PIEZOELECTRIC ACTUATOR DESIGN OPTIMISATION FOR SHAPE CONTROL OF SMART COMPOSITE PLATE STRUCTURES

Van Ky Quan NGUYEN
BE (Aeronautical)

A thesis submitted in fulfilment
of the requirements for the degree of
Doctor of Philosophy

School of Aerospace, Mechanical and Mechatronic Engineering
The University of Sydney

March 2005

Abstract

Shape control of a structure with distributed piezoelectric actuators can be achieved through optimally selecting the loci, shapes and sizes of the piezoelectric actuators and choosing the electric fields applied to the actuators. Shape control can be categorised as either static or dynamic shape control. Whether it is a transient or gradual change, static or dynamic shape control, both aim to determine the loci, sizes, and shapes of piezoelectric actuators, and the applied voltages such that a desired structural shape is achieved effectively.

This thesis is primarily concerned with establishing a finite element formulation for the general smart laminated composite plate structure, which is capable to analyse static and dynamic deformation using non-rectangular elements. The mechanical deformation of the smart composite plate is modelled using a third order plate theory, while the electric field is simulated based on a layer-wise theory. The finite element formulation for static and dynamics analysis is verified by comparing with available numerical results. Selected experiments have also been conducted to measure structural deformation and the experimental results are used to correlate with those of the finite element formulation for static analysis. In addition, the Linear Least Square (LLS) method is employed to study the effect of different piezoelectric actuator patch pattern on the results of error function, which is the least square error between the calculated and desired structural shapes in static structural shape control.

The second issue of this thesis deals with piezoelectric actuator design optimisation (PADO) for quasi-static shape control by finding the applied voltage and the configuration of piezoelectric actuator patch to minimise error function, whereas the piezoelectric actuator configuration is defined based on the optimisation technique of altering nodal coordinates (size/shape optimisation) or eliminating inefficient elements in a structural mesh (topology optimisation). Several shape control algorithms are developed to improve the structural shape control by reducing the error function. Further development of the GA-based voltage and piezoelectric actuator design optimisation method includes the constraint handling, where the error function can be optimised subjected to energy consumption or other way around. The numerical examples are presented in order to verify that the proposed algorithms are applicable to quasi-static shape control based on voltage and piezoelectric actuator design optimisation (PADO) in terms of minimising the error function.

The third issue is to use the present finite element formulation for a modal shape control and for controlling resonant vibration of smart composite plate structures. The controlled resonant vibration formulation is developed. Modal analysis and LLS methods are also employed to optimise the applied voltage to piezoelectric actuators for achieving the modal shapes. The Newmark direct time integration method is used to study harmonic excitation of smart structures. Numerical results are presented to induce harmonic vibration of structure with controlled magnitude via adjusting the damping and to verify the controlled resonant vibration formulation.

Acknowledgements

The success of the work contained in this thesis would not have been possible without the support and companionship of those peoples around me during my PhD program. I am grateful to my supervisor, Professor Liyong Tong, for his advice, support, numerous interesting ideas, wisdom, encouragement and patience. Thanks to the staff members of the School of Aerospace, Mechanical and Mechatronic Engineering at The University of Sydney, especially Dr. Douglass Auld for his technical support for things related to the computer network and resources. Thanks also to Ms Yvonne Witting, Administrative Officer of Postgraduate Study, for her tireless efforts in the university bureaucracy. On a personal note, thanks to my fellows at the Structural Optimisation Research Group Australia (SORGA) and the Composite and Smart Structures Group (CSSG) members and friends for interesting discussions. In addition to this, specifically thanks Dr. Clinton Chee for his references and Dr. Dongchang Sun for his assistance in the experimental tests. I would like to thank Mr. Phu Duc Nguyen and Mr. Michael Wood for their helping and friendship. I would like to acknowledge the support for provisions of all the necessary research tools by The University of Sydney, and the financial support of the Australian Research Council via a Discovery Project (DP No.0210716) awarded to my supervisor, Professor Liyong Tong. Also thanks for the support of Australian Research Council and National Natural Science Foundation of China for the ARC Linkage International Award (ARC LX0348548), which has funded my valuable half-year visiting research time at the State Key Laboratory for Analysis of Industrial Equipment, at the Dalian University of Technology, China. Special thanks to Professor Yuanxian Gu for his supports and provided ideas during my visiting research at the Dalian University of Technology, China. I would like to thank all members of my family. And of course, thank you, Ms. Kieu Anh Tran, for your understanding and constant support throughout the course of this research, and for your love. Finally, I dedicate this work to the memory of my grandparents, who passed away during my PhD study.

Preface

This thesis is to submit in fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering, to be awarded by The University of Sydney. The goal to achieve in this thesis is to focus on three main issues, namely, the development and validation of finite element solutions for smart composite plate structure with non-rectangular elements; development of piezoelectric actuator design optimisation techniques/methods for quasi-static shape control of smart composite plate structures and effectively control of resonant vibration of smart composite plate structures. The whole thesis is covered in 8 chapters.

In Chapter 1, an overview of the current state of smart structure and its applications in shape control problem using piezoelectric actuators. The pertinent references are cited for establishing the concepts of analytical and finite element formulation including piezoelectric materials effect and shape control algorithms. Also, various representative state-of-art structural optimisation techniques are briefly reviewed. In view of current status of shape control problem, the scope of research is proposed.

In Chapter 2, the finite element formulation for the general laminate composite plate structure is extended to analyse static and dynamic deformation of the smart structures with non-rectangular elements. Its mechanical deformation is modelled using a third order plate theory, while the electric field is simulated based on a layer-wise theory. Static and dynamic finite element formulations are verified by comparing with available numerical results. Experimental facilities have been set up to test structural deformation under piezoelectric material effects in order to validate the finite element formulation for static deformation. Before going into development control algorithms, a set of numerical results are presented to investigate the influence of the configuration of piezoelectric actuators on error function for static shape control. The results are obtained for the optimum values of the electric field in the piezoelectric actuators to achieve the desired shape using the LLS method.

In Chapter 3, piezoelectric actuator design optimisation (PADO) algorithms are developed for static shape control of smart structures. These algorithms incorporate an iterative process of the linear least square (LLS) method, and genetic algorithm (GA) into finite element analysis, which have been developed in Chapter 2, known as coupled alternating algorithm (CAA) and coupled concurrent algorithm (CCA) of GA+LLS (GALLS). Voltage distribution and piezoelectric actuator configuration can be optimised to attain a desired structural shape based on GA, CAA and GALLS. To demonstrate the capability of these algorithms, numerical examples are presented and compared to the LLS method, where only voltage optimisation is performed. The proposed algorithms including PADO show improvement in achieving the desired structural shape.

Chapter 4 presents shape control of smart composite plate structures under additional constraints beyond the structural behaviour constraint. Two main issues are investigated here. One is to minimise the error function (Lnm) subjected to the energy consumption constraints. Another to minimise the total energy consumption subjected to the error function constraint. Both issues aim to control the structure to attain the desired structural shape within a given constraint condition. Genetic Algorithm (GA) is employed to optimise the applied voltages in actuators and the geometries of piezoelectric actuators subjected to given additional constraints. For each issue, numerical results are presented to demonstrate the effect of given additional constraint on the results of objective function. Also, a comparison between single criterion optimisation (voltage optimisation only) method and multiple criteria optimisation (voltages and geometrical of piezoelectric actuator optimisation) method is given to indicate the better optimisation method.

Chapter 5 presents a new evolutionary algorithm for the determination of an active piezoelectric actuator configuration in the shape control of smart composite plate structures. The LLS and the features of evolutionary structure optimisation (ESO) are employed in order to find the voltage distribution and the active piezoelectric actuator configuration for achieving the desired structural shapes subjected to the removal of active piezoelectric material element based on the error function sensitivity number. On the basis of FE analysis, the error function sensitivity number including electro-mechanical effect is derived to compute the change in error function due to removing the active piezoelectric materials. Evolutionary piezoelectric actuator design optimisation

(EPADO) is proposed with a set of numerical examples to verify EPADO at a given applied voltage.

On the basic concepts of CAA and GALLS methods, in Chapter 6, LLS and EPADO are merged to form two new algorithms, named Alternative Voltage & Evolutionary Piezoelectric Actuator Design Optimisation (AVEPADO) and Voltage & Evolutionary Piezoelectric Actuator Design Optimisation (VEPADO) for the solution of voltage and piezoelectric actuator design optimisation problems. Within these algorithms, the piezoelectric material distribution and the applied voltages are optimised simultaneously. A comparison is given for the results obtained through AVEPADO and VEPADO in order to indicate the better algorithm in terms of the performance to attain the optimal solution.

In Chapter 7, the modal analysis and the LLS techniques are employed to optimise the applied voltage to achieve the modal shapes. The Newmark direct time integration method is used to study the harmonic excitation of smart structures. Numerical results are presented to show the optimal values of electrical fields in the actuator to achieve the desired modal shape and to induce the harmonic vibration of structure with controlled magnitude via damping adjustments.

Finally, Chapter 8 presents a summary of the results and achievements of this thesis. An overview of some thoughts on some future directions is also presented.

List of Publications

1. Nguyen Q. and Tong L., 2002, “Shape Control of Smart Composite Plate Structures with Non-Rectangular Shaped PZT Actuator”, *Proceeding of the Third Australasian Congress on Applied Mechanics,* Sydney, Australia, February 20th-22nd, pp. 421-426.

2. Nguyen, Q. and Tong, L., 2002, “Shape Control of Smart Composite Plates Structures Based on Actuator Shape Optimisation”, *Recent Advances in Computational Sciences and Engineering-Proceeding of the International Conference on Scientific and Engineering Computation*, Singapore, December 3rd-5th, Ed. H.P. Lee and K. Kumar, Imperial College Press, London, pp.434-437.

3. Nguyen Q. and Tong L., 2004, “Shape Control of Smart Composite Plate Structures with Non-Rectangular Piezoelectric Actuator”, *Composite Structures,* Vol. 66, No. 1-4, pp. 207-214.

4. Nguyen Q. and Tong L., 2004, “Modal Shape Control of Smart Composite Plates Using Piezoelectric Actuators”, *Proceeding of The Sixth World Congress on Computation Mechanics in conjunction with The Second Asian Pacific Congress on Computational Mechanics*, Beijing, China, September 5th-10th, Tsinghua University Press & Springer-Verlag.

5. Nguyen Q., Tong L. and Gu Y., “Evolutionary Piezoelectric Actuators Design Optimisation for Static Shape Control of Smart Plates”, *Computer Methods in Applied Mechanics and Engineering*, (Under Review).

6. Nguyen Q. and Tong L., “Voltage and Evolutionary Piezoelectric Actuator Design Optimisation for Static Shape Control of Smart Plate Structures”, *Materials and Design*, (in-press).

Notations

SYMBOLS

General variables

c	Mechanical stiffness
c_b	Bending stiffness matrix
c_s	Shear stiffness matrix
d	Piezoelectric strain constant
D	Electric displacement
e	Piezoelectric stress constant
e_b	Piezoelectric bending stress matrix
e_s	Piezoelectric shear stress matrix
E	Electric field
E_i, E_o	Electric field sub-matrices
(E_{xx}, E_{yy}, E_{zz})	Electric potential derivative
F_v	Volume load matrix
F_s	Surface load matrix
F_p	Point load matrix
(K, P, W)	Kinetic energy, potential energy and work done, respectively
Lnm	Error function
Q_s	Electrical surface charge matrix
(u_o, v_o, w_o)	Displacement components on the mid-plane
(U, V, W)	Total displacement components
ε	Mechanical strain
ε_b	Bending strain matrix
ε_s	Shear strain matrix
$(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz})$	Normal strains
$(\gamma_{xy}, \gamma_{yz}, \gamma_{zx})$	Shear strains
$(\zeta_x, \zeta_y, \zeta_z, \phi_x, \phi_y)$	Higher order terms in the Taylor's series expansions

σ	Mechanical stress
ϕ	Electric potential
ϕ_j	Electric potential within the jth layer
χ	Electric permittivity
χ_i, χ_o	Electric permittivity sub-matrices
(ψ_x, ψ_y, ψ_z)	Rotational
ρ	Density

Finite element variables

B_{bu}	Bending strain-displacement matrix
B_{su}	Transverse shear strain-displacement matrix
$B_{\phi o}$, $B_{\phi i}$	Matrices defining an electric field
C_{uu}	Proportional damping matrix
D	Definition matrices of some variable with derivatives
F_g	Mechanical force vector
J	Jacobian matrix
K_{uu}	Structural stiffness matrix
$K_{u\phi}$, $K_{\phi u}$	Piezoelectric stiffness matrices
$K_{\phi\phi}$	Dielectric stiffness matrix
M_{uu}	Mass matrix
N	Interpolation function
N_u	Interpolation function matrix [11x88]
N_ϕ	Interpolation function matrix [(nlayer+1)x8(nlayer+1)]
Q	2D transformation matrix – rotation about z-axis
Q_g	Electric charge vector
R	Transformation between engineering and infinitesimal strain tensor
T, T_{3D}	3D transformation matrix – rotation about z-axis
u_e	Nodal displacement matrix [1x88]
V	Volume
(x, y)	Global planar coordinate system
(ξ, η)	Local planar coordinate of element

ϕ_e	Nodal electric potential matrix [8(nalyer+1)]
Star (*)	Represents the shape functions N_i (i = 1,…, 8).
Dot (.)	Represents zeros in the matrix

Static shape control variables

B_c, B_a	Allowable and actual value of the additional constraints
C^w	Influence coefficient matrix
E_c, E_a	Allowable and actual value of total energy constraint
e^s	Number of removal piezoelectric material pieces
e^r	Number of active piezoelectric material pieces in the current design
Lnm^a, Lnm^c	Allowable and actual value of error fucntion
Lnm^n, En, ϕ_n^k	Optimal values obtained without constraint handling
N	Total number of nodes of the FE model
N_p	Total number of piezoelectric actuator patches
R	Resistance conductor
RMR	Removal material rate
S	Configuration design variables vector
V	Design variable vector
V_L, V_U	Lower and upper bounds of the design variable vectors
v	Volume
w^d, w^c	Desired and calculated displacements matrices, respectively
w_i^d, w_i^c	Desired and calculated displacements of the ith node, respectively
w^d_{max}	Maximum desired displacement
X	Geometric design variables vector
X_L, X_U	Lower and upper bounds of geometric design variable vectors
ΔLnm	Error function sensitivity number
ϕ	Voltage design variables vector
ϕ_c, ϕ_a	Allowable and actual value of voltage constraint
ϕ_L, ϕ_U	Lower and upper bounds of voltage design variable vectors
τ	Error tolerance

Dynamic modal shape control variables

f	Constant factor
t	Time
V	Eigenvectors
α, β	Rayleigh's damping coefficients
α^*, β^*	Newmark's parameters
λ	Eigenvalues
Ω	Frequency input
ω	Mechanical natural frequencies

ACRONYMS

AVEPADO	Alternatively Voltage and Evolutionary Piezoelectric Actuator Design Optimisation
BVD	Build-up Voltage Distribution
DBSC	Displacement Based Shape Control
CA	Coupled Algorithms
CAA	Coupled- Alternating Algorithm
CCA	Coupled- Concurrent Algorithm
CLPT	Classical Laminate Plate Theory
CDBSC	Curvature-Displacement Based Shape Control
DOF	Degree of Freedom
EPADO	Evolutionary Piezoelectric Actuator Design Optimisation
ERF	Electro Rheological Fluids
ESO	Evolutionary Structure Optimisation
FE	Finite Element
FEA	Finite Element Analysis
FSDT	First-order Shear Deformation Theory
GA	Genetic Algorithm
GALLS	Genetic Algorithm and Linear Least Square
HOT	Higher Order Theory
LLS	Linear Least Square

MRF	Magneto-Rheological Fluid
OC	Optimal Criteria
PZT	Piezoceramics (Lead Zirconate Titanate)
PADO	Piezoelectric Actuator Design Optimisation
PVDF	Polyvinylidene Fluoride
SA	Simulated Annealing
SDBSC	Slope-Displacement Based Shape Control
SLP	Sequential Linear Programming
SMA	Shape Memory Alloy
TOD	Third Order Displacement
TODL	Third Order Displacement Layerwise
TODL-FE	Third Order Displacement Layerwise – Finite Element
VEPADO	Voltage and Evolutionary Piezoelectric Actuator Design Optimisation
1D, 2D, 3D	One, Two, Three Dimension

Table of Contents

2. Finite Element Formulation and Verifications