

Analysis of Some Linear and Nonlinear Time Series Models

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Abstract

This thesis considers some linear and nonlinear time series models. In the linear case, the analysis of a large number of short time series generated by a first order autoregressive type model is considered. The conditional and exact maximum likelihood procedures are developed to estimate parameters. Simulation results are presented and compare the bias and the mean square errors of the parameter estimates. In Chapter 3, five important nonlinear models are considered and their time series properties are discussed. The estimating function approach for nonlinear models is developed in detail in Chapter 4 and examples are added to illustrate the theory. A simulation study is carried out to examine the finite sample behavior of these proposed estimates based on the estimating functions.

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