philosophy, the candidate claims originality for the following work:

(i) The method of analysis of groups containing piles of different length and diameter is claimed as original although it is based on the method developed by Prof. H.G. Poulos for symmetrical pile groups. The Reverse Plastic slip analysis is claimed as original, as is the inclusion of the Novak and El Sharnouby approximation.

(ii) The complete experimental program is claimed as original.

Two supporting papers and a technical note which were based on the work presented in this thesis were written in conjunction with Professor H.G. Poulos. They were:


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The calculations were performed in the C.A. Hawkins Computing Laboratory at a terminal to a multi-user PRIME 9750 minicomputer thanks to the Civil Engineering Graduates Association for providing the funds to purchase the original system and subsequent improvements. This thesis was typed by the author on a VAX 11/780 minicomputer using the \LaTeX{} typesetting program and was printed by an Apple laserwriter. The author is grateful to the Department of Electrical Engineering for making these facilities available.

The tests were performed in the Geomechanics Laboratory. My thanks go to the laboratory staff in general and to Bob Fraser in particular for the assistance given during the experimental program.

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Special thanks go to my friend and fellow research student Richard Barnes who made the last four years a most enjoyable experience by introducing me to and sharing with me the joys of canoeing.
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NOTATION

All symbols are defined where they first appear in the text. In most cases only one meaning has been assigned to each symbol, but where this is not the case, the interpretation will be evident from the context. For convenience, the notation is given below.

Variables

\( A_r \)  
cross-sectional area of pile

\( A_b \)  
surface area of base element

\( c_u \)  
undrained cohesion

\( d, D \)  
pile diameter

\( D \)  
degradation factor

\( D_r \)  
skin friction degradation factor

\( D' \)  
degradation factor prior to last full reversal of slip

\( E_p \)  
Young's modulus of pile

\( E_s \)  
Young's modulus of soil

\( f_s \)  
ultimate skin friction

\( h \)  
distance between centre of element and base

\( j' \)  
mirror image element of element \( j \)

\( K_c \)  
cyclic stiffness

\( k \)  
compressibility factor of bearing stratum

\( L \)  
length of pile

\( l_i \)  
length of element \( i \)

\( N \)  
number of load cycles

\( N_B \)  
number of base elements

\( N_D \)  
number of discontinuity elements

\( N_S \)  
number of shaft elements

\( P \)  
applied load

\( P_c \)  
cyclic load

\( P_o \)  
mean load

\( P_{u}, P_{us} \)  
static ultimate load

\( P_{uc} \)  
post-cyclic load capacity

\( p_j \)  
shear stress on element \( j \)

\( p_b \)  
vertical stress on base

\( R \)  
elasticity factor

\( R_s \)  
settlement ratio
$S$  deflection of pile
$s$  pile spacing
$X_i$  cross-sectional area of element $i$
$X$  representative load level
$z$  depth
$\alpha$  interaction factor due to adjacent pile
$\delta$  pile settlement, pile group settlement
$\delta_{acc}$  accumulated displacement
$\delta_c$  cyclic displacement amplitude
$\eta$  group efficiency factor
$\lambda$  degradation rate parameter
$\rho_p$  displacement of pile
$\rho_{base}$  displacement of pile tip
$\rho_c$  displacement of pile cap
$\rho_{cs}$  cyclic slip displacement
$\tau$  limiting shear stress
$\tau_a$  ultimate skin friction for static loading
$\tau_{ac}$  ultimate skin friction after cyclic loading
$v$  Poisson's ratio
$\omega$  displacement, relative movement

Vectors

$\{A\}$  elemental surface areas
$\{p\}$  interaction stresses
$\{\delta\}$  global displacements
$\{\sigma\}$  global interaction stresses
$\{s,p\}$  soil displacements
$\{s,\rho\}$  pile displacements
$\{1\}$  values of unity

Matrices

$[AD][FE]$  axial compression of the pile
$[H]$  global soil influence factors
$[s,I]$  soil displacement influence factors
$[PM]$  pile action coefficients
$[\Pi]$  global pile action coefficients
INTRODUCTION and
LITERATURE REVIEW

1.1 THE PROBLEM

Piles have been used as a means of supporting marine structures for centuries. The original offshore oil platforms many decades ago were no more than marine structures cut off from land. The extension of these structures into deeper and deeper waters was gradual, particularly in North America, and confidence in design methods for the foundations gradually built up.

With the discovery of the oil fields under the North Sea, however, platforms have had to be anchored to the sea floor in water depths of up to 300 metres. This has represented a sudden departure from previous experience due to the hostile marine environments. For example, the North Rankin A natural gas platform, located 130 km offshore from Karatha in Western Australia, has had to be designed to withstand a 100 year cyclone with 23 m waves and 215 km/hr winds. It is impractical to install individual piles to resist these loadings and so pile groups must be used.
Most of the jacket-type platforms in use at the North Sea have a piled foundation. Such piles are subject to a permanent static load component due to the self-weight of the structure and to a cyclic load component due to wave action which produces a regular repeated load. The need to study the effects of repeated loading stems from the observation that, in general, when a foundation is loaded and then unloaded, there is a net change in the state of stress at the end of the cycle compared to the original state. Cyclic loading is not only confined to offshore structures, it also occurs onshore due to the effects of wind, wave, earthquake, ice, tide and external load changes. In the case of lightweight structures, such as electric power transmission structures, these transient loads often exceed the dead loads used for static design criteria.

There appears to have been little research done into the interaction between cyclically loaded pile groups and the surrounding soil. No generally-applicable analysis of the load-settlement behaviour of a cyclically-loaded pile group appears to have been developed. None of the commonly-applied design rules used in offshore pile design directly accounts for cyclic loading effects. Thus, there may be a very inconsistent safety level in the different types of piled offshore structures designed in accordance with current practice.

The major component of wave loading is in the vertical direction, resulting in axial loads being transferred to the pile groups. Therefore this study will look at the axial behaviour of pile groups. The major design considerations for the pile group are:

- the ultimate axial capacity of the pile group after it has been subjected to prior cyclic axial loading, and

- the load-settlement behaviour after it has been subjected to prior cyclic axial loading.

The purpose of this thesis is to study, both theoretically and experimentally, the response to cyclic axial loading of offshore pile groups.
1.2 LITERATURE REVIEW

1.2.1 General

A quantitative analysis of the settlement behaviour of axially loaded pile foundations has only been available in the last 20 years. Previously, most methods of predicting the settlement of pile foundations relied on more or less arbitrary assumptions of the way in which the piles transfer their load to the soil.

The literature review presented in this thesis covers the development of the quantitative analysis of single piles and the way in which different researchers have modified the analysis to take into account group action and cyclic loading. The experimental work on which the researchers have based or verified their analyses is also presented.

1.2.2 Theoretical

The theoretical work is divided into static loading and cyclic loading response. The static loading of single piles is treated first and then the analyses are extended to pile groups. O'Neill (1983) has presented a useful summary of the state of the art in analytical techniques for group action. He categorises the analyses into a number of classes which will be adopted for this review.

The cyclic loading of single piles is discussed and a number of different methods are presented to account for the effects of cycling. Finally the effects of cyclic loading on the response of pile groups are reviewed.

(a) Static Loading of Single Piles

The static loading of single piles has been the subject of much research however this review will not try to cover the complete analyses of single piles. A number of methods have been used to study the static behaviour of single
piles. These methods include the boundary element method, the finite element method and the load-transfer (t-z) method. The boundary element method is well suited to the solution of pile problems since it involves only a discretization of the boundaries and so the number of equations to be solved is much smaller than in the finite element method. Thus for three-dimensional foundation engineering problems it is the most efficient numerical method. Piles are usually analysed using Mindlin’s (1936) three-dimensional elasticity equations for the displacement field generated by a static point load applied in the interior of the elastic half-space.

Poulos and Davis (1968) used this method to analyse the behaviour of a single axially loaded incompressible floating pile in an ideal elastic soil. The pile was divided into n equal cylindrical elements each acted upon by a uniform shear loading p and a circular base having a uniform vertical stress ps. The shear stress on a pile element was represented as a vertical stress uniformly distributed around the outer circumference of the element. The vertical displacement of the soil was given by

\[ \{s\rho\} = \frac{d}{E_s} [sI] \{p\} \]  \hspace{1cm} (1.1)

where

\[ \{s\rho\} \quad \text{= soil displacement vector} \]
\[ [sI] \quad \text{= matrix of soil displacement influence factors obtained by double integration of the Mindlin equation} \]
\[ \{p\} \quad \text{= interaction stress vector.} \]

By considering the axial compression of each element of the pile the vertical displacement of the pile was given by

\[ \{p\rho\} = \rho_{base}\{1\} + [AD][FE]\{p\} \]  \hspace{1cm} (1.2)

where

\[ \{p\rho\} \quad \text{= pile displacement vector} \]
\[ \rho_{base} \quad \text{= displacement of pile tip} \]
\[ \{1\} \quad \text{= vector of values of unity} \]
\[ [AD][FE] = \text{axial compression matrix} \]
\[ \{p\} = \text{interaction stress vector}. \]

The stresses were solved for by equating the pile and soil displacements to ensure compatibility at the pile soil interface to give

\[
\frac{d}{E_s} \{sI\} \{p\} = \rho_{\text{base}} \{1\} + [AD][FE] \{p\} \tag{1.3}
\]

The effects of slip between the pile and the soil were taken into account by assuming that slip would occur when the shear stress at an element reached the adhesive strength between the pile and the soil. Any element which had failed could take no additional load and so the increase in load was redistributed among the remaining elastic elements. This procedure could be repeated until all the elements in the pile had failed. In this way the load-settlement relationship up to failure could be obtained.

Poulos and Mattes (1969) extended the previous work to include the analysis of end-bearing piles and to take into account the relative compressibility of the pile. To allow for the reduction in the soil displacement due to the presence of the bearing stratum, a mirror image element \(j'\) of an element \(j\) was introduced. Element \(j'\) was acted upon by a shear stress \(kp_j\) acting in the opposite direction to \(p_j\) where \(0 \leq k \leq 1\). The value of \(k\) depended on the compressibility of the bearing stratum. Therefore the vertical displacement of the soil (Eq. 1.1) became

\[
\{s \rho\} = \frac{d}{E_s} \{p\} [sI - k_s I'] \tag{1.1}
\]

The relative compressibility of the pile was incorporated into the analysis by the addition of a Hookean elongation term into the pile displacement equation due to the applied load \(P\). This additional term was

\[
\rho \rho = \frac{Ph}{E_p A_p}
\]

where
\[
h = \text{distance between centre of element and base} \]
\[
A_p = \text{cross-sectional area of pile} \]
\( E_p \) = Young's modulus of pile.

The analysis was further extended to the case of non-homogeneous soils in Poulos (1979a). Although Mindlin's equation was no longer strictly applicable if the soil was non-homogeneous, it was found that it could be utilized approximately by taking \( E_s \) as the mean of the moduli at the influenced and influencing elements. The soil displacement equation for a homogeneous soil (Eq. 1.1) was replaced by

\[
\{s, \rho\} = \bar{d} \left[ \frac{s}{E_s} \right] \{p\}
\]

assuming

\[
E_s = \frac{E_{si} + E_{sj}}{2}
\]

where

\( E_{si} \) = soil modulus at the influenced element \( i \)
\( E_{sj} \) = soil modulus at the influencing element \( j \).

The finite element method is the most powerful of the theoretical methods. Balaam, Poulos and Booker (1975) have adopted a finite element analysis for an axially loaded pile in which the pile and soil were analysed as separate bodies. Equilibrium and displacement compatibility at the pile soil interface was then imposed to obtain a solution for the settlement of the pile. This method overcame the need to introduce special joint elements at the pile-soil interface, as had been used by Ellison, D'Appolonia and Thiers (1971). The analysis could take account of slip at the pile-soil interface and local yield within the soil. A comparison was made between load-settlement curves to failure from this finite element analysis and that of Poulos and Mattes (1969). The agreement was quite good, although at loads approaching failure the settlements given by the finite element analysis were somewhat greater.

Everett and McMillan (1975) reported finite element investigations on a test pile. They used an elastic axi-symmetric finite element analysis which was conducted in four stages to take account of the failure in friction which was expected to occur between the shaft and the soil in the upper strata as the load
was increased. Comparison of results from this model with the full-scale test pile gave good agreement as is shown in Fig. 1.1. Other finite element analyses on single piles have been carried out by Desai (1974) and Ellison et al. (1971).

The load-transfer method utilizes soil data measured from field tests on instrumented piles and laboratory tests on model piles. The relevant soil data required in this method are curves relating the ratio of the adhesion (or load transfer) and the soil shear strength to the pile movement. A number of curves may be required to describe the load transfer along the whole length of the pile.

(b) Static Loading of Pile Groups

The analytical techniques used to study the static loading of pile groups, as outlined by O'Neill (1983), include the boundary element method, the modified boundary element method, the hybrid model and the finite element method. Poulos (1968) and Poulos and Mattes (1971) extended their boundary element analysis of a single pile to consider the settlement interaction between two identical floating piles and to the case of a general floating pile group. As long as all the piles in a group were identical, 2-pile interaction factors, denoted by \( \alpha \), could be used to analyse the group. The interaction factor was defined as the additional displacement at the top of a pile due to an equally loaded identical adjacent pile, i.e.

\[
\alpha = \frac{\text{additional displacement due to adjacent pile}}{\text{displacement of pile under its own load}}
\]

The interaction factor was a function of the distance between the two piles and could be obtained by integration of the Mindlin equation for vertical displacement. For the case of 2-pile groups and symmetrical pile groups (i.e. groups in which the piles were spaced equally around the circumference of a circle and each pile displaced equally while also carrying an equal amount of load), the use of interaction factors rendered an extremely efficient solution because the size of the matrix to be solved was only the same as that of a single pile. For a general pile group of \( m \) piles, the displacement of any pile \( k \) in the group was

\[
\rho_k = \rho_1 \sum_{i \neq k} P_i \alpha_{kj} + \rho_1 P_k
\]  

(1.4)
where
\[ \alpha_{kj} = \text{value of } \alpha \text{ for two piles corresponding to the spacing between pile } k \] and pile \( j \),
\[ P_j = \text{load in pile } j \]
\[ \rho_1 = \text{displacement of a single pile under unit load.} \]

The solution of this equation for a general pile group, however, required a larger matrix to be solved.

Several other researchers have tackled the pile group problem using the boundary element method. El Sharnouby and Novak (1984) developed a simple, efficient method for the analysis of large pile groups subjected to vertical loads, horizontal loads and moments under static or cyclic loading. The simplification came in the calculation of the displacement influence factors. Instead of obtaining them by double integration of the Mindlin equation, nodal soil flexibility coefficients were generated by the application of point loads applied and located such that the resultant flexibility coefficients were almost the same as those obtained from continuously distributed shears. The group was then analysed directly, avoiding the use of interaction factors. This approach is further investigated in section 2.4.

The interaction factor method is generally satisfactory provided only the load distribution at the pile heads is required and all piles in the group are of the same diameter, length and stiffness. Therefore the method can be applied only to symmetrical pile groups. Direct boundary element models, however, involve the simultaneous solutions for pile-soil-pile interaction between all piles in a group. They tend to be more accurate than 2-pile interaction solutions because they take account of the trend for redistribution of axial loads towards the pile tips as the number of piles in a group increases.

Butterfield and Banerjee (1971) described a direct boundary element analysis of general groups under axial loading in which the rigid pile base and radial deformation compatibility conditions could be included. This analysis was more rigorous than the Poulos (1968) analysis of groups involving the use of interaction factors.
The above method was combined with a three-dimensional frame analysis and a simplified integral representation for the soil domain in Banerjee and Driscoll (1976) which could be used to analyse vertical pile groups of any geometry subjected to any combination of vertical loads, horizontal loads and moments. Although this method allowed for the effect of the interaction between the piles it was a linear analysis and could not allow for slip or local pile-soil failure along the pile. However it was a step towards a less approximate method for analysis of soil structure interaction problems.

Randolph and Wroth (1979) introduced an approximate analytical model for the computation of vertical deformation of pile groups which was a modification of the boundary element model. The method was based on the superposition of the displacement fields of individual piles within the group, considering the average behaviour down the pile shafts separately from that beneath the level of the pile bases. The piles were coupled through the soil, using a plane strain assumption rather than using the three-dimensional solutions used in the boundary element model, and thus simplifying the solution. The soil could be homogeneous, or its stiffness could increase linearly with depth. However, the method was again confined to linear elastic soil behaviour.

O’Neill, Ghazzaly and Ha (1977) proposed a hybrid model in which the axial and lateral responses of individual piles were modelled using the load-transfer (t-z) method for axial loading and the ‘p-y’ method for lateral loading. The interaction between the piles through the soil was effected using Mindlin’s solution for a point load within a homogeneous, isotropic elastic half-space. Non-linear pile response and non-homogeneous soil profiles could be handled, and it was possible to consider general three-dimensional groups subjected to all six components of load on the pile-cap.

Chow (1986) presented an approach for the analysis of linear and non-linear response of vertically loaded pile groups. The soil behaviour of individual piles was modelled using hyperbolic load-transfer curves while the pile-soil-pile interaction was based on Mindlin’s solution. Comparison of this method with the more rigorous boundary element approach for pile groups in a homogeneous,
isotropic elastic half-space gave generally good agreement.

Clausen, Aas and Hasle (1981) have written a computer program which combined a three-dimensional linear elastic structural analysis program with the non-linear hybrid model for pile groups, similar to that described by O'Neill et al (1977) above. This analysis could handle three-dimensional geometry with six degrees of freedom for both structure and piles. Compatability between superstructure and piles at the interface was ensured and pile group interaction effects were included as an integral part of the solution. The program included static loading conditions only.

The finite element method is the most powerful of the theoretical methods but because of the three-dimensional nature of the pile group problem it is also the most expensive. Pressley and Poulos (1986) have used a total stress non-linear finite element analysis to examine the mechanisms of group behaviour and their variation with pile spacing. The groups were represented by an equivalent axially-symmetric model as shown in Fig. 1.2. It was shown that, at close spacings, the block failure mechanism occurs, with significant plastic zones being developed below the group and full pile-soil slip only being developed along the outer piles. This is illustrated for a closely-spaced 9-pile group in Fig. 1.3.

Ottaviani (1975) used three-dimensional finite elements to study the maximum shear stress distribution on a horizontal cross-section just above the piles’ base of a 3x3 pile group, assuming for simplicity, linear soil behaviour. He found that the stresses in the soil were much smaller inside the perimeter of the pile group than immediately outside. Most of the above analyses assumed a symmetric distribution of shear stress around the periphery.

(c) Cyclic Loading of Single Piles

Poulos extended his boundary element analysis for static loading to include cyclic loading by a number of different methods. To simplify the computations and give a better insight into the progress of cyclic degradation Poulos (1979b) introduced the concept of a ‘degradation factor’ which related the value of a
soil parameter after cyclic loading to the corresponding value for static loading. For example, the degradation factor for ultimate skin friction $D_r$ was defined as

$$D_r = \frac{\tau_{ac}}{\tau_s}$$

where

$\tau_{ac}$ = ultimate skin friction on a pile element after it has been subjected to cyclic loading

$\tau_s$ = ultimate skin friction for static loading.

Poulos (1979b) outlined an effective stress approach in which cyclic loading was assumed to develop excess pore pressures in the soil adjacent to the pile with a constant reduction in soil modulus and skin friction. However, further developments by Poulos (1981a) suggested that consideration of cyclic degradation of the soil in terms of total stress could be more appropriate for piles in clay. In his analysis, the response of a pile was ascertained after a number of cycles $N$ of uniform magnitude (maximum $P_{max}$, minimum $P_{min}$) was applied to the pile and the soil parameters were then adjusted to reflect the effects of cyclic loading at the end of the load sequence.

An alternative to the ‘single step’ analysis for cyclic loading, described above, was the ‘cycle-by-cycle’ analysis presented by Poulos (1983). For this approach each of the $N$ cycles was modelled in sequence. After each cycle, the degradation factors were determined and used to adjust the ultimate skin friction, ultimate base resistance and soil modulus for the next cycle of load. Because the analysis simulated each cycle of loading, the behaviour of the pile at all stages of the cycling process could be determined.

The cycle-by-cycle analysis allowed for the consideration of ‘parcels’ of load of different amplitude as described by Poulos (1982). This procedure was useful for assessing the effects of storm wave loading on offshore piles and pile groups.

Finally, it was recognised that, although cyclic degradation of skin friction and base resistance dominated under two-way cyclic loading, under one-way cyclic loading it was probable that failure would occur by the accumulation of
permanent displacements. These displacements are discussed in section 5.4.8. The calculation of permanent displacements was incorporated into the analysis in Poulos (1987) by modifying the expression for incremental soil displacements.

Three different methods for the analysis of non-linear behaviour of single piles using load-transfer (t-z) curves, as described in section 1.2.2 (a), have been presented in the literature. In the first method (Matlock and Foo, 1980) a general, versatile numerical procedure was presented to facilitate the examination of an axially loaded pile, under both static and cyclic loadings. The method of analysis included a fully non-linear, hysteretic, degrading soil model; soil degradation was assumed to occur if a full reversal of yielding in both directions occurred. This method of soil degradation was adopted by Poulos (1983) for his cycle-by-cycle analysis.

In the second method (Karlsrud, Nadim and Haugen, 1986) an attempt was made to model cyclic degradation of piles under both one-way and two-way loading. Most existing methods did not take into account the accumulation of permanent displacements which could lead to failure in one-way cyclic loading. The method involved the construction of relevant strain contours. The strain contours were constructed from the results of cyclic direct simple shear tests. Load-transfer curves for spring elements were then directly evaluated from the strain contours and an iterative procedure was used to calculate the shear stress distributions.

In the third method Randolph (1988) discussed a form of load-transfer curve which consisted of a parabola up to peak skin friction, \( \tau_p \), followed by strain softening down to residual skin friction, \( \tau_r \), after a specified additional (absolute) displacement, \( \Delta w_{res} \). The form of the curve, which has been incorporated into a computer code, RATZ (Randolph, 1986) is shown in Fig. 1.4. Under cyclic loading, accumulated 'plastic' displacements (arising from the non-linear form of the unload-reload response) were treated as equivalent to monotonic plastic displacement in calculating the degree of degradation. This load-transfer algorithm was able to separate cyclic shear stress regimes where failure would ultimately occur, from a region which was 'safe'.

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(d) Cyclic Loading of Pile Groups

All of the above methods for the analysis of single piles under cyclic loading can also be used for pile groups. Poulos (1982) has used his analysis to study pile groups subjected to cyclic loading and has found that group effects tend to increase displacements and cause more severe degradation. He described an example which indicated that the maximum cyclic load (per pile) which could be applied decreased significantly as the number of piles in a group increased.

Randolph's (1988) RATZ solution accounted for interaction effects within groups of piles by altering the displacement to peak skin friction which was calculated from the specified shear modulus of the soil. This displacement was one of the parameters describing the load-transfer curve. However, it is unclear from the paper what the group effects are.

1.2.3 Experimental Data

A number of researchers have carried out model pile tests subjected to static and cyclic loading in both sand and clay. However, very few large-scale tests or tests on pile groups have been performed. Although the properties of sand and clay are very different, their responses to cyclic loading have many similarities and so both have been included in this review.

(a) Static Loading of Pile Groups

A series of tests on model pile groups in remoulded soft clay reported by Matlock, Bogard and Cheang (1982) indicated that a pile group could displace up to 4.5 times that of a single pile before fully plastic soil response was developed. The primary effect of group action on axial pile behaviour was to increase the group displacement at any value of load per pile as compared with a single pile.

O'Neill (1983) has presented a useful summary of the vertical efficiencies
for model pile groups in both sand and clay. His figures are reproduced in Fig. 1.5 and he noted the following trends from the figure: (a) group efficiency \( \eta \) in loose sands always exceeds unity, with highest values occurring at spacing-to-diameter ratios, \( s/d \), of about 2, and generally higher \( \eta \) occurring with increasing numbers of piles. Block failure affects \( \eta \) only below \( s/d \approx 1.5 \) to 2.0. (b) \( \eta \) in dense sands may be either greater or less than unity, although the trend is towards \( \eta > 1 \) for \( 2 < s/d < 4 \). (c) \( \eta \) in clay is always less than unity. Block failure may occur in square groups at \( s/d \) values of less than about 2.

(b) Cyclic Loading of Single Piles

Karlsrud et al. (1986) conducted an extensive series of static and cyclic axial pile load tests on large-scale test piles in an overconsolidated clay deposit. They found that cyclic failure in one-way cyclic loading was associated with a large and sudden increase in accumulated displacements. This increase in accumulated displacements occurred before any significant increase in cyclic displacements. For symmetrical two-way loading, the cyclic displacement amplitude governs the failure, whereas in non-symmetrical two-way loading, both the cyclic displacement amplitude and accumulated displacement increase towards failure. Fig. 1.6 shows the difference in failure trends between one-way and symmetrical two-way loading.

From their observations they proposed the following failure criterion for their tests. Complete cyclic failure was defined by the following combined criteria:

- a total accumulated displacement during cycling of 1.0 - 1.5mm
- an 'effective' rate of displacement exceeding 0.5 mm/min
- an increase in displacement amplitude of 0.5 to 1.0 mm relative to the first cycle.

However these criteria may not be appropriate for larger-size piles e.g. offshore
platform piles.

Many of the model tests on piles have concentrated on the reduction of skin friction due to cyclic loading. Steenfelt, Randolph and Wroth (1981) carried out two-way cyclic loading tests to failure and found that this led to a dramatic reduction in shaft adhesion, over 18 cycles, by a factor of \(\sim 4\) in compression and \(\sim 2\frac{1}{2}\) in tension. Poulos (1981b) found that for two-way cyclic loading both the magnitude of cyclic displacement and the number of cycles influenced the loss of skin friction. Charlie, Turner and Kulhawy (1985) found, in a review of available field and laboratory data, that all but the very lowest levels of repeated loading (less than 25\% of the static capacity in the few reported studies) showed continuing deformation without any limit.

Some ideas have been put forward as to the mechanism of the cyclic degradation. Poulos (1981b) commented that the reduction in skin friction because of load reversal could be largely a result of particle re-orientation at the pile-soil interface. However Matlock et al. (1982) reported a series of tests on model pile segments in saturated soft clay, subjected to two-way displacements, in which the changes occurring in the load-deformation behaviour during cyclic degradation indicated that the mechanism of degradation was related to the concentration of reversed-shear slip on a thin cylindrical shear zone, or slip surface, located some finite distance from the pile wall. A mechanistic interpretation of the behaviour is shown in Fig. 1.7.

A number of test results indicate a threshold level of cyclic load. Steenfelt et al. (1981) reported that in performing cyclic loading there appeared to be a threshold load level below which two-way cyclic loading had no detrimental effect on pile capacity. Above this threshold there was evidence that dramatic reductions in the available shaft adhesion could occur. Poulos (1981a) conducted a series of small-scale model pile tests in four different remoulded soft saturated clays. In these tests only the side resistance was mobilised. The results indicated little reduction in ultimate load capacity at 1000 cycles for two-way repeated loads unless the half amplitude of cyclic load \(P_e\) exceeded about 60-70\% of the ultimate static load capacity. A threshold for no reduc-
tion in static capacity was a repeated loading of 20% of the static capacity. The results of another series of model tests carried out by Poulos (1981b) on a pile shaft in clay showed that, for cyclic displacements of less than about ±0.2% of the pile diameter, little or no loss of skin friction occurred, but for larger cyclic displacements, a marked decrease in load capacity was observed. It was also observed from this series that the skin friction was not seriously affected unless the cyclic load exceeded about 40% of the static ultimate load, $P_{us}$. For cyclic loads in excess of about 40% of $P_{us}$, a very marked decrease of skin friction was found to occur.

It has generally been found that one-way cycling may lead to skin friction degradation, but that two-way cycling leads to much more severe degradation. In their review of repeated axial load tests on deep foundations, Charlie et al. (1985) have found that the available laboratory and field data suggested that two-way repeated loads generally led to capacity reduction while one-way loads had less effect on capacity.

Karlsrud et al. (1986) found that for one-way loading the cyclic capacity of test piles in an overconsolidated clay deposit was 75 to 85% of the ultimate static capacity. For two-way cyclic loading however, the minimum degraded capacity that was finally reached was only 42% of the ultimate static capacity. An ‘interaction diagram’ relating normalised cyclic load to normalised mean load has been plotted from their extensive test series and is shown in Fig. 1.8. It is evident from this diagram that the cyclic capacity was much smaller after two-way cyclic loading than after one-way cyclic loading. The results of cyclic direct simple shear tests that were carried out on remoulded re-consolidated clay were also plotted on the diagram and were in good agreement with the cyclic pile test results.

Poulos (1979b) found that for two-way cycling of laboratory model piles jacked into remoulded clay, failure during cycling occurred at cyclic load levels of between 56% and 62% of the ultimate static load. This behaviour during two-way cycling fitted in with the mechanism of cyclic degradation proposed by Matlock et al. (1982) mentioned earlier i.e. that the enforced reversals of
slip in the clay near the pile due to cyclic loading would produce more severe
degradation than one-directional slip.

Poulos (1981b) has looked at the effect of rate of loading on pile load ca-
pacity. The results of two series of tests on a pile shaft in clay indicated an
increasing ultimate load capacity with increasing rate. This is illustrated in
Fig. 1.9. Charlie et al. (1985) reported tests by Kraft, Cox and Verner (1981)
who found that if the loading rate was increased by about three orders of mag-
nitude the ultimate pile capacity could be increased by 40% to 75% which is
in agreement with Poulos' findings.

It has generally been found that skin friction degradation in silica sand is
more severe than that in clay. Poulos (1984) has also found from laboratory
model pile tests that skin friction degradation in calcareous sand is even more
severe than that in silica sand. Fig. 1.10 shows a comparison of skin friction
degradation in silica sand and in calcareous sand. At a normalised cyclic de-
flation of $\rho_c/\rho_un = 4.0$, where $\rho_un$ is the displacement required to develop the
ultimate static skin friction, the normally-consolidated silica sand had reached
its minimum skin friction degradation factor of 0.615, whereas the skin friction
degradation factor for the tests in calcareous sand was 0.75 and decreased to
reach a minimum value of 0.50.

Van Weele (1979) observed substantial increases in settlement due to cyclic
loading of full-scale piles in medium-dense saturated sand. Above a repeated
load of approximately 60% of the static capacity, each cycle of repeated load
caused additional settlements. Poulos (1984) found that skin friction degra-
dation of model piles in calcareous sand depended on the amplitude of cyclic
displacement. Significant cyclic degradation of skin friction would not occur
unless the cyclic axial displacement exceeded the static displacement required
to cause full shaft slip.

Poulos (1982) reported that in small-scale laboratory tests in medium-dense
silica sand the skin friction could drop to 60% of the static value after only 10
load cycles if the cyclic displacement was 10% of the pile diameter. However in
calcareous sand Poulos (1984) reported that for large relative cyclic displace-
ments a reduction in skin friction of about 50% could occur. Poulos and Chan (1986) also reported a reduction in skin friction of about 50% and Lu (1986) reported model pile tests in which the skin friction had dropped to only 35% of the static value after performing a displacement-controlled two-way test in which the pile was cycled to ±14% of its diameter for 200 cycles.

Chan and Hanna (1980) performed one-way cyclic loading tests on laboratory model piles in dry medium dense sand to large numbers of cycles. They found that pile behaviour was highly dependent on the magnitude of the repeated load and that pile failure could occur with cyclic loads of 30% of the ultimate static load. They also found that one of the effects of repeated loading was to cause a redistribution of loads from the shaft to the base as can be seen in Fig. 1.11.

Poulos (1982) found that the general characteristics of cyclic degradation of skin friction in silica sand, with its dependence on the cyclic displacement, were similar to those for piles in clay and the majority of degradation appeared to take place within the first ten cycles. Poulos (1984) also reported a series of tests in calcareous sand which indicated that the majority of cyclic degradation occurred within the first few cycles, and that beyond about ten cycles, very little further degradation occurred.

Finally, Poulos (1984) found that neither the soil modulus nor the ultimate base bearing capacity appeared to be seriously influenced by cyclic loading.

(c) Cyclic Loading of Pile Groups

There have been very few tests carried out on pile groups subjected to cyclic loading. A series of tests on model pile groups in remoulded soft clays have been reported by Matlock et al. (1982). After 80 load cycles of two-way cyclic load in which the 6-pile group was cycled to a displacement of 25% of the pile diameter, the cyclic capacity of the group was reduced to about 34% of the ultimate static capacity. The shear transfer variation among the piles in the group is shown in Fig. 1.12. This loss in capacity was much greater than that for the single piles.
O’Neill, Hawkins and Mahar (1982) described the results of one-way repeated compression loading of full size piles driven into saturated overconsolidated clay. A 9-pile group and two single piles were loaded to failure in compression three times. Overall side resistance degradation during compression testing was 23% in the group piles compared to 9% in the single piles. The degradation in side resistance was thought to be due to side shear stress reversals. The successive loadings to failure also had the effect of increasing the tip resistance, especially in the pile groups.

1.2.4 Shortcomings in Knowledge of Pile Groups

The literature review carried out exposes a number of shortcomings in knowledge on the subject of pile groups. There has been some work done on single piles, both theoretical and experimental, but a comparatively small amount of work done on the pile group problem, especially groups subjected to cyclic loading.

Very few of the theoretical analyses can analyse general non-symmetrical pile groups. The Poulos (1968) analysis can only handle symmetrical pile groups. The Banerjee and Driscoll (1976) analysis can handle pile groups of any geometry but cannot allow for slip along the pile. The analyses of O’Neill et al. (1977) and Clausen et al. (1981) can handle general three-dimensional groups and non-linear pile response but are unsuitable when considering cyclic loading effects. The Ottaviani (1975) finite element analysis is too expensive for most uses.

The biggest shortcoming in the experimental work is the dearth of tests on pile groups, especially those subjected to cyclic loading. Until a wide range of test results are available it is difficult to confidently predict the cyclic response of offshore pile groups.
1.3 OUTLINE OF THESIS

A numerical analysis of pile groups under both static and cyclic axial loading is presented in Chapter 2, which extends the previously developed boundary element model of Poulos. The solution procedure does not use interaction factors to approximate the effect of a loaded pile on the response of neighbouring piles but analyses the group directly, effecting a complete simultaneous solution for pile-soil-pile interaction between all piles in the group. This analysis can handle groups of dissimilar piles, which may be of non-uniform diameter, arranged in any configuration. The theory can predict the settlement of the pile cap, the axial deflection and the distribution of pile-soil shear stress and load along the pile at each load increment.

A Reverse Plastic slip analysis is developed which takes into account the effects of slip between the pile and the soil by excluding negative plastic work. This method calculates the actual slip movements at each element which has slipped and is more efficient than the method outlined by Poulos and Davis (1968). Allowance is made for individual pile loads to be input, instead of the total group load. Under cyclic loading, consideration is given to cyclic degradation of skin friction, base resistance, Young's modulus and the accumulation of permanent displacements.

A number of different pile groups are analysed in Chapter 3. Under static loading, consideration is given to the effect of bells, piles of unequal length, piles of different diameter, a defective pile in the group and imperfect alignment. The mechanism of cyclic degradation in such groups is then discussed and a number of different groups under cyclic loading are analysed.

Chapter 4 describes two series of tests on model piles and pile groups which have been carried out in overconsolidated clay. The first series, performed in a large vessel, involves 26 static tests and 52 cyclic tests on single piles and 14 static tests and 30 cyclic tests on 2-pile, 4-pile or 8-pile groups. In each test, axial loads have been applied to the pile group, and group settlements, group loads and pile strains have been measured. The second series, performed
on a single pile in a small vessel, has been carried out to determine the cyclic
degradation characteristics of the clay. The test procedures and the observed
response of the pile groups are described.

Comparisons are made between the theoretically predicted and experimentally observed responses to cyclic loading of single piles in Chapter 5 and pile
groups in Chapter 6. Finally, a summary of conclusions is presented in Chapter
7.
Figure 1.1 Load Deflection Curve from Finite Element Analysis
(Everett and McMillan, 1975)
Figure 1.2  Modelling of Pile Group (Presley and Poulos, 1986)
Figure 1.3 3x3 - 1.5D Group showing Plastic Zones and Slipped Zones (Pressley and Poulos, 1986)
Figure 1.4 Load Transfer Curve (Randolph, 1986)
Figure 1.5  Vertical Efficiencies for Model Pile Groups (O’Neill, 1983)
Figure 1.6 Load Displacement Curves for some Selected Cycles during a One-way and Two-way Test (Karlsrud, Nadim and Haugen, 1986)
Figure 1.7  Mechanistic Interpretation of Reversed Shear Slip Behaviour (Matlock, Bogard and Cheang, 1982)
Figure 1.8 Interaction Diagram giving Number of Load Cycles to Failure as Function of Cyclic Load Level
(Karlsrud, Nadim and Haugen, 1986)
Figure 1.9 Effect of Strain Rate on Ultimate Load Capacity of Pile Shaft (Poulos, 1981b)
Figure 1.10  Skin Friction Degradation Factors for Piles in Silica Sand and Calcareous Sand (Poulos, 1982 and Poulos, 1984)
Figure 1.11 Load Transfer and Pile Movements
(Chan and Hanna, 1980)
Figure 1.12  Shear Transfer Variation Among Piles in the Group  
(Matlock, Bogard and Cheang, 1982)
THE ANALYSIS
OF PILE GROUPS

2.1 INTRODUCTION

The analysis of groups in which the piles are symmetrically located around the circumference of a circle has been described by Poulos (1968), and the concept of interaction factors, to allow consideration of general pile group configurations, has been developed from this analysis. However, the use of interaction factors becomes questionable for cases in which not all piles in the group are identical, and for such cases, a more general analysis is necessary.

In this chapter a numerical analysis of pile groups containing dissimilar piles, arranged in any configuration, is presented. The extension of the static loading analysis to consider cyclic loading is also discussed. Several improvements to Poulos' original analysis are presented, including:

(i) a more efficient method of evaluating the analytical and numerical integrations, suggested by El Sharnouby and Novak (1984);

(ii) the calculation of permanent displacements under cyclic loading;
(iii) the adoption of a 'Reverse Plastic Slip' model which is more efficient and more reliable than the analysis described in section 2.2 and also calculates the actual movements when slip occurs.

A number of computer programs have been written to perform the above analyses and details of these programs are presented.

2.2 STATIC LOADING

Fig. 2.1 shows a general pile group containing non-identical piles, in which a typical pile $i$ is divided into $m_i$ elements, which may include cylindrical shaft elements, annular base elements, and annular elements at discontinuities in shaft diameter. The surrounding soil is assumed to be an elastic continuum having a constant Poisson's ratio $\nu_s$ and a Young's modulus $E_s$, at any element $j$. Each element is acted upon by an appropriate interaction stress or traction at the pile-soil interface.

Consideration is given to compatibility between the displacements of the pile and the soil at each element, but allowance is also made for the possibility of pile-soil slip or soil yield by specifying limiting values of the interaction stress at each element.

For a single uniform section pile in a homogenous soil, the incremental soil displacements at each element may be written as:

$$\{s\Delta \rho\} = \frac{d}{E_s}[sI]\{\Delta p\}$$

(2.1)

where

$$\{s\Delta \rho\} = (m_i) \text{ incremental soil displacement vector}$$

$$[sI] = (m_i \times m_i) \text{ matrix of soil displacement influence factors}$$

$$\{\Delta p\} = (m_i) \text{ incremental interaction stress vector, } m_i \text{ is the number of elements in pile } i.$$  

The elements of $[sI]$ may be obtained by double integration of the Mindlin (1938) equation, as described by Poulos and Davis (1968).
Equation 2.1 can be extended to consider a non-uniform section pile in a non-homogeneous soil mass, giving

$$\{s\Delta \rho\} = \left[\frac{d}{E_s}I\right] \{\Delta p\}$$  \hspace{1cm} (2.2)

where $E_s$ is the mean value of modulus, as described by Poulos (1979a). For example, the value of $E_s$ in equation 2.2 is assumed to be:

$$E_s = \frac{E_{si} + E_{sj}}{2}$$

where

- $E_{si} = \text{soil modulus at the influenced element } i$
- $E_{sj} = \text{soil modulus at the influencing element } j$.

Equation 2.2 can further be extended to a general group of $n$ piles as follows:

$$\begin{bmatrix}
\{s\Delta \rho_1\} \\
\vdots \\
\{s\Delta \rho_i\} \\
\vdots \\
\{s\Delta \rho_n\}
\end{bmatrix} =
\begin{bmatrix}
\text{I}_{11} & \cdots & \text{I}_{1j} & \cdots & \text{I}_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\text{I}_{i1} & \cdots & \text{I}_{ij} & \cdots & \text{I}_{in} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\text{I}_{n1} & \cdots & \text{I}_{nj} & \cdots & \text{I}_{nn}
\end{bmatrix}
\begin{bmatrix}
\Delta p_1 \\
\vdots \\
\Delta p_j \\
\vdots \\
\Delta p_n
\end{bmatrix}$$  \hspace{1cm} (2.3)

where

- $\{s\Delta \rho_i\} = (m_i)$ column vector for soil displacements of pile $i$
- $\text{I}_{ij} = (m_i \times m_j)$ soil influence factor matrix for effect of pile $j$ on pile $i$, multiplied by $d_j/E_s$
- $\Delta p_j = (m_j)$ column vector of incremental interaction stresses on pile $j$
- $m_i = \text{number of elements in pile } i$
- $m_j = \text{number of elements in pile } j$.

Equation 2.3 may be written more compactly as:

$$\{s\delta\} = [H]\{\sigma\}$$  \hspace{1cm} (2.3a)

where $\{s\delta\}$ and $\{\sigma\}$ represent the global soil displacement and interaction stress increment vectors, and $[H]$ represents the global soil influence factor matrix.
By considering the axial compression of each pile due to the pile head load and the interaction stresses, the following expression may be derived for the incremental displacements of the piles themselves:

\[
\begin{pmatrix}
    p \Delta \rho_1 \\
    \vdots \\
    p \Delta \rho_i \\
    \vdots \\
    p \Delta \rho_n
\end{pmatrix}
= \begin{pmatrix}
    \Delta \rho_{b1} \\
    \vdots \\
    \Delta \rho_{bi} \\
    \vdots \\
    \Delta \rho_{bn}
\end{pmatrix} + \begin{pmatrix}
    PM_i & 0 & \cdots & 0 \\
    0 & \ddots & & 0 \\
    \vdots & & \ddots & \vdots \\
    0 & 0 & \cdots & PM_n
\end{pmatrix} \begin{pmatrix}
    \Delta p_1 \\
    \vdots \\
    \Delta p_j \\
    \vdots \\
    \Delta p_n
\end{pmatrix}
\]  

(2.4)

where

\( p \Delta \rho_i = (m_i) \) column vector of incremental displacements along pile \( i \)

\( \Delta \rho_{bi} \) = vector of \((m_i)\) values of the pile base incremental displacements

\( PM = (m_j \times m_i) \) matrix of pile action coefficients

\( \Delta p_j = (m_j) \) column vector of incremental interaction stresses on pile \( j \)

or

\[
\{ p \delta \} = \{ \Delta \rho_b \} + [\Pi] \{ \sigma \}
\]  

(2.4a)

where \( \{ p \delta \} \) and \( \{ \sigma \} \) are the global pile displacement and interaction stress increment vectors, \( \{ \Delta \rho_b \} \) is the vector of pile base incremental movements, and \( [\Pi] \) is the global pile action coefficient matrix.

The derivation of equation 2.4 and the definitions of the various vectors and matrices are given in Appendix A.

If conditions at the pile-soil interfaces remain elastic, i.e. slip or local yield does not occur at any of the elements, then the pile incremental displacements from equations 2.3a and 2.4a may be equated to give

\[
[H - \Pi] \{ \sigma \} - \{ \Delta \rho_b \} = 0
\]  

(2.5)

The vectors \( \{ \sigma \} \) and \( \{ \Delta \rho_b \} \) are of order \( n_i \), and \( [H - \Pi] \) is an \( n_i \times n_i \) matrix, where \( n_i \) is the total number of elements in the entire pile group.

Vertical equilibrium of each pile requires that, for a pile \( i \),

\[
\{ A_i \} \{ \Delta p_i \} = \Delta P_i
\]  

(2.6)
where

\[ \{ A_i \} = (m_i) \text{ row vector of elemental surface areas} \]
\[ \{ \Delta p_i \} = (m_i) \text{ column vector of incremental interaction stresses on pile } i \]
\[ \Delta P_i = \text{ increment of load on pile } i. \]

Thus for a group of \( n \) piles, \( n \) such equations can be written.

Also, for vertical equilibrium of the entire group,

\[ \sum_{i=1}^{n} \Delta P_i = \Delta P_G \quad (2.7) \]

where \( \Delta P_G = \text{ load increment applied to the pile group} \).

Finally, if the piles are connected to a rigid cap and concentric axial loading is applied, the incremental displacement of the head of each pile will be equal to the incremental displacement of the pile cap \( \Delta p_c \). For each pile, the incremental pile head displacement will be equal to the incremental displacement of the uppermost node (located half-way along the top element) plus the elastic compression of the upper half of the top element. Thus, \( n \) further equations may be obtained. The derivation of these equations is given in Appendix B.

In all, the following equations therefore exist:

(i) \( n_t \) displacement compatibility equations (equation 2.5)
(ii) \( n \) pile equilibrium equations (equation 2.6)
(iii) 1 group equilibrium equation (equation 2.7)
(iv) \( n \) equations for the rigid pile cap condition.

These may be solved simultaneously for the following unknowns:

(i) \( n_t \) incremental displacements
(ii) \( n \) incremental base movements
(iii) the incremental settlement of the pile cap
(iv) \( n \) incremental pile head loads.

The incremental pile displacements may then be computed from equation 2.3 or 2.4.
End-bearing piles can be considered by adopting the method used by Poulos and Mattes (1969). This method takes the base stratum into account when calculating the matrix of soil influence factors by introducing a mirror-image element $j'$ of element $j$, loaded by an equal and opposite shear stress. This new influence matrix replaces the original $[s]$ in the floating pile analysis and the equations are solved as for the floating pile case.

To allow for pile-soil slip or yield of an element, an upper limit $\tau_{\text{max},j}$ to the value of the interaction stress $p_{ij}$ is specified at each element. It is assumed that yield will occur when the shear stress at a point reaches the value $\tau_{\text{max},j}$. If the solution indicates that $p_{ij} > \tau_{\text{max},j}$, the displacement compatibility equation for element $j$ (in equation 2.5) is then replaced by the condition:

$$\Delta p_{ij} = \tau_{\text{max},j} - p_{ij}$$

(2.8)

where $p_{ij} = \text{total pile-soil stress at the previous load increment}$. The solution is then recycled until, for all elements, the interaction stress is less than or equal to the specified limiting stress.

Similarly a lower limit $\tau_{\text{min},j}$ to the value of the interaction stress is specified at each element. If $p_{ij} < \tau_{\text{min},j}$ an analogous procedure is adopted. Generally $\tau_{\text{min}} = -\tau_{\text{max}}$.

### 2.3 PROGRAM GAPFIL

The computer program GAPFIL has been written to evaluate the above analysis for unsymmetrical pile groups subjected to static loading. This program is based on program TAPILE (Poulos, 1978), which analyses single piles under static loading.

The program computes the settlement of the pile cap, the axial deflections along the piles, and the distribution of pile-soil shear stress and load along the piles. The group may be subjected to a series of incremental loads.

The group may contain piles of different length, diameter, Young's modulus, and cross-sectional area arranged in any configuration. The individual piles may contain discontinuities, i.e. different diameters. The soil may be non-homogeneous.
The user has some control over the speed versus accuracy of the analysis by inputing the required number of steps in the integration around the element used to determine \( \{ \Delta \rho \} \). The importance of each integration is directly related to the distance between influencing and influenced elements. Allowance is made for the input of four different numbers of steps in the integration around the element for the following four spacings between influencing and influenced elements:

(i) \( \text{spacing} \leq 5D \) do \( \text{lim}1 \) steps in the integration
(ii) \( 5D < \text{spacing} \leq 10D \) do \( \text{lim}2 \) steps in the integration
(iii) \( 10D < \text{spacing} \leq 25D \) do \( \text{lim}3 \) steps in the integration
(iv) \( 25D < \text{spacing} \) do \( \text{lim}4 \) steps in the integration

where \( D \) is the diameter of the influenced element.

For example, for a faster, less accurate solution, the following may be adopted:

\[
\text{lim}1 = 15 \\
\text{lim}2 = 12 \\
\text{lim}3 = 10 \\
\text{lim}4 = 8.
\]

For a more accurate but slower solution, suitable values are:

\[
\text{lim}1 = 40 \\
\text{lim}2 = 20 \\
\text{lim}3 = 15 \\
\text{lim}4 = 10.
\]

Table 2.1 gives an indication of the speed versus accuracy of different combinations of integration steps.

In addition, the program allows individual pile loads to be input, instead of the total group load, if required. In this case, there is one less equation because the group equilibrium equation (equation 2.7) is no longer required. The \( n \) incremental pile head loads are no longer unknown and are replaced
as unknowns by the \( n \) incremental settlements of the piles. There is also one less unknown, the incremental settlement of the pile cap. Therefore, the \( n \) equations for the rigid pile cap condition are replaced by \( n \) equations for the incremental settlements of the piles.

For the case of known individual pile loads the following equations exist:

(i) \( n \) displacement compatibility equations (equation 2.5)
(ii) \( n \) pile equilibrium equations (equation 2.6)
(iii) \( n \) equations for the incremental pile settlements.

These may be solved simultaneously for the following unknowns:

(i) \( n \) incremental displacements
(ii) \( n \) incremental base movements
(iii) \( n \) incremental pile settlements.

2.4 NOVAK AND EL SHARNOUBY APPROACH

One of the most time-consuming parts of the preceding analysis is the evaluation of the extensive analytical and numerical integrations along and around the pile associated with the determination of the elements of \([\mu I]\). They can be obtained by double integration of the Mindlin equation, as described by Poulos and Davis (1968).

However, El Sharnouby and Novak (1984) have suggested an approach which reduces considerably the computing time, allowing for a rapid analysis of larger groups. Their approach generates the nodal soil flexibility coefficients by the application of discrete point loads, applied and located such that the resultant flexibility coefficients are almost the same as those obtained from continuously distributed shears.

The calculation of the displacement factors for pile elements is given in Appendix C. Also shown in Appendix C is how the Novak and El Sharnouby analysis has been incorporated into the program GAPFIL enabling a practical solution for larger, non-symmetrical pile groups.
A comparison of the CPU time required to analyse a problem and the top movement obtained from the analysis is shown in Table 2.2 for different combinations of the original integration method and the Novak and El Sharnouby method of calculating the soil flexibility coefficients.

It is shown in Table 2.2 that a third analysis, which incorporates the Novak and El Sharnouby simplification only for the non-diagonal elements of the GAPFIL analysis, gives the most satisfactory results. This simplification reduces the CPU time by 75%, yet loses only 1% accuracy in the process. This analysis is incorporated in the program GAPFIN, which is a modification of the program GAPFIL.

2.5 REVERSE PLASTIC SLIP ANALYSIS

In the elastic stages of the analysis for both single piles and pile groups, some of the pile shaft elements may carry a small negative stress under static axial loading. These elements are generally found at the base of the shaft for the single pile analysis and towards the top of the shaft for the pile group analysis. At higher load levels, the load redistributes itself so that these elements carry a positive stress, as expected.

Occasionally, this small negative stress is larger than the limiting tensile skin friction for the shaft element \( \tau_{\text{min}} \), and so the element slips in tension. Having slipped, this element is then ‘locked’ in that position and cannot ‘unlock’ itself. At higher load levels the element which had slipped in tension remains plastic and is excluded from the resulting load redistribution, whereas a fully rigorous solution would have shown that the element returns to an elastic state.

Therefore, the final solution, although it is a valid solution, i.e. it satisfies the equations set out in section 2.2, it is not the correct solution to this problem. It is the solution to a different problem involving a different stress path. In fact, there are any number of valid solutions, but only one which is the correct solution for the particular stress path input.

Davis, Ring and Booker (1974) have shown the importance of following a

\footnote{This tensile stress which is larger than the limiting tensile skin friction is generally not a physical stress but the result of ill-conditioning or similar phenomena in the equations.}
particular load path. The solution to an elasto-plastic problem is dependent
on load history because of the irreversible and non-linear nature of the plastic
straining. They have shown that the correct load path can be traced if the
solution process includes means of excluding negative plastic work. Negative
plastic work implies the physically impossible situation that the slip movement
of the element is in the opposite direction to the applied stress. Any elements
which undergo negative plastic work should be assumed to be elastic, to al-
low these elements to elastically unload rather than plastically unload, thus
of corresponding to the actual physical behaviour.

Therefore an analysis was developed, termed the 'Reverse Plastic Slip' anal-
ysis, which enables the correct solution to be found (i.e. the valid solution which
also follows the given stress path). This analysis is also more efficient be-cause
in the early stages of slipping only a small matrix need be solved for the num-
ber of slipped elements, whereas the previous analysis requires the complete
matrix to be solved for every load increment.

2.5.1 The Concept Behind the Analysis

The behaviour of a single element is examined during loading to failure in
compression, reversal of loading to failure in tension, followed by reversal of
loading to failure in compression again. No cyclic degradation is assumed at
this stage. The stress-settlement curve is shown in Fig. 2.2.

During the initial part of the curve, up to point (1) in Fig. 2.2, the element
behaves elastically. There is no relative movement between the pile surface
and the soil surface. At point (1) the stress on the element, \( p_i \), has reached the
upper limit of stress for that element \( \tau_{max} \). From point (1) to point (2) the
element can carry no more load and so it continues to deflect with no increase
in stress. There is therefore slip between the pile surface and the soil surface.
Relative movement \( \omega \) is defined to be the relative slip between the pile and the
soil.

\[
\omega = s\rho - p\rho
\]  
(2.9)
where \( p_\rho \) = settlement of pile surface  
\( s_\rho \) = settlement of soil surface.

If the direction of loading is reversed, the element moves from point (2) to point (3) and the direction of deflection of the pile and soil also changes. Again there is no relative displacement between the pile surface and the soil surface until the stress on the element \( p_i \) reaches the lower limit of stress for that element \( \tau_{\text{min}} \). This occurs at point (3). From point (3) to point (4) the element can carry no more load and so it continues to deflect with no increase in stress. Again there is relative slip between the pile and the soil.

It is more convenient to change the axes for this representation of element response to \( p \) versus \( \omega \), as shown in Fig. 2.3. Fig. 2.3 clearly shows that from point (0) to point (1) and from point (2) to point (3) the element behaves elastically, from point (1) to point (2) failure occurs in the positive direction and from point (3) to point (4) failure occurs in the negative direction. Therefore, the condition for elastic behaviour at element \( i \) is \( \Delta \omega_i = 0 \), and the condition for slip (i.e. failure) is \( \Delta p_i = 0 \).

Now if \( \Delta \omega_i = 0, \quad \Delta p_i \neq 0 \)  
and if \( \Delta p_i = 0, \quad \Delta \omega_i \neq 0 \)

The sign convention is arranged to give  
\[ p \cdot \Delta \omega \geq 0 \]
so that plastic work is positive everywhere. If there is negative plastic work at any element, then the solution is incorrect and the element should be reset to the elastic response conditions.

In the case described in section 2.5 in which an element slips in tension and then cannot unlock the slip, at higher load levels the stress on that element is positive and the movement of that element is negative and so that element is in a state of negative plastic work. The element should be reset to the elastic response conditions and unlocked from its slipped state. That element can then
carry its share of the redistributed load.

2.5.2 The Analysis

For a single pile, the incremental soil displacements at each element, from equation 2.2, are

\[
\{\lambda \Delta \rho\} = \left[ \frac{d}{E_s} I_s \right] \{\Delta p\}
\]

and the incremental pile displacements, from equation 2.4, are

\[
\{\rho \Delta \rho\} = \{\Delta \rho_b\} + [PM]\{\Delta p\}
\]

Now by definition

\[
\omega = \lambda \rho - \rho \rho
\]

therefore

\[
\{\Delta \omega\} = \{\lambda \Delta \rho\} - \{\rho \Delta \rho\}
\]

(2.10)

\[
\{\Delta \omega\} = \left[ \frac{d}{E_s} I_s \right] \{\Delta p\} - \{\Delta \rho_b\} - [PM]\{\Delta p\}
\]

\[
= \left[ \frac{d}{E_s} I_s - PM \right] \{\Delta p\} - \{\Delta \rho_b\}
\]

(2.11)

where

\[
[\rho I] = \left[ \frac{d}{E_s} I_s - PM \right]
\]

Vertical equilibrium of each pile from equation 2.6 gives

\[
\{A\}\{\Delta p\} = \{\Delta P\}
\]

Combining equations 2.11 and 2.6 gives

\[
\begin{bmatrix}
\rho I & -1 \\
A & 0
\end{bmatrix}
\begin{bmatrix}
\Delta p \\
\Delta \rho_b
\end{bmatrix}
= \begin{bmatrix}
\Delta \omega \\
\Delta P
\end{bmatrix}
\]

(2.12)

inverting equation 2.12 gives

\[
\begin{bmatrix}
\Delta p \\
\Delta \rho_b
\end{bmatrix}
= \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta P
\end{bmatrix}
\]

(2.13)
Now for elastic conditions $\Delta \omega = 0$ therefore

$$\{\Delta p\} = [K_{12}][\Delta P]$$ \hspace{1cm} (2.14)

$\{p\Delta p\}$ and $\{s\Delta p\}$ can then be calculated from equations 2.2 and 2.4 and they should be the same while the response is elastic.

2.5.3 Application of Analysis to Single Pile

As can be seen this analysis can be used only for load increments in which each element is either fully elastic or fully plastic (i.e. full slip). Any load increment which would involve the transition of any element from an elastic state to a plastic state or vice versa must be divided into smaller increments.

The following steps should be taken to implement the analysis:

1. The load is applied and the above analysis is used to solve for the stresses from equation 2.14 assuming all elements behave elastically.

2. A check is done to see if all elements do, in fact, behave elastically. A quantity, $R_i$, is calculated for each element, $i$, where

$$R_i = \frac{\sigma_{max}_{i} - \sigma_{initial}_{i}}{\Delta p_i}$$

and the minimum value of $R$ for all elements, $R_{\text{min}}$, is found. If a particular element is behaving elastically, then $R_i \geq 1$. If $R_i < 1$ then the element will have become plastic. The critical value of $R$, $R_c$, is equal to $R_{\text{min}}$ and occurs at the critical element, $i_c$, i.e. the element at which slip first occurs.

If $R_c \geq 1$ then the initial solution is correct and all elements are elastic. The deflections can be found using equations 2.2 and 2.4.

If $R_c < 1$ then the load must be divided into smaller increments. $\{\Delta p\}$ and $\Delta P$ must be factored by $R_c$ to obtain a valid solution, i.e. a solution which satisfies the equations given in section 2.2.
3. The remainder of the initial load increment (after subtracting $R_c \Delta P$) is now applied and the relative displacement at element $i_c, \Delta \omega_{i_c}$, is calculated as follows. Expanding equation 2.13 for a single pile gives

$$
\begin{align*}
\begin{bmatrix}
\Delta p_1 \\
\Delta p_2 \\
\vdots \\
\Delta p_{i_c} \\
\vdots \\
\Delta p_n \\
\Delta \rho_b
\end{bmatrix}
&= 
\begin{bmatrix}
k_{11} & k_{12} & \ldots & k_{1i_c} & \ldots & k_{1n} & k_{1n+1} \\
k_{21} & k_{22} & \ldots \\
\vdots \\
k_{i_c1} & k_{i_c2} & \ldots & k_{i_ci_c} & \ldots \\
k_{n1} & \ldots & \ldots & k_{nn} \\
k_{n+11} & \ldots & \ldots & k_{n+1n+1}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega_1 \\
\Delta \omega_2 \\
\vdots \\
\Delta \omega_{i_c} \\
\vdots \\
\Delta \omega_n \\
\Delta P_b
\end{bmatrix}
\end{align*}
$$

(2.15)

where

- $n$ is the number of elements in the pile
- $\Delta P_b$ is the increment of load on the pile.

Before element $i_c$ reaches the failure stress, the following conditions are satisfied:

- $\Delta p_1, \Delta p_2, \ldots, \Delta p_{i_c}, \ldots, \Delta p_n, \Delta \rho_b$ are unknown
- $\Delta \omega_1 (= 0), \Delta \omega_2 (= 0), \ldots, \Delta \omega_{i_c} (= 0), \ldots, \Delta \omega_n (= 0), \Delta P$ are known

and therefore it is straightforward to solve for the unknowns.

When element $i_c$ reaches the failure stress it can carry no more load, so that $\Delta p_{i_c} = 0$ and it begins to slip i.e. $\Delta \omega_{i_c} \neq 0$. Now the following conditions are satisfied:

- $\Delta p_1, \ldots, \Delta p_{i_c-1}, \Delta p_{i_c+1}, \ldots, \Delta p_n, \Delta \rho_b$ and $\Delta \omega_{i_c}$ are unknown
- $\Delta \omega_1 (= 0), \ldots, \Delta \omega_{i_c-1} (= 0), \Delta \omega_{i_c+1} (= 0), \ldots, \Delta \omega_n (= 0), \Delta P$ and $\Delta p_{i_c}$ are known.
Equation 2.15 can be rearranged as follows to solve for the unknowns:

$$
\begin{pmatrix}
\Delta p_1 - k_{i,c} \Delta \omega_i \\
\vdots \\
\Delta p_{i,c-1} - k_{i,c-1} \Delta \omega_i \\
-k_{i,c} \Delta \omega_i \\
\Delta p_{i,c+1} - k_{i,c+1} \Delta \omega_i \\
\vdots \\
\Delta p_n - k_{n,c} \Delta \omega_i \\
\Delta p \_ - k_{n+1,c} \Delta \omega_i \\
\end{pmatrix}
= 
\begin{pmatrix}
\Delta \omega_1 \\
\vdots \\
\Delta \omega_{i,c-1} \\
\Delta p_{i,c} \\
\Delta \omega_{i,c+1} \\
\vdots \\
\Delta \omega_n \\
\Delta P_t \\
\end{pmatrix}
$$

(2.16)

From equation $i_c$ in the above matrix c

$$
-k_{i,c} \Delta \omega_i = k_{i,c} \Delta \omega_1 + \cdots + k_{i,c-1} \Delta \omega_i - \Delta p_{i,c} + k_{i,c+1} \Delta \omega_{i+1} + \\
\cdots + k_{i,n} \Delta \omega_n + k_{i,n+1} \Delta P_t
$$

Now

$$
\Delta \omega_1 = \Delta \omega_2 = \cdots = \Delta \omega_{i-1} = \Delta \omega_{i+1} = \cdots = \Delta \omega_n = 0
$$

and $\Delta p_{i,c} = 0$

therefore

$$
-k_{i,c} \Delta \omega_i = k_{i,n+1} \Delta P_t
$$

i.e.

$$
\Delta \omega_i = -\frac{k_{i,n+1}}{k_{i,c}} \Delta P_t
$$

(2.17)

where $n + 1$ is the position of the equilibrium equation in equation 2.16.
Thus the relative slip displacement of element $i_c$ can easily be found without having to invert any matrices.

This solution for $\Delta \omega_{i_c}$ is for a single pile, but the solution for a pile group can be obtained in exactly the same way, giving the following equation:

$$\Delta \omega_{i_c} = -\frac{k_{i_n+1}}{k_{i_c}} \Delta P_{tot}$$

where $\Delta P_{tot}$ is the total load on the group.

4. Now substituting this value of $\{\Delta \omega\}$ into equation 2.15, a solution for $\{\Delta p\}$ may be obtained. An example of this procedure is given in Appendix D.

5. Calculate $R_{min}$ again, as in step 2, to see if all the remaining elements are still behaving elastically. If $R_c < 1$, then at least one of the elements is not elastic and so $\{\Delta p\}$ and $\Delta P$ should be factored by $R_c$.

6. **Check for Negative Plastic Work.**

To determine whether this is the correct solution so far a check is made of the plastic work done on each element. Plastic work is calculated as

$$p_i \Delta \omega_i$$

If plastic work is positive for each element then the solution so far is correct. Add the factored solution obtained in step 5 to the solution for the previous load increment, reapply the remaining load and continue with step 7. However, if plastic work is negative for any element, then that element should be reset to elastic element conditions and the solution recalculated. To recalculate the solution, add no load and resolve for the slip at the failed elements $\{\Delta \omega\}$ as shown in step 7.

7. **Calculating $\{\Delta \omega\}$.**

Assuming, for example, there are two unknown slip displacements $\Delta \omega_i$ and $\Delta \omega_j$, the slip at the failed elements, $\{\Delta \omega\}$, is solved for as follows.

From equation 2.15 there are two unknown slip displacements $\Delta \omega_i$ and $\Delta \omega_j$. The remainder of the slip movements are zero. Selecting equations
\[ \Delta p_i = k_{i1}\Delta \omega_1 + k_{i2}\Delta \omega_2 + \cdots + k_{in}\Delta \omega_n + k_{i(n+1)}\Delta P \]

\[ \Delta p_j = k_{j1}\Delta \omega_1 + k_{j2}\Delta \omega_2 + \cdots + k_{jn}\Delta \omega_n + k_{j(n+1)}\Delta P \]

Now \( \Delta \omega_i \) and \( \Delta \omega_j \) are the only non-zero slip movements so these equations reduce to

\[ \Delta p_i = k_{i1}\Delta \omega_i + k_{i2}\Delta \omega_j + k_{i(n+1)}\Delta P \]

\[ \Delta p_j = k_{j1}\Delta \omega_i + k_{j2}\Delta \omega_j + k_{j(n+1)}\Delta P \]

But \( \Delta p_i = \Delta p_j = 0 \) because these slipped elements cannot carry any more load giving

\[
\begin{pmatrix}
0 \\
0 
\end{pmatrix} = 
\begin{bmatrix}
k_{ii} & k_{ij} \\
k_{ji} & k_{jj}
\end{bmatrix}
\begin{pmatrix}
\Delta \omega_i \\
\Delta \omega_j
\end{pmatrix} + \Delta P
\begin{pmatrix}
k_{i(n+1)} \\
k_{j(n+1)}
\end{pmatrix}
\]

therefore

\[
\begin{pmatrix}
\Delta \omega_i \\
\Delta \omega_j
\end{pmatrix} = -\Delta P
\begin{bmatrix}
k_{ii} & k_{ij} \\
k_{ji} & k_{jj}
\end{bmatrix}^{-1}
\begin{pmatrix}
k_{i(n+1)} \\
k_{j(n+1)}
\end{pmatrix}
\]

This solution for the unknown slip movements is valid for any number of unknown slip movements e.g. for \( m \) unknown slip movements the solution is

\[
\begin{pmatrix}
\Delta \omega_i \\
\Delta \omega_j \\
\vdots \\
\Delta \omega_m
\end{pmatrix} = -\Delta P
\begin{bmatrix}
k_{ii} & k_{ij} & \cdots & k_{im} \\
k_{ji} & k_{jj} & \cdots & k_{jm} \\
\vdots & \vdots & \ddots & \vdots \\
k_{mi} & k_{mj} & \cdots & k_{mm}
\end{bmatrix}
\begin{pmatrix}
k_{i(n+1)} \\
k_{j(n+1)} \\
\vdots \\
k_{m(n+1)}
\end{pmatrix}
\]

8. Return to step 4 and continue the cycle until the whole load increment is applied and \( R_c \geq 1 \).

2.5.4 Advantages of the Analysis

The main advantage of this method over the previous analysis (e.g. Poulos and Davis, 1968) is that, in all cases, it should find the correct solution. In some cases, especially those involving piles of non-uniform diameter, the previous analysis allows some elements to slip in the opposite direction to that of loading. These elements are then ‘locked’ in that position and cannot carry their share.
of load as the load increases. Any element which locks cannot unlock unless the direction of load is reversed.

Another considerable advantage is the efficiency of this method. The previous analysis requires the whole matrix to be inverted when each load increment is applied, and additionally if any elements slip. The number of times the matrix needs to be inverted depends upon the total number of elements which slip and the number which slip within each cycle of the solution. In the worst case the matrix would need to be inverted once for each element.

In comparison, the modified 'Reverse Plastic Slip' analysis requires the whole matrix to be inverted only once, at equation 2.13. If any elements slip, then only the unknown slip movements of those elements need be solved for. For example, if three elements slip then a $[3 \times 3]$ matrix is solved for the three slip movements. These small matrices take only a fraction of the time to solve than would be taken to solve the whole matrix. Consequently, the solution using this analysis is much more efficient than that using the previous analysis.

A third advantage with this modified analysis is that the actual slip movements are calculated. These may be a useful parameter in determining degradation, as described later in this chapter.

2.5.5 Program GAPFIX

The computer program GAPFIX has been written based on the above analysis. This program is based on GAPFIN but with the body of the program being substantially modified.

Both GAPFIX and GAPFIN give similar results at low load levels, but at higher load levels, approaching failure, GAPFIN sometimes gives results which are clearly incorrect, while GAPFIX gives results which look reasonable. This generally occurs with groups containing piles of unequal length or on piles containing discontinuity elements.

An example of such a group is shown in Fig. 2.4. This group contains two identical piles with bells and inserts. The skin friction of the insert is consider-
ably higher than that of the remainder of the pile representing a stronger soil or rock layer below the level of the bells.

Under static loading conditions both programs give identical results, as expected. At low loads, the GAPFIN results are much more erratic in the pile insert than the GAPFIX results. Fig. 2.5 compares the stress distribution from both the GAPFIN and GAPFIX analyses at a load level of 81% of ultimate load. At this load all the shaft elements have slipped in either the positive or negative direction. Results from the GAPFIN analysis are clearly incorrect because alternate elements all the way down the pile are slipping in opposite directions. Many of the elements are 'locked' slipping in the negative direction. Results from the GAPFIX analysis are believable with nearly all the elements, including all the insert elements, slipping in the positive direction. As a result the piles in the GAPFIN analysis cannot carry any more load, while those in the GAPFIX analysis can continue carrying load right up until the ultimate failure load.

2.6 CYCLIC LOADING

The extension of the static loading analysis to consider cyclic loading has been discussed by Poulos (1983). Cyclic degradation of skin friction, base resistance and soil modulus can be taken into account via appropriate degradation factors, the degradation factor being the ratio of the property after cycling to the property for cyclic loading. Thus, for example, for the ultimate skin friction the degradation factor $D_r$ is defined as

$$D_r = \frac{f_s(\text{post-cyclic})}{f_s(\text{static})}$$

where

$f_s(\text{post-cyclic})$ = ultimate skin friction on a pile element after it has been subjected to cyclic loading

$f_s(\text{static})$ = ultimate skin friction for static loading.

Normally, the degradation factors will be equal to or less than 1.0.
A variety of degradation criteria can be postulated but the simplest degradation model is that proposed by Matlock and Foo (1980) in which it is assumed that, at any element of the pile, the property in question (e.g. skin friction) decreases by some proportion when a full reversal of slip (or yield) occurs at that element. This model can be described as follows, in terms of the degradation factor:

\[ D = (1 - \lambda)(D' - D_{\text{min}}) + D_{\text{min}} \]  \hspace{1cm} (2.19)

where

- \( D \) = degradation factor after full reversal of slip or yield
- \( D' \) = degradation factor prior to the last full reversal of slip or yield
- \( D_{\text{min}} \) = minimum value of degradation factor
- \( \lambda \) = degradation rate parameter, expressing the extent of degradation for each full reversal of slip.

The cyclic load analysis of the group follows the same method as for a single pile, as is described by Poulos (1983). The group is analysed for the maximum group load \( P_{\text{max}} \), and then for the minimum load \( P_{\text{min}} \). For elements at which full reversal of slip has occurred, the degradation factors for limiting skin friction are computed from equation 2.19. The values of limiting skin friction for the next cycle are determined by multiplying the static values by the appropriate degradation factor. Similar definitions apply to the degradation factor \( D_b \) for ultimate base resistance, and the degradation factor \( D_E \) for Young's modulus. However attention here is mainly confined to \( D_r \), because tests by Poulos (1984) indicate that there appears to be relatively little effect of cyclic loading on either the soil modulus or the ultimate base bearing capacity.

The next cycle of loading is then applied and the procedure repeated until the entire sequence of cyclic loading has been simulated. In this way, the minimum and maximum group deflections can be computed, together with the distribution of limiting stress along each pile and hence, the ultimate group load capacity after cycling.
2.7 PERMANENT DISPLACEMENTS

The analysis described above cannot cope adequately with accumulation of deflection, $\delta_{acc}$. The only way this analysis can cope with accumulation in deflection is to degrade the modulus as demonstrated in Fig. 2.6, but laboratory tests indicate that the modulus may not degrade in this way; rather, there may often be an accumulation of permanent displacement, with little or no reduction in cyclic stiffness of the pile.

This problem can be overcome by the addition of another term into the equation, equating the soil and pile displacements, as outlined by Poulos (1987)

$$\left[\frac{d}{E_s} I_s - P M\right] \{\Delta p\} - \{\Delta \rho_b\} = -\{\Delta \delta_{acc}\} \quad (2.20)$$

where $\{\Delta \delta_{acc}\} =$ permanent displacement.

Poulos (1987) has suggested the following expression for permanent displacements

$$\delta_{acc} = BX^n N^m$$

where

$B = \text{constant}$

$m,n = \text{exponential parameters}$

$X = \text{representative load level}$

$$X = \sqrt{\frac{P_0 P_{\text{max}}}{P_u^2}}$$

$P_0 = \text{mean value of load}$

$P_{\text{max}} = \text{maximum value of load}$

$P_u = \text{static ultimate load}$

To obtain $\Delta \delta_{acc}$ it is necessary to differentiate

$$\delta_{acc} = BX^n N^m$$

54
\[
\frac{d\delta_{acc}}{dX} = BN^m n X^{n-1} \\
\frac{d\delta_{acc}}{dN} = \delta_{acc} \frac{n}{X} dX \\
\frac{d\delta_{acc}}{dN} = BX^m m N^{m-1} \\
\delta_{acc} = \delta_{acc} \frac{m}{N} dN
\]

therefore

\[\Delta \delta_{acc} = \delta_{acc} \left(\frac{n}{M} dX + \frac{m}{N} dN\right)\]

For each particular load case considered, the mean load \(P_0\) is fixed and a number of cycles of constant-amplitude loading is applied. During cycling neither \(P_0\) nor \(P_{\text{max}}\) change, therefore \(X\) is constant and so \(dX = 0\), therefore

\[\Delta \delta_{acc} = \delta_{acc} \frac{m}{N} dN\]  
(2.21)

This expression for \(\Delta \delta_{acc}\) can then be substituted into equation 2.20.

Permanent displacements are incorporated into the analysis as follows:

1. For each load set \(X\) is calculated once and \(\delta_{acc}\) is calculated initially as the elastic settlement of the first cycle.

2. For each load cycle each element is checked to see if slip has occurred in either direction. The elements in which slip has not occurred are given the permanent displacement (equation 2.21) in equation 2.20. The elements in which slip has occurred are not given the permanent displacement because the stresses are known and so are not solved for using equation 2.20 but using equation 2.8.

3. Then for each load cycle the incremental permanent displacement is multiplied by \(m/N\) and the total permanent displacement is increased by this increment.

4. The next load cycle is then applied.
2.8 PROGRAM GAPCYC

The computer program GAPCYC has been written using the above analysis to analyse groups of dissimilar piles which may be of non-uniform diameter under cyclic loading. This program is based on a combination of program GAPFIN, described in Appendix C, and program TAPCYC, written by Poulos (1985), which analyses a single pile or symmetrical pile group under cyclic loading. The program incorporates the analysis of permanent displacements.

2.9 SUMMARY

In this chapter a numerical analysis of pile groups under both static and cyclic axial loading has been presented. A number of computer programs have been written to implement the analysis. All of these programs can handle groups of dissimilar piles, which may be of non-uniform diameter. They all determine the settlement of the pile cap, the axial deflection, and the distribution of pile-soil shear stress and load along the pile at each load increment. They include the effects of slip between the pile and the soil. The basic features of each program are given in Table 2.3.

The analysis presented in this chapter includes:

- allowance for individual pile loads to be input, instead of the total group load

- the Novak and El Sharnouby approach to calculating the soil flexibility coefficients, which cuts down considerably on computing time

- the Reverse Plastic Slip analysis which:-
  1. avoids the problem of following an incorrect load path by excluding negative plastic work, enabling the correct solution to be found
  2. calculates the actual slip movements at each element which has slipped
  3. is more efficient than the previous method because if any elements slip, only the unknown slip movements need be solved for and so a much smaller matrix needs to be inverted
• consideration of cyclic degradation of skin friction, base resistance and Young’s modulus.

• the incorporation of permanent displacement accumulation under cyclic loading.

The utilization of these programs will be described in Chapters 3, 5 and 6.
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<th>lim3</th>
<th>lim4</th>
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Table 2.1 Speed versus Accuracy in GAPFII.
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<th>Method</th>
<th>CPU time (as a proportion of CPU time in GAPFIL solution)</th>
<th>top movement (as a proportion of movement in GAPFIL solution)</th>
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<tr>
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<td>i.e. integration of all side and base elements</td>
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</tr>
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<td>2 integration for diagonal side elements,</td>
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<td>Novak and El Sharmouby method for other side elements,</td>
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<td>integration for all base elements</td>
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<td>4 Novak and El Sharmouby method for all side elements,</td>
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<td>integration for all base elements</td>
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Table 2.2 Comparison of CPU time and top movement for different analyses of soil flexibility coefficients
<table>
<thead>
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<th>Program</th>
<th>Features</th>
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</thead>
</table>
| GAPFIL  | - Analyses groups of dissimilar piles subjected to static loading.  
- Individual pile loads may be input. |
| GAPFIN  | - Analyses groups of dissimilar piles subjected to static loading.  
- Uses the Novak and El Sharnouby approach to simplify the evaluation of the integrations by the application of discrete point loads to obtain the flexibility coefficients.  
- Individual pile loads may be input. |
| GAPFIX  | - Analyses groups of dissimilar piles subjected to static loading using the reverse plastic slip analysis. This method calculates the actual slip movements of each slipped element and is more efficient and more reliable than the GAPFIL analysis.  
- Individual pile loads may be input.  
- Uses the Novak and El Sharnouby approach to evaluate the integrations. |
| GAPCYC  | - Analyses groups of dissimilar piles subjected to cyclic loading.  
- Calculates the distribution of ultimate skin friction and the ultimate end-bearing capacity of the pile.  
- Uses the Novak and El Sharnouby approach to evaluate the integrations.  
- Allows for degradation of skin friction, end bearing and Young’s modulus during cycling. |

Table 2.3 Basic Features of Programs used for Analysis
Figure 2.1  Two Dissimilar Piles in a Group
Figure 2.2  Stress-Settlement Curve for a Single Element, $i$
Figure 2.3 Stress Slip Curve for a Single Element, $i$
<table>
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<tr>
<th>Length (m)</th>
<th>Diameter (m)</th>
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<th>$E_s$ (MPa)</th>
<th>$\pm f_s$ (MPa)</th>
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</tr>
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</table>

Figure 2.4 Geometry of Problem Analysed
Figure 2.5 Comparison of Stress versus Depth for the Two Analyses
(a) Actual load-settlement curve showing accumulation in deflection

(b) Degradation of modulus in original analysis to cope with accumulation in deflection

Figure 2.6 Modelling Permanent Displacements
3

THE BEHAVIOUR OF PILE GROUPS

3.1 INTRODUCTION

Most offshore pile groups are designed to have identical piles arranged in a symmetrical configuration. However, a number of circumstances, planned or unplanned, may arise in which a group contains dissimilar piles, e.g.

- construction difficulties may result in one or more of the piles having some defect or weakness or being misaligned during installation
- an existing group may need to be upgraded by the installation of bells on some of the piles, or by increasing the length of some of the piles (e.g. by installation of grouted inserts) or by the addition of extra piles which may be of different length or diameter to the original piles.

In this chapter some verification of the analysis presented in Chapter 2 is presented by comparing the results of the analysis to those obtained from a boundary element analysis by Butterfield and Banerjee (1971). Also, a number of different pile groups are analysed under static loading and consideration is given to:
(i) effect of bells
(ii) piles of unequal length
(iii) piles of different diameter
(iv) a defective pile in the group
(v) imperfect alignment.

The mechanism of cyclic degradation in such groups is then discussed, using the relatively simple cyclic degradation criterion developed by Matlock and Foo (1980). Finally, the effect of the length of additional piles is considered by analysing a two-pile group in which two further piles are added whose length may differ from the original piles.

The programs used for the analyses are described in Chapter 2.

3.2 VERIFICATION OF ANALYSIS

Comparisons have been made between the results of the present analysis and those obtained from a boundary element analysis by Butterfield and Banerjee (1971) for groups of five, six and seven piles. These results have been obtained without using the superposition approximation of Poulos (1968) for the non-symmetrical pile groups. These results allow checking only of the elastic load distribution between identical piles in a group given by the two different methods of analysis. A comparison of solutions from the two analyses is presented in Fig. 3.1. The solutions for the elastic stiffness of the group have been found to be virtually identical. It can be seen that the present analysis predicts a slightly more uniform distribution of load within the group, particularly for more slender piles, but the difference between the solutions is generally less than 10%. The differences could possibly be due to differences in the area ratio of the piles or the thickness of the soil layer.

No independent solutions for non-linear group behaviour or cyclic response are available for comparison, although the present more general solution for cyclic response gives identical results to those of Poulos (1983) for a symmetrical pile group.
The main reason why the present direct analysis is more accurate than the 2-pile interaction factor approach is because the stress distribution down the pile changes as the number of piles increases. The stress distributions for two different pile spacings and two different soil stiffnesses are plotted in Fig. 3.2 for a single pile, and for 2-pile, 4-pile and 8-pile groups. It can be seen that there is a trend for redistribution of axial loads towards the pile tips as the number of piles in the groups increases. The 2-pile interaction factor approach assumes the same stress distribution as for a 2-pile group, irrespective of how many piles are in the group, and this is clearly not the case.

Previously, one of the more common methods available for analysing a non-symmetrical pile group under static loading was by the use of interaction factors. The interaction factors are either input for various dimensionless centre-to-centre spacings or are computed by the computer program. The program then evaluates the matrices of interaction factors for the specified group by interpolation of the computed or input interaction factors. Therefore the matrix of interaction factors is not exact. A number of programs adopt this method of analysing a pile group under static loading, one of which is DEFPIL (Poulos, 1980). A comparison of the two methods has been made for a 5-pile group and a 6-pile group.

For the 5-pile group, the load and stress distribution results for the four corner piles given by GAPFIL and by DEFPIL do not differ by very much, however the load and stress distributions down the middle pile are quite different. These distributions are shown in Fig. 3.3. It can be seen that the DEFPIL analysis predicts a fairly uniform distribution of load down the pile, while the GAPFIL analysis predicts very little load being carried in the top half of the pile, and in fact more than 50% of the load which is carried in skin friction is carried in the bottom 20% of the pile shaft. As a result the skin friction in the upper section of the pile is much smaller in the GAPFIL analysis than in the DEFPIL analysis, and in the lower section of the pile it is much larger in the GAPFIL analysis than in the DEFPIL analysis. The difference in load distributions is because the DEFPIL analysis assumes the same stress distribution as for a 2-pile group, however the middle pile is surrounded on all four
sides by another pile, and so in the more accurate GAPFIL analysis the load is redistributed towards the pile tip.

Table 3.1 shows a comparison between the two analyses for a 5-pile and a 6-pile group. The same trends are evident with the stress distribution on the inner pile of the 6-pile group, although the differences between the two methods are not as great because the inner piles in the 6-pile group are not surrounded by piles on all four sides. This Table indicates that, depending on the geometry of the pile group, the deflections can differ by 20%, the pile stress by 260% and the pile loads by 10% between the two analyses.

3.3 INTERACTION FACTORS FOR TWO DISSIMILAR PILES

Two-pile interaction factors have been widely used to analyse pile group settlements (Poulos, 1968; Randolph and Wroth, 1979). Available solutions for interaction factors are confined to two identical piles. The present more general analysis was used to study the interaction between two dissimilar piles.

A study has been made of the effect of a second unloaded pile close to a loaded pile. The effect has been studied at different pile spacings and with different lengths and diameters of the two piles.

In the first analysis, two piles of equal diameter but different lengths, \( L_1 \) and \( L_2 \) with \( L_1 = 2L_2 \), were studied. The effect of the loaded pile on the unloaded pile is shown in Fig. 3.4, where the spacing-to-diameter ratio of the two piles is plotted against the deflection of the unloaded pile normalised with respect to the deflection of a single pile, i.e. \( s/d \) vs \( S_{\text{unloaded}}/S_{\text{single}} \) where

\[
S_{\text{single}} = \frac{S_{\text{single1}} + S_{\text{single2}}}{2}
\]

i.e. the average deflection of a pile of length \( L_1 \) and a pile of length \( L_2 \) loaded as a single pile.

As expected, the stress in the soil due to the loaded pile causes the unloaded pile to deflect. This deflection increases as both the pile spacing and the length of the loaded pile decrease.
From Fig. 3.4, the following conclusions may be reached for the interaction between two dissimilar piles:

(i) for two piles \(k\) and \(l\) of different diameter,

\[
\alpha_{kl} \approx \alpha_{ll}
\]

where \(\alpha_{kl}\) = interaction factor for pile \(k\) due to pile \(l\)
\(\alpha_{ll}\) = interaction factor for two piles the dimensions of pile \(l\).

(ii) for two piles \(k\) and \(l\) of different lengths \(L_k\) and \(L_l\),
if \(L_k > L_l\)

\[
\alpha_{kl} \approx \frac{\alpha_{kk} + \alpha_{ll}}{2}
\]

if \(L_k < L_l\)

\[
\alpha_{kl} \approx \alpha_{ll}
\]

where \(\alpha_{kk}, \alpha_{ll}\) = interaction factors for two piles having the dimensions of piles \(k\) and \(l\) respectively.

The accuracy of approximate analyses of group behaviour using the above interaction factors has yet to be thoroughly investigated. However, in view of the potential inaccuracy of the group analysis using interaction factors (see Section 3.2), it would appear desirable to use the more complete analysis to analyse groups containing dissimilar piles.
3.4 STATIC PERFORMANCE OF GROUPS WITH DISSIMILAR PILES

The effects of various forms of dissimilarity within an offshore pile group on its static response have been studied by analysing a hypothetical 8-pile group, in which the piles are arranged symmetrically in a circular configuration, with a pitch circle diameter of 10m. The ‘standard’ piles are steel tubes 100m long and 1.5m in diameter, with a wall thickness of 50mm. As shown in Fig. 3.5, four different soil profiles have been considered:

(i) a ‘stiff’ homogeneous soil having constant soil modulus with depth of $E_s = 100$ MPa (case A)
(ii) a ‘weak’ homogeneous soil having constant soil modulus with depth of $E_s = 10$ MPa (case B)
(iii) a ‘stiff’ soil whose modulus increases linearly with depth so that, at depth $z$, $E_s = 2.0z$ MPa (case C)
(iv) a ‘weak’ soil whose modulus increases linearly with depth such that, $E_s = 0.2z$ MPa (case D).

The limiting static skin friction and end-bearing pressure associated with each profile are shown in Fig. 3.5.

These cases bound the range of properties likely to be encountered in practice.

Solutions are presented for the effect of dissimilar piles on the settlement and load distribution within the group at relatively low load levels (using a purely elastic analysis), and at a load of 90% of the ultimate load capacity of the group, in which case slip occurs along most of the length of the piles.

The theoretical solutions presented below have all been derived from the complete group analysis described in Chapter 2. The solutions are only valid when no cap rotations or lateral translations are involved.
3.4.1 Effect of Bells

Analyses have been carried out for three groups:

(a) a group containing all ‘standard’ piles
(b) a group in which two of the piles (diametrically opposite) are belled
(c) a group in which all piles are belled.

The bells are assumed to be of 3 m diameter. Table 3.2 shows the percentage reduction in settlement due to using the bells, as compared with the standard group. Little benefit is derived under elastic conditions unless the soil is very weak (case D), since little of the applied load is transferred to the pile tip. As expected, greater benefit is derived by having all the piles belled.

At the higher load level, the settlement reduction due to the bells is somewhat greater, but still not significant.

Table 3.3 gives the distribution of load in the group with bells, and demonstrates that there is little or no difference in the loads transmitted to the belled and ‘standard’ unbelled piles.

Fig. 3.6 gives the load distribution down the piles for a ‘weak’ soil (case B) and a ‘stiff’ soil (case C) and shows that there is not much difference between the two soils and between the different groups.

Clearly, bells would be more effective if the stratum on which they were founded was substantially stiffer than the overlying soil.

3.4.2 Piles of Unequal Length

Analyses have been carried out for an 8-pile group in which four of the piles are 100 m long and four (every second pile) are 50 m long. Table 3.4 shows the increase in settlement, as a percentage of the settlement of the standard group containing eight 100 m piles. At low load levels, within the purely elastic range, the use of shorter piles results in only a minor increase in settlement. However, at loads approaching failure, there are considerable increases in settlement.
Table 3.5 shows the load distribution within the group and demonstrates that there can be significant non-uniformity of loads at both load levels, particularly for soil profiles B and D. Fig. 3.7 illustrates that this non-uniformity occurs because groups in weak soils tend to carry most of their load in the lower half of the piles while those in stiff soils tend to carry most of their load towards the top of the piles. Therefore, at working loads there is not such a marked non-uniformity of loads for the ‘stronger’ soil profiles (A and C).

The deflection results for the unequal-length group were also compared to those for a group containing eight 50 m long piles. For the homogeneous soils the deflection of the unequal-length group was slightly less than the average deflection of the long 100 m control group and the short 50 m group.

However, in the ‘Gibson’ soils, where soil strength is proportional to depth, the deflection of an unequal-length group is almost the same as that of the control group and significantly less than for the short group. In soil D, the deflection of the unequal-length group is less than half that of the short group under working loads. This suggests that in Gibson soils, especially weak ones, unequal-length pile groups may be the best solution in order to achieve control of deflections and economy of materials.

3.4.3 Piles of Different Diameter

Analyses have been carried out on an 8-pile group in which four of the piles have 1.0 m diameter and four (every second pile) have 0.5 m diameter, all piles being 50 m in length. Table 3.6 shows the increase in settlement, as a percentage of the settlement of a standard group containing eight 50 m piles of 1.0 m diameter. At low load levels, within the purely elastic range, the use of smaller diameter piles results in only a very small increase in settlement, as was found for the piles of unequal length. However, at loads approaching failure, there are large increases in settlement particularly for the weakest soil, case D, for which settlement increased by 104%.

Table 3.7 shows the load distribution within the group and demonstrates that for each load level, the load distribution between the piles is similar for all
the soil profiles. At working loads, the disparity in pile loads between the two pile types is only small, however at loads approaching failure the larger diameter piles carry considerably more load because they have a larger capacity.

3.4.4 A Defective Pile in the Group

The possible consequences of having a defective pile in the group were investigated by carrying out an analysis of an 8-pile group in which one pile had a modulus of one quarter of the value for the other seven piles. This led to a minor increase in settlement and a redistribution of load among the piles in the group. Table 3.8 summarizes the settlement increases, as a percentage of the settlement of the standard group. In all cases, the increase in settlement is less than 10%.

The load carried by the defective pile and the two adjacent piles is summarized in Table 3.9. At working loads, the defective pile generally carries only one-third to one-half of the average load, with the majority of the load being redistributed to the adjacent piles. As failure is approached, the load carried by the defective pile increases (it should be noted that the ultimate load capacity of the defective pile is assumed here to be the same as for the other piles).

3.4.5 Non-Perfect Alignment

The possible consequences of misalignment of a pile during installation were investigated by carrying out an analysis in which one of the piles was positioned next to the adjacent pile, with a gap of only 0.5 m between these two piles. This had a negligible effect on the group settlement and led to a minor redistribution of load among the piles in the group. In all cases, the increase in settlement over the settlement of the standard group is less than 1%.

The load carried by the misaligned pile, the pile adjacent to it and the pile directly opposite it, is summarized in Table 3.10. At working loads, the misaligned pile generally carries about 90% of the average load, the adjacent pile carries only slightly less than the average load and the load is redistributed
fairly evenly among the other piles. As failure is approached, the misaligned pile is forced to carry its share of the load and so there is very little redistribution of load.

3.4.6 Effect of Unloaded Piles in the Group

The analysis was used to analyse a proposal to increase the capacity of an existing pile group. This would normally be a very difficult and costly task. However it was proposed to drive in a circle of piles just outside the pile cap of an existing 8-pile group. It was hoped these piles would carry some of the load being transferred through the soil and hence reduce the load on the original piles.

The results of the analysis were disappointing. The additional piles had very little effect on the original pile group. Some of the groups analyzed had a slight increase in deflection while others had a slight decrease but no marked change in deflection. This trend was evident whether the additional piles were close to the original piles or at a fair distance from them.

A more detailed look at the results offered an explanation. It showed that the lower halves of the additional piles were carrying load and were under compressive stress. However the upper halves were in tension which was loading the pile. This tension and compression in the pile was necessary to maintain equilibrium which required the resultant load on the pile to be zero. The effect of these stresses in the additional piles was to increase the stress in the upper half of the original piles and to decrease the stress in the lower half. However the maximum stress in the original piles was near the top of the piles and so the maximum stress was increased.
3.5 CYCLIC RESPONSE OF GROUPS WITH DISSIMILAR PILES

Cyclic response analyses were carried out for three different pile groups:

(a) the 'standard' 8-pile group shown in Fig. 3.5 containing all 100 m long piles
(b) an 8-pile group in which every alternate pile is only 50 m in length
(c) an 8-pile group in which every alternate pile is 1.0 m in diameter.

Only soil profile C in Fig. 3.5 was considered.

In all cases, the mean load $P_0$ was fixed at 20% of the ultimate static failure load of the standard group in compression, 20 cycles of constant-amplitude loading were applied, and various values of the cyclic load amplitude $\pm P_c$ were considered. In the Matlock and Foo degradation model used (Equation 2.19) $D_{min}$ was taken to be 0.5 and $\lambda$ was 0.25 for the degradation of skin friction. No degradation of soil modulus or base resistance was considered.

3.5.1 Piles of Unequal Length

Fig. 3.8 plots the post-cyclic compressive load capacity, $P_{uc}$, against the cyclic load amplitude, $P_c$, for the standard 8-pile group and for the group containing four 50 m long piles. Both $P_{uc}$ and $P_c$ are normalised with respect to the static compressive load capacity $P_u$ of the group. For the standard group, no cyclic degradation occurs until the cyclic load reaches about 71% of the ultimate compressive load; almost instantaneous degradation then occurs and the pile fails during cycling. In contrast to this very sudden cyclic degradation and failure, degradation in the group containing the shorter piles commences at much lower cyclic load amplitudes, but proceeds much more gradually. Failure during cycling occurs at a somewhat lower cyclic load ($P_c/P_u = 0.65$) than for the standard group ($P_c/P_u = 0.71$).

Fig. 3.9 shows the distribution of skin friction along the 50 m and 100 m piles.
in the non-standard group for two cyclic load levels. For a cyclic load of 0.35 $P_u$, considerably larger skin friction is generated in the shorter piles than in the longer piles, with full slip occurring along the lower part of the pile when the maximum load is applied. For a cyclic load of 0.64 $P_u$ (just before failure during cycling occurs), substantial cyclic degradation of skin friction occurs over most of the length of the shorter piles, and the longer piles then carry considerably more load. The progressive nature of the cyclic degradation is therefore due to the early degradation of the shorter piles followed by the gradual degradation along the longer piles. This analysis explains the two 'humps' which occur in Fig. 3.5 for the group containing the different-length piles. The first 'hump' occurs when the short piles begin to degrade while the second, which leads to failure of the group during cycling, occurs when the long piles begin to degrade.

Fig. 3.10 shows the variation of load in the longer and shorter piles as the cyclic load level is increased. The marked redistribution of load from the shorter to the longer piles at $P_c/P_u = 0.36$ reflects the onset of degradation of skin friction along the shorter piles.

3.5.2 Piles of Unequal Diameter

Fig. 3.11 plots the normalised post-cyclic load capacity, $P_{uc}/P_u$, against the cyclic load amplitude, $P_c/P_u$, for the standard 8-pile group and for the non-standard group containing four 1.0 m diameter and four 1.5 m diameter piles. For the latter case, cyclic degradation commences at a lower cyclic load level (about 0.6) than the standard group (0.71), and is slightly more gradual, with failure during cycling occurring at a cyclic load level of about 0.63.

Fig. 3.12 shows the distribution of skin friction along the 1.0 m diameter and 1.5 m diameter piles in the non-standard group for two cyclic load levels. For a cyclic load of 0.59 $P_u$, considerably larger skin friction is generated in the smaller-diameter piles than in the larger-diameter piles. All the elements are approaching the maximum interface shear stress in the smaller-diameter piles with full slip occurring along the lower part of the pile. However in the larger-diameter piles the skin friction generated is generally only 50-60% of the
maximum interface shear stress. For a cyclic load of 0.63 \( P_u \) (just before failure during cycling occurs), substantial cyclic degradation of skin friction occurs in the smaller diameter piles, and the larger diameter piles carry considerably more load.

The variation of the maximum pile loads with increasing cyclic load is shown in Fig. 3.13. At working loads, there is only a small difference between the loads carried in the 1.0 m and 1.5 m diameter piles. However, once degradation commences, the smaller diameter piles degrade first, and there is a rapid redistribution of load to the 1.5 m diameter piles, and failure during cycling occurs if the cyclic load is increased to 0.63 \( P_u \).

The general characteristics of behaviour of groups containing different diameter piles are therefore similar to those of groups containing different length piles, although the effects of dissimilarity of diameter may be less severe than dissimilarity of length.

### 3.6 EFFECT OF LENGTH OF ADDITIONAL PILES

The effect of the length of additional piles has been studied by analysing a hypothetical 2-pile group in stiff clay in which two further piles are added whose length may be different from that of the original piles. The effect of the length of the additional piles is examined in relation to:

(i) the static load capacity of the group;
(ii) the settlement and load distribution under static loading;
(iii) the load capacity after cyclic loading.

Fig. 3.14 shows the problem analysed and is meant to represent the case of a group of bored piles in stiff clay. The piles are concrete 15 m long and 0.6 m in diameter. The piles are arranged symmetrically in a circular configuration with a pitch circle diameter of 2.4 m, the original piles being opposite each other.

Fig. 3.15 shows the effect of length of the added piles on the static load.
capacity of the group, and Fig. 3.16 illustrates the effect of added pile length on group settlement and load distribution. Clearly, very little benefit is gained from the additional piles unless they are at least half the length of the original piles.

Fig. 3.17 shows the load distributions in the original and additional piles, and reveals that the short added piles are subjected to negative friction along at least part of their length, thus explaining why such piles do not improve the group performance.

Fig. 3.18 shows the influence of the amplitude of cyclic load on the group load capacity after cyclic loading. For any particular length of pile, a 'critical' cyclic load can be found at which the group fails during cycling, after ten cycles of load. As shown in Fig. 3.19, for groups of piles of equal length, this critical cyclic load is about 70% of the static load capacity. However, when the group contains piles of unequal length, the critical cyclic load may fall to about 50% of the static load capacity.

Fig. 3.20 summarises the effect of length of added piles on the safety factor under both static and cyclic loading. Clearly, the use of short added piles does not increase the safety factor under cyclic loading to the same extent as under static loading. However, in this case, the use of additional piles of length equal to the length of the original piles (15 m) results in the same factor of safety under both static and cyclic loading, since no degradation of skin friction occurs then.

For the particular case considered, the results suggest that, under static working loads, the use of short additional piles results in very little improvement in the performance of a pile group. This occurs because the long piles carry almost all the load, while the short piles may be subjected to negative friction. Under cyclic loading, short additional piles result in a smaller increase in the safety factor than under static loading, and the critical level of cyclic load (relative to the static load capacity) is less than for groups with equal-length piles. The overall conclusion is that groups with piles of equal length perform more efficiently than groups containing different length piles.
3.7 SUMMARY

In this chapter verification of the analysis presented in Chapter 2 has been carried out by comparing the results of this analysis with those from an independent similar analysis for symmetrical pile groups under elastic loading. The two results are in good agreement.

A comparison has also been carried out between this analysis and a 2-pile interaction factor analysis. The stress and load distribution results from the two analyses for the middle pile in a 5-pile group differ considerably. This is because the 2-pile interaction factor approach assumes the same stress distribution as for a 2-pile group irrespective of how many piles are in the group and so does not take into account the trend for redistribution of axial loads towards the pile tips as the number of piles in a group increases. The interaction factor analysis of groups therefore becomes less accurate as the size of the group increases.

Analyses have been carried out on a typical 8-pile offshore group to investigate the effects of a variety of dissimilarities on the static and cyclic response of the group, in a variety of soil profiles. Under static loading, the effects of having some piles belled, different length and diameter piles, a defective pile in the group and a misaligned pile in the group have been investigated. In all cases, the effect on settlement at working loads is relatively minor as compared with the case of a group containing all 'standard' piles, while the settlement at loads approaching failure is generally influenced to a greater extent. The greatest effects on settlement and load distribution generally occur for the more compressible soil profiles (case B and D).

Under cyclic loading, degradation of skin friction commences at considerably lower load levels in groups containing piles of different length or diameter, and the degradation process is more gradual as the cyclic load is increased. The results suggest that, under static loading, the effects of having some shorter or defective piles in a group may not be serious at normal working load levels. However if the group is to be subjected to significant cyclic loading, the use of pile groups containing dissimilar piles (in particular, piles of considerably different length) should be avoided.
<table>
<thead>
<tr>
<th></th>
<th>DEFPIG analysis</th>
<th>GAPFIL analysis</th>
<th>percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>top movement</td>
<td>0.0722</td>
<td>0.0742</td>
<td>2.8</td>
</tr>
<tr>
<td>base movement</td>
<td>0.0647</td>
<td>0.0672</td>
<td>3.7</td>
</tr>
<tr>
<td>outer pile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>top stress</td>
<td>0.00208</td>
<td>0.00212</td>
<td>1.7</td>
</tr>
<tr>
<td>load</td>
<td>0.341</td>
<td>0.342</td>
<td>0.1</td>
</tr>
<tr>
<td>base movement</td>
<td>0.0631</td>
<td>0.0694</td>
<td>9.1</td>
</tr>
<tr>
<td>inner pile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>top stress</td>
<td>0.00119</td>
<td>0.000836</td>
<td>-42</td>
</tr>
<tr>
<td>load</td>
<td>0.218</td>
<td>0.217</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(a) 6-pile group

<table>
<thead>
<tr>
<th></th>
<th>DEFPIG analysis</th>
<th>GAPFIL analysis</th>
<th>percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>top movement</td>
<td>0.250</td>
<td>0.217</td>
<td>-15</td>
</tr>
<tr>
<td>base movement</td>
<td>0.227</td>
<td>0.194</td>
<td>-17</td>
</tr>
<tr>
<td>outer pile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>top stress</td>
<td>0.00760</td>
<td>0.00724</td>
<td>-4.9</td>
</tr>
<tr>
<td>load</td>
<td>1.119</td>
<td>1.103</td>
<td>-1.4</td>
</tr>
<tr>
<td>base movement</td>
<td>0.220</td>
<td>0.202</td>
<td>-8.9</td>
</tr>
<tr>
<td>inner pile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>top stress</td>
<td>0.00328</td>
<td>0.000905</td>
<td>-262</td>
</tr>
<tr>
<td>bottom stress</td>
<td>0.00516</td>
<td>0.0143</td>
<td>64</td>
</tr>
<tr>
<td>load</td>
<td>0.526</td>
<td>0.588</td>
<td>11</td>
</tr>
</tbody>
</table>

(b) 5-pile group

Table 3.1 Comparison of DEFPIG and GAPFIL
<table>
<thead>
<tr>
<th>Group</th>
<th>Soil Profile Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>2 bells</td>
<td>0.3</td>
</tr>
<tr>
<td>8 bells</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a) All pile elements behave elastically

<table>
<thead>
<tr>
<th>Group</th>
<th>Soil Profile Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>2 bells</td>
<td>0.4</td>
</tr>
<tr>
<td>8 bells</td>
<td>1.5</td>
</tr>
</tbody>
</table>

(b) Pile Group subjected to a load of 0.9 $P_u$ of standard group

Table 3.2  Percentage Reduction in Settlement  
Due to Bells
<table>
<thead>
<tr>
<th>Soil Profile Case</th>
<th>Load/Average Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belled</td>
<td>1.00  1.02  1.00  1.03</td>
</tr>
<tr>
<td>Standard</td>
<td>1.00  0.99  1.00  0.99</td>
</tr>
</tbody>
</table>

(a) All pile elements behave elastically

<table>
<thead>
<tr>
<th>Soil Profile Case</th>
<th>Load/Average Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belled</td>
<td>1.00  1.03  1.00  1.07</td>
</tr>
<tr>
<td>Standard</td>
<td>1.00  0.99  1.00  0.97</td>
</tr>
</tbody>
</table>

(b) Pile Group subjected to a load of 0.9 $P_u$ of standard group

Table 3.3 Load Distribution in Group with Two Bells
<table>
<thead>
<tr>
<th>Loading</th>
<th>Soil Profile Case</th>
<th>Group</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td></td>
<td>4×100 m piles,</td>
<td>5.6</td>
<td>6.4</td>
<td>6.5</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4× 50 m piles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>near failure</td>
<td></td>
<td>4×100 m piles,</td>
<td>21.0</td>
<td>2.0</td>
<td>28.4</td>
<td>29.1</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td></td>
<td>4× 50 m piles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4  Percentage Increase in Settlement due to Shorter Piles in Group
<table>
<thead>
<tr>
<th>Loading</th>
<th>Soil Profile Case</th>
<th>Load/Average Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pile Type</td>
<td>A</td>
</tr>
<tr>
<td>Elastic</td>
<td>100 m pile</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>50 m pile</td>
<td>1.00</td>
</tr>
<tr>
<td>near failure</td>
<td>100 m pile</td>
<td>1.49</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td>50 m pile</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 3.5 Load Distribution in Group with Shorter Piles
<table>
<thead>
<tr>
<th>Loading</th>
<th>Group</th>
<th>Soil Profile Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>4× D=1.0 m piles,</td>
<td></td>
<td>1.9</td>
<td>1.3</td>
<td>0.7</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>4× D=0.5 m piles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>near failure</td>
<td>4× D=1.0 m piles,</td>
<td></td>
<td>43</td>
<td>10</td>
<td>24</td>
<td>104</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td>4× D=0.5 m piles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6 Percentage Increase in Settlement due to Piles of Smaller Diameter
<table>
<thead>
<tr>
<th>Loading</th>
<th>Soil Profile Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>D = 1.0 m pile</td>
<td>1.08</td>
<td>1.07</td>
<td>1.03</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>D = 0.5 m pile</td>
<td>0.92</td>
<td>0.93</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>near failure</td>
<td>D = 1.0 m pile</td>
<td>1.26</td>
<td>1.26</td>
<td>1.29</td>
<td>1.33</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td>D = 0.5 m pile</td>
<td>0.74</td>
<td>0.74</td>
<td>0.71</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 3.7  Load Distribution within a Group containing Piles of Different Diameter
<table>
<thead>
<tr>
<th>Soil Profile Case</th>
<th>Percentage Increase in Settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>A</td>
</tr>
<tr>
<td>Elastic</td>
<td>3.6</td>
</tr>
<tr>
<td>near failure</td>
<td>7.3</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.8  Percentage Increase in Settlement due to a Defective Pile**
<table>
<thead>
<tr>
<th>Loading</th>
<th>Soil Profile Case</th>
<th>Load/Average Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>standard pile</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>(adjacent to</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>defective pile)</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>defective pile</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.40</td>
</tr>
<tr>
<td>near failure</td>
<td>standard pile</td>
<td>1.10</td>
</tr>
<tr>
<td>(90% of ultimate</td>
<td>(adjacent to</td>
<td>1.10</td>
</tr>
<tr>
<td>load)</td>
<td>defective pile)</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>defective pile</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 3.9  Load Distribution in Group with Defective Pile
<table>
<thead>
<tr>
<th>Loading</th>
<th>Soil Profile Case</th>
<th>Soil Profile Case</th>
<th>Soil Profile Case</th>
<th>Soil Profile Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>misaligned pile</td>
<td>0.88</td>
<td>0.88</td>
<td>0.95</td>
</tr>
<tr>
<td>Elastic</td>
<td>adjacent pile</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>opposite pile</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>near failure</td>
<td>misaligned pile</td>
<td>1.00</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>(90% of ultimate load)</td>
<td>adjacent pile</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>opposite pile</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3.10  Load Distribution in Group with Misaligned Pile
Figure 3.1 Load-Displacement Curves
Figure 3.2 Effect of Number of Piles on Stress Distribution
Figure 3.3  Load and Stress Distributions in 5-Pile Group - Middle Pile
\[ \alpha_A = \frac{S_{\text{unloaded}}}{S_{\text{single}_2}} \]

\[ \alpha_B = \frac{S_{\text{unloaded}}}{S_{\text{single}_1}} \]

\[ \alpha_C = \frac{\alpha_A + \alpha_B}{2} = \frac{S_{\text{unloaded}}}{S_{\text{single}_1} + S_{\text{single}_2} / 2} \]

Figure 3.4 Interaction Factors for 2 Piles of Different Length
Arrangement of piles in group

"Standard" piles are steel tubes
L = 100m
d = 1.5m
Wall thickness t = 50mm

<table>
<thead>
<tr>
<th>Case</th>
<th>$E_s$ (MPa)</th>
<th>$\tau_a$ (kPa)</th>
<th>$p_b$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>2z</td>
<td>1.8</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>2z</td>
<td>1.8</td>
</tr>
<tr>
<td>C</td>
<td>2z</td>
<td>2z</td>
<td>1.8</td>
</tr>
<tr>
<td>D</td>
<td>0.2z</td>
<td>2z</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Distributions of Soil Modulus $E_s$, Limiting Skin Friction $\tau_a$ and End Bearing Pressure $p_b$

Figure 3.5 Cases Analysed for Static Response
Figure 3.6 Load Distribution for Belled Groups
Figure 3.7 Load Distribution in Unequal-Length Group
at $P=60\%P_a$
Figure 3.8  Effect of Piles of Different Length on Group Load Capacity After Cycling
Figure 3.9  Distribution of Maximum Shear Stress on Piles During Cycling. Group Containing Four Shorter Piles
Figure 3.10  Pile Loads in Groups with Four Shorter Piles
\[ P = 0.2 P_0 + P_c \]

**Figure 3.11** Effect of Piles of Different Diameter on Group Load Capacity After Cycling
Figure 3.12  Effect of Piles of Different Diameter on Load Distributions
Figure 3.13  Pile Loads in Groups Containing Piles of Different Diameter
Figure 3.14  Problem Analysed
Figure 3.15  Effect of Length of Additional Piles on Ultimate Static Load Capacity
Figure 3.16 Settlement and Load Distribution in Pile Groups
- Static Loading
Figure 3.17  Static Load Distribution in Piles, Total Load = 3.0 MN
Figure 3.18  Variation of Load Capacity After Cycling With Cyclic Load Amplitude
Figure 3.19 Effect of Length of Additional Piles on Critical Cyclic Load
Figure 3.20  Effect of Piles on Factor of Safety
EXPERIMENTAL METHODS

4.1 INTRODUCTION

Two series of carefully controlled tests on model pile groups were carried out in overconsolidated clay. The first series, performed in a large vessel, was carried out in an attempt to verify some of the theoretical results described in Chapter 3 and also to obtain degradation data on which to base the theory. The second series, performed in a small vessel, was carried out to determine the degradation characteristics of the clay.

In the first series, tests were conducted on single piles and on 2-pile, 4-pile and 8-pile groups, some of which contained piles of unequal-length. In each test, axial loads were applied to the pile group and group settlements, group loads and pile strains were measured.

In the second series, tests were conducted on a single pile in a vessel with flexible upper and lower boundaries provided by rubber membranes. The effect of cyclic loading on static ultimate skin friction was investigated.
This chapter describes each piece of apparatus, the preparation of the clay, the installation of the piles and the testing procedures.

4.2 LARGE TEST VESSEL

4.2.1 The Vessel

The vessel in which the clay was consolidated and the tests carried out is pictured in Fig. 4.1 and shown diagrammatically in Fig. 4.2. The body of the vessel was a thick steel tube with an internal diameter of 500 mm and a depth of 480 mm. This was large enough to accommodate an 8-pile group, containing 300 mm long piles, and ensured that the vessel boundary was at a reasonable distance from the outside of the pile group. The upper portion of the vessel during consolidation of the clay is shown in Fig. 4.2 and consisted of two sheets of 0.2 mm filter paper, a 1.5 mm rubber membrane, spacing ring, domed lid and stiffening ring. This top assembly was secured to the base with twenty Allen screws. O-rings above and below the spacing ring prevented leakage of clay or water. The design of the pressure vessel has been described by Wiesner (1977).

Drains on either side at the top of the vessel and in the centre of the base plate were protected with wire gauze and covered with filter paper to permit drainage of water from the clay under pressure. As the clay consolidated, water flowed out of the drains at the top and the bottom of the vessel and this expelled water was continuously measured in separate measuring cylinders to monitor the progress of consolidation. Preliminary tests carried out using this system of drainage proved unsatisfactory because only 28% of the expelled water was making its way to the top drains and the whole consolidation process was taking an unduly long time. A top centre drain was constructed which allowed the water to be drawn off from the centre of the vessel, passed through a hole in the rubber membrane, and carried to a tapping in the spacing ring by a plastic tube. This drainage method proved more successful with 55% of the expelled water passing through the top drain and the total consolidation time
decreased by 60%. The average consolidation time using the centre drain was 25 days.

4.2.2 Pile Groups

The model pile groups consisted of a 19 mm rigid steel pile cap and tubular aluminium piles. A typical 8-pile group is pictured in Fig. 4.3.

The model piles were made from 25 mm external diameter aluminium tubing with a wall thickness of 1.2 mm. A 45° conical plug was fixed to the toe of each pile and a 33 mm diameter collar fixed to the pile head. The collar was threaded so that it could be screwed into the pile cap. Two pile lengths were used, the long piles being 310 mm in length and the short piles 160 mm long.

One of the long piles and one of the short piles were instrumented with electrical resistance strain gauges, wired individually in a quarter bridge configuration. Gauges were positioned at four locations along the long pile and at three locations along the short pile as shown in Fig. 4.4. Details of the wiring of the strain gauges have been described by Pressley (1982). The strain gauges were calibrated by loading the instrumented pile in a testing machine. The calibration setup is shown in Fig. 4.5.

The pile cap had 17 threaded holes in a square grid with a spacing of 50 mm between adjacent holes. The loading plunger was screwed into the central hole for testing. A number of different configurations of pile groups were tested. The standard group on which most group tests were conducted was a 4-pile group with a centre-to-centre spacing of 71 mm between adjacent piles. Tests were also carried out on an 8-pile group in a square configuration with 50 mm spacing, and on a 2-pile group with 100 mm spacing. The layout of the test vessel with the 8-pile group is shown in Fig. 4.6. The pile group is not in the centre because two other 8-pile groups were tested in the same test bed. All the above groups contained only long piles however a few additional tests were carried out on pile groups containing a combination of long and short piles.

A number of tests were also conducted on single piles made of perspex to
investigate the effects of pile compressibility. The perspex piles were made from 25 mm diameter perspex rod bored to give a wall thickness of 1.4 mm. This was the smallest wall thickness which could be obtained without too many problems. The piles were 310 mm in length and contained a conical plug made of perspex and an aluminium collar. The outer dimensions of the piles were identical to those of the aluminium piles.

4.2.3 Properties of Clay

The model tests were carried out using a refined C/10 kaolin. This material was selected because it consolidates quickly, does not creep excessively, is easy to handle and can be reused over and over again after water is added and mixed in. It has been used successfully in model pile tests by a number of previous researchers. The kaolin has liquid and plastic limits of 46 and 33 respectively. After loading to 200 kPa for several weeks and then subsequent unloading the clay was found to have an undrained cohesion of 32 kPa from unconfined undrained triaxial tests and an average moisture content of 37%.

4.2.4 Preparation of Clay

The clay was prepared in galvanised steel bins in which enough water was added to the consolidated clay from the previous test to bring the moisture content up to 59%. After soaking for a few days the clay was mechanically mixed using a low speed power drill fitted with a propeller until a uniform mixture was formed. Moisture content samples were taken from the centre of the mixture and the bins were then sealed and allowed to stand for 24 hours, after which any water required to bring the moisture content up to 60% was added to the clay and it was again thoroughly mixed.

The clay was poured into the test vessel with the aid of an immersion vibrator and it was vibrated at several stages of the filling operation to remove as much entrapped air as possible. When the vessel was filled the clay was domed slightly in the centre as pictured in Fig. 4.7 to allow for the 'dishing' of the top surface during consolidation with the centre drain. The top surface
was subjected to vigorous tapping with a trowel to eliminate air voids and then
smoothed with a long spatula.

The two filter papers were moistened and placed over the clay so that they
covered the clay but did not project onto the O-ring groove. An extra piece of
filter paper and a small piece of wire gauze were placed on the centre of the
filter papers so that the centre drainage assembly, made of brass, did not pierce
the filter papers. The rubber membrane, with the two-piece drainage assembly
screwed onto it through a hole in its centre, was placed over the filter papers
so that the drain sat on the wire gauze. The spacing ring was positioned on
the rubber membrane and the plastic tube from the drainage assembly was
connected to the tapping on the spacing ring. The domed lid and the stiffening
ring were placed on top and the whole assembly was screwed onto the vessel.

The top section of the vessel was filled with water through a hole in the
domed lid and a cap was screwed into the hole. Pressure was applied by the
water to the rubber membrane to consolidate the clay. The pressure was applied
via an air-water exchange pot using a Norgren (0-400 kPa) pressure regu-later
coupled to a compressed air supply through a Budenburg standard test gauge
(see Fig. 4.2). The water expelled from the drains was measured to monitor
the progress of consolidation and to indicate when topping-up of the vessel was
required.

After two days at an effective pressure of 60 kPa, the clay surface had gener-
ally settled about 60 mm due to expulsion of pore water. The vessel was then
topped up with clay which had been again thoroughly mixed at a moisture
content of 60%. After a further five days at pressures gradually increasing to
150 kPa the vessel was topped up again. The clay was further consolidated at
200 kPa for a minimum of 15 days. A typical consolidation curve is shown in
Fig. 4.8 showing all stages of consolidation. When the pressure was released
after consolidation the ends of the drainage tubes were immersed in water to
ensure that no air was sucked into the clay. Finally, the lid was removed and
the surface of the clay smoothed to be level with the top of the vessel.
4.2.5 Setting up for Testing

The single piles and the pile groups were jacked into the clay using the rapid hand feed gearing of a Wykeham Farrance constant rate of penetration loading machine. The single pile was attached to the loading machine via a three-piece connection which allowed the pile to be unscrewed from the loading machine without the pile turning. This three-piece connection is shown in Fig. 4.9. The pile group was attached via a similar connection which screwed into the central hole on the pile cap. The group was first assembled by screwing the individual piles into the pile cap. The three-piece connection was then screwed into both the loading machine and the pile cap. The whole pile group was then jacked into the clay.

The strain gauges in the pile were connected to the channels of a data acquisition system so that the loads in the strain gauges could be recorded while the pile was being jacked in. The long piles were jacked to a penetration of 300 mm into the clay so that the top strain gauge was just at the clay surface. Installation of the piles took approximately 20 minutes. The vessel was then covered with several rubber sheets to prevent the clay surface drying out and left for 24 hours to allow dissipation of any excess pore pressures set up during jacking. The group was inserted in exactly the same way.

Instrumentation on the pile group consisted of a DCDT deflection transducer to measure settlement, a 10 kN proving ring to measure group load and strain gauges to measure loads in the pile. The tests were controlled using a Hewlett Packard 3052A Data Acquisition System. The complete system consisted of a computer, two disc drives, a plotter, a printer, a digital voltmeter, a scanner and a 5 V power supply.

The body of the deflection transducer was clamped to a fitting attached to the proving ring while the central core rested on a magnetic clamp. This was clamped to a small steel channel section which spanned the vessel and was securely clamped to both sides with G-clamps. The transducer was calibrated by placing it in a micrometer and connecting it to the data acquisition system. The micrometer was turned through ±25 mm with a voltage reading taken
every 0.5 mm.

Vertical load was applied to the centre of the pile cap through a plunger driven by an electric motor through a variable speed gearbox. The loading machine was mounted on a 2.3 m high loading frame and could be positioned over any pile or pile group in the vessel by a combination of movement of the loading machine along the frame and movement of the vessel on its trolley across the frame (see Fig. 4.1).

Tests were carried out to see whether deliberate misalignment of the pile affected the results. A test was carried out in which the pile and the loading machine were aligned as closely as possible. The pile was loaded to failure in compression, then to failure in tension and then back to zero load. A second test was carried out in which the pile and the loading machine were deliberately misaligned by about 5 mm (in the test series the piles would never have been misaligned by more than 2 mm) and the same load sequence was applied. The resulting load-deflection curve was the same shape as the curve for the aligned pile, the only difference being that the ultimate load was slightly less which was to be expected due to the previous loading and unloading of the pile which reduced the skin friction slightly.

The load was transferred from the loading machine, through the proving ring, to the pile cap. The load transducer on the proving ring was calibrated on the data acquisition system and the dial gauge on the proving ring was calibrated independently in a tension and compression loading rig. Fig. 4.10 shows the position of the transducers during testing.

4.2.6 Testing

The data acquisition system was used to control the tests. By means of a computer program, written by T.S. Hull, the system could convert transducer readings and strain gauge readings to load and deflection values, plot and print the values as well as control the direction of loading of the pile group. The tests were load-controlled so that, when the group had carried a specified load, the loading machine stopped or changed the direction of loading.
The first step in carrying out a test was to manually wind the loading plunger down until contact was made with the pile group. The three-piece connection between the proving ring and the pile cap was then screwed together so that the group could be loaded in tension and in compression. Finally the gears were engaged ready for testing.

In some of the early single pile tests the gears were engaged with the hand feed on coarse adjustment and so a preload of up to 60 N was applied to the piles. In most of the tests however there was no such preload on the piles.

Also, in some of the early tests, when the load was released the deflection continued to increase and it was suspected that this was due to a crooked pile. A further test was carried out in which dial gauge readings were recorded on either side of the proving ring. The resulting load-deflection curve, shown in Fig. 4.11, indicates that the proving ring was rotating and therefore the pile was crooked. It was noted that the piston in the loading machine was wound down nearly to its full extension and so had no support to hold the pile vertical and was wobbling. A brass extension piece was made up so that in all future tests the piston was nearly fully enclosed in the loading machine. This ensured that the pile remained vertical.

In the group tests in which all the piles in the group were identical, only one instrumented pile was used. For the group tests using different length piles, one long instrumented pile and one short instrumented pile were used.

The minimum time between insertion and the commencement of testing was 24 hours. Each test generally consisted of three stages:

(i) static axial failure of the pile group; failure here was defined as the load to cause a movement of 5% of the pile diameter i.e. 1.25 mm;
(ii) the pile group was subjected to cyclic loading between specified limits of load for ten cycles;
(iii) after the last cycle, the group was subsequently loaded to failure to determine the influence of cycling on load capacity.

The group was then unloaded, disconnected from the loading machine and left to recover for 24 hours.
The rate of loading was 1.5 mm per minute, which was the fastest speed on the loading machine.

Most of the groups were subjected to two tests. In the first test the cyclic load was small enough to ensure that failure did not occur during cycling. In the second test the cyclic load was chosen with the expectation that failure would occur within ten load cycles. In a few of the tests failure did not occur when the piles were subjected to the expected failure load and so a third test was carried out in which the pile group failed during cyclic loading.

Two types of failure were identified:

(i) catastrophic failure in which the group was unable to attain the cyclic load in one of the ten load cycles; and

(ii) failure due to accumulated deflection, in which the group attained the cyclic load but in doing so underwent a large permanent displacement. The group was defined to have failed due to accumulated deflection when the accumulated displacement exceeded 0.4 mm after up to ten cycles.

The tests were controlled by a computer program using the Hewlett Packard Data Acquisition System. The controlling program allowed for specification of values of failure deflection, cyclic load and frequency of readings as well as load and deflection limits so that a plot of load versus displacement could be made as the test was progressing. The direction of loading could be changed either manually or by the computer program when the required cyclic load was reached. A test usually lasted between 15 minutes and two hours.

The tests were identified by a number and letter code. The first letter or group of letters indicated the type of group being tested. The letter code was as follows;

S  single pile
G  4-pile group containing equal-length piles
UG 4-pile group containing unequal-length piles
BG  8-pile group (Big Group) containing equal-length piles
2G  2-pile group containing unequal-length piles
SS  short single pile
SP  single pile made of perspex.

This letter code was followed by a number code which indicated the number of the group or single pile being tested. Finally a letter code indicated the type of test being carried out on that particular group. The final letter code was as follows;

A  jacking in the group
B  the first test on the group which usually did not result in failure during cycling
C  the second test on the group which usually resulted in failure during cycling
D  the third test which was carried out only if neither of tests B or C had resulted in failure during cycling
E  jacking the group out of the clay.

For example, test number G8C represents the eighth 4-pile equal-length group tested. It was cycled to loads which would induce failure during cycling.

The 4-pile unequal-length groups contained two long piles and two short piles, each positioned at diagonally opposite corners of a square. This group is shown in Fig. 4.12 immediately before insertion.

In all, tests were carried out in eight clay beds. Table 4.1 gives a summary of the tests carried out in each bed. At the conclusion of testing, while the clay was being removed, tube samples were taken from the top and the bottom of the vessel. Unconfined undrained triaxial tests, unconfined compression tests and moisture content determinations were then carried out.

After the 4-pile groups were removed it was noted that cracks up to 60 mm
in length had generally formed between the piles. While the clay was being removed, the depth of these cracks was measured to be up to 5 mm only. The 8-pile groups generally had more severe cracking. After test BG1D, which failed due to accumulated deflection by block failure, 20 mm deep cracks had formed between the piles on three sides and the clay in the centre of the group had risen by 10 mm.

4.3 SMALL TEST VESSEL

Tests were performed in a small vessel to determine the degradation characteristics of the clay independently.

4.3.1 The Vessel

The small vessel in which the clay was consolidated and the pile tested is shown diagramatically in Fig. 4.13. The internal diameter was 180 mm and the depth 256 mm. The lid and the base of the vessel were grooved so that a rubber O-ring could be fitted to give a water-tight seal. Two holes in the side of the vessel were located near the top and near the base to allow two-way drainage. Both the lid and the base of the vessel contained water which could be subjected to identical pressure by a common pressure source. Rubber membranes separated the pressurised water from the clay and the pile. The pressure was supplied using a Norgren pressure regulator coupled to a compressed air supply through a Budenburg standard test gauge and an air-water exchange pot. The rubber membrane was secured to the pile with a screw-in sealing nut. A central hole in the lid enabled a loading plunger to be screwed into the nut. The plunger was located in a guide fitted with linear bearings to minimize friction and effect a seal.

A special feature of this vessel was its ability to rotate on two arms protruding from the main body of the vessel and be turned upside down. This enabled the base of the vessel to be filled with water.

Tests in this vessel were carried out on 20 mm diameter, 256 mm long solid aluminium piles. They had a tapped hole at the pile head and flat toes.
4.3.2 Preparation of Clay

The clay used for the small vessel tests was the same C/10 kaolin used for the large vessel tests and it was prepared in much the same way but on a smaller scale.

The two drain holes at the top and bottom of the vessel were protected with gauze and covered with filter paper to permit drainage of water. A base plate was fitted to the base of the vessel. The base plate had a 20 mm diameter, 2 mm deep recess in its centre in which a small circular piece of sponge was placed. This ensured the pile was not damaged while being inserted when it reached the base plate. The base plate was covered with a circular sheet of plastic which prevented the clay from sticking to it.

The clay was thoroughly mixed and scooped into the small vessel. It was vibrated by hand using a plastic scoop at several stages of the filling operation to remove entrapped air. The clay was domed up about 20 mm in the centre and then smoothed over.

A piece of filter paper was moistened and placed over the clay and the rubber membrane was placed over it. The lid was then positioned on top and screwed onto the vessel. The top section was filled with water and then the hole in the lid was sealed with a cap. To ensure that all the air was removed from the top section there was a small hole in the lid. An effective pressure of 20 kPa was applied and as soon as water started to discharge from the hole, it was plugged with a screw. The pressure was increased to 200 kPa and the expelled water from the drains was continuously measured in a measuring cylinder.

After approximately 28 hours the pressure was released, the water in the top section was siphoned out and the lid removed. The vessel was topped up with clay which had been again thoroughly mixed. After a further five days consolidating at 200 kPa the vessel was topped up again. After another two days the pile was inserted.

To insert the pile the pressure was released and the lid removed. The clay surface was smoothed to be level with the top of the vessel using a long spatula.
A pile guide of length 90 mm, shown in Fig. 4.14, was clamped to either side of the vessel with G-clamps. This ensured that the pile was inserted vertically into the centre of the vessel. The pile was inserted slowly by hand into the clay until the head of the pile was pushed into the pile guide. An extension piece 20 mm in diameter and 110 mm in length with a groove machined 85 mm from its base was then screwed into the pile and the extension piece was pushed into the pile guide until the groove disappeared which meant the pile had reached the base plate. The pile guide and the extension piece were removed and a rubber membrane with a 9.5 mm diameter hole in its centre was placed over the vessel. The head of the pile was milled to a diameter of 10 mm for a depth of 1 mm, providing a neat seat for the rubber membrane to sit on. The membrane was secured to the pile with the screw-in sealing nut. The loading plunger was screwed into the sealing nut and the top section again filled with water. The loading plunger guide was slipped over the loading plunger and screwed onto the lid.

The vessel was rotated to the upside down position and the base plate removed. A rubber membrane was placed over the clay and the pile base. A spacing ring was inserted between the base plate and the vessel to allow pressurised water to occupy the space. The tip of the pile thus rested on the base membrane (with perhaps a few mm of clay in between), so that the tip resistance would be negligible, with the entire resistance of the pile being derived from skin friction.

The vessel was rotated back to the upright position and a 200 kPa pressure applied. This pressure was maintained for six days or until no more water was expelled from the vessel under two-way drainage. This procedure was designed to produce clay with the same properties as that produced in the large vessel. After testing the clay was found to have an average moisture content of 36%. This was slightly less than the average value for the large vessel (37%).
4.3.3 Test Procedure

Instrumentation on the small vessel consisted of a DCDT deflection transducer to measure settlement and a 10 kN proving ring to measure axial load. The data recording system for these tests consisted of a computer, two disc drives, a plotter, an analog to digital converter interfaced to the computer and a power supply for the transducers. The pile tests were controlled and the data recorded by a program written by K.F. Chan and modified by C.Y. Lee on a TRS 80 computer. The data was sampled at predetermined discrete times and stored (if required) on a floppy disc.

After consolidation of the clay was complete the drainage taps were turned off so that no further water was expelled. In one of the tests, the drainage taps were left open by mistake, which resulted in an increase in strength of the clay from the 10th to the 100th cycle of 24%. This increase was a result of the test being drained, rather than undrained as the rest of the tests were. There was a redistribution of stress within the clay due to the drainage.

The deflection transducer was fitted over the loading plunger in a clamp and clamped to the plunger while the inner core rested on the vessel lid. The proving ring was connected to the loading machine which was positioned over the vessel. The vessel was levelled using a spirit level and then positioned directly in line with the loading machine. The loading machine was then wound down by hand and clamped to the loading plunger.

Two movable triggers were attached to the loading machine and positioned above the maximum and below the minimum possible values of deflection for each test. If there was a malfunction in the program or the loading machine which prevented the load from being reversed at the appropriate time, the triggers were set off and the loading machine stopped. Fortunately there were no such malfunctions in this test series.

Each test generally consisted of three stages:

(i) static axial failure of the pile and unloading to zero load; as before, failure was defined as the load to cause a movement of 5%
of the pile diameter i.e. 1.0 mm;

(ii) the pile was then subjected to cyclic loading between specified limits of deflection for 100 cycles;

(iii) after the last cycle, the group was again loaded to failure to determine the influence of cycling on load capacity.

The pile was then unloaded manually, disconnected from the loading machine and the clay was removed from the vessel. The whole cycle of consolidating the clay and testing took two weeks.

The rate of loading was 1.5 mm per minute. The whole test was controlled by the controlling program which allowed for specification of values of failure deflection, cyclic deflection, number of cycles and the cycle numbers for which storage was required on the floppy disc. Another program was used to plot the load versus displacement curves for any of the cycles which were stored. Fig. 4.15 shows a typical load versus displacement curve for a small vessel test.

There were two identical small vessels which were used concurrently for this series of tests.

4.4 SUMMARY

In this chapter two test series on model piles and pile groups in overconsolidated clay have been described. The first test series consisted of tests on single piles and on 2-pile, 4-pile and 8-pile groups in a large test vessel under both static and cyclic loading. The second series involved testing a single pile in a vessel with flexible upper and lower boundaries under cyclic loading. The apparatus used, the preparation of the clay, the installation of the piles and the testing procedures have been described.

The results of the tests under both static and cyclic loading will be compared with the theoretical analyses. The single pile results will be described in Chapter 5 and the pile group results will be described in Chapter 6.
<table>
<thead>
<tr>
<th>Bed No.</th>
<th>Description of Tests</th>
<th>Test Nos</th>
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<tbody>
<tr>
<td>1</td>
<td>5 single pile tests</td>
<td>S1-S5</td>
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<tr>
<td>2</td>
<td>7 single pile tests</td>
<td>S6-S12</td>
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<tr>
<td>3</td>
<td>3 4-pile group tests</td>
<td>G1-G3</td>
</tr>
<tr>
<td></td>
<td>1 single pile test</td>
<td>S13</td>
</tr>
<tr>
<td>4</td>
<td>3 4-pile group tests</td>
<td>G4-G6</td>
</tr>
<tr>
<td>5</td>
<td>3 4-pile group tests</td>
<td>G7-G9</td>
</tr>
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<td></td>
<td>3 single pile tests</td>
<td>S14-S16</td>
</tr>
<tr>
<td>6</td>
<td>1 8-pile group test</td>
<td>BG1</td>
</tr>
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<td></td>
<td>4 single pile tests</td>
<td>S17-S20</td>
</tr>
<tr>
<td>7</td>
<td>2 4-pile group tests</td>
<td>UG1-UG2</td>
</tr>
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<td></td>
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<td>2G1</td>
</tr>
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<td>1 single pile test</td>
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<td>1 short single pile test</td>
<td>SS1</td>
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<tr>
<td>8</td>
<td>1 8-pile group test</td>
<td>BG2</td>
</tr>
<tr>
<td></td>
<td>4 perspex single pile tests</td>
<td>PS1-PS4</td>
</tr>
</tbody>
</table>

Table 4.1 Summary of Tests carried out in each Clay Bed
FIG. 4.1  TEST VESSEL WITH 4-PILE GROUP INSERTED
Figure 4.2 Test Vessel For Consolidation of Clay
FIG. 4.3  8-PILE GROUP
Figure 4.4 Details of Instrumented Model Piles
Figure 4.5  Calibration Set-up for Single Pile
Figure 4.6  Layout of 8-Pile Group in Test Vessel
Figure 4.8 Consolidation Curve for the Clay
Figure 4.9  Three-Piece Connection Attaching Pile to Loading Machine
FIG. 4.10  PROVING RING SET UP FOR TESTING 8-PILE GROUP
FIG. 4.12  4-PILE UNEQUAL-LENGTH GROUP
Figure 4.13  Small Test Vessel
FIG. 4.14 SMALL TEST VESSEL WITH PILE GUIDE ATTACHED
5

SINGLE PILE
TEST RESULTS

5.1 INTRODUCTION

Single pile tests were carried out both in the large test vessel and the small test vessel described in Chapter 4. In all, 26 static tests and 52 cyclic tests were carried out in the large vessel, and ten cyclic tests were carried out in the small vessel. Most of the single pile tests were carried out using the long pile with only a few being carried out using the short pile.

In this chapter results for both static and cyclic loading are presented and compared with the theoretical analyses. Consideration is given to

(i) pile installation
(ii) ultimate load
(iii) load-settlement behaviour
(iv) load distribution
(v) pile head stiffness
(vi) skin friction degradation
(vii) the interaction between cyclic load and mean load
(viii) the effect on pile load capacity of mean load
(ix) the effect on pile load capacity of cyclic load
(x) cyclic stiffness
(xi) cyclic displacement
(xii) accumulated deflection
(xiii) residual loads.

A schedule of the tests carried out is given in Table 5.1. The table lists the tests in chronological order and includes the clay bed number, the specific test number, the static ultimate load, the mean load normalised with respect to the static ultimate load, the normalised cyclic load, the skin friction degradation factor, an indication of whether or not the pile failed during cycling, the cyclic deflection, the accumulated deflection after ten cycles, the pile head stiffness and any general remarks on the test.

5.2 PILE INSTALLATION

Analysis of the pile installation data gives an indication of the distribution of skin friction down the pile during and after jacking. Fig. 5.1 shows a typical result in which the load in the pile is plotted against depth of penetration as the pile is progressively inserted. For each penetration depth, the load in the gauges which were still above the clay was equal, and the load in the gauges which had entered the soil decreased uniformly down the pile. However, there was some decrease in load in the upper part of the pile as the pile approached final penetration. The results therefore indicate a fairly uniform skin friction distribution down the pile throughout installation and confirm the assumptions of a uniform skin friction employed in section 5.3.

After the jacking force was removed the distribution of residual load was measured, and there was, as expected, a small compressive load in the upper part of the pile and a small tensile load in the lower part of the pile.

Fig. 5.2 illustrates further the skin friction distribution with depth. The
load in the four strain gauges is plotted against penetration depth. The load in the fourth (i.e. lowest) strain gauge was initially 130 N when the pile tip only was inserted and then reduced to 80 N when the pile was fully inserted. This suggested that the soil at a depth of 300 mm was slightly softer than at the surface. This was confirmed by the undrained triaxial tests. The average value of undrained cohesion at the top of the vessel was \( c_u = 34 \) kPa and in the middle of the vessel the average value was \( c_u = 30 \) kPa.

The load in the third gauge increased uniformly until a depth of 130 mm which is exactly when it would have come in contact with the clay. After this depth the load decreased uniformly due to the clay being slightly weaker at greater depths.

Similarly the load in the second gauge increased uniformly to a depth of 260 mm where it came in contact with the clay and then decreased. The load in the first gauge increased fairly uniformly to 300 mm. This again confirms the validity of the assumption of uniform skin friction assumed in the previous analysis.

5.3 STATIC LOADING

A static load test was carried out immediately before each of the cyclic load tests. This was primarily to establish the static load-settlement behaviour, load distribution and pile head stiffness so that these characteristics could be compared with the corresponding values after cyclic loading.

Solutions were obtained using the theory described in Chapter 2 to model the static load tests. In the analysis, ten shaft elements were used to represent the pile and the soil was assumed to have a uniform skin friction. The computations were carried out on a PRIME minicomputer using the program TAPFIN, which is a modification of the program TAPFIL written by Prof. H.G. Poulos, and incorporates the Novak and El Sharnouby formulation described in section 2.4. The theoretical solutions are plotted for comparison with the experimental results in the relevant static loading plots.
5.3.1 Ultimate Load

The static ultimate pile capacity, $P_u$, was defined as the compressive load to cause a movement of 5% of the pile diameter. The average value was calculated from the results of the 15 $B$ series static tests. The $B$ series tests involved a static test in which the pile was loaded to failure in compression and then unloaded to zero load, followed by a cyclic test in which the pile was subjected to cyclic loading between specified limits of load for ten cycles, as described in section 4.2.6. The values are set out in Table 5.1. The average static ultimate pile capacity of the long pile was 470 N and the standard deviation was 50 N.

The capacity of the piles in the $C$ series, which represented the second test to be carried out on each pile, did not change significantly from the initial ultimate capacity in any of the tests. However, the $C$ series tests all showed that the peak load was developed after about 0.15 mm movement, after which the load dropped slightly before it started to rise again up to the residual load at a deflection of 1.25 mm. In some tests, the peak capacity was slightly greater than the residual static ultimate capacity.

This peak-residual behaviour did not manifest itself all the way down the pile. The two strain gauges in the upper half of the pile showed a definite 'peak', yet the two in the lower half of the pile showed no such peak. This could represent the point where the skin friction along the upper half of the pile has reached its maximum and all the elements are slipping. The cycling in the previous $B$ series test on that pile may have weakened the clay and reduced the skin friction causing the upper half of the pile to begin slipping at this lower peak load.

The average ultimate load for the perspex piles was 380 N compared to 470 N for the aluminium piles and the standard deviation for the perspex piles was 47 N.

To calculate the skin friction of the piles it was necessary to determine how much of the ultimate load was carried in skin friction and how much was carried in end-bearing. In seven of the early tests the piles were loaded to failure in compression and then loaded to failure in tension. In each case the load
carried in compression was greater than that carried in tension. The difference in the two loads was assumed to be the difference between the compressive end-bearing capacity and the tensile end-bearing capacity.

The load carried in tension was measured after a gap had formed between the pile base and the clay so as not to include any tensile base resistance. The point where a gap formed was easily detectable from the load-settlement curve as the tensile capacity first decreased and then remained constant. In the seven tests, the average tensile load was 93% of the average compressive load. Therefore, in all calculations of skin friction the load carried as skin friction was assumed to be 93% of the total compressive load carried. The compressive end-bearing capacity was assumed to be 7% of the total compressive load and the tensile end-bearing capacity was assumed to be 20% of the compressive end-bearing capacity i.e. 1.4% of the total compressive load carried.

The skin friction was calculated as the static ultimate load, excluding the load carried by the base, divided by the surface area of the pile in contact with the soil (excluding the pile base). For the aluminium piles the average skin friction was 18.6 kPa, the compressive end-bearing capacity was 67.0 kPa and the tensile end-bearing capacity was 13.4 kPa. For the perspex piles the average skin friction was 15.0 kPa, the compressive end-bearing capacity was 54.2 kPa and the tensile end-bearing capacity was 10.8 kPa.

The ultimate load is plotted against bed number in Fig. 5.3, and undrained cohesion in Fig. 5.4 to give some indication of the variability of the results. The ultimate load varied from 396 N to 586 N for the aluminium piles and from 299 N to 420 N for the perspex piles. For any particular bed of clay the range did not generally exceed 60 N which indicates that the clay beds were fairly homogeneous. No results are shown for clay Bed Number 1 or for clay Bed Number 4. Results from Bed Number 1 are unreliable because initially problems were encountered with the pile slipping and also with the pile being crooked; clay Bed Number 4 contained only pile group tests i.e. no single pile tests were performed. Fig. 5.4 indicates that there is a roughly linear correlation between ultimate load and undrained cohesion.
5.3.2 Load Settlement Behaviour

The soil modulus, $E_s$, was backfigured from the experimental results. The initial part of the load-settlement curves for tests on five different single piles was plotted. All the curves were very close and an average load-settlement curve was constructed from these curves. A tangent was fitted to the initial elastic part of this curve and for this tangent the value of settlement/load, $S/P$, was calculated to be 0.081 mm/kN. Theoretical results were produced for a single pile under static loading using different values of $E_s$. For each value of $E_s$, the value of $S/P$ at the load level $P = 0.2P_u$ was calculated. At this load level the theory indicated that the pile was behaving elastically. Fig. 5.5 shows a plot of $E_s$ versus $S/P$ calculated from the theory. The dashed line shows the experimental value and so the intersection of the two curves gives an experimental value of $E_s = 10$ MPa.

The same backfiguring process was adopted for the perspex pile and the experimental value was found to be $E_s = 3$ MPa. The values for the two piles should have been the same and the reason for the significant difference in $E_s$ between the two pile types is not clear.

Typical load-settlement curves for the two piles are plotted in Fig. 5.6 together with the corresponding two theoretical curves calculated using the above soil moduli. The shapes of the theoretical load-settlement curves were quite different from the experimental curves. The latter showed a very small elastic region up to about 10% of the ultimate load, then a gradual departure from linearity up to 93% of the ultimate load as the clay began to slip and then a definite point of failure. The theoretical curve showed a much more elastic response with elastic behaviour up to about 90% of the ultimate load and then a marked nonlinearity to failure as the elements slipped.

In contrast to the aluminium pile tests, all the perspex pile static tests had a definite point in the load-settlement curves where the pile slipped and any further load was carried as end-bearing. This point generally occurred at 90% of the ultimate load when the deflection was about 0.3 mm. The curve tended to plateau until the deflection was 0.35 mm and then it began to rise again.
5.3.3 Load Distribution

The measured load distribution down the pile from five static load tests has been plotted and compared to that obtained from theory. The load distributions are plotted in Fig. 5.7 for load levels of 40%, 70% and 90% of the ultimate load.

All the measured load distribution curves show excellent agreement with the theory. The experimental values at all four depths are generally within 10% of those predicted. The load distribution down the pile is almost perfectly linear at all three load levels indicating that the load is carried evenly down the pile up to failure.

5.3.4 Pile Head Stiffness

The pile head stiffness for the single pile tests was defined to be

\[ \frac{P_u/2}{\delta(atP_u/2)} \]

where \( P_u \) is the static ultimate load and \( \delta \) is the pile settlement at a particular load. The pile head stiffnesses ranged from 4.23 kN/mm to 7.36 kN/mm for the aluminium piles and from 1.2 kN/mm to 1.95 kN/mm for the perspex piles. This large range in stiffness was in spite of the fact that there was only a relatively small variation in stiffness in the undrained cohesion between the various test beds.

Pile head stiffness versus bed number is plotted in Fig. 5.8 and shows that the range in stiffness for a particular bed is smaller than the total range. It was also noted that the pile head stiffness was independent of the order in which tests were carried out in a particular bed of clay.

As expected the stiffness of the perspex piles was considerably less than that of the aluminium piles because perspex is about 20 times more compressible than aluminium.

The fairly close range of values of ultimate load indicates that the variability in the pile head stiffnesses is unlikely to be due to variability in the soil.
It could be due to misalignment of the pile and the loading machine i.e. the loading machine may not have been directly vertically above the pile, or the variable time delay between inserting the pile and loading. Most of the static tests were carried out 24 hours after the pile was inserted but some were carried out up to three days after inserting the pile. The limited accuracy of pile head deflection measurement could also have contributed to the pile head stiffness variability.

5.4 CYCLIC LOADING

Many earlier studies of cyclic pile behaviour have dealt with the degradation of pile capacity under displacement-controlled type cyclic loading. In this study attempts were made to relate the loss of pile capacity after cycling to the normalised cyclic loads.

The main objective of the cyclic loading tests was to examine the degradation characteristics of the soil and to determine how reliably degradation of pile capacity under cyclic loading could be predicted using the theory developed in Chapter 2. The degradation was thought to be dependent on the mean load, $P_o$, and the cyclic load, $P_c$. A typical set of load-settlement data is shown in Fig. 5.9. It shows the initial static test, in which the pile is loaded in compression to determine ultimate load, the ten load cycles and finally the loading to failure after cycling. In this particular test there was very little skin friction degradation.

The loss of pile capacity was characterised via the skin friction degradation factor, introduced in section 2.6, which was defined here as:

$$D_r = \frac{f_s\text{(post-cyclic)}}{f_s\text{(static)}}$$

where

- $f_s\text{(static)}$ = static skin friction which is calculated as the initial static ultimate load excluding the load carried by the base as determined in section 5.3.1, divided by the surface area of the pile in contact with the soil excluding the pile
base.

\( f_s(\text{post-cyclic}) \) post-cyclic skin friction which is calculated as the ultimate load after cyclic loading excluding that carried by the base, divided by the contact surface area excluding the base.

Because the two contact surface areas are the same, and the total load carried excluding the base load is assumed to be 93% of the respective ultimate load for both cases, it is more convenient to calculate \( D_r \) as

\[
D_r = \frac{P_{sc}}{P_u}
\]

A total of 52 cyclic load tests were carried out on the single piles with a wide range of values of mean load and cyclic load. Table 5.1 provides a summary of all the cyclic tests carried out. Table 5.1(a) and Table 5.1(b) give results for the aluminium pile tests while Table 5.1(c) gives results for the perspex pile tests.

5.4.1 Skin Friction Degradation - Small Vessel Tests

The small vessel tests were carried out as an independent method of obtaining the soil-pile parameters for cyclic loading. In particular, a relationship was sought between the skin friction degradation factor, \( D_r \), and the normalised cyclic deflection, \( \delta_c/d \). The unique design of the apparatus used ensured that all the degradation was due to skin friction.

Ten tests were carried out in the small vessel at cyclic deflections ranging from 0.1 mm to 4 mm. A test plot in which the cyclic deflection was 4 mm is shown in Fig. 5.10. In this particular test, in which the pile was subjected to large cyclic deflections, all of the degradation occurred within the first ten cycles. In fact, during the following 90 cycles the capacity of the pile actually increases slightly. However, in the remainder of the tests, as the cyclic deflection decreased there was less degradation in the first ten cycles and more in the following 90 cycles.
The results are summarised in Fig. 5.11 in which the skin friction degradation factor is plotted against normalised cyclic deflection. There appears to be a critical displacement, before which there is negligible skin friction degradation and after which the degradation increases markedly. This critical displacement is termed the cyclic slip displacement, \( \rho_{cs} \). In this test series \( \rho_{cs}/D = 0.01 \). In tests in which the cyclic deflection exceeded this value, as the cyclic deflection increased, so too did the skin friction degradation for 100 cycles.

The Matlock and Foo degradation parameters, as discussed in section 2.6, were calculated for a number of the tests. The minimum degradation factor, \( D_{\text{min}} \), was calculated from the load capacity after 100 cycles as this was assumed to be close to the minimum value. The skin friction degradation factor can be read from Fig. 5.11 for any value of cyclic deflection. This factor decreased as the cyclic deflection increased. The degradation rate parameter, \( \lambda \), was calculated using the relationship given in equation 2.19

\[
D_r = (1 - \lambda)(D' - D_{\text{min}}) + D_{\text{min}}
\]

(where \( D' = \) degradation factor at previous cycle). \( \lambda \) tended to decrease as the cyclic deflection increased i.e. at low cyclic deflections where the minimum degradation factor was small, only a few load cycles were needed to reach this value but at higher cyclic deflections where the minimum degradation factor was larger, more and more load cycles were needed to reach this value.

A plot of the skin friction degradation factor against the number of cycles is shown in Fig. 5.12 for two of the tests. The Matlock and Foo model fits the experimental curves fairly well. A summary of the tests, giving the Matlock and Foo parameters, is shown in Table 5.2. This Table indicates that a rough estimate of the degradation parameters can be obtained for any particular cyclic deflection.

A comparison with results from the large vessel tests reveals that most of those tests in which \( \delta_c \leq 0.25 \text{ mm} \), (i.e. \( \delta_c/d \leq 0.011 \)) did not fail during the first ten cycles and most tests in which \( \delta_c/d > 0.011 \) did fail. The point where \( \delta_c/d = 0.011 \) is the point where degradation began to occur in the small vessel tests and so the two test series are in excellent agreement.
For this analysis cyclic stiffness was defined to be

\[ K_c = \frac{P_u/2}{\delta_{P=P_u/2} - \delta_{P=0}} \]

where

\[ \delta_{P=0} = \text{deflection of the pile when the load on the pile is zero} \]
\[ \delta_{P=P_u/2} = \text{deflection of the pile when the load on the pile is half the static ultimate load in compression.} \]

The cyclic stiffness was examined for the small vessel tests to see if there was any change as the number of cycles increased. It was found that there is little change in cyclic stiffness in the first ten cycles and, in fact, there is only a small decrease in cyclic stiffness in the first 100 cycles.

5.4.2 Interaction Between Cyclic Load and Mean Load

A summary of the tests carried out is presented in an ‘interaction diagram’ shown in Fig. 5.13. The interaction diagram relates the skin friction degradation factor to the cyclic and mean loads which are normalised with respect to the reference static capacity, \( P_u \), from the respective initial static tests. The tests which failed are clearly shown and for those tests which did not fail the skin friction degradation factors after ten cycles are given.

In obtaining the theoretical solutions for comparison with the experimental data, the degradation model proposed by Matlock and Foo, discussed in section 2.6, required two input parameters; the minimum degradation factor, \( D_{\text{min}} \), and the degradation rate parameter, \( \lambda \). These two parameters were obtained independently from the results of a series of tests in which the pile was repeatedly loaded to failure in compression and then loaded to failure in tension for up to 30 cycles. From the results the skin friction degradation factor, \( D_r \), was calculated at each cycle and so a plot of \( D_r \) versus \( N \), where \( N \) is the number of load cycles, was made and is shown in Fig. 5.14.

The degradation parameters \( \lambda \) and \( D_{\text{min}} \) were obtained from this plot using equation 2.19. This model fitted the experimental points reasonably well.
However the experimental points degraded more severely in the first few cycles than the model and then the rate of degradation decreased in the experimental results as compared to the model as can be seen in Fig. 5.14. The degradation parameters which best fit the experimental results from the large test vessel for which $\delta_c/d = 0.05$ were the degradation rate parameter $\lambda = 0.3$ and the minimum degradation factor $D_{\text{min}} = 0.69$. These parameters were used in all the theoretical calculations for comparison with the experimental results.

These values can be compared with the parameters obtained from the small vessel tests for the different cyclic deflections shown in Table 5.2. It can be noted that tests involving large cyclic deflections up to $\delta_c/d = 0.2$ resulted in a $D_{\text{min}}$ as low as 0.465 after 100 load cycles. The value of the degradation factor, $\lambda$, which would have been predicted for a cyclic deflection of $\delta_c = 1.25$ mm is $\lambda = 0.1$ and this is lower than the calculated value. The calculated value of the minimum degradation factor is also lower than the value predicted from the small vessel tests of $D_{\text{min}} = 0.85$. This indicated that degradation in the large test vessel is more severe than that in the small vessel and most of the degradation occurs in the first five cycles in the large vessel compared to the first 30 cycles in the small vessel. This could be due to the close proximity of the side restraint in the small vessel which would reduce the deflections by confining the soil and hence reduce the degradation.

Using the parameters $\lambda = 0.3$ and $D_{\text{min}} = 0.69$ the failure envelope was calculated using program TAPCYC, as discussed in section 2.8, and is plotted with the experimental results in Fig. 5.13. Also plotted on the graph is the limiting theoretical relationship between cyclic and mean load for no degradation. Tests represented by points which fall below this line are predicted to have no degradation.

In the analysis, the skin friction contributed 88% to the static ultimate load, and the end-bearing resistance in compression contributed the remaining 12%. The base resistance in tension was assumed to provide 2% of the ultimate load and so the ultimate tensile load was 90% of the ultimate load in compression.

Fig. 5.13 shows that the theoretical prediction of the failure envelope shows
excellent agreement with the test results. In fact, in all of the tests in which the pile failed, failure was predicted. The tests in which the mean load is compressive appear to be more predictable than for a tensile mean load. The tests in which the mean load is tensile all withstood higher cyclic loads than predicted. This could be because the tensile capacity of the pile is underestimated. Of the tests which fall below the limiting theoretical relationship between cyclic and mean load for no degradation, most suffered a maximum of 10% degradation.

Fig. 5.15 shows a similar plot for the perspex pile. The theoretical failure envelope curve, which was calculated using the soil modulus found from static results for the perspex pile, is very close to the curve for the aluminium piles although the line of no degradation is higher. The agreement with the test results for the perspex pile is excellent, even when the mean load is tensile.

5.4.3 Effect of Cyclic Load on Pile Load Capacity

During the tests, the cyclic load emerged as a possible dominating factor in predicting degradation. To investigate this possibility a plot of skin friction degradation factor versus normalised cyclic load is shown in Fig. 5.16. The theoretical failure curve, calculated from the theory in Chapter 2 assuming \( P_o = 0 \) is shown as a dashed line. This theoretical failure curve follows the same path for all values of \( P_o \), however the failure point (marked by an ‘X’ in Fig. 5.16) changes for different \( P_o \). The failure point for a particular value of \( P_o \) occurs at the lowest value of \( P_c \) for which catastrophic failure occurs. The experimental points represent a wide range of values of \( P_o \).

The test results show fairly good agreement with theory. The theoretical failure curve follows the somewhat scattered experimental points reasonably well although the degradation is slightly underestimated in the region \( 0.5 \leq P_c/P_u \leq 0.7 \). The theoretical failure point predicts the experimental test failures very well. It occurs at just the point on the failure curve when most of the tests are failing. The diagonal line, representing \( D_r = P_c/P_u \), represents the boundary between failure during cycling and either non-failure or failure due to accumulated deflection. Any test points which fall to the right of
this boundary indicate that the pile failed during cycling. The tests in which failure occurred and which fall to the left of the diagonal line failed due to accumulated deflection.

A trend which emerged from Fig. 5.16 for the particular pile considered in kaolinite is that, up to a certain value of cyclic load, there is no degradation at all. This value is the theoretical onset of degradation and occurs for these tests at $P_c/P_u = 0.59$. After this load the degradation increases as the cyclic load increases up to a critical value of cyclic load $P_c/P_u = 0.78$; beyond this value all tests fail during cycling. It is not known if this conclusion is valid for soils which may be more susceptible to degradation or to different conditions of relative pile-soil stiffness and pile length.

5.4.4 Effect of Mean Load on Pile Load Capacity

Fig. 5.17 is a plot of skin friction degradation factor, $D_r$, versus normalised mean load, $P_0/P_u$, for a particular value of cyclic load, $P_c/P_u = \pm 0.7$. Fig.5.17(a), for the aluminium pile, shows good agreement with the theoretical line although the tests which had a tensile mean load did not fail as predicted. These results seem to be independent of mean load. Fig. 5.17(b), for the perspex pile, again shows good agreement with the theoretical line and for this pile the tests which were predicted to fail did so. All in all, these tests confirmed that the amount of degradation of skin friction was more or less independent of mean load.

5.4.5 Load Distribution

In some of the tests which suffered catastrophic failure the load-settlement curve developed a characteristic peak-residual shape. In those cases the load peaked and began to decrease before it reached the cyclic load. In some tests the loss in capacity after ten cycles was as much as 15% of the peak load. This is shown in Fig. 5.18 for test S11C. This pile failed catastrophically at cycle 3 when it developed a peak-residual shape. In each subsequent cycle the peak load decreased, the loss in capacity after reaching the peak load increased and the cyclic deflection required for the pile to reach the cyclic load increased.

This plot also shows quite clearly the load carried by the pile base in tension. When the pile reached its tensile capacity (shown as negative load in Fig. 5.18)
a gap formed between the pile base and the clay which was in contact with it and the plot shows a corresponding loss of tensile capacity. This loss in capacity decreased by the same value in every cycle and so this value corresponds to the tensile base capacity of the pile.

Load distribution curves for this test are plotted in Fig. 5.19. They illustrate that in the first few cycles before the onset of failure the skin friction along the lower third of the pile reduced at every cycle and the base end-bearing recovered the lost capacity so that there was virtually no net loss in capacity. After failure, when the pile exhibited a peak-residual behaviour, the skin friction at peak load in the lower third of the pile had decreased dramatically to only 25% of its original value. Both the skin friction in the remainder of the pile and the end-bearing increased slightly but not by enough to reach the cyclic load. The load then decreased to its minimum value because the skin friction in the lower third of the pile continued to decrease, so much so that it became negative. The subsequent increase in the capacity of the pile was due entirely to an increase in end-bearing.

Load distribution curves at different cycles of test S12C, which failed at cycle 8, are plotted in Fig. 5.20. Also shown are the theoretical load distributions. They show that the behaviour down the pile is modelled well by the program TAPCYC. As the pile skin friction degrades, more of the load is transferred to end-bearing on the pile base until at cycle eight, the base load reaches the base load capacity, at which stage the pile is deemed to have failed.

5.4.6 Cyclic Stiffness

The small vessel tests, discussed in section 5.4.1, have illustrated that there is little change in cyclic stiffness in the first ten cycles and, in fact, there is only a small decrease in cyclic stiffness in the first 100 cycles.

Fig. 5.21 shows a plot of normalised cyclic stiffness versus normalised cyclic load for the large vessel tests. It indicates a trend towards a larger decrease in cyclic stiffness of the pile as the cyclic load increases. This is as expected because, as the cyclic load increases, increasing degradation occurs. For two-way
loading, for which piles can fail catastrophically, (i.e. the pile deflects by 5% of its diameter and is unable to attain the cyclic load) the stiffness decreases rapidly with increasing cyclic load.

5.4.7 Cyclic Displacement

The cyclic displacement amplitude, $\delta_c$, is plotted against normalised cyclic load amplitude, $P_c/P_u$, in Fig. 5.22(a). These data points cover both one-way and two-way tests. The small scatter in these points for $\delta_c < 0.3$ mm indicates that the cyclic displacement amplitude is almost uniquely related to the cyclic load amplitude, and is nearly independent of the mean load.

Fig. 5.22(b) shows a similar plot for the perspex pile, and this shows the same trends as the previous plot although the cyclic deflection in the tests which did not fail is approximately twice that of the aluminium piles at corresponding load levels and the cyclic displacement generally reaches 0.5 mm before failure begins.

In Fig. 5.23 cyclic displacement is plotted against skin friction degradation factor, $D_r$. It shows that in each test where the cyclic displacement exceeded 0.27 mm, failure occurred within ten cycles. It also confirms that, as the cyclic displacement amplitude increases, the skin friction degradation also increases.

5.4.8 Accumulated Deflection

Fig. 5.24 shows the accumulated displacements, $\delta_{acc}$, after ten cycles plotted against normalised mean load. There is clearly a trend towards increasing positive accumulated deflections as the mean load increases in compression and increasing negative accumulated deflections as the mean load increases in tension. The accumulated deflections also tend to increase as the cyclic load increases.

Attempts were made to predict the accumulated deflection by relating it to the maximum load
i.e. \( P_{\text{max}} = (P_c + P_o) \) if \( P_o \geq 0 \)
\( = (P_c - P_o) \) if \( P_o < 0 \)

Fig. 5.25 shows that there is very little accumulated deflection until \( P_{\text{max}} \) exceeds 80% of the ultimate load. It was difficult to find a relationship between the accumulated deflection and the maximum load. The curve

\[
\delta_{\text{acc}} = 0.01 \tan\left(\frac{\pi}{2} \frac{P_{\text{max}}}{P_u}\right)
\]

is plotted on the figure as this was found to give the best fit with the data points.

The accumulated deflection was also plotted against a quantity \( X \) in Fig. 5.26, where

\[
X = \frac{\sqrt{P_o P_{\text{max}}}}{P_u}
\]

as developed by Poulos (1987), and a rough linear relationship was found.

The tests which failed due to accumulated deflection generally involved one-way cyclic loading or non-symmetrical two-way loading while failure in the tests involving symmetrical two-way loading was governed by the cyclic load amplitude. The tests which failed in this way after only ten load cycles generally had a mean load \( P_o \leq 0.15 \ P_u \) and a cyclic load \( P_c \geq 0.80 \ P_u \).

5.4.9 Residual Loads

The residual loads were compared before and after cycling and the results are plotted in Fig. 5.27. They generally do not exceed 15% of the ultimate load. After ten cycles the residual loads in the top section of the pile increase slightly and those in the middle section of the pile decrease significantly. This demonstrates that the skin friction along the lower part of the pile has reduced due to cyclic loading.
5.5 SUMMARY

In this chapter the results of tests on different single piles subjected to both static and cyclic loading have been presented along with results using the theoretical model.

The main conclusions are

• in displacement controlled tests there is a critical displacement, the cyclic slip displacement, before which there is negligible skin friction degradation and after which the degradation increases markedly

• the Matlock and Foo model predicted the post-cyclic load capacity fairly well for both the aluminium and the perspex piles

• post-cyclic load capacity is independent of mean load

• post-cyclic load capacity is dependent on cyclic load. Up to a certain value of cyclic load there is no degradation at all; after this load the degradation increases as the cyclic load increases up to a critical value of cyclic load; beyond this value all tests fail during cyclic loading

• both static and cyclic load distributions can be predicted fairly accurately using the theory presented

• in the model tests, skin friction degradation starts at the base of the pile and works its way up the shaft (this was found to be a function of pile stiffness); during cycling the loss of capacity due to skin friction degradation is recovered through an increase in end-bearing capacity

• the cyclic displacement amplitude is almost uniquely related to the cyclic load amplitude under elastic loading conditions

• accumulated deflection increases as the cyclic load increases and as the mean load increases

• residual loads are relatively small and tend to decrease in the lower sections of the pile after cycling, indicating that the skin friction along the lower part of the pile reduces due to cyclic loading.
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<th>$P_c/P_u$</th>
<th>$D_r$</th>
<th>Failed?</th>
<th>$\delta_0$ (mm)</th>
<th>$\delta_{acc}$ (mm)</th>
<th>Pile Head Stiffness</th>
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Table 5.1(a) Summary of Test Results
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<th>$P_c/P_u$</th>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>S18C</td>
<td>0.37</td>
<td>0.60</td>
<td>0.96</td>
<td>yes</td>
<td>0.13</td>
<td></td>
<td>&gt; 0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>S19B</td>
<td>450</td>
<td>-0.50</td>
<td>0.38</td>
<td>0.95</td>
<td>no</td>
<td>0.04</td>
<td>-0.088</td>
<td>7.03</td>
<td>catastrophic failure</td>
</tr>
<tr>
<td>6</td>
<td>S19C</td>
<td>0.04</td>
<td>0.80</td>
<td>0.84</td>
<td>yes</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>S20B</td>
<td>438</td>
<td>0.08</td>
<td>0.71</td>
<td>0.93</td>
<td>0.17</td>
<td></td>
<td>0.02</td>
<td>6.64</td>
<td>N = 30 cycles</td>
</tr>
<tr>
<td>6</td>
<td>S20C</td>
<td>-0.30</td>
<td>0.69</td>
<td>0.86</td>
<td>no</td>
<td>0.14</td>
<td></td>
<td>-0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>SS1B</td>
<td>460</td>
<td>0.13</td>
<td>0.57</td>
<td>0.74</td>
<td>yes</td>
<td>0.40</td>
<td>-0.70</td>
<td>2.13</td>
<td>short pile</td>
</tr>
<tr>
<td>7</td>
<td>SS1C</td>
<td>0</td>
<td>$P_c$</td>
<td>0.65</td>
<td>yes</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>S21C</td>
<td>0</td>
<td>0.88</td>
<td></td>
<td>no</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1(b) Summary of Test Results
<table>
<thead>
<tr>
<th>Test Series</th>
<th>Test Number</th>
<th>$P_u$ (N)</th>
<th>$P_a/P_u$</th>
<th>$P_c/P_u$</th>
<th>$D_r$</th>
<th>Failed?</th>
<th>$\delta_c$ (mm)</th>
<th>$\delta_{acc}$ (mm)</th>
<th>Pile Head Stiffness</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>SP1B</td>
<td>407</td>
<td>0.01</td>
<td>0.73</td>
<td>0.87</td>
<td>no</td>
<td>0.50</td>
<td>-0.26</td>
<td>1.77</td>
<td>perspex pile</td>
</tr>
<tr>
<td>8</td>
<td>SP1C</td>
<td>0.12</td>
<td>0.75</td>
<td>0.97</td>
<td>no</td>
<td>0.45</td>
<td>-0.02</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP1D</td>
<td>0.06</td>
<td>0.73</td>
<td>0.88</td>
<td>no</td>
<td>0.46</td>
<td>-0.30</td>
<td>1.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP2B</td>
<td>388</td>
<td>0.45</td>
<td>0.50</td>
<td>1.0</td>
<td>no</td>
<td>0.26</td>
<td>0.07</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP2C</td>
<td>0.23</td>
<td>0.73</td>
<td>0.97</td>
<td>yes</td>
<td>1.0</td>
<td>2.8</td>
<td>1.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP3B</td>
<td>420</td>
<td>-0.26</td>
<td>0.54</td>
<td>0.84</td>
<td>no</td>
<td>0.30</td>
<td>-0.32</td>
<td>1.96</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP3C</td>
<td>-0.09</td>
<td>0.72</td>
<td>failed</td>
<td>yes</td>
<td>0.50</td>
<td>5.7</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP4B</td>
<td>299</td>
<td>0.06</td>
<td>0.79</td>
<td>1.0</td>
<td>no</td>
<td>0.35</td>
<td>0.02</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SP4C</td>
<td>0.07</td>
<td>0.88</td>
<td>1.02</td>
<td>yes</td>
<td>0.41</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1(c) Summary of Test Results for Perspex Pile
<table>
<thead>
<tr>
<th>Test Number</th>
<th>( \delta_c ) (mm)</th>
<th>( \delta_c/d )</th>
<th>( D_r )</th>
<th>( \lambda )</th>
<th>( D_{min} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>±0.1</td>
<td>0.005</td>
<td>0.903</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>±0.2</td>
<td>0.01</td>
<td>0.996</td>
<td>0.5</td>
<td>0.995</td>
</tr>
<tr>
<td>3</td>
<td>±0.4</td>
<td>0.02</td>
<td>0.968</td>
<td>0.5</td>
<td>0.97</td>
</tr>
<tr>
<td>7</td>
<td>±0.6</td>
<td>0.03</td>
<td>0.857</td>
<td>0.1</td>
<td>0.86</td>
</tr>
<tr>
<td>8</td>
<td>±1.0</td>
<td>0.05</td>
<td>0.913</td>
<td>0.15</td>
<td>0.91</td>
</tr>
<tr>
<td>1</td>
<td>±1.0</td>
<td>0.05</td>
<td>0.821</td>
<td>0.075</td>
<td>0.82</td>
</tr>
<tr>
<td>5</td>
<td>±2.0</td>
<td>0.1</td>
<td>0.682</td>
<td>0.06</td>
<td>0.68</td>
</tr>
<tr>
<td>6</td>
<td>±4.0</td>
<td>0.2</td>
<td>0.467</td>
<td>0.4</td>
<td>0.465</td>
</tr>
<tr>
<td>experimental (large vessel)</td>
<td>±1.25</td>
<td>0.05</td>
<td>0.3</td>
<td></td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 5.2  Comparison between Matlock and Foo parameters calculated from results of small vessel tests and those calculated from experimental results from the large vessel
Figure 5.1 Typical Load Distributions During Installation
Figure 5.2 Penetration Depth versus Load in each Strain Gauge
Figure 5.3  Variability of Ultimate Load in Clay Beds
Figure 5.4 Correlation Between Ultimate Load and Undrained Cohesion
Figure 5.5  Backfiguring the Soil Modulus
Figure 5.6  Comparison of Theoretical and Experimental Load-Settlement Curves
Figure 5.7  Measured and Calculated Load Distributions For Static Loading
Figure 5.8  Variability of Pile Head Stiffness in Clay Beds
Figure 5.9  A Typical Load-Settlement Curve
Figure 5.11  Skin Friction Degradation Factor for Model Piles in Small Test Vessel
Figure 5.12  Skin Friction Degradation Curves for Two Small Vessel Tests
Figure 5.13 Interaction Diagram for Aluminium Pile

- Test did not fail after 10 cycles
- Indicates test failed during cycling
Note: Numbers indicate value of $D_r$ after 10 cycles
Figure 5.14  Comparison of Experimental and Theoretical Skin Friction Degradation Factors
Figure 5.15 Interaction Diagram for Perspex Pile

- Test did not fail after 10 cycles
- × Indicates test failed during cycling

Note: Numbers indicate value of $D_T$ after 10 cycles
Figure 5.16 Effect of Cyclic Load on Skin Friction Degradation
Figure 5.17  Effect of Mean Load on Degradation Factor
Figure 5.18 Load-Settlement Curve Showing Failure During Cycling
Figure 5.19 Load Distributions for Test S11C
Figure 5.20  Load distributions for Test S12C which Failed at Cycle 8

Note: There is no theoretical curve drawn for N=8 because failure occurred before cycle 8.
Figure 5.21  Effect of Cyclic Load on Cyclic Stiffness
Figure 5.22  Effect of Cyclic Load on Cyclic Displacement
Figure 5.23 Effect of Cyclic Displacement on Skin Friction Degradation Factor
Figure 5.24  Accumulated Deflection for Single Pile
Figure 5.25  Accumulated Deflection versus Maximum Load
Showing Proposed Theoretical Relationship

\[ \delta_{acc} = 0.01 \tan \left( \frac{\pi}{2} \frac{P_{max}}{P_u} \right) \]
Figure 5.26  Relationship Between Accumulated Deflection and $X$

$$X = \sqrt{P_0 \frac{P_{\text{max}}}{2}}$$

$$P_u$$
Figure 5.27 Residual Load Distributions
PILE GROUP

TEST RESULTS

6.1 INTRODUCTION

Pile group tests were carried out in the large test vessel on a number of different pile groups, as described in Chapter 4. The number of tests carried out on each particular group is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Static tests</th>
<th>Cyclic tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-pile group</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>8-pile group</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4-pile unequal-length group</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2-pile unequal-length group</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

In this chapter results for both static and cyclic loading are presented and compared to both the single pile results and the theoretical analyses. Consideration is given to...
(i) load-settlement behaviour
(ii) settlement ratio
(iii) load distribution
(iv) pile head stiffness
(v) interaction between cyclic load and mean load
(vi) effect of cyclic load on pile load capacity
(vii) effect of mean load on pile load capacity
(viii) cyclic stiffness
(ix) cyclic displacement
(x) accumulated deflection
(xi) residual loads.

A schedule of the tests carried out is given in Table 6.1.

The theoretical solutions presented in this chapter were obtained using a number of different programs. The static analysis of the 4-pile group was carried out using program TAPFIN; the static analyses of the remainder of the groups, all of which were not symmetrical, were carried out using program GAPFIX. The cyclic analysis of the 4-pile group was carried out using program TAPCYC, and the cyclic analyses of the remainder of the pile groups were carried out using program GACPYC. It was more efficient to use four programs for the analyses, each better suited to the particular problem considered, than to use one large multi-purpose program. All of these programs are described in detail in Chapter 2.

The parameters adopted for the theoretical analysis are the same as those used for the single pile analysis, discussed in section 5.3. Ten shaft elements were used to represent the pile and the soil was assumed to have a uniform skin friction. The value of soil modulus adopted of $E_s = 10$ MPa was the same value chosen for the single pile tests.
6.2 STATIC LOADING

6.2.1 Load versus Settlement

Typical load-settlement responses for the four different pile groups tested are shown in Fig. 6.1. They show that as the number of piles increased, the slope of the initial portion of the curves decreased thus indicating the presence of group interaction. Also plotted in Fig. 6.1 are the respective theoretical load-settlement curves assuming $E_s = 10$ MPa. The theory fits less well as the group size increases because the load-settlement response becomes more nonlinear. The fit for the 2-pile and the 4-pile groups is fairly good however for the 8-pile group a smaller value of $E_s$ would probably give a better fit.

The average static ultimate load for the 4-pile groups was 1.86 kN and the standard deviation (S.D.) was 0.42 kN. This can be compared to the average value for single piles of 0.470 kN. The efficiency factor, $\eta$, is defined as

$$\eta = \frac{\text{ultimate load capacity of group}}{\text{sum of ultimate load capacities of individual piles}}$$

For the 4-pile group $\eta = 1.86/(4 \times 0.470) = 0.99$, so that grouping has virtually no effect on load capacity at this spacing.

6.2.2 Settlement Ratio

The settlement ratio was defined as

$$R_s = \frac{\text{settlement of pile group}}{\text{settlement of single pile carrying same average load as a pile in the group}}$$

For the unequal-length pile groups the settlement of a single pile carrying the same average load as a pile in the group was calculated as the average settlement of a long single pile and a short single pile, each carrying the same average load as a pile in the group. The settlement ratio results for the four groups tested are shown in Fig. 6.2 where the load level is the actual load on the group divided by the static ultimate load of the group. The theory is only valid for normal working loads up to about 50% of ultimate, within which the group is still behaving reasonably elastically; for higher load levels, when slip occurs, the assumptions made in the group analysis may be inaccurate.
Overall the agreement between theory and experiment is not good. As the size of the group increases the settlement ratio calculated from theory tends to underestimate the experimental value. Also, as the load level increases the theoretical settlement ratio remains almost constant while the experimental value increases. This occurs because the theoretical response remains elastic up until a load level of at least 0.8 of ultimate whereas the experimental response for the single pile remains elastic up until a load level of only 0.15 of ultimate. As the size of the pile group increases the maximum load level at which the group response is elastic decreases, and this results in an increase in settlement ratio.

The agreement between experiment and theory for the 2-pile and the 4-pile groups is better because for both these groups and the single pile the measured load-settlement curves depart from theory at about the same rate and so the settlement ratio is not very different from that predicted. However, for the 8-pile group the measured settlement is so much greater than that predicted, even under very small loads, that the settlement ratio is also much greater than that predicted. Better agreement could be obtained by using a smaller value of $E_s$ and perhaps also by using a non-linear analysis. It can be seen from Fig. 6.2 that the experimental load-settlement curve is quite non-linear and the theoretical curve using $E_s = 10$ MPa does not fit it at all well.

6.2.3 Load Distribution

The measured load distribution down the pile has been plotted and compared to that obtained from theory for all the different pile groups. The load distributions are plotted in Figs. 6.3 to 6.6 for load levels of 40%, 70% and 90% of the static ultimate load of the group.

Results for the 4-pile group show excellent agreement between experiment and theory. The experimental values are all within 10% of those predicted. Similarly the theoretical results for the 8-pile group give good agreement with the measurements in the early stages of loading. In the latter stages, the agreement is quite good although the experimental results indicate that the
pil carries most of its load in the lower two thirds of the pile and very little in the upper third, particularly for the mid-side piles. Therefore the measured load in the middle section of the piles is underestimated by the theory by up to 20%.

The load distribution in the unequal-length groups cannot be predicted as closely. The theoretical curves for both the 4-pile unequal-length groups and the 2-pile unequal-length groups are almost identical; however they do not match the experimental points very closely. The main difference is that the short piles carry about 30% more load than predicted by the theory. In fact in many of the tests the short piles carried just under half of the total load on the group while the long piles carried just over half. The long piles were predicted to carry 65% of the total load on the group under elastic conditions, decreasing to 60% when the group was loaded to a load level which was close to failure. These values seem quite reasonable and the reason for the discrepancy with the measured loads is not clear.

The experimental loads decreased uniformly with depth for both the long and the short piles in both groups although slightly less load is carried in the upper section of both piles. In the long piles, the theory predicts that only 30% of the load would be carried in the upper half of the piles, most of the load being carried below the level of the tip of the short piles. Therefore the loads in the long piles were overestimated by the theory by an average of 32%.

6.2.4 Pile Head Stiffness

The average Pile Head Stiffness per pile for the pile group tests was defined to be

\[
\left( \frac{P_u/2}{\delta(\text{at } P_u/2)} \right) / N
\]

where

- \( P_u \) = static ultimate load of the group
- \( \delta \) = pile group settlement at a particular load
- \( N \) = number of piles in the group.
The pile head stiffness was measured during the initial static loading test and so was independent of the cyclic load level of the test.

The pile head stiffnesses for the different pile groups are shown in Fig. 6.7. As can be seen from the figure there is a wide range in pile head stiffness in each of the groups and this range is due to the variability between the tests. It is not due to variability between the clay beds because within each clay bed there is a large range of stiffnesses. Also seen from the figure is the decrease in pile head stiffness as the number of piles in the group increased. This was expected because the pile group settlement increased as the size of the group increased and pile head stiffness is inversely proportional to settlement.

6.3 CYCLIC LOADING

Cyclic loading tests were carried out to determine how reliably degradation of pile group capacity under cyclic loading could be predicted using the theory developed in Chapter 2. In all, 30 cyclic load tests were carried out on different groups. Tables 6.1(a) and 6.1(b) provide a summary of all the cyclic load tests carried out.

In the presentation of the cyclic test results, the theoretical solutions are also plotted to determine how well the theory incorporating the Matlock and Foo degradation model could predict the cyclic response of the groups.

The same degradation model was used for the pile groups as for the single piles. The single pile degradation data of $\lambda = 0.30$ and $D_{min} = 0.69$ was used to predict the group response for all the different pile groups. The theoretical degradation curve using single pile data is shown with the experimental curve for the 4-pile group in Fig. 6.8. As expected, the experimental curve shows more degradation than the theoretical curve because degradation of the pile group is more severe than single pile degradation due to pile group interaction.
6.3.1 Interaction Between Mean Load and Cyclic Load

A summary of the tests carried out on each particular group is presented in an interaction diagram. The interaction diagram is defined in section 5.4.2. The theoretical failure envelope, using the Matlock and Foo degradation parameters, is also plotted on each diagram as is the limiting theoretical relationship between cyclic and mean load for no degradation. Tests represented by points which fall below this line are predicted to suffer no degradation. The results for the 4-pile group are shown in Fig. 6.9, for the 8-pile group in Fig. 6.10, and for the 4-pile unequal-length group in Fig. 6.11.

Results for the 4-pile group show excellent agreement between the theoretical prediction and the experimental results. The outcome of every test is close to the predicted outcome. Of the tests which fall below the theoretical line of no degradation, only one suffers slight degradation. It is also noted that there is no noticeable variation in the results between the three clay test beds used for the 4-pile group tests. Unlike the single pile tests, the 4-pile group tests appear to be equally predictable whether the mean load is compressive or tensile. As expected, the failure envelope for the 4-pile group falls inside that of the single pile and generally the 4-pile group fails at slightly lower relative cyclic loads than the single pile, indicating that interaction has some effect on the cyclic response of piles.

Results for the 8-pile group, in Fig. 6.10, again show excellent agreement with the theoretical prediction. All four tests behaved as predicted; the two tests which fall outside the theoretical failure curve failed and the two tests which fall inside the theoretical failure curve did not fail. As expected, both the failure envelope and the line of no degradation for the 8-pile group were lower than those of the 4-pile group.

The relatively few results for the 4-pile unequal-length groups (Fig. 6.11) show good agreement with the theoretical prediction. It should be noted that there are two features of the theoretical curve which stand out because they are markedly different from the curves for the single pile tests and for the other group tests. Firstly, the limiting theoretical relationship between cyclic
and mean load for no degradation is much lower. It occurs at a value of \( P_c/P_u = 0.28 \) for the 4-pile unequal-length group compared to \( P_c/P_u = 0.48 \) for the 8-pile group which was the lowest value for the symmetric groups. This means that tests in the range \( 0.28 \leq P_c/P_u \leq 0.48 \) which are predicted to have no degradation for single pile tests and symmetrical pile group tests are, in fact, predicted to degrade for the 4-pile unequal-length group tests. This is because all four piles do not degrade evenly. The shaft elements at the bottom of the long pile begin to slip at much lower loads than those which would cause slip in the short piles. In all other symmetric pile groups, all the piles in the group begin to slip at approximately the same load.

The second feature is the presence of two peaks in the failure envelope, one at a tensile mean load of \( P_0/P_u = -0.10 \) and the other at a compressive mean load of \( P_0/P_u = 0.10 \). For the other pile groups and the single pile, there is only one such peak in the envelope, at a compressive mean load of \( P_0/P_u = 0.10 \). One of the peaks represents the load combination at which the long piles were failing and the other represents the load combination at which the short piles were failing.

Overall the failure envelope for the 4-pile unequal-length group was approximately 0.2 \( P_u \) below that of the symmetrical 4-pile group.

It was noted that the limiting theoretical relationship between cyclic and mean load for no degradation, as described in section 5.4.2, could be predicted from the theoretical static load results for each respective group. It occurred at approximately the same load which caused the first element to slip under static loading. As expected, the single pile carried the highest load level of 0.66 \( P_u \) before the first element slipped, followed by the 4-pile group at 0.52 \( P_u \) and the 8-pile group at 0.49 \( P_u \). The 4-pile unequal-length group carried the smallest load level of only 0.33 \( P_u \) before elements of the long pile began to slip.

### 6.3.2 Effect of Cyclic Load on Pile Load Capacity

The effect of cyclic load on pile load capacity is shown in Fig. 6.12 for the 4-pile group. In this figure, the skin friction degradation factor, \( D_r \), defined in
section 5.4, is plotted against the normalised cyclic load, $P_c$. The theoretical line, shown as a dashed line, gives an excellent line of best fit between the somewhat scattered experimental points. The points which fall to the right of the diagonal line indicate that the pile failed during cycling. These points also lie beyond the theoretical 'failure point' marked by an 'X' in the figure. It can be noted that all the experimental points which fall below the theoretical onset of degradation i.e. $P_c/P_u \leq 0.565$, did not fail during the cyclic loading test. A clear trend emerging from Fig. 6.12 is that, up to a certain value of cyclic load, there is no degradation at all, after which the degradation increases as the cyclic load increases up to a critical value of cyclic load $P_c/P_u = 0.75$, beyond this value all tests fail during cycling. This trend is consistent with the theoretical analysis.

6.3.3 Effect of Mean Load on Pile Load Capacity

Fig. 6.13 plots the post-cyclic load capacity against the normalised mean load for the 4-pile group. For each of the three values of cyclic load considered the experimental value of skin friction degradation is very close to the predicted theoretical value, and furthermore, the experimental value of post-cyclic load capacity is independent of the mean load, as is predicted by the theory. This finding agrees with the single pile results in which the post-cyclic load capacity was also found to be independent of the mean load. This is true of the experimental results regardless of whether or not the test failed during cycling. The two end points on the theoretical curves mark the theoretical failure points beyond which failure is predicted to occur. All points in Fig. 6.13 indicating tests in which failure occurred lie either beyond the failure points or very close to them, and all these failures occurred due to accumulated deflection.

6.3.4 Load Distribution

Load distributions for typical tests for the three different pile groups are plotted in Figs. 6.14 to 6.18. The load distributions during the first load cycle and also during the tenth load cycle are plotted so that any redistribution
of load down the pile during cycling can be traced. The load distribution at ultimate load, both before and after cycling, is also plotted, so that any loss in pile capacity after cycling can be traced to either skin friction degradation along particular sections of the pile or to degradation of end-bearing capacity at the pile base.

Fig. 6.14 shows the load distributions for test G9B on the 4-pile group. This particular test did not fail during cycling. The plot shows that there is no indication of degradation during cycling which is in complete agreement with the theoretical prediction. However, on loading to failure the load capacity after cycling is reduced by 26%. This loss in capacity is the result of a loss in skin friction which occurs uniformly down the pile. It can be seen that the ultimate load after cycling has decreased down almost to the value of the cyclic load and so it can be assumed that if the cyclic load was slightly larger then the group would have failed. This agrees with the theoretical failure envelope. As it stands, the test point at $P_0/P_u = -0.01$ and $P_c/P_u = 0.73$ lies just inside the failure envelope in Fig. 6.9 and the test was predicted not to fail. However, if the cyclic load had been increased to $P_c/P_u = 0.75$ it would then lie just outside the failure envelope and would therefore be predicted to fail.

Fig. 6.15 shows the load distributions for test G8C on the 4-pile group which failed during cycling. The plot shows that during cycling there was some loss in skin friction in the lower half of the pile and a corresponding increase in base end-bearing to recover the lost capacity and maintain the cyclic load. In this test $(P_0 + P_c)/P_u = 0.90$ and so only a small amount of degradation was required to cause failure. This degradation in skin friction can be seen in the plot of ultimate load after cycling and was fairly uniform along the pile.

The pattern of degradation for the 8-pile group was slightly different to that of the 4-pile group. The load distribution curves for test BG2B on the 8-pile group are shown in Fig. 6.16. This test did not fail during cycling. The plot shows that again there was virtually no degradation during cycling, as found for the 4-pile group. However the plot of ultimate load after cycling shows severe skin friction degradation along the lower half of the pile. The load carried by
the upper third of the pile doubled after cycling, the middle third of the pile halved the load that it was carrying, and the load carried by the lower third of the pile decreased to almost zero. The load carried in end-bearing by the base remained constant. This degradation was much more severe than any found in the 4-pile group tests. This pattern of degradation agrees with the theoretical results. In all of the load cases analysed for the 8-pile group the skin friction degradation starts to occur at the base of the pile and work its way up the shaft as either the load or the number of cycles increase.

In this figure results are plotted for the corner pile in the 8-pile group and it was found that this pile carried approximately the same load throughout the load cycling i.e. the load did not tend to be redistributed between the corner piles and the mid-side piles as cycling progressed. The theoretical curves plotted in Fig. 6.17 indicate that initially the corner piles carry more load than the mid-side piles, but as cycling progresses and the corner piles begin to degrade the load is shared fairly evenly between all the piles. Also, in this test there was more load carried in end-bearing than was predicted from the theory.

Load distributions for the 4-pile unequal-length group are shown in Fig. 6.18 for test UG2B which did not fail during cycling. As was found for the other pile groups there were no signs of degradation during cycling although it was noted that, as cycling proceeded, the load carried by the short pile increased slightly and that carried by the long pile decreased slightly. This trend was also found in the theoretical results.

The load carried by the short piles was found to be 10% greater than that predicted by the theory; in fact the short piles carried 40% of the total load while the long piles carried only 60% of the total load. Consequently the skin friction along the short piles was greater than that along the long piles, because the long piles have twice the shaft area of the short piles, resulting in more degradation of skin friction along the short piles.

On loading to failure after cycling skin friction degradation was found in both the long and the short piles. In the long piles the skin friction degradation
occurred almost exclusively in the lower half of the pile i.e. below the level of the base of the short pile, while in the short pile the degradation was fairly uniform along the length of the pile. The degradation was more severe in the short pile than in the long pile.

The results of the single pile tests show that the load carried by end-bearing tends to increase during cycling, while that carried by skin friction decreases. However, the pile groups tested showed that, during cycling, the load carried in end-bearing and in skin friction tends to remain more or less constant. After cycling the loss of capacity of the piles appeared to be due entirely to skin friction degradation, while the load carried as end-bearing remained constant.

6.3.5 Cyclic Stiffness

The pile group tests showed a greater change in cyclic stiffness in the first ten cycles than did the single pile tests. A plot of the average cyclic stiffness per pile versus number of cycles for a typical single pile, a typical 4-pile group and a typical 8-pile group are shown in Fig. 6.19, and demonstrate that there is a substantial decrease in cyclic stiffness of the groups as the number of cycles increases for the first ten cycles. This trend was also found in the theoretical results for the pile groups.

6.3.6 Cyclic Displacement

The cyclic displacement, $\delta_c$, is plotted against normalised cyclic load amplitude, $P_c/P_u$, for the 4-pile group in Fig. 6.20(a). As with the single pile tests, there is only a small scatter in the tests which did not fail during cycling. There is a roughly linear relationship between the cyclic load and the cyclic displacement (for $\delta_c \leq 0.4$ mm) i.e. the section of the curve in which most tests did not fail during cycling. Again this indicates that the cyclic displacement is nearly independent of the mean load. The cyclic displacement for the 4-pile group is approximately 50% more than that for the single pile at a corresponding load level.
Fig. 6.20(b) shows a similar plot for the 8-pile group although there are only a limited number of test points. The cyclic displacement for the 8-pile group is three times that of the 4-pile group at a corresponding load level. The cyclic displacement generally exceeds 1.0 mm before failure occurs during cycling, compared to 0.4 mm for the 4-pile group. The theoretical results indicate that the increase in cyclic displacement of the 8-pile group over the 4-pile group should only be 26%. These results follow on from the discrepancy between experiment and theory for the 8-pile group under static loading. The measures suggested in section 6.2.2 for reducing $E_a$ in the analysis and perhaps adopting a non-linear analysis may be required to accurately predict the cyclic displacements of the 8-pile group.

6.3.7 Accumulated Deflection

Accumulated deflection versus normalised mean load for the 4-pile group is plotted in Fig. 6.21. Although the test points are fairly scattered the same trends emerge as found for the single pile tests. There is increasing positive accumulated deflection as the mean load increases for each level of cyclic load, and also as the cyclic load increases. When the mean load is tensile, the limited number of data points indicate that there is increasing negative accumulated deflection as the cyclic load increases.

6.3.8 Residual Loads

Residual loads for the 4-pile group and the 8-pile group, both before and after cycling, are plotted in Figs 6.22 and 6.23. The plots do not show any particular trends. In some tests, the residual loads decrease after cycling while in other tests they increase after cycling. The random pattern of the residual loads suggests that they may be governed more by factors such as the time delay after installation that the readings were taken, than by the cyclic loading.

Residual loads for the 4-pile unequal-length group are shown in Fig. 6.24. There is a definite trend towards reduced residual loads after cycling in both the long piles and the short piles. Also noted from this Figure is that the
residual loads in the long piles are tensile, while those in the short piles are compressive. The three tests shown here were all performed on the same pile group and there is a trend towards reduced residual loads in each successive test. This indicates that the previous test history of the group has some effect in determining the residual loads. This trend was also noted for the 8-pile group tests.

Generally, however, the residual loads in all the pile group tests are fairly small. The maximum load is less than 15% of the ultimate static load capacity and so the residual loads probably do not have a great influence on the cyclic response of the group.

6.4 SUMMARY

In this chapter the results of tests on different pile groups subjected to both static and cyclic load have been presented along with the results using the theoretical model.

The main conclusions are

- post cyclic load capacity can be predicted fairly well for different pile groups using single pile data
- post cyclic load capacity is independent of mean load
- the effect of cyclic loading on pile interaction increases as the number of piles in the group increases
- in the 8-pile group, skin friction degradation starts to occur at the base of the pile shaft and works its way up the shaft
- in the unequal-length groups, skin friction degradation is more severe in the short pile than in the long pile
- accumulated deflection tends to increase as both the mean load and the cyclic load increase
the residual loads in all the groups are relatively small and in the unequal-length group they tend to reduce after cycling, demonstrating that the skin friction has reduced due to cyclic loading.

the settlement of the 8-pile group is underestimated by the theory and so for this group the parameters used and/or the theory may need adjusting in order to predict the settlement more accurately.

Unfortunately the computer codes did not model the load-deformation behaviour of the model pile groups very well. This is probably because pile load-deformation is non-elastic, even at relatively small relative displacements and so when these are amplified into the group domain by pile-soil-pile interaction a magnified non-linear effect for the group results. Other factors which could contribute to the discrepancy include

- experimental effects associated with jacking the piles into the clay mass as a group instead of one at a time (soil deformations during jacking could have produced imperfect contact with some of the piles)

- the fact that soil modulus backcalculated from single pile tests for use in the group presumes that the soil within and around the group remains 'elastic'. Soil disturbance during installation could alter the soil modulus significantly.

- the pore pressures developed due to insertion of the piles may not have had time to completely dissipate in the 24 hours given between insertion and static axial failure of the group.
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<th>Test Number</th>
<th>Number of Piles</th>
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<th>$P_o/P_u$</th>
<th>$P_c/P_u$</th>
<th>$D_r$</th>
<th>Failed?</th>
<th>$\delta_c$ (mm)</th>
<th>$\delta_{acc}$ (mm)</th>
<th>Pile Head Stiffness</th>
<th>Remarks</th>
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Table 6.1(a) Summary of Test Results for Groups
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<th>No. of piles</th>
<th>$P_u$ (N)</th>
<th>$P_o/P_u$</th>
<th>$P_c/P_u$</th>
<th>$D_r$</th>
<th>Failed</th>
<th>$\delta_c$ (mm)</th>
<th>$\delta_{ac}$ (mm)</th>
<th>Pile Head Stiffness</th>
<th>Remarks</th>
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<td>0.80</td>
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Table 6.1(b) Summary of Test Results for Groups
Figure 6.1 Comparison of Experimental and Theoretical Load-Settlement Responses
Figure 6.2  Comparison of Experimental and Theoretical Settlement Ratios
Figure 6.3 Static Load Distributions for 4-Pile Group
Figure 6.4 Static Load Distributions for 8-Pile Group
Figure 6.5 Static Load Distributions for 4-Pile Unequal-Length Group
(a) Pile Loaded to $0.4P_u$

(b) Pile Loaded to $0.7P_u$

(c) Pile Loaded to $0.9P_u$

Figure 6.6  Static Load Distributions for 2-Pile Unequal-Length Group
Figure 6.7  Pile Head Stiffness for $P/P_0 = 0.5$
Figure 6.8  Comparison of Experimental and Theoretical Degradation Factors for 4-Pile Group
Figure 6.9 Interaction Diagram for 4-Pile Group

- Test did not fail after 10 cycles
- Indicates test failed during cycling

Note: Numbers indicate value of $D_T$ after 10 cycles
Figure 6.10 Interaction Diagram for 8-Pile Group

- Test did not fail after 10 cycles
- Indicates test failed during cycling

Note: Numbers indicate value of $D_T$ after 10 cycles
Figure 6.11 Interaction Diagram for 4-Pile Unequal-Length Group
Figure 6.12  Effect of Cyclic Load on Skin Friction Degradation for 4-Pile Group
Figure 6.13  Effect of Mean Load on Post-Cyclic Load Capacity for 4-Pile Group
Figure 6.14  Load Distributions for 4-Pile Group

\[ P_0/P_u = -0.01 \quad P_c/P_u = 0.73 \]
Figure 6.15  Load Distributions for 4-Pile Group
Figure 6.16  Load Distributions for 8-Pile Group
Figure 6.17 Load Distributions for Corner Pile of 8-Pile Group
Figure 6.18 Load Distributions for 4-Pile Unequal-Length Group

\[ \frac{P_o}{P_u} = 0.01 \quad \frac{P_c}{P_u} = 0.73 \]
Figure 6.19  Cyclic Stiffness for $P_c/P_u = 0.70$
Figure 6.20 Effect of Cyclic Load on Cyclic Displacement
Figure 6.21  Accumulated Deflection for 4-Pile Group
Figure 6.22  Residual Load Distributions for 4-Pile Group
Figure 6.23  Residual Load Distributions for 8-Pile Group
Figure 6.24 Residual Load Distributions for 4-Pile Unequal-Length Group
CONCLUSIONS

The aim of this thesis has been to investigate, by analytical and experimental means, the behaviour of pile groups when subjected to cyclic axial loading.

In Chapter 1, a literature review was presented which revealed a number of gaps in knowledge on the subject of pile groups. They include:

(i) there has been very little work done on pile groups subjected to cyclic loading
(ii) very few of the theoretical analyses can analyse general, non-symmetrical groups, allowing for slip along the pile and cyclic loading effects
(iii) there is a dearth of tests on pile groups, especially those subjected to cyclic loading.

Hence, the remainder of the thesis was devoted to examining some of these deficiencies in knowledge.

In Chapter 2, a numerical analysis of pile groups under both static and cyclic axial loading was presented, which extended the previously developed boundary element model of Poulos. The solution procedure did not use interaction factors to approximate the effect of a loaded pile on the response of neighbouring piles but analysed the group directly, effecting a complete simultaneous solution for pile-soil-pile interaction between all piles in the group. This analysis can handle
groups of dissimilar piles, which may be of non-uniform diameter, arranged in any configuration. The settlement of the pile cap and the axial deflection and the distribution of pile-soil shear stress and load along the pile can be determined at each load increment up to failure, including the effects of slip between the pile and the soil.

A Reverse Plastic slip analysis was developed which can take into account the effects of slip between the pile and the soil by excluding negative plastic work. This method calculated the actual slip movements at each element which had slipped and was more efficient than the method outlined by Poulos and Davis (1968). Allowance was made for individual pile loads to be input, instead of the total group load. Under cyclic loading, consideration was given to cyclic degradation of skin friction, base resistance, Young's modulus and the accumulation of permanent displacements.

In Chapter 3, analyses were carried out on a typical 8-pile offshore group to investigate the effects of a variety of dissimilarities on the static and cyclic response of the group, in a variety of soil profiles. Under static loading, the effects of having some piles belled, different length and diameter piles, a defective pile in the group and a misaligned pile in the group were investigated. In all cases, the effect on settlement at working loads was relatively minor as compared with the case of a group containing all 'standard' piles, while the settlement at loads approaching failure was generally influenced to a greater extent. The greatest effects on settlement and load distribution generally occurred for the softer or more compressible soil profiles.

Under cyclic loading, degradation of skin friction commenced at considerably lower load levels in groups containing piles of different length or diameter, and the degradation process was more gradual as the cyclic load increased. The results suggested that, under static loading, the effects of having some shorter or defective piles in a group may not be serious at normal working load levels. However if the group was to be subjected to significant cyclic loading, the use of pile groups containing dissimilar piles (in particular, piles of considerably different length) should be avoided.
In Chapter 4 the results of two series of tests on model piles and pile groups in overconsolidated clay were presented. The first series, performed in a large vessel, involved 26 static tests and 52 cyclic tests on single piles and 14 static tests and 30 cyclic tests on 2-pile, 4-pile or 8-pile groups. In each test, axial loads were applied to the pile group, and group settlements, group loads and pile strains were measured. The second series, performed on a single pile in a small vessel with flexible upper and lower boundaries, was carried out to determine the cyclic degradation characteristics of the clay. The test procedures and the observed response of the pile groups were described.

Comparisons were made between the theoretically predicted and experimentally observed responses to cyclic loading of single piles in Chapter 5 and pile groups in Chapter 6. The parameters which the theory requires are Young's modulus for the clay and the Matlock and Foo parameters, i.e. the degradation rate parameter $\lambda$ and the minimum value of the degradation factor $D_{\text{min}}$. Young's modulus may be obtained by matching the initial slope of the static load-settlement curve with that obtained from theory. The theoretical value can then be backfigured. The values of the Matlock and Foo degradation parameters may be obtained by repeatedly loading a pile to failure in compression and then loading to failure in tension for several cycles. The skin friction degradation factor $D_r$ can be calculated at each cycle and so a plot of $D_r$ versus the number of load cycles can be made. The degradation parameters $\lambda$ and $D_{\text{min}}$ may be obtained from this plot using Equation 2.19.

For the aluminium and perspex single pile tests and the different pile group tests the post-cyclic load capacity was predicted fairly well by the Matlock and Foo model using single pile data. Interaction diagrams were constructed relating the normalised mean and cyclic loads to the normalised post-cyclic load after ten load cycles and theoretical failure lines were calculated. The theoretical failure lines gave conservative estimates of failure after ten cycles and so provided good lower bounds. In fact, not one of the tests which lay inside the theoretical failure line, failed. The maximum loss of capacity for the single piles was 30% and that for the pile groups was 40%.
For tests on single piles and pile groups the post-cyclic load capacity was independent of mean load but dependent on cyclic load. Up to a certain value of cyclic load there was no degradation at all; after this load the degradation increased as the cyclic load increased up to a critical value of cyclic load; beyond this value all tests failed during cyclic loading. The test results also indicated that two-way cycling leads to much more serious skin friction degradation than one-way cycling. Also, in displacement-controlled tests there was a critical displacement, before which there was negligible skin friction degradation and after which the degradation increased markedly. This critical displacement was found to be approximately equal to the displacement required to cause full slip under static loading.

The effect of cyclic loading on pile interaction in the model tests increased as the number of piles in the group increased. This behaviour was in agreement with the theory. In the model pile tests for the single piles and the 8-pile groups, skin friction degradation started to occur at the base of the pile shaft and worked its way up the shaft. This behaviour was found to be a function of pile stiffness. However, the loss of capacity during cycling due to skin friction degradation was recovered through an increase in end-bearing capacity. In the unequal-length groups, skin friction degradation was more severe in the short pile than in the long pile.

The pile load distributions, under both static and cyclic loading, could be predicted fairly accurately using the theory presented, for load levels right up to failure.

For both the single piles and the pile groups the accumulated deflection tended to increase as both the mean load and the cyclic load increased. In the model tests the cyclic displacement amplitude was almost uniquely related to the cyclic load amplitude for normal working load levels.

The results of the model tests indicated that the theory can provide satisfactory predictions of the cyclic response of offshore pile groups and particularly that the behaviour of pile groups of up to eight piles can be predicted using the results of single pile tests.
7.1 FUTURE RESEARCH

Further experimental investigation of the influence of cyclic loading on pile groups is required. Tests on larger-scale piles would be particularly useful because they would not suffer from possible scaling effects as much as smaller-scale tests. The largest group size tested in this experimental programme was an 8-pile group, however in practice larger groups are used. Therefore, there is a need for tests on larger groups. There is also a need for the experimental loading to model the actual loading of offshore pile groups. This could be done by including “storm-loading” sequences in the loading programme.

Another problem which requires further research is the method of determining the parameters used in the theoretical analysis. A simple method of determining the parameters needs to be developed. This method could perhaps use the results of a full-scale single pile test, in-situ soil tests or laboratory tests.

To overcome the problem of the non-elastic load-deformation behaviour of the groups it is suggested that a multiple-slip interface model could be adopted and this suggestion is left to future investigators.
REFERENCES


APPENDIX A

DERIVATION OF PILE DISPLACEMENT EQUATIONS FOR SINGLE PILE (from Poulos, 1979b)

Referring to Fig. 2.1, for any shaft element $i$, the incremental axial compression $\delta \rho_i$ is

$$\delta \rho_i = \frac{\Delta P_i l_i}{E_i X_i} \quad (A1)$$

where

$\Delta P_i$ = incremental average load in element $i$

$l_i$ = length of element $i$

$E_i$ = Young's modulus of pile element $i$

$X_i$ = cross-sectional area of element $i$

$\Delta P_i$ can be expressed as

$$\Delta P_i = \sum_{k=1}^{NB} \Delta p_{bk} A_{bk} + \sum_{j=1}^{NS} \Delta p_{sj} A_j T_{ij} + \sum_{l=1}^{ND} \Delta p_{dl} A_{dl} G_{il} \quad (A2)$$

where

$\Delta p_{bk}$ = incremental normal stress on element $k$ of base

$A_{bk}$ = surface area of base element $k$

$\Delta p_{sj}$ = incremental shear stress on shaft element $j$

$A_j$ = surface area of element $j$

$T_{ij}$ = 1 for $j > i$

0.5 for $j = i$

0 for $j < i$

$\Delta p_{dl}$ = incremental normal stress on discontinuity element $l$

$A_{dl}$ = surface area of element $l$

$G_{il}$ = 1 for $z_l \geq z_i$

0 for $z_l < z_i$

$z_i$ = distance from top of pile to bottom of element $i$
NB = number of base elements
NS = number of shaft elements
ND = number of discontinuity elements.

For nodes on the shaft and base of the pile, the incremental compression vector \( \{ \delta \rho \} \) can be expressed as

\[
\{ \delta \rho \} = [FE]\{ \Delta p \}
\]  \hfill (A3)

where \([FE]\) is an \((NS + NB)\) by \(NT\) matrix in which

\[
FE(i,j) = \frac{A_j T_{ij} l_i}{E_i X_i} \quad \text{for}\quad i = 1 \text{ to } NS,
\]

\[ j = 1 \text{ to } NS \]

\[
FE(i,j) = \frac{A_j l_i}{E_i X_i} \quad \text{for}\quad i = 1 \text{ to } NS,
\]

\[ j = NS+1 \text{ to } NS+NB \]

\[
FE(i,j) = \frac{A_j l_i G_{ij}}{E_i X_i} \quad \text{for}\quad i = 1 \text{ to } NS,
\]

\[ j = NS+NB+1 \text{ to } NT \]

\[
FE(i,j) = 0 \quad \text{for}\quad i = NS+1 \text{ to } NS+NB, \text{ and all } j
\]

NT = NS+NB+ND

\( \{ \Delta p \} \) = vector of incremental interaction stress.

The total axial incremental deflection vector for all nodes on the pile is then

\[
\{ \Delta \rho_p \} = \Delta \rho_b \{1\} + [AD]\{ \delta \rho \}
\]  \hfill (A4)

where

\( \Delta \rho_b \) = incremental displacement of pile tip

\( \{1\} \) = vector of values of unity

\([AD]\) = NT by \((NS+NB)\) matrix, in which

(a) for \( i = 1 \text{ to } NS, \text{ and } j = 1 \text{ to } NS, \)

\[
AD(i,j) = 1 \quad \text{for } j > i
\]

0 \quad \text{for } j < i

0.5 \quad \text{for } j = i
(b) for \( i = 1 \) to \( NS \), and \( j = NS+1 \) to \( NS+NB \),

\[
AD(i,j) = 0
\]

(c) for \( i = NS+1 \) to \( NS+NB \), and all \( j \),

\[
AD(i,j) = 0
\]

(d) for \( i = NS+NB+1 \) to \( NT \), and \( j = 1 \) to \( NS \),

\[
AD(i,j) = 1 \quad \text{for} \quad z_j > z_i \\
0 \quad \text{for} \quad z_j < z_i
\]

(e) for \( i = NS+NB+1 \) to \( NT \), and \( j = NS+1 \) to \( NS+NB \),

\[
AD(i,j) = 0
\]

Thus, from equations A3 and A4,

\[
\{\Delta \rho_p\} = \Delta \rho_b\{1\} + [AD][FE]\{\Delta p\} \quad \text{(A5)}
\]

\[
\{\Delta \rho_p\} = \Delta \rho_b\{1\} + [PM]\{\Delta p\}
\]

where \([PM] = [AD][FE]\)

In the computer program, the numbering sequence is as follows:

(i) elements 1 to \( NS \) are shaft elements

(ii) elements \( NS+1 \) to \( NS+NB \) are base elements

(iii) elements \( NS+NB+1 \) to \( NT \) are discontinuity elements.
APPENDIX B

DERIVATION OF PILE BOUNDARY CONDITION EQUATIONS

If the piles in the group are connected to a rigid pile cap and concentric axial loading is applied, the incremental displacement of the head of each pile will be equal to the incremental displacement of the pile cap $\Delta \rho_c$. The base movement plus the elastic compression of the pile is equal to the movement of the pile cap. It is possible to calculate the amount of movement each element undergoes in terms of the incremental load on the pile and the incremental stress. Therefore there are $n$ boundary condition equations, one for each pile:

$$\rho_j = \sum_{i=1}^{m_j} \frac{\Delta P_i l_i}{E_i X_i} - \rho_c = 0$$

where

$\Delta P_i = $ incremental average load in element $i$

$l_i = $ length of element $i$

$E_i = $ Young's modulus of pile element $i$

$X_i = $ cross-sectional area of element $i$

$m_j = $ number of elements in pile $j$

where the load at the node midway down the top element is calculated first as:

$$P_1 = \Delta P_j - \Delta p_1 \frac{A_1}{2}$$

and midway down the $i$th element

$$P_i = \Delta P_j - \Delta p_1 A_1 - \Delta p_2 A_2 - \cdots - \Delta p_{i-1} A_{i-1} - \Delta p_i \frac{A_i}{2}$$

where

$\Delta P_j = $ incremental load on pile $j$

$A_i = $ surface area of element $i$
In this way the deflection is summed up all the way down the pile to give:

\[ [B] \{ \Delta p \} + \{ \rho_b \} - \{ \rho_2 \} + [C] \{ \Delta P_i \} = 0 \]

where

\[ [B] = \text{coefficients for } \{ \Delta p \} \]
\[ [C] = \text{coefficients for } \{ \Delta P \}. \]
APPENDIX C

CALCULATION OF DISPLACEMENT FACTORS FOR PILE ELEMENTS

The geometry of a typical cylindrical pile element is shown in Fig. C1. El Sharnouby and Novak (1984) have found that the displacement can be very nearly obtained from two equal point loads which act on the cylinder surface at a distance of \( \delta_i/4 \) from the soil element ends, which are related to the reference node O by an angle of 40° as shown in Fig. C1. Thus, the horizontal distance between the reference node, O, and the points of load application is \( d_j \sin 40° \).

The vertical displacement due to a vertical point load are given by Mindlin’s equation as:

\[
\rho_z = \frac{P}{16\pi G(1 - \nu)} \times \left[ \frac{3 - 4\nu}{R_1} + \frac{8(1 - \nu)^2 - (3 - 4\nu)}{R_2} + \frac{(z - c)^2}{R_1^3} + \frac{(3 - 4\nu)(z + c)^2 - 2cz}{R_3^3} + \frac{6cz(z + c)^2}{R_3^4} \right]
\]

where the geometry involved is shown in Fig. C2. Now

- \( c = z_j - 0.25\delta_j, z_j + 0.25\delta_j \) (for the 2 point loads)
- \( z = z_i \)
- \( r = d_j \sin 40° \) (for the pile itself)
- \( P = 0.5P_j \)
- \( = 0.5 p_j \pi d_j \delta_j \)

also \( G = \frac{E}{2(1+\nu)} \)

Therefore, for each point load

\[
\rho_z = \frac{0.5p_j \pi d_j \delta_j}{16\pi E(1 - \nu)} [K]
\]

\[
\rho_z = \frac{p_j d_j \delta_j}{E \frac{(1 + \nu)}{16(1 - \nu)}} [K]
\]

where \([K]\) is the kernel.
For adjacent piles the approach is very similar. The geometry of adjacent piles is shown in Fig. C3. For simplicity, the displacements are computed on the axis of adjacent piles

\[ r^2 = s^2 + r_0^2 - 2sr_0 \cos 80^\circ \]

But, when \( s = 0 \), \( r = d \sin 40^\circ \).

El Sharnouby and Novak have also carried out a similar analysis for the pile base. They found that the displacement produced by a vertical unit load distributed uniformly over the base can be generated at the centre of the base by the unit load concentrated as a point load at a distance from the base centre equal to \( r = d_b/\pi \). The geometry of the pile base is shown in Fig. C4.

The same Mindlin equation applies:

\[ \rho_z = \frac{P}{16\pi G(1 - \nu)} [K] \]

where \([K]\) is the kernel, as before.

Here

\[ P = p_b \frac{\pi d_b^2}{4} \]

Therefore

\[ \rho_z = \frac{p_b \frac{\pi d_b^2}{4}}{16\pi \frac{E}{2(1 - \nu)^2}} [K] \]

\[ \rho_z = \frac{p_b}{32E} \frac{d_b^2 (1 + \nu)}{(1 - \nu)} [K] \]

(a) For elements on the pile itself,

\[ c = L_i \]

\[ z = z_i \]

\[ r = (0.5 - \frac{1}{\pi})d_b \]

\[ = (0.5 - 0.3183)d_b \]

\[ = 0.1817d_b \]

(b) For the effect of base on base, multiply by a factor of \( \frac{\pi}{4} \).

(c) For elements on other piles, use \( r = s \).
This analysis has been incorporated into the pile group program GAPFIL enabling a practical solution for larger, non-symmetrical pile groups. The resulting program is called GAPFIN. This approach was incorporated into the program as follows:

The soil displacements are given in equation 2.2 by:

\[ \{s \Delta \rho \} = \left[ \frac{d}{E_s} I_s \right] \{ \Delta \rho \} \]

Thus the \( I_{ij} \) values are as follows:

(a) **Shaft element influences** (\( j = 1 \) to NS, where NS is the total number of shaft elements in the group)

\[ I_{ij} = \sum_{k=1}^{a} \frac{1}{16} \frac{(1 + \nu)}{(1 - \nu)} d_i \delta_j K_{ij,k} \]

where

\[
K_{ij,k} = \frac{3 - 4\nu}{R_1} + \frac{8(1 - \nu)^2 - (3 - 4\nu)}{R_2} + \frac{(z - c_k)^2}{R_1^2} + \frac{(3 - 4\nu)(z + c_k)^2 - 2cz}{R_2^2} + \frac{6c_k(z + c_k)^2}{R_2^2}
\]

\[ z = z_i \]
\[ r = d_j \sin 40^\circ \] (for the pile itself, i.e. \( s = 0 \))
\[ c_k = z_j - 0.25 \delta_j \] (\( k=1 \))
\[ = z_j + 0.25 \delta_j \] (\( k=2 \))
\[ R_1 = (r^2 + (z - c_k)^2)^{\frac{3}{2}} \]
\[ R_2 = (r^2 + (z + c_k)^2)^{\frac{3}{2}} \]

For other piles, \( s > 0 \), as above, except

\[ r = (s^2 + 0.25 d_j^2 - s d_j \cos 80^\circ)^{\frac{3}{2}} \]
(b) **Base element influences** \((j = NS + 1)\)

\[
I_{ib} = F_i \frac{(1 + \nu)}{32(1 - \nu)} d_i^2 K_{ib}
\]

where

\[
K_{ib} = \frac{3 - 4\nu}{R_1} + \frac{8(1 - \nu)^2 - (3 - 4\nu)}{R_2} + \frac{(z - c)^2}{R_1^3} + \frac{(3 - 4\nu)(z + c)^2 - 2cz}{R_2^3} + \frac{6cz(z + c)^2}{R_2^5}
\]

\[
c = L_j \\
z = z_i \\
r = 0.1817d_i \text{ (for the pile itself, i.e. } s = 0) \\
R_1 = (r^2 + (z - c)^2)^{\frac{1}{2}} \\
R_2 = (r^2 + (z + c)^2)^{\frac{1}{2}} \\
F_i = 1.0 \text{ (} i \neq b \text{) } \\
= \pi/4 \text{ (} i = b \text{) }
\]

For other piles, i.e. \(s > 0\), \(r = s\), \(F_i = 1.0\).
Figure C.1  Geometry of a Typical Cylindrical Pile Element
Figure C.2 Basic Geometry for Mindlin's Equation

Figure C.3 Geometry of Adjacent Piles
Figure C.4  Geometry of Pile Base
APPENDIX D

EXAMPLE OF SOLUTION FOR
RELATIVE DISPLACEMENT OF PILE GROUP

Assume the pile group has two piles, each containing two elements. Equation 2.12 becomes:

\[
\begin{bmatrix}
i_{11} & i_{12} & i_{13} & i_{14} & -1 & 0 & 0 & 0 & 0 \\
i_{21} & i_{22} & i_{23} & i_{24} & 0 & -1 & 0 & 0 & 0 \\
i_{31} & i_{32} & i_{33} & i_{34} & -1 & 0 & 0 & 0 & 0 \\
i_{41} & i_{42} & i_{43} & i_{44} & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
B_1 & 0 & 0 & 0 & 1 & 0 & -1 & B_3 & 0 \\
0 & B_2 & 0 & 0 & 0 & 1 & -1 & 0 & B_4 \\
C_1 & 0 & C_2 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & C_1 & 0 & C_2 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\Delta p_1 \\
\Delta p_2 \\
\Delta p_3 \\
\Delta p_4 \\
\Delta \rho_{b1} \\
\Delta \rho_{b2} \\
\Delta \rho_c \\
\Delta P_1 \\
\Delta P_2
\end{bmatrix}
= 
\begin{bmatrix}
\Delta \omega_1 \\
\Delta \omega_2 \\
\Delta \omega_3 \\
\Delta \omega_4 \\
\Delta P_{tot}
\end{bmatrix}
\]  \hspace{1cm} (D1)

where

\([B] = \text{coefficients for } \{\Delta p\} \text{ in the boundary condition equation given in appendix B}\)

\([C] = \text{coefficients for } \{\Delta P\} \text{ in the boundary condition equation given in appendix B}\)

\{\Delta \rho_b\} = \text{displacement of pile base}

\{\Delta \rho_c\} = \text{displacement of pile cap.}
When equation D1 is inverted it becomes:

\[
\begin{bmatrix}
\Delta p_1 \\
\Delta p_2 \\
\Delta p_3 \\
\Delta p_4 \\
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_c \\
\Delta P_1 \\
\Delta P_2
\end{bmatrix}
= 
\begin{bmatrix}
k_{11} & k_{12} & \cdots \\
k_{21} & k_{22} & \\
\vdots & k_{33} & \\
\vdots & \vdots & \\
\end{bmatrix}
\begin{bmatrix}
\Delta \omega_1 \\
\Delta \omega_2 \\
\Delta \omega_3 \\
\Delta \omega_4 \\
\Delta P_{\text{tot}} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  
(D2)

Assuming element (3) fails and rearranging so that \( \Delta \omega_3 \) can be solved for equation D2 becomes:

\[
\begin{bmatrix}
\Delta p_1 - k_{13} \Delta \omega_3 \\
\Delta p_2 - k_{22} \Delta \omega_3 \\
-k_{33} \Delta \omega_3 \\
\Delta p_4 - k_{43} \Delta \omega_3 \\
\Delta \rho_1 - k_{33} \Delta \omega_3 \\
\Delta \rho_2 - k_{33} \Delta \omega_3 \\
\Delta \rho_c - k_{33} \Delta \omega_3 \\
\Delta P_1 - k_{33} \Delta \omega_3 \\
\Delta P_2 - k_{33} \Delta \omega_3
\end{bmatrix}
= 
\begin{bmatrix}
k_{11} & k_{12} & 0 \\
k_{21} & k_{22} & 0 \\
0 & k_{33} & -1 & k_{34} & k_{35} & \cdots \\
0 & k_{44} & & \\
0 & & \cdots & \\
0 & & & \\
0 & & & \\
0 & & & \\
0 & & & \\
\end{bmatrix}
\begin{bmatrix}
\Delta \omega_1 \\
\Delta \omega_2 \\
\Delta p_3 \\
\Delta \omega_4 \\
\Delta P_{\text{tot}} \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]  
(D3)

From equation 3 in the above matrix

\[-k_{33} \Delta \omega_3 = k_{31} \Delta \omega_1 + k_{32} \Delta \omega_2 - \Delta p_3 + k_{34} \Delta \omega_4 + k_{35} \Delta P_{\text{tot}}\]

Now \( \Delta \omega_1 = \Delta \omega_2 = \Delta \omega_4 = 0 \) and \( \Delta p_3 = 0 \)

therefore

\[-k_{33} \Delta \omega_3 = k_{35} \Delta P_{\text{tot}}\]

\[\Delta \omega_3 = -\frac{k_{35}}{k_{33}} \Delta P_{\text{tot}}\]

i.e. the same as for the single pile.