Selling Mechanisms and The Australian Housing Market

by

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Statement of Originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes. I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

Peyman Khezr

Abstract

This thesis examines selling mechanisms relevant mainly to auctions and applicable in the context of the housing market. In the first two chapters a context is described in which the seller of an object has private information about its value that is important to potential buyers. If the seller is unable to reveal this information to the buyers at no cost, the problem of adverse selection arises. Among other examples, auctions of arts, wines, and residential properties are most relevant to the current study. The sellers in these markets observe some private characteristics of their objects that are important to buyers but not revealable to them at no cost. In the first chapter we study some common selling mechanisms in this setting. Specifically, we study an ascending auction with two different reserve price regimes for the seller: first, disclosing the reserve price at the beginning of the auction; and second, keeping the reserve price secret and reserving the right to accept or reject the auction price after the bidding ends. We also study the common posted-price mechanism for the purposes of comparison. Throughout this chapter the assumption is that the seller chooses the mechanism from the *ex ante* point of view-that is, before observing her signal. Thus, the choice of mechanism itself does not reveal any further information to the buyers. The results in the first chapter suggest that in a one-shot game the seller can realise a higher *ex ante* expected payoff by choosing the secret reserve price regime than the other two mechanisms. At the end this chapter a dynamic setting is studied to examine the possibility of an extension of these static results to a dynamic case. Most of the results for the one-shot game are extendable to the proposed dynamic game.

In the second chapter we study an informed seller's best interest among the two previously mentioned reserve price regimes at the *interim* stage-that is, after the seller has observed her private information. We study how the seller's expected payoff could change if she observes the signal and then chooses the reserve price mechanism. In this case the choice of mechanism itself could reveal some information to buyers. The results show the conditions under which an informed seller, after observing her signal, chooses to keep the reserve price secret or discloses the reserve price.

The last two chapters focus specifically on the housing market. The third chapter adds to the theoretical literature of the housing market by proposing a more realistic selling mechanism applicable to this market: the one in which the seller posts a price to attract potential buyers to make a counteroffer. This game is studied in a dynamic setting with the possibility of more than one potential buyer arriving at each period. In the event that one buyer arrives, the seller engages in negotiation with that buyer; in the event that multiple buyers arrive, the seller runs an auction with a reserve price. This explains why sometimes sale prices are higher than the asking price and at the same time proposes a role for the asking price in this market. Other small variations of this mechanism are also studied for the purposes of comparison.

The final chapter is an empirical study of the Sydney housing market. We use comprehensive data on the Sydney housing market composed of 25,489 observations for properties sold in the Sydney region in 2011. We consider the fact that both the seller of the property and the real estate agent have a common goal: to sell the property at the highest possible price in the shortest amount of time. The analysis is divided into two major parts. First, we estimate a two-stage least square model to analyse which parameters affect time on the market for a property. Second, we propose a probit model that estimates the parameters that affect a revision in list prices. The results suggest that overpricing increases time spent on the market, and properties with a revised list price stay on the market for a longer time.

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Chapter 1

Selling Mechanisms With an Informed Seller

1 Selling Mechanisms With an Informed Seller

1.1 Introduction

The aim of this chapter is to compare selling mechanisms commonly used when the seller of an item has private information about its quality that is payoff relevant to prospective buyers but is unable to reveal that information to the buyers at no cost.

Examples of economic settings in which the quality of an object for sale is uncertain and the seller has private information unknown to prospective buyers are legion. Indeed, Akerlof's classic paper on the "lemons problem" Akerlof (1971), which introduced the problem of asymmetric information in economics, is concerned with precisely such a setting. The owner of an object often has information about attributes that affect the quality and desirability of the object from her experience of owning and using it. For example, the seller of a lot of wine offered at an auction would typically have private information of the conditions under which the wine was stored. And, of course, the owner of a house would typically have a detailed knowledge of the specifications the house that has an obvious bearing on the valuations of prospective buyers. In the context of auctions, the environment described is a special case of interdependent valuations introduced in Milgrom and Weber (1982a), in which the valuations of the bidders may depend on the information of other agents. However, the particular case in which the interdependence is only through the seller's information provides structure that can be exploited and can be especially relevant in many settings-such as in auctions of wines or residential property.

The choice of mechanism for selling an object typically rests either with an institution (such as an auction house) or with the seller of the object. For art, antiques, and wine, established auction houses have, over the centuries, designed the rules of the mechanism: for example, an ascending auction with a disclosed reserve or an undisclosed reserve in which the seller has the right of refusal. Because the auction house typically gets a fixed share of the revenue generated at the auction, one can presume that the rules are designed to maximise the seller's expected revenue from an ex ante perspective-that is, before the seller learns her information. Of course, when the seller sets a reserve (whether disclosed or undisclosed), she does so knowing this information. Thus, in an auction designed by an auction house with a disclosed reserve price, the particular value of the reserve acts as a signal but the mechanism itself does not. In contrast, the owner of a house who chooses a mechanism to sell the house does so at the interim stage, after she learns the information. Therefore, this is a design problem with an informed principal.

This chapter assumes that the form of the mechanism-whether a posted price or an auction with a disclosed or a secret reserve-is chosen by the auction house before the realisation of the seller's signal. It is supposed that the mechanism is chosen to maximise expected revenue from an *ex ante* perspective. The posted price or the reserve price (whether disclosed or secret) for an item in an auction is of course chosen by the seller at the interim stage when the seller's private signal is known to the seller. Thus, the value of the posted or reserve price acts as a signal in the corresponding mechanism, but the choice of the mechanism itself does not.

Cai, Riley, and Ye (2007) observe that when a seller lacks access to a technology for costless and credible revelation of her information, an announced reserve price at an auction can act as a credible device to signal the seller's information. This is because the marginal cost of a higher reserve price-a lower probability of sale-is lower for a seller who has a superior signal and, therefore, a higher use value for the item. They characterise the unique separating equilibrium in such a setting. We treat their work as the theoretical benchmark for the environment that is the concern of this paper. However, reserve prices are almost never disclosed in real-world auctions.[1] We compare this benchmark model with the two most commonly observed selling mechanisms: the mechanism in which the seller simply posts a price and the mechanism in which the seller conducts an auction with a secret reserve price and has the right to retain the item. These three mechanisms differ in the degree to which they reveal the seller's information.

We compare the three mechanisms first in a static model (i.e., when the seller faces a known finite number of prospective buyers, each of whom receives an independent private signal but also cares about the seller's private signal). We then compare the mechanisms when such prospective buyers arrive over time. We show that in the static context, the posted-price mechanism generates lower expected revenue than an auction with a disclosed reserve; in contrast, an auction with a secret reserve price may generate higher or lower expected revenue than one with a disclosed reserve price, depending on the type of the seller. We also show how, in the

[1] Several studies have documented this: see, for example, Ashenfelter (1989), Cassidy (1967), and Hendricks and Porter (1988). This can also be readily confirmed by visiting the websites of traditional auction houses like Christie's and Sotheby's, which explicitly state that reserve prices for items are kept secret.

dynamic context, the optimal reserve price and posted price may change over time and affect the seller's expected revenue.

The two papers most closely related to the model described in this chapter are Cai, Riley, and Ye (2007), already mentioned, which focuses on a second-price auction with a disclosed reserve price, and Jarman and Sengupta (2012), which describes an auction with a secret reserve price in an environment similar to the one considered here. Jarman and Sengupta (2012) characterise the bidding function and the seller's expected revenue in the secret reserve regime and demonstrate that the seller's *ex ante* expected revenue can be greater than that in the unique signalling equilibrium characterised in Cai, Riley, and Ye (2007). We discuss and present their results for the analysis of the static model and further examine the posted-price mechanism for the proposed setting. Then we compare the results with those for the dynamic model.

One of the very first studies with a model closely related to the one proposed here is Milgrom and Weber (1982a). Although their model is more general in terms of the affiliation of buyers' signals, some of their results are useful for the present study. The results suggest that an English auction generates higher average prices than a second-price auction. The authors also suggest that with risk-neutral bidders, a second-price auction results in higher average prices than a Dutch or a first-price auction. They show that when the information is verifiable, the seller's best strategy is to pre-commit to revealing the information. Thus, a seller in their model always increases the expected price in any of these auctions by establishing a policy to evaluate the quality of the object for the buyers. In contrast, the main focus of the present study is the case when the seller's information is not verifiable to buyers without a cost. Therefore, if a seller conceals private information, she may not reduce the expected revenue by that policy as long as there is no costless access to that information.

Auctions in dynamic settings have received less attention in the literature than those in static contexts. Wang (1993) compares auction and posted-price mechanisms in a dynamic context. Wang's dynamic model is quite different from the one studied here. Most important, in his model, the seller's valuation is common knowledge and there is no role for signalling. Under the conditions of the independent private values model, he suggests when the dispersion of buyers' valuations around the mean is higher, an auction is the preferred selling mechanism, even if it is costlier than a posted-price mechanism. In his model the seller does not discount future income, and consequently the time of the sale is not important for the seller. Wang (1998) compares the same selling methods for the case of correlated values in a single-period model. The main result is in line with that in Wang (1993), that is, the higher the dispersion of the values, the better the match for an auction mechanism.

Bergemann and Said (2010) is a comprehensive survey on dynamic auctions focused mainly on revenue maximisation and efficiency. Most recent studies have focused on models in which buyers have independent and private valuations for the object and the seller's valuation for the object is known. Moreover, other assumptions of dynamic auction models could change the results compared to a static model. For instance, the arrival of buyers and whether it is random or known could potentially change the outcome of a given model. In any dynamic model the amount of patience of buyers can also change the outcome of that model. The seller's discount rate is also important and could explain how the time when an object is being sold is important for the seller. Kremer and Skrzypacz (2007) study the effect of the revelation of information on trade in a dynamic signalling setting. Their setup includes a privately informed seller who faces potential buyers arriving over time and offering prices to the seller. The seller can reject the offers and delay the selling process. They suggest that with noisy signals a trade may not happen before the revelation of the information. In fact, in their model the external revelation of information strongly indicates both parties' outcomes. Therefore, agents may commit to a costly signalling even if this requirement is hardly achievable. Said (2012) studies the revenue maximisation and efficiency of auctions in a dynamic setup. In his model the seller will sell a set of objects to patient buyers before a deadline because the objects expire at a specific time. From the efficiency point of view, he suggests that a sequence of ascending auctions is efficient in such a setup. The extended results show that the optimal mechanism is a sequence of ascending auctions with asynchronous price clocks, mainly because the buyers are *ex ante* heterogeneous.

In Section 1.2 the static model is introduced followed by the study of each three mechanism we explained above in Sections 1.3-1.5. In Section 1.6 an example is studied for the revenue comparison of each selling mechanism in the static model. The dynamic model is introduced in Section 1.7 followed by the analysis of each selling mechanism in the dynamic case. In Section 1.9 the possibility of extending is discussed with concluding statements for this chapter.

1.2 The Model

A seller with an indivisible object to sell faces a set $N = \{1, \ldots, n\}$, $n \geq 2$, of potential buyers. The seller privately observes a signal s, drawn from a known distribution G with support $[0, \bar{s}]$, assumed twice differentiable with a continuous density g. Each buyer i has a private signal x_i for the object independently and identically distributed according to the distribution function F on $[0, \bar{x}]$, twice differentiable with continuous density f; moreover, each x_i is statistically independent of the seller's signal s. The valuation of each buyer i for the object is given by $v : [0, \bar{x}] \times [0, \bar{s}] \to \mathbb{R}_+$, a symmetric, continuous, and increasing function of her individual private signal x_i as well as the seller's signal s. Consequently, in this environment, a buyer cares not only about her own signal but also about the seller's signal. The seller's own valuation for the object is given by $v_0 : [0, \bar{s}] \to \mathbb{R}_+$, a continuous and increasing function of her own signal. It is further assumed that the *hazard rate*[2] function of F is increasing.

In this chapter, a mechanism is chosen before the realisation of the seller's signal. There are three possible selling mechanisms: a posted-price regime (PP), an ascending auction in which a reserve price is disclosed before the bidding starts—a *disclosed-reserve* regime (DR), and an ascending auction in which a reserve price is never disclosed but the seller retains the right to refuse the highest bid and keep the object—a *secret-reserve* regime (SR). Although the choice of the mechanism itself precedes the realisation of the seller's private signal, the decision of the reserve price (in the case of a disclosed reserve) or the decision to accept the highest bid (in the case

[2]The *hazard rate* function of F is defined by $\lambda(x) = \frac{f(x)}{1-F(x)}$

of a secret reserve) is made after the seller observes her signal.

The next three subsections describe the behaviour of the buyers and the seller's expected revenue for each of the three regimes.

1.3 Disclosed Reserve Price (DR)

Under this regime, the reserve price is announced at the beginning of the auction before the bidding starts. First, the seller observes her private signal s and then publicly announces a reserve price $r(s)$ and commits to it. Second, the bidding proceeds as in an open ascending auction with a reserve price $r(s)$. After observing the reserve price, buyers indicate whether they are willing to participate in the auction. Then the auctioneer starts to raise the price continuously (starting from the reserve price). Each bidder is active until the maximum price at which he or she is willing to buy the object. The price stops rising when there is only one active bidder left; this price is called the *auction price*. That bidder wins the object and pays the auction price.

For this regime most of the results in Cai, Riley, and Ye (2007) are directly applicable to the present analysis.[3] They assume that the function J, defined below, is strictly increasing in x,

$$J(s,x) = v(s,x) - \frac{\partial v(s,x)}{\partial x} \frac{F_{(2)}(x) - F_{(1)}(x)}{f_{(1)}(x)}. \tag{1}$$

$F_{(1)}$ and $F_{(2)}$ are the first and the second highest order statistics. Since the

[3]These authors study a second-price auction with a disclosed reserve price, but in this set-up in which bidders' valuations do not depend on the signals of other bidders, the second-price auction is strategically equivalent to an ascending auction.

signals of the bidders are independent, the above becomes

$$J(s,x) = v(s,x) - \frac{\partial v(s,x)}{\partial x}\frac{1-F(x)}{f(x)}. \tag{2}$$

This assumption is a slight generalisation of the assumption in Myerson (1981), in the context of independent private valuation models, that *virtual valuation* is strictly increasing in x. In the present model, the assumption that J in (2) is strictly increasing in x is satisfied as long as the hazard rate of F is strictly increasing and the valuation function is concave with respect to x. Cai, Riley, and Ye (2007) demonstrate that the seller can use the reserve price to credibly signal her type to potential buyers.

Following the seller's disclosure of the reserve price, let $\hat{s}$ represent a bidder's belief of the seller's signal. Then it follows from Milgrom and Weber (1982a) that it is a Bayesian-Nash equilibrium for each buyer i to bid-that is, stay active until the price reaches-$v(x_i, \hat{s})$: that is the bidder's expected value, given the belief that the seller's signal is $\hat{s}$. Each bidder enters the auction if her expected value is greater than the reserve price. In this situation the reserve price is a potential signal given the fact that a higher reserve increases the probability of no sale. Cai, Riley, and Ye (2007) suggest in this environment there are several pooling equilibria, but by the intuitive criterion discussed by Cho and Kreps (1987)[4] those equilibria can be ruled out, and as Riley (1979) show one can focus on the

[4]In this signalling game like most of the others, there are several perfect Bayesian equilibria. The intuitive criterion is basically an equilibrium refinement method to reduce the set of equilibria. In this method by restricting out-of-equilibrium beliefs to those which are reasonable we can eliminate many unintuitive equilibria. In particular, we can eliminate any PBE if there is a type which can make profitable deviation and other players assign zero probability to this deviation because of the belief that the action is equilibrium dominated.

unique separating equilibrium of this signalling game in which the lowest seller type chooses a reserve price such that it is optimal under the scenario of complete information.

Define $m(s)$ as the minimum buyer type who enters the auction given a belief that the seller's signal is $\hat{s}$, then in equilibrium the seller sets the reserve price equal to the expected value of the lowest buyer type who enters the auction, that is, $r = v(m(s), s)$. Having the reserve price and the bidding strategies, a seller with signal s who reports type $\hat{s}$ would have a net expected payoff equal to

$$\begin{aligned} U^{DR}(s, \hat{s}, m(\hat{s})) = & \Big[F_{(2)}\big(m(\hat{s})\big) - F_{(1)}\big(m(\hat{s})\big)\Big]\big(v(m(\hat{s}), \hat{s}) - v_0(s)\big) \\ & + \int_{m(\hat{s})}^{\bar{\omega}} [v(x, \hat{s}) - v_0(s)] f_{(2)}(x) dx. \end{aligned} \tag{3}$$

$F_{(1)}$ and $F_{(2)}$ are the first and second highest order statistics. $F_{(2)}(m(s)) - F_{(1)}(m(s))$ is the probability that there is only one (highest) bidder with an expected value higher than the reserve price.

If there is full information, then s is directly observable, so $\hat{s} = s$. Then the seller chooses $m(s)$ to maximise $U^{DR}(s, s, m)$. Let $m^*(s)$ be the optimal full information minimum type. Then according to $\frac{\partial U^{DR}}{\partial m(s)}$,

$$m^*(s) = \begin{cases} 0 & \text{if} \quad v_0(s) < J(s, 0) \\ J^{-1}(v_0(s)) & \text{if} \quad J(s, 0) \leq v_0(s) < J(s, \bar{\omega}) \\ \bar{\omega} & \text{if} \quad v_0(s) \geq J(s, \bar{\omega}), \end{cases} \tag{4}$$

where J^{-1} is the inverse of J.

Focusing on the incentive compatible direct mechanism, we must have

$$U^{DR}(s, s, m(s)) = \max_{\hat{s}} U^{DR}(s, \hat{s}, m(\hat{s})). \tag{5}$$

Furthermore, for $m(.)$ to characterise a separating equilibrium we must have $m'(s) > 0$ for every $s \in [0, \bar{s}]$. By Theorem 1 in Cai, Riley, and Ye (2007), the following differential equation characterises the unique separating equilibrium of the signalling game.[5]

$$\frac{ds}{dm} = -\frac{D_3 U^{DR}(s, s, m(s))}{D_2 U^{DR}(s, s, m(s))}. \tag{6}$$

To find the *ex ante* expected profit for the seller, one first needs to find the change in U^{DR} when s changes, which is

$$\begin{aligned} D_1 U^{DR}(s, s, m(s)) &= D_2 U^{DR} + (D_3 U^{DR} m'(s)) \\ &\quad - v_0'(s)[1 - F_{(1)}(m(s))] \\ &= -v_0'(s)[1 - F_{(1)}(m(s))] < 0. \end{aligned} \tag{7}$$

By the envelope theorem, the first line of (7) becomes zero. The fundamental theorem of calculus can be used to obtain another expression for the seller's payoff, that is,

$$U^{DR}(s, s, m(s)) = U^D(0, 0, m(0)) - \int_0^s [1 - F_{(1)}(m(t))] v_0'(t) dt. \tag{8}$$

After taking expectation and rearranging the integrals in (8), the *ex ante* expected payoff for the seller becomes

$$\mathbb{E}_s[U^{DR}(s, s, m(s))] = U^D(0, 0, m(0)) - \int_0^{\bar{s}} [1 - G(t)][1 - F_{(1)}(m(t))] v_0'(t) dt. \tag{9}$$

[5] Given a function $g : A \to \mathbb{R}$, where $A \subset \mathbb{R}^n$, $D_i g(x_1, \ldots, x_i, \ldots, x_n)$ represents the partial derivative of g with respect to its i-th argument evaluated at the point $(x_1, \ldots, x_n)$.

1.4 Secret Reserve Price (SR)

Under this regime, the game has two stages. First, bidders start bidding in an open ascending auction like the one explained in section 1.3 but with a slight difference: that is, the price increases from zero rather than the reserve price. Second, the seller observes the auction price or the price when there is only one active bidder left, and simply accepts or rejects it. If the seller accepts the auction price, the highest bidder wins and pays this price; otherwise, the seller retains the object. Jarman and Sengupta (2012) examine bidding behaviour under this regime. Given an auction price p at the end of an ascending auction, the seller's optimal decision is to accept p if and only if it is greater than or equal to her value.

Define $w(x_i, s_0)$ as the expected value of bidder i given that the seller's signal is less than s_0. Focusing on the symmetric equilibrium of the game, given that other players play β^{SR}, the optimal strategy for player i is to stay active until the price is equal to her expected value conditional on being accepted by the seller-that is, being higher than the seller's value. If the expected value is lower than the highest possible seller's value in the interval, $w(x_i, \bar{s}) < v_0(\bar{s})$, then bidders bid their conditional expected value; otherwise, they bid their unconditional expected value, $w(x_i, \bar{s})$.

Following proposition 1 in Jarman and Sengupta (2012), the equilibrium bidding function is as follows:

$$\beta_{SR}(x_i) = \begin{cases} p = w(x_i, v_0^{-1}(p)) & \text{if} \quad w(x, \bar{s}) < v_0(\bar{s}) \\ w(x_i, \bar{s}) & \text{if} \quad \text{Otherwise.} \end{cases} \tag{10}$$

Let $\hat{m}(s)$ be the minimum buyer type with a positive probability of clearing the seller's reserve price. Since the seller accepts any auction price greater than her valuation, the undisclosed reserve price is basically the seller's valuation for the object. One can define $\hat{m}(s)$ as follows:

$$\hat{m}(s) = \inf\{x : \beta_{SR}(x) \geq v_0(s)\}. \tag{11}$$

If there is any bid higher than the seller's value, $\hat{m}(s)$ shows the lowest bidder's signal with an expected value higher than the seller's value; otherwise, the highest bid will not be accepted by the seller, simply because it is lower than the seller's value.

Using the equilibrium bidding function, one can now derive the net expected payoff for the seller for this regime:

$$U^{SR}(s, \hat{m}(s)) = \int_{\hat{m}(s)}^{\bar{\omega}} [\beta_{SR}(x) - v_0(s)] f_{(2)}(x) dx. \tag{12}$$

To obtain the *ex ante* expected payoff for the seller, one needs to know how the *interim* expected profit changes when s changes. The derivative of (12) with respect to s is

$$\begin{aligned} D_1 U^{SR}(s, \hat{m}(s)) &= -[\beta_{SR}(\hat{m}(s)) - v_0(s)] f_2(\hat{m}(s)) \hat{m}'(s) \\ &\quad - v_0'(s)[1 - F_2(\hat{m}(\bar{\omega})) + F_2(\hat{m}(\bar{\omega})) - F_2(\hat{m}(s))] \\ &= -v_0'(s)[1 - F_2(\hat{m}(s))] < 0. \end{aligned} \tag{13}$$

By the definition of $\hat{m}(s)$ the first line becomes zero. Thus, the higher the seller's signal, the lower the net expected profit for the seller. According to the fundamental theorem of calculus and using the result in (13), one can calculate the seller's *ex ante* expected profit as follows:

$$U^{SR}(s, \hat{m}(s)) = U^{SR}(0, \hat{m}(0)) - \int_0^s [1 - F_2(\hat{m}(x))] v_0'(x) dx. \tag{14}$$

Taking expectation from (14) over s and rearranging the integrals results in the following expression for the *ex ante* expected payoff for the seller:

$$\mathbb{E}[U^{SR}(s, \hat{m}(s))] = U^{SR}(0, \hat{m}(0)) - \int_0^{\bar{s}} [1 - G(s)][1 - F_2(\hat{m}(s))] v_0'(s) ds. \tag{15}$$

Comparing the expressions in (9) and (15) we can show the conditions under which the *ex ante* expected payoffs from these two different reserve price regimes can dominate each other. In section 1.6 in an example, we compare the seller's expected payoffs for these two regimes.

1.5 Posted-Price (PP)

This section considers a setting in which the seller chooses a price p for selling an object to N potential buyers. The seller offers the object at price p as a take-it-or-leave-it offer. For a buyer to accept the seller's offer, she must have an expected valuation higher than p for the object; otherwise, the buyer declines the offer. If multiple buyers accepts to buy the object at p, one of them wins the object randomly.

The initial condition that needs to be satisfied for the optimal p according to the seller's point of view is that the posted price has to be greater than or equal to the seller's valuation; otherwise, there is no rational explanation for the seller to sell the object. If $p \geq v_0(s)$, then $s \leq v_0^{-1}(p)$. Thus, each buyer's expected value for the object with respect to the realisation of the seller's signal is $v(\tilde{s}, x_i) = E[V_i|S = \tilde{s}, s \leq v_0^{-1}(p)]$. According to the buyers' valuations, only buyers with valuation $v(\tilde{s}, x_i) \geq p$ are willing to accept the price. So a buyer accepts p if and only if her expected value is higher than p given the seller's signal is $\tilde{s}$. Let $\tilde{m}(s)$ be the optimum buyer type who is willing to accept the posted price given the seller's signal $\tilde{s}$. $\tilde{m}(.)$ is a strictly increasing function. It can be shown in equilibrium that by setting the price equal to the expected value of this optimum type the seller basically optimises her expected payoff from the signalling game. The expected value of the optimum buyer who is willing to accept p is $v(\tilde{s}, \tilde{m}(s))$. Then the net expected payoff for the seller with signal s who reports $\tilde{s}$ becomes

$$U^{pp}(s, \tilde{s}, \tilde{m}(s)) = [v(\tilde{s}, \tilde{m}(\tilde{s})) - v_0(s)]\big(1 - F_{(1)}(\tilde{m}(\tilde{s}))\big), \tag{16}$$

where $F_{(1)}$ is the highest order statistics and $\big(1 - F_1(\tilde{m}(\tilde{s}))\big)$ is the probability that the highest value among bidders is lower than the posted price.

Differentiating (16) with respect to $\tilde{s}$ and $\tilde{m}(s)$ results in

$$D_2U^{pp}(s, \tilde{s}, \tilde{m}(s)) = \frac{\partial v(\tilde{s}, \tilde{m}(s))}{\partial \tilde{s}}(1 - F_{(1)}(\tilde{m})) \tag{17}$$

$$\begin{aligned} D_3U^{pp}(s, \tilde{s}, \tilde{m}(s)) =& \frac{\partial v(\tilde{s}, \tilde{m}(s))}{\partial \tilde{m}(s)}(1 - F_{(1)}(\tilde{m})) \\ & - f_{(1)}(\tilde{m})(v(\tilde{s}, \tilde{m}(s)) - v_0(s)) \\ & = f_{(1)}(m)\big(v_0(s) - J_1(\tilde{s}, m)\big). \end{aligned} \tag{18}$$

where

$$J_1 = v(\tilde{s}, \tilde{m}(s)) - \frac{\partial v(\tilde{s}, \tilde{m}(s))}{\partial \tilde{m}(s)} \frac{1 - F_{(1)}(\tilde{m})}{f_{(1)}(\tilde{m})}. \tag{19}$$

Because D_3U^{pp} is increasing in s and D_2U^{pp} is independent of s, we can verify that the single crossing condition holds here as well. For the $\tilde{m}(.)$ function to characterise a separating equilibrium it must be the one in which $U^{pp}(s, s, \tilde{m}(s)) = \max_{\tilde{s}} U(s, \tilde{s}, \tilde{m}(\tilde{s}))$. Differentiating that with respect to $\tilde{s}$ and considering the fact that in the separating equilibrium $\tilde{s} = s$ reveals

$$D_2U^{pp}(s, s, \tilde{m}(s)) + D_3U^{pp}(s, s, \tilde{m}(s))m'(s) = 0. \tag{20}$$

Proposition 1.1. *Differential equation* $\tilde{m}'(s) = -\frac{D_2U^{pp}(s,s,\tilde{m}(s))}{D_3U^{pp}(s,s,\tilde{m}(s))}$ *characterises the unique separating equilibrium of the posted-price mechanism.*

Proof. See Appendix.

The solution reveals the optimum buyer type that maximises the seller's expected payoff. To find an expression for the seller's *ex ante* expected payoff one needs to differentiate (16) with respect to s when $s = \tilde{s}$. We have

$$D_1U^{pp}(s, s, \tilde{m}(s)) = (D_2U^{pp}) + (D_3U^{pp})(\tilde{m}'(s)) - v_0'(s)(1 - F_{(1)}(\tilde{m})). \tag{21}$$

Again, by a standard envelope theorem argument it can be shown that the first two arguments in (21) become zero. By (21) and the fundamental theorem of calculus one can find the *ex ante* expected payoff for the seller, which is

$$\mathbb{E}[U^{pp}(s, s, \tilde{m}(s))] = U^{pp}(0, 0, \tilde{m}(s)) - \int_0^{\bar{s}} (1 - G(s))(1 - F_1(\tilde{m}(s))v_0'(s)ds. \tag{22}$$

In the next section we compare the expected payoffs from the three selling mechanisms described here and show how they change when the seller type changes.

1.6 Example

As an example, suppose the valuations of the seller and the prospective buyers are linear functions of the signals. The seller's valuation is a linear function of her signal $v_0(s) = \gamma s$ for $\gamma > 0$. Buyers are symmetric, and their valuations are also simple linear functions of the buyers' own signals and the seller's signal: $v(s, x_i) = s + x_i$. Suppose all signals are independent and distributed uniformly on $[0, 1]$. Now it is possible to calculate and compare the seller's payoffs from each mechanism described in the previous section.

1.6.1 Disclosed Reserve Price (DR)

In this regime, the seller discloses the reserve price at the beginning of the auction upon observing her signal. This is a signalling game, and we are looking for a unique separating equilibrium of this game in which the seller reveals her true type via the reserve price. First use (3) to express the seller's expected payoff and rewrite it for the present example, that is,

$$\begin{aligned} U^{DR}(s, \hat{s}, m(\hat{s})) = & \Big[F_{(2)}\big(m(\hat{s})\big) - F_{(1)}\big(m(\hat{s})\big)\Big]\big(m(\hat{s}) + \hat{s} - \gamma s\big) \\ & + \int_{m(\hat{s})}^{1} [x + s - \gamma s] f_{(2)}(x) dx. \end{aligned} \tag{23}$$

Function $J(.)$, which is

$$J(s, x) = s + x - \frac{(1 - F(x))}{f(x)}, \tag{24}$$

is now equal to $J(x) = s + 2x - 1$, which is strictly increasing in x.

If $m^*(s)$ is the optimal reserve price for the case of complete information, then equation (4) becomes

$$m^*(s) = \begin{cases} 0 & \text{if} \quad \gamma s < J(s,0) \\ \frac{1}{2}((\gamma-1)s+1) & \text{if} \quad J(s,0) \leq \gamma s < J(s,1) \\ 1 & \text{if} \quad \gamma s \geq J(s,1). \end{cases} \tag{25}$$

Because γ is greater than or equal to zero and $s \in [0,1]$, then for every $0 < \gamma \leq 1$

$m^*(s) = \frac{1}{2}((\gamma-1)s+1)$.

To calculate the minimum buyer type one needs to solve the differential equation from (6), which is as follows:

$$s(m) = (1 - F_{(1)}(m))^{\gamma-1}\left[\int_{\underline{m}}^{m} f_{(1)}(x)(1-F_{(1)}(x))^{-\gamma}J(x)dx\right]. \tag{26}$$

According to (Cai, Riley, and Ye, 2007), for every $0 < \gamma \leq 1$ this is a solution for the separating equilibrium. One can solve this differential equation for a given γ and use the result to calculate the seller's expected payoff. When $N = 2$ it is

$$s(m) = (1-m^2)^{\gamma-1}\int_{\frac{1}{2}}^{1}\frac{(4x^2-2x)}{(1-x^2)^{\gamma}}dx. \tag{27}$$

The inverse of (27) gives the optimal m that maximises the following expected payoff function.

$$U^{DR}(s,s,m) = \int_{m}^{1} 2(s+x-\gamma s)(1-x)dx + (2m-2m^2)(m+s-\gamma s). \tag{28}$$

1.6.2 Secret Reserve Price (SR)

In this regime, there is no extra information available to the bidders, because there is no announced reserve price. According to the bidding function for this regime

$$\beta_{SR}(x) = \begin{cases} \frac{2\gamma}{2\gamma-1}x & \text{if} \quad x \leq \gamma - \frac{1}{2} \\ x + \frac{1}{2} & \text{if} \quad x \geq \gamma - \frac{1}{2}. \end{cases} \tag{29}$$

Calculation of the minimum buyer type who enters the auction is much more straightforward in this case. To solve $\hat{m}(s)$ numerically one can use (11), start with a given s in the interval, and calculate the minimum buyer type for a given γ. After calculating $\hat{m}(s)$ one can substitute the result into the seller's expected payoff, which is the following equation:

$$U^{SR}(s, \hat{m}(s)) = \int_{\hat{m}}^{1} [\beta_{SR}(x) - \gamma s](2 - 2x)dx. \tag{30}$$

The main difference here is the bidding function, which can be either conditional or unconditional. After calculating $\hat{m}$ for a given γ one needs to find the related bidding function and then substitute it into the seller's expected payoff equation. In the event that both of the bidding functions are relevant, the expected payoff will be two different integrals.

1.6.3 Posted-Price (PP)

Because the seller's valuation is equal to $v_0(s) = \gamma s$, she is willing to sell the object if and only if $\gamma s \leq p$. After the seller announces the posted price, the buyer's expected value for the object becomes $v(\tilde{s}, x_i) = E[V_i|s =$

$\tilde{s}, s \leq \frac{p}{\gamma}]$.

According to the buyers' expected valuations, only buyers with expected value $v(\tilde{s}, x_i) \geq p$ are willing to buy. It was shown that in equilibrium a seller posts a price equal to the expected value of the optimum buyer type who is willing to buy, that is, $s + \tilde{m}(s)$. To calculate the optimum buyer type one needs to calculate the differential equation in (6) by differentiating the seller's payoff from the posted price with respect to $\tilde{s}$ and m.

$$D_2U^{PP} = 1 - F_{(1)}(m) \tag{31}$$

$$D_3U^{PP} = (\gamma s - \tilde{s} - \tilde{J}(m))f_{(1)}(m), \tag{32}$$

where

$$\tilde{J}(m) = m - \frac{1 - F_{(1)}(m)}{f_{(1)}(m)}$$

.

Now the differential equation is equal to

$$s'(m) = \frac{(\gamma s - \tilde{s} - \tilde{J}(m))f_{(1)}(m)}{1 - F_{(1)}(m)}. \tag{33}$$

Solving this differential equation results in

$$s(m) = (1 - F_{(1)}(m))^{\gamma - 1}\left[\int_{\underline{m}}^{m} f_{(1)}(x)(1 - F_{(1)}(x))^{-\gamma}\tilde{J}(x)dx\right]. \tag{34}$$

In this example $\underline{m}(s) = \frac{1}{2}$. The integral in (34) can be solved numerically for a given γ to find the value of s. If $n = 2$, then the seller's expected payoff according to (16) is

$$U^{pp}(s, s, \tilde{m}(s)) = (s + \tilde{m} - \gamma s)(1 - \tilde{m}^2). \tag{35}$$

1.6.4 Payoff Comparison

In this section we compare the expected payoffs for the seller from each of the three mechanisms described. In signalling games it is generally not possible to find an analytic solution for $s(m)$. So first fix any γ, then start with the smallest m in the interval and solve for $s(m)$. After finding a numerical solution for $s(m)$ one can find the expected payoff for the seller.

Figures 1 and 2 show the expected payoffs when γ changes. When $\gamma = 0.33$, the secret reserve price mechanism (top curve) dominates other two, but when γ increases to 1, then the disclosed reserve price or posted-price mechanisms may dominate the secret reserve mechanism. However, in the static model, a disclosed reserve price always dominates a posted price. If there is a higher cost for running an auction than posting a price-which is normally the case in the real world-then one can rationalize the common use of the posted-price mechanism.

1.7 Dynamic Model

In auction theory analysis, models are commonly assumed to be static, like the one in the previous sections. One of the possible extensions of this model is to assume that there is more than one period of time in which the seller can sell the object. The main objective of this section is

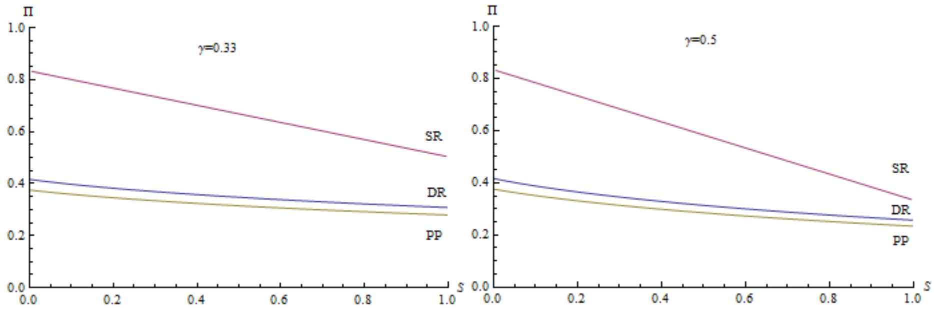

Figure 1: Payoff comparison 1

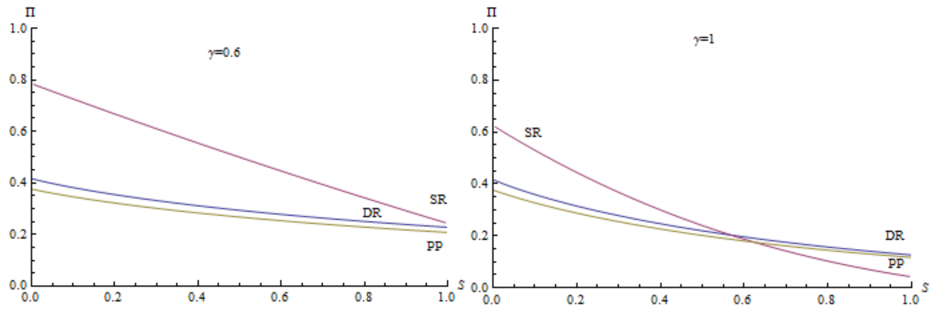

Figure 2: Payoff comparison 2

to analyse how the signalling equilibrium or the secret reserve equilibrium strategy may change if the seller has more than one period of time to sell the object.

To characterise a time horizon for the model, assume that the seller keeps the object in the market for a finite number of periods. If the seller cannot sell it to potential buyers, then she retains the object at its value. The finite-horizon model can also justify the cost of running an auction at each period. The two possible mechanisms for the seller are a posted price or an ascending auction. Once the seller decides to sell via auction or posted price, she cannot change this decision, so she has to retain the same method of sale until the end of the game. This assumption is mainly due to advertising and menu costs, as every time a seller decides to sell via a mechanism she needs to inform buyers through an advertisement.

Assume that if a seller advertises one method of sale, she has to wait long enough to change it in the future. The valuation structures and the signals are exactly the same as in section (1.2). The only difference is that now there are multiple periods in the game. There are T periods in the game, where time is discrete and $t \in \{1, 2, .., T\}$. Assume that at each period t, n potential buyers arrive to the seller. In the present model buyers are impatient so they arrive and leave the market quickly. Further assume that at the end of each period all arrived buyers leave and the seller, if she waits for the next period, faces a new set of buyers. Suppose T is exogenously defined and known to both parties at the beginning of the game as the maximum amount of time for the object to be displayed for sale. T can also be defined endogenously as the time in which the seller's discounted expected payoff becomes equal to her value for the object, but in that case, then the total number of periods itself could be a potential revelation of the seller's information. Therefore, for simplicity assume that T is known and defined exogenously. The seller discounts the future at a rate δ, so the time in which she sells the object is also important. Furthermore, assume that buyers know the history of prices in the game, such as posted prices or disclosed reserve prices.

1.7.1 Dynamic Posted-Price: Constant

In this section we consider a variation of posted-price selling in which the seller cannot change the price during the game. Suppose the seller, before observing her private signal, decides to post a uniform price to sell the object. At this stage, T, which is the maximum number of periods the seller is going to stay in the market, is known, as is the number of bidders at each period. The seller chooses a price with respect to the total number of bidders who arrive until the end of period T, which is $N = nT$. The analysis would be similar to that for the static model if there were no discounting, but because the seller cares when the object sells, this is a different problem.

Because each arriving buyer knows that the seller will never post a price less than her value, the expected value of the object to each buyer is $v(\tilde{s}, x_i) = E[V_i | s = \tilde{s}, s \leq v_0^{-1}(p)]$.

Define $\tilde{m}^D(s)$ as the optimum buyer type who is willing to buy the object at the posted price. If the seller posts a price equal to the expected value of the optimum buyer type who is willing to buy, then the posted price becomes $p^D(\tilde{s}) = v(\tilde{s}, \tilde{m}^D(s))$.

Having the structure of the posted price, the seller maximises her expected payoff with respect to $\tilde{m}^D(s)$. The discounted payoff of every period t becomes

$$U_t^{PD}(s, \tilde{s}, \tilde{m}^D(s)) = (p^D(\tilde{s}) - v_0(s))(1 - F_{(1)}(\tilde{m}^D))\delta^t, \qquad (36)$$

where $F_{(1)}$ is the first-order statistics of n buyers and $1 - F_{(1)}(\tilde{m}^D)$ is the probability that at least one has a value higher than the optimum bidder at that period. Therefore, the present value of the discounted expected payoff for the seller becomes

$$U^{PD}(s, \tilde{s}, \tilde{m}^D(s)) = \sum_{t=1}^{T} U_t^{PD}(s, \tilde{s}, \tilde{m}^D(s))[F_{(1)}(\tilde{m}^D)]^{t-1}. \qquad (37)$$

Substituting (36) into (37) results in

$$U^{PD}(s,\tilde{s},\tilde{m}^D(s)) = (p^D(\tilde{s}) - v_0(s))(1 - F_{(1)}(\tilde{m}^D))\frac{1 - \delta^T F_{(1)}(\tilde{m})^T}{1 - \delta F_{(1)}(\tilde{m})}\delta. \quad (38)$$

The single crossing condition must hold to continue with the possibility of signalling. So far it cannot be said for certain that signalling is possible in this environment, as the indifference curve between $\tilde{m}^D$ and $\tilde{s}$ does not necessarily have a decreasing slope. In section (1.8) we show how the single crossing condition may hold in this environment.

In some markets, like the housing market, sellers may prefer not to change the posted price while they are advertising their object. This may not be an optimal decision for all sellers, but because it occurs in the real. One can make a similar analysis for an infinite-horizon model and check whether the single crossing condition holds and signalling is possible. In fact, a constant posted price makes more sense in those models than in finite-horizon models. In the next section we examine a case in which the seller revises the posted price at each period.

1.7.2 Dynamic Posted-Price: Variable

Suppose at each period the seller revises the posted price specifically for that period. At the beginning of the game the seller observes her signal and then chooses the posted price for period 1. If the object does not sell, the seller can revise the price for the next period. This process continues until period T, which is the final period of the game. At each period, buyers observe the price and decide whether to buy. At the end of each period buyers leave the market, and if the seller has not sold the object, she faces a new set of buyers in the next period. Each buyer i at period

t, after observing the posted price, determines her own expected value for the object given that the seller's signal is $\tilde{s}$, that is, $v(\tilde{s}, x_i) = E[V_i | s = \tilde{s}, s \leq v_{0t}^{-1}(p_t^D)]$, where v_{0t} is the seller's reservation value at period t and is equal to her value at the final period.

At any stage of the game the buyers know the history of the prices as well as T. If the seller posts a price equal to the expected value of the optimum buyer type at each period, then the expected payoff for the seller who posts $p_1^D(s)$ at period 1 becomes

$$U_1^{PD}(s, \tilde{s}, \tilde{m}_1^D(\tilde{s})) = (p_1^D(\tilde{s}) - v_0(s))(1 - F_1(\tilde{m}_1^D)) + \delta F_1(\tilde{m}_1^D) U_2, \tag{39}$$

that is, the probability that the object will be sold at the first period times the net income, plus the discounted expected payoff for the next period if the object is not sold at the first period. To explain the equilibrium one can use the backward induction method by starting from the last period. At the beginning of the final period, because there is no more chance for the seller to sell the object in the future, the expected payoff becomes

$$U_T^{PD}(s, \tilde{s}, \tilde{m}_T^D(s)) = (p_T^D(s, \tilde{s}) - v_0(s))(1 - F_1(\tilde{m}_T^D)). \tag{40}$$

$\tilde{m}_T^D$ is derived exactly the same as in the static model with n buyers, and therefore the equilibrium posted price in the final period is the same as the posted price in the separating equilibrium of the static game. Going one period back, the seller chooses a price to maximise the combined revenue of two periods. This process gives a set of posted prices for each period that maximises the total expected payoff for the seller from the dynamic game. Thus, at every period the seller has the following expected payoff by setting a price for that period:

$$U_t^{PD}(s, \tilde{s}, \tilde{m}_t^D(\tilde{s})) = (v(\tilde{s}, \tilde{m}_t^D(s) - v_0(s))(1 - F_1(\tilde{m}_t^D)) + \delta F_1(\tilde{m}_t^D)U_{t+1}, \tag{41}$$

where F_1 is the first order statistics.

Because $\frac{\partial U_t^{PD}}{\partial \tilde{m}_t^D}$ is increasing in s there is a decreasing slope indifference curve in $\tilde{m}_t^D$ and $\tilde{s}$ plane. Therefore, the single crossing condition holds in every period and signalling is possible. If (41) is differentiated with respect to the second and the third elements, one can form the same differential equation as in (20) now for each period t and calculate the optimum buyer type. At the end of this chapter, in an example, we solve this model for a case with only two periods to show how the price could change.

There are some observations about the separating equilibrium. Here, the seller revises the price at each period, and the optimal decision consists of T posted prices. Intuitively, one can argue that the seller starts with a high posted price and reduces it at every period if she cannot sell the object. Since the number of buyers is similar at each period, if a given seller does not sell the object until period t, the seller has a lower expectation for the remaining periods than what she had before t. Therefore, the seller reduces the price to increase the chance of selling the object in the remaining periods. For the seller to signal her true type in a separating equilibrium buyers must know the full price history and the number of periods in which the seller has been active. If the buyers were patient, then the argument would be different and the seller's optimal decision would change.

1.7.3 Dynamic Auction With Disclosed Reserve

In this section we examine the dynamic game for a seller who chooses an ascending auction with a disclosed reserve price. The seller faces T periods, and at each period n buyers arrive and leave at the end of the period. Therefore, at each period the seller only faces the buyers who have arrived at the market at that period. Although the number of arrivals at every period is equal, it may not be optimal for the seller to keep the reserve price constant if she has a chance to sell the object in the next periods. At each period, upon observing the reserve price, buyers make their believes $\hat{s}$ for the seller's signal. Then each buyer i stays active until the price equals her expected value, given that the seller's signal is $\hat{s}$.

The seller knows that at every period $t < T$, if the object has not sold, there is another chance to sell in the next period. At period T there is no more chance for the seller to run another auction if the object has not sold, so the seller sets a reserve price the same as in a one-shot game. Therefore, at the final period there is the same optimal decision as in the static game. Define $m_t(.)$ as the minimum buyer type who enters the auction given the reserve price at that period. At the first period the seller's expected payoff becomes

$$\begin{aligned} U_{t=1}^{DRD}(s, \hat{s}, m_1) = & \int_{m_1}^{\bar{\omega}} [v(t, \hat{s}) - v_0(s)] f_{(2)}(t) dt \\ & + [F_{(2)}(m_1(\hat{s})) - F_{(1)}(m_1(\hat{s}))] \big(v(m_1(\hat{s}), \hat{s}) - v_0(s)\big) \\ & + F_{(1)}(m_1(\hat{s}))(\delta U_{t=2}), \end{aligned} \tag{42}$$

where $F_{(1)}$ and $F_{(2)}$ are the distribution of the first- and second-order statistics of the buyers' signals and, because the number of buyers is the same at each period, are the same for every period. The last element in (42) is the probability that no one has a value higher than the reserve price multiplied by the discounted expected utility of the next period. In equilibrium the seller posts a reserve price equal to the expected value of the minimum buyer type at that period, which is not the same for every period. One first needs to differentiate (42) with respect to $\hat{s}$ and $m_1(\hat{s})$, which results in

$$\begin{aligned} D_2U_{t=1}^{DRD}(s,\hat{s},m_1) = \frac{\partial v}{\partial \hat{s}}(F_{(2)}(m_1) - F_{(1)}(m_1)) + \int_m^{\bar{\omega}} \frac{\partial v}{\partial \hat{s}} dF_2(x) \\ + \delta F_{(1)}(m_1)\frac{\partial (U_{t=2})}{\partial \hat{s}} \end{aligned} \tag{43}$$

$$D_3U_{t=1}^{DRD}(s,\hat{s},m_1) = f_1(m_1)(\delta U_{t=2} - J(\hat{s},m_1)) + F_1(m_1)\frac{\partial(\delta U_{t=2})}{\partial m_1}. \tag{44}$$

The last element in (42) is independent of s, so $\frac{\partial U_{t=1}^{DRD}}{\partial m_1(\hat{s})}$ is increasing in s, and, similar to the static model, the single crossing condition holds. To find an expression for the equilibrium of this game, start from the final period and optimise the seller's behaviour backward until the first period. In the final period the seller's expected payoff is

$$\begin{aligned} U_T^{DRD}(s,\hat{s},m_T) = \int_{m_T}^{\bar{\omega}} [v(t,\hat{s}) - v_0(s)] f_{(2)}(t)dt \\ + [F_{(2)}(m_T(\hat{s})) - F_{(1)}(m_T(\hat{s}))]\big(v(m_T(\hat{s}),\hat{s}) - v_0(s)\big). \end{aligned} \tag{45}$$

The same differential equation as in (6) characterises the separating equilibrium of this period, and $m_T(.)$ is the same as $m(.)$ in the static model. One period before T, the seller knows the discounted expected payoff for the next period and thus chooses m_{T-1} such that it maximises her expected payoff from both periods. This backward method continues until the first period, where the seller calculates $m_1(.)$ by (43) and (44).

Therefore, at every period t, the following differential equation characterises the separating equilibrium for that period, and a set of T differential equations characterises the separating equilibrium of the game:

$$\frac{ds}{dm_t} = -\frac{D_3 U_t^{DRD}(s, s, m_t(s))}{D_2 U_t^{DRD}(s, s, m_t(s))}. \tag{46}$$

In section 1.8 in an example with two periods and linear values we show how the seller optimises her payoff by disclosing a reserve price at each period.

1.7.4 Dynamic Auction With Secret Reserve

For the sake of comparison, suppose the seller decides to sell the object via an ascending auction with a secret reserve price, like the one in the static model, but within T periods of time. Assume that when the seller decides to use this mechanism she has not observed her signal yet. The game has the same sequence as the static version, except that here if the seller does not accept the auction price at any period $t < T$, she has another chance in the upcoming period to run another auction.

Because there is more than one period in this model, the seller's optimal decision at each period is to accept the auction price if it is higher than the reservation value. Like with the other dynamic auction, backward induction can be used to characterise the seller's optimal decision. Starting from the last period, the seller's optimal decision is to accept any offer greater than her valuation for the object, as there are no more periods left. At period $T - 1$, the optimal decision is to accept the highest bid if it is higher than the seller's value and the expected payoff from the last period, and so on until the first period. Therefore, the seller's optimal reserve price for this dynamic game is no longer her value for the object, and it varies for every period. At every period t, the seller's secret reserve price R_t, or the minimum price she will accept for the object, is

$$R_t = Max(v_0(s), \delta U_{t+1}), \tag{47}$$

which is the maximum discounted expected payoff for the next period and her value. This necessarily becomes the seller's value at the last period. If $\hat{m}_t(.)$ as is the minimum buyer type who is willing to bid in the auction at every period t, then

$$\hat{m}_t(s) = \inf\{x : \beta_{SR}(x) \geq R_t\}. \tag{48}$$

Having the minimum buyer type at each period, the seller's expected payoff at period 1 is equal to

$$U^{SRD}_{t=1}(s, \hat{m}(s)) = \int_{\hat{m}_1(s)}^{\bar{\omega}} [\beta_{SR}(x) - v_0(s)] f_{(2)}(x)dx + F_{(2)}(\hat{m}_1(s))(\delta U_{t=2}). \tag{49}$$

The same method used in the previous section can be used here to characterise the equilibrium of the game. Starting from the last period, the seller's optimal reserve price and the minimum buyer type are the same as in the static model with n bidders. Continuing this method until period 1 gives a set of T reserve prices and minimum buyer type functions that characterises the equilibrium of this game. The argument for the secret reserve is much simpler than the one for the disclosed reserve. In fact, the key change here compared to the same method of sale in the static model is the set of different reserve prices. Because the seller does not disclose any information at any period, buyers are not able to update their beliefs about the seller's signal. Of course, one may assume that buyers are not aware of the auction prices for the previous periods. Therefore, the bidding function is the same, but instead of the seller's value we have the seller's reserve price for each period.

1.8 Example: Dynamic Model

In this section we compare the three selling mechanisms for a dynamic game for a linear valuation example. For simplicity suppose there are only two periods, and two buyers arrive at each period and then leave at the end of the period. The seller discounts the future at δ. There are two choices of mechanism for the seller-ascending auction and posted price-and two different reserve price regimes for the auction. All other assumptions are as in section (1.6). For simplicity we assume at every period n two buyers arrive and leave at the end of the period.

Suppose the seller decides to post a price at period 1 and does not change it until the end of the game. According to (38)

$$U^{PD}(s, \tilde{s}, \tilde{m}(s)) = (p^D(\tilde{s}) - v_0(s))(1 - F_{(1)}(\tilde{m}))\frac{1 - \delta^T F_{(1)}(\tilde{m})^T}{1 - \delta F_{(1)}(\tilde{m})}\delta, \tag{50}$$

which for this example becomes

$$U^{PD}(s, \tilde{s}, \tilde{m}(s)) = (\tilde{s} + \tilde{m}(s) - v_0(s))(1 - F_{(1)}(\tilde{m}))(1 + \delta F_{(1)}(\tilde{m}))\delta. \tag{51}$$

Differentiating the seller's payoff with respect to $\tilde{s}$ and $\tilde{m}$ produces

$$D_2U^{PD}(s, \tilde{s}, \tilde{m}(s)) = \frac{\partial U^{PD}}{\partial \tilde{s}} = (1 - F_{(1)}(\tilde{m}))(1 + \delta F_{(1)}(\tilde{m}))\delta \tag{52}$$

$$
\begin{aligned}
D_3U^{PD}(s,\tilde{s},\tilde{m}(s))\frac{\partial U^{PP}}{\partial \tilde{m}} =&(1-F_{(1)}(\tilde{m}))(1+\delta F_{(1)}(\tilde{m}))\delta \\
&-f_{(1)}(\tilde{m})(\tilde{s}+\tilde{m}(s)-v_0(s))(1+\delta F_{(1)}(\tilde{m}))\delta \\
&\delta^2 f_{(1)}(\tilde{m})(\tilde{s}+\tilde{m}(s)-v_0(s))(1-F_{(1)}(\tilde{m})).
\end{aligned} \tag{53}
$$

Because D_3U is increasing in s, the single crossing condition holds here. After some algebra it can be shown that the following differential equation characterises a separating equilibrium of this game:

$$
m'(s) = \frac{f_{(1)}(\tilde{m})(\tilde{s}+\tilde{m}(s)-v_0(s))}{(1-F_{(1)}(\tilde{m}))} - \frac{\delta f_{(1)}(\tilde{m})(\tilde{s}+\tilde{m}(s)-v_0(s))}{1+\delta F_{(1)}(\tilde{m})} - 1. \tag{54}
$$

As mentioned before, keeping the posted price constant is not optimal for the seller. We study this problem mainly to show how the seller's behaviour might change if she has to keep the price constant for both periods.

The next step is to consider the case in which the seller revises the price if the object does not sell in the first period. As shown in section (1.7.2), the seller's expected payoff at the final period is the same as in the static game. For this example, because there are only two periods, the seller's expected payoff at period 2 becomes

$$
U_2^{PD}(s,\tilde{s},\tilde{m}_2(s)) = (\tilde{s}+\tilde{m}_2(s)-v_0(s))(1-F_1(\tilde{m}_2)). \tag{55}
$$

The derivation of $\tilde{m}_2$ is exactly as in section (1.6.3), so the following equation gives the inverse function

$$
s(m_2) = (1-F_{(1)}(m))^{\gamma-1}\left[\int_{\underline{m}}^{m} f_{(1)}(x)(1-F_{(1)}(x))^{-\gamma}\tilde{J}(x)dx\right], \tag{56}
$$

where $\underline{m}(s) = \frac{1}{2}$. The integral in (56) can be solved numerically for any given $0 < \gamma \leq 1$ to find the value of s. For $n = 2$ the seller's expected payoff becomes

$$U_2^{PD}(s, s, \tilde{m}(s)) = (s + \tilde{m}_2 - \gamma s)(1 - \tilde{m}_2^2). \tag{57}$$

Having the payoff and $\tilde{m}_2$ for period 2, one can write the expected payoff for period 1 as follows:

$$U_1^{PD}(s, \tilde{s}, \tilde{m}_1(s)) = (\tilde{s} + \tilde{m}_1(s) - v_0(s))(1 - F_1(\tilde{m}_1)) + F_1(\tilde{m}_1)(\delta U_2^{PD}). \tag{58}$$

Now suppose the seller decides to sell via an ascending auction with a disclosed reserve price. Starting from the second period, because it is the last period, the seller's expected payoff is

$$\begin{aligned} U_{t=2}^{DRD}(s, \hat{s}, m_2) =& \gamma s(F_{(1)}(m_2) - 1) + \hat{s}(1 - F_{(1)}(m_2)) + \\ & m_2(F_{(2)}(m_2) - F_{(1)}(m_2)) + \int_{m_2}^{1} x dF_{(2)}(x), \end{aligned} \tag{59}$$

which is the same as in the single-period model. Thus, all other steps for calculating the minimum buyer type are straightforward. At the first period the seller's payoff becomes

$$\begin{aligned} U_{t=1}^{DRD}(s, \hat{s}, m_1) =& \gamma s(F_{(1)}(m_1) - 1) + m_1(F_{(2)}(m_1) - F_{(1)}(m_1)) \\ & + \hat{s}(1 - F_{(1)}(m_1)) + \int_{m_1}^{1} x dF_{(2)}(x) + F_1(m_1)(U_{t=2}^{DRD}). \end{aligned} \tag{60}$$

Differentiating (60) with respect to m_1 and $\hat{s}$ produces

$$\frac{\partial U_{t=1}^{DRD}}{\partial m_1} = (\gamma s - \hat{s} - J(m_1))f_1(m_1) + f_1(m_1)(U_{t=2}^{DRD}) \tag{61}$$

$$\frac{\partial U_{t=1}^{DRD}}{\partial \hat{s}} = 1 - F_1(m_1) + F_1(m_1)(1 - F_1(m_2)). \tag{62}$$

Thus,

$$s'(m_1) = -\frac{(\gamma s - \hat{s} - J(m_1))f_1(m_1) + f_1(m_1)(U_{t=2}^{DRD})}{1 - F_1(m_1) + F_1(m_1)(1 - F_1(m_2))}. \tag{63}$$

The solution to the differential equation in (63) gives the minimum buyer type who is willing to bid in period 1. If $a = U_{t=2}^{DRD}$ and $b = F_1(m_2)$, then the solution is

$$s(m_1) = (1 - bF_{(1)}(m_1))^{\frac{\gamma-1}{b}} \left[\int_{\underline{m}}^{m_1} f_{(1)}(x)(1 - bF_{(1)}(x))^{\frac{1-\gamma-b}{b}} (J(m_1) + a)(x)dx \right]. \tag{64}$$

For any given $0 < \gamma \leq 1$ the solution to the above integral gives the inverse function of the minimum buyer type in the first period, which is obviously different than the one in the second period. This results in different equilibrium reserve prices for each period, as shown before.

1.9 Conclusion

When the seller of an object has private information about its value and this information is important to potential buyers, as long as there is no costless method of communicating this information the problem of adverse selection arises. In this case, the seller's revelation of information is not credible to the buyers. In this environment, signalling is one credible method of revealing information. Here we have studied three different selling mechanisms with different degrees of information revelation. We show that under some conditions signalling is possible for two of these mechanisms, and there exists a unique separating equilibrium in which the seller reveals her true signal via a reserve price or a posted price. We show that in a one-shot static model, it is optimal for some sellers, *ex ante* and before observing the signal, not to reveal any information. In fact, if the seller's value is a function that results in a smaller value than her actual signal, the secret reserve auction, which includes no information revelation, can dominate the other two mechanisms, from the seller's point of view. However, if the value of the seller's signal is higher, there is usually less of a chance having a higher *ex ante* expected revenue by not revealing her true type.

For the dynamic setting, we present a finite-horizon model in which the seller has more than one chance to sell the object. In the model, a number of buyers arrive at the beginning of each period and leave at the end. We show under what conditions signalling is still possible and how the seller's signalling strategy changes. For instance, in an auction with a disclosed reserve price, the seller starts with a high reserve price in the first period and gradually reduces this reserve until the last period, when the reserve price is the same as in the static model. Buyers have the complete history of the reserve prices and the number of periods in this model.

If the seller decides not to reveal any information, then her secret reserve price is no longer equal to her value. It is equal to the maximum of the seller's value and the expected payoff from the remaining periods. Thus, even if the seller decides not to disclose the reserve price, her reservation value is higher at the beginning of the game, and in the last period it is equal to the seller's value for the object. In fact, in the dynamic model the auction with a secret reserve has an advantage in terms of the menu cost, as the seller does not reveal any price and does not need to publicly revise the price.

We have also studied two versions of the dynamic posted-price mechanism, one in which the seller keeps the price constant at every period and the other in which the seller revises the price at each period. Keeping the price constant is not the optimal strategy for the seller in this environment.

We have focused here on markets in which buyers arrive and leave on average sooner than sellers, such as the housing market. In these markets, given the higher cost of an auction compared to a posted price, if the length of the period in which the buyers leave becomes smaller, the number of auctions has to increase to continue the signalling game. The price must be revised more often in this case. Therefore, some sellers may decide to change not the posted price but the selling mechanism. In the third chapter we examine a selling mechanism in which the seller may not revise the posted price but engages in negotiation with interested buyers.

In conclusion, if a seller expects a high number of buyers to arrive at each period, and buyers are more patient, then running an auction is generally more preferable than posting a price. However, if there is only a small number of buyers at each period, and the buyers are impatient, then running an auction may not be the best option considering the cost.

Finally, throughout this chapter, the assumption has been that the seller observes her signal after the selling mechanism has been chosen, so the mechanism chosen does not reveal any information to buyers. In fact, the analysis becomes more interesting if the seller, after observing her signal, chooses one mechanism from a set of available mechanisms to sell the object. The next chapter studies the case in which the seller observes her signal and then decides which selling mechanism to choose in a one-shot game.

Chapter 2

Interim Analysis of Auctions With Informed Sellers

2 Interim Analysis of Auctions With Informed Sellers

2.1 Introduction

In the previous chapter we examined some representative selling mechanisms from the point of view of an informed seller. The environment is such that the seller of an indivisible object has private information about the attributes of the object. Potential buyers care about this information, but there is no costless method for the seller to reveal the information. We argued that in this situation signalling is a credible method for the seller to reveal this private information. We studied an auction game format in which the seller signals her true type in a unique separating equilibrium, that is, an open ascending auction with a disclosed reserve price. We also studied a variation of this same auction in which the seller does not disclose the reserve price but retains the right to accept or reject the auction price after bidding finishes. As we discussed, after the seller observes her signal, any subsequent actions on the seller's part could potentially act as a signal. In the previous chapter we considered a case in which the seller observes her signal after choosing the selling mechanism or the reserve price regime. However, if the seller observes her signal before choosing the selling method, then the selling method itself could potentially reveal some information about the seller's private signal.

One of the very first studies of this problem in the mechanism design literature is Myerson (1983). Myerson claims that there exists a set of the principal's neutral optima of unblocked mechanisms, and any neutral optima is an equilibrium and in the core. In general, it may be possible to find a set of mechanisms that are the principal's optima and in the core, but here we look at this problem from another point of view. Suppose the principal or the seller does not have the option to design a mechanism but does have a choice of some variety of selling methods. Then the question is how the seller optimises her revenue in this setting. The answer to this question is strongly related to the selling methods that are available to the seller. We focus here on two reserve price regimes examined in the previous chapter and characterise the seller's revenue-maximising decisions.

In most of the classic literature on auction theory, the seller does not have private information about the object, and privately informed bidders compete with one another. Myerson (1981) shows that in those types of environments-known as independent private value models-the optimal mechanism is a sealed-bid second-price auction with an optimally chosen reserve price. But in many real-world examples sellers have some private information about the objects they are selling. We differentiate our model by relaxing two main assumptions of the IPV models. First, the seller has a private signal for the object and a value that is increasing in her signal. Second, buyers care not only about their own private signal but also about the seller's signal in an increasing manner. Therefore, the results of IPV models no longer apply in this setting, mainly because of the interdependence of the buyers' values.

The related literature is much the same as for the previous chapter. To our knowledge, Kremer and Skrzypacz (2004) is the only paper closely related to this chapter. It is a working paper with the same objective as we have here. They study an informed seller's best interest within various standard auctions. Their results indicate that when sellers are informed, different types choose different auction formats. High types prefer English auctions and low types prefer sealed-bid auctions. Their model is divided into two parts-private values and common values-which is different from the current study. Moreover, we entirely focused on the open ascending auction, whereas in their model the seller can choose from among some standard auctions. We mainly study the reserve price regime choice rather than the auction choice.

Skreta (2011) studies the optimal level of information disclosure from an informed seller's point of view. The results indicate that for independent private value models information disclosure is irrelevant, and the revenue-maximising mechanism results in the same revenue as the full information optimal mechanism. The informed seller in their model has information about the individual buyers' signals, and buyers observe only their own private signals. This paper is different from the current study in the sense that in the present model the seller has a private signal for the object that is payoff relevant for her and that the buyer's value increasingly depends on that signal.

The model is discussed in Section 2.2. We begin the analysis this chapter with a motivating example in Section 2.3 to describe the possible equilibria of the game. Then we extend it to consider some results for linear valuation models, and finally we generalise the model in Section 2.4 and study the conditions in which the results may still hold.

2.2 Mechanisms and the Reserve Price Regimes

Consider almost the same model as in the previous chapter: The seller of an indivisible object faces $N = \{2, \ldots, n\}$ potential buyers. The seller observes a private signal s which is drawn from a distribution G with support $[0, 1]$, twice differentiable with a continuous density g. Each buyer i privately observes a signal x_i for the object that is independently and identically distributed on $F \in [0, 1]$, twice differentiable with continuous density f. Each buyer i's valuation is a symmetric and continuous function with the format $v : [0, 1] \times [0, 1] \to \mathbb{R}_+$. This function is an increasing function of the buyer's private signal as well as the seller's signal. The seller has a value for the object with the functional format $v_0 : [0, 1] \to \mathbb{R}_+$ which is a continuous and increasing function of her own signal. This environment has a special case of interdependent values as discussed in the previous chapter. Therefore, in this setting the English auction and the open ascending auction are strategically equivalent. The results may stand for both auctions, but we fix the standard setting of the model to the open ascending auction. We further assume that the *hazard rate* function of $F(.)$ is increasing and that the valuation functions are weakly concave.

Consider a seller who is willing to sell her object in an open ascending auction with two different reserve price regimes like in the previous chapter: first, disclosing the reserve price at the beginning of the auction and before the bidding starts; and second, keeping the reserve price secret forever and revealing no extra information to the buyers. If the seller decides to disclose the reserve price, she has to commit to it, meaning that the seller will accept any auction price that is higher than the reserve price.

Otherwise, if she chooses not to disclose the reserve price, then when there is only one active bidder left, it is the seller's choice to accept or reject the auction price. Call the first regime the disclosed reserve price (DR) and the second one the secret reserve price (SR).

The steps of the game are as follows: First the seller observes her signal, and then she chooses between the two reserve price regimes. The seller then announces the reserve price regime publicly. If the seller has chosen DR she also announces the reserve price; if it is SR the seller announces no further information. Finally, the bidding starts and continues until there is only one bidder left. At this point the stages are exactly the same as in the previous chapter. For the DR the last active bidder wins the object and pays the auction price and for the SR the seller either accepts or rejects the auction price. If she accepts the bidder wins the object and pays the auction price, otherwise the seller retains the object. We start with a motivating example to explain the game with only two bidders in a simple way.

2.3 A Motivating Example

As an example, consider a case in which the seller's valuation for the object is equal to her signal, i.e., $v_0(s) = s$, and the buyers' valuations are symmetric and linear with the following format: $v(x_i, s) = x_i + s$. Also assume that all signals are distributed uniformly from $[0, 1]$. Figure 3 shows the *interim* expected payoffs from both regimes according to equations (3) and (12) in the previous chapter when there are only two buyers.

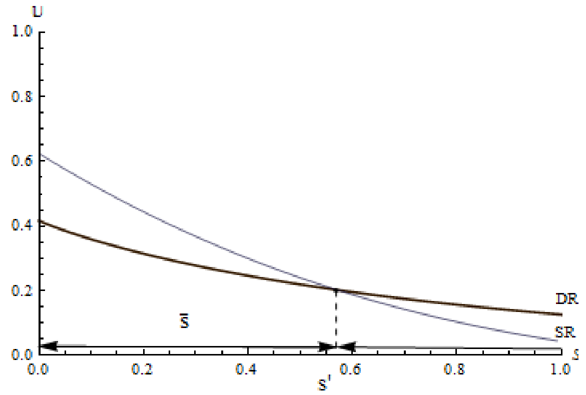

Figure 3: *Interim* payoffs: DR vs SR

2.3.1 Interim Equilibrium Analysis

In this section we analyse the seller's behaviour at the *interim* level. A seller, after observing her signal, must choose between the two reserve price regimes described previously. For the purpose of analysis, suppose the valuations are like the above example. According to Figure 3, the seller's expected revenue has been shown for any given signal. Now suppose the seller chooses the reserve price regime after observing her signal. In this situation the choice of regime itself reveals information to the buyers-that is, the chosen regime must have an expected revenue for the seller at least as high as the other regime. This information could affect the bidding behaviour of the buyers. We continue with this argument, which will be useful for the equilibrium analysis.

According to Figure 3, at the beginning of the game, if the seller observes a signal less than s', then she knows her expected payoff will be higher if she chooses not to disclose the reserve price. But buyers also know that if the seller chooses to keep the reserve price secret, then her signal must be less than s'. Let's focus on a marginal seller who has a signal less than s'. This seller knows that at that signal her *interim* expected payoff will be higher if she chooses the secret reserve. But because the seller chooses the reserve price regime after observing her signal, buyers will figure out that any seller who chooses a secret reserve regime must have a signal less than s'. Thus, buyers form an expectation of the seller's signal between $[0, s']$, which is, for example, $\bar{s}$ and strictly less than the seller's actual signal. The bidding function changes according to the new expectation for the seller's signal, and the expected payoff shifts to the bottom left (Figure 4). Thus, all sellers with signals between $[s'', s']$ are better off choosing to reveal their type via a reserve price.

Now buyers know that a seller with a signal between $[s'', s']$ will also choose a disclosed reserve price because of higher expected payoffs. Thus, if a seller with a signal slightly less than s'' chooses a secret reserve, buyers will bid according to an expectation of that signal being between $[0, s'']$. This will result in even lower bids, and the expected payoff will shift further to the bottom left. Continuing this argument leads to the conclusion that all seller types are better off choosing to disclose their reserve price at the beginning of the game except type $s = 0$, who is indifferent to either regime.

Proposition 2.1. *With the linear valuations and uniform signals on* $[0, 1]$ *with only two buyers, all seller types with positive signals will choose the disclosed reserve price to sell their object after observing their signal.*

Proof. See Appendix

This result gives an advantage to the regime in which the seller dis-

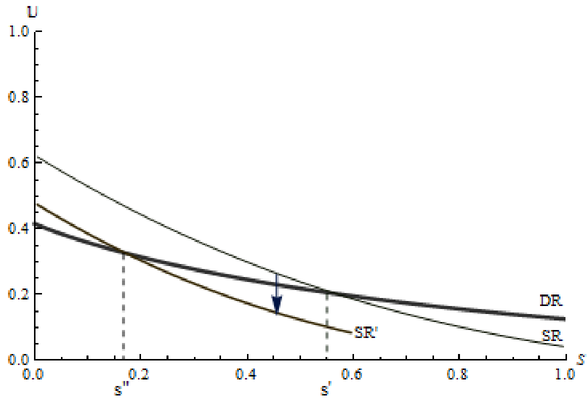

Figure 4: *Interim* payoffs: DR vs SR

closes her information, although most times in the real world the opposite situation arises: Sellers with private information try to keep it secret if costless access to the information is not possible. Several assumptions we made are to simplify the calculations and may not hold in general. We first relax the assumption of two bidders and increase the number of bidders to observe the effects on the previous results.

Figure 5 shows the *interim* payoff for the seller when the number of bidders increases.

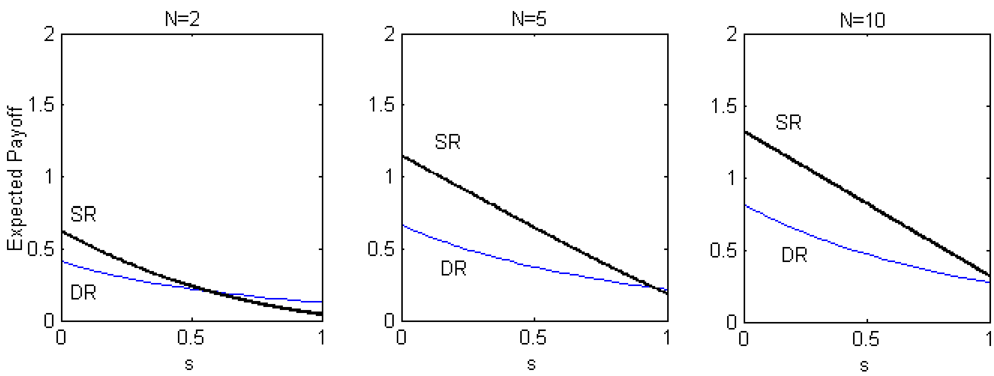

Figure 5: *Interim* payoffs: DR vs SR

According to Figure 5, the previous argument is true as long as there is an intersection between the expected payoffs from the two regimes. When the number of bidders increases to 10, the expected payoff in the secret reserve regime dominates that in the disclosed reserve price regime for all seller types in the entire interval. In this situation, sellers are better off keeping the reserve price secret. And because all types choose the secret reserve, buyers' expectations for the seller's signal will not be affected by the choice of regime, and therefore the bidding function will not change.

Observation 1. If one regime dominates the other for all signals, then at the interim level the choice of mechanism does not reveal any new information to buyers.

This is because the optimal decision for all seller types will be the same before and after observing the signals, and therefore it does not reveal any extra information to buyers. One can now conclude that with the linear valuations and with signals uniformly distributed on $[0,1]$, all seller types in the interval will have a higher payoff by not revealing the reserve price when the number of bidders is large enough, that is, more than 10.

2.4 Disclosed Versus Secret Reserve Price

To generalise the previous results it is necessary to investigate how the *interim* expected payoffs from both regimes change when the seller's signal changes. Differentiating (3) with respect to s and using the Envelope Theorem results in

$$\begin{aligned}D_1U^{DR}(s,\hat{s},m(\hat{s})) &= D_2U^{DR} + (D_3U^{DR}m'(s)) \\ &\quad - v_0'(s)[1-F_1(m(s))] \\ &= -v_0'(s)[1-F_1(m(s))] < 0.\end{aligned} \tag{65}$$

Thus, the expected payoff from this regime is strictly decreasing in the seller's signal. Differentiating (65) another time with respect to s results in

$$D_{11}U^{DR}(s,\hat{s},m(\hat{s})) = -v_0''(s)[1-F_1(m(s))] + m'(s)f_1(m(s))v_0'(s) > 0. \tag{66}$$

The second term in (66) is clearly positive, and the first term is also positive as long as the seller's valuation function is weakly concave. Therefore, the net expected payoff for the seller is strictly decreasing and convex in her signal.

Using the fundamental theorem of calculus and the result in (65), one can represent the seller's expected payoffs in another useful way, that is,

$$U^{DR}(s,s,m(s)) = U^{DR}(0,0,m(0)) - \int_0^s [1-F_1(m(x))]v_0'(x)dx. \tag{67}$$

Differentiating the seller's expected payoff in the secret reserve price regime in (12) with respect to s gives

$$\begin{aligned}D_1U^{SR}(s,\hat{m}(s)) &= -[\beta_{SR}(\hat{m}(s)) - v_0(s)]f_2(\hat{m}(s))\hat{m}'(s) \\ &\quad - v_0'(s)[1-F_2(\hat{m}(1)) + F_2(\hat{m}(1)) - F_2(\hat{m}(s))] \\ &= -v_0'(s)[1-F_2(\hat{m}(s))] < 0.\end{aligned} \tag{68}$$

By the definition of $\hat{m}(s)$, the first term becomes equal to zero. Thus, the net expected payoff from the secret reserve price regime is also strictly decreasing in the seller's signal. The second differentiation would also result in

$$D_{11}U^{SR}(s, \hat{m}(s)) = -v_0''(s)[1 - F_2(\hat{m}(s))] + \hat{m}'(s)f_2(\hat{m}(s))v_0'(s) > 0. \qquad (69)$$

Again, as long as the seller's valuation function is concave, the second-order derivative of the seller's expected payoff with respect to s is strictly positive. Furthermore, the fundamental theorem of calculus can be applied to the result in (68) to find another useful way to represent the seller's expected payoff from the secret reserve price regime:

$$U^{SR}(s, \hat{m}(s)) = U^{SR}(0, \hat{m}(0)) - \int_0^s [1 - F_2(\hat{m}(x))]v_0'(x)dx. \qquad (70)$$

Lemma 2.1. *The minimum buyer type who bids in the disclosed reserve price auction $m(s)$ is greater than or equal to the minimum buyer type who bids in the secret reserve price auction $\hat{m}(s)$.*

Proof. The proof is straightforward. By the definition of $\hat{m}(.)$ in (11), the minimum buyer type who bids in the secret reserve auction is the one with an expected value higher than the seller's value, whereas by the definition of $m(.)$ function, the minimum buyer type who bids in the disclosed reserve auction is the one with an expected value equal to the reserve price, which is at least as high as the seller's value. □

The result in Lemma (2.1) suggests that on average more bidders bid in the SR auction than the equilibrium of the DR auction. This is because bidders have no extra information about the seller's signal in the SR auction, and there are more types with positive probability to clear the reserve price. This is the main reason why SR becomes more interesting to the seller with a higher number of bidders. In DR a higher number of bidders requires a higher reserve price[6], but in SR the reserve price is independent of the number of bidders.

Proposition 2.2. *When each regime is considered separately, a seller with a signal equal to zero has a higher expected payoff from the secret reserve price regime than the disclosed reserve price regime.*

Proof. See Appendix

This proposition considers each regime separately as if the other regime did not exist. This is because of the equilibrium analysis and is helpful for finding the seller's optimal decision at the interim level. In fact, if both regimes are analysed together, the only thing that might change is the buyers' beliefs about the seller's signal, not their equilibrium bidding function.

Proposition 2.3. *When DR and SR are the only two selling options, if the highest seller type in the interval has a higher expected payoff from DR than SR, then in equilibrium all seller types after observing their signals are better off choosing the DR regime except the lowest type, which is indifferent to either regime.*

Proof. See Appendix

[6]This was proven in Cai, Riley, and Ye (2007), Theorem 2, and the result is applicable to the present DR auction

This result is for cases in which some types, including the highest seller type, have a higher expected payoff by disclosing their reserve price. Then it can be concluded that all other types will also disclose the reserve price. Indeed, this scenario occurs when the two expected payoffs intersect each other once in the interval. We next show the necessary and sufficient condition for a seller to choose the secret reserve price regime after observing her signal.

Proposition 2.4. *When DR and SR are the only two selling options, a seller with a positive signal s, after observing her signal, chooses SR to sell the object if and only if the highest seller type in the interval has a higher expected payoff from SR than DR.*

Proof. See Appendix

The result in proposition 2.4 rules out situations with multiple intersections between two expected payoffs, such as the one in Figure 6. In fact, because $m(.)$ is strictly increasing in s, as long as the expected payoff from DR is higher than that from SR for one signal, it will be higher for all higher signals. Thus, the payoffs will never intersect more than once during the interval. Now a seller with a positive signal, after observing her signal, only cares about the expected payoffs of the highest type in the interval when deciding whether to choose a secret reserve price or disclosed reserve price.

2.5 Conclusion

In this chapter we have studied the behaviour of an informed seller facing an ascending auction with two different reserve price regimes. Studying

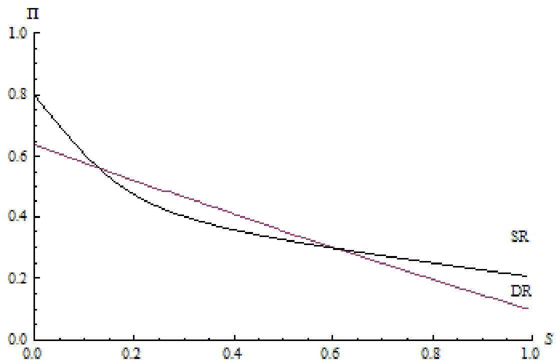

Figure 6: An impossible outcome

each regime separately suggests that one results in an equilibrium in which the seller reveals her true type, via the reserve price, and the other results in no information disclosure by the seller. These two extremes result from either disclosing the reserve price or keeping it secret. We show the conditions under which all seller types choose the disclosed reserve price regime. Because the net expected payoffs from both regimes are strictly decreasing in the seller's signal, as long as the highest seller type has a higher expected payoff from revealing her information, then lower types must make the same decision in the *interim* stage. Otherwise, buyers update their information by the choice of mechanism and lower their expectations for the seller's signal, which results in a lower payoff for the seller. We also show the necessary and sufficient conditions under which all seller types choose the secret reserve price. The results suggest that an informed seller chooses a secret reserve price if and only if the highest seller type in the interval has a higher payoff from the secret reserve price than the disclosed reserve price. Every equilibria studied in this chapter suggests that signalling is not possible with the choice of the reserve price regime. Therefore, given the distributions of the signals and the number of bidders, all seller types choose the same reserve price regime to sell the object after observing the signal.

In this chapter, only two selling methods were available to the seller; other researchers can extend this analysis to more options for the seller, for instance standard auctions like those studied by Kremer and Skrzypacz (2004). The certain conclusion, however, is that as long as there are only two selling mechanisms available to the seller and one is dominated by the other, the seller will make the same choice after observing her signal as before observing it. It appears from the analysis that the extraction of a general result is very complicated for more than two selling mechanisms. However, we suggest that a parallel analysis of standard auctions is worth pursuing.

Chapter 3

Asking Price and the Housing Market

3 Asking Price and the Housing Market

3.1 Introduction

There are several markets in which the seller of an indivisible object posts a price to attract potential buyers to make an offer. In these markets, sometimes the posted price acts as a commitment device from the seller, and sometimes it is just a guide for further negotiations. Examples of such markets are legion, but the one most relevant to the present study is the housing market. In several countries, such as the United States and Australia, one of the most popular selling methods in the housing market is when the seller of a property posts an asking price to attract offers from potential buyers over time. After negotiation between the seller and buyers, the sale price for the object is sometimes lower than the asking price, sometimes the same, and sometimes higher. So what is the role of the asking price in this specific market? Some researchers (Horowitz (1992) and Chen and Rosenthal (1996a), among others) argue that the asking price acts as a ceiling price or a commitment device from the monopolist. Yet these theoretical models do not result in a convincing argument for those cases in which the sale price is higher than the asking price. Indeed, a theoretical model that explains the role of the asking price and at the same time predicts all possible outcomes with respect to that asking price has not been studied before. The purpose of this analysis is to examine a selling mechanism that has a role for asking price and that at the same time can result in sale prices higher than the asking price. In this case the asking price no longer functions as a ceiling price or a commitment device from the seller.

Several researchers have attempted to study the behaviour of sellers and buyers in the housing market from a theoretical point of view, and some have also tested their models empirically. From a theoretical point of view, the behaviour of parties in the housing market has been mainly studied using search theory. Some papers have studied one-sided search models, in which only one party, mainly the seller, is searching for potential traders, whereas others have attempted to study two-sided search models. Two-sided search models are more complicated in terms of the equilibrium analysis because there are two active parties in the game. This makes it even more difficult to empirically study two-sided search models.

Yinger (1981) is one theoretical study of the real estate market. This paper studies the search behaviour of a real estate broker when there is uncertainty about the number of buyers and available listings. The role of the real estate broker in this market is to find matches between buyers and sellers. Yinger (1981) also studies the behaviour of real estate brokers in the Multiple Listing Service in the United States. They claim the Multiple Listing Service increases the efficiency of the market and reduces commissions. Some studies have mainly focused on the strategic role of the asking price in the housing market. Horowitz (1992) attempts to model and estimate the behaviour of a seller in the housing market. He considers an infinite-horizon stationary search framework in which a seller posts an asking price and waits for offers from potential buyers who arrive over time. The asking price in this model acts as a ceiling price or a commitment device from the seller. Thus, at any time during the game, if a buyer asks to buy the object at the asking price, the seller accepts the offer. Horowitz (1992) finds the optimal asking price and the reservation price when the price offers are drawn from a known distribution. Consequently, it is not optimal for a seller to vary the asking price over time, which can explain why a seller who has not sold her house for a long time may not change the asking price. Finally, he estimates the parameters of the model using data on list price, transaction price, and time on the market. There are some limitations to his model. First, sellers in his model are identical, so he characterises the behaviour of only one seller. Second, it is not possible to have more than one buyer at any given time, and it is also not possible to have a sale price higher than the asking price. Another limitation concerns the exogenous rate of offers arriving to a seller, which can not characterise the search behaviour of the buyers in the housing market.

Yavas and Yang (1995) study the strategic role of the asking price in a single-period model. In their theoretical study they examine how the choice of list price affects the broker's incentive to search and the length of time the property is on the market. Their study also attempts to show empirically the effect of higher asking prices on time on the market.

Chen and Rosenthal (1996a) and Chen and Rosenthal (1996b) are two theoretical attempts to show the optimal behaviour of a monopolist using an asking price mechanism to attract buyers. They assume that, in an infinite-horizon setup, the asking price is the seller's commitment device to attract potential buyers to incur the search cost. Chen and Rosenthal (1996a) show the optimal reservation price and asking price of a seller in an environment in which the buyers pay a cost to inspect the object and after the inspection bargain with the seller over a share of the surplus (if any). They also study duopolistic competition. Yet there are some limitations to their model. First, they only study a monopolistic case with the possibility of shared bargaining powers between parties. In duopolistic cases the seller has complete bargaining power, and the objects are identical. Second, at any period of time only one buyer can make an offer; thus, it is not possible to have a sale price higher than the asking price. Chen and Rosenthal (1996b) argue that under some specific assumptions this asking price is the optimal mechanism within the class of incentive compatible mechanisms. The critical assumption is that the seller can extract the entire surplus in the bargaining game. In other words, the seller has all the bargaining power.

Arnold (1999) analyses not only the search behaviour but also the bargaining game between the seller and potential buyers. In his model, the asking price, which is chosen by the seller, can influence the number of buyers who want to inspect the house. Yet there is another role for the asking price in this work as well: as the initial offer in the bargaining game. Arnold (1999) introduces a different bargaining game than the one in Chen and Rosenthal (1996b). In Arnold (1999), the outcome of bargaining no longer is a fixed share of the surplus but depends on the discount rates of the buyers and the seller. He claims that because this change makes the seller's surplus a non linear function of the total surplus, unlike in Chen and Rosenthal (1996b), the comparative statistics analysis will also change.

To our knowledge Carrillo (2012) is the only empirical study of a two-sided search model in the housing market. He presents an environment in which both sellers and buyers search for potential traders. He introduces an asking price mechanism like the one in Chen and Rosenthal (1996a,b) and Horowitz (1992) as a ceiling price and a commitment device from the seller. In his model there is simple negotiation between two parties, in which the potential buyers have a random chance to make a one-time take-it-or-leave-it counteroffer to the asking price. Carrillo (2012) argues that a buyer's optimal counteroffer, given that she has a chance to make one, is the seller's reservation price. He solves the buyer-seller search problem and finds the condition for the seller's optimal reservation and asking prices. To estimate the model, he uses an arbitrary function as a starting point to solve the baseline model and to show the convergence of equilibrium. Finally, he estimates the parameters of the model using the maximum likelihood method. In this study it is not possible to have a final price higher than the asking price. There is also no possibility of multiple buyers arriving at any stage of the game. The aim of this paper is to answer how the amount of information on the house and the real estate agent's commission can change the outcome.

Albrecht, Gautierz, and Vroman (2012) is a working paper that models buyers' and sellers' direct search behaviour in the housing market. It is unique in the literature because it considers the possibility of multiple offers from buyers at any stage of the game given the seller's asking price. Their model explains cases in which the house is sold below, above, or at the asking price. In their model a buyer can accept the asking price or make a counteroffer. If a seller receives more than one request at the asking price, she runs a second-price auction with the asking price as the reserve price. In the first part of the paper the authors assume that all sellers are homogenous, keeping the ratio of the number of sellers and buyers exogenous. Then any configuration of an asking price higher than the seller's reservation value forms an equilibrium. Therefore, in the case of homogenous sellers there is no specific role for the asking price. They also study the efficiency of this model with free entry. In their heterogeneous seller model there are two types of sellers in the market: low and high. In this model, they show that under some conditions there exists a separating equilibrium at which sellers signal their type via the asking price; thus, the asking price plays a signalling role in the heterogeneous seller model.

Wang (2011) studies a game in which a seller posts a price and buyers may pay the price or bargain. Wang (2011) studies how a seller signals the quality of her house through the list price. His results suggest that in the separating equilibrium high-quality sellers signal with higher prices, and the higher prices induce more bargaining. In his setting buyers need to pay a cost to realise their type and the quality of the house. In the separating equilibrium buyers infer the true type by the list price.

The nature of the environment under study here often results in a negotiation between a buyer and the seller. But because this is a dynamic game and the seller may be uncertain about future demand, it is sometimes the case that more than one potential buyer arrives in some periods. If the seller is aware of the fact that she might have more than one interested buyer in some periods, she can set the asking price to optimise both events. For example, consider a seller who hires a real estate agent to sell her property and pays the agent a fixed commission. The seller will not negotiate directly with the buyers, and hence before the property goes to market she will need to specify two prices to the real estate agent: first, the advertisement price, or the asking price; and second, the minimum price she will accept in the event of any negotiation with buyers, or the reservation price.[7] The agent is not allowed to sell the house at a lower price than the reservation price. However, it might be the case that she sells the house for more than the advertisement price.

Before defining the selling mechanism and explaining the model it is important to note some facts about the arrival of buyers in a dynamic game. If one assumes that there is at most one potential buyer at each period who negotiates with the seller, it is conceivable to support the role of the asking price as a ceiling price, such as in Chen and Rosenthal (1996a,b) and Horowitz (1992). Although this is the case for most transactions in the housing market, sometimes transaction prices are higher than the asking prices. A single-buyer-arrival assumption does not support these situations. In Section 3.2 we relax some of the assumptions of the standard model common in the literature on the housing market to include cases in which the transaction price is above the asking price. This is not possible without multiple buyers arriving in at least some periods. We then examine a selling method with an asking price and analyse whether there is still a role for the asking price in this situation. In Section 3.3 a more general model is introduced to examine the possible extension of the previous results. Section 3.4 concludes the results in this chapter and discuss the possibility of further extensions.

[7]The reservation price is different from the reserve price of an auction.

3.2 A Model With a Maximum of Two Bidders

The seller of an indivisible object posts an asking price p_a to sell the object in an infinite sequence of time until it is sold. The seller discounts the future at the rate δ. The seller *ex ante* believes that at each period with some probability ρ_1 there will be only one buyer; with probability ρ_2 there will be two buyers; and with probability $1 - \rho_1 - \rho_2$ there will be no potential buyers, so she has to wait for the next period. Arriving buyers leave at the end of each period, and the seller faces a new set of buyers in the next period. Suppose each buyer i's value for the object is a random variable V_i, independently and identically distributed according to $F(.)$ on the interval $[0, \bar{v}]$, and $F(.)$ is continuous and differentiable with density f. After arriving, each buyer realises the match-specified value v of V. We assume there is no cost for the realisation of v. Furthermore, we assume that the *hazard rate* function of $F(.)$ is increasing.

The selling mechanism is as follows. The seller posts an asking price at period zero. Then at each period of the game, if there is only one buyer, the buyer has the option to buy the object at the asking price or make a counteroffer, which would trigger a bargaining game between the seller and the buyer. If there are two buyers, the seller runs a sealed-bid second-price auction with a reserve price, from which one of three possible outcomes results:

- If both buyers have values lower than the reserve price, the seller waits for the next period.
- If only one buyer has a value higher than the reserve price, then that buyer wins the object and pays the reserve price.

- If both buyers have values higher than the reserve price, then the highest bidder wins and pays the second highest bid.

Suppose the seller's outside value for the object is zero. Define the seller's reservation price p_r, which is the minimum price that she will accept to sell the object at any stage of the game. Suppose that at this stage the reserve price in the auction game is equal to the reservation price p_r. In fact, this assumption does not necessarily maximise the seller's expected revenue, but if the seller is restricted to choosing only two prices, this is what will eventually happen. The seller's problem is to choose an optimal asking price and reservation price to maximise her expected payoff for the game.

We next define the bargaining game as follows. If the buyer's value is v and the seller's reservation price is p_r, then the transaction price resulting from bargaining is between v and p_r. To simplify the game like the one in Chen and Rosenthal (1996b), suppose that a fixed fraction θ of the surplus $v - p_r$ goes to the seller and the remainder goes to the buyer. This happens as long as the expected transaction price is lower than the asking price. Therefore, if a buyer has a value high enough that the outcome of bargaining would result in a transaction price higher than the asking price, then she buys the object at the asking price. Therefore, we can define the transaction price as follows:

$$p = \begin{cases} \theta v + (1-\theta)p_r & \text{if} \quad p_r < v < p_l \\ p_a & \text{if} \quad p_l < v, \end{cases} \tag{71}$$

where $p_l = (p_a - (1-\theta)p_r)/\theta$. Here we suppose that at the time of bargaining v is revealed to the seller. Although this is a common assumption in the literature [8], even if the values were unknown we could argue that there exists a θ such that the outcomes are the same as the full information case. In fact, the assumption of *ex post* complete information simplifies the bargaining game.

We define the game as follows. After the seller posts the asking price, she observes the number of buyers arriving at each period. If there is only one buyer, then they negotiate according to the aforementioned bargaining process. If there are two buyers, the seller runs a second-price auction with a reserve price equal to p_r. Clearly, p_r might not be the best reserve price for the auction, but because we assume that the seller can only optimise the situation with two prices-namely, the asking price and the reservation price-then p_r itself becomes the reserve price for the auction. We will also study the case in which the seller can separately identify an optimal reserve price for any possible auction, but in reality this might not be an option for the seller, although it could result in a higher expected payoff. We are also going to examine a case in which the seller combines the asking price and the reservation price into a single price that is a take-it-or-leave-it offer if only one buyer arrives and the reserve price for the auction if multiple buyers arrive. In this case the seller does not engage in negotiation with the buyer.

In the model with only two prices (an asking price and a reservation price), although buyers only observe the asking price, according to the seller's optimal decision in equilibrium, the reservation price would also

[8] See Arnold and Lippman (1995), Albrecht, Anderson, Smith, and Vroman (2007) or Chen and Rosenthal (1996b).

be revealed to them. We continue with the seller's problem to find an optimal asking price and the reservation price. At each period t the seller's expected payoff is as follows:

$$\begin{aligned} U_t^e =& \rho_1 \big[\int_{p_r}^{p_l} (\theta v + (1-\theta) p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \big] \\ &+ \rho_2 \big[\int_{p_r}^{\bar{v}} v f_2(v) dv + (F_2(p_r) - F_1(p_r)) p_r \big] \\ &+ \big[\rho_1 F(p_r) + \rho_2 F_1(p_r) + (1 - \rho_1 - \rho_2) \big] \delta U_{t+1}^e. \end{aligned} \tag{72}$$

According to the model, ρ_1 and ρ_2 are exogenous, and δU_{t+1}^e is the discounted expected payoff from going to the next period. Because the model is infinite horizon, the seller's expected profit is independent of time. This stationary model implies that the reservation price and the asking price are also independent of time [9]. Thus, there exists a steady state in which the expected payoffs converge to a payoff independent of time:

$$U_t^e = U_{t+1}^e = U^e. \tag{73}$$

In this case the seller's optimal decision is to set the reservation price equal to the discounted reservation value, i.e.,

$$p_r^* = \delta U^e. \tag{74}$$

Substituting this condition in (72), then we have

[9]This is a standard approach in infinite-horizon search models (see Lippman and McCall (1976) for a survey).

$$
\begin{aligned}
U^e =&\rho_1\big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)\big]\\
&+\rho_2\big[\int_{p_r}^{\bar{v}}vf_2(v)dv+(F_2(p_r)-F_1(p_r))p_r\big]\\
&+\big[\rho_1F(p_r)+\rho_2F_1(p_r)+(1-\rho_1-\rho_2)\big]p_r.
\end{aligned}
\tag{75}
$$

The seller chooses a reservation price p_r^* and an asking price p_a^* to maximise the total value of the search. We obtain the following expression for the optimal asking price by differentiating (75) with respect to p_a:

$$\rho_1(1-F(p_l))=0. \tag{76}$$

Proposition 3.1. *The optimal asking price and the reservation price are a pair (p_a^*, p_r^*) that solve (76) and (75) simultaneously.*

Proof. See Appendix

The results for proposition(3.1) suggest that the seller sets the asking price in such a way that the buyer with the highest value in the interval is indifferent about entering the negotiation or buying at the asking price as long as she is the only buyer.

3.2.1 Example

Suppose buyers' values are distributed uniformly from [0,1]. ρ_1 and ρ_2 are equal to 0.4. Also assume that $\theta = 0.5$, and the discount factor is 0.9. Then the optimal asking price and the reservation price are

$$p_a^* = 0.72 \quad \text{and} \quad p_r^* = 0.45.$$

Figure 7 and 8 show how the asking price and the reservation price change when θ changes. When the seller has all of the bargaining power, she sets the asking price at the highest level and extracts the entire surplus in the bargaining game. In fact, this situation has the highest expected payoff for the seller when the distributional assumptions and the discount factor are kept the same.

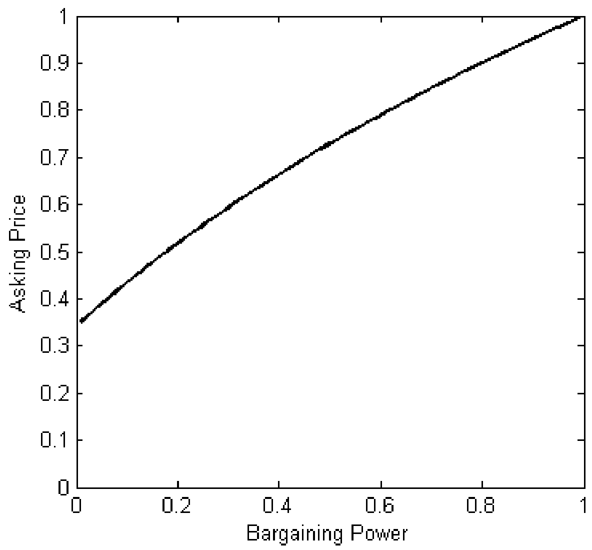

Figure 7: Asking Prices for Different Bargaining Powers

3.2.2 Optimally Chosen Reserve Price

As we mentioned before, if the seller has the option to choose a reserve price for the auction, she might have a higher expected payoff. In this case the seller optimises the expected payoff with respect to three prices-an asking price p_a, a reservation price p_r, and a reserve price r-for the auction event only. The seller chooses the optimal reserve price, and the equation in (72) becomes

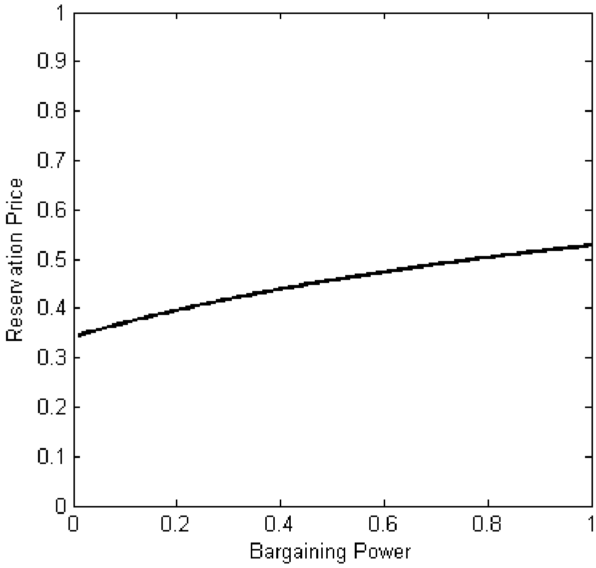

Figure 8: Reservation Prices for Different Bargaining Powers

$$\begin{aligned} U_t^e =& \rho_1\big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)\big] \\ &+\rho_2\big[\int_r^{\bar{v}}vf_2(v)dv+(F_2(r)-F_1(r))r\big] \\ &+\big[\rho_1F(p_r)+\rho_2F_1(r)+(1-\rho_1-\rho_2)\big]\delta U_{t+1}^e. \end{aligned} \tag{77}$$

Considering the same argument for the stationary infinite-horizon models results in the following steady-state expected payoff:

$$\begin{aligned} U^e =& \rho_1\big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)\big] \\ &+\rho_2\big[\int_r^{\bar{v}}vf_2(v)dv+(F_2(r)-F_1(r))r\big] \\ &+\big[\rho_1F(p_r)+\rho_2F_1(r)+(1-\rho_1-\rho_2)\big]p_r. \end{aligned} \tag{78}$$

As mentioned before, because the expected payoff is independent of time, the seller's optimal decision is to set $p_r^* = \delta U^e$.

Lemma 3.1. *The optimal reserve price for the auction is* $r^* = p_r + \frac{1-F(r)}{f(r)}$.

Proof. Differentiating (78) with respect to r equal to zero gives

$$-rf_1(r) + F_2(r) - F_1(r) + p_r f_1(r) = 0$$

$$-r + p_r + \frac{F_2(r) - F_1(r)}{f_1(r)} = 0$$

$$r^* = p_r + \frac{1 - F(r)}{f(r)}.$$

□

In fact, lemma 3.1 suggests that the optimal reserve price is the continuation value plus the inverse of the *hazard rate* function, which is the same as the reserve price for the optimal auction. This is not surprising because the expected payoffs are independent of time and there is a sealed-bid second-price auction.

Proposition 3.2. *As long as (*76*) and lemma (*3.1*) hold, the optimal reservation price* p_r^* *is the one that solves (*78*).*

Proof. (76) and lemma (3.1) are the result of the first-order conditions of maximising (78). The second-order condition is satisfied as long as the *hazard rate* function of $F(.)$ is increasing, which is an assumption of the present model. Thus, r^* and p_a^* maximise (78). Substituting the condition in (74) into (78) results in

$$\begin{aligned}\frac{p_r}{\delta} =& \rho_1 \big[\int_{p_r}^{p_l} (\theta v + (1-\theta)p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \big] \\ &+ \rho_2 \big[\int_r^{\bar{v}} v f_2(v) dv + (F_2(r) - F_1(r))r \big] \\ &+ \big[\rho_1 F(p_r) + \rho_2 F_1(r) + (1 - \rho_1 - \rho_2) \big] p_r. \end{aligned} \tag{79}$$

With an argument the same as Proposition 3.1, we can show that there exists a p_r that solves this equation. □

For the example in (3.2.1), if we calculate an optimally chosen reserve price, then we have

$$p_a^* = 0.78 \quad \text{and} \quad p_r^* = 0.54 \quad \text{and} \quad r^* = 0.57.$$

If the seller is able to identify a reserve price separately for a possible auction, then she will raise the optimal asking price compared to if she uses the same reservation price at the negotiation and for the auction. The seller's expected payoff for the game will also rise in this case because now she chooses the reserve price to maximise the revenue from the case in which she faces two buyers.

3.2.3 Comparison With an Optimal Auction

As mentioned previously, in the types of markets being analysed here, sellers, after posting a price, may accept a counteroffer. In these markets the posted price is not necessarily the lower bound of the transaction price. For the purpose of this analysis we introduce another selling mechanism, in which the seller may never negotiate with buyers on the asking price. Suppose a seller advertises a price p at the beginning of the game. If there are multiple buyers, the seller runs a second-price auction with the reserve price p, and if there is only one buyer, then p is a take-it-or-leave-it offer to that buyer. All other assumptions are the same as in the previous section. The seller's expected payoff at any given period becomes

$$\begin{aligned} U_t^o = &\rho_1 \left[p(1 - F(p))\right] \\ &+ \rho_2 \Big[\int_p^{\bar{v}} v f_2(v) dv + (F_2(p) - F_1(p))p \Big] \\ &+ [\rho_1 F(p) + \rho_2 F_1(p) + (1 - \rho_1 - \rho_2)] \delta U_{t+1}^o. \end{aligned} \tag{80}$$

If a price p maximises U_t^o, it will also maximise U_{t+1}^o, because the seller is facing exactly the same problem in each period. Therefore, in the steady state $U_t^o = U_{t+1}^o = U^o$, which results in an optimal price that is independent of time. Then the expected payoff becomes

$$U^o = \frac{\rho_1[p(1 - F(p))] + \rho_2[\int_p^{\bar{v}} v f_2(v) dv + (F_2(p) - F_1(p))p]}{1 - \delta[\rho_1 F(p) + \rho_2 F_1(p) + (1 - \rho_1 - \rho_2)]}. \tag{81}$$

The differentiation of (81) with respect to p gives the price that maximises the expected revenue for the seller. For the example in (3.2.1), the optimal price is equal to 0.5 and the expected payoff is 0.48. There exists a θ in which the expected payoff to the seller for the game defined in the previous section is higher than the expected payoff of this game. In fact, the greater the seller's negotiation power, the greater the chance that she accepts any counteroffer and enter the negotiation process, as the seller knows that in the bargaining game she can extract more surplus on average.

3.3 A More General Model

In this section we relax the assumption of a maximum of two buyers arriving at each period to generalise the results. In particular, we assume that the probability that $n \in \{1, 2, ...\}$ buyers arrive at each period is distributed geometrically, and is independent of time, with the probability of each success equal to ρ, where $0 < \rho \leq 1$. In this situation the seller expects any number of buyers, but with lower probabilities for higher numbers of arrivals. Keeping all other assumptions as in section 3.2 we define the game as follows. The seller posts an asking price p_a at period zero before the game starts. Then at every period after buyers arrive, according to the probability distribution explained previously, the seller observes the number of buyers. If there is only one buyer, the buyer can either offer to buy the object at the asking price or make a counteroffer, which would result in a bargaining game like the one explained in section 3.2. If there is more than one buyer, then the seller runs a sealed-bid second-price auction with a reserve price equal to the reservation price.

The seller's expected payoff at each period is independent of time in this model as well:

$$\begin{aligned} U^m =& \rho\Big[\int_{p_r}^{p_l} (\theta v + (1-\theta)p_r)dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) + F(p_r)\delta U^m\Big] \\ &+ \sum_{n=2}^{\infty} \left[\int_{p_r}^{\bar{v}} v f_2^{(n)}(v)dv + (F_2^{(n)}(p_r) - F_1^{(n)}(p_r))p_r + F_1^{(n)}(p_r)\delta U^m\right](1-\rho)^{n-1}\rho. \end{aligned} \tag{82}$$

This equation can be rewritten as follows:

$$
\begin{aligned}
U^m =&\rho\Big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)+F(p_r)\delta U^m\Big] \\
&+\sum_{n=2}^{\infty}\left[\int_{p_r}^{\bar{v}}nF^{n-1}(v)J(v)dF(v)+F^n(p_r)\delta U^m\right](1-\rho)^{n-1}\rho,
\end{aligned}
\tag{83}
$$

where $J(v) = v - \frac{1-F(v)}{f(v)}$.

Again, the seller's optimal decision is to set the reservation price such that $p_r = \delta U^m$, which is the minimum price for which the seller agrees to sell the object at any period of time. It is possible to find the sum of the series in the second term of the right-hand side of (83), but it is not necessary for the analysis at this stage. The optimal reservation price needs to satisfy (83), and the asking price needs to satisfy the first-order condition of (83) with respect to the asking price. Because the first-order condition of maximising the bargaining outcome with respect to the asking price is independent of the number of bidders, the general model has the same equation for the optimal asking price as the two-buyer model. Of course, the optimal asking price itself will not be the same for these models because the reservation price will not be the same because of the effect of ρ.

If the seller could choose a separate reserve price for the auction, again it would be independent of the number of buyers. Indeed, in the general model the reservation price is affected by the arrival rate of the buyers and changes the optimal asking price according to the first-order condition of maximising (83). With the same argument as in Proposition 3.1, it is possible to show that there exists a p_r that satisfies (83).

3.4 Conclusion

In some markets, like the housing market, there is a seller of an indivisible object and uncertainty about the number of potential buyers interested in buying the object. In these markets the problem becomes even more complicated when the seller faces more than one period over time. The objective of the seller is to sell the object at the highest possible price in the shortest amount of time. Running an auction is generally a costly activity, and if there is little probability of there being more than one buyer, the auction becomes even less interesting. A selling mechanism has been proposed in which, before the game starts, the seller has to choose two prices: the asking price and the reservation price. The asking price is the price that is publicly announced in an advertisement, and the reservation price is the lowest price the seller will accept for the object, in this case her property. Buyers arrive according to a random process. Initially the number of arriving buyers was restricted to a maximum of two at each period. That is, we assume with some exogenous probabilities that there is a chance that there will be one buyer, two buyers, or zero interested buyers at each period and that these are the only three possibilities. If there is only one buyer, the seller may engage in negotiation with her with the asking price as the ceiling price for the negotiation and the reservation price as the floor price. If there are two buyers, then the seller runs an auction with a reserve price equal to the reservation price. Under some conditions this mechanism can result in a higher expected payoff for the seller than when the seller chooses a uniform price as a take-it-or-leave-it offer if there is only one buyer and the same price as the reserve price for two buyers. Indeed, sellers with greater bargaining power may prefer this mechanism and accept counteroffers, but sellers with less bargaining power may prefer the uniform price case. We also studied a mechanism in which the seller can choose the reserve price for a possible auction separately. Of course, this mechanism can do better than the proposed mechanism with the reservation price as the reserve price of a possible auction, but in practice the seller may not be able to propose three different prices to the selling agent.

In a more general model the assumption of a maximum of two buyers was relaxed to reflect any number of bidders. The setting suggests that the number of bidders is geometrically distributed, with lower probabilities for higher numbers of arrivals. This model shows that the optimality condition for the asking price is almost the same as that for the model with a maximum of two buyers. The analysis also shows the existence of an optimal reservation price for the general model.

The current chapter has considered the assumptions of the independent private value models. One possible extension of this work is to create a setting similar to that used in the first two chapters to determine the possibility of signalling for the asking price mechanism. Another possible extension involves adding the buyer search to the game. A buyer may need to pay a cost to realise her private signal or a part of her value. In that case a two-sided search model of buyers and sellers in the housing market would provide a more general role for the asking price in these markets.

Chapter 4

The Sydney Housing Market: An Empirical Study

4 The Sydney Housing Market: An Empirical Study

4.1 Introduction

Buying a house is typically the largest single investment an individual makes in his or her lifetime. Selling a house is likewise a momentous decision. Quantitatively speaking, the housing market constitutes a significant component of the overall economy. For instance, the value of the properties sold in 2011 in the state of New South Wales, Australia, was approximately $71 billion [10]. Yet the theory and empirical study of the housing market have not attracted much analytical scrutiny. In the previous chapter theoretical models of the housing market were examined to explain some unique characteristics of this market. In this chapter the main focus is on the empirical aspects of the Australian housing market.

As mentioned in the previous chapter, the most popular selling mechanism in the housing market is when the seller of a property advertises an asking price and engages in subsequent negotiations with potential buyers over time. This selling method is called the asking price mechanism (or sometimes the private treaty), and it was examined extensively in the previous chapter. In 2011 in Sydney, Australia, almost 90% of properties were sold via the asking price mechanism[11]. Although auctions are another method of selling houses in Australia, they are not as popular as the asking price mechanism. One reason for this has to do with the cost of running an

[10] According to the Australian Property Monitors.

[11] The corresponding percentages for 2010, 2009, and 2008 were 89%, 92.5%, and 91% respectively, according to data from the Australian Property Monitors.

auction, which includes hiring an auctioneer[12]. The other reason could be related to the nature of these markets, in which buyers are less patient than sellers. Over time a seller cannot run one time auction for all interested buyers. Because with the asking price mechanism the seller only needs to advertise a price and wait for potential buyers, this method of selling is less costly than running an auction.

In the U.S. housing market, the asking price method is the most popular selling method in the real estate market. In fact, almost all of the literature on the housing market focuses on the asking price mechanism. However, online auctions are becoming more popular for selling property in North America. In contrast, in Australia auctions have been used to sell residential properties for a long time. Today, auctions are popular in Australia mainly for properties with special characteristics, such as a view or location, for which the seller expects enough interested buyers at a single time to run an auction. However, the current chapter empirically examines the use of the more popular asking price mechanism in Sydney, Australia.

In this method, the seller of a property normally hires a real estate agent to assist her in the selling process. Both parties have similar goals: to sell the property at the highest possible price in the shortest amount of time. They come up with an estimate of the value of the property and calculate the initial list price. Failing to sell the property is the worst possible outcome, and to prevent that from happening they may decide to revise the list price if the house still has not sold after a period of time. The empirical study in this chapter tries to answer the following questions: How does the choice of the initial list price affect time on the market?

[12]Today, the cost of running an auction in Sydney, Australia, may exceed $10,000.

Why do some sellers revise their list price during the selling period and others don't? Finally, what types of properties have a greater chance of undergoing a change in price during the selling period?

Empirical models of the housing market have been significantly understudied compared to models of similar markets. The housing market has been described using search theory (either one-sided or two-sided search models). These models can be divided into two major categories: first, models that result in a stationary equilibrium in the search market, with uniform prices independent of time; second, non-stationary search models in which prices are revised at each period with respect to the outcomes of previous periods. In this chapter we review the empirical stationary models, which have received more attention in the literature and are more relevant to the current study. One of the reasons that non-stationary models have attracted less attention could be related to the lack of detailed data on revised prices in the housing market. Most of the data available reflect only the initial asking price (or the list price) and the transaction price (or the sale price[13]).

One of the first attempts to empirically study search models of the housing markets is by Horowitz (1992). He considers an infinite-horizon stationary search framework in which a seller posts a time-invariant asking price and waits for offers from potential buyers who arrive over time. The asking price in his model acts as a ceiling price, and at any time, if a buyer asks to buy the object at the asking price, the seller must accept the offer; the reservation price is the lowest price that the seller accepts for the

[13]The terms *asking price* and *list price* as used in this chapter have the same meaning. The same is true for *transaction price* and *sale price.*

object. Thus, the transaction price could lie on an interval starting with the reservation price and ending with the asking price. Horowitz (1992) estimates the model on data from Baltimore, Maryland, which include the asking prices, house characteristics, time on the market, and transaction prices for 1,196 observations. He suggests that the model he presents predicts transaction prices better than hedonic price regression outcomes. However, his model cannot conclude accurate predictions of time on the market.

Yavas and Yang (1995) also study how the asking price affects the length of time the seller remains in the market. They call this time on the market. Their empirical study is based on 270 observations listed by the Multiple Listing Service of the School College district of Pennsylvania in 1991. Their results suggest that for mid-price properties, a higher asking price can significantly increase time on the market. In contrast, the asking price has no significant effect on time on the market for low-price and high-price properties.

Jud, Winkler, and Kissling (1995) study the liquidity of the housing market from another perspective. Instead of time on the market they look at the spread between the asking and transaction prices. Their data consist of 3,597 observations on list prices, sale prices, and dates of sale in Greensboro, North Carolina, for a period of 25 months starting in April 1991. Their empirical results show that the spread between the list price and the transaction price is positively related to the list price and the cost of search. They also show that the price spread is negatively related to the standard deviation of price offers.

Haurin, Haurin, Nadauld, and Sanders (2010) study a search model in which list prices are important in two respects. First, they are an upper bound of the transaction price. Second, they affect the arrival rates of offers. Their study draws on data from property sales in central Ohio from 1997 to 2005. Their estimation results suggest that greater variance in the distribution of offers results in higher list and seller reservation prices. They also find that on average atypical properties take longer to sell and that the sellers of such properties set higher asking prices.

Bin (2004) empirically studies the prediction of housing sales prices. Bin (2004) uses the hedonic price approach with both parametric and semi-parametric regressions. The study uses a data set with 2,595 observations of housing sales in Pitt County, North Carolina, within a 3-year period. The results show that semi-parametric regression gives a better prediction of sale prices than parametric regression. Anglin and Wiebe (2013) question the relation between the change in list prices and sale prices. They use data that include houses that have been sold twice in a period of time by two different sellers. Their findings suggest that a single seller, even though only a small part of the market, can affect the sale price of her property by a change in asking price, when all other factors are the same. Their findings are consistent the literature, but empirically the data are unique, as they are comparing the exact same properties with different sellers.

In all of these studies, asking prices and reservation prices were independent of time throughout the search. This could be because of limited data on revised asking prices in most of the cases. Some other literatures with more detailed data focus on the revision of asking prices in the market.

Knight (2002) focuses on price revisions during the selling process. In most studies such price revision is not observed in the data; however, Knight (2002) combines information on the list price, sale price, and time on the market with the asking price revisions. The data consist of 3,490 observations in Stockton, California, of houses that sold between January 1997 and December 1998. The data include only one change in the list price (if there was a revision). If there was more than one revision, the data contain the final revised list price. The findings suggest that properties with unusual characteristics are least likely to have a revised price. However, properties that are vacant and those with high initial mark-ups are most likely to have a revised list price. Another important suggestion of their work is that miscalculating the initial list price is costly for the seller in terms of time and money. The larger the change in the list price, the longer the selling time and the lower the transaction price.

In Section 4.2 the data is described. Section 4.3 discuss the empirical results of the estimation of two different models to analyse the parameters that affect the time on the market as well as price revision. In Section 4.4 we conclude this chapter and discuss the limitation.

4.2 Data

The data for the current study include 25,489 properties sold via the asking price (or private treaty) in the Sydney region, state of New South Wales, Australia, in 2011[14]. The region has been divided geographically into 164 postal areas. Each postcode could potentially include one or more

[14]Data were collected by the Australian Property Monitors. Their main resource is government data on housing market transactions as well as real estate agents.

suburbs, but in terms of the geographical distribution of dwellings, it is one of the most accurate units of analysis for the region. For each property we observe the first advertised price (or the list price or asking price), the final list price, the transaction price, time on the market, and property characteristics. The data also include neighbourhood characteristics for each postal area according to the Australian Bureau of Statistics and census data, such as mean household income, median age, median weekly rent, and average household size. Table 1 shows some descriptive statistics of the data.

Table 1: Descriptive Statistics for Housing Transactions (Asking Price Mechanism) in Sydney, New South Wales, 2011 (Postcodes 2000-2200)

	Mean	Median	St. Dev.	Min.	Max.
First Asking Price (AP1) (thousand $)	599.82	520	325.84	100	4,250
Final Asking Price (AP2) (thousand $)	589.35	510	321.37	100	4,250
Transaction Price (TP) (thousand $)	582.74	507	317.51	107	4,000
Days on the Market (TOM)	68.9	45	67.8	1	365
Home Characteristics					
Property Type (Type) (House = 1, Unit = 0)	0.5	1	0.49	0	1
Number of Bedrooms (Beds)	2.62	3	1.06	0	7
Number of Bathrooms (Bath)	1.53	1	0.65	0	7
Number of Parking Spaces (Park)	1.34	1	0.79	0	9
Study	0.12	0	0.33	0	1
Central Air Conditioner	0.29	0	0.45	0	1
Balcony	0.32	0	0.46	0	1
Pool	0.08	0	0.27	0	1
Alarm	0.10	0	0.30	0	1
Neighbourhood Characteristics in the Postcode					
Mean Household Income (thousand $)	64.93	59.2	20.8	40.7	176.3
Log of Population	10.05	10.08	0.67	7.34	11.46
Median Age	35.46	35	3.34	27	53
Number of Private Dwelling	11,070	9,383	7,101	584	32,922
Average Household Size	2.57	2.68	0.43	1.32	4.51
Median Monthly Mortgage Repayment ($)	2,376	2,410	397	1,506	3,500
Median Weekly Rent ($)	401	400	92	175	780
Average Motor Vehicles per Dwelling	1.5	1.5	0.34	0.5	2.6

Number of observations = 25,489

In Sydney, when real estate agents advertise a property for sale, they determine the method of sale, which is usually an auction or an asking price (private treaty) mechanism. In 2011, auctions were used to sell approximately 10% of properties in the region, which have been excluded from the present data. Here the main focus is cases in which the seller advertises a list price (asking price) for the property and engages in negotiation with potential buyers. Sellers may sell their homes above the list price, at the list price, or below it. In fact, if the first advertised list price is used as the base list price, then approximately 25% of the present properties were sold above the list price, 7% at the list price, and the remaining 68% below the initial list price. Compared to other data in the literature, the present data are special in terms of the high number of observations and the inclusion of revised list prices. One interesting observation concerns the change in list price: In the current data, approximately 39% of homes saw a decline in list price over time, meaning that the first list price was higher than the final list price. For 51% of homes the list price remained constant, and for 10% it increased over time.

According to Table 1, the average initial and final list prices are higher than the average transaction price. However, the average of initial list prices is lower than the average of final list prices. The current data are limited to houses or units. Other types of properties have been excluded from the data, although they represent less than 1% of the total number of properties. Figure 9 shows a histogram of transaction prices in thousands of dollars. The distribution of the prices is used to divide properties into different categories with respect to their values.

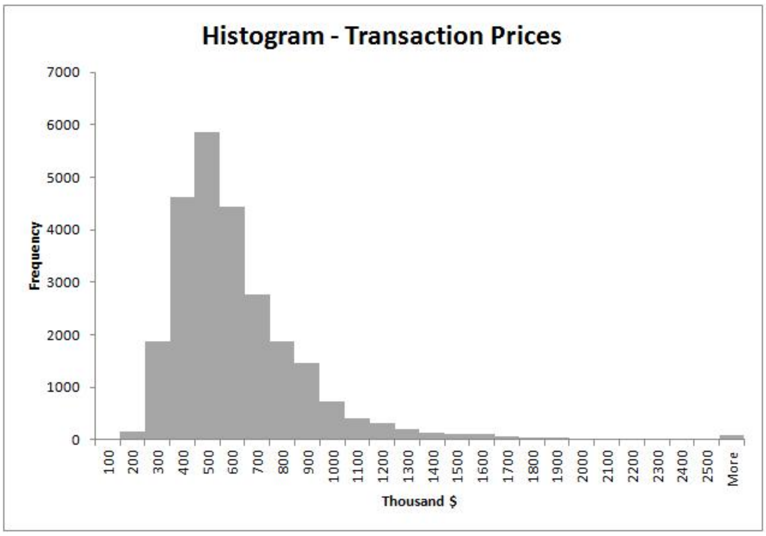

Figure 9: Histogram

4.3 Empirical Analysis

Different parameters can influence the process of selling a property. Observable in the current data, besides the physical characteristics of the property, is the pricing strategy of the seller. A seller can either increase the initial list price, keep it constant, or reduce it over time. All three cases are reflected in the data. As mentioned before, the current data include the initial list price, final list price, and transaction price. To determine how the list prices differ from the transaction price for a property, we run a least squares regression over the property characteristics and the neighbourhood characteristics in which the dependent variables are either the logarithm of the initial list price, final list price, or transaction price. Table 2 shows the results of this ordinary least squares regression.

The results in Table 2 suggest that, on average, physical and neighbourhood characteristics have almost the same effect on the three prices. The signs of all of the coefficients are as expected. For instance, if the

Table 2: OLS regressions Over Property Characteristics

	Dependent Variables		
	LOG of Transaction Price	LOG of First Ad Price	LOG of Last Ad Price
Constant	0.217	0.339	0.342
	(0.08)	(0.08)	(0.08)
Log of Days on the Market	-0.013	0.008	-0.005
	(0.001)	(0.001)	(0.001)
Property Type (House = 1, Unit = 0)	0.190	0.193	0.190
	(0.004)	(0.004)	(0.004)
Number of Bedrooms	0.161	0.163	0.162
	(0.002)	(0.002)	(0.002)
Number of Bathrooms	0.098	0.097	0.099
	(0.002)	(0.002)	(0.002)
Number of Parking Spaces	0.058	0.058	0.058
	(0.002)	(0.002)	(0.002)
Study	0.043	0.043	0.041
	(0.004)	(0.004)	(0.004)
Balcony	0.027	0.027	0.027
	(0.003)	(0.003)	(0.003)
Pool	0.034	0.037	0.037
	(0.005)	(0.005)	(0.005)
Alarm	0.012	0.008	0.009
	(0.004)	(0.004)	(0.004)
Log of Mean Household Income ($)	0.272	0.279	0.276
	(0.010)	(0.010)	(0.010)
Log of Population	-0.132	-0.126	-0.125
	(0.005)	(0.005)	(0.005)
Log of Median Age	0.219	0.191	0.195
	(0.019)	(0.020)	(0.020)
Average Household Size	-0.302	-0.292	-0.291
	(0.004)	(0.004)	(0.004)
Log of Median Weekly Rent ($)	0.483	0.450	0.457
	(0.009)	(0.010)	(0.010)
R^2	0.73	0.72	0.72
S.E. of Regression	0.22	0.22	0.22

Number of observations = 25,489.
Standard errors are in parentheses.
All coefficients are significant at the 1% level.

property type is a house, one might expect a higher sale price for it on average. The negative coefficient for the variable logarithm of days on the market in the first regression suggests that the more days a property is on the market, the lower the sale price. Mean household income has a positive effect on the prices, whereas the population and the average household size negatively affect the prices. The results in Table 2 are very general but surprisingly consistent with what is expected for the housing market. It is possible to conclude from this general regression that on average, except for time on the market, characteristics of a property affect all three prices in the same manner.

4.3.1 Time On the Market

The main objective of a seller or a real estate agent is to sell a property at the highest possible price in the shortest possible time. List prices play an important role in this process. When all other characteristics are the same, a higher list price may result in more days spent waiting for interested buyers. However, a higher list price could also potentially signal the unobserved attributes of a property to buyers and thus increase the expected sale price. In this section we examine the effect of overpricing on time on the market. A semi-log hedonic model is used to analyse this effect. The analysis has two stages. First, we regress the transaction price over fairly chosen property attributes. Second, we use the predictions in the first regression to estimate the effect of overpricing and price revision on time on the market. This analysis is similar to the approach in Yavas and Yang (1995); however, it features a change in the list price and does not consider seasonal effects. Thus, the second equation, which characterises the variables that affect time on the market, is quite different. The two-stage model in the current study is based on the following equations:

$$Log(p) = \Gamma(X) \tag{84}$$

$$Log(TOM) = \Psi(Pratio, Pchange, Prevised). \tag{85}$$

In the first stage, equation (84) regresses the logarithm of the sale price over a vector of property and neighbourhood characteristics. In the second stage, equation (85) uses the results from (84) to form variable *Pratio*, which is the log of the predicted sale price over the log of the initial list price. Variable *Pchange* is the percent change in the list price, and *Prevised* is a dummy variable with value 1 if the list price has been revised and 0 otherwise. The two-stage model is estimated once over the total number of observations and once over four different quartiles with respect to the sale price. Properties are divided into four categories with respect to their sale prices. For each quartile the transaction prices are restricted as the best indication of the values of the properties. Tables 3 and 4 show the results of the first-stage regression for each quartile, and Table 5 shows the results of the same regression over the complete sample. As the results for the complete sample show, all signs for the independent variables are as expected. For each quartile separately there is a lower R^2, which is expected, and most of specifications, such as the number of bedrooms and bathrooms, have significant coefficients with signs as expected.

The results from the second-stage estimation are shown in Tables 6 to 10. For the estimation on the complete sample (Table 6), all coefficients are significant at the 1% level. The negative sign for *Pratio* suggests that, on average, when the list price is closer to the predicted sale price, one expects a lower time on the market. The positive coefficient for the dummy variable *Prevised* suggests that, on average, if the list price has been revised, then the property has been on the market for a longer time. At this stage all revised list price properties are considered, even those with an increase in the list price. In the next section we specifically examine the change in list price. Finally, the negative coefficient for *Pchange* suggests that the higher the percent change in the list price, the lower the time on the market.

All of the estimates for the quartiles have the same sign as the complete sample. Except for one coefficient (*Pratio* for the second quartile), all others are significant at the 1% or 5% level. The nonsignificant *Pratio* coefficient in the second quartile suggests that the difference between the list price and the estimated sale price has no influence on the length of the sale for these properties.

Our results are different from the similar studies such as the results in Yavas and Yang (1995). Their results suggest that there is a significant difference among the subgroups and two of their subgroups have insignificant coefficients. In the current estimation all variables except one are significant and there is no distinction in terms of the signs of the variables in every quartile as well as the complete sample. It is important to mention that Yavas and Yang (1995) considered the seasonal effects in their estimation model while here the focus is on more important variables such as *Prevised* and *Pchange*.

Table 3: First-Stage Regression Transaction Prices, Q1 and Q2

Dependent Variable: LOG(P)	$107 < P \leq 400$		$400 < P \leq 500$	
Variable	Coefficient	t-Statistic	Coefficient	t-Statistic
CONSTANT	1.786	(11.2)**	5.415	(94.7)**
PROPTYPE	0.098	(17.5)**	0.013	(4.66)**
BEDS	0.119	(31.8)**	0.020	(12.7)**
BATHS	0.060	(12.2)**	0.011	(6.33)**
PARKING	0.036	(10.9)**	0.009	(7.28)**
STUDY	0.021	(2.13)*	0.012	(4.10)**
AIRCON	0.016	(3.63)**	0.000	(-0.15)
BALCONY	0.044	(10.7)**	0.000	(0.15)
POOL	-0.069	(-7.76)**	0.001	(0.62)
ALARM	-0.019	(-2.38)**	0.007	(2.57)**
LOG MEANINCOME	0.014	(0.76)	0.010	(1.78)
LOGPOP	-0.083	(-11.7)**	-0.019	(-6.30)**
LOG MEDIANAGE	0.438	(14.4)**	0.035	(3.00)**
HOUSEHOLDSIZE	-0.074	(-9.70)**	-0.026	(-8.93)**
LOG MEDIANWEEKLYRENT	0.424	(25.2)**	0.087	(12.7)**
Total number	6671		5866	
R-squared	0.40		0.10	
Adjusted R-squared	0.40		0.10	
S.E. of regression	0.14		0.06	

PROPTYPE: Property type (1 = house, 0 = unit).
BEDS: Number of bedrooms.
BATHS: Number of bathrooms.
PARKING: Number of parking spaces.
STUDY: 1 = property has a study, 0 = otherwise.
AIRCON: 1 = property has central air conditioner 1, 0 = otherwise.
BALCONY: 1 = property has a balcony, 0 = otherwise.
POOL: 1 = property has a pool, 0 = otherwise.
ALARM: 1 = property has an alarm, 0 = otherwise.
MEANINCOME: Mean income in the neighbourhood.
LOGPOP: Log of the population in the neighbourhood.
MEDIANAGE: Median age in the neighbourhood.
HOUSEHOLDSIZE: Average size of households in the neighbourhood.
MEDIANWEEKLYRENT: Median weekly rent in the neighbourhood.
** Significant at the 1% level.
* Significant at the 5% level.

Table 4: First-Stage Regression Transaction Prices, Q3 and Q4

Dependent Variable: LOG(P)	$500 < P \leq 650$		$650 < P$	
Variable	Coefficient	t-Statistic	Coefficient	t-Statistic
C	5.603	(96.6)**	2.191	(12.1)**
PROPTYPE	0.028	(9.10)**	0.165	(18.1)**
BEDS	0.024	(14.4)**	0.070	(15.7)**
BATHS	0.017	(9.11)**	0.084	(16.3)**
PARKING	0.007	(5.23)**	0.044	(11.8)**
STUDY	0.006	(2.18)*	0.019	(2.83)**
AIRCON	-0.002	(-1.05)	-0.015	(-2.38)*
BALCONY	-0.002	(-1.03)	-0.012	(-1.83)
POOL	0.001	(0.50)	0.090	(10.1)**
ALARM	-0.005	(-1.87)	0.038	(4.43)**
LOG MEANINCOME	-0.007	-1.14	0.455	(26.1)**
LOGPOP	-0.020	(-6.13)**	-0.042	(-3.42)**
LOG MEDIANAGE	0.091	(7.09)**	-0.268	(-6.56)**
HOUSEHOLDSIZE	-0.058	(-17.7)**	-0.280	(-28.2)**
LOGMEDIANWEEKLYRENT	0.102	(15.2)**	0.119	(6.22)**
Total number	6027		6925	
R-squared	0.14		0.39	
Adjusted R-squared	0.14		0.39	
S.E. of regression	0.069		0.234	

** Significant at the 1% level.
* Significant at the 5% level.

Table 5: First-Stage Regression, Complete Sample

Dependent Variable: LOG(P)		
Variable	Coefficient	t-Statistic
C	0.136	(1.59)
PROPTYPE	0.190	(43.5)**
BEDS	0.161	(67.3)**
BATHS	0.097	(33.1)**
PARKING	0.058	(27.3)**
STUDY	0.039	(8.92)**
AIRCON	-0.001	(-0.56)
BALCONY	0.025	(7.46)**
POOL	0.032	(6.17)**
ALARM	0.010	(2.15)*
LOG MEANINCOME	0.274	(26.7)**
LOGPOP	-0.131	(-24.3)**
LOG MEDIANAGE	0.225	(11.3)**
HOUSEHOLDSIZE	-0.303	(-64.6)**
LOG MEDIANWEEKLYRENT	0.483	(48.6)**
Total number	25,489	
R-squared	0.73	
Adjusted R-squared	0.73	
S.E. of regression	0.22	

Table 6: Second-Stage Regression, Complete Sample

Dependent Variable: LOG(DURATION)			
Variable	Coefficient	S.E.	t-Statistic
C	3.26	0.00	(411.5)**
LOG(PRATIO)	-21.0	0.68	(-30.7)**
PREVISED	0.80	0.01	(65.4)**
PCHANGE	-0.68	0.16	(-4.24)**
Included observations	25,489		
R-squared	0.26		
Adjusted R-squared	0.26		
S.E. of regression	0.90		

Table 7: Second-Stage Regression, Q1 $P \leq 400$

Dependent Variable: LOG(DURATION)			
Variable	Coefficient	S.E.	t-Statistic
C	3.40	0.04	(80.1)**
LOG(PRATIO)	-0.39	0.15	(-2.54)*
PREVISED	0.95	0.02	(39.3)**
PCHANGE	-1.94	0.21	(-8.87)**
Included observations	6,671		
R-squared	0.24		
Adjusted R-squared	0.24		
S.E. of regression	0.92		

Table 8: Second-Stage Regression, Q2 $400 < P \leq 500$

Dependent Variable: LOG(DURATION)			
Variable	Coefficient	S.E.	t-Statistic
C	3.15	0.05	(52.5)**
LOG(PRATIO)	-0.06	0.17	(-0.35)
PREVISED	0.89	0.02	(34.5)**
PCHANGE	-3.40	0.29	(-11.5)**
Included observations	5,866		
R-squared	0.24		
Adjusted R-squared	0.24		
S.E. of regression	0.92		

Table 9: Second-Stage Regression, Q3 $500 < P \leq 650$

Dependent Variable: LOG(DURATION)			
Variable	Coefficient	S.E.	t-Statistic
C	3.27	0.01	(204)**
LOG(PRATIO)	-28.2	1.54	(-18.2)**
PREVISED	0.70	0.02	(28.7)**
PCHANGE	-1.21	0.35	(-3.44)**
Included observations	6,027		
R-squared	0.28		
Adjusted R-squared	0.28		
S.E. of regression	0.87		

Table 10: Second-Stage Regression, Q4 $650 < P$

Dependent Variable: LOG(DURATION)			
Variable	Coefficient	S.E.	t-Statistic
C	3.38	0.01	(228)**
LOG(PRATIO)	-33.0	1.53	(-21.5)**
PREVISED	0.67	0.02	(28.6)**
PCHANGE	-0.60	0.34	(-1.75)
Included observations	6,925		
R-squared	0.27		
Adjusted R-squared	0.27		
S.E. of regression	0.87		

4.3.2 Change in List Price

As mentioned before, there was a change in the list price for almost half of the observations in the data. The analysis in this section focuses on the reasons for such changes in price. The current data contain the first list price (advertised price) and the last list price for the selling period. If these two prices are not the same for any observation, the property is considered to have undergone a revision in list price. In the previous section this was connected to time on the market. However, this may be only one of many reasons for a change in list price, because sometimes a property is on the market for a long time but does not revise its initial list price. Likewise, some properties revise their list price after being on the market for only a very short period of time. This may reflect a poor estimate of market demand. In fact, if the initial list price is calculated based on inaccurate future demand, further revision of the price will be necessary to sell the property faster or to increase the sale price.

To test whether the change in the list price in the data is statistically significant, we run the following simple ordinary least squares regression model:

$$Log(AP1) = \beta_0 + \beta_1 Log(AP2) + \epsilon \qquad (86)$$

where $AP1$ is the initial list price and $AP2$ is the final list price.

If the results show that the two list prices are different from each other, then this can guide further analysis. Thus, the null hypothesis is $\beta_1 = 1$, which reflects a non-significant difference between the two prices. The test results in Table 11 show that the null hypothesis is rejected and that the two prices are significantly different from each other.

Table 11: Comparison of the First and Final List Prices

Dependent var Log(AP1)	Test Results	St. Dev.
Constant	0.5	(0.004)**
Log(PA2)	0.99	(0.0006)**
Wald test (F-statistic)	62.7**	

** Significant at the 1% level.

Some literature on the theory of dynamic pricing suggests that revising the price after each period if the object has not sold and if a price change is permitted could be the optimal strategy of a seller in equilibrium [15]. Generally speaking, the price sequence in those equilibriums is such that the seller starts with a relatively higher price and reduces it over time if

[15] Read (1988) and Lazear (1986).

the object has not sold. As we mentioned before, in 10% of the cases in the current study there is an increase in the list price, which is not consistent with these theoretical models. When a seller or a real estate agent decides on the list price, he or she typically considers an estimate of the future demand in the market. The houses in the present data were on the market for more than 3 months, on average, before being sold. In 3 months several factors can change the optimal asking price of a seller. In fact, committing to a fixed price is not necessarily an optimal strategy for a seller in this environment. Because the data have only two points in the list price trend, the empirical analysis is restricted. We thus choose to run a probit estimation to test which variables increase the chance of a price revision.

In the probit model the dependent variable has a value of 1 if a property undergoes a revision in list price and 0 otherwise. The probit model is run over some fairly chosen dependent variables. To proxy market thickness a method like Knight (2002) is used-that is, two dummy variables for properties with low price $0 < p < 300$ and high price $p > 900$. These properties have a thin market, and theory suggests that there is less of a chance of a property of this kind undergoing a price revision[16]. This is mainly because in a thin market there is less information for a seller to learn after a failed sale and consequently less chance of a price revision. Another dummy variable is for properties with price spreads close to the average $450 \leq p \leq 650$. These properties are traded in a relatively thicker market.

Table 12 shows the results of the probit estimation when the dependent

[16] Knight (2002), Haurin (1988).

Table 12: Probit Model (Dep. var. = 1 if the list price is revised, 0 = otherwise)

Variable	Estimate	St. Dev.
Constant	-5.33	(0.396)**
Log(Time on the Market)	0.676	(0.009)**
Property Type	0.118	(0.019)**
Low-Price Property	0.053	(0.035)
Mid-Price Property	0.027	(0.019)
High-Price Property	-0.281	(0.032)**
Log(Median Weekly Rent)	-0.108	(0.058)*
Log(Mean Household Income)	0.301	(0.048)**

∗ Significant at the 5% level.
∗∗ Significant at the 1% level.
Low price = $0 < p < 300$, mid price = $450 \leq p \leq 650$,
and high price = $p > 900$.

variable is equal to 1 for every observation that has a revision in the list price, regardless of whether it was an increase or decrease in price. As the results in Table 12 suggest, a property with a longer time on the market has a greater chance of undergoing a price revision. This is consistent with the results in the previous section. The type of property can also influence the price revision. The results show that if the property is house, there is a greater chance that the price will be revised during the selling period. On average, houses have more unique characteristics than units, which results in more uncertain expected demand, mainly because of a higher dispersion in buyers' valuations. Haurin (1988) suggests that if there are widely varying beliefs about the value of an object, the seller might wait longer in the market for a better offer. Knight (2002) extends this thinking by suggesting that a seller of this kind may also be more hesitant to revise the price. The present results suggest that the initial intuition holds, but the extension does not necessarily apply to the data. The negative coefficient for the high-price properties suggests that when the market is thin, there is less of a chance that the price will be revised. However, the coefficients for the low- and mid-price properties are not significant. Other results suggest that the higher the rent of the properties in the neighbourhood, the less of a chance of a price revision; and the richer the neighbourhood, the greater the chance that the price will be revised. One intuition behind this could be the fact that properties in richer neighbourhoods are more expensive and have higher values on average. These properties have some characteristics which increases the dispersion of the valuation of buyers and as we discussed earlier the literature suggest in these situations we have higher chance of price revision.

In Table 13 the same estimation is performed, but only for those properties with a reduction in their list price. All coefficients have the same sign as in Table 12. Moreover, the low-price properties have a significant and positive coefficient. Because the market for low-price properties is considered thin, this result no longer supports the greater chance of no price revision in a thin market.

4.4 Conclusion

In this chapter the empirical parameters of the Sydney housing market were examined. A two-stage regression model was used to analyse time on the market. In the first stage the logarithm of the sale price was regressed over various characteristics of properties. In the second stage the predicted

Table 13: Probit Model (Dep. var. = 1 if the list price is reduced, 0 = otherwise)

Variable	Estimate	St. Dev.
Constant	-4.46	(0.403)**
Log(Time on the Market)	0.695	(0.009)**
Property Type	0.149	(0.019)**
Low-Price Property	0.095	(0.036)**
Mid-Price Property	0.0004	(0.019)
High-Price Property	-0.311	(0.033)**
Log(Median Weekly Rent)	-0.106	(0.058)*
Log(Mean Household Income)	0.187	(0.049)**

∗ Significant at the 5% level.
∗∗ Significant at the 1% level.
Low price = $0 < p < 300$, mid price = $450 \leq p \leq 650$,
and high price = $p > 900$.

sale price was used in a regression in which the logarithm of time on the market was the dependent variable. The results suggest that, on average, overpricing increases time on the market. Thus, when the initial list price is higher, the seller can expect to take more time to sell the property regardless of whether he or she revises the price. Properties that underwent a price revision spent more time in the market, on average. Those sellers who reduced the initial list price more spent less time in the market. All of these results are consistent with what is expected for the housing market and more significant than similar findings in the literature mainly because of the better data set.

In the second stage of the estimation we ran a probit regression to analyse which variables affect revisions in list price. In the probit model the dependent variable was equal to 1 if the property had a revised list price, and 0 if not. The results suggest that the type of property affects the likelihood of a price revision; that is, houses on average have a greater chance than units of undergoing a price revision in the selling period. The first probit model shows support for less of a chance of a price revision in a thin market; however, this result does not hold when only properties with a reduction in their list price are considered.

Although the data used for the current study are much more comprehensive and detailed than those used in similar studies, this work still has some limitations. The current data have the initial and the final list prices, while sometimes the list price changes more than one time in the selling period. More detailed information on the sequence of list price changes can direct us to a better understanding of the subsequent effects. The data could also be extended to include the number of offers a given seller has received within the selling time. Sometimes sellers receive multiple offers at the same time, which could potentially be very important in terms of time on the market as well as deciding to revise the price. Further research could analyse the effect of the number of offers on time on the market as well as the final sale price. This information could give researchers a more detailed explanation of why the initial list price sometimes changes after only a short amount of time on the market and why it sometimes does not change even after a long time on the market.

5 Appendix

Proof of Proposition 1.1 First we are going to rewrite the differential equation as follows

$$\frac{ds}{d\tilde{m}} = -\frac{D_3 U^{pp}(s, s, \tilde{m}(s))}{D_2 U^{pp}(s, s, \tilde{m}(s))} \tag{87}$$

substituting D_3 and D_2 from the differentiation of the expected payoffs we have

$$\frac{ds}{d\tilde{m}} = \frac{f_1(\tilde{m})(v_0(s) - J_1(s, \tilde{m})}{\frac{\partial v(s,\tilde{m}(s))}{\partial s}(1 - F_1(\tilde{m}))} \tag{88}$$

The single crossing condition has been verified in the text. Therefore, as long as the *hazard rate* function is increasing and the valuation function is concave, it is possible to follow the arguments in Riley (1979) or Cai, Riley, and Ye (2007) to show that there is a unique solution going through the optimum of the seller with type zero in the full information situation that is strictly increasing.

We also need to show the incentive compatibility of this unique solution is satisfied, that is, no type would deviate from $\tilde{m}(s)$. The solution to the above differential equation gives us the $\tilde{m}$ that a seller with signal s chooses to maximize $U^{pp}(s, s(\tilde{m}), \tilde{m})$. In fact, as long as the single crossing condition holds the $\tilde{m}$ that maximises this payoff is the one that satisfies $s(\tilde{m}) = s$. This argument is similar to the one in Theorem 1 Cai, Riley, and Ye (2007). □

Proof of Proposition 2.1.

When all signals distributed uniformly on [0,1], we can use equations (3) and (12) to find an expression for the expected payoffs.

$$U^{DR} = \int_m^1 (2x - 2x^2) + (2m^2 - 2m^3) \tag{89}$$

$$s(m) = \int_{0.5}^m (4x^2 - 2x)/(1 - x^2) \tag{90}$$

$$\beta_{SR}(x) = \begin{cases} 2x & \text{if} \quad x \leq \frac{1}{2} \\ x + \frac{1}{2} & \text{if} \quad x \geq \frac{1}{2} \end{cases} \tag{91}$$

$$U^{SR} = \int_{\hat{m}(s)}^{0.5} (2x - s)(2 - 2x)dx + \int_{0.5}^1 (x + 0.5 - s)(2 - 2x)dx \tag{92}$$

It is easy to check that both U^{DR} and U^{SR} are decreasing when the seller's signal increases. In fact, we show this is true in general in (65) and (68). Since at $s = 0$, U^{SR} is greater than U^{DR} and at $s = 1$, $U^{DR} > U^{SR}$ then by continuity there must be a signal $\dot{s}$ in which $U^{DR} = U^{SR}$. There is no point for the sellers with signals higher than $\dot{s}$ to choose the secret reserve regime. Since the buyers know all the types greater than $\dot{s}$ never chooses the secret reserve price, they update their information regarding the seller's signal and the expected payoffs to the secret reserve shifts to the left. Continuing this argument the same as the motivating example it is possible to conclude that no positive type chooses the secret reserve price regime. Having this strategy for all sellers' types with positive signals, then there is no profitable deviation for any positive type $\ddot{s} < \dot{s}$, since the buyers' expectation for that type is zero. $\square$

Proof of Proposition 2.2.

When $s = 0$ by choosing an auction with disclosed reserve price in equilibrium, the seller, signals her true type with the reserve price. However, in the secret reserve regime the seller does not reveal any information. Each buyer's expected value in the separating equilibrium of DR is a function of the seller's true type $v(0, x_i)$ while in the SR it is a function of the expectation of s rather than zero. Both conditional and unconditional expectations for the buyers' values in the SR are greater or equal to zero. Since valuation function is increasing in s then the expected value of each buyer in the SR is greater or equal than the one in DR, and any bid in SR clears the reserve price, which is $v_0(0)$ in this situation. The payment rule is the same for both auctions, therefore the expected payoff for SR is greater or equal to DR for $s = 0$. This argument is only true for the lowest type in the interval. □

Proof of Proposition 2.3. According to (65) and (68) we know the seller's net expected payoffs from both regimes are strictly decreasing when s increases. From the Proposition (2.2), we also know that the lowest signal has a higher expected payoff from the secret reserve regime. If the highest type has a higher expected payoff from the DR, then by continuity there must be one intersection between both payoffs. Consider that both expected payoffs are weakly convex, therefore there must be exactly one intersection. Then with an argument the same as the one in Proposition 2.1 or the motivating example, we can conclude no type except the lowest type would choose the SR. □

Proof of Proposition 2.4.

First we need to show the net expected payoffs from the DR and the SR at most intersect once during the interval. We know for $s = 0$ the expected payoffs from the SR is higher than the DR. From (65) and (68) we know both payoff are strictly decreasing. At any signal $\dot{s}$, if the expected payoffs from SR and DR intersect, the SR cuts DR from the top. Comparing the slope of these two payoffs at that point, the absolute value of the SR slope must be higher than the DR slope. Therefore, we must have

$$v_0'(\dot{s})[1 - F_1(m(\dot{s}))] < v_0'(\dot{s})[1 - F_2(\hat{m}(\dot{s}))] \tag{93}$$

As long as this inequality hold for $\dot{s}$ we have,

$$F_1(m(\dot{s})) > F_2(\hat{m}(\dot{s})) \tag{94}$$

To have another intersection since both payoffs are strictly decreasing and convex, this inequality needs to be reverse at least once in the interval. For every $s > \dot{s}$, since $m(.)$ is an strictly increasing function and $\hat{m}(.)$ is the same, then the left hand side of (94) increases while the right hand side remain the same. Thus this inequality will always hold for every $s > \dot{s}$ and two expected payoffs will never intersect twice in the interval.

If part: If the SR has a higher expected payoff for the highest seller's type in the interval, then by Proposition 2.2 and the fact that both payoffs are strictly decreasing in s and convex, they must never intersect with each other and all other types must also have a higher expected payoffs from the SR. Therefore, all types must choose the secret reserve price regime.

Only if part: Suppose a seller with positive signal s chooses the SR regime, and the highest type in the interval prefers the DR regime. Then by the above argument there must be exactly one intersection between the two payoffs and according to Proposition 2.3, all types must choose the DR regime which is a contradiction. □

Proof of Proposition 3.1.

Since $p_l = (p_a - (1-\theta)p_r)/\theta$, setting $\bar{v} = 1$ without loss of generality, the first order condition of maximizing the expected payoff with respect to p_a in (76) would be,

$$p_a^* = \theta + (1-\theta)p_r^* \tag{95}$$

For any $0 \leq \theta \leq 1$, this implies that $p_a^* \geq p_r^*$. Rewriting (75), we have,

$$\begin{aligned}\frac{p_r}{\delta} =&\rho_1\big[\int_{p_r}^{1}(\theta v + (1-\theta)p_r)dF(v) + p_r F(p_r)\big]\\ &+ \rho_2\big[\int_{p_r}^{\bar{v}} v f_2(v)dv + (F_2(p_r) - F_1(p_r))p_r + p_r F_1(p_r)\big]\\ &+ (1-\rho_1-\rho_2)p_r\end{aligned} \tag{96}$$

At $p_r = 0$ the left hand side is equal to zero and the right hand side is positive. At $p_r = 1$ the left hand side is greater than one (since $\delta < 1$) while the right hand side at every possible scenario the maximum is p_r. Therefore, by continuity there exist at least one p_r that satisfies (96). At this stage we do not find the uniqueness condition in general and if more than one p_r satisfies (96) then p_r^* would be the highest one. $\square$

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